Fault estimation and active fault tolerant control for linear parameter varying descriptor systems

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Summary

Starting with the baseline controller design, this paper proposes an integrated approach of active fault tolerant control (AFTC) based on proportional derivative extended state observer (PDESO) for linear parameter varying (LPV) descriptor systems. The PDESO can simultaneously provide the estimates of the system states, sensor faults and actuator faults. The L_2robust performance of the closed-loop system to bounded exogenous disturbance and bounded uncertainty is achieved by a two-step design procedure adapted from the traditional observer based controller design. Furthermore, a linear matrix inequality (LMI) pole-placement region and the L_2 robustness performance are combined into a multi-objective formulation by suitably combing the appropriate LMI descriptions. A parameter-varying system example is given to illustrate the design procedure and the validity of the proposed integrated design approach.

Key words: Active fault tolerant control, robust fault estimation, linear parameter varying systems, descriptor systems, observer-based design, integrated control and estimation design

1 INTRODUCTION

The design of feedback mechanisms which can be applicable to non-linear systems must be done very carefully with regard to the form of system non-linearity and the restrictions that can be imposed by using system linearization methods. However, a linearization approximation in the form of a time-varying system can be a basis for suitable representation of the original non-linear system. An early approach to achieve a representation of a time-varying system has been to use the so-called gain-scheduling design methods which are still in use today e.g. for flight control systems, principally due to their apparent simplicity[1]. In gain-scheduling, sets of model parameters, gains etc. are pre-stored in the control systems and a selection mechanism is used to switch between the various models according to the understood non-linear behaviour.

An attractive alternative to gain-scheduling is to represent the non-linear system with a linear parameter varying (LPV) model that depends on a set of measured or estimated parameters [1, 2]. The main advantage of LPV models is that they allow powerful linear design tools to be applied even to some complex non-linear systems, whilst also guaranteeing global stability over the entire working envelope [2-8]. LPV modelling of monitored systems has been considered for fault diagnosis or fault estimation in [9-17], and fault tolerant control (FTC) [18-22].

On the other hand, descriptor systems have attracted significant attention in the control community as a consequence of their flexibility for modelling real systems with constraints[23-32]. In particular, the LPV formulation of a descriptor system can have powerful analysis and design properties [33-37]. For example in [37] an unknown input observer (UIO) design procedure is generalized to an LPV descriptor system. However, few studies have been concerned with LPV approaches to the joint problems of state and fault estimation for descriptor systems. This is despite an interesting result on fault estimation by [38].

In addition, to achieve desired performance of the overall system, each part or subsystem should be considered carefully. For instance, in the observer based controller design, the controller and observer will inject uncertainties to each other; hence the overall performance may be degraded if the two parts cannot work in harmony.

In the paper, a proportional derivative extended state observer (PDESO) for LPV descriptor systems is proposed for simultaneous state and fault estimation. With the proposed PDSEO, an integrated design approach is proposed to achieve the required closed-loop control robustness to bounded exogenous disturbance and bounded modelling uncertainty using a two-step design procedure adapted from the observer-based controller design approaches in [39, 40]. A multi-objective optimization framework for the design strategy can be achieved by combing the LMI design descriptions with suitably chosen design parameters.

This remainder of the paper is organized as follows: Section 2 introduces the concept of LPV descriptor systems following design approaches for pole-placement design and H_∞ optimization. In the light of the duality of linear descriptor systems, Section 3 proposes a procedure to design a PDESO for LPV descriptor systems. Based on the well-known observer-based state feedback control structure, Section 4 proposes an integrated AFTC scheme with the PDESO. A numerical example is used to illustrate the design of the proposed AFTC system in Section 5 and the conclusion is given in Section 6.

Notations: $\mathbb{R}$ denotes the real number set. . $\mathcal{C}$ denotes the complex number set. I_q denotes the identity matrix with dimensions of $q \times q$. $Ker(E)$ means the null space of E. s denotes the Laplace variable. P>0 denotes that P is a symmetric positive matrix. $\otimes$ denotes the Kronecker product. $\star$denotes the symmetric part of a matrix, e.g. $M + \star = M + M^T, N + M + \star = N + M + M^T$ where N is a symmetric matrix.

2 BASELINE CONTROLLER FOR LPV DESCRIPTOR SYSTEMS

Following the LPV descriptor system formulation given in [35, 36, 41], consider a system with sensor and actuator faults given as:

$$E\dot{x} = A\big(\theta(t)\big)x + \mathrm{B}u + Rd \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ and $d \in \mathbb{R}^d$are the state vector, the input vector and disturbance vector or modelling uncertainties, respectively. d denotes the "lumped total effect" of exogenous disturbance and modelling uncertainties. $B, E,$ and R are known constant matrices. $rank(E) = r \leq n$. $A\big(\theta(t)\big)$ is a known continuous function of a time-varying parameter vector $\theta(t)$ which satisfies:

$$\theta(t) = \left[\theta_1(t), \cdots \theta_{n_\theta}(t)\right]^T \in \Theta, \forall\, t \geq 0$$

where Θ is a compact set. Consider a parameter-dependent state feedback controller as:

$$u = K(\theta)x \tag{2}$$

The closed-loop system can be obtained as:

$$E\dot{x} = (A(\theta) + BK(\theta))x + Rd \tag{3}$$

2.1 Quadratic admissibility

Not only stability, but also impulse-free system behaviour should be considered in the design and analysis of descriptor systems. Thus, the following Definition must be stated.

Definition 1 (Admissibility): A pair (E, A) is admissible if it is regular and has neither impulsive modes nor unstable finite modes. Alternatively, a pair is admissible if it is impulsive free and stable. Recall that a pair (E, A) is regular if $det(sE - A)$ is not identically equal to zero.

For a linear time-invariant descriptor system, it is well known [28] that the following Lemma exists:

Lemma 1: A pair (E, A) is admissible if and only if there exists matrices $P \in \mathbb{R}^{n\times n}$ and $S \in \mathbb{R}^{(n-r)\times(n-r)}$ such that:

$$A(PE^T + USV^T) + (PE^T + USV^T)^T A^T < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively.

From the above Lemma, it is noted that the admissibility of a pair (E, A) means the stability and impulse-free behaviour of the pair (E, A).

Following the definition of quadratic stability for standard LPV systems [8], the definition of quadratic admissibility for LPV systems is given as follows:

Definition 2: (Quadratic admissibility): An LPV descriptor system pair $(E, A(\theta))$ is said to be quadratically admissible if there exist matrices $P \in \mathbb{R}^{n\times n}$ and $S \in \mathbb{R}^{(n-r)\times(n-r)}$ such that for all $\theta \in \Theta$:

$$A(\theta)(PE^T + USV^T) + (PE^T + USV^T)^T A^T(\theta) < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively.

2.2 Pole-placement design

Following the well-known definition of an LMI region [8, 42], two Lemmas to handle pole-placement of descriptor systems are presented below based on the results given in [28].

Definition 3 (LMI region): A subset $\mathfrak{D}$ of the complex plane is called an LMI region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{q\times q}$, and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{q\times q}$ such that:

$$\mathfrak{D} := \{z \in \mathcal{C} : f_{\mathfrak{D}}(z) < 0\} \quad (4)$$

with characteristic function given as:

$$f_{\mathfrak{D}}(z) := \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl}z + \beta_{lk}\bar{z}]_{1\leq k,l\leq q} \quad (5)$$

Following this, the concept of $\mathfrak{D}$ stability is defined as follows:

Definition 4 ($\mathfrak{D}$ stability): A descriptor system is said to be $\mathfrak{D}$ stable if all the finite eigenvalues of the system belong to an LMI region $\mathfrak{D}$.

Considering the possibility of infinite modes of a descriptor system, an admissibility and $\mathfrak{D}$ stability result for LTI descriptor systems given in [28] is presented as follows:

Lemma 2: The pair (E, A) is admissible and $\mathfrak{D}$ stable if and only if there exist matrices $P > 0, P \in \mathbb{R}^{n\times n}$ and $S \in \mathbb{R}^{(n-r)\times(n-r)}$ such that:

$$\alpha \otimes (EPE^T) + \beta \otimes (APE^T) + I_q \otimes (AUSV^T) + \star < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively.

Definition 5 (Quadratic admissibility and $\mathfrak{D}$ stablility): An LPV descriptor system is said to be quadratically admissible and $\mathfrak{D}$ stable if there exist matrices $P > 0, P \in \mathbb{R}^{n\times n}$ and $S \in \mathbb{R}^{(n-r)\times(n-r)}$ such that for all $\theta \in \Theta$:

$$\alpha \otimes (EPE^T) + \beta \otimes (A(\theta)PE^T) + I_q \otimes (A(\theta)USV^T) + \star < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively.

It is worth mentioning that in a general case, the poles of an LPV descriptor system cannot reflect the stability of the LPV descriptor system. However, comparing Definition 5 and Definition 2, it can be seen that, the quadratic admissibility and $\mathfrak{D}$ stablility reduce to the quadratic admissibility for the case $\alpha = 0$, and $\beta = 1$. Hence, the requirement of **quadratic admissibility and $\mathfrak{D}$ stablility** in an LMI region of an LPV descriptor system can be considered instead of a **quadratic admissibility** condition.

The following Lemma 3 [28] provides a state feedback design approach for an LPV descriptor using LMI.

Lemma 3: The pair $(E, A(\theta) + BK(\theta))$ is quadratically admissible and $\mathfrak{D}$ stable if there exist matrices $> 0, P \in \mathbb{R}^{n\times n} S \in \mathbb{R}^{(n-r)\times(n-r)}$, $L(\theta) \in \mathbb{R}^{m\times n}$ and $H(\theta) \in \mathbb{R}^{m\times(n-r)}$ such that for all $\theta \in \Theta$:

$$\alpha \otimes (EPE^T) + \beta \otimes (A(\theta)PE^T + BL(\theta)E^T) + I_q \otimes (A(\theta)USV^T + BH(\theta)V^T) + \star < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$ respectively. Then the controller gain $K(\theta)$ is given by:

$$K(\theta) = (L(\theta)E^T + H(\theta)V^T)(PE^T + USV^T)^{-1}.$$

The solution of the above LMI may lead to a singular S, which in turn leads to a singular $(PE^T + USV^T)$. The way to avoid this singularity is to replace S by $S + \mu I$ where μ is a small constant number to obtain a non-singular $(PE^T + USV^T)$, whilst still satisfying Lemma 3.

2.3 L_2robustness optimization

Following the design procedure within the L_2 framework discussed in Section 3.2 in [28], an L_2 performance variable is defined as:

$$z = C_{zx}x \tag{6}$$

which leads to the transfer function:

$$G(\theta, s) = C_{zx}(sE - A(\theta) - BK(\theta))^{-1}R \tag{7}$$

The defined $G(\theta, s)$ is a measurement of the influence of disturbance on system states of (1) and (6) in H_∞ framework.

Definition 6 (QuadraticL_2 performance): A LPV descriptor system of (1) and (6) has quadratically L_2 performance if there exist matrices $P \in \mathbb{R}^{n\times n}$ $S \in \mathbb{R}^{(n-r)\times(n-r)}$ such that for all $\theta \in \Theta$:

$$\begin{bmatrix} A(\theta)(PE^T + USV^T) + \star & R & (PE^T + USV^T)^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively.

Lemma 3 [28] is summarised below to handle the L_2 design problem of a LPV descriptor system.

Lemma 4: The system pair $(E, A(\theta) + BK(\theta), R, C_{zx})$ is quadratically admissible and satisfies $\|G(\theta, s)\|_2 < \gamma$ if there exist matrices $P > 0, P \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{(n-r)\times(n-r)}$, $L(\theta) \in \mathbb{R}^{m\times n}$ and $H(\theta) \in \mathbb{R}^{m\times(n-r)}$ such that for all θ:

$$\begin{bmatrix} \Delta & R & (PE^T + USV^T)^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0$$

with:

$$\Delta = A(\theta)(PE^T + USV^T) + B(L(\theta)E^T + H(\theta)V^T) + \star$$

where U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$ respectively. Then the gain $K(\theta)$ is given by:

$$K(\theta) = (L(\theta)E^T + H(\theta)V^T)(PE + USV^T)^{-1}.$$

It can be seen that the number of LMIs is infinite for arbitrary θ in the conditions given in Lemmas 3 & 4. Fortunately, for polytopic or affine LPV systems, the above requirement for an infinite set of LMIs can be transformed to finite dimensional LMIs, with ease of solution using the MATLAB LMITOOL box [43]. It is worth pointing out that combination of Lemmas 3 & 4 with suitable parameters can be used to achieve a multi-objective design.

3 DESIGN OF PROPORTIONAL DERIVATIVE EXTENDED STATE OBSERVER

For the FTC framework, the system of (1) and (2) can be modified to consider sensor and actuator faults as:

$$E\dot{x} = A(\theta(t))x + \mathrm{B}u + F_a f_a + R_1 d_u \tag{8}$$

$$y = \mathrm{C}x + F_s f_s + R_2 d_s \tag{9}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m and\ y \in \mathbb{R}^h, f_a \in \mathbb{R}^q, f_s \in \mathbb{R}^{p-q}$ and $\begin{bmatrix} d_u \\ d_s \end{bmatrix} \in \mathbb{R}^d$ are the state vector, the input vector and measured output vector, actuator fault vector, sensor fault vector and disturbance, respectively. B, C, D, F_a, F_s, E, R are known constant matrices. $rank(E) = r \leq n$. $A(\theta(t))$ is known continuous function of a time-varying parameter vector $\theta(t)$ which satisfies:

$$\theta(t) = \left[\theta_1(t), \cdots \theta_{n_\theta}(t)\right]^T \in \Theta, \forall\, t \geq 0$$

whereΘ is a compact set. Clearly, the above system can be rewritten as:

$$E\dot{x} = A(\theta)x + \mathrm{B}u + F_f f + Rd \tag{10}$$

$$y = \mathrm{C}x + D_f f + \mathrm{D}d \tag{11}$$

where:

$$f = \begin{bmatrix} f_a \\ f_s \end{bmatrix}, D_f = \begin{bmatrix} 0 & F_s \end{bmatrix}, F_f = \begin{bmatrix} F_a & 0 \end{bmatrix}, f \in R^p, d = \begin{bmatrix} d_u \\ d_s \end{bmatrix}, \mathrm{D} = \begin{bmatrix} 0 & R_2 \end{bmatrix}, \mathrm{R} = \begin{bmatrix} R_1 & 0 \end{bmatrix},$$

That is because of the parameter-independent Lyapunov function adopted through the above Lemmas which could lead to some conservative solutions with reduced computational complexity. A parameter-dependent Lyapunov function must be adopted if no results are found satisfying the above conditions.

Lemma 5: The LPV descriptor system $(E, A(\theta), C)$ is observable if the following conditions hold for all θ:

A1): $rank\left(\begin{bmatrix} sE - A(\theta) \\ C \end{bmatrix}\right) = n,$ ⍰$s > 0, s\ is\ finite$?????????

A2): $rank\left(\begin{bmatrix} E \\ C \end{bmatrix}\right) = n$

The following condition is assumed for the design of the PDESO LPV representations of the descriptor system:

A3): $rank\begin{bmatrix} A(\theta) & F_f \\ C & D_f \end{bmatrix} = n + p$

3.1 Duality of descriptor systems

The duality of a square descriptor system is presented briefly, followed by a systematic design approach via designing a dual observer instead of the original one. Consider a linear square descriptor system as:

$$E\dot{x} = A(\theta)x + \mathrm{R}d_x \tag{12}$$

$$y_x = \mathrm{C}x \tag{13}$$

The dual of system (12) and (13) is given as:

$$E^T\dot{z} = A^T(\theta)z + C^T d_z \tag{14}$$

$$y_z = \mathrm{R}^T z \tag{15}$$

It is already well known that the original system of (12)-(13) and its dual system of (14) and (15) share the same stability property. The goal here is to show a duality property in terms of H_∞ performance. Given the transfer function from the disturbances to outputs as:

$$G(\theta, s) = C(sE - A(\theta))^{-1}R$$

$$G_{dual}(\theta, s) = R^T(sE^T - A^T(\theta))^{-1}C^T$$

Then it follows that:

$$\|G(\theta, s)\|_\infty = \|G^T(\theta, s)\|_\infty = \|R^T(sE^T - A^T(\theta))^{-1}C^T\|_\infty = \|G_{dual}(\theta, s)\|_\infty$$

From the above relationship, it can be seen that the dual system of (14) and (15) shares the same robustness properties with the original system of (12) and (13). Based on this observation, the following focuses on the PDESO observer design with the dual system ($E^T \quad A^T(\theta)$) instead of the original system ($E \quad A(\theta)$), whilst in the robust design the dual system ($E^T \quad A^T(\theta) \quad R^T \quad C^T$) is considered instead of the original system ($E \quad A(\theta) \quad C \quad R$).

3.2 The PDESO structure

Fault signal f in the system of (8) and (9) or the system of (10) and (11) are assumed to be slowly-varying. The original system can be augmented first if the unknown input signal f has fast variation using a multi-augmentation technique first introduced in [44]. If the q^{th} derivative of the f, i.e. $f^{(q)}$ is slowly time-varying. The fault signal can be written as:

$$\left.\begin{array}{c} \dot{f} = \delta_1 \\ \dots \\ \dot{\delta}_{q-1} = f^{(q)} \end{array}\right\} \tag{16}$$

Furthermore, the system can be reorganized in matrix form as:

$$\begin{bmatrix} E & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{f} \\ \dot{\delta}_1 \\ \dots \\ \dot{\delta}_{q-1} \end{bmatrix} = \begin{bmatrix} A(\theta) & F_f & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x \\ f \\ \delta_1 \\ \dots \\ \delta_{q-1} \end{bmatrix} + \begin{bmatrix} \mathrm{B} \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} u + \begin{bmatrix} R \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} d$$

$$+[0 \quad 0 \quad 0 \quad \dots \quad 1]^T f^{(q)} \tag{17}$$

$$y = [\mathrm{C} \quad D_f \quad 0 \quad \dots \quad 0][x \quad f \quad \delta_1 \quad \dots \quad \delta_{q-1}]^T + Dd \tag{18}$$

For the case f slowly time-varying, a sensor-noise free LPV descriptor system of (8) and (9) can be augmented via introducing a variable $\omega = R_2 d_s$ as follows:

$$E_a \dot{x}_a = A_a(\theta) x_a + B_a u + R_a d_D \tag{19}$$

$$y = C_a x_a \tag{20}$$

where:

$$x_a = \begin{bmatrix} x \\ \omega \\ f \end{bmatrix}, E_a = \begin{bmatrix} E & 0 & 0 \\ 0 & I_h & 0 \\ 0 & 0 & I_p \end{bmatrix}, A_a = \begin{bmatrix} A(\theta) & 0 & F_f \\ 0 & -\rho I_h & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R_1 & 0 \\ 0 & I_h \\ 0 & 0 \end{bmatrix}$$

$$C_a = [C \quad I_h \quad D_f], d_D = \begin{bmatrix} d_u \\ \rho R_2 d_s + R_2 \dot{d}_s \end{bmatrix}$$

Lemma 6: The augmented system (E_a, A_a, C_a) is observable with Assumptions A1)-A3).

Proof:

$$rank\left(\begin{bmatrix} E_a \\ C_a \end{bmatrix}\right) = rank\left(\begin{bmatrix} E & 0 & 0 \\ 0 & I_h & 0 \\ 0 & 0 & I_p \\ C & I_h & D_f \end{bmatrix}\right) = rank\left(\begin{bmatrix} E \\ C \end{bmatrix}\right) + p + h = n + p + h$$

$$rank\left(\begin{bmatrix} sE_a - A_a(\theta) \\ C_a \end{bmatrix}\right) = rank\left(\begin{bmatrix} sE - A(\theta) & 0 & F_f \\ 0 & sI_h - \rho I_h & 0 \\ 0 & 0 & sI_p \\ C & I_h & D_f \end{bmatrix}\right)$$

$$= \begin{cases} rank\begin{bmatrix} A(\theta) & F_f \\ C & D_f \end{bmatrix} = n + p + h & s = 0 \\ rank\left(\begin{bmatrix} sE - A(\theta) \\ C \end{bmatrix}\right) + p + h = n + p + h & Re(s) > 0 \end{cases} = n + p + h$$

Hence, the augmented system is observable. ■

Then an LPV observer in the following form is proposed:

$$E_a\dot{\hat{x}}_a = A_a(\theta)\hat{x}_a + B_a u + L_p(\theta)(\hat{y} - y) + L_d(\theta)\left(\dot{\hat{y}} - \dot{y}\right) \quad (21)$$

$$\hat{y} = C_a\hat{x}_a \quad (22)$$

where $L_p(\theta), L_d(\theta)$ are to be determined and:

$$L_p(\theta) = \begin{bmatrix} L_x(\theta) \\ L_\omega(\theta) \\ L_f(\theta) \end{bmatrix}, L_d(\theta) = \begin{bmatrix} L_{xd}(\theta) \\ L_{\omega d}(\theta) \\ L_{fd}(\theta) \end{bmatrix} \quad (23)$$

3.3 The PDESO design approaches

Define $e_{x\omega} = \begin{bmatrix} x \\ \omega \end{bmatrix} - \begin{bmatrix} \hat{x} \\ \hat{\omega} \end{bmatrix}$, $e_f = f - \hat{f}$, and $e_{x\omega f} = \begin{bmatrix} e_{x\omega} \\ e_f \end{bmatrix}$, then the augmented state estimation error system can be obtained as:

$$E_o(\theta)\dot{e}_{x\omega f} = A_o(\theta)e_{x\omega f} + R_a d_D \quad (24)$$

with:

$$A_o(\theta) = A_a(\theta) + L_p(\theta)C_a, E_o(\theta) = E_a - L_d(\theta)C_a$$

Now define the performance function as:

$$z_{e\omega f} = C_{ze\omega f}e_{x\omega f} \quad (25)$$

The dual system of the system given in (24) and (25) can be obtained as:

$$(E_a^T - C_a^T L_d^T(\theta))\dot{z} = (A_a^T(\theta) + C_a^T L_p^T(\theta))z + C_{ze\omega f}^T d_z \quad (26)$$

$$y_z = \mathrm{R}_a^T z \quad (27)$$

With a suitable augmentation, the dual system of (26) and (27) can be transformed to

$$\begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T(\theta) + C_a^T L_p^T(\theta) & -E_a^T + C_a^T L_d^T(\theta) \end{bmatrix}\begin{bmatrix} z \\ z_a \end{bmatrix} + \begin{bmatrix} 0 \\ C_{ze\omega f}^T \end{bmatrix} d_z \quad (28)$$

$$y_z = C_{ea}\begin{bmatrix} z \\ z_a \end{bmatrix} \quad (29)$$

The introduction of $z_a = \dot{z}$ may introduce impulsive modes because the continuity of z does not imply the continuity of $\dot{z}$. The impulsive modes in the time response of a descriptor system may be highly detrimental to the system operation. However, as pointed out in [45], the following statements are equivalent:

1) $rank\ (E_o^T) = n$
2) The system $\left(\begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & I_{n+p+h} \\ A_o^T & -E_o^T \end{bmatrix}\right)$ is impulsive-free.

Generally, $rank(E_o^T) = n$ is a very restrictive condition. However, in the PD-ESO design, the requirement can always be satisfied as previously proved by Lemma 6. If the following are defined:

$$E_{ea} = \begin{bmatrix} I_{n+p+\mathrm{h}} & 0 \\ 0 & 0 \end{bmatrix}, A_{ea} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T(\theta) & -E_a^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}, R_{ea} = \begin{bmatrix} 0 \\ C_{ze\omega f}^T \end{bmatrix}$$

$$L_{ea}(\theta) = \begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix}, C_{ea} = \begin{bmatrix} R_a^T & 0 \end{bmatrix}$$

System (28) and (29) can be organized as:

$$E_{ea}\begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = (A_{ea}(\theta) + B_{ea}L_{ea}(\theta))\begin{bmatrix} z \\ z_a \end{bmatrix} + R_{ea}d_z \quad (30)$$

$$y_z = C_{ea}\begin{bmatrix} z \\ z_a \end{bmatrix} \quad (31)$$

The problem of stabilizing the original system with PDESO is transferred to a state feedback problem which can be easily solved with the theory presented in Section 2. Here the following two Theorems are proposed to design a descriptor system PDESO within the LPV framework.

Theorem 1: There is a quadratically admissible and $\mathfrak{D}$ stable PDESO in the form of (21) and (22) if there exist matrices $P > 0, P \in \mathbb{R}^{(2n+2p+2h)\times(2n+2p+2h\times}$, $S \in \mathbb{R}^{(n+p+h)\times(n+p+h)}$, $L(\theta) \in \mathbb{R}^{h\times(n+h+p)}$ and $H(\theta) \in \mathbb{R}^{h\times(n+h+p)}$ such that:

$$\alpha \otimes (E_{ea}^T P E_{ea}) + \beta \otimes (A_{ea}(\theta) P E_{ea}^T + B_{ea} L(\theta) E_{ea}^T) + I_q \otimes (A_{ea}(\theta) U_{ea} S V_{ea}^T + B_{ea} H(\theta) V_{ea}^T) + \star < 0 \quad (32)$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $Ker(E_{ea})$ and $Ker(E_{ea}^T)$, respectively. Then PDESO gains can be calculated as:

$$\begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix} = (L(\theta)E_{ea}^T + H(\theta)V_{ea}^T)(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1} \quad (33)$$

Theorem 2: The error system of (24) and (25) is quadratically admissible and satisfies $\left\| C_{ze\omega f}(sE_o - A_o(\theta))^{-1}R_a \right\|_2 < \gamma$ if there exist matrices $P \in \mathbb{R}^{(2n+2p+2h)\times(2n+2p+2h\times}$, $P > 0, S \in \mathbb{R}^{(n+p+h)\times(n+p+h)}$, $L(\theta) \in \mathbb{R}^{h\times(n+h+p)}$ and $H(\theta) \in \mathbb{R}^{h\times(n+h+p)}$, such that:

$$\begin{bmatrix} \Delta & R & (PE_{ea}^T + U_{ea}SV_{ea}^T)^T C_{ze\omega f}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (34)$$

with:

$$\Delta = A_{ea}(\theta)(PE_{ea}^T + U_{ea}SV_{ea}^T) + B_{ea}(L(\theta)E_{ea}^T + H(\theta)V_{ea}^T) + \star$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $Ker(E_{ea})$ and $Ker(E_{ea}^T)$, respectively. Then, the PDESO gain can be calculated as:

$$\begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix} = (L(\theta)E_{ea}^T + H(\theta)V_{ea}^T)(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1} \quad (35)$$

Remark 1: Let $\xi = E_a\hat{x}_a - L_d(\theta)(\hat{y} - y)$, then we can obtain an implementation of (23)-(24) as given in the following form:

$$\dot{\xi} = A_a(\theta)\hat{x}_a + B_a u + L_p(\theta)(\hat{y} - y) \quad (36)$$

$$\hat{x}_a = E_o^{-1}(\theta)\xi - E_o^{-1}(\theta)L_d(\theta)y \quad (37)$$

$$\hat{y} = C_a\hat{x}_a \quad (38)$$

where $E_o(\theta) = E_a - L_d(\theta)C_a$. The derivatives of outputs are not appeared in the modified PDESO and only original coefficient matrices are utilized.

Remark 2: Calculating $(E_a - L_d(\theta)C_a)^{-1}$ on line would be a disaster with the increase of matrix dimensions. However, the inversion can be calculated off-line once L_d is parameter independent, which is practical. Inspired by the specific structure of E_{ea}, with the following partitioning:

$$L(\theta) = \begin{bmatrix} L_1(\theta) & L_2(\theta) \end{bmatrix}, P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_4 \end{bmatrix}$$

Then it follows that:

$$L(\theta)E_{ea}^T = \begin{bmatrix} L_1(\theta) & 0 \end{bmatrix}, H(\theta)V_{ea}^T = \begin{bmatrix} 0 & H(\theta) \end{bmatrix}$$

$$PE_{ea}^T = \begin{bmatrix} P_1 & 0 \\ P_2^T & 0 \end{bmatrix}, U_{ea}SV_{ea}^T = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}, \begin{bmatrix} P_1 & 0 \\ P_2^T & S \end{bmatrix}^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -S^{-1}P_2^TP_1^{-1} & S^{-1} \end{bmatrix}$$

$$L(\theta)E_{ea}^T + H(\theta)V_{ea}^T = [L_1(\theta) \quad H(\theta)], (PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -S^{-1}P_2^TP_1^{-1} & S^{-1} \end{bmatrix}$$

Hence, it can be obtained that:

$$L_d^T(\theta) = H(\theta)S^{-1} \tag{39}$$

Setting H to be parameter independent, a constant gain L_d^T can be obtained. Hence, the calculation of $(E + L_dC_a)^{-1}$ can be carried out off-line, therefor the modified PDESO is reliable for practical application.

4 AN INTEGRATEDACTIVE FAULT TOLERANT CONTROL DESIGN APPROACH

4.1 The AFTC structure

The entire AFTC system structure is shown in Figure 1 for the LPV descriptor system of (8) and (9) to handle both sensor and actuator faults. In the structure, sensor faults are hidden by the previously developed PDESO. For actuator faults, the strategy is to compensate for the actuator fault effect using an additional term. Then, based on the estimated fault signals, a controller is designed in the form:

$$u_{FTC} = K(\theta)\hat{x} - K_f\hat{f} \tag{40}$$

where $K(\theta)\,\hat{x}$ is designed to satisfy the nominal performance demand and $K_f\hat{f}$ is used to compensate for the fault influence. Then the closed-loop LPV descriptor system is obtained as:

$$E\dot{x} = A(\theta)x + BK(\theta)\hat{x} - BK_f\hat{f} + F_ff + Rd \tag{41}$$

It is further assumed that:

A4): $rank[B \quad F_f] = rank[B]$

This assumption means that there exists a matrix K_f which satisfies $F_f = BK_f$. That is $K_f = B^{\dagger}F_f$, where $B^{\dagger}$ denotes the generalized inverse (pseudo-inverse) of B.

From a practical point of view, fault estimation and compensation can be a reasonable strategy for fault accommodation, depending on the characteristics of the expected faults. For example, an actuator offset or actuator loss of effectiveness can be considered as a suitable fault scenario for this form of compensation-based AFTC, as considered in a winding machine application in [46], or for a three tank system in [47], or friction compensation in [20], and wind turbine actuator fault compensation in [21]. Hereafter, it is assumed that under certain fault conditions this compensation strategy is reasonable and is hence adopted in this research.

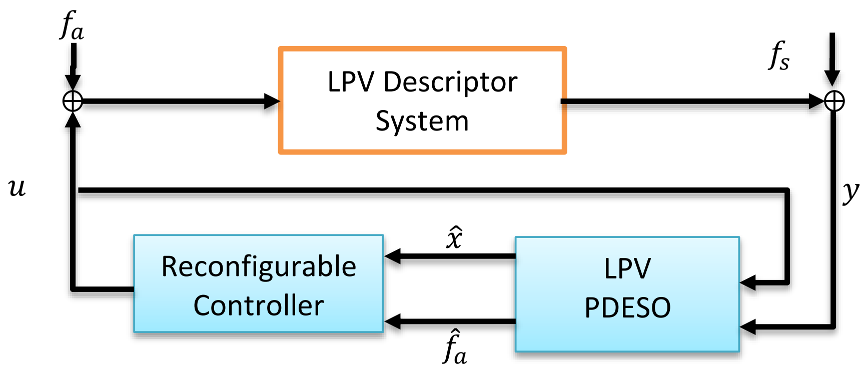

Figure 1: The structure of AFTC for LPV descriptor systems with PDESO

4.2 The integrated design approach

Although the robustness of the closed-loop system can be obtained to some degree if the observer and controller are designed separately, the performance of the closed-loop is not guaranteed and hence performance degradation should be considered [48]. Moreover, attention should be paid to the effect of imperfect fault estimation on the closed-loop stability and/or robustness performances due to estimation delay or disturbances. However, the solution for an integrated design cannot be obtained in all cases.

From an engineering point of view, a two-step procedure was first introduced by [39, 40] to achieve observer-based control designs for uncertain systems. In this approach, the controller gain is designed first via solving a pure LMI set, and then an observer gain is designed based on the obtained controller to achieve global stability and robustness via the solution of a new LMI set. This approach is practical as each step has some connections with the real requirements.

With the strategy proposed in Section 4.1, sate equation of the closed-loop system is transformed to:

$$E\dot{x} = (A(\theta) + BK(\theta))x + BK(\theta)e_x + F_a e_{fa} + Rd \tag{42}$$

where $e_{fa} = f_a - \hat{f}_a, e_x = x - \hat{x}$ are the estimation errors. Combing the error system of (24) and (25), the closed-loop system is obtained as:

$$\mathcal{E}(\theta)\begin{bmatrix}\dot{x}\\ \dot{e}_{x\omega f}\end{bmatrix} = \mathcal{A}(\theta)\begin{bmatrix}x\\ e_{x\omega f}\end{bmatrix} + \mathcal{R}d_D \tag{43}$$

$$z = \mathcal{C}\begin{bmatrix}x\\ e_{x\omega f}\end{bmatrix} \tag{44}$$

where $\mathcal{C} = [C_{zx} \quad C_{ze}]$ is a weighting matrix and:

$$\mathcal{E}(\theta) = \begin{bmatrix} E & 0 \\ 0 & E_o(\theta) \end{bmatrix}, \qquad \mathcal{A}(\theta) = \begin{bmatrix} A(\theta) + BK(\theta) & [BK(\theta) \quad F_f] \\ 0 & A_o(\theta) \end{bmatrix}, \qquad \mathcal{R} = \begin{bmatrix} R \\ R_a \end{bmatrix}$$

The dual system matrices of $(\mathcal{E}(\theta), \mathcal{A}(\theta), \mathcal{C}, \mathcal{R})$ would be $(\mathcal{E}_d(\theta), \mathcal{A}_d(\theta), \mathcal{C}_d, \mathcal{R}_d)$ where:

$$\mathcal{E}_d(\theta) = \begin{bmatrix} E^T & 0 \\ 0 & E_o^T(\theta) \end{bmatrix}, \; \mathcal{A}_d(\theta) = \begin{bmatrix} A^T(\theta) + K^T(\theta)B^T & 0 \\ [BK(\theta) \quad F_f]^T & A_o^T(\theta) \end{bmatrix}, \mathcal{C}_d = \begin{bmatrix} R \\ R_a \end{bmatrix}^T$$

$$\mathcal{R}_d = [C_{zx} \quad C_{ze}]^T$$

By incorporating the design with a descriptor system LPV PDESO structure, an integrated design scheme is proposed to the augmented dual system to achieve the desired performance of the original system. The augmented system can be organized as:

$$\mathcal{E}_{da}\dot{z} = \mathcal{A}_{da}(\theta)\mathrm{z} + \mathcal{R}_{da}d_z \tag{45}$$

$$y_{da} = \mathcal{C}_{da}\mathrm{z} \tag{46}$$

where:

$$\mathcal{E}_{da} = \begin{bmatrix} E^T & 0 \\ 0 & E_{ea} \end{bmatrix}, E_{ea} = \begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{R}_{da} = \begin{bmatrix} C_{zx}^T \\ R_{ea}^T \end{bmatrix}, R_{ea}^T = \begin{bmatrix} C_{ze}^T \\ 0 \end{bmatrix}$$

$$\mathcal{A}_{da}(\theta) = \begin{bmatrix} A^T(\theta) + K^T(\theta)B^T & 0 \\ \Delta(\theta) & A_{ea}(\theta) \end{bmatrix}$$

$$A_{ea}(\theta) = \begin{bmatrix} 0 & I_{n+p+h} \\ A_o^T(\theta) & -E_o^T(\theta) \end{bmatrix}, \Delta(\theta) = \begin{bmatrix} 0 \\ K^T(\theta)B^T \\ F_f^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}$$

$$L_{ea}(\theta) = \begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix}, \mathcal{C}_{da} = [R^T \quad C_R^T], C_R^T = [0 \quad R_a^T]$$

Then the transfer function can be obtained as $G_{da}(\theta, s) = \mathcal{C}_{da}(s\mathcal{E}_{da} - \mathcal{A}_{da}(\theta))^{-1}\mathcal{R}_{da}$. Based on the Bound Real Lemma for a descriptor system given in Section 3.2, the closed-loop system $(\mathcal{E}_{da}, \mathcal{A}_{da}(\theta))$ is admissible for all θ and $\|G_{da}(\theta, s)\|_2 < \gamma$ if there exist $\mathcal{P} > 0, \mathcal{S}$ with compatible dimensions such that:

$$\begin{bmatrix} (\mathcal{P}\mathcal{E}_{da}^T + U_{da}\mathcal{S}V_{da}^T)\mathcal{A}_{da}(\theta) + \star & (\mathcal{P}\mathcal{E}_{da}^T + U_{da}\mathcal{S}V_{da}^T)\mathcal{R}_{da} & \mathcal{C}_{da}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \tag{47}$$

where U_{da} and V_{da} are full column rank and contain the basis vectors for $Ker(\mathcal{E}_{da})$ and $Ker(\mathcal{E}_{da}^T)$, respectively. Noting the specific structure of $\mathcal{E}_{da}$, U_{da} and V_{da} can be specified in the following structure:

$$U_{da} = \begin{bmatrix} U & 0 \\ 0 & U_{ea} \end{bmatrix}, V_{da} = \begin{bmatrix} V & 0 \\ 0 & V_{ea} \end{bmatrix}$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $Ker(E_{ea})$ and $Ker(E_{ea}^T)$, U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$, respectively. Furthermore, by choosing Lyapunov functions with the structure as follows:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \tag{48}$$

Eq. (47) can be re-organized as:

$$\begin{bmatrix} \Delta_{11}(\theta) & \star & \star & \star \\ \Delta(\theta)(P_1E^T + US_1V^T) & \Delta_{22}(\theta) & \star & \star \\ C_{zx} & R_{ea} & -\gamma & \star \\ R^TP_1E^T + R^TUS_1V^T & C_R^TW_2E_{ae}^T + C_R^TU_{ae}S_2V_{ae}^T & 0 & -\gamma \end{bmatrix} < 0 \tag{49}$$

with:

$$\Delta_{11}(\theta) = (A^T(\theta) + K^T(\theta)B^T)(P_1E^T + US_1V^T) + \star$$

$$\Delta(\theta) = [BK(\theta) \quad F_a \quad 0_{n\times(p-q)}]$$

$$\Delta_{22}(\theta) = A_{ea}(\theta)P_2E_{ea}^T + A_{ea}(\theta)U_{ea}S_2V_{ea}^T + B_{ea}Y(\theta)E_{ea}^T + B_{ea}H(\theta)V_{ea}^T + \star$$

Then PDESO gain can be calculated as:

$$L_{ea} = (Y(\theta)E_{ae}^T + H(\theta)V_{ae}^T)(W_2E_{ae}^T + U_{ae}S_2V_{ae}^T)^{-1} \tag{50}$$

It can be seen that Eq. (49) is an LMI if $K(\theta)$ is known. Hence, the two-step procedure [39, 40]for traditional observer based controller design is adapted here to solve the above problem with the following algorithm:

Algorithm 1:

Step 1: Solve state feedback $K(\theta)$according to Lemmas 1 & 2 with common determine variables.

Step2: With obtained $K(\theta)$, solve the following LMI problem: $\min_{P,S,Q,W} \gamma$, subject to (49)

After the PDESO observer gain is obtained, the AFTC can be implemented with the structure depicted in Figure 1.

5 CASE STUDY

In this Section, a numerical example shows the procedure of the integrated design for the descriptor system with an AFTC structure with PDESO and state feedback control, within an LPV framework.

5.1 Case study model

Consider the following descriptor system example modified from [36, 41]:

$$E\dot{x} = A(\theta)x + \mathrm{B}u + F_a f_a + Rd$$

$$y = \mathrm{C}x$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A(\theta) = \begin{bmatrix} \theta & 2 & 1 \\ 1 & -1 & 0 \\ \theta & -\theta & -2-\theta \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = [2 \quad 1 \quad 0]^T, F_a = [2 \quad 1 \quad 0]^T, R = [0.1 \quad 0.1 \quad 0]^T$$

where $\theta \in [-1.5 \quad 1.5]$. In this study, only an actuator fault is considered to illustrate of the design procedure. The disturbance is assumed to be a zero-mean Gaussian distributed band-limited white noise, parameterized with noise power=0.001 and sample time=0.01.

5.2 Controller and observer design

It can be verified that the observability and controllability are satisfied and hence exist a solution for the integrated AFTC scheme.

Step 1: As there is only one varying parameter, it is necessary and sufficient to use two vertices to cover all the possible systems in a convex set. Therefore, two state-feedback gains are calculated as:

$$K_1 = [0.30089 \quad -3.2382 \quad -0.54922];\ K_2 = [-2.6155 \quad -0.9643 \quad -2.8214]$$

In this design, the finite eigenvalues are constrained to satisfy $-5 < Re(\lambda) < -0.5$ to get a suitably fast time response whilst restricting excessively fast responses.

Step2: In this step, the fault signal is augmented twice in order account for more complex possible faults than just constant faults. The finite eigenvalues are constrained to satisfy $-5 < Re(\lambda) < -1.5$. However, the convergence rate can be adjusted to satisfy the time response requirements. Considering the discussion in Section 3.3, a constant derivative gain is designed. With the obtained controller gains in last step, a set of LMIs are solved and the observer gains are calculated as:

$$L_{p1} = \begin{bmatrix} 39.056 & 1.1212 & 90.334 & 0.25934 & 0.68652 \\ -154.14 & -29.329 & -229.38 & -149.75 & -202.34 \end{bmatrix}^T$$

$$L_{p2} = \begin{bmatrix} 37.376 & 1.0861 & 90.196 & 0.0044804 & 0.35453 \\ -153.86 & -29.172 & -226.5 & -149.07 & -201.41 \end{bmatrix}^T$$

$$L_d = \begin{bmatrix} 9.8783 & 0.88448 & 28.82 & -3.437 & -2.7463 \\ -42.219 & -9.5686 & -73.954 & -26.931 & -31.457 \end{bmatrix}^T$$

The obtained robustness performance for the integrated system is $\gamma = 1.2992$.

To show the advantage of proposed integrated design approach, we design another observer gain without considering the influence of state feedback. Based on the approach described in Section 3.3, a PDESO is designed satisfying the same eigenvalue requirements. The PDESO gains obtained are as follows:

$$L_{p1s} = \begin{bmatrix} -60.508 & -32.825 & 7.572 & -171.961 & 225.013 \\ -26.701 & 16.569 & -133.498 & 86.668 & 110.810 \end{bmatrix}^T$$

$$L_{p2s} = \begin{bmatrix} -61.896 & -32.780 & 7.524 & -171.915 & 224.939 \\ -26.710 & 16.610 & -131.182 & 86.852 & 111.024 \end{bmatrix}^T$$

$$L_{ds} = \begin{bmatrix} -20.104 & -10.960 & 1.737 & -41.457 & 42.632 \\ -3.068 & 6.767 & -44.680 & 27.020 & 25.323 \end{bmatrix}^T$$

5.3 For comparison reason, the robustness performance of the overall system with separated designed gains are calculated as $\gamma = 1.5918$. Although the numbers in the gains L_{p1s}, L_{p2s}, L_{ds} are not larger than those in L_{p1}, L_{p2}, L_d, the overall system robustness performance ($\gamma = 1.2992$) with integrated gain design gains is much better than that ($\gamma = 1.5918$) with separate gain designs. Here, there is a suggestion that the integrated design approach may have significant advantages over the approach involving separate designs.Simulation results

A MATLAB/Simulink based simulation is carried out to evaluate the proposed design scheme. The original system is implemented based on an input-output equivalence [36]. The observer is implemented with the equivalence form proposed in Remark 1.

From the simulation results Figures 2 & 3, it can be seen that the proposed LPV AFTC scheme is applicable with a step fault signal injected from 1s. First, both the actuator fault and system states are estimated accurately. Secondly, the system states of the AFTC system converge to zero again after the fault has occurred, as shown in Figure 3.

To illustrate the advantage of the proposed scheme, the two scenarios of whether or not the AFTC is activated are considered where a sinusoid fault signal is considered injected from 0s. The simulation results are given in Figures 4 & 5. It is clear that the AFTC can improve the system performance dramatically. As the system dynamics will influence the observer performance, the fault estimation performs much better after 20s when the AFTC activated, as shown in Figure 4.

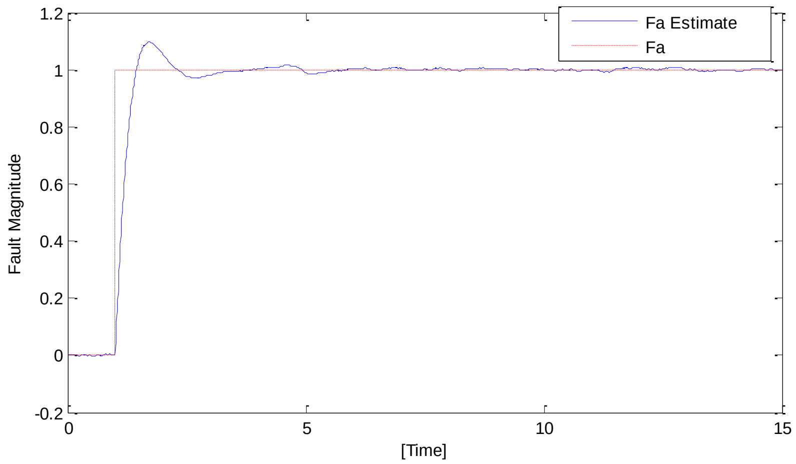

Figure 2: Fault signal f_a and its estimate $\hat{f}_a$ with AFTC activated

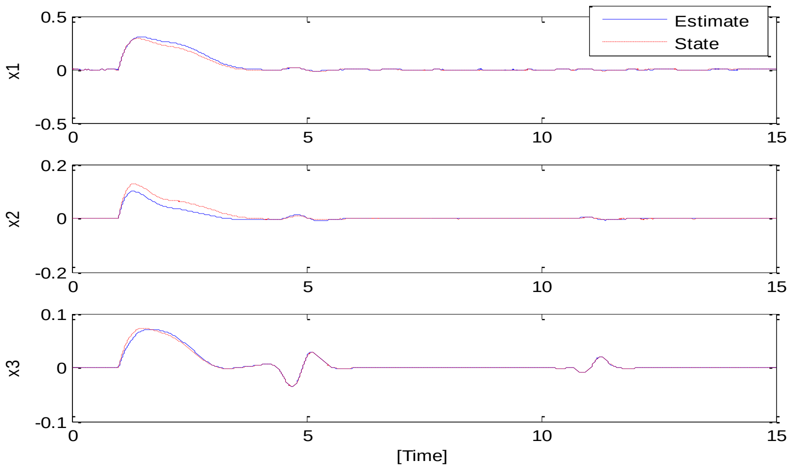

Figure 3: System states and their estimates with the AFTC activated

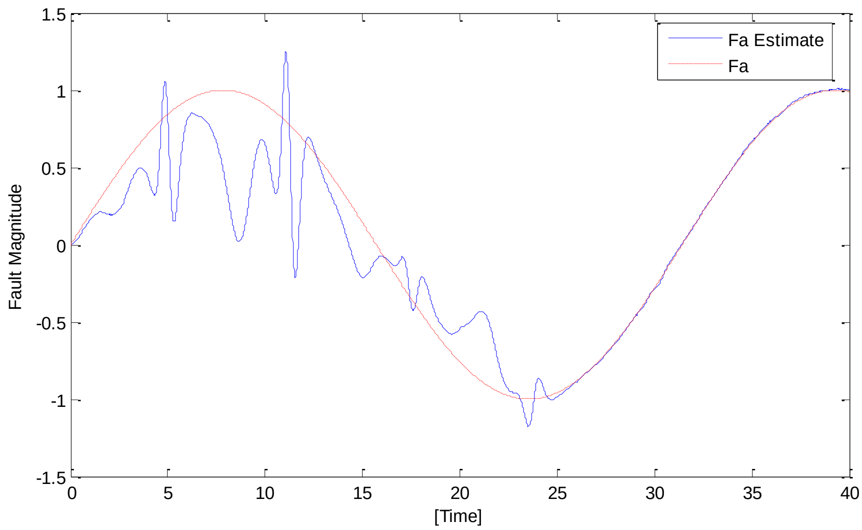

Figure 4: Fault signal f_a and fault estimate $\widehat{f}_a$ with AFTC activated at 20s

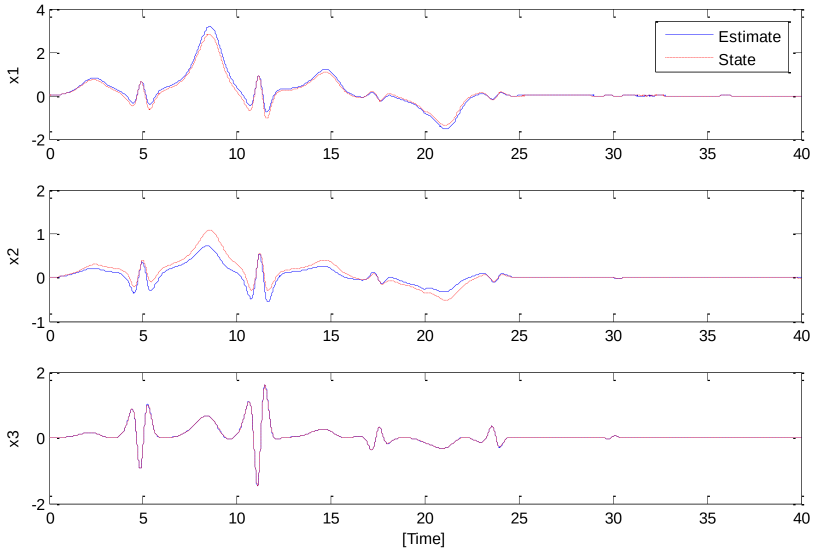

Figure 5: States & their estimates before and after the AFTC activated at 20s

6 CONCLUSION

The paper is focussed on the extension of a classical wind turbine control system operating below rated wind speed. The extension involves an AFTC system structure with the property of fault-tolerance againstFollowing the introduction of some basic concepts for LPV descriptor systems, an approach to PDESO design within an LPV formulation is proposed for linear descriptor systems with parameter variation. An observer based AFTC structure is adopted in the paper. An integrated design algorithm is proposed taking into account the interaction of the two subsystems- the reconfigurable controller and the fault and state observer. Using a two-step design procedure, the overall robustness performance of the AFTC system can be guaranteed within an L_2 framework.

The pole-placement constraints and robustness to disturbance are considered both in the controller and observer designs. The observer poles have been assigned using the LMI approach. Furthermore, the robustness to the exogenous disturbances has also been optimized using the LMI-based L_2 procedure.As these objectives are achieved using LMIs, a combined multi-objective performance is also optimized in the LMI framework. A numerical tutorial example illustrates the design procedure and shows the usefulness of the integrated design.

This work only considers parameter-independent Lyapunov function analysis and design within quadratic admissibility and quadratic L_2 performance. Future work can consider a parameter-dependent Lyapunov function approach as a potential option to reduce the conservatism and improve the AFTC performance even further. Also, real system application studies of the proposed approach will be interesting to explore the capability of this LPV descriptor system strategy.

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