A new strategy for integration of fault estimation within fault-tolerant control

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Abstract

The problem of active fault tolerant control (FTC) of dynamical systems involves the process of fault detection and isolation/fault estimation (FDI/FE) used to either make a decision as to when and how to change the control, based on FDI or to compensate the fault in the control system via FE. The combination of the decision-making/estimation and control gives rise to a bi-directional uncertainty in which the modelling and fault uncertainties and disturbances all affect the quality and robustness of the FTC system. This leads to the FTC requirement for an integrated design of the FDI/FE and control system reconfiguration. This paper focuses on the FTC approach using FE and fault compensation within the control system in which the design is achieved by integrating together the FE and FTC controller modules. The FE is based on a modified reduced-/full-order unknown input observer and the FTC system is constructed by sliding mode control using state/output feedback. The integrated design is converted into an observer-based robust control problem solved via H_∞ optimization with a single-step LMI formulation. The performance effectiveness of the proposed integrated design approach is illustrated through studying the control of an uncertain model of a DC motor.

Key words: Fault-tolerant control, fault estimation, sliding mode control, additive/multiplicative faults, robust control

1 Introduction

Due to high demands on the reliability, safety, and acceptable performance of automatic systems, such as aircraft, nuclear power plants, robotic systems, and chemical plants, fault detection and isolation(FDI) and fault-tolerant control (FTC) have become important research theory and application topics in the control community (Patton, 1997; Blanke et al., 2006; Zhang & Jiang, 2008; Ding, 2009; Isermann, 2011; Zhang et al., 2014; Feng & Patton, 2014; Wang, 2015; Su et al., 2015).

1.1 Integration of FD/FTC

It has been known for some time that the FDI performance for closed-loop systems is affected by the controller when the system has modelling uncertainty (Nett et al., 1988; Kilsgaard et al., 1996; Niemann & Stoustrup, 1997; Patton, 1997; Suzuki & Tomizuka, 1999; Zhou & Ren, 2001; Khosrowjerdi et al., 2004; Henry & Zolghadri, 2005; Blanke et al., 2006; Zhang & Jiang, 2006; Weng et al., 2008; Davoodi et al., 2011, 2014a,b; Zhong & Yang, 2015). However, all these studies focus on the integration of control and FDI and do not consider the integrated design of FDI within an active FTC system. The term *active* FTC here means that the controller changes in an active way according to the effects that faults have on the control reconfiguration. The passive form is just an extension of robust control in which the faults are considered as an additional form of uncertainty affecting the closed-loop system.

The design integration is a hard challenge since the reconfiguration and FDI roles have a bi-directional uncertainty which is more complex when compared with integration of FDI within a closed-loop system i.e. without an FTC function. The complexity arises from the joint multi-objectives of robust closed-loop stability, robust residual performance (requiring optimal fault detection thresholds), and robust fault tolerance with stable reconfiguration, generally operating in the presence of variable time delay and uncertainty.

Furthermore, the FTC systems that use robust FDI are exceedingly difficult to design and implement because of (a) discrete-event structure with complex decision, (b) variable and unknown time delay, and (c) a control re-

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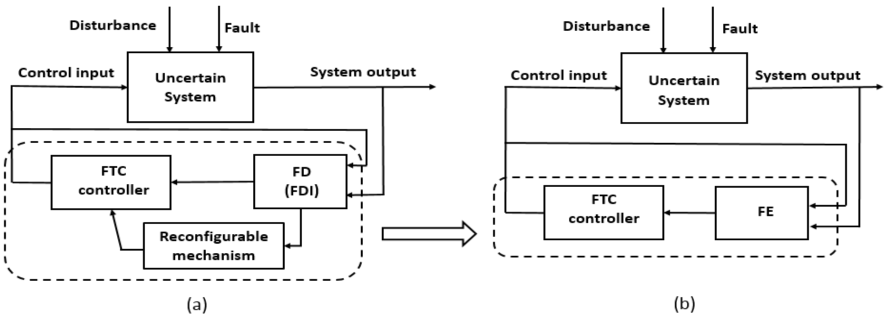

Fig. 1. Integration of (a) FD/FTC and (b) FE/FTC

configuration that is very complex if not impossible in some cases (Yang et al., 2009; Cieslak et al., 2015). This approach to FTC is one of the most difficult problems of adaptive control and in general is not suitable for practical application. As shown in Fig. 1, a systematic and easily implementable integrated FTC design approach based on FE rather than on the traditional residual-based FDI is of interest in this paper.

1.2 Integration of FE/FTC

As a powerful alternative to using the traditional FDI approach in FTC systems, it is attractive to consider a direct reconstruction of the fault signal through FE, once it occurs. The FE function intrinsically includes the fault detection and fault isolation roles and the more complex FDI role is thus obviated. In this approach the reconstructed or estimated fault signals are used directly in the control system to compensate for the effects of the faults. Several approaches to FE design for FTC have been proposed, e.g. based on: adaptive observer (Kaboré & Wang, 2001; Jiang et al., 2006), sliding mode observer (SMO) (Edwards & Tan, 2006), extended state observer (ESO) (Gao & Ding, 2007), unknown input observer (UIO) (Odgaard & Stoustrup, 2012), and moving horizon estimation (Feng & Patton, 2014). A combination of SMO and ESO was also proposed in Shi et al. (2015).

The direct use of FE without the need for a reconfigurable mechanism brings significant convenience and application potential to the subject of FTC system design. This approach can also facilitate the development of robust methods of truly integrated FE/FTC design, taking account of the bi-directional uncertainty. However, the available literature for the integrated design of FE/FTC is limited. Jiang et al. (2006) dealt with the FE based fault accommodation for non-linear systems with actuator faults. Rodrigues et al. (2014) proposed an observer-based FTC design for LPV descriptor systems with actuator faults. Cheng et al. (2011) considered this problem including disturbance and uncertainty, focusing on a reliable satellite attitude control system subject to sensor faults. However, their approach used the so-called passive FTC without FE, i.e. based on robust control and cannot be included in the active approach to FTC that is important in this current study.

1.3 Contribution

Inspired by the above background, this paper develops a novel integrated FTC design for uncertain linear systems with additive/multiplicative faults and disturbance. Compared to the relevant existing literature, the main contributions of this paper are:

• *Reduced-/full-order UIOs without rank condition are proposed to achieve FE.* Although there are many existing FE observers as listed above, in the adaptive observer the faults are estimated with finite error, and for the estimation of time-varying faults, the observer has a proportional-integration (PI) structure with carefully chosen learning rate. The canonical form SMO (Edwards & Tan, 2006) requires several state transformations as well as *a priori* knowledge of the upper bounds of the faults and it is difficult to reconstruct sensor and actuator faults simultaneously. The ESO reconstructs the faults in a polynomial form with *a priori* knowledge of orders. The moving horizon estimation is a highly complex on-line optimization problem. The existing UIOs are obtained after satisfying a well-known rank condition (Chen & Patton, 1999). In this study, reduced-/full-order UIOs without rank condition are modified from Chen & Patton (1999) and Xiong & Saif (2003) and combined with the ESO to achieve either (a) combined state and fault estimation for time-varying faults, or (b) simultaneous time-varying faults and system states for the output feedback case. The proposed UIOs do not require state transformation and fault information (upper bounds and fault characteristics), or on-line computation. Another new property of the reduced-order UIO for FE is that the estimation of the system states is not necessary, leading to the design of an observer with reduced dimension.

• *Both the cases of state/output feedback sliding mode FTC are studied.* Considering its potential robustness to uncertainty and disturbance, SMC has recently been used extensively for FTC design (e.g., Alwi & Edwards (2008); Xiao et al. (2012); Zhao et al. (2014); Huang & Patton (2015)). Few works consider unmatched norm-bounded system uncertainty and FE design and little attention has been paid to the output feedback case. Here, sliding mode FTC designs for both state and output feedback cases are developed for systems subject to unmatched norm-bounded uncertainty.

• *A novel integrated FE/FTC design strategy is proposed.* For systems with additive or multiplicative faults, an integrated FE/FTC strategy is developed by designing together the FE and FTC controller via H_∞ optimization with a single-step LMI formulation.

1.4 Structure and notation

The paper is organized as follows. Section 2 gives the problem formulation. The integrated FE/FTC designs with additive faults using state/output feedback are considered in Sections 3 and 4, respectively. The integrated design for systems with multiplicative faults is outlined in Section 5. Section 6 provides an illustrative example, and Section 7 concludes the study. The symbol † represents the pseudo inverse, $\|\cdot\|$ represents the Euclidean norm, $\mathrm{He}(X) = X + X^\top$, and $\star$ represents the symmetric part of a matrix.

2 Problem formulation

In order to illustrate the concept of FE/FTC integration in a simple and effective way, the following class of linear systems with system state uncertainty is considered

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(t)) + Bu(t) + F_a f_a(t) + Dd(t)\\ y(t) &= Cx(t) + F_s f_s(t)\end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^p$ stand for the state, control input, and system output, respectively. $f_a \in R^q$ and $f_s \in R^{q_1}$ denote respectively the actuator and sensor faults, which might be viewed as actuator/sensor offsets in physical systems (Isermann, 2011). $d \in R^l$ denotes the external disturbance. $\Delta A(t)$ represents the unknown unmatched system matrix uncertainty. The matrices A, B, F_a, D, C, and F_s are known constant matrices of compatible dimensions. Without loss of generality, assume that $q \le m$ and $q_1 \le p$.

Assumption 2.1 *The pair (A, C) is observable, the pair (A, B) is controllable, and $rank(B, F_a) = rank(B) = m$.*

Assumption 2.2 *The uncertainty matrix $\Delta A(t)$ is norm-bounded (energy bounded) with the form*

$$\Delta A(t) = M_0 F_0(t) N_0$$

where M_0 and N_0 are known matrices with appropriate dimensions, and $F_0(t)$ is an unknown matrix satisfying $F_0^\top(t)F_0(t) \le I$.

Assumption 2.3 *The faults and disturbance satisfy $\|f_a\| \le \bar{f}_a$, $\|f_s\| \le \bar{f}_s$, and $\|d\| \le d_0$ with unknown positive scalars $\bar{f}_a$, $\bar{f}_s$, and d_0, respectively. Moreover, f_a and f_s have norm-bounded first time derivatives.*

Remark 2.1 *Assumption 2.1 provides some standard requirements of controlled systems, while $rank(B, F_a) = rank(B)$ ensures f_a to be in the range space of the control u so that the fault effect is compensated through the control action. Assumption 2.2 gives a general representation of a system unmatched uncertainty matrix using H_∞ optimization. Assumption 2.3 implies that the considered faults and disturbance are norm-bounded with unknown upper bounds, this is useful for practical application.*

Augmenting the system (1) into

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \Delta\bar{A}\bar{x} + \bar{D}\bar{d}\\ y &= \bar{C}\bar{x}\end{aligned} \tag{2}$$

where $\bar{x} = [x^\top\ f_a^\top\ f_s^\top]^\top$, $\bar{d} = [d^\top\ \dot{f}_a^\top\ \dot{f}_s^\top]^\top$, and

$$\bar{A} = \begin{bmatrix} A & F_a & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \Delta\bar{A} = \begin{bmatrix}\Delta A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \bar{B} = \begin{bmatrix} B\\ 0\\ 0\end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} D & 0 & 0\\ 0 & I_q & 0\\ 0 & 0 & I_{q_1}\end{bmatrix},\ \bar{C} = [C\ 0\ F_s].$$

The following UIO is proposed to estimate the augmented state $\bar{x}$

$$\begin{aligned}\dot{\xi}_o &= M_o\xi_o + G_o u + L_o y\\ \hat{\bar{x}} &= \xi_o + H_o y\end{aligned} \tag{3}$$

where $\xi_o, \hat{\bar{x}} \in R^{n+q+q_1}$ denote respectively the observer system state and the augmented state estimate. The design matrices M_o, G_o, L_o, and H_o are of appropriate dimensions.

Denote $\Xi = I_{n+q+q_1} - H_o\bar{C}$ and $L_o = L_1 + L_2$, so that

$$\begin{aligned}\dot{e}_o = &(\Xi\bar{A} - L_1\bar{C})e_o + (\Xi\bar{A} - L_1\bar{C} - M_o)\xi_o\\ &+(\Xi\bar{B} - G_o)u + [(\Xi\bar{A} - L_1\bar{C})H_o - L_2]y + \chi_a\end{aligned} \tag{4}$$

where $\chi_a = \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d}$.

Necessary conditions for the stability and unbiasedness of the error system (4) are

$$M_o \text{ is Hurwitz} \tag{5}$$
$$\Xi\bar{A} - L_1\bar{C} - M_o = 0 \tag{6}$$
$$\Xi\bar{B} - G_o = 0 \tag{7}$$
$$(\Xi\bar{A} - L_1\bar{C})H_o - L_2 = 0. \tag{8}$$

With (6) - (8), (4) becomes

$$\dot{e}_o = (\Xi\bar{A} - L_1\bar{C})e_o + \chi_a. \tag{9}$$

The disturbance and uncertainty can be totally decoupled when $\chi_a = 0$. However, this decoupling is usually not possible in practice so that the term χ_a affects the estimation performance. Define e_x, e_{f_a}, and e_{f_s} as the estimation errors of the state, actuator fault, and sensor fault, respectively. Let $e_o = [e_x^\top \; e_{f_a}^\top \; e_{f_s}^\top]^\top$ and $H_o = [H_1; H_2; H_3]$. Note that

$$\chi_a = \begin{bmatrix} (I_n - H_1C)(Dd + \Delta Ax) - H_1F_s\dot{f}_s \\ -H_2C(Dd + \Delta Ax) + \dot{f}_a - H_2F_s\dot{f}_s \\ -H_3C(Dd + \Delta Ax) + (I_{q_1} - H_3F_s)\dot{f}_s \end{bmatrix}. \tag{10}$$

It is observed from (9) and (10) that the model mismatch between the observer and the control system (i.e., system uncertainty ΔAx, disturbance d, and fault modelling errors $\dot{f}_a$ and $\dot{f}_s$) affects the state and fault estimation performance. If the actuator and sensor faults occur simultaneously, the sensor fault modelling error will also affect the actuator fault estimation performance.

A general form of active FTC controller using FE is

$$u = K_a\hat{\bar{x}} \tag{11}$$

where the compatible matrix $K_a = [K_x \; K_f]$ includes the nominal controller gain K_x and the fault compensation gain K_f. According to Assumption 2.1, it can be chosen that $K_f = B^\dagger F_a$.

Substituting (11) into (1) yields the closed-loop system

$$\dot{x} = (A + BK_x)x - BK_ae_o + \Delta Ax + Dd. \tag{12}$$

As shown in (12), the system uncertainty, disturbance, and estimation errors all affect the FTC performance.

It follows from (9) and (12) that there exists a bi-directional robustness interaction between the observer and the control system, which is summarized in Fig. 2. In most cases, the bi-directional interaction exists commonly in the designs of observer-based FE within FTC systems due to the existence of the inevitable model mismatch and estimation error. This bi-directional interaction breaks down the well-known Separation Principle and thus puts forward the necessity and importance of the integrated design of FE/FTC, which motives us to study the following problem.

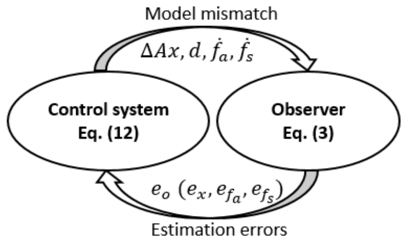

Fig. 2. Bi-directional interaction within FTC systems

Problem 2.1 *Given the system (1) with uncertainty, disturbance, and faults, it is required to design together the following FE and FTC modules to guarantee system stability after the fault occurrence: (i) Observer: estimate the faults for the state feedback case, and simultaneously the faults and states for the output feedback case; (ii) Sliding mode FTC controller: state/output feedback.*

Note that **Problem 2.1** is an observer-based robust control problem, which for solution requires a bilinear matrix inequality (BMI) that cannot be solved directly using the LMI toolbox. To obviate this BMI problem, Shi & Patton (2015) proposed a two-step method for integrated FE/FTC design for LPV descriptor systems with actuator/sensor faults and disturbance. However, this two-step approach can only obtain a suboptimal solution of the integrated design, and the feasible controller gains obtained in the first step cannot guarantee the solvability of the observer designed in the second step.

A robust observer-based control design was proposed in Lien (2004) for uncertain linear systems with equality constraint solved by a single-step LMI formulation. More recently, a new observer-based control design approach was proposed in Kheloufi et al. (2013) without equality constraint with the help of the Young relation. However, as commented in Wang & Jiang (2014), this new approach has no superiority over the one in Lien (2004), which is thus used in this study.

3 Integrated FE/FTC design: state feedback

Provided that all the system states are available, then only the fault estimation is needed for the FTC design.

3.1 FE design

Note that the observability of (2) is equivalent to that of (1). The reduced-order UIO in Xiong & Saif (2003) is modified here to estimate the faults f_a and f_s, i.e., $z = L\bar{x}$ with $L = [0\ I_{q+q_1}] \in R^{(q+q_1)\times(n+q+q_1)}$

$$\begin{aligned} \dot{\xi}_s &= M\xi_s + Gu + Ry \\ \hat{z} &= \xi_s + Hy \end{aligned} \tag{13}$$

where $\xi_s, \hat{z} \in R^{q+q_1}$ denote the observer state and the estimate of z, respectively. The design matrices M, G, R, and H are of appropriate dimensions.

Define $\varepsilon = \xi_s - T\bar{x}$, it follows that

$$\begin{aligned} \dot{\varepsilon} &= M\varepsilon + (MT + R\bar{C} - T\bar{A})\bar{x} + (G - T\bar{B})u - T\Delta\bar{A}\bar{x} \\ &\quad - T\bar{D}\bar{d} \\ e_s &= \varepsilon + (T + H\bar{C} - L)\bar{x}. \end{aligned} \tag{14}$$

Theorem 3.1 *There exists a stable and unbiased generalized observer (13) for the system (2) when $\Delta\bar{A}\bar{x} = 0$ and $\bar{d} = 0$, if it holds that*

$$M \text{ is Hurwitz} \tag{15}$$
$$MT + R\bar{C} - T\bar{A} = 0 \tag{16}$$
$$T + H\bar{C} - L = 0 \tag{17}$$
$$G - T\bar{B} = 0. \tag{18}$$

Proof 3.1 *With (16) - (18) and $\Delta\bar{A}\bar{x} = 0$ and $\bar{d} = 0$, the error system (14) becomes*

$$\begin{aligned} \dot{\varepsilon} &= M\varepsilon \\ e_s &= \varepsilon. \end{aligned}$$

Since M is Hurwitz, $\lim_{t\to\infty} e_s(t) = 0$.

Define a full-row rank matrix: $S = [L^\dagger\ (I_{n+q+q_1} - L^\dagger L)] = [S_1\ S_2]$ with the property that $S_2 S_1 = 0$ and $\mathrm{rank}(S) = \mathrm{rank}(S_1) + \mathrm{rank}(S_2) = n + q + q_1$.

Lemma 3.1 *The matrix equation*

$$\Lambda\Omega = \Psi \tag{19}$$

with $\Omega = \begin{bmatrix} \bar{C}S_2 \\ \bar{C}\bar{A}S_2 \end{bmatrix}$, $\Psi = L\bar{A}S_2$, and the determined matrix Λ, is solvable if it holds that

$$rank \begin{bmatrix} L\bar{A} \\ \bar{C} \\ \bar{C}\bar{A} \\ L \end{bmatrix} = rank \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \\ L \end{bmatrix}. \tag{20}$$

Proof 3.2 *Post-multiplying both sides of (20) with a full row-rank matrix $[S_2\ S_1]$ gives*

$$rank \begin{bmatrix} \Psi \\ \Omega \end{bmatrix} = rank(\Omega).$$

Thus, the matrix equation (19) is solvable.

Lemma 3.2 *The pair (M_2, M_1), where $M_1 = L\bar{A}S_1 - \Psi\Omega^\dagger\Gamma$, $M_2 = (I_{2p} - \Omega\Omega^\dagger)\Gamma$, and $\Gamma = \begin{bmatrix} \bar{C}S_1 \\ \bar{C}\bar{A}S_1 \end{bmatrix}$, is detectable, if it holds that, $\forall s \in \mathcal{C},\ Re(s) \geq 0$,*

$$rank \begin{bmatrix} sL - L\bar{A} \\ \bar{C} \\ \bar{C}\bar{A} \end{bmatrix} = rank \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \\ L \end{bmatrix}. \tag{21}$$

Proof 3.3 *See Appendix A.*

It follows from (17) that $T = L - H\bar{C}$, and substituting this into (15) yields

$$M(L - H\bar{C}) + R\bar{C} - (L - H\bar{C})\bar{A} = 0.$$

Now denote $T_1 = R - MH$, it follows that

$$L\bar{A} - ML = [T_1\ H] \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \end{bmatrix}. \tag{22}$$

Post-multiplying both sides of (22) by S yields

$$M = L\bar{A}S_1 - [T_1\ \ H] \begin{bmatrix} \bar{C}S_1 \\ \bar{C}\bar{A}S_1 \end{bmatrix} \tag{23}$$

$$L\bar{A}S_2 = [T_1\ \ H] \begin{bmatrix} \bar{C}S_2 \\ \bar{C}\bar{A}S_2 \end{bmatrix}. \tag{24}$$

Rearranging (24) as $[T_1\ H]\Omega = \Psi$, which is solvable according to Lemma 3.1 with a general solution

$$[T_1\ H] = \Psi\Omega^\dagger + Z(I_{2p} - \Omega\Omega^\dagger) \tag{25}$$

where $Z \in R^{(q+q_1)\times 2p}$ is an arbitrary matrix.

It follows from (23) and (25) that

$$M = M_1 - ZM_2,\ H = H_1 + ZH_2 \tag{26}$$

where $H_1 = \Psi\Omega^\dagger\Gamma_1$, $H_2 = (I_{2p} - \Omega\Omega^\dagger)\Gamma_1$, and $\Gamma_1 = [0\ I_p]^\top$.

The matrices M_1 and M_2 in (26) are known from Lemma 3.2. By designing Z to make M Hurwitz, H can be obtained. Further using $T_1 = R - MH$ gives R and using Theorem 3.1 gives G.

However, since there exist uncertainty and disturbance in the system (1), i.e., $\Delta\bar{A}\bar{x} \neq 0$ and $\bar{d} \neq 0$, the error system (14) should be made robustly stable. Denote $\bar{H}_1 = H_1\bar{C} - L$ and substitute $M = M_1 - ZM_2$ and $T = L - H\bar{C}$ into (19), it follows that

$$\dot{e}_s = (M_1 - ZM_2)e_s + (\bar{H}_1 + ZH_2\bar{C})(\Delta\bar{A}\bar{x} + \bar{D}\bar{d}). \quad (27)$$

Thus, by designing the arbitrary matrix Z to ensure (27) to be robustly stable, the observer (13) for the system (1) with uncertainty and disturbance can be obtained.

Remark 3.1 *The proposed reduced-order UIO (with an order of $q + q_1$) is interesting in three respects: (i) The traditional UIOs (Chen & Patton, 1999; Xiong & Saif, 2003; Odgaard & Stoustrup, 2012) decouple the disturbance with the satisfaction of a rank condition, i.e., $rank(\bar{C}\bar{D}) = rank(\bar{D})$, which is restrictive and often cannot be satisfied. H_∞ optimization is employed here to attenuate the disturbance and the arbitrary matrix Z is obtained using LMI tools; (ii) In contrast to the majority of existing FE approaches (with an order of $n+q+q_1$), the fault estimation is achieved without extra effort to estimate the system states which are available for FTC design; (iii) Note that (20) and (21) are two sufficient conditions for the existence of a solution to Theorem 3.1 as well as the proposed reduced-order UIO. However, since $L\bar{A} = 0$ and $s \in \mathcal{C},\ Re(s) \geq 0$, these two conditions are always satisfied.*

3.2 FTC design

The sliding surface for the system (1) is designed as

$$s_1 = N_1 x \quad (28)$$

where $s_1 \in R^m$ and $N_1 = B^\dagger - Y_1(I_n - BB^\dagger)$ with $B^\dagger = (B^\top B)^{-1}B^\top$ and an arbitrary matrix $Y_1 \in R^{m\times n}$. Differentiating s with respect to time gives

$$\dot{s}_1 = N_1(A + \Delta A)x + u + N_1F_af_a + N_1Dd. \quad (29)$$

Design the control input as

$$u = u_{l_1} + u_{n_1} \quad (30)$$

where the linear feedback component is $u_{l_1} = -K_sx - E_1\hat{f}_a$ with a design matrix $K_s \in R^{m\times n}$ and $E_1 = B^\dagger F_a$. The nonlinear component u_{n_1} is

$$u_{n_1} = \begin{cases} -\rho_{s_1}(t)\frac{s_1}{\|s_1\|}, & s_1 \neq 0 \\ 0, & s_1 = 0 \end{cases}$$

with $\rho_{s_1}(t) = \hat{\eta}_{s_1} + \varphi_{s_1} + \varepsilon_{s_1}$. $\varphi_{s_1} > 0$ is a constant scalar and $\varepsilon_{s_1} > 0$ is some small scalar. The scalar $\hat{\eta}_{s_1}$ is introduced to estimate the unknown scalar $\eta_{s_1} = \|N_1D\|d_0 + \|E_1\|(\bar{f}_a + \|\hat{f}_a\|)$ using an update law

$$\dot{\hat{\eta}}_{s_1} = \sigma_1\|s_1\|,\ \hat{\eta}_{s_1}(0) \geq 0$$

with a learning rate $\sigma_1 > 0$ to be designed.

Define the estimation error of η_{s_1} as $\tilde{\eta}_{s_1} = \eta_{s_1} - \hat{\eta}_{s_1}$. Consider a Lyapunov function

$$V_{s_1} = \frac{1}{2}(s_1^\top s_1 + \frac{1}{\sigma_1}\tilde{\eta}_{s_1}^2).$$

It follows from (29) and (30) that

$$\begin{aligned} \dot{V}_{s_1} &= s_1^\top \dot{s}_1 - \frac{1}{\sigma_1}\tilde{\eta}_{s_1}\dot{\hat{\eta}}_{s_1} \\ &\leq (\omega_{s_1}\|x\| + \eta_{s_1} - \rho_{s_1}(t))\|s_1\| - \tilde{\eta}_{s_1}\|s_1\| \\ &\leq (\omega_{s_1}\|x\| - \varphi_{s_1} - \varepsilon_{s_1})\|s_1\|. \end{aligned}$$

where $\omega_{s_1} = \|N_1A - K_s\| + \|N_1M_0\|\|N_0\|$. By choosing $\varphi_{s_1} > \omega_{s_1}\phi_{s_1}$ with some scalar $\phi_{s_1} > 0$, it follows that the reaching and sliding conditions are satisfied, i.e., $s_1^\top\dot{s}_1 \leq -\varepsilon_{s_1}\|s_1\|$, in the subset $\Omega_{s_1} = \{x : \|x\| \leq \phi_{s_1}\}$. Thus, the controller (30) ensures that if $x(0) \in \Omega_{s_1}$, then for all $t > \|s_1(0)\|/\varepsilon_{s_1}$, $s_1 = \dot{s}_1 = 0$.

Now consider the system stability analysis corresponding to the sliding mode. Suppose that the system has already been controlled to remain within the sliding mode (28). Substituting the equivalent control

$$u_{eq_1} = -(N_1Ax + N_1Dd) + u_{l_1} \quad (31)$$

into (1) gives the closed-loop system

$$\dot{x} = (\Theta_1A - BK_s)x + \Delta Ax + F_1e_s + \Theta_1Dd \quad (32)$$

where $\Theta_1 = I_n - BN_1$ and $F_1 = [F_a\ 0]$.

Thus, by designing K_x such that (32) is robustly stable, then the system (1) is maintained on the sliding mode with the equivalent control (31).

3.3 Integrated synthesis

The augmented closed-loop system consisting of (27) and (32) is

$$\begin{aligned} \dot{x} &= (\Theta_1A - BK_s)x + \Delta Ax + F_1e_s + D_1\bar{d} \\ \dot{e}_s &= (M_1 - ZM_2)e_s + (\bar{H}_1 + ZH_2\bar{C})(\Delta\bar{A}\bar{x} + \bar{D}\bar{d}) \\ y_c &= y - F_s\hat{f}_s \\ z_s &= C_{sx}x + C_{se}e_s \end{aligned} \quad (33)$$

where $z_s \in R^r$ is the measured output and $D_1 = [\Theta_1D\ 0]$.

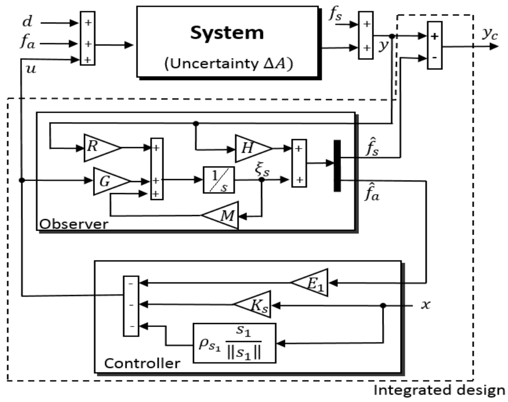

Fig. 3. Integrated FE/FTC design: state feedback case

Theorem 3.2 *Under Assumptions 2.1 - 2.3, given a positive scalar γ_s, the augmented closed-loop system (33) is stable with H_∞ performance $\|G_{z_s\bar{d}}\|_\infty < \gamma_s$, if there exist symmetric positive definite matrices P and Q, and matrices $\hat{P}$, R_1, and R_2 such that*

$$PB = B\hat{P} \tag{34}$$

$$\begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} & 0 & C_{sx}^\top \\ \star & \chi_{22} & \chi_{23} & 0 & \chi_{25} & C_{se}^\top \\ \star & \star & -\gamma_s^2 I & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{35}$$

where $\chi_{11} = He(P\Theta_1 A - BR_1) + 2N_0^\top N_0$, $\chi_{12} = PF_1$, $\chi_{13} = PD_1$, $\chi_{14} = PM_0$, $\chi_{22} = He(QM_1 - R_2M_2)$, and $\chi_{23} = (Q\bar{H}_1 + R_2H_2\bar{C})\bar{D}$, $\chi_{25} = (Q\bar{H}_1 + R_2H_2\bar{C})\bar{M}_0$, then the gains are given by $K_s = \hat{P}^{-1}R_1$ and $Z = Q^{-1}R_2$.

Proof 3.4 *See Appendix B.*

Remark 3.2 *Note that the equality constraint (34) is difficult to solve using the LMI toolbox. However, by using the method presented in Corless & Tu (1998), for a positive scalar β_s, it can be converted into the following optimization problem and solved using the LMI toolbox:*

$$\text{Minimize } \beta_s$$
$$\text{subject to (35) and } \begin{bmatrix} \beta_s I & PB - B\hat{P} \\ \star & \beta_s I \end{bmatrix} > 0.$$

The proposed state feedback based integrated FE/FTC design is summarized in Fig. 3.

4 Integrated FE/FTC design: output feedback

Section 3 presents a state feedback based integrated FE/FTC strategy with the assumption that the system states are fully available. However, this is often not the case in practical applications. This section considers an output feedback based integrated FE/FTC strategy, for which purpose one more assumption is made.

Assumption 4.1 $\text{rank}(CB) = \text{rank}(B)$.

4.1 FE design

The observer (3) is proposed to estimate simultaneously the state and faults, i.e., the augmented state $\bar{x}$. It follows from Section 2 that with (6) - (8) the error system is

$$\dot{e}_o = (\Xi\bar{A} - L_1\bar{C})e_o + \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d}. \tag{36}$$

Thus, by designing H_0 and L_1 such that the error system (36) is robustly stable, the observer (3) can be obtained.

4.2 FTC design

A sliding surface using the output feedback information for the system (1) is designed as

$$s_2 = N_2 y_c \tag{37}$$

where $s_2 \in R^m$, $N_2 = (CB)^\dagger - Y_2(I_p - CB(CB)^\dagger)$ with an arbitrary matrix $Y_2 \in R^{m\times p}$ and $(CB)^\dagger = ((CB)^\top CB)^{-1}(CB)^\top$. $y_c = y - F_s\hat{f}_s = Cx + F_s e_{f_s}$ with the sensor fault estimation error e_{f_s}.

Differentiating s_2 with respect to time gives

$$\dot{s}_2 = N_2C((A+\Delta A)x + F_af_a + Dd) + N_2F_s\dot{e}_{f_s} + u. \tag{38}$$

Design the control input as

$$u = u_{l_2} + u_{n_2} \tag{39}$$

where the linear component is $u_{l_2} = -K_o\hat{x} - E_2\hat{f}_a$ with a design matrix $K_o \in R^{m\times n}$ and $E_2 = B^\dagger F_a$. $\hat{x}$ and $\hat{f}_a$ are the estimates of the system state and actuator fault, respectively. The nonlinear component u_{n_2} is

$$u_{n_2} = \begin{cases} -\rho_{s_2}(t)\frac{s_2}{\|s_2\|}, & s_2 \neq 0 \\ 0, & s_2 = 0 \end{cases}$$

with $\rho_{s_2}(t) = \hat{\eta}_{s_2} + \varphi_{s_2} + \varepsilon_{s_2}$. $\varphi_{s_2} > 0$ is a design constant and $\varepsilon_{s_2} > 0$ is some small constant. The scalar $\hat{\eta}_{s_2}$ is used to estimate the unknown scalar $\eta_{s_2} = \|N_2CD\|d_0 +$

$\|E_2\|(\|\hat{f}_a\| + \bar{f}_a) + \|K_o\|\|e_x\| + \|N_2\|(\|\dot{\hat{f}}_s\| + \|\dot{f}_s\|)$ with an update law

$$\dot{\hat{\eta}}_{s_2} = \sigma_2 \|s_2\|, \;\; \hat{\eta}_{s_2}(0) \geq 0$$

where $\sigma_2 > 0$ is a given constant scalar.

Define the estimation error of η_{s_2} as $\tilde{\eta}_{s_2} = \eta_{s_2} - \hat{\eta}_{s_2}$. Consider a Lyapunov function

$$V_{s_2} = \frac{1}{2}(s_2^\top s_2 + \frac{1}{\sigma_2}\tilde{\eta}_{s_2}^2).$$

It follows from (38) - (39) that

$$\dot{V}_{s_2} \leq (\omega_{s_2}\|x\| - \varphi_{s_2} - \varepsilon_{s_2})\|s_2\|.$$

where $\omega_{s_2} = \|N_2CA - K_o\| + \|N_2M_0\|\|N_0\|$. Choosing $\varphi_{s_2} > \omega_{s_2}\phi_{s_2}$ with a given scalar $\phi_{s_2} > 0$, then the reaching and sliding condition is satisfied, i.e., $s_2^\top \dot{s}_2 \leq -\varepsilon_{s_2}\|s_2\|$, in the subset $\Omega_{s_2} = \{x : \|x\| \leq \phi_{s_2}\}$. Thus, the controller (39) ensures that if $x(0) \in \Omega_{s_2}$, then for all $t > \|s_2(0)\|/\varepsilon_{s_2}$, $s_2 = \dot{s}_2 = 0$.

Consider the analysis of the system stability on the sliding mode. Given the equivalent control

$$u_{eq_2} = -(N_2CAx + N_2CDd) + u_{l_2} \tag{40}$$

and denote $\Theta_2 = I_n - BN_2C$ and $F_2 = [BK_o \; F_a \; 0]$. It follows from (1) that (40) can maintain the system on the sliding mode by designing K_o such that the following closed-loop system is robustly stable

$$\dot{x} = (\Theta_2 A - BK_o)x + F_2 e_o + \Delta A x + \Theta_2 D d. \tag{41}$$

4.3 Integrated synthesis

The augmented closed-loop system consisting of (36) and (41) is

$$\begin{aligned} \dot{x} &= (\Theta_2 A - BK_o)x + F_2 e_o + \Delta A x + D_2 \bar{d} \\ \dot{e}_o &= (\Xi \bar{A} - L_1 \bar{C})e_o + \Xi \Delta \bar{A} \bar{x} + \Xi \bar{D} \bar{d} \\ y_c &= y - F_s \hat{f}_s \\ z_o &= C_{ox}x + C_{oe}e_o \end{aligned} \tag{42}$$

where $z_o \in R^r$ is the measured output and $D_2 = [\Theta_2 D \; 0]$.

Theorem 4.1 *Under Assumptions 2.1 - 2.3 and 4.1, given a positive scalar γ_o, the augmented closed-loop system (42) is stable with H_∞ performance $\|G_{z_o\bar{d}}\|_\infty < \gamma_o$, if there exist symmetric positive definite matrices P_o, Q_1, Q_2, and Q_3, and matrices $\hat{P}_o$, X_1, X_2, X_3, X_4, X_5, X_6, and X_7 such that*

$$P_o B = B\hat{P}_o \tag{43}$$

$$[\bar{\Xi}]_{10\times 10} < 0 \tag{44}$$

where $\bar{\Xi}$ is a symmetric block matrix whose (i,j) block element is represented by $\bar{\Xi}_{i,j}$. For $1 \leq i \leq j \leq 10$,
$\bar{\Xi}_{1,1} = He(P_o\Theta_2 A - BX_1) + 2N_0^\top N_0, \bar{\Xi}_{1,2} = BX_1$,
$\bar{\Xi}_{1,3} = P_oF_a, \bar{\Xi}_{1,5} = P_o\Theta_2 D, \bar{\Xi}_{1,8} = P_0M_0, \bar{\Xi}_{1,10} = C_{ox}^\top$,
$\bar{\Xi}_{2,2} = He(Q_1A - X_2CA - X_3C)$,
$\bar{\Xi}_{2,3} = Q_1F_a - X_2CF_a - A^\top C^\top X_4^\top - C^\top X_5^\top$,
$\bar{\Xi}_{2,4} = M_3F_s - A^\top C^\top X_6^\top - C^\top X_7^\top$,
$\bar{\Xi}_{2,5} = Q_1D - X_2CD$, $\bar{\Xi}_{2,7} = -X_2F_s$,
$\bar{\Xi}_{2,9} = (Q_1 - X_2C)M_0$, $\bar{\Xi}_{2,10} = C_{ex}^\top$,
$\bar{\Xi}_{3,3} = He(-X_4CF_a)$, $\bar{\Xi}_{3,4} = -X_5F_s - F_a^\top C^\top X_6^\top$,
$\bar{\Xi}_{3,5} = -X_4CD$, $\bar{\Xi}_{3,6} = Q_2$, $\bar{\Xi}_{3,7} = -X_4F_s$,
$\bar{\Xi}_{3,9} = -X_4CM_0$, $\bar{\Xi}_{3,10} = C_{efa}^\top$,
$\bar{\Xi}_{4,4} = He(-X_7F_s)$, $\bar{\Xi}_{4,5} = -X_6CD$, $\bar{\Xi}_{4,7} = Q_3 - X_6F_s$,
$\bar{\Xi}_{4,9} = -X_6CM_0$, $\bar{\Xi}_{4,10} = C_{efs}^\top$,
$\bar{\Xi}_{5,5} = \bar{\Xi}_{6,6} = \bar{\Xi}_{7,7} = -\gamma_o^2 I$, $\bar{\Xi}_{8,8} = \bar{\Xi}_{9,9} = \bar{\Xi}_{10,10} = -I$,
$\bar{\Xi}_{i,j}$ are null for all the others.
Then the gains are given by $K_o = \hat{P}_o^{-1}X_1$, $H_1 = Q_1^{-1}X_2$, $H_2 = Q_2^{-1}X_4$, $H_3 = Q_3^{-1}X_6$, $L_{11} = Q_1^{-1}X_3$, $L_{12} = Q_2^{-1}X_5$, and $L_{13} = Q_3^{-1}X_7$.

Proof 4.1 *Denote $C_{oe} = [C_{ex} \; C_{efa} \; C_{efs}]$, $Q_o = diag(Q_{1(n\times n)}, Q_{2(q\times q)}, Q_{3(q_1\times q_1)})$, $L_1 = [L_{11}; L_{12}; L_{13}]$, and $H_o = [H_1; H_2; H_3]$, and consider the Lyapunov functions $V_{x_o} = x^\top P_o x$ and $V_{e_o} = e_o^\top Q_o e_o$. The proof is similar to that of Theorem 3.2, and thus is omitted here.*

According to Remark 3.2, Theorem 4.1 can be further converted into an optimization problem with a positive scalar β_o. The proposed output feedback based integrated FE/FTC design is summarized in Fig. 4.

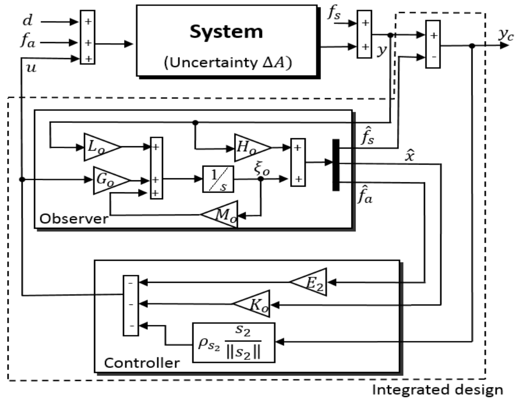

Fig. 4. Integrated FE/FTC design: output feedback case

5 Integrated FE/FTC design: multiplicative faults

The previous Sections focus on the integrated FE/FTC designs for systems with actuator/sensor faults. The considered faults are added to the system state and output, i.e., additive faults, resulting in changes in the mean values of the system states and outputs. Besides additive faults, multiplicative faults which are defined as component faults (even some kinds of actuator and sensor faults are in the form of multiplicative faults, e.g., partial loss of actuator effectiveness) also need to be discussed, since they affect the stability and degrade the performance of the post-fault system. Several works have been published on some topics related to multiplicative faults, e.g., multiplicative fault modelling and diagnosis (Ding, 2008), and multiplicative fault estimation (Wang & Daley, 1996; Tan & Edwards, 2004; Gao & Duan, 2012).

Consider an uncertain linear systems in the form of

$$\begin{aligned} \dot{x} &= (A + \Delta A(t))x + Bu + F_m f_m + Dd \\ y &= Cx \end{aligned} \tag{45}$$

where $F_m \in R^{n \times q_m}$ and other terms are defined in (1). The fictitious multiplicative fault $f_m \in R^{q_m}$ is

$$f_m = B_m \sum_{i=1}^{q_m} \theta_i \phi_i(A, B, x, u) \tag{46}$$

where $B_m = F_m^{\dagger} - (F_m^{\dagger} F_m - I_{q_m})W$ with an arbitrary matrix $W \in R^{q_m \times n}$. $\theta_i \in R^1$, $i = 1, 2, \cdots, q_m$, are time-varying scalar functions denoting the multiplicative faults, and $\phi_i(A, B, x, u) \in R^{n \times 1}$, $i = 1, 2, \cdots, q_m$ are known functions related to A, B, x, and u.

The formulation (46) represents a wide class of multiplicative faults, e.g.,

$$\begin{aligned}
&\sum_{i=1}^{q_A} \theta_{Ai} A_i x = F_{m_A} \left(B_{m_A} \sum_{i=1}^{q_A} \theta_{Ai} A_i x \right) = F_{m_A} f_{m_A}, \\
&\sum_{i=1}^{q_B} \theta_{Bi} B_i u = F_{m_B} \left(B_{m_B} \sum_{i=1}^{q_B} \theta_{Bi} B_i u \right) = F_{m_B} f_{m_B}, \\
&\sum_{i=1}^{q_A} \theta_{Ai} A_i x + \sum_{i=1}^{q_B} \theta_{Bi} B_i u \\
&= F_m \left(B_m \sum_{i=1}^{q_A} \theta_{Ai} A_i x + B_m \sum_{i=1}^{q_B} \theta_{Bi} B_i u \right) = F_m f_m
\end{aligned}$$

where A_i, $i = 1, 2, \cdots, q_A$, and B_i, $i = 1, 2, \cdots, q_B$, denote the known matrices related to A and B.

In the literature (Wang & Daley, 1996; Tan & Edwards, 2004; Ding, 2008; Gao & Duan, 2012), the effort was put into the estimation of θ_i, $i = 1, 2, \cdots, q_m$. However, few works have been published on FTC design for systems with multiplicative faults. Provided that the aim is to achieve acceptable closed-loop system performance, the purpose of FTC design is to compensate for the effect of the multiplicative faults, whatever their sources or size. This can be achieved even if the fictitious multiplicative fault f_m cannot reflect the real fault location and size. In this respect, the integrated FE/FTC design of the system (45) along with multiplicative fault can be achieved through the designs proposed in Sections 3 - 4 with minor modification, by estimating and compensating the fictitious multiplicative fault f_m with the chosen F_m satisfying $\text{rank}(B, F_m) = \text{rank}(B) = m \leq n$.

Remark 5.1 *The considered system (1) is required to satisfy the matching condition, i.e., $rank(B, F_a) = rank(B)$. However, when $rank(B, F_a) \neq rank(B)$ but $rank(B) = m \leq n$, the actuator fault f_a can be handled in the following way: Denote $F_a f_a = (BB^{\dagger} + B^{\perp} B^{\perp^{\dagger}}) F_a f_a$ where $B^{\perp} \in R^{n \times (n-m)}$ spans the null space of B and $BB^{\dagger} + B^{\perp} B^{\perp^{\dagger}} = I_n$. Using the proposed design strategy, the matching part $BB^{\dagger} F_a f_a$ of the actuator fault can be estimated and compensated, while the unmatched part $B^{\perp} B^{\perp^{\dagger}} F_a f_a$ can be treated as a disturbance covered by H_∞ optimization.*

Remark 5.2 *In this paper, the existing nonlinear constraints are converted into linear ones by introducing equality constraints. Although this facilitates the solution of the formulated optimization problem, the equality constraints impose restrictions on the controlled system models. As discussed in Lien (2004) and Kheloufi et al. (2013), necessary conditions for the feasibility of the obtained LMIs are: (1) The system (1) is stabilizable and detectable; and (2) The matrix B is full-column rank. These two conditions are satisfied for most controlled systems. However, more conservativeness might be imposed on the optimization problem in some special cases, e.g., for the DC motor model studied in Section 6, the symmetric positive definite matrices P and P_0 are required to be diagonal as the matrix B is of the form $B = [B_1; 0]$ where B_1 is a none null matrix of appropriate dimension.*

Remark 5.3 *Recall that by using an adaptive law in the proposed controller to estimate the unknown scalar related to the faults and disturbance, of which a priori knowledge of the upper bounds are not required. This adaptive updating requires some on-line computation. However, except for the adaptive gains all the other controller and observer gains are pre-determined off-line, mainly by solving single-step LMIs. Moreover, the proposed design procedure is quite straightforward and easy to follow. The proposed integrated design strategies are with acceptable computational complexity and can be applicable in practice.*

6 Case Study

Considering the stabilization control for a DC motor with the state space model

$$\dot{x} = (A + \Delta A)x + Bu + Dd$$
$$y = Cx \tag{47}$$

with states $x = [i_a \ w]^\top$, control input $u = v_a$, disturbance $d = -\frac{T_l}{J_i}$, output y, and

$$A = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_v}{L_a} \\ \frac{K_m}{J_i} & -\frac{B_0}{J_i} \end{bmatrix}, \ B = \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix}, \ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \Delta A = \begin{bmatrix} 0 & \sigma_v \\ \sigma_m & 0 \end{bmatrix}.$$

where i_a, w, and v_a denote the armature current, the angular velocity, and the armature voltage, respectively. R_a is the armature resistance and L_a is the inductance. K_v and K_m are the voltage and motor constants which are supposed to have parameter variations $|\sigma_v| \leq 0.06$ and $|\sigma_m| \leq 0.06$, respectively. J_i is the moment of inertia and B_0 is the friction coefficient. T_l is the unknown load torque. Our purpose is to regulate the output y, i.e., the armature current and angular velocity, to be zero.

Taken from Bélanger (1995) the parameters of the DC motor: $R_a = 1.2$, $L_a = 0.05$, $K_v = 0.6$, $K_m = 0.6$, $J_i = 0.1352$, and $B_0 = 0.3$. The parameter variations and disturbance are assumed to be $\sigma_v = \sigma_m = -0.01$ and $d = 0.01\sin(t)$, respectively. Denote $|\sigma_v| \leq \alpha_v$ and $|\sigma_m| \leq \alpha_m$ with two positive scalars α_v and α_m. According to Assumption 2.2, it can be chosen that

$$M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ F_0 = \begin{bmatrix} \frac{\sigma_v}{\alpha_v} & 0 \\ 0 & \frac{\sigma_m}{\alpha_m} \end{bmatrix}, \ N_0 = \begin{bmatrix} 0 & \alpha_v \\ \alpha_m & 0 \end{bmatrix}$$

where $\alpha_v = 0.01$ and $\alpha_m = 0.01$.

6.1 *Integrated FE/FTC design with additive faults*

As considered by Isermann (2011), there might be additive faults during the operation of the DC motor system (47). Consider here an offset fault of the armature current and angular velocity sensors, i.e., sensor fault f_s, and a voltage sensor gain fault of v_a, i.e., actuator fault f_a. It follows from (1) that the model (47) now becomes

$$\dot{x} = (A + \Delta A)x + Bu + F_a f_a + Dd$$
$$y = Cx + F_s f_s \tag{48}$$

where

$$F_a = \begin{bmatrix} \frac{1}{10L_a} \\ 0 \end{bmatrix}, \ f_a = \begin{cases} 0.5, & 0 \leq t \leq 1.5 \\ 1, & 1.5 < t \leq 3 \\ 0.2, & 3 < t \leq 3.5 \\ 0.6, & 3.5 < t \leq 4 \\ 1, & t > 4 \end{cases},$$

$$F_s = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \ f_s = \begin{cases} 0, & 0 \leq t \leq 1 \\ 0.1\sin(0.5(t-1)), & t > 1 \end{cases}.$$

6.1.1 *State feedback case*

Assumptions 2.1 - 2.3 are satisfied for the system (48). Given $C_{sx} = C_{se} = I_2$ and $Y_1 = [0.5; 0.5]$. Solving Theorem 3.2 with $\beta_s = 0.001$ and $\gamma_s = 0.65$ gives

$$K_x = [45.8131 - 0.1965], \ M = \begin{bmatrix} -5.0124 & -7.2313 \\ -0.5645 & -4.8486 \end{bmatrix},$$
$$G = \begin{bmatrix} -50.1241 \\ -5.6447 \end{bmatrix}, \ R = \begin{bmatrix} 41.9302 & 23.8482 \\ 1.5662 & 1.4904 \end{bmatrix},$$
$$H = \begin{bmatrix} 2.5062 & 0.8147 \\ 0.2822 & 0.5463 \end{bmatrix}.$$

For comparison, closed-loop system simulations using the separated design and the proposed integrated design are performed with $\varepsilon_{s_1} = 0.5$, $\phi_{s_1} = 0.001$, $\varphi_{s_1} = 0.1$, $\sigma_1 = 2$, $x(0) = [0.5; 0.5]$, and $\hat{\eta}_{s_1}(0) = 0$.

Figs. 5 and 6 show the simulation results for the fault estimation and the time response of the closed-loop system outputs, respectively. Using the proposed integrated FTC design, the armature current and the angular velocity of the DC motor are regulated to be asymptotically stable. Although the separated design can stabilize the system, it suffers from worse estimation and control performance, i.e., larger overshoot and much longer settling time. Moreover, the proposed integrated design achieves better fault estimation performance.

6.1.2 *Output feedback case*

Assumptions 2.1 - 2.3 and 4.1 are satisfied for the system (48). Given $C_{ox} = I_2$, $C_{oe} = [1\ 0\ 1\ 1; 0\ 1\ 1\ 1]$, and $Y_2 = I_2$. Solving Theorem 4.1 with $\beta_o = 0.001$ and $\gamma_o = 0.55$ gives the gains

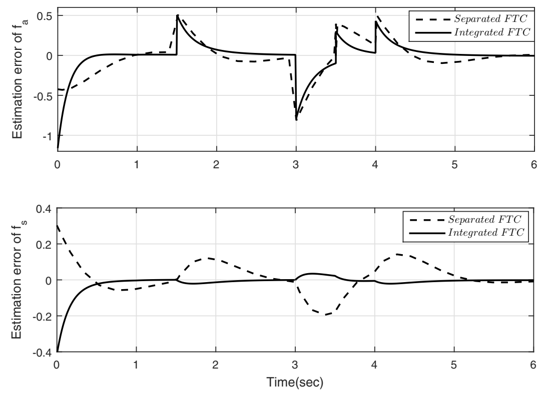

Fig. 5. FE performance: state feedback case

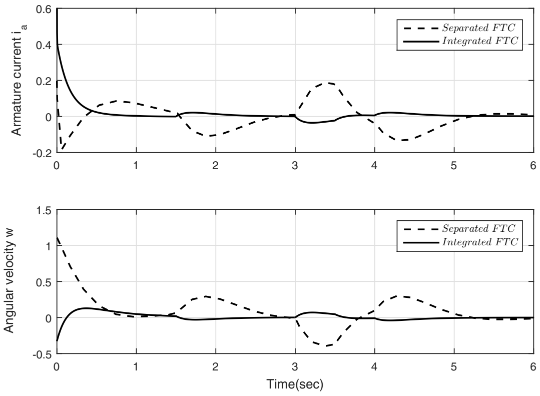

Fig. 6. FTC performance: state feedback case

$$K_x = [11.3306 - 0.0097],\ G = \begin{bmatrix} 14.6394 \\ -29.3091 \\ -211.9200 \\ 14.1988 \end{bmatrix},$$

$$M = \begin{bmatrix} -83.6656 & 76.0492 & 1.4639 & 235.4760 \\ 166.2354 & -152.9879 & -2.9309 & -471.6437 \\ 12.8172 & -16.9832 & -21.1920 & -84.2490 \\ 8.8649 & -44.7627 & 1.4199 & -98.7734 \end{bmatrix},$$

$$L = \begin{bmatrix} 3.3142 & 0.0505 \\ -6.6387 & -0.1019 \\ 36.5627 & 44.9315 \\ -4.1485 & -3.5423 \end{bmatrix},\ H = \begin{bmatrix} 0.2680 & 0.0325 \\ 1.4655 & 0.9361 \\ 10.5960 & 4.2211 \\ -0.7099 & 0.0431 \end{bmatrix}.$$

Simulations are performed with $\varepsilon_{s_2} = 0.5$, $\phi_{s_2} = 0.001$, $\varphi_{s_2} = 0.1$, $\sigma_2 = 1$, $x(0) = [0.5; 0.5]$, and $\hat{\eta}_{s_2}(0) = 0$.

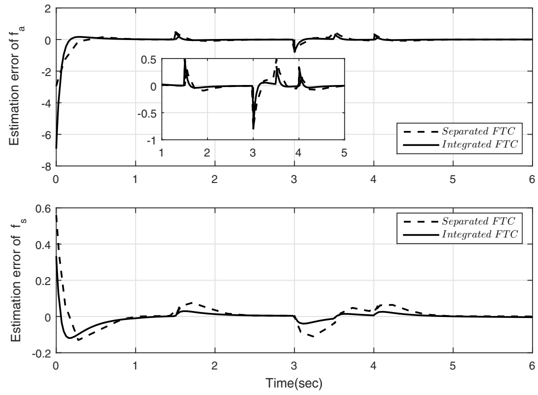

Fig. 7. FE performance: output feedback case

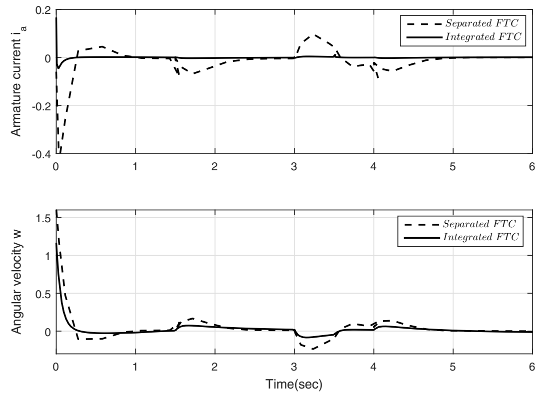

Fig. 8. FTC performance: output feedback case

Figs. 7 and 8 show the simulation results for the fault estimation and time responses of the closed-loop system outputs, respectively. The phenomena observed from the results are similar to those of the state feedback case.

6.2 Integrated FE/FTC design with multiplicative faults

Suppose that there exists partial loss of actuator effectiveness (multiplicative actuator fault) in the system (47), then the faulty model is represented as

$$\begin{aligned} \dot{x} &= (A + \Delta A)x + B(1-\theta)u + Dd \\ y &= Cx \end{aligned} \tag{49}$$

where the scalar $\theta \in [0, 1]$ denotes the extent of the loss of actuator effectiveness. If $\theta = 0$, the actuator is healthy. If $\theta = 1$, the actuator is totally loss of effectiveness, which cannot be handled by control design and is out of the scope of this paper. If $\theta \in (0, 1)$, the actuator loses a ratio of θ of its effectiveness. The matrices A, ΔA, B,

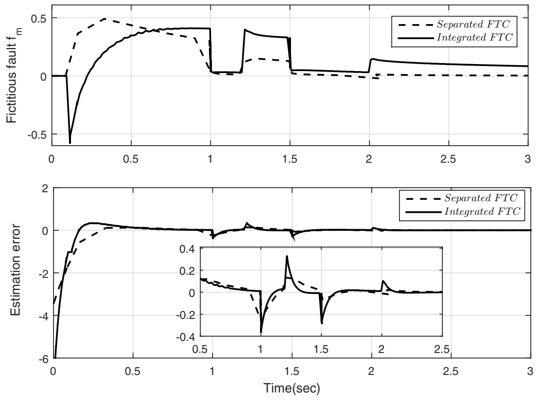

Fig. 9. Fictitious fault estimation performance

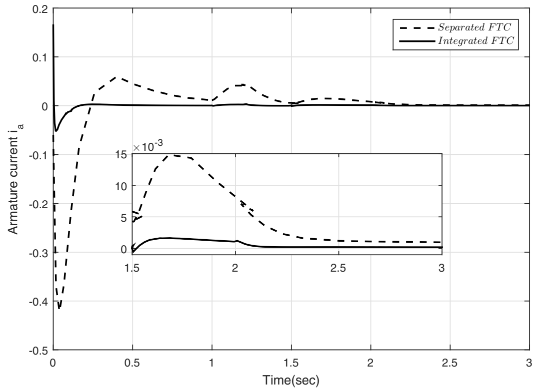

Fig. 10. FTC performance of i_a: multiplicative fault case

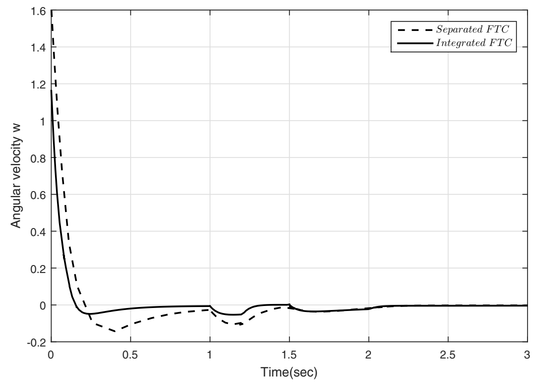

Fig. 11. FTC performance of w: multiplicative fault case

D, and C are defined the same as those in (48) and

$$\theta = \begin{cases} 0, & 0 \le t \le 0.1 \\ -0.1(1-e^{-t})+1, & 0.1 < t \le 1 \\ 0.1, & 1 < t \le 1.2 \\ 0.99, & 1.2 < t \le 1.5 \\ 0.2, & 1.5 < t \le 2 \\ 0.8, & t > 2 \end{cases}.$$

According to Section 5, the partial loss of actuator effectiveness can be represented by a fictitious fault

$$B(-\theta)u = F_m f_m,\ F_m = \begin{bmatrix} \frac{1}{10L_a} \\ 0 \end{bmatrix},\ f_m = -10\theta u.$$

Note that the integrated FE/FTC design of (49) is similar to that of (48) with $f_s = 0$ and by replacing $F_a f_a$ with $F_m f_m$. Thus, the proposed design strategy for the additive fault case is easily amenable to cover this multiplicative case. Without loss of generality and to consider a more practically realizable situation, only the output feedback case is studied, for which the observer/controller gains in Section 6.1.2 can be applied.

Simulations are performed with the same initial conditions as those in Section 6.1.2, and similar results are shown in Figs. 9 - 11. Depending on both the time-varying multiplicative fault θ and the control inputs u, the fictitious faults f_m are different for the separated and integrated FTC designs. However, as observed in Fig. 9, the fictitious fault estimation performance of the integrated FTC design is better than that of the separated FTC design. For the FTC performance shown in Figs. 10-11, the integrated FTC design also has quicker response and smaller overshoot than the separated one.

Summing up, the simulation results for the DC model example with system uncertainty, external disturbance, and faults, show that the proposed integrated design successfully demonstrates superior FE/FTC performance compared with the separated design by taking account of the bi-directional robustness interaction between the observer and the control system.

7 Conclusion

A new strategy of integration of FE within FTC for linear systems with unmatched uncertainty along with additive/multiplicative faults and disturbance is proposed. The presented approach is to design together the FE and FTC, using an observer-based robust control method achieved by H_∞ optimization with a single-step LMI formulation. Both the cases of state and output feedback FTC are discussed.

Simulation and comparison of the stabilization control for a DC model shows that the proposed integrated design approach leads to a better FE and FTC performance compared with the separated design approach.

The strategy developed is amenable to extension to cover time-varying and uncertain nonlinear systems via multiple-model approaches (e.g. LPV or T-S fuzzy modelling). The limitations of this paper are (1) The proposed design for the multiplicative fault case cannot obtain the estimate of the real multiplicative faults, which are sometimes important and useful for system maintenance and security, and (2) The uncertainties on the other system matrices are not taken into account. Thus, it remains an open question as to how to develop alternative strategies to estimate and compensate the real multiplicative faults and handle systems with uncertainties on all the system matrices.

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A Proof of Lemma 3.2

Post-multiplying the left hand side of (21) by a full row-rank matrix $[S_1\ S_2]$ gives

$$
\begin{aligned}
&\mathrm{rank}\left(\begin{bmatrix} sL - L\bar{A} \\ \bar{C} \\ \bar{C}\bar{A} \end{bmatrix} [S_1\ S_2]\right) \\
&= \mathrm{rank}\begin{bmatrix} sI_{q+q_1} - L\bar{A}S_1 & -\Psi \\ \Gamma & \Omega \end{bmatrix} \\
&= \mathrm{rank}\left(\begin{bmatrix} I_{q+q_1} & \Psi\Omega^\dagger \\ 0 & (I_{2p} - \Omega\Omega^\dagger) \\ 0 & \Omega\Omega^\dagger \end{bmatrix}\right. \\
&\times \left.\begin{bmatrix} sI_{q+q_1} - L\bar{A}S_1 & -\Psi \\ \Gamma & \Omega \end{bmatrix}\right) \\
&= \mathrm{rank}\begin{bmatrix} sI_{q+q_1} - M_1 & 0 \\ M_2 & 0 \\ \Omega\Omega^\dagger\Gamma & \Omega \end{bmatrix} \\
&= \mathrm{rank}\begin{bmatrix} sI_{q+q_1} - M_1 \\ M_2 \end{bmatrix} + \mathrm{rank}(\Omega). \qquad \text{(A.1)}
\end{aligned}
$$

Similarly, the right hand side of (21) is

$$
\begin{aligned}
\mathrm{rank}\left(\begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \\ L \end{bmatrix} [S_1\ S_2]\right) &= \mathrm{rank}\begin{bmatrix} \bar{C}S_1 & \bar{C}S_2 \\ \bar{C}\bar{A}S_1 & \bar{C}\bar{A}S_2 \\ I_{q+q_1} & 0 \end{bmatrix} \\
&= q + q_1 + \mathrm{rank}(\Omega). \qquad \text{(A.2)}
\end{aligned}
$$

By (A.1) and (A.2), it holds that

$$\mathrm{rank}\begin{bmatrix} sI_{q+q_1} - M_1 \\ M_2 \end{bmatrix} = q + q_1.$$

Thus, the pair (M_2, M_1) is detectable.

B Proof of Theorem 3.2

Consider a Lyapunov function $V_{e_s} = e_s^\top Q e_s$ with a symmetric positive definite matrix $Q \in R^{(q+q_1)\times(q+q_1)}$. Denote $W_1 = \bar{H}_1 + ZH_2\bar{C}$ and $\chi_{s_1} = He(e_s^\top QW\Delta\bar{A}\bar{x})$ and $\bar{M}_0 = [M_0^\top\ 0]^\top$, so that

$$
\begin{aligned}
\chi_{s_1} &= -[\bar{M}_0^\top W_1^\top Q e_s - F_0N_0x]^\top[\bar{M}_0^\top W_1^\top Q e_s - F_0N_0x] \\
&\quad + e_s^\top QW_1\bar{M}_0\bar{M}_0^\top W_1^\top Qe_s + x^\top N_0^\top F_0^\top F_0 N_0 x \\
&\le e_s^\top QW_1\bar{M}_0\bar{M}_0^\top W_1^\top Qe_s + x^\top N_0^\top N_0 x.
\end{aligned}
$$

Then it follows that

$$
\begin{aligned}
\dot{V}_{e_s} &= e_s^\top He(Q(M_1 - ZM_2))e_s + He(e_s^\top QW_1\Delta\bar{A}\bar{x}) \\
&\quad + He(e_s^\top QW_1\bar{D}\bar{d}) \\
&\le e_s^\top[He(Q(M_1 - ZM_2)) + QW_1\bar{M}_0\bar{M}_0^\top W_1^\top Q]e_s \\
&\quad + x^\top N_0^\top N_0 x + He(e_s^\top QW_1\bar{D}\bar{d}). \qquad \text{(B.1)}
\end{aligned}
$$

Further consider $V_x = x^\top Px$ with a positive definite matrix $P \in R^{n\times n}$. Denote $\chi_{s_2} = He(x^\top P\Delta Ax)$, then

$$
\begin{aligned}
\chi_{s_2} &= -[M_0^\top Px - F_0N_0x]^\top[M_0^\top Px - F_0N_0x] \\
&\quad + x^\top PM_0M_0^\top Px + x^\top N_0^\top F_0^\top F_0N_0x \\
&\le x^\top PM_0M_0^\top Px + x^\top N_0^\top N_0x.
\end{aligned}
$$

Similarly, it can be shown that

$$
\begin{aligned}
\dot{V}_x &= x^\top He[P(\Theta_1 A - BK_s) + PM_0M_0^\top P + N_0^\top N_0]x \\
&\quad + He(x^\top PF_1e + x^\top PD_1\bar{d}). \qquad \text{(B.2)}
\end{aligned}
$$

Let $\xi_s = [x^\top\ e_s^\top]^\top$, the H_∞ performance $\|G_{z_s\bar{d}}\| < \gamma_s$ is

$$J = \int_0^\infty (\xi_s^\top\xi_s - \gamma_s^2\bar{d}^\top\bar{d})dt < 0. \qquad \text{(B.3)}$$

Denote $V_s = V_{xs} + V_{es}$, then under zero initial conditions, it follows that

$$J = \int_0^\infty (\xi_s^\top\xi_s - \gamma_s^2\bar{d}^\top\bar{d} + \dot{V}_s)dt - \int_0^\infty \dot{V}_s dt$$

$$\leq \int_0^\infty (\xi_s^\top \xi_s - \gamma_s^2 \bar{d}^\top \bar{d} + \dot{V}_s) dt.$$

Now, a sufficient condition of (B.3) is

$$J_1 = \xi_s^\top \xi_s - \gamma_s^2 \bar{d}^\top \bar{d} + \dot{V}_s < 0. \tag{B.4}$$

It follows from (B.4) with (B.1) and (B.2) that

$$J_1 \leq \begin{bmatrix} \xi_s \\ \bar{d} \end{bmatrix}^\top \begin{bmatrix} J_{11} & \chi_{12} & \chi_{13} \\ \star & J_{22} & \chi_{23} \\ \star & \star & -\gamma_s^2 I \end{bmatrix} \begin{bmatrix} \xi_s \\ \bar{d} \end{bmatrix} < 0 \tag{B.5}$$

where $J_{11} = \chi_{11} + PM_0M_0^\top P + C_{sx}^\top C_{sx}$, $\chi_{11} = He(P(\Theta_1 A - BK_x)) + 2N_0^\top N_0$, $\chi_{12} = PF_1$, $\chi_{13} = PD_1$, $J_{22} = \chi_{22} + C_{se}^\top C_{se}$, $\chi_{22} = He(Q(M_1 - ZM_2)) + QW_1\bar{M}_0\bar{M}_0^\top W_1^\top Q$, and $\chi_{23} = QW_1\bar{D}$.

By the Schur complement, (B.5) holds if

$$\begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} & PM_0 & 0 & C_{sx}^\top \\ \star & \chi_{22} & \chi_{23} & 0 & QW_1\bar{M}_0 & C_{se}^\top \\ \star & \star & -\gamma_s^2 I & 0 & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0. \tag{B.6}$$

Note that the constraint (B.6) is non-linear and cannot be solved directly using the LMI toolbox. However, it can be further modified into linear constraints (34) and (35) by denoting $PB = B\hat{P}$, $R_1 = \hat{P}K_s$, and $R_2 = QZ$.

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