Commodity futures returns: More memory than you might think!

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Abstract

This paper investigates long-range dependence in fourteen commodity and three other financial futures returns series from 1993-2009 and shows that long memory is a pervasive phenomenon in contrast to the extant evidence. Utilizing a semi-parametric wavelet-based estimator with time windows, the results provide overwhelming evidence of time-varying long-range dependence in all futures returns series. Structural break tests indicate multiple regimes of dependence, in the majority of which the persistence parameter is statistically significant. The results also provide evidence of predominantly negative parameters values which are known as anti-persistence. The latter is consistent with investor overreaction to shocks and suggests temporary departures from market efficiency.

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1. Introduction

Although early studies of commodity price movements generally supported the random walk hypothesis (Larson, 1960), later research demonstrated that this hypothesis can often be rejected by the inappropriate assumption of normal distributions of commodity prices. The original work of Mandelbrot (1963, 1966) showed that commodity price changes are not normally distributed, but fat-tailed, or leptokurtic. Since then, many others have supported his argument (see, inter alios, Stevenson and Bear, 1970; Dusak, 1973). Recently, researchers also found that some commodity returns can be characterised by a long memory component (Barkoulas et al., 1999; Crato and Ray, 2000; Elder and Jin, 2009).[1] Long memory or long term dependence is a special form of dynamics that describes the correlation structure of a series at long lags. If a series exhibits long memory, then it is characterised by distinct but non-periodic cyclical patterns. Long memory is interesting because the associated dependence in the first and /or second moments of the distribution can lead to a potentially predictable component in the series. This may enable investors to exploit such predictability and earn speculative profits, thereby casting doubt on the random walk and weak-form efficiency hypotheses.

The extant research has primarily addressed the problem of whether a series exhibits long memory by estimating a single 'static' value of the long memory parameter employing the popular semi-parametric Geweke and Porter-Hudak (1983) (hereafter GPH) estimator. This single value describes the global long-range dependence and implicitly assumes a stable environment in financial markets. For example, Elder and Jin (2009) examined 15 commodity daily futures return series from 1974 to 2006. Using wavelet-based global estimators, they found limited evidence of fractional integration in metal futures whereas

[1] Early evidence of long memory has also been found in stock returns (see *inter alios*, Greene and Fieltitz, 1977; Lo, 1991; Barkoulas and Baum, 1996) and in exchange rates (see, *inter alios*, Cheung, 1993; Baillie and Bollerslev, 1989 and 1994).

agricultural commodities exhibited significant anti-persistence in approximately half the cases they examined. Anti-persistence is interesting as it indicates a series that is, as Elder and Jin comment, “choppier than white noise,” and thus implies that investors may overreact to shocks or new information.

More recently, researchers such as Cajueiro and Tabak (2004) and Batten and Szilagyi (2007), suggest that one needs to allow parameters to evolve over time properly to model these dynamic systems. This can provide insights into the causes of any observed variation in dependence over time and any intertemporal deviations from efficiency by decomposing the measure of dependence into its underlying components. Fernandez (2010) employed a time-varying approach to assess long-range dependence in commodity markets instead of relying on single static measures. Employing a dataset of 20 commodity daily futures return series from 1991 to 2008, a series of long memory parameters were estimated using a rolling window and the rejection percentage of the null hypothesis of no long-range dependence is recorded. At conventional significance levels, the majority of the rejection percentages are less than 10%; however, these percentages are occasionally found to be much higher and indicate evidence of both persistence and anti-persistence.

This paper continues the examination of commodity futures returns for evidence of time-varying long memory. Our sample of daily data covers the period 1993 to 2009 and comprises 17 daily futures prices, of which 14 are commodities (including storable and non-storable agricultural commodities), as well as one stock index and two major exchange rates for comparative purposes. The results indicate that 9 out of 17 of futures returns display evidence of long memory over the full sample period based on static estimation, suggesting that long memory matters but is not pervasive. Analogously to Elder and Jin (2009), all of the significant results point to anti-persistence as the global long memory phenomenon.

Our first substantive contribution is to utilize the semi-parametric wavelet estimator developed by Jensen (1999) to produce time-varying long memory results for each asset series. Presenting these graphically, albeit a simple approach, has considerable advantages over the prior approach of recording the rejection percentage of the null of no long-range dependence. In particular, a rejection percentage, in aggregating the number of rejections, does not show the intertemporal location and persistence of any inefficiency. Interestingly, the graphs suggest important fluctuations in long memory for all return series. The typical pattern indicates longer periods of low anti-persistence that are occasionally punctuated by shorter periods of high inefficiency. This nuance is particularly important in the case of commodities that showed no long memory under static estimation, whereas all now reveal long memory parameters that appear significant in some periods but not in others. The observed variability in the long memory parameter estimates suggests that a formal test of time-varying parameters is required.

Our second contribution is that the Bai and Perron (2003) test results indicate multiple structural breaks in the persistence parameter. They indicate that all return series exhibit at least one structural break in the persistence parameter and eight contracts exhibit at least 2 structural breaks. Moreover, the persistence parameter is significant in most regimes for series. These novel findings formally establish that persistence is time-varying in commodity and other asset futures markets, long memory is far more pervasive than previously thought and consequently, significant periods of inefficiency exist. More importantly, the pragmatic implications of such time-varying dependence may widely impact on different markets such as the options market for futures.

The final contribution is a tentative interpretation of the main result of anti-persistence. The Tang and Xiong (2012) financialization of commodities hypothesis posits that commodities are behaving like other financial assets mainly as a result of increased index investment. This implies that they may display the typical under- and overreaction patterns followed by stocks. Such patterns typically stem from biased beliefs about pricing on the part of the representative investors as in the Barberis et al. (1998) and the Daniel et al. (1998) models, or alternatively they could arise from the behaviour of naive relative to informed investors as in the Hong and Stein (1999) model. The prevalent anti-persistence patterns in commodity futures returns are consistent with market inefficiency caused by investor overreaction to shocks or news which is relatively quickly reversed.

The rest of this paper is organized as follows. Section 2 presents the fractional integration testing methodology and section 3 describes the data and the analyses the results. Section 4 discusses and interprets our results while Section 5 concludes.

2. Methodology

2.1 Fractional integration

The introduction of the autoregressive fractionally integrated moving average model (ARFIMA) by Granger and Joyeux (1980) and Hosking (1981) allows the modelling of persistence or long memory via estimation of the differencing or memory parameter *d*. A time series y_t follows an ARFIMA (p,d,q) process if

$$\Phi(L)(1-L)^d y_t = \mu + \Theta(L)\varepsilon_t, \quad \varepsilon_t \sim iid(0,\sigma^2) \tag{1}$$

where L is the backward-shift operator, and $\Phi(L)$ and $\Theta(L)$ are autoregressive and moving average polynomials, respectively, with roots outside the unit circle. The fractional differencing lag operator $(1-L)^d$ is defined by the binomial expansion

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \tag{2}$$

where $\Gamma(\bullet)$ is the gamma function. An ARFIMA process is said to be stationary and invertible when -0.5 < d < 0.5. For such processes, the autocorrelation function for a stationary process exhibits geometric decay, whereas that for a long memory process exhibits slow hyperbolic decay and the autocorrelation coefficients are of the same sign as d. When $1/2 \le d < 1$, the relevant series is non-stationary, the unconditional variance growing at a more gradual rate than when $d = 1$, but mean reverting.

The memory parameter d can be estimated by several approaches. The most widely used technique is the log-periodogram GPH estimator because of its semi-parametric nature (Geweke and Porter-Hudak, 1983; Robinson, 1995a).[2] This requires only weak assumptions on the short-memory process ε_t in equation (1). Geweke and Porter-Hudak demonstrate that for frequencies near zero, d can be consistently estimated from the least squares regression

$$\log(I(\lambda_j)) = \beta_0 - d\log\left\{4\sin^2(\lambda_j/2)\right\} + \varepsilon_j \qquad j = l+1, l+2 ... m \tag{3}$$

where $I(\lambda_j)$ is the sample spectral density of y_t evaluated at the frequencies $\lambda_j = 2\pi j/T$, T is the number of observations and m is small compared to T. One of the advantages of the GPH technique is that hypotheses about d can be tested using standard t-statistics (Hassler et al., 2006). For the stationary range, $-1/2 < d < 1/2$, Robinson (1995a) shows that the GPH estimate is consistent and asymptotically normally distributed. Velasco (1999a) shows that the estimate of d is consistent for $1/2 < d < 2$ and asymptotically normally distributed for $1/2 < d < 7/4$ when the data are differenced.

2.2 Wavelet estimator of the fractional integration parameter

The wavelet ordinary least squares (OLS) estimator of the fractional integration parameter d was introduced by Jensen (1999). It is worth noting that there is a similarity between the wavelet OLS estimator and the popular semi-parametric GPH estimator. The GPH estimator utilizes Fourier analysis to decompose a time series or signal into low frequency and high

[2] Other semi-parametric estimators like the Gaussian Semi-Parametric (GSP) estimator (Robinson, 1995b and Velasco, 1999b) are studied in Robinson and Henry (1999). We employ the GPH technique because of its wide application in the literature.

frequency components. In other words, a time series can be expressed as a linear combination of sine and cosine functions in the frequency domain. Wavelet analysis, on the other hand, utilizes functional transforms, known as wavelets, to decompose a signal into various frequencies called scales, where the scale is inversely related to frequency, and is localised in time.

Although the wavelet estimator can be most simply described as functional transforms in the spirit of Fourier analysis, it has some distinct advantages. Firstly, Fourier analysis transforms a time series from the time domain to the frequency domain. Therefore it does not preserve information in the time domain. By contrast, wavelet analysis transforms the time series into different frequencies (scales) and is localized in time. Therefore, wavelets can zoom in on series behaviour at a particular point in time, whilst they can also zoom out to reveal any long and smooth features of a process. Secondly, since few economic series follow the smooth rhythmic cycles suggested by the sine and cosine functions underlying Fourier analysis, spectral analysis cannot always adequately capture abrupt changes or cusps in a signal. In contrast, the basis functions underlying wavelet analysis typically do not oscillate indefinitely and are not generally smooth. Wavelet basis functions have finite oscillations that can be scaled and shifted to capture events that are local in time. Finally, wavelets have been found to be useful in finance for dealing with multi-scale problems, in parameter estimation and in noise removal.[3]

Consider a real-valued function $\psi(t)$, which satisfies two basic properties $\int_{-\infty}^{\infty} \psi(t)d(t) = 0$ and $\int_{-\infty}^{\infty} \psi(t)^2 d(t) = 1$, so that ψ is a square integrable function $L^2(\Re)$ with finite oscillations that diminish to zero as $t \to \pm\infty$. $\psi(t)$ is a wavelet if it also satisfies the admissibility condition (see Percival and Walden, 2000). The wavelet $\psi(t)$ can be scaled and translated by integers j and k

[3] We would like to thank an anonymous reviewer for this insight.

$$\psi(t)_{j,k} = 2^{-j/2}\psi(2^{-j}t-k) \qquad (4)$$

where *j* represents *scaling* parameter and *k* represents *translation* (or *shift*) parameter. The set of scaled and translated wavelets $\psi(2^{-j}t-k)$ form an orthonormal basis for the set of square integrable function $L^2(\Re)$. The normalization factor $2^{-j/2}$ is maintained at different scales. $\psi(t)$ is called the mother wavelet, which is *mother* to all scales and translations of ψ in (4).

The wavelet transform of a signal (x) usually utilizes two orthogonal functions: the mother wavelet function $\psi(t)$ and a scaling function $\varphi(t)$.[4] We follow Elder and Jin (2007) in ignoring the scaling function for heuristic purposes. The wavelet coefficients ($w_{j,k}$) that link the original series to the scaled and translated wavelets can be obtained through the projection of the original series $x(t)$ onto a sequence of the wavelet basis functions

$$x(t) = \sum_j \sum_k w_{j,k} 2^{-j/2}\psi(2^{-j}t-k) \qquad (5)$$

where *j, k* are integer indices for the finite or infinite sum. In practice, the wavelet defined in (5) restricts the sample size to a factor of two. To avoid boundary effects associated with the evaluation of (5), the sample size must be a power of two, or if not, the sample must be trimmed with zeros. We thus choose a sample with 2^{12} or 4096 observations.

Several different wavelets have been proposed in the mathematics literature including the Daubechies (1988) family of wavelets.[5] The Daubechies wavelet is very popular in empirical time series analysis since it has compact support. Such an important characteristic allows wavelets to more parsimoniously describe functions with cusps and spikes (Lien and Shrestha, 2007; Power and Turvey, 2010). We therefore apply the Daubechies wavelet.

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[4] The scaling function is also called a *father* wavelet.

[5] Mallat (1999), who has been a pioneer in the wavelet literature, provides a broad perspective on optimizing the design of the wavelet. His approach involves the usage of an effective pyramid algorithm for computing the fast wavelet transform. Orthonormal wavelets are considered to work particularly well with the fast wavelet transform computation such as Haar, Daubechies, Coiflet and Symmlet wavelets. Other wavelets such as Meyer wavelets may not be best suited for such fast computation due to their non-orthogonal bases. We thank an anonymous reviewer for this point.

Similar to the GPH estimator that captures low-frequency spectral behaviour, the wavelet estimator captures high scale wavelet behaviour. That is, the wavelet scales which contribute the most to the series' variance, are associated with the wavelet coefficients with the largest variance. Thus, the wavelet coefficients sample variance can be used to provide a parametric estimate of the fractional integration parameter *d*. More specifically, the GPH estimator can be viewed as a regression of the log sample spectrum on the log frequency whereas the wavelet OLS estimator can be viewed as a regression of the (normalized) log wavelet scale spectrum on a log scale.

To express the above algebraically, consider that Jensen (1999) demonstrates that the wavelet coefficients $w_{j,k}$ in Equation (5), associated with a mean zero series ARFIMA (*0*, *d*, *0*) process $x(t)$ with $|d| < 0.5$, are distributed approximately $N(0, \sigma^2 2^{-2d(J-j)})$. If we denote the variance of wavelet coefficients at scale *j* by $\text{var}(w_{j,\cdot}) = \sigma^2 2^{-2d(J-j)}$, then after taking logarithms, an estimate of the fractional integration parameter *d* can be obtained by applying ordinary least squares to

$$\ln[\text{var}(w_{j,\cdot})] = \ln[\sigma^2] + d\ln[2^{-2(J-j)}] \tag{6}$$

where the sample variance of the wavelet coefficients at scale *j* is simply the sum of the squared wavelet coefficients at scale *j* normalized by the number of wavelet coefficients

$$\hat{\text{var}}(w_{j,\cdot}) = \frac{1}{2^{J-j}} \sum_{k=0}^{2^{(J-j)}-1} w_{j,k}{}^2 \tag{7}$$

Jensen (1999) demonstrates that the wavelet OLS estimate $\hat{d}$ is a consistent estimator of the fractional integration parameter *d*. Additionally, Jensen also demonstrates through Monte Carlo experiments that the wavelet OLS estimator of d in (6) has approximately four to six times smaller MSE than the familiar GPH estimator.

2.3 Time-varying long memory parameters

Some recent literature argues that market efficiency seems to evolve over time and implies that a single long memory parameter cannot be representative of an entire data series. For comparisons sake, we therefore adopt two different approaches to evaluate the wavelet OLS estimates. The first approach is to estimate a static long memory parameter over the full sample of $N = 4096$ observations. The second approach employs the "rolling sample" method following Cajueiro and Tabak (2004) and Batten et al. (2005).

To be clear about the latter approach, let r_t be the logarithm of futures returns. The wavelet coefficients of r_t can be obtained by setting $x(t) = r_t$ in (5). Next, consider a sub-sample of continuously compounded asset returns $\{r_1, r_2, ..., r_n\}$, where the sub-sample consists of n observations. Following Cajueiro and Tabak (2004), we choose an approximately 4 year time-window[6] where n = 1024 (i.e., 2^{10}). Specifically, we then calculate the wavelet OLS estimate for the initial period of 1024 observations and then roll the sample one data point forward, eliminating the first observation and including the next one, repeating this procedure until the end of the series. In this case, we would have (N – n +1) or 3073 wavelet OLS estimates. This methodology, in effect, creates a series of long memory parameter values, allowing any changes in those values to be assessed over time.

2.4 Testing for structural breaks

To test whether time-varying long memory parameters contain multiple structural breaks, we apply the Bai and Perron approach (1998, 2003a, 2003b, 2004). Specifically, consider the m-breaks in mean model[7]

$$y_t = \mu_j + \varepsilon_t \tag{8}$$

[6] Cajueiro and Tabak (2004) argue that a 4 year time-window reflects political and business cycles in most countries and it is sufficient to give precise estimates.

[7] Choi and Zivot (2007) argue that structural breaks in the mean have a natural interpretation due to the direct effect of an economic shock to the forward premium.

where $j = 1,...,m+1$ and μ_j is the mean level of y_t in the j^{th} regime. Additionally, the indices ($T_1,...,T_m$) denote the breakpoints for the different regimes and we adopt the convention that $T_0 = 0$ and $T_{m+1} = T$. These breakpoints can be estimated via the following objective function

$$(\hat{T}_1,...,\hat{T}_m) = \arg\min_{T_1,...,T_m} S_T(T_1,...,T_m), \tag{9}$$

where for each *m*-partition ($T_1,...,T_m$) the least squares estimates μ_j are generated by minimizing the sum of the squared residuals

$$S_T(T_1,...,T_m) = \sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (y_t - \mu_j)^2 , \tag{10}$$

giving μ_j ($T_1,...,T_m$) as the mean estimates associated with the given *m*-partition that minimizes S_T ($T_1,...,T_m$). Bai and Perron (2004) suggest the use of a specific dynamic programming algorithm to solve the minimization problem in equation (9).

Bai and Perron (1998) propose a group of test statistics to choose the number of mean breaks (*m*). Let $SupF_T(l)$ be the *F* statistic for testing the null hypothesis of no structural breaks ($m = 0$) versus the alternative that there are breaks ($m = l$). Two "double maximum" statistics can now be proposed to determine if a structural break has occurred, both testing the null hypothesis of no structural breaks against the alternative of an unknown number of breaks (where L is an upper bound). First, $UD\max = \max_{1 \le l \le L} SupF_T(l)$ and second, the weighted double maximum statistic $WD\max = \max_{1 \le l \le L} w_l \cdot SupF_T(l)$, which applies different weights to the individual $SupF_T(l)$ so that the marginal *p*-values are equal across values of *l*. Lastly, Bai and Perron also propose using $SupF_T(l+1 \mid l)$ to test the null hypothesis of *l* breaks against the alternative of *l* +1 and derive appropriate critical values for each test statistic.

On the basis of several Monte Carlo simulations, Bai and Perron (2004) recommend the following estimation strategy to assess whether a structural change has occurred. In a preliminary step, use the double maximum statistics to detect if at least one break is present. If the double maximum statistics are significant, next select the number of structural breaks using the $SupF_T(l+1|l)$ statistics sequentially starting with l = 1. This procedure will be stopped when selection process rejects the largest value of l.

The Bai and Perron framework has been widely employed in empirical analysis of multiple structural breaks[8] due to some useful characteristics. Firstly, it estimates unknown multiple break points in a dynamic linear regression model using the least-squares principle. Secondly, the Bai and Perron (1998, 2003a) method allows for general specifications when test statistics are calculated. In particular, specifications can allow for autocorrelation and heteroskedasticity in the regression model residuals, as well as different moment matrices for the regressors in the different regimes. To allow for all these features, the most general Bai and Perron (1998, 2003a) specification[9] is adopted in this paper.

3. Data and empirical analysis

3.1 Data

Our data set consists of seventeen daily futures prices to undertake a comparative analysis across a range of assets. These comprise Soybeans, Corn, Wheat, Cocoa, Sugar, Cotton, Heating oil, Gold, Silver, Copper, Live cattle, Feeder cattle, Hogs, Pork bellies, S&P 500, $/Pound and $/Yen. All the data are taken from DataStream International. Our data cover different sectors of futures markets such as agricultural, energy and metal commodities as

[8] See, *inter alios*, Guo and Wohar (2006), Choi and Zivot (2007) and Kellard and Sarantis (2008).
[9] Following Kellard and Sarantis (2008), we set cor_u = 1, het_u = 1, π = 0.15 using the notation of Bai and Perron (2004). π is an arbitrary small trimming value which sets the maximum number of breaks allowed in the series. When π = 0.15, the maximum of 5 breaks is allowed. We set the maximum breaks L = 5, as suggested by Choi and Zivot (2007). Note that the Bai and Perron (1998, 2003a, b) statistics are computed using the GAUSS program available from Pierre Perron's home page at http://econ.bu.edu/perron/.

well as stocks and currencies. The choice of these contracts is based on the recent studies of long memory in commodity futures markets (see, *inter alios*, Barkoulas et al., 1999; Crato and Ray, 2000; Elder and Jin, 2009; Fernandez, 2010). Since the storability characteristics of commodities may significantly affect the price discovery performance, both storable and non-storable commodity data are collected.

Daily futures contract settlement prices spanning the period October 1993 to December 2009 are obtained from DataStream yielding 4,096 observations per contract. The futures prices are those from the nearest contract but contracts are rolled over to the next contract on the first business day of the contract month. Yang et al. (2001) argue that it is appropriate to use the nearby futures contract since it is typically the most liquid and the most actively traded. Futures return series are calculated using the settlement price for days *t* and *t*-1. Specifically, the continuously compounded daily returns are defined as $r_t = \ln F_t - \ln F_{t-1}$. Most of the return distributions are negatively skewed and display a high degree of excess kurtosis. All return series also appear extremely non-normal from the Jarque and Bera (1987) test results. These significant deviations from normality (combined with our later results on long memory) may be an indication of non-linear dynamics (Fang et al., 1994).

We implement the augmented Dickey-Fuller (ADF) unit root test results for futures returns where the number of lags is chosen through general-to-specific testing at the 5 percent significance level as recommended by Ng and Perron (2001). The unit root null is clearly rejected at all conventional significance levels. The results from the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) test for stationarity indicate that return series are stationary at the 5% significance level.[10]

3.2 Baseline static long memory results

[10] The summary statistics and ADF and KPSS results are available from the authors upon request.

The static or global fractional integration parameter d is estimated for the futures return series by the wavelet OLS estimator d_{WOLS} and for the sake of comparison, the popular GPH estimator d_{GPH}.[11] Given that all return series are stationary, we test the null of no long memory ($H_0 : d = 0$) against the alternative of fractional integration ($H_0 : d \neq 0$). The Daubechies (1988) wavelet is considered to be the most suitable wavelet method for economic or financial series (Jensen, 1999; Tkacz, 2001).[12] This wavelet is employed with six smoothing parameters (i.e., Daubechies-6). In particular, Jin et al. (2006) argue that, although there is no metric for selecting an "optimal" value, greater distortion may be caused by large smoothing parameters due to boundary effects. Table 1[13] reports the wavelet and GPH estimates for the fractional integration parameters over the entire 1993-2009 sample.

[Insert Table 1 about here]

Some interesting findings emerge. First, the Daubechies-6 wavelet OLS estimator results strongly support fractional integration for many commodity futures returns. The null of no long memory is rejected at the 5% level for 8 commodity returns (Wheat, Cocoa, Sugar, Heating oil, Silver, Live cattle, Hogs and Pork bellies). Such results are consistent with recent work by Elder and Jin (2009) who analogously report evidence of long memory in commodity futures returns using a wavelet OLS estimator.

The GPH results with an estimation window of $N^{0.5}$ indicate that the null can be rejected at the 5% for 5 of the 14 commodity return series (Sugar, Copper, Live cattle, Hogs and Pork bellies). Although a little less supportive than the wavelet analysis, these results still

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[11] The computations are implemented using OX 4.1 and Matlab 7.8.

[12] Since the Daubechies (1988) family of wavelets has more desirable properties for time series such as improved frequency localization and the ability to represent continuous signals (see Section 2.2 for detailed discussions), it is commonly utilized in the analysis of economic and financial series.

[13] As the wavelet OLS estimator is employed under the assumption of normal standard errors, we also used test statistics robust towards autocorrelation and heteroskedasticity by utilising the Newey-West (1987) estimator in the wavelet OLS regression. The details of the results are not reported, as they are quantitatively similar to those of the wavelet OLS with normal standard errors, but they are available upon request.

contrast with Crato and Ray (2000) who found no evidence of long memory. In any case and as noted earlier, Jensen (1999) demonstrates that the wavelet estimator has a significantly smaller MSE than the GPH estimator. Jin et al. (2006) argue that this may imply considerably lower sampling variability which may translate into greater power against the null of no fractional integration.

Table 1 indicates that little evidence of long memory in the stock index is found both from the (Daubechies-6) wavelet or GPH estimator. These findings are line with the Barkoulas et al. (1999) and Crato and Ray (2000) results for returns on US stock index futures series. In contrast, the foreign exchange rate futures series display persistence using the wavelet OLS estimator, but not with the GPH estimator. Interestingly, Jin et al. (2006) have also reported similar results for the British Pound and Japanese Yen. Finally, most significant estimates of *d* from both wavelet and GPH estimators are negatively signed and, as such, are suggestive of anti-persistence. Again, our results are consistent with much of the extant literature including Jin et al. (2006), Elder and Jin (2009) and Fernandez (2010).[14] We return to the interpretation of anti-persistence later.

3.3 Time-varying long memory results

Following Tabak and Cajueiro (2007), we use the Daubechies-6[15]wavelet OLS estimator for overlapping rolling windows of 1024 observations each across the entire sample. Figures 1-17[16] depict the time-varying long memory parameter and corresponding 95% confidence intervals (CI).

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[14] Our results can also be interpreted in terms of the Hurst exponent H. The negative estimate of *d* indicates that a negatively correlated long memory process can be characterized by a Hurst exponent in the interval (0, 0.5). The positive estimate of *d* shows that the Hurst exponent falls in the interval (0.5, 1), suggesting positively correlated long-range dependence.

[15] We also estimate the time-varying long memory parameters with the Daubechies-4, 8 and Haar wavelets. They indicate that our results are robust (the results are not reported but are available from the authors).

[16] Since the wavelet OLS estimator is employed under the assumption of normal standard errors we also re-estimate time-varying long memory parameters with the Daubechies-6 wavelet OLS estimators with HAC

[Insert Figures 1-17 about here]

The date on the *x*-axis shows the beginning of the sample window used in the estimation. Thus, for October 1993, the wavelet OLS estimate was evaluated for the sample beginning in October 1993 and ending four years later (i.e., October 1997), and so forth. Overall, the typical pattern in our figures indicates longer periods of zero or low anti-persistence that are occasionally punctuated by shorter, sharper periods of even greater anti-persistence.

The dynamics of the wavelet estimates for the commodity futures returns series (Figures 1-14) are striking and show dramatic fluctuations over time in their long memory parameters. Although the $\hat{d}$ point estimates for a few commodities such as Soybeans and Corn occasionally display positive persistence ($\hat{d} > 0$), the majority of estimates $\hat{d}$ tend to move between zero and negative dependence. In other words, the parameters $\hat{d}$ exhibit anti-persistence ($\hat{d} < 0$) for non-trivial periods of time, suggesting that these markets are inefficient in these periods due to under- and overreaction patterns. In other periods, the $\hat{d}$ are insignificantly different from zero, indicating the markets are locally efficient. Moreover, it is found that the time-varying $\hat{d}$ estimates for some series (e.g., Soybeans, Corn, Wheat) appear to have an upward shift whilst others (e.g., Cocoa, Feeder cattle and Hogs) exhibit a downward shift over time. The plots in Figures 15-17 also presents that long memory parameters vary over time for the S&P 500, $/Pound and $/Yen futures returns. They suggest such markets are less efficient in the early and middle of the sample period, but tend to increase in efficiency after the late 1990s.

Although no long memory is found in the previous analysis with a fixed window, Figures 2, 6, 8 and 15 for Corn, Cotton, Gold and the S&P 500 futures returns, respectively,

standard errors. The results are similar to the graphs obtained from the Daubechies-6 wavelet OLS with normal standard errors. Details are available upon request.

show that the long memory parameters are significant in some time periods but not in others. This indicates a formal test of time-varying parameters is required.

We test stability formally by applying the Bai and Perron test of multiple structural breaks to the time-varying persistence parameter (*d*) and the results are reported in Table 2.

[Insert Table 2 about here]

The main points to emerge are as follows. First, the UDmax and WDmax provide strong evidence of structural changes in all the time-varying persistence parameters implying that *d* has at least two distinct regimes over the sample period. Second, the persistence parameter exhibits multiple (i.e., at least 2) structural breaks for all softs and grain (excluding Soybeans) contracts, and the Heating oil, Hogs and $/Pound contracts. These findings shed new light on the recent studies by showing that long memory parameters vary significantly over time.

Table 3 gives details of the regimes implied by the structural break test and the associated persistence parameters with *t*-statistics that use robust standard errors.

[Insert Table 3 about here]

The results are striking. First, the persistence parameters are statistically significant in almost all regimes for all commodities. There is only a handful of exceptions in the whole sample: *d* is insignificant in just 8 out of 44 regimes but all commodities exhibit persistence in at least one regime. This result contrasts sharply with the results in Table 1 which assumed one static persistence parameter over the full sample period and where only 8 series exhibited significant evidence at the 5% critical value. Once one allows for structural breaks, the persistence parameter becomes statistically significant within regimes for all series making it a pervasive feature of futures contracts. This is an extremely striking result since it implies deviations from market efficiency for all return series.

Second, almost all the persistence parameters are significantly negative, indicating anti-persistence. Only 1 (Soybeans Regime 2) out of the 44 persistence parameters is significantly positive while 3 others are insignificantly different from zero. This is in line with the graphical evidence in Figures 1-17. Anti-persistence is consistent with patterns of overreaction to news or shocks as indicated by the blips in Figures 1-17.

Third, the timing of the breaks is also interesting. Some 6 contracts experienced their first significant break in 1995-96 and these range from Gold to Live cattle and Wheat. A further 7 series had their first break in the late 1990s (e.g., S&P 500, heating oil and hogs). Finally, there is a clustering of breaks in 2002-2004. Some 4 commodity futures series experienced their second structural break in 2002-03, including Cocoa, Heating oil, Hogs and Wheat. The Corn and Wheat series experienced their third structural break in 2004. This would be consistent with the Tang and Xiong (2012) financialisation of commodities hypothesis. They argue that after the collapse of equity market in 2000, institutional investors and wealthy individuals discovered that a small negative correlation between commodity returns and stock returns could be used to reduce portfolio risk. However, significant commodity index investment started to flow into commodity markets after 2002 and all the above commodities are members of the two well-known, traded commodity indexes.[17]

4. Discussion and interpretation of the results[18]

4.1 Key results

Our results highlight three distinctive features of persistence in futures returns. Static measures of long memory dependence in futures returns indicate only limited evidence of persistence. By contrast, our first key finding is that the rolling sample wavelet method results indicate that persistence is pervasive across futures returns. These results indicate departures from efficiency in all series.

[17] These are the SP-GSCI and DJ-UBS indexes.

[18] We are grateful to two anonymous reviewers whose helpful comments and suggestions inspired a substantial redrafting of this section and, in particular, the new sub-section.

The second key finding is negative dependence or anti-persistence with $d < 0$. This has negative auto-covariances and unbounded variance at high spectral frequencies. Figures 1-17 show a general pattern of longer periods of zero or low anti-persistence punctuated by shorter, sharper periods of even greater anti-persistence. Campbell et al. (1997) state that a series with $d < 0$ is still called long-range dependent since its autocorrelations decay much more slowly than those of more conventional time series even though d eventually collapses to zero. Thus if the negative long-range dependence exhibits an increasing past trend, it is followed by a decreasing future trend and vice versa, consistent with overreaction in returns.

Our results are in line with recent studies on financial asset returns. Mulligan (2000), Muniandy et al. (2001) and Jin et al. (2006) find foreign exchange returns exhibit anti-persistence. Mulligan (2000) argues that the more negative persistence in exchange rates, the less stable the economy. A negatively dependent series in exchange rates should also have much shorter cycle lengths than random walks. They argue that one source of negative persistence behaviour may be suboptimal policy rules that delay intervention. Batten and Szilagyi (2007) argue that a negative long memory parameter for a foreign currency links to episodes of one currency decline/appreciation, or vice versa. They also claim that a negatively dependent series would be consistent with the long-term actions of arbitrageurs whose attempts to profit from a deviation of covered interest parity cause disequilibria to reverse. Avramov et al. (2006) argue that the presence of negative autocorrelations in individual security returns makes it difficult for investors to profit from predictability. However, Mulligan and Lombardo (2004) find strong evidence of anti-persistence in some maritime equities and argue that this is because market participants habitually overreact to new information, and never learn not to.

Evidence of anti-persistence in commodity markets has also been reported in recent research (Elder and Jin, 2009; Fernandez, 2010). Elder and Jin (2009) found evidence of anti-persistence in grain and meat commodity futures returns while Fernandez report this phenomenon in the 20 DJ-AIG commodity futures indices. Like Mulligan and Lombardo (2004), Fernandez (2010) interprets these price dynamics as implying that returns tend to overreact to new information. Therefore, commodity futures returns would have considerable periodic high-frequency variation. Our negative persistence estimates for commodity futures returns provide additional evidence on this. Moreover, the finding may also imply that investors' overreaction to new information causes commodity futures prices to temporarily swing away from their fundamental values, which leads to a violation of weak form efficiency. Several studies such as Irwin et al. (2009), Tang and Xiong (2012) and Singleton (2014) argue that commodities experienced a price bubble in the run up to 2008, i.e., the prices drifted above their fundamental values. Our results indicate that investor overreaction may play an important role in creating such a phenomenon. As anti-persistence in the futures returns are more established, it would be interesting to further explore its impact on pricing forecasting and optimal hedging. Overall, given the much lower MSE for the wavelet estimator, our results suggest that anti-persistence in futures returns is more prevalent than previously recognized.

The third key finding is that persistence fluctuates dramatically over time. The Bai and Perron structural break tests confirm the existence of at least two regimes for all return series and the persistence parameters are significant in most regimes. Thus, long-range dependence in futures returns is best conceptualized as time-varying. It tends to shift between positive and negative values (i.e., $d > 0$ and $d < 0$) even if it is invariably negative on average in all commodity/asset regimes. Alvarez-Ramirez et al. (2008) obtained a similar finding in US stock markets. A possible explanation for this may lie in changes in market participants' behaviour. Manzan and Westerhoff (2005) develop a behavioural exchange rate model in which speculators' perception of news is based on the psychological principle and argue that over- and under reaction to news drives the evolution of the exchange rate via the agents' orders. More specifically, Daniel et al. (1998) suggests that investor overconfidence drives overreaction (and this manifests itself as negative autocorrelation of short-term returns at long lags), where the overconfidence itself is represented, via biased self- attribution, as a function of investments outcomes. Our results may also imply investors' behaviour in commodity futures markets is analogous to those in other financial markets. In other words, commodity futures prices display positive persistence and anti-persistence over different time periods due to under- and overreaction respectively. This new finding challenges the focus of previous studies only on a single value of long memory parameter, as our statistical evidence clearly shows that long memory parameters vary over time periods for all futures returns.

4.2 Explaining our results

Given that financialization implies that commodities behave more like traditional financial assets (see Tang and Xiong, 2012 and Singleton, 2014), we suggest some possible financial covariates to examine potential sources for the uncovered time-varying long range dependence in commodity futures returns. Firstly, recent work (see Rostek, 2014) has suggested that lower volatility is commensurate with overreaction, and likewise, higher volatility is linked to underreaction. For a possible behavioural explanation, we posit that a positive shock to volatility may reduce biased self-attribution, given investment outcomes are less certain. In the Daniel et al. (1998) framework, this would lead to less investor overconfidence and hence decrease overreaction. This provides a possible link between time-varying long memory and futures volatility.

Secondly, researchers such as Singleton (2014), Hong and Yogo (2012), Kellard et al. (1999) and Fama and French (1987) suggest the futures basis (difference between the nearby futures price and spot price) may explain some of the behavior of commodity market returns.

This is because variation in the basis is caused by variation in expectations about the future spot price, the expected risk premium or investor sentiment. Some prior studies have found a negative association between returns and the basis showing (Singleton, 2014) and other a positive association (Kellard et al., 1999). A negative sign suggests that such markets are consistent with the theory of backwardation.

We estimate the following model to assess the effect of the basis and futures volatility on futures returns time-varying long memory parameters:

$$d_{f,t} = \alpha_i + \beta_i(f_t - s_t) + \lambda_i(r_t^2) + v_t, \quad v_t = \begin{cases} \varepsilon_t \\ \varepsilon_t + \theta_i \varepsilon_{t-1} \end{cases} \tag{11}$$

where $d_{f,t}$ is the time-varying value of d for futures returns, $r_t = f_t - f_{t-1}$ and v_t is either white noise or an MA(1) error process.[19] The least-squares results are presented in Table 4:

[Table 4 around here]

The results show that the majority of λ_i coefficients (10 out of 17) on the futures volatility are significantly positive except those on sugar (significantly negative). This overwhelmingly positive relationship between persistence and futures volatility suggests that investors overreact less in volatile markets, as might be expected when investment outcomes are less certain and confidence is lower. Furthermore, most of the β_i basis coefficients in Table 4 are also statistically significant. As in the more traditional return-basis relationship, the estimated parameter signs for the long memory-basis association are also mixed. Specifically, they are negative for eight (predominantly financial and livestock) futures markets, suggesting that persistence is higher (underreaction or less overreaction) the greater the degree of backwardation in these markets. By contrast, the coefficient sign is positive in six (predominantly grain and metals) markets, implying less anti-persistence as contango increases.

[19] To match with the length of the time-varying d series, the basis and futures volatility are selected from 27/10/1997 to 31/12/2009, providing 3073 observations.

Other transformations of volatility and the basis may also provide explanatory power for long memory in futures returns. Recent papers (Elder and Jin, 2007; Coakley et al., 2011) provide evidence of long memory in both our suggested covariates. A natural extension of model (12) is therefore to estimate the following regression:

$$d_{f,t} = a_i + \beta_i \, d_{i,t} + v_t, \qquad v_t = \begin{cases} \varepsilon_t \\ \varepsilon_t + \theta_i \varepsilon_{t-1} \end{cases} \qquad (12)$$

where $d_{i,t}$ is the is the time-varying value of d for futures volatility and the basis. The results are presented in Tables 5 and 6:

[Tables 5 and 6 around here]

Table 5 indicates a significantly positive relationship between the time-varying persistence parameters of futures returns and those of futures volatility for all but cocoa. Table 6 shows significant relationships (8 positive and 7 negative) with the futures basis persistence. The results newly demonstrate that long range dependence parameters in commodity futures returns, future volatility and the basis are time-varying and correlated with each other.

Finally, given commodity futures are well known as a hedging instrument to reduce commodity price risk, we also test whether the effect of hedging pressure[20] is one of the drivers of persistence/anti-persistence in commodity futures returns. First, a joint error-correction and multivariate GARCH (EC-BEKK) approach that accounts for the effect of the basis and heteroscedasticity is applied to estimate time-varying, optimal hedging ratios (OHRs). Second, we regress the time-varying long memory parameter d of the futures return on the dynamic time-varying hedge ratios (b*):

$$d_{f,t} = a_i + \beta_i b_t^* + v_t, \qquad v_t = \begin{cases} \varepsilon_t \\ \varepsilon_t + \theta_i \varepsilon_{t-1} \end{cases} \qquad (13)$$

The results are presented in Table 7:

[Table 7 around here]

[20] We thank an anonymous reviewer for this suggestion.

The results suggest that all β_i are statistically significant with eight positive and eight negative coefficients (one insignificant). The positive coefficients indicate that hedging pressure increases persistence in commodity futures returns and again implies irrational behavior in the form of over/underreaction. Our finding is in line with those in Basu and Miffre (2013) who find that hedging-pressure and commodity futures links are significant.

To sum up, the time-varying found persistence in commodity futures returns can be linked to relevant drivers. Volatility has a positive relationship with time-varying persistence and this might be explained within a Daniel et al. (1998) type behavioural framework, where increased volatility makes investment outcomes less certain and therefore overconfidence and overreaction are diminished. Both the futures basis and the hedge ratio have significant but mixed sign relationships with futures return persistence. This sign difference might be due to heterogeneous agents and limits to arbitrage and could be the subject of future research.

5. Conclusions

This paper investigates evidence of time-varying long memory in commodity futures returns by applying a rolling version of the semi-parametric wavelet estimator developed by Jensen (1999) to a large sample of daily futures returns from 1993 to 2009. The results reveal that the long memory is pervasive and that the parameters vary over time. Structural break tests show that all sample commodity and asset series exhibit at least one significant structural break or two regimes in the persistence parameter. Moreover, the persistence parameter itself is statistically significant in the majority of regimes for series. These results contrast sharply with the findings of no long memory established for many commodities using static estimation, both in this paper and the extant literature. Long memory in futures returns is more pervasive and more variable than you might think!

Our results are striking because, to our knowledge, this is the first time that a significant and pervasive deviation from market efficiency has been formally identified for so many commodities and other markets. Moreover, most series display evidence of negative fractional integration ($-0.5 < d < 0$), indicating that commodity futures return series are characterised by anti-persistence. These results are in agreement with other recent studies that find evidence of anti-persistence in the spot stock market and exchange rate returns and in commodity futures returns (Jin et al., 2006; Elder and Jin, 2009). Anti-persistence is consistent with investor overreaction to news and shocks, a return pattern that have been posited in the behavioural finance literature.

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Table 1
Wavelet OLS and GPH estimates of daily futures returns

	$\hat{d}$ D-6	$\hat{d}$ GPH($N^{0.5}$)
Soybeans	-0.103*	0.045
	(-1.68)	(0.51)
Corn	-0.056	0.023
	(-1.56)	(0.26)
Wheat	-0.172**	-0.049
	(-2.04)	(-0.54)
Cocoa	-0.105**	0.054
	(-2.01)	(0.60)
Sugar	-0.070**	0.185**
	(-2.37)	(2.07)
Cotton	-0.031	-0.036
	(-1.45)	(-0.40)
Heating oil	-0.068**	0.117
	(-2.65)	(1.31)
Gold	-0.093	-0.113
	(-1.46)	(-1.27)
Silver	-0.067**	-0.141
	(-3.14)	(-1.57)
Copper	0.011	0.220**
	(0.36)	(2.46)
Live cattle	-0.126**	-0.220**
	(-2.87)	(-2.46)
Feeder cattle	-0.015	0.054
	(-0.72)	(0.61)
Hogs	-0.311**	-0.302**
	(-3.09)	(-3.38)
Pork bellies	-0.188**	-0.176**
	(-2.99)	(-1.99)
S&P 500	0.004	0.074
	(0.13)	(0.83)
$/Pound	-0.069**	0.036
	(-1.97)	(0.40)
$/Yen	-0.052*	0.053
	(-1.94)	(0.60)

Notes: 1. $\hat{d}$ *D-6* column represents the Daubechies-6 wavelet OLS estimate for the entire 1993-2009 sample. *d^GPH* column represents the GPH estimate for the entire 1993-2009 sample.

2. The values in parentheses are *t*-statistics.
** indicates that the null of d = 0 is rejected at the 5% level; * indicates at the 10% level.

Table 2
Bai and Perron statistics for tests of multiple structural breaks in the time-varying persistence parameter *d*

	UDmax[a]	WDmax(5%)[b]	$F(1\|0)$[c]	$F(2\|1)$[d]	$F(3\|2)$[e]	$F(4\|3)$[f]	$F(5\|4)$[g]
Soybeans	49.93***	55.96**	49.93***	6.75	2.18	-	-
Corn	62.50***	113.26**	45.43***	43.83***	27.38***	6.68	6.84
Wheat	53.37***	84.76**	46.60***	48.79***	18.18***	6.17	-
Cocoa	26.25***	44.38**	22.65***	10.52***	2.68	2.32	2.69
Sugar	94.88***	208.20**	27.16***	38.36***	8.16	13.44**	0.46
Cotton	31.29***	37.18**	8.14*	42.20***	4.13	4.20	-
Heating oil	42.84***	56.16**	13.76***	30.75***	5.37	5.35	7.16
Gold	35.87***	35.87**	35.87***	4.17	1.61	2.01	1.14
Silver	10.08**	13.99**	10.08**	4.45	9.49**	4.73	0.10
Copper	10.85**	11.58**	10.85**	6.43	2.71	0.69	-
Live cattle	34.82***	34.82**	34.82***	8.90*	3.19	3.29	-
Feeder cattle	38.63***	39.97**	38.63***	6.67	2.83	13.10**	-
Hogs	51.38***	61.05**	47.71***	18.31***	4.68	-	-
Pork bellies	19.53***	28.17**	10.27**	3.27	38.65***	1.87	0.04
S&P 500	38.69***	38.81**	38.69***	2.13	2.51	0.67	-
$/Pound	97.74***	97.74**	97.74***	21.57***	7.54	2.64	1.38
$/Yen	12.79***	17.40**	12.79***	4.10	1.62	1.05	-

Notes: 1. The time-varying persistence parameter *d* is estimated using the Daubechies-6 wavelet OLS estimator for overlapping rolling windows of 1024 observations each across the entire sample.
2. [a] 10, 5 and 1 percent critical values are 7.46, 8.88 and 12.37, respectively.
3. [b]Critical value is 9.91.
4. [c]10, 5 and 1 percent critical values are 7.04, 8.58 and 12.29, respectively.
5. [d]10, 5 and 1 percent critical values are 8.51, 10.13 and 13.89, respectively.
6. [e]10, 5 and 1 percent critical values are 9.41, 11.14 and 14.80, respectively.
7. [f]10, 5 and 1 percent critical values are 10.04, 11.83 and 15.28, respectively.
8. [g]10, 5 and 1 percent critical values are 10.58, 12.25 and 15.76, respectively.
9. *, **, *** indicate 10%, 5% and 1% significance, respectively.

Table 3: Bai and Perron regime means and end dates for the time-varying persistence parameter *d*

		Regime 1	Regime 2	Regime 3	Regime 4
Soybeans	Mean (t-ratio)[1]	-0.0924(-6.42)	0.0327(3.17)		
	End date[2]	25/04/2000			
Gold	Mean(t-ratio)	-0.0752 (-7.52)	-0.1528(-18.41)		
	End date	05/03/1996			
Silver	Mean(t-ratio)	-0.1667(-21.93)	-0.0975(-4.78)		
	End date	07/11/2003			
Copper	Mean(t-ratio)	-0.0593(-6.74)	0.0174(0.81)		
	End date	03/01/2003			
Live cattle	Mean(t-ratio)	-0.2011(-20.52)	-0.1212(-13.03)		
	End date	20/11/1996			
Feeder cattle	Mean(t-ratio)	-0.0099(-0.51)	-0.0802(-5.81)		
	End date	03/08/1995			
Pork bellies	Mean(t-ratio)	-0.0607(-5.62)	-0.1501(-6.79)		
	End date	24/09/1999			
S&P 500	Mean(t-ratio)	-0.1831(-14.3)	-0.0734(-6.07)		
	End date	22/06/1998			
$/Yen	Mean(t-ratio)	-0.0197(-2.03)	-0.0686(-7.15)		
	End date	21/12/2000			
Cocoa	Mean(t-ratio)	-0.1278(-5.65)	-0.0488(-5.42)	-0.17(-10.83)	

	End date	18/06/1996	25/06/2002		
Sugar	Mean(t-ratio)	-0.1144(-4.89)	-0.0103(-0.79)	-0.286(-12.33)	
	End date	08/03/1996	10/12/1999		
Cotton	Mean(t-ratio)	-0.1124(-8.26)	-0.0142(-1.67)	-0.1162(-8.80)	
	End date	26/10/1998	04/10/2002		
Heating oil	Mean(t-ratio)	-0.0468(-4.63)	-0.1781(-16.19)	-0.0343(-1.46)	
	End date	12/01/2000	15/09/2003		
Hogs	Mean(t-ratio)	-0.0317(-3.27)	-0.1031(-7.47)	-0.181(-15.21)	
	End date	24/04/2000	29/08/2003		
$/Pound	Mean(t-ratio)	-0.162(-12)	-0.246(-20.5)	-0.0624(-7.09)	
	End date	31/08/1995	28/07/1997		
Corn	Mean(t-ratio)	0.0061(0.32)	-0.1929(-14.5)	-0.0594(-5.55)	0.0113(1.36)
	End date	09/09/1996	19/04/2000	13/02/2004	
Wheat	Mean(t-ratio)	-0.0407(-2.99)	-0.1771(-22.71)	-0.0862(-7.12)	-0.0011(-0.07)
	End date	16/10/1995	28/01/2002	13/02/2004	

Notes: 1. The time-varying persistence parameter *d* is estimated using the Daubechies-6 wavelet OLS estimator for overlapping rolling windows of 1024 observations each across the entire sample.
2. The mean is the average time-varying long memory parameters (*d*) in each regime; *t*-ratios are reported in parentheses.
3. The end date shows when each regime ends.
4. Breaks are identified at the 5% significance level using robust standard errors (see Table 2).

Table 4: Persistence and the basis and volatility

We report the results of estimating:

$$d_{f,t} = \alpha_i + \beta_i(f_t - s_t) + \lambda_i(r_t^2) + v_t, \qquad v_t = \begin{cases} \varepsilon_t & (1) \\ \varepsilon_t + \theta_i \varepsilon_{t-1} & (2) \end{cases}$$

where $d_{f,t}$ is the time-varying value of *d* for futures returns, $f_t - s_t$ is the basis and r_t^2 is the futures volatility. The basis and futures volatility are selected from 27/10/1997 to 31/12/2009 to match the length of the time-varying *d* series. This provides some 3073 observations. As equations (1) and (2) give qualitatively similar results, only the results from (2) are reported.

	α_i	β_i	λ_i	θ_i	R^2
Soybeans	-0.044**	0.326**	0.041**	0.745**	0.589
t-statistic	*(-16.29)*	*(4.83)*	*(3.57)*	*(61.88)*	
Corn	-0.114**	0.53**	0.008	0.765**	0.612
t-statistic	*(-24.8)*	*(10.14)*	*(0.68)*	*(65.65)*	
Wheat	-0.14**	0.326**	0.021*	0.687**	0.606
t-statistic	*(-60.38)*	*(21.13)*	*(1.79)*	*(52.17)*	
Cocoa	-0.092**	0.177**	0.013	0.744**	0.582
t-statistic	*(-23.65)*	*(2.84)*	*(1.62)*	*(61.46)*	
Sugar	-0.098**	0.018	-0.001**	0.794**	0.623
t-statistic	*(-24.02)*	*(0.62)*	*(-3.25)*	*(72.14)*	
Cotton	-0.08**	-0.03	0.009	0.704**	0.537
t-statistic	*(-23.87)*	*(-0.63)*	*(1.05)*	*(54.87)*	
Heating oil	-0.083**	-0.373**	0	0.699**	0.555
t-statistic	*(-41.29)*	*(-7.4)*	*(0.06)*	*(54.03)*	
Gold	-0.138**	-0.047	0.041*	0.654**	0.484
t-statistic	*(-73.41)*	*(-0.38)*	*(1.68)*	*(47.82)*	
Silver	-0.155**	0.214*	0.026**	0.62**	0.451
t-statistic	*(-75.09)*	*(1.8)*	*(2.74)*	*(43.72)*	
Copper	-0.046**	0.298	0.026**	0.72**	0.558
t-statistic	*(-28.46)*	*(2.81)*	*(3.05)*	*(56.88)*	
Live cattle	-0.139**	-0.402**	0.106**	0.679**	0.531
t-statistic	*(-70.44)*	*(-9.65)*	*(3.84)*	*(51.3)*	
Feeder cattle	-0.075**	-0.133**	0.038	0.716**	0.526
t-statistic	*(-38.61)*	*(-2.33)*	*(0.84)*	*(56.86)*	
Hogs	-0.078**	-0.084**	0.002	0.742**	0.581
t-statistic	*(-41.95)*	*(-4.48)*	*(0.98)*	*(61.22)*	
Pork bellies	-0.082**	-0.115**	0	0.696**	0.548
t-statistic	*(-44.79)*	*(-9.23)*	*(-0.12)*	*(53.73)*	
S&P 500	-0.102**	-4.627**	0.046**	0.684**	0.573
t-statistic	*(-45.13)*	*(-14.9)*	*(3.52)*	*(51.42)*	
$/Pound	-0.109**	-0.662*	0.268**	0.68**	0.517
t-statistic	*(-47.31)*	*(-1.84)*	*(2.26)*	*(51.18)*	
$/Yen	-0.038**	-0.344**	0.07**	0.777**	0.597
t-statistic	*(-22.76)*	*(-1.98)*	*(2.5)*	*(68.24)*	

Notes: 1. *, ** indicate 10% and 5% significance, respectively.

2. Hogs are not included due to lack of the spot rates as wavelet estimation requires 4096 observations.

Table 5: Time-varying long memory in futures returns and volatility

We report the results of estimating:

$$d_{f,t} = a_i + \beta_i\, d_{i,t} + v_t, \qquad v_t = \begin{cases} \varepsilon_t & (1) \\ \varepsilon_t + \theta_i \varepsilon_{t-1} & (2) \end{cases}$$

where $d_{f,t}$is the time-varying d for the futures returns and $d_{i,t}$ is the time-varying d of the futures volatility. As equations (1) and (2) give qualitatively similar results, only the results from (2) are reported.

	a_i	β_i	θ_i	R^2
Soybeans	-0.095**	0.379**	0.728**	0.635
t-statistic	*(-27.55)*	*(20.82)*	*(58.73)*	
Corn	-0.08**	0.297**	0.765**	0.615
t-statistic	*(-34.5)*	*(11.43)*	*(65.82)*	
Wheat	-0.149**	0.439**	0.672**	0.627
t-statistic	*(-63.92)*	*(25.94)*	*(50.11)*	
Cocoa	-0.1**	-0.007	0.747**	0.58
t-statistic	*(-35.48)*	*(-0.34)*	*(62.27)*	
Sugar	-0.119**	0.26**	0.756**	0.636
t-statistic	*(-39.16)*	*(11.19)*	*(60.06)*	
Cotton	-0.072**	-0.255**	0.679**	0.578
t-statistic	*(-37.48)*	*(-17.67)*	*(51.11)*	
Heating oil	-0.115**	0.327**	0.683**	0.596
t-statistic	*(-47.24)*	*(19.38)*	*(51.75)*	
Gold	-0.133**	-0.038**	0.654**	0.485
t-statistic	*(-44.28)*	*(-1.97)*	*(47.92)*	
Silver	-0.169**	0.084**	0.624**	0.451
t-statistic	*(-32.38)*	*(2.95)*	*(44.2)*	
Copper	-0.058**	0.105**	0.727**	0.562
t-statistic	*(-23.17)*	*(6.57)*	*(58.71)*	
Live cattle	-0.151**	0.17**	0.681**	0.525
t-statistic	*(-68.07)*	*(8.23)*	*(51.58)*	
Feeder cattle	-0.067**	-0.052**	0.717**	0.526
t-statistic	*(-18.38)*	*(-2.48)*	*(56.92)*	
Hogs	-0.075**	0.151**	0.745**	0.591
t-statistic	*(-39.88)*	*(9.64)*	*(61.69)*	
Pork bellies	-0.102**	0.177**	0.692**	0.552
t-statistic	*(-45.34)*	*(10.45)*	*(53.05)*	
S&P 500	-0.155**	0.205**	0.698**	0.567
t-statistic	*(-43.4)*	*(13.32)*	*(53.96)*	
$/Pound	-0.163**	0.462**	0.651**	0.586
t-statistic	*(-51.54)*	*(23.04)*	*(47.55)*	
$/Yen	-0.042**	0.024*	0.778**	0.597
t-statistic	*(-21.47)*	*(1.67)*	*(68.71)*	

Notes: 1. *, ** indicate 10% and 5% significance, respectively.

Table 6: Time-varying long memory in futures returns and the basis

We report the results of estimating:

$$d_{f,t} = a_i + \beta_i\, d_{i,t} + v_t, \qquad v_t = \begin{cases} \varepsilon_t & (1) \\ \varepsilon_t + \theta_i \varepsilon_{t-1} & (2) \end{cases}$$

where $d_{f,t}$ is the time-varying *d* for the futures returns and $d_{i,t}$ is the time-varying *d* for the basis. As equations (1) and (2) give qualitatively similar results, only the results from (2) are reported.

	a_i	β_i	θ_i	R^2
Soybeans	0.242**	-0.443**	0.731**	0.632
t-statistic	*(17.3)*	*(-19.96)*	*(59.41)*	
Corn	0.025	-0.136**	0.773**	0.602
t-statistic	*(1.17)*	*(-4.67)*	*(67.45)*	
Wheat	-0.021*	-0.129**	0.707**	0.557
t-statistic	*(-1.75)*	*(-7.96)*	*(55.31)*	
Cocoa	0.022**	-0.222**	0.738**	0.604
t-statistic	*(2.42)*	*(-13.7)*	*(60.63)*	
Sugar	-0.514**	0.672**	0.748**	0.731
t-statistic	*(-43.75)*	*(35.84)*	*(62.25)*	
Cotton	-0.133**	0.089**	0.702**	0.541
t-statistic	*(-13.68)*	*(5.42)*	*(54.58)*	
Heating oil	-0.196**	0.218**	0.698**	0.561
t-statistic	*(-17.09)*	*(9.9)*	*(54.02)*	
Gold	-0.138**	-0.057**	0.654**	0.485
t-statistic	*(-74.8)*	*(-2.84)*	*(47.93)*	
Silver	-0.137**	-0.124**	0.622**	0.456
t-statistic	*(-38.59)*	*(-5.97)*	*(44.05)*	
Copper	-0.055**	0.023**	0.724**	0.557
t-statistic	*(-13.87)*	*(2.61)*	*(58.11)*	
Live cattle	-0.234**	0.161**	0.682**	0.523
t-statistic	*(-18.56)*	*(7.38)*	*(51.67)*	
Feeder cattle	0.062**	-0.301**	0.695**	0.573
t-statistic	*(8.18)*	*(-18.57)*	*(53.55)*	
Pork bellies	-0.161**	0.18**	0.696**	0.553
t-statistic	*(-22.92)*	*(10.85)*	*(53.65)*	
S&P 500	-0.225**	0.267**	0.682**	0.604
t-statistic	*(-42.31)*	*(22.03)*	*(51.61)*	
$/Pound	-0.199**	0.264**	0.675**	0.531
t-statistic	*(-20.71)*	*(9.87)*	*(50.68)*	
$/Yen	-0.036**	-0.01	0.778**	0.596
t-statistic	*(-12.64)*	*(-1.39)*	*(68.72)*	

Notes: 1. *, ** indicate 10% and 5% significance, respectively.

2. Hogs are not included due to lack of the spot rates as wavelet estimation requires 4096 observations.

Table 7: Time-varying futures return persistence and optimal hedging ratios

We report the results of estimating:

$$d_{f,t} = a_i + \beta_i\, b_t^* + v_t, \qquad v_t = \begin{cases} \varepsilon_t & (1) \\ \varepsilon_t + \theta_i \varepsilon_{t-1} & (2) \end{cases}$$

where $d_{f,t}$ is the time-varying *d* for the futures returns and b_t^* are the time-varying, optimal hedging ratios. As equations (1) and (2) give qualitatively similar results, only the results from (2) are reported.

	a_i	β_i	θ_i	R^2
Soybeans	0.035**	-0.083**	0.744**	0.59
t-statistic	*(3.34)*	*(-6.76)*	*(61.68)*	
Corn	-0.12**	0.054**	0.771**	0.602
t-statistic	*(-11.17)*	*(4.51)*	*(67.02)*	
Wheat	-0.255**	0.155**	0.692**	0.583
t-statistic	*(-28.36)*	*(16.23)*	*(52.87)*	
Cocoa	-0.075**	-0.031**	0.744**	0.581
t-statistic	*(-7.05)*	*(-2.51)*	*(61.44)*	
Sugar	-0.04**	-0.108**	0.785**	0.629
t-statistic	*(-4.73)*	*(-7.59)*	*(69.67)*	
Cotton	-0.058**	-0.029**	0.704**	0.538
t-statistic	*(-7.3)*	*(-3.13)*	*(54.92)*	
Heating oil	-0.003	-0.09**	0.699**	0.553
t-statistic	*(-0.19)*	*(-6.43)*	*(54.14)*	
Gold	-0.156**	0.031*	0.655**	0.484
t-statistic	*(-14.99)*	*(1.8)*	*(47.99)*	
Silver	-0.151**	-0.005	0.623**	0.45
t-statistic	*(-9.59)*	*(-0.26)*	*(44.19)*	
Copper	-0.094**	0.052**	0.723**	0.558
t-statistic	*(-7.54)*	*(3.93)*	*(58.06)*	
Live cattle	-0.128**	-0.094**	0.683**	0.517
t-statistic	*(-34.17)*	*(-4.23)*	*(51.89)*	
Feeder cattle	-0.08**	0.027**	0.716**	0.527
t-statistic	*(-31.27)*	*(3.13)*	*(56.84)*	
Hogs	-0.078**	-0.167*	0.742**	0.579
t-statistic	*(-40.14)*	*(-1.64)*	*(61.18)*	
Pork bellies	-0.085**	-0.009*	0.702**	0.537
t-statistic	*(-40.62)*	*(-1.91)*	*(54.61)*	
S&P 500	-0.444**	0.344**	0.702**	0.552
t-statistic	*(-10.95)*	*(8.1)*	*(54.67)*	
$/Pound	-0.386**	0.356**	0.669**	0.547
t-statistic	*(-20.22)*	*(14.72)*	*(49.89)*	
$/Yen	-0.201**	0.183**	0.776**	0.602
t-statistic	*(-8.29)*	*(6.67)*	*(68.27)*	

Notes: 1. *, ** indicate 10% and 5% significance, respectively.

Notes: Figures 1-17 depict the time-varying long memory parameter *d* for 17 futures returns with the corresponding 95% confidence intervals (CI). The parameter is estimated using the Daubechies-6 wavelet OLS estimator for overlapping rolling windows of 1024 observations each across the entire sample.

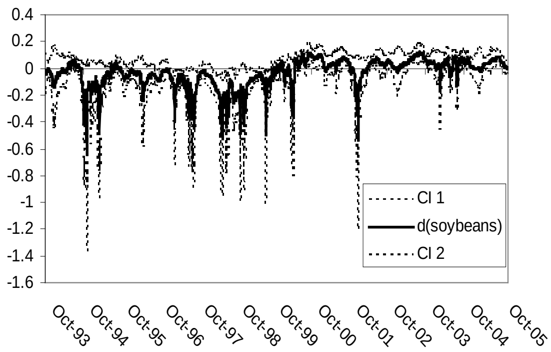

Fig 1. The time-varying long memory parameter *d* for Soybeans

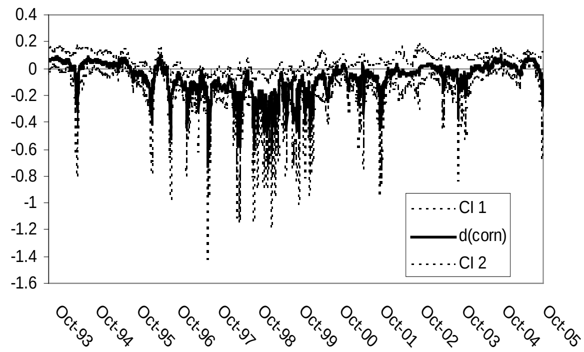

Fig 2. The time-varying long memory parameter *d* for Corn

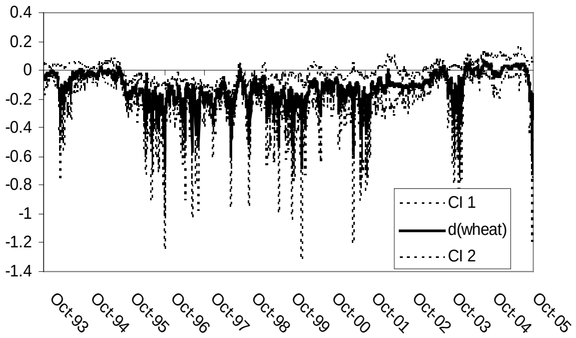

Fig 3. The time-varying long memory parameter *d* for Wheat

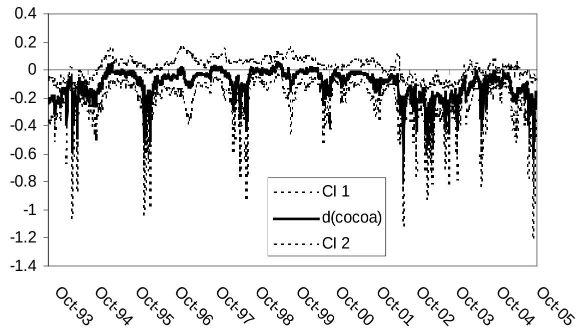

Fig 4. The time-varying long memory parameter *d* for Cocoa

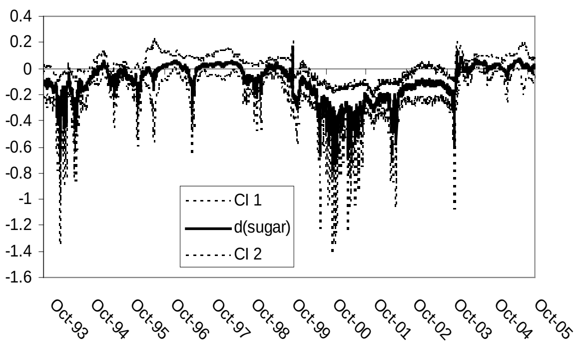

Fig 5. The time-varying long memory parameter *d* for Sugar

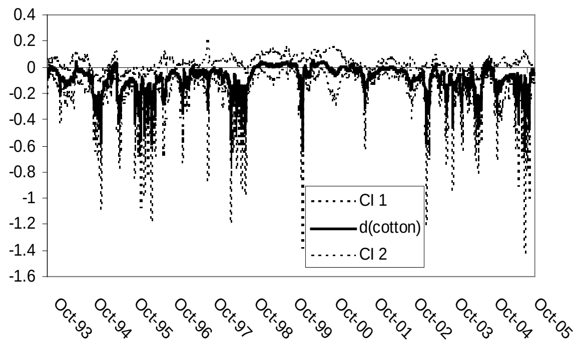

Fig 6. The time-varying long memory parameter *d* for Cotton

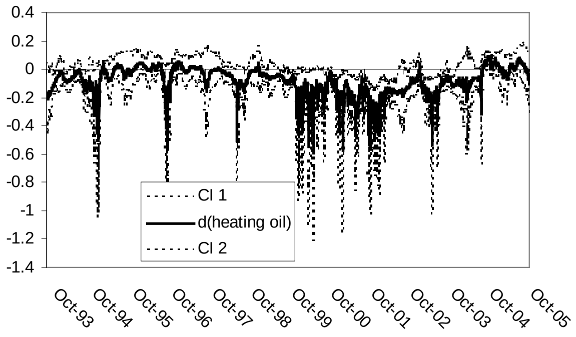

Fig 7. The time-varying long memory parameter *d* for Heating oil

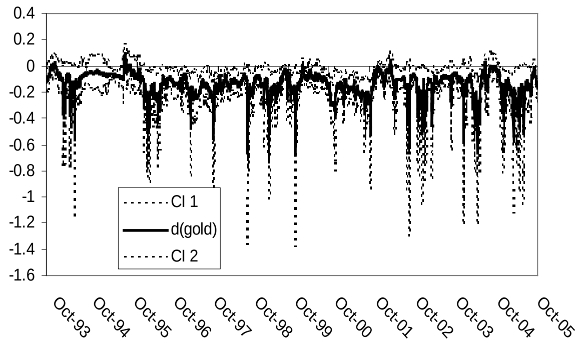

Fig 8. The time-varying long memory parameter *d* for Gold

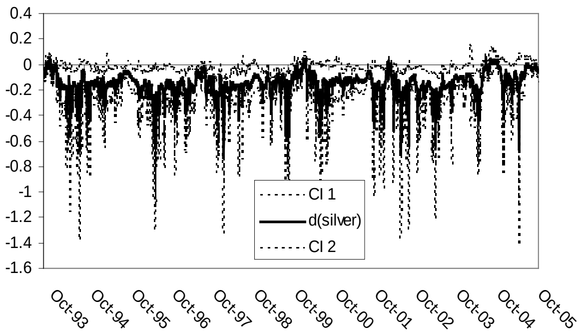

Fig 9. The time-varying long memory parameter *d* for Silver

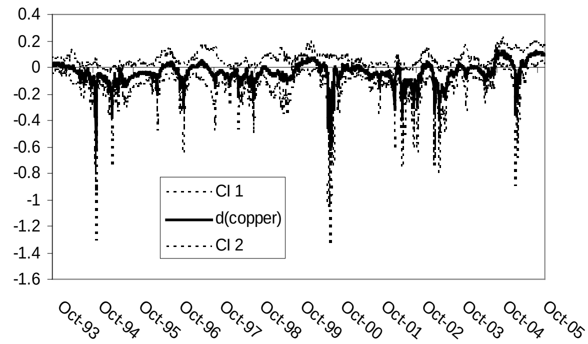

Fig 10.The time-varying long memory parameter *d* for Copper

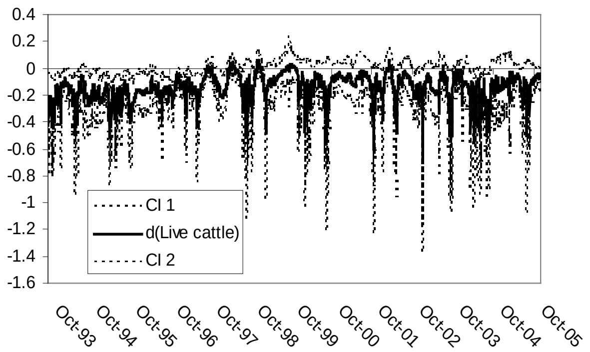

Fig 11.The time-varying long memory parameter *d* for Live cattle

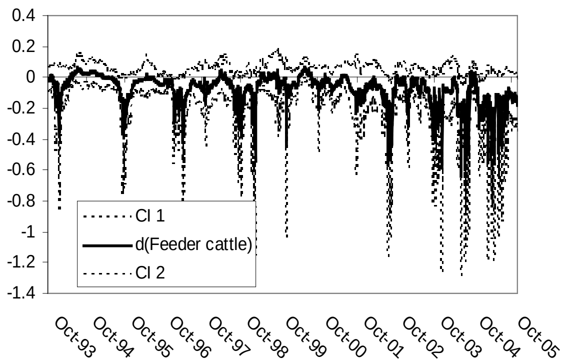

Fig 12.The time-varying long memory parameter *d* for Feeder cattle

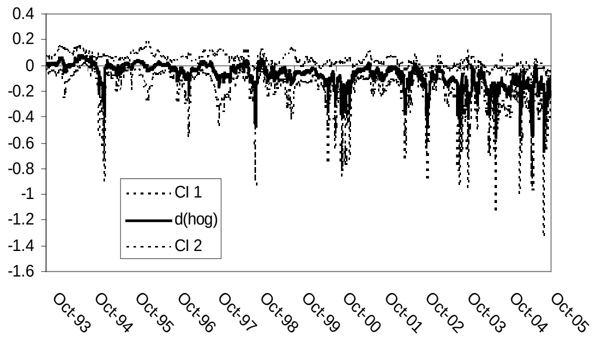

Fig 13.The time-varying long memory parameter *d* for Hog

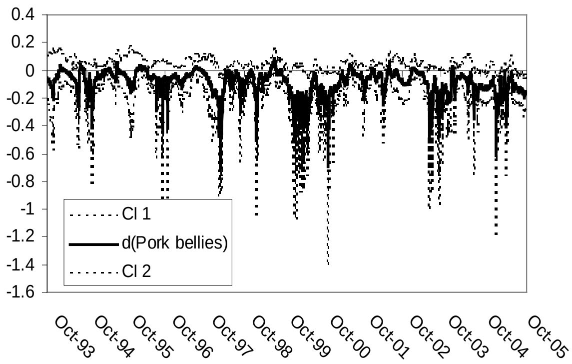

Fig 14.The time-varying long memory parameter *d* for Pork bellies

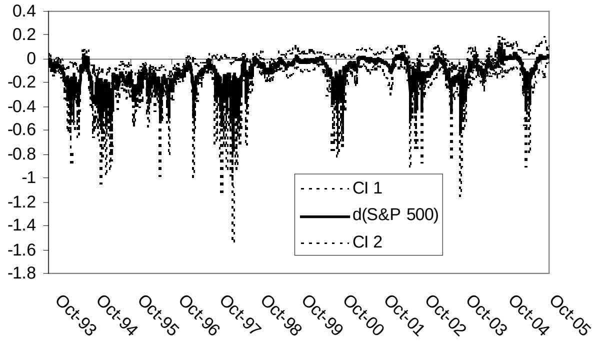

Fig 15.The time-varying long memory parameter *d* for S&P 500

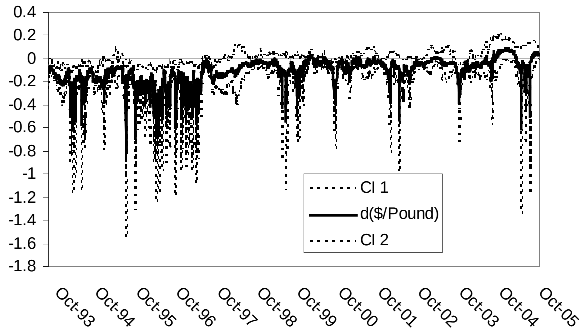

Fig 16.The time-varying long memory parameter *d* for $/Pound

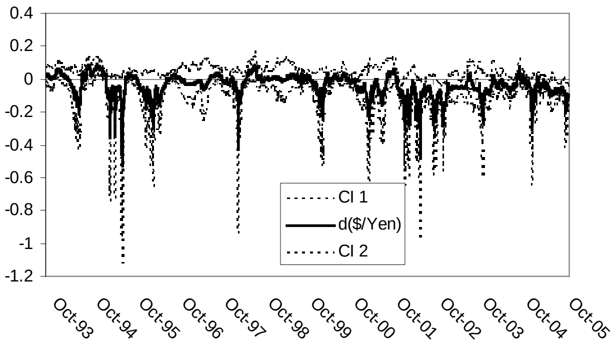

Fig 17. The time-varying long memory parameter *d* for $/Yen