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A Relaxed Solution to Unknown Input Observers for State and Fault Estimation

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Abstract: A lot of effort has been devoted to the unknown input observer (UIO) research over the past years. However, the strong disturbance decoupling assumption (manifested as some rank constraint) is often implicitly embedded in much of the existing UIO work. With the purpose of state and fault estimation, this fact motivates us to investigate the viability of the UIO research when the strong disturbance decoupling is not possible, i.e., a "degenerate" problem of UIO decoupling exists. Inspired by the scheme of reducing the effect of external disturbance on estimation error, this paper incorporates the relaxed UIO (R_xUIO) concept by means of the H_∞, H_2, and mixed H_2/H_∞ techniques. Necessary and sufficient conditions for the existence of different R_xUIOs are presented in the tractable linear matrix inequality (LMI) form. Numerical experiments are presented to illustrate the effectiveness of the suggested method.

Keywords: Fault Diagnosis, Fault Estimation, Unknown Input Observer, Linear Matrix Inequality.

1. INTRODUCTION

The increasing complexity of industrial systems greatly push the practical need for the ability of diagnostic alert and fault tolerance. Unknown input observers (UIOs) play an important role in the fault diagnosis community. For example, Odgaard and Stoustrup (2012) made use of them to accommodate the faults of rotor and generator speed sensors in a wind turbine. Over the past decades, a great deal of research effort has been devoted to the UIO theory.

For instance, Chen et al. (1996) and Chen and Patton (1999) avoided the complicated canonical form transformation by solving a group of algebraic matrix equations to acquire UIO parameters. Later, the UIO parameters were resolved by Amato and Mattei (2002) through H_∞ performance index in the linear matrix inequality (LMI) framework. Using the robust H_∞ technique, Pertew et al. (2005) explored the issue of synthesizing UIOs for the nonlinear Lipschitz system. As for the nonlinear descriptor system case, Koenig (2006) dealt with the synthesis of UIOs based on the LMI approach. In addition, Filasová and Krokavec (2009) investigated the UIO synthesis for the linear discrete-time system using the LMI technique. Moreover, Hamdi et al. (2012) designed a tractable procedure to evaluate UIO parameters of PI form for the popular linear parameter varying descriptor model, whereas Chadli and Karimi (2013) built on some extra relaxation variables to reduce the conservatism in the UIO solution of the T-S fuzzy model. As far as model uncertainty is concerned, Lungu and Lungu (2014) took advantage of eigenstructure assignment to strengthen the robustness of UIO estimators. The aforementioned research findings (just name a few) highly enrich our knowledge of the UIO-based estimation theory.

However, much of the existing work on UIO research has some sort of requirement on the ranks of system coefficient matrices. For example, the existence of a UIO estimator for a linear time invariant (LTI) system requires that the rank of the multiplication of measurement coefficient matrix and disturbance coefficient matrix equal that of the disturbance coefficient matrix (i.e., $rank(CE) = rank(E)$, see explanations in Subsection 3.1). When these rank-related conditions are not satisfied, it will be impractical to devise a UIO estimator in terms of current strong disturbance decoupling principle, and a so-called degenerate UIO decoupling problem arises. In this context, it is the rank conservatism that motivates us to relieve the constraint imposed by the traditional UIO technique.

This paper is organized as follows. Section 2 briefly describes the problem of interest in this paper. Then the main results are presented in Section 3. Section 4 gives a tutorial example to illustrate the potential of the suggested method. Finally, Section 5 concludes this paper.

2. PROBLEM DESCRIPTION

Consider the LTI model of the form:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^p$ is the control input, $w(t) \in \mathbb{R}^d$ is the external disturbance, $y(t) \in \mathbb{R}^q$ is the sensor measurement, and the matrices A, B, C, and E are of compatible dimension. Meanwhile, a full-order UIO can be readily formulated as

$$\dot{q} = Fq + GBu + Ky \tag{3}$$
$$\hat{x} = q + Hy \tag{4}$$

where $q(t) \in \mathbb{R}^n$ is the observer state, $\hat{x}(t) \in \mathbb{R}^n$ is the state estimate, and matrices F, G, K and H are the UIO parameters. Unlike many of UIO papers, this work does not assume that E has full-column rank, i.e., $rank(E) = d$. Our problem is how to evaluate F, G, K and H for the system in (1)–(2) when the rank of CE is degenerate, that is, $rank(CE) < rank(E)$.

3. RELAXED SOLUTION TO UIO DESIGN

3.1 Traditional UIO Revisit

For the purpose of discussion convenience, estimation error is defined as $e(t) = x(t) - \hat{x}(t)$. Obviously, $e(t)$ is expected to converge to zero as $t \to \infty$. First of all, it is easy to obtain from (1)–(4) that

$$\begin{aligned} \dot{e}(t) = {} & [(I_n - HC)A - K_1C]e(t) \\ & +[(I_n - HC)A - F - K_1C]q(t) \\ & +(I_n - G - HC)Bu(t) \\ & +\{[(I_n - HC)A - K_1C]H - K_2\}y(t) \\ & +(I_n - HC)Ew(t), \end{aligned} \tag{5}$$

where $K_1 + K_2 = K$ and I_n is the $n \times n$ identity matrix. In terms of strong disturbance decoupling principle, the traditional UIO theory requires that F, G, K, and H fulfill

$$A - HCA - F - K_1C = 0 \tag{6}$$
$$I_n - G - HC = 0 \tag{7}$$
$$(A - HCA - K_1C)H - K_2 = 0 \tag{8}$$
$$(I_n - HC)E = 0. \tag{9}$$

Remark 1. The key step in solving UIO parameters lies in (9). Chen et al. (1996) revealed that there exists a solution for H in (9) if and only if $rank(CE) = rank(E)$. Unfortunately, less attention is paid to devising a UIO estimator in case of $rank(CE) \neq rank(E)$ (or $rank(CE) < rank(E)$). In this context, it is still necessary to explore the UIO design issue under the condition that the preceding rank-equality constraint does not hold. In this paper, the concept of relaxed UIOs is introduced to address the rank-degenerate UIO problem.

3.2 H_∞ R$_\mathrm{x}$UIO Design

Definition 1. A UIO in (3)–(4) is referred to as an H_∞ R$_\mathrm{x}$UIO if the H_∞ norm $\|T_{we}\|_\infty$ of the transfer matrix between the disturbance $w(t)$ and the estimation error $e(t)$ is less than some γ, where $\gamma \in \mathbb{R}^+$.

Theorem 1. For a given $\gamma \in \mathbb{R}^+$, the system in (1)–(2) has a γ-suboptimal H_∞ R$_\mathrm{x}$UIO if and only if there exist a symmetric positive-definite matrix $P \in \mathbb{R}^{n\times n}$ and matrices $Y \in \mathbb{R}^{n\times q}$ and $V \in \mathbb{R}^{n\times q}$ such that

$$\begin{bmatrix} A^TP + PA - \Xi(Y,V) & PE - YCE & I_n \\ E^TP - E^TC^TY^T & -\gamma I_d & 0_{d\times n} \\ I_n & 0_{n\times d} & -\gamma I_n \end{bmatrix} \prec 0 \tag{10}$$

where $\Xi(Y,V) = YCA + A^TC^TY^T + VC + C^TV^T$.

Proof. The above analysis of (6)–(9) implies that as long as the value of H becomes known, it will be trivial to get the values of F, G, and K from (6)–(8). Hence we can first focus on the solution to H. Under those circumstances, the model about estimation error can be adapted to

$$\dot{e}(t) = (A - HCA - K_1C)e(t) + (E - HCE)w(t) \tag{11}$$
$$z(t) = e(t). \tag{12}$$

Since $rank(CE) \neq rank(E)$, it is impossible to seek an H that makes $E - HCE = 0$. However, we can resort to the use of the bounded-real lemma by Boyd et al. (1994) to reduce the effect of $w(t)$ on $e(t)$ using the H_∞ technique. According to this lemma, the H_∞ norm $\|T_{wz}\|_\infty$ is less than γ if and only if there exists a symmetric positive-definite matrix P fulfilling

$$\begin{bmatrix} PA + A^TP - \Xi(PH, PK_1) & P(E - HCE) & I_n \\ (E - HCE)^TP & -\gamma I_d & 0_{d\times n} \\ I_n & 0_{n\times d} & -\gamma I_n \end{bmatrix} \prec 0 \tag{13}$$

where $\Xi(M,N) = A^TC^TM^T + MCA + NC + C^TN^T$. Using the variable changes of $Y = PH$ and $V = PK_1$ can simplify (13) to (10). So the proof is finished.

Corollary 2. The system in (1)–(2) has an optimal H_∞ R$_\mathrm{x}$UIO if and only if the following optimization problem is solvable:

$$\begin{aligned} \min\ & \gamma \\ s.t.\ & (10),\ P \succ 0,\ \gamma > 0 \end{aligned} \tag{14}$$

where $P \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times q}$, and $V \in \mathbb{R}^{n\times q}$.

3.3 H_2 R$_\mathrm{x}$UIO Design

Definition 2. A UIO in (3)–(4) is referred to as an H_2 R$_\mathrm{x}$UIO if the H_2 norm $\|T_{we}\|_2$ of the transfer matrix between the disturbance $w(t)$ and the estimation error $e(t)$ is less than some ρ, where $\rho \in \mathbb{R}^+$.

Theorem 3. For a given $\rho \in \mathbb{R}^+$, the system in (1)–(2) has a ρ-suboptimal H_2 R$_\mathrm{x}$UIO if and only if there exist symmetric positive-definite matrices $P \in \mathbb{R}^{n\times n}$ and $W \in \mathbb{R}^{d\times d}$ and matrices $Y \in \mathbb{R}^{n\times q}$ and $V \in \mathbb{R}^{n\times q}$ such that

$$A^TP + PA - \Xi(Y,V) + I_n \prec 0 \tag{15}$$
$$\begin{bmatrix} -W & E^TP - E^TC^TY^T \\ PE - YCE & -P \end{bmatrix} \prec 0 \tag{16}$$
$$trace(W) < \rho^2 \tag{17}$$

where $\Xi(Y,V) = A^TC^TY^T + YCA + C^TV^T + VC$.

Proof. Following a similar procedure to that in the proof of Theorem 1, we can readily prove the correctness of this result using the LMI formulation for H_2 theory. For the sake of space, the details are omitted.

Corollary 4. The system in (1)–(2) has an optimal H_2 R$_\mathrm{x}$UIO if and only if the following optimization problem is solvable:

$$\begin{aligned} \min\ & \rho \\ s.t.\ & (15), (16), (17) \\ & P \succ 0,\ W \succ 0,\ \rho > 0 \end{aligned} \tag{18}$$

where $P \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times q}$, $V \in \mathbb{R}^{n\times q}$, and $W \in \mathbb{R}^{d\times d}$.

3.4 Mixed H_2/H_∞ $\mathrm{R_x}$UIO Design

Definition 3. A UIO in (3)–(4) is referred to as a mixed H_2/H_∞ $\mathrm{R_x}$UIO if the transfer matrix between the disturbance $w(t)$ and the estimation error $e(t)$ satisfies $\|T_{we}\|_2 < \rho$ and $\|T_{we}\|_\infty < \gamma$, where $\rho \in \mathbb{R}^+$, $\gamma \in \mathbb{R}^+$, and $\| \bullet \|_2$ and $\| \bullet \|_\infty$ represent the H_2 and H_∞ norms, respectively.

Theorem 5. For a given pair of γ and ρ, the system in (1)–(2) has a mixed H_2/H_∞ $\mathrm{R_x}$UIO if and only if there exist symmetric positive-definite matrices $P \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times n}$, and $W \in \mathbb{R}^{d\times d}$, and matrices $X \in \mathbb{R}^{n\times q}$, $Y \in \mathbb{R}^{n\times q}$, $U \in \mathbb{R}^{n\times q}$, and $V \in \mathbb{R}^{n\times q}$ such that

$$\begin{bmatrix} PA + A^TP - \Xi(X,U) & PE - XCE & I_n \\ E^TP - E^TC^TX^T & -\gamma I_d & 0_{d\times n} \\ I_n & 0_{n\times d} & -\gamma I_n \end{bmatrix} \prec 0 \quad (19)$$

$$A^TQ + QA - \Xi(Y,V) + I_n \prec 0 \quad (20)$$

$$\begin{bmatrix} -W & E^TQ - E^TC^TY^T \\ QE - YCE & -Q \end{bmatrix} \prec 0 \quad (21)$$

$$trace(W) < \rho^2 \quad (22)$$

$$P^{-1}X = Q^{-1}Y \quad (23)$$

$$P^{-1}U = Q^{-1}V. \quad (24)$$

where $\Xi(M,N) = MCA + A^TC^TM^T + NC + C^TN^T$.

Proof. It is trivial to reach this result through integrating the proofs of Theorems 1 and 3.

Remark 2. It is worth noting that the last two constraints in Theorem 5 ($P^{-1}X = Q^{-1}Y$ and $P^{-1}U = Q^{-1}V$) are not convex on P, Q, Y, and V, because of inclusion of the inverses of P and Q. In general, it is also challenging to directly solve (19)–(24). One commonly used approach is to introduce an extra constraint of $P = Q$, which can smoothly eliminate the last two nonconvex constraints in (23)–(24) at the expense of some conservatism.

Corollary 6. For a given pair of γ and ρ, if there exist symmetric positive-definite matrices $P \in \mathbb{R}^{n\times n}$ and $W \in \mathbb{R}^{d\times d}$ and matrices $Y \in \mathbb{R}^{n\times q}$ and $V \in \mathbb{R}^{n\times q}$ such that (10) and (15)–(17) hold, then the system in (1)–(2) has a mixed H_2/H_∞ $\mathrm{R_x}$UIO.

Remark 3. Like the optimization scheme adopted in Corollary 2 and 4, it is equally possible to incorporate a convex objective function $h(\gamma,\rho)$ of γ and ρ into Corollary 6.

Corollary 7. A sub-optimized mixed H_2/H_∞ $\mathrm{R_x}$UIO exists for the system in (1)–(2) when the convex optimization problem is solvable:

$$\begin{aligned} &\min \;\; h(\gamma,\rho) \\ &s.t. \;\; (10), (15), (16), (17) \\ &\qquad P \succ 0,\; W \succ 0,\; \gamma > 0,\; \rho > 0 \end{aligned} \quad (25)$$

where $h(\gamma,\rho)$ is a convex function on γ and ρ.

Remark 4. For the convenience of LMI evaluation, we can change the upper bound of the trace of W from ρ^2 to λ in Corollary 4 and 7, but this will not have a significant impact on estimation accuracy from the computational perspective. Meanwhile, it should also be pointed out that the final value of $\|T_{we}\|_2$ in Corollary 4 and 7 should be taken as the square root of the optimized λ, i.e., $\sqrt{\lambda}$.

Remark 5. Although numerical solutions to the feasibility problems in Theorem 1 and 3 might exhibit some variety for different initial conditions or distinct off-the-shelf software, the objective of the feasible LMI formulation of $\mathrm{R_x}$UIO is to pave the way for an optimization design process, e.g., Corollary 2, 4, and 7. Analyzing the effect of variation in the solution to the feasibility problem in Theorem 1 or 3 and seeking a multiobjective optimized design are beyond the scope of this paper.

4. ILLUSTRATIVE EXAMPLE

4.1 Experimental Setup

To validate the effectiveness of the proposed method, consider the following tutorial model parameters:

$$A = \begin{bmatrix} -2 & -2 & 0 \\ -1 & -1 & -2 \\ 2 & -2 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$B = [-2\;\; -2\;\; -1]^T \qquad E = [-2\;\; 0\;\; 1]^T.$$

According to Chen and Patton (1999), it is impossible to design a conventional UIO for this illustrative model because of the degenerate rank condition: $rank(CE) = 0 < rank(E) = 1$. However, it is possible to take advantage of the $\mathrm{R_x}$UIO technique to restrain the influence of external disturbance on the estimation error to the maximum extent, for example in observer design. In order to give a fair evaluation, this paper only presents simulation results of the optimized $\mathrm{R_x}$UIOs and merely considers the regulation problem. As far as the mixed H_2/H_∞ $\mathrm{R_x}$UIO design is concerned, an intuitive choice of $h(\gamma,\rho)$ is $h(\gamma,\rho) = \alpha\gamma^2 + (1-\alpha)\rho^2$, where $\alpha \in [0,1]$. In this paper, α is simply chosen as 0.5 (see Subsection 4.3). During the numerical simulation, white noise with mean 0 and variance 0.02 is used to serve as external disturbance $w(t)$. For the sake of space, we merely focus on one actuator fault. Under this condition, the faulty system model is of the form:

$$\dot{x}(t) = Ax(t) + B(u(t) + f(t)) + Ew(t) \quad (26)$$

$$y(t) = Cx(t). \quad (27)$$

It is assumed that $f(t)$ is slowly varying (i.e., $\dot{f} \approx 0$). To accomplish this degenerate state and fault estimation, we can directly apply the $\mathrm{R_x}$UIO method to the following augmented model system

$$\dot{x}_g(t) = A_gx_g(t) + B_gu(t) + E_gw(t) \quad (28)$$

$$y(t) = C_gx_g(t). \quad (29)$$

where $x_g(t) = \begin{bmatrix} x(t) \\ f(t) \end{bmatrix}$, $A_g = \begin{bmatrix} A & B \\ 0_{p\times n} & 0_{p\times p} \end{bmatrix}$, $B_g = \begin{bmatrix} B \\ 0_{p\times p} \end{bmatrix}$, $C_g = [C\;\; 0_{q\times p}]$, and $E_g = \begin{bmatrix} E \\ 0_{p\times d} \end{bmatrix}$. At the same time, a multi-stepwise fault $f(t)$ is added into the system process operation over all the experiments to relieve the assumption on the constancy of $f(t)$.

4.2 Simulation Results

Figure 1 shows the multi-stepwise fault used in this paper. It can be seen from Fig. 1 that this fault exhibits a time-varying feature and hence weakens the preceding assumption of $\dot{f} = 0$, to some extent. Table 1–3 list the computed $\mathrm{R_x}$UIO parameters by the optimized H_∞, H_2, and mixed H_2/H_∞ techniques, respectively. A comparison can be made between the corresponding $\mathrm{R_x}$UIO parameters. Compared with the H_2 and mixed H_2/H_∞

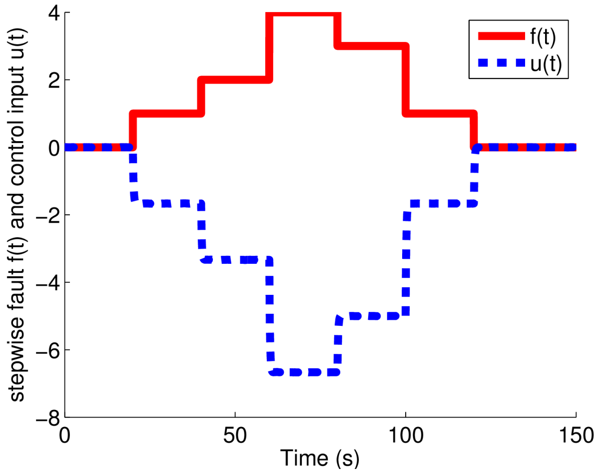

Fig. 1. A multi-stepwise fault signal $f(t)$ and its corresponding control input $u(t)$

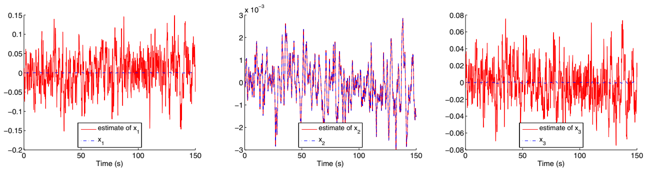

Fig. 2. H_∞ R_xUIO for state estimation under the fault-free condition

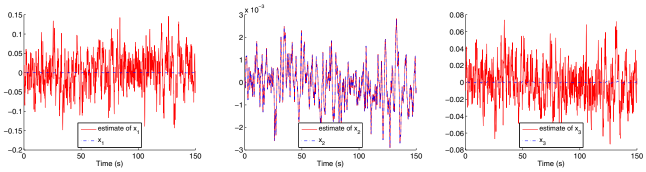

Fig. 3. H_2 R_xUIO for state estimation under the fault-free condition

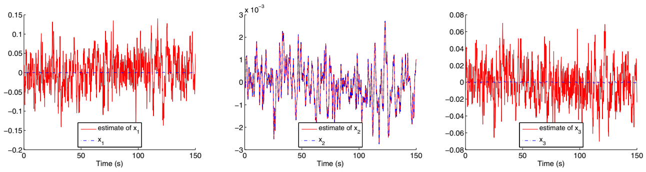

Fig. 4. Mixed H_2/H_∞ R_xUIO for state estimation under the fault-free condition

R_xUIO, the H_∞ R_xUIO has much larger magnitude of parameters, particularly for the terms G, K, and H. This might attribute to the "notorious" high gain characteristic of the optimal H_∞ technique.

Figures 2–4 depict the performance of state estimation under the fault-free condition. We can find out from Figs. 2–4 that all the proposed R_xUIOs can achieve a perfect estimation of the second component x_2 of x, but the random white noise has some moderate impact on the x_1 and x_3's estimates. For the fault-free case, it is not easy to differentiate which R_xUIO has the best performance in

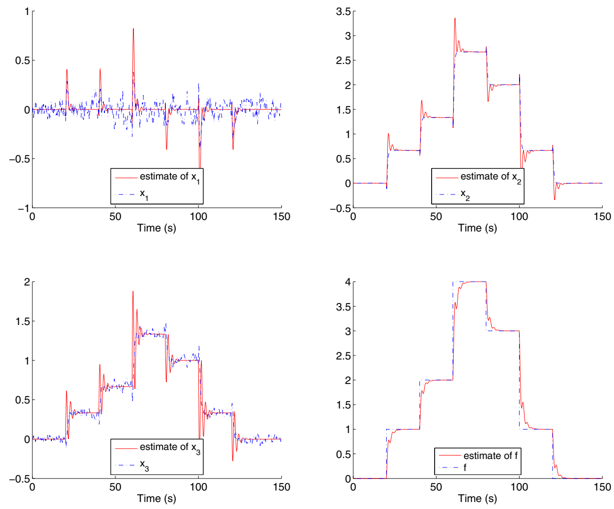

Fig. 5. H_∞ R_xUIO for state and fault estimation

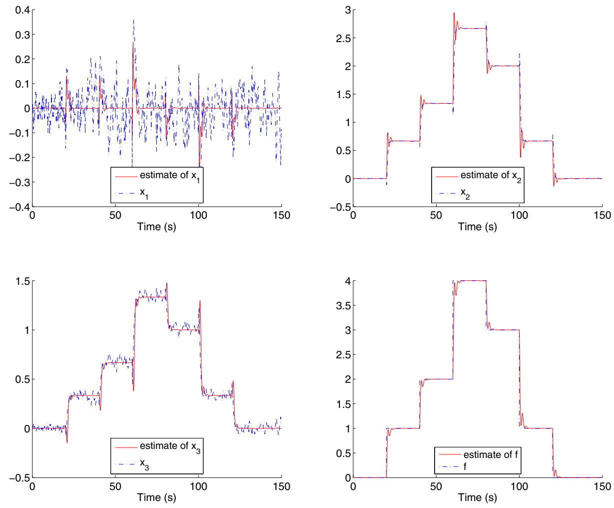

Fig. 6. H_2 R_xUIO for state and fault estimation

terms of estimation accuracy. In this context, we continue to conduct the actuator fault experiment.

Figures 5–7 delineate both state and fault estimation when a stepwise fault arises in the actuator. It can be noted that both the white noise and the actuator fault lead to a noticeable inaccuracy in the estimate of x_1, but the estimation for x_2, x_3, and f is at an acceptable level. The nonstepwise profile of x_1 in Figs. 5–7 should be ascribed to the uncontrollability of x_1 in the augmented model. Figure 8 illustrates the location of poles of the three evaluated R_xUIOs. The estimated faults by the H_∞, H_2, and mixed H_2/H_∞ R_xUIOs are collectively compared in Fig. 9. The quantitative indices for measuring disturbance suppression are shown in Fig. 10, where γ_∞ for the H_∞ R_xUIO, γ_2 for the H_2 R_xUIO, $\gamma_{m\infty}$ for the H_∞ component of the mixed H_2/H_∞ R_xUIO, and γ_{m2} for the H_2 component of the mixed H_2/H_∞ R_xUIO. Surprisingly, we can notice that the collectively optimized $\gamma_{m\infty}$ and γ_{m2} components in the mixed H_2/H_∞ R_xUIO are comparable with the corresponding separately minimized γ_∞ and γ_2 in the H_∞ and H_2 R_xUIOs. A careful examination of

Table 1. Parameters of the optimized H_∞ $\mathrm{R_xUIO}$

F	G	K	H
$\begin{bmatrix} -1.6359 & 17.7098 & 0.7282 & 0.9893 \\ 0.3472 & 1.9540 & 0.6943 & 1534.6 \\ 0.5499 & -12.2393 & -0.9001 & -14.5366 \\ 0.2187 & 1534.5 & 0.4373 & -3.1280 \end{bmatrix}$	$\begin{bmatrix} 5.1621 & -9.8190 & 8.3243 & 0 \\ -336.0957 & -95.1033 & -672.1915 & 0 \\ 0.5529 & 5.6624 & 2.1059 & 0 \\ -14.4011 & 30.3662 & -28.8022 & 1 \end{bmatrix}$	$\begin{bmatrix} 2756.9 & 1616.9 \\ 110780 & -44303 \\ -2881.7 & -723.0573 \\ -224280 & -145790 \end{bmatrix}$	$\begin{bmatrix} -23.8001 & 9.8190 \\ 143.8892 & 96.1033 \\ 10.7719 & -5.6624 \\ 75.1334 & -30.3662 \end{bmatrix}$

Table 2. Parameters of the optimized H_2 $\mathrm{R_xUIO}$

F	G	K	H
$\begin{bmatrix} -1.7881 & -0.9894 & 0.4238 & -0.5599 \\ 0.9893 & -0.9593 & 1.9785 & -1.6849 \\ 0.4237 & -1.9788 & -1.1527 & -1.1197 \\ 0.5598 & 1.6850 & 1.1195 & -0.9407 \end{bmatrix}$	$\begin{bmatrix} 0.7273 & -0.1747 & -0.5454 & 0 \\ -0.0639 & 0.9703 & -0.1278 & 0 \\ -0.0030 & 0.0659 & 0.9939 & 0 \\ 0.1985 & 0.0734 & 0.3969 & 1.0000 \end{bmatrix}$	$\begin{bmatrix} -1.9644 & 0.4717 \\ -2.5586 & 0.5100 \\ 1.5032 & 0.1045 \\ 5.8262 & -2.8060 \end{bmatrix}$	$\begin{bmatrix} -0.0767 & 0.1747 \\ 0.0044 & 0.0297 \\ 0.1349 & -0.0659 \\ -0.0516 & -0.0734 \end{bmatrix}$

Table 3. Parameters of the optimized mixed H_2/H_∞ $\mathrm{R_xUIO}$

F	G	K	H
$\begin{bmatrix} -1.6748 & -1.1767 & 0.6504 & -0.7121 \\ 1.1745 & -0.9615 & 2.3490 & -3.9215 \\ 0.6504 & -2.3534 & -0.6992 & -1.4241 \\ 0.7103 & 3.9222 & 1.4205 & -0.3731 \end{bmatrix}$	$\begin{bmatrix} 0.7917 & -0.2273 & -0.4166 & 0 \\ -0.2086 & 2.3779 & -0.4171 & 0 \\ 0.0523 & 0.1075 & 1.1045 & 0 \\ -0.5557 & 1.2979 & -1.1113 & 1.0000 \end{bmatrix}$	$\begin{bmatrix} -7.0360 & 2.7487 \\ -18.7544 & 6.2637 \\ -10.2561 & 5.2463 \\ 11.1600 & -6.7975 \end{bmatrix}$	$\begin{bmatrix} -0.2463 & 0.2273 \\ 2.9643 & -1.3779 \\ 0.1628 & -0.1075 \\ 3.1514 & -1.2979 \end{bmatrix}$

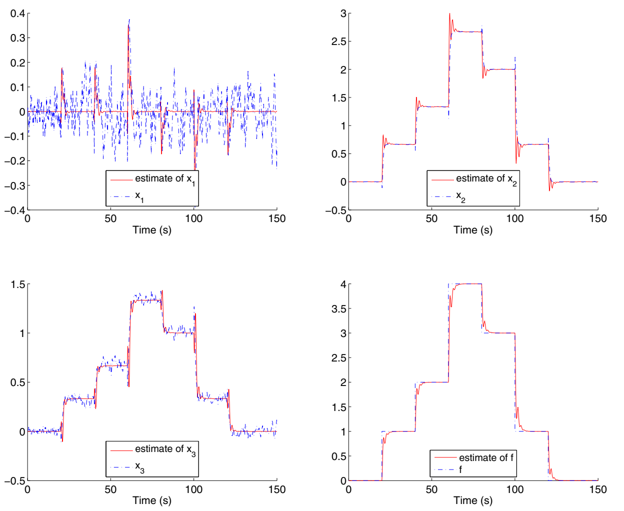

Fig. 7. Mixed H_2/H_∞ $\mathrm{R_xUIO}$ for state and fault estimation

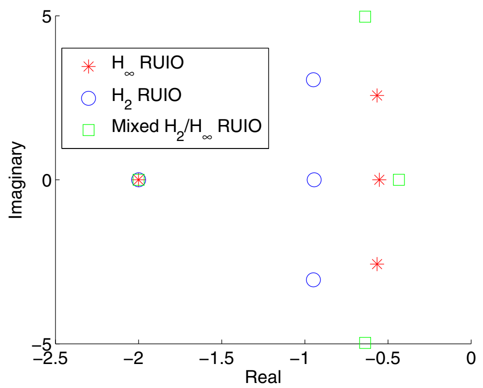

Fig. 8. The location of poles of the three $\mathrm{R_xUIOs}$

Figs. 5–9 reveals that the H_2 $\mathrm{R_xUIO}$ is relatively superior to the other two $\mathrm{R_xUIOs}$, particularly from the perspective of fault estimation (with the purpose of fault tolerant control). Moreover, the mixed $\mathrm{R_xUIO}$ is just slightly better

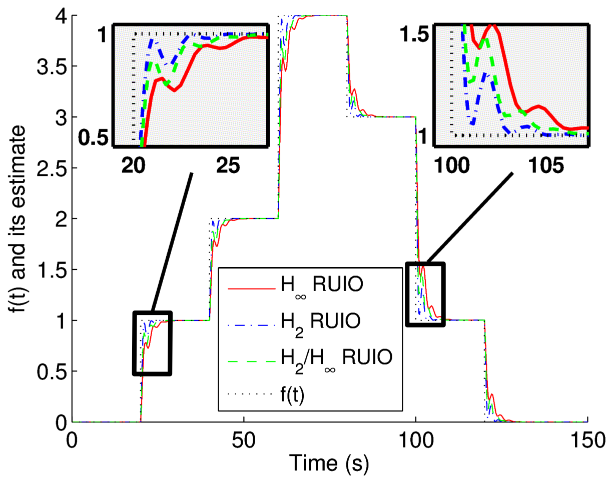

Fig. 9. A comparison of the estimated stepwise fault

than the H_∞ $\mathrm{R_xUIO}$. At this moment, the mixed H_2/H_∞ $\mathrm{R_xUIO}$ does not bring any notable advantage over the single H_2 $\mathrm{R_xUIO}$.

4.3 Discussion

In the preceding experiments, the mixed H_2/H_∞ $\mathrm{R_xUIO}$ laid an equal emphasis on $\|T_{we}\|_\infty$ and $\|T_{we}\|_2$ ($\alpha = 0.5$). A natural question arises of what effect the weight α has upon fault estimation in the mixed $\mathrm{R_xUIO}$. Figure 11 illustrates the effect of different α's (small, medium, and large) on fault estimation and zooms in the estimation dynamics. It is easy to see from Fig. 11 that the choice of $\alpha = 0.5$ corresponds to the best performance from the viewpoint of estimation transient. Hence $\alpha = 0.5$ is an appropriate value. Figure 12 exemplifies the relation between γ_{m2} and $\gamma_{m\infty}$ with an ascending α. The claim on $\alpha = 0.5$ can also be empirically justified from Fig. 12. Simulation experience indicates that a good starting point for the value of α in mixed $\mathrm{R_xUIO}$ can be selected from the interval $[0.4, 0.6]$. The $\mathrm{R_xUIO}$ idea here is also extensible to the popular linear parameter varying model case.

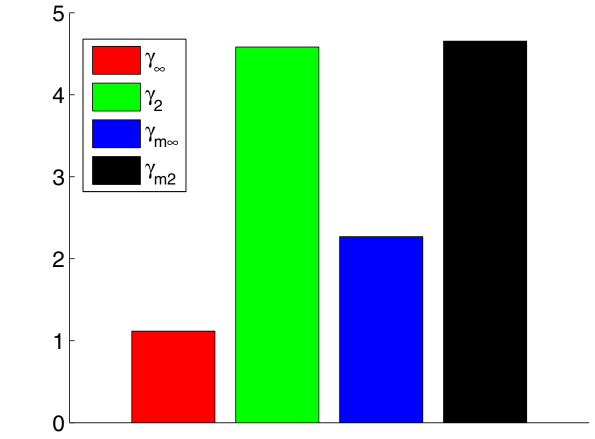

Fig. 10. Optimized γ parameters

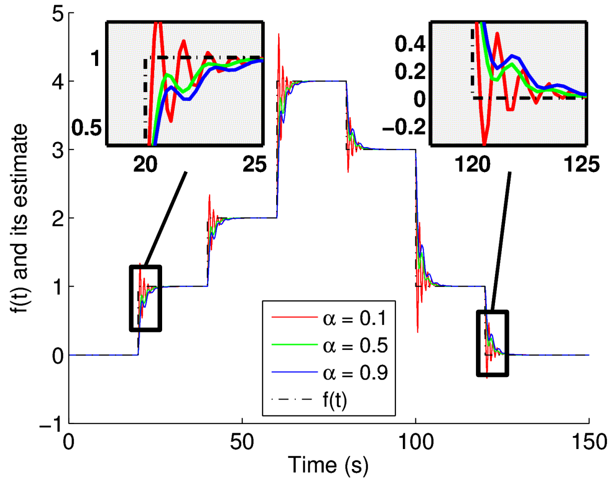

Fig. 11. Effect of α on stepwise fault estimation

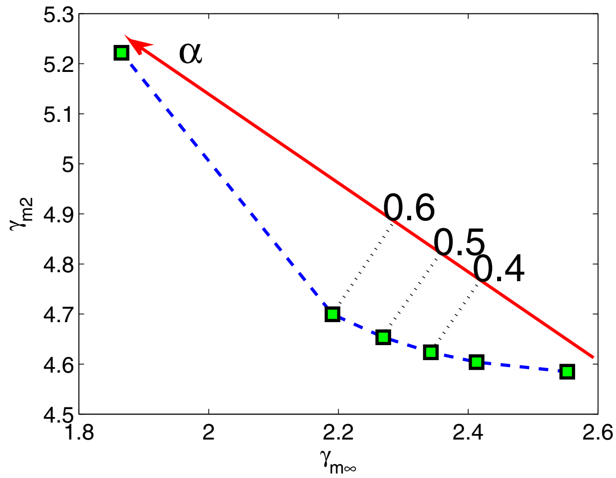

Fig. 12. γ parameters of the mixed H_2/H_∞ $\mathrm{R_xUIO}$ with an ascending α (arrow direction)

5. CONCLUSION

This paper has proposed the $\mathrm{R_x}$UIO concept to deal with the case when the conventional UIO is impractical due to the strict rank limitation. The suggested method is still applicable when the strong disturbance decoupling is infeasible. The application of this given work to the issue of (rank) degenerate state and fault estimation validates the encouraging potential of the proposed method. Our future work will take into account the pole location constraints associated with estimation transients and make a comparison with Luenberger-type observers under deterministic disturbance.

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