Modeling and Control of Anterior-Posterior and Medial-Lateral Sways in Standing Posture

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Abstract

To study essential anterior-posterior and medial-lateral sways of the stance caused by rotational movements about the ankle and hip joints, a mathematical model is developed for the 3D postural kinematics and dynamics. The model is in the form of nonlinear differential-algebraic equations corresponding to a biomechanical system with holonomic constraints. A nonlinear feedback control law is further derived for stabilizing the upright stance, whilst eliminating internal torques induced by the constraints on postural movements. Numerical simulations of the model parametrized with experimental data of human body segments illustrate the performance of postural balancing with the proposed control. This work is an essential step towards a much improved understanding of constrained geometry and balancing control of 3D human standing dynamics.

Keywords: biomechanics, postural dynamics, 3D modeling, nonlinear control, Numerical simulation

1. Introduction

The idea of modeling the human body from a biomechanical perspective dates back more than a century, and has been studied intensively. Understanding and prevention of falls in the fast-growing global population of elderly people (WHO report 2008) can benefit from studies on balancing upright

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stance (Darkin and Bolton, 2018). The naturally evolved human postural control mechanism is very complex, involving interactions between body dynamics, musculoskeletal and neurosensory systems. This work focuses on modeling of 3D standing dynamics and synthesis of nonlinear balancing control.

The majority of the research on balancing the upright stance (Maurer and Peterka, 2005; Pinter et al., 2008; Suzuki et al., 2016; Chumacero-Polanco, et al., 2019) have used single, double or triple inverted pendulum models. These 2D biomechanical models describe the dynamics of anterior-posterior sways, but cannot reveal the essential 3D nature of postural movements. More often than not, postural sways are not solely back and forth or sideways motions, and due to interconnections of the 3D postural dynamics, they cannot be fully described by two separate 2D models for anterior-posterior and medial-lateral sways respectively.

The complexity of dynamics of combined anterior-posterior and medial-lateral postural sways has long been recognized (Winter, 1995). An experimental method was proposed to obtain a general 3D robotic kinematic model of human postures (Desjardins et al., 2002). A kinematic model describes displacements and velocities, but not dynamics of postural movements. Developed using symbolic computation tools, the 3D model of sit-to-stand postural dynamics (Mughal and Iqbal, 2010, 2013) described a large number of joint variables and included some constraints imposed on the postural movements, but ignored the induced constraint forces imposed on the postural dynamics.

In investigations of humanoid robots, various multiple-link 3D models have been developed for examining contact tasks in walking and running. Hybrid aspects of models and control of 3D bipedal robotic walking were highlighted in a comprehensive survey (Grizzle, et al., 2014), where handling of the double-support phase was mentioned as one of a few open problems in modeling and control of 3D robotic walking. Being about 20% of the gait, this is an important phase during walking, and sagittal geometries and dynamics of this phase were examined (Mu and Wu, 2006; Hamed et al, 2012). Although open stance balancing is not same as that in the double-support phase during walking or sit-to-stand transiting, treatments of constraints in modeling should be similar in these cases.

The current investigation on modeling and nonlinear control synthesis of 3D standing dynamics focuses on specification of the postural geometry, identification of a complete set of constraints imposed on postural movements, separation of free and constrained dynamic modes, synthesis of feedback control, and verification of the stability of postural dynamics. To explore all these theoretical aspects, this study derives an essential model which describes the standing dynamics with a minimal number of joint rotations and is not integrated with musculoskeletal and neurosensory models.

Having a closed chain of body segments pivoting at the ankles, the standing posture is viewed as a parallel robot and its kinematics can be obtained using a modeling method in robotics (Sciavicco and Siciliano, 2000). Compared with the well-known parallel mechanism of the Gough-Stewart platform (Liu, et al., 2000; Khalil and Guegan 2004) as a hexapod, the standing posture has the upper body as the platform supported by two legs on the base at the level of ankles. A special treatment of kinetic and potential energies of the posture and application of the Lagrange-d'Alembert principle (Bloch, 2003) will naturally result in a set of differential-algebraic equations describing constrained dynamics of the standing posture. As a case-by-case process, specification of the constraints imposed on postural movements is the key in applying the general modeling method.

In the first step of nonlinear control synthesis, the current work uses a decomposition of postural dynamics to determine a manifold where the postural movements are free of constraints. The decomposition also facilitates numerical simulations of the postural dynamics. The method of decomposition was first used in the synthesis of stabilization and tracking control for constrained robots (McClamroch and Wang, 1988), and adopted in the observer design for linear mechanical systems with holonomic and non-holonomic constraints (Hou et al., 1993).

As widely recognized, adding damping to the unstable postural dynamics by proportional and derivative feedback of joint angles (Hill, 1970) is an essential control strategy. Based on the developed model of 3D postural dynamics and with verified stability, a nonlinear proportional and derivative feedback control is derived in the current work for balancing the standing posture. Using linearized 2D postural models, linear feedback controls (Hemami and Wyman, 1979; Kuo, 1995) were designed for human stance balancing. For controlling the sit-to-stand posture, a linear controller was derived from the linearized 3D dynamics model by Mughal and Iqbal (2010). Design of linear controllers based on linearized models has the advantage of being supported by a range of synthetic methods, but the stability is local. Under the Lipschitz condition, a stabilizing controller designed on the basis of a linearized model operates satisfactorily in the vicinity of an equilibrium specified by joint variables taking particular values, but has no guarantee on postural stability when operates away from the vicinity even if a switching logic is incorporated with multiple linear controllers.

2. Modeling

The most essential 3D movement of the upright stance is a combination of anterior-posterior and medial-lateral postural sways caused by rotations of legs and upper body about the ankle and hip joints respectively. The rotations correspond to variations of three angles on each body side. The postural dynamics with the six interactive rotations will be shown to have only two degrees of freedom. Rotations of thighs about knee joints are not considered to avoid introducing a further angle for the knee joint and another for the hip joint on each body side.

2.1. Kinematics of Displacements and Velocities

Modeling of human postural kinematics is fairly straightforward following the Denavit-Hartenberg (D-H) convention in robotics (Craig, 2005). It starts with assignment of joint frames, then describes the relationships between them, and ends up with specification of linear and angular velocities of body segments.

2.1.1. Geometry setup with frame assignment

Figs. 1 and 2 show two axes for each of the six joint frames plus the reference. In essence, a joint frame is attached to a body segment at the joint linking this and previous segments. Its z-axis is chosen to be the rotation axis of this segment with respect to the other. Once all z-axes of joint frames are specified, the perpendicular to the current and next z-axes defines the current x-axis, and the y-axis is determined by the right-hand rule. In this way, all joint frames are specified.

Denoted by F_0 with X-Y-Z axes is the reference frame fixed to the ground. F_w^u with x_w^u-y_w^u-z_w^u axes represents further three frames assigned to ankle and hip joints on each body side, for $u = l, r$ and $w = a, f, h$. For instance, attached to the leg at the ankle joint on the left, F_a^l and F_f^l relate leg rotations about z_a^l and z_f^l to F_0 and F_a^l respectively, while, attached to the pelvis at the left hip joint, F_l^h relates the upper-body rotation about z_f^l to F_f^l.

For simplicity and clarity, the two body sides are assumed to be symmetric about the sagittal plane when the posture is in the home position. Angular offset $\beta_0 = \sin^{-1} \frac{d-d_l}{h_l}$ between the x-axes of ankle frames F_a^u and F_f^u on either body side indicates how oblique the legs are to the ground when the posture is in the home position, where as indicated in Fig. 1, $2d$ is the distance between the legs, $2d_l$ the distance between the origins of left and right hip frames, and h_l the total leg length.

2.1.2. Positions and orientations

The postural movements in the sagittal plane (XY plane) are caused by rotations of the legs about z_a^l and z_a^r axes, and in the frontal plane (XZ plane) about z_f^l and z_f^r axes, and rotations of the upper body about z_h^l and z_h^r axes. The angles associated with these rotations are denoted by variables α_u, $\bar{\beta}_u$ and β_u for $u = l, r$, and collectively by vector

$$q = \begin{bmatrix} \alpha_l & \alpha_r & \bar{\beta}_l & \bar{\beta}_r & \beta_l & \beta_r \end{bmatrix}', \tag{1}$$

where $'$ stands for the transpose.

The four D-H parameters associated with the joint frames are summarized in Table 1 with a_i and α_i being respectively the distance and twist angle between z-axes of frames F^u_{i-1} and F^u_i, while d_i and θ_i being the same but between x-axes of the two frames. F^u_1, F^u_2 and F^u_3 are alternative notations of F^u_a, F^u_f and F^u_h respectively.

Table 1: D-H parameters, where positive and negative signs are taken for $u = l, r$ respectively.

	a_i	α_i	d_i	θ_i	
1	e	0	$\pm d$	α_u	F^u_a
2	0	$\pm\pi/2$	0	$\bar{\beta}_u - \beta_0$	F^u_f
3	h_l	0	0	β_u	F^u_h

With short notations $s(\cdot) = \sin(\cdot)$ and $c(\cdot) = \cos(\cdot)$, the rotation matrix and position vector

$$R^u_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i\, c\alpha_i & c\theta_i\, c\alpha_i & -s\alpha_i \\ s\theta_i\, s\alpha_i & c\theta_i\, s\alpha_i & c\alpha_i \end{bmatrix}, \quad \rho^u_i = \begin{bmatrix} a_i \\ -d_i\, s\alpha_i \\ d_i\, c\alpha_i \end{bmatrix} \tag{2}$$

relate the orientation and position of F^u_i to F^u_{i-1}. Consequently,

$$\bar{R}^u_i = \bar{R}^u_{i-1} R^u_i\,, \qquad \bar{\rho}^u_i = \bar{\rho}^u_{i-1} + \bar{R}^u_{i-1}\rho^u_i \tag{3}$$

relate F^u_i to F_0 due to the chain of frame associations, with $\bar{R}^u_0 = I$ (identity matrix) and $\bar{\rho}^u_0 = 0$ (zero vector).

2.1.3. Linear and angular velocities, and CoMs

Linear and angular velocities of F^u_i with respect to F_0 are derived from (3) as (see, e.g. Sciavicco and Siciliano, 2000)

$$v^u_i = \frac{\partial \bar{\rho}^u_i}{\partial q}\dot{q}\,, \qquad \omega^u_i = \omega^u_{i-1} + \bar{R}^u_i \dot{\Theta}_i \tag{4}$$

with $\omega^u_0 = 0$ and $\dot{\Theta}_i = \begin{bmatrix} 0 & 0 & \dot{\theta}_i \end{bmatrix}'$. As illustrated in Fig. 3, the CoM of the leg with respect to F^u_f is $p_f = \begin{bmatrix} h_{cl} & 0 & 0 \end{bmatrix}'$, and that of the upper body with respect to F^u_h is

$$p_h = \begin{bmatrix} h_{cu}c_0 + d_l s_0 & -d_l c_0 & 0 \end{bmatrix}' \tag{5}$$

with short notations $s_0 = \sin\beta_0$ and $c_0 = \cos\beta_0$. Referring to (3), the two CoMs with respect to F_0 are

$$p_f^u = \bar{\rho}_2^u + \bar{R}_2^u p_f \,, \qquad p_h^u = \bar{\rho}_3^u + \bar{R}_3^u p_h \,, \tag{6}$$

and from (4), their linear velocities are $\dot{p}_f^u = \frac{\partial p_f^u}{\partial q}\dot{q}$ and $\dot{p}_h^u = \frac{\partial p_h^u}{\partial q}\dot{q}$.

2.2. Geometric Constraints

Described by (2)-(4) along with the frame specifications in Figs. 1 and 2, and the D-H parameters in Table 1, the kinematics of segmental positions and orientations are subject to constraints due to the connectivity of the two body sides.

The direction vectors of $\bar{R}_3^u = \begin{bmatrix} r_x^u & r_y^u & r_z^u \end{bmatrix}$ defined in (3) must satisfy

$$(r_x^l)' r_x^r = \cos 2\beta_0 \,, \tag{7}$$

$$(r_y^l)' r_y^r = \cos(\pi - 2\beta_0) \,, \tag{8}$$

$$(r_z^l)' r_z^r = \cos\pi \tag{9}$$

because there is no movement between F_h^l and F_h^r as they are attached to the same body segment. In view of (5), with respect to F_h^l, the origin of F_h^r is

$$\rho_h = \begin{bmatrix} 2d_l s_0 & -2d_l c_0 & 0 \end{bmatrix}' . \tag{10}$$

Its coordinates with respect to F_0 from one body side must be equal to those from the other, namely, according to (3) and (6),

$$\bar{\rho}_3^l + \bar{R}_3^l \rho_h = \bar{\rho}_3^r \,. \tag{11}$$

Fully describing constraints on the variations of joint variables, the six equations in (7)-(9) and (11) have nevertheless some redundancy. Long yet elementary manipulations of these equations lead to the simplification

$$\alpha_r = \alpha_l, \tag{12}$$

$$\beta_r = -\beta_l - \bar{\beta}_r - \bar{\beta}_l, \tag{13}$$

$$\beta_l = \tan^{-1}\frac{a_0 + s_{l,0}}{c_{l,0}} - \sin^{-1} b_0 \frac{b_1 + s_{l,0}}{\sqrt{b_2 + s_{l,0}}} - \bar{\beta}_l, \tag{14}$$

$$\bar{\beta}_r = \tan^{-1}\frac{c_{l,0}}{a_0 + s_{l,0}} - \sin^{-1} b_3 \frac{b_4 + s_{l,0}}{\sqrt{b_2 + s_{l,0}}} + \beta_0 \tag{15}$$

with $s_{l,0} = \sin(\bar{\beta}_l - \beta_0)$, $c_{l,0} = \cos(\bar{\beta}_l - \beta_0)$, and

$$b_0 = \frac{\sqrt{dh_l}}{2d_l}, \quad b_1 = \frac{d_l^2 + d^2}{dh_l}, \quad b_2 = \frac{h_l^2 + 4d^2}{4dh_l}, \tag{16}$$

$$b_3 = \sqrt{\frac{d}{h_l}}, \quad b_4 = \frac{h_l^2 + 2d^2 - 2d_l^2}{2dh_l}, \quad a_0 = \frac{2d}{h_l}. \tag{17}$$

In (12)-(15), angles α_r, β_r, β_l and $\bar{\beta}_r$ are expressed explicitly as functions of α_l and $\bar{\beta}_l$, which means that the postural movements have only two degrees of freedom.

2.3. Postural Geometry

Analysis of the postural geometry also leads to constraints (12)-(15). Since the knees are assumed to be fully extended and locked, and rotational movement of the hips have only one degree of freedom, (12) is obvious. The convex quadrilateral shown in Fig. 4 has the origins of F_f^u and F_h^u as its vertices denoted by O_f^u and O_h^u for $u = l, r$. The interior angles associated with O_f^u and O_h^u are $\gamma_f^u = \pi/2 + \bar{\beta}_u - \beta_0$ and $\gamma_h^u = \pi/2 + \beta_u + \beta_0$. Given any particular interior angle, say γ_f^l, the two adjacent interior angles can be uniquely determined by elementary triangle geometry, and the remaining interior angle can be obtained from identity $\gamma_f^l + \gamma_f^r + \gamma_h^l + \gamma_h^r = 2\pi$. This implies that $\bar{\beta}_l$ uniquely determines angles β_l, β_r, and $\bar{\beta}_r$ due to the one-to-one correspondence between angles in $\{\gamma_f^l, \gamma_f^r, \gamma_h^l, \gamma_h^r\}$ and $\{\bar{\beta}_l, \bar{\beta}_r, \beta_l, \beta_r\}$. Hence, (13)-(15) are indirectly confirmed. Theoretically α_l can vary in $[-\pi/2, \pi/2]$, and the quadrilateral geometry implies that $\bar{\beta}_l$ varies within

$$\left[\beta_0 - \sin^{-1}\frac{d_l^2 + d_l h_l + d^2}{dh_l + 2dd_l}, \quad \beta_0 + \sin^{-1}\frac{d_l^2 + d_l h_l - d^2}{dh_l}\right]. \tag{18}$$

2.4. Postural Dynamics

An application of classical non-holonomic mechanics to particularly formulated kinetic and potential energies of the posture leads to a mathematical model of postural dynamics.

2.4.1. Kinetic and potential energies

Induced by linear and angular velocities, the leg kinetic energy is, for $u = l, r$,

$$K_f^u = \frac{1}{2} m_f^u (\dot{p}_f^u)' \dot{p}_f^u + \frac{1}{2} (\omega_1^u)' \bar{R}_1^u I_1^u (\bar{R}_1^u)' \omega_1^u + \frac{1}{2} (\omega_2^u)' \bar{R}_2^u I_2^u (\bar{R}_2^u)' \omega_2^u \,, \tag{19}$$

and that of the upper body is, for $u = l$ or r,

$$K_h^u = \frac{1}{2} m_h (\dot{p}_h^u)' \dot{p}_h^u + \frac{1}{2} (\omega_3^u)' \bar{R}_3^u I_3^u (\bar{R}_3^u)' \omega_3^u, \tag{20}$$

where, m_f^u is the total mass of the leg on side u, m_h the total mass of the upper body, $\bar{R}_i^u$ the rotation matrix in (2), ω_i^u the angular velocity in (4), p_f^u and p_h^u the CoMs in (6). I_i^u is the inertia tensor of the leg on side u for $i = 1, 2$, and that of the upper body for $i = 3$, about frame $\bar{F}_i^u$ which is aligned with F_i^u but originated at p_f^u for $i = 1, 2$, and at p_h^u for $i = 3$.

The total kinetic energy of the posture has two equivalent expressions: $K = K_f^l + K_f^r + K_h^l$ and $K = K_f^l + K_f^r + K_h^r$. Averaging the two gives

$$K = K_f^l + K_f^r + \frac{1}{2}(K_h^l + K_h^r) = \frac{1}{2} \dot{q}' M(q) \dot{q} \tag{21}$$

with positive-definite inertia matrix $M(q) = M_l(q) + M_r(q)$, where $M_u(q)$ can be readily determined from (19)-(20) in view of (4) and (6). Similarly, the total potential energy of the posture is given by

$$P = P_f^l + P_f^r + \frac{1}{2}(P_h^l + P_h^r) \tag{22}$$

with $P_f^u = m_f^u g_0' p_f^u$, $P_h^u = m_h g_0' p_h^u$, and $g_0 = \begin{bmatrix} g & 0 & 0 \end{bmatrix}'$, where g is the gravitational acceleration. The special formulations of the total kinetic and potential energies in (21) and (22) will automatically bring all angular variables in (1) into equations of the postural dynamics.

2.4.2. Differential-algebraic equations

Following the Lagrange-d'Alembert principle of non-holonomic mechanics (Bloch, 2003), the postural dynamics are described by the differential-algebraic equations

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) &= \tau + F'\lambda \,, & (23) \\ f(q) &= 0 \,. & (24) \end{aligned}$$

Here, vector τ is the excitation torque; with $F = \frac{\partial f(q)}{\partial q}$ and Lagrange multiplier λ, $F'\lambda$ is the internal torque vector induced by constraint (24) which is the compact notation of (12)-(15). The centrifugal and Coriolis matrix and gravitational torque vector are respectively

$$C(q,\dot{q})\dot{q} = \dot{M}(q)\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}\dot{q}'M(q)\dot{q}\,, \qquad G(q) = \frac{\partial P(q)}{\partial q} \tag{25}$$

with inertia matrix $M(q)$ defined in (21) and potential energy $P(q)$ in (22). To ensure the skew symmetry of $\dot{M}(q) - 2C(q,\dot{q})$, the components of $C(q,\dot{q})$ are specified as (Sciavicco and Siciliano, 2000)

$$c_{ij} = \frac{1}{2}\dot{m}_{ij} + \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i}\right)\dot{q}_k, \tag{26}$$

where m_{ij} is the (i,j) element of $M(q)$, n is the total number of joint variables with $n = 6$ in this study.

2.4.3. Decomposition of dynamic equations

Define a change of variables as $p = p(q)$ with

$$p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \qquad p_1 = \begin{bmatrix} \alpha_l \\ \bar{\beta}_l \end{bmatrix}, \qquad p_2 = f(q)\,. \tag{27}$$

Its inverse $q = p^{-1}(p_1)$ is simply given by algebraic equation (24) with p_1 in (27). Also, the Jacobian matrix of $p(q)$ and its inverse can be determined as

$$\frac{\partial p}{\partial q} = \begin{bmatrix} \bar{F} \\ F(q) \end{bmatrix}, \qquad \left(\frac{\partial p}{\partial q}\right)^{-1} = \begin{bmatrix} F_1(p_1) & F_2(p_1) \end{bmatrix}, \tag{28}$$

where the first block partition is of 2 and 4 rows, and the second of 2 and 4 columns. Importantly, (27) with (28) brings the system (23)-(24) into

$$M_1(p_1)\ddot{p}_1 + C_1(p_1,\dot{p}_1)\dot{p}_1 + G_1(p_1,\dot{p}_1) = F_1'(p_1)\tau\,, \tag{29}$$

$$A_1(p_1,\dot{p}_1)\dot{p}_1 + A_2(p_1,\dot{p}_1) = A_3(p_1)\tau + \lambda, \tag{30}$$

$$p_2 = 0 \tag{31}$$

with, by dropping the arguments of the matrices and vectors for simplicity,

$$M_1 = F_1' M F_1, \quad C_1 = F_1' C F_1, \quad G_1 = F_1' G + F_1' M \dot{F}_1 \dot{p}_1, \tag{32}$$

$$A_1 = F_2' M \dot{F}_1 + F_2' C F_1 - F_2' M F_1 M_1^{-1} C_1, \tag{33}$$

$$A_2 = F_2' G - F_2' M F_1 M_1^{-1} G_1, \quad A_3 = F_2' - F_2' M F_1 M_1^{-1} F_1'. \tag{34}$$

The decomposed system (29)-(31) consists of free dynamics for p_1, explicit Lagrangian multiplier λ, and trivial constraint $p_2 = 0$. In general, interactions between the anterior-posterior and medial-lateral sways exist because $M_1(p_1)$, $C_1(p_1, \dot{p}_1)$ and $G_1(p_1)$ are normally not with any particular structures so that dynamics of α_l and $\bar{\beta}_l$ described by (29) are independent from each other.

3. Control Synthesis

The control task is set to regulate p_1 to zero, which ensures q converging to zero, namely the posture is back to the upright home position. This is due to $q = p^{-1}(q_1) = 0$ for $q_1 = 0$, which can be verified directly from the constraints in (12)-(15), or indirectly from the analysis of postural geometry in Section 2.3. Besides, it is desirable, as the human body probably does, to eliminate the internal torques $F'\lambda$ induced by the constraints.

Consider a Lyapunov function candidate

$$v(p_1, \dot{p}_1) = \frac{1}{2} \dot{p}_1' M_1(p_1) \dot{p}_1 + \frac{1}{2} p_1' K_p \, p_1 \,, \tag{35}$$

where the first term is the kinetic energy associated with p_1 movement, while the second term with K_p being an arbitrary positive-definite constant matrix is related to the potential energy of the posture. In view of skew symmetry of $F_1'(p_1)\,(M(q) - 2C(q, \dot{q}))\,F_1(p_1)$, the time derivative of v is

$$\begin{aligned} & \frac{1}{2} \dot{p}_1' \dot{M}_1(p_1) \dot{p}_1 + \dot{p}_1' M_1(p_1) \ddot{p}_1 + p_1' K_p \, \dot{p}_1 \\ = \; & \frac{1}{2} \dot{p}_1' F_1'(p_1) \dot{M}(q) F_1(p_1) \dot{p}_1 + \dot{p}_1' F_1'(p_1) M(q) \dot{F}_1(p_1) \dot{p}_1 + \\ & p_1' K_p \, \dot{p}_1 + \dot{p}_1' (F_1'(p) \tau - F_1'(p_1) C(q, \dot{q}) F_1(p_1) \dot{p}_1 - \\ & F_1'(p_1) G(q) - F_1'(p_1) M(q) \dot{F}_1(p_1) \dot{p}_1) \\ = \; & p_1' K_p \, \dot{p}_1 + \dot{p}_1' \, (F_1'(p_1) \tau - F_1'(p_1) G(q)) \end{aligned} \tag{36}$$

which suggests a feedback control τ satisfying

$$F_1'(p_1)\tau = F_1'(p_1)G(p^{-1}(p_1)) - K_p\, p_1 - K_d\, \dot{p}_1 \tag{37}$$

so that $\dot{v}(p_1, \dot{p}_1) = -\dot{p}_1' K_d\, \dot{p}_1 \leq 0$, where the equality holds only if $\dot{p}_1 = 0$ when K_d is chosen as an arbitrary positive-definite constant matrix. In such a case, the system (29) with a control satisfying (37) is reduced to $K_p\, p_1 = 0$ which is possible only if $p_1 = 0$. This verifies the stability of the controlled system by virtue of LaSalle's theorem (Khalil, 2002).

As a consequence of $M(q)$ being positive-definite and $F_1(p_1)$ having full column rank, $M_1(p_1)$ is positive-definite. Considering (28), (30), (37) and invertibility of matrix

$$\begin{bmatrix} F_1' \\ A_3 \end{bmatrix} = \begin{bmatrix} I & 0 \\ -F_2' M F_1 M_1^{-1} & I \end{bmatrix} \begin{bmatrix} F_1' \\ F_2' \end{bmatrix}, \tag{38}$$

a feedback control

$$\begin{aligned} \tau &= \begin{bmatrix} F_1' \\ A_3 \end{bmatrix}^{-1} \begin{bmatrix} F_1' G - K_p p_1 - K_d \dot{p}_1 \\ A_1 \dot{p}_1 + A_2 \end{bmatrix} \\ &= (\bar{F}' + F' F_2' M F_1 M_1^{-1})(F_1' G - K_p\, p_1 - K_d\, \dot{p}_1) + F'(A_1\, \dot{p}_1 + A_2) \end{aligned} \tag{39}$$

is determined, which regulates p_1 and hence q to zero, whilst, due to $\lambda = 0$, eliminates torque $F'\lambda$ induced by constraint (24).

4. Model Parametrization

For simulation studies of postural control, the mathematical model (23)-(24) needs to be parametrized with mass distributions and moments of inertia of the body segments.

4.1. Assembly of segment data

Consider an object consisting of segments a and b with no movement between them. Segment v for $v = a, b$, has the known mass and inertia tensor (m_v, I_v) with I_v referring to frame F_v. (R_a, p_a) relates F_a to reference frame F_0, and

(R_b^a, p_b^a) relates F_b to F_a. Let F_v's origin p_v be the CoM of segment v with respect to F_0. Clearly

$$R_b = R_a R_b^a, \quad p_b = p_a + R_a p_a^b \tag{40}$$

relate F_b to F_0. The total mass and CoM of the object are respectively

$$m_0 = m_a + m_b, \qquad p_0 = \frac{m_a p_a + m_b p_b}{m_0}. \tag{41}$$

The inertia tensor of the object about F_0 is

$$I_0 = R_a I_a R_a' + m_a \Omega'(p_a)\Omega(p_a) + R_b I_b R_b' + m_b \Omega'(p_b)\Omega(p_b) \tag{42}$$

with

$$\Omega(r) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}, \qquad r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{43}$$

4.2. Segment parameters

Fig. 5 shows the segmental frames and averaged longitudinal CoM positions for 7 out of the 14 body segments from the 6 specimens (Chandler, et al. 1975), where unused feet, and due to the assumption on bilateral body symmetry, the arm, hand and leg on the left are not shown. Tables 2 and 3 summarize mass and inertial parameters used in this study. The data are given with respect to the first segmental frame in each table, where symbol $*$ stands for symmetric elements of inertia tensors. The inertia tensor of an original segment is calculated from the segmental principal moments of inertia and the rotation matrix relating the principal axes to the segmental frame, while the parameters of the combined segments are calculated using (41) and (42).

5. Numerical Simulations

Based on the free dynamics described in (29) and control torque in (39), simulations have been carried out in the MATLAB/SIMULINK environment

Table 2: Data for legs

	Mass (kg)	CoM (cm)	Inertia tensor ($kg\,cm^2$)
Thigh	6.52	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1224.61 & 15.55 & -62.64 \\ * & 1250.19 & 75.07 \\ * & * & 268.43 \end{bmatrix}$
Shank	2.69	$\begin{bmatrix} 0 \\ 0 \\ 42.41 \end{bmatrix}$	$\begin{bmatrix} 393.54 & 3.60 & 7.40 \\ * & 390.37 & 6.76 \\ * & * & 29.54 \end{bmatrix}$
Combined	9.21	$\begin{bmatrix} 0 \\ 0 \\ 12.39 \end{bmatrix}$	$\begin{bmatrix} 5043.28 & 19.15 & -55.24 \\ * & 5065.69 & 81.83 \\ * & * & 297.97 \end{bmatrix}$

Table 3: Data for upper body

	Mass (kg)	CoM (cm)	Inertia tensor ($kg\,cm^2$)
Torso with head	37.98	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 21172.01 & -2687.27 & -648.47 \\ * & 20744.19 & -458.77 \\ * & * & 4071.38 \end{bmatrix}$
Right arm with hand	3.35	$\begin{bmatrix} 0 \\ 21.92 \\ 15.75 \end{bmatrix}$	$\begin{bmatrix} 1108.61 & 1.85 & -15.67 \\ * & 1112.37 & 3.78 \\ * & * & 36.85 \end{bmatrix}$
Left arm with hand	3.35	$\begin{bmatrix} 0 \\ -21.92 \\ 15.75 \end{bmatrix}$	$\begin{bmatrix} 1108.61 & -1.85 & 15.67 \\ * & 1112.37 & 3.78 \\ * & * & 36.85 \end{bmatrix}$
Combined	42.41	$\begin{bmatrix} 0 \\ 0 \\ 2.49 \end{bmatrix}$	$\begin{bmatrix} 28022.01 & -2687.27 & -679.81 \\ * & 24382.46 & -458.77 \\ * & * & 7304.34 \end{bmatrix}$

to produce responses of angles α_l and $\bar{\beta}_l$ to the feedback control. Lagrange multiplier λ is determined from (30) and the remaining four angles in q are recovered from the explicit expressions in (12) to (15). The numerical solver of differential equations is a variable-step ode45 with relative tolerance of 10^{-3}.

The mass and inertia parameters of the legs and upper body are summarized in Section 4.2. The height of the feet is $e = 6\,\mathrm{cm}$, and the distance between the origins of left and right hip frames is half of the buttock breadth $2d_l = 17.42\,\mathrm{cm}$. The distance between the legs is set to be $2d = 20.32\,\mathrm{cm}$ (8in).

In all simulations, unless indicated differently, the controller gains are chosen as $K_p = \mathrm{diag}(300, 300)$ and $K_d = \mathrm{diag}(100, 100)$, and the initial conditions as $\alpha_l(0) = -\bar{\beta}_l(0) = 5°$, and $\dot{\alpha}_l(0) = \dot{\bar{\beta}}_l(0) = 0$.

5.1. Angular displacements and control torques in the absence or presence of disturbances

To consider the response of the postural control system to disturbances, $w = \begin{bmatrix} w_1 & w_2 \end{bmatrix}'$ is added to the control torque in the second bracket of (39), with $w_i = 2\sin(0.6\pi t + \phi_i)\,\mathrm{Nm}$ for $i = 1, 2$, where phases ϕ_1 and ϕ_2 are two statistically independent Gaussian random variables with zero mean and variance equal to 0.1. The addition of disturbances in this way avoids violating the constraints in (24). The controlled standing posture does not shown any unexpected responses when the amplitude of sinusoidal noise increases from 2 to 10 and phase variance from 0.1 to 10, and simulation plots of those cases are spared for brevity.

Fig. 6 shows the angular displacements of the sways in the two cases: presence and absence of disturbances. The control torques applied to the joints are shown in Fig. 7. In the absence of disturbances, the control torques associated with anterior-posterior sways around the ankle joints converge to zero, while those associated with medial-lateral sways around the ankle and hip joints converge to constant values. This is due to the control strategy of eliminating the internal torques induced by the postural geometric constraints. It implies that gravity generates torques on the joints associated with medial-lateral sways, even if the posture is in the home position and without any influence from disturbances. Although this may be obvious, it illustrates how simulations can provide insights into actions and reactions of human postural control, especially with practical difficulties in directly measuring all joint torques in experiments.

5.2. Effects of controller gains on postural sways

In theory any positive definite matrices can be chosen as the gain matrices K_p and K_d for the control defined in (37) or (39). This means that by simply taking $K_p = \text{diag}(k_p, k_p)$ and $K_d = \text{diag}(k_d, k_d)$, k_p and k_d can be any positive numbers. Fig. 8 shows how anterior-posterior and medial-lateral sways respond to the postural control with different gains in the absence of disturbances. The oscillating frequency and convergent speed of the postural sways depend on combined effects of the controller gains. Despite having no effect on stability, to resemble real postural sways of human bodies, positive gains k_p and k_d should take values in certain ranges. Determination of these ranges requires a joint endeavour in simulation and experiment studies of standing postures.

5.3. Effects of controller time delays on postural sways

In real applications, postural balancing through feedback control is affected by time delays in the neurosensory signaling pathways and in the muscle contractile process. Denote the feedback control specified in (37) by $\bar{\tau}(t)$. A simple way of considering effects of time delays is to assume that the postural dynamics described by (29) are driven by the time-delayed control $\bar{\tau}(t - t_0)$ with $t_0 > 0$. Unlike in the case of the control with arbitrary positive gains k_p and k_d, stability of the standing postural dynamics under the delayed control actions is generally not guaranteed in theory. Nevertheless, simulation studies of a postural control system model help investigations of time delay effects. As expected and shown in Fig. 9, a small delay in control slightly affects postural sways, but with a large delay the control cannot balance the stance.

5.4. Interactions between anterior-posterior and medial-lateral sways

It would be theoretically and practically significant if anterior-posterior and medial-lateral sways of the standing posture were independent from each other. As pointed out in Section 2.4.3, the dynamics model (29) does not show having a particular structure leading to sway decoupling. In fact, simulations of the postural control system in the absence of disturbances shows that the magnitude ratio of the (2,1) and (1,1) elements of the inertia matrix M_1 varies between 0.02 and 0.14, and that of centrifugal and Coriolis matrix C_1 is about 0.3. While indicating sway interactions through inertial accelerations and centrifugal and Coriolis forces, this does not however necessarily imply inherent coupling of the sways because the proposed nonlinear control has not been particularly designed to introduce coupling or decouple the sway dynamics. If the sway dynamics were naturally decoupled, an uncontrolled standing posture with only one non-zero initial sway angle would fall in that sway direction without inducing the other sway. This does not happen as shown in Fig. 10, where an anterior-posterior sway induces a medial-lateral sway during a free fall, and vice versa.

6. Concluding Remarks

Modeling and control of the standing posture in the 3D environment is considerably more complicated than its 2D counterpart. The difficulties lie in exact specification of the postural geometry, identification of the imposed constraints, separation of free and constrained dynamic modes, synthesis of feedback control, and verification of the stability of postural dynamics. All these theoretical aspects have been covered in this investigation of anterior-posterior and medial-lateral sways of the standing posture. The key findings are the explicit expressions for constraints on postural movements, discovery of the manifold dynamics (i.e. the subsystem without constraints), and a nonlinear feedback control stabilizing the stance. This study has built a good foundation for adding further complexity to the model through inclusion of more joint variables and integration with musculoskeletal and neurosensory models.

In parametrization of a postural model with mass centers and inertial parameters given in body segmental frames, an axis about which a segment rotates needs to be specified with respect to the segmental frame. This study has used the mass distribution and inertial data of body segments from six male cadavers reported by Chandler, et al. (1975), where a segmental rotation axis becomes known to be parallel to a particular axis of the segmental frame. Use of many other datasets for postural model parametrization could however be a problem. This is because the information on segmental rotation axes is often not provided, for instance in the original datasets (McConville et al. 1980; Young et al. 1983) and their adjustments (Dumas, et al. 2007, 2015) of the 31 male and 46 female living specimens. Moreover, for combining multiple segments into one when there is no movement among them, it needs to know how a pair of adjacent segmental frames are related to each other, but this information cannot be deduced from these datasets. Among many other datasets, the dataset of Chandler et al (1975) is not without deficiency as it implies, for several segments, the segmental mass center is located on the straight line connecting the proximal and distal centroids of the segment.

In terms of the shapes and ranges of responses of joint angles and torques, the basic simulation results in Section 5.1 are consistent with other simulated and experimental results (e.g. Cahouëta, et al., 2002; Ferry et al., 2007; Bonneta et al., 2011; Moraux et al., 2013), and the additional simulation results in Sections 5.2-5.4 show further interesting and expected behaviours of postural dynamics. The derived control with verified postural stability also confirms that both stiffness and damping effects are needed in the feedback control mechanism as suggested before (e.g. Hill, 1970; Maurer and Peterka, 2005; among others). Beside ensuring postural stability, the nonlinear control developed in the current work also nullifies internal torques. The internal torques are passive torques induced by the constraints on movements of body segments due to connectivity of two body sides and act as additional torques on these segments, while the excitation torques are active torques generated from muscle-tendon units by neurosensory stimulus. In theory, postural movements normally induce internal torques affecting segmental dynamics. The proposed active control cancels out these passive torques. It is however unknown whether or not elimination or reduction of internal torques happens in real postural dynamics.

Within the framework undertaken in this study and with further complexity added to the developed essential model, many interesting and challenging problems in 3D postural control can be rigorously studied. A good mathematical model is essential for simulations and control synthesis. Theoretical analysis and simulation studies of a postural dynamics model integrated with musculoskeletal and neurosensory models can bring insights into roles of muscle stimulus, force-length relationships, neurosensory signalling played in postural control. To derive a model with more joint variables, the general modeling method developed in this study can be followed. The key of this generalization is to determine whether or not the six position and orientation constraints imposed on postural movements are redundant, and, if they are, simplification of them is needed to remove the redundancy. An integration of musculoskeletal and neurosensory models into the postural dynamics model only requires replacement of the excitation torque by the products of muscle-tendon forces and their moment arms. The integration is not considered as a significant challenge, but synthesis of an overall nonlinear feedback control for the combined system probably is.

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