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Mahes Visvalingam
Reader Emeritus
Dept of Computer Science
University of Hull
HULL HU6 7RX, UK

Explorations in Digital Cartography

DISCUSSION PAPER 2

The Visvalingam algorithm
metrics, measures and heuristics

by

Mahes Visvalingam

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ABSTRACT

This paper provides the background necessary for a clear understanding of forthcoming papers relating to the Visvalingam algorithm for line generalisation, for example on the testing and usage of its implementations. It distinguishes the algorithm from implementation-specific issues to explain why it is possible to get inconsistent but equally valid output from different implementations. By tracing relevant developments within the now-disbanded Cartographic Information Systems Research Group (CISRG) of the University of Hull, it explains why a) a partial metric-driven implementation was, and still is, sufficient for many projects but not for others; b) why the Effective Area (EA) is a measure derived from a metric; c) why this measure (EA) may serve as a heuristic indicator for in-line feature segmentation and model-based generalisation; and, d) how metrics may be combined to change the order of point elimination. The issues discussed in this paper also apply to the use of other metrics. It is hoped that the background and guidance provided in this paper will enable others to participate in further research based on the algorithm.

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1. Introduction

The Visvalingam algorithm for line generalisation described in Visvalingam and Whyatt (1993) demonstrated that it was possible to achieve caricatural generalisation, even if some features of the algorithm may not suit all purposes. The algorithm is versatile because it can be driven with a variety of metrics and adapted for use in diverse applications. Some of the datasets used by Visvalingam and her co-workers can be downloaded from https://hydra.hull.ac.uk/resources/hull:9040. To save others wasting time on trying to get results which are identical to ours, this paper explains why variations in implementation are inevitable and sometimes necessary to suit different purposes.

Some students used partial implementations of the algorithm successfully in their undergraduate and postgraduate projects at the University of Hull. Given that Visvalingam's algorithm is now regarded as a classic, this paper traces its origin and its uses in the programme of research undertaken by Visvalingam and her students within the now-disbanded Cartographic Information Systems Research Group of the University of Hull (CISRG, 2014). This history of the development and use of the algorithm facilitates the explanation of why a) a partial metric-driven implementation was, and still is, sufficient for many projects but not for others; b) why the Effective Area is a measure derived from a metric; c) why this measure serves as a heuristic indicator to facilitate model-based generalisation; and, d) how metrics may be combined to change the order of point elimination. The issues discussed in this paper also apply to the use of other metrics. It is hoped that the background and guidance provided in this paper will enable others to participate in further research and in the testing of the growing number of open source and commercial implementations. Given the allowance for variability, the testing should not be concerned with pedantic details but with whether the implementation is consistent with the intent of the algorithm.

2. Background

2.1 The Visvalingam Algorithm

The Visvalingam algorithm is very simple. It consists of repeated elimination of the point which is least significant in a given line and treating the remainder as the new input line. An algorithm is an abstract statement and it does not specify implementation details. For example, this algorithm does not specify the measure of significance nor how least should be defined. These decisions can be varied to meet different purposes so long as the implementation is consistent with the specification.

2.2 Metrics, Measures and Heuristics

There is a great deal of confusion over the terms metrics and measures. Even within the National Institute of Standards and Technology, DARPA ICV (1999) and SAMATE (2014) offer opposite meanings. Such confusion arises partly because of varying usage of these words within different disciplines. Software Engineering is a relatively new discipline compared with Statistics. This paper favours the definitions provided in DARPA ICV (1999) since they correspond to usage in statistics and its applications.

Chirhocub (2010) explained that "*A metric is the 'how' of measurement, that is,… the method by which one assigns numerical values based on the concept of distance, i.e. length, area, volume, progress, etc".* He noted that distance is the metric but the value is not. He also explained why **all metrics are measures** – but not all measures are metrics, using concepts in basic statistics.

The word measure is used rather loosely to refer to the taking of measurements, to the metrics; and in a more technical sense. In statistics, measures are derived through interpretation of one or more qualitative or quantitative metrics and/or other measures. The DARPA Report (1999, p 35) proposed that “*a metric is an observable value, while a measure associates meaning to that value by applying human judgement.*“

In statistics, we have Measures of Central Tendency (e.g. mean and median), Measures of Dispersion (variance and standard deviation) and Measures of Significance whose interpretation depends on associated probability distributions.

The word heuristic also has multiple meanings. In computing, heuristic refers to a rule of thumb, used often to speed up processing when exhaustive searching is impractical. More generally, it serves *to indicate or point out; stimulating interest as a means of furthering investigation … based on experimentation, evaluation, or trial-and-error methods…* it *refers to experience-based techniques for problem solving, learning, and discovery that givs a solution which is not guaranteed to be optimal* (The Free Dictionary, undated). This meaning is also used in other disciplines, e.g. in medicine.

2.3 Effective Area versus Least Areal Displacement

As stated in Visvalingam and Whyatt (1993), any metric can be used to assess the points’ importance. This was demonstrated by Visvalingam and Brown (1999) who used the algorithm for deriving decogons (decorative patterns) from fractal Koch curves.

Visvalingam chose to iteratively drop the point which resulted in the least areal displacement (AD in Pseudocode 1) but pointed out that this was one of many options. Areal displacement was chosen because some other metrics, such as for shape, only start to have an impact when the size of a feature exceeds a perceptual limit. It also takes account of the relationship of the distance between the points and their angles.

Areal displacement (**AD**) is the metric used to assess importance
Compute AD for all internal points of the input line
While there are more than the required number of points {
  Flag the point with lowest AD for removal
  Recalculate AD for the two neighbouring points
}
Output the co-ordinates of the retained points.

Pseudocode 1: Expression of the Visvalingam algorithm for filtering individual coastlines

Visvalingam and Whyatt (1993) only focused on the filtering of individual coastlines and used Whyatt’s program (Whyatt, 1991), which corresponds to Pseudocode 1. This approach has some merits and some limitations as explained in Section 3 below. So, in the paper by Visvalingam and Whyatt (1993), Visvalingam provided a fuller specification which corresponds to Pseudocode 2.

Visvalingam coined the term Effective Area (EA) to stress that it is a heuristic measure rather than a prescriptive metric, as explained fully in Section 3.3.2.

Statements 5 and 6 in Pseudocode 2 stipulate that if a point's EA is less than or equal to (<=) that of the last eliminated point, then its EA should be set to that of the latter. This conditional operator gives better results with some types of data. It makes no difference to the coastline data used in previous projects but produces more consistent results when processing fractals. Statement 5 ensures that points will be filtered in the same order as in Pseudocode 1, i.e. in a way which is consistent with the algorithm. The EA of the dropped point is recorded (statement 7) so that the polyline can be filtered repeatedly on this value (or its corresponding rank) in subsequent interactive applications. It is useful to note the sign of EA since this can be exploited in some applications, such as in terrain sketching (See Section 3.3.1). The modification of the EA in statement 5 is quite important since it can lead to a cascade of points being eliminated with the same EA or rank on thin elongated features.

Let previous = 0.0 [1]

Let EA = AD for all internal points of the input line [2]

While there are internal points { [3]

  Find the point with the least EA [4]

  [The condition in statement 5 below is preferable to the condition used in past papers, which was:

  ***If** (EA of this point < previous)…]*

  If (EA of this point <= previous) EA = previous [5]

    Else previous = EA [6]

  Record the EA of this point and note that it has already been dropped. [7]

  Recalculate the EA = AD for the two neighbouring points [8]

} [9]

Pseudocode 2: Original specification of the Visvalingam algorithm

Lines 5 to 7 in Pseudocode 2 are not an integral part of the basic generalisation algorithm. However, they facilitate the filtering of both single, and also multiple, independently processed polyline(s). The various implications and applications of the original specification (Pseudocode 2) are explained by tracing its use within the CISRG, using illustrations based on data for Humberside derived from Ordnance Survey 1:50000 maps. This free-to-use data is available at https://hydra.hull.ac.uk/resources/hull:9040.

2.4 The least important point

Section 2.3 explained that the algorithm may be driven with any metric. Experience favoured the use of EA for generalising coastlines, but Visvalingam and Brown pointed out that the offset value was better for approximating lines; they also demonstrated the effect of using other metrics. In addition, the algorithm and Pseudocodes 1 & 2 are not prescriptive as to the precise method to use when choosing the least important point for the following reasons. Visvalingam and Whyatt (1990) scrutinised the RDP algorithm (named after Ramer (1972) and Douglas and Peucker, 1973). They found that different implementations, and even the same program run on different computers, could produce different results depending on the specific test condition used, the direction of parsing, machine rounding errors and other factors. They also pointed out that cartographic data are inexact and are only representative. Given that digitising errors are much larger than rounding errors, they can make prescriptive stipulations of how to choose a point from a set of equal-valued candidates somewhat pedantic.

Visvalingam and Whelan (2014, Appendix 1), noted that the output from Mapshaper's Visvalingam option is not identical to ours. Visvalingam and Brown (1999) had previously demonstrated how Visvalingam's algorithm is sensitive to the direction in which a line is parsed. Indeed, if the same line is input in start-to-end and then in end-to-start order, it is possible to get different results. All versions should output similar, even if not identical, generalisations of coastlines. The only restriction is that the output must not be so inconsistent with the intent of the algorithm that it looks inappropriate for its intended purpose. This can happen if the point is picked from a candidate set with equal-values without due regard to its position along the line. This may not matter when coastlines are only simplified to a modest extent.

Visvalingam's implementation searches the array of EA from start to end, and eliminates the point chosen with the test condition ***If (EA < minEA) minEA = EA***. This is no more and no less arbitrary than using ***If (EA <= minEA) minEA = EA***. It is being stated here for the benefit of those who wish to compare their results with our published figures and for no other reason.

3. Implementations to suit different purposes

The basic algorithm and the specification as stated in Pseudocode 2 were developed on an ICL Perq personal workstation running the PNX operation system. This platform enabled speedy, suck-and-see interactive development of ideas and heuristics, using C, Fortran, and the Graphical Kernel System (see Visvalingam 1987a and b). The Graphical Kernel System (GKS) was then the international standard for 2D graphics (Hopgood et al, 1984). Visvalingam was fortunate to have sole personal use of this system through the award of a UK Science and Engineering Research Council (SERC) grant from 1983-86 to Graham Kirby (in Computer Science) and herself (then an Honorary Research Associate in Geography) at the University of Hull. When this project commenced, powerful UNIX-based single user workstations with high resolution A4 bit-mapped screens were just becoming widely available. It was still not possible to undertake explorations of multivariate data through cross-linked graphical interfaces as anticipated by Visvalingam and Kirby (1984) and Visvalingam (1985). However, the ability to run concurrent interactive programs enabled Visvalingam to manually cross-reference data and displays to study the impact of changes to specification. Other CISRG members, including research students, initially had to rely on batch processing on a multiuser mainframe computer and use a Calcomp pen plotter and later Postscript for quality figures.

The various implementations used within the multidisciplinary CISRG reflected the requirements of projects allocated to students, which took account of their disciplinary strengths, skills in programming, the computing resources at their disposal and their desire to learn new programming languages and other skills.

3.1 Whyatt's implementation

Whyatt's implementation, which corresponds to Pseudocode 1, continues to provide an excellent starting point for short undergraduate student projects since it is easy to implement. It focuses on the gist of Visvalingam's algorithm; namely the iterative elimination of the least important point. Even in the late 1980s, it was not unusual to run the input data repeatedly through filter programs to calculate and output the required subset specified by a filter value or the number (percentage) of points to be retained. Whyatt only had access to a Sequent Symmetry multitasking computer, running DYNIX (a version of Unix). His research focused on comparing algorithms using a few carefully selected isolated coastlines. Each coastline was repeatedly run thorough his Fortran program to retain a specified number of points for offline drawing of maps on a Calcomp plotter (Whyatt, 1991, Appendix of Program Listings). Note that Pseudocode 1 uses the metric AD, which is the calculated Areal Displacement. AD is not the same as EA as explained in Section 3.3. Figure 1a shows 8 out of 2227 points in the Humberside coastline.

3.2 The rank order of points

A minor modification could be made to Pseudocodes 1 & 2 to attribute a rank to each point to indicate the order in which points are eliminated by Visvalingam's algorithm. The tagged data output can then be repeatedly filtered on this rank value by a separate process. Figure 1a shows that filtering on rank gives the same output as the process described in section 3.1 using Pseudocode 1.

The rank attribute is much more efficient and convenient for retaining the number (or percentage) of points in some applications. It was only mentioned briefly by Visvalingam and Whyatt (1993) although it has been widely used within the CISRG. It is especially useful for comparing algorithms, metrics, weights etc. All CISRG projects which compared the RDP, Visvalingam and Bendsimplify algorithms used the rank filter since the metrics themselves are not directly comparable. The Bendsimplify algorithm was studied by Visvalingam and Herbert, 1999. The comparison of weighted and unweighted EAs by Vinsvalingam and Whelan (2014) also compared subsets filtered with rank.

When (***EA of a point <= previous***) in Statement 5 of Pseudocode 2, the rank can also be reset, if needed, to that of the previously eliminated point so that individual lines could be filtered on rank or on EA in a consistent way. Ranks could be specific to individual lines and/or be global to a set of lines, to suit the implementation and its target applications. Rank is not used in applications which rely on multiple scale-related filters (See Section 3.3.1). Like the choice of the metric and the least important point, this is a purpose-driven implementation issue. Within the CISRG, ranks were specific to separately processed individual polylines since EA was used to filter multiple lines across all applications.

3.3 Why and how the EA measure was conceived and used for filtering

Visvalingam had already noted that some in-line shapes can result in the lack of a monotonic relationship between rank and AD; ADs can sometimes become smaller with increasing rank. Visvalingam was impressed by Wade's solution to a similar problem in his implementation of the RDP algorithm and adapted his approach in her implementation. Wade's Fortran program is listed in Whyatt and Wade (1988). He wrote this in his first year of postgraduate research (1983/84) on behalf of the Market Analysis Division of CACI, the collaborating partner part-funding his SERC CASE (Science and Engineering Research Council Collaborating Awards in Science and Engineering) studentship. In Section 2.4, we explained how different implementations of the RDP algorithm can

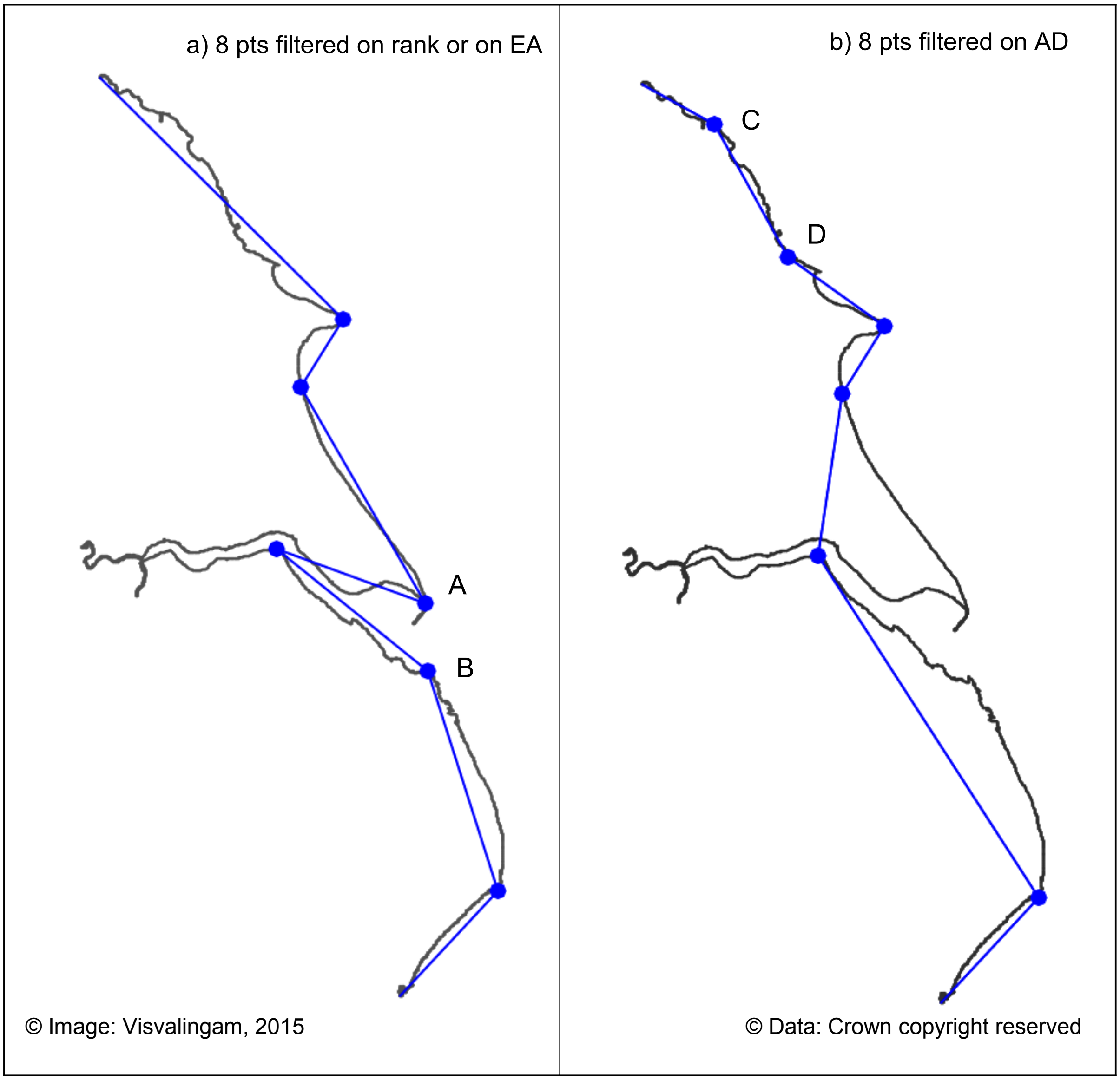

Figure 1 : The effective area (EA) is not the same as areal displacement (AD)

produce different results for a variety of reasons, including the treatment of special geometric cases. Peucker (1975, p 511) had already noted that offset values can increase with recursive segmentation of a line by the RDP algorithm. This was ignored in earlier implementations by others since the recursive selection of points was terminated when the required number of points was ouptut or when offset values fell below a specified threshhold.

The GIMMS mapping package (Waugh and McCalden, 1983) was the first to tag points. Whyatt and Wade (1988) noted how its GENERAL command avoided the need for repeated calculation of offsets. Using the GENERAL command, it was possible to assign points to nominal, scale related classes for subsequent filtering. However, GIMMS did not address the lack of a monotonic relationship between the rank and offset values.

In his independent implementation of the RDP algorithm, Phil Wade tagged each point with the offset distance which led to its selection to enable subsequent interactive filtering by an independent process. When an offset distance was greater than that of the previously chosen point, Wade demoted the former's value to that of the latter as explained with illustrations in Visvalingam and Whyatt (1990). This ensured that his output was consistent with the spirit of the original specification of the RDP algorithm.

With his and the sponsor's permission, his program was used in undergraduate projects to evaluate the RDP algorithm. These preliminary investigations, especially by Ian Jenkinson, revealed that the RDP algorithm had significant limitations, and this was confirmed by Whyatt's PhD project, which used Wade's implementation of the RDP algorithm.

Returning to Visvalingam's algorithm, maps filtered using AD (Section 3.1) and rank (Section 3.2) can differ. As already noted, filtering individual lines on rank will give identical results consistent with the algorithm and Pseudocodes 1 & 2. Filtering on AD can remove points in the wrong order and produce unacceptable results. So, Visvalingam adapted Wade's technique and promoted AD to EA when the AD of the current point is smaller than that of the last point to be eliminated. As illustrated in Figure 1, points C & D have larger ADs than points A and B. So, filtering on AD gives Figure 1b which is inappropriate. The intended output can be obtained by filtering on EA.

Visvalingam coined the term EA to stress that it is not AD, but a **heuristic** measure based on the metric AD. It was derived through experience and indicates the presence of a potentially meaningful geometric entity. As is true of many heuristics, it does not guarantee optimal results in all circumstances. EA not only facilitates filtering on a scale-related value, it also a) facilitates the filtering of maps consisting of several independently tagged lines; and b) the segmentation of lines to facilitate model-based filtering as explained below.

3.3.1 Filtering maps and terrain with several polylines

Visvalingam produced a Fortran version of her program for use by Peter Williamson in his four-month Dissertation project in part fulfilment of an MSc conversion course in IT and Manufacture in 1991/92. His project work was also done on the mainframe computer running Unix. It involved the generalisation of data for a set of road line segments on a 1:1250 large-scale Ordnance Survey plan. Figures with single and multiple lines were used to compare the performance of the RDP and Visvalingam algorithms (Visvalingam and Williamson, 1995).

Since EA is based on AD, which is a scale-related metric, it can be used to filter multiple lines. Programs written by Wade and Visvalingam were used by Williamson to filter the lines and the results were compared using Williamson's C program using GKS. Alan Whitaker wrote Postscript programs for producing the high quality figures for publication.

Visvalingam's implementation was adapted by Dowson for the research on terrain sketching undertaken by Visvalingam and Dowson (1998). This project demonstrated that it is possible to use EA to identify important breaks of slope in multiple terrain profile sections for producing algorithmic sketches of the terrain. EA is the **heuristic measure** of significance (i.e. the indicator of significant convex (+EA) and concave (-EA) breaks of slope). Different EA values were used to filter core cells for different types of breaks of slope. The core cells were then extended to form the profile stokes which made up the P-stroke sketch. Ranks have little use in this sort of application. Terrain sketching and coastline generalisation use Visvalingam's algorithm in different ways. This is another reason for avoiding pedantic specifications in the algorithm and in the Pseudocode(s)

Other research students translated this version into programming languages of their choice. Brown's C translation was used in undergraduate and MSc coursework and included metrics other than EA as reported in Visvalingam and Brown (1999). As acknowledged in Visvalingam and Brown (1999), Michael Harasimiuk and Roger Whyatt (students on the taught MSc Programme on Computer Graphics and Virtual Environments in 1995/96) were the first to use an albeit erroneous triadic Koch curve to compare algorithms using Brown's and Wade's programs. Whelan's Java translation of Dowson's program was used by Whelan and Visvalingam (2003) for the P-strokes which complemented the novel Formulated Silhouettes.

Visvalingam ensured that all implementations were fit for their purpose and that they produced consistent results within the scope for variation as noted in Sections 2.4 and 3.3. Variations in output do not imply a lack of conformance with Visvalingam's specification nor that an implementation is inappropriate. A forthcoming paper will illustrate the difficulties involved in testing and certifying that an implementation conforms to the specification, given the allowances for variation.

3.3.2 Model-based filtering

Cartographers eliminate features – not points. The detection of features within coastlines remains a research challenge. So, Visvalingam started with triangles and found that repeated elimination of the smallest triangle using EA can point to the presence of inline structures, such as inflections and thin elongated features like rivers and spits. Unfortunately, points with the same offset value in Wade's implementation of the RDP algorithm did not depict features in a manner which aided their delineation. Sections of lines belonging to features can be eliminated or segmented with EA. It looked as if the visual clues provided by EA could be heuristically exploited by a separate process to delineate the implied geomorphic features.

If lines could be segmented into their constituent inline features, they can be structured for intelligent feature-based filtering as suggested in Visvalingam and Whyatt (1993). The feasibility of this was only demonstrated in year 2000 as described in Visvalingam and Whelan (2014, Section 5.5). Visvalingam anticipated that long lines could be segmented and topologically structured to be processed by the Disassociative Area Model; Wade (1986) conceived and programmed this model for extracting the hierarchic area topology from link-and-node structured Ordnance Survey topographic data for the Administrative Areas of England Wales (see also, Wade et al, 1986; Visvalingam et al, 1986; Kirby et al, 1989). Linear data can be topologically structured on other criteria and not just on administrative boundaries. Figure 1 in Visvalingam (1990) represented the model of Digital Cartography within which linear data was to be topologically structured during the Digital Mapping phase for subsequent Visual Mapping both offline and interactive. The automation of this remains a research challenge.

4. Complex measures, such as weights

As we noted in Section 2.2, measures can be derived through interpretation of several qualitative or quantitative metrics and/or measures. Visvalingam and Whyatt suggested that the EA measure could be weighted to change the rank order of points, for example to take account of shape. AD iand EA in Pseudocodes 1 & 2 respectively have a standard weight of 1. Zhou and Jones (2004) proposed a complex formula for using weighted EAs (which they shortened to WEA) to take account of shape. In

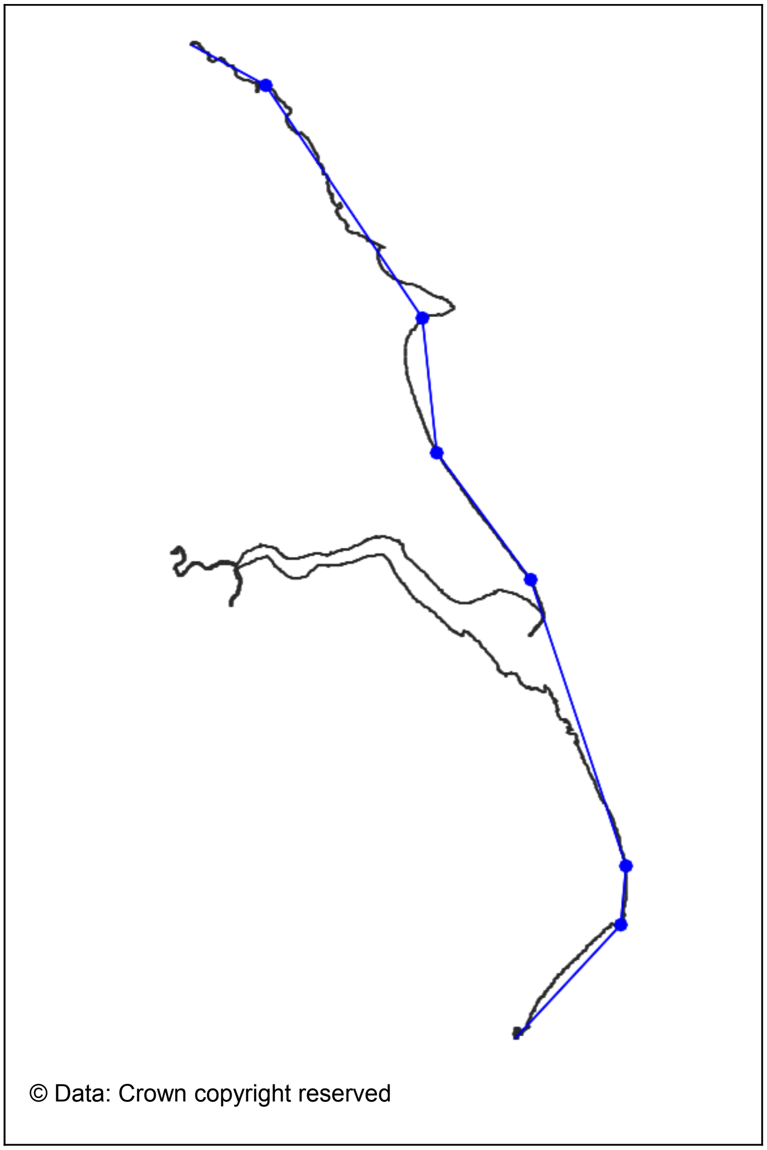

Figure 2: Last 8 points exported by Mapshaper v 0.2.0 using EA = EA * (1-cosine (theta))

Mapshaper 0.2.0 Bloch (2014a) used a simpler scheme and down weighted the EA of acute angles only with ***if (theta < 90) EA = EA * (1 – cosine (theta))*** to eliminate spiky detail. Visvalingam and Whelan (2014) cited his reasons for doing this and demonstrated how WEA can amplify the angular truncation of features and produce inappropriate caricatural generalisations of this coastline. Figure 2 provides a much more pleasing depiction of the shape of the coast, compared with Figure 1a, if the features were unimportant. The same figure was also output for the last 8 points by Mapshaper version 0.2.17, which features the weighting optionD by Bloch (2014c).

5. Summary

This paper has explained why the specification of Visvalingam's algorithm and sample Pseudocode(s) avoided implementation-specific details since these can be purpose-oriented. It explains why Pseudocode 1 is sufficient for expressing the algorithm but why Pseudocode 2 with the concept of EA is more useful. When EA is used for filtering multiple lines, with one or more filter values, it is just an implementation-specific driver. The scope it offers for inferring geometric features makes it a **heuristic indicator** – a rule-of-thumb based on a metric. The subsequent detection of in-line structures (still a research challenge) can support intelligent model-based generalisation. The indicator may be based on multiple metrics, for example when it is weighted to eliminate spiky detail in simplification and typification. It explains why variations in implementation are inevitable and sometimes necessary to suit different purposes.

Acknowledgements

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