Distributed Leader-Follower Formation Control of Quadrotors Swarm Subjected to Disturbances

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***Abstract* - This paper proposes a novel control scheme for a group of quadrotors aircrafts that form a leader-follower configuration and are subjected to nonlinear behavior with lumped disturbances. For each aircraft, a distributed formation control law is designed. The desired geometrical pattern is achieved and the reference formation trajectory is tracked using the synthesized fixed-time position control robust law. Considering the overall feedback system, the presented work also presents a rigorous stability analysis of the system. Moreover, to characterize the control performance, simulations are conducted in a realistic ROS/Gazebo environment. Compared with the relevant literature, the proposed scheme demonstrates superior performance in practice because (i) convergence-time of the agents does not depend on their initial positions; (ii) chattering problem of switching control methods is avoided; (iii) zero error in steady-state is obtained while ensuring robustness.**

Index Terms* – *Continuous sliding mode control, Fixed-time control, Robust distributed formation control, Multi-agent system.

I. Introduction

The formation control of quadrotors swarm is considered as one of the most interesting research problem in Unmanned Aerial Vehicles (UAV) community [1]. It has a large range of important applications including but not limited to; search and rescue, package delivery, and fire monitoring [2] [3] [4] [5]. A quadrotor aircraft is a six Degrees of Freedom (DoF) Multi-Input Multi-Output (MIMO) underactuated system with highly nonlinear and strongly coupled dynamics [6]. The performance and operation of a quadrotor's system is prone to serious effects as a result of disturbances [7]. Therefore, the control design for the formation of networked quadrotors is a complex task. The design of the formation control algorithm becomes more challenging since the distributed control relies only on the available information of the neighboring entities to achieve the desired behavior.

The robust fixed-time distributed formation control of multi-quadrotors is one of the persistent cooperative control problems in the related scientific community. Particularly, accuracy, strong robustness and fast convergence are prominent features of a formation control law. Therefore, a modern control technique is vital to design a reliable and robust distributed control scheme to achieve superior performance for the formation of the networked quadrotors.

Scientific literature reports various studies that are focused on the formation control of the multi-quadrotor system. Compared with control methods providing asymptotic stability, the finite-time control exhibits better control performance, such as strong disturbance rejection capability, high precision and fast convergence rate [8]. Considering finite-time control approaches for formation control of multi-quadrotors, a distributed formation control method is addressed in [9] and [10]. Prospects of local communication for formation control is explored in [11].

The key benefit of a finite-time stable system is its ability to demonstrate superior performance compared to an asymptotically stable system. However, the initial conditions deviating the system's dynamics from the equilibrium point lead to increase the convergence time of finite-time control as discussed in [12]. One way of dealing this problem is to introduce an extension of finite-time stabilization [13]. Fixed-time stability offers a provision of defining and adjusting a settling time that is uniformly bounded. Also, the states of the system can be stabilized in fixed-time regardless of the values of the initial conditions. These distinguishing features are instrumental in providing deep insight for formation control of multi-agent systems that require the agents to demonstrate fixed-time convergence to the desired trajectory.

Another related avenue in formation control is stability analysis of fixed-time control laws. Research in [14] reports a formation of underwater Remotely Operated Vehicles (ROV) that autonomously work in leader-follower fashion. Work in [15] discusses a mobile robotic platform having nonholonomic constraints with a focus on design of a multivariable fixed-time control scheme for formation configuration. Another work involving nonholonomic constraints on multiple robots is reported in [16] where a fixed-time control for formatting tracking is investigated. For the multi-quadrotor formation control, a consensus algorithm and bi-limit homogeneity theory are exploited in [17] to develop a fixed-time formation control. Nevertheless, disturbances in the position subsystem are not taken into account and the presented disturbance observer limitedly considers estimated convergence time. Moreover, the control algorithm has not been verified through real hardware implementation.

As per the authors' knowledge, only a few reported works focus on the distributed fixed-time controller design of multi-quadrotors formation. Therefore, a novel distributed control scheme is proposed in the present paper. The key contributions of this research can be summarized as:

(i) Unlike asymptotic control, i.e., infinite-time convergent control, and finite-time convergent control methods, the tracking errors in the present work are shown to be stable in fixed-time regardless of the agents' initial position.

Moreover, the work thoroughly analyzes the stability of the closed-loop feedback control system.

(ii) Owing to continuous nature of the control signal, the chattering phenomenon inherently found in discontinuous and switching control methods is avoided in this work.

(iii) Extensive simulation experiments in a reliable and realistic ROS/Gazebo environment are conducted. A comparative study is exhaustively made to highlight the attained improvement.

This paper is organized into five sections: Section II introduces fundamental concepts and formulates the control problem. Section III derives the design of the control law and analyzes stability of the system. Section IV presents results of simulation experiment with a critical discussion on the control performance achieved. Finally, conclusion and potential avenues of further research are discussed in Section V.

Throughout this paper, the following notations are used. $\lceil x \rfloor^{\alpha}$ is given by $\lceil x \rfloor^{\alpha} := |x|^{\alpha}\mathrm{sign}(x)$ where $\alpha \in \mathbb{R}_{+}$ and $\forall x \in \mathbb{R}$. The standard signum function is denoted by $\mathrm{sign}(\cdot)$. For the sake of brevity, $\mathrm{s}x := \sin x$ and $\mathrm{c}x := \cos x$.

II. Preliminaries and Problem Statement

A. Preliminaries

Consider a system given below with an initial condition $x(0) = x_0$,

$$\dot{x}(t) = f(x), x \in \mathbb{R}^n \tag{1}$$

where the nonlinear function $f\colon \mathrm{D} \times \mathbb{R}_{+} \to \mathbb{R}^n$ is continuous on an open neighborhood $\mathrm{D} \subseteq \mathbb{R}^n$ of the origin. The equilibrium point of the system (1) is taken as the origin.

Lemma 1. ([18]). Let $V(x(t))\colon \mathbb{R}^n \to \mathbb{R}$ is a positive definite and a continuous Lyapunov function with its derivative satisfying the condition $\dot{V}(x(t)) \leq -\lambda V^{\alpha} - \mu V^{\gamma}$, where $\lambda, \mu, \alpha > 1,$ and $\gamma < 1$ are positive constants, then the origin of system represented in (1) demonstrate stability in fixed-time. Let T_0 denotes the function corresponding to the settling-time. T_0 is uniform w.r.t the initial condition $x(0) \in \mathbb{R}^n$ and is bounded by $T^{\star}$ as $T_0 \leq T^{\star}(\alpha, \gamma, \lambda, \mu) := (1/\lambda(\alpha - 1)) + (1/\mu(1 - \gamma))$.

Lemma 2. ([18]). If the positive constants $k_j > 0, (j = \overline{1, n})$ make the n^{th} order polynomials $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ and $s^n + 3k_n s^{n-1} + \cdots + 3k_2 s + 3k_1$ be Hurwitz in terms of Laplace operator (s), the origin of the system (2)

$$\begin{cases} \dot{x}_i = x_{i+1}, \;\; i = \overline{1, n-1} \\ \dot{x}_n = -\sum_{j=1}^{n} k_j \left(\lceil x_j \rfloor^{\alpha_{1,j}} + \lceil x_j \rfloor + \lceil x_j \rfloor^{\alpha_{2,j}} \right). \end{cases} \tag{2}$$

is fixed-time stable equilibrium, where $\alpha_{1,j}$ and $\alpha_{2,j}$ are found based on the bi-limit homogeneity reasoning i.e.: $\alpha_{1,n-k} = \frac{\alpha}{(k(1-\alpha)+1)}$ and $\alpha_{2,n-k} = \frac{(2-\alpha)}{(k(\alpha-1)+1)}$, where $k = \overline{0, n-1}$ and $\alpha \in (\epsilon, 1), \epsilon \in \left(\frac{n-2}{n-1}, 1\right)$.

B. Problem Statement

1). Quadrotor Dynamic Model

The acceleration dynamics corresponding to each quadrotor agent subjected to disturbances is given as [2] [18]

$$\begin{cases} \ddot{x}_i = -m_i^{-1}\left((\mathrm{c}\Phi_i \mathrm{s}\theta_i \mathrm{c}\psi_i + \mathrm{s}\Phi_i \mathrm{s}\psi_i) u_{i,z} - k_{i,x}\dot{x}_i + d_{i,x}^{\mathrm{ext}} \right), \\ \ddot{y}_i = -m_i^{-1}\left((\mathrm{c}\Phi_i \mathrm{s}\theta_i \mathrm{s}\psi_i - \mathrm{s}\Phi_i \mathrm{c}\psi_i) u_{i,z} - k_{i,y}\dot{y}_i + d_{i,y}^{\mathrm{ext}} \right), \\ \ddot{z}_i = -m_i^{-1}\left((\mathrm{c}\Phi_i \mathrm{c}\theta_i) u_{i,z} - k_{i,z}\dot{z}_i + d_{i,z}^{\mathrm{ext}} \right) + \mathrm{g}. \end{cases} \tag{3}$$

where $i = \overline{0, N}$, m_i is the quadrotor's net mass, $\mathrm{g} = 9.81\mathrm{m/s^2}$ is the gravity constant, $k_{i,x}, k_{i,y}, k_{i,z}$ are aerodynamic drag coefficients, $\dot{P}_i := [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T$, and $\dot{P}_i := \Upsilon_i = [\nu_{i,x}, \nu_{i,y}, \nu_{i,z}]^T$, where Υ_i is the translational speed. The terms $d_{i,\eta}^{\mathrm{ext}} := [d_{i,\Phi}^{\mathrm{ext}}, d_{i,\theta}^{\mathrm{ext}}, d_{i,\psi}^{\mathrm{ext}}]^T$ and $d_{i,P}^{\mathrm{ext}} := [d_{i,x}^{\mathrm{ext}}, d_{i,y}^{\mathrm{ext}}, d_{i,z}^{\mathrm{ext}}]^T$ denote the external time-varying disturbances acting on the rotational and translational accelerations of the quadrotor.

Assumption 1. Owing to practical difficulties in identification of the aerodynamic coefficients $K_{i,a} = \mathrm{diag}(k_{i,x}, k_{i,y}, k_{i,z})$, the unmodeled dynamics is the drag force $F_{i,a}$ which is defined as

$$d_{i,P}^{\mathrm{unc}} := \begin{bmatrix} d_{i,x}^{\mathrm{unc}} \\ d_{i,y}^{\mathrm{unc}} \\ d_{i,z}^{\mathrm{unc}} \end{bmatrix} = F_{i,a} = \begin{bmatrix} -k_{i,x}\nu_{i,x}/m_i \\ -k_{i,y}\nu_{i,y}/m_i \\ -k_{i,z}\nu_{i,z}/m_i \end{bmatrix}. \tag{4}$$

Thus, from (3), the following second-order system with disturbances can be assigned to the translational dynamics of the leader and follower quadrotors as

$$\begin{cases} \dot{X}_{i,3}(t) = X_{i,4}(t), \\ \dot{X}_{i,4}(t) = f_{i,P}(X_{i,4}, t) + F_{i,P}(X_{i,1}, t) + d_{i,P}(t), \\ \mathcal{Y}_{i,2}(t) = X_{i,4}(t). \end{cases} \tag{5}$$

Here $i = \overline{0, N}$, where $X_{i,P} := [X_{i,3}, X_{i,4}]^T \in \mathbb{R}^{3\times 2}$ is the states vector, $X_{i,3} := P_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3, X_{i,4} := \dot{P}_i = \Upsilon_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T = [\nu_{i,x}, \nu_{i,y}, \nu_{i,z}]^T \in \mathbb{R}^3$. The controlled outputs vector is $\mathcal{Y}_{i,2} := [x_i, y_i, z_i]^T \in \mathbb{R}^3$. The uncertain function $d_{i,P}(t) := [d_{i,x}, d_{i,y}, d_{i,z}]^T = d_{i,P}^{\mathrm{ext}} + d_{i,P}^{\mathrm{unc}} \in \mathbb{R}^3$ summarizes the total lumped disturbances, where $\|\dot{d}_{i,P}(t)\| \leq l_P$ and $0 < l_P < \infty$. The functions $f_{i,P}$ and $F_{i,P}$ are defined as:

$$f_{i,P} := F_{i,a} = -K_a X_{i,4}. \tag{6}$$

$$F_{i,P} := \begin{bmatrix} F_{i,x} \\ F_{i,y} \\ F_{i,z} \end{bmatrix} = \begin{bmatrix} -u_{i,z} m_i^{-1}(\mathrm{c}\Phi_i \mathrm{s}\theta_i \mathrm{c}\psi_i + \mathrm{s}\Phi_i \mathrm{s}\psi_i) \\ -u_{i,z} m_i^{-1}(\mathrm{c}\Phi_i \mathrm{s}\theta_i \mathrm{s}\psi_i - \mathrm{s}\Phi_i \mathrm{c}\psi_i) \\ -u_{i,z} m_i^{-1}(\mathrm{c}\Phi_i \mathrm{c}\theta_i) + \mathrm{g} \end{bmatrix}. \tag{7}$$

The control formulation in the present research involves designing a robust distributed nonlinear law for multiple quadrotors with a leader-follower structure. The formation pattern is determined by the desired relative position from the i^{th} follower quadrotor to the leader quadrotor 0, where $\Delta_{i0,P} := [\Delta_{i0,x}, \Delta_{i0,y}, \Delta_{i0,z}]^T, i = \overline{1, N}$.

Definition 1. Considering a multi-quadrotor system consisting of $N + 1$ agents; N followers $i \in \{1, \ldots, N\}$ with one leader labeled as 0. The dynamics expressing position of the agents is given in (5). Let the formation tracking errors along the x, y and z axes be defined as

$$e_{i,0}^P(t) \coloneqq P_i - P_0 - \Delta_{i0,P}. \quad (8)$$

where $e_{i,0}^P(t) \coloneqq [e_{i,0}^x, e_{i,0}^y, e_{i,0}^z]^T \in \mathbb{R}^3, i = \overline{1,N}$. The control problem is to design a distributed formation law $u_{i,P} \coloneqq [u_{i,x}, u_{i,y}, u_{i,z}]^T \in \mathbb{R}^3$ to ensure the following features:

(i) Fast convergence to the desired formation pattern

(ii) Robustness against lumped disturbances

(iii) The tracking error of formation lead to the origin in a fixed-time, i.e., for $\forall e_{i,0}^P(t), \forall i = \overline{1,N}, \exists T_f > 0$, such that

$$\lim_{t \to T_f} e_{i,0}^P(t) = 0, \quad \forall t > T_f \text{ or } \lim_{t \to T_f} P_i - P_0 = \Delta_{i0,P} \quad (9)$$

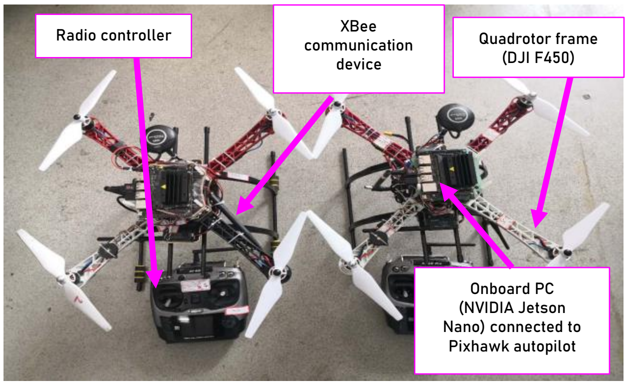

Fig. 1. Quadrotor aircraft. Two identical experimental platforms are established and other platforms are being set-up to further conduct real outdoor flight experiments of the formation control.

III. Control Design and Stability Analysis

The control of formation in a multi-agent system can be realized by robustly tracking the position references and stabilizing the attitude for each quadrotor. The present research exploits hierarchical control by adopting an inner-outer loop structure. The inner-loop is based on a PID controller while the outer-loop implements a distributed control law for robust tracking of the position. The inputs to the outer-loop are the states of the leader quadrotor and the relative position deviations between the leader and each follower $\Delta_{i0,P}$. The outputs of the outer-loop are the velocities' setpoints which are sent to the Pixhawk autopilot in 'offboard mode'.

A. Position Control Design

The overall disturbances that affect the position subsystem are attenuated by the finite-time observer (FTO) designed in our previous work [2]. From (8), the formation tracking error and its dynamics are obtained as

$$\begin{cases} e_{i,0}^P \coloneqq P_i - P_0 - \Delta_{i0,P}, \\ \epsilon_{i,0}^P \coloneqq \dot{e}_{i,0}^P. \end{cases}, \quad i = \overline{1,N} \quad (10)$$

where $P_0 = [x_0, y_0, z_0]^T$ denotes the position of the leader and $P_i = [x_i, y_i, z_i]^T$ refers to the position of followers. $\epsilon_{i,0}^P \coloneqq \dot{e}_{i,0}^P = [\epsilon_{i,0}^x, \epsilon_{i,0}^y, \epsilon_{i,0}^z]^T$. By differentiating (10), we get

$$\begin{cases} \epsilon_{i,0}^P = \dot{e}_{i,0}^P, \\ \dot{\epsilon}_{i,0}^P = \ddot{e}_{i,0}^P = \ddot{P}_i - \ddot{P}_0. \end{cases} \quad (11)$$

To improve the tracking performance of the position system, this work proposes the following continuous nonsingular terminal sliding manifold for the position of each follower (5)

$$s_i^P(t) \coloneqq \dot{\epsilon}_{i,0}^P(t) + \sum_{j=1}^n k_j^P \left(\lceil e_{i,j}^P(t) \rfloor^{\alpha_{1,j}} + \lceil e_{i,j}^P(t) \rfloor + \lceil e_{i,j}^P(t) \rfloor^{\alpha_{2,j}} \right). \quad (12)$$

where the nonnegative parameters $k_j^P, \alpha_{1,j}$ and $\alpha_{2,j}$ are chosen based on Lemma 2. Besides, since the position system in (5) is a second-order system, we have $n = 2$, hence $j = 1,2$. Therefore, for $j = 1$, $e_{i,1}^P \coloneqq e_{i,0}^P$ and for $j = 2$, $e_{i,2}^P \coloneqq \epsilon_{i,0}^P$. The control input needed to ensure the reaching phase of s_i^P and the sliding motion on $s_i^P = 0$ is designed as

$$F_{i,P} \coloneqq u_{i,eq} + u_{i,r} \quad (13)$$

where $u_{i,eq}$ and $u_{i,r}$ are parts corresponding to equivalent control and reaching control respectively. The $u_{i,eq}$ control keeps the variables on the sliding manifold whereas the $u_{i,r}$ control ensures the fast fixed-time convergence. $u_{i,eq}$ can be determined from the sliding motion $s_i^P = 0$. Therefore, when $s_i^P = 0$, the sliding surface becomes as

$$s_i^P = \dot{\epsilon}_{i,0}^P + \sum_{j=1}^n k_j^P \left(\lceil e_{i,j}^P \rfloor^{\alpha_{1,j}} + \lceil e_{i,j}^P \rfloor + \lceil e_{i,j}^P \rfloor^{\alpha_{2,j}} \right) = 0. \quad (14)$$

Thus, the equivalent control can be obtained as

$$u_{i,eq} \coloneqq -f_{i,P} - d_{i,P} + \ddot{P}_0 - \sum_{j=1}^n k_j^P \left(\lceil e_{i,j}^P \rfloor^{\alpha_{1,j}} + \lceil e_{i,j}^P \rfloor + \lceil e_{i,j}^P \rfloor^{\alpha_{2,j}} \right). \quad (15)$$

The $u_{i,r}$ control is selected to make sure that the states' trajectories reach the sliding manifold in fixed-time. The $u_{i,r}$ control is proposed as

$$\dot{u}_{i,r} \coloneqq -k_i^P \left(\lceil s_i^P \rfloor^{\xi} + \lceil s_i^P \rfloor^{\varepsilon} \right). \quad (16)$$

where $k_i^P, \xi, \varepsilon \in \mathbb{R}_+$ are positive constants. Finally, using (15), (16), and considering Assumption 1 ($f_{i,P} = 0$), the required virtual control protocol $F_{i,P}^d$ is given as

$$F_{i,P}^d \coloneqq -\hat{d}_{i,P} + \ddot{P}_0 - \sum_{j=1}^n k_j^P \left(\lceil e_{i,j}^P \rfloor^{\alpha_{1,j}} + \lceil e_{i,j}^P \rfloor + \lceil e_{i,j}^P \rfloor^{\alpha_{2,j}} \right) - u_{i,r}. \quad (17)$$

From the control point of view, $F_{i,P}^d$ are the velocities' setpoints to be sent to the Pixhawk autopilot.

Theorem 1. Considering the feedback-loop system comprising of (5) and the distributed control laws (17), the formation tracking errors are guaranteed to demonstrate fixed-time stabilization to the origin.

Proof. Firstly, we investigate the fixed-time reaching of the sliding manifold. Secondly, convergence of the tracking errors along the sliding surface in fixed-time is proved.

Step 1. By substituting $\dot{\epsilon}_{i,0}^P$ by its expression into the sliding surface $s_i^P(t)$ in (12), we get

$$s_i^P = f_{i,P} + F_{i,P} + d_{i,P} - \ddot{P}_0 + \sum_{j=1}^n k_j^P \left(\lceil e_{i,j}^P \rfloor^{\alpha_{1,j}} + \lceil e_{i,j}^P \rfloor + \lceil e_{i,j}^P \rfloor^{\alpha_{2,j}} \right) = 0. \quad (18)$$

Then, by substituting (17) into s_i^P (18), we get

$$s_i^P = u_{i,r} + d_{i,P} - \hat{d}_{i,P}, \quad (19)$$

Moreover, the FTO estimates the disturbances in finite-time T_0, hence $d_{i,P} \equiv \hat{d}_{i,P}$, and thereby

$$s_i^P = u_{i,r}, \quad \text{for } t \geq T_0 \quad (20)$$

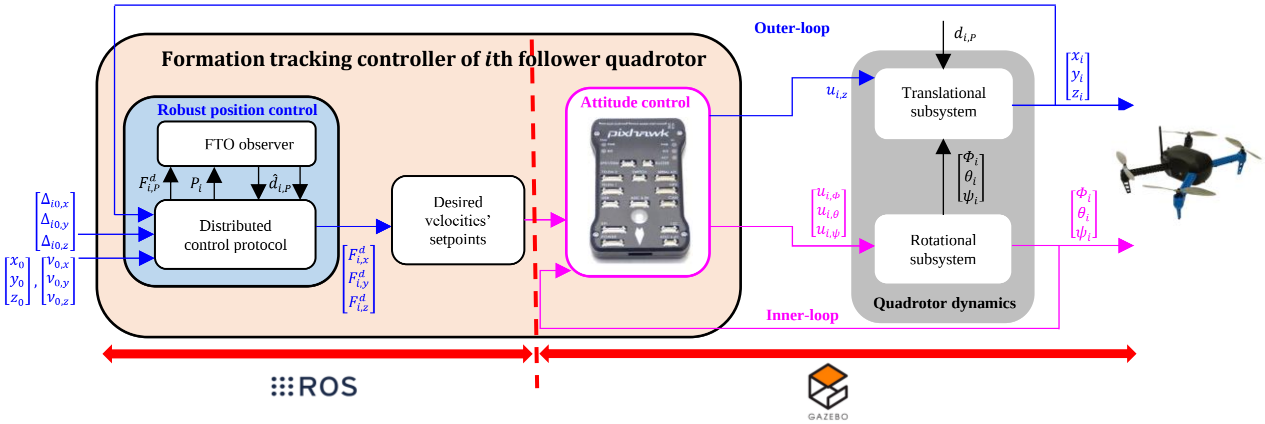

Fig. 2. Proposed control scheme for each follower quadrotor - Block diagram showing the integration of controller with quadrotor dynamics

By differentiating (20) and using (16), we get

$$\dot{s}_i^P = \dot{u}_{i,r} = -k_i^P\left(\lceil s_i^P \rfloor^{\xi} + \lceil s_i^P \rfloor^{\varepsilon}\right) \tag{21}$$

Consider the candidate Lyapunov function given in (22)

$$V \coloneqq \textstyle\sum_{i=1}^N |s_i^P|, \tag{22}$$

where $|s_i^P| \coloneqq s_i^P \operatorname{sign}(s_i^P)$. The derivative of (22) w.r.t. time is

$$\begin{aligned}\dot{V} &= \textstyle\sum_{i=1}^N \operatorname{sign}(s_i^P)\dot{s}_i^P,\\ &\le -\textstyle\sum_{i=1}^N k_i^P\left(\lceil s_i^P \rfloor^{\xi} + \lceil s_i^P \rfloor^{\varepsilon}\right)\end{aligned} \tag{23}$$

Then, we can get

$$\begin{aligned}\dot{V} &\le -N^{1-\xi}\underline{k}\left(\textstyle\sum_{i=1}^N |s_i^P|\right)^{\xi} - \underline{k}\left(\textstyle\sum_{i=1}^N |s_i^P|\right)^{\varepsilon},\\ &\le -N^{1-\xi}\underline{k}\, V^{\xi} - \underline{k}V^{\varepsilon}\end{aligned} \tag{24}$$

where $\underline{k} > 0$ and $\underline{k} \coloneqq \min\{k_1^P, k_2^P, \cdots, k_N^P\}$. Let $N^{1-\xi}\underline{k}, \bar{\mu} \coloneqq \underline{k}$. Thus, it follows from Lemma 1 that the settling time to guarantee the fixed-time reaching of the sliding manifold $s_i^x = 0$ is $T_r \le T_{\max} \coloneqq \frac{1}{N^{1-\xi}\underline{k}(\xi-1)} + \frac{1}{\underline{k}(1-\varepsilon)}$, which is not a function of the initial conditions.

Step 2. When the sliding motion occurs (i.e., $s_i^P \equiv 0$), it yields

$$\ddot{P}_i = \ddot{P}_0 - \textstyle\sum_{j=1}^n k_j^P\left(\left\lceil e_{i,j}^P \right\rfloor^{\alpha_{1,j}} + \left\lceil e_{i,j}^P \right\rfloor + \left\lceil e_{i,j}^P \right\rfloor^{\alpha_{2,j}}\right). \tag{25}$$

Recalling the formation errors' dynamics from (11)

$$\begin{cases}\dot{e}_{i,0}^P = \epsilon_{i,0}^P,\\ \dot{\epsilon}_{i,0}^P = \ddot{P}_i - \ddot{P}_0,\end{cases} \tag{26}$$

Substituting (25) into the errors' dynamics, one gets

$$\begin{cases}\dot{e}_{i,0}^P = \epsilon_{i,0}^P,\\ \dot{\epsilon}_{i,0}^P = -\textstyle\sum_{j=1}^n k_j^P\left(\left\lceil e_{i,j}^P \right\rfloor^{\alpha_{1,j}} + \left\lceil e_{i,j}^P \right\rfloor + \left\lceil e_{i,j}^P \right\rfloor^{\alpha_{2,j}}\right).\end{cases} \tag{27}$$

From Lemma 2, we can conclude that the tracking error dynamics given in (27) can be stabilized to zero in fixed-time during the sliding motion $s_i^x = 0$. Hence, there exists a constant T_s independent of initial conditions such that $e_{i,0}^P \to 0$ and $\epsilon_{i,0}^P \to 0$ for all $t \ge T_f \coloneqq T_0 + T_r + T_s$. Finally, since $e_{i,0}^P \to 0$, so, we obtain $P_i \to P_0 - \Delta_{i0,x}$ thus concluding the proof.

IV. Simulation Results and Discussions

Numerical simulations are conducted to characterize the control performance. A comparative study is performed to quantify the relative effectiveness of the presented control scheme. A pseudo-real world scenario is selected based on a realistic environment in Gazebo connected with ROS. The controller has been implemented in ROS using C++ code.

The parameters corresponding to a physical quadrotor system include: $m_i = 1.70$ kg, $J_{i,xx} = 0.0232$ kg m^2, $J_{i,yy} = 0.0249$ kg m^2, $J_{i,zz} = 0.0342$ kg m^2, $l = 0.225$ m . The formation consists of four followers marked as 'Follower 1-4' lead by a leader named as 'Leader 0'. A directed connected graph is used to describe the communication paradigm among the agents. The desired formation pattern in the xy plane and the communication topology are illustrated in Fig. 3. The reference formation shape is set to be a pentagon in the xy plane. The relative position deviations between the leader 0 and the followers 1-4 are: $\Delta_{10} = [-3,3,0]^T, \Delta_{20} = [-3,-3,0]^T, \Delta_{20} = [-3,-3,0]^T, \Delta_{30} = [-6,-3,0]^T, \Delta_{40} = [-6,3,0]^T$. The leader is subject to tracking a circular reference trajectory given by

$$[x_d, y_d, z_d]^T = [2\sin(0.05t), 2\cos(0.05t), 2.8]^T \tag{28}$$

The selected parameters of the position controller are: $k_1^x = k_2^x = 2, k_1^y = k_2^y = 1.4, k_1^y = k_2^y = 1.2,\ k_i^x = k_i^y = k_i^z = 0.2$. A value of $\alpha = 0.85$ is taken to tune the exponents of the position controller.

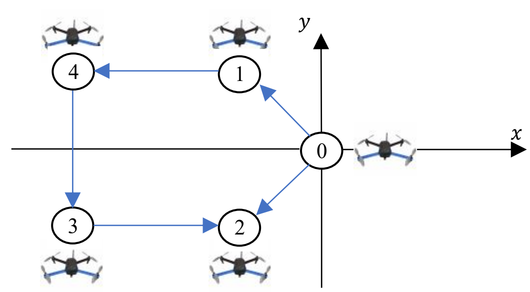

Fig. 3. Desired formation pattern and communication topology graph among the networked quadrotor system.

The formation control is analyzed in simulation to investigate the effect of internal model uncertainties, parametric variations and time-varying externally applied wind disturbances. These disturbances are defined as follows. Uncertainties of +50% and +30% are introduced in $J_{i,xx}, J_{i,yy}, J_{i,zz}$(moments of inertia) and m_i(total mass) respectively. According to Assumption 1, the unmodeled dynamics in the control law are present as disturbances. The wind model provided by Gazebo is used to simulate external disturbances.

A. *Results of the Proposed Robust Formation Controller*

The formation tracking results of our proposed controller are presented in Fig. 4-7. The way in which the quadrotors' swarm form the 3D pattern and follow the reference flight trajectory is presented in the 3D state-space in Fig. 4. It is evident from the figure that the formation is well maintained with accurate trajectory tracking. Moreover, no collision happened between the agents. Fig. 5 shows the translational position of all quadrotors while Fig. 6 displays the evolution of the tracking errors. As illustrated in Fig. 5-6 that the presented distributed control law allows to achieve the required formation pattern in a short time. This pattern is then maintained in the presence of disturbances while robustly tracking the desired time-varying Cartesian trajectory. The tracking errors are found to be converging to the origin where they steadily maintain their values in the close vicinity. The profiles of the velocities of the quadrotors are illustrated in Fig. 7. A consensus between the velocities of the followers and the leader is shown in the figure, hence the formation can be maintained.

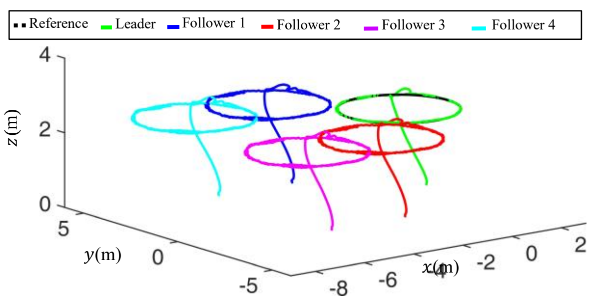

Fig. 4. 3D formation tracking of the quadrotors.

B. *Results of the Distributed PID Controller*

The 3D formation tracking of the quadrotors is depicted in Fig. 8. The formation tracking errors can be seen in Fig. 9, which shows the inability of the PID controller in terms of accurate tracking of the position trajectory, where the follower quadrotors are significantly affected by the disturbances. Fig. 9 shows that the tracking error can reach almost 0.2 m for the followers on the y axis for instance, which is very large compared to our proposed robust controller.

For the sake of quantitatively comparing the results obtained, Integral Square Error (ISE) index is used, i.e., ISE $:= \int_{t_0}^{t_f} e_{i,0}^{P}(\tau)^2 \ d\tau$, where t_0 and t_f represent the initial and the final time instants respectively, $i = \overline{1,3}$. The ISE is calculated and is presented in Table I where the bold text indicates the best performances. It is clear from the table that the presented controller demonstrates the smallest ISE value for all position states of the four followers. It is apparent that the PID law, being a linear control strategy, shows high sensitivity to disturbances and shows an inadequate response with large tracking errors.

Remark 1. We have conducted further comparative simulations which are not depicted here for the sake of brevity. It is observed that the settling-time of the finite-time convergent controller proposed in [19] increases with the increase in the values of the initial conditions. In contrast to that, in the present work, the formation errors reach the origin in the same settling-time for different initial positions of the quadrotors as evidenced by the proposed fixed-time convergent controller. Additionally, the authors of [20] have shown that their fixed-time controller can guarantee a settling-time of 12s which is larger than the settling-time of our controller (11.53 s).

TABLE 1
ISE PERFORMANCE INDEX FOR THE FORMATION TRACKING CONTROL

Agent	Control technique	Performance index ISE		
		x_i	y_i	z_i
Follower 1	PID	10.08	29.30	1.76
	Proposed	**0.07**	**0.03**	**0.03**
Follower 2	PID	5.88	12.20	2.76
	Proposed	**0.29**	**0.07**	**0.09**
Follower 3	PID	10.35	13.03	1.39
	Proposed	**0.12**	**0.08**	**0.10**
Follower 4	PID	11.08	13.20	2.17
	Proposed	**0.18**	**0.11**	**0.08**

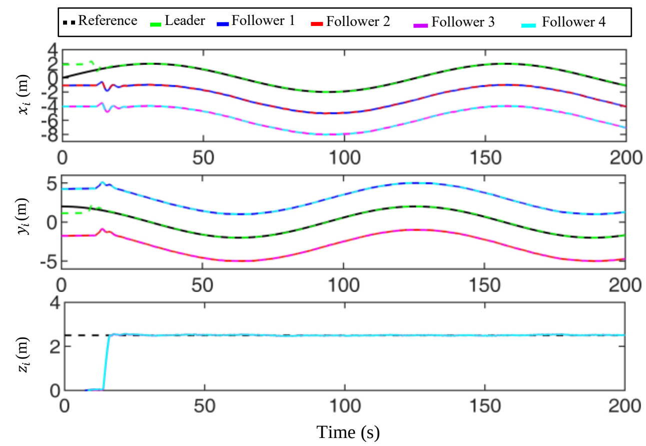

Fig. 5. Profiles of the position of the quadrotors (x_i, y_i, z_i).

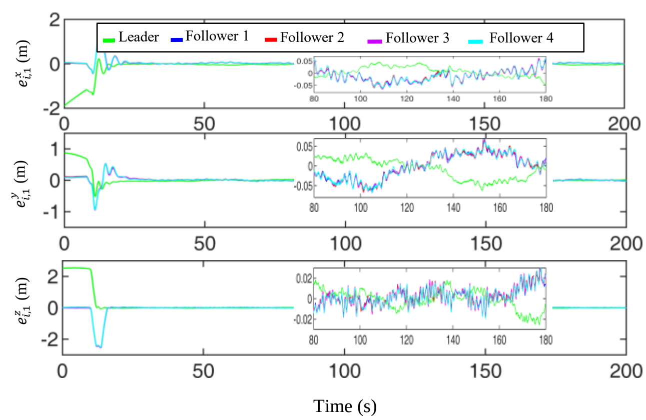

Fig. 6. Evolution of the formation tracking errors $(e_{i,1}^{x}, e_{i,1}^{y}, e_{i,1}^{z})$.

V. CONCLUSION

This research presents the robust formation tracking control scheme for a team of quadrotors subjected to disturbances. A distributed control protocol has been designed. The proposed flight control system demonstrated formation tracking in the presence of lumped disturbances in a fixed-time. Smooth nature of the control signal permits avoiding chattering phenomenon. Lyapunov theorem has been applied for rigorous stability analysis of the feedback loop system. Numerical simulations have demonstrated the superiority of the designed formation control algorithm. Potential future work includes addressing communication delays. Also, a real outdoor flight experiment is anticipated to be conducted in near future.

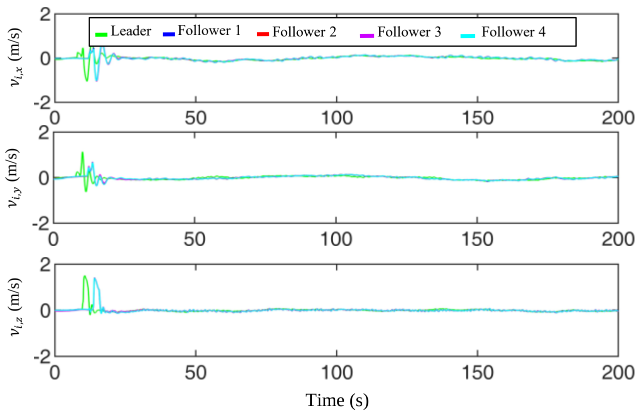

Fig. 7. Profiles of the velocities of the quadrotors $(v_{i,x}, v_{i,y}, v_{i,z})$.

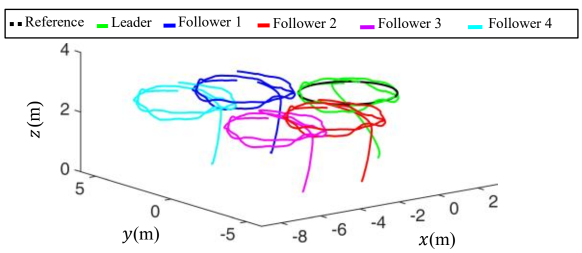

Fig. 8. 3D formation tracking of the quadrotors: PID.

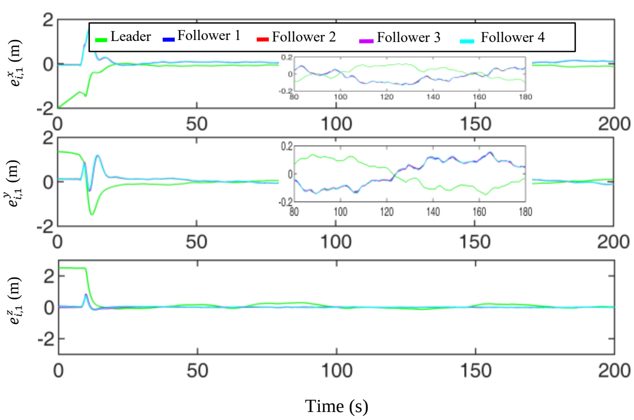

Fig. 9. Evolution of the formation tracking errors $(e^x_{i,1}, e^y_{i,1}, e^z_{i,1})$: PID.

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