Managing performance expectations in association football

Abstract

Motivated by excessive managerial pressure and sackings, together with associated questions over the inefficient use of scarce resources, we explore realistic performance expectations in association football. Our aim is to improve management quality by accounting for information asymmetry. Results highlight uncertainty caused both by football's low-scoring nature and the intensity of the competition. At a deeper level we show that fans and journalists are prone to under-estimate uncertainties associated with individual matches. Further, we quantify reasonable expectations in the face of unevenly distributed resources. In line with the statactivist approach we call for more rounded assessments to be made once the underlying uncertainties are adequately accounted for. Managing fan expectations is probably impossible though the potential for constructive dialogue remains.

Keywords: Alpha effect, Association football, Information Asymmetry, Pareto Principle, Quantitative Methods, Statactivist

1. Introduction

Innovation is playing an increasingly prominent role in sport business management as a result of many changes taking place in the sport landscape. This process is particularly characterised by increased competition (Ratten, 2017). Association football is the world's most popular sport played by approximately 250 million players in over 200 countries and dependencies. Association football is renowned for intense levels of competition culminating in excessive managerial pressure and sackings (Cooper and Johnston, 2012; Calvin, 2015) together with nonlinear reward structures (Carlsson-Wall et al., 2016). One of the main reasons for sackings is the asymmetry of information between fans and football managers (see e.g. Dolles et al., 2014). However, managerial sackings constitute a complex and multi-facetted subject.

Sackings represent the ultimate sanction for perceived poor performance. Hope (2003) and Bell et al. (2013a) outline economic models that may help to determine when sackings are appropriate. However, Flint et al. (2016) question whether recent episodes may have exceeded accepted ethical and legal standards. There are also large question marks surrounding the inefficient allocation of scare resources (Flint et al., 2014). The website offthepitch.com reports that in the years 2015-2019 English Premier League clubs paid £130 million in compensation to sacked managers. Typically, managerial sackings seem to lead to lower mean performance but a higher variance (Audas et al., 2002; d'Addona and Kind, 2014) leading to suggestions that some managers have been sacked on the basis of a "gamble for resurrection" (d'Addona and Kind, 2014). There is a clear danger that managers may be forced out due to bad luck rather than bad performance (ter Weel, 2011). This is an issue we explore in depth in Sections 5-6 below. There have been dramatic increases in the number of sackings over time (d'Addona and Kind, 2014). This chronic insecurity has been variously associated with added commercial and fan pressures, increased player power and new overseas ownership structures (Flint et al., 2016). There have even been links made between managerial sackings and stock-price increases of listed clubs (Bell et al., 2013b). Risk factors for sackings include manager age (d'Addona and Kind, 2014), managerial inexperience, transfer-fee spending and a short length of time remaining on their contract (ter Weel, 2011).

In this context the expectations of sports fans and spectators are particularly important as a determinant of both quality and purchase intentions (Cronin et al., 1992; Brady et al., 2002). However, a self-fulfilling prophecy may also be at play whereby customer perceptions have a greater impact upon purchase intentions than the actual level of service quality delivered (Cronin et al., 1992; Brady et al., 2002). Within all sports, especially association football, the performance expectations of fans and spectators may thus require careful management. As discussed below the proper analyses must variously account for randomness, intense competition (Sections 4-6) and resource imbalances (Section 6). The specific case of agency between fans (principal) and team managers (agent) can help to explain the imbalanced reactions and conflicts that arise in the face of uncertain events (Beccerra and Gupta, 1999). Managers also have a role in reducing this information asymmetry and better communication in the press about performance could allow for a more balanced and realistic formation of expectations.

In this paper we use quantitative methods to address the issue of (reasonable) performance expectations, to reduce asymmetry of information and to improve management quality. A particular problem is that such applications are commonly associated with a self-defeating managerialist perspectives (see e.g. Burrows, 2012). Here, in contrast, we wish to pursue a "statactivist" approach (Samuel, 2014; Erickson et al., 2020) whereby the process of quantifying phenomena can help to reveal prevailing injustices by reducing information asymmetry. Our particular motivations are the intense pressures and unfair sackings currently facing managers (see e.g. Cooper and Johnston, 2012) together with questions over the inefficient allocation of resources given the large sums of money paid in compensation packages to sacked managers. It is also instructive to investigate the role that pure chance may continue to play in such episodes.

The increased recent emphasis on data analytics within sports innovation (Ratten, 2017) necessitates the need for additional quantitative modelling and forecasting of team performance. The use of quantitative methods within this environment is inevitable given both the mass of information available and the depth of analysis required (Baker and McHale, 2015). Given the widespread use of sports betting markets it is in principle extremely easy for stakeholders such as fans or journalists to gain access to calculated real-time outcome probabilities. However, this actually makes the models and the lines of enquiry adopted in this paper even more pertinent. Past work shows that expert football tipsters display both poor forecasting performance (Forrest and Simmons, 2000) and an inability to incorporate all relevant publicly available information (Andersson et al., 2009). Further, as discussed in Section 5, football stakeholders are prone to an allegiance bias that means they systematically over-estimate probabilities associated with desirable outcomes (Edmans et al., 2007; Bernile and Lyandres, 2011).

Our paper contributes to the wider literature on outcome uncertainty and results forecasting. Previous approaches to forecasting football matches include mathematical modelling (Boshnakov et al., 2017; Owen, 2011), machine learning (Constantinou et al., 2012; Baboota and Kaur, 2019) and betting markets (Vlastakis et al., 2009; Angelini and De Angelis, 2019). Indeed, the Poisson model covered in this paper arises as an important special case of the model in Boshnakov et al. (2017). However, theoretical elements of this Poisson model remain of independent interest in its own right (see e.g. the discussion in Fry et al., 2021). In this paper we develop new theoretical models to provide an extended theoretical treatment of the managerial and performance issues covered in Hope (2003). From the perspective of managerial applications results obtained are interesting and important in their own right. However, our specific innovation in Section 6 involves invoking models inspired by Pareto power-law models in statistical physics. Popularised by the so-called 80/20 rule this can help us to examine the performance implications of extreme discrepancies in the level of team resources. To date this seems to have been underexplored in the literature. Our model thus remains analytically tractable without making the rather extreme simplifying assumption that all teams are equally strong.

Our contribution to sports innovation management is as follows. Given the recent growth in sports betting discussed above there are thus contributions to wider sports innovation themes such as data analytics and technology innovation (Ratten, 2017) and e-service innovation (Chuang and Lin, 2015). Association football is particularly interesting in this regard as it has previously been resistant to the introduction of data analytics and new technologies (Ratten, 2017). Amid growing interest embryonic quantitative approaches to association football are discussed in Anderson and Sally (2013) and Kuper and Szymanksi

(2014). Football's transparent tournament structures and extreme competitiveness also make it particularly amenable to an analytical treatment. We discuss new ways of conceptualising network effects in sports innovation when competition is either perfect (Section 4) or otherwise distorted by large resource imbalances (Section 6). We introduce new models to quantify forms of explicit and tacit knowledge (allegiance bias, resource imbalances).

Sports innovation activity can increase when there is a more favourable environment that includes both less fear of failure and a better understanding of the innovation process (Ratten, 2017). It is clear that the intense pressures and unfair sackings currently facing managers (see e.g. Cooper and Johnston, 2012) may reduce management quality and raise questions about the inefficient allocation of resources given the large sums of money paid in compensation packages to sacked managers. However, this pales into insignificance against the backdrop of ethical and legal violations associated with recent episodes (Flint et al., 2016). A further aim is therefore that by promoting greater understanding, and strengthening the relationship between fans and other stakeholders (Beccarini and Ferrand, 2006), we may provide innovative ways of creating and nurturing the fan experience.

A further motivation behind this study is the theory of explicit versus tacit knowledge in sports innovation management (Ratten, 2017). Whist the analogy is not exact it remains useful. Explicit knowledge is easily codified and more readily transferable. In contrast, tacit knowledge requires additional experience and skill and involves the complex interplay between multiple types of information. Examples discussed in Sections 5-6 include psychological effects and the effects of resource imbalances. While both approaches remain valuable, there may be important differences as to the level of intuition involved in each case. Under an explicit-knowledge-based approach it is possible to construct scenarios whereby a systematic and quantitative way of working leads to intuitive findings. Here, only a limited amount of information is required for fans to form reasonable expectations. See Sections 3-4. However, using a tacit-knowledge-based approach, it is also possible to construct scenarios that lead to counter-intuitive insights. Therefore, in this case, asymmetry of information is important and fans may require additional information in order to form realistic expectations. See Sections 5-6. This, in turn, is suggestive of a need for more communication and a more relational way of working between management and stakeholders.

The importance of our contribution is twofold. Firstly, we develop quantitative methods to discuss intuitive and counter-intuitive aspects of team performance across both individual matches and season-long competitions. Our ultimate aim is to reduce asymmetry of information and improve management quality. Such an approach is broadly in line with theories of explicit (intuitive) and tacit (counter-intuitive) knowledge in sports innovation management (Ratten, 2017). Our methods share their roots with classical applied probability (Feller, 1966) and physics (Proakis, 1983) models. Nonetheless, we discuss the straightforward statistical implementation of our models using standard software such as MS excel. Secondly, in line with the statactivist agenda (Erickson, 2020; Samuel, 2014), and as a counter-point to intense managerial pressures (Carlsson-Wall et al., 2016; Cooper and Johnston, 2012), we call for more rounded judgements, and a more relational way of assessing team performance, together with a need to allocate scarce financial resources more efficiently. Our results highlight both the role played by pure chance and the extremely competitive nature of elite sport (Ratten, 2017).

The layout of this paper is as follows. Section 2 reviews the literature. Intuitive explicit-knowledge-based approaches to the analysis of individual matches and season-long tournaments are discussed in Sections 3-4. Counter-intuitive tacit-knowledge-based approaches to the analysis of individual matches and season-long tournaments are discussed in Sections 5-6. Section 7 concludes and discusses the opportunities for further research. An Appendix discussing the implementation of these models in MS Excel is contained at the end of the paper.

2. Literature review

2.1. Expectations in soccer and related sports

Fan expectations contain both a mixture of emotional and rational components (Beccarini and Ferrand, 2006). It is hard to overstate the depth of the emotion involved. Numerous studies point to a relationship between soccer results and stock market performance (see e.g. Bernile and Lyandres, 2011; Berument and Ceylan, 2012; Demir and Danis, 2011). An association between American football results and domestic violence is outlined in Card and Dahl (2011).

Behavioural expectations may have a profound impact on match outcomes and, perhaps more importantly, how results are interpreted by fans and pundits alike. Bartling et al. (2015) show that the behaviour of professional soccer players and coaches depends significantly on whether or not their team is behind the expected match outcome. In a related vein the home advantage that persists throughout soccer may be linked to self-fulfilling expectations and the seemingly more offensive strategies typically followed by home-team managers (Staufenbiel et al., 2015). When it comes to interpreting results evidence from teams listed on stock markets suggests investors are overly optimistic about their teams' prospects. This has been termed "allegiance bias" (Edmans et al., 2007; Bernile and Lyandres, 2011). The poor forecasting performance of "expert" tipsters in soccer is discussed in Forrest and Simmons (2000). There are further suggestions that soccer experts may be both over-confident and fail to appreciate the predictive value of publicly-available information (Andersson et al., 2009).

2.2. Managing performance in association football

Given its high profile and the huge amounts of money involved, academic work on elite-level sport, such as professional football, must include financial and economic considerations (Buraimo et al., 2018; Skirstad and Chelladurai, 2011). Therefore, research on professional sport performance has been heavily linked to the field of economics and principally the concepts of uncertainty of outcome, competitive balance and profit and utility maximisation (e.g. Wilson et al, 2013; Plumley et al, 2017). However, beyond a purely economic analysis, there is perhaps also a need to use such quantitative tools more humanely. Can we use such methods to take a more rounded view of empirical sporting performance given the intense levels of competition involved? A recent example of this kind of analysis is contained in Bell et al. (2013a) who consider the relative performance of football managers in England relative to the amount of money spent on transfer fees.

Contemporary sport, especially at the elite level, presents a complex challenge for management (Wilson et al., 2018). The joint nature of production means that the product sport delivers to participants and fans is idiosyncratic (Smith and Stewart, 2010). Whilst professional sport is in large part just another form of business, it has a range of special features that demand a customised set of practices to ensure its effective operation (Smith and Stewart, 2010). As such, professional sport performance can be seen as a natural phenomenon that can be theorised with the use of probability laws and empirically tested to provide optimised business recommendations. In addition the use of performance management can improve objective setting in football management through more systematic strategic planning (Winand et al., 2010).

Therefore, evaluating sporting performance is of independent interest. This is particularly true given some of the peculiarities of the challenges associated with professional sports (Rika et al., 2016). There is a long history of statistical modelling applied to sports analytics and to football in particular. For instance, the roots of the Poisson model adopted in this paper can be traced back to Maher (1982). In addition, the application of probabilistic models to football betting (Angelini and De Angelis, 2019; Dixon and Pope, 2004) is an important discipline in its own right and performance prediction has improved with the rise of big data modelling (Haigh, 2009; Vlastakis et al., 2009). Therefore, association football is particularly interested in performance management and there is already an established academic literature albeit one that has tended to focus upon purely statistical aspects (see e.g. Maher, 1982; Dixon and Coles, 1997; Dixon and Robinson, 1998). As we demonstrate below football's transparent tournament structures and extreme competitiveness also make it especially amenable to a quantitative treatment. See also (Fujimara and Sugiahra, 2005).

2.3. Managing expectations in sports organisations

The issue of customer expectations within sports organisations is multi-faceted yet relatively under-explored (Robinson, 2006). Sports teams have certain specific challenges relative to other service-sector organisations. In a challenging environment there remains the possibility that dialogue will facilitate the development of realistic expectations (Robinson, 2004).

The analysis of spectators of professional sports has long shown that customers rate their expectations higher than their perceptions of the quality of service they receive (Theodorakis et al., 2001). Rising expectations are problematic for all service providers (Robinson, 2004). However, these difficulties are accentuated within sports due to the additional emotional investments involved (Robinson, 2006). Moreover, there is also the concept of "fandom" and the added importance that being a fan has on many organisations (Van Leeuwen et al., 2002).

There is a long-standing need within organisations to manage expectations in order to improve perceptions of service quality (Boulding et al., 1993). There is particular concern about the gap between expectations and performance (Burns et al., 2003). Ojasalo (2001) makes a crucial distinction between realistic and unrealistic expectations. Whilst the control of customer expectations may be unrealistic (Robinson, 2004) the potential for influence does exist (O'Neil and Palmer, 2003).

3. Explicit-knowledge-based approach to the analysis of individual matches

In this section we discuss aspects of unpredictability relating to individual matches. In particular, using an explicit-knowledge-based approach, we can derive the intuitive result that draws are relatively common in association football due to the game's low-scoring nature. In this case the asymmetry of information is less severe. However, even in the absence of an information asymmetry, these observations are important with respect to managing expectations lest managers become subject to unfair criticism for the results of one-off matches.

Consider the following statistical model of a football match between Teams X and Y discussed in Fry et al. (2021). Suppose the number of goals scored by Teams X and Y are independent and Poisson distributed with parameters λp and $\lambda(1-p)$ respectively. Under this interpretation λ gives the expected number of goals in each match. The parameters p and $1-p$ denote the probability that if a goal is scored it is scored by Team X or by Team Y. In empirical applications λ can be estimated using extensive historical match data. The parameters p and $1-p$ can be estimated using estimated relative team strengths amid further corrections for home advantage.

Under this model it can be shown (see e.g. Fry et al., 2021) that the probability of a draw can be calculated as

$$Pr(\text{Draw}) = e^{-\lambda} I_0(2\lambda\sqrt{p(1-p)}), \tag{1}$$

where $I_0(\cdot)$ denotes the modified Bessel function of the first kind (Abramowitz and Stegun, 1968). In MS excel this can be calculated using the function `BESSELI`.

As an illustration Figure 1 plots the probability of a draw given in equation (1) as a function of the expected number of goals per game assuming that both teams are of equal strength. Reallistic figures for elite soccer leagues are that the expected number of goals per game is between 2.5-3.5 (Fry et al., 2021). In this case we might expect roughly one quarter of all games to end in a draw. Similar probability estimates are obtained in Cain and Haddock (2006). Figure 2 plots how the probability of a draw depends on the relative strengths of the two teams assuming an average of 3.0 goals per game. Results in Figure 2 show relatively high probabilities of a draw due to pure chance alone – even when there appears to be a mismatch in terms of the underlying quality of the two teams. These results can be summarised in the following proposition:

[INSERT FIGURES 1-2 ABOUT HERE]

Proposition 1. *Soccer's low-scoring nature makes the outcome of individual matches highly uncertain. Further, the probability that a match ends in a draw remains relatively high even in the presence of a sizeable mismatch in quality between the two teams.*

4. Explicit-knowledge-based approach to the analysis of season-long competitions

In this section, using an explicit-knowledge-based approach, we discuss points targets in perfectly competitive soccer leagues. Our aim is two-fold. Firstly, we want to highlight the limits on team attainment caused by the intensity of the competition. Secondly, results are also useful in demonstrating that final season outcomes may be subject to considerable variation between one year and the next. History is replete with examples where teams' final-season points totals exceed certain rules of thumb yet still got relegated.

Throughout football folklore rules of thumb abound that in leagues such as the English Premier League and the French Ligue 1 teams need to secure around 40 points to avoid relegation. In other leagues such as the English Championship the rule of thumb is that 50 points are required to avoid relegation. In this section we show how these rules of thumb emerge as a natural consequence of very high levels of competition. In line with the simplicity of this approach there appears to be little or no information asymmetry in this case.

Consider a soccer league of n teams of equal strength who play each other home and away once only. Each team thus plays $2(n-1)$ games over the course of a season. Three points are awarded for a win, one point for a draw and 0 points for a defeat. The objective for team X is to secure enough points over an entire league season so that X defeats a randomly chosen opponent, team Y say, with probability q. For example, the English Premier league consists of $n = 20$ teams. The target to avoid relegation is to defeat at least 3 teams thus finishing 16+1=17th place or higher. This gives $q = (3 \text{ defeated teams})/(19 \text{ rival teams})$. Let X_i denote the number of points scored by team X in game i. We have that $Pr(X_i = 3) = Pr(X_i) = 0 = w$, $Pr(X_i = 1) = 1 - 2w$. It follows that $E[X_i] = 1 + w$. In empirical work and from equation (1) we may estimate

$$1 - 2w = Pr(\text{Draw}) = e^{-\lambda} I_0(\lambda);\ w = \frac{1 - e^{-\lambda} I_0(\lambda)}{2}, \tag{2}$$

where λ is the average number of goals scored in historical matches.

Given that there are $2n - 2$ matches across the league season the expected season points total for team X is $(2n - 2)(1 + w)$. The number of points scored by team X compared to team Y is given by

$$X - Y = \sum_{i=1}^{2n-4} X_i - Y_i + W_{2n-3} + W_{2n-2}, \tag{3}$$

where the W_i represent direct head-to-head matches between the two teams and take the values ± 3 with probability w and 0 with probability $1 - 2w$, the distribution of the X_i and Y_i is as described above and all the random variables in equation (3) are mutually independent. We have that

$$E[X - Y] = 0;\ \text{Var}[X - Y] = \sigma^2 = w[20n - 4] + (8 - 4n)w^2. \tag{4}$$

Using a normal distribution approximation if $Pr(X - Y \leq z) = q$, it follows that

$$\frac{X - Y}{\sigma} \approx \Phi^{-1}(q);\ X \approx Y + \sigma \Phi^{-1}(q), \tag{5}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse CDF of a standard normal random variable. Equations (4-5) lead to the following approximation formulae for end-of-season points targets:

Proposition 2. *Points targets in perfectly competitive leagues:*

$$Target = (1 + w)2(n - 1) + \sqrt{w^2[8 - 4n] + w[20n - 4]}\,\Phi^{-1}(q), \tag{6}$$

where w is defined in equation (2), q is the probability of beating a randomly chosen opponent (outlined above) and Φ^{-1} denotes the inverse CDF of a standard normal random variable.

As an illustration, we apply equation (6) to infer the points targets that would be required to avoid relegation throughout the English football pyramid using data from the last fully-completed 2018/19 league season at the time of writing. The results are shown below in Table 1. Further adjustments to this model to account for a small number of disproportionately strong teams (as would typically seem to be the case in several major European leagues) are discussed below in Section 6.

[INSERT TABLE 1 ABOUT HERE]

5. Tacit-knowledge-based approach to the analysis of individual matches

In this section we use a tacit-knowledge-based approach to show how conventional wisdom is prone to under-estimate the competitiveness of elite-level football. This constitutes a critical source of information asymmetry and sustained communication efforts may be required in order to adequately resolve this. As a numerical example we follow Dyte and Clark (2000) and Suzuki et al. (2010) in considering applications to recent international tournaments. Historical records show that prior to the 2016 championship there had been an average of 2.46 goals per game in European Championship finals matches. We use data from FIFA's Coca Cola world rankings (data correct as of June 2nd 2016) as a proxy measure of team quality to estimate the value of p in the above. This follows a similar approach using FIFA team ratings in Dyte and Clarke (2000).

England's worst performance of all time? England's much derided elimination from EURO 2016 has been described as a "national sporting embarrassment" (McNulty, 2016). However, how does this supposedly shock defeat compare to the output of our model? Perhaps this is not quite the surprise result many people perceive? Prior to Euro 2016 England were world-ranked 11th with 1069 points. In contrast, Iceland were world-ranked 34th with 751 points. Thus, we calculate the relative probability of an England goal as $p = 1069/(1069 + 751) = 0.587$. Under the model in Section 3 and assuming that each team is equally likely to win on a penalty shootout we have that

$$\begin{aligned}
Pr(\text{X wins after 90 minutes}) &= Q_0(\sqrt{2\lambda p}, \sqrt{2\lambda(1-p)}), \\
Pr(\text{X wins after extra-time}) &= \frac{1}{2}Q_0\left(\sqrt{\frac{2\lambda p}{3}}, \sqrt{\frac{2\lambda(1-p)}{3}}\right) \\
&+ \frac{1}{2}\left(1 - Q_0\left(\sqrt{\frac{2\lambda(1-p)}{3}}, \sqrt{\frac{2\lambda p}{3}}\right)\right) \\
&= Pr(\text{X aet}), \\
Pr(\text{X wins after extra-time and penalties}) &= Q_0(\sqrt{2\lambda p}, \sqrt{2\lambda(1-p)}) \\
&+ e^{-\lambda} I_0(2\lambda\sqrt{p(1-p)})Pr(\text{X aet}),
\end{aligned} \tag{7}$$

where $Q_0(\cdot)$ denotes the Marcum Q-function (Nuttall, 1975) and $I_0(\cdot)$ denotes the modified Bessel function of the first kind (Abramowitz and Stegun, 1968). For full details see Fry et al. (2021). From equation (7) the probability of various scores are shown below in Table 2. Results suggest that England's defeat by Iceland may not be the national embarrassment many perceive. The most probable match score prior to kick-off is 1-1. According to our model the probability of a draw after 90 minutes is 0.265, the probability of an England win is 0.470, and the probability of an Iceland win is 0.265. Note that whilst an England win is the most probable pre-match outcome the estimated probability that this outcome is observed, 0.470, is deceptively low. The effect appears linked to the much-discussed "allegiance bias" which results in inflated probability estimates for desirable outcomes (Edmans et al., 2007; Bernile and Lyandres, 2011). Using equation (7) the probability of an overall England win (including extra time and penalties) is estimated to be 0.616 and the probability of an overall Iceland win is estimated to be 0.384. Thus, according to our model there is a roughly 40% chance prior to kick-off that Iceland will beat England. Though England are clear pre-match favourites their defeat is far from unforeseeable. The example thus serves as a cautionary tale against under-estimating the competitiveness of elite-level football. We summarise these results in the following proposition:

[INSERT TABLE 2 ABOUT HERE]

Proposition 3. *Journalists and fans are liable to under-estimate the competitiveness of elite-level football.*

6. Tacit-knowledge-based approach to the analysis of season-long competitions

In this section our motivation is intensive managerial pressure (Cooper and Johnston, 2012; Calvin, 2015). This includes managers whose performance relative to the level of transfer fee spend has been exceptional (Bell et al., 2013a). The model of perfect competition in Section 4 is both conceptually useful and may give useful insights into issues such as the avoidance of relegation. However, using a tacit-knowledge-based approach, an improved analysis is needed to take into account imperfect levels of competition. For example, as in several major leagues throughout Europe, competitions may be dominated by a small number of disproportionately strong teams. Failure to account for these imperfections are liable to result in further under-estimates of the intensity of the competition – especially at the elite level. This imperfect competition constitutes a further source of information asymmetry which, in applications, would also require careful management and sustained communication.

In this section we consider adjustments to the model in Section 4 whereby instead of all teams being equal leagues are instead dominated by a small number of disproportionately strong teams. In related contexts these dominant outliers are termed Dragon Kings in Sornette and Ouillon (2012). We thus account for the failure of the perfect competition model by incorporating a version of the Pareto Principle (see e.g. Box and Meyer, 1986) which states that as a stylised empirical fact across a variety of different economic and social networks approximately 80% of the effects can be attributed to around 20% of the cases.

Motivated by network models in statistical physics (Newman, 2018) we propose the following extension to the Pareto Principle – the α effect. The original

Pareto Principle corresponds to the special case of $\alpha = 0.2$. The perfect competition model in Section 4 corresponds to the case $\alpha = 0$. However, alternative values of α may provide more insight in particular situations. In particular, a value of $\alpha = 0.3$ may be appropriate for the English Premier League which, in recent seasons, has seen the emergence of 6/20 disproportionately strong teams. This leads to the reasonable working assumption that in the English Premier League 70% of the top playing talent is concentrated amongst the top 6 teams. Similarly, if we consider that the Spanish La Liga is dominated by three disproportionately top teams (Barcelona, Real Madrid, Atletico Madrid) a value of $\alpha = 3/20$ may be appropriate. In this case this would lead to the working assumption that in La Liga 85% of the top playing talent is concentrated amongst the top 3 teams.

The layout of this section is as follows. Section 6.1 discusses ways of conceptualising differences between elite and non-elite teams. Section 6.2 discusses revised performance targets for elite and non-elite teams in the face of imperfect competition. Section 6.3 discusses numerical applications to the English Premier League and the Spanish La Liga.

6.1. Conceptualising differences between elite and non-elite teams

In this section we conceptualise differences between elite and non-elite teams as follows. Suppose that standard teams have a team strength normalised to 1. In contrast, elite teams have a team strength of T. In line with the observations above it follows that

$$\begin{aligned}
\text{Total team strengths} &= \text{Elite team strengths} + \text{Non-elite team strengths} \\
&= n\alpha T + n(1-\alpha) \qquad (8) \\
\text{Proportion of elite team strengths} &= 1-\alpha = \frac{n\alpha T}{\alpha T + n(1-\alpha)} \\
\text{Elite team strength} &= T = \frac{(1-\alpha)^2}{\alpha^2}. \qquad (9)
\end{aligned}$$

Suppose that a goal is scored in a game between either two standard teams or between two elite teams. In common with the model in Section 4 we assume that in both cases each team is equally likely to score. In contrast, suppose that a goal is scored in a match between an elite team and a non-elite team. In this case we want to account for the potential mis-match that occurs but also account for the empirical observation in Clarke and Norman (1995) that random effects such as chance and home advantage may account for roughly 0.5 goals in every game.

In line with findings in Clarke and Norman (1995) suppose that goals are scored by pure chance with probability $0.5/\lambda$. This can happen equi-probably in favour of either team. If the goal scored is not down to pure chance it follows from equation (9) that the probability the goal is scored by the elite team is

$$Pr(\text{Elite team scores}) = \frac{\text{Elite team strength}}{\text{Total team strength}} = \frac{T}{T+1} = \frac{(1-\alpha)^2}{1-2\alpha+2\alpha^2}. \quad (10)$$

Suppose a goal is scored. The probability that the elite team scores is given by

$$\begin{aligned} Pr(\text{Elite team scores}) &= Pr(\text{Elite}|\text{Pure Chance})Pr(\text{Pure chance}) \\ &+ Pr(\text{Elite}|\text{ Not Pure Chance})Pr(\text{Not Pure chance}), \\ Pr(\text{Elite team scores}) &= \frac{1}{2\lambda}.\frac{1}{2} + \left(\frac{2\lambda-1}{2\lambda}\right)\frac{(1-\alpha)^2}{1-2\alpha+2\alpha^2} \\ &= \frac{2\alpha-1+4\lambda(1-2\alpha+\alpha^2)}{4\lambda(1-2\alpha+2\alpha^2)}. \quad (11) \end{aligned}$$

Similarly, the probability that the non-elite team scores can be calculated as

$$Pr(\text{Non-elite team scores}) = \frac{1-2\alpha+4\lambda\alpha^2}{4\lambda(1-2\alpha+2\alpha^2)}.$$

It follows from equation (11) that

$$\begin{aligned} Pr(\text{Elite team wins}) &= Q_0\left(\sqrt{\frac{2\alpha-1+4\lambda(1-2\alpha+\alpha^2)}{2(1-2\alpha+2\alpha^2)}}, \sqrt{\frac{1-2\alpha+4\lambda\alpha^2}{2(1-2\alpha+2\alpha^2)}}\right) = w_+ \\ Pr(\text{Non-elite team wins}) &= Q_0\left(\sqrt{\frac{1-2\alpha+4\lambda\alpha^2}{2(1-2\alpha+2\alpha^2)}}, \sqrt{\frac{2\alpha-1+4\lambda(1-2\alpha+\alpha^2)}{2(1-2\alpha+2\alpha^2)}}\right) = w_- \\ Pr(\text{Draw}) &= 1-w_+-w_-. \quad (12) \end{aligned}$$

6.2. Revised performance targets for elite and non-elite teams

In this section we quantify the idea of different mini-leagues within the same league. Consider, for example, the English Premier league with α set equal to

$6/20 = 0.3$. Consider two non-elite teams X and Y who are competing to avoid relegation. The task of avoiding relegation then reduces to finishing ahead of 3 of the other 13 non-elite teams rather than finishing ahead of 3 of the 19 other teams in the division. In this case this suggests setting $q = 3/13$ would be more appropriate than setting $q = 3/19$. The points total scored by team X relative to team Y is given by

$$\begin{aligned} X - Y &= \text{Points against other non-elite teams} \\ &+ \text{Points in head-to head matches} + \text{Points against elite teams} \\ &= \sum (X_i - Y_i) + \sum W_i + \sum (V_i - U_i), \end{aligned} \tag{13}$$

where the X_i and Y_i take the values 3 with probability w, 1 with probability $1 - 2w$ and 0 otherwise, the W_i take the values ± 3 with probability w and 0 otherwise, and the U_i and V_i take the values 3 with probability w_-, 1 with probability $1 - w_- - w_+$ and 0 otherwise. Moreover, all these random variables are mutually independent. It follows from equation (13)

$$\begin{aligned} E[X] &= (2n - 2n\alpha - 2)(1 + w) + 2n\alpha(2w_- + 1 - w_+), \\ \text{Var}[X - Y] &= [20(n - \alpha) - 4]w + [8 - 4n + 4\alpha]w^2 + 4n\alpha[4w_- + w_+ - (2w_- - w_+)^2]. \end{aligned} \tag{14}$$

Equation (14) thus leads to the following points objective for non-elite teams:

Proposition 4 (Points targets in imperfectly competitive leagues for non-elite teams).

$$\begin{aligned} \textit{Target} &= (2n - 2n\alpha - 2)(1 + w) + 2n\alpha(2w_- + 1 - w_+) \\ &+ \sqrt{[20(n - \alpha) - 4]w + [8 - 4n + 4\alpha]w^2 + 4n\alpha[4w_- + w_+ - (2w_- - w_+)^2]}, \end{aligned}$$

where w is the probability of beating a fellow non-elite team and $\Phi^{-1}(\cdot)$ denotes the inverse CDF of a standard normal random variable.

Similarly, in a contest between two elite teams X and Y the points total scored by team X relative to team Y is given by

$$X - Y = \text{Points against other elite teams}$$

$$
\begin{aligned}
&+ \quad \text{Points in head-to-head matches} + \text{Points against non-elite teams} \\
&= \quad \sum(X_i - Y_i) + \sum W_i + \sum(V_i - U_i), \qquad (15)
\end{aligned}
$$

where the X_i and the Y_i take the values 3 with probability w, 1 with probability $1 - 2w$ and 0 otherwise, the W_i take the values ± 3 with probability w and 0 otherwise, and the U_i and V_i take the values 3 with probability w_+, 1 with probability $1 - w_+ - w_-$ and 0 otherwise. Moreover, all the random variables are mutually independent. It follows from equation (15) that

$$
\begin{aligned}
E[X] &= (2n\alpha - 2)(1 + w) + 2n(1 - \alpha)[2w_+ + 1 - w_-] \qquad (16) \\
\mathrm{Var}(X - Y) &= [20n\alpha - 4]w + [8 - 4n\alpha]w^2 + 4n(1 - \alpha)[4w_+ + w_- - (2w_+ - w_-)^2].
\end{aligned}
$$

Equation (16) thus leads to the following points objective for elite teams:

Proposition 5 (Points targets in imperfectly competitive leagues for elite teams).

$$
\begin{aligned}
\mathit{Target} &= (2n\alpha - 2)(1 + w) + 2n(1 - \alpha)[2w_+ + 1 - w_-] \\
&+ \sqrt{[20n\alpha - 4]w + [8 - 4n\alpha]w^2 + 4n(1 - \alpha)[4w_+ + w_- - (2w_+ - w_-)^2]},
\end{aligned}
$$

where w is the probability of beating a fellow elite team and $\Phi^{-1}(\cdot)$ denotes the inverse CDF of a standard normal random variable.

6.3. Numerical examples

In this section we construct performance targets for two major European football leagues (the English Premier League and the Spanish La Liga) under the assumptions of both perfect and imperfect levels of competition. For estimating the number of points needed to win the league we use the method of Hyndman and Fan (1996) to estimate the number of points needed to win the league and choose $q = (n - 1 - 1/3)/(n - 1 + 1/3)$, where n is the number of teams in the league. Results are shown below in Table 3.

[INSERT TABLE 3 ABOUT HERE]

Results shown in Table 3 indicate that imperfect competition leads to dramatic increases in the number of points required to finish high up in the league in both countries. In contrast, results suggest that as a result of imperfect levels of competition fewer points may ultimately be needed to avoid relegation. Results also suggest that imperfect levels of competition may make it harder to compare the results of different eras. For example, results suggest that the effort required to score 32 points in a current Premier League season may have been sufficient to stave off relegation in previous seasons.

7. Conclusions and further work

Motivated by data analytics and technology innovation (Ratten, 2017) we use probabilistic and physics-based models to contribute to long-standing problems in outcome uncertainty and results forecasting (see e.g. Hope, 2003). Our particular innovation lies in new physics-based models to investigate the performance implications of extreme discrepancies in the level of team resources. This simultaneously provide new ways of conceptualising network effects in sports innovation alongside new ways of quantifying forms of explicit and tacit knowledge.

Our motivation stems from elements of increased competition (Ratten, 2017) together with the intense managerial pressure and sackings that characterise modern football (Cooper and Johnston, 2012; Calvin, 2015; Flint et al., 2014). Recent episodes may have violated acceptable ethical and legal standards (Flint et al., 2016). There are also question marks surrounding the inefficient use of scarce resources. Throughout football substantial sums of money have been paid in compensation packages to sacked managers. Even managers with elite performance records relative to the amount of transfer fee spend (Bell et al., 2013a) have been dismissed. We thus follow a statactivist approach and seek greater fairness, justice and understanding once some of the relevant quantitative issues involved are adequately understood (Samuel, 2014; Erickson et al., 2020).

Managing expectations is a long-standing problem within service organisations (Boulding et al., 1993). The level of emotion involved (Robinson, 2006) coupled with the nature of fandom (Van Leeuwen et al., 2002) means that these pressures are further intensified within sports. Whilst controlling expectations is probably impossible (Robinson, 2004) the potential for dialogue and influence does exist (O'Neil and Palmer, 2003). Recent research also underscores the importance of relational ways of working (see e.g. Brown et al., 2017). An important distinction also needs to be made between realistic and unrealistic expectations (Ojasalo, 2001). A potential information asymmetry exists that stems from the intense, albeit imperfect, level of the competition.

However, it is important to note that even when distinguishing between realistic and unrealistic expectations findings may range from the intuitive (explicit-knowledge-based approach) to the counter-intuitive (tacit-knowledge-based approach). Using an explicit-knowledge-based approach it can be shown that some of the uncertainty associated with match outcomes (especially draws) is due to football's low-scoring nature. See Section 3. Some much discussed aspects of leagues, such as the number of points typically required to avoid relegation, can also be linked to the intensity of the level of competition. See Section 4.

However, on occasion, the implications of quantitative analyses can be more complex. Using a tacit-knowledge-based approach we show that journalists and fans are liable to under-estimate the uncertainty associated with one-off matches. An "allegiance bias" is much discussed in the literature and suggests stakeholders are prone to over-estimate the probabilities associated with desirable outcomes (Edmans et al., 2007; Bernile and Lyandres, 2011). See Section 5. In Clarke and Norman (1995) it is shown that random factors such as home advantage can account for a non-trivial amount of the total goals scored (around 0.5 goals per game). Moreover imperfect levels of competition may further increase the standards required in order to meet certain performance objectives. This is especially true if, as is the case with major European leagues such as those in England and Spain, leagues are dominated by a small number of disproportionately strong teams. See Section 6. Using physics-based approaches (the alpha-effect and the Pareto principle) we quantify the uncertainties within such leagues where, as in other social and economic networks (Newman, 2018), resources are unequally distributed. Such inequalities may also cause particular difficulties in comparing the results from different seasons.

Future work will analyse other sports such as rugby and cricket which are higher scoring in nature but generally thought to be less competitive (Szymanski, 2003). Future work will also examine the more reasoned use of performance metrics. This is particularly important given the danger that overtly managerialist procedures can cause in other contexts (see e.g. Burrows, 2012) coupled with a potential failure to adequately account for the complexity of social problems (Law and Urry, 2004). In this case this is caused both by the extremely competitive nature of elite football coupled with the role that lady luck will continue to play in one-off matches.

Appendix: Implementation in MS Excel

Our models require the numerical evaluation of certain mathematical functions. This can be achieved in the standard implementation of MS excel as follows. We have the elementary formula $I_0(x) = \texttt{BESSELI(x, 0)}$ and $\Phi(x) = \texttt{NORM.S.DIST(x, 1)}$. An approximate numerical formula for $Q_0(a, b)$ can be constructed as follows.

As discussed in Annamalai and Tellambura (2008) Q_0 can be written in terms of the Cumulative Distribution Function of a non-central χ^2 random variable:

$$Q_0(\alpha, \beta) = 1 - F_{2,\alpha^2}(\beta^2) - e^{-\frac{\alpha^2+\beta^2}{2}} I_0(\alpha\beta), \tag{17}$$

where $F_{2,\alpha^2}(\cdot)$ denotes the Cumulative Distribution Function (CDF) of the non-central χ^2 distribution with 2 degrees of freedom and non-centrality parameter α^2. From Mai (2015) and Sankaran (1959) a numerical approximation for the Cumulative Distribution Function of a non-central χ^2 distribution can be constructed as

$$F_{\delta,\alpha}(x) \approx \Phi\left(\frac{\left(\frac{x}{\alpha+\delta}\right)^h - \left(1 + hp\left(h - 1 - \frac{(2-h)mp}{2}\right)\right)}{h\sqrt{2p}(1 + mp/2)}\right), \quad (18)$$

where

$$\begin{aligned} h &= 1 - \frac{2(\delta+\alpha)(\delta+3\alpha)}{3(\delta+2\alpha)^2}, \\ p &= \frac{\delta+2\alpha}{(\delta+\alpha)^2}, \\ m &= (h-1)(1-3h). \end{aligned}$$

Combining equations (17-18) leads to the following approximate numerical formula for $Q_0(a,b)$:

$$Q_0(\alpha,\beta) \approx 1 - \Phi\left(\frac{\left(\frac{\beta^2}{2+\alpha^2}\right)^h - \left(1 + hp\left(h - 1 - \frac{(2-h)mp}{2}\right)\right)}{h\sqrt{2p}(1 + mp/2)}\right) - e^{-\frac{\alpha^2+\beta^2}{2}} I_0(\alpha\beta), \quad (19)$$

where

$$\begin{aligned} h &= \frac{2+4\alpha^2+3\alpha^4}{6+12\alpha^2+6\alpha^4}, \\ p &= \frac{2+2\alpha^2}{(2+\alpha^2)^2}, \\ m &= (h-1)(1-3h). \end{aligned}$$

Details of the excel calculations used in this paper can be obtained as an Appendix from the authors by request.

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Figures and Tables

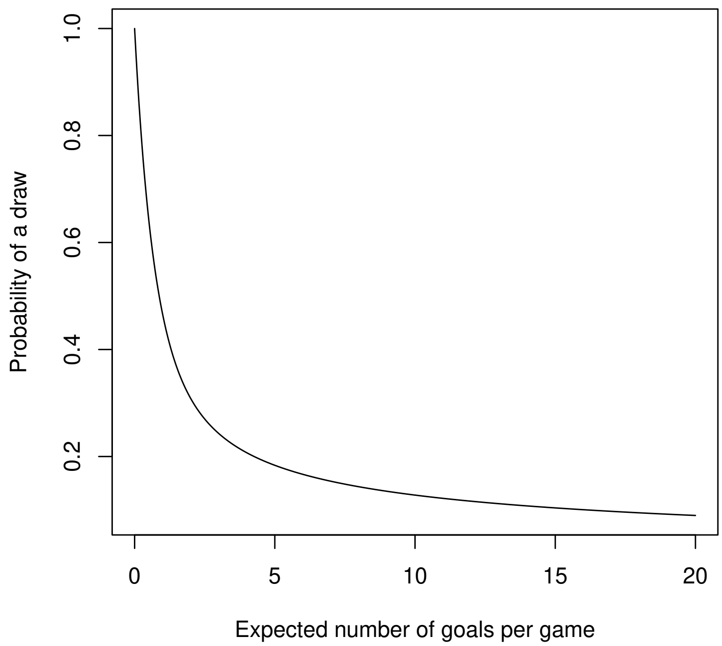

Figure 1: How the probability of a draw depends on the average number of goals per game assuming teams of equal strength.

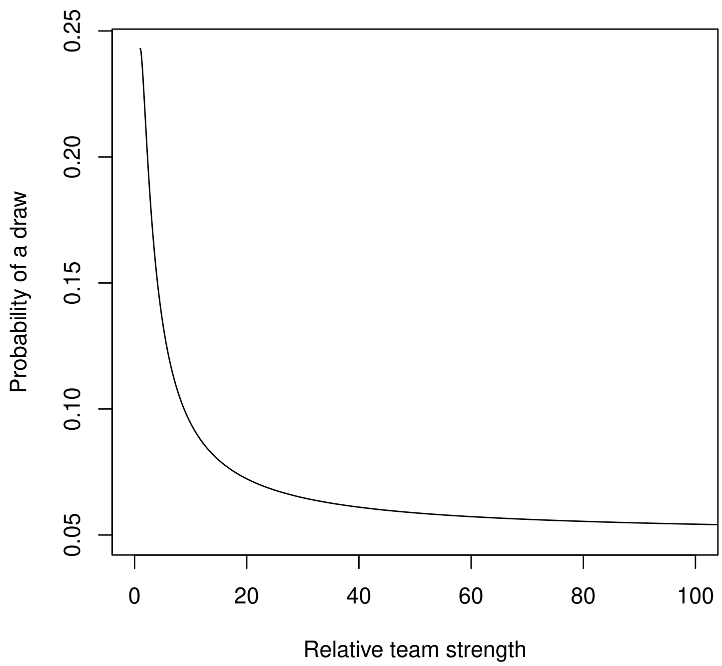

Figure 2: How the probability of a draw depends on the relative team strengths assuming an average number of 3.0 goals scored per game. Relative team strength= $\max\left\{\frac{p}{1-p}, \frac{1-p}{p}\right\}$.

League	**No of. teams**	λ	q	**Points Target**
Premier League	20	2.82	3/19	41
Championship	24	2.67	3/23	49
League 1	24	2.65	4/23	51
League 2	24	2.55	2/23	46
National League	24	2.49	4/23	51
National League (North)	22	2.91	3/21	45
National League (South)	22	2.82	3/21	45
Isthmian Premier	22	2.91	3/21	45
Northern Premier	22	2.85	3/21	45
Southern Premier Central	22	2.75	3/21	45
Southern Premier South	22	3.45	3/21	45

Table 1: Proposed points targets to avoid relegation throughout the English football league pyramid rounded to the next largest whole number. Average number of goals per game (λ) taken from the last fully completed 2018/19 season at the time of writing.

Goals scored	**Iceland 0**	**Iceland 1**	**Iceland 2**	**Iceland 3**
England 0	0.085	0.087	0.044	0.015
England 1	0.123	0.125	0.064	0.022
England 2	0.089	0.091	0.046	0.016
England 3	0.043	0.044	0.022	0.007

Table 2: Probability of various scores for England v. Iceland in EURO 2016

Parameters	English Premier League	
	Perfect Competition	**Imperfect competition**
λ	2.82	2.82
α	–	0.3
w	0.374	0.374
w_+	–	0.741
w_-	–	0.089
Objective		
Champions	$q = 56/58$, Target=74 points	$q = 14/16$, Target=93 points
Champions League Qualification	$q = 16/19$, Target=65 points	$q = 2/5$, Target=79 points
Europa League Qualification	$q = 13/19$ Target=59 points	$q = 38/40$, Target=62 points
Avoid relegation	$q = 3/19$, Target=41 points	$q = 3/13$, Target=32 points
Parameters	**Spanish La Liga**	
	Perfect Competition	**Imperfect competition**
λ	2.59	2.59
α	–	0.15
w	0.368	0.368
w_+	–	0.823
w_-	–	0.038
Objective		
Champions	$q = 56/58$, Target=74 points	$q = 5/7$, Target=99 points
Champions League Qualification	$q = 16/19$, Target=64 points	$q = 47/49$, Target=66 points
Europa League Qualification	$q = 13/19$, Target=58 points	$q = 13/16$, Target=56 points
Avoid relegation	$q = 3/19$, Target=41 points	$q = 3/16$, Target=35 points

Table 3: Suggested performance targets for English and Spanish football leagues assuming both perfect and imperfect levels of competition. Average number of goals per game λ taken from the last fully completed 2018/2019 league season.