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Output feedback sliding mode FTC for a class of nonlinear inter-connected systems

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Abstract: This paper is concerned with the challenge of developing a fault-tolerant control (FTC) scheme for an inter-connected decentralised system in which the individual subsystems are linear but the inter-connections are non-linear functions of the subsystem states and controls. It is assumed that the subsystems are disturbed by matched faults. The purpose of the decentralised control is to de-couple the subsystems with global and local control objectives as well as de-coupling the effects of uncertainties and faults. The paper describes the LMI-based sliding mode control (SMC) design, including Lemmas and proofs were appropriate and the main properties of the design approach, control objectives, stability, fault-tolerance and robustness are outlined. Results are given to illustrate the properties of the control design, meeting the desired objectives of stability, local and global control performance, subsystem de-coupling and fault-tolerance for a 3 electrical machine inter-connected system with non-linear inter-connections that are functions of machine rotor angle deviations.

***Keywords**:* Sliding mode control; Decentralized control; Inter-connected systems; Fault-tolerant control; Static output feedback

1. INTRODUCTION

The design of decentralized control of interconnected systems has received considerable attention due to the need for increased reliability as well economic and information constraints (Bakule, 2008). Interest is in decentralized control using only local information at subsystem levels, containing much simpler architecture than centralized control. Some studies consider decentralized control in the presence of uncertainties (Šiljak & Stipanovic, 2000).

In most real systems state variables are not available or are costly to measure and output feedback control methods are required. For de-centralized control very few output feedback strategies have been proposed. Some examples are based on decentralized observer based control (Zhu & Pagilla, 2007). However, the design of decentralized observer-based feedback for nonlinear interconnected system is challenging since the Separation Principle is usually not applicable to systems influenced by uncertainties and/or faults. Static output feedback control is an alternative way to deal with this problem. For example (Cao *et al*, 1998) proposed an ILMI method to stabilize linear interconnected system without uncertainties. (Zečević & Šiljak, 2004) proposed an LMI method to deal with the nonlinear interaction. (Yan et al, 2004) show how a decentralized system can be designed and stabilized using SMC with rejection of matched disturbance. The main contribution of this paper is the development of a decentralized static output feedback SMC for interconnected systems with nonlinear interconnection, making use of Lyapunov stability parameterized in terms of LMI constraints solved efficiently by convex optimization tools. The interaction terms are assumed to satisfy quadratic constraints as proposed by (Šiljak & Stipanovic, 2000) to not only restrict the effects of the interconnection nonlinearities but also limit the effects of subsystem uncertainties.

The method proposed uses a simple LMI approach to calculate the linear control which will not only stabilize the aggregate system states of but also simultaneously maximize the interconnection bounds. Then by pre-structuring a symmetric positive definite (s.p.d.) matrix, an SMC is constructed. In the general case matched uncertainty (disturbance) or faults are rejected using the SMC. For simplicity here matched subsystem faults are considered (without the effect of disturbance). It is shown that the SMC algorithm shows good compatibility to combine with other static output feedback design methods.

The design problem is formulated in Section 2. Section 3 introduces the new static output feedback control method is introduced. In Section 4, an efficient SMC algorithm is developed and discussed. Section 5 outlines a design example of 3 inter-connected electrical machines with nonlinear rotor angle misalignment and steam valve faults giving rise to a need for good FTC action as well as satisfactory individual machine and overall system performance in the presence of faults. Section 6 gives the conclusion discussion.

2. PROBLEM FORMULATION

Consider a state space system with N subsystems with nonlinear interconnection:

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i(u_i(t) + f_i(t)) + h_i(x,t) \\ y_i(t) &= C_i x_i(t)\end{aligned} \tag{1}$$

where the state vector $x_i \in \mathbb{R}^{n_i}$, the controls are $u_i \in \mathbb{R}^{m_i}$ the output signals $y_i \in \mathbb{R}^{p_i}$, i = *1,2,..N*, .and the condition $m_i \le p_i < n_i$ is fulfilled. The triple (A_i, B_i, C_i) represents known constant matrices of appropriate dimensions with B_i and C_i full rank. The $h_i(x,t)$ represent unknown subsystem interactions. The functions $f_i(t)$ represent bounded unknown matched faults.

A decentralized asymptotically stabilising static output feedback control is to be determined for the system (1) using SMC, subject to the following.

Assumptions: (A_i, B_i) and (A_i, C_i) are controllable and observable, respectively. Any invariant zeros of (A_i, B_i, C_i) lie in the left-half complex plane. Moreover, $rank(C_i B_i) = m_i$ for $i = 1, \ldots, N$. The i-th subsystem fault $f_i(t)\colon \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \to \mathbb{R}^m$ is bounded by a known constant f_{iu} and a known function $\varphi_i(y,t)$ for $i = 1, \ldots, N$:

$$\|f_i(t)\| < f_{iu}\|u_i\| + \phi_i(y,t) \tag{2}$$

The functions $h_i(x,t)$ are assumed to satisfy the well-known quadratic constraints used for inter-connected systems (Siljak & Stipanovic, 2002; Swarnakar, Marquez, and Chen, 2007):

$$h_i^T(x,t)h_i(x,t) \le \alpha_i^2 x^T H_i^T H_i x \tag{3}$$

$\alpha_i > 0$ are bounding interconnection parameters and H_i are fixed matrices. (2) & (3) are used to bound the faults and interactions. Interconnection functions are assumed to be continuous (Siljak and Stipanovic, 2002; Shafai *et al*, 2011). The state system of N interconnected subsystems with matched faults is:

$$\begin{aligned}\dot{x}(t) &= \mathrm{A}x(t) + B(u(t) + f(t)) + h(x,t) \\ y &= Cx\end{aligned} \tag{4}$$

where $A = diag(A_1, \ldots, A_N)$, and $B = diag(B_1, \ldots, B_N), C = diag(C_1, \ldots, C_N)$.

$f(t) = [f_1^T(t), \ldots, f_N^T(t)]^{\mathrm{T}}$ is the overall system fault vector. $h(x,t) = [h_1^T(x,t), \ldots, h_N^T(x,t)]^{\mathrm{T}}$ are the nonlinear interconnection functions. The aggregate system state vector is:

$$x(t) = [x_1^T(t) \quad .. \quad x_N^T(t)]^T \in \mathbb{R}^n, n = n_1 + .. + n_N$$

The aggregate system control vector is $u = [u_1^T(t) \quad \cdots \quad u_N^T(t)]^T \in \mathbb{R}^m, m = m_1 + \cdots + m_N$.

The aggregate system output vector is:

$$\mathrm{y} = [y_1^T(t) \quad \cdots \quad y_N^T(t)]^T \in \mathbb{R}^p, p = p_1 + \cdots + p_N.$$

Assume that $h(x,t)$ is bounded as follows:

$$h^T(x,t)h(x,t) \le x^T\left(\sum_{i=1}^{N}\alpha_i^2 H_i^T H_i\right)x = x^T H^T H x \tag{5}$$

The aim is to design a control law to reject matched subsystem faults, giving good FTC performance via SMC design with controls $u_i = u_{i,0} + u_{i,1}$. $u_{i,0} = K_i y_i = K_i C_i x_i$ are the linear continuous controls. $u_{i,1}$ are the discontinuous SMC designed to reject the faults and matched uncertainties. Sections 3 and 4 introduce the design methods.

3. STATIC OUTPUT FEEDBACK DESIGN

According to SMC theory, the switching control can reject the matched disturbance completely. Thus, in the static output feedback design procedure, stabilization of the system without matched disturbance is the main problem. Here a novel LMI static output feedback design is given.

A Lyapunov function for (4) without $f(t)$ is:

$$V(x) = x^T P x \tag{6}$$

where P is an s.p.d. matrix. The main static output feedback design challenge was pointed out by (Benton & Smith, 1998). Considering the derivative of (6) along the system trajectory, gives:

$$\begin{aligned}\dot{V}(x) &= x^T(PA + A^T P + PBKC + C^T K^T B^T P)x \\ &\quad + x^T P h + h^T P x\end{aligned} \tag{7}$$

To obtain a quadratic form, use the following result:

$$X^T Y + Y^T X \le X^T X + Y^T Y \tag{8}$$

It follows that:

$$x^T P h + h^T P x \le x^T P P x + h^T h = x^T\left(PP + H^T H\right)x$$

Then it gives:

$$\begin{aligned}\dot{V}(x) &\le x^T(PA + A^T P + PBKC + C^T K^T B^T \\ &\quad + PP + H^T H)x\end{aligned} \tag{9}$$

If matrices K and P can be found satisfying:

$$PA + A^T P + PP + H^T H + PBKC + C^T K^T B^T P < 0 \quad (10)$$

Or equivalently finding matrices K, X satisfying (Boyd, 1993):

$$AX + XA^T + BKCX + XC^T K^T B^T + I + XH^T HX < 0 \quad (11)$$

Inequalities (10) & (11) are not convex for P and K and are coupled since $X = P^{-1}$, they are impossible to solve using conventional LMIs, a main obstacle in static output feedback design. The method removes the need to consider coupled inequalities.

(A_i, B_i) are controllable, hence consider the state feedback $u_{sf} = K_0 x$ which can stabilize the overall system with $(A + BK_0)$ stable. Hence (9) becomes:

$$\begin{aligned} \dot{V}(x) = & \\ & x^T \begin{bmatrix} P(A + BK_0 - BK_0 + BKC) \\ +(A + BK_0 - BK_0 + BKC)^T P \end{bmatrix} x + \\ & x^T Ph + h^T Px \le \begin{bmatrix} x \\ (KC - K_0)x \end{bmatrix}^T \\ & \begin{bmatrix} \beta_0 + PP + H^T H & PB \\ B^T P & 0 \end{bmatrix} \begin{bmatrix} x \\ (KC - K_0)x \end{bmatrix} \end{aligned} \quad (12)$$

here $\beta_0 = P(A + BK_0)^T + (A + BK_0)^T P$.

Define $S = KC - K_0$, and consider:

$$x^T \begin{bmatrix} I & S \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} S & -I \end{bmatrix} \begin{bmatrix} I \\ S \end{bmatrix} x = 0 \quad (13)$$

A $G \in \mathbb{R}^{m \times m}$ also exists satisfying (12) due to $[S \quad -I][I \quad S]^T = 0$. Thus, by adding the LHS of (12) and its transpose to (11):

$$\dot{V}(x) \le \begin{bmatrix} x \\ (KC - K_0)x \end{bmatrix}^T \begin{bmatrix} \beta_0 + PP + H^T H & PB + S^T G^T \\ B^T P + GS & -G - G^T \end{bmatrix} \begin{bmatrix} x \\ (KC - K_0)x \end{bmatrix} \quad (14)$$

Thus, by computing P, G, K satisfying:

$$\begin{bmatrix} \beta_0 + PP + H^T H & PB + S^T G^T \\ B^T P + GS & -G - G^T \end{bmatrix} < 0 \quad (15)$$

The no fault aggregate system is Lyapunov stable.

(15) is not convex, and hence the Schur Complement Lemma with (5) gives new LMIs:

Minimize $\sum_{i=1}^{N} \gamma_i$ subject to $P = diag(P_i) > 0$, $L = diag(L_i), G = diag(G_i), i = 1, \dots, N$

$$\begin{bmatrix} \beta_0 & PB - K_0^T G^T + C^T L^T & P & H_1^T & \cdots & H_N^T \\ B^T P + LC - GK_0 & -G - G^T & 0 & 0 & \cdots & 0 \\ P & 0 & -I & 0 & \cdots & 0 \\ H_1 & 0 & 0 & -\gamma_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N & 0 & 0 & 0 & \cdots & -\gamma_N \end{bmatrix} < 0$$

(16)

where ; $L = GK$. If the LMIs (16) are feasible, the output feedback gain K can be obtained by:

$$K = G^{-1} L$$

(16) is equivalent to the following two inequalities:

$$x^T \beta_0 x + x^T Ph + h^T Px < 0 \quad (17)$$

$$x^T \beta_C x + x^T Ph + h^T Px < 0 \quad (18)$$

where $\beta_C = P(A + BKC) + (A + BKC)^T P$.

To prove this use the "Elimination Lemma":

Lemma 1 (Gahinet and Apkarian, 1994) Given a symmetric matrix $\Psi \in \mathbb{R}^{m \times m}$, and Γ, W of column dimension m, consider the problem of finding a Θ of compatible dimension such that:

$$\Psi + \Gamma^T \Theta^T W + W^T \Theta \Gamma < 0 \quad (19)$$

The columns of $\tilde{\Gamma}$ and $\widetilde{W}$ form bases of the null spaces of Γ &W, then (19) is solvable for Θ iff:

$$\tilde{\Gamma}^T \Psi \tilde{\Gamma} < 0 \text{ and } \tilde{W}^T \Psi \tilde{W} < 0 \quad (20)$$

Lemma 2 For (4), the following are equivalent:

1. There exists an s.p.d block diagonal matrix $P = diag(P_1, \dots, P_N), P_i \in \mathbb{R}^{n_i \times n_i}$, a non-singular block diagonal matrix $G = diag(G_1, \dots, G_N), G_i \in \mathbb{R}^{m_i \times m_i}$, a state feedback gain matrix $K_0 \in \mathbb{R}^{m_i \times n_i}$ and an output feedback gain matrix $K = diag(K_1, \dots, K_N), K_i \in \mathbb{R}^{m_i \times p_i}$, hence:
2. There exists an s.p.d block diagonal matrix $P = diag(P_1, \dots, P_N), P_i \in \mathbb{R}^{n_i \times n_i}$, a state feedback gain matrix $K_0 \in \mathbb{R}^{m_i \times n_i}$ and an output feedback gain $K = diag(K_1, \dots, K_N), K_i \in \mathbb{R}^{m_i \times p_i}$, hence:

$$\beta_0 + PP + H^T H < 0 \quad (21)$$

$$\beta_C + PP + H^T H < 0 \quad (22)$$

Proof *Define*: $\Psi = \begin{bmatrix} \beta_0 + PP + H^T H & PB \\ B^T P & 0 \end{bmatrix}$ where P, K_0 follow the Lemma description. Since: $\begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} S & -I \end{bmatrix} = 0$:

$$\begin{aligned} & \Psi + \begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} S & -I \end{bmatrix} + \begin{bmatrix} S \\ -I \end{bmatrix} G^T \begin{bmatrix} 0 & I \end{bmatrix} \\ & = \begin{bmatrix} \beta_0 + PP + H^T H & PB + S^T G^T \\ B^T P + GS & -G - G^T \end{bmatrix} \end{aligned} \quad (23)$$

using the Elimination Lemma:

$$\Psi + \begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} S & -I \end{bmatrix} + \begin{bmatrix} S \\ -I \end{bmatrix} G^T \begin{bmatrix} 0 & I \end{bmatrix} < 0$$

is solvable for G if and only if:

$$\begin{bmatrix} I & 0 \end{bmatrix} \Psi \begin{bmatrix} I \\ 0 \end{bmatrix} = \beta_0 + PP + H^T H < 0$$

$$\begin{bmatrix} I & S^T \end{bmatrix} \Psi \begin{bmatrix} I \\ S \end{bmatrix} = \beta_C + PP + H^T H < 0$$

A G satisfying (13) is freely available, hence the solvability of (15) is equivalent to the solvability of (21) & (22) ■

The choice of $K_0 = -B^T P_0$ can be obtained by solving the following ARE problem:

$$A^T P_0 + P_0 A - P_0 B B^T P_0 = -\varepsilon I \tag{24}$$

where $\varepsilon > 0$ is arbitrarily small.

Assume K_0 a freely adjustable design parameter by adjusting ε in (24). A small ε leads to a large γ_i if the LMI problem (16) is feasible. Other methods could also be used to design K_0. There may be a stabilizing K_0 for which (16) is infeasible with large γ_i (Benton & Smith, 1998). The existence of an admissible K_0 is still an open problem. The proposed method establishes a relation between state feedback and static output controls, i.e. finding an s.p.d. matrix P such that inequalities (21) & (22) hold simultaneously. The LMI feasibility problem (16) depends on the stabilizing gain K_0.

4. SLIDING MODE CONTROL DESIGN

The above method focuses on the decentralized system without matched faults. However, the SMC switching will reject the matched fault and hence the linear control is used to stabilize the system considering zero faults. Here the SMC system is described and the stability of the system with faults is proved. The sliding surface must be designed as a function of y (i.e. Cx) and not x, as follows:

$$\sigma = \begin{bmatrix} \sigma_1^T & \dots & \sigma_N^T \end{bmatrix}^T = diag(F_1, ..., F_N) \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \tag{25}$$

$F = diag(F_1, \dots, F_N)$ is an $m \times p$ block diagonal matrix such that $F_i C_i B_i, i = 1, \dots, N$ are non-singular and the reduced $(n-m)$ th-order equivalent system dynamics restricted to:

$Fy = FCx = 0$, are asymptotically stable.

To form an SMC, use the well-known equation $B^T P = FC$ (Walcott & Zak, 1987), where F is used to design the sliding surface $\sigma(t) = Fy(t)$. The sliding surface for each subsystem is designed by:

$$\sigma_i(t) = F_i y_i(t) \tag{26}$$

Pre-defining the structure of $P_i = \tilde{B}_i W_{1,i} \tilde{B}_i^T + C_i^T W_{2,i} C_i$ of the LMI (16), where $\tilde{B}_i$ is the orthogonal complement of the B_i, the F_i can be obtained as $F = B_i^T C_i^T W_{2,i}$. Hence,

$P = diag(P_i) = \tilde{B} W_1 \tilde{B}^T + C^T W_2 C$, both W_1 & W_2 are block diagonal matrices.

Theorem 4.1. For the overall system in the form of (4), obtain $W_{1,i}, W_{2,i}, K_i$ by solving (16) and design sliding surface $\sigma_i(t) = F_i y_i(t) = B_i^T C_i^T W_{2,i} y_i(t)$ using the decentralized control law:

$$u_i = \begin{cases} K_i y_i - \rho_i \dfrac{\sigma_i}{\|\sigma_i\|}, & \sigma_i \neq 0 \\ K_i y_i, & \sigma_i = 0 \end{cases} \tag{27}$$

The aggregate system is insensitive to matched uncertainty/faults and is quadratically stable.

Proof: It is easy to show that

$$\dot{\sigma}(t) = FC\dot{x}(t) = FCAx(t) + FCB(u(t) + f(t)) \tag{28}$$

It is required to prove that in the sliding surface, the system is stable and insensitive to the matched faults with $\sigma = 0$ and $\dot{\sigma} = 0$.

Define the Lyapunov function:

$$V(x) = \sum_{i=1}^{N} x_i^T P_i x_i \tag{29}$$

The time derivative of (29) is given by:

$$\begin{aligned} \dot{V}(x) &\le x^T (PA + A^T P + PBKC + C^T K^T B^T P + PP + \\ &\quad H^T H)x + \sum_{i=1}^{N} \left(2x_i^T P_i B_i f_i - 2x_i^T P_i B_i \rho_i \frac{\sigma_i}{\|\sigma_i\|} \right) \\ &\le x^T \Sigma x + 2\sum_{i=1}^{N} \left[x_i^T C_i^T F_i^T \left(\|f_i\| - \rho_i \frac{\sigma_i}{\|\sigma_i\|} \right) \right] \\ &\le x^T \Sigma x + 2\sum_{i=1}^{N} \left[\|\sigma_i\| \left(\|f_i\| - \rho_i \right) \right] \end{aligned} \tag{30}$$

Since $\Sigma < 0$ is already proved by LMI of step 5), if $\rho_i > \|f_i\|, i = 1, \dots, N$ it can be claimed that the system is quadratically stable as $\dot{V}(x) < 0$. In this case, ρ_i can be chosen as $\rho_i = f_{iu}\|u\| + \varphi_i(y,t) + \eta_i, i = 1, \dots, N$ where $\eta_i, i = 1,, N$ are positive constants chosen by the designer. With appropriate $\rho_i > \|f_i\|$, the sliding reachability follows from the Lyapunov functions:

$$V(\sigma) = \frac{1}{2} \sum_{i=1}^{N} \left[\sigma_i^T (F_i C_i B_i)^{-1} \sigma_i \right] \tag{31}$$

$F_i C_i B_i = B_i^T P_i B_i$ satisfies the s.p.d constraint. Hence, the time derivative of (30) is:

$$\begin{aligned} \dot{V} &= \sigma_i^T \left[(F_i C_i B_i)^{-1} F_i C_i \left[(A_i + B_i K_i C_i) x_i + h_i \right] + f_i - \rho_i \frac{\sigma_i}{\|\sigma_i\|} \right] \\ &\le \|\sigma_i\| \left\| (F_i C_i B_i)^{-1} F_i C_i \left[(A_i + B_i K_i C_i) x_i + h_i \right] \right\| + \|\sigma_i\| \left(\|f_i\| - \rho_i \right) \end{aligned} \tag{32}$$

Using $\rho_i = f_{iu}\|u\| + \varphi_i(y,t) + \eta_i, i = 1,\dots,N$, can be rewritten as:

$$\dot{V} \le \sum_{i=1}^{N}\left\{\|\sigma_i\|\left[(F_iC_iB_i)^{-1}F_iC_i\left[(A_i + B_iK_iC_i)x_i + h_i\right] - \eta_i\right]\right\} \quad (33)$$

Let $0 < \tilde{\eta}_i < \eta_i$. Following closed-loop subsystem stability, sliding domains are reached according to:

$$\Omega_i = \left\{\begin{array}{l} x_i : \|(F_iC_iB_i)^{-1}F_iC_i\|\|(A_i + B_iK_iC_i)x_i + h_i\| \\ < \eta_i - \tilde{\eta}_i \end{array}\right\}, \quad (34)$$

$i = 1,\dots,N$

$\dot{V} \le \sum_{i=1}^{N}(-\tilde{\eta}_i\|\sigma_i\|)$ implies that all sliding surfaces $\sigma_i = 0$ can be reached in finite time, remaining there subsequently. ■

The SMC is designed to reject matched faults. It does not influence the static output gain K design procedure, this is an attractive alternative to the state feedback "Integral sliding mode control" (ISMC) (Utkin and Shi, 1996), eliminating the reaching phase whilst not affecting the state feedback design procedure (Castaños and Fridman, 2006). The only study about output feedback ISMC requires several sliding surfaces in a step by step estimation strategy (Bajarano *et al*, 2007). The proposed method provides a simple computational way to reject matched faults within the local and aggregate systems, without requiring that the reachability phase be removed. A suitable decentralised system output feedback algorithm is:

1. Solve the Algebraic Riccati equation:

$A^TP_0 + P_0A - P_0BB^TP_0 = -\varepsilon I$ where $\varepsilon > 0$ is arbitrarily small.

2. Set $K_0 = -B^TP_0$ and matrix structures:

   s.p.d matrix

$P = diag(P_i), P_i = \tilde{B}_iW_{1,i}\tilde{B}_i^T + C_i^TW_{2,i}C_i \in \mathbb{R}^{n_i \times n_i}$ non-singular matrix

$G = diag(G_1,\dots,G_N), G_i \in \mathbb{R}^{m_i \times m_i}$ and

$K = diag(K_1,\dots,K_N), K_i \in \mathbb{R}^{m_i \times p_i}$ .Compute G , L W_2 by solving the LMI problem:

   Minimize $\sum_{i=1}^{N}\gamma_i$ subject to $P > 0$ together with (16)

3. Design the static output feedback matrix $K_i = G_i^{-1}L_i$ and sliding surface function $\sigma_i(t) = F_iy_i(t) = B_i^TC_i^TW_{2,i}y_i(t)$.

Step 1 determines a stabilising state feedback by suitable choice of ε. Step 2 calculates the static output feedback and sliding surface gains K_i and F_i, and maximum subsystem uncertainty gains γ_i.

5. POWER SYSTEM NONLINEAR MODEL

The multi-machine power system has been widely used to illustrate the decentralized methods (Guo *et al*, 2000; Siljak *et al*, 2002; Tlili and Braiek, 2009, etc). System interactions are nonlinear, with interesting challenges for decentralised FTC. A *3*-machine example power system with steam valve control has *3* interconnected subsystems as:

Let $x_i = [\Delta\delta_i(t) \quad \omega_i(t) \quad \Delta P_{m_i}(t) \quad \Delta X_{e_i}(t)]^T$ denote the state vector of a machine (Tlili and Braiek, 2009). The *i*-th machine dynamics, $i = 1,\dots,3$, can be represented by (1), where:

$h_i(x,t) = \sum_{j=1,j\ne i}^{N} p_{ij}G_{ij}g_{ij}(x_i,x_j)$ is a nonlinear function characterizing the interactions. The system parameters are given as:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.625 & 39.27 & 0 \\ 0 & 0 & -2.86 & 2.86 \\ 0 & -0.637 & 0 & -10.0 \end{bmatrix},$$

$$A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.294 & 30.80 & 0 \\ 0 & 0 & -2.86 & 2.86 \\ 0 & -0.637 & 0 & -10.0 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10.0 \end{bmatrix},$$

$$C_i^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, G_{ij} = \begin{bmatrix} 0 \\ \alpha_{ij} \\ 0 \\ 0 \end{bmatrix}, i,j = 1,2,3$$

$$g_{ij}(x_i,x_j) = \sin\left(\delta_i(t) - \delta_j(t)\right) - \sin(\delta_{i0} - \delta_{j0})$$

$$p_{ij} = 1, \qquad i,j = 1,2,3, i \ne j$$

$$\alpha_{11} = \alpha_{22} = \alpha_{33} = 0;\ \alpha_{12} = \alpha_{13} = 27.49;$$

$$\alpha_{21} = \alpha_{23} = \alpha_{31} = 23.1$$

For the *i*-th machine the physical variables are defined as $\Delta\delta_i(t) = \delta_i(t) - \delta_{i0}$; $\Delta P_{m_i}(t) = P_{m_i}(t) - P_{m_{i0}}$; $\Delta X_{e_i}(t) = X_{e_i}(t) - X_{e_{i0}}$; $\delta_i(t)$ is the rotor angle (radians); $\omega_i(t)$ is the relative speed; $P_{m_i}(t)$ is the mechanical power in pu; $X_{e_i}(t)$ is the steam valve position pu; $\delta_{i0}, P_{m_{i0}}, X_{e_{i0}}$ are nominal values of $\delta_i(t), P_{m_i}(t), X_{e_i}(t)$. All the parameters are given in (Guo, Hill and Wang, 2000).

The algorithm in Section 4 is used with the choice of $\varepsilon = -0.01$ to derive the static output and sliding surface gains K_i , F_i (excluded for brevity).

Figure 1 shows the subsystem 1 state responses when a matched step fault of magnitude 0.5 occurs in this subsystem at $t = 15s$. The 4 output feedback state responses without sliding mode shows that subsystem 1 (hence subsystems 2 and 3) is/are affected significantly by the interconnections.

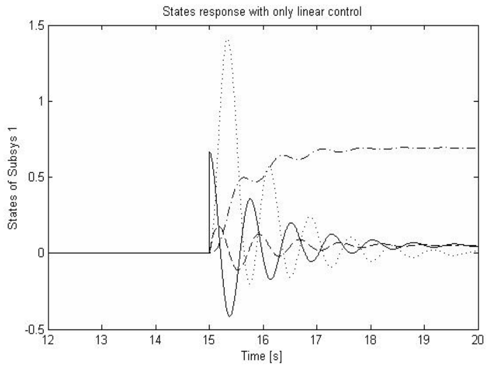

Figure 1. Subsytem 1 state responses to step fault at t = 15 s, without sliding mode component.

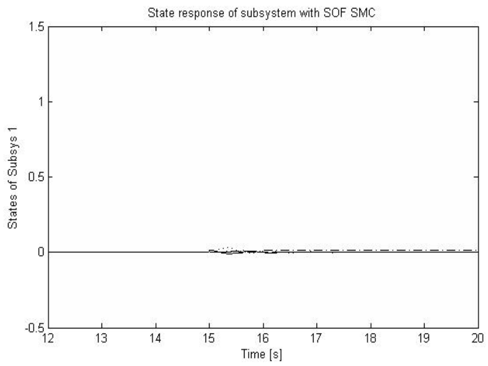

Figure 2. Subsystem 1 state responses to step fault at t = 15s. with SMC

Figure 2 shows that applying the SMC design (with the same linear control as in Figure 1) the fault is effectively rejected, implementing an FTC scheme. With proper choice of sliding surface gains ρ_i, the matched interactions can also be removed. The system gains are chosen as $\rho_i = 10, i = 1,2,3.$

6. CONCLUSION

In this study a novel static output feedback SMC strategy for decentralized nonlinear interconnected systems is developed with application to robust stabilization of a 3-machine power system. Sufficient conditions for the quadratic stability of the proposed output feedback control are characterized in terms of LMI constraints. A state feedback control law is first determined to formulate this LMI problem. By solving a simple LMI problem, both static output feedback and sliding gains are obtained. SMC switching gains are used to reject matched fault signals as well as any matched interconnection uncertainties in FTC scheme for decentralized systems. The interconnected generator systems example with steam valve controls confirms the availability and efficiency of the approach, robustly stabilizing the system despite the presence of a step fault in the steam valve for subsystem 1.

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