SAME SAME BUT DIFFERENT – STYLIZED FACTS OF CTA SUB STRATEGIES

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Abstract. Using a unique dataset of daily returns of 89 programmes of Commodity Trading Advisors (CTA), we investigate the distributional properties of CTA strategies including trend following, fundamental and contrarian strategies. We find that daily data exhibits strong features of fat-tail, volatility clustering, and long memory in volatility. This is different from previous studies which are often based on monthly data. Our study contributes to the literature of stylized facts of financial markets, it also provides insights to practitioners because the information from monthly data might be misleading.

Keywords: Commodity Trading Advisors; trend following; fundamental strategy; contrarian strategy; Stylized facts

JEL Classification: C58; G14; G23

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1. Introduction

Commodity Trading Advisors (CTAs) are managers of Managed Futures that comprise a diverse collection of active Hedge Fund trading strategies specializing in extremely liquid, transparent, exchange-traded futures markets and deep foreign exchange markets, so they can scale in and out of positions on a daily basis. CTAs is one of the largest Alternative Investment categories. According to Barclay Hedge/CTA, the industry's assets under management (AUM) increased more than tenfold over the last two decades, i.e. from BUSD 25 in 1994 to BUSD 351 in 2016. In 2016, CTAs was the third largest asset class within the Alternative Investment space. Only Fixed Income (BUSD 564 AUM) and Multi-Strategy (BUSD 360 AUM) were bigger. The rapid growth of CTAs as an industry is due to the growth of the futures markets in the 1970s and the development of technology. There are several benefits of allocating to Managed Futures for investors: 1) They exhibit low correlation to traditional asset classes. 2) They also add diversification to a portfolio of Hedge Fund managers (Kat, 2002). 3) They are able to provide investors with a positive crisis alpha, since they have a high degree of adaptability in their investment style and can go both long and short. Additionally, they have the possibility of investing across an entire spectrum of assets, allowing them to capture the upside and downside of many markets. Furthermore, futures market have lower and symmetric costs of trading compared to spot markets or other derivative markets (Kaminski, 2011). 4) They are transparent and liquid. Futures contracts are transparent, unlike collateralized debt obligations or over the counter swaps. In addition, the markets that Managed Futures trade - such as energy and foreign exchange - given their massive size, are among the most liquid in the world. Since the instruments that Managed Futures trade tend to be exchange-listed futures or extremely deep cash-forward markets, price risk is the main risk associated with CTAs, in contrast to more non-directional Hedge Fund strategies that have credit and liquidity exposures. Bhaduri and Art (2008) reveal the underestimation value of liquidity, and, Hedge Funds that trade illiquid instruments have underperformed Hedge Funds that trade liquidity asset. All of the above make CTAs unique in contrast to other Hedge Funds strategies (Kaminski and Mende, 2011; Hamill *et al.*, 2016).

Traditionally, CTAs have been technical trend following managers with a medium- to long-term view, but as the space has grown and evolved the types of strategies have broadened as well. Whereas academia mostly distinguishes between two types of traders, i.e. systematic funds (mechanical/computer-driven trading) vs. discretionary funds (real-time decision making by the manager), the industry has divided the space into many more sub strategy categories such as contrarian/short-term trading, liquid global macro or fundamental trading, volatility traders, sector specialists etc. These sub strategies are all active in the same liquid futures markets. That being said, even today, the CTA universe is clearly dominated by diversified systematic trend following managers (Krohn *et al.*, 2017). Trend following intends to capture market trends that are commonly related to serial correlation in price changes. A trend is a series of asset prices that move persistently in one direction on a given time duration, where price changes exhibit positive serial correlation. A trend follower attempts to identify developing price patterns with this property and trade in the direction of the trend if and when this occurs. The life cycle of a trend starts as an initial under reaction to the asset price movement caused by news, supply shocks or demand shifts. Anchoring and insufficient adjustment (e.g. Edwards, 1968 and Tversky and Kahneman, 1974), the disposition effect (e.g. Shefrin and Statman, 1985 and Frazzini, 2006), and non-profit-seeking market participant who fight trends (e.g. Silber, 1994) may lead to actions that slow down the process of price discovery. Once a trend has started, the trend then over-extends due to herding effects (e.g. De Long *et al.*, 1990 and Bikhchandani *et al.*, 1992), confirmation bias and representativeness (e.g. Wason, 1960 and Tversky and Kahneman, 1974), and risk management (e.g. Garleanu and Pedersen, 2007). At the end, the trend reverses. One of the main challenges for directional Managed Futures strategies is to minimize losses associated with the ending of trends, so-called give-back losses, and to preserve capital in range bound markets that do not exhibit trends, so-called whipsaw losses.

Like any investment, using Managed Futures requires proper due diligence on the part of potential investors. Because Managed Futures are low/un-correlated with traditional asset classes, they can provide impressive diversification to investors' portfolios, for example, during the financial crisis of 2008; but during years of the stock market rebound, such as 2009-2013, gains do not automatically translate into them. In addition, as an Alternative Investment, Managed Futures have a higher fee structure than traditional mutual funds. Therefore, there is the need for careful screening when looking for the right manger. CTAs, as well as other Hedge Funds, primarily market themselves towards investors using monthly performance figures that are voluntarily reported to different databases such as Barclay Hedge/CTA or TASS Lipper. Table A.1 shows the eleven most often used CTAs return indices. Over 60% of the indexes are reported in monthly basis. Furthermore, benchmark indices are comprised of equal-weighted average returns of up to over 400 reporting managers.[1] For the average CTA investors it is thus impossible to replicate the

[1]The Barclay CTA Index is a leading industry benchmark of representative performance of commodity trading advisors. There are currently 532 programs included in the calculation of the Barclay CTA Index for 2016. The Index is equally weighted and rebalanced at the beginning

performance of such a broad CTA index. He or she typically has to make do with investing in 1-3 CTA managers at a time. Moreover, many institutional investors have learned to acknowledge the benefits of daily transparency, i.e. daily access to a fund's performance and positioning, especially during times of crisis like Brexit for example. In fact, an investor's live investment experience is often made up of the daily return fluctuations and not so much of the monthly return stream that was the basis for the investment decision. The need for higher resolution is even greater today than, for example, 20-30 years ago as crises occur and spread at a much higher frequency due to technological progress (among other things) like the flash crash in May 2010. Unfortunately, due to a certain degree of secrecy on behalf of the managers and some investors' lack of sophistication, potential clients often (have to) assess risk and reward of investing by means of analyzing the abovementioned monthly data. The problem is that a program's performance characteristics on a daily basis can be substantially different from the reported monthly figures, especially during times of crisis and/or when volatility spikes. Some CTA investors investing in multiple managers apply a strategy balancing concept combining, let us say, one or more trend following programs with fundamental trading strategies in order to smooth the overall return stream of their investment. These investors face the problem of different distribution characteristics of different types of CTA strategies, i.e. sometimes strategy diversification works, sometimes it does not. Finally, CTAs themselves often combine different trading strategies with each other in order to be able to profit from different profit sources at different times and, thus, improve their long-term performance. Thus, what - if any - are the general differences with regards to risk and performance depending on data frequency? Do different CTA strategies behave differently? Obviously, daily figures are scarcer due to their proprietary nature and subsequently also academic research using daily returns. To our knowledge there exist no other studies on the differences between daily and monthly returns for CTAs.

This study analyses the difference between daily and monthly return distributions for CTAs using a unique dataset combining monthly figures retrieved from Barclay Hedge/CTA which is the most complete individual database (Joenväärä *et al.*, 2012) and a non-public dataset of daily returns series, provided by 89 managers from January 1990 to April 2014. We discuss the different empirical distributions and implications for investors with regards to strategy space in general and regarding

of each year. To qualify for inclusion in the CTA Index, an advisor must have four years of prior performance history. Additional programs introduced by qualified advisors are not added to the Index until after their second year. These restrictions, which offset the high turnover rates of trading advisors as well as their artificially high short-term performance records, ensure the accuracy and reliability of the Barclay CTA Index.

sub strategies in particular. Is it possible to generalize with regards to a particular CTA sub strategy? What would an average investor have to expect when investing in a certain type of manager? A wide range of methodologies are applied in this study including ACF, GARCH, EGARCH, GJR, Hurst index estimation, and multifractal models. We find that the return distributional properties of CTAs are non-normal, and do exhibit stylized facts including fat tails and skewed, volatility clustering, as well as long memory in volatility, which had been documented pervasively cf. Lo (1991), Ding et al. (1993), Liu et al. (2007), Zheng et al. (2018).

In related literature, Groth (2009) finds a high degree of non-normality and long memory of CTA returns in daily basis. Gregoriou and Rouah (2003), like most of the studies use public available monthly CTA data, finds random walk behavior of monthly CTA returns. There are a number of literatures on Managed Futures. Rollinger (2012) and Tee (2012) find that adding Managed Futures to investors' portfolios reduced their portfolio standard deviation to a greater degree and more quickly than did Hedge Funds alone, and without the undesirable side effects on skewness and kurtosis. There are other literatures modelling trend. Fung and Hsieh (2001) apply lookback straddle to capture the general characteristics of the entire family of trend following strategies. Fung and Hsieh (1997b) found these returns to exhibit option-like features, they tended to be large and positive during the best and worst performing months of the world equity markets. On the empirical study of trend following strategy, for a large set of futures and forward contracts Moskowitz, Ooi, and Pedersen (2012) find that the trend following strategy based on excess returns over the past 12 months persists for between one and 12 months and then partially reverses over longer time horizons. Asness, Moskowitz and Pedersen (2013) highlight that strategy combines value, momentum and trend following strategies is more profitable than each in isolation. He *et al.* (2017) reinforces this insight in both a theoretical and empirical frameworks.

The rest of the paper is organized as follows: Section 2 outlines the dataset; Section 3 reports the empirical results, Section 4 provides conclusion of the paper.

2. Data

The data set in this study consists of two parts generated in three steps, i.e. first, daily net (of fees) US dollar-nominated return series of 89 CTA programs from Jan-90 to Apr-14 collected by RPM Risk & Portfolio Management AB, a CTA specialist investment manager based in Stockholm, Sweden. In a second step, daily track records are paired with the according monthly performance and AUM figures retrieved from the Barclay Hedge/CTA database. Due to its confidential nature, the data was first anonymized by RPM before being made available to the researchers. Furthermore, we are restricted from reporting details on individual funds and their distributions. Finally, the dataset is supplemented with the in-house strategy classification from RPM distinguishing between systematic and discretionary traders on the one hand and the three main strategy categories, i.e. trend following, short-term trading, and fundamental, on the other hand.[2] This "external" classification avoids any self-reporting biases. With regards to sub strategies, the sample contains 43 trend following programs, 27 (contrarian) short-term traders, and 19 liquid global macro or fundamental managers. The whole sample includes only seven discretionary managers, i.e. two discretionary trend followers and five discretionary fundamental traders. Therefore, we abstain from analyzing this group specifically and rather incorporate the few discretionary managers in their respective sub strategy group, i.e. trend following and fundamental. These ratios are generally in line with the overall number of sub strategies in the Barclay Hedge/CTA database (Krohn et at. 2017).

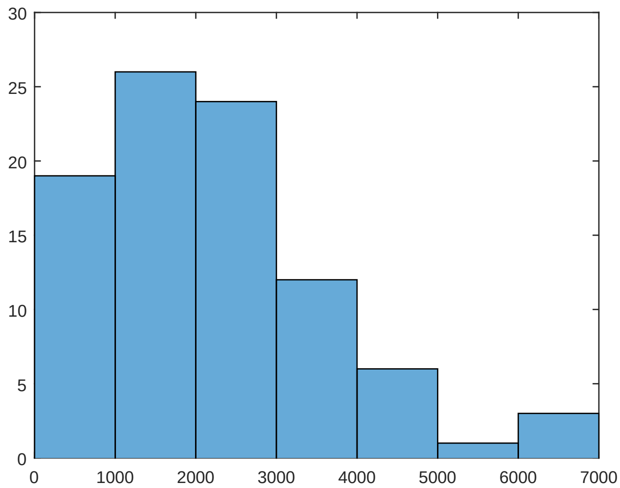

Figure 2.1. Distribution of sample size.

To reduce the uncertainty in the statistical analysis we have focused on programs with a track record spanning more than two years of trading. Sample sizes vary from 530 to 6,130 trading days, averaging 2,127 observations (median is 1,970), Figure 2.1 depicts the distribution of sample sizes. No pro forma results are used. As of the time of writing, i.e. 2017Q1, the dataset includes "dead and alive" funds. Some of

[2]Trend following seeks to profit from large market moves in financial markets (trends) using technical indicators (moving averages, momentum, volatility breakouts etc.) to extrapolate direction of asset price movements over future period. Short-term trading aims at exploiting short-term price inefficiencies typically through technical analysis (trend and countertrend). Fundamental strategies aim at capturing price trends before they occur. This is done by analyzing wide range of fundamental data and econometric modeling in order to derive intrinsic (relative) value of security.

the managers are well established in the industry, belonging to the ten largest CTAs with track records spanning over several decades. Other managers can be classified as emerging managers with regards to age and size (See Figure A.1 for distribution of AUM in May-14). Thus, overall, a broad range of managers and trading styles is included in our representative dataset. Acquiring daily return figures for dissolved funds is difficult and, thus, the number of simultaneously reporting programs increases with time. The scarce availability of daily performance figures for dissolved funds could introduce a survivorship bias. Several articles investigate survivorship bias for CTAs and Hedge Funds, see Diz (1999) and Fung and Hsieh (1997). Capocci (2004) used the Barclay Hedge/CTA database, consisting of 1,892 individual funds, to investigate survivorship bias and dissolution frequencies from Jan-85 until Dec-02. The study concludes that the annual survivorship bias amounts to 5.4% over the entire period. In our study, 13 out of the reporting 89 funds have been liquidated at some point during the sample period, i.e. a death ratio of 14.6%. Thus, since most managers in this study are still active the results are most likely subject to a minor survivorship bias, but this impact is not further investigated as we are interested in the relative differences between monthly and daily performance figures and not so much in absolute performance of individual funds or groups of funds.

3. Empirical Findings

To empirically investigate the CTA returns across trend following, fundamental and contrarian strategies, we start with descriptive statistics, then we study fat tail behavior, volatility clustering and long memory in volatility.

3.1. **Summary statistics.** Our empirical analysis on the returns of CTAs starts with descriptive statistics including mean, standard deviation, skewness, kurtosis, and Jarque-Bera normality test. Table 3.1 gives a summary of CTA returns of the full sample and all sub strategies. The average daily US dollar-nominated CTA return net of fees and in excess of risk-free rate is 0.0346% with a standard deviation of 0.0086. It is slightly positive skewed with high kurtosis. Looking at sub strategies' daily performance, the estimation on four moments of CTA returns gives a different picture. Contrarian strategies provide the best daily return of 0.036%, while the average return of trend following is 0.0344% with the largest standard deviation of 0.0096. Trend following returns are negatively skewed of -0.1382 on a daily basis, while contrarian strategies returns are positively skewed of 0.2324. This may reflects that trend following managers suffer bigger losses in market downturns, while contrarian strategies gain more on upturns. All CTA returns have high kurtosis with magnitude significantly greater than three, together with studentized range statistics (which is the range divided by the standard deviation), they indicate a more frequent appearance of extreme events and a more peaked distribution with more mass in the tails, i.e., the characteristic fat-tailed behavior compared with a normal distribution. Figure A.2, A.3 and A.4 plot the distributions of mean, skewness and kurtosis of all strategies and sub strategies, they further confirm the non-normal feature of CTA returns across strategies. Indeed, the Jarque-Bera normality test statistic is far beyond the critical value, which suggest that CTA returns are far from normal distributions. Figure 3.1 shows the kernel estimates of probability density functions of returns of trend following, fundamental strategies, and short-term contrarian strategies. Trend following shows the thickest fat tail and smallest peaked distribution, while contrarian strategies have a thinnest tail and highest peaked distribution, and fundamental have a distribution between them. All of the above indicate that daily CTA returns are non-normal distributed. Table 3.2 reports the summary statistics for monthly returns. Apparently, similar to the stylized of stock monthly returns, the monthly CTA returns are also non-normal distributed, but to a much lesser degree comparing to the CTA daily returns.

TABLE 3.1. Summary statistics of r_t.

	mean(%)	std.	skew.	kurt.	min	max	stud. range	J-B
All	0.0346	0.0086	0.0035	8.0374	-0.0502	0.0510	11.626	4993
STG 1	0.0344	0.0096	-0.1382	7.3048	-0.0538	0.0533	11.084	2495
STG 2	0.0330	0.0066	-0.0012	5.8122	-0.0349	0.0344	10.329	1151
STG 3	0.0360	0.0085	0.2324	10.770	-0.0554	0.0590	13.404	11676

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

TABLE 3.2. Summary statistics of monthly r_t.

	mean	std.	skew.	kurt.	min	max	stud. range	J-B
All	0.0068	0.0399	0.3104	5.0000	-0.1028	0.1433	5.9573	153.7889
STG 1	0.0076	0.0465	0.2334	4.5015	-0.1153	0.1652	5.9341	38.4408
STG 2	0.0057	0.0336	0.1663	4.9738	-0.0973	0.1184	5.8526	51.3539
STG 3	0.0064	0.0347	0.5067	5.6997	-0.0889	0.1289	6.0551	376.3546

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

It is also interesting to look at relationship between CTA returns of different strategies and the market return represented by the S&P 500, and the market volatility represented by the VIX. We divide the S&P 500 returns equally into five groups from the lowest 20% to the highest 20%, the first bar in Figure 3.2 from the left is the average returns of the lowest 20% of S&P 500 returns, followed by returns of all strategies, STG1, STG2, and STG3 at the time, so on and so forth for others. We

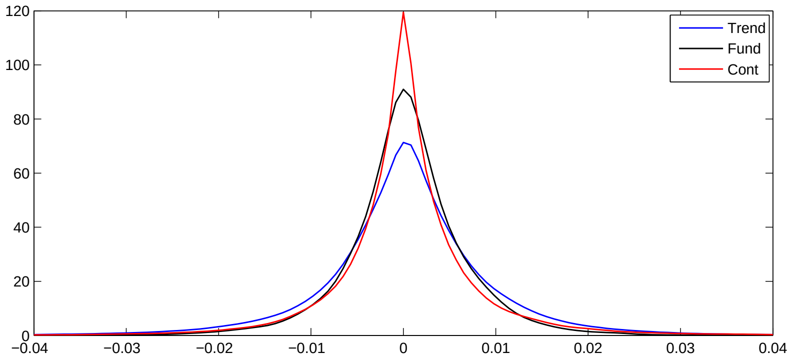

FIGURE 3.1. Probability density functions of trend following, fundamental, and contrarian strategy

observe that in each state of the market, on average the CTA returns are all positive. Trend following strategy performs very well in market down turns but not so good in upturns. The performance of fundamental strategy improves along the market sates, and the performance of contrarian strategy is stable and slightly better when the whole market is in good state. We do the same for the changes of VIX and CTA returns, and plot Figure 3.3. We observe that the CTA returns are not sensitive to changes of VIX. For each category of strategy, the trend following strategy suffers when VIX experiences big decrease, but performs very well when VIX experiences big increases. The fundamental strategy perform well but returns decrease when changes of VIX increase, and the performance of the contrarian strategy is u-shaped across changes of VIX.

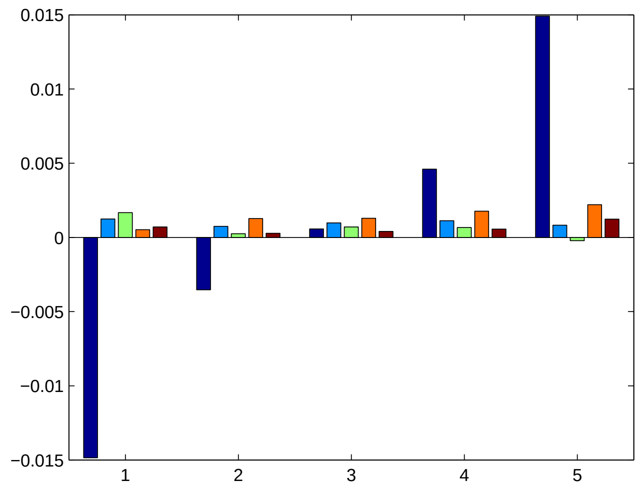

FIGURE 3.2. CTA returns and the S&P 500 returns

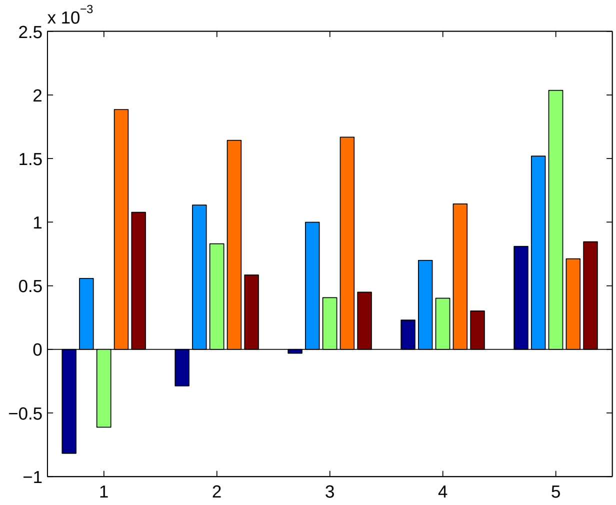

FIGURE 3.3. CTA returns and the changes of VIX

3.2. **Fat tail behavior.** The above descriptive statistics and Figure 3.1 indicate fat tail behavior of CTA returns. Here we turn to further quantify the tail behavior. In general, if f_{normal} is the probability density function of a normal distribution with mean μ and variance σ^2, then we have $\log f_{normal}(x) \sim -\frac{1}{2\sigma^2}x^2$ as $x \to \pm\infty$. A random variable X is said to follow a power-law or Pareto distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ if $Pr[X > x] = (x/\beta)^{-\alpha}$, for $x \geq \beta$. In this case, $\log f_{Pareto}(x) \sim -(\alpha + 1)\log(x)$ as $x \to +\infty$. Hence the difference of the tail behavior between the normal and Pareto distribution is significant.

The estimations of tail indices have been studied in great detail in extreme value theory. More precisely, let $X_1, X_2, ..., X_n$ be a sequence of observations from some distribution function F, with its order statistics $X_{1,n} \leq X_{2,n} \leq ... \leq X_{n,n}$. As an analogue to the central limit theorem, we know that, on average, if the maximum $X_{n,n}$, suitably centered and scaled, converges to a non-degenerate random variable, then there exist two sequences $\{a_n\}$ $(a_n > 0)$ and $\{b_n\}$ such that

$$\lim_{n\to\infty} \Pr\left(\frac{X_{n,n} - b_n}{a_n} \leq x\right) = G_\gamma(x), \tag{3.1}$$

where $G_\gamma(x) := \exp(-(1 + \gamma x)^{-1/\gamma})$ for some $\gamma \in R$ and x such that $1 + \gamma x > 0$. Note that for $\gamma = 0$, $-(1+\gamma x)^{-1/\gamma} = e^{-x}$. If (3.1) holds, then we say that F is in the max-domain of attraction of G_γ and γ is called the extreme value index. In Pareto distribution, the tail index $\gamma := 1/\alpha$ measures the thickness of the tail distribution, the bigger the γ, the heavier the tail. The estimation of γ has been thoroughly studied, see Beirlant *et al.* (2006) for a detailed account. We focus on one of the most often used estimator, the Hill estimator. The Hill index is defined by

$$H_{k,n} = \frac{1}{k}\sum_{j=1}^{k} \log X_{n-j+1,n} - \log X_{n-k,n}.$$

This estimator is consistent for $k \to \infty$, $k/n \to 0$ as $n \to \infty$, and under extra conditions, $\sqrt{k}(H_{k,n} - \gamma)$ is asymptotically normal with mean 0 and variance γ^2.

The Hill index relies on the average distance between extreme observations and the tail cutoff point to extrapolate the behavior of the tails into the broader part of the distribution. In practice, the behavior of the Hill index depends heavily on the choice of cutoff point k. This choice involves a tradeoff between bias and variance, which is well known in non-parametric econometrics. If k is chosen conservatively with few order statistics in the tail, then the tail estimate is sensitive to outliers in the distribution and has a high variance. On the other hand if the tail includes observations in the central part of the distribution, the variance is reduced but the estimate is biased upward. So, we estimate the tail index over a range of tail sizes. We apply three cut-off points in this study: namely, 1.0%, 1.5% and 2% of the total observations. Results for the Hill tail index estimates are reported in Table 3.3. The column 'Negative' estimates left-tail of distributions while 'Positive' is for the right-tail of distributions. The Hill tail index estimates are provided on the first, second, third and fourth rows in Table 3.3 representing all strategies, trend following, fundamental strategies and contrarian strategies; with cut-off points ranging from 1% to 2% of the total observations. A tail index of zero is equivalent to the tail density of the normal distribution. The results reveal the values of the Hill tail estimates are significantly different from zero which indicates that all CTA return distributions have fat tails. For instance, the Hill estimators of positive return at 1% cut-off point ranges from 0.2327 to 0.2658. In general, the contrarian strategies have the thickest tail and they experienced more extreme returns, the fundamental strategies have the thinnest tail and they experienced fewer extreme returns, and trend following are between them.

3.3. **Volatility clustering.** Volatility clustering is a salient feature in financial time series, which reflects observations that in markets big (small) movements are more likely to be followed by big (small) movements. Engle (1982) develops ARCH model and Bollerslev (1986) develops GARCH model for volatility clustering. Nelson (1991) further extends it to the exponential GARCH (EGARCH) model, which is a GARCH variant that models the logarithm of the conditional variance process. In addition to modeling the logarithm, the EGARCH model has additional leverage terms to capture asymmetry in volatility clustering. Glosten *et al.* (1993) proposes the GJR-GARCH model. The GJR model is a GARCH variant that includes leverage terms for modeling asymmetric volatility clustering. In the GJR formulation,

TABLE 3.3. Hill index for returns of the whole sample, trend following, fundamental and contrarian strategy.

	Negative			Positive		
	1%	1.5%	2%	1%	1.5%	2%
All	0.2451	0.2671	0.2798	0.2327	0.2495	0.2662
STG 1	0.2374	0.2646	0.2809	0.2277	0.2425	0.2636
STG 2	0.2260	0.2388	0.2440	0.1970	0.2264	0.2380
STG 3	0.2707	0.2910	0.3033	0.2658	0.2769	0.2903

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

large negative changes are more likely to be clustered than positive changes. To quantify volatility clustering in CTA returns, we conduct ARCH test developed by Engle (1982), and we then estimate the GARCH-t, EGARCH and the GJR models.

TABLE 3.4. ARCH effect test

	test stats	p-value	Sig.%
All	74.72	0.0349	88.76
STG 1	65.91	0.0139	86.05
STG 2	66.96	0.0788	84.21
STG 3	94.21	0.0372	96.30

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

Table 3.4 reports the ARCH test results, it shows clear evidence of ARCH effects which are strongest for contraian and comparable for trend following and fundamental strategies. The GARCH estimates in Table 3.5 provide further evidence of volatility clustering, and also show evidence of fat tail. For the EGARCH estimates, Table 3.6 reports the results, we see that the GARCH and ARCH coefficients are all positive, and the leverage coefficient, as expected, is negative for the whole sample and the contraian strategy. However, the leverage effect is not significant for the trend following and the fundamental strategies. GJR estimates in Table 3.7 show similar patterns of ARCH, GARCH, and leverage effects.

3.4. **Long memory in volatility.** Apart from the stylized facts of fat tail and volatility clustering, another well known stylized fact of financial return series is that the returns themselves contain little serial correlation, but the absolute returns $|r_t|$ and the squared returns r_t^2 do have significantly positive serial correlation over long lags. For example, Ding *et al.* (1993) investigate autocorrelations (ACs) of returns (and their transformations) of the daily S&P 500 index over the period 1928 to 1991 and find that the absolute returns and the squared returns tend to have

Table 3.5. GARCH $(1, 1) - t$ estimates

	a	b	α_0	α_1	β_1	DoF
All	0.0215	0.0344	0.0748	0.1914	0.7624	5.842
	(0.0147)	(0.0243)	(0.0210)	(0.0369)	(0.0361)	(1.174)
STG 1	0.0244	0.0541	0.0539	0.1811	0.7821	6.037
	(0.0165)	(0.0239)	(0.0171)	(0.0349)	(0.0302)	(1.101)
STG 2	0.0276	-0.0078	0.0343	0.1307	0.7915	7.812
	(0.0137)	(0.0272)	(0.0141)	(0.0278)	(0.0502)	(2.273)
STG 3	0.0124	0.0328	0.1367	0.2506	0.7105	4.146
	(0.0125)	(0.0231)	(0.0321)	(0.0465)	(0.0355)	(0.518)

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

Table 3.6. EGARCH estimates

	a	b	α_0	β_1	α_1	θ	DoF
All	0.0233	0.0363	-0.0699	0.9041	0.3213	-0.0014	5.8527
	(0.0144)	(0.0235)	(0.0323)	(0.0302)	(0.0598)	(0.0375)	(1.1643)
STG 1	0.0297	0.0402	-0.0418	0.9461	0.3009	0.0394	6.2121
	(0.0158)	(0.0230)	(0.0174)	(0.0160)	(0.0621)	(0.0412)	(1.1903)
STG 2	0.0289	-0.0078	-0.1600	0.7961	0.2375	0.0000	7.6226
	(0.0137)	(0.0266)	(0.0662)	(0.0759)	(0.0472)	(0.0286)	(1.9462)
STG 3	0.0092	0.0612	-0.0513	0.9131	0.4130	-0.0674	4.0348
	(0.0126)	(0.0221)	(0.0321)	(0.0205)	(0.0652)	(0.0378)	(0.5500)

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

very slow decaying autocorrelations, and further, the sample autocorrelations for the absolute returns are greater than those for the squared returns at every lag up to at least 100 lags. This kind of AC feature indicates the long-range dependence in volatility. The autocorrelations for the CTA returns are plotted in Figure 3.4, which clearly support the findings in Ding *et al.* (1993).

Besides the visual inspection of ACs of r_t, r_t^2 and $|r_t|$, one can also construct models to estimate the decay rate of the ACs of r_t, r_t^2 and $|r_t|$. For instance, we can semiparametrically model long memory in a covariance stationary series x_t, $t = 0$, ± 1, ..., by $s(\omega) \approx c_1 \omega^{1-2H}$ as $\omega \to 0^+$, where $0 < c_1 < \infty$, $s(\omega)$ is the spectral density of x_t, and ω is the frequency. Note that $s(\omega)$ has a pole at $\omega = 0$ for $0.5 < H < 1$ (when there is a long memory in x_t). When the value of H is close to 1, it reveals the greater degree of persistence or long-range dependence. For $H \geq 1$, the process is not covariance stationary. For $H = 0.5$, $s(\omega)$ is positive and finite

TABLE 3.7. GJR estimates

	a	b	α_0	β_1	α_1	γ	DoF
All	0.0231	0.0343	0.0530	0.7785	0.1797	0.0109	6.0485
	(0.0149)	(0.0240)	(0.0146)	(0.0331)	(0.0397)	(0.0471)	(1.3156)
STG 1	0.0281	0.0523	0.0387	0.8016	0.1795	-0.0184	6.2748
	(0.0167)	(0.0235)	(0.0127)	(0.0268)	(0.0393)	(0.0448)	(1.1995)
STG 2	0.0284	-0.0072	0.0337	0.7936	0.1376	-0.0211	8.0235
	(0.0139)	(0.0271)	(0.0137)	(0.0489)	(0.0358)	(0.0413)	(2.5589)
STG 3	0.0116	0.0349	0.0892	0.7311	0.2122	0.0801	4.2982
	(0.0126)	(0.0226)	(0.0187)	(0.0318)	(0.0434)	(0.0554)	(0.5704)

Note: All refers to full sample; STG1 refers to trend following strategy; STG2 refers to fundamental strategy; STG3 refers to contrarian strategy.

indicating uncorrelated series such as random walk. For $0 < H < 0.5$, we have short memory, negative dependence, or antipersistence. The ACs can be described by $\rho_k \approx c_2 k^{2(H-1)}$, where c_2 is a constant and $2(H-1)$ corresponds to the hyperbolic decay index.

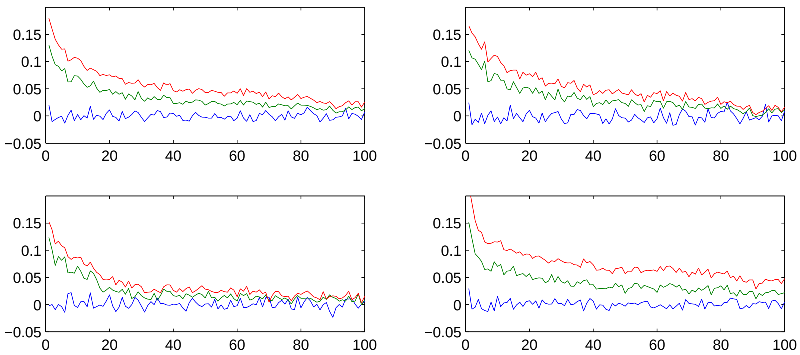

FIGURE 3.4. ACFs of the whole sample, trend following, fundamental and contrarian strategies.

There are several techniques in the literature to estimate the Hurst index H, for example, Hurst (1951); Hurst *et al.* (1965) introduce the re-scaled range statistical analysis to estimate the Hurst exponent. This analysis can display long-run correlations in random process. However, Lo (1991), Teverovsky *et al.* (1999), Weron and Przybylowsciz (2000) and Weron (2002) argued that this approach lacked robustness since it is very sensitive to the presence of short memory, heteroskedasticity, outliers, and multiple scale behavior. Lo (1991) modifies the rescaled range statistical analysis by using autocovariance estimator instead of that of the standard deviation. There are other ways to estimate the Hurst index, for instance, Geweke and Porter-Hudak (1983) use the periodogram regression, Peng *et al.*, (1994) uses the multi-affine analysis, Ivanova and Ausloos (1999) applies multi-fractal/multi-affine analysis, Ausloos (2000) uses the detrended fluctuation analysis, Ellinger (2000) applies the moving-average analysis technique, and Percival and Walden (2006) uses the wavelet transform module maxima method; but still, most of them suffer from sensitivity and robustness as discussed above.

Recently, Di Matteo (2007) proposes a generalized Hurst exponent approach which deals with the sensitivity issues to any type of dependence in the data. The approach is computationally straight forward and simple to apply. The Generalized Hurst exponent estimation provides a natural, unbiased, statistically and computationally efficient analyzing tool for empirical studies. This method examines the scaling properties of the data directly via the computation of the q-order moments of the distribution of the increments. The q-order moments are much less sensitive to the outliers than the maxima/minima and different exponents' q are associated with different characterizations of the multi-scaling complexity of the signal. This method allows us to distinguish between uni-scaling and multi-scaling process. In the case of uni-scaling process, the scaling behavior is determined by the unique constant H that consists with the Hurst exponent where $qH(q)$ is liner ($H(q) = H$). In the case of multi-scaling process, $H(q)$ depends on q where $qH(q)$ is non-liner.

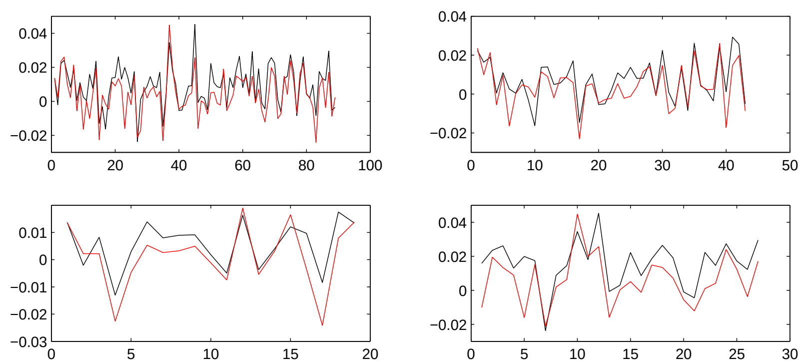

FIGURE 3.5. Hurst index plot for returns of the whole sample, trend following, fundamental and contrarian strategies.

Figure 3.6 shows the curves of $qH(q)$ as a function of q not linear in q, but significantly bending below the linear trends. This reveals the returns of CTAs exhibit evidence of multi-scaling behavior which is a sign of deviation from the Brownian, fractional Brownian, Levy and fractional Levy models. The same behavior holds for the case of all CTAs, trend following, fundamental strategies, and contrarian strategies.

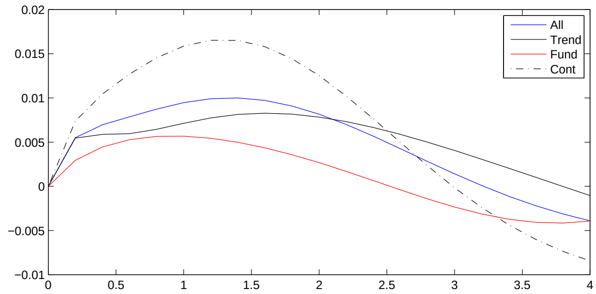

FIGURE 3.6. $(q, qH(q))$ plot.

3.5. **Markov-switching multifractal model.** Financial markets volatility displays some similarities to fluid turbulence. For example, both turbulence and financial fluctuations are characterized by intermittency at all scales. A cascade of energy flux is known to occur from the large scale of injection to the small scale of dissipation, cf. Mandelbrot (1974) and Harte (2001). In statistical physics, such "cascades" are modeled by multiplicative operations on probability measures.

Mandelbrot et al. (1997) first introduced the multifractal apparatus into financial markets, adapting the approach of Mandelbrot (1974) to an asset-pricing framework. This multifractal model of asset returns (MMAR) assumes that asset returns r_t follow a compound process, in which an incremental fractional Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, the practical applicability of MMAR suffers from the non-causal nature of the time transformation and non-stationarity due to the inherent restriction to a bounded interval. These limitations have been overcome by the development of an iterative version of the MF models, including the Markov-switching multifractal model (MSM), cf. Calvet and Fisher (2004) and Lux (2008). In this approach, asset returns are modeled as:

$$r_t = \sigma \left(\prod_{i=1}^{k} M_t^{(i)} \right)^{1/2} \cdot \epsilon_t, \tag{3.2}$$

with ϵ_t drawn from a standard Normal distribution $N(0, 1)$ and instantaneous volatility being determined by the product of k volatility components or multipliers $M_t^{(1)}$, $M_t^{(2)}$ ..., $M_t^{(k)}$, and a constant scale parameter σ. Each volatility component is renewed at time t with probability γ_i depending on its rank within the hierarchy of multipliers or remains unchanged with probability $1 - \gamma_i$. Calvet and Fisher (2004) propose to specify transition probabilities as

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^{i-1})}, \tag{3.3}$$

with parameters $\gamma_1 \in (0, 1)$ and $b \in (1, \infty)$; and Lux (2008) assumes $\gamma_i = 2^{(k-i)}$. Both specifications guarantee convergence of the discrete-time multifractal process to a limiting continuous-time version with random renewals of the multipliers. Additionally, $E[M_t^{(i)}]$ or $E[\sum M_t^{(i)}]$ equal to some arbitrary value is usually imposed for the sake of normalizing the time-varying components of volatility. Both Calvet and Fisher (2004) and Lux (2008) assume a Binomial distribution with parameters m_0 and m_1 for the volatility components, which means, whenever there is volatility component updated, either m_0 or m_1 will be drawn. In addition, they further assume $m_1 = 2 - m_0$ and thus guaranteeing an expectation of unity for all $M_t^{(i)}$.

With this rather parsimonious approach, one preserves the hierarchical structure of MMAR while dispensing with its restriction to a bounded interval. While this model is asymptotically "well-behaved" (i.e. it shares all the convenient properties of Markov-switching processes) it is still capable of capturing some important properties of financial time series, namely, volatility clustering and the power-law behaviour of the autocovariance function of absolute moments:

$$Cov(|r_t|^q, |r_{t+\tau}|^q) \propto \tau^{2d(q)-1}. \tag{3.4}$$

The Markov-switching MF model is rather characterized by only 'apparent' long-memory with an approximately hyperbolic decline of the autocorrelation of absolute powers over a finite horizon and exponential decline thereafter. In particular, approximately hyperbolic decline as expressed in eq. (3.4) holds only over an interval $1 \ll \tau \ll b^k$ with b the parameter of the transition probabilities of eq. (3.3) and k the number of hierarchical levels.

Various approaches have been employed to estimate multi-fractal models. The parameters of the combinatorial MMAR have been estimated via an adaption of the scaling estimator and frequency spectrum approach of statistical physics. However, this approach has been shown to yield very unreliable results (cf. Lux (2004)). A broad range of more rigorous estimation methods have been developed, including maximum likelihood (ML) estimation by Calvet and Fisher (2004) and simulation based ML by Calvet et al (2006), and GMM (Generalized Method of Moments) by Lux (2008).

In this paper we adopt the GMM approach formalized by Hansen (1982), which has become one of the most widely used methods of estimation for models in economics and finance. With analytical solutions of a set of appropriate moment conditions provided, the vector of parameters, say β, can be obtained through minimizing the differences between analytical moments and empirical moments:

$$\widehat{\beta}_T = \arg\min_{\beta \in \Theta} \bar{M}_T(\beta)' W_T \bar{M}_T(\beta). \tag{3.5}$$

Θ is the parameter space, and in our case the parameters to be estimated $\beta \in \{m_0, \sigma\}$. $\bar{M}_T(\beta)$ stands for the vector of differences between sample moments and analytical moments, and W_T is a positive definite weighting matrix, which controls over-identification when applying GMM. Implementing Eq. (3.5), one typically starts with the identity matrix; then the inverse of the covariance matrix obtained from the first round estimation is used as the weighting matrix in the next step; and this procedure continues until the estimates converge. We use Newey-West covariance matrix within our study.

As is well-known, $\hat{\beta}_T$ is consistent and asymptotically Normal if suitable 'regularity conditions' are fulfilled (sets of which are detailed, for example, in Harris (1999)). $\hat{\beta}_T$ then converges to

$$T^{1/2}(\hat{\beta}_T - \beta_0) \sim N(0, \Xi), \tag{3.6}$$

with covariance matrix $\Xi = (\bar{F}_T' \bar{V}_T^{-1} \bar{F}_T)^{-1}$ in which β_0 is the true parameter vector, $\hat{V}_T^{-1} = Tvar\bar{M}_T(\beta)$ is the covariance matrix of the moment conditions, $\hat{F}_T(\beta) = \frac{\partial \bar{M}_T(\beta)}{\partial \beta}$ is the matrix of first derivatives of the moment conditions, and $\bar{V}_T$ and $\bar{F}_T$ are the constant limiting matrices to which $\hat{V}_T$ and $\hat{F}_T$ converge.

We have estimated the Markov-switching multifractal model with three trading strategies returns, the sample sizes for STG 1 trend following, STG 2 fundamental and STG 3 contrarian strategies are 42, 19 and 27 respectively. We observe estimates of $m_0 = 1$ for all time series are significant except with one return series in trend following strategy. We present the empirical GMM estimates for the three different investment strategies as summarized in Table 3.8, including the summary statistics for overall samples estimates. We observe m_0 of each strategies are apparenty deviate from 1,[3] and the varieties across three different strategies, namely, results on trend following strategy showing less degree of long memory, while results from contrarian strategy showing the highest m_0 estimates.

4. Conclusion

Previous work finds random walk behaviour of monthly CTA returns. In this paper we use a unique dataset of daily returns of 89 programmes of Commodity Trading Advisors (CTA), we investigate the distributional properties of CTA strategies including trend following, fundamental and contrarian strategies. We find that daily CTA return behaves quite differently from that of monthly return. Daily CTA return exhibits strong features of fat-tail, volatility clustering, and long memory in volatility. Our findings are robust to different measures of fat-tail, volatility clustering and long memory in volatility. Our study contributes to the literature of stylized facts of financial markets, it also provides insights to practitioners because

[3] $m_0 = 1$ is the borderline cases in multifractal processes which the volatility process collapses to a constant, and therefore implies long memory does not exist.

TABLE 3.8. Empirical estimates for returns of the three trading strategies returns.

	all		STG 1		STG 2		STG 3	
	m_0	σ	m_0	σ	m_0	σ	m_0	σ
mean	1.363	0.009	1.331	0.010	1.362	0.007	1.415	0.008
s.d.	0.134	0.004	0.121	0.004	0.139	0.002	0.136	0.004
min	1.102	0.002	1.102	0.005	1.131	0.004	1.173	0.002
max	1.730	0.020	1.597	0.019	1.568	0.010	1.730	0.020

Note:This table reports the statistics of the Markov-switching multifractal models estimates for overall strategies, and three separate trading strategies, respectively. N is the sample size for each trading strategy, $ret1$ refers to returns from trend following strategy; $ret2$ refers to returns from fundamental strategy; $ret3$ refers to returns from contrarian strategy

the information from monthly data might be misleading. Investors are usually provided with monthly CTA performance data, they should be cautious when making investment decision and selecting programmes of CTA.

Appendix A. Additional Tables and Figures

Table A.1. Most often used CTAs return indices.

	Rebalance Frequency	Constituents Reformed	Reporting Frequency
Altegris 40 Index	Monthly	Monthly	Monthly
Barclay BTOP50 Index	Annually	Annually	Daily
Barclay CTA Index	Annually	Annually	Monthly
Barclay Systematic Trader Index	Annually	Annually	Monthly
CISDM CTA Equal Weighted Index	Monthly	Monthly	Monthly
CISDM CTA Equal Weighted Index	Monthly	Monthly	Monthly
Credit Suisse Managed Futures Hedge Fund Index	Monthly	Quarterly	Monthly
ISTOXX Efficient Captial Managed Futures 20 Index	Monthly	Annually	Daily
Newedge CTA Index	Annually	Annually	Daily
Newedge CTA Trend Index	Annually	Annually	Daily
STARK 300 Trader Index	Monthly	Monthly	Monthly
STARK Systematic Trader Index	Monthly	Monthly	Monthly

TABLE A.2. Sample divided by sub strategy.

	Trend following	Short-term trading	Fundamental
Systematic	43	27	19
Discretionary	2	0	5

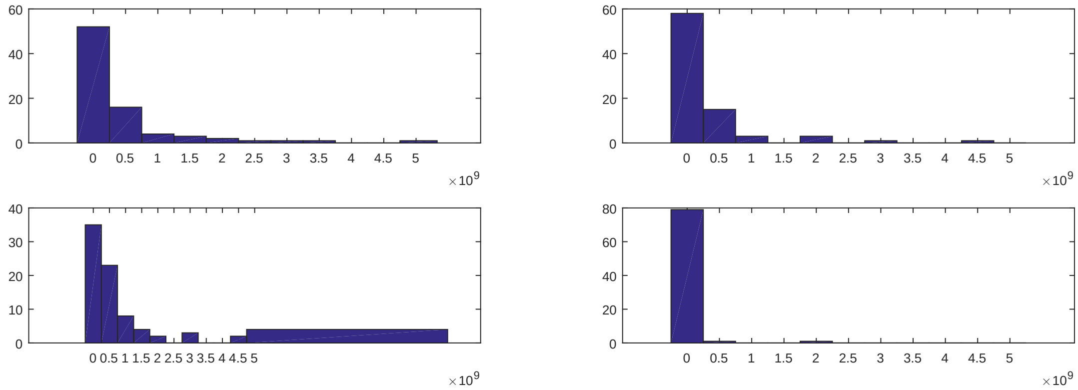

FIGURE A.1. Distribution of AUM of the whole sample, trend following, fundamental and contrarian strategy.

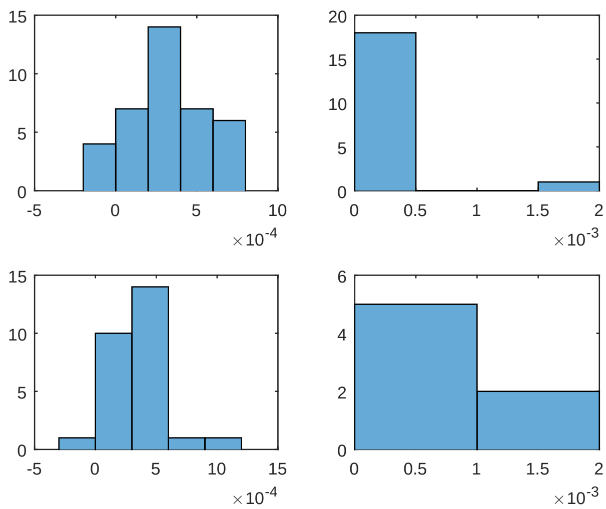

FIGURE A.2. Distribution of mean of returns of the whole sample, trend following, fundamental and contrarian strategy.

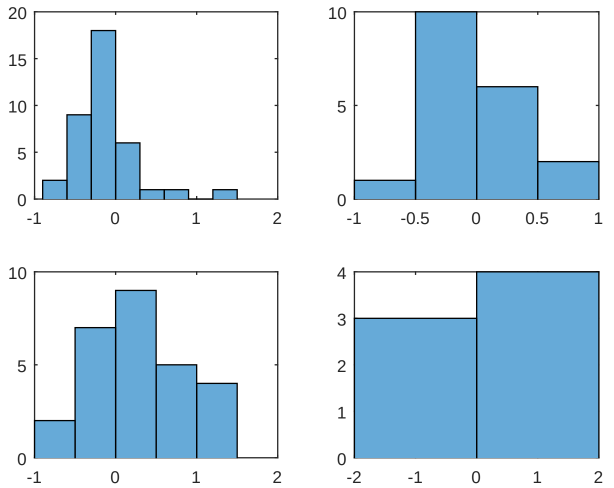

FIGURE A.3. Distribution of skewness of returns of the whole sample, trend following, fundamental and contrarian strategy.

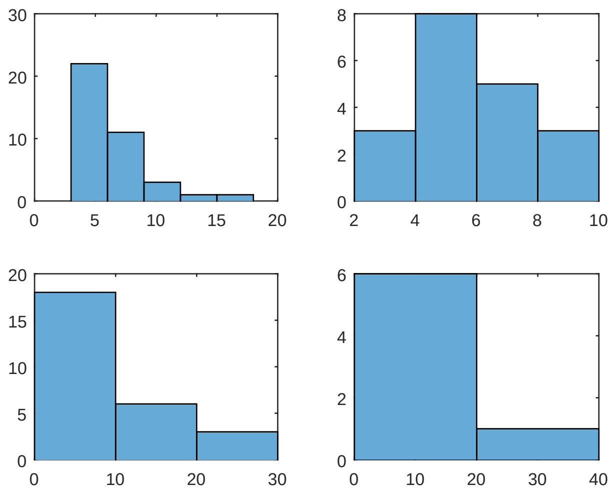

FIGURE A.4. Distribution of kurtosis of returns of the whole sample, trend following, fundamental and contrarian strategy.

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