Convergence among nations that share the same preferences and technologies is a key result of the closed-economy neoclassical growth framework that has received substantial support in the data. However, Heckscher-Ohlin versions of the two-sector neoclassical growth model predict that nations that differ in their capital-labor ratios may not converge to the same steady state, even if they are identical in all other aspects. This is a puzzling result that warns us about potential dangers of international trade. In this paper we show that when land, an input in fixed supply, is introduced into the model, international trade in goods no longer limits the capacity of poor nations to catch up with the advanced world.

JEL Classification: O41, F43.
Convergence in a Dynamic Heckscher-Ohlin Model with Land

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Abstract

Convergence among nations that share the same preferences and technologies is a key result of the closed-economy neoclassical growth framework that has received substantial support in the data. However, Heckscher-Ohlin versions of the two-sector neoclassical growth model predict that nations that differ in their capital-labor ratios may not converge to the same steady state, even if they are identical in all other aspects. This is a puzzling result that warns us about potential dangers of international trade. In this paper we show that when land, an input in fixed supply, is introduced into the model, international trade in goods no longer limits the capacity of poor nations to catch up with the advanced world.

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1 Introduction

Convergence across economies, or the lack of it, is a main research agenda in the economic growth literature. The question asked is whether income per capita in poor countries or regions tends to grow faster than in rich ones. The answer to this question has clearly important implications for the expected evolution of welfare levels in developing nations. The closed-economy neoclassical growth framework helps in that direction by offering an important insight. It predicts that economies that have the same fundamentals and only differ in the initial capital-labor ratios will eventually share the same levels of output per capita. This has become the well known conditional convergence hypothesis.\(^1\)

Open economy frameworks, however, challenge the above prediction. Among them, we find the dynamic Heckscher-Ohlin model. First introduced by Oniki and Ozawa (1965) and Bardhan (1965), it represents a widely used setup to analyze trade and growth issues that embeds the standard Heckscher-Ohlin theory of trade into the neoclassical growth model. One of its important predictions is that developing countries that are identical to developed nations in all aspects except for the capital stock level can remain permanently poorer, as Chen (1992) and Atkeson and Kehoe (2000) show. This result, besides being in stark contrast to the predictions of closed economy models, also warns us about the dangers of international trade.

In this paper, we show that the above lack-of-convergence prediction among economies that share the same technologies and preferences heavily depends on the characteristics of production inputs. In particular, when land is introduced into the model, international trade in goods does not limit the capacity of poor nations to catch up with the advanced world.\(^2\)

More specifically, we consider a dynamic Heckscher-Ohlin model similar to Atkeson and Kehoe’s (2000); the only difference is the inclusion of a third input, land. The world is composed of a large number of small open economies that posses identical preferences and production technologies. Individuals are infinitely lived and make decisions so as

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\(^1\)Empirically, the conditional convergence hypothesis has performed surprisingly well. Starting with the seminal work of Barro (1991) and Barro and Sala-i-Martin (1992), it has been tested in many instances, finding important support. In particular, after controlling for measures of education, policies and institutions, poor areas tend to grow faster than rich ones.

\(^2\)Land is an important production input. See Smith (1776), Schultz (1967), Galor and Weil (2000), and Caselli and Coleman (2001), among many others.
to maximize the discounted value of their future utility stream. Each country has
the production structure of the standard two-sector neoclassical growth model with
consumption and investment goods. All firms, regardless of the sector, employ land,
capital and labor, and have different input intensities – thus meaning that each sector
produces with different relative input proportions. The two commodities are traded
internationally, but inputs cannot cross borders.

Countries in the model only differ in the date when they start the development
process. Some of them, which we call early-bloomers, have already reached their steady
states, while others begin to develop, the late-bloomers. Examples of early-bloomers
would include high income nations such as the U.S., the E.U. economies or Japan.
Examples of late-bloomers could be fast growing Asian, African and Latin-American
countries such as China, Mozambique and Brazil. In terms of the model, the distinction
simply implies that late-bloomers possess a capital-labor ratio that is below the steady-
state level of the early-bloomers; this could occur, for example, if late-bloomers adopt
the early-bloomer’s institutions and enjoy a sudden increase in total factor productivity.

It is the property that land is an input in fixed supply that causes our main re-
sult that income levels of identical late-blooming and early-blooming nations always
converge. More specifically, in the more standard two-sector model without land, in-
ternational trade allows a developing nation end up relatively specialized in the less
capital-intensive sector. This outcome occurs because as soon as the economy’s cap-
ital stock is enough to make the return to capital converge to the one of the rest of
the world, the developing nation stops converging. But with the less capital intensive
sector dominating production, the late-blooming country remains permanently poorer.
However, because land is in fixed supply, only one pair of labor allocations and, there-
fore, one capital-labor ratio can accommodate the long-run rental rate on capital in
our framework, thus making differences in steady-state income levels between identical
economies impossible.

Our result then rests on the equalization of steady-state interest rates across economies.
This happens even though the model presents a Heckscher-Ohlin structure with two
goods and three inputs, and so the Factor Price Equalization Theorem (Samuelson
1948 and 1949) does not hold. Two comments are in order here. First of all, factor
price equalization (FPE) is a feature of the steady state because the infinite horizon
structure implies that the interest rate along the balanced growth path is exclusively
pinned down by preferences, which eliminates the degree of freedom in factor prices.
Second of all, outside the steady state, the model does not predict FPE; during the
transition, factor prices vary with the physical capital stock.

Other papers that use multi-sector models of international trade and growth include
Ventura (1997), Mountford (1998), Galor and Mountford (2006, 2008), Guilló and
Perez-Sebastian (2007), and Bajona and Kehoe (2010). Although none of them focus
on whether the timing of development matters for convergence, one of the results in
Bajona and Kehoe is related to our findings. They show that, for a given elasticity
of substitution between traded goods, convergence in relative incomes depends on the
pattern of trade over time. In our model two small economies that have the same land
endowment will end up having the same long run income independently of their trade
patterns along the adjustment path.

The paper proceeds as follows. Section 2 presents the model. Sections 3 describes
the steady state in which the early bloomers are located, and the development path
followed by a later-blooming nation. The main question posed by the paper – does the
timing of development matter? – is addressed in section 4. Section 5 concludes.

2 The Economy

Consider a world economy consisting of a large number of small open economies that
share the same preferences and production technologies, but can differ in the input
endowment. There are two goods, and three factors of production. As in Oniki and
Ozawa (1965), the economy manufactures consumption and investment goods. Pro-
duction of these goods needs capital, labor and land inputs, which can freely move
across sectors. For concreteness, we assume that consumption-goods production is less
capital intensive and more land intensive than the investment sector.\footnote{This is
without loss of generality. It can be shown that our main results also hold if consumption
goods are less land intensive and/or more capital intensive than investment goods.} There is free
trade in goods, but international movements of inputs are prohibited. All markets are
perfectly competitive. Population is constant and its size equals $L$ in each nation.

2.1 Consumers

The economy is populated by Infinitely-lived agents that discount future utility with the
factor $\rho$, and have preferences only over consumption. In particular, their preferences
are given by
\[ \sum_{t=0}^{\infty} \rho^t \left( \frac{c_t^{1-\sigma} - 1}{1-\sigma} \right), \quad \rho \in (0, 1), \quad \sigma > 0. \]  

(1)

Individuals offer labor services and rent capital and land to firms. The total amount of land in the economy is fixed over time, equals \( N \), and is uniformly distributed across all individuals. Since in each period international trade must be balanced, consumers in each country face the following budget constraint
\[ c_t + p_t x_t = r_{kt} k_t + r_{nt} n_t + w_t, \]  

(2)

where the evolution of capital is governed by
\[ k_{t+1} = (1 - \delta) k_t + x_t. \]  

(3)

In the above expressions, \( c_t \) is the per capita demand for consumption goods; \( x_t \) is the per capita demand for investment goods; \( r_{kt}, r_{nt}, \) and \( w_t \) are the rental rates on capital, land, and labor, respectively; \( n_t \) and \( k_t \) denote the amount of the natural input and capital owned by the individual at date \( t \). The consumption good is the numeraire, and \( p_t \) gives the price of the investment product.

Consumers in each country will maximize (1) subject to (2) and (3), taking as given the world output prices and the domestic rental rates for production factors. The Euler equation corresponding to this dynamic programming problem is standard:
\[ \frac{c_{t+1}}{c_t} = \left[ \frac{p_{t+1}}{p_t} \frac{r_{kt+1}}{r_{kt+1} + 1 - \delta} \right]^{1/\sigma}. \]  

(4)

It says that the growth rate of consumption depends on the present-utility value of the rate of return to saving. This return reflects that giving up a unit of present consumption allows buying \( 1/p_t \) units of the investment good today that, after contributing to the production process, will convert themselves tomorrow in \( (1 + r_{kt+1}/p_{t+1} - \delta) \) units that can be sold at a price \( p_{t+1} \).

2.2 Firms

In each nation, a sufficiently large number of profit-maximizing firms manufacture the consumption good \( (Y_{ct}) \) and the investment good \( (Y_{xt}) \) employing the following technologies:
\[ Y_{ct} = AK_{ct}^\alpha N_{ct}^\beta L_{ct}^{1-\alpha-\beta} = AL_{ct}^\alpha k_{ct}^\beta n_{ct}, \quad \alpha, \beta \in (0, 1). \]  

(5)
\[ Y_{xt} = BK_{xt}^{\theta}N_{xt}^{\gamma}L_{xt}^{1-\theta} = BL_{xt}^{\theta}k_{xt}^{\theta}n_{xt}^{\gamma}, \quad \theta, \gamma \in (0, 1). \]  

Above, \( K_{it}, N_{it} \) and \( L_{it} \) denote, respectively, the amount of capital and labor devoted in period \( t \) to the production of good \( i \); and \( n_{it} = N_{it}/L_{it} \), \( k_{it} = K_{it}/L_{it} \), for all \( i = c, x \). Production of \( c \)-goods is less capital intensive but more land intensive. Which means that \( c \)-products are manufactured employing a lower capital-labor ratio but a higher land-labor ratio than investment goods. In order to ensure this asymmetry in factor proportions, we need to assume that \( \theta (1 - \beta) > \alpha (1 - \gamma) \) and \( \gamma (1 - \alpha) < \beta (1 - \theta) \).

Firms in each country will maximize profits taking as given world prices and the domestic rental rates on production factors. From the production functions (5) and (6), production efficiency implies that

\[ r_{kt} = \alpha A k_{ct}^{-\alpha} n_{ct}^{\beta} = p_t \theta B k_{xt}^{-\theta} n_{xt}^{\gamma}, \tag{7} \]
\[ r_{nt} = \beta A k_{ct}^{\alpha} n_{ct}^{-\beta} = \gamma p_t B k_{xt}^{-\theta} n_{xt}^{-\gamma}, \tag{8} \]
\[ w_t = (1 - \alpha - \beta) A k_{ct}^{\alpha} n_{ct}^{\beta} = (1 - \theta - \gamma) p_t B k_{xt}^{\theta} n_{xt}^{\gamma}. \tag{9} \]

Of course, these equalities will hold only for the technologies that coexist in equilibrium. More specifically, domestic firms will produce both goods if the international price \( p_t \) is in the interval \( (p_{t}^{\text{min}}, p_{t}^{\text{max}}) \). If investment goods are too cheap, \( p_t \leq p_{t}^{\text{min}} \), the economy will specialize in the production of consumption goods; whereas if \( x \)-products are sufficiently expensive, \( p_t \geq p_{t}^{\text{max}} \), consumption goods will not be manufactured. It is easy to show that the minimum and maximum price that define this diversification interval are, respectively:

\[ p_{t}^{\text{min}} = A \left( \frac{\alpha}{\theta} \right) \left( \frac{\beta}{\gamma} \right) \left( \frac{1 - \alpha - \beta}{1 - \theta - \gamma} \right)^{1-\theta-\gamma} k_t^{\alpha-\theta} n_t^{\beta-\gamma} \tag{10} \]
\[ p_{t}^{\text{max}} = A \left( \frac{\alpha}{\theta} \right) \left( \frac{\beta}{\gamma} \right) \left( \frac{1 - \alpha - \beta}{1 - \theta - \gamma} \right)^{1-\alpha-\beta} k_t^{\alpha-\theta} n_t^{\beta-\gamma} \tag{11} \]

These threshold prices depend on the nation’s per capita endowments of capital and land; hence, we can write them as \( p_{t}^{\text{min}}(k_t; n_t) \) and \( p_{t}^{\text{max}}(k_t; n_t) \).

Let us focus on the diversified-production equilibrium, and define the relative factor prices \( \omega_{kt} = w_t/r_{kt} \) and \( \omega_{nt} = w_t/r_{nt} \). The efficiency conditions in production (7) and (9) determine the optimal allocations of capital as a function of the relative factor

\[ \text{Equation (10) [(11)] is obtained forcing profits of } x \text{-goods } [c \text{-goods}] \text{ firms to be positive when input prices are given by optimality conditions (7) to (9) for the } c \text{-goods } [x \text{-goods}]. \]
price:

\begin{align*}
k_{xt} &= \left(\frac{\theta}{1 - \theta - \gamma}\right)\omega_{kt}, \quad (12) \\
k_{ct} &= \left(\frac{\alpha}{1 - \alpha - \beta}\right)\omega_{kt}. \quad (13)
\end{align*}

Similarly, (8) and (9) imply that

\begin{align*}
n_{xt} &= \left(\frac{\gamma}{1 - \theta - \gamma}\right)\omega_{nt}, \quad (14) \\
n_{ct} &= \left(\frac{\beta}{1 - \alpha - \beta}\right)\omega_{nt}. \quad (15)
\end{align*}

Hence, the assumptions that \( \theta (1 - \beta) > \alpha (1 - \gamma) \) and \( \gamma (1 - \alpha) < \beta (1 - \theta) \) ensure that \( k_{xt} > k_{ct} \) and \( n_{xt} < n_{ct} \), respectively.

2.3 Trade, market clearing and income per capita

Define \( E_{zt} \) as net exports of \( z \)-goods at date \( t \). Commodity trade between countries is free, and requires that in each economy the following equalities hold:

\[ Y_{ct} = c_{t}L + E_{ct}, \quad (16) \]

and

\[ Y_{xt} = x_{t}L + E_{xt}. \quad (17) \]

In addition, balanced trade implies that the (absolute) value of net exports must be the same for the two goods but with opposite signs; that is,

\[ E_{ct} = -p_{t}E_{xt}. \quad (18) \]

Let us denote the labor share in the production of good \( i \) by \( l_{it} = L_{it}/L \). Notice that because consumers are alike, the amount of capital and land owned by each individual will equal the country’s capital-labor and land-labor ratios, respectively. Hence, the constraints on labor, capital and land within a region can be written as follows:

\begin{align*}
l_{ct} + l_{xt} &= 1, \quad (19) \\
l_{ct}k_{ct} + l_{xt}k_{xt} &= k_{t}, \quad (20) \\
l_{ct}n_{ct} + l_{xt}n_{xt} &= n \quad (21)
\end{align*}
We are now ready to derive two main expressions in our analyses. Both refer to economies that show diversified production, that is, economies for which \( p_t \in (p_t^{\min}, p_t^{\max}) \). Using technologies (5) and (6), along with FOC (9) and market clearing condition (19), we can write a nation’s GDP level per capita as

\[
y_t = l_{ct}y_{ct} + p_l l_{xt}y_{xt} = \frac{u_l}{1-\theta-\gamma} \left[ 1 + l_{ct} \left( \frac{\alpha + \beta - \theta - \gamma}{1-\alpha-\beta} \right) \right];
\]

where \( y_{ct} = Y_{ct}/L_{ct} \), and \( y_{xt} = Y_{xt}/L_{xt} \). Conditions (7), and (12) to (15), in turn, provide the next expression for the price of investment goods:

\[
p_t = \frac{A}{B} \left( \frac{\alpha}{\theta} \right)^{\alpha} \left( \frac{\beta}{\gamma} \right)^{\beta} \left( \frac{1-\alpha-\beta}{1-\theta-\gamma} \right)^{1-\alpha-\beta} k_{xt}^{\alpha-\theta} n_{xt}^{\beta-\gamma}.
\]

Notice that all the above expressions apply to all economies, regardless of their level of development. Next, we describe the equilibrium focusing first on the early-bloomers and then on the late-bloomers.

### 3 Early- and Late-Bloomers

We consider two types of economies: early- and late-bloomers. The former label corresponds to advanced nations that have already reached the steady state. Late-bloomers, in turn, denote underdeveloped countries that possess a capital labor ratio well below the steady-state level; this could be, for example, a consequence of an increase in total factor productivity due to the adoption of the early-bloomer’s institutions and policies.

To be more precise, suppose that all but one of our small-open countries have already reached the steady-state. Assume as well that all these early-blooming countries share the same endowments. In equilibrium, identical countries make the same choices. So the equilibrium for these economies will be the same as the equilibrium for a single large and closed economy, and it will not be affected by the behavior of the small (still developing) country. Therefore, their net exports equal zero and goods-market clearing conditions (16) and (17) become

\[
c_t = Al_{ct}k_{ct}^{\alpha} n_{ct}^{\beta},
\]

\[
x_t = Bl_{xt}k_{xt}^{\theta} n_{xt}^{\gamma}.
\]

On the steady-state equilibrium, early-bloomers diversify production, and variables in per capita terms, relative employment of inputs and prices will remain invariant.
Denoting by an asterisk (*) steady-state outcomes, the consumers’ optimality condition (4) implies that the interest rate in terms of investment goods at steady state is exclusively pinned down by consumers’ preferences:

\[ r^*_k = p^* (\rho^{-1} + \delta - 1) \]  

(26)

In addition, the appendix shows that the relative price is given by

\[ p^* = \frac{A}{B} \left( \frac{\alpha}{\theta} \right)^{\beta} \left( \frac{1}{1 - \theta - \gamma} \right)^{1 - \alpha - \beta} \left( \frac{1}{1 - \gamma} \right)^{1 - \alpha - \beta} \frac{2 - \delta}{1 - \theta} \frac{n}{1 - \theta} \right] \]  

(27)

The result is quite intuitive. As \( c \) production becomes more profitable than \( x \) manufacturing because land is more abundant, the economy devotes relatively more resources to the production of consumption goods, making investment products relatively scarcer and, as a consequence, more expensive. It must be the case that \( p^* = p^{\min}(k^*_c; n^*_c) = p^{\max}(k^*_x; n^*_x) \) and that \( n^*_c > n^*_x \) and \( k^*_c < k^*_x \) by assumption. These conditions will prove useful later in Section 4.5

After describing the equilibrium in which advanced nations are located, we turn to the late-bloomer. Consider the other small nation with the same land endowment as the early bloomers that starts developing later with an initial capital stock \( k_0 < \min \{k^*_c, k^*_x\} \), and is still moving along the adjustment path. This developing country faces the steady-state relative output price obtained above for the developed world – i.e., \( p_t = p^* \) for all \( t \). From here on, the asterisk (*) denotes the international diversified-production equilibrium for the early-bloomers, whereas we remove the time subscript to denote the steady state values for the less developed country.

Along the adjustment path, the small developing country will accumulate capital until its domestic rate of return falls down to the rate \( r^*_k \), the one that prevails in the rest of the world, because preferences are the same across nations.6 At the same time, capital accumulation will bring an increase in the prices of the other two inputs, land and labor. The pattern of production along the transition will be, in turn, determined by expressions (10) and (11). In the steady-state diversified production equilibrium,  

5An expression for \( y^* \) can be obtained from (22) using (19), (9), (29), (30) and (32).

6In numerical analysis of the model, we have found that, for a wide set of reasonable parameter values, its transition is characterized by a one-dimensional stable saddle-path that implies that the adjustment path is asymptotically stable and unique.
equations (7) to (9), and (26) imply that the long-run capital-labor ratio in the investment sector will equal the one of the rest of the world, \( k_x = k_x^* \). This equality and expressions (12) to (15) and (26) guarantee that, in the long run, international factor-price equalization holds, and that the country will be using the same techniques as the rest of the developed nations; that is, \( k_c = k_c^* \), \( w = w^* \), \( n_c = n_c^* \), \( n_x = n_x^* \) and \( r_n = r_n^* \).

4 Does the Timing of Development Matter?

Now, we answer the key question of whether the time when an economy starts its development path towards the steady state affects its long-run performance. Put differently, we ask whether the initial relative capital stock is a determinant of the long-run income level in the model with land. For that purpose, it is interesting to briefly recall why late-blooming nations can remain permanently poor in the more standard dynamic Heckscher-Ohlin model.\(^7\)

Suppose that land is not present in the model, that is, \( \beta = \gamma = 0 \). Because \( k_0 < \min\{k_c^*, k_x^*\} \), the developing nation starts its development path specialized in \( c \)-goods production. From (5), this means that output per capita is given by \( y_t = Ak_t^{\alpha} \). As explained above, the economy will continue accumulating capital until \( r_k t = r_k^* \), which is pinned down by preferences. Notice that this equality will hold as soon as \( k = k_c^* \), and that for this capital stock the specialization condition \( p^* \leq p^\text{min}(k) \) will hold because \( p^* = p^\text{min}(k_c^*) \). Hence, \( k < k^* \in (k_c^*, k_x^*) \), and \( y < y^* \).

To fully understand the reason, consider a late-bloomer that starts its development path diversifying production with \( k_0 \in (k_c^*, k_x^*) \). This economy is already in long-run equilibrium. It just needs to choose \( l_x \) such that \( (1 - l_x)k_c^* + l_x k_x^* = k_0 \), and the return to capital already equals the one of the rest of the world. But if \( k_0 < k^* \) then \( y < y^* \); that is, the late bloomer remains permanently poorer.

Let us go back to our case, and consider strictly positive values of \( \beta \) and \( \gamma \). In the long-run, all economies regardless of their timing of development will face the same equilibrium price \( p^* \) and the same interest rate \( r_k^* \). Suppose first that the late-bloomer is diversifying production at the steady state. The late-bloomer will end up using the same production techniques as the rest of the world, \( k_i = k_i^* \) and \( n_i = n_i^* \) \((i = c, x)\). In addition, because the developing economy has the same land endowment

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\(^7\)See Chen (1992) and Atkeson and Kehoe (2000) for additional details.
as early-blooming nations, it will have the same long-run labor allocation, $l_c = l'_c$, by equilibrium conditions (19) and (21), and the same long-run income $y$ by (22).

The only scenario in which the developing economy can remain permanently poorer is, therefore, long-run specialization. Because $k_0 < \min \{k^*_c, k^*_x\}$, the developing nation starts its development path specialized in $c$-goods. Long-run international interest-rate equalization means that $r_k = r^*_k$, and then $k = k^*_c(n/n^*_c)^{\beta/(1-\alpha)}$ by (7). The issue is whether this capital stock is consistent with the specialization condition $p^* \leq p_{\text{min}}$. Remember that we can write $p^* = p^\text{min}(k^*_c; n^*_c)$ by (10). Hence, the last inequality becomes $p^\text{min}(k^*_c; n^*_c) \leq p^\text{min}(k; n)$, which implies that $k \leq k^*_c(n/n^*_c)^{(\beta-\gamma)/(\theta-\alpha)}$. The assumption that $c$-goods are more land intensive, in turn, implies that $n_{ct} > n_{xt}$ for all $t$; as a result, $n/n^*_c < 1$. Simple algebra, then, leads to the conclusion that the two conditions for $k$ are compatible if and only if $\beta (1-\theta) \leq \gamma (1-\alpha)$. The same assumption requires, however, that $\beta (1-\theta) > \gamma (1-\alpha)$, which is a contradiction. Therefore, the developing economy always enters the diversification cone before arriving at long-run equilibrium and, because of that, converges to the income levels of the identical early bloomers. All you need is that both types of nations that differ only in the timing of development face the same prices at steady state.

5 Conclusion

The closed-economy neoclassical growth model predicts that welfare levels in nations that share the same technologies and preferences should converge, and that the initial physical capital stock should not be an important determinant of long-run income levels. The dynamic two-sector Heckscher-Ohlin model, however, can challenge this prediction: a late-blooming nation that starts its development process with a relatively low capital stock can end up permanently poorer. We have shown that when land is introduced in the latter framework two small open economies with identical land-labor endowments will converge to the same long-run income independently of their timing of development.

It is the property that land is in fixed supply that drives this result. The infinite horizon framework of international trade and economic growth implies that the return on capital in poor and rich countries is equalized at steady state. When land is not present, this makes possible that a late-blooming economy with a lower capital stock than the rest of the world reaches a long-run equilibrium in which it remains perma-
nently poorer. The reason is that it can use the world-wide optimal techniques by simply allocating more labor than rich nations to the less capital intensive activity. Adding land as a factor of production, however, eliminates this possibility: a fixed supply of land implies that along the balanced-growth path identical economies have to allocate the same amount of labor across activities.
References


A Obtaining expression (27)

From (19), (20), (12) and (13), it is possible to write \( k_t \) as a function of \( k_{xt} \) and \( l_{xt} \):

\[
k_t = k_{xt} \left[ (1 - l_{xt}) \frac{(1 - \theta - \gamma)\alpha}{\theta(1 - \alpha - \beta)} + l_{xt} \right]. \tag{28}
\]

From (19), (21), (14) and (15) it follows that

\[
n = n_{xt} \left[ (1 - l_{xt}) \frac{(1 - \theta - \gamma)\beta}{\gamma(1 - \alpha - \beta)} + l_{xt} \right]. \tag{29}
\]

It is also possible relating \( n_{xt} \) and \( k_{xt} \). In particular, equation (7) implies that

\[
n_{xt} = \left[ \frac{r_{kt} k_{xt}^{(1-\theta)}}{p_t B B} \right]^{1/\gamma}. \tag{30}
\]

Using (25) and (7), we can write output of investment goods as a function of the interest rate and capital, the resulting expression for investment output must satisfy the steady state condition of (3):

\[
\delta k^* = \frac{r^* k^*}{p^* \theta} l^*_x k^*_x. \tag{31}
\]

Then using (28) we can solve for the steady state labor allocation in the investment sector:

\[
l^*_x = \frac{\delta (1 - \theta - \gamma) \alpha}{(1 - \alpha - \beta) (\rho^{-1} - 1) + \delta ((1 - \beta) (1 - \theta) - \alpha \gamma)}. \tag{32}
\]

Substituting this result into (29) we can solve for \( n^*_x \), which yields from (29) and (26) the value of \( k^*_x \). Finally, taking the resulting expressions for \( n^*_x \) and \( k^*_x \) into (23) give (27).
Appendix only for referees

Getting $p_t^{\text{min}}$ and $p_t^{\text{max}}$ Suppose production of consumption goods is positive. Profits of investment products are equal to

$$\Pi_{xt} = p_t B K_{xt}^\theta (E_t L_{xt})^\gamma (E_t N_{xt})^{1-\theta-\gamma} - r_{kt} K_{xt} - \bar{r}_{nt} (E_t N_{xt}) - \bar{w}_t (E_t L_{xt}).$$

At the maximum manufacturing profits are

$$(p_t B)^{\frac{1}{\theta-\gamma}} \left( \frac{\theta}{r_{kt}} \right)^{\frac{\theta}{\theta-\gamma}} \left( \frac{\gamma}{\bar{r}_{nt}} \right)^{\frac{\gamma}{\theta-\gamma}} (E_t L_{xt}) \left[ (1 - \theta - \gamma) - \bar{w}_t \left( \frac{\left( \frac{\theta}{\gamma} \right)^\theta (\frac{\bar{r}_{nt}}{\gamma})^\gamma}{p_t B} \right)^{\frac{1}{\theta-\gamma}} \right]$$

(33)

So domestic firms will enter the market of manufactures if and only if profits are positive:

$$p_t B > \left( \frac{\bar{w}_t}{1 - \theta - \gamma} \right)^{1-\theta-\gamma} \left( \frac{r_{kt}}{\theta} \right)^\theta \left( \frac{\bar{r}_{nt}}{\gamma} \right)^\gamma$$

(34)

Getting the equilibrium prices from the optimality conditions for agricultural goods given in (7), (8) and (9), we obtain expression (10).

Suppose now that production of $x$-goods is positive. Following the same steps, it follows that domestic firms will enter the market of $c$-products if and only if profits are positive

$$A > \left( \frac{\bar{w}_t}{1 - \alpha - \beta} \right)^{1-\alpha-\beta} \left( \frac{r_{kt}}{\alpha} \right)^\alpha \left( \frac{\bar{r}_{nt}}{\beta} \right)^\beta$$

(35)

Getting the equilibrium prices from the optimality conditions for manufactures given in (7), (8) and (9), and changing the direction of inequality, we obtain expression (11).