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Integrated Fault Estimation and Fault Tolerant Control: A Joint Design

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Abstract: Over the past decades, substantial progress has been made in fault detection and diagnosis (FDD) and fault tolerant control (FTC) research. Recently, some attention has been paid to the integrated design of fault estimation (FE) and active FTC in a two-stage manner: estimation first and then compensation. Different from the preceding two-stage work, this paper deals with the joint design problem of FE and active FTC in an attempt to resolve them simultaneously. The joint design is accomplished by borrowing the H_2, H_∞, and mixed H_2/H_∞ concepts in the robust control field to quantitatively evaluate the fault tolerant performance. Necessary and sufficient conditions for the existence of a joint solution are formulated in the linear matrix inequality (LMI) language. A numerical tutorial example is used to illustrate the effectiveness of the proposed method.

Keywords: Fault Diagnosis, Fault Estimation, Active Fault Tolerant Control, Linear Matrix Inequality

1. INTRODUCTION

Fault detection and diagnosis (FDD) and fault tolerant control (FTC) are two closely interrelated subjects in the context of accommodating a variety of faults in industrial systems. In particular, active FTC (Patton (1997) and Zhang and Jiang (2008)) has been drawing an increasing attention in academia, industry, and government agencies. The FDD unit delivers to the FTC module valuable diagnostic information that is needed to reconfigure dynamic system operation acceptably when the system encounters a failure or multiple faults.

Researchers got used to address FDD and FTC isolatedly in earlier active FTC research, since it was often assumed that the FDD task has been perfectly accomplished. The focus of active FTC at that time was on control reconfiguration. Few attempts have been made to investigate the interconnection between FDD and FTC. Until recently, some emphasis has been laid on integrated design of fault detection and normal feedback control (under the health condition). For instance, Wang and Yang (2009) studied for linear parameter varying (LPV) systems the integrated design of a dynamic (detector/controller) system that can generate detection residual and control input simultaneously. Interested readers can refer to the survey paper by Ding (2009) and the references therein for more information. It should be noted that these so-called integrated research papers are usually concerned with residual generation and normal control law design.

On the other hand, some attempts have been made to conduct fault estimation (FE) for the purpose of compensating the adverse effect of system faults on control performance directly (e.g., see Odgaard et al. (2006)). Zhang et al. (2010) explored the integrated design issue of FE and dynamic output feedback (DOF) FTC for Lipschitz nonlinear discrete-time systems. Later, Sami and Patton (2013) addressed actuator and sensor FE and DOF active FTC for Takagi-Sugeno continuous-time systems in an integrated manner. Recently, using linear matrix inequality (LMI) technique, Tabatabaeipour and Bak (2014) presented an integrated solution to actuator FE and state feedback FTC for piecewise linear systems by minimizing the suggested input-to-state stability gain. In the meantime, Shi and Patton (2014) studied for LPV descriptor systems the integrated issue of proportional derivative extended state (system states and sensor and actuator faults) estimation and active FTC at two steps. However, in these pioneering investigations, FE and active FTC are carried out not simultaneously but separately in a two-stage way: estimation first and then compensation. In addition, the assumption in previous work that the coefficient matrices of control inputs and actuation faults are colinear or "matched" (i.e., $rank([B\ E]) = rank(B)$, see Section 2) makes it nontrivial to compensate the estimated faults when using existing two-stage approaches for the noncolinear case.

In order to avoid a confusion with the preceding two-stage integrated design, we introduce the concept of the "joint" design problem to indicate that FE and active FTC are resolved simultaneously. *Fault tolerant robustness, the ability of a system to retain its normal operation performance in the presence of faults and the extent to*

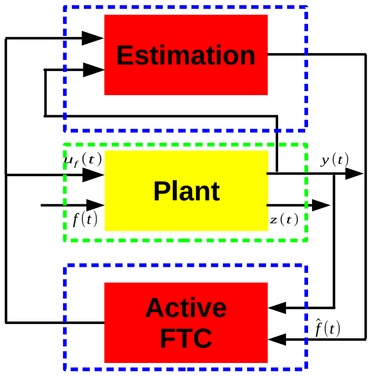

Fig. 1. Schematic structure of the joint design

which the impact of the interaction between FE error and active FTC capability can be decoupled, is explicitly used to address this issue through the H_2, H_∞, and mixed H_2/H_∞ concepts in the robust control field. Figure 1 illustrates the schematic structure for the joint FE/FTC issue. To the best of our knowledge, it is still unclear how to devise for a dynamic system FE and active FTC jointly, with taking into consideration their possible interaction and imperfect compensation. It is this fact that motivates us to perform this research. The benefit of the joint design is to provide an opportunity to address FE and active FTC in a unified framework and quantitatively consider the interaction between FE and FTC in a systematic way.

The remainder of this paper is organized in the following manner. First of all, the problem of interest is briefly formulated in Section 2. Next, Section 3 describes the details of our solution to the joint design problem. Then in Section 4, a numerical example is used to illustrate the effectiveness of the proposed method. Finally, this paper is concluded in Section 5.

2. PROBLEM FORMULATION

Consider the linear continuous-time dynamic model:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \tag{1}$$
$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the vector of system states, $u(t) \in \mathbb{R}^p$ is the vector of control inputs, $f(t) \in \mathbb{R}^p$ is the vector of unmatched actuating faults (E is not colinear with B, i.e., $rank([B\ E]) \neq rank(B)$), and $y(t) \in \mathbb{R}^q$ is the vector of measured outputs. Here the matrices A, B, C, and E are of compatible dimensions. Although only actuator faults are considered, the inclusion of some auxiliary states (Sami and Patton (2013)) can generate an augmented model that is of the form (1)–(2) for the sensor fault case. The focus of this paper is thus on actuator faults.

Our objective is to seek a mechanism to simultaneously design a fault estimator and a fault tolerant controller in an integrated framework. It should be pointed out that the requirement of the noncolinearity between E and B does not mean that the proposed joint design in the subsequent section is only suited for unmatched faults. Instead, the joint design is valid as well for the model with matched faults (i.e., $rank([B\ E] = rank(B))$, although much simpler solutions exist for the matched case, e.g., by using sliding mode control (Edwards and Spurgeon (1998)) without the help of FE.

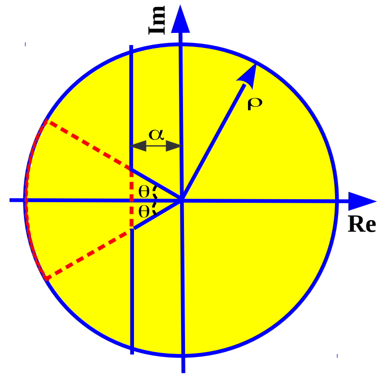

Fig. 2. Convex LMI region enclosed by the dashed line

3. SUGGESTED METHOD

3.1 Fault Estimator Structure

For the ease of mathematical manipulation, it is temporarily assumed that $f(t)$ is slowly varying, *i.e.*, $\dot{f}(t) \approx 0$. Let $\hat{A} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\hat{C} = [C\ 0]$, and $\eta(t) = \begin{bmatrix} x(t) \\ f(t) \end{bmatrix}$. Under this circumstance, a fault-augmented model can thus be constructed:

$$\dot{\eta}(t) = \hat{A}\eta(t) + \hat{B}u(t) \tag{3}$$
$$y(t) = \hat{C}\eta(t) \tag{4}$$

As far as this virtual system in (3)–(4) is concerned, a Lunberger observer can be introduced of the form:

$$\dot{\hat{\eta}}(t) = \hat{A}\hat{\eta}(t) + \hat{B}u(t) + L(y(t) - \hat{y}(t)) \tag{5}$$
$$\hat{y}(t) = \hat{C}\hat{\eta}(t) \tag{6}$$

where $\hat{\eta}(t)$ and $\hat{y}(t)$ represent the estimates of $\eta(t)$ and $y(t)$, respectively. In this paper, L is of great interest. Here we require that the observer poles reside in a convex-shaped LMI region $\Xi(\rho, \alpha, \theta)$ (Chilali and Gahinet (1996)) enclosed by the dashed line in Fig. 2.

3.2 Fault Tolerant Tracking Controller Structure

In the practical fault tolerant control, it is highly desirable that the tracked measurement variables will not be affected too much when some fault arises in the system. In this paper, the tracked variables are denoted as $y_r(t) \in \mathbb{R}^l$. Suppose that $y_r(t)$ can be mathematically expressed as $y_r(t) = S_c y(t)$, where S_c is a matrix for component extraction from $y(t)$. In order to eliminate steady tracking error, the integral of tracking error is defined as $e_I(t) = \int_0^t (r(\tau) - y_r(\tau))d\tau$, where $r(\tau)$ is the vector of reference command signals. The interested fault tolerant tracking controller is of the form: $u_f(t) = [K_1\ K_2\ K_3]\left[x^T(t)\ f^T(t)\ e_I^T(t)\right]^T$. Let $\bar{A} = \begin{bmatrix} A & E & 0 \\ 0 & 0 & 0 \\ -S_r C & 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$, $\bar{E} = \begin{bmatrix} 0 \\ 0 \\ I_l \end{bmatrix}$, $\bar{C} = [0\ 0\ I_l]$, $K = [K_1\ K_2\ K_3]$, and $\zeta(t) = \begin{bmatrix} \eta(t) \\ e_I(t) \end{bmatrix}$. From the fault tolerant tracking perspective, if $e_I(t)$ is chosen as the performance evaluation signal, then we can easily construct the following model:

$$\dot{\zeta}(t) = \bar{A}\zeta(t) + \bar{B}u_f(t) + \bar{E}r(t) \tag{7}$$
$$z(t) = \bar{C}\zeta(t) \tag{8}$$
$$u_f(t) = K\zeta(t), \tag{9}$$

where I_l represents the $l \times l$ identity matrix. The fault tolerant controller has a state-feedback form and takes into account potential occurring faults and tracking errors in a straightforward manner.

3.3 Fault Estimator and Fault Tolerant Controller: A Joint Design

In this paper, the values of L and K are needed to be solved in a joint way. Using the well-known H_2, H_∞, and mixed H_2/H_∞ techniques, this paper proposes three integrated design mechanisms to accommodate possibly occurring faults of the system described by (1)–(2) in the beginning of control system design.

LMI Formulation of H_2 Joint Design

Definition 1. The joint design for the system (1)–(2) of a fault estimator (5)–(6) whose poles reside in the convex region $\Xi(\rho, \alpha, \theta)$ in Fig. 2 and a fault tolerant controller (9) such that $\|T_{zr}\|_2 < \gamma_2$ is called an H_2 FE/FTC joint design problem, where $\|T_{zr}\|_2$ denotes the H_2 norm of the transfer-function matrix from $r(t)$ to $z(t)$ and $\gamma_2 \in \mathbb{R}^+$.

Theorem 1. For a given $\gamma_2 \in \mathbb{R}^+$, an H_2 FE/FTC joint design problem is solvable if and only if there exist symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that

$$P\hat{A} - Y\hat{C} + \hat{A}^T P - \hat{C}^T Y^T + 2\alpha P \prec 0 \tag{10}$$
$$\begin{bmatrix} -rP & P\hat{A} - Y\hat{C} \\ \hat{A}^T P - \hat{C}^T Y^T & -rP \end{bmatrix} \prec 0 \tag{11}$$
$$\begin{bmatrix} \sin\theta(P\hat{A} + \hat{A}^T P & \cos\theta(P\hat{A} - \hat{A}^T P \\ - Y\hat{C} - \hat{C}^T Y^T) & - Y\hat{C} + \hat{C}^T Y^T) \\ \cos\theta(\hat{A}^T P - P\hat{A} & \sin\theta(P\hat{A} + \hat{A}^T P \\ + Y\hat{C} - \hat{C}^T Y^T) & - Y\hat{C} - \hat{C}^T Y^T) \end{bmatrix} \prec 0 \tag{12}$$
$$\bar{A}Q + Q\bar{A}^T + \bar{B}Z + Z^T\bar{B}^T + \bar{E}\bar{E}^T \prec 0 \tag{13}$$
$$trace(\bar{C}Q\bar{C}^T) < \gamma_2^2. \tag{14}$$

If the LMIs (10)–(14) are feasible, then the fault estimator gain L in (5) and the fault tolerant controller gain K in (9) can be simultaneously synthesized as $L = P^{-1}Y$ and $K = Q^{-1}Z$ in an H_2 suboptimal sense.

Proof. On the one hand, according to the work by Chilali and Gahinet (1996), the poles of the estimator in (5)–(6) lie in the convex region $\Xi(\rho, \alpha, \theta)$ if and only if there exists a symmetric positive-definite matrix $P \succ 0$ such that

$$P(\hat{A} - L\hat{C}) + (\hat{A} - L\hat{C})^T P + 2\alpha P \prec 0 \tag{15}$$
$$\begin{bmatrix} -\gamma P & P(\hat{A} - L\hat{C}) \\ (\hat{A} - L\hat{C})^T P & -\gamma P \end{bmatrix} \prec 0 \tag{16}$$
$$\begin{bmatrix} \sin\theta(P(\hat{A} - L\hat{C}) & \cos\theta(P(\hat{A} - L\hat{C}) \\ + (\hat{A} - L\hat{C})^T P) & - (\hat{A} - L\hat{C})^T P) \\ \cos\theta((\hat{A} - L\hat{C})^T P & \sin\theta(P(\hat{A} - L\hat{C}) \\ - P(\hat{A} - L\hat{C})) & + (\hat{A} - L\hat{C})^T P) \end{bmatrix} \prec 0 \tag{17}$$

Introduce an auxiliary variable Y as $Y = PL$. Substituting the definition of Y into (15)–(17) yields (10)–(12).

On the other hand, we can express the fault-tolerant closed-loop system as

$$\dot{\zeta}(t) = (\bar{A} + \bar{B}K)\zeta(t) + \bar{E}r(t) \tag{18}$$
$$z(t) = \bar{C}\zeta(t) \tag{19}$$

According to the LMI formulation for the H_2 performance index (Boyd et al. (1994)), the H_2 norm $\|T_{zr}\|_2$ of the transfer-function matrix from $r(t)$ to $z(t)$ is less than γ_2 if and only if there exists a symmetric positive-definite matrix $Q \succ 0$ such that

$$(\bar{A} + \bar{B}K)Q + Q(\bar{A} + \bar{B}K)^T + \bar{E}\bar{E}^T \prec 0 \tag{20}$$
$$trace(\bar{C}Q\bar{C}^T) < \gamma_2^2 \tag{21}$$

Define $Z = KQ$. Then this definition will naturally lead to (13)–(14). So the proof is finished.

Corollary 2. If there are $\gamma_2 > 0$, symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that

$$\begin{array}{ll} \min & \gamma_2 \\ s.t. & (10), (11), (12), (13), (14) \end{array} \tag{22}$$

then an optimized fault tolerant controller (9) (in an H_2 sense) and a robust fault estimator (5)–(6) can be simultaneously constructed. The interested gains for the controller and estimator are $K = Q^{-1}Z$ and $L = P^{-1}Y$, respectively, and $\|T_{zr}\|_2 < \gamma_2$.

LMI Formulation of H_∞ Joint Design

Definition 2. The joint design for the system (1)–(2) of a fault estimator (5)–(6) whose poles lie in the convex region $\Xi(\rho, \alpha, \theta)$ in Fig. 2 and a fault tolerant controller (9) such that $\|T_{zr}\|_\infty < \gamma_\infty$ is called an H_∞ FE/FTC joint design problem, where $\|T_{zr}\|_\infty$ denotes the H_∞ norm of the transfer-function matrix from $r(t)$ to $z(t)$ and $\gamma_\infty \in \mathbb{R}^+$.

Theorem 3. For a given $\gamma_\infty \in \mathbb{R}^+$, an H_∞ FE/FTC joint design problem is solvable if and only if there exist symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that (10)–(12) and

$$\begin{bmatrix} \bar{A}Q + Q\bar{A}^T + \bar{B}Z + Z^T\bar{B}^T & \bar{E} & Q\bar{C}^T \\ \bar{E}^T & -\gamma_\infty I & 0 \\ \bar{C}Q & 0 & -\gamma_\infty I \end{bmatrix} \prec 0 \tag{23}$$

hold. If the LMIs (10)–(12) and (23) are feasible, then the fault estimator gain L in (5) and the fault tolerant controller gain K in (9) can be simultaneously synthesized as $L = P^{-1}Y$ and $K = Q^{-1}Z$ in an H_∞ suboptimal sense.

Proof. With the help of the LMI technique for the H_∞ formulation (Boyd et al. (1994)), it is easy to derive this result by means of a similar procedure to that in the proof of Theorem 1. For the sake of space, the details are omitted.

Corollary 4. If there are $\gamma_\infty > 0$, symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that

$$\begin{array}{ll} \min & \gamma_\infty \\ s.t. & (10), (11), (12), (23) \end{array} \tag{24}$$

then an optimized fault tolerant controller (9) (in an H_∞ sense) and a robust fault estimator (5)–(6) can be simultaneously constructed. The interested gains for the controller and estimator are $K = Q^{-1}Z$ and $L = P^{-1}Y$, respectively, and $\|T_{zr}\|_\infty < \gamma_\infty$.

LMI Formulation of Mixed H_2/H_∞ Joint Design

Definition 3. The joint design for the system (1)–(2) of a fault estimator (5)–(6) whose poles are within the convex region $\Xi(\rho, \alpha, \theta)$ in Fig. 2 and a fault tolerant controller (9) such that $\|T_{zr}\|_2 < \gamma_2$ and $\|T_{zr}\|_\infty < \gamma_\infty$ is called a mixed H_2/H_∞ FE/FTC joint design problem, where $\|T_{zr}\|_2$ and $\|T_{zr}\|_\infty$ denote the H_2 and H_∞ norms of the transfer-function matrix from $r(t)$ to $z(t)$, respectively, and $\gamma_2 \in \mathbb{R}^+$ and $\gamma_\infty \in \mathbb{R}^+$.

Theorem 5. For a given pair of $\gamma_2 \in \mathbb{R}^+$ and $\gamma_\infty \in \mathbb{R}^+$, a mixed H_2/H_∞ FE/FTC joint design problem is solvable if and only if there exist symmetric positive-definite matrices $P \succ 0$, $Q \succ 0$, and $R \succ 0$ and matrices X, Y, and Z such that

$$P\hat{A} - X\hat{C} + \hat{A}^T P - \hat{C}^T X^T + 2\alpha P \prec 0 \quad (25)$$

$$\begin{bmatrix} -rP & P\hat{A} - X\hat{C} \\ \hat{A}^T P - \hat{C}^T X^T & -rP \end{bmatrix} \prec 0 \quad (26)$$

$$\begin{bmatrix} \sin\theta(P\hat{A} + \hat{A}^T P & \cos\theta(P\hat{A} - \hat{A}^T P \\ - X\hat{C} - \hat{C}^T X^T) & - X\hat{C} + \hat{C}^T X^T) \\ \cos\theta(\hat{A}^T P - P\hat{A} & \sin\theta(P\hat{A} + \hat{A}^T P \\ + X\hat{C} - \hat{C}^T X^T) & - X\hat{C} - \hat{C}^T X^T) \end{bmatrix} \prec 0 \quad (27)$$

$$\begin{bmatrix} \bar{A}Q + Q\bar{A}^T + \bar{B}Y + Y^T\bar{B}^T & \bar{E} & Q\bar{C}^T \\ \bar{E}^T & -\gamma_\infty I & 0 \\ \bar{C}Q & 0 & -\gamma_\infty I \end{bmatrix} \prec 0 \quad (28)$$

$$\bar{A}R + R\bar{A}^T + \bar{B}Z + Z^T\bar{B}^T + \bar{E}\bar{E}^T \prec 0 \quad (29)$$

$$trace(\bar{C}R\bar{C}^T) < \gamma_2^2 \quad (30)$$

$$Q^{-1}Y = R^{-1}Z. \quad (31)$$

If the constraints (25)–(31) are feasible, then the fault estimator gain L in (5) and the fault tolerant controller gain K in (9) can be simultaneously synthesized as $L = P^{-1}X$ and $K = Q^{-1}Y$ or $K = R^{-1}Z$ in a mixed H_2/H_∞ suboptimal sense.

Proof. By combining Theorems 1 with 3, we can smoothly obtain (25)–(30). In order to make sure the consistency of K across the mixed H_2 and H_∞ settings, it is natural to incorporate (31). So the proof is finished.

Remark 1. In practice, Equation (31) poses a formidable challenge to numerical optimization due to its involvement of the inverses of Q and R, which give rise to nonconvexity. Fortunately, this issue can be handled in a conservative manner by imposing the equality of Q and R.

Theorem 6. For a given pair of $\gamma_2 \in \mathbb{R}^+$ and $\gamma_\infty \in \mathbb{R}^+$, if there exist symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that (10)–(14) and (23) hold, then it is viable to conduct a mixed H_2/H_∞ FE/FTC joint design for the system (1)–(2). When the constraints (10)–(14) and (23) are feasible, the fault estimator gain L in (5) and the fault tolerant controller gain K in (9) will be of the form $L = P^{-1}Y$ and $K = Q^{-1}Z$ in a mixed H_2/H_∞ suboptimal sense.

Proof. It is trivial to yield this result through integrating Theorem 5 and the requirement of $Q = R$.

Corollary 7. If there are $\gamma_2 > 0$, $\gamma_\infty > 0$, symmetric positive-definite matrices $P \succ 0$ and $Q \succ 0$ and matrices Y and Z such that

Table 1. Optimized quantitative H_2 and/or H_∞ indexes for evaluating fault tolerant tracking

	γ_2/γ_∞
H_2 Joint Design	0.0043
H_∞ Joint Design	0.0042
Mixed Joint Design	0.0300/0.0277

$$\begin{array}{ll} \min & \gamma_\infty^2 + \gamma_2^2 \\ s.t. & (10), (11), (12), (13), (14), (23) \end{array} \quad (32)$$

then an optimized fault tolerant controller (9) and a robust fault estimator (5)–(6) can be jointly evaluated in a mixed H_2/H_∞ sense. The interested gains for the controller and estimator are $K = Q^{-1}Z$ and $L = P^{-1}Y$, respectively, and $\|T_{zr}\|_2 < \gamma_2$ and $\|T_{zr}\|_\infty < \gamma_\infty$.

4. NUMERICAL EXAMPLES

To illustrate the power of the integrated design approach, a tutorial model of the form (1)–(2) is considered whose parameters are as follows:

$$A = \begin{bmatrix} 1 & -2 & -2 \\ 2 & 1 & 1 \\ 1 & -2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & -1 \\ 1 & -2 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Note that $rank([B\ E]) = 2$ and $rank(B) = 1$ and imperfect compensation arises. During simulation experiments, a stepwise fault is introduced to mitigate the assumption of f being approximately constant, viz., $\dot{f} \approx 0$. In the meantime, an LMI region of the shape enclosed by the dashed line in Fig. 2 on the complex plane that has $\alpha = 2$, $\rho = 3$, and $\theta = 36°$ is chosen for robust FE by means of regional pole placement. In this tutorial example, the tracked variable of interest is the second measured output, i.e., $S_r = [0\ 1\ 0]$.

Table 1 summarizes the optimized quantitative H_2 and/or H_∞ indexes for evaluating fault tolerant tracking capability. For the sake of space, the corresponding observer and controller gains are omitted. It can be noted that the optimal values of H_2 and H_∞ indexes are comparable when independently being optimized. It is thus reasonable to assign equal weights to H_2 and H_∞ indexes in the mixed joint design. On the other hand, it should be pointed out that the current joint optimization design methods will give rise to drastically high controller gains (and dramatic variation in the control input implicitly), albeit their desirable perfect fault tolerant tracking performance. An additional constraint (e.g., control saturation) need to be incorporated into the optimization design, but this is beyond the scope of this paper. Instead, we resort to the use of suboptimal joint design algorithms to exemplify the suggested method. Table 2 lists the observer and controller gains by the suboptimal joint design approaches corresponding to $\gamma_2/\gamma_\infty = 0.4$.

Figure 3 shows the stepwise fault and its estimate by the three joint design methods. It takes an approximate two seconds to give an accurate estimate of the occurring fault. All three joint designs can provide an acceptable performance for the FE task. It can be noted from Fig. 3 that the

Table 2. Gains of the suboptimal fault estimator and fault tolerant controller at $\gamma_2/\gamma_\infty = 0.4$

Method	L	K
H_2 Joint Design	$\begin{bmatrix} 1.1678 & 0.8322 & 3.0000 \\ 2.1152 & -4.2407 & -4.3560 \\ 0.2814 & -2.7424 & -2.0237 \\ 0.8239 & -4.1193 & -4.9431 \end{bmatrix}$	$\begin{bmatrix} 6.2924 & -12.8205 & 3.9500 & 1.7537 & 43.8252 \end{bmatrix}$
H_∞ Joint Design	$\begin{bmatrix} 1.1686 & 0.8314 & 3.0000 \\ 2.1163 & -4.2445 & -4.3609 \\ 0.2820 & -2.7474 & -2.0295 \\ 0.8254 & -4.1272 & -4.9526 \end{bmatrix}$	$\begin{bmatrix} 4.0650 & -6.5691 & 3.0762 & 3.0871 & 13.7278 \end{bmatrix}$
Mixed Joint Design	$\begin{bmatrix} 1.1698 & 0.8302 & 3.0000 \\ 2.1153 & -4.2368 & -4.3521 \\ 0.2804 & -2.7415 & -2.0219 \\ 0.8230 & -4.1148 & -4.9378 \end{bmatrix}$	$\begin{bmatrix} 33.4599 & -56.7316 & 33.3148 & 8.7038 & 340.6045 \end{bmatrix}$

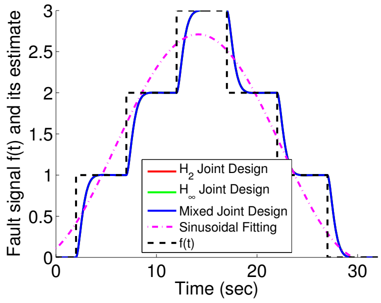

Fig. 3. Estimated stepwise fault

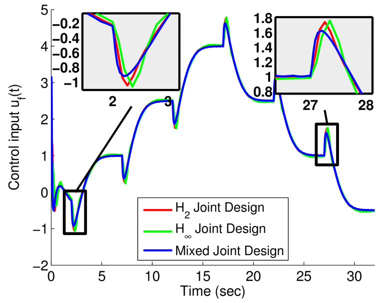

Fig. 4. Control input for the stepwise fault

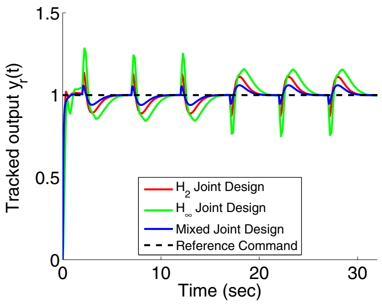

Fig. 5. Tracked output when a stepwise fault occurs

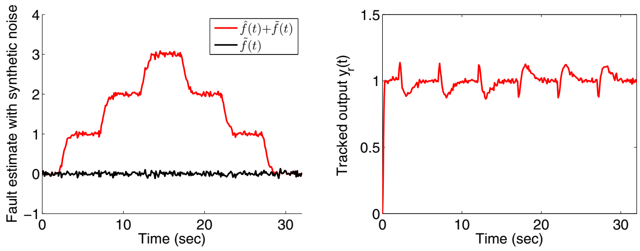

Fig. 6. Effect of synthetic error $\tilde{f}(t)$ in FE on FTC tracking for the H_2 joint design

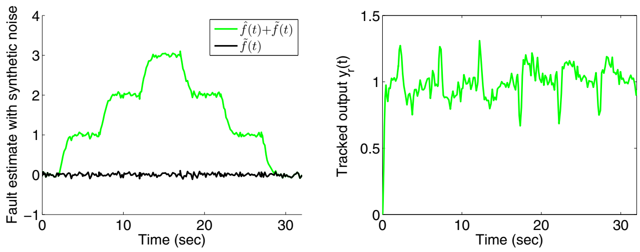

Fig. 7. Effect of synthetic error $\tilde{f}(t)$ in FE on FTC tracking for the H_∞ joint design

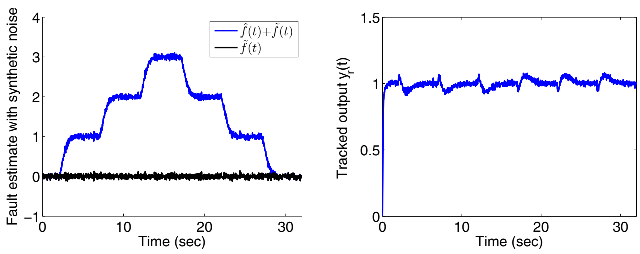

Fig. 8. Effect of synthetic error $\tilde{f}(t)$ in FE on FTC tracking for the mixed H_2/H_∞ joint design

fault can be approximately fitted by the sinusoidal signal $1.34 + 1.37sin(0.19t - 1.12)$. The low 0.03 Hz frequency helps explain why the FE is still working when the fault assumption is violated. If one expects to further reduce the time needed to estimate the fault, this can be done by retuning α, ρ, and θ. However, this issue is not the focus of this paper. Figure 4 shows the corresponding control input when the stepwise fault arises. It seems that the fault tolerant control signal $u_f(t)$ by the mixed H_2/H_∞ joint design is more preferable, since the transitional change is slightly moderate. Meanwhile, Figure 5 delineates the generated diagram for the tracked output. It can be seen from Fig. 5 that under the same level of fault tolerance (viz., $\gamma_2 = \gamma_\infty$), the mixed H_2/H_∞ design is superior to the single H_2 or H_∞ joint approach (this might be ascribed to that the mixed design assembles the respective advantages of the H_2 and H_∞ schemes), and the H_2 joint design is slightly better than the H_∞ design method.

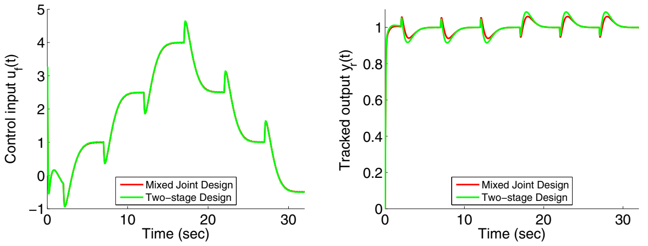

Fig. 9. A comparison of joint design and conventional two-stage scheme

In order to figure out the effect of FE erroneous disturbance on FTC tracking performance, we introduce a random signal $\tilde{f}(t)$ that is distributed normally with mean zero and variance σ^2: $\tilde{f}(t) \sim \mathcal{N}(0, \sigma^2)$. In this paper, σ is chosen as $\sigma = 0.04$. This synthetic signal is forcefully added into $\hat{f}(t)$ to emulate the FE process with stochastic error. Figure 6–8 shows the impact of synthetic FE error on the tracked output. Compared with the corresponding error-free cases, the H_2 and mixed H_2/H_∞ joint methods seem to be more robust relative to the H_∞ design, since a little error in FE can lead to the "spike" fluctuation of the tracked output for the H_∞ joint method.

For the fault tolerant control form $u_f(t) = u^*(t) - B^* E \hat{f}(t)$ (Jiang et al. (2006), Zhang et al. (2010), Tabatabaeipour and Bak (2014)), where $u^*(t)$ represents some control law obtained by an independent design under the ideal normal condition and B^* satisfies $BB^*E = E$, it is impossible to achieve perfect compensation in this example since E is not colinear with B. On the other hand, our experiments indicate that the coefficient of $f(t)$ in the resolved fault tolerant controller gain will be -1 when $B = E$—this coincides with the conventional two-stage strategy: $u_f(t) = u^*(t) - \hat{f}(t)$ (Sami and Patton (2013)). Moreover, we normally devised a mixed H_2/H_∞ state feedback controller as well, whose gain is $[32.0444\ -50.5600\ 21.6624\ 313.6474]$, disregarding the impact of $f(t)$. This ordinary controller, compensated by $B^\dagger E \hat{f}(t)$ where $B^\dagger$ is the Moore-Penrose pseudoinverse of B, is connected with the observer derived from the joint design. Figure 9 makes a comparison between the mixed joint design and the conventional two-stage strategy in control input and tracking output. Although it appears that no significant difference in control input is noticeable from Fig. 9, yet the mixed joint design outperforms the two-stage design in command tracking.

5. CONCLUSION

This paper has suggested a holistic design scheme for dealing with the issues of fault estimation and fault tolerant tracking control jointly. The contribution lies in the formation of the fault tolerant robustness concept that leads to the H_2, H_∞, and mixed H_2/H_∞ joint design LMI formulations for tackling FE and FTC in a unified framework. "Separation" principle in state feedback control weakens the joint design recommendation to some extent, but it is possible to generalize the joint thought to other feasible combinations of state observers and static/dynamic controllers or to nonlinear system models. Our experience indicates that the proposed design is not suitable for high frequency faults and more work is needed for fault modeling. To make full use of the power of the joint design scheme, it is necessary to incorporate parameter uncertainty and other constraints into the proposed framework. Because of the limitation of space, more example illustrations will be presented in an extended paper. All these will be explored in our future research work.

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REFERENCES

Boyd, S., Ghaoui, L.E., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics (SIAM).

Chilali, M. and Gahinet, P. (1996). H_∞ design with pole placement constraints: an LMI approach. *IEEE Transactions on Automatic Control*, 41(3), 358–367.

Ding, S. (2009). Integrated design of feedback controllers and fault detectors. *Annual Reviews in Control*, 33(2), 124–135.

Edwards, C. and Spurgeon, S.K. (1998). *Sliding Mode Control: Theory and Applications*. CRC Press, London.

Jiang, B., Staroswiecki, M., and Cocquempot, V. (2006). Fault accommodation for nonlinear dynamic systems. *IEEE Transactions on Automatic Control*, 51(9), 1578–1583.

Odgaard, P., Stoustrup, J., Andersen, P., Wickerhauser, M., and Mikkelsen, H. (2006). A fault tolerant control scheme for CD players to handle surface defects. *Control Engineering Practice*, 14(12), 1495–1509.

Patton, R.J. (1997). Fault-tolerant control systems: The 1997 situation. In *Proc. of IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes*, 1033–1054. Kingston upon Hull, UK.

Sami, M. and Patton, R.J. (2013). Active fault tolerant control for nonlinear systems with simultaneous actuator and sensor faults. *International Journal of Control, Automation, and Systems*, 11(6), 1149–1161.

Shi, F. and Patton, R.J. (2014). Fault estimation and active fault tolerant control for linear parameter varying descriptor systems. *International Journal of Robust and Nonlinear Control*, Early View, doi: 10.1002/rnc.3266.

Tabatabaeipour, S.M. and Bak, T. (2014). Robust observer-based fault estimation and accommodation of discrete-time piecewise linear systems. *Journal of the Franklin Institute*, 351(1), 277–295.

Wang, H. and Yang, G. (2009). Integrated fault detection and control for lpv systems. *International Journal of Robust and Nonlinear Control*, 19(3), 341–363.

Zhang, K., Jiang, B., and Shi, P. (2010). Observer-based integrated robust fault estimation and accommodation design for discrete-time systems. *International Journal of Control*, 83(6), 1167–1181.

Zhang, Y. and Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control*, 32(2), 229–252.