Microeconomics: Learning through Games and Simulations (551375)

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Abstract

A concise knowledge of microeconomic theories is essential in order to understand the complex mechanism of allocation of resources efficiently. Right decisions based on such analysis not only enhances wellbeing of households who maximise their utility subject to budgent constraints but also promotes economic growth as firms produce at their optimal level. Good understanding of markets is essential to understand how finance, fiscal and monetary policies affect decsions of households and firms. Strategic interaction of economic agents can be more comprehensible with detailed understanding of input-output relations and general equilibrium process by which relative prices determine the volume of demand and supply in goods and factor markets. These underpin trade and exchanges in a very competitive world to be assessed by theories of cooperative and non-cooperative games, bargaining, signalling and mechanism design under asymmetric information. Clear understanding of advanced microeconomics is essential for evaluation of economic policies decisions made by households, firms and government in modern economies in normal as well as exceptional circumstances, such as those created by COVID-19 pandemic and unique events like Brexit. This requires a problem solving approach to micro economic analysis as presented compactly in this workbook.

JEL Classification: D, E
Keywords: micro, macro, game theory, trade

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1 Introduction

Microeconomics is a study on how relative prices of commodities and factors of production are determined and how they allocate scarce economic resources making it consistent to maximum of the individual and social welfare in an economy. It focuses on economic behaviour of individuals and firms. It shows how the demands and supplies of each commodity and market prices are consistent to preferences of individuals, technology of firms and economic policies of the government. A rational individual maximises satisfaction from consumptions of goods and services choosing optimal bundles of goods and services given his/her budget constraint. A firm maximises profit by choosing the optimal combinations of inputs allowed by a production technology. Ultimately, the prices of commodities and factors of productions are determined in perfectly or imperfectly competitive markets based on the underlying demand and supply factors. Many ideas of classical economists summarised in "invisible hand" of prices and "self interest" of Adam Smith for a rational economic agent were followed and further improved in derivations of Walras, Marshall, Edgeworth, Hicks, Samuelson, Nash and many other economists essentially form the core principles of microeconomics.

This workbook will introduce quite advanced microeconomic concepts, and employ quite sophisticated mathematical techniques. The major objective of this workbook is to provide an overview of the advanced principles of microeconomics using constrained optimisation process to derive the demand and supply sides of the markets. These models are represented in diagrams and equations. It starts by examining the budget constraints and the role of relative prices in determining their shapes and slopes. Then will proceed on how the demand for goods and services are derived from subjective or psychological preferences of individuals match to their objective budget constraints. Duality analysis will base on the indirect utility and expenditure functions. It will derive income and substitution effects of changes in prices. Elasticities of demand with respect of prices and income as well as the consumer surplus and welfare impacts of taxes and subsidies to individual commodities will be assessed and examined.

The next section will be on production, cost minimisation and supply sides of the market. Major focus in this part will be on how firms choose optimal combinations of inputs to minimize cost or to maximize profits. The average and marginal product using a total production function along with iso-cost and iso-quant will be used to show how firms choose optimal inputs given factor prices and the underlying total, average and marginal costs functions.

Then it will examine price and output decisions of firms and market demand and supply under the perfectly competitive markets and imperfectly competitive markets such as duopoly, oligopoly and monopolistic competition with imperfect information. Similarly it will introduce to input-output models, linear programming and the general equilibrium for the pure exchange economy with Edgeworth box diagrams and with production. General equilibrium effects of taxes and trade policies will be studied.

Economic activities of consumers, producers, governments and nations or regions are interdependent. Game theory provides tools to study the strategic interactions among such economic agents where decisions taken by one individual depend on actions taken by others. Each game has a number of players who choose a set of strategies and rules. Optimal choices available to one depend on choices made by others when pay-offs are clearly defined for each player strategy pairs. Strategic modelling like this started with classics such as Cournot (1838), Bertrand (1883), Edgeworth (1925) von Neumann and Morgenstern (1944), Nash (1950). It is developing very fast in recent years following works of Kuhn (1953), Shapley (1953), Selten (1965), Aumann (1966), Scarf (1967), Shapley and Shubic (1969), Harsanyi (1967), Spence (1974), Kreps (1990), Fundenberg and

Tirole (1991) and Binmore (1992). The game theory sections will briefly introduce to the strategic interdependence of firms using simple cooperative and non-cooperative games, mechanism design and auction.

2 L1: Derivation of A Demand Function

Consumption Set

Let $\mathbf{x} = (x_{1,}x_{2,}.....x_n)$ be quantities of n commodities in nonnegative orthant of $X \in \mathbb{R}^n$. The consumption set X fulfills following properties.

1. $0 \neq X \subseteq \mathbb{R}^n$
2. X is closed
3. X is convex
4. $0 \subseteq X$

Let $B \in X$ be a feasible set such that $x^* \succsim x$ for all $x \in B$.

These concepts date back to Pareto (1896), Marshall (1890), Slutskey (1915), Hicks (1939), Debreau (1959) and others.

Axioms of Consumer Choice

- **Axiom 1: Completeness**

If $x_{1,\text{ and }} x_{2,}$ are both in X , $x_{1,}x_2 \subseteq X$ either $x_1 \succsim x_2$ or $x_1 \precsim x_2$. Consumer can compare.

- **Axiom 2: Transitivity**

For $x_{1,}x_2, x_3 \subseteq X$ if $x_1 \succsim x_2$ $x_2 \succsim x_3$ then $x_1 \succsim x_3$. Consumer is consistent.

- **Axiom 3: Continuity**

Preference relations $\succsim x_i$ $\precsim x_i$ are closed in $\mathbb{R}^n$.

- **Axiom 4: Monotonicity**

For $x_0 \subseteq \mathbb{R}^n$ and for all $\varepsilon > 0$ there exists some $x \in B_\varepsilon(x_0) \cap \mathbb{R}^n$ such that $x > x_0$.

- **Axiom 5: Convexity**

If $x^1 \succsim x^0$ then $tx^1 + (1-t)x^0 \succsim x^0$ for all $t \in [0,1]$.

2.0.1 A utility function and its properties

Where preference relations are complete, transitive, continuous, monotonous and convex then there exists a real valued utility function

$$u : \mathbb{R}^n_+ \Longrightarrow \mathbb{R} \tag{1}$$

and this utility function has following properties

$u(x)$ is strictly increasing if and only if $\succsim$ is strictly monotonic.

$u(x)$ is quasi-concave if and only if $\succsim$ is convex.

$u(x)$ is strictly quasi-concave if and only if $\succsim$ is strictly convex.

2.0.2 What is optimisation?

- To maximise or minimise subject of constraints
  - to maximise the utility or profit or social welfare as function of one or several variables.
  - to minimise the cost
- Linear and non-linear programming are applied in order to find the optimal solutions.
  - Linear programming if the objective functions and constraints are linear
  - non-linear programming if they are non-linear
- By duality theorem every maximisation problem has a corresponding minimisation problem:
  - utility maximisation corresponds to expenditure minimisation; profit maximisation corresponds of cost minimisation

2.1 Primal problem for Consumer's Optimisation

Derivation of a demand function:

$$max\ u = x_1 x_2 \text{ subject to } 2x_1 + 4x_2 = a \tag{2}$$

$$L(x_1, x_2) = x_1 x_2 + \lambda [a - 2x_1 - 4x_2] \tag{3}$$

$$\frac{\partial L(x_1, x_2)}{\partial x_1} = x_2 - 2\lambda = 0 \tag{4}$$

$$\frac{\partial L(x_1, x_2)}{\partial x_2} = x_1 - 4\lambda = 0 \tag{5}$$

$$\frac{\partial L(x_1, x_2)}{\partial \lambda} = a - 2x_1 - 4x_2 = 0 \tag{6}$$

From the first two FOCs $\frac{x_1}{x_2} = 2$. Then put this into the last FOC to get: $x_1 = \frac{a}{4}$; $x_2 = \frac{a}{8}$;, $\lambda = \frac{a}{16} \Longrightarrow u^* = x_1 x_2 = \frac{a}{4} \times \frac{a}{8} = \frac{a^2}{32}$. By an envelop theorem evaluating the indirect utility function and Lagrange multiplier at the optimal solution: $\frac{\partial u^*}{\partial a} = \frac{a}{16} = \frac{\partial L(x_1,x_2)}{\partial a} = \lambda$. QED. If consumer income $a = 200$ then $x_1 = \frac{a}{4} = \frac{200}{4} = 50$; $x_2 = \frac{a}{8} = \frac{200}{8} = 25$. Then $u^* = x_1 x_2 = 50 \times 25 = 1250$.

2.1.1 Properties of a demand function

- Continuous
  - demand is $x_1 = \frac{a}{2P_1}$ and $x_2 = \frac{a}{2P_2}$ and $u = \frac{a}{2P_1}\frac{a}{2P_2} = \frac{a^2}{4P_1P_2}$; $\frac{\partial u}{\partial P_1} \neq 0$
- Homegenous of degree zero in (p, a)
  - Increase both a and p by the same proportion and does not have impact on demand
- Strictly increasing in
  - $\frac{\partial u}{\partial a} > 0$
- Decreasing in p
  - $\frac{\partial u}{\partial p} < 0$
  - Quasiconvex in p and y.
  - Roy's identity

$$x_i\left(p^0, y^0\right) = -\frac{\frac{\partial v\left(p^0,y^0\right)}{\partial p_i}}{\frac{\partial v(p^0,y^0)}{\partial y}} ....i = 1..m \tag{7}$$

2.1.2 Marshallian demand functions (primal problem)

Consumer's Optimisation

$$L = x_1^{\alpha}x_2^{\beta} + \lambda\left[m - p_1x_1 - p_2x_2\right] \tag{8}$$

$$\frac{\partial L}{\partial x_1} = \alpha x_1^{\alpha-1}x_2^{\beta} - \lambda p_1 = 0 \tag{9}$$

$$\frac{\partial L}{\partial x_2} = \beta x_1^{\alpha}x_2^{\beta-1} - \lambda p_2 = 0 \tag{10}$$

$$\frac{\partial L}{\partial \lambda} = m - p_1x_1 - p_2x_2 = 0 \tag{11}$$

Marshallian demand functions

$$x_1 = \frac{\alpha m}{p_1}; x_2 = \frac{\beta m}{p_2}$$

Indierct utility function

$$V(x_1, x_2) = \left(\frac{\alpha m}{p_1}\right)^{\alpha}\left(\frac{\beta m}{p_2}\right)^{\beta} = m\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha}p_2^{-\beta} \tag{12}$$

Expenditure function

$$m = \alpha^{-\alpha}\beta^{-\beta}p_1^{\alpha}p_2^{\beta}\overline{u} \tag{13}$$

Properties of a demand function

- Continuous
  - demand is $x_1 = \frac{\alpha m}{p_1}$ and $x_2 = \frac{\beta m}{p_2}$ and $u = m\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha}p_2^{-\beta}$; $\frac{\partial u}{\partial P_1} \neq 0$
- Homegenous of degree zero in (p, m)
  - Increase both m and p by the same proportion θ as $x_1 = \frac{\alpha\theta m}{\theta p_1} = \frac{\alpha m}{p_1}$ and $x_2 = \frac{\alpha\theta m}{\theta p_2} = \frac{\beta m}{p_2}$. Equal propotional change in m and p does not have any impact on demand. It stays the same as before. This is why this demadn function is homegenous of degree zero in (p, m).
- Strictly increasing in income
  - $\frac{\partial u}{\partial M} > 0$
- Decreasing in p
  - $\frac{\partial u}{\partial p} < 0$

Roy's identity: Marshallian deman function can be obtained from the indirct utility function as follows:
$x_1 = -\frac{\frac{\partial V(x_1,x_2)}{\partial p_1}}{\frac{\partial V(x_1,x_2)}{\partial M}}$; $x_2 = -\frac{\frac{\partial V(x_1,x_2)}{\partial p_2}}{\frac{\partial V(x_1,x_2)}{\partial M}}$
Proof:
$-\frac{\frac{\partial V(x_1,x_2)}{\partial p_1}}{\frac{\partial V(x_1,x_2)}{\partial M}} = -\frac{-\alpha m\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha-1}p_2^{-\beta}}{\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha}p_2^{-\beta}} = \frac{\alpha m}{p_1}$ and $-\frac{\frac{\partial V(x_1,x_2)}{\partial p_2}}{\frac{\partial V(x_1,x_2)}{\partial M}} = -\frac{-\beta m\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha-1}p_2^{-\beta-1}}{\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha}p_2^{-\beta}} = \frac{\beta m}{p_2}$
Compensated demand function can be obtained taking derivation of expenditure function wrt price:
$m = \alpha^{-\alpha}\beta^{-\beta}p_1^{\alpha}p_2^{\beta}\overline{u} \Longrightarrow \frac{\partial m}{\partial p_1} = \alpha^{1-\alpha}\beta^{-\beta}p_1^{\alpha-1}p_2^{\beta}\overline{u} = x_1^c \quad \frac{\partial m}{\partial p_2} = \alpha^{-\alpha}\beta^{1-\beta}p_1^{\alpha}p_2^{\beta-1}\overline{u} = x_2^c$

$$\frac{\partial V}{\partial p_i} = \frac{\partial L}{\partial p_i} = -\lambda^* x_1(p_1, p_2, m). \tag{14}$$

2.1.3 Slutskey Equation:

Total effect of price change = substituion effect and income effect

$$\frac{\partial x_1}{\partial p_1} = \left(\frac{\partial x_1}{\partial p_1}\right)_{Cmp} - \frac{\partial x_1}{\partial E}\frac{\partial E}{\partial p_1} \tag{15}$$

Compensated demand

$$x_1 = \overline{u}p_1^{-\frac{1}{2}}p_2^{\frac{1}{2}} \Longrightarrow \left(\frac{\partial x_1}{\partial p_1}\right)_{cmp} = -\frac{1}{2}\overline{u}p_1^{-\frac{3}{2}}p_2^{\frac{1}{2}} = -\frac{1}{2}\left(\frac{E}{2p_1^{\frac{1}{2}}p_2^{\frac{1}{2}}}\right)p_1^{-\frac{3}{2}}p_2^{\frac{1}{2}} = -\frac{1}{4}Ep_1^{-2} \tag{16}$$

$$E = 2\overline{u}p_1^{\frac{1}{2}}p_2^{\frac{1}{2}} \Longrightarrow \frac{\partial E}{\partial p_1} = \overline{u}p_1^{-\frac{1}{2}}p_2^{\frac{1}{2}} = x_1 \tag{17}$$

Given the Marshalian demand $x_1 = \frac{E}{2p_1}$

$$\frac{\partial x_1}{\partial E} = \frac{1}{2p_1} \tag{18}$$

Slutskey decomposition:

$$\begin{aligned}\frac{\partial x_1}{\partial p_1} &= \left(\frac{\partial x_1}{\partial p_1}\right)_{Cmp} - \frac{\partial E}{\partial p_1}\frac{\partial x_1}{\partial E} = -\frac{1}{4}Ep_1^{-2} - \overline{u}p_1^{-\frac{1}{2}}p_2^{\frac{1}{2}}\left(\frac{1}{2p_1}\right) \\ &= -\frac{1}{4}Ep_1^{-2} - \left(\frac{E}{2p_1^{\frac{1}{2}}p_2^{\frac{1}{2}}}\right)p_1^{-\frac{1}{2}}p_2^{\frac{1}{2}}\left(\frac{1}{2p_1}\right) = -\frac{1}{4}Ep_1^{-2} - \frac{1}{4}Ep_1^{-2} = -\frac{E}{2p_1^2}\end{aligned} \tag{19}$$

First part is substitution effect and the second part is income effect.
If $E = 800$; $p_1 = 4$
substitution effiect is -$\frac{E}{4p_1^2} = \frac{800}{2\times 4^2} = \frac{800}{4\times 4\times 4} = -12.5$ and the income effect is also -$\frac{E}{4p_1^2} = \frac{800}{2\times 4^2} = \frac{800}{4\times 4\times 4} = -12.5$
Both reinforce each other and total effect is -25.

2.1.4 Expenditure functions with the CES utility functions

$$\underset{x_1,x2}{min}\, E = p_1x_1 + p_2x_2 \tag{20}$$

Subject to

$$u = \left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}} \tag{21}$$

$$L\left(x_1, x_2, \lambda\right) = p_1x_1 + p_2x_2 + \mu\left[u - \left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}}\right] \tag{22}$$

$$\frac{\partial L}{\partial x_1} = p_1 - \mu\,\frac{1}{\rho}\left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}-1}\rho x_1^{\rho-1} = 0 \tag{23}$$

$$\frac{\partial L}{\partial x_2} = p_2 - \mu\,\frac{1}{\rho}\left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}-1}\rho x_2^{\rho-1} = 0 \tag{24}$$

Expenditure functions with the CES utility functions

$$\frac{\partial L}{\partial \mu} = u - \left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}} = 0 \tag{25}$$

$$\frac{p_1}{p_2} = \left(\frac{x_1}{x_2}\right)^{\rho-1} \tag{26}$$

$$x_1 = x_2\left(\frac{p_1}{p_2}\right)^{\frac{1}{\rho-1}} = x_2p_1^{\frac{1}{\rho-1}}p_2^{-\frac{1}{\rho-1}} \tag{27}$$

$$u = \left(x_1^\rho + x_2^\rho\right)^{\frac{1}{\rho}} = \left[x_2^\rho\left(\frac{p_1}{p_2}\right)^{\frac{\rho}{\rho-1}} + x_2^\rho\right]^{\frac{1}{\rho}} = x_2\left[\left(\frac{p_1}{p_2}\right)^{\frac{\rho}{\rho-1}} + 1\right]^{\frac{1}{\rho}} \tag{28}$$

$$x_2 = u\left[\left(\frac{p_1}{p_2}\right)^{\frac{\rho}{\rho-1}}+1\right]^{-\frac{1}{\rho}} = u\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} p_2^{\left(-\frac{\rho}{\rho-1}\right)\left(-\frac{1}{\rho}\right)} \tag{29}$$

Expenditure functions with the CES utility functions

$$x_2 = u\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} p_2^{\frac{1}{\rho-1}} = \frac{u \cdot p_2^{\frac{1}{\rho-1}}}{\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} \tag{30}$$

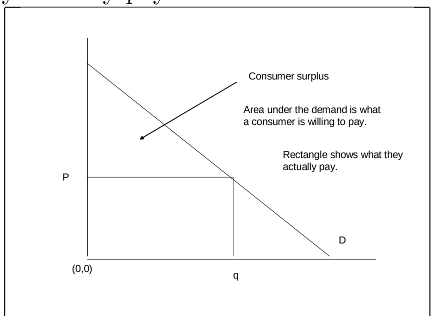
Putting x_2 in x_1

$$x_1 = x_2 p_1^{\frac{1}{\rho-1}} p_2^{-\frac{1}{\rho-1}} = u\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} p_2^{\frac{1}{\rho-1}} p_1^{\frac{1}{\rho-1}} p_2^{-\frac{1}{\rho-1}} \tag{31}$$

$$x_1 = u\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} p_1^{\frac{1}{\rho-1}} = \frac{u \cdot p_1^{\frac{1}{\rho-1}}}{\left[p_1^{\frac{\rho}{\rho-1}}+p_2^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} \tag{32}$$

3 L2: Consumer Surplus

Draw a demand curve and show the difference between how much consumers want to pay and what they actually pay.



Consumer surplus: definition

3.1 Consumer surplus and equivalent and compensating variations

Consider expenditure function and indirect utility function considered earlier as

$$E = p_1 x_1 + p_2 x_2 = p_1 \left(\overline{u} p_1^{-\frac{1}{2}} p_2^{\frac{1}{2}} \right) + p_2 \left(\overline{u} p_1^{\frac{1}{2}} p_2^{-\frac{1}{2}} \right) = 2 \overline{u} p_1^{\frac{1}{2}} p_2^{\frac{1}{2}} \tag{33}$$

$$\overline{u} = \frac{E}{2 p_1^{\frac{1}{2}} p_2^{\frac{1}{2}}} \tag{34}$$

Now consider the p_1 falls to $p_1^{'}$ Consumer welfare increases from $\overline{u}$ to $\overline{u}^{'}$. Note that $\overline{u}^{'} > \overline{u}$ and $p_1^{'} < p_1$. Then the equivalent variation and compensating variations are defined as follows:

$$EV = E\left(p_1^{'}, p_2, \overline{u}^{'}\right) - E\left(p_1, p_2, \overline{u}^{'}\right)$$

$$CV = E\left(p_1^{'}, p_2, \overline{u}\right) - E\left(p_1, p_2, \overline{u}\right)$$

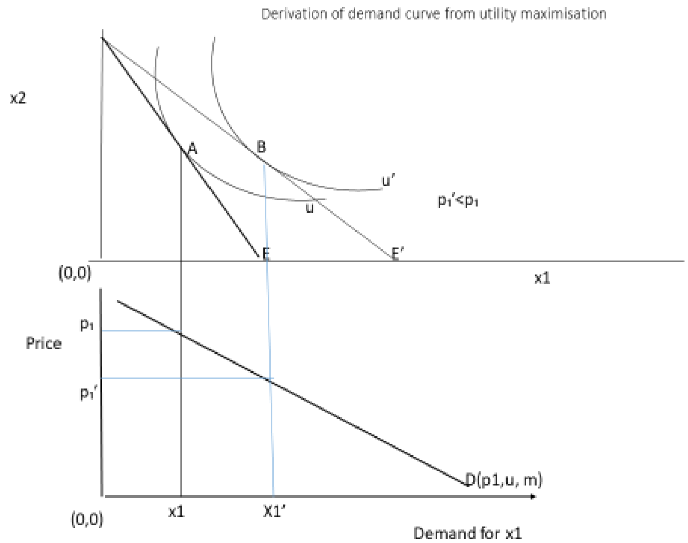
The money metric utility can be measured by an integrals

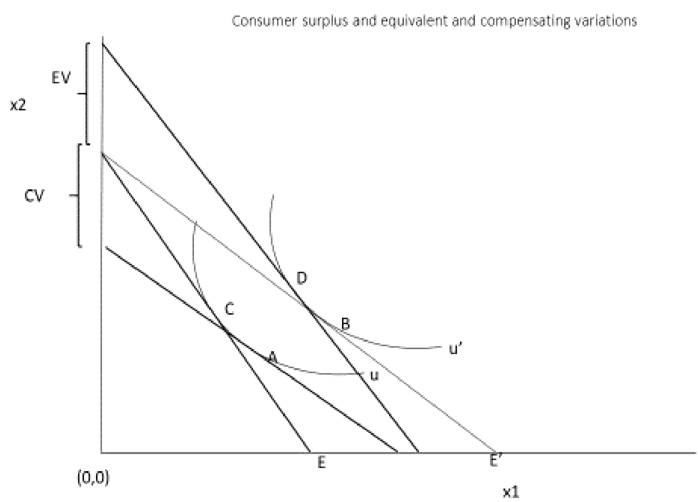
$$EV = \int_{p_1}^{p_1^{'}} \frac{\partial E\left(p_1^{'}, p_2, \overline{u}^{'}\right)}{\partial p_1} dp_1$$

$$CV = \int_{p_1}^{p_1^{'}} \frac{\partial E\left(p_1^{'}, p_2, \overline{u}\right)}{\partial p_1} dp_1$$

or in terms of the demand function:

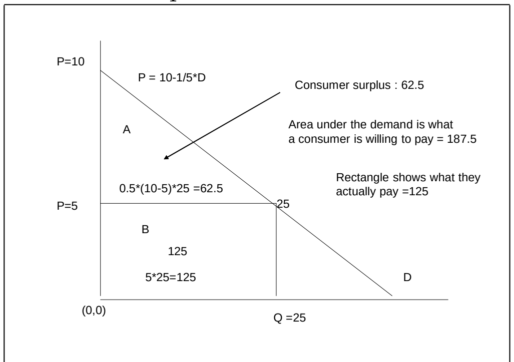
$$EV = \int_{p_1}^{p_1^{'}} x_1^{*}\left(p_1^{'}, p_2, \overline{u}^{'}\right) dp_1$$


Derivation of demand curve from utility maximisation
x2
A
B
u'
u
p1'<p1
E
E'
(0,0)
x1
Price
p1
p1'
D(p1,u, m)
(0,0)
x1
X1'
Demand for x1


Consumer surplus and equivalent and compensating variations
EV
x2
CV
D
C
B
u'
A
u
E
E'
(0,0)
x1

$$CV = \int_{p_1}^{p_1'} x_1^* \left(p_1', p_2, \overline{u}'\right) dp_1$$

Calculate the amount of consumer surplus for a product whose demand equals $D = 50 - 5P$ when the market price is 5?



Consumer surplus: calculation

Consider a demand curve $D = a - bP$; calculate consumer surplus for following scenarios

Table 1: Paramerters of demand function

Scenarios of parameters for CS					
	Sc1	Sc2	Sc3	Sc4	Sc5
b	5	4	3	2	1
a	50	50	50	50	50
P	5	5	5	5	5

Answer for these are as follows (see also Consumer-surplus.xls);

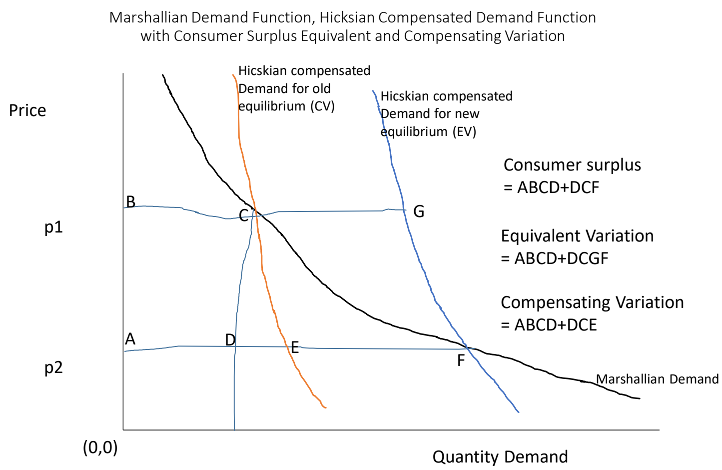
Table 2: Computation of consumer surplus

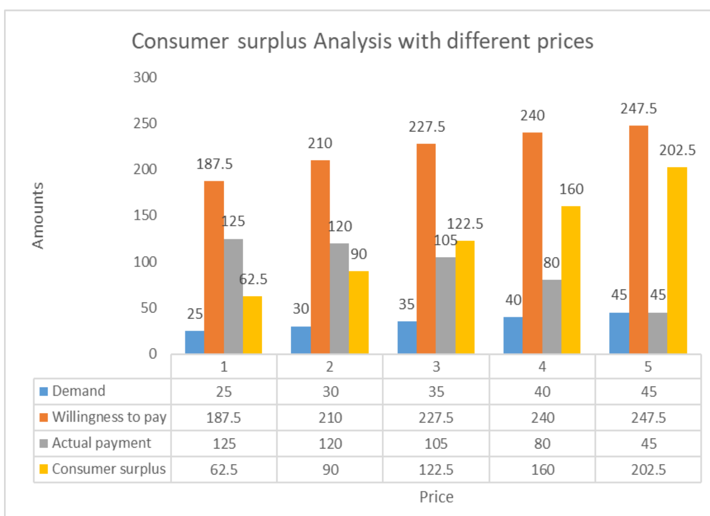
Scenarios for consumer surplus with different slopes					
	Sc1	Sc2	Sc3	Sc4	Sc5
Demand	25	30	35	40	45
Willingness to pay	187.5	262.5	379.2	600	1237.5
Actual pay	125	150	175	200	225
Consumer surplus	62.5	112.5	204.2	400.0	1012.5

Technical innovation reduces prices of computers. How does it improve the consumer surplus?

3.2 Burden of Taxes in Partial Equilibrium Analysis

Burden of Taxes in Partial Equilibrium Analysis (it depends on elasticities)





	1	2	3	4	5
Demand	25	30	35	40	45
Willingness to pay	187.5	210	227.5	240	247.5
Actual payment	125	120	105	80	45
Consumer surplus	62.5	90	122.5	160	202.5

Table 3: Paramerters of demand function

Scenarios of parameters					
	Sc1	Sc2	Sc3	Sc4	Sc5
b	5	5	5	5	5
a	50	50	50	50	50
P	5	4	3	2	1

Table 4: Computation of consumer surplus

Scenarios for comsumer surplus under various rate technical progress					
	Sc1	Sc2	Sc3	Sc4	Sc5
Demand	25	30	35	40	45
Willingness to pay	187.5	210.0	227.5	240.0	247.5
Actual pay	125	120	105	80	45
Consumer surplus	62.5	90.0	122.5	160.0	202.5

Consider linear demand and supply model

$$D = 150 - 3P \tag{35}$$

$$S = 30 + 2P \tag{36}$$

Equilibrium D =S implies P=24 and Q = 78.

Now let there be a sales tax in this commodity so that consumers pay more $\left(P^D\right)$ and suppliers get less $\left(P^S\right)$ because of the tax wedge.

$$P^D = P^S + t \tag{37}$$

where t is tax imposed per unit. Let $t = 2$.

$$D = 150 - 3P^D = 150 - 3\left(P^S + 2\right) \tag{38}$$

Burden of Taxes in Partial Equilibrium Analysis (it depends on elasticities)

$$D = 150 - 3\left(P^S + 2\right) = S = 30 + 2P^S \tag{39}$$

$P^S = 22.8$; $P^D = P^S + t = 24.8$ Q = 75.6

Table 5: Impact of sales tax on equilibrium

	No tax case	Tax case
P	24	24.8
Q	78	75.6

Deadweight loss of taxes =loss of consumer surplus+loss of producer surplus =
$= 0.5(0.8{\times}2.4) + 0.5\left(1.2 \times 2.4\right) = 0.96 + 1.44 = 2.4$

Elasticity of demand : $\frac{\partial Q}{\partial P}\frac{P}{Q} = -3 \times \frac{24}{78} = 0.92$ Elasticity of supply: $\frac{\partial Q}{\partial P}\frac{P}{Q} = 2 \times \frac{24}{78} = 0.61$. Thus more burden is taken by producers, as supply is less elastic. (see TAXBEN model of IFS; www.ifs.org.uk).

Impacts of Tax Reforms

Hicksian Compensating and Equivalent Variations (changes in money metric utility):

Table 6: Summary of Equivalent and Compensating Variation

	Fall in Price	Rise in Price
EV	+	-
CV	-	+

Need to consider income and substitution effects in individual markets and take account of multiple rounds of knock on effects to measure the impacts of tax changes more accurately.

A general equilibrium model is needed to evaluate tax reforms.

EV is measured in after tax prices and CV measured in before tax prices.

General equilibrium impacts of such taxes are generally much higher than the partial equilibrium impacts. Compute a large scale general equilibrium tax model with micro-consistent data from the input-output tables using GAMS or MPSGE or MATLAB. Demo version of GAMS (dowloadable from http://www.gams.com can solve small models.

3.3 Income and substitution effects of tax changes: Changes in money metric utility due to taxes: a numerical example

$$Max \;\; U = X_1^{0.4} X_2^{0.6} \tag{40}$$

- Subject to

$$p_1.X_1 + p_2.X_2 = 150 \tag{41}$$

Lagrangian optimisation:

$$L\left(X_1, X_2, \lambda\right) = X_1^{0.4} X^{0.6} + \lambda \left[150 - p_1.X_1 - p_2.X_2\right] \tag{42}$$

- For base equilibrium assume that $p_1 = 3$ and $p_2 = 2$.
- Optimal demand for goods X_1

$$X_1 = \frac{0.4\,(150)}{p_1} = \frac{60}{3} = 20; \quad X_2 = \frac{0.6\,(150)}{p_2} = \frac{90}{2} = 45 \tag{43}$$

$$U_0 = X_1^{0.4} X_2^{0.6} = (20)^{0.4}\,(45)^{0.6} = 32.53 \tag{44}$$

Now assume that there is a subsidy in X_1 of £1 and price reduces from 3 to 2; $p_1 = 2$.

Equivalent Variation What is the Hicksian Equivalent and compensating variations of price change? What are the income and substitution effects of this price change?

First find out how much money is required at new prices to guarantee the original utility by solving

$$U_0 = \left(\frac{0.4\,(m')}{2}\right)^{0.4}\left(\frac{0.6\,(m')}{2}\right)^{0.6} = 32.53 \tag{45}$$

$$U_0 = \left(\frac{0.4\,(m')}{2}\right)^{0.4}\left(\frac{0.6\,(m')}{2}\right)^{0.6};\ m' = \frac{2\,(32.53)}{0.4^{0.4}\times 0.6^{0.6}} = 127.49 \tag{46}$$

- Equivalent variation (money to be taken away when prices fall)

$$EV = 150 - 127 = 22.51 \tag{47}$$

Compensating Variation

- For compensating variation first compute the demand in new prices and utility

$$X_1 = \frac{0.4\,(150)}{p_1} = \frac{60}{2} = 30;\quad X_2 = \frac{0.6\,(150)}{p_2} = \frac{90}{2} = 45 \tag{48}$$

$$U_1 = X_1^{0.4}X_2^{0.6} = (30)^{0.4}\,(45)^{0.6} = 38.26 \tag{49}$$

$$U_1 = \left(\frac{0.4\left(m''\right)}{3}\right)^{0.4}\left(\frac{0.6\,(m'')}{2}\right)^{0.6} = 38.26 \tag{50}$$

$$m'' = \frac{(38.26)\times 3^{0.4}\times 2^{0.6}}{0.4^{0.4}\times 0.6^{0.6}} = 176.39 \tag{51}$$

$$CV = 150 - 176.39 = -26.39 \tag{52}$$

Summarising the Money Metric Utility Changes Due to Taxes

Table 7: Summary of Equivalent and Compensating Variation

	Fall in Price	Rise in Price	Fall in Price	Basis of evaluation
EV	+	-	22.51	New Price-Old Utility
CV	-	+	-26.39	OLD Price- New Utility

Substitution Effect : 2.5 =10-7.6; Income effect:7.6=22.5/3 and total effect: 10.

This is partial equilibrium result - general equilibrium impacts must take interaction with all other markets. Ultimate impact can be much higher or much lower than this. It need to bring production, income distribution sides into account.

4 L3: Production, Cost, Profit and Supply functions

4.1 Production and supply function: basics

1. Let us consider a production function for a fruit firm operating in the competitive market is given by

$$y = 2\sqrt{l} \tag{53}$$

where y is output and l is labour input. Product price is p and input price is w. What is the cost function for this firm? What is its profit function? What is its supply function? What is the demand function for labour? What are the properties of the these production, profit and cost functions?

Since this is a one input production funtion the cost function can derived direcly from the production technology as:

$$l = \frac{y^2}{4} \tag{54}$$

Producer pay wage to supply this commodity:

$$c = wl = w\frac{y^2}{4} \tag{55}$$

The profit is the difference between the revenue and cost of the firm as given by the profit function:

$$\pi = py - c = py - w\frac{y^2}{4} \tag{56}$$

The supply function for commodity y is derived using the first orcer condition of the profit function as:

$$\frac{\partial \pi}{\partial y} = p - w\frac{y}{2} = 0 \Longrightarrow y = \frac{2p}{w} \tag{57}$$

Supply is positively related to prices and negatively to the input cost, in this case the wage rate. Demand for labour:

$$l = \frac{1}{4}y^2 = \frac{1}{4}\left(\frac{2p}{w}\right)^2 \tag{58}$$

4.1.1 Properties of a profit function

1. Increasing in p
2. decreasing in w
3. homogenous of degree one in p and w
4. concave in y and convex w

These properites satisfy in this example.

4.1.2 Supply function

This supply function is homegeous of degree zero in price and wage, $y = \frac{2p}{w}$ as there is no change in level of output when price and wage increase by the same amount. It is increasing in p and decreasing in w.

Profit function is concave as its second derivative wrt to output is negative, $\frac{\partial^2 \pi}{\partial y^2} = -\frac{w}{2} < 0$;

Production function is also concave. $\frac{\partial y}{\partial l} = l^{-\frac{1}{2}} \Longrightarrow \frac{\partial^2 y}{\partial l^2} = -\frac{1}{2} l^{-\frac{3}{2}} < 0$.

Cost function is convex: $\frac{\partial c}{\partial y} = w\frac{y}{2} \Longrightarrow \frac{\partial^2 c}{\partial y^2} = \frac{w}{2} > 0$.

Demand function for labour is also homegeous of degree zero in price and wage as $l = \frac{1}{4}\left(\frac{2p}{w}\right)^2$.

4.2 Popular production functions

Popular production functions where output (y) is expressed as functions of inputs (x_i):

- Cobb-Douglas: $y = x_1^{\alpha} x_2^{1-\beta}$
- CES: $y = \left(x_1^{\rho} + x_2^{\rho}\right)^{\frac{1}{\rho}}$
- Nested: $x_4 = \left(x_1^{\rho} + x_2^{\rho}\right)^{\frac{1}{\rho}}$ and then $y = x_4^{\alpha} x_3^{1-\beta}$
- generalised Leontief: $Y = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\sqrt{x_i x_j}$; $a_{ij} = a_{ji}$
- Translog: $\ln Y = a_0 + \sum_{i=1}^{n} a_i \ln x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \ln x_i \ln x_j$; $a_{ij} = a_{ji}$

A tanslog production function adds squares and product terms to the regual production function as:

$$\ln Y = a_0 + \sum_{i=1}^{n} a_i \ln x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \ln x_i \ln x_j \;; \quad a_{ij} = a_{ji}$$

This function is popular as it allows a large number of substitution posibilities among inputs. Prove that this function becomes a constant return to scale when $\sum_{i=1}^{n} a_i = 1$ and $\sum_{j=1}^{n} a_{ij} = 0$.

Generalised Leontief function:

$$Y = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\sqrt{x_i x_j} \;; \quad a_{ij} = a_{ji}$$

Nested production function shows how composite inputs are used with other inputs (very popular in the CGE and macro modelling):

Let V be the CES composite of labour and capital

$$V = \left[\alpha L^{\rho} + (1-\alpha) K^{\rho}\right]^{\frac{1}{\rho}} \tag{59}$$

then let E be energy input in production. Then Y is prouced using V and E as:

$$Y = V^{\alpha} E^{1-\beta}$$

This is one level nest. There can be many levels of nests in the production process.

4.3 Producer optimisation: profit maximization

Producers maximise profit given the prices of their products $p > 0$ supplying $y \geq 0$ for all input price $w \geq 0$ and output levels $y \in \mathbb{R}^n_+$ with technology of production $(f(x))$:

$$\pi(p, w) = p.y - w.x \quad \text{s.t..} \ f(x) \geq y \tag{60}$$

Constrained optimisation

$$L(x, \lambda) = py - w.x + \lambda[f(x) - y] \tag{61}$$

$$\frac{\partial L(x, \lambda)}{\partial x_1} = w_1 - \lambda f'(x_1) = 0 \tag{62}$$

$$\frac{\partial L(x, \lambda)}{\partial x_n} = w_n - \lambda f'(x_n) = 0 \tag{63}$$

$$y - f(x) = 0; \frac{\frac{\partial f(x,)}{\partial x_j}}{\frac{\partial f(x,)}{\partial x_i}} = \frac{w_j}{w_i} \tag{64}$$

$$MRTS_{j,i} = \frac{\frac{\partial f(x,)}{\partial x_j}}{\frac{\partial f(x,)}{\partial x_i}} = \frac{w_j}{w_i} \tag{65}$$

Optimality requires that rations of marginal productivities of inputs x_j and x_i ; $\frac{\frac{\partial f(x,)}{\partial x_j}}{\frac{\partial f(x,)}{\partial x_i}}$ equals ratios of inputs prices of w_j and w_i, $\frac{w_j}{w_i}$.

4.4 Cost Mininimisation with CES production Function

Consider a problem of a producer

$$\min_{x_1, x_2,} w_1.x_1 + w_2.x_2 \tag{66}$$

subject to

$$(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} \geq y \tag{67}$$

Show that solution is

$$c(w, y) = y\left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{68}$$

Constraint cost minimisation

$$\mathcal{L} = w_1.x_1 + w_2.x_2 + \lambda\left[y - (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}}\right] \tag{69}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = w_1 - \lambda\frac{1}{\rho}(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}-1}\rho x_1^{\rho-1} = 0 \tag{70}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = w_2 - \lambda \frac{1}{\rho}(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}-1} \rho x_2^{\rho-1} = 0 \tag{71}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = y - (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} = 0 \tag{72}$$

Optimal input demands

$$\frac{w_1}{w_2} = \frac{\lambda \frac{1}{\rho}(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}-1} \rho x_1^{\rho-1}}{\lambda \frac{1}{\rho}(x_1^\rho + x_2^\rho)^{\frac{1}{\rho}-1} \rho x_2^{\rho-1}} = \left(\frac{x_1}{x_2}\right)^{\rho-1} \tag{73}$$

$$x_1 = x_2 \left(\frac{w_1}{w_2}\right)^{\frac{1}{\rho-1}} \tag{74}$$

Substituting this into the production function

$$y = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} = \left[x_2^\rho \left(\frac{w_1}{w_2}\right)^{\frac{\rho}{\rho-1}} + x_2^\rho\right]^{\frac{1}{\rho}} \tag{75}$$

$$y = x_2 \left[\left(\frac{w_1}{w_2}\right)^{\frac{\rho}{\rho-1}} + 1\right]^{\frac{1}{\rho}} \tag{76}$$

Optimal input demands

$$x_2 = y \left[\left(\frac{w_1}{w_2}\right)^{\frac{\rho}{\rho-1}} + 1\right]^{-\frac{1}{\rho}} = y w_2^{\frac{1}{\rho-1}} \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \tag{77}$$

$$\begin{aligned} x_1 &= x_2 \left(\frac{w_1}{w_2}\right)^{\frac{1}{\rho-1}} = y w_2^{\frac{1}{\rho-1}} \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \left(\frac{w_1}{w_2}\right)^{\frac{1}{\rho-1}} && (78) \\ &= y w_1^{\frac{1}{\rho-1}} \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} && (79) \end{aligned}$$

Substitute these values in the cost function

$$c = w_1.x_1 + w_2.x_2 \tag{80}$$

Cost function

$$c = w_1.y w_1^{\frac{1}{\rho-1}} \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} + w_2.y w_2^{-\frac{1}{\rho-1}} \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \tag{81}$$

$$c = y \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \left(w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right) \tag{82}$$

$$c\,(w, y) = y \left[w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \ ; \ QED \tag{83}$$

4.4.1 Translog cost function

A translog cost function adds squares and product terms to above function to make costs flexible to production as:

$$\ln C = \ln Y + a_0 + a_1 \ln r + a_2 \ln w + a_3 (\ln r)^2 + a_4 (\ln w)^2 + a_5 \ln r \ln w$$

This is homogenous of degree 1 when $a_1 + a_2 = 1$ and $a_3 = a_4 = a_5 = 0$.
Cobb-Douglas is a special case of this translog function when $a_3 = a_4 = a_5 = 0$.
Input shares in translog cost functions depend on both w and r:

$$s_w = \frac{\partial \ln C}{\partial \ln w} = a_2 + 2a_4 (\ln w) + a_5 \ln r$$
$$s_r = \frac{\partial \ln C}{\partial \ln r} = a_1 + 2a_3 (\ln r) + a_5 \ln w$$

input shares are constant when $a_4 = 0; a_5 = 0$.

4.4.2 Properties of a short-run supply function

Short run supply function

$$\pi = pL^{\alpha}\overline{K}^{\beta} - wL - r\overline{K} \tag{84}$$

$$\frac{\partial \pi (p, w)}{\partial L} = \alpha p L^{\alpha-1}\overline{K}^{\beta} - w = 0 \Longrightarrow L = \left[\frac{\alpha}{w} p \overline{K}^{\beta}\right]^{\frac{1}{1-\alpha}} \tag{85}$$

Supply function

$$Y = \left[\frac{\alpha}{w} p \overline{K}^{\beta}\right]^{\frac{\alpha}{1-\alpha}} \overline{K}^{\beta} \tag{86}$$

Supply slopes positively with output price and negatively with input price as

$$\frac{\partial Y}{\partial p} > 0; \quad \frac{\partial Y}{\partial w} < 0$$

Q1 Production function for a firm operating in the competitive market is given by

$$y = 2\sqrt{l} \tag{1.1}$$

where y is output and l is labour input. Product price is p and input price is w.

1. **Determine the cost function for this firm.**
2. **What is its profit function?**
3. **Determine its supply function.**
4. **What is its demand function for labour?**
5. **Discuss properties of the production, profit and cost functions.**

5 L4: Markets

5.1 Perfect Competition

Markets bring consumers and producers together in determining the relative prices in which commodities are bought and sold. Demand functions derived from the utility maximisation by consumers and supply functions derived from the profit maximisation by producers interact in determining prices. When all markets clear simultaneously allocations become Pareto efficient and fulfil the first and second theorems of the welfare economics; 1) every competitive equilibrium is Pareto optimal 2) every Pareto optimal allocation is consistent to competitive equilibrium allocation. Efficient markets are good for the social welfare by the first theorem and benevolent dictators maximise social welfare in the second theorem. Smith (1776), Marshall (1890), Pigou (1932), Hicks (1939), Shoven and Whalley (1984), Dawes and Thaler (1988), Jarrell, Brickley and Netter (1988), Elster (1989), Simon (1991), North (1991), Katz and Shapiro (1994), Markusen (1995), and Porter and van der Linde (1995) provide general introduction to markets. This section focuses on partial equilibrium and general equilibrium will be discussed in the next section.

Let $I = (1, 2, ..., I)$ be index set of consumers and market demand is sum of their individual demand; P is prices of other commodities.

$$q^d(p) = \sum\nolimits_{i \in J} q^i(p, P, y) \tag{87}$$

Let $J = (1, 2, ..., J)$ be index set of suppliers and market supply is sum of their supplies

$$q^s(p) = \sum\nolimits_{j \in J} q^j(p, P, y) \tag{88}$$

Partial equilibrium is given by the price of the this commodity that clears this market holding everything else constant

$$q^d(p^*) = q^s(p^*) \tag{89}$$

- GAMS programme: demand_supply_2.gms

Exercise

1. Professor David Dong has just written the first textbook in Economics. Market research suggests that the demand curve for the book is, $Q = 2000 - 100p$, where p is the price. It will cost £1000 to set the book in type. This set up cost is necessary before any copy is printed. In addition to the set up cost, there is a marginal cost of £4 per book printed.

(a) The total revenue function for the book is $R(Q) =$?
(b) The total cost function for producing the book is $C(Q) =$?
(c) The marginal revenue function for the book is $MR(Q) =$?
(d) The marginal cost function for the book is $MC(Q) =$?
(e) The profit maximising quantity of books is $Q =$?
Perfect competition: an example
demand curve for the book is

$$Q = 2000 - 100p \tag{90}$$

Inverse demand curve

$$p = 20 - \frac{Q}{100} \tag{91}$$

Revenue function

$$R = pQ = \left(20 - \frac{Q}{100}\right) Q = 20Q - \frac{Q^2}{100} \tag{92}$$

Cost curve

$$C = 1000 + 4Q \tag{93}$$

Marginal revenue curve

$$MR = \frac{\partial R}{\partial Q} = 20 - \frac{2Q}{100} = 20 - \frac{Q}{50} \tag{94}$$

$$MC = \frac{\partial C}{\partial Q} = 4 \tag{95}$$

Profit maximising output is found setting the equilibrium condition $MR = MC$

$$MR = MC \Longrightarrow 20 - \frac{Q}{50} = 4 \Longrightarrow Q = 800$$

Market price

$$p = 20 - \frac{Q}{100} = 20 - \frac{800}{100} = 12 \tag{96}$$

demand and supply functions in two interdependent markets, the equilibrium prices and quantities could be found by solving the simultaneous equation system as:

Market 1:

$$X_1^d = 10 - 2p_1 + p_2 \tag{97}$$

$$X_1^S = -2 + 3p_1 \tag{98}$$

Market 2:

$$X_2^d = 15 + p_1 - p_2 \tag{99}$$

$$X_2^S = -1 + 2p_2 \tag{100}$$

Equilibrium in both markets implies:

$X_1^d = X_1^S$ implies $10 - 2p_1 + p_2 = -2 + 3p_1$
$X_1^d = X_1^S$ implies $15 + p_1 - p_2 = -1 + 2p_2$

This in matrix notation:

$$\begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix} \tag{101}$$

Application of Matrix in solving equations

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 16 \end{bmatrix} \tag{102}$$

Determinant
$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} = (5 \times 3 - (-1)(-1)) = 15 - 1 = 14;$
Cofactor transpose:
$C' = \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}' = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$
Solution by matrix inversion:

$$\begin{aligned} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} &= \frac{1}{14} \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 12 \\ 16 \end{bmatrix} \\ &= \frac{1}{14} \begin{pmatrix} (3 \times 12) + (1 \times 16) \\ (1 \times 12) + (5 \times 16) \end{pmatrix} = \begin{pmatrix} \frac{52}{14} \\ \frac{92}{14} \end{pmatrix} = \begin{pmatrix} \frac{26}{7} \\ \frac{46}{7} \end{pmatrix} \end{aligned} \tag{103}$$

Cramer's Rule is easier

$$p1 = \frac{\begin{vmatrix} 12 & -1 \\ 16 & 3 \end{vmatrix}}{\begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{36+16}{15-1} = \frac{26}{7}; \quad p2 = \frac{\begin{vmatrix} 5 & 12 \\ -1 & 16 \end{vmatrix}}{\begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{80+12}{15-1} = \frac{46}{7} \tag{104}$$

Market 1:

$$LHS = 10 - 2p_1 + p_2 = 10 - 2\left(\frac{26}{7}\right) + \left(\frac{46}{7}\right) = \frac{64}{7} = -2 + 3p_1 = \frac{64}{7} = RHS \tag{105}$$

Market 2:

$$LHS = 15 + p_1 - p_2 = 15 + \frac{26}{7} - \frac{46}{7} = \frac{85}{7} = -1 + 2p_2 = \frac{85}{7} = RHS \tag{106}$$

QED.
Extension to N-markets is obvious. Matrix makes solving large models much easier.

5.2 Policy relevance of consumer surplus

Understanding consumer surplus is important to understand the practice of price discrimination by firms with market power or for tax-subsidy policy of government. It is also important to study marketing or advertisements and changes in preferences, even for analysis of merger and acquisition decisions of small or large local, national or multinational corporations.

Why taxes make economic system inefficient? What are the deadweight loss of taxes?

Take regular demand and supply functions $D = a - bP$ and $S = -c + dP$, and find the equilibrium price and quantity demanded and supplied.

$$P = \frac{a+c}{b+d};\ q = \frac{ad+bc}{b+d}$$

Now impose a tax on the commodity. This creates a wedge between the price received by suppliers and price paid by consumers. This distorts the market equilibrium and allocation. Slightly reformulate above equations reflecting tax rate t paid by consumers as follows:

$P^D = P^S + t$

again demand and supply are equal with tax distorted prices as $D = a - bP^D$ and $S = -c + P^S$. Then

$$\begin{aligned} a - bP^D &= -c + P^S \iff a - b\left(P^S + t\right) = -c + P^S \iff P^S = \frac{a+c-bt}{b+d} \\ P^D &= P^S + t \iff P^D = \frac{a+c-bt}{b+d} + t \iff P^D = \frac{a+c+dt}{b+d} \end{aligned}$$

Obviously price paid by consumers is higher than price received by producers. Middle slice is taken by government.

The equilibrium demand in tax distorted market is:

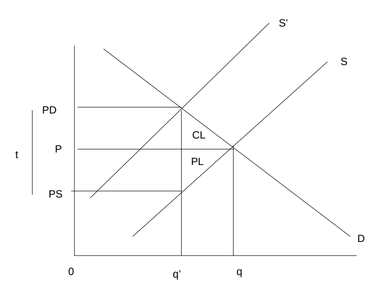
$$D = a - bP^D \iff D = a - b\left(\frac{a+c+dt}{b+d}\right) \iff D = \frac{ad-bc-bdt}{b+d}$$

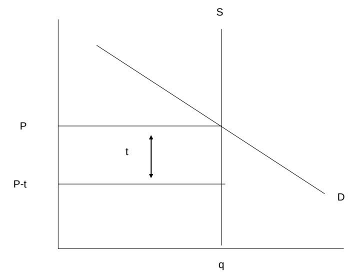
This tax distorted equilibrium is sub-optimal relative to no tax equilibrium because price paid by consumers is higher than the price received by the suppliers and consumer welfare is smaller compared to no tax scenario. The exact price depends very much on demand and supply side parameters, a, b, c,. d and t. Following figure shows this.

The loss to consumer because of taxes is CL and to producers is PL.. Total deadweight loss is sum of these two. This represents distortions or inefficiency due to taxes.

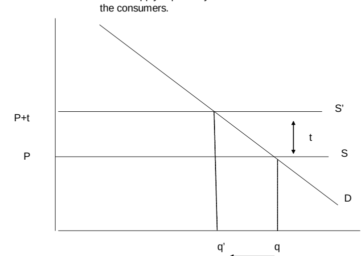
Who bears this excess burden? This depends on the elasticity of demand and supply.

When the supply is perfectly inelastic, the producers bear the burden of whole taxes and when the supply is perfectly elastic all burden is passed onto consumers.





When supply is perfectly elastic all burden of tax falls on to the consumers.



5.3 Welfare impacts of Tax Reforms:

Impacts of Tax Reforms: Hicksian Compensating Variations

- Base utility $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, with budget $m = p_1 x_1 + p_2 x_2$ if $m = 100$; $(p_1, p_2) = (1, 1)$ demand $\left(x_1 = \frac{m}{2p_1}, x_2 = \frac{m}{2p_2}\right) = (50, 50)$

$u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = 50.$

- Now there is tax on good 1 and new prices are $(p_1, p_2) = (2, 1)$ income does not change.

new demand $(x_1, x_2) = (25, 50)$.

- How much income need to be compensated to this consumer to maintain at the old level of utility?

$u_0 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \left(\frac{m'}{2p_1}\right)^{\frac{1}{2}} \left(\frac{m'}{2p_2}\right)^{\frac{1}{2}} = 50$. Here $m' = 2\sqrt{2} \times 50$ =141.4
CV =141.4-100=41.4. Compensating variation is positive for a price rise is positive.

- How much money should be taken away from the consumer in the original prices to make him/her achieve the utility level after the price change.

$u_0 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = (25)^{\frac{1}{2}} (50)^{\frac{1}{2}} = 35.35; \left(\frac{m'}{2}\right)^{\frac{1}{2}} \left(\frac{m'}{2}\right)^{\frac{1}{2}} = 35.35; \Longrightarrow m'$ =70.7
EV= 70.7-100 = -29.3.
Equivalent variation in negative for a rise in price level is negative.
This consumer would have got 35.35 utility by paying 70.7 if prices were (1,1) as before.

5.4 Mirrlees' Theory of Optimal Taxation

Mirrlees' (1971) Theory of Optimal Taxation

- Society has distributions of highly skilled and non-skilled people.
- Highly productive people earn more and pay higher taxes.
- There is an incentive compatible mechanism of tax design to ensure this.
- Highly productive individuals have an incentive to work hard even though their net of tax income may not be proportional to their labour.
- Incentive compatible consumption maximises the social welfare and tax system can be designed to obtain this.

See:

1) Mirrlees J., S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles, and J. Poterba (2010) Dimensions of tax design: the Mirrlees review, Oxford: Oxford University Press

2) Mirrlees J., S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles, and J. Poterba (2011) Ttax by Design: the Mirrlees review, Oxford: Oxford University Press

6 L5: Markets with Imperfect Competition

6.0.1 Cournot, Stackelberg and Cartel: which is better of consumer welfare?

Market demand function where two firms produce q_1, q_2 levels of output and sell them at the market price P as:

$$P = 30 - q_1 - q_2 \tag{107}$$

Cost function

$$C_i = 6q_i \tag{108}$$

Profit:

$$\Pi_i = Pq_i - C_i \tag{109}$$

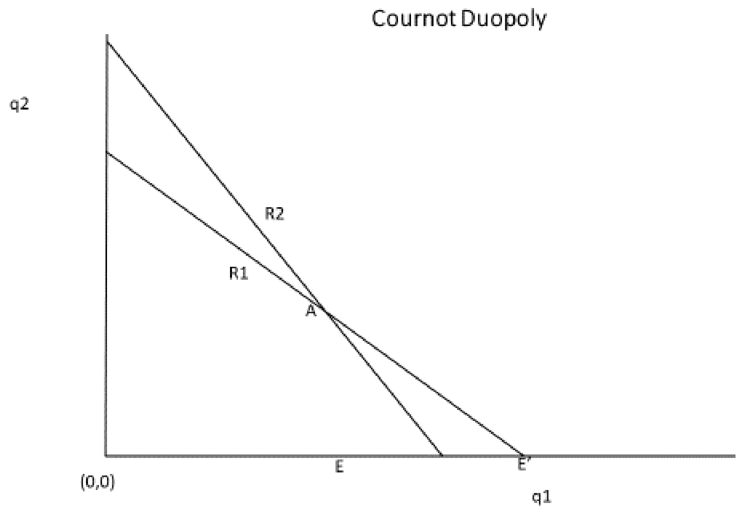
Cournot set up Profit of firm 1

$$\Pi_1 = (30 - q_1 - q_2)\, q_1 - 6q_1 \tag{110}$$

$$\Pi 2 = (30 - q_1 - q_2)\, q_2 - 6q_2 \tag{111}$$

Reaction functions:

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow 2q_1 + q_2 = 24 \Longrightarrow q_1 = 12 - \frac{q_2}{2} \tag{112}$$



$$\frac{\partial \Pi_2}{\partial q_2} = 0 \Rightarrow q_1 + 2q_2 = 24 \Longrightarrow q_2 = 12 - \frac{q_1}{2} \tag{113}$$

Cournot solution is symmetric

$$q_1 = q_2 = 8 \tag{114}$$

$$\begin{aligned} P &= 30 - 8 - 8 = 14 &(115)\\ \Pi_1 &= \Pi_2 = 64 &(116)\end{aligned}$$

Consumer surpluses from both firms

$$CS_1 = CS_2 = \frac{1}{2} \times 16 \times 8 = 64 \tag{117}$$

Total welfare under duopoly

$$W = \Pi_1 + \Pi_2 + CS_1 + CS_2 = 64 + 64 + 64 + 64 = 256 \tag{118}$$

6.0.2 Price-Leadership by firm 1 in Stackelberg equilibrium

When the Firm 1 is the leader and firm 2 is the follower. Leader incorporates follower reaction function into its profit function.

$$\begin{aligned} \Pi_1 &= (30 - q_1 - q_2)\, q_1 - 6q_1 = 30q_1 - q_1^2 - q_1 q_2 - 6q_1 \\ &= 30q_1 - q_1^2 - q_1 \left(12 - \frac{q_1}{2}\right) - 6q_1 = 12q_1 - \frac{1}{2}q_1^2 &(119)\end{aligned}$$

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow q_1 = 12 \tag{120}$$

$$q_2 = 12 - \frac{q_1}{2} = 6 \tag{121}$$

$$P = 30 - q_1 - q_2 = 30 - 12 - 6 = 12 \tag{122}$$

$$\Pi_1 = Pq_1 - 6q_1 = 12 \times 12 - 6 \times 12 = 72 \tag{123}$$

$$\Pi_2 = Pq_2 - 6q_2 = 12 \times 6 - 6 \times 6 = 36 \tag{124}$$

Consumer surpluses from both firms

$$CS_1 = \frac{1}{2} \times 18 \times 12 = 108 \tag{125}$$

$$CS_2 = \frac{1}{2} \times 18 \times 6 = 54 \tag{126}$$

Total welfare under price leadership

$$W = \Pi_1 + \Pi_2 + CS_1 + CS_2 = 72 + 36 + 108 + 54 = 270 \tag{127}$$

Collusion (cartel): maximise industry profit

$$\Pi = PQ - C = (30 - Q)\,Q - 6Q = 30Q - Q^2 - 6Q \tag{128}$$

$$\frac{\partial \Pi}{\partial Q} = 24 - 2Q = 0 \Longrightarrow Q = 12 \tag{129}$$

$$P = 30 - Q = 30 - 12 = 18 \tag{130}$$

Industry profit

$$\Pi = PQ - C = 18 \times 12 - 6 \times 12 = 144 \tag{131}$$

If both firm are equally strong they will share output and profits in half. $q_1 = q_2 = 6$ and $\Pi_1 = \Pi_2 = 72$

Welfare under cartel

$$CS_1 = \frac{1}{2} \times 12 \times 6 = 36 \tag{132}$$

$$CS_2 = \frac{1}{2} \times 12 \times 6 = 36 \tag{133}$$

$$W = \Pi_1 + \Pi_2 + CS_1 + CS_2 = 72 + 72 + 36 + 36 = 216 \tag{134}$$

Table 8: Cournot, Stackleberg and Cartel: Comparison

	Cournot	Stakleberg	Cartel	Perfect Competition
P	14	12	18	6
q_1	8	12	6	24
q_2	8	6	6	24
Π_1	64	72	72	0
Π_2	64	36	72	0
CS_1	64	108	72	288
CS_2	64	36	36	288
C_1	48	72	36	144
C_2	48	36	36	144
TW	256	270	216	576

Under the perfect competition P = MC =6; Q=30-6=24 for each firm. Number of firms is indeterminate but assume only there were two firms. Each will supply 24 and generate consumer surplus 288. Total welfare would be 576, about 2.25 times higher than in the monopoly. Actual welfare gain can be a lot more than this if more firms operate in the market. This is the argument for regulating monopolies and liberalising the market. Consumers can buy goods at lower prices and can have a big consumer surplus under the competitive markets compared to those under the monopolies.

- GAMS programme: cartel.gms; cournot.gms;Pdiscrimn.gms

6.0.3 Monopolistic competition and Trade

Perfect competitions and monopolies are two extreme possibilties of market conditions. Actual markets have elements of both of these. Ever sicne Chamberlin (1933) developed this concept in his book "Theory of Monopolistic Competition", this type of market has been very popular in economic analysis. The literature on brand loyalty and product differentiation characterise the main form of the monopolistic competition. There are plenty of examples in the market; for instance: ipod, CD, DVD, diskettes PCs in information technology induestry. There varieties of product in the soft drinks market such as Coke, Pepsi, Fanta, Tango, Sprite, 7 Up, Dr. Pepper or think of brands of cars such as BMW, Voxhaul, Poeguet, Chrisler, Ford, GM, Toyota, Nissan, Hyundai, Fiat or the Cosmetics, Shoes ,Watches, Camera, Fast food,Yoghurt, Aspirins,Pens, or books in microeconomics or macroeconomics. After Chamberlin (1933) authors like Sweezy (1939), Hall and Hitch (1939), Stigler (1947, 1948), Peck (1961), Osborne (1974), Reid (1981), Maskin and Tirole (1988), Bhaskar (1988) have contributed significantly in developing the theory of monopolistic competition.

Similar to a firm under the monopoly, a typical firm under the monopolistic compition has its own downward sloping demand and so has some monopolistic power in pricing. It faces competition from firms producing close substitutes. If it charges higher prices it loses markets to other producers. Free entry implies zero profit for the incumbent firms. Firms do not produce at the most efficient point. Therefore they producce at less efficient point than firms in perfectly competitive markets. A number of authors including Sweezy (1939) have developed the concepts of kink in demand to explain such behaviour. It is called a model of price and quantity rigidity. If a firm reduces its own price rival firms will reduce their prices, when a typical firm raises its own price none of the others will raise their prices. A firm reduces its own price when another firm reduces it but does not raise its own price when any other firm raise the price of their products. For instance Vaxhaul and Toyota are close substitute. If Toyota company lowers price of its cars Vaxhaul will also lower it. In contrast if Toyota raises it price Vaxhaul will not raise its own price. There are many such example across various market. In general a firm is reluctant to change its own price as it does not want to stir and disturb the other firms in the market by sending wrong signals. Both price and quantity are fixed. This kind of behaviour also occurs in the factor markets particularly in labour markets.

Prices (P) and quantities (X) are fixed, firms do not follow $MR = MC$ principle.

Price and revenue:

$$P = a - X; \quad R = PX = aX - X^2 \tag{135}$$

Average cost:

$$AC = 20 - 2X + 0.1X^2 \tag{136}$$

Total cost:

$$TC = 20X - 2X^2 + 0.1X^3 \tag{137}$$

Marginal revenue (MR):

$$\frac{\partial R}{\partial X} = a - 2X \tag{138}$$

Marginal cost (MC)

$$\frac{\partial TC}{\partial X} = 20 - 4X + 0.3X^2 \tag{139}$$

Two conditions required in the monopolistic competition 1) MR = MC and 2) AR =AC

MR = MC implies

$$a - 2X = 20 - 4X + 0.3X^2 \Longrightarrow 20 - a = 2X - 0.3X^2 \tag{140}$$

AR =AC implies

$$a - X = 20 - 2X + 0.1X^2 \Longrightarrow 20 - a = X - 0.1X^2 \tag{141}$$

From both of these equations

$$2X - 0.3X^2 = X - 0.1X^2 \Longrightarrow X = 5 \tag{142}$$

Now price can be determined

$$a = 20 - X + 0.1X^2 = 20 - 5 + 0.1 \times 25 \Longrightarrow 17.5 \tag{143}$$

$$P = a - X = 17.5 - 5 = 12.5 \tag{144}$$

6.0.4 Monopolistic competition in an industry with two firms

There are two firms in a market, I and II. The marked demand and cost functions faced by each is as following

$$P_1 = 105 - 2q_1 - q_2; \qquad C_1 = 5q_1^2 \tag{145}$$

$$P_2 = 35 - q_1 - q_2; \qquad C_2 = q_2^2 \tag{146}$$

When firm I raises price firm II does not raise its price and gets more profit by supplying more but charging the same price. When firm I reduces price firm II also reduces price and produces same as before but gets less profit.

The base line Cournot duopoly equilibrium:

$$\Pi_1 = P_1 q_1 - C_1 = (105 - 2q_1 - q_2)\, q_1 - 5q_1^2 = 105q_1 - q_2 q_1 - 7q_1^2 \tag{147}$$

$$\Pi_2 = P_2 q_2 - C_2 = (35 - q_1 - q_2)\, q_2 - q_2^2 = 35q_2 - q_1 q_2 - 2q_2^2 \tag{148}$$

Reaction functions

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow 105 - q_2 - 14q_1 = 0 \Longrightarrow 14q_1 + q_2 = 105 \tag{149}$$

$$\frac{\partial \Pi_2}{\partial q_2} = 0 \Rightarrow 35 - q_1 - 4q_2 = 0 \Longrightarrow q_1 + 4q_2 = 35 \tag{150}$$

Solving two reaction functions

$$56q_1 + 4q_2 = 420 \tag{151}$$

$$q_1 + 4q_2 = 35 \tag{152}$$

$$55q_1 = 385 \Longrightarrow q_1 = \frac{385}{55} = 7; \; q_2 = 7 \tag{153}$$

$$P_1 = 105 - 2q_1 - q_2 = 105 - 2 \times 7 - 7 \quad = 84 \tag{154}$$

$$P_2 = 35 - q_1 - q_2 = 35 - 7 - 7 = 21 \tag{155}$$

$$C_1 = 5q_1^2 = 5 \times 7^2 = 245 \tag{156}$$

$$C_2 = q_2^2 = 7^2 = 49 \tag{157}$$

$$\Pi_1 = P_1 q_1 - C_1 = 84 \times 7 - 5 \times 7^2 = 588 - 245 = 343 \tag{158}$$

$$\Pi_2 = P_2 q_2 - C_2 = 21 \times 7 - 7^2 = 147 - 49 = 98 \tag{159}$$

Now consider that firm I raises its price by 2 but the firm II does not react.
$P_1 = 84 + 2 = 86$ but $P_2 = 21$
First get the reaction function of firm II that does not change its price, i.e. or
$P_2 = 35 - q_1 - q_2 = 21 \Longrightarrow q_2 = 14 - q_1$
Use this reaction function of II into the price function of I to get output of firm I.
$P_1 = 105 - 2q_1 - q_2 = 105 - 2q_1 - (14 - q_1) \Longrightarrow 86 \;= 91 - q_1 \Longrightarrow q_1 = 5$
Using II's reaction function

$$q_2 = 14 - q_1 = 14 - 5 = 9 \tag{160}$$

$$\Pi_1 = P_1 q_1 - C_1 = 86 \times 5 - 5 \times 5^2 = 430 - 125 = 305 \tag{161}$$

$$\Pi_2 = P_2 q_2 - C_2 = 21 \times 9 - 9^2 = 189 - 81 = 108 \tag{162}$$

$$C_1 = 5q_1^2 = 5 \times 5^2 = 125 \tag{163}$$

$$C_2 = q_2^2 = 9^2 = 81 \tag{164}$$

If the duopolist I reduces price by 2 the firm II will also follow the suit.
$P_1 = 84 - 2 = 82$ but $P_2 = 21 - 2 = 19$
Given above demand functions
Firm I reduces its price by 2 i.e.

$$P_1 = 105 - 2q_1 - q_2; \;\Longrightarrow 82 = 105 - 2q_1 - (14 - q_1) \;\Longrightarrow q_1 = 9 \quad C_1 = 5q_1^2 \tag{165}$$

Get firm II's reaction function from and use this reaction function of II into the price function of I to get output of firm I.

$$P_2 = 35 - q_1 - q_2; \Longrightarrow \; 19 = 35 - 9 - q_2 \;\Longrightarrow q_2 \;= 7 \quad C_2 = q_2^2 \tag{166}$$

Note firm II wants to maintain the old level of output by reducing its price

$$\Pi_1 = P_1 q_1 - C_1 = 82 \times 9 - 5 \times 9^2 = 738 - 405 = 333 \tag{167}$$

$$\Pi_2 = P_2 q_2 - C_2 = 19 \times 7 - 7^2 = 133 - 49 = 84 \tag{168}$$

$$C_1 = 5q_1^2 = 5 \times 9^2 = 405 \tag{169}$$

$$C_2 = q_2^2 = 7^2 = 49 \tag{170}$$

Summary of the Monopolistic Competition Model

Table 9: Firm Behaviour in Monopolistic Competition

	Baseline Cournot	When I raises P1 by 2	When I reduces P1 by 2
P_1	84	86	82
P_2	21	21	19
q_1	7	5	9
q_2	7	9	7
Π_1	343	305	333
Π_2	98	108	84
R_1	588	430	738
R_2	147	189	133
C_1	245	125	405
C_2	49	81	49

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6.1 Natural Monopoly

MR = MC rule leads to negative profit under the natural monopoly

Price

$$P = 100 - X; \quad R = PX = 100X - X^2 \tag{171}$$

Average cost

$$AC = 50 - 0.125X \tag{172}$$

Total cost

$$TC = AC \times X = 50X - 0.125X^2 \tag{173}$$

Marginal revenue (MR)

$$\frac{\partial R}{\partial X} = 100 - 2X \tag{174}$$

Marginal cost (MC)

$$\frac{\partial TC}{\partial X} = 50 - 0.25X \tag{175}$$

MR = MC does not produce efficient quantity

$$100 - 2X = 50 - 0.25X \Longrightarrow 1.75X = 50 \Longrightarrow X = \frac{50}{1.75} = 28.57 \tag{176}$$

$$P = 100 - X = 100 - 28.57 = 71.43 \tag{177}$$

$$AC = 50 - 0.125X = 50 - 0.125\,(28.57) = 46.43 \tag{178}$$

$$\Pi = PX - C = 71.43 \times 28.57 - 46.43 \times 28.57 = 714.25 \tag{179}$$

If P = MC natural monopolist will make negative profit

$$P = 100 - X = 50 - 0.25X \Longrightarrow\Longrightarrow 0.75X = 50 \Longrightarrow X = \frac{50}{0.75} = 66.7 \tag{180}$$

$$P = 100 - X = 100 - 66.7 = 33.33 \tag{181}$$

$$\Pi = PX - 50X + 0.125X^2 = 66.7\times 33.33 - 50\times 66.7 + 0.125\left(66.7^2\right) = 2222 - 2778.88 = -556.9 \tag{182}$$

Two part tariff in natural monopoly Let the natural monopolist set price equal to the average cost but let them allow any loss to be made by second tariff to each consumer. P = AC

$$P = 100 - X = 50 - 0.125X \Longrightarrow X = \frac{50}{0.857} = 58.14 \tag{183}$$

$$P = 100 - X = 100 - 58.14 = 41.86 \tag{184}$$

$$\Pi = PX - 50X + 0.125X^2 = 58.14\times 41.86 - 50\times 58.14 + 0.125\left(58.14^2\right) = 2433.74 - 2907 + 422.5 = -50.76 \tag{185}$$

If there are 1000 customers each will pay tariff to make up the loss equal to 50.76/1000= 0.05 increase price to them by that margin.

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6.1.1 Bertrand Game of price competition

There are two firms and the market price is

$$P = 130 - (q_1 + q_2)\,; \quad C = 10q_i \tag{186}$$

Firm I is considering three pricing strategies. First $P = 10.02$ or $P = 10.01$ or $P = 10$.

"cut-throat" price competition implies that if firm I charges 10.02 then firm II can also set $P = 10.02$ or charge less $P = 10.01$ to get all customers or set $P = 10$ to drive firm I out from the market. If both agree to charge $P = 10.02$ then

$10.02 = 130-(q_1 + q_2) \implies (q_1 + q_2) = 130-10.02 = 119.98$. Then total profit is $0.02\times119.98 = 2.3996$ and each makes 1.1998.

Similarly if they agree to charge $P = 10.01$ then $10.01 = 130 - (q_1 + q_2) \implies (q_1 + q_2) = 130 - 10.01 = 119.99$.Then total profit is $0.01\times119.98 = 1.1999$ and each makes 0.5995.

Instead if both charge $P = 10$ Then $10.0 = 130 - (q_1 + q_2) \implies (q_1 + q_2) = 130 - 10 = 120$. There will zero profit; none will make any profit. Price war has resulted in perfect competition outcome. Draw these points in a diagram.

Exercise

If the inverse demand function for a firm is

$$P = a - q_A - q_B \tag{187}$$

and the cost of production is

$$C_i = cq_i \tag{188}$$

Prove that Cournot reaction functions are given by

$$q_A = \frac{a - c - q_B}{2} \tag{189}$$

$$q_B = \frac{a - c - qA}{2} \tag{190}$$

Cournot Nash equilibrium

$$q_A = \frac{a-c}{3} \tag{191}$$

$$q_B = \frac{a-c}{3} \tag{192}$$

If firm A is stackelberg quantity leader and considers firm B's reaction function $\left(q_B = \frac{a-c-qA}{2}\right)$while making its price and output decisions, prove that

$$q_A = \frac{a-c}{2} \tag{193}$$

$$q_B = \frac{a-c}{4} \tag{194}$$

Bertrand competition Demand for product of firm A depends on price it charge and price charged by the rival firm B

$$q_A = a - P_A + bP_B \tag{195}$$

$$q_B = a - P_B + bP_A \tag{196}$$

$$\Pi_A = (a - P_A + bP_B)\,P_A - c\,(a - P_A + bP_B) \tag{197}$$

$$\Pi_B = (a - P_B + bP_A)\,P_B - c\,(a - P_B + bP_A) \tag{198}$$

Reaction functions

$$\frac{\partial \Pi_A}{\partial P_A} = 0 \Rightarrow (a - 2P_A + bP_B) + c = 0 \Longrightarrow P_A = \frac{a + bP_B + c}{2} \tag{199}$$

$$\frac{\partial \Pi_B}{\partial P_B} = 0 \Rightarrow (a - 2P_B + bP_A) + c = 0 \Longrightarrow P_B = \frac{a + bP_A + c}{2} \tag{200}$$

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6.2 Monopolistic competition and trade

1. Consider a firm in monopolistically competitive industry (refer to chapter 6 of Krugman and Obstfeld (2000))

$$Q = A - B \cdot P \tag{201}$$

Prove that its marginal revenue is given by

$$MR = P - \frac{Q}{B} \tag{202}$$

(a) If the cost function is $C = F + cQ$ then prove that the average cost declines because of the economy of scale.

(b) Further assume that givens the total sales of the industry (S), the output sold by a firm (Q), number of firms, its own price and average prices of firms are given by

$$Q = S\left[\frac{1}{N} - b\left(P - \overline{P}\right)\right] \tag{203}$$

show that the average cost rises to number of firms in the industry when all firms charge same price .
$AC = \frac{n.F}{s} + c$

[hint: $Q = \frac{S}{N}$] More firms in the industry less each will sell and hence higher the AC.

1. (a) Prove that price charged by a particular firm declines with the number of firms

$$P = c + \frac{1}{b \cdot n} \tag{204}$$

(b) Determine the number of firms and price in equilibrium. Explain entry exit behavior prices when number of firms are below or above this equilibrium point. [draw price and average cost curve against number of firms in the industry.]

(c) Collusive and strategic behaviors may limit above conclusions. Discuss.

(d) Apply above model to explain international trade and its impact on prices and number of firms in a particular industry.

(e) Use this model to explain interindustry and intra-industry trade.

(f) Use monopolistic competition model to analyse consequences of dumping practices in international trade.

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6.3 Nonlinear pricing and price discrimination

Firms with market power engage in three types of price discrimination. First degree price discrimination occurs when firms know willingness to pay of its customers perfectly. They will charge a price specific to each customer according to that customer's willingness to pay for the product. All consumer surplus will be extracted by the firm in this case. Now draw a deman curve, show the market price and consumer surplus and series of prices that firm charges to each customer.

Then second degree price discrimination occur when firm does not know the types of customers but sets prices by the quantity of product bought by the cosumer. One very good example of this types of price discrimination is for cappuccino coffee in a Cafe.

Table 10: Non-linear price of Capuccino coffee in a Cafe

	Q (shots)	Price	Average price
Primio	2	2.20	1.10
Midio	3	2.45	0.82
Massimo	4	2.65	0.66

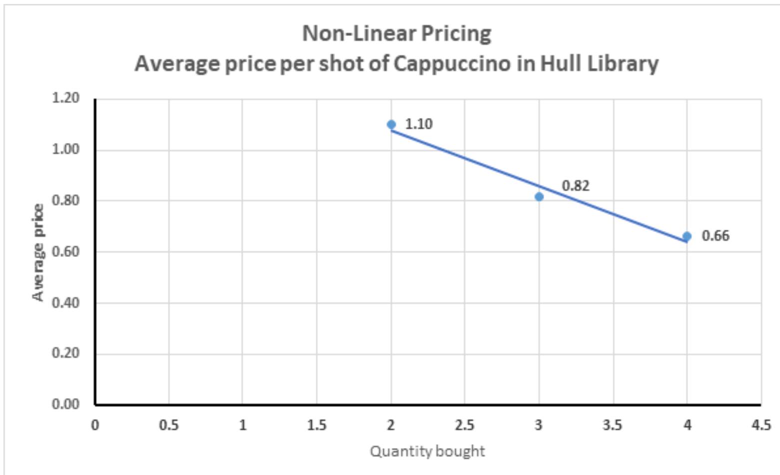
Third degree price discrimination occurs when a firm charges different prices for the different segments of the markets. Rail companies or phone companies have peak and off-peak prices, short haul or long distance rates. Traders charge different prices for domestic and foreign customers. Firms charge higher prices on less elastic segments of the market and lower prices in more elastic segments of the market.

7 L6: Game Theory : Introduction

- Economic activities of consumers, producers, governments and nations or regions are interdependent.
- Game theory provides tools to study the strategic interactions among such economic agents where decisions taken by one individual depend on actions taken by others.
- Each game has a number of players who choose a set of strategies and rules. .Optimal choices available to one depend on choices made by others.
- Pay-offs are clearly defined for each player strategy pairs.
- Strategic modelling like this started with classics such as Cournot (1838), Bertrand (1883), Edgeworth (1925) von Neumann and Morgenstern (1944), Nash (1950). It is developing very fast in recent years following works of Kuhn (1953), Shapley (1953),Selten (1965) Aumann (1966) Scarf (1967), Shapley and Shubic (1969), Harsanyi (1967), Spence (1974), Kreps (1990), Fundenberg and Tirole (1991) and Binmore (1992).

Elements of a Game

- Rational Players
- Strategies


Non-Linear Pricing
Average price per shot of Cappuccino in Hull Library
Average price
Quantity bought
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
1.10
0.82
0.66

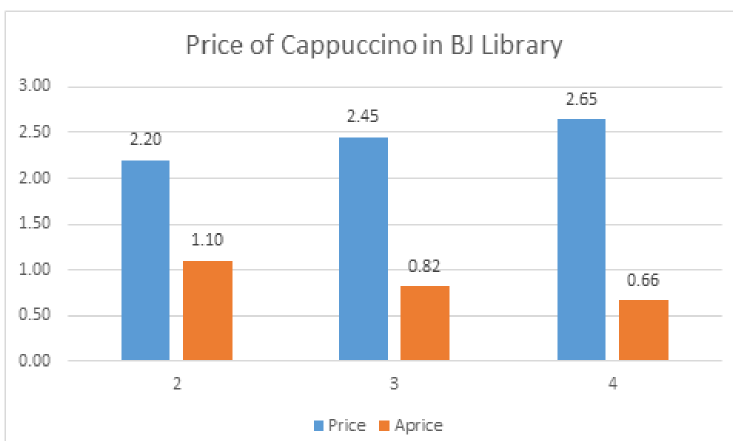

Price of Cappuccino in BJ Library
3.00
2.50
2.00
1.50
1.00
0.50
0.00
2.20
1.10
2.45
0.82
2.65
0.66
2
3
4
Price
Aprice

Table 11: Structure of a Game

		Player A	
		Strategy 1	Strategy 2
Player B	Strategy 1	$(\Pi^R_{1,1}, \Pi^C_{1,1,})$	$(\Pi^R_{1,2}, \Pi^C_{1,2})$
	Strategy 2	$(\Pi^R_{2,1}, \Pi^C_{2,1,})$	$(\Pi^R_{2,2}, \Pi^C_{2,2})$

- Payoff matrix

$\Pi^R_{1,1}$ is pay-off to row player if he plays strategy 1 and the column player plays strategy 1.

Players like to maximise their own pay-off given opponent's strategy; B will choose strategy 1 or 2 that maximises his/her payoff looking at the choice of player A.

Most games have equilibrium from which players do not have any incentive to move away.

Types of Games

Table 12: Zero Sum Game

		Player A	
		Strategy 1	Strategy 2
Player B	Strategy 1	$(10, -10)$	$(-10, 10)$
	Strategy 2	$(-10, 10)$	$(10, -10)$

zero sum game: one's gain = loss of another ; sports ; market shares

- two or many players; Chess, football
- Cooperative Games: Global climate change; bargaining game
- Non-cooperative Games: two or many players

Competition and Collusion

oligopoly/competition between opposing political parties, countries

- Single period of multiple period: static and dynamic
- Full information or incomplete information :Firms and consumers; government and public;Among individuals, clubs, parties; nations

7.1 Solution of Games

7.1.1 Solution of Games by the Dominant Strategy

- Dominant strategy

Dominant strategy is to advertise for both A and B. With a slight change

Dominant strategy is to advertise for A but B has no dominant strategy.

Table 13: Advertisement Game

		Player A	
		Advert	Dont Advert
Player B	Advert	$(10, 5)$	$(15, 0)$
	Dont Advert	$(6, 8)$	$(10, 2)$

Table 14: Advertisement Game

		Player A	
		Advert	Dont Advert
Player B	Advert	$(10, 5)$	$(15, 0)$
	Dont Advert	$(6, 8)$	$(\mathbf{20}, 2)$

7.1.2 Solution of Games by Nash Equilibrium (Prisoner's Dilemma)

Punishment structure for a crime

Finding Nash solution (underscore the best strategy to a player i given the choice of the opponent.

Nash Equilibrium: Prisoner's Dilemma

- Fact: both players did a crime together. Police suspects and arrest both of them.
- Playing non cooperatively each convicts another. Game results in Nash solution (confess, Confess) = $(-5, -5)$; Each ends up with 5 years in prison.
- By confessing, each gives evidence to the police to determine the highest possible punishment.
- If they had cooperated remaining silent, police would not have enough evidence.
- Each would have been given only two years of prison $(-2, -2)$. This is Pareto optimal outcome, "where no one could be made better off without making someone worse-off".
- Cooperation is better but each think that other player will cheat and therefore doesn't cooperate. Therefore stay longer in jail.
- There are many example of prisoner's dilemma game in real world -pricing and output in a cartel, pollution, tax-revenue.

7.1.3 Solution by the mixed strategy

Solution by the mixed strategy

This game does not have equilibrium in pure strategy. Player B will play H is A plays H but A will play T if B plays H. If A plays T it is optimal to play T for B, then it is optimal for B to play H. Game goes in round in circle again.

It can be solved my the mixed strategy.

Flip the coin to randomise the chosen strategies. If each played H or T half of the times optimal payoff is zero to both players. Probability of playing H or T is 0.5.

Solution by mixed strategy

B plays Top p times and Bottom $(1-p)$ times if A plays Left . B plays Top p times and Bottom $(1-p)$ times if A plays Right.

Table 15: Prisoners' Dilemma Game

		Player A	
		Confess	Dont Confess
Player B	Confess	$(-5,-5)$	$(-1,-10)$
	Dont Confess	$(-10,-1)$	$(-2,-2)$

Table 16: Prisoners' Dilemma Game

		Player A	
		Confess	Dont Confess
Player B	Confess	$(-\underline{5},-\underline{5})$	$(-\underline{1},-10)$
	Dont Confess	$(-10,-\underline{1})$	$(-2,-2)$

B likes to be equally well off no matter what A plays.
Solution by the mixed strategy
Expected pay-off for B if A plays Left

$$E\left(\Pi_{B,L}\right) = 50p + 90(1-p) \tag{205}$$

Expected pay-off for B if A plays Right

$$E\left(\Pi_{B,R}\right) = 80p + 20(1-p) \tag{206}$$

Making these two payoffs equal

$$50p + 90(1-p) = 80p + 20(1-p) \Longrightarrow 100p = 70 \tag{207}$$

$$p = 0.7 \tag{208}$$

B plays Top 70 % of times and Bottom 30% of times.
Subsidy Game Between the Airbus and Boeing
If both Boeing and Airbus produce a new aircraft each will lose -10. If Airbus does not produce and only Boeing produces Boeing will make 100 profit. If Airbus does not produce Airbus can make 100 but then Boeing will decide to produce even at a loss of 10 so that Airbus does not enter in that market.
Subsidy Game Between the Airbus and Boeing
EU countries want Airbus to produce, they change this by subsidising 20 to Airbus.
Producing new aircraft is dominant strategy for Airbus now, no matter whether Boding produces or not.
Entry Deterrence Game
Inflation and unemployment game between public and private sectors
Higher payoff is good.
First element represents payoff to the row-player (Government). Second element represents payoff to the column-player (private sector).
Nash solution is $(H,H) = (4,4)$ Cooperative solution would have been better with $(L,L) = (5,5)$.

Table 17: Game of matching penny: mixed strategy

		Player A	
		Head	Tail
Player B	Head	$(1,-1)$	$(-1,1)$
	Tail	$(-1,1)$	$(1,-1)$

Table 18: Competitive Game

		Player A	
		Left	Right
Player B	Top	$(50,-50)$	$(80,-80)$
	Bottom	$(90,-90)$	$(20,-20)$

7.1.4 Cost of Cheating and discount factor

Cooperative solution would have been better with $(L, L) = (5, 5)$ but distrusting each other results in $(H, H) = (4, 4)$.

If the game is plaid repeatedly what will be value of the game? It is given by the discounted present value of the game for any discount rate $0 < \delta < 1$:

$$PV(cooperate) = 5 + 5\delta + 5\delta^2 + ... + 5\delta^n = \frac{5}{1-\delta} \tag{209}$$

However, there is an incentive to cheat to get 6 instead of 5. when one player deviates from the cooperative strategy this way another will found out being cheated next period. Then he/she will punish the cheater by playing non-cooperatively next period. So the value of game :

$$PV(cheat) = 6 + 4\delta + 4\delta^2 + ... + 4\delta^n \tag{210}$$

Cost of Cheating

$$PV(cheat) = 6 + 4\delta + 4\delta^2 + ... + 4\delta^n \tag{211}$$

Taking the sum

$$PV(cheat) = 6 + \delta\left(4 + 4\delta + 4\delta^2 + ... + 4\delta^n\right) \tag{212}$$

$$PV(cheat) = 6 + 4\frac{\delta}{(1-\delta)} \tag{213}$$

Whether a person cheats or not depends on discount factor

$$\frac{5}{1-\delta} = 6 + 4\frac{\delta}{(1-\delta)} \quad or 5 = 6(1-\delta) + 4\delta \quad -1 = -2\delta; \quad \delta = \frac{1}{2} \tag{214}$$

Table 19: Subsidy Game

		Airbus	
		Produce	Don't produce
Boeing	Produce	$(-10,-10)$	$(100,0)$
	Don't produce	$(0,100)$	$(0,0)$

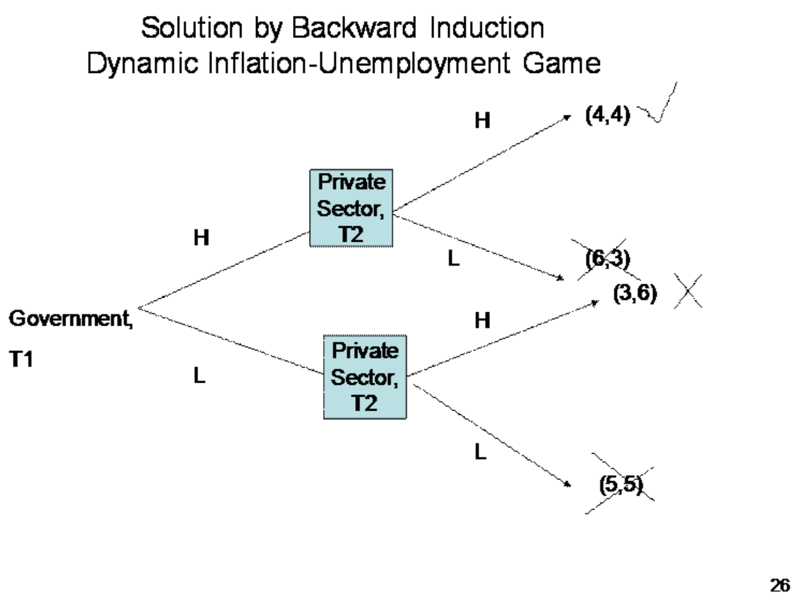
Table 20: Subsidy Game

		Airbus	
		Produce	Don't produce
Boeing	Produce	$(-10,10)$	$(100,0)$
	Don't produce	$(0,120)$	$(0,0)$

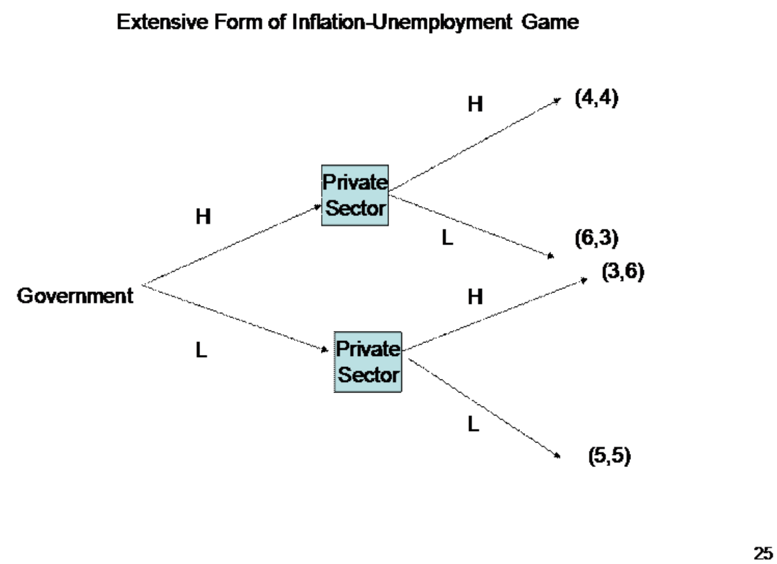
7.1.5 Extensive form of the game

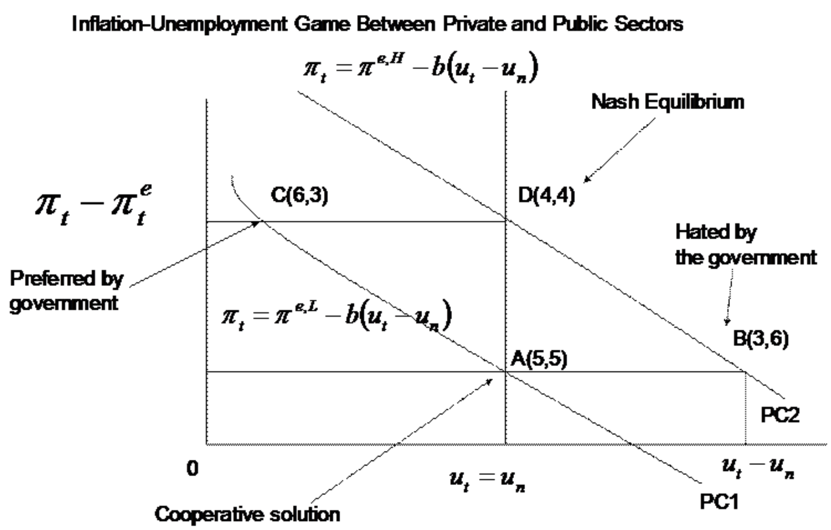
7.1.6 Solution by Backward Induction

Solution by Backward Induction (Is there any first movers advantage?)



Inflation and unemployment game in a diagram
Inflation and unemployment game in a diagram





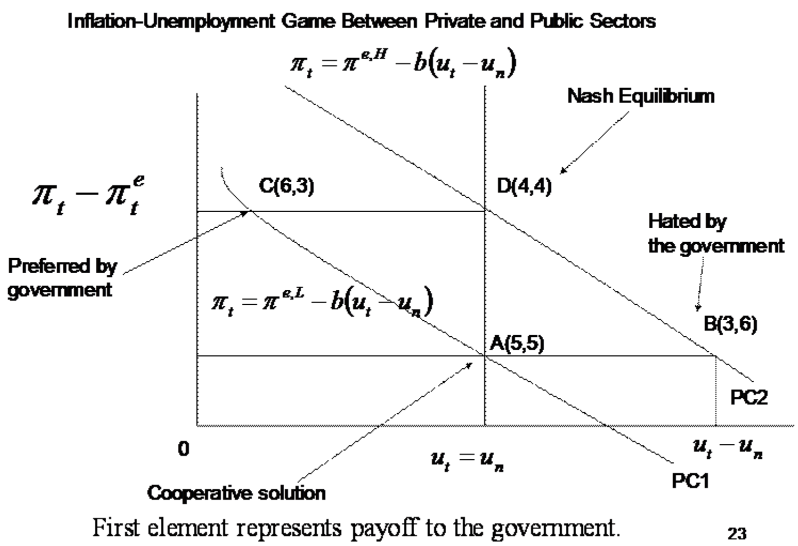
First element represents payoff to the government.

Table 21: Subsidy Game

		Entrant	
		Enter	Dont Enter
Incumbent	Enter	$(-10,-10)$	$(100,0)$
	Dont Enter	$(0,100)$	$(0,0)$

Table 22: Subsidy Game

		Entrant	
		Enter	Dont Enter
Incumbent	Enter	$(-10,10)$	$(100,0)$
	Dont Enter	$(0,120)$	$(0,0)$

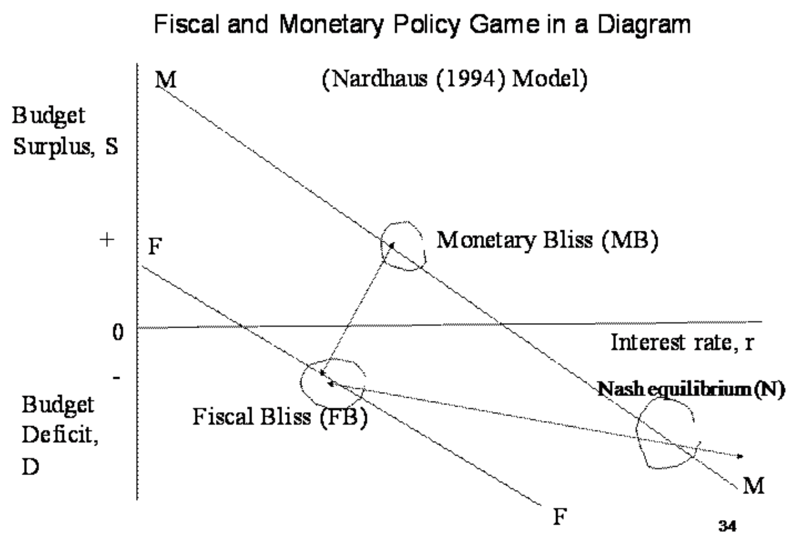


First element represents payoff to the government.

Economic policy game between the fiscal and monetary authority

Table 23: Inflation and unemployment game

		Private Sector	
Government		H	L
	H	$(4,4)$	$(6,3)$
	L	$(3,6)$	$(5,5)$



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8 L7: Repeated Games

Cooperative Solution

Market demand for a product is

$$P = 130 - (q_1 + q_2) \tag{215}$$

Cost of production for each of two firms is .

$$C_i = 10q_i \tag{216}$$

If played infinite number of time two firms form a cartel and monopolise the market.

Each will supply only 30, set market price to monopoly level at £70 and divide total profit £3600 equally; each getting £1800.

This is shown by (1800,1800) point in the diagram.

It pays to cooperate in the long run; it is sub-game perfect equilibrium.

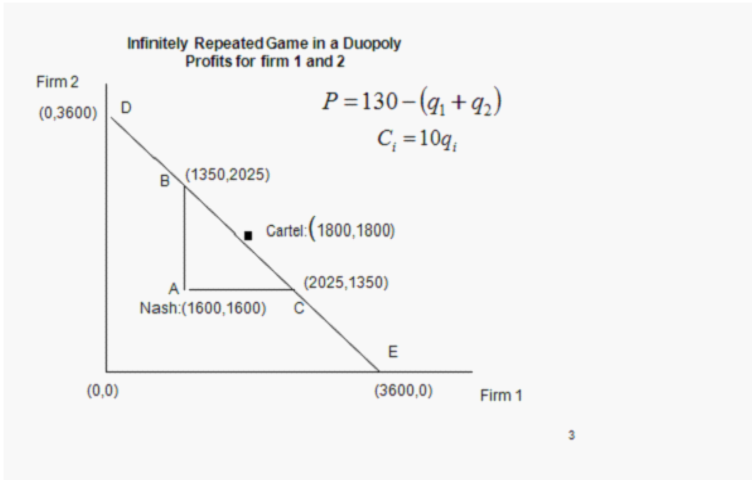
$$\Pi = (130 - Q)\,Q - 10Q = 130Q - Q^2 - 10Q \tag{217}$$

$$\frac{\partial \Pi}{\partial Q} = 130 - 2Q - 10 = 0 \Longrightarrow Q = \frac{120}{2} = 60 \tag{218}$$

$P = (130 - Q) = 130 - 60 = 70; C = 10Q = 10 \times 60 = 600;$
$\Pi = PQ - C = 60 \times 70 - 600 = 3600$
Non-Cooperative Nash Equilibrium

- If any one firm cheats and tries to supply more in order to get more profit; it will be found out by another firm.
- It will react to this.
- Game will be non-cooperative with resulting in a Cournot Nash equilibrium.
- with each firm producing 40 units, market price of 50 and each getting £1600 profits.
- $\Pi_1 = (130 - (q_1 + q_2))\,q_1 - 10q_1$ and $\Pi_2 = (130 - (q_1 + q_2))\,q_2 - 10q_2$
- with reaction functions $2q_1 + q_2 = 120$ and $q_1 + 2q_2 = 120$
- Total supply is 80, each supplying 40 and making profit of 1600 and market price 50.

Trigger Strategy and Perpetual Punishment



- If firm 1 plays Cournot game but firm 2 still plays cartel and supply just 30.
- Then from the firm 1' reaction function . $2q_1 + q_2 = 120$

$$q_1 = 60 - \frac{1}{2}q_2 = 60 - \frac{1}{2}(30) = 45 \tag{219}$$

- If firm 1 supplies 45, market price will be .

$$P = 130 - (q_1 + q_2) = 130 - 45 - 30 = 55 \tag{220}$$

- This makes profit margin of firm 1 to be 45 and its profit .$\Pi_1 = (55q_1 - 10q_1) = 45q_1 = 45 \times 45 = 2025$
- Firm 2 will find out that firm 1 has cheated.
- It will also produce according to its reaction curve.
- Thus the Nash equilibrium will result with each firm producing 40 and earning 1600 profit for the rest of the periods and the market price will be 50.

For whom is it profitable to Cheat?

Does firm 1 gain or lose by deviation from the agreement. For this evaluate the infinite series of profits in deviation and in compliance with agreement.

Present value of profit in case of cheating

$\Pi_1 = \left[2025 + 1600\delta + 1600\delta^2 + ... + ...\right]$

$\delta\Pi_1 = \delta\left[2025 - 1600 + 1600 + 1600\delta + 1600\delta^2 + ... + ...\right]$

(Note just with − and + 1600)

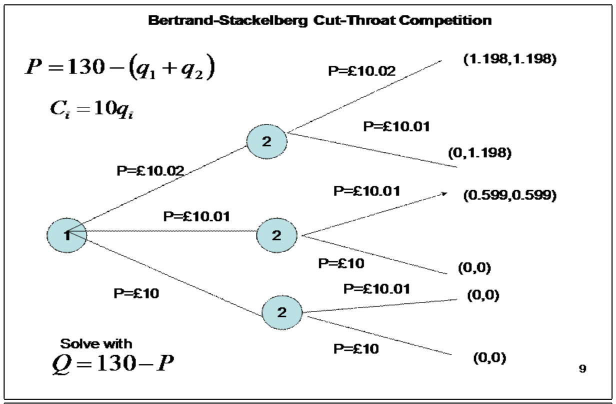
Using operator to maintain a constant payoff from the game

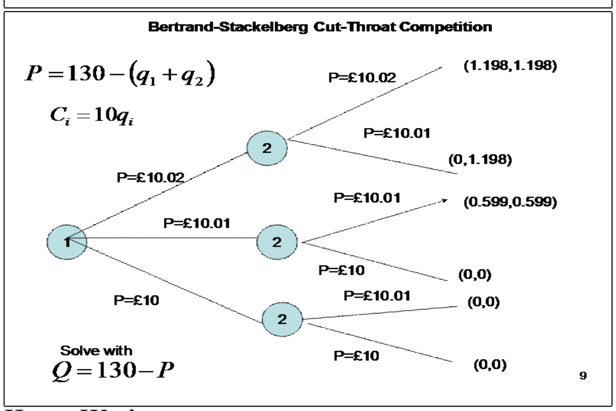
$(1-\delta)\,\Pi_1 = (1-\delta)\left[425 + \frac{1600}{(1-\delta)}\right] = [425\,(1-\delta) + 1600] = 425 - 425\delta + 1600 = 2025 - 425\delta$

By comparing profits with and without cheating

$2025 - 425\delta < 1800$ or ; $425\delta > 2025 - 1800$; $\delta > \frac{225}{425} \Longrightarrow \delta > \frac{9}{17}$

Whether the firm 1 will stick to agreement or not depends on whether its discount factor if greater than $\delta > \frac{9}{17}$. For discount factor $\delta < \frac{9}{17}$ it is beneficial to stick to the agreement, which is very high, about 53 percent.





Home Work

- Show above results in a diagram
- Illustrate repeated game for multiple periods using branch nodes
- Workout Bertrand type competition for above game and illustrate "cut-throat" price competition in a diagram.

9 L8: Bargaining Game

- The very common example for bargaining game is splitting a pie between two individuals.
- The sum of the shares of the pie claimed by both cannot exceed more than 1, otherwise each will get zero.

- If we denote these shares by θ_i and θ_j then $\theta_i + \theta_j \leq 1$ is required for a meaningful solution of the game where each get $\theta_i \geq 0$ and $\theta_j \geq 0$ payoff. When $\theta_i + \theta_j > 1$ then and $\theta_i = 0$ and $\theta_j = 0$.
- Standard technique to solve this problem is to use the concept of Nash Product

9.0.7 Nash Product in Bargaining Game

$$\max U = (\theta_i - 0)(\theta_j - 0) \tag{221}$$

- subject to

$$\theta_i + \theta_j \leq 1 \tag{222}$$

- or by non-satiation property $\theta_i + \theta_j = 1$
- Using a Lagrangian function

$$L(\theta_i, \theta_j, \lambda) = (\theta_i - 0)(\theta_j - 0) + \lambda [1 - \theta_i - \theta_j] \tag{223}$$

First Order Conditions
First order conditions of this maximization problem are

$$\frac{L(\theta_i, \theta_j, \lambda)}{\partial \theta_i} = \theta_j - \lambda = 0 \tag{224}$$

$$\frac{L(\theta_i, \theta_j, \lambda)}{\partial \theta_j} = \theta_i - \lambda = 0 \tag{225}$$

$$\frac{L(\theta_i, \theta_j, \lambda)}{\partial \lambda} = 1 - \theta_i - \theta_j = 0 \tag{226}$$

From the first two first order conditions $\theta_j - \lambda = \theta_i - \lambda$ implies $\theta_j = \theta_i$ and putting this into the third first order condition $\theta_j = \theta_i = \frac{1}{2}$. This is called focal point.

Thus Nash solution of this problem is to divide the pie symmetrically into two equal parts. Any other solution of this not stable. Roy Gardner (2003) and Charles Holt (2007) have a number of interesting examples on bargaining game.

9.0.8 Application of Bargaining Game

Money to be divided between two players

$$M = u_1 + u_2 \tag{227}$$

- The origin of this bargaining game is the disagreement point $d(0,0)$, the threat point.
- Here the utility of player one (u_1) is plotted against the utility of player two u_2 and the line $u_1 u_2$ is the utility possibility frontier (UPF).
- Starting of bargaining can be $(0, M)$ or $(M, 0)$ where one player claims all but other nothing.

- But this is not stable.
- Offers and counter offers will be made until the game is settled at $u^* = \left(\frac{1}{2}M, \frac{1}{2}M\right)$ where each player gets equal share.

Numerical Example of Bargaining Game Suppose there is 1000 in the table to be split between two players. What is the optimal solution from a symmetric bargaining game if the threat point is given by d(0,0)? Using a Lagrangian function for constrained optimisation

$$L\left(u_{1,}u_2, \lambda\right) = u_1 u_2 + \lambda\left[1000 - u_1 - u_2\right] \tag{228}$$

First order conditions of this maximization problem are

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial u_1} = u_2 - \lambda = 0 \tag{229}$$

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial u_2} = u_1 - \lambda = 0 \tag{230}$$

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial \lambda} = 1000 - u_1 - u_2 = 0 \tag{231}$$

From the first two first order conditions $u_2 - \lambda = u_1 - \lambda$ implies $u_2 = u_1$ and putting this into the third first order condition $u_2 = u_1 = \frac{1000}{2} = 500$. This is called focal point.

The Nash bargaining solution is the values of u_1 and u_2 that maximise the value of the Nash product $u_1 u_2$ subject to the resource allocation constraint,$u_1 + u_2 = 1000$.

This bargaining solution fulfils four different properties: 1) symmetry 2) efficiency 3) linear invariance 4) independence of irrelevant alternatives (IIA).

Symmetry implies that equal division between two players and efficiency implies no wastage of resources $u_1 + u_2 = M$ or maximisation of the Nash product, $u_1 u_2$.

Linear invariance refers to the location of threat point as can be shown in a bankruptcy game say dividing 50000. If u^* is a solution to the bargaining game then $u^* + d$ is a solution to the bargaining problem with disagreement point d.

$$L\left(u_{1,}u_2, \lambda\right) = \left(u_1 - d_1\right)\left(u_2 - d_2\right) + \lambda\left[50000 - u_1 - u_2\right] \tag{232}$$

Suppose the player 1 has side payment $d_1 = 15000$

$$L\left(u_{1,}u_2, \lambda\right) = \left(u_1 - 15000\right)\left(u_2 - d_2\right) + \lambda\left[50000 - u_1 - u_2\right] \tag{233}$$

First order conditions of this maximization problem are

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial u_1} = u_2 - \lambda = 0 \tag{234}$$

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial u_2} = u_1 - 15000 - \lambda = 0 \tag{235}$$

$$\frac{L\left(u_{1,}u_2, \lambda\right)}{\partial \lambda} = 50000 - u_1 - u_2 = 0 \tag{236}$$

- From the first two first order conditions $u_2 - \lambda = u_1 - 15000 - \lambda$
- implies $u_2 = u_1 - 15000$ and
- putting this into the third first order condition
- $u_2 + 15000 = u_1; u_2 = \frac{50000-15000}{2} = 17500; u_1 = 15000 + u_2 = 32500.$
- Then $u_1 + u_2 = 17500 + 32500 = 50000.$

Risk and Bargaining A risk averse person looses in bargaining but the risk neutral person gains. Suppose the utility functions of risk averse person is given by$u_2 = (m_2)^{0.5}$ but the risk neutral person has a linear utility $u_1 = m_1$.

$m_1 + m_2 = M$

$.u_1 + u_2^2 = 100$

Using a Lagrangian function for constrained optimisation

$$L(u_1, u_2, \lambda) = u_1 u_2 + \lambda \left[100 - u_1 - u_2^2\right] \tag{237}$$

First order conditions of this maximization problem are

$$\frac{L(u_1, u_2, \lambda)}{\partial u_1} = u_2 - \lambda = 0 \tag{238}$$

$$\frac{L(u_1, u_2, \lambda)}{\partial u_2} = u_1 - 2\lambda u_2 = 0 \tag{239}$$

$$\frac{L(u_1, u_2, \lambda)}{\partial \lambda} = 100 - u_1 - u_2^2 = 0 \tag{240}$$

From the first two first order conditions $\frac{u_2}{u_{1,}} = \frac{\lambda}{2\lambda u_2}$ implies $u_1 - 2u_2^2$ and putting this into the third first order condition $.3u_2^2 = 100$; $u_2^2 = \frac{100}{3} = 33.3$; $u_2 = 5.77$

$u_1 = 2u_2^2 = 2(5.77)^2 = 66.6$

$u_1 + u_2^2 = 66.6 + 33.3 = 100$

Thus the risk nuetral player's utility is 66.7 and risk averse player's utility is only 5.7.

Morale: do not reveal anyone if you are risk averse, otherwise you will lose in the bargaining.

9.1 Coalition Game: Coalition Possibilities

2^N -1 rule for possible coalition

Consider Four Players A,B,C,D

A, B, C, D

AB, AC, AD

BC, BD, CD

ABC, ABD,ACD, BCD,

ABCD

16 -1=15

9.1.1 Problem 9: Strategic Models and Optimal Tax

Q1. Only two firms supply products in a certain market in which the market demand for the product is:

$$P = 150 - (q_1 + q_2) \tag{241}$$

Cost of production for each of the two firms is .

$$C_i = 11q_i \quad for \quad i = 1, 2 \tag{242}$$

a) **What is the total profit when these two firms collude?**
b) **What is the output in Cournot equilibrium? What kind of game is this?**

Q2. Consider a firm in monopolistically competitive industry

$$Q = A - B \cdot P \tag{243}$$

Prove that its marginal revenue is given by

$$MR = P - \frac{Q}{B} \tag{244}$$

(a) If the cost function is $C = F + cQ$ then prove that the average cost declines because of the economy of scale.

(b) Further assume that the output sold by a firm, number of firms, its own price and average prices of firms are given by

$$Q = S\left[\frac{1}{N} - b\left(P - \overline{P}\right)\right] \tag{245}$$

show that the average cost rises to number of firms in the industry when all firms charge same price.
$AC = \frac{n.F}{s} + c$

(c) Prove that price charged by a particular firm declines with the number of firms
$P = c + \frac{1}{b \cdot n}$

(d) Determine the number of firms and price in equilibrium. Explain entry exit behavior and prices when number of firms are below or above this equilibrium point.

(e) Collusive and strategic behaviors may limit above conclusions. Discuss.

(f) Apply above model to explain international trade and its impact on prices and number of firms in a particular industry.

(g) Use this model to explain interindustry and intra-industry trade.

(h) Use monopolistic competition model to analyse consequences of dumping practices in international trade.

Q3. Nature left 1000 pounds on the table to be split between two players. What is the optimal solution from a symmetric bargaining game if the threat point is given by d(0,0)?

Q4. Use of Game Theory for Analysis of Macroeconomic Policy

Consider an inflation-unemployment strategic policy game between the government and the private sector. Both sectors can play high (H) or low (L) inflation rate strategy. The H and L for private sector denotes their expectation of high or low inflation depending on their perception about the actual policy to be taken by the government. Government sector's H or L strategy refer to the choice of actual rate of inflation by the government. The first element in the pay-off matrix is the gain for the government (row player) and the second element refers to the gain for the private sector (column player).

Pay-Off Matrix for Inflation-unemployment Policy Game

		Private Sector	
		H	L
Public Sector	H	$4, 4$	$6, 3$
	L	$3, 6$	$5, 5$

(246)

a. Represent this policy game using a set of expectation augmented Phillips curves. An equation for such a curve can be written as where and are actual and the expected rates of inflation, and are the actual and the natural rate of unemployment respectively.

b. What is the pay-off for the private and the government sectors in the Nash equilibrium? Show that the non-cooperative Nash equilibrium is Pareto inferior to the cooperative solution in this game. Argue why policy of cooperation is not credible.

c. Write this game in an extensive form assuming that the government sector makes its decision first on whether to choose high or low rate of actual inflation and the private sector then selects its strategic move looking at the actions of the government. Solve this game using the backward induction technique?

d. What would be the solution of this game if it is played infinite number of times? Show how the deviation from the co-operative solution either by the private or the government sector player would be punished by another sector player. What would be the discount rate consistent with equilibrium of the game?

Q5. Consider a two person zero sum (TPZS) framework. Such game can be given by a matrix such as

		Strong Hand	
		H	L
Strong Leg	H	$-10, 10$	$10, -10$
	L	$10, -10$	$-10, 10$

(247)

a) Explain this TPZS game. Solve it by using minmax = maxmin method.

b) Find a mixed strategy for strong-leg and strong-hand

c) What is the value of the game?

d) Why this game is not realistic in modern world?

Q6. Prove that optimal tax rate is independent of market structure, optimal tax rate is the same whether market is under monopoly or oligopoly.

Profit function of a monoplist with taxes

$$\Pi = PQ - TC - T \tag{248}$$

$$P = a - bQ \tag{249}$$

total cost with marginal cost c and fixed cost f

$$TC = cQ + f \tag{250}$$

Tax revenue

$$T = tQ \tag{251}$$

9.1.2 Problem 12: Bargaining and Cooperative Game

Q1. Find the Nash equilibrium in the prisoner's dilemma game given below.

Table 24: Prisonar's Dilemma Game

		Player A	
		Confess	Don't Confess
Player B	Confess	$(-7,-7)$	$(-1,-10)$
	Don't Confess	$(-10,-1)$	$(-2,-2)$

[Negative sign indicates bad payoff; -10 is worse than -7].
What would have been cooperative and the Pareto optimal solution?

Q2. One common example for a bargaining game is splitting a pie between two individuals, i and j. The total amount to be divided is 1. Their shares in this pie are given by θ_i and θ_j respectively and they should not claim more than what is on the table, i.e. $\theta_i + \theta_j \leq 1$. This implies a meaningful solution of the game requires $\theta_i \geq 0$ and $\theta_j \geq 0$. If the sum of claims is more than what is on the table each gets zero i.e. when $\theta_i + \theta_j > 1$ then $\theta_i = 0$ and $\theta_j = 0$.

Thus the Nash bargaining problem is given by

$$\max U = (\theta_i - 0)(\theta_j - 0) \tag{252}$$

subject to

$$\theta_i + \theta_j = 1 \tag{253}$$

Formulate the constrained optimisation of this problem. Find the optimal values of θ_i and θ_j that satisfy the Nash equilibrium.

Q3. In Spence's model of signalling, type 1 workers are less productive than type 2 workers. Workers signal their productivity type by choosing years of education to maximise their utility. As given below the utility of a worker is positively related to the wage rate (w) and negatively to the effort for education (e) but it is less costly for more productive workers to get education.

$$u_t(w_t, e) = 42\sqrt{w_t} - k_t e^{1.5} \text{ with } k_1 = 3; \;\; k_2 = 1 \;\; w_1 = e; \; w_2 = 2e \tag{254}$$

Given the values of k_t and the above utility function find the optimal choice of e for each type of worker.

Q4. Consider a game in which player B has top and bottom strategies and player A has left and right strategies as following.

Probability of playing Top by B is p and playing Bottom is $(1-p)$ if A plays Left . Similarly probability of B playing Top is p and playing Bottom $(1-p)$ if A plays Right. B likes to be equally well off no matter what A plays.

Find the optimal probability p of playing Top by player B solving this game by the mixed strategy.

Table 25: Game of Mixed Strategy

		Player A	
		Left	Right
Player B	Top	$(30, -30)$	$(70, -70)$
	Bottom	$(80, -80)$	$(10, -10)$

9.2 Principal Agent Games

Popular Principal Agent Games

Table 26: Principal Agent Games

Principal	Agent	Action
Shareholders	CEO	Profit maximisation
Landlord	Tenants	work effort
People	Government	Political power
Manager	Workers	Work effort
Central Banks	Banks	Quality of credit
Patient	Doctor	Intervention
Owner	Renter	Maintenance
Insurance company	Policy holder	Careful behavior

10 L9: General Equilibrium and Welfare Analysis

What is general equilibrium?

- Households and firm optimise subject to their constraints
  - Utility maximisation by households and profit maximisation by firms
- System of prices when all markets clear simultaneously (all goods and factor markets)

$$D\left(p_1 \;\; p_2 \quad p_3, ...p_n\right) = S\left(p_1 \;\; p_2 \quad p_3, ...p_n\right) \tag{255}$$

  Excess demand is zero in equilibrium.

- Income of agents equals their expenditure
- Imports equals exports in an open economy model
- Saving equals investment in a dynamic economy model
- Public spending accounts are balanced in model with public sector
- General equilibrium is obtained by the price system when economy is in perfect harmony.
- Consider one of the easiest possible example of a general equilibrium model with production

- Equilibrium is a point of rest, where the opposing forces remain in balance.
- Theoretically there has been much work, since the time of Adam Smith and Walras to Arrow-Debreau-Hahn-McKinzie for finding whether it exists, or is unique or is stable along with analysis of Pareto efficiency for a centralised or decentralised economy.
- In abstract level existence of equilibrium or Walras' law is proved using a **unit simplex** and **Brouwer's fixed point theorem** in which the uniqueness is guaranteed by the choice of preferences and technology and trade functions that fulfil **continuity, concavity or convexity or twice differentiability properties**.
- In applied policy work, numerical methods are adopted to find the solutions of these models as the explicit analytical solutions are possible only for very small scale models that hardly represent highly complicated mechanism in a modern economy.

Price system, excess demands and equilibrium General equilibrium in an economy is given by a system of relative prices that clear all goods and factor markets. It is often stated in terms of vectors of prices, demand and supply and excess demand functions for inputs and outputs.

- Given the vector of prices, $p = (p_{1,}\, p_{1,}, ..., p_{j,}...p_n)$ demand for commodities are expressed in terms of the price vector $X_j^d = X_j^d\,(p) = X_j^d\,(p_{1,}\, p_{1,}, ..., p_{j,}...p_n)$ and
- supply functions defined similarly $X_j^s = X_j^s\,(p) = X_j^s\,(p_{1,}\, p_{1,}, ..., p_{j,}...p_n)$ and
- the excess demand functions $E\,(p) = X_j^d\,(p) - X_j^s\,(p)$ reflect the gap between demand and supply for each commodity for $j = 1, 2, \ldots\ldots.n$. Economy has n excess demand functions.
- The general equilibrium is a price vector, p^*, such that $p^* \geqslant 0$, when $E\,(p^*) \geq 0$;if $E\,(p^*) < 0$ then $p^* = 0$

Properties of excess demand function Excess Demand functions for general equilibrium analysis

Properties of excess demand function

1. The excess demand functions are single valued continuous function.
2. bounded from below $E\,(p) \geq b$ for all p and
3. it is homogenous of degree zero in all prices $E\,(\alpha p) = p$ for all α.
4. only relative price matter and satisfies the Walras' law; $p.E\,(p) = \sum_{i=1}^{n} p_i.E_i\,(p) = 0$ for all $p \geq 0$

- If the excess demand functions satisfy above properties then, the existence of the general equilibrium is guaranteed by fixed point theorems; it is unique and stable as well.

Fixed Point Theorems: Existence of general equilibrium

- The fixed equilibrium point is found by continuous transformation of the nonempty convex set onto itself $p^* \longrightarrow E(p^*) \longrightarrow \ p^*$. Given the properties of demand and supply functions equilibrium exists and is stable and unique.

Fixed point and contraction mapping

Let a set S be in $\mathbb{R}^n$ and B be the set of all bounded functions from S into $\mathbb{R}^m$.Contraction mapping is $T : B \rightarrow B$. There exist a unique function φ^* in B such that $\varphi^* = T(\varphi^*)$

Brouwer's fixed point theorems

For K a non-empty compact (closed and bounded) convex set in $\mathbb{R}^n$, let f be continuos mapping of K into itself. Then f has a fixed point $f(x^*) = x^*$.

A continuous function from a compact convex set into itself has a fixed point.

Brouwer fixed point theorem on existence of general equilibrium Graphical Illustration of Brouwer fixed point theorem

10.0.1 Existence, uniqueness and stability of general equilibrium

Existence of general equilibrium

- Prices can be normalised to make their sum equal to one using the homogeneity assumption as:

$$S = \left\{ p / \sum_{i=1}^{n} p_i = 1, p \geq 0 \right\} \tag{256}$$

- Consider a set of the excess demand functions evaluated at p. Update or adjust this price according to following rules for each of j commodities:

$$\left\{ \begin{array}{c} \overline{p}_j = p_j \quad \text{if} \ \ E(p_j) = 0 \\ \overline{p}_j = p_j + \Delta \quad \text{if} \ \ E(p_j) > 0 \\ \overline{p}_j = p_j - \Delta \quad \text{if} \ \ E(p_j) < 0 \end{array} \right\} \ \text{for } j = 1, 2, ..., N \tag{257}$$

Existence of general equilibrium

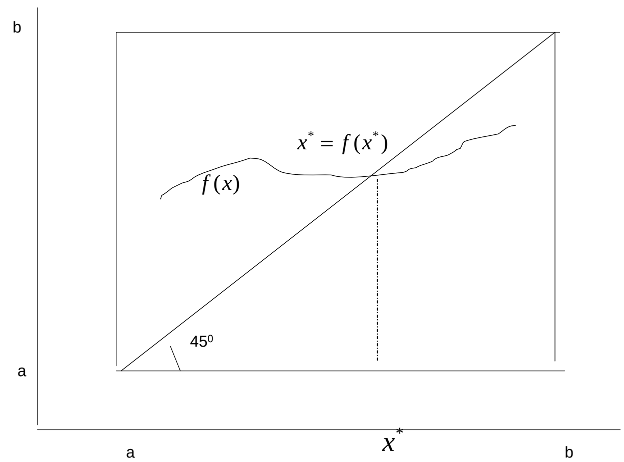
- Here Δ represents a very small positive constant. Following above rule in each iteration find new prices as:

$$p \longrightarrow E(p_j) \Longrightarrow \overline{p} \tag{258}$$

$\overline{p}$ remains unchanged if excess demand is zero, $E(p) = 0$; $\overline{p}$ rises if, $E(p) > 0$ and $\overline{p}$ falls if $E(p) < 0$ and if $E(p) < 0$ then $p = 0$.

- The fixed equilibrium point is found by continuous transformation of the nonempty convex set onto itself

$$p^* \longrightarrow E(p^*) \Longrightarrow p^* \tag{259}$$



10.1 Two fundamental theorems of welfare economics

Definition: An allocation-price pair (p, x) is a Walrasian equilibrium if (1) allocation is feasible (2) each agent is making an optimal choice permited by the budget set.

$\sum_i x_i = \sum_i \omega_i$ and if x_i' is preferred by agent to x_i then $px_i' > p\omega_i$.

First theorem of welfare reconomics: If (x, p) is a Walrasian equilibiurm, then x is Pareto efficient.

Proof by contradiction: If x_i' is feasible and preferred by the economic agents then $px_i' > p\omega_i$. Then summing over all individuals

$$p\sum_i^N \omega_i = p\sum_i^N x_i' > \sum_i^N p\omega_i \tag{260}$$

This is a contradiction. x_i' is not feasible.

Second theorem of welfare reconomics (revealed preference proof): If x_i^* is pareto efficient allocation the (p, x^*) is competitive Walrasian equilibrium.

Proof: Since x_i^* is in consumers budget set, it must be true that $x_i' \succeq_i x_i^*$. As x_i^* is pareto efficient $x_i^* \sim_i x_i'$ Thus x_i' is as optimal as x_i^* Hence (x', p) is a Walrasian equilibrium.

11 L10: General Equilibrium Theory

11.1 Two Good Pure Exchange General Equilibrium Model

- Households, h = A B.
- Two goods X_1 and X_2
- Endowments of two goods ω_1^A ω_2^A ω_1^B ω_2^B
- Objective of each is to maximise life time utility subject to budget constraints w r t X_1 and X_2

- Equilibrium relative price determines the optimal allocation; It is Pareto Optimal.

Main Features of Applied General Equilibrium Model
Three conditions
Demand = supply ; n markets, n-1 relative prices
Income = expenditure
Firms maximise profit: zero economic profit in competitive markets
Relative Prices
references and technology parameters determine relative prices in equilibrium.
Relative prices are determined by forces of demand and supply.
Numeraire or anchor price; normalised to 1.
Preferences and technology parameters determine relative prices in equilibrium.
Relative prices are determined by forces of demand and supply.
Numeraire or anchor price; normalised to 1.
Markets allocations depend on relative prices.
Demand for a commodity depends on preferences and income.
Income of a household is determined by her endowment and price of that endowment.
Exchange or trade of goods is mutually beneficial.
Each consumer/ producer optimises in equilibrium.
Problem of representative households
For household A

$$Max \quad U(X_1^A, X_2^A) = \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{1-\alpha_A} \tag{261}$$

Subject to the budget constraint:

$$P_1 X_1^A + P_2 X_2^A = P_1 \omega_1^A + P_2 \omega_2^A = I^A \tag{262}$$

For household B

$$Max \quad U(X_1^B, X_2^B) = \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{1-\alpha_B} \tag{263}$$

Subject to the budget constraint:

$$P_1 X_1^B + P_2 X_2^B = P_1 \omega_1^B + P_2 \omega_2^B = I^B \tag{264}$$

Intertemporal budget constraint
Lagrangian for constrained optimisation for Household A :

$$\mathcal{L}^A = \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{1-\alpha_A} + \lambda \left(P_1 \omega_1^A + P_2 \omega_2^A - P_1 X_1^A - P_2 X_2^A\right) \tag{265}$$

Lagrangian for constrained optimisation for Household V :

$$\mathcal{L}^B = \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{1-\alpha_B} + \lambda \left(P_1 \omega_1^B + P_2 \omega_2^B - P_1 X_1^B - P_2 X_2^B\right) \tag{266}$$

First order conditions for optimisation
For household A and B

$$\frac{\partial \mathcal{L}^A}{\partial X_1^A} = \alpha_A \left(X_1^A\right)^{\alpha_A - 1} \left(X_2^A\right)^{1-\alpha_A} - \lambda P_1 = 0 \tag{267}$$

$$\frac{\partial \mathcal{L}^A}{\partial X_2^A} = (1-\alpha_A) \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{-\alpha_A} - \lambda P_2 = 0 \tag{268}$$

$$\frac{\partial \mathcal{L}^A}{\partial \lambda} = P_1\omega_1^A + P_2\omega_2^A - P_1X_1^A - P_2X_2^A = 0 \tag{269}$$

$$\frac{\partial \mathcal{L}^B}{\partial X_1^B} = \alpha_B \left(X_1^B\right)^{\alpha_B - 1} \left(X_2^B\right)^{1-\alpha_B} - \lambda P_1 = 0 \tag{270}$$

$$\frac{\partial \mathcal{L}^B}{\partial X_2^B} = (1-\alpha_B) \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{-\alpha_B} - \lambda P_2 = 0 \tag{271}$$

$$\frac{\partial \mathcal{L}^B}{\partial \lambda} = P_1\omega_1^B + P_2\omega_2^B - P_1X_1^B - P_2X_2^B = 0 \tag{272}$$

Demand and market clearing conditions
For household A

$$X_1^A = \frac{\alpha_A I^A}{P_1}; \quad X_2^A = \frac{(1-\alpha_A) I^A}{P_2} \tag{273}$$

For household B

$$X_1^B = \frac{\alpha_B I^B}{P_1}; \quad X_2^B = \frac{(1-\alpha_B) I^B}{P_2} \tag{274}$$

Market clears each period

$$X_1^A + X_1^B = \omega_1^A + \omega_1^B \tag{275}$$

$$X_2^A + X_2^B = \omega_2^A + \omega_2^B \tag{276}$$

Market clearing Prices
Obtained from the market clearing conditions

$$\frac{\alpha_A I^A}{P_1} + \frac{\alpha_B I^B}{P_1} = \omega_1^A + \omega_1^B \tag{277}$$

$$\frac{(1-\alpha_A) I^A}{P_2} + \frac{(1-\alpha_B) I^B}{P_2} = \omega_2^A + \omega_2^B \tag{278}$$

Walrasian numeraire: $P_1 = 1$. with this specification

$$I^A = \omega_1^A \qquad I^B = P_2\omega_2^B \tag{279}$$

Equilibrium Relative Price and Proof of Walras' Law

Table 27: Parameters in Pure Exchange Model

	Household A	Household B
Endowments	$\{\omega_1^A, \omega_2^A\} = \{100, 0\}$	$\{\omega_1^B, \omega_2^B\} = \{0, 200\}$
Preference for X_1 (α)	0.4	0.6
Preference for X_2 $(1-\alpha)$	0.6	0.4

$$\begin{aligned} \frac{\alpha_A I^A}{P_1} + \frac{\alpha_B I^B}{P_1} &= \alpha_A I^A + \alpha_B I^B \\ &= \alpha_A \omega_1^A + \alpha_B P_2 \omega_2^B = \omega_1^A \\ 0.4\,(100) + 0.6\,(200)\,P_2 &= 100; \quad P_2 = 0.5 \end{aligned}$$

$$I^A = \omega_1^A = 100 \qquad I^B = P_2 \omega_2^B = 0.5\,(200) = 100 \tag{280}$$

Table 28: Parameters in Pure Exchange Model

	Household A	Household B
Endowments	$\{\omega_1^A, \omega_2^A\} = \{100, 0\}$	$\{\omega_1^B, \omega_2^B\} = \{0, 200\}$
Prices	1	0.5
Demand for X_1 (α)	40	60
Demand for for X_2 $(1-\alpha)$	120	80
Utility	77.3	67.3
Income	100	100

Theoretical observations

- Relative prices of goods, income and consumption change when preferences (alpha, beta) change.
- Change in the relative income affects the level of utility and welfare of households
- Household A can make household B worse off by increasing the demand of good 1 that he owns (or supplying less to the market).
- Household B can increase his relative income and reduce the relative price of good 1 by increasing the demand for good 2 (reducing its supply).
- Relative prices and allocations depend on preferences and endowments.

Homework
Do sensitivity analysis (solve model for various parametric specifications

1) by changing endowments $\{\omega_1^A, \omega_2^A\} = \{100, 50\}$ and $\{\omega_1^B, \omega_2^B\} = \{200, 150\}$

2) by changing preferences $\{\alpha_A,\, \alpha_B\} = \{0.50, 0.50\}$; $\{\alpha_A,\, \alpha_B\} = \{0.750, 0.30\}$

3) introduce VAT of 20 percent in commodity 1. Assume that revenue collected is spent entirely by the government and does not add to any utility for the household.

11.2 Two period general equilibrium model

- Households, A and B.
- They live today and tomorrow.
- They are endowed with goods in both periods.
- Objective of each is to maximise life time utility subject to budget constraints in period 1 and 2.
- Financial market allows lending and borrowing.
- Equilibrium interest rate is price that determines intertemporal allocations.

Problem of representative households

$$Maximise \quad U(C_1^i, C_2^i) = \ln C_1^i + \beta \ln C_2^i \qquad \text{i = A, B} \tag{281}$$

Subject to
First period budget constraint:

$$C_1^i + b^i = \omega_1^i \tag{282}$$

Second period budget constraint:

$$C_2^i = b^i (1 + r) + \omega_2^i \tag{283}$$

here C_1^i and C_2^i are consumption in period 1 and 2 by household i = A, B.
ω_1^i and ω_2^i are endowments in period 1 and 2 of household i = A, B; r is the interest rate and β is the discount factor.
Intertemporal budget constraint
From (283)

$$b^i = \frac{C_2^i}{1+r} - \frac{\omega_2^i}{1+r} \tag{284}$$

substituting (284) in (282) gives the intertemporal budget constraint

$$C_1^i + \frac{C_2^i}{1+r} = \omega_1^i + \frac{\omega_2^i}{1+r} \tag{285}$$

Lagrangian for constrained optimisation is

$$\mathcal{L} = \ln C_1^i + \beta \ln C_2^i + \lambda \left(\omega_1^i + \frac{\omega_2^i}{1+r} - C_1^i - \frac{C_2^i}{1+r} \right) \tag{286}$$

First order conditions for optimisation

$$\frac{\partial \mathcal{L}}{\partial C_1^i} = \frac{1}{C_1^i} - \lambda = 0 \tag{287}$$

$$\frac{\partial \mathcal{L}}{\partial C_2^i} = \frac{\beta}{C_2^i} - \frac{\lambda}{1+r} = 0 \tag{288}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \omega_1^i + \frac{\omega_2^i}{1+r} - C_1^i - \frac{C_2^i}{1+r} = 0 \tag{289}$$

Now dividing (**??**) by (**??**) gives the marginal rate of substitution between current and future consumption

$$C_2^i = \beta\,(1+r)\,C_1^i \tag{290}$$

Demand and market clearing conditions

From (397)

$C_1^i + \frac{C_2^i}{1+r} = \omega_1^i + \frac{\omega_2^i}{1+r}$ Putting (290) in it gives $C_1^i + \frac{\beta(1+r)C_1^i}{1+r} = (1+\beta)\,C_1^i = \omega_1^i + \frac{\omega_2^i}{1+r}$ The demand for consumption in period 1 is

$$C_1^i = \frac{1}{(1+\beta)}\left(\omega_1^i + \frac{\omega_2^i}{1+r}\right) \tag{291}$$

Similarly the demand for consumption in period 2 is obtained by putting (291) in (290)

$$C_2^i = \beta\,(1+r)\,C_1^i = \frac{\beta\,(1+r)}{(1+\beta)}\left(\omega_1^i + \frac{\omega_2^i}{1+r}\right) \tag{292}$$

Market clears each period

$$C_1^A + C_1^B = \omega_1^A + \omega_1^B \tag{293}$$

$$C_2^A + C_2^B = \omega_2^A + \omega_2^B \tag{294}$$

Market clearing interest rate

Put (291) and (292) in (293)

$$C_1^A + C_1^B = \frac{1}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right) + \frac{1}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right) \tag{295}$$

$$= \omega_1^A + \omega_1^B \tag{296}$$

$$\frac{1}{(1+\beta)}\left[\omega_1^A + \omega_1^B + \frac{\omega_2^A}{1+r} + \frac{\omega_2^B}{1+r}\right] = \omega_1^A + \omega_1^B \tag{297}$$

$$\frac{1}{1+r}\left[\omega_2^A + \omega_2^B\right] = (1+\beta)\left(\omega_1^A + \omega_1^B\right) - \left(\omega_1^A + \omega_1^B\right) = \beta\left(\omega_1^A + \omega_1^B\right) \tag{298}$$

$$1+r = \frac{1}{\beta}\left[\frac{\omega_2^A + \omega_2^B}{\omega_1^A + \omega_1^B}\right]; \qquad r = \frac{1}{\beta}\left[\frac{\omega_2^A + \omega_2^B}{\omega_1^A + \omega_1^B}\right] - 1 \tag{299}$$

is the value of income in terms of utils

Proof of Walras' Law

$$\lambda = \frac{1}{C_1^i} = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right)} = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right)} \tag{300}$$

By Walras law when one market clears other market automatically clears. Check this by putting (**??**) and (292) in (294)

$$C_2^A + C_2^B = \frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right) + \frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right) \tag{301}$$

$$= \omega_2^A + \omega_2^B \tag{302}$$

$$\frac{\beta(1+r)}{(1+\beta)}\left[\omega_1^A + \omega_1^B + \frac{\omega_2^A}{1+r} + \frac{\omega_2^B}{1+r}\right] = \omega_2^A + \omega_2^B \tag{303}$$

Proof of Walras' Law

$$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^A + \omega_1^B\right) = \left(\omega_2^A + \omega_2^B\right) - \frac{\beta}{(1+\beta)}\left[\omega_2^A + \omega_2^B\right] \tag{304}$$

$$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^A + \omega_1^B\right) = \left[1 - \frac{\beta}{(1+\beta)}\right]\left(\omega_2^A + \omega_2^B\right) = \frac{1}{(1+\beta)}\left(\omega_2^A + \omega_2^B\right) \tag{305}$$

$$(1+r) = \frac{1}{\beta}\left[\frac{\omega_2^A + \omega_2^B}{\omega_1^A + \omega_1^B}\right]; \quad r = \frac{1}{\beta}\left[\frac{\omega_2^A + \omega_2^B}{\omega_1^A + \omega_1^B}\right] - 1 \tag{306}$$

QED

Summary of results

Table 29: Summary of two period general equilibrium model

	Individual A	Individual B
Endowments	$\left\{\omega_1^A, \omega_2^A\right\}$	$\left\{\omega_1^B, \omega_2^B\right\}$
Equilibrium interest rate	$r = \frac{1}{\beta}\left[\frac{\omega_2^A+\omega_2^B}{\omega_1^A+\omega_1^B}\right] - 1$	
Life time income	$\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right)$	$\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right)$
Consumption in period 1	$\frac{1}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right)$	$\frac{1}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right)$
Consumption in period 2	$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right)$	$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right)$
Saving/borrowing period 1	$S_1^A = \omega_1^A - C_1^A$	$S_1^\beta = \omega_1^\beta - C_1^\beta$
Saving/borrowing period 2	$S_2^A = \omega_2^A - C_2^A$	$S_2^\beta = \omega_2^\beta - C_2^\beta$
Life time utility	$U(C_1^A, C_2^A) = \ln C_1^A + \beta \ln C_2^A$	$U(C_1^B, C_2^B) = \ln C_1^B + \beta \ln C_2^B$
Shadow price	$\lambda = \frac{1}{C_1^i} = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^A+\frac{\omega_2^A}{1+r}\right)}$	$\lambda = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^B+\frac{\omega_2^B}{1+r}\right)}$

Summary of results

Table 30: Parameters

	Individual A	Individual B
Endowments	$\{\omega_1^A, \omega_2^A\} = \{50, 100\}$	$\{\omega_1^B, \omega_2^B\} = \{150, 200\}$
Discount rate	$\beta = 0.9$	$\beta = 0.9$

Table 31: Solution

	Individual A	Individual B
Equilibrium interest rate	$r = \frac{1}{\beta}\left[\frac{\omega_2^A+\omega_2^B}{\omega_1^A+\omega_1^B}\right] - 1 = 0.667$	
Life time income	$\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right) = 110$	$\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right) = 270$
Consumption in period 1	$\frac{1}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right) = 57.895$	$\frac{1}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right) = 142.105$
Consumption in period 2	$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right) = 86.842$	$\frac{\beta(1+r)}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right) = 213.158$
Saving/borrowing period 1	$S_1^A = \omega_1^A - C_1^A = -7.895$	$S_1^\beta = \omega_1^\beta - C_1^\beta = 7.895$
Saving/borrowing period 2	$S_2^A = \omega_2^A - C_2^A = 13.158$	$S_2^\beta = \omega_2^\beta - C_2^\beta = -13.158$
Life time utility	$U(C_1^A, C_2^A) = \ln C_1^A + \beta \ln C_2^A = 8.076$	$U(C_1^B, C_2^B) = \ln C_1^B + \beta \ln C_2^B = 9.782$
Shadow price	$\lambda = \frac{1}{C_1^i} = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^A + \frac{\omega_2^A}{1+r}\right)} = 0.0017$	$\lambda = \frac{1}{\frac{1}{(1+\beta)}\left(\omega_1^B + \frac{\omega_2^B}{1+r}\right)} = 0.007$

Homework

Do this exercise when

Table 32: Parameters

	Individual A	Individual B
Endowments	$\{\omega_1^A, \omega_2^A\} = \{100, 50\}$	$\{\omega_1^B, \omega_2^B\} = \{200, 150\}$
Discount rate	$\beta = 0.9$	$\beta = 0.9$

12 L11: General equilibrium with production

12.1 Main Features of an Applied General Equilibrium Model

Remarks about the history of General Equilibrium Theory

"L. Walras first formulated the state of the economic system at any point of time as the solution of the system of simultaneous equations representing the demand for goods by consumers, the supply of goods by producers and equilibrium condition that supply equal demand on every market. It was assumed that each consumer acts so as to maximize his utility, each producer acts so as to maximize his profit, and perfect competition prevails, in the sense that each producer and consumer regard the price paid and received as independent of his own choices. Walras did not, however, give any conclusive arguments to show that equations, as given, have a solution." **Arrow and Debreu (1954)**

"If the price system is such as to make these demands and supplies equal, we have a position of equilibrium. If not, some prices at least will be bid up or down." Hicks (1939) **(Value and Capital p. 59)**

see https://www.ifs.org.uk/green-budget

"The Walrasian model proves an ideal framework for appraising the effects of policy changes on resource allocation and for assessing who gains and loses, policy impacts not well covered by empirical macro models" **Shoven and Whalley (1984)**

Main Features of an Applied General Equilibrium Model

- Three conditions

Demand = supply ; n markets, n-1 relative prices
Income = expenditure
Firms maximise profit: zero economic profit in competitive markets

- Relative Prices (see price system.xls)

Preferences and technology parameters determine relative prices in equilibrium.
Relative prices are determined by forces of demand and supply.
Numeraire or anchor price; normalised to 1.

- Markets allocations depend on relative prices.

Demand for a commodity depends on relative prices.
Income of a household is determined by her endowment and price of that endowment.

- Exchange or trade of goods is mutually beneficial.

Each consumer/ producer optimises in equilibrium.

12.2 Simplest General Equilibrium Production

Consider a general equilibrium model with taxes in which a representative household maximises utility subject to its budget constraint and the firm maximises profit subject to a technology constraint as given below.

$$\max \quad U = C \cdot L \tag{307}$$

Subject to

$$p\,(1+t)\,C + wL = w\overline{L} \tag{308}$$

The firm's profit maximisation problem is:

$$\max \quad \pi = p.Y - w.LS \tag{309}$$

Subject to

$$Y = LS \tag{310}$$

You may select $p = 1$ as a numeraire.

Find expressions for the wage rate, consumption, output, labour supply and demand for labour consistent with the general equilibrium.

12.2.1 Analysis and derivation for the General equilibrium with production

Lagrangian for household optimisation:

$$L(C, L, \lambda) = C \cdot L + \lambda \left[p(1+t)C + wL - w\overline{L} \right] \tag{311}$$

Household optimisation: first order conditions:

$$\frac{L(C, L, \lambda)}{\partial C} = L + \lambda p(1+t) = 0 \tag{312}$$

$$\frac{L(C, L, \lambda)}{\partial L} = C + \lambda w = 0 \tag{313}$$

$$\frac{L(C, L, \lambda)}{\partial \lambda} = p(1+t)C + wL - w\overline{L} = 0 \tag{314}$$

Above three FOC equations (312) - (314) can be solved for three variables:

$$MRS_{CL} = \frac{\frac{L(C,L,\lambda)}{\partial C}}{\frac{L(C,L,\lambda)}{\partial L}} \Longrightarrow \frac{L}{C} = \frac{p(1+t)}{w} \tag{315}$$

$$L = \frac{p(1+t)}{w} C \tag{316}$$

Putting (700) into (314)

$$p(1+t)c + wL - w\overline{L} = p(1+t)C + w\frac{p(1+t)}{w}C - w\overline{L} = 0 \tag{317}$$

$$C = \frac{1}{2}\frac{w\overline{L}}{p(1+t)} \tag{318}$$

$$L = \frac{p(1+t)}{w}C = \frac{p(1+t)}{w}\frac{1}{2}\frac{w\overline{L}}{p(1+t)} = \frac{1}{2}\overline{L} \tag{319}$$

- Demand for goods is low with higher taxes and prices, high with higher wage rate and labour endowment; high with the higher share of spending on goods and services.
- Given these preferences the demand for leisure is half of the labour endowment.

Supply Side of the General Equilibrium Model:
Firms' profit maximisation problem

$$\max \quad \pi = p.Y - w.LS \tag{320}$$

Subject to

$$Y = LS = \frac{1}{2}\overline{L} \tag{321}$$

Consumers pay tax not the producers. In no tax case, given this production technology and demand side derivations labour demand equals

Market clearing

Labour market

$$L + LS = \overline{L} \tag{322}$$

Goods market

$$C = Y \tag{323}$$

Let total labour endowment $\overline{L}$ be 200. Then labour supply is

$$LS = \overline{L} - L = \overline{L} - \frac{1}{2}\overline{L} = \frac{1}{2}\overline{L} = 100 \Longrightarrow Y = 100 = C;\ p = w = 1 \tag{324}$$

$$U = 100 \cdot 100 = 10000 \tag{325}$$

Progressive tax system for fairness Mirrleeian idea on progressive taxation to achieve equity

R amount of revenue is to be raised:

$$R = t_1 B\left(y_1\right) + t_2 B\left(y_2\right)$$

t_1 is tax rate in for low income $B\left(y_1\right)$ benefit from low income; t_2 is tax rate for high income, $B\left(y_1\right)$ benefit from high income;

$$dR = t_1 MB_1 dy_1 + t_2 MB_2 dy_2 = 0$$

Let increase in income of low income person be equal to fall in income of high income ($dy_1 = -dy_2$)

$$t_1 MB_1 dy_1 = t_2 MB_2 dy_2 = 0; \Longrightarrow \frac{t_1}{t_2} = \frac{MB_2}{MB_1} < 1 \Longrightarrow t_1 < t_2$$

Tax system should be progressive to achieve vertical and horizontal equity.

budget: http://www.hm-treasury.gov.uk/; Green Budget: http://www.ifs.org.uk/; http://www.ukpublicspending.co.uk/index.php

Common beliefs on efficient tax and spending policies in UK

" The core of our proposal is for a progressive, neutral tax system; that minimises economic distortions and is a right tool for achieving distributional objectives"

"There are taxes that are fairer, less damaging, and simpler than those that we have now. To implement them will take a government, ..., willing to put long term strategy ahead of short term tactics".

" .. the cost of not doing so are very large. .. Economic welfare could be improved by many billions of pounds if the taxation of income, expenditure, profits, environmental externalities and saving were reformed..."

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12.2.2 Problem 10: General Equilibrium Model: Pure Exchange

Q1. Consider a pure exchange general equilibrium model for an economy with individuals A and B with the set of parameters in the table given below.

Lagrangian for constrained optimisation for Household A :

$$\mathcal{L}^A = \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{1-\alpha_A} + \lambda \left(P_1\omega_1^A + P_2\omega_2^A - P_1X_1^A - P_2X_2^A\right) \tag{326}$$

Lagrangian for constrained optimisation for Household B :

$$\mathcal{L}^B = \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{1-\alpha_B} + \lambda \left(P_1\omega_1^B + P_2\omega_2^B - P_1X_1^B - P_2X_2^B\right) \tag{327}$$

Table 33: Parameters in Pure Exchange Model

	Household A	Household B
Endowments	$\left\{\omega_1^A, \omega_2^A\right\} = \{100, 0\}$	$\left\{\omega_1^B, \omega_2^B\right\} = \{0, 200\}$
Preference for X_1 (α)	0.6	0.4
Preference for X_2 $(1-\alpha)$	0.4	0.6

You may assume Walrasian numeraire: $P_1 = 1$ with this specification, and implied incomes for A and B are:

$$I^A = \omega_1^A \qquad I^B = P_2\omega_2^B \tag{328}$$

Derive the demand functions for both A and B individuals and find the relative price that clears the markets for both X_1 and X_2 .

Q2. Consider a pure exchange economy in which the utility of households A and B are given by $U^A = \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{1-\alpha_A}$ and $U^B = \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{1-\alpha_B}$. Here U^A and U^B are levels of utilities of household A and B respectively, and α_A and α_B denote preferences of these households for the consumption of good 1. Similarly X_1^A and X_2^A, and X_1^B and X_2^B are consumptions of good 1 and good 2 by household A and B respectively. Only household A has an endowment of good 1 and it is $\omega_1^A = 100$; and only household B has an endowment of good 2 and it is $\omega_2^B = 200$; is α_A 0.4 and α_B is 0.6.

a. Represent the initial endowment position of goods A and B of these two households using the Edgeworth box diagram with a number of indifference curves for each.

b. Formulate the Lagrangian function for constrained optimisation for A and B.

c. Provide the first order conditions necessary for optimisations by both households.

d. Derive demand functions for both products by both households.

e. State the market clearing conditions for both goods.

f. Use good 1 as a numeraire. Find the relative price of good 2 that clears both markets and is consistent with maximization of utility (satisfaction) by both households given their budget constraints.

g. Determine the income of each household.

h. Evaluate optimal demands X_1^A and X_2^A, and X_1^B and X_2^B for those endowments and preferences.

i. Check whether your solutions satisfy the market-clearing conditions required for a general equilibrium.

j. What are the levels of utility for A and B at equilibrium?

k. Represent the general equilibrium (optimal quantities, relative prices) in another Edgeworth box diagram.

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12.3 Tutorial 8: Taxes, welfare and general equilibrium

12.3.1 Example 1

An individual gets utility (u) by consuming cashew nuts (x_1) and peanuts (x_2) as given by $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. This person has 200 to spend (m) between these two goods, thus the budget constraint is $m = p_1 x_1 + p_2 x_2$. Prices of cashew nuts and peanuts were $(p_1, p_2) = (2, 2)$ last month, giving the base line demand as $\left(x_1 = \frac{m}{2p_1}, x_2 = \frac{m}{2p_2}\right) = (50, 50)$ and the utility from this consumption was $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = 50$.

Now an increase in VAT on cashew nuts raises its price to 4 but there is no change in the price of peanuts, $(p_1, p_2) = (4, 2)$. Income does not change and stays the same at 200. The new demand for cashew nuts and peanuts implied by their prices are $(x_1, x_2) = (25, 50)$. This person has become worse off because of higher prices due to increase in taxes. **Calculate the Hicksian compensating and equivalent variations of this price change.**

Answers

- Base utility $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, with budget $m = p_1 x_1 + p_2 x_2$ if $m = 100$; $(p_1, p_2) = (1, 1)$ demand $\left(x_1 = \frac{m}{2p_1}, x_2 = \frac{m}{2p_2}\right) = (50, 50)$

$u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = 50$.

- Now there is tax on good 1 and new prices are $(p_1, p_2) = (2, 1)$ income does not change.

new demand $(x_1, x_2) = (25, 50)$.

- How much income need to be compensated to this consumer to maintain at the old level of utility?

$u_0 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \left(\frac{m'}{2p_1}\right)^{\frac{1}{2}} \left(\frac{m'}{2p_2}\right)^{\frac{1}{2}} = 50$. Here $m' = 2\sqrt{2} \times 50$ =141.4
CV =141.4-100=41.4. Compensating variation is positive for a price rise is positive.

How much money should be taken away from the consumer in the original prices to make him/her achieve the utility level after the price change.
$u_0 = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = (25)^{\frac{1}{2}} (50)^{\frac{1}{2}} = 35.35$; $\left(\frac{m'}{2}\right)^{\frac{1}{2}} \left(\frac{m'}{2}\right)^{\frac{1}{2}} = 35.35; \Longrightarrow m'$ =70.7
EV= 70.7-100 = -29.3.
Equivalent variation in negative for a rise in price level is negative.
This consumer would have got 35.35 utility by paying 70.7 if prices were (1,1) as before.

13 L12: Two Country Ricardian Trade Model

Production Possibility Frontier (PPF): Ricardian Trade and General Equilibrium

Two Country Ricardian Trade Model

There are two countries producing two goods (2×2)

Production possibility frontier (PPF) of country 1:

$$Y_{1,1} + Y_{1,2}^2 = 2000 \tag{329}$$

Production possibility frontier (PPF) of country 2

$$Y_{2,1}^2 + Y_{2,2} = 2000 \tag{330}$$

Country 1 has comparative advantage in producing good 1 and country 2 has it in producing good 2.

13.1 Preferences and budget constraints

Preferences:

$$U_1 = X_{1,1}^{\alpha_1} X_{1,2}^{1-\alpha_1} \tag{331}$$

and

$$U_2 = X_{2,1}^{\alpha_2} X_{2,2}^{1-\alpha_2} \tag{332}$$

Let $\alpha_1 = 0.4$ and $\alpha_2 = 0.6$.

Budget constraints

$$I_1 = P_1 X_{1,1} + P_2 X_{1,2} \tag{333}$$

and

$$I_2 = P_1 X_{2,1} + P_2 X_{2,2} \tag{334}$$

13.2 Demand for goods and Market clearing

Demand for goods by countries and equilibrium price

$$X_{1,1} = \frac{\alpha_1 I_1}{P_1}; \quad X_{2,1} = \frac{\alpha_2 I_2}{P_1} \tag{335}$$

For country B

$$X_{1,2} = \frac{(1-\alpha_1) I_1}{P_2}; \quad X_{2,2} = \frac{(1-\alpha_2) I_2}{P_2} \tag{336}$$

Global market clearing:

$$X_{1,1} + X_{2,1} = Y_1 \tag{337}$$

$$X_{1,2} + X_{2,2} = Y_2 \tag{338}$$

where $Y_1 = Y_{1,1} + Y_{2,1}$ and $Y_2 = Y_{1,2} + Y_{2,2}$. Let numeraire $P_1 = 1$

By Walras' law if one market clears another automatically clears (Solving for the market 1):

$$0.4\,(2000) + 0.6\,(2000)\,P_2 = 2000 \Longrightarrow P_2 = 1 \tag{339}$$

13.2.1 Equilibrium Allocations

Income in both countries

Income in both countries:

$$I_1 = P_1 Y_1 = 1.2000 = 2000; \quad I_2 = P_2 Y_2 = 1.2000 = 2000 \tag{340}$$

Equilibrium allocations

$$X_{1,1} = \frac{\alpha_1 I_1}{P_1} = 0.4\,(2000) = 800 \tag{341}$$

$$X_{2,1} = \frac{\alpha_2 I_1}{P_1} = 0.6\,(2000) = 1200 \tag{342}$$

$$X_{1,2} = \frac{(1-\alpha_1)\,I_1}{P_2} = 0.6\,(2000) = 1200 \tag{343}$$

$$X_{2,2} = \frac{(1-\alpha_2)\,I_2}{P_2} = 0.4\,(2000) = 800 \tag{344}$$

13.3 Utilities and welfare

Utilities and welfare in the global eocnomy

GlobaL market clearing:

$$X_{1,1} + X_{2,1} = Y_1; \quad 800 + 1200 = 2000 \tag{345}$$

$$X_{1,2} + X_{2,2} = Y_2; \;\; ; \;\; 1200 + 800 = 2000 \tag{346}$$

Welfare of representative households from trade:

$$U_1 = X_{1,1}^{\alpha_1} X_{1,2}^{1-\alpha_1} = 800^{0.4} 1200^{0.6} = 1020.34 \tag{347}$$

and

$$U_2 = X_{2,1}^{\alpha_2} X_{2,2}^{1-\alpha_2} = 1200^{0.6} 800^{0.4} = 1020.34 \tag{348}$$

Both country have the same utility level because of preference structure and specialisation

Complete Specialization Solution of Ricardian Trade Model 1

- There are two countries indexed by j, producing two goods, manufacturing and services.
- Each of them have an option to be self reliant or to trade on the basis of comparative advantage.

Table 34: Solution of Ricardian Trade Model

	Specialisation	
	country A	country B
Supply of X1	2000	0
Supply of X2	0	2000
Preference for X_1 : (α_i)	0.4	0.6
Preference for X_2 : $(1-\alpha_i)$	0.6	0.4
Price of good X_1	1	1
Price of good X_2	1	1
Demand for X_1	800	1200
Demand for for X_2	1200	800
Utility	1020.34	1020.34
Income	2000	2000

- Under the ISI regimes countries favoured to be self reliant and infant industries were protected by tariffs and non-tariff barriers. After numerous rounds of trade negotiations under GATT/WTO over the years, all countries now have realised that the autarky solutions like this are economically inefficient. In contrast
- trade is mutually beneficial for trading nations and raises welfare in both countries. Aim of this section is to illustrate on these statements analytically and numerically with a small and transparent example.
- For this it is assumed that each country j specialises in commodities that it is more efficient and engages in trade.
- The exchange rate is determined by the relative prices of two commodities in the global market.

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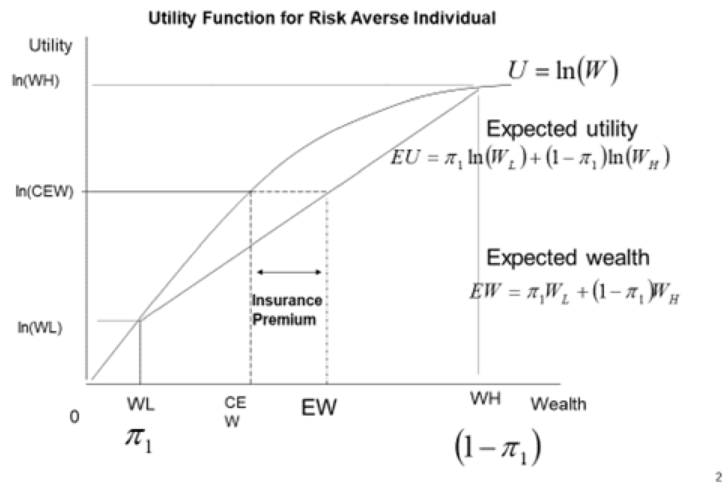
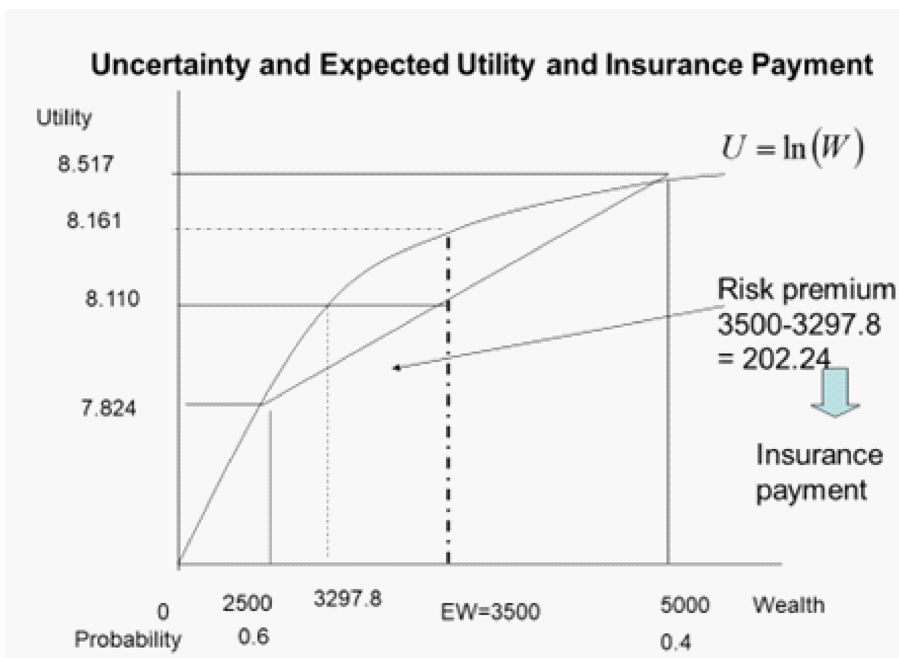
14 L13: Expected utility from expected wealth

Questions

- An individual has a car worth £10000. Probability of accident is 0.01 percent and the car will be useless if it meets any accident. How much insurance should this person pay?
- John has a house worth 200,000. Probability of fire is 0.05 percent. House is worthless after the fire. How much insurance should John pay for the fire insurance?
- Jane has a business worth 1 million. Probability of bankruptcy is 0.02 percent. How much insurance should Jane pay to protect against such bankruptcy?
- Crops are worth 50000. Probability of storm or flood is 0.01 percent. Crop is completely destroyed if a storm or flood occurs. How much insurance is ideal in this business?
- What is the risk adjusted return in a given portfolio of stocks (CAPM)?
- Future is uncertain; two states - high wealth and low wealth.
- Contingent wealth in high state is W_H and in low state is W_L .
- Probability of high wealth π_H and low wealth π_L.
- Utilities from high wealth $u(W_H)$ and low wealth $u(W_L)$.
- Expected wealth $EW = \pi_L W_L + \pi_H W_H$.
- Expected utility $EU = \pi_L u(W_H) + \pi_H u(W_L)$.
- Faced with uncertainty people maximise expected utility (von-Neumann-Morgentern preferences).
- People are ready to pay some amount to insure themselves against possible risks.


Utility Function for Risk Averse Individual
Utility
ln(WH)
ln(CEW)
ln(WL)
$U = \ln(W)$
Expected utility
$EU = \pi_1 \ln(W_L) + (1-\pi_1)\ln(W_H)$
Expected wealth
$EW = \pi_1 W_L + (1-\pi_1) W_H$
Insurance Premium
0
WL
CE W
EW
WH
Wealth
π_1
$(1-\pi_1)$
2


Uncertainty and Expected Utility and Insurance Payment
Utility
8.517
8.161
8.110
7.824
$U = \ln(W)$
Risk premium
3500-3297.8
= 202.24
Insurance
payment
0
2500
3297.8
EW=3500
5000
Wealth
Probability
0.6
0.4

Preferences of risk averse consumer
Utility functions of risk averse individual

$$U(W) = ln(W) \tag{349}$$

$$U(W) = \sqrt{W} = W^{\frac{1}{2}} \tag{350}$$

Expected utility theorem: utilities under uncertainty are additively separable (von-Neumann-Morgenstern Utility)

$$Max \quad EU = \pi_{_H}.u\left(W_H\right) + \pi_{_L}.u\left(W_L\right) \tag{351}$$

Utility from expected wealth

$$U\left(EW\right) = \ln\left(\pi_{_H}W_{_H} + \pi_{_L}W_{_L}\right) = \ln\left(EW\right) \tag{352}$$

Certainty equivalent wealth

$$CEW = \exp\left(EU\right) \tag{353}$$

Maximum insurance that person is ready to pay to cover risk:

$$Insurance = EW - CEW \tag{354}$$

Table 35: Uncertainty of Income and Wealth

	High	Low
Probability	0.75	0.25
Income	5000	1000
Expected Income	3750	250

Expected wealth $EW = \pi_L W_L + \pi_H W_H = 0.75 \times (5000) + 0.25\,(1000) = 4000$
Do people maximize expected wealth? No.
They maximize expected utility.
Maximum insurance against risk and a measure of risk aversion

$$\begin{aligned} EU &= \pi_{_H}.u\left(W_H\right) + \pi_{_L}.u\left(W_L\right) = \pi_{_H}.\ln\left(W_H\right) + \pi_{_L}.\ln\left(W_L\right) && (355) \\ &= 0.75 \times \ln\left(5000\right) + 0.25\ln\left(1000\right) = 6.388 + 1.727 = 8.115 && (356) \end{aligned}$$

Certainty equivalent wealth

$$CEW = \exp\left(EU\right) = \exp(8.115) = 3344.26 \tag{357}$$

Maximum insurance that person is ready to pay to cover risk:

$$Insurance = EW - CEW = 4000 - 3344.26 = 655.74 \tag{358}$$

After paying 655.74 for the insurance company, this person can be sure that no matter high or low state 3344.26 is guaranteed. Can sleep well! Risk pooling is possible. If 100 people ensure like this revenue of insurance company is 65575; only 25 percent people claim ($2344.26 \times 25 = 5860.6$). Profit to the insurer is 65575 - 58606 = 6969.

14.0.1 Measure of risk aversion

Arrow-Pratt (1964) measure of risk aversion

$$r(W) = -\frac{U''(W)}{U'(W)} > 0 \tag{359}$$

Risk lovers

$$r(W) = -\frac{U''(W)}{U'(W)} < 0 \tag{360}$$

Risk neutral

$$r(W) = -\frac{U''(W)}{U'(W)} = 0 \tag{361}$$

Measure of risk aversion for logarithmic preference

$$U(W) = ln(W) \tag{362}$$

Arrow-Pratt (1964) measure of risk aversion

$$r(W) = -\frac{U''(W)}{U'(W)} = -\frac{-\frac{1}{W^2}}{\frac{1}{W}} = \frac{1}{W} > 0 \tag{363}$$

with Cobb-Douglus type preferences

$$U(W) = W^{\frac{1}{2}} \tag{364}$$

$$r(W) = -\frac{U''(W)}{U'(W)} = -\frac{-\frac{1}{2} \times \frac{1}{2}W^{-\frac{1}{2}-1}}{\frac{1}{2}W^{-\frac{1}{2}}} = \frac{1}{2}\frac{1}{W} > 0 \tag{365}$$

Maximum insurance against risk and a measure of risk aversion
Risk lovers

$$U(W) = \exp(aW) \tag{366}$$

$$r(W) = -\frac{U''(W)}{U'(W)} = -\frac{a^2W^2\exp(aW)}{aW\exp(aW)} = -aW < 0 \tag{367}$$

Risk neutral

$$U(W) = aW \tag{368}$$

$$r(W) = -\frac{U''(W)}{U'(W)} = \frac{0}{a} = 0 \tag{369}$$

14.0.2 St Petersberg Paradox (Bernoulli Game)

People are ready to play a small amount for a lottery but do not want to risk a huge amount in it. People care about utility.

How much should one pay to play a game that promises to pay 2^n if the head turns up in the nth trial? Answer 1.39. How?

Expected payoff is infinite

$$E(\pi) = \pi_1 \cdot 2 + \pi_2 \cdot 2^2 + \pi_3 \cdot 2^3 + ... + \pi_n \cdot 2^n = 1 + 1 + 1 + ... + 1 = \infty \tag{370}$$

$$\pi_1 = \frac{1}{2} > \pi_2 = \frac{1}{2^2} > \pi_3 = \frac{1}{2^3} > ...... > \pi_n = \frac{1}{2^n} \tag{371}$$

but the **Expected Utility** is finite here

$$E(u) = \pi_1 \cdot \ln(2) + \pi_2 \cdot \ln\left(2^2\right) + \pi_3 \cdot \ln\left(2^3\right) + ... + \pi_n \cdot \ln(2^n) < \infty \tag{372}$$

$$E(u) = \frac{1}{2} \cdot \ln(2) + \frac{1}{2^2} \cdot \ln\left(2^2\right) + \frac{1}{2^3} \cdot \ln\left(2^3\right) + ... + \frac{1}{2^n} \cdot \ln(2^n) < \infty \tag{373}$$

$$E(u) = \sum_{i=1}^{\infty} \frac{1}{2^i} \cdot i \cdot \ln(2) = \ln(2) \sum_{i=1}^{\infty} \frac{i}{2^i} = \ln(2) \cdot 2 = 1.39 \tag{374}$$

People buy lotteries for small amount but not for more (**Allais Paradox**)

14.0.3 St Petersberg Paradox (Bernoulli Game)

$$E(u) = \frac{1}{2} \cdot \ln(2) + \frac{1}{2^2} \cdot \ln\left(2^2\right) + \frac{1}{2^3} \cdot \ln\left(2^3\right) + ... + \frac{1}{2^n} \cdot \ln(2^n) < \infty \tag{375}$$

$$E(u) = \sum_{i=1}^{\infty} \frac{1}{2^i} \cdot i \cdot \ln(2) = \ln(2) \sum_{i=1}^{\infty} \frac{i}{2^i} = \ln(2) \cdot 2 = 1.39 \tag{376}$$

People are ready to pay small amount to buy lotteries but do not want to risk large sums (**Allais Paradox**)

14.1 Asset Markets

Utility is derived from return and risk (return is measured by mean and risk by standard deviation) as:

$$U(W) = U(\mu_w, \sigma_W) \tag{377}$$

Average return from portfolio of risky and risk free assets (for $0 < x < 1$ $\sum_{s=1}^{S} \pi_s = 1$)

$$r_x = \sum_{s=1}^{S} \left(x m_s + (1-x) r_f\right) \pi_s = \sum_{s=1}^{S} x m_s \pi_s + (1-x) r_f \sum_{s=1}^{S} \pi_s \tag{378}$$

$$r_x = x r_m + (1-x) r_f \tag{379}$$

Variance of the portfolio is

$$\sigma_x^2 = \sum_{s=1}^{S} \left[\left(x m_s + (1-x) r_f \right) - r_x \right]^2 \pi_s = \sum_{s=1}^{S} \left(x m_s - x r_m \right)^2 \pi_s = x^2 \ \sigma_m^2 \tag{380}$$

14.2 Price of risk

Price of risk

$$\sigma_x = x \ \sigma_m \tag{381}$$

price of risk from return-risk diagram

$$p = \frac{r_m - r_f}{\sigma_m} \tag{382}$$

Marginal rate of substitution between return and risk should equal this price ratio

$$MRS_{\mu,\sigma} = \frac{\partial U(W)/\partial \sigma}{\partial U(W)/\partial \mu} = \frac{r_m - r_f}{\sigma_m} \tag{383}$$

$$\beta_i = \frac{risk \ \text{ of asset i}}{risk \ \text{ of stock market}} \tag{384}$$

amount of risk $\beta_i \sigma_m$;cost of risk $\beta_i \sigma_m p$

$$\text{Risk adjustment } = \beta_i \sigma_m p = \beta_i \sigma_m \frac{r_m - r_f}{\sigma_m} = \beta_i \left(r_m - r_f \right) \tag{385}$$

Risk adjusted returns should be equal in all assets

$$r_i - \beta_i \left(r_m - r_f \right) = r_j - \beta_j \left(r_m - r_f \right) \tag{386}$$

If one of the assets is the risk free asset

$$r_i - \beta_i \left(r_m - r_f \right) = r_f - \beta_f \left(r_m - r_f \right) \tag{387}$$

$\beta_f = 0$

$$r_i = r_f + \beta_i \left(r_m - r_f \right) \tag{388}$$

This the theory behind the capital asset price model (CAPM).

14.2.1 Homework

- An individual has a car worth £10000. Probability of accident is 0.01 percent and the car will be useless if it meets any accident. How much insurance should this person pay?
- John has a house worth 200,000. Probability of fire is 0.05 percent. House is worthless after the fire. How much insurance should John pay for the fire insurance?
- Jane has a business worth 1 million. Probability of bankruptcy is 0.02 percent. How much insurance should Jane pay to protect against such bankruptcy?

- Crops are worth 50000. Probability of storm or flood is 0.01 percent. Crop is completely destroyed if a storm or flood occurs. How much insurance is ideal in this business?

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14.2.2 Problem 11: Uncertainty and insurance

Q1. **What are the measures of risk aversion for consumers with following utility functions? Which of these consumers is risk-averse, which one is risk neutral and which one is a risk lover?**

(a) Logarithmic utility in wealth: $U(W) = ln(W)$

(b) Cobb-Douglus type utility: $U(W) = W^{\frac{1}{2}}$

(c) Linear utility: $U(W) = aW$

(d) Exponential utility: $U(W) = \exp(aW)$

Q2. The amount of wealth in the good state is W. If a bad event occurs there will be a loss (L) and the probability of a loss is p.

The owner of the property can insure for amount (q) paying premium rate (m). The expected utility maximisation problem of the individual is implicitly written as:

$$\max_q EU = p.u\left(W - L - mq + q\right) + (1 - p)\,u\left(W - mq\right) \tag{389}$$

The profit maximising condition of the insurance company with perfect competition in the insurance market is:

$$p\left(1 - m\right)q - \left(1 - p\right)mq = 0 \tag{390}$$

A risk averse consumer likes to get the same marginal utility whether in the good or bad state

$$u'\left(W - L - mq + q\right) = u'\left(W - mq\right) \tag{391}$$

Prove that the optimal premium rate equals the probability of loss (p), and that it is optimal for an individual facing uncertainty in this way to purchase full insurance.

Q3. Utility function of an individual is given by $U(W) = ln(W)$, where U is the utility and W is the level of wealth. Is this a risk loving, risk averse or risk neutral individual?

a. Draw this utility function in a diagram in space and explain the economic meaning underlying the curvature of the utility function.

b. Find expected utility of this individual if probability of high wealth is and that of low wealth is . Show what is the certainty equivalent income and the amount of insurance that this person is ready to pay against income uncertainty.

c. Probability of getting high wealth of 5000 is 0.4 against 0.6 probability of getting low wealth of 2500. What is the expected wealth of this person?

d. What is the utility of expected wealth?

e. What is the value of expected utility form high and low values of wealth?

f. Find the certainty equivalent income and the maximum amount that this individual will be ready to pay for the insurance?

Q4. Utility from wealth for a person living in Fairfield village is given by $u = ln(W)$where U is the utility and W is the level of wealth. This person has a prospect of good income of 4000 with probability 0.4 and prospect of low income of 1000 with probability of 06. How much would this person pay to insure against such income uncertainty?

15 L14: Choice under Uncertainty and Insurance

Uncertainty of Good Times and Bad Times

- Future is uncertain; can be good or bad; two states.
- Contingent consumption in good times C_g and in bad times C_b
- Probability of good times π_g and of bad times π_b
- Prices of good times p_g and of bad times p_b
- Utilities from contingent consumption in good times $u(C_g)$ and in bad times $u(C_b)$
- Budget constraint $.I = P_g C_g + P_b C_b$

Consumer problem under uncertainty

Expected utility theorem: utilities under uncertainty are additively separable (von-Neumann-Morgenstern Utility)

$$Max \quad EU = \pi_g u(C_g) + \pi_b u(C_b) \tag{392}$$

Subject to

$$I = P_g C_g + P_b C_b \tag{393}$$

Lagrangian for constrained optimisation

$$\mathcal{L} = \pi_g u(C_g) + \pi_b u(C_b) + \lambda \left[I - P_g C_g - P_b C_b \right] \tag{394}$$

First order conditions for optimisation

$$\frac{\partial \mathcal{L}}{\partial C_g} = \pi_g u'(C_g) - \lambda P_g = 0 \tag{395}$$

$$\frac{\partial \mathcal{L}}{\partial C_b} = \pi_b u'(C_b) - \lambda P_b = 0 \tag{396}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_g C_g - P_b C_b = 0 \tag{397}$$

Dividing (395) by (396) gives the marginal rate of substitution between good and bad times

$$\frac{\pi_g u'(C_g)}{\pi_b u'(C_b)} = \frac{P_g}{P_b}; \qquad \frac{P_g}{P_b} = \frac{\pi_g}{\pi_b} \tag{398}$$

Fair market for contingent goods implies ratio of prices in good and bad states equals ratio of respective probabilities.

Utility and allocation in good and bad times

$$\frac{u'(C_g)}{u'(C_b)} = 1 \tag{399}$$

$$u'(C_g) = u'(C_b) \tag{400}$$

Since preference are symmetric over the states

$$C_g = C_b \tag{401}$$

consumer likely to fully insure against any risk; like to have same consumption in both good and bad states.

Represent above result in a diagram with certainty line. budget line and indifference curve $u(C_g, C_b)$.

It is possible that individuals like to consume a bit more in good times and a bit less in bad times.

15.1 Optimal Demand for Insurance

There is certain wealth (W), if an event occurs there will be a loss (L). probability of loss is (p).

Owner of the property can insure for amount (q) paying premium (m)

Expected utility maximisation problem is

$$\max_q EU = p.u(W - L - mq + q) + (1 - p) u(W - mq) \tag{402}$$

Choose q to maximise EU using the first order condition as:

$$\frac{\partial EU}{\partial q} = p.u'(W - L - mq + q)(1 - m) - (1 - p) u'(W - mq) m = 0 \tag{403}$$

Optimal condition

$$\frac{u'(W - L - mq + q)}{u'(W - mq)} = \frac{(1-p)}{p} \frac{m}{(1-m)} \tag{404}$$

Profit function of the insurance company

$$\Pi = (1 - p) mq - p(1 - m) q \tag{405}$$

Assume perfect competition in the insurance business, profit is zero

$$p(1 - m) q - (1 - p) mq = 0 \tag{406}$$

The premium rate equals the probability of loss in equilibrium

$$p = m \tag{407}$$

This is actuarially fair insurance.

Insert (**??**) into (403)

$$p.u'(W - L - mq + q)(1 - p) - (1 - p) u'(W - mq) p = 0 \tag{408}$$

$$u'(W - L - mq + q) = u'(W - mq) \tag{409}$$

Optimal demand for insurance
For risk averse consumer $u''(W) < 0$

$$W - L - mq + q = W - mq \tag{410}$$

$$q = L \tag{411}$$

Consumer completely insures (q) against the loss (L).

15.1.1 Risk spreading and risk diversification

- Risk can be spread among individuals. Imagine a society with 1000 individuals each endowed with £35000.
- Each faces a risk of losing £10000 with probability of 1 percent.
- Only 10 person in aggregate face this risk. It is a big loss for each individual as it can happen to each of them.
- Now they create an insurance market. Each contributes 100 to mitigate this uncertainty. This creates 100,000 insurance fund.
- This is enough to ensure each for any eventual loss.
- Every one will be certain (ensured) to have 34,900.: endowment minus insurance contribution.
- This is an example of risk spreading. Risk is spread (divided) among all. Each pays 100 to ensure against loss of 10000.

Risk spreading and risk diversification

- Risk can be diversified by choosing an appropriate portfolio.
- Consider an excellent example from Varian (2010) on sunglasses and raincoat.
- You have 100 to invest. Probability of rain or shine is equally likely.
- You can invest only in sunglasses or raincoats or split 50/50 in each. Value of sunglass investment will double if it is sunny or down by half if it is rainy. Similarly value of investment will be double if rainy and down by half if sunny.
- If invested all in one then at the end of the day the expected value is 0.5(50)+0.5(200)=125.
- There is considerable risk. If case of splitting 50/50 the expected value of investment is [0.5(25) + 0.5(100)]+[0.5(100) + 0.5(25)]=125.
- Thus 125 is guaranteed no matter rainy or shiny. Diversification has ensured 125.
- Do not put all your eggs in one basket.

15.1.2 Homework

A person has wealth worth £35000. There is 1 percent probability of loss of 10,000. This individual is risk neutral.

1) What is expected wealth without insurance?

2) This person can buy insurance equal to amount K to cover insurance by paying γK insurance premium, where γ is the premium rate. Write individuals budget in case of accident and in case of no accident.

3) Write the expected utility function of this person. Assume that person receives utility from the wealth that he has.

4) What is expected profit of the insurance company?

5) Prove that premium rate equals the probability of the event.

6) Prove that consumption is same in both states with insurance.

L) Prove that it is optimal to fully insurance against the loss and that is actuarially fair insurance.

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16 L15: Signalling and Principal Agent Model: Asymmetric (incomplete) Information

Impacts of Asymmetric (incomplete) Information on Markets

- Equilibrium is inefficient relative to full information case
- Signalling can improve the efficiency: warranty and guarantee
- Screening: revealing the risk type of agent
- Credit history from credit card companies
- Government can improve the market by setting high standards of business contracts or bailing out troubled ones (Northern Rock, Bear Stearns, Lehman Brothers)

- Right regulations – Financial Services Authority, Fair trade commissions; Office of standards;
- Bank of England

Moral hazard (hidden action)

- Probability of bad event is raised by the action of the person
  People who have theft insurance are likely to have\ low quality locks

that are easy to break (in cars, houses, bicycle (car)) most likely to claim insurances

- Remedy: deductible amount; to ensure that some customers take care in security.

Adverse Selection (hidden information) Problem

- Uncertainty about the quality of good or services
  - honest borrowers less likely to borrow at higher interest rates.
  - low quality items crowd out high quality items
  - risky borrowers drive out gentle borrowers in the financial market.
- Theft insurance; health insurance;
  - people from safe area are less likely to buy theft insurance; only
  - unsafe customers end up buying theft insurance
  - healthy people are less likely to buy health insurance

Adverse Selection (hidden information) Problem

- Uncertainty about the quality of good or services
  - honest borrowers less likely to borrow at higher interest rates.
  - low quality items crowd out high quality items
  - risky borrowers drive out gentle borrowers in the financial market.
- Theft insurance; health insurance;
  - people from safe area are less likely to buy theft insurance; only
  - unsafe customers end up buying theft insurance
  - healthy people are less likely to buy health insurance

Asymmetric information in Used Car Market -Akerlof's Model of Asymmetric Information

- Sellers know exactly quality of cars but buyers do not.
- Equilibrium is affected when sellers have more information than buyers.

- Market has plums: good cars and lemons: bad cars
- Seller knows his quality of cars but buyers do not
- Market for good cars disappear because of existence of bad cars in the market.
- Demand for high quality car falls and demand for low quality cars rise.
- Ultimately only low quality cars remain in the market.

Asymmetric information in Used Car Market -Signalling solves the Problem

- signals: warranty and Guarantee
- Providing warranty less costly for high quality cars as they last long.
- Warranty is costly for low quality cars as they frequently break down.
- Buyers can decide whether a car is good or bad looking at the warranty and pay appropriately.
- Right signalling can remove inefficiency due to incomplete information.
- Markets for both types of car can operate efficiently by right signals of warranty and Guarantee

Pooling, Separating and Mixed Equilibrium

- Complete market failure

pooling equilibrium (same price for good and bad cars; good cars disappear from the market)

- Complete market success

Separating equilibrium where players act as they should according to the signal (prices according to quality)

- Partial market success

(both good and bad cars are bought, some feel cheated)
Near Market failure (mixed strategies) Bayesian updating mechanism at work
Education Level- A Signal of Productive Worker

- An employer does not know is more productive and who is less productive
- It pays the same wage rate to both productive and unproductive workers
- market is inefficient, it drives out more productive workers.
- Workers can signal their quality by the level of educational attainment, then market may work well.
- Less costlier for high quality worker to get education.
- costlier for low quality worker to get the specified education.
- so the low quality worker gets no education, but the higher quality worker gets education.
- Employers pay according to the level of education.
- Education works as a signalling device and makes the market efficient.
- Education separates the equilibrium.

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16.0.3 Signalling and Incentives

1. Education as a signal of productivity

Level of education signals quality of a worker. Given the cost of education it is easier for a high quality worker to complete a degree than for a low quality worker. In an efficient market potential employers take level of education as a signal in hiring and deciding wage rates paid to its employees. Spence (1973) model was among the first to illustrate how to analyse principal agent and role of signalling in the job market.

Pooling equilibrium Consider a situation where there are N individuals applying to work. In absence of education as the criteria of quality employers cannot see who is a high quality worker and who is a low quality worker. Employers know that θ proportion of workers is of high quality and (1-θ) proportion is of bad quality. Therefore they pay each worker an average wage rage as:

$$\overline{w} = \theta w_h + (1-\theta)\, w_l \tag{412}$$

Every worker gets the average wage rate ; there is no wage premium for higher quality in pooling equilibrium. If more productive worker is worth 40000 and less productive worker is worth 20000 and θ =0.5 then the average wage rate will be 30000;

$\overline{w} = \theta w_h + (1-\theta)\, w_l = 0.5\,(40000) + 0.5\,(20000) = 30000.$

Let c denote the cost of education. It is worth for high quality worker to go to school only if the wage difference having and not having education is greater than the cost of education which is given by

$$w_h - \overline{w} = w_h - [\theta w_h + (1-\theta)\, w_l] \tag{413}$$

Simplification of this condition implies a signalling condition

$$\theta > 1 - \frac{c}{w_h - w_l} \tag{414}$$

Going to school is not worth if the wage premium of school is less than the cost of education

$$w_h - w_l < c \tag{415}$$

If the cost of education is 15000, the net of education wage for high quality worker is 25000 which is less than wage at the pooling equilibrium. Therefore no signalling occurs in this case. Employers pay average wage rate to each worker without any consideration of their abilities. This pooling equilibrium remains inefficient as productive workers do not have enough incentive to put their full efforts at work.

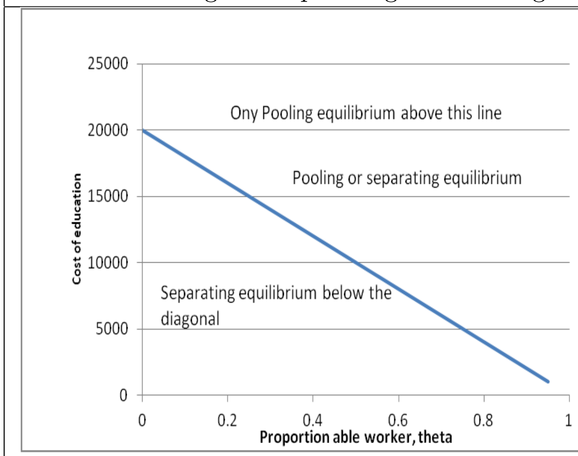
Separating equilibrium It is worthwhile for more productive worker to signal if

$$w_h - w_l > c \tag{416}$$

This is possible if the cost of education is 5000; then wage net of education cost for high quality is 35000 which is above the pooling wage rate. This makes sense to signal by choosing higher education. Signalling is optimal in this case; θ fraction of workers will signal by going to education.

Aggregate labour cost will be the same but wages will be paid according to the productivity of workers as reflected by the level of education of workers.

Figure: Separating and Pooling Equilibrium



Excel calculations.

While making a hiring decision employers take level of education as a signal of quality of workers.

Government Policy and Signalling

It is important to have optimal amount of signalling – too little or too much signalling generates inefficient result. Empirical finding on signalling is mixed. Public policy could be designed to generate right amount of signalling as following:

1. It can create separating equilibrium by subsidizing education of more able workers. It can ban on wasteful signalling by banning schools that do not produce good workers.
2. High education provides signals, employers pay according to this signal, this will affect the distribution of wages.

16.0.4 Spence model of education

- Players consisting of {workers, firms and nature}.
- There are two types of workers [$t = \{1, 2\}$].
- Type 1 is less productive and type 2 more productive.

- Employer does not know which one is low or high quality worker but sees level of education
- Nature decides whether a worker is high or low productivity type.
- Level of education signals the quality of worker

Spence model of education: Preferences over wage and level of education

- Workers choose level of education according to their beliefs about its impact on wage offer: $w_t(e)$.
- Utility from wage and education is given by $u_t(w,e)$.
- Utility is rising in wage received $\frac{\partial u_t(w,e).}{\partial w}$
- Utility falls in work efforts $\frac{\partial u_t(w,e).}{\partial e} < 0$
- It is costly to get education.
- The utility function satisfies the single-crossing property

Spence model of education: Problem of Choosing Right Level of Education

- $$\underset{e}{\text{Max}} \; u_t(w,e) = f(w(e)) - k_t g(e) \qquad k_t > 0 \;\; t = 1,2 \tag{417}$$
- k_t. indicates the cost of education for the worker type t.
- It is more expensive for less productive worker to produce education signal $k_1 > k_2$
- More Specifically

$$u_t(w,e) = 42\sqrt{w} - k_t e^{1.5} \quad k_1 = 2; \;\; k_2 = 1 \;\; w_1 = e; \; w_2 = 2e \tag{418}$$

Level of education chosen by less productive worker

- In perfect information equilibrium, firms pay according to the marginal productivity
- Wage of less productive worker: $w_1 = e$;
- The type 1 worker's optimisation problem

$$\underset{e}{\text{Max}} \; u_t(w,e) = 42\sqrt{w} - k_t e^{1.5} = 42\sqrt{e} - 2e^{1.5} \tag{419}$$

$$\frac{\partial u_t(w,e).}{\partial e} = 42\frac{1}{2\sqrt{e}} - 3e^{\frac{1}{2}} = 0 \tag{420}$$

$$42\frac{1}{2\sqrt{e}} = 3e^{\frac{1}{2}} \Longrightarrow e_1^* = \frac{42}{6} = 7 \tag{421}$$

- It is optimal for the less productive worker to takes only seven years of education

Level of education chosen by more productive worker

- Wage of less productive worker: $w_2 = 2e$;
- The type 1 worker's optimisation problem

$$\underset{e}{\text{Max}} \;\; u_t\left(w,e\right) = 42\sqrt{w} - k_t e^{1.5} = 42\sqrt{2e} - e^{1.5} \tag{422}$$

$$\frac{\partial u_t\left(w,e\right).}{\partial e} = 42\frac{1}{2\sqrt{2e}} \times 2 - 1.5e^{\frac{1}{2}} = 0 \tag{423}$$

$$42\frac{1}{\sqrt{2e}} = 1.5e^{\frac{1}{2}}; 42\frac{1}{1.5\sqrt{2}} = e \Longrightarrow e_2^* = \frac{42}{2.121} = 19.8 \tag{424}$$

- It is optimal for the more productive worker to takes 19.8 years of education.

Government Policy and Signalling

- It is important to have optimal amount of signalling – too little or too much signalling generates inefficient result. Empirical finding on signalling is mixed. Public policy could be designed to generate right amount of signalling as following
- It can create separating equilibrium by subsidizing education of more able workers. It can ban on wasteful signalling by banning schools that do not produce good workers.
- High education provides signals, employers pay according to this signal, this will affect the distribution of wages.

Education Level- A Signal of Productive Worker Consider a level of education e^*

$$c_1 e^* \geq c_2 e^* \Longrightarrow c_1 \geq c_2 \tag{425}$$

Cost of education of unproductive worker is much higher

$$c_2 e^* < \left(a_2 - a_1\right) < c_1 e^* \tag{426}$$

Cost of education relative to productivity of low and high quality workers for education e^*

$$\frac{\left(a_2 - a_1\right)}{c_1} < e^* < \frac{\left(a_2 - a_1\right)}{c_2} \tag{427}$$

16.0.5 Problem 13: Signalling and mechanism

1. Let there be two bidders bidding b_1 and b_2 but with true values v_1 and v_2. The highest bidder wins the auction at the price of the second-highest bid. The expected value for bidder 1 is then given by $prob\,(b_1 > b_2)\,(v_1 - b_2)$. **Prove that honesty is the best policy in this game.**

2. A22 is a taxi company in a certain city. There are two options for owners of the company. Option one is to lend all taxis to taxi drivers on a fixed fee (F) basis. Option two is to collide with the taxi drivers for maximisation of joint profit which could be divided between taxi drivers and owners according to their mutual agreement. The market demand and cost functions for this company are given as:

$$P = 24 - 0.5q; \qquad C = 12q \tag{428}$$

3. **Prove the solutions of output, price, revenue, cost and profit are the same whether this taxi company operates under the fixed fee (F) contract or under the joint profit maximisation agreement.**

   [Hints: revenue: $R = P.q$ profits: $\Pi\,(q) = P.q - C$ vs. $\Pi\,(q) = P.q - F - C]$.

4. Productivity of a worker with the level of education e^* is a_2 and it is a_1 without education e^* ; i.e. productivity difference is $a_2 - a_1$ between educated and non-educated workers.

$$c_1 e^* \geq c_2 e^* \Longrightarrow c_1 \geq c_2 \tag{429}$$

   Show how the cost of education relative to the productivity differences $a_2 - a_1$ is lower for the high quality worker than for the low quality worker.

5. Level of education signals quality of a worker. Spence (1973) model was among the first to illustrate how to analyse principal agent and role of signalling in the job market.Consider a situation where there are N individuals applying to work. In absence of education as the criteria of quality employers cannot see who is a high quality worker and who is a low quality worker. Employers know that θ proportion of workers is of high quality and (1-θ) proportion is of bad quality. Therefore they pay each worker an average wage rate as:

$$\overline{w} = \theta w_h + (1 - \theta)\, w_l \tag{430}$$

6. more productive worker is worth 70000 and less productive worker is worth 30000 and θ =0.5 then the average wage rate will be 50000. Prove separating equilibrium is more efficient than the pooling equilibrium and that it is worth for high quality workers to signal their quality by the standard of their education.

7. Owners of a company are concerned about a project that would earn them £600,000 if successful. Probability of success is 60 percent if the manager puts in normal effort. This probability can rise to 80 percent if the manager puts in extra effort. The manager will put in extra effort only if an additional payment of £50,000 is made above the basic salary of £100,000.

However, it is difficult for owners to monitor whether the manager is putting in extra effort even if they pay an additional amount of £50,000.

a) Is it profitable for owners to pay an extra £50,000 for the manager? Why does such an extra payment not automatically guarantee higher probability of profit?

b) Design incentive compatibility and participation constraints in terms of the basic salary and a bonus so that the manager puts in extra effort in return for extra payments.

c) Based on above information what is the minimum payment required by the manager to put in extra effort? Do owners find it profitable to pay such extra payments as an incentive device?

d) Consider now the case where managers can be of low or high productivity type. How can the level of education of a prospective manager signal to the employers whether he or she is of high productivity type?

e) How can owners signal to the manager that they cannot be fooled by a manager who pretends to the owners of putting in extra effort while actually putting in only normal efforts? [hint: survey of customers]

17 L16: Mechanism Design Game

17.0.6 Mechanism to ensure high efforts by a CEO: Signalling for managing a company

- Owners of a company are concerned about a project that would earn them 600,000 if successful.
- Probability of success with normal effort from the manager is 60 percent and this can increase up to 80 percent if the manager puts extra efforts.
- The basic salary of the manager is 100,000. He would put extra efforts only if he is paid additional amount of at least 50,000. Owners cannot monitor whether the manager is putting high or low efforts.

a) Is it profitable to pay extra for the manager?
Profit without paying extra: 0.6 * 600,000 - 100,000 = 260, 000
Profit with extra incentive payment: 0.8 * 600,000 - 150,000 = 330, 000
Extra payment can make up to 70,000 with probability of 0.8.
Once extra payment is made how can owners make sure that he puts extra efforts? This requires evaluation of incentive compatibility and participation constraints.
Mechanism to ensure high efforts by a CEO
a) Incentive compatibility constraint

$$(s + 0.8b) - (s + 0.8b) \geqslant 50,000 \tag{431}$$

$$0.2b \geqslant 50,000 \tag{432}$$

b = 250,000
b) Participation constraint:

$$(s + 0.8b) \geqslant 150,000 \quad (433)$$

$$s = 150,000 - 0.8b; \quad s = 150,000 - 0.8\,(250,000) = -50,000 \quad (434)$$

It is not possible to hire manager with negative salary. At most managers can be conditioned to bonus payment but with zero salary.

Mechanism to ensure high efforts by a CEO

$$(0 + 0.8b) \times (250,000) \geqslant 150,000 \quad (435)$$

$$200,000 \geqslant 150,000 \quad (436)$$

Pay 200,000 and the manager will put maximum effort.
c) Is it profitable to pay extra 200,000 as an incentive payment?
Profit with incentive payment
0.8 * 600,000 - 200,000 = 280, 000
Profit without incentive payment
0.6 * 600,000 - 100,000 = 260, 000
Thus profit increases by 20,000 with the incentive payments.
Reference: Dixit A and S Skeath (1998) Games of Strategy, New York Norton.

17.0.7 Why Mechanism Design for Price Discrimination: Low Cost Airlines Example

- Economy and Business Class Ticket Problem for Airlines (Based on Dixit et. al. (2009))
- Two types of travellers: economy and business
- Assume 100 travellers and 70 of them economy type tourists and 30 business type first class.

	Cost of	Reservation Price		Airline's Profit	
	the Airlines	Tourists	Business	Tourists	Business
Economy	100	140	225	40	125
First Class	150	175	300	25	150

- Economy class tickets cost less than the business class.
- Business traveller is ready to pay higher price than economy class for both economy and first class but the airlines cannot separate them out.
- Business traveller may well buy economy class ticket rather then business class.
- Airlines likes to build a mechanism so that business class customers buy business class tickets and economy class customers buy economy class ticket.
- What is the profit to the airlines if it knows reservation prices of tourists and business group of travellers?
- How would this profit change if business type buy the economy class ticket?

- What is the incentive compatible price that the airlines can offer to the business group?
- What would happen if the split between the business and economy class is 50/50? What will be the optimal reaction of the airlines?

Incentive Compatible Mechanism
Profit in an ideal scenario (perfect price discrimination; if the airlines knew each customer type)

$$\begin{aligned} 30(300-150)+(140-100)(70) &= 30\times 150+40\times 70 \\ &= 4500+2800=7300 \end{aligned} \tag{437}$$

Business travellers have consumer surplus of 225 -140 = 85 in economy class ticket. For this all 30 of may decide to buy economy class ticket. Then the profit of the airlines when the airlines fails to screen customers will be

$$(140-100)(100)=4000 \tag{438}$$

Airlines should give consumer surplus of 85 to business traveller and charge them (300-85) = 215. This will alter their profit

$$\begin{aligned} 30(215-150)+(140-100)(70) &= 30\times 65+40\times 70 \\ &= 1950+2800=4750 \end{aligned} \tag{439}$$

Incentive Compatible and Participation Constraints

- Airline initially does not have enough information on types of customers
- It should design incentive compatible pricing scheme so that business class travellers do not defect to economy class.
- This requirement is contained in the incentive compatible constraint. If it charges 240 for the business class then the their consumer surplus will be equal (300-240) = 60 from business class travel and (225-165)=60
- However 140 is the maximum the tourist class traveller is ready to pay. If the airline raises price to 165 they will lose all tourist travellers. Mechanism requires fulfillment of the participation constraint.
- Airlines should operate taking account of the participation constraint of tourists and incentive compatible constraint of the business travellers.
- $X < 140$ is the participation constraint; incentive compatible constraint is $225 - X < 300 - Y$ or $Y < X + 75$

Mechanism when the composition of travellers change
Charging 215 for the business class and 140 for the economy class is the
solution to the mechanism design problem.
30(215-150)+(140-100) (70) =30$\times$ 65+40$\times$ 70 =1950+2800=4750

- Suppose the composition of travellers changes to 50% of each. Profit with the above price mechanism

$$\begin{aligned} 50(215-150)+(140-100)(50) &= 50\times 65+40\times 50 \\ &= 3250+2000=5250 \end{aligned} \tag{440}$$

- It is more profitable to scrap the tourist class tickets instead and charge the business class its full reservation price

$$50(300-150)=50\times 150=7500 \tag{441}$$

- There are relatively few customers but all are willing to pay higher price. There is no problem of screening as the airlines now does not serve to the tourist class at all.

17.0.8 Mechanism design in renting lands

Proposition 1: Results of fixed fee contract and joint profit maximisation are equivalent

Proposition 2: Hire contract is incentive incompatible and leads to production inefficiency

Proposition 3: Moral hazard problem and production inefficiency exists in revenue sharing contingent contract

Proposition 4: Profit sharing contract is efficient and free of moral hazard problem

Price and cost

$$P=24-0.5q \qquad C=12q \tag{442}$$

Revenue

$$R=P.q \tag{443}$$

Mechanism design in renting lands

Under the joint profit maximisation agreement

$$\Pi(q)=P.q-C=(24-0.5q)\,q-12q=24q-0.5q^2-12q \tag{444}$$

Under the fixed fee contract tenant maximises

$$\Pi(q)=P.q-C-F=(24-0.5q)\,q-12q-F=24q-0.5q^2-12q-F \tag{445}$$

Under both these arrangements

$$\Pi'(q)=24-q-12=0 \tag{446}$$

$$q=12;\quad p=18;\; R=216;\quad C=144;\; \Pi(q)=72 \tag{447}$$

- This is the total profit. It is divided between the tenant and the landlord by their mutually agreed arrangement. Under the fixed fee contract landlord may fix the amount that he needs at 48.

- Then the residual 24 profit goes to the tenant.
- This arrangement achieves production efficiency, is incentive compatible, fulfils the participation constraint and motivates to put the optimal effort and solves the moral hazard problem.

17.0.9 Hire contract

- Landowner can hire workers in fixed fee basis, say 12 per unit of output a.
- This does not motivate tenant to work because his cost per a is also 12 and so does not make any profit. Landlord has to raise payment to tenant to say 14 to motivate him to work.
- Then the profit maximisation problem of the landlord will be

$$\Pi(q) = P.q - C = (24 - 0.5q)\, q - 14q = 24q - 0.5q^2 - 14q \tag{448}$$

$$\Pi'(q) = 24 - q - 14 = 0 \tag{449}$$

$$q = 10; \quad p = 19; \; R = 190; \quad C = 120; \Pi_{LL}(q) = 50; \Pi_T(q) = 20 \tag{450}$$

The tenant has incentive to overproduce whenever is paid more than 12.
Revenue sharing contract

- Let the landlord enter into a revenue sharing contract whereby she gets $\frac{1}{4}$ th of the revenue and leavening $\frac{3}{4}$ of revenue to the tenant who also bears all production cost. The profit function of the tenant is now modified as

$$\Pi(q) = \frac{3}{4}P.q - C = \frac{3}{4}(24 - 0.5q)\, q - 12q \tag{451}$$

$$\Pi'(q) = 6 - \frac{3}{4}q = 0 \Rightarrow q = \frac{4}{3} \times 6 = 8 \tag{452}$$

$$\begin{aligned} q &= 8; \quad p = 20; \; R = 160; \quad C = 96; \Pi_T(q) = \frac{3}{4}(160) \\ &= 120; \Pi_{LL}(q) = \frac{1}{4}(160) = 40 \end{aligned} \tag{453}$$

Profit of tenant = 120 - 96 =24

This level of production is not incentive compatible for the land-lord who would be interested in maximising revenue by producing 24

Profit sharing contract

- Now let us assume the landlords and tenants enter into a profit sharing deal, say 1/3rd of profit goes to the tenant and 2/3rd to the landlord.

$$\frac{1}{3}\Pi(q) = \frac{1}{3}(P.q - C) = \frac{1}{3}\left(24q - 0.5q^2 - 12q\right) \tag{454}$$

$$\Pi'(q) = 4 - \frac{1}{3}q = 0 \quad \Rightarrow q = 3 \times 4 = 12 \tag{455}$$

$$\begin{aligned} q &= 12; \;\; p = 18; \; R = 216; \;\; C = 144; \; \Pi(q) = 72; \\ \Pi_{LL}(q) &= 48; \Pi_T(q) = 24 \end{aligned} \tag{456}$$

There are many other situations, including optimal tax designs, optimal price discrimination, fund management, management of theme-park, renting of buildings, collection of taxes or tariffs, union-management contracts, where these types of models have been applied.

Incentive compatible game on renting a piece of agricultural land
If a worker puts x amount of effort, the land produces $y = f(x)$
Then the land owner pays worker $s(y)$.
The land owner wants to maximise profit $\pi = f(x) - s(y) = f(x) - s(f(x))$
Worker has cost of putting effort $c(x)$ and has a reservation utility, $\overline{u}$
The participation constraint is given by . $s(f(x)) - c(x) \geq \overline{u}$
Including this constraint maximisation problem becomes
$max \;\; \pi = f(x) - s(f(x))$
subject to
$sf(x) - c(x) \geq \overline{u}$
Solution: marginal productivity equals marginal efforts $f'(x)) - c'(x)$
Incentive compatible game on renting a piece of agricultural land
(a) renting the land where the workers pays a fixed rent R to the owner and takes the residual amount of output, at equilibrium

$$f(x^*) - c(x^*) - R = \overline{u} \tag{457}$$

(b) Take it or leave it contract where the owner gives some amount such as

$$B - c(x^*) = \overline{u} \tag{458}$$

(c) hourly contract

$$s(f(x)) = wx + K \tag{459}$$

(d) sharecropping, in which both worker and owner divide the output in a certain way.

In (a)-(c) burden of risks due to fluctuations in the output falls on the worker but it is shared by both owner and worker in (d).

Which of these incentives work best depends on the situation.

Problem

1. Given the market demand and cost functions

$$P = 24 - 0.5q \qquad C = 12q \tag{460}$$

Prove following four propositions regarding efficient contract.

Proposition 1: Results of fixed fee contract and joint profit maximisation are equivalent

Proposition 2: Hire contract is incentive incompatible and leads to production inefficiency

Proposition 3: Moral hazard problem and production inefficiency exists in revenue sharing contingent contract

Proposition 4: Profit sharing contract is efficient and free of moral hazard problem

18 L17: Auction Game and Efficiency Conditions and Regulation

18.1 Types of Auction: English and Dutch Auctions

A public way of finding a high value buyers of objects. Good and services are bought sold in auctions such as auctions of popular arts, auto-actions or other auctions in the eBays (https://www.ebay.co.uk/). Large public projects for constrution of highways or operations of public networks are also supplied in actions. Auction is a mechanism to find values put in an object by customers. Financial auctions are popular in traditional charities or communities. Nobel prize 2020 was awarded to Paul Milgram and Robert Wilson “for improvements to auction theory and inventions of new auction formats."

- First price, sealed-bid: person who bids the highest amount gets the good.
- Second-price, Sealed-bid: Each submit a bid. Higher bidder wins and pays second-highest bid for the good.
- Dutch Auction: Seller begins from very high price and reduces it until someone raises a hand.
- English Auction: Begins with very low price, bigger drops out by raising a hand.
- Which one of these four mechanism is good for the seller??

Auction: Vickrey-Clerk-Grove (VCG) mechanism
Three factors that influece the outcome of an Auction:

the first is the auction’s rules, or format. Are the bids open or closed? How many times can participants bid in the auction? What price does the winner pay – their own bid or the second-highest bid?

The second factor relates to the auctioned object. Does it have a different value for each bidder, or do they value the object in the same way?

The third factor concerns uncertainty. What information do different bidders have about the object’s value?

- Milgrom, P. and Milgrom, P.R., 2004. Putting auction theory to work. Cambridge University Press.
- Wilson, R., 1979. Auctions of shares. The Quarterly Journal of Economics, pp.675-689.
- Honesty is the best policy in Vickery auction; truth telling is the winning strategy.

Proof

- Let there be two bidders bidding b_1 and b_2 but with true values v_1 and v_2. Highest bidder wins the auction at the price of the second-highest bid. English auctions and second-highest sealed-bid auctions are equivalent.

Expected value for bidder 1 is then given by

$$prob\,(b_1 > b_2)\,(v_1 - b_2) \tag{461}$$

- If $(v_1 > b_1)$ it is in the best interest of bidder 1 to raise the probability of winning $prob\,(b_1 > b_2)$, this can happen when $(v_1 = b_1)$
- Similarly If $(v_1 < b_2)$ then it is in the interest of bidder 1 to make $prob\,(b_1 < b_2)$ as small as possible. It happens when .$(v_1 = b_1)$ Thus the truth telling is the best interest in such auction.

Auction: Financing Mechanism for Public Goods

- Let x be a public good such as streetlight or road; $x = 1$ if it is provided $x = 0$ if not.
- If state knew that how much each person is willing to pay for this it could bill efficiently.
- Each would pay according to the value they put in such public good. Unfortunately it is impossible to know preferences of individuals.
- Individuals do not tell true value when asked that how much they are ready to pay for this. Let N individuals be indexed by i. Then the utility from the public good to an individual i is given by $U_i(x)$.
- There is free rider problem with public goods. Individuals may underreport their utility thinking that others will pay higher for it if they act like this but they will have opportunity of full benefit.
- Under Veckrey-Clark-Grove mechanism it is in the best interest of individuals to tell the truth.

Under Grove mechanism each individual is asked to reported his her utility; which is $r_i(x)$. . Then the state chooses x* that maximises the sum of reported utilities $R = \sum\limits_{i=1}^{N} r_i(x)$. Each individual receives a side-payment $R_i = \sum\limits_{j \neq 1}^{N} r_i(x)$..

With side payment the total utility of an individual is

$$U_i(x) + \sum_{i=1}^{N} r_i(x) \tag{462}$$

State chooses x to maximise

$$r_i(x) + \sum_{i=1}^{N} r_i(x) \tag{463}$$

Therefore it is in the best interest of individual to tell the truth $U_i(x) = r_i(x)$. All agents tell truth like this and this mechanism generates efficient outcome (see Varian HR (2010):36).

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18.2 Regulation Theory and Practice

18.2.1 Theory of Regulation

- Good understanding of microeconomic theories will lead to better policies and regulations for the efficient functioning of the market economy.
- These policies particularly focus on competition, adoption of better technology, governance and information, correcting externality and good environment, social insurance, more equal distribution of income and identification of cases for government intervention.
- For recent policies see relevant web page of the government such as in the Department for Business Innovation & Skills https://www.gov.uk/government/organisations/competition-and-markets-authority.

Literature on Regulation

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- Bain (1951), Mayson (1939), Lerner (1934),Marshall (1890)

Introduction to regulation

- Tirole (2014) in his Nobel lecture states producer deliver goods to costumers but policy makers should be aware that firms may provide low quality goods at higher prices.
- This must be checked by developing a business model specific to firms based empirical analysis or laboratory experiments. Markets fail to provide quality goods if unchecked.
- Theory of industrial regulations starts from Cournot and Du Point in 19th century; Sherman Act 1890 and Structure Conduct Performance (SCP) hypothesis.
- Chicago school led by Stigler, Demeltz and Posner in general favoured the competitive market without any specific theoretical doctrine for regulation.

- Collective efforts by Fundenberg (1991), Markin (1994), Laffront (1997), Jaskow (1996), Rochet and Tirole (1997) Rey (1998), Lerner (1934) combined game theory and information economics in designing optimal regulations. Regulatory authorities, for electricity, telecommunication, railways, airlines and road transports, postal offices, financial institutions and ports sprang up in Europe as well as in America.
- The regulators paid attention to the cost of firms, prices they charge and the rate of return analysis in regulating these industries. Particularly they compared trade-offs between the lower prices and rate of return.
- Anti-trust laws were designed to prevent horizontal and vertical mergers and to protect patents and innovations.
- Industries controlling the bottleneck inputs such as railway tracks or postal services were allowed to integrate their downstream services in order provide cheaper commodities to the final producers based on cost plus or fixed price contracts to avoid adverse selection and moral hazard problems in the research and development and innovations.
- Authorities also could auction monopoly rights.
- Regulators can design incentive compatible mechanism so the it is not in the interest of the firms with market power to their full extent following Ramsey Boiteux pricing strategy.
- Such incentive contracts can generate superior outcome as firms most often have more information about their customers than the regulators particularly in two sided markets.
- Regulators should not intervene in affecting the price structure and should practice fair reasonable and non-discriminating rates (FRAND) in anti-trust regulation for efficient to make a better world.
- Better understanding of the cost and demand sides of industries is essential for better regulations.

18.2.2 Measures of concentration and performance

- Structure Conduct and Performance (SCP) paradigm
- Number of firms (n), buyers and sellers, nature of products and entry barriers
- Concentration curves, concentration ratios (cumulative market share: $CR_x = \sum_{i=1}^{x} S_i$), Hefindahl-Hirschman Index (HHI) : $HHI = \sum_{i=1}^{n} S_i^2$, Hannah and Kay Index (1977): $HK = \left(\sum_{i=1}^{n} S_i^{\alpha}\right)^{\frac{1}{1-\alpha}}$, Entropy Index: $E = \sum_{i=1}^{n} S_i \log\left(\frac{1}{S_i}\right)$, Variance of the logarithms of firm size: $V = \frac{1}{N}\sum_{i=1}^{n}\left(\log S_i\right)^2 - \frac{1}{N}\sum_{i=1}^{n}\left(\log S_i\right)^2$, Gini Coefficient
- Welfare measure: Harberger's welfare loss

$$Dwl = \frac{1}{2}\Delta q \Delta p = \frac{1}{2}\left(\frac{\Delta p \times e \times q}{p}\right)\Delta p = \frac{1}{2}\left(\Delta p\right)^2 e\frac{q}{p} \quad \because e = \frac{\Delta q \times p}{q \times \Delta p}$$

18.2.3 Why research need to be subsidized?

- Consider an economy with production function $Y = 10(L - F)$, where F is fixed labour, L is labour, w the wage rate.
- Then the cost of production is $C = wL$ and the cost function by substituting L from the production function: $C = w\left(\frac{Y}{10} + F\right)$.
- Under the marginal cost pricing rule: $\frac{\partial C}{\partial Y} = \frac{w}{10} = P$.
- Average cost declines with production: $\frac{C}{Y} = \left(\frac{w}{10} + \frac{wF}{Y}\right)$
- but the producers experience negative profit: $\pi = R - C = \frac{w}{10}Y - w\left(\frac{Y}{10} + F\right) = -wF < 0$
- They will not undertake this project on their own. Government need to subsidise to produce optimal amount of research. This example was based on Jones (2002) Introduction to Economic Growth.

Mark-up: basis for regulation

$$TR_i = P_i Y_i \ ; \quad \sigma = -\frac{\partial Y_i}{\partial P_i}\frac{P_i}{Y_i} \tag{464}$$

$$\begin{aligned} MR_i &= \frac{\partial\left(TR_i = P_i Y_i\right)}{\partial Y_i} = P_i + \frac{\partial P_i}{\partial Y_i}Y_i \\ &= P_i\left(1 + \frac{\partial P_i}{P_i}\frac{Y_i}{\partial Y_i}\right) = P_i\left(1 - \frac{1}{\sigma}\right) \end{aligned} \tag{465}$$

$$MR_i = MC_i \Longrightarrow P_i = \frac{MC_i}{\left(\frac{\sigma-1}{\sigma}\right)} = \frac{\sigma}{\sigma - 1}MC_i$$

Here $\frac{\sigma}{\sigma-1}$ is the measure of the mark up.

18.2.4 Why regulation? Welfare effects of monopoly

Why regulation? Welfare effects of monopoly

$$TR = PQ \ ; \quad e = -\frac{\partial Q}{\partial P}\frac{P}{Q} \tag{466}$$

$$\begin{aligned} MR &= \frac{\partial\left(TR = PQ\right)}{\partial Q} = P + \frac{\partial P}{\partial Q}Q \\ &= P\left(1 + \frac{\partial P}{P}\frac{Q}{\partial Q}\right) = P\left(1 + \frac{1}{e}\right) \end{aligned} \tag{467}$$

$$MR = MC \Longrightarrow P\left(1 + \frac{1}{e}\right) = MC \Longrightarrow \frac{P - MC}{P} = -\frac{1}{e}$$

$$MR = MC \Longrightarrow P\left(1 + \frac{1}{e}\right) = MC \Longrightarrow e = -\frac{P}{P - MC} = -\frac{P}{\Delta P}$$

$$MR = MC \Longrightarrow P\left(1+\frac{1}{e}\right) = MC \Longrightarrow e = -\frac{P}{P-MC} = -\frac{P}{\Delta P}$$

$$\Delta Q = \frac{\Delta P}{P}Qe \Longrightarrow \frac{Q}{\Delta Q} = 1$$

Profit of the firm:

$$\pi = (P - c\ )\,Q = \Delta P Q$$

Welfare of price changes (a la Harberger):

$$W = \frac{1}{2}\Delta P\Delta Q = \frac{1}{2}\Delta P Q = \frac{\pi}{2}$$

Thus welfare cost of monopoly is half of its profit.

18.2.5 Optimal advertising

What is the optimal intensity of advertising:

$$\pi = PQ - cQ - A \quad \text{and} \quad Q = f\,(P, A)$$

$$\frac{\partial \pi}{\partial P} = Q + P\frac{dQ}{\partial P}\ - \frac{dC}{\partial Q}\frac{dQ}{\partial P} = 0$$

$$\frac{\partial \pi}{\partial A} = P\frac{dQ}{\partial A} - \frac{dC}{\partial Q}\frac{dQ}{\partial A} - 1 = 0$$

Dividing the first FOC by $\frac{dQ}{\partial P}Q$.

$$\frac{\partial \pi}{\partial Q} = \frac{Q}{P}\frac{dP}{\partial Q} + 1\ - \frac{dC}{P\partial Q} = 0 \ \Longrightarrow \frac{P\partial Q - dC}{P\partial Q} = -\frac{Q}{P}\frac{dP}{\partial Q}$$

$$\frac{P - \frac{dC}{\partial Q}}{P} = -\frac{Q}{P}\frac{dP}{\partial Q} = -\frac{1}{e}$$

The second FOC:

$$\left(P - \frac{dC}{\partial Q}\right)\frac{dQ}{\partial A} - 1 = 0 \ \Longrightarrow \frac{dQ}{\partial A} = \frac{1}{\left(P - \frac{dC}{\partial Q}\right)}$$

Using above results

$$P\frac{dQ}{\partial A} = \frac{P}{\left(P - \frac{dC}{\partial Q}\right)} = -e$$

This results in Dorfman-Steiner condition for the optimal advertisement intensity for a period:

$$P\frac{dQ}{\partial A}\frac{A}{Q} = -e\frac{A}{Q} \ \Longrightarrow \frac{A}{PQ} = \frac{e_a}{e}$$

Overtime these are discounted by r and the depreciation rate (δ)

$$\frac{A}{PQ} = \frac{e_a}{e\left(r+\delta\right)}$$

18.3 Efficiency conditions of the market system

18.3.1 Efficiency in consumption

Marginal rate of substitution between two products should equal price ratios for a certain consumer

$$\frac{U_x}{P_x} = \frac{U_y}{P_y} = ... = \frac{U_n}{Pn} \tag{468}$$

Allocation is Pareto efficient if it is not possible to make one person better off without making another worse off.

$$L = U(X,Y) + \lambda\left[T(X,Y)\right] \tag{469}$$

$$\frac{\partial L}{\partial X} = \frac{\partial U}{\partial X} + \lambda\frac{\partial T}{\partial X} = 0 \tag{470}$$

$$\frac{\partial L}{\partial Y} = \frac{\partial U}{\partial Y} + \lambda\frac{\partial T}{\partial Y} = 0 \tag{471}$$

$$\frac{\partial L}{\partial \lambda} = \lambda\left[T(X,Y)\right] = 0 \tag{472}$$

$$\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} = \frac{\frac{\partial T}{\partial X}}{\frac{\partial T}{\partial Y}} \tag{473}$$

$$MRS_{X,Y} = -\frac{\partial Y}{\partial X} = RPT_{X,Y} \tag{474}$$

This is the optimal point in the production possibility frontier. See the trade model.

18.3.2 Efficiency in production

If it is not possible to more of one good without reducing the production of another good.

$$X = f_1\left(K_1, L_1\right) + f_2\left(K_2, L_2\right) \tag{475}$$

$$K_1 + K_2 = K \tag{476}$$

$$L_1 + L_2 = L \tag{477}$$

$$X = f_1\left(K_1, L_1\right) + f_2\left(K - K_1, L - L_1\right) \tag{478}$$

$$\frac{\partial X}{\partial K_1} = \frac{\partial f_1}{\partial K_1} + \frac{\partial f_2}{\partial K_2} = \frac{\partial f_1}{\partial K_1} - \frac{\partial f_2}{\partial K_2} = 0 \tag{479}$$

$$\frac{\partial X}{\partial L_1} = \frac{\partial f_1}{\partial L_1} + \frac{\partial f_2}{\partial L_2} = \frac{\partial f_1}{\partial L_1} - \frac{\partial f_2}{\partial L_2} = 0 \tag{480}$$

Marginal productivity of capital and labour inputs are same across both sectors:

$$\frac{\partial f_1}{\partial K_1} = \frac{\partial f_2}{\partial K_2} \tag{481}$$

$$\frac{\partial f_1}{\partial L_1} = \frac{\partial f_2}{\partial L_2} \tag{482}$$

18.3.3 Efficiency of Trade (Exchange)

If it is possible to increase welfare of one country without harming another country.

$$\left(MRS_{x,y} = \frac{U_x}{U_y} = \frac{P_x}{P_y}\right)_1 = ... = \left(MRS_{x,y} = \frac{U_x}{U_y} = \frac{P_x}{P_y}\right)_N \tag{483}$$

18.3.4 Efficiency in public goods

When $i = 1, ...N$ individuals live in a society then, the social marginal utility of public goods is the sum of the utilities from public good for individuals

$$SMU_P = SMU_P^1 + SMU_P^2 + ... + SMU_P^N \tag{484}$$

$$SMRS_{P,G} = \frac{SMU_P}{MU_G^i} = \frac{SMU_P^1}{MU_G^i} + \frac{SMU_P^2}{MU_G^i} + ... + \frac{SMU_P^N}{MU_G^i} \tag{485}$$

$$SMRS_{P,G} = RPT_{P,G} \tag{486}$$

Rate of transformation of private to public goods should equal the social rate of substitution of private to public goods.

19 L18: Externality

- What would happen to public parks if city councils do not maintain? Personal and social benefits of a beautiful garden?
- Why private market does not produce efficient amount of education and health?
- Why will market produce excessive amount of water, air or noise pollution?
- Why many cities in England are introducing congestion charges?

19.0.5 Positive Externality

A classic example of positive externality: bees pollinate apple trees and they get materials for honey from apples. For instance cost of producing apples is $C_a = a^2$ and cost of producing honey $C_h = h^2 - a$,

Private market solution

Firms maximise own profit independently:

$$\Pi_a = P_a a - a^2 \tag{487}$$

by marginal cost pricing rule and $\frac{\partial \Pi_a}{\partial a} = P_a - 2a = 0 \Longrightarrow P_a = 2a$ and hence supply of apples

$$a = \frac{P_a}{2} \tag{488}$$

Similarly

$$\Pi_h = P_h h - h^2 + a \tag{489}$$

Supply of honey by the private market
$\frac{\partial \Pi_h}{\partial h} = P_h - 2h = 0 \Longrightarrow P_h = 2h$ .and

$$h = \frac{P_h}{2}. \tag{490}$$

Private market does not consider positive externality.

Now consider a social planner that produces both to maximise joint profit:

$$\Pi = P_a a - a^2 + P_h h - h^2 + a \tag{491}$$

Then optimal apple supply is :
$\frac{\partial \Pi_a}{\partial a} = P_a - 2a + 1 = 0 \Longrightarrow P_a = 2a - 1$ .and

$$a = \frac{P_a}{2} + \frac{1}{2} \tag{492}$$

Optimal honey supply is
$\frac{\partial \Pi_h}{\partial h} = P_h - 2h = 0 \Longrightarrow P_h = 2h$ .and

$$h = \frac{P_h}{2} \tag{493}$$

It is optimal to produce more apples taking account of its positive externality.

19.0.6 Negative Externality

Negative externality production of electricity and pollution and food production

Electricity production using coal generates electricity as well as pollution. This pollution raises production cost in the food industry.

Cost of electricity production when the environment is not taken into account

$C_e = e^2 - (x-3)$ and its profit function is: $\Pi_e = P_e e - e^2 - (x-3)^2$

the cost of food production $C_f = f^2 + 2x$ and its profit $\Pi_f = P_f f - f^2 - 2x$. Pollution adds extra cost in food production.

Private market solution

$\Pi_e = P_e e - e^2 - (x-3)^2$
$\frac{\partial \Pi_e}{\partial e} = P_e - 2e = 0 \Longrightarrow P_e = 2e$ and hence supply of electricity

$$e = \frac{P_e}{2} \tag{494}$$

$\Pi_f = P_f f - f^2 - 2x$
$\frac{\partial \Pi_f}{\partial f} = P_f - 2f = 0 \Longrightarrow P_f = 2f$ and hence supply of electricity

$$f = \frac{P_f}{2} \tag{495}$$

Here pollution is produced more than optimal.

$$\frac{\partial \Pi_e}{\partial x} = 2(x-3) = 0 \Longrightarrow x = 3. \tag{496}$$

Socially optimal solution :

$$\Pi = \Pi_e + \Pi_e = P_e e - e^2 - (x-3)^2 + P_f f - f^2 - 2x \tag{497}$$

$\frac{\partial \Pi_e}{\partial e} = P_e - 2e = 0 \Longrightarrow P_e = 2e$ and hence supply of electricity

$$e = \frac{P_e}{2} \tag{498}$$

$\frac{\partial \Pi_f}{\partial f} = P_f - 2f = 0 \Longrightarrow P_f = 2f$ and hence supply of electricity

$$f = \frac{P_f}{2} \tag{499}$$

$$\frac{\partial \Pi}{\partial x} = 2(x-3) + 2 = 0 \Longrightarrow x = 2. \tag{500}$$

Social solution generates less pollution than the market solution.
A Pigovian tax on procucts generating negative externalities produces efficient soltuion.

19.0.7 Samuelson and Nash on Sharing Public Good

Consider a case where two friends share a public good $x = x_1 + x_2$ but consume private good y_i.

$$\max \; u_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} \tag{501}$$

subject to

$$10x_1 + y_1 = 300 \tag{502}$$

$$\mathcal{L}_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} - \lambda [300 - 10x_1 - y_1] \tag{503}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2}(x_1+x_2)^{-\frac{1}{2}} y_1^{\frac{1}{2}} - 10\lambda = 0 \tag{504}$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = \frac{1}{2}(x_1+x_2)^{\frac{1}{2}} y_1^{-\frac{1}{2}} - \lambda = 0 \tag{505}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 300 - 10x_1 - y_1 = 0 \tag{506}$$

From the first two FOC

$$\frac{y_1}{(x_1+x_2)} = 10 \Longrightarrow y_1 = 10(x_1+x_2) \tag{507}$$

Putting this y_1 back in the budget constraint
$300 - 10x_1 - y_1 = 300 - 10x_1 - 10(x_1+x_2) = 0$

$$x_1 = 15 - \frac{x_2}{2} \tag{508}$$

As the problem is symmetric dimilar proces for individual 2 we get

$$\mathcal{L}_2 = (x_1+x_2)^{\frac{1}{2}} y_2^{\frac{1}{2}} - \lambda\left[300 - 10x_2 - y_2\right] \tag{509}$$

$$x_2 = 15 - \frac{x_1}{2} \tag{510}$$

$$x_1 = 15 - \frac{x_2}{2} = 15 - \frac{1}{2}\left(15 - \frac{x_1}{2}\right) \Longrightarrow x_1 = \frac{4}{3}\frac{1}{2} \times 15 = 10 \tag{511}$$

$$x_1 = x_2 = 10 \Longrightarrow x_1 + x_2 = 20 \tag{512}$$

$$y_1 = 10(x_1+x_2) = 10 \times 20 = 200 \tag{513}$$

Utility level in non-cooperative Nash scenario is:

$$u_1 = (x_1+x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} = (20)^{\frac{1}{2}} 200^{\frac{1}{2}} = 63.2 = u_2 \tag{514}$$

Under the Samuelsonian rule

$$\frac{\frac{\partial \mathcal{L}}{\partial x_1}}{\frac{\partial \mathcal{L}}{\partial y_1}} + \frac{\frac{\partial \mathcal{L}}{\partial x_2}}{\frac{\partial \mathcal{L}}{\partial y_2}} = MRS_1 + MRS_2 = MRT \tag{515}$$

$$\frac{\frac{1}{2}(x_1+x_2)^{-\frac{1}{2}} y_1^{\frac{1}{2}}}{\frac{1}{2}(x_1+x_2)^{\frac{1}{2}} y_1^{-\frac{1}{2}}} + \frac{\frac{1}{2}(x_1+x_2)^{-\frac{1}{2}} y_2^{\frac{1}{2}}}{\frac{1}{2}(x_1+x_2)^{\frac{1}{2}} y_2^{-\frac{1}{2}}} = \frac{y_1}{x} + \frac{y_2}{x} = \frac{p_x}{p_y} = \frac{10}{1} = 10 \tag{516}$$

$$y_1 + y_2 = 10x \tag{517}$$

Combined budget constraint of both persons:

$$10x + y_1 + y_2 = 600 \tag{518}$$

$$10x + 10x = 600 \Longrightarrow x = 30 \tag{519}$$

$$y_1 + y_2 = 10x = 10 \times 30 = 300 \tag{520}$$

If the private good is equally devided each gets 150.

$$u_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} = (30)^{\frac{1}{2}} 150^{\frac{1}{2}} = 67.1 = u_2 \tag{521}$$

Table 36: Inefficienty of competitive equilibrium in case of positive externality

	Nash (CE)	Optimal (Samuelson)
x	20	30
y_1	200	150
y_2	200	150
u	63.2	67.1

Key questions: Pollution controls are less important in developing countries such as China and India. How does it affect the global environment?

Readings: VAR 35

19.0.8 Sameulson's Theorem on Public Good

Provision of public goods: two consumers, and public and private goods, but not clear how they should pay for it; valuation of each person is different.

Proposition: Pareto optimality requires that sum of the marginal rate of substitution between private and public goods by two individuals should equal the marginal cost of provision of public goods (see two citizen public good model).

Consumers consume private (x) and public goods (G)

$$max \quad u_1 = u_1(x_1, G) \tag{522}$$

subject to a given level of utility for the second consumer

$$max \quad \overline{u}_2 = u_2(x_2, G) \tag{523}$$

and the resource contraint

$$x_1 + x_2 + c\,(G) = w_1 + w_2 \tag{524}$$

Constrained optimisation for this is

$$L = u_1(x_1, G) - \lambda\,[\overline{u}_2 - u_2(x_2, G)] - \mu\,[x_1 + x_2 + c\,(G) - w_1 - w_2] \tag{525}$$

$\frac{\partial L}{\partial x_1} = \frac{\partial u_1(x_1,G)}{\partial x_1} - \mu = 0 \Longrightarrow \mu = \frac{\partial u_1(x_1,G)}{\partial x_1};; \frac{\partial L}{\partial x_1} = -\lambda \frac{\partial u_2(x_2,G)}{\partial x2} - \mu = 0 \Longrightarrow -\frac{\partial u_2(x_2,G)}{\partial x2} = \frac{\mu}{\lambda}$

$$\frac{\partial L}{\partial G}=\frac{\partial u_1(x_1,G)}{\partial G}+\lambda\frac{\partial u_2(x_2,G)}{\partial G}-\mu\frac{\partial c(G)}{\partial G}=0;\Longrightarrow\frac{1}{\mu}\frac{\partial u_1(x_1,G)}{\partial G}-\frac{\lambda}{\mu}\frac{\partial u_2(x_2,G)}{\partial G}=\frac{\partial c(G)}{\partial G}$$

$$\frac{\frac{\partial u_1(x_1,G)}{\partial G}}{\frac{\partial u_1(x_1,G)}{\partial x_1}}+\frac{\frac{\partial u_2(x_2,G)}{\partial G}}{\frac{\partial u_2(x_2,G)}{\partial x_2}}=\frac{\partial c\left(G\right)}{\partial G}; \qquad MRS_1+MRS_2=MC(G)...Q.E.D. \tag{526}$$

19.0.9 Theory of second best

When the optimal point is not achievable, other points in the efficiency frontier are not necessarily optimal. (draw a diagram to prove).

20 L19: Social Welfare Function and Redistribution

Social welfare function is aggregation of individual welfare functions. There are mainly three theories on how to aggregate to the social welfare functions from the individual welfare functions. First is the classical social welfare function proposed by Bentham (1832) in his book Introduction of Principles of Morals and Legislation. It involves sum of utilities of individual utilities (U_i, $i=1,...,N$) to get a social welfare function (W):

$$W=U_1+U_2+.....+U_N$$

It is popular as utilitarian approach to social welfare function. The second and more recent approach is due to Rawls (1971) in his book A Theory of Justice.

$$W=\min\left(U_1,U_2,.....,U_N\right)$$

The development of a society should be measured in terms of the living standard of the least well-off individual of the society but not from the sum of the utilities as proposed by Bentham.

Then the third approach to the social welfare function is explained in terms of the Arrow's impossibility theorem.

Arrow (1951) and Sen (1968) have and others have popularised social choice theory.

Social Choice and Arrows Impossibility Theorem Let there be $i=1$, N individuals in the society and each one of their ranking about alternatives (for allocating limited resources) is given by $R_1,.R_2,.....,$ R_N. Then the social welfare function F is defined as

$$R=F(R_1,.R_2,....., \quad R_N)$$

This F function possesses four properties: U P I D; Preferences relations are complete and transitive.

U: unrestricted domain, every one is included in it (Complete).

P: Pareto optimality, if $x\succ y$ by each individual then society should also prefer it.

I: Independence of irrelevant alternatives, if the choice is between x and y then another choice say z should not matter for choices between x and y.

D: No dictatorship; no single individual should determine the outcome of the society.

Arrow proved that it is impossible to satisfy all above conditions.

Let there be social states A, B, and C. Each individual in the society can rank these states according to their desirability. Does society wide scale exist to record these individual preferences. Suppose Society consists of two individuals Smith and Jonnes.

Smith's preference AP_sB and BP_sC then rationality or transitivity implies AP_sC.

Jones preferences CP_JA and AP_JB then rationality or transitivity implies CP_JB.

Since BP_sC and CP_JB it must be that $(C\ I\ B)$. But this will violate no dictator conditions see AP_sB, AP_sC but CP_JB. Therefore no social welfare function exists that fulfills all six axioms (U, P, I and D conditions).

Condorcet Paradox of majority voting Let three students, A ,B and C form a society. Let their 1st, and 3rd ranking over three alternatives x, y and z (reading, writing, speaking) be as follows.

ranks	A	B	C
1st	x	y	z
2nd	y	z	x
3rd	z	x	y

for A $x \succ y \succ z$ for B $y \succ z \succ x$ for C $z \succ x \succ y$

Individually choice of each of A, B, C is transitive but it is not for the society; x wins between options x and y; then y beats z between y and z; and finally z wins y between y and z. While pairwise $x \succ y$; $y \succ z$ transitivity implies $x \succ z$ for A but $z \succ x$ for individual C. $x \succ z$ first then $z \succ x$. This is called Condorcet paradox of voting.

Social Welfare Function: a problem There are two people living in an economy. For simplicity assume that a fixed amount of output of 200 is produced each year. Entire output is consumed in the same year. Utility of individual 1 and 2 is represented by $U_1 = \sqrt{Y_1}$ and $U_2 = \frac{1}{2}\sqrt{Y_1}$.

1. (a) What is the utility received by each individual if the output is divided equally between these two people? What is the output received by each if it is distributed so that each of them gets the same amount of the utility?

   (b) What is the distribution of output that maximises the total utility for the whole economy?

   (c) If person 2 needs utility 5 in order to survive how should the output be distributed?

   (d) Suppose that the authorities like to maximise the social welfare function $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$, how should the output be distributed between them?

2. (a) An economy is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. They aim to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. Preferences for type 1 are given by $U_1 = \sqrt{Y_1}$and for type 2 by $U_2 = \sqrt{Y_2}$. In a given year total output, Y, was 1000 billion pounds.

   (b) What is the distribution of output between type 1 and type 2 that maximises the social welfare index? What is the maximum value of the social welfare index of this economy?

(c) What would have been the allocation if policy makers had given equal weight to the utility of both types of people in the economy such as $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$. By how much does the welfare index change in this case than compared to the social welfare in (a) above?

(d) How would the social welfare index change in (a) if a tax rate of 20 percent is imposed in consumption and the tax receipts are not given back to any of these consumers? How much would the value of social welfare index be in this case?

e. Assume that the policy makers still hold the welfare function to be $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$. How would the social welfare index change in (c) if all tax receipts are transferred to type 2 people?

20.1 Social Welfare Function

Distribution of income can be a result of the social choice. If the policy makers assign different weights to utility of different types of individuals in the economy it results in patter of income distribution that is different when policy makers tread every individuals equally. In general it is good to reward more productive workers than to lazier one. For instance, consider an economy that is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. Let us assume that they want to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. Utility of more productive type is three times worth more than less productive ones. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. If the preferences for type 1 are given by $U_1 = \sqrt{Y_1}$ and for type 2 by $U_2 = \sqrt{Y_2}$ and the total output, Y, is 1000 billion pounds. Four scenarios are considered and the optimal allocations and social welfare are presented in Tables below.

Table 37: Parameters in consumption of the two sector model

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 1		
	Income	Utility	U. function: $\sqrt{Y_i}$
Type 1 individuals	750	$\sqrt{750} = 27.4$	$\sqrt{Y_1} = \sqrt{750} = 27.4$
Type 2 individuals	250	$\sqrt{250} = 15.8$	$\sqrt{Y_2} = \sqrt{250} = 15.8$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 27.4^{\frac{3}{4}} \times 15.8^{\frac{1}{4}} = 23.9$		
	Y=1000; $\alpha_1 = \frac{1}{2}$; $\alpha_2 = \frac{1}{2}$; Economy 2		
Type 1 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_1} = \sqrt{500} = 22.4$
Type 2 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_2} = \sqrt{500} = 12.8$
Social Welfare	$W = (U_1)^{\frac{1}{2}} (U_2)^{\frac{1}{2}} = 22.4^{\frac{1}{2}} \times 22.4^{\frac{1}{2}} = 22.4$		

Let us consider four scenarios of social welfare. It is maximized at 23.9 when policy makers put weight $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$ and there are not taxes. Social welfare index diminishes to 22.2 in economy 2 where policy makers put equal weight to productive and non-productive workers. Social welfare decreases even further to 21.1 if 20 percent tax is imposed and no transfer is returned any of these

Table 38: Parameters in consumption of the two sector model

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 3 (20 percent tax away)		
	Income	Utility	U. function: $\sqrt{Y_i}$
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	250	$\sqrt{0.8 \times 250} = 14.1$	$\sqrt{Y_2} = \sqrt{200} = 14.1$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 24.4^{\frac{3}{4}} \times 14.4^{\frac{1}{4}} = 21.3$		
	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 3 (Tax revenue to poor)		
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	400	$\sqrt{400} = 20$	$\sqrt{Y_2} = \sqrt{400} = 20$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 24.4^{\frac{3}{4}} \times 20^{\frac{1}{4}} = 23.2$		

households. It slightly improves to 22.4 if all tax revenue is given back to the poor household. Tax economy is Pareto inferior to the no tax economy. More elaborated analysis is in Bhattarai, Haughton and Tuerck (2015). This is more comprehensive theory of income distribution and welfare that can accommodate wide ranging concerns relating to social justice and inequality.

Social welfare with inequal weights in utilities
Preferences for type 1 are given by $U_1 = \sqrt{Y_1}$and for type 2 by $U_2 = \sqrt{Y_2}$.
Social welfare function to be $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$.
Total production of the economy $1000 - Y_1 - Y_2$

$$L = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}} + \lambda \left[1000 - Y_1 - Y_2\right] = \left(\sqrt{Y_1}\right)^{\frac{3}{4}} \left(\sqrt{Y_2}\right)^{\frac{1}{4}} - \lambda \left[1000 - Y_1 - Y_2\right] \tag{527}$$

$$L = Y_1^{\frac{3}{8}} Y_2^{\frac{1}{8}} - \lambda \left[1000 - Y_1 - Y_2\right] \tag{528}$$

Social welfare with inequal weights in utilities

$$\frac{\partial L}{\partial Y_1} = \frac{3}{8} Y_1^{-\frac{5}{8}} Y_2^{\frac{1}{8}} - \lambda = 0 \tag{529}$$

$$\frac{\partial L}{\partial Y_2} = \frac{1}{8} Y_1^{\frac{3}{8}} Y_2^{-\frac{7}{8}} - \lambda = 0 \tag{530}$$

$$\frac{\partial L}{\partial \lambda} = 1000 - Y_1 - Y_2 = 0 \tag{531}$$

$$\frac{3}{8} Y_1^{-\frac{5}{8}} Y_2^{\frac{1}{8}} = \frac{1}{8} Y_1^{\frac{3}{8}} Y_2^{-\frac{7}{8}} \tag{532}$$

$$3Y_2 = Y_1 \tag{533}$$

Social welfare with inequal weights in utilities

$$1000 = Y_1 + Y_2 = 3Y_2 + Y_2 \Longrightarrow Y_2 = \frac{1000}{4} = 250 \tag{534}$$

$$Y_1 = 3Y_2 = 3\,(250) = 750 \tag{535}$$

Index of social welfare in this economy is

$$W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}} = 750^{\frac{3}{8}} 250^{\frac{1}{8}} = 23.9 \tag{536}$$

Social welfare with equal weights in utilities

$$L = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}} + \lambda\,[1000 - Y_1 - Y_2] = \left(\sqrt{Y_1}\right)^{\frac{1}{1}} \left(\sqrt{Y_2}\right)^{\frac{1}{2}} - \lambda\,[1000 - Y_1 - Y_2] \tag{537}$$

$$L = Y_1^{\frac{1}{4}} Y_2^{\frac{1}{4}} - \lambda\,[1000 - Y_1 - Y_2] \tag{538}$$

$$\frac{\partial L}{\partial Y_1} = \frac{1}{4} Y_1^{-\frac{3}{4}} Y_2^{\frac{1}{4}} - \lambda = 0 \tag{539}$$

$$\frac{\partial L}{\partial Y_2} = \frac{1}{4} Y_1^{\frac{1}{4}} Y_2^{-\frac{3}{4}} - \lambda = 0 \tag{540}$$

Social welfare with equal weights in utilities

$$\frac{\partial L}{\partial \lambda} = 1000 - Y_1 - Y_2 = 0 \tag{541}$$

$$\frac{1}{4} Y_1^{-\frac{3}{4}} Y_2^{\frac{1}{4}} = \frac{1}{4} Y_1^{\frac{1}{4}} Y_2^{-\frac{3}{4}} \tag{542}$$

$$Y_2 = Y_1 \tag{543}$$

$$1000 = Y_1 + Y_2 \Longrightarrow Y_2 = \frac{1000}{2} = 500 \tag{544}$$

$$Y_1 = Y_2 = 500 \tag{545}$$

Index of social welfare in this economy is

$$W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}} = 500^{\frac{1}{4}} 500^{\frac{1}{4}} = 22.4 \tag{546}$$

Social welfare with non-recycling of tax

$$L = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}} + \lambda\,[1000 - Y_1 - Y_2] = \left(\sqrt{0.8Y_1}\right)^{\frac{3}{4}} \left(\sqrt{0.8Y_2}\right)^{\frac{1}{4}} - \lambda\,[1000 - Y_1 - Y_2] \tag{547}$$

$$L = 0.8 \times Y_1^{\frac{3}{8}} Y_2^{\frac{1}{8}} - \lambda\,[1000 - Y_1 - Y_2] \tag{548}$$

$$\frac{\partial L}{\partial Y_1} = 0.8 \times \frac{3}{8} Y_1^{-\frac{5}{8}} Y_2^{\frac{1}{8}} - \lambda = 0 \tag{549}$$

$$\frac{\partial L}{\partial Y_2} = 0.8 \times \frac{1}{8} Y_1^{\frac{3}{8}} Y_2^{-\frac{7}{8}} - \lambda = 0 \tag{550}$$

$$\frac{\partial L}{\partial \lambda} = 1000 - Y_1 - Y_2 = 0 \tag{551}$$

Social welfare with non-recycling of tax

$$0.8 \times \frac{3}{8} Y_1^{-\frac{5}{8}} Y_2^{\frac{1}{8}} = 0.8 \times \frac{1}{8} Y_1^{\frac{3}{8}} Y_2^{-\frac{7}{8}} \tag{552}$$

$$3Y_2 = Y_1 \tag{553}$$

$$1000 = Y_1 + Y_2 = 3Y_2 + Y_2 \Longrightarrow Y_2 = \frac{1000}{4} = 250 \tag{554}$$

$$Y_1 = 3Y_2 = 3\,(250) = 750 \tag{555}$$

Index of social welfare in this economy is

$$W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}} = (0.8 \times 750)^{\frac{3}{8}} (0.8 \times 250)^{\frac{1}{8}} = (600)^{\frac{3}{8}} (200)^{\frac{1}{8}} = 21.4 \tag{556}$$

Social welfare with non-recycling of tax
If all tax is given to person 2.

$$Y_1 = 600; \; Y_2 = 400 \tag{557}$$

$$W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}} = 600^{\frac{3}{8}} 400^{\frac{1}{8}} = 23.3 \tag{558}$$

20.1.1 Social Welfare Function

Distribution of income can be a result of the social choice. If the policy makers assign different weights to utility of different types of individuals in the economy it results in patter of income distribution that is different when policy makers tread every individuals equally. In general it is good to reward more productive workers than to lazier one. For instance, consider an economy that is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. Let us assume that they want to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. Utility of more productive type is three times worth more than less productive ones. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. If the preferences for type 1 are given by $U_1 = \sqrt{Y_1}$and for type 2 by $U_2 = \sqrt{Y_2}$ and the total output, Y, is 1000 billion pounds. Four scenarios are considered and the optimal allocations and social welfare are presented in Tables below.

Let us consider four scenarios of social welfare. It is maximized at 23.9 when policy makers put weight $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$ and there are not taxes. Social welfare index diminishes to 22.2 in economy

Table 39: Parameters in consumption of the two sector model

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 1		
	Income	Utility	U. function: $\sqrt{Y_i}$
Type 1 individuals	750	$\sqrt{750} = 27.4$	$\sqrt{Y_1} = \sqrt{750} = 27.4$
Type 2 individuals	250	$\sqrt{250} = 15.8$	$\sqrt{Y_2} = \sqrt{250} = 15.8$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 27.4^{\frac{3}{4}} \times 15.8^{\frac{1}{4}} = 23.9$		
	Y=1000; $\alpha_1 = \frac{1}{2}$; $\alpha_2 = \frac{1}{2}$; Economy 2		
Type 1 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_1} = \sqrt{500} = 22.4$
Type 2 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_2} = \sqrt{500} = 12.8$
Social Welfare	$W = (U_1)^{\frac{1}{2}} (U_2)^{\frac{1}{2}} = 22.4^{\frac{1}{2}} \times 22.4^{\frac{1}{2}} = 22.4$		

Table 40: Parameters in consumption of the two sector model

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 3 (20 percent tax away)		
	Income	Utility	U. function: $\sqrt{Y_i}$
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	250	$\sqrt{0.8 \times 250} = 14.1$	$\sqrt{Y_2} = \sqrt{200} = 14.1$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 24.4^{\frac{3}{4}} \times 14.4^{\frac{1}{4}} = 21.3$		
	Y=1000; $\alpha_1 = \frac{3}{4}$; $\alpha_2 = \frac{1}{4}$; Economy 3 (Tax revenue to poor)		
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	400	$\sqrt{400} = 20$	$\sqrt{Y_2} = \sqrt{400} = 20$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 24.4^{\frac{3}{4}} \times 20^{\frac{1}{4}} = 23.2$		

2 where policy makers put equal weight to productive and non-productive workers. Social welfare decreases even further to 21.1 if 20 percent tax is imposed and no transfer is returned any of these households. It slightly improves to 22.4 if all tax revenue is given back to the poor household. Tax economy is Pareto inferior to the no tax economy. More elaborated analysis is in Bhattarai, Haughton and Tuerck (2015). This is more comprehensive theory of income distribution and welfare that can accommodate wide ranging concerns relating to social justice and inequality.

20.2 Redistribution: Why is the Share of Labour declining?

Who benefits from the process of economic growth? Do low income people (or poor) benefit as much as rich people? Do capitalists gain more than workers? These issues of size and functional distribution of income is discussed widely in the literature on distribution of income (Picketty (2014), Jenkins (1995), Atkinson (1970), Kuznet (1955)). Share of labour income has gradually declined in the global economy (Figure 1) and as discussed in Karabarbounis and Neiman (2014). Bachman et al. (2016) apply a dynamic CGE model of the US economy to investigate how tax and transfer system affects the distribution of income in the US economy under Trump and Cruz tax-transfer scenarios and find the share of labour income not only depends on tax and transfer system but also on the substitutability of capital and labour in production.

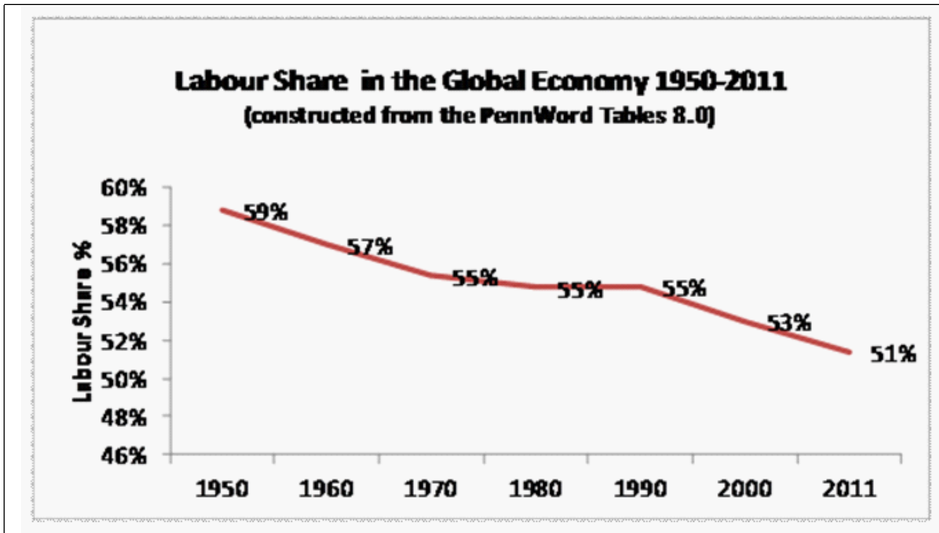


Figure 1: Share of labour income in the global economy

Gini coefficient an indicator of the inequality of income. Its value range between zero and one; zero for perfect equality and one for perfect inequality. In majority of countries in the Western Europe it has increased from around 0.29 in 1960 to 0.39 in recent years mainly because of declining share of labour. OECD (2015) provides a time series evidence on growing inequality among the OECD and emerging economies and its adverse impacts on economic growth (Figure 2).

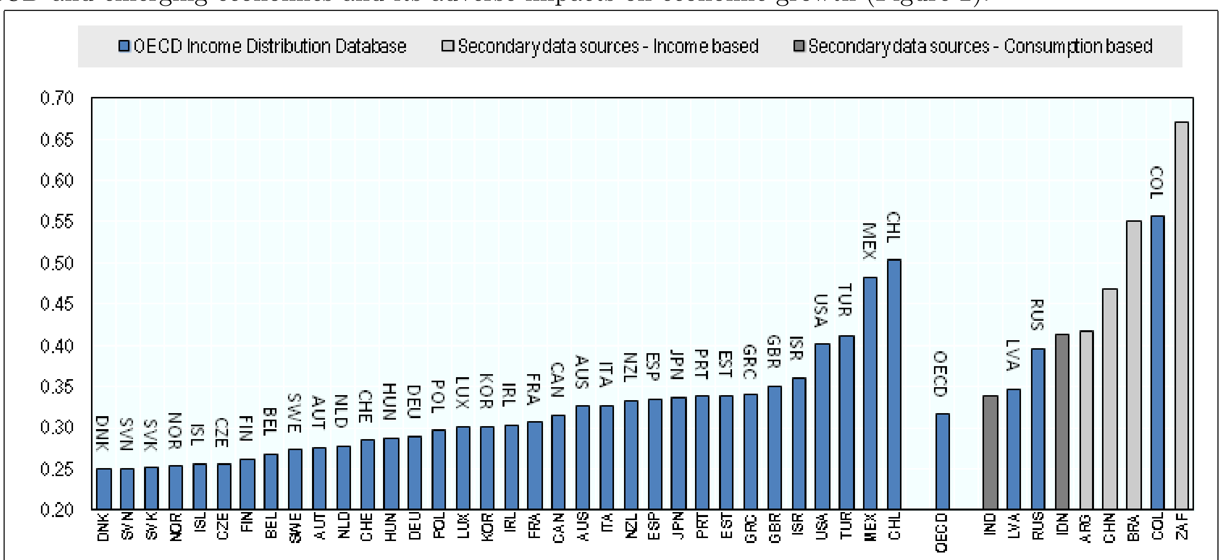


Figure 2: Gini coefficients among the OECD and emerging economies (OECD)

Should there be more redistribution or more growth? It is obvious that no one solution fits to all circumstances. Institutions and culture vary by countries as do the endowments of labour and capital as well as of the natural resources. Policies should be designed according to economic and social institutions of a country (see debates in the World Economic Forum in Davos, http://www.weforum.org/). See Penn World Tables for the dataset at:

http://www.rug.nl/research/ggdc/data/penn-world-table. Picketty suggests global tax on property to reduce such inequality. This idea is close to socialists' approach to income distribution. Capitalists would argue for more equality by raising productivity of workers through additional accumulation of physical capital, development of human capital by investing in education and skills. They favour taxes on consumption rather than in income, indirect tax than the direct tax.

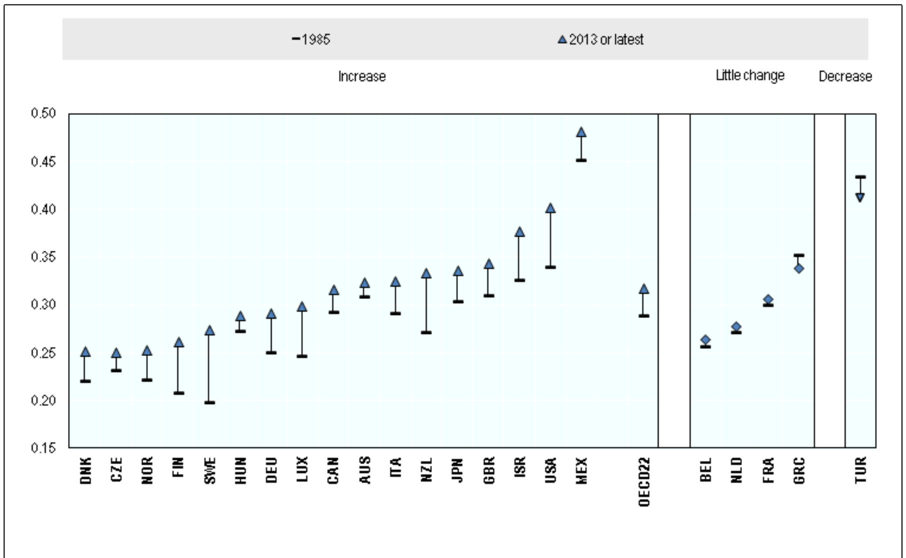


Figure 3: Rising Gini coefficients among the OECD countries (OECD)

Objective of this paper is to provide a general overview of these theories with recent evidences on the share of labour in income and to show how these theories enhance our understanding on the role of income distribution in production, consumption and social welfare in an economy.

20.3 Theories on share of labour in total income

The shares of capital and labour in national income vary considerably both over time and across countries. Picketty (2014) has formed dataset on income distribution between capital and labour for last 200 years for the advanced countries of Europe as well as the US (see http://topincomes.parisschoolofeconomics.eu/). He found that trade openness and technological innovation have a positive and significant effect on labour shares. Similarly foreign direct investments (FDI) inflows and mechanization seem to be negative drivers in it . He also looks into a number of variables including the level of economic development, education, and the strength of the regulations in the labour market.

Share of labour in income was an important issue in the functional distribution of income. It was widely discussed by classical and neoclassical economists, Marx, Kaldor, Hansen, Hahn, Hicks in terms of marginal productivity theory of distribution. Factors are paid according their marginal productivity in their theories. Recently there are new theories of bargaining of income and wages (Mortensen and Pissarides (1994)). Bhattarai, Haughton and Tuerck (2015a, 2015b) find significant impact of fair taxes and corporate income taxes on growth and distribution of income in the US economy.

It was believed that the share of labour was relatively constant, between 60 to 70 percents of GDP up to 1980s (Parente and Prescott (2002)). However many recent studies find that there is a general pattern of reduction in the share of GDP going to labour around the world, in particular from the mid-1980s onwards as shown in Figure 1 above. Seminal works on labour share are include Hicks (1932), Kuznet (1955), Kennedy (1964), Hahn (1972), Cowling, Molho and Oswald (1981), Lavoie and Stockhammer (2014), O'Mahony and Timmer (2009), Stockhammer, Onaran and Ederer (2009), Stockhammer and Onaran (2004) and Elsby, Hobijn, Şahin (2013) and Picketty (2014). What is the optimal amount labour share $(1-\alpha)$ that maximises the economic growth? This issue is yet far from settled. A sort summary on important theories is provided here for a concise understanding of these topics.

20.4 Neoclassical theory of functional distribution of income

The neoclassical theory of functional distribution can simply be represented in a employment-wage diagram. Market sets the wage rate (w) where the demand for labour intersects to the supply of labour. Area of rectangle represents wage bill. The area below the demand curve is part of production that goes to employers as profit.

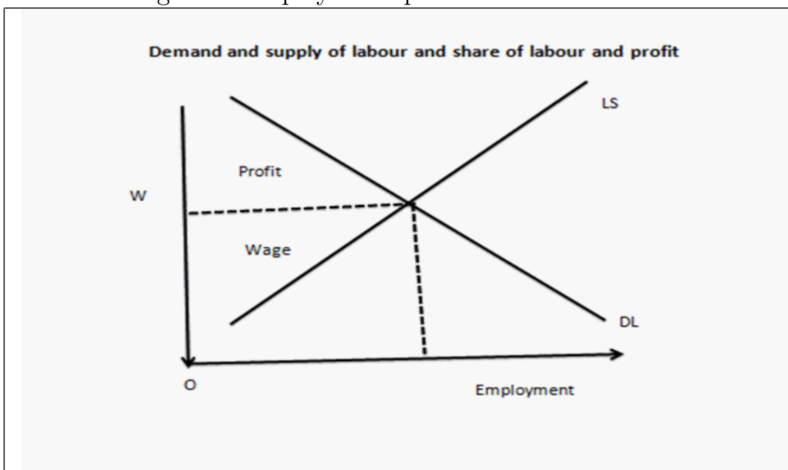


Figure 3: Wage and profit in the neoclassical distribution model

Marginal productivity declines with more employment due to the law of diminishing returns to labour. Upward sloping labour supply function represents psychological and other costs as working more hours becomes more difficult. Workers are ready to put in more efforts only if they are paid more. The equilibrium point shown by the intersection between the demand and supply of labour also shows the functional distribution of income in wages and profits. It is obvious when profits are high when wages are low. There are always incentives among owners to pay low wage in order to minimise cost and to raise their profits. Given the α and β shares of capital and labour in total output (Y), division of national income between capital (rK) and labour (wL) occurs according to the marginal productivity of these inputs as shown in following equations:

$$\frac{Y}{A} = K^{\alpha} L^{\beta} \Longrightarrow Y = AK^{\alpha} L^{\beta} \tag{559}$$

$$Y = rK + wL \Longrightarrow \frac{rK}{Y} + \frac{wL}{Y} = 1 \tag{560}$$

$$\frac{rK}{Y}+\frac{wL}{Y}=\frac{\alpha AK^{\alpha-1}L^{\beta}K}{AK^{\alpha}L^{\beta}}+\frac{\beta AK^{\alpha}L^{\beta-1}L}{AK^{\alpha}L^{\beta}}=(\alpha+\beta)=1 \tag{561}$$

Technology of production is characterised by the elasticity of substitution (σ) between labour and capital that measures the degree of response of capital labour ratio to the wage rental ratios. In a simple Cobb-Douglas function this elasticity of substitution is 1.

$$\sigma=\frac{d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)}{d\left(\frac{w}{r}\right)/\left(\frac{w}{r}\right)}=\frac{d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)}{d\left(\frac{(1-\alpha)AK^{\alpha}L^{-\alpha}}{\alpha AK^{\alpha-1}L^{1-\alpha}}\right)/\left(\frac{(1-\alpha)AK^{\alpha}L^{-\alpha}}{\alpha AK^{\alpha-1}L^{1-\alpha}}\right)}=\frac{d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)}{d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)}=1 \tag{562}$$

A CES production function allows other values of σ , a higher value of σ represents more capital intensive technology and a lower value of σ indicates more labour intensive technology. Productivity of labour rises with more capital, whether wage rate rises depends on how the distribution of income occurs between capital and labour. Thus rising income inequality among the global economy reflects more capital intensity production and more return to capital than to the labour. Increase in human capital through education can enhance the labour share and more investment in education and skills is important in raising the value of β, share of labour in the national income.

20.4.1 Investors, marginal productivity of capital and tax credit

$$\Pi=\frac{F(K)}{(1+r)}-P_1^K K+\frac{(1-\delta)\,P_2^K K}{(1+r)} \tag{563}$$

$$\frac{\partial\Pi}{\partial K}=\frac{F'(K)}{(1+r)}-P_1^K+\frac{(1-\delta)\,P_2^K}{(1+r)}=0 \tag{564}$$

$$MPK=(1+r)\,P_1^K-(1-\delta)\,P_2^K=0 \tag{565}$$

$$MPK=\left[(1+r)-(1-\delta)\left(1+\pi^k\right)\right]P_1^K \tag{566}$$

$$MPK\simeq\left[r+\delta-\pi^k\right]P_1^K \tag{567}$$

Investors, marginal productivity of capital and tax credit: Problem of a Car Company

A car manufacturer sells each car at 8000 and pays 2000 for capital equipment per car. The nominal interest rate is 6%, appreciation of value of capital stock (capital gain) is 3% and the depreciation of capital stock is 3% per year.

The production function for this company is given by with $.Y=K^{\alpha}$ and $\alpha=0.75$

What is the optimal capital stock for this manufacturer? (hint).

$$MRPK=P.\alpha K^{\alpha-1}\simeq\left[r+\delta-\pi^k\right]P_1^K \tag{568}$$

$$8000.\,(0.75)\,K^{0.5-1}\simeq[0.06+0.03-0.03]\cdot 2000 \tag{569}$$

solve for K

$$6000.K^{-0.25}\simeq[0.06]\cdot 2000 \tag{570}$$

$$K = \left(\frac{3}{0.06}\right)^4 = 50^4 = 6,250,000 \quad (571)$$

Show how the investment tax credit affect the optimal capital stock employed by a firm using an appropriate diagram.

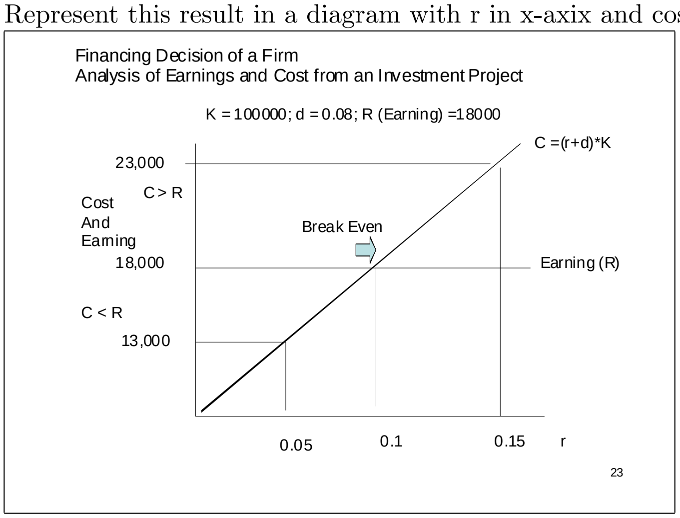
20.4.2 Investment problem

A certain project costs 100,000. This project brings annual earning equal to 18000. Depreciation rate is 8% and the market interest vary as shown below? When is this investment profitable?

Table 41: Discount factor over time

	$Scen1$	$Scen2$	$Scen3$
r	0.05	0.1	0.15
δ	0.08	0.08	0.08
Cost	13000	18000	23000
Revenue	18000	18000	18000
	5000	0	-5000

Represent this result in a diagram with r in x-axix and cost and revenue in y-axis.



Return on a portfolio (example based on Martin Anthony Mathematics for Economics and Finance, Oxford University Press).

- Consider a portfolio of three assets

$$Y = (y_1, y_2 \quad y_3) \quad (572)$$

$$Y = \begin{pmatrix} 5000 & 1000 & 4000 \end{pmatrix} \quad (573)$$

- Return on two states of nature on each asset

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \\ r_{3,1} & r_{3,2} \end{bmatrix} = \begin{bmatrix} 1.25 & 0.95 \\ 1.05 & 1.05 \\ 0.9 & 1.15 \end{bmatrix} \tag{574}$$

Return on Portfolio

$$\begin{aligned} Y \cdot R &= (y_1, y_2 \;\; y_3) \begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \\ r_{3,1} & r_{3,2} \end{bmatrix} \\ &= \begin{pmatrix} 5000 & 1000 & 4000 \end{pmatrix} \begin{bmatrix} 1.25 & 0.95 \\ 1.05 & 1.05 \\ 0.9 & 1.15 \end{bmatrix} \\ &= (10900, \; 10400) \end{aligned} \tag{575}$$

Arbitrage Portfolio

$$Y = \begin{pmatrix} 5000 & \text{-10000} & 5000 \end{pmatrix} \tag{576}$$

$$Y \cdot R = \begin{pmatrix} 5000 & \text{-10000} & 5000 \end{pmatrix} \begin{bmatrix} 1.25 & 0.95 \\ 1.05 & 1.05 \\ 0.9 & 1.15 \end{bmatrix} = = (250, \; 0) \tag{577}$$

Which of the following portfolios is better?
Y is borrow from bank and invest in two assets

$$Y = \begin{pmatrix} 1000 & \text{-5000} & 4000 \end{pmatrix} \tag{578}$$

$$Z = \begin{pmatrix} 0 & \text{-5000} & 4000 \end{pmatrix} \tag{579}$$

When the state contingent returns are given by

$$R = \begin{bmatrix} 0.95 & 0.90 & 1.0 \\ 1.1 & 1.1 & 1.1 \\ 1.2 & 1.15 & 1.25 \end{bmatrix} \tag{580}$$

$$Y \cdot R = \begin{pmatrix} 250 & 0 & 500 \end{pmatrix} \tag{581}$$

$$Z \cdot R = \begin{pmatrix} 500 & 250 & 750 \end{pmatrix} \tag{582}$$

Obviously Z portfolio is better than Y.

20.4.3 Essentials of capital asset price model

Cost of capital to a firm (r) varies according to risk it has

$$r = r_f + \beta \left(r_m - r_f \right) \tag{583}$$

r_f = risk free rate (i e. treasury bill rate); r_m = return on portfolio; $\beta \left(r_m - r_f \right)$ = risk premium

$$\beta = \frac{\sigma_{i,m}}{\sigma_m^2} \tag{584}$$

β varies enormously among companies and over time.

Gains from picking a certain stock $= \left(r - r_f \right) - \beta \left(r_m - r_f \right)$

Risk is measured by the variance of return $\left(\sigma^2 \right) = E \left(\overline{r} - r \right)^2$

Essence of Financial system: Intertemporal Balance of Households, Firms, Government and the Economy

20.5 Marxian theory the surplus value (S)

The notion of surplus value (S) is a key concept in the Marxian theory. This theory attributes that all value is created by labour. Only labour generates value. Capital is made by labour in the past. The capitalists own the capital and they pay only the subsistence wage to the labour. Each unit of labour creates more output than requires for its subsistence but the surplus value, the gap between the output and wage. goes to capitalists who own the firm and employ the labour. In other words the surplus value (S) per unit of output represents the amount by which price (P) of a commodity is above the wage (W):

$$S = P - W$$

Wage share in output is denoted simply by the ratio wage to the price ($\frac{W}{P}$) is the output minus the the share of the surplus value $s = \frac{S}{P}$. Thus:

$$\frac{W}{P} = (1 - s); \quad s = \frac{S}{P}$$

Capitalists squeeze on wages by increasing the surplus value ratio (s); more strictly following the iron law of wage. More is the surplus value less is paid to the labour. More unequal becomes the income distribution. Labour produces more than the iron wage requited for its subsistence but the capitalists do not pay more that what is necessary for survival. Development of economies in advanced economies avoided class between capitalists and workers by adopting more egalitarian social security system funded by the tax-transfer system and provision for pensions and tax credits to low income groups in the last century, particularly after the World War II. Waves privatisation and deregulation since mid 1980s have gradually caused increases in the surplus value and reductions in the share of labour. This has resulted in significant increase in inequality in each country among the OECD and other economies as shown in Figure 1 - 3 above.

21 Competitive and Monopsonistic Labour Market

In equilibrium labour demand should be equal to labour supply

$L^D = L^S \Longrightarrow 450 - 50\left(\frac{w}{p}\right) = 100\left(\frac{w}{p}\right)$

$\frac{w}{p} = 3 \quad L^D = L^S = 300$

Now if the minimum real wage rate is fixed at 4 then the labour demand will fall to $L^D = 450 - 50\left(\frac{w}{p}\right) = 450 - 50\,(4) = 250$; Labour supply will increase to $L^S = 100\left(\frac{w}{p}\right) = 100\,(4) = 400$; When labour demand is only 250 wage rate from the labour supply curve is $450 - 50\left(\frac{w}{p}\right) = 400$ $\Longrightarrow \left(\frac{w}{p}\right) = 1$. Therefore government has to pay subsidy of 3 for each furloughed worker.

If the minimum real wage is set at $\left(\frac{w}{p}\right) = 4$; the demand for labour will be $L^D = 450 - 50\left(\frac{w}{p}\right) = 450 - 50\,(4) = 250$; and the supply of labour will be $L^S = 100\left(\frac{w}{p}\right) = 100\,(4) = 400$ therefore 400 -250 = 150 workers will be unemployed.

Now all these information can be presented in one diagram.

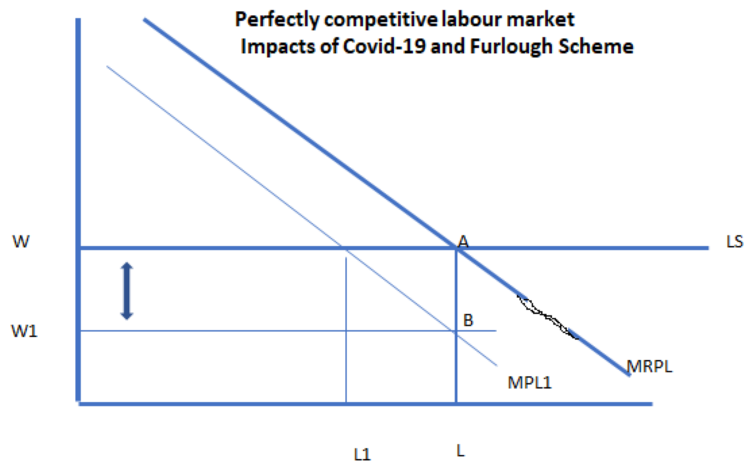
21.0.1 Monopsoy labour market

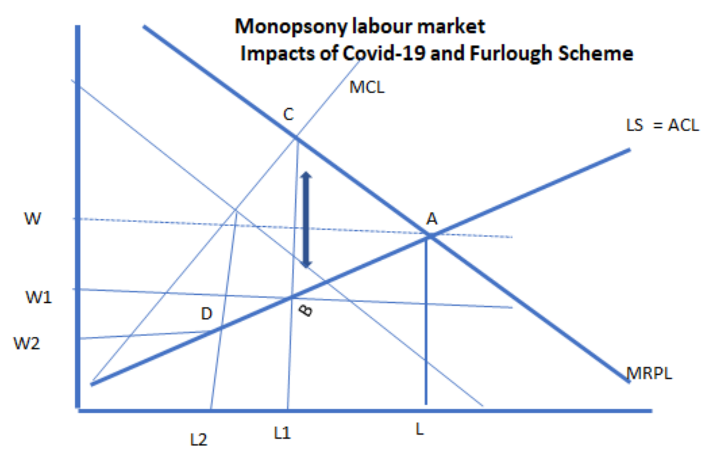
Monopsony labour market is a situation where there is a single employer for many potential workers contesting for the job market. A university, like Hull, is a single employer of hundreds of academics. A regional or local hospital is a single employers of many doctors in a certain city. Local superstore like, ASDA and Tesco also could be a single employer for retail jobs in certain cities. The wage and employment behavior in the labour market with monopsony is very different than a perfectly competitive labour market as illustrated above.

As usual the demand curve for labour is given by the marginal revenue product (MRP) of labour; it is a downward sloping function determined by the underlying diminishing marginal productivity of the labour and the product price. Normally, supply of labour, that relates wage to employment, is given by an upward sloping function representing utility maximisaiton from leisure and consumption of households. This function also represents the average cost of labour showing how wage rises as the levels of employment rise. A monopsonistic employer looks for marginal cost of labour while employing one additional worker. Increase in total wage cost is greater than the wage cost of marginal worker as every worker need to be paid the marginal wage rate. Look at the following table:

Table 42: Average and marginal cost of labour in monopsony

	1st	2nd	3rd	4th	5th
L	0	1	2	3	4
W	0	3	4	5	6
L×W	0	3	8	15	24
MLC	0	3	5	7	9


Perfectly competitive labour market
Impacts of Covid-19 and Furlough Scheme
W
W1
A
B
LS
MRPL
MPL1
L1
L


Monopsony labour market
Impacts of Covid-19 and Furlough Scheme
MCL
C
LS = ACL
W
A
W1
D
B
W2
MRPL
L2
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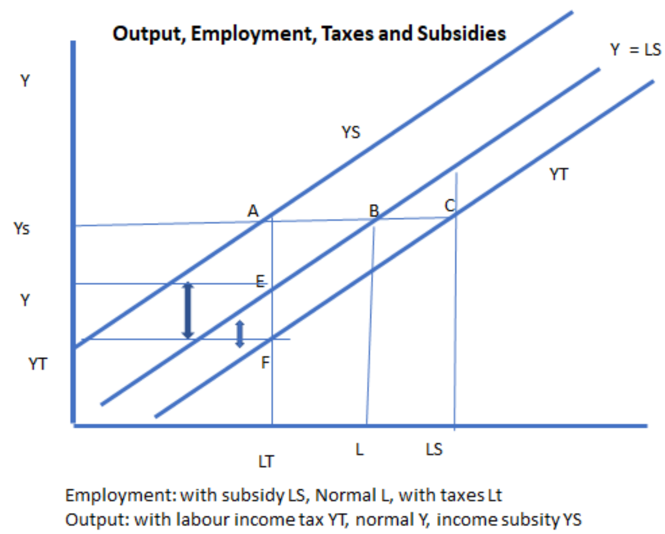

21.1 Labour Market and Search and Matching Model

Producers use labour to produce goods and services. A production function shows how labour complements with other inputs in production and the marginal productivity of labour shows the additional unit of output produced by each additional unit of labour. Thus demand for labour is derived from the demand for output. On the supply side every working age person has 168 hours a week, 720 hours per months or 8760 hours per year of time endowment which can be allocated between work and leisure. How many hours does one work and how much is spent in free time really depends upon the preference between consumption and leisure on one hand and the job vacancies on the other. In theory, flexibility of real wages guarantees equality between demand and supply in the labour in a competitive labour market. However, the labour market is far from a perfectly competitive market. Firms exercise monopoly powers, acting as monopsonists in the labour market or use their market power in order to retain more effective workers. Hiring decisions of firms also are dependent on the aggregate demand. Firms hire more workers during expansion but are reluctant of recruit any workers during the contraction. A significant number of workers become unemployed as a consequence.

Given a production function that related output (Y_t) to capital (K_t), technology (A_t) and labour (L_t)

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} \qquad\qquad 0 < \alpha < 1 \tag{585}$$

Wage rate should be paid according to the marginal productivity of labour as:

$$w_t = (1-\alpha) K_t^{\alpha} \left(A_t L_t\right)^{-\alpha} A_t \tag{586}$$

Supply of labour occurs through the utility maximising behavior of the household.

$$\max \sum_{t=0}^{\infty} \beta^t U\left(c_t, 1-l_t\right) \tag{587}$$

subject to

$$c_t + k_{t+1} = w_t l_t + (1 + r_t) k_t \tag{588}$$

This results in labour supply to be:

$$\overline{L}_t - L_t = \overline{L}_t - \frac{(1-\alpha)}{(1-\alpha) + (1-\alpha\beta) b} \tag{589}$$

Income patterns over time are different for different individuals. Some people start at a very low level of earnings and experience a rapid rise in income as they gain more job specific experience. Others may have a steady and stagnant income process over years. Still others may even have to face declining growth in income. What are the factors that lead to higher income growth rates and what are the factors that setback the process of income growth has been an issue of great interest among the labour economists.

The years of schooling and job market experience are the most important factors associated with higher income levels. Given other things constant, generally it is believed that an individual with greater number of schooling years earns more than a person with a few years or no schooling. Similarly a person with greater experience earns higher income. Both schooling and experience are perceived to be the main factors enhancing productivity of an individual.

There are a number of factors that set back the income process. Gender bias has been an area of continuous research in the labour economics. Females earn less than male either because of a structural breaks in their career for family reasons or due to gender discrimination in the labour market. Similarly there are cross regional variation in the income process.

As discussed in Pissarides (2013) and in Bhattarai and Dixon (2014) "the phenomenon of equilibrium unemployment results from the interaction among N number of firms and unions (representing H number of households) which bargain over wages and employment".

Matching and bargaining functions across all N industries are key elements determining equilibrium unemployment . The Matching function (Beveridge curve) gives equilibrium conditions in the labour market balancing entry and exit from unemployment by aggregating sector and skill specific vacancies $\left(V_{i,t}^h\right)$ and unemployment $\left(UN_{i,t}^h\right)$ with job creation as:

$$M_t = M(V_t, UN_t) = V_t^{\gamma_t} UN_t^{(1-\gamma_t)} \tag{590}$$

where M_t, V_t and UN_t denote the aggregate number of matching, vacancies and unemployment respectively among job seekers at time t and aggregate variables are geometric means of household level variables[1]. The matching parameter γ_t is between zero and one and varies over time. It can be adjusted for prosperous period when there are more vacancies than job seekers or in recession when there are more unemployed than vacancies. In steady state it should be about 0.5 to reflect the balance between job creation and job destruction. Heterogeneity in the labour market is reflected by sector and skill specific $M_{i,t}^h$, $V_{i,t}^h$ and $UN_{i,t}^h$. These capture the labour market conditions where production sectors suffer from shortages of certain skills while facing abundance of other skills. In each case job seekers and employers bargain over expected earnings by maximising the Nash-product $\left(NP_{i,t}^h\right)$ of the bargaining game over the difference between the earnings from work $(W_{i,t}^h)$ than in being unemployed $(UN_{i,t}^h)$ and earnings to firms from filled $\left(J_{i,t}^h\right)$ and vacant jobs $\left(V_{i,t}^h\right)$.

[1] $V_t = \prod_{i=1}^{N} V_{i,t}^h; UN_t = \prod_{i=1}^{N} UN_{i,t}^h; M_t = \prod_{i=1}^{N} M_{i,t}^h = \prod_{i=1}^{N} M\left(V_{i,t}^h, UN_{i,t}^h\right).$

$$NP_{i,t}^h = \left(W_{i,t}^h - UN_{i,t}^h\right)^{\theta_b^h} \left(J_{i,t}^h - V_{i,t}^h\right)^{1-\theta_b^h} \tag{591}$$

Market imperfections in the labour market create opportunity of gains from bargains which is divided between firms and workers as indicated by parameter θ_b^h that can assume any value between zero and one, reflecting the relative strength of unions (workers) over firms in such bargains. Symmetric solution of this satisfies joint profit maximisation condition as:

$$\left(W_{i,t}^h - UN_{i,t}^h\right) = \theta_b^h \left(J_{i,t}^h + W_{i,t}^h - V_{i,t}^h - UN_{i,t}^h\right) \tag{592}$$

In aggregate the job search model can be explained using three simple equations as summarised by Pissarides (1979, 2000).

First, for each skill type h the dynamics of unemployment depends on the rate of job destruction, $\lambda_t^h \left(1 - un_t^h\right)$, and the rate of job creation, $\theta_t^h q\left(\theta_t^h\right) un_t^h$ as $\Delta un_t^h = \lambda_t^h \left(1 - un_t^h\right) - \theta_t^h q\left(\theta_t^h\right) un_t^h$. The steady state equilibrium implied by this is:

$$un_t^h = \frac{\lambda_t^h}{\lambda_t^h + \theta_t^h q\left(\theta_t^h\right)}; \quad un_T = \frac{\lambda_T}{\lambda_T + \theta_T q\left(\theta_T\right)} \tag{593}$$

where λ_t^h is the rate of idiosyncratic shock of job destruction of household type h and θ_t^h is the ratio of vacancy to the unemployment and $q\left(\theta_t^h\right)$ is the probability of filling a job with a suitable candidate through the matching process explained in (590). Then un_T is the equilibrium unemployment rate average across all households expressed in terms of averages of λ_t^h θ_t^h and $q\left(\theta_T\right)$ given by λ_T, θ_T and $q\left(\theta_T\right)$ respectively.

Secondly the upward sloping wage curve in (θ_t^h, w_t^h) space shows positive links between the reservation wage (z_t^h) the price of product p and cost of hiring $(\theta_t^h c_t^h)$ implying higher wage rates for tighter labour markets as:

$$w_{i,t}^h = z_t^h \left(1 - \theta_b^h\right) + \theta_b^h p_t \left(1 + \theta_t^h c_t^h\right) \tag{594}$$

Finally there is a downward sloping job creation curve $w_t^h = p_t - \left(r_t + \lambda_t^h\right) \frac{p_t c_t^h}{q\left(\theta_t^h\right)}$, where p_t is the price of product, w_t^h the wage rate, and $\left(r_t + \lambda_t^h\right) \frac{p_t c_{h_t}}{q\left(\theta_t^h\right)}$, is the cost of hiring and firing. It shows the possibility of job creation at lower wage rates and creation of fewer jobs at higher wage rates. The optimal job creation (demand for labour curve) occurs when firms balance the marginal revenue product of labour to wage and hiring and firing costs (see some details in Bhattarai and Dixon (2014)).Following the market signals of demand and relative prices and costs of inputs, profit maximising firms create vacancies for specific tasks and hire workers when they find suitable candidates for these jobs. Similarly there are workers seeking jobs that match their skills and others who quit jobs and join the pool of unemployed who may choose to quit jobs and become unemployed. Market specific idiosyncratic shocks cause such entries and exits in the labour market. Equilibrium unemployment and wage rates result from a Nash-bargain between workers and firms. Whether the rate of unemployment falls or rises depends on the relative proportion of entry and exit into the labour market.

21.2 Human capital theory of income share

Recently authors economists Becker, Mincer, Lucas, Aghion, Helpman, Jones, Weale, Temple and Blanchard have emphasized on the human capital theory of income distribution. Education provides skills and make people more productive. Higher productivity translates into higher wage rates. Individuals who invest more on education and skills earn more than others who have not invested in them. This can be illustrated with a simple model of life time income (LI) with and without university education as given below:

$$\begin{aligned} LI &= \left[1 + (1+g) + (1+g)^2 + ... + (1+g)^n\right] Y_0 \\ &= Y_0 \left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right] \end{aligned} \tag{595}$$

Life time income with university education

$$\begin{aligned} LI &= Y_0 \left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right] - 3 \times C \\ &= 30000 \left[\frac{1-(1.04)^{42+1}}{1-(1.04)}\right] - 3 \times 15000 = 3,255,371 \end{aligned} \tag{596}$$

Life time income without university education

$$Y_0 \left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right] = 17000 \left[\frac{1-(1.02)^{45+1}}{1-(1.02)}\right] = 1,263,620 \tag{597}$$

Extra life time income comes from the university education. Difference in income made the university level education in the life time of an individual thus is the difference between these two levels of income; £3,255,371-£1,263,620=£1,991,751. Thus university education makes one better off by nearly 2 million pounds. Studies of Jenkins (1995, 1996) illustrate on such differences. Econometrically these studies estimate a standard earning function from the labour market dataset such as the Annual Population Survey (APS). In Mincerian tradition earnings depend on qualifications and status of health and many other conditions as shown in a regression table below.

$$w_{i,t} = a_i + \beta_i S_{i,t} + \gamma_i A_{i,t} + \psi_i A_{i,t}^2 + \lambda_i G_{i,t} + \delta_i R_{i,t} + \pi_i P_{i,t} \pi + \theta_t \Delta_t + \varepsilon_{i,t}$$

where $w_{i,t}$ is the wage rate of individual i in year t; $S_{i,t}$ is years of schooling; $A_{i,t}$ is age of individual i in time t; $G_{i,t}$ is the gender of an individual, $R_{i,t}$ is regional location, Δ_t is wave t, $P_{i,t}$ is professional background of individual i. Coefficients of such earning functions are estimated using cross section or panel dataset.

Bargaining between unions of workers and firms also is important as taxes on income and consumption and unemployment benefits (see Mirrlees et al. (2010) or the Green Budgets from the IFS for the UK for more extensive analysis on these issues).

Thinks of millions of workers in the economy. They work for earnings; in Mincerian traditions earnings depend on qualifications and status of health and many other conditions as shown in a regression table below.

$$w_{i,t} = a_i + \beta_i S_{i,t} + \gamma_i A_{i,t} + \psi_i A_{i,t}^2 + \lambda_i G_{i,t} + \delta_i R_{i,t} + \pi_i P_{i,t} \pi + \theta_t \Delta_t + \varepsilon_{i,t}$$

where $w_{i,t}$ is the wage rate of individual i in year t; $S_{i,t}$ is years of schooling; $A_{i,t}$ is age of individual i in time t; $G_{i,t}$ is the gender of an individual, $R_{i,t}$ is regional location, Δ_t is wave t, $P_{i,t}$ is professional background of individual i. Coefficients of such earning functions are estimated using cross section or panel dataset. For instance using the cross section of the APS:

In addition to above variable earning differ by location of the labour markets. Local, regional, national, urban, rural, global labour markets function differently. Earning also vary by professions. Teachers, lawyers, doctors, engineers, scientists, artists have different levels of earning. Skilled workers are paid more than unskilled or semi-skilled workers. Labour market institutions mater. Job prospects are less in the rigid and opaque labor markets than in flexible and transparent labour markets. Labour earning also vary by the term of employment. Earnings are less in short term compared to medium term and long term employments. There are professions where labour supply occurs in inter-generational setting.

21.3 Global Empirical Evidence on Declining Labour Share

We construct panel data set for 127 countries for year 1990 to 2011 for labour income share (labshare), consumption share (consshare), capital share (capshare), government consumption share (Govconshare), import share (impshare), exports share (expshare) and real trade share (Rtrdshare). It is clear that average share of labour is declining for each decade as shown in Table 7. Labour shae in income was about 59 percent of GDP and it has declined by 9 pecent point to 51.4 percent by 2011. The dispersion in these shares have increased as the standard deviation has reduced from 0.116 to 0.137. Maddison project have more data investigate. EU KLEMs dataset also provides such information.

Table 43: Average labour share by decades

Years	Average share	Stand Dev	Countries
1950	0.587773529	0.116112995	48
1960	0.570278757	0.128005175	87
1970	0.55353534	0.146336493	107
1980	0.547496719	0.136702051	109
1990	0.548241206	0.138618056	127
2000	0.530072014	0.133766665	127
2011	0.513952814	0.136878886	127
Data source: Penn World Tables v8; Maddison dataset			

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Source	SS	df	MS		
				Number of obs =	368525
				F(17,368507) =	8.76
Model	139225393	17	8189728.98	Prob > F =	0.0000
Residual	3.4450e+11	368507	934864.212	R-squared =	0.0004
				Adj R-squared =	0.0004
Total	3.4464e+11	368524	935198.879	Root MSE =	966.88

GRSEXP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ILLDAYS2	.5135184	3.158899	0.16	0.871	-5.67783	6.704866
ILLDAYS1	2.180592	2.085396	1.05	0.296	-1.906723	6.267907
ILLDAYS3	-4.435447	3.853808	-1.15	0.250	-11.9888	3.117903
ILLDAYS4	8.066717	4.485219	1.80	0.072	-.7241797	16.85761
ILLDAYS5	-5.238917	3.738412	-1.40	0.161	-12.56609	2.08826
ILLDAYS6	-1.988442	6.544168	-0.30	0.761	-14.81482	10.83793
ILLDAYS7	-.1315924	8.270102	-0.02	0.987	-16.34075	16.07756
QUAL_14	4.705906	5.354454	0.88	0.379	-5.788664	15.20048
QUAL_15	-6.997297	16.48718	-0.42	0.671	-39.31169	25.3171
QUAL_16	-12.89791	11.85998	-1.09	0.277	-36.14313	10.3473
QUAL_17	97.47657	34.51317	2.82	0.005	29.83178	165.1214
QUAL_18	-13.84225	32.57728	-0.42	0.671	-77.69276	50.00826
QUAL_19	3.716753	4.476542	0.83	0.406	-5.057138	12.49064
QUAL_2	13.91868	12.39392	1.12	0.261	-10.37304	38.2104
QUAL_20	32.4236	7.656584	4.23	0.000	17.41692	47.43028
QUAL_21	43.351	4.285564	10.12	0.000	34.95142	51.75058
QUAL_22	-1.453274	6.644386	-0.22	0.827	-14.47607	11.56953
_cons	-5.248379	52.42735	-0.10	0.920	-108.0044	97.50769

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22 L20: Experimental Economics,Tax-Trasfer and IO examples

Experimental economics is a branch of economics that studies human behavior in a controlled laboratory setting or out in the field, rather than just as mathematical models. It uses scientific experiments to test what choices people make in specific circumstances, to study alternative market mechanisms and test economic theories.

https://scholar.princeton.edu/kahneman/lectures

https://kahneman.socialpsychology.org/

https://www.investopedia.com/terms/e/experimental-economics.asp

Together, with behavioral economics – which has established that people are a lot less rational than traditional economics had assumed experimental economics is being also being used to investigate how markets fail, and explore anticompetitive behavior.

The field was pioneered by Vernon Smith, who won the Nobel Prize in Economics in 2002, for developing a methodology that allows researchers to examine the effects of policy changes before they are implemented, and help policymakers make better decisions.

https://www.nobelprize.org/prizes/economic-sciences/2002/smith/biographical/

Smith's early experiments focused on theoretical equilibrium prices and how they compared to real-world equilibrium prices. He found that even though humans suffer from cognitive biases, traditional economics still can make accurate predictions about the behavior of groups of people. Groups with biased behavior and limited information still reach the equilibrium price by becoming 'smarter' through their spontaneous interaction.

Experimental economics is used to help understand how and why markets function like they do. These market experiments, involving real people making real choices, are a way of testing whether theoretical economic models actually describe market behavior, and provide insights into the power of markets and how participants respond to incentives – usually cash.

The field was pioneered by Vernon Smith, who won the Nobel Prize in Economics in 2002, for developing a methodology that allows researchers to examine the effects of policy changes before they are implemented, and help policymakers make better decisions.

https://www.weforum.org/agenda/2015/07/what-has-experimental-economics-taught-us/

Banerjee and Duflo (2020).

Abhijit Banerji,Esther Duflo and Michael Kremer won the Nobel prize in eocnomics in 2019 for applying experimental economics techniques including Radom Control Trial (RCT) method for analysis of effectiveness of measures to reduce poverty or to improve cognitive skill of students.

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22.1 Experimental Approaches to Poverty Alleviation

Poverty is measured relatively and absolutely. Adam Smith (1776) was absolutist, for him it meant being ashamed to appear in public due to inability to afford necessary things according to the custom and standards of the country - "... A linen shirt is strictly speaking not a necessity of life. The Greeks and Romans lived very comfortably though they had no linen. But in the present time ... a creditable day-laborer would be ashamed to appear in public without a linen shirt...". Marx followed Adam Smith in thinking that necessary wants of the workers were products of historical development that depended to a great extent on the degree of civilization of the country (Marx and Engles (1848)). Rowntree (1902) in a study of minimum living standards for a respectable life in York in Britain considered a family to be living in poverty if its total earnings were insufficient to obtain the minimum necessaries for the maintenance of merely physical needs; the minimum requirements of protein and calories, housing, thermal comforts, basic health and education. Based on expenditure on porridge and skim milk for breakfast, bread and cheese for lunch, vegetable broth, bread, cheese, dumpling for dinner, and bread and porridge for supper he set a poverty threshold and determined the number of poor households below that line (Glennester, Huills, Piachaud and Webb (2004)). Beveridge (1942) took these facts into account while designing social insurance programs. Orshansky (1965) did similar study for the United States. Townsend (1969) revisited them for the UK. Sen (1976), Foster and Shorrocks (1985), Basu (1985), Vaughan (1987), Preston (1995), Shorrocks (1995) and Chakravarty (1997), Davidson and Duclos (2000) then focused on theoretical issues relating to definition and relative and absolute measurement of poverty. Sen's (1976) note on inadequacy of the head count ratio and a need for poverty gap to measure the depth of poverty that would fulfill axioms of monotonicity, transfer, relative equity and ordinal rank to capture severity of deprivation of households under the poverty line has been extended to cases of temporary and chronic poverty adjusting the poverty gap for its duration by Foster-Greer-Thorbecke (1984). Importance of Pigou-Dalton transfer axiom and first and second order stochastic dominance properties indicated by Atkinson (1970 and 1987) were implemented by Beckerman (1979) and using subjective method by Praag, Goedhard and Papeyn (1980) and Jenkins (1991) and Jenkins and Lambert (1997) showing how transfers could eliminate poverty among low income households. Blundell et al. (2000) applied that to evaluate the impacts of income tax credit in consumption and income inequality in UK .Banerjee and Duflo (2007, 2008) contain several studies based on random control trials on impacts of programme intervention among treatment and control groups.Glewwe and Kremer (2006) took experimental approach to schooling to reduce poverty.

There is no magic wand to eliminate massive poverty that exists around the world. It depends on strategic interactions among players of the poverty game. Effective implementation of poverty alleviation programmes requires thinking of strategies and actions available to major players in the poverty game - poor themselves who are often considered beneficiaries of aids, grants and transfers; rich individuals who bear the burden of taxes to pay for those transfers; the governments that are involved not only in measuring the depth of poverty and setting up objectives, targets and programmes that aim to eliminate poverty but also are subject to distortions arising from corruption and misuse of public money; and the global community that can provide a natural and fair playing fields for these players. Designing an effective incentive structure in dynamic contexts and balancing economic and political power over the benchmark equilibrium path of these economies is essential.

Ideally high income individuals would like to see the end of poverty as has been campaigned by public and private sectors in advanced countries in recent years but it is logical for them to expect that poor who receive benefits make good efforts to get out of the poverty trap by investing their time and resources in education, skill and training and health care and economically productive initiatives with a clear foresight of progress over a horizon rather than doles for daily consumptions. Government, made of representatives of both poor and rich people, may propose very ideal programmes, rules and regulations but they become ineffective in reducing poverty without active cooperation from tax payers and the transfer recipients. It may create nany-state syndrome and drag on overall growth as in many EU economies in 1990s.

Inclusion of strategic and behavioral aspects of poverty game discussed above is apparently missing from the existing literature and forms the major content of the current paper. A numerical example is provided in the next section to summarise basic concepts existing in the literature to set a background for a dynamic cooperative and non-cooperative game of poverty in section III and a brief reference to the dynamic multi-household general equilibrium model in section IV with conclusions and references in the last section.

22.2 A Numerical Example on the Measurement and Transfer for Alleviation of Poverty

Consider an economy inhabited by N number of individuals where income of each is denoted by for each $i = 1,2, \ldots ,N$. Income vary among individuals for economic, social, political, cultural or many other less obvious reasons; $y_i \neq y_j$ for all $\forall_i$. A strict ordering implies, $y_1 < y_2 < y_3 < ... < y_N$ with corresponding ordering of welfare with lower income individuals having lower level of welfare. Infinite numbers of income configurations (distributions) are possible which often are summarised by their mean and variances as in Jenkins (1991) or Preston (1995). Distributions, with lower variances are more equal than with higher ones.

Poverty line relates income of individuals to average incomes, $\overline{y} = \frac{1}{N}\sum_{i=1}^{n} y_i$; the ratio of people below this line in relation to N individuals in society is the head count measure of poverty Many countries adopt one half of the average income as a cut-off point for absolute poverty line;$z = \frac{1}{2}\overline{y}$, which is then used to come up with either the head count ratio, which is the ratio of number of people below the poverty line divided by the total number of individuals in the population or the income gap ratio more preferably measure of the depth of poverty which indicates the deficiency of income in relation to poverty line $I = \frac{\sum_{i=1}^{n}(y_i - z)}{z.n}$. Sen (1976) requires further modification to poverty gap to capture the income inequality to achieve monotonicity as well as the transfer axiom to reflect increase in poverty when resources are transferred from poorest poor to less poor persons as:

$$P = H.J + (1 - I)G \tag{598}$$

here P is a composite index of poverty, H the headcount ratio, I the income gap ratio, G the Gini coefficient; higher values of H, I, and G mean higher degree of poverty.

22.2.1 Numerical example of poverty alleviation

Numerical example in Table 1 and associated Figure 1 can illustrate these concepts more accurately and effectively.

Column y gives the income by households, N the number of households in each income category, cy and cp are cumulative income and population; yshre and cyshre columns present income share of each decile and cumulative shares; pshre and cpshre columns present income share of each decile and cumulative shares; area under the Lorenz curve can be approximated using triangles and rectangles.

The total income is 1000; with 10 households, average income is 100. Area under the Lorenz curve is 0.236, that between the Lorenz curve and equality line is 0.264; this implies a Gini coefficient of 0.528; higher G reflecting more unequal distribution.

By the headcount ratio seventy percent of population is poor if the accepted poverty line is set at the average income $\overline{y}$=100 but only 40 percent is poor when absolute poverty line is established as the half of this average income $z = \frac{1}{2}\overline{y} = 50$ as only four individuals are below the poverty line. As stated above this head count ratio does not indicate the depth of poverty as it ignores the income gap ratio,$I = \frac{\sum\limits_{i=1}^{n}(y_i - z)}{z.n} = \frac{40+3=+20+10}{50\times 4} = \frac{100}{200} = 0.5$. In terms of Sen povety index in this economy is $P = H.J + (1-I)G = 0.5 \times 0.4 + (1-0.5) \times 0.528 = 0.2 + 0.264 = 0.464$.

and is illustrated in Figure 1 (actual income distribution from BHPS is n Figure 2):

Table 44: Measuring and Alleviating Poverty in a hypothetical economy

y	N	cy	cp	yshre	cyshre	pshare	cpshare	triangle	Rectangle	Area	ygap
10	1	10	1	0.01	0.01	0.1	0.1	0.0005	0.000	0.0005	-90
20	1	30	2	0.02	0.03	0.1	0.2	0.0010	0.001	0.0020	-80
30	1	60	3	0.03	0.06	0.1	0.3	0.0015	0.003	0.0045	-70
40	1	100	4	0.04	0.10	0.1	0.4	0.0020	0.006	0.0080	-60
50	1	150	5	0.05	0.15	0.1	0.5	0.0025	0.010	0.0125	-50
60	1	210	6	0.06	0.21	0.1	0.6	0.0030	0.015	0.0180	-40
90	1	300	7	0.09	0.30	0.1	0.7	0.0045	0.021	0.0255	-10
100	1	400	8	0.10	0.40	0.1	0.8	0.0050	0.030	0.0350	0
200	1	600	9	0.20	0.60	0.1	0.9	0.0100	0.040	0.0500	100
400	1	1000	10	0.40	1.00	0.1	1.0	0.0200	0.060	0.0800	300

Figure 1

Poverty with Income Inequallity

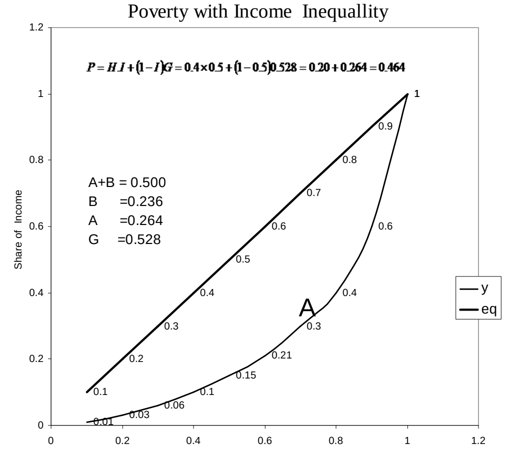
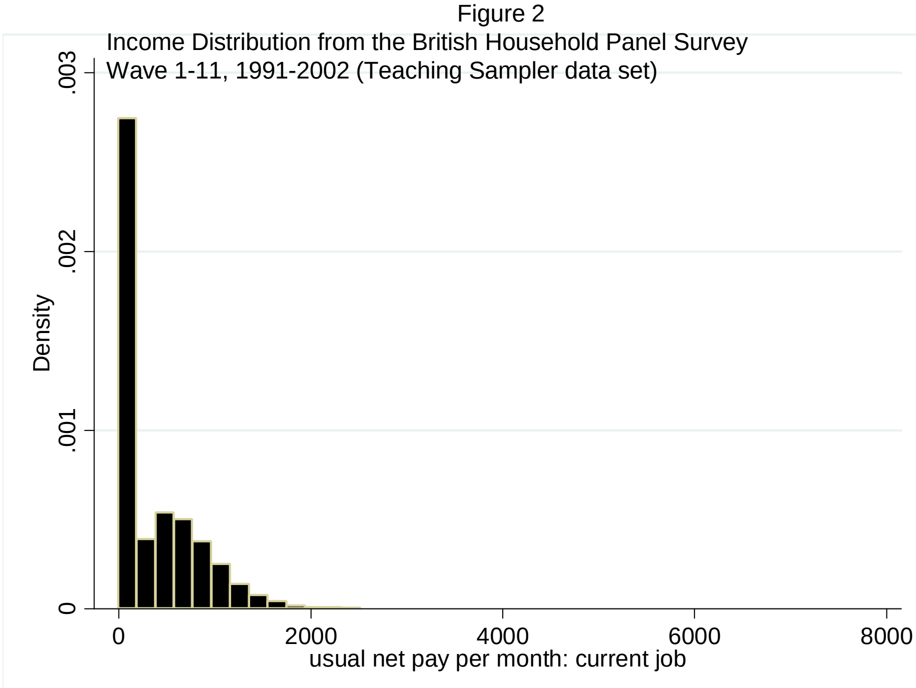


Figure 2



The elimination of the absolute poverty in this example requires transfers of 100 to poor individuals with $T_1 = 40$ for the poorest household $T_1 = 30$ and $T_1 = 20$, and $T_1 = 10$ accordingly to other three households below the poverty line. This transfer can be funded by a 10 percent and 20 percent tax on the income of 9th and 10th deciles raising 20 and 80 respectively. This brings H to zero and I to 1 making P to zero.

22.2.2 Refinement of measures

It is obvious that the value of poverty index in above example as in real life is influenced by the choice of the poverty line; when income is perfectly equally distributed no one is below the poverty line; H is zero and G also is zero with no poverty, $P = 0$; but these are extreme cases only of theoretical possibility. In a real world situation, values of P range between zero and one, $0 < P < 1$, with higher P indicating to the higher level of poverty. Empirically this poverty index varies across countries and over time according to characteristics of income distribution functions; it would have a larger value if the income distribution was more unequal or when the poverty line is set at the higher level. Thus the relative measure of poverty is sensitive to the choice of the poverty line; it is high in an economy when the mean of the income is taken as a poverty line than when only the half of the mean income is taken for it and when it is more unequal than its comparator. More fundamentally the degree and depth of poverty can be changed by influencing the choices of individuals and households and by adopting economic programmes that are more efficient and generate better outcomes. This means when looked from this relative sense there are poor in every economy, it can never be abolished. Variations in relative poverty arise from the basic structure of the socio-economic model adopted by the country. For this reason Sen focused on minimum capability view in his subsequent works. Foster-Greer-Thorbecke (1984) have normalised poverty gap ratio to capture the severity of poverty $FGT = \left(\frac{1}{N}\right)\sum\limits_{h=1}^{N}\left(1-\frac{y_i}{z}\right)^2$ for $y_i < z$ and see the importance of duration of the poverty spell in the poverty index by average poverty index given by the mean gap over time and individuals $A_{FGT}(T) = \left(\frac{1}{N}\right)\sum\limits_{t=1}^{T}\sum\limits_{h=1}^{N}\left(1-\frac{y_{i,t}}{z}\right)^2$; $N = nT$ and chronic measures of poverty as $C_{FGT}(T) = \left(\frac{1}{N}\right)\sum\limits_{t=1}^{T}\sum\limits_{h=1}^{N}\left(1-\frac{Y_{i,t}^*}{z}\right)^2$ taking gap from the permanent income $Y_{i,t}^*$. Three 'I's Incidence, intensity and inequality (TIP) measure in Jenkins and Lambert (1997) obtained by cumulative ranking of people from poorest to richest is another smarter way of visualising poverty graphically across time, countries or regions or households.

It is often argued that poverty can be eliminated by means of tax and transfer as illustrated in the numerical example in Table 1 above and in Beckerman (1979). Broader questions arise regarding the impact of such transfer programme. First relates to its impact on labour supply of rich and poor individuals. Higher taxes may discourage rich individuals to work and transfer receipts may reduce the need to work to earn for living for the poor. Secondly, higher taxes may discourage incentives of saving and investment. Third, modality of transfer payment may be crucial for long term growth. Providing in kind transfer in the form of education and health spending may be better than cash transfers to empower productive capacity of the poor. Fourth, in addition to transfer payments governments need to provide public goods for the entire population. As everyone consumes these public goods these should be provided by taxing on income of both rich and the poor. Alleviation of poverty can better be studied in terms of a strategic model as illustrated in the next section.

22.3 Readings in Experimental Economics

DECISION MAKING: ERRORS AND BIASES

- Introduction

Kahneman, Daniel, Jack L. Knetsch and Richard H. Thaler. 1991. "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias." Journal of Economic Perspectives 5 (1): 193-206.

Kahneman, D., 2003. A psychological perspective on economics. American economic review, 93(2), pp.162-168.

- Risk and Uncertainty

Tversky, Amos and Daniel Kahneman. 1974. "Judgment under Uncertainty: Heuristics and Biases." Science 184 (4157): 1124-1131.

- Prospect Theory

Kahneman, Daniel and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decision under Risk." Econometrica 47 (2): 263-292.

Tversky, Amos and Daniel Kahneman. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." Journal of Risk and Uncertainty 5: 297-323.

- Loss Aversion

Kahneman, Daniel, Jack L. Knetsch, and Richard Thaler. 1990. "Experimental Tests of the Endowment Effect and the Coase Theorem." Journal of Political Economy 98 (6): 1325-1348.

- Mental Accounting

Thaler, Richard. 1999. "Mental Accounting Matters." Journal of Behavioral Decision Making12: 183-206. (PDF)

- Time and Choice

Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting." Quarterly Journal of Economics 443-477.

O'Donoghue, Ted and Matthew Rabin. 1999. "Doing It Now or Later." American Economic Review 89 (1): 103-124.

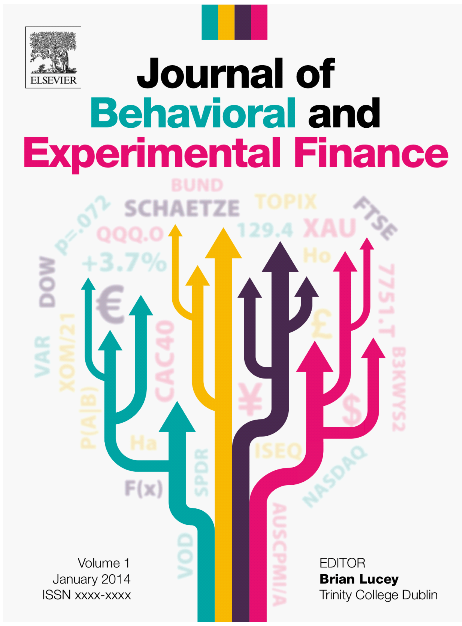
- SOME APPLICATIONS

Bertrand, Marianne, Sendhil Mullainathan and Eldar Shafir. 2004. "A Behavioral Economics View of Poverty. AEA Papers and Proceedings 94 (2): 419-423.

Camerer, Colin, Samuel Issacharoff, George Loewenstein, Ted O'Donoghue, and Matthew Rabin. "Regulation for Conservatives: Behavioral Economics and the Case for Asymmetric Paternalism." University of Pennsylvania Law Review 151 (3): 1211-1254.

Madrian, Brigitte C. and Dennis F. Shea. 2001. "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior." Quarterly Journal of Economics Vol. CXVI (4): 1149-1187.

Thaler, Richard and Shlomo Benartzi. 2004. "Save More Tomorrow: Using Behavioral Economics to Increase Employee Saving." Journal of Political Economy 112 (1): S164-187.



First four books for basic background knowledge in experimental economics:

1. Daniel Kahneman's Thinking, Fast and Slow: Still the best overview of behavioural science.

2. Richard Thaler's Misbehaving: A pretty good history of behavioural economics.

3. Michael Lewis's The Undoing Project gives a good accessible overview of Kahneman and Tversky's work

4. The Behavioral Foundations of Public Policy edited by Eldar Shafir, probably the best book.

5. Kagel, J.H. and Roth, A.E. eds., 2016. The handbook of experimental economics, volume 2: the handbook of experimental economics. Princeton university press.

https://www.thecompleteuniversityguide.co.uk/courses/economics/the-ten-most-influential-economists-of-all-time/

23 Input-Output Model

An example of input-Output Table

Structure of an input-output table (snap-shot of the economy for a given time)

$$\begin{bmatrix} IO & F \\ VA & Trasfers \end{bmatrix} \tag{599}$$

Leontief coefficients

Input-Output Model: Structural Equations

$$X_1 = X_{11} + X_{12} + F_1 \tag{600}$$

$$X_2 = X_{21} + X_{22} + F_2 \tag{601}$$

Table 45: Leontief Coefficients

	Intermediate demand		Final Demand	Total
	X_1	X_2	F	Y
X_1	10	20	70	100
X_2	30	20	150	200
Labour input	40	50		90
Capital input	20	110		130
Total	100	200	220	

Table 46: Leontief Technology and Primary Input Coefficients

	Intermediate demand	
	X_1	X_2
X_1	0.1	0.1
X_2	0.3	0.1
Labour input	0.4	0.25
Capital input	0.2	0.55
Total	1.0	1.0

$$a_{11} = \frac{X_{11}}{X_1};\; a_{12} = \frac{X_{12}}{X_2}; a_{21} = \frac{X_{21}}{X_1};\; a_{22} = \frac{X_{22}}{X_2}; \tag{602}$$

$$X_1 = a_{11} X_1 + a_{12} X_2 + F_1 \tag{603}$$

$$X_2 = a_{21} X_1 + a_{22} X_2 + F_2 \tag{604}$$

Input-Output Model

$$X_1 - a_{11} X_1 - a_{12} X_2 = F_1 \tag{605}$$

$$-a_{21} X_1 + X_2 - a_{22} X_2 = F_2 \tag{606}$$

$$\begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{607}$$

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix}^{-1} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{608}$$

$$X = (I - A)^{-1} F \tag{609}$$

Employment (labour income)

$$L = l_1 \times X_1 + l_2 \times X_2 \tag{610}$$

Capital stock (capital income)

$$K = k_1 \times X_1 + k_2 \times X_2 \tag{611}$$

Solution of the input - output model by Cramer's Rule

$$|A| = \begin{vmatrix} (1 - a_{11}) & -a_{12} \\ -a_{21} & (1 - a_{22}) \end{vmatrix} = \left(1 - a_{_{1,1}}\right) \times \left(1 - a_{_{2,2}}\right) - a_{21}a_{12} \tag{612}$$

$$X_1 = \frac{\begin{vmatrix} F_1 & -a_{12} \\ F_2 & (1 - a_{22}) \end{vmatrix}}{\begin{vmatrix} (1 - a_{11}) & -a_{12} \\ -a_{21} & (1 - a_{22}) \end{vmatrix}} = \frac{F_1\left(1 - a_{22}\right) + a_{12}F_2}{\left(1 - a_{_{1,1}}\right) \times \left(1 - a_{_{2,2}}\right) - a_{21}a_{12}} \tag{613}$$

$$X_2 = \frac{\begin{vmatrix} (1 - a_{11}) & F_1 \\ -a_{21} & F_2 \end{vmatrix}}{\begin{vmatrix} (1 - a_{11}) & -a_{12} \\ -a_{21} & (1 - a_{22}) \end{vmatrix}} = \frac{F_2\left(1 - a_{11}\right) + a_{21}F_1}{\left(1 - a_{_{1,1}}\right) \times \left(1 - a_{_{2,2}}\right) - a_{21}a_{12}} \tag{614}$$

Numerical Example of Input Output Model

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{bmatrix} (1 - 0.1) & -0.1 \\ -0.3 & (1 - 0.1) \end{bmatrix}^{-1} \begin{pmatrix} 70 \\ 150 \end{pmatrix} \tag{615}$$

$$X_1 = \frac{\begin{vmatrix} 70 & -0.1 \\ 150 & 0.9 \end{vmatrix}}{\begin{vmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{vmatrix}} = \frac{63 + 15}{0.81 - 0.03} = \frac{78}{0.78} = 100 \tag{616}$$

Numerical Example of Input Output Model

$$X_2 = \frac{\begin{vmatrix} 0.9 & 70 \\ -0.3 & 150 \end{vmatrix}}{\begin{vmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{vmatrix}} = \frac{135 + 21}{0.81 - 0.03} = \frac{156}{0.78} = 200 \tag{617}$$

Solutions reproduce the benchmark data. Model is calibrated.
Solving the Input-Output Model by Matrix Inverse

$$X = (I - A)^{-1} F \tag{618}$$

$$(I - A)^{-1} = \begin{bmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{bmatrix}^{-1} = \frac{1}{|I - A|} adj\,(I - A) \tag{619}$$

$$adj\,(I - A) = C' \tag{620}$$

For C cofactor matrix. For this cross the row and column corresponding to an element and multiply by $(-1)^{i+j}$

$$C=\left[\begin{array}{cc} |1-a_{22}| & -|a_{21}| \\ -|a_{12}| & |1-a_{11}| \end{array}\right]=\left[\begin{array}{cc} 0.9 & 0.3 \\ 0.1 & 0.9 \end{array}\right] \tag{621}$$

$$C'=\left[\begin{array}{cc} 0.9 & 0.3 \\ 0.1 & 0.9 \end{array}\right]'=\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right] \tag{622}$$

Inverse of A

Inverse of the Leontief technology matrix is the major element of the Input-Output model

$$\begin{aligned} (I-A)^{-1} &= \\ \frac{1}{\left(1-a_{1,1}\right)\times\left(1-a_{2,2}\right)-a_{21}a_{12}}\left[\begin{array}{cc} 1-a_{22} & -a_{12} \\ -a_{21} & 1-a_{11} \end{array}\right] &= \\ \frac{1}{0.81-0.03}\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right] &= \\ \frac{1}{0.78}\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right] &= \left[\begin{array}{cc} \frac{0.9}{0.78} & \frac{0.1}{0.78} \\ \frac{0.3}{0.78} & \frac{0.9}{0.78} \end{array}\right] \end{aligned}$$

$$X=(I-A)^{-1}F=\frac{1}{0.78}\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right]\left(\begin{array}{c} 70 \\ 150 \end{array}\right) \tag{623}$$

Inverse of A

$$\begin{aligned} X &= (I-A)^{-1}F=\frac{1}{0.78}\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right]\left(\begin{array}{c} 70 \\ 150 \end{array}\right) \\ &= \frac{1}{0.78}\left(\begin{array}{c} 63+15 \\ 21+135 \end{array}\right)=\frac{1}{0.78}\left(\begin{array}{c} 78 \\ 156 \end{array}\right)=\left(\begin{array}{c} 100 \\ 200 \end{array}\right) \end{aligned} \tag{624}$$

Model is calibrated

Impact analysis

$$\left(\begin{array}{c} \Delta X_1 \\ \Delta X_2 \end{array}\right)=\left[\begin{array}{cc} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{array}\right]^{-1}\left(\begin{array}{c} \Delta F_1 \\ \Delta F_2 \end{array}\right) \tag{625}$$

$$\Delta X=(I-A)^{-1}\Delta F \tag{626}$$

Impact Analysis

If the final demand of sector X_1 changes by 15 percent

$$\begin{aligned} \left(\begin{array}{c} \Delta X_1 \\ \Delta X_2 \end{array}\right) &= \frac{1}{0.78}\left[\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right]\left(\begin{array}{c} 70\times 0.15 \\ 150\times 0 \end{array}\right) \\ &= \frac{1}{0.78}\left(\begin{array}{cc} 0.9 & 0.1 \\ 0.3 & 0.9 \end{array}\right)\left(\left(\begin{array}{c} 10.5 \\ 0 \end{array}\right)\right) \\ \left(\begin{array}{c} \Delta X_1 \\ \Delta X_2 \end{array}\right) &= \frac{1}{0.78}\left(\begin{array}{c} 9.45 \\ 3.15 \end{array}\right)=\left(\begin{array}{c} 12.11 \\ 4.03 \end{array}\right) \end{aligned} \tag{627}$$

Employment (labour income)

$$\Delta L = l_1 \times \Delta X_1 + l_2 \times \Delta X_2 = 0.4 \times 100 + 0.25 \times 200 = 40 + 50 = 90 \tag{628}$$

Capital stock (capital income)

$$\Delta K = k_1 \times \Delta X_1 + k_2 \times \Delta X_2 = 0.2 \times 100 + 0.55 \times 200 = 20 + 110 = 130 \tag{629}$$

- A 15 pecent change in the final demand of sector will change gross output of both sector.
- change in capital and labour demand could be found out by using the capital and labour coefficients.
- Backward and forward linkages cause this to happen.
- Real world input- output model can be easily computed using Matrix routines in Excel.

References

[1] Attanasio, O.P., 2009. Expectations and perceptions in developing countries: their measurement and their use. American Economic Review, 99(2), pp.87-92.

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[9] OECD IO database: http://stats.oecd.org/index.aspx

24 Linear Programming

Linear Programming: Maximisation Problem

1. Solve the following linear programming problem using a simplex method. What are the optimal value of R, X_1 and X_2?

$$\max \ R = 10X_1 + 5X_2 \tag{630}$$

Subject to

$$25X_1 + 10X_2 \leq 1000 \tag{631}$$

$$20X_1 + 50X_2 \leq 1500 \tag{632}$$

where $X_1 \geq 0$ and $X_2 \geq 0$;

2. Write the dual of the above problem. Show that optimal solution of dual is equivalent to optimal solution of the primal problem.

3. Show that LP problem given above is a special case of non-linear problem.

Linear Programming: Simplex Algorithm for Maximisation

Table 47: Simplex Table 1

	R	X_1	X_2	S_1	S_2	Constant	Ratios
Row0	1	-10	-5	0	0	0	
Row1	0	25	10	1	0	1000	40
Row2	0	20	50	0	1	1500	75

Basic feasible solution

R	X_1	X_2	S_1	S_2
0	0	0	1000	1500

Linear Programming: Simplex Algorithm for Maximisation

Table 48: Simplex Table 2

	R	X_1	X_2	S_1	S_2	Constant	Ratios
Row0	1	0	-1	2/5	0	400	
Row1	0	1	2/5	1/25	0	40	100
Row2	0	0	42	-4/5	1	700	16.7

Basic feasible solution

R	X_1	X_2	S_1	S_2
400	40	0	0	16.7

Linear Programming: Simplex Algorithm for Maximisation

Table 49: Simplex Table 3

	R	X_1	X_2	S_1	S_2	Constant
Row0	1	0	0	8/21	1/42	17500/42
Row1	0	1	0	1/21	-1/105	700/21
Row2	0	0	1	-2/105	1/42	700/42

Basic feasible solution

R	X_1	X_2	S_1	S_2
17500/42	700/21	700/42	0	0

Linear Programming: Duality

Every maximisation problem has corresponding minimisation problem. The revenue maximisation problem above has equivalent to the cost minimisation problem.

Primal

$$\max \; R = 10X_1 + 5X_2 \tag{633}$$

Subject to

$$\begin{bmatrix} 25 & 10 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \leq \begin{bmatrix} 1000 \\ 1500 \end{bmatrix}; \; X_1 \geq 0; X_2 \geq 0 \tag{634}$$

This is equivalent to minimising the cost

$$Min \; C = 1000Y_1 + 1500Y_2 \tag{635}$$

subject to:

$$\begin{bmatrix} 25 & 20 \\ 10 & 50 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \geq \begin{bmatrix} 10 \\ 5 \end{bmatrix}; \; Y_1 \geq 0; Y_2 \geq 0 \tag{636}$$

Linear Programming: Fundamental Theorems of Duality

Two fundamental theorems of duality:

(1) Optimal values of the primal and the dual objective functions are always identical, provided that optimal feasible solution does exist.

(2) If a certain choice variable in a linear programme is optimally nonzero then the corresponding dummy variable should be equal to zero. Similarly if a certain choice variable in a linear programme is optimally zero then the corresponding dummy variable in the linear programme should be non-zero.

Lagrangian for the constrained optimisation (linear program (LP) as a special case of non-linear program (NLP))

$$L = 10X_1 + 5X_2 + \mu_1 \left[1000 - 25X_1 - 10X_2\right] + \mu_2 \left[1500 - 20X_1 - 50X_2\right] \tag{637}$$

Solution of the nonlinear program will be equivalent to the solution of the non-linear program:

$$\frac{\partial L}{\partial X_1} = 10 - 25\mu_1 - 20\mu_2 = 0 \tag{638}$$

$$\frac{\partial L}{\partial X_2} = 5 - 10\mu_1 - 50\mu_2 = 0 \tag{639}$$

$$\frac{\partial L}{\partial \mu_1} = 1000 - 25X_1 - 10X_2 = 0 \tag{640}$$

$$\frac{\partial L}{\partial \mu_2} = 1500 - 20X_1 - 50X_2 = 0 \tag{641}$$

$1000 = 25X_1 + 10X_2 \Longrightarrow 200 = 5X_1 + 2X_2$
$1500 = 20X_1 + 50X_2 \Longrightarrow 150 = 2X_1 + 5X_2$

From these two
$1000 = 25X_1 + 10X_2$
$300 = 4X_1 + 10X_2$

Then $700 = 21X_1$

$$X_1 = \frac{100}{3} = \frac{700}{21} \tag{642}$$

$150 = 2X_1 + 5X_2 \Longrightarrow 150 = 2\left(\frac{100}{3}\right) + 5X_2 \Longrightarrow 5X_2 = 150 - \frac{200}{3} = \frac{250}{3};$

$$X_2 = \frac{50}{3} = \frac{700}{42} \tag{643}$$

$$R = 10X_1 + 5X_2 = 10\left(\frac{700}{21}\right) + 5\left(\frac{700}{42}\right) = \frac{14000 + 3500}{42} = \frac{17500}{42} = 416.67 \tag{644}$$

Find the shadow prices

$$25\mu_1 + 20\mu_2 = 10 \tag{645}$$

$$10\mu_1 + 50\mu_2 = 5 \tag{646}$$

$$5\mu_1 + 4\mu_2 = 2 \tag{647}$$

$$2\mu_1 + 10\mu_2 = 1 \tag{648}$$

$$10\mu_1 + 8\mu_2 = 4 \tag{649}$$

$$10\mu_1 + 50\mu_2 = 5 \tag{650}$$

$$\mu_2 = \frac{1}{42} \tag{651}$$

$$2\mu_1 + 10\mu_2 = 1 \Longrightarrow \mu_1 = \frac{1}{2}\left(1 - \frac{10}{42}\right) = \frac{16}{42} = \frac{8}{21} \tag{652}$$

Linear Programming: Minimisation Problem

1. One family wants to find the minimum expenditure with optimal amounts of vegetarian (X_1), meat (X_2) and fat (X_3) contents in its food mix for a month. Per unit market price of these items is £5, £3 and £2 respectively. Suppose that nutritionists recommend 1000 units of carbohydrate, 1000 units of protein and 200 units of fat. One unit of vegetable item gives 5 units of carbo, 3 units of protein and 0.3 units of fat; one unit of meat item gives 3 units of carbo, 6 units of protein and 1 unit of fat; one unit of dairy product gives 2 units of carbo, 2 units of protein and 5 units of fat. Using a simplex method find the optimal amounts of vegetarian (X_1), meat (X_2) and fat (X_3) items that fulfils the nutrition constraints that minimises food expenditure for this family.

Hint: formulate the problem as follows:

$$\min \ E = 5X_1 + 3X_2 + 2X_3 \tag{653}$$

Subject to

$$5X_1 + 3X_2 + 2X_3 \geq 1000 \tag{654}$$

$$3X_1 + 6X_2 + 2X_3 \geq 1000 \tag{655}$$

$$0.3X_1 + 1X_2 + 5X_3 \geq 200 \tag{656}$$

1. where $X_1 \geq 0$, $X_2 \geq 0$ and $X_3 \geq 0$

   What are the optimal values of X_1 X_2,and X_3? What is the optimal expenditure?

Basic feasible solution

E	X_1	X_2	X_3	
1000	127.06	87.43	51.2	

2. Solve the following minimisation problem using a simplex method.

$$\min \ C = 0.6X_1 + X_2 \tag{657}$$

   Subject to

$$10X_1 + 4X_2 \geq 20 \tag{658}$$

$$5X_1 + 5X_2 \geq 1500 \tag{659}$$

$$2X_1 + 6X_2 \geq 12 \tag{660}$$

   where $X_1 \geq 0$ and $X_2 \geq 0$

   What are the optimal values of X_1 and X_2?

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25 L21: Game of Poverty

Allocations should be incentive compatible for rich and poor households and the governments to interact cooperatively in the poverty alleviation game. The solutions of the game when cooperative strategies are incentive compatible for them are consistent to poverty alleviation objectives while the catastrophic results may occur when non-cooperative strategies are become optimal for individual players. In a utility or welfare maximising world, model results will be based on comparison of expected welfare continent on their strategies (Vaughan (1987), Pryatt (1987), Desai and Shah (1988), Myles (2001)).

Limitations of one time transfers to end poverty have made alleviation of poverty one of the major global agenda in recent years (Millennium Development Goals (MDG), G8 meeting and Live

8 concerts 2005; poverty alleviations strategies of many developing economies including the OECD, China and India). As mentioned above poverty is not only the problem of developing economies but also of advanced economies. Effective implementation of these require strategic thinking among three major players in the poverty game; poor themselves who are often considered beneficiaries of aids, grants and transfers, rich individuals who bear the burden of taxes to pay for those transfers and the government that is involved not only in determining the depth of poverty and setting objectives, targets and programmes that aim to eliminate poverty but also is subject to corruption and misuse of public money. This effectively involves designing an effective incentive structure in the economy and the balance of economic and political power among these three players.

Ideally high income individuals would like to see the end of poverty as has been campaigned by public and private sectors in advanced countries in recent year. In the mean time they also expect that poor who receive benefit should make good efforts to get out of the poverty trap by investing their time and resources in education, skill and training and health care taking a longer time view rather than taking transfers to pay only for current spending. Government, made of representatives of both poor and rich people, might bring very sound and ideal programmes and propose rules and regulations but they become ineffective in removing poverty if there is not enough cooperation from tax payers and the recipients of the aid. A small game theoretic model is presented here to explain the dynamic situation of poverty. The solutions differ when all players use cooperative strategy and when they play a non-cooperative strategy. In a utility or welfare maximising world, model results will be based on comparison of expected welfare in each strategy.

25.1 Model of the Poverty Game

There are three players in the poverty game -poor, rich and government; each has three strategies available to it to play, s, l, and k , cooperation, indifference and non cooperation. The outcome of the game is the strategy contingent income for poor and rich, $y_t^p(s,l,k)$ and $y_t^R(s,l,k)$ with the probability of being in particular state like this is given by $\pi_t^p(s,l,k)$ and $\pi_t^R(s,l,k)$ respectively and tax and transfer profiles associated to them. The state-space of the game rises exponentially with the length of time period t. The objective of these rich and poor households is to maximize the expected utility that is assumed to be concave in income. The government can influence this outcome by choices of taxes and transfers that can be liberal, normal or conservative. More specifically, following propositions should hold in this poverty alleviation game.

Proposition 1: The state contingent expected money metric utility of poor is less than that of rich, which can be expressed as:

$$\sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\pi_t^p(s,l,k)\delta_t^p u\left(y_t^p(s,l,k)\right) < \sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\pi_t^R(s,l,k)\delta_t^R u\left(y_t^R(s,l,k)\right) \tag{661}$$

where $\pi_t^p(s,l,k)$ gives the probability of choosing one of strategies by poor given that the rich and the government has chosen l and k strategies. Utility is derived from income as given by $u\left(y_t^p(s,l,k)\right)$ and $\delta_t^p = \frac{1}{(1+r_t^p)}$ is the discount factors for poor and $\delta_t^R = \frac{1}{\left(1+r_t^R\right)}$ the discount factor for rich.

Proposition 2: Transfer raises money metric expected utility of poor and reduces the utility of rich.

$$\begin{aligned}&\sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\left[\pi_t^p(s,l,k)\delta_t^p u\left(y_t^p(s,l,k)\right)+\sum_{t}^{T}T_t^p(s,l,k)\right]\\ <\ &\sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\left[\sum_{t}^{T}\pi_t^R(s,l,k)\delta_t^R u\left(y_t^R(s,l,k)\right)-\sum_{t}^{T}T_t^R(s,l,k)\right]\end{aligned} \tag{662}$$

Proposition 3: Incentive compatibility requires that

$$\sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\left[\pi_t^p(s,l,k)\delta_t^p u\left(y_t^p(s,l,k)\right)+\sum_{t}^{T}T_t^p(s,l,k)\right] > \sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\pi_t^p(s,l,k)\delta_t^p u\left(y_t^p(s,l,k)\right) \tag{663}$$

and

$$\sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\sum_{t}^{T}\pi_t^R(s,l,k)\delta_t^R u\left(y_t^R(s,l,k)\right) > \sum_{s=1}^{s}\sum_{l=1}^{l}\sum_{k=1}^{k}\left[\sum_{t}^{T}\pi_t^R(s,l,k)\delta_t^R u\left(y_t^R(s,l,k)\right)-\sum_{t}^{T}T_t^R(s,l,k)\right] \tag{664}$$

Proposition 4: Growth requires that income of both poor and rich are rising over time:

$$T_t^p(s,l,k) < T_{t+1}^p(s,l,k) < T_{t+1}^p(s,l,k) < \ldots.. < T_{t+T}^p(s,l,k) \tag{665}$$

$$Y_t^p(s,l,k) < Y_{t+1}^p(s,l,k) < Y_{t+1}^p(s,l,k) < \ldots.. < Y_{t+T}^p(s,l,k) \tag{666}$$

$$Y_t^R(s,l,k) < Y_{t+1}^R(s,l,k) < Y_{t+1}^R(s,l,k) < \ldots.. < Y_{t+T}^R(s,l,k) \tag{667}$$

Proposition 5: Termination of poverty requires that every poor individual has at least the level of income equal to the poverty line determined by the society. When the poverty line is defined one half of the average income this can be stated as:

$$Y_t^p(s,l,k) \geqslant \frac{1}{2}\left(\frac{1}{N}\sum_{h=1}^{N}Y_t^h(s,l,k)\right) \tag{668}$$

Above five propositions comprehensively incorporate all possible scenarios in the poverty game mentioned above. Propositions 2-5 present optimistic scenarios for a chosen horizon T. Testing above propositions in a real world situation is very challenging exercise. It requires modelling of the entire state space of the economy. Moreover in real situation consumers and producers are heterogeneous regarding their preferences, endowments and technology and economy is more complicated than depicted in the model above. In essence it requires a general equilibrium set up of an economy where poor and rich households participate freely in economic activities taking their share of income received from supplying labour and capital inputs that are affected by tax and transfer system as illustrated in the next section.

26 Overlapping Generation Model

26.1 Two Period Overlapping Generation Model

- Assume an economy, inhibited by two generations, young and old.
- Young ones work, earn , consume and save and old ones stay at home in retirement and consume out of their past savings. Economy is continuum of generations such as $g_{i,t}$where $i = 1, 2, ...N$ refers to the generations and t refers to the time period.
- Each agent is assumed to live for two periods - as a young and as an adult. For instance, person in generation 1, $g_{1,1}$ is born and young in $t = 1$ and becomes older in $t = 2$ and is succeeded by $g_{2,1}$ who is young in $t = 2$, becomes old one in period $t = 3$ and dies at the end of that period.
- In this manner new generations continuously replace the old generations but the economy continues with these two types of people forever. Behavior of each type is similar to their types in earlier periods; young ones work, earn, save part of their income and make families and get children and old ones retire and consume their savings and leave some bequest to their children.

Three Period Overlapping Generation Model

- Three types of people exist every year in the economy: young ones, adults and old ones.
- Young ones go to the school, adults work, and old ones stay home in retirement.
- In g11 first subscript refers to the generation and second subscript to the period.
- Person in g11 is born in period 1, becomes adult in period 2 and becomes older in
  period 3 and dies in period 4
- Economy continues with these three types of people forever.
- Behavior of each type is very different. Young ones borrow to fund their education;
- adult ones work, earn and save part of their income and make families and get children;
- old ones retire and consume their saving and leave some bequest to their children.

26.1.1 Specification of an Overlapping Generation Model

- The simplest version of this model can be explained in fifteen equations as following (see Samuelson, 1958; Auerbach and Kotlikoff,1987 for details).

Period1	g_{11}	g_{01}	g_{02}									
Period2		g_{21}	g_{11}	g_{01}								
Period3			g_{31}	g_{21}	g_{11}							
Period4				g_{41}	g_{31}	g_{21}						
Period5					g_{51}	g_{41}	g_{31}					
Period6						g_{61}	g_{51}	g_{41}				
Period7							g_{71}	g_{61}	g_{51}			
Period8								g_{81}	g_{71}	g_{61}		
Period9									g_{91}	g_{81}	g_{71}	
Period10										g_{101}	g_{91}	g_{81}

Production is function of capital, labour and technology and is subject to constant return to scale with here $\alpha + \beta = 1$.

$$Y_t = AK_t^{\alpha} L_t^{\beta} \tag{669}$$

In terms of income per effective worker:

$$y_t = Ak^{\alpha} \tag{670}$$

Market clears in each period, whatever is produced is either consumed or invested.

$$Y_t = C_t + I_t \tag{671}$$

Equilibrium conditions in Overlapping Generation Model
Aggregate consumption is total of the consumption of young and old

$$C_t = N \cdot c_{yt} + N \cdot c_{ot} \tag{672}$$

Wage income is given by the labour share in production

$$W_t = \beta AK_t^{\alpha} L_t^{\beta} \tag{673}$$

Interest rate equals the marginal product of capital

$$r_t = \alpha AK_t^{\alpha-1} L_t^{\beta} \tag{674}$$

Agents consume θ fraction of their income in period 1

$$c_{yt} = \theta w_t \tag{675}$$

Equilibrium conditions in Overlapping Generation Model
save $(1-\theta)$ share of w_t and invest it in assets for consumption at the old age:

$$a_t = (1-\theta)\, w_t \tag{676}$$

$$c_{ot} = a_t\,(1+r_t) = (1-\theta)\, w_t\,(1+r_t) \tag{677}$$

Law of accumulation of capital stock, with no depreciation is:

$$K_{t+1} = K_t + I_t \tag{678}$$

From 671 and 669

$$C_t = AK_t^{\alpha} L_t^{\beta} - I_t \tag{679}$$

Then substituting **??** and 672 in 679

$$Nc_{yt} + Nc_{ot} = AK_t^{\alpha} L_t^{\beta} - K_{t+1} + K_t \tag{680}$$

Capital Accumulation in Overlapping Generation Model
Further substituting 675 and 676 for consumption of young and old

$$N\theta w_t + N\,(1-\theta)\, w_t\,(1+r_t) = AK_t^{\alpha} L_t^{\beta} - K_{t+1} + K_t \tag{681}$$

substituting 673

$$AK_t^{\alpha} L_t^{\beta} - K_{t+1} + K_t = \theta\beta AK_t^{\alpha} L_t^{\beta} + (1-\theta)\,(1-\beta)\, AK_t^{\alpha} L_t^{\beta}\,(1+r_t) \tag{682}$$

By further re-arrangement

$$K_{t+1} - K_t = AK_t^{\alpha} L_t^{\beta} - \theta\beta AK_t^{\alpha} L_t^{\beta} - (1-\theta)\,(1-\beta)\, AK_t^{\alpha} L_t^{\beta}\,(1+r_t) \tag{683}$$

Parameters and results in Overlapping Generation Model

Table 50: Parameters of the Two Period OLG Model

Parameter	α	β	θ	K_0	k_0	N	τ_l	τ_k
Value	0.5	0.5	0.5	300	3	100	0.1	0.1

Table 51: Results of the Two Period OLG Model

Variables	k	K	Y	w	r	cy	$c0$	S	I
Solution without tax									
Initial condition	1.5	150	1229.3	7.91	2.26	3.95	4.89	245.3	245.3
Steady State	5.79	598	1710	11.98	0.857	5.99	11.12	0	0
Solution with tax									
Initial condition	1.5	150	1229.3	7.12	2.03	3.56	4.55	205.7	205.7
Steady State	3.7	370.7	1480	9.3	1.08	5.2	8.2	0	0

Summary of the OLG model

- This is a first order differential equation in K_t and can be solved iteratively using a numerical method starting from initial condition where K_0 is given. System converges to the steady state when $K_{t+1} = K_t$.

- A numerical method is adopted to solve the model using Excel for tax and no tax scenarios. Labour income and capital income taxes distort the first order conditions $(1-\tau_k)r_t = \alpha A K_t^{\alpha-1} L_t^{\beta}$ and $(1-\tau_l)W_t = \beta A K_t^{\alpha} L_t^{\beta}$.

- This raises the cost of capital and labour to the producer and reduces the capital stock and output as the level of welfare of the households. Net investment and savings are zero in the steady state. Solutions of the model for parameter values in tables given above

- .

- As expected capital and labour income taxes have significantly reduced the capital stock, output, wage rate, saving and investment and consumption of young and old in the model.

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References

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27 Preliminary Baseline Problems

27.1 Problem Set 1

1. Find the partial derivatives ($\delta z/\delta x$, $\delta z/\delta y$) for the following functions:

$z = 3x^2 + 4y^2$
$z = 4 + 2x + 3x^4$
$z = 10xy$
$z = 4x^{1/3}y^{2/3}$
$z = 100x^{1/2}y^{1/2}$
$z = 7x^2y^3$

2. Explain the meaning of concave and quasi-concave functions. What is their importance in solving maximisation and constrained maximisation problems?

3. Consider the demand function $q = 100 - 2p$ where q stands for quantity demanded and p for price.

(i) Find the elasticity of demand when $p = 40$, $p = 25$, $p = 20$
(ii) Find the total revenue, the average revenue and the marginal revenue functions.

4. Consider the demand function $q = \frac{100}{p}$.

(i) Find the elasticity of demand when $p = 40$, $p = 25$, $p = 20$.
(ii) Find the total revenue, the average revenue and the marginal revenue functions.

5. Consider the following two utility functions:

(i) U = 2x + y and (ii) U = Minimum (2x, y), where x and y are the two goods consumed.
Draw the indifference curves in a graph paper (or, if you wish, in a plain paper) corresponding to the two utility functions. How would you describe the relationship between the goods x and y in these two cases?

27.2 Problem Set 2

1. Explain concept of cross-elasticity of demand and its use in economics.

2. Explain why the elasticity of demand is often used to classify goods as luxury goods or essential goods.

3. Consider a consumer consuming only apples (x) and bananas (y).

(i) Suppose the prices of apples and bananas are given by $p_x = 2$ and $p_y = 5$. The income of the consumer is 100. Write down his budget equation.
(ii) Suppose the consumer's utility function is given by: $U = 10x^{1/4}y^{3/4}$.
Find the marginal utilities of apples (MU_x) and bananas (MU_y).
How many apples and bananas will the consumer consume?

4. Using the concepts of income effect and substitution effect, explain why an increase in wages may induce workers to work for less hours.

27.3 Problem Set 3

1. In the competitive market, the demand function and the supply function of yoyos are given by,

$$q^d = 900 - 4p$$

$$q^s = -60 + 2p$$

(a) Find the equilibrium price and quantity.

(b) Evaluate the consumers' surplus, the producers' surplus and the total surplus in the yoyo market.

(c) Suppose the government imposes a sales tax, $t = 60$, per unit of output sold. Find the loss of welfare (loss of total surplus) from the imposition of the sales tax.

2. Discuss the relationship between the elasticity of demand and the Incidence of Taxation (i.e., how the burden of taxation is distributed between the consumers and the producers).

27.4 Problem Set 4

1. Explain the following three concepts:

(i) Consumers' Surplus
(ii) Equivalent Variation
(iii) Compensating Variation.
How they are related to each other?

2. Consider the production Function, $Q = 90L^{1/3}K^{2/3}$.

(i) Find the marginal productivities of labour and capital (MPL and MPK).
(ii) Show that the production function satisfies constant returns to scale.
(iii) Show that $Q = MPL.L + MPK.K$
(iv) Prove that $MPL/MPK = \frac{1}{2}(K/L)$. What is the significance of this result?

27.5 Problem Set 5

1. Explain the relationship between Long-run Average Cost Curve (LRAC) and the corresponding Short-run Average Cost Curve (SRAC) in the presence of economies of scale as well as under constant returns to scale.

2. Explain why the Short-run Marginal Cost Curve should pass through the lowest point of the SRAC curve.

3. A competitive firm has the following short-run cost function (Total Cost):

$$C(Y) = Y^3 - 8Y^2 + 30Y + 5$$

(a) The firm's marginal cost function is $MC(Y) =$?
(b) The firm's average variable cost function is $AVC(Y) =$?
[NOTE, the total variable cost is $C(Y) - C(0)$]
(c) Find the level of output at which the average variable cost is minimised.
(d) What is the minimum price at which the firm will produce any output?
What is the minimum level of output which the firm will ever supply?
(e) At what price the firm will supply exactly six units of output?

27.6 Problem Set 6

1. David New is a watchmaker who is about to start operating in a perfectly competitive market. His total costs are given by:

$$C = 100 + Q^2$$

where Q is the level of output.
(a) If the price of watches is £60, how many watches should he produce to maximize his profit?
(b) What will the profit level be?
(c) What is the minimum price at which David finds it profitable to operate in the long-run?

2. A firm's total revenue function is given by $R = aQ - 2Q^2$. Is this a perfectly competitive firm? Explain.

3. Is it desirable for a firm to produce output even though it is losing money? Explain your answer carefully.

4. Can a perfectly competitive firm ever maximize profit by operating at a point on the downward-sloping portion of its marginal cost curve?

5. Why is the equality between price and marginal cost regarded as the criterion of

efficiency?

27.7 Problem Set 7

1. Professor David Dong has just written the first textbook in Economics. Market research suggests that the demand curve for the book is, $Q = 2000 - 100p$, where p is the price. It will cost £1000 to set the book in type. This set up cost is necessary before any copy is printed. In addition to the set up cost, there is a marginal cost of £4 per book printed.

(a) The total revenue function for the book is $R(Q) =$?
(b) The total cost function for producing the book is $C(Q) =$?
(c) The marginal revenue function for the book is $MR(Q) =$?
(d) The marginal cost function for the book is $MC(Q) =$?
(e) The profit maximising quantity of books is $Q =$?
[hints (a) R(Q) = 20Q - (Q^2/100) (b) C(Q) = 1000 + 4Q, (c) MR(Q) = 20 - (Q/50) (d) MC(Q) = 4 Q = 800]

2. A monopolist faces a demand curve given by $Q = 50 - 0.5p$ and a total cost curve $C = 640{+}20Q$. What is the profit maximising price? What is the quantity of output the monopolist will sell?

[hints:$P = 60, Q = 20$]

3. (a) Explain how a price discriminating monopolist, selling his product in two different markets, sets his prices.

(b) How a perfectly price discriminating monopolist decides the quantity of output to be sold? Explain the efficiency implication of perfect price discrimination.

27.8 Problem Set 8

1. The flowerpot industry is a duopoly. The two producers, Bill and Ben, face the following market demand curve:

$Q = 200 - 2p$
The total cost function for Bill is: $C_1 = 5Q_1$ and for Ben is: $C_2 = 0.5Q_2^2$.
(a) Assume Cournot behaviour. Derive the reaction function and the equilibrium output for each duopolist. What is the market price? What is the profit for each producer?
(b) Suppose Bill and Ben collude to maximize joint profit. How much output will each of them produce?

27.9 Problem Set 9 : Basics of Game theory

1. Draw a pay-off matrix for a market only with two producers. Explain strategic choices of for each player.
2. Formulate a two by two game which can be solved using a dominant strategy for one or both players.
3. Formulate a game which can be solved using a Nash equilibrium method. Explain why any other solution is not a stable solution.

27.10 Problem Set 10: General equilibrium under pure exchange

10.1 Use an Edgeworth box diagram to illustrate how two individuals A and B can benefit from exchanging goods X and Y in a market economy.
10.2 Derive production possibility frontier for an economy using Edgeworth diagram where the labour and capital inputs are used producing goods X and Y.
10.3 What drives the prices of these goods. Can you specify a model?

27.11 Problem Set 11: Externality

11.1 Give five examples of negative and positive externalities in consumption and in production.
11.2 Illustrate a market for pollution? Explain why market fails provide correct amount of clean air?
11.3 Explain positive externality of internet or education? Justify cases for subsidising research on the ground of positive externality.

27.12 Problem Set 12: short notes

1. Write short note in any four of the following

a) Equivalent and compensating variation measures of price change.
b) Normal, inferior and superior goods.
c) Aggregate deadweight loss of taxes in market model of demand and supply.
d) Axioms of utility theory and cardinal and ordinal measures of utility Assumptions and limitations of perfect competition.
e) Every game has a solution in mixed strategy.
f) Role of bargaining in cooperative games.
g) Normal and extensive form of a game.
h) Utility from wealth for a person living in Fairfield village is given by $U = \ln(W)$, where U is the utility and W is the level of wealth. This person has a prospect of good income of 4000 with probability 0.4 and of bad prospect of low income of 1000 with probability of 0.6. How much would this person pay to insure against income uncertainty?
i) A consumer lives for two periods and has income of 400 and 800 in the first and second periods respectively. He/she values consumption of both periods equally. What would be present value of consumption in the first and the second periods at the following interest rates:
(i) at zero rate of the real interest (ii) at 10 percent rate of real interest?

28 Assignment (optional)

Write an essay in 1500 words in any one of the following topics. Support your statements with some derivations based on economic theories and evidenced from the real world.

1. Does Brexit lead to increase in food and fuel prices in the UK?
2. Welfare costs of market imperfections
3. Burden of taxes on consumers and producers
4. Two application of game theory for policy analysis

This essay accounts for 20 percent of the module marks. Write in your own words referring to existing economic theories and evidences available to you. Be critical, analytical and precise. Submit the electronic copy of essay through Turnitin and a hard copy through the undergraduate office. Class ID and password and the Turnitin procedure are given in the module handbook ready to be downloaded from the resources folder in eBridge site for this module.

This is expected to be a professional piece of work and must contain a model and analysis. The elements of marks will broadly be based on the originality of the motivation to the question (15%), statement of the relevant model (15%), derivations (15%), analysis based on derivations (15%), application of the model (40%). Students are allowed to ask any question on the chosen topic in any teaching sessions.

You are allowed and encouraged to use any material covered in lectures or tutorials presented in this workbook and placed in the resources folder in the eBridge in this module.

Tentative list of articles for each topic is listed below. Howerver, students should try to most recent articles as the study progresses. Policy documents from the central bank and the treasury or finance ministries or planning agencies could be used for analysis.

28.1 Topic: Economic Impacts of COVID-19

Topic 1:Contrast impacts of COVID-19 on price and output of firms in service sector such travel and tourism and in comparison of firms in the manufacturing sector. Use COVID-19 shocks to demand for and supply of products under perfectly competitive or imperfectly competitive markets.

Topic 2: What were the impacts of COVID-19 on labour demand, labour supply and wages. Apply a simple general equilibrium model with production to assess this issue.

Topic 3:What are the impacts of COVID-19 on investment and capital stocks of production firms in production and service sectors of the economy?

Topic 4: Apply theory of cooperative and non-cooperative games to analyse interactions at local, regional or global level during COVI-19 pendemic.

Topic 5: Lock-down under COVI-19 pendemic has shown how to achieve zero emission target by 2020. Discuss using solutions of a model.

28.1.1 Useful Webinar during COVID-19 Lockdown-1

- Royal Economic Society: https://www.res.org.uk/resource-library-page/covid-19.html
- RPEC https://biblio.repec.org/entry/iab.html
- Bank of England https://www.bankofengland.co.uk/coronavirus
- Office of National Statistics: https://www.ons.gov.uk/peoplepopulationandcommunity/healthandsocialcare/conditi
- London Business School:

https://www.london.edu/campaigns/executive-education/pandemic-webinarsPrinceton

- Bendheim Center for Finance

https://bcf.princeton.edu/event-directory/covid19/

- IMF Webinars:

https://www.imf.org/external/mmedia/viewlive.aspx?eventID=7454&sm=true

- OECD webinars:

https://oecdtv.webtv-solution.com/

- ONS, OBR, NIESR, Fiscal Studies, Bank of England, UK Parliament, LEPs

- Tax Foundation

https://taxfoundation.org/covid19-economic-recovery/

- World Health Organistion

https://www.who.int/emergencies/diseases/novel-coronavirus-2019

- COBRA: https://www.instituteforgovernment.org.uk/our-work/coronavirus
- BBC

https://www.bbc.co.uk/news/coronavirus

- Our World in Data Oxford

https://ourworldindata.org/

- John Hopkins University

https://coronavirus.jhu.edu/data

- CDC

https://www.cdc.gov/coronavirus/2019-nCoV/index.html

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28.2 Topic: Economic Impacts of Brexit

Focus your essay on one of the following five aspects of Brexit.

28.2.1 Topic 1:Does Brexit lead to increase in food and fuel prices in the UK?

28.2.2 Topic 2: Impact of Brexit on the labour market and migration

28.2.3 Topic 3: Impact of Brexit on capital market

28.2.4 Topic 4: Imapct of Brexit on innovation and productivity of firms

28.2.5 Topic 5: Application of game theory for analysis Brexit options

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29 Popular Databases

Constructing Data for Analysis: Step by Step Guidelines

Connect to http://www.esds.ac.uk/international/ Choose direct links to macro data

Important Steps for extracting data

I. World Bank Data (World Bank data Indicator

1. Click on Direct Links to Macro Data
2. Choose World Bank Data
3. Select University of Hull
4. Put Athense user name and pass word;
5. Complete the registration process required by data
6. Select World Bank Development Indicators
7. Select Year 1960 -2008 (all can be selected by a tick mark)
8. Select a country (e.g. South Africa)
9. Select a series (e. g. Population between 15-64; and population growth rate) ; can search for population
10. Click on show Table
11. Adjust row and column dimensions of the table by moving around icons
12. Download data in *.CSV (MS-DOS) format
13. Open the data file just created
14. Make some time series graph
15. Next time; add few more variables like DGP per capita constant 2000 dollars; Gross fixed capital formation % of GDP; Final consumption Expenditure as a % GDP; Current account balance as a % of GDP; General Government final consumption % of GDP; GDP constant 2000 $

II. IMF World Economic Outlook (WEO) data

1. Steps 1 -5 as above
2. Select IMF WEO data
3. Select World Economic Outlook
4. Select Euro Aria

5. Select crude oil price, output gap, unemployment rate, inflation average consumer price)
6. Select all years 1991-2010
7. Show Table; Download the data; Open and Excel.
8. Do macro analysis.

III. Eurostat New Cornos (Data for EU countries)

1. Steps 1 -5 as above
2. Select Eurostat New Cornos
3. Select Economy and Finance
4. Eurostat
5. Exchange rates
6. Nominal effective exchange rates
7. Real effective exchange rates

IV. Datastream

29.1 Econometric and Statistical Software

- Excel
- OX-GiveWin/PcGive/STAMP
- Eviews
- Shazam
- microfit
- RATS
- LIMDEP
- GAUSS
- STATA/SPSS
- **http://www.feweb.vu.nl/econometriclinks/;** https://www.aeaweb.org/rfe/

1. Excel Spreadsheets are very user friendly and could be used for algebraic calculations and statistical analyses for many kinds of economic models. First prepare an analytical solution by hand then use Excel formula to compute. Excel has constrained optimiser routine at tool/goal seek and solver commend. It also contains matrix routines to get determinants of matrices and to multiply and invert them using multiple cell options. Koop (2007) is a brilliant text for analysis of economic data using excel. Koop G (2007) Analysis of Economic Data, Wiley, UK.

2. OX-GiveWin/PcGive/STAMP (www.oxmetrics.net) is a very good econometric software for analysing time series and cross section data. This software is available in all labs in the network of the university by sequence of clicks Start/applications/economics/givewin. Following steps are required to access this software.

a. save the data in a standard excel file. Better to save in *.csv format .

b. start give win at start/applications/economics/givewin and pcgive (click them separately)

c. open the data file using file/open datafile command.

d. choose PcGive module for econometric analysis.

e. select the package such as descriptive statistics, econometric modelling or panel data models.

d. choose dependent and independent variables as asked by the menu. Choose options for output.

e. do the estimation and analyse the results, generate graphs of actual and predicted series.

A Batch file can be written in OX for more complicated calculations using a text editor such as pfe32.exe. Such file contains instructions for computer to compute several tasks in a given sequence.

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29.2 Mathematical software

4. GAMS is good particularly in solving linear and non-linear problems. It has widely been used to solve general equilibrium models with many linear or non-linear equations on continuous or discrete variables. It comes with a number of solvers that are useful for numerical analysis. For economic modelling it can solve very large scale models using detailed structure of consumption, production and trade arrangements on unilateral, bilateral or multilateral basis in the global economy where the optimal choices of consumers and producers are constrained by resources and production technology or arrangements for trade.

It is a user friendly software. Any GAMS programme involves

- declaration of set, parameters, variables, equations,
- initialisation of variables and
- setting their lower or upper bounds and
- solving the model using Newton or other methods for linear or non-linear optimisation
- and reporting the results in tables or graphs (e.g. ISLM.gms).

Full version of GAMS/MPSGE program is good for large scale standard general equilibrium models. GAMS programme can be downloaded from demo version of GAMS free from www.gams.com/download).

The check whether the results are consistent with the economic theory underlying the model such as ISLM-ASAD analysis for evaluating the impacts of expansionary fiscal and monetary policies. Use knowledge of growth theory to explain results of the Solow growth model from Solow.gms.

Consult GAMS and GAMS/MPSGE User Manuals, GAMS Development Corporation, 1217 Potomac Street, Washington D.C or www.gams.com or www.mpsge.org for GAMS/MPSGE.

For other relevant software visit: http://www.feweb.vu.nl/econometriclinks/ or https://www.aeaweb.org/rfe/

29.3 MATLAB

MATLAB is widely used for solving models. It has script and function files used in computations.

Both have *.m extensions. Its syntax are case sensivite. Solving a system of linear equations and handling matrices

Example 1

Write a programme file matrix.m like the following and try run.

```
% now solve a linear equation
% 5x1 + 2x2 =20
% 3x2 + 4x2 =15
k =[5 2;3 4];
n = [20 15];
kk = inv(k)
x = kk*n'
```

One more example of system of equation and factorisation of matrices

```
A=[1 2 3; 3 3 4; 2 3 3]
b=[1; 2; 3]
%solve AX=b
X = inv(A)*b
%eigen value and eigenvectors of A
[V,D]=eig(A)
%LU decomposition of A
[L,U]=lu(A)
%orthogonal matrix of A
[Q,R]=qr(A)
%Cholesky decomposition (matrix must be positive definite)
%R = chol(A)
%Singular value decomposition
[U,D,V]=svd(A)
```

Contents.m for list of files in MATLAB demo. MATLAB demo available in http://www.youtube.com/.

30 Sample exam papers

30.1 Class test Samples 2018

30.2 Class Test 1 (Any two)

Q1. Consider a market for a product A with linear demand (D) and supply (S) functions in terms of market price (P) as follows:

$$D = 150 - 3P \tag{1.1}$$

$$S = 30 + 2P \tag{1.2}$$

1. **What is the equilibrium price where demand equals supply ($D = S$) in this market? [15]**

2. **What amount of this product is bought and sold at equilibrium price? What is the consumer surplus? [15]**

3. **Now let there be a sales tax (t) on this commodity. Then suppliers get P^S but the consumers pay, $P^D = P^S + t$. Note that P^S is less than P, and P^D is above P. The difference between P^S and P^D is the amount of tax or the tax wedge (t) in market for this commodity after the imposition of this unit tax. Let $t = 2$. Find the prices, P^S and P^D and quantity bought and sold in tax distorted equilibrium. [20]**

4. **Draw a diagram to compare equilibrium before and after the imposition of tax rate t. Indicate the overall deadweight loss and and its components in terms consumer and producer surpluses because of this sales tax in this diagram. [20]**

5. **Using above solutions calculate the exact amount of consumer and producer surpluses lost due to this tax. [15]**

6. **Find price elasticities of demand and supply at the pre-tax equilibrium price P. Show that the burden of tax (deadweight loss) is higher for the less elastic part of the market. [15]**

Q2. Consider a general equilibrium model with taxes in which a representative household maximises utility (U) from consumpiton (C) and leisure (L) subject to its budget constraint as:

$$\max \quad U = C \cdot L \tag{2.1}$$

subject to

$$p\,(1+t)\,C + wL = w\overline{L} \tag{2.2}$$

Here p is price, t is the tax rate and w is the wage rate.

A representative firm maximises profit (π) producing Y susing labour input (LS) subject to its technology constraint as:

$$\max \quad \pi = p.Y - w.LS \tag{2.3}$$

subject to

$$Y = LS \tag{2.4}$$

Market clearing in the labour and goods markets imply:

$$LS + L = \overline{L};\ Y = C + G \tag{2.5}$$

For simplicity assume that all of tax revenue is spent on public spending (G) as a result of a balaced budget policy of the government. Assume that endowment of labour $\overline{L}$ is 200; tax rate t to be 20 percent, $t = 0.2$. You may select price of commodities as a numeraire, $p = 1$. Wage (w) is set according to the marginal productivity of labour.

1. **What is the wage rate that makes supply eqaul demand for the labour market equilibrium? [25]**
2. **Find the level of output consistent to that labour market equilibrium.[25]**
3. **What are the levels of consumption by the representative household and spending by the government? [25]**
4. **What are the levels of utility in this economy with or without tax distortions? Comment. [25]**

Q3. Only two firms ($i = 1, 2$) supply products in a certain market in which the market demand for the product is:

$$P = 150 - (q_1 + q_2) \tag{3.1}$$

Cost of production for each firms is:

$$C_i = 10q_i \quad for \quad i = 1, 2 \tag{3.2}$$

1. **What are the levels of output, market price and total profit when these two firms collude? [40]**
2. **Find levels of output, market price and profit of each firm under a Cournot duopoly? [40]**
3. **Compare consumer and producer surpluses based on your solutions under the collusion and the Cournot duopoly. [20]**

Q4. The amount of wealth in the good state is W. If a bad event occurs there will be a loss (L) and the probability of a loss is p.

The owner of the property can insure for amount (q) paying premium rate (m). The expected utility maximisation problem of the individual is implicitly written as:

$$\max_{q} EU = p.u\left(W - L - mq + q\right) + (1 - p)\, u\left(W - mq\right) \tag{4.1}$$

The profit maximising condition of the insurance company with perfect competition in the insurance market is:

$$p\,(1-m)\,q-(1-p)\,mq=0 \tag{4.2}$$

A risk averse consumer likes to get the same marginal utility whether in the good or bad state

$$u'\,(W-L-mq+q)=u'\,(W-mq) \tag{4.3}$$

Answer following questions studying this model carefully.

1. **Prove that the optimal premium rate (m) equals the probability of loss (p). Explain. [50]**
2. **Prove that it is optimal for an individual facing uncertainty in this way to purchase full insurance $[L=q]$. How much does the insurer pay to the insurance company? Discuss your findings. [50]**

30.3 Class Test 2 (Any two)

Q1. Consider a consumer's utility (U) maximisation problem given by a Cobb-Douglas utility function as below:

$$\underset{x_1,x_2}{Max} U=x_1^{\alpha}x_2^{\beta},\ \alpha+\beta=1;\text{s.t } E=p_1x_1+p_2x_2 \tag{1.1}$$

here $p_1$and p_2 are prices of commodities $x_1$and x_2 and E denotes the total expenditure (amount of money) available.

1. **Derive Marshallian demand functions for both $x_1$and x_2 and associated indirect utility function. [20]**
2. **Prove that this demand function is homogenous of degree zero in price and income, increasing in income and decreasing in price. [20]**
3. **Formulate the dual of this problem and derive the expenditure function (E). [20]**
4. **By Shepherd's Lemma, $\frac{\partial E}{\partial p_i}=x_i(p_1,p_2,E)$. Evaluate this using the expenditure fuction derived above. Draw the compensated (Hicksian) and uncompensated (Marshallian) demand functions for x_1 in a well labeled diagram. [20]**
5. **Decompose the total price effect into income and substitution effects using the Slutskey equation. [20]**

Q2. One common example for a bargaining game is splitting a pie between two individuals, i and j. The total amount to be divided is 1. Their shares in this pie are given by θ_i and θ_j respectively and they should not claim more than what is on the table, i.e. $\theta_i+\theta_j\leq 1$. This implies a meaningful solution of the game requires $\theta_i\geq 0$ and $\theta_j\geq 0$. If the sum of claims is more than what is on the table each gets zero i.e. when $\theta_i+\theta_j>1$ then $\theta_i=0$ and $\theta_j=0$.

Thus the Nash bargaining problem is given by

$$\max U = (\theta_i - 0)(\theta_j - 0) \tag{2.1}$$

subject to

$$\theta_i + \theta_j = 1 \tag{2.2}$$

Study this model carefully and answer the following questions:

1. **Formulate the constrained optimisation of this problem. [10]**
2. **Find the optimal values of θ_i and θ_j that satisfy the Nash equilibrium. [25]**
3. **Suppose individual i has some bargaining power against individual j and puts a threat that he/she walks away if a certain amount ϕ is taken out for her/him before bargaining begins. Modify the above constrained optimisation model to accommodate such threat point in the bargain. [35]** (600481)
(question continued..)
4. **Discuss four properties (symmetry, efficiency, linear invariance and independence of irrelevant alternatives) that apply to bargaining game like this. [30]**

Q3. Consider a pure exchange general equilibrium model for an economy with two goods X_1 **and** X_2 and two individuals A and B. A Lagrangian function for constrained optimisation for Household A is given by $\mathcal{L}^A$:

$$\mathcal{L}^A = \left(X_1^A\right)^{\alpha_A}\left(X_2^A\right)^{1-\alpha_A} + \lambda\left(P_1\omega_1^A + P_2\omega_2^A - P_1X_1^A - P_2X_2^A\right) \tag{3.1}$$

Similarly Lagrangian function for constrained optimisation by Household B is given by $\mathcal{L}^B$:

$$\mathcal{L}^B = \left(X_1^B\right)^{\alpha_B}\left(X_2^B\right)^{1-\alpha_B} + \lambda\left(P_1\omega_1^B + P_2\omega_2^B - P_1X_1^B - P_2X_2^B\right) \tag{3.2}$$

Here X_j^h is demand for good $j = 1,2$ by individuals $h = A, B$; and α_i and $1-\alpha_i$ denote the share income spent on good 1 by household h and $1-\alpha_i$ is their spending on good 2. ω_j^h represent the endowment of commodity $j = 1,2$ of household $h = A, B$.

A Set of parameters required to solve this model are given in the table below.

Table 52: Parameters in Pure Exchange Model

	Household A	Household B
Endowments	$\{\omega_1^A, \omega_2^A\} = \{100, 0\}$	$\{\omega_1^B, \omega_2^B\} = \{0, 200\}$
Preference for X_1 (α)	0.6	0.4
Preference for X_2 $(1-\alpha)$	0.4	0.6

You may assume Walrasian numeraire; this means price of good 1 is 1; $P_1 = 1$. With this specification implied incomes (I^h) for households A and B are:

$$I^A = \omega_1^A \; ; \quad I^B = P_2\omega_2^B \tag{684}$$

Study this model carefully and answer following questions:

1. **What are the demand functions of households A and B for goods 1 and 2? [25]**
2. **What is the relative price that clears the markets for both X_1 and X_2? [25]**
3. **What is the utility of households A and B? [25]**
4. **Represent your result in a suitable Edgeworth-box diagram. [25]**

31 Sample Final Exam Papers

31.1 Sample 1 (Any three)

Q1. Consider a utility maximisation problem of a consumer with the CES utility function as follows:

$$\max_{x_1,x2} u = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} \tag{1.1}$$

Subject to

$$M = p_1 x_1 + p_2 x_2 \tag{1.2}$$

Here in equation (1.1) the level of utility (u) is obtained by consuming goods x_1 and x_2 given the value of substitution parameter ρ. Then (1.2) gives the budget constraint of the consumer where the income M is either spent on x_1 and x_2 taking prices p_1 and p_2 as given.

1. **Write a Langrangian function for constrained optimisation and associated first order conditions. [20]**
2. **Derive demand function for x_1 and x_2. [20]**
3. **Prove that these demand functions are homogenous of degree zero in prices p_1 and p_2 and income (M). [20]**
4. **Derive the indirect utility function from the Marshallian demand functions for x_1 and x_2 derived above.[20]**
5. **Derive the expenditure function using the indirect utility function. [10]**
6. **What is Shepherd's lemma? Is expression**

$$\frac{\partial M}{\partial p_i} = x_i(p_x, p_y, M) \tag{1.3}$$

true for compensated demand function for either for x_1 and x_2 based on duality principle? [10]

Q2. Market demand curve for mobile phones is given by

$$Q = 10 - \frac{1}{5}P \tag{2.1}$$

Two firms exist in the market to supply this product. Their cost functions are

$$C_1 = 4q_1 \; and \; C_2 = 5q_2. \tag{2.2}$$

1. **Write the profit function for each these two firms in this mobile phone market. [10]**
2. **Assume that thse firms operate under the Cournot duopoly model. Each firm takes account of level of output supplied by another firm while determining its own quantity to be supplied to the market. Determine reaction functions of both firms and represent them in one diagram. [20]**
3. **How much will each produce? What will be the market price? How much profit will each make from selling mobile phones? [15]**
4. **What will be the consumer surplus from each of them? Show these consumer surpluses diagrams for each of these two firms.[15]**
5. **Assume that firm 1 is a leader and incorporates the ouput reaction function of its follower in its profit function. Modify it's profit function. Determine the level of output that firm 1 (the leader) will supply in the market? [15]**
6. **What will be the output of the firm 2, which is the follower firm in this mobile phone market? [5]**
7. **What will be the market price and levels of profit of firm 1 and firm 2? [10]**
8. **What would have been the level of output, market price and profit if they had formed a collusive cartel? [10].**

Q3. Shareholders in a large corporation (such as the Royal Bank of Scotland) perceive that profit earned by the corporation depends largely on the level of effort put in by the Chief Executive Officer (CEO). Therefore they agree to provide a good amount of bonus to the CEO above his salary. The general public is skeptical of the effectiveness of a bonus scheme like this. In this context think of a project that would earn £800,000 profit if successful. The probability of success is 60 percent if the CEO puts in normal effort. This probability can rise to 90 percent if the CEO puts in extra effort. The CEO will put in extra effort only if an additional payment of £50,000 is made above the basic salary of £100,000. However, it is difficult for shareholders to monitor whether the CEO is putting in extra effort even if they pay an additional amount of £50,000.

1. **Is it profitable for owners to pay an extra £50,000 for the CEO? Why does such an extra payment not automatically guarantee a higher probability of profit? [20]**

2. **Design incentive compatibility and participation constraints in terms of the basic salary and a bonus so that the CEO puts in extra effort in return for an extra payment. [20]**

3. **Based on the above information, what is the minimum payment required by the CEO to put in extra effort? Do owners find it profitable to make such an extra payment as an incentive device? [20]**

4. **Consider now the case where the CEO can be of low or high productivity type. How can the level of education of a prospective CEO signal to the shareholders whether he or she is of high productivity type? [20]**

5. **How can shareholders signal that they cannot be fooled by a CEO who pretends (to the owners) to be putting in extra effort while actually putting in only normal effort? [20]**

Q4. Consider a firm in a monopolistically competitive industry facing a demand for its products (Q) as a function of price (P) in the market as:

$$Q = A - B \cdot P \tag{4.1}$$

Here A is an intercept and B is the slope of the demand function.

6. **Prove that its marginal revenue (MR) is given by**

$$MR = P - \frac{Q}{B} \tag{4.2}$$

[15]

7. **If the cost function is $C = F + cQ$ with fixed cost F and variable cost c, then prove that the average cost (AC) declines because of the economy of scale. [15]**

8. **Given the size of the market (S) assume that the output sold by a firm (Q), number of firms (N), its own price (P) and average prices of firms $\left(\overline{P}\right)$ are given by**

$$Q = S\left[\frac{1}{N} - b\left(P - \overline{P}\right)\right] \tag{4.3}$$

Note $Q = \frac{S}{N} + S.b.\overline{P} - S.b.P$; $\frac{S}{N} + S.b.\overline{P} = A$; $S.b. = B$ and $MR = MC \iff P - \frac{Q}{B} = c$. **Prove that price charged by a particular firm declines with the number of firms , $P = c + \frac{1}{b \cdot n}$. [15]**

9. **Show that the average cost rises to number of firms in the industry when all firms charge same price; $AC = \frac{n.F}{s} + c$. [15]**

10. **Determine the number of firms and price in equilibrium using $P = AC$. Explain entry and exit behavior and prices when number of firms are below or above this equilibrium point. How do collusive and strategic behaviors may limit these conclusions. Discuss. [20]**

11. **Apply above model to explain interindustry and intra-industry international trade and its impact on prices and number of firms in a particular industry.[20]**

Q5 Utility function of an individual is given by

$$U(W) = ln(W) \tag{5.1}$$

where U is the utility and W is the level of wealth. This individual will have high wealth (W_H) in good state but faces risk of loss and being in low wealth (W_L) in a bad state.

1. **Draw this utility function in a diagram in (W, U) space and explain the economic meaning underlying the curvature of the utility function. [10]**
2. **Is this a risk loving, risk averse or risk neutral individual?** Determine **this using the Arrow-Pratt (1964) measure of risk aversion. [15]**
3. **Let probability of high wealth (π_H) be 0.4 and that of low wealth (π_L) be 0.6. Further let the high wealth in good state be 5000 and low wealth in bad state be 2500. What is the expected wealth of this person? [15]**
4. **Represent expected utility line from the low to high wealth in the above diagram. How does it compare to the utility from weath without uncertainty? [10]**
5. **Show what is the certainty equivalent wealth and the amount of insurance that this person is ready to pay against wealth uncertainty in this diagram. Explain. (15]**
6. **Discuss whether the expected utility from expected wealth is higher or lower than the certainty equivalent wealth for this person? [10]**
7. **Find the certainty equivalent wealth and the maximum amount that this individual will be ready to pay for the insurance? [15]**
8. **Represent all results in a well labeled diagram. [10]**

Q6. Level of education signals quality of a worker. Spence (1973) model was among the first to illustrate how to analyse principal agent and role of signalling in the job market. Consider a situation where there are N individuals applying to work. In absence of education as the criteria of quality employers cannot see who is a high quality worker and who is a low quality worker. Employers know that θ proportion of workers is of high quality and $(1-\theta)$ proportion is of bad quality. Therefore they pay each worker an average wage rate as:

$$\overline{w} = \theta w_h + (1-\theta)\, w_l \tag{6.1}$$

more productive worker is worth 40000 and less productive worker is worth 20000 and θ =0.5 then the average wage rate will be 30000. Let c the cost of being high quality worker be 15000.

1. **What are the wage rates in pooling and separating equilibrium? Is the separating equilibrium is more efficient than the pooling equilibrium? Discuss why. [25]**

2. **Is it worth for high quality workers to signal their quality by the standard of their education if the cost of education is 5000? [25]**

3. **Consider four different levels of cost of eduction, c, being at 5000, 1000, 15,000 and 20,000. Determine in which of these cases is it appropriate for the good workers to signal their quality by taking higher education. [25]**

4. **Represent above results in a well labelled diagram appropriately. Explain. [25]**

31.2 Sample 2(Any three)

Q1. An individual gets utility (u) by consuming oats (x_1) and nuts (x_2) as given by $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. This person has 200 to spend (m) between these two goods, thus the budget constraint is $m = p_1 x_1 + p_2 x_2$. Prices of oats and nuts were $(p_1, p_2) = (2, 2)$ last month, giving the base line demand as $\left(x_1 = \frac{m}{2p_1}, x_2 = \frac{m}{2p_2}\right) = (50, 50)$ and the utility from this consumption was $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = 50$.

Now an increase in VAT on nuts raises its price to 4 but there is no change in the price of oats, $(p_1, p_2) = (2, 4)$. Income does not change and stays the same at 200. The new demand for oats and nuts implied by their prices are $(x_1, x_2) = (50, 25)$. This person has become worse off because of higher prices due to increase in taxes.

1. **Illustrate the demand for oats and nuts in a suitable diagram. [33]**

2. **Calculate the Hicksian compensating and equivalent variations of this price change. [33]**

3. **Illustrate Marshallian demand function along with compensated demands at both levels of utilities. [34]**

Q2. Consider a bargaining game of allocating the assets of a firm between two investors i and j. The total amount to be divided is 1,000,000. The amounts received by investors i and j are given by θ_i and θ_j respectively but the sum of the claims should not be more than what is available, i.e. $\theta_i + \theta_j \leq 1,000,000$. This implies a meaningful solution of the game requires $\theta_i \geq 0$ and $\theta_j \geq 0$. If the investors are greedy and their claims sum to more than the value of the asset, $\theta_i + \theta_j > 1,000,000$ then each gets nothing, $\theta_i = 0$ and $\theta_j = 0$. Neither of these two investors has any threat point in this bargain. Thus the Nash product maximising bargaining problem for them is given by:

$$\max U = (\theta_i - 0)(\theta_j - 0) \tag{685}$$

subject to

$$\theta_i + \theta_j = 1,000,000 \tag{686}$$

1. **Formulate the constrained optimisation form of this bargaining problem. Explain. [30]**
2. **Find the optimal amounts θ_i and θ_j and the Nash product in equilibrium. Discuss. [30]**
3. **Discuss the efficiency, symmetry, linear invariance and independence of irrelevant alternatives (IIA) properties of such a solution. [40]**

Q3. Passenger car market is operating under a monopolistic competition. A particular firm under this market will cut down it own prices if any another firm reduces its price but will not raise its price if another firm raises its price. For simplicity assume that there two firms in the market and their inverse demand and cost functions are as following:

Demand and cost function of firm I:

$$P_1 = 105 - 2q_1 - q_2; \quad C_1 = 5q_1^2 \tag{3.1}$$

Demand and cost function of firm II:

$$P_1 = 35 - q_1 - q_2; C_2 = q_2^2 \tag{3.2}$$

1. **Find the Cournot duopoly equilibrium as a base line for comparison. What are output, price and profit of each firm under this market conditions. [20]**
2. **Now firm I raises price of its own cars by 2 but the firm II does not change its own price. What will be the prices, output and profit of each firm? [15]**
3. **If the firm I reduces its price of cars by 2, how much firm II reduce its price to maintain its market share? What are the profits, level of output and prices for each firm? [15]**
4. **Put every result of the calculation in one table and explain the underlying factors behind these results.[15]**
5. **A firm under the monopolistic competition operates under less than full capacity setting price equal to its average cost in addition to $MR = MC$ rule. Explain. [15]**
6. **In what sense this is a monopoly? In what sense a competitive market? Illustrate discussion with some real world examples. [20]**

Q4. An economy is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. They aim to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. Preferences for type 1 are given by $U_1 = \sqrt{Y_1}$ and for type 2 by $U_2 = \sqrt{Y_2}$. In a given year total output, Y, was 1000 billion pounds.

1. **What is the distribution of output between type 1 and type 2 that maximises the social welfare index? What is the maximum value of the social welfare index of this economy? [25]**

2. **What would have been the allocation if policy makers had given equal weight to the utility of both types of people in the economy such as $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$. By how much does the welfare index change in this case than compared to the social welfare in (a) above? [25]**

3. **How would the social welfare index change in sub-question (1) above if a tax rate of 20 percent is imposed in consumption and the tax receipts are not given back to any of these consumers? How much would the value of social welfare index be in this case? [25]**

4. **Assume that the policy makers still hold the welfare function to be $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$. How would the social welfare index change in (c) if all tax receipts are transferred to type 2 people? [25]**

Q5. **What are the measures of risk aversion for consumers with following utility functions? Which of these consumers is risk-averse, which one is risk neutral and which one is a risk lover? [50]**

1. (a) Logarithmic utility in wealth: $U(W) = ln(W)$
   (b) Cobb-Douglus type utility: $U(W) = W^{\frac{1}{2}}$
   (c) Linear utility: $U(W) = aW$
   (d) Exponential utility: $U(W) = \exp(aW)$

2. The amount of wealth in the good state is W. If a bad event occurs there will be a loss (L) and the probability of a loss is p.

The owner of the property can insure for amount (q) paying premium rate (m). The expected utility maximisation problem of the individual is implicitly written as:

$$\max_q EU = \ p.u\left(W - L - mq + q\right) + \left(1 - p\right) u\left(W - mq\right) \tag{687}$$

The profit maximising condition of the insurance company with perfect competition in the insurance market is:

$$p(1-m)q-(1-p)mq=0 \tag{688}$$

A risk averse consumer likes to get the same marginal utility whether in the good or bad state

$$u'(W-L-mq+q)=u'(W-mq) \tag{689}$$

Prove that the optimal premium rate equals the probability of loss (p), and that it is optimal for an individual facing uncertainty in this way to purchase full insurance. [50]

Q6. Consider a pure exchange economy in which the utility of households A **and** B are given by $U^A=\left(X_1^A\right)^{\alpha_A}\left(X_2^A\right)^{1-\alpha_A}$ and $U^B=\left(X_1^B\right)^{\alpha_B}\left(X_2^B\right)^{1-\alpha_B}$. Here U^A and U^B are levels of utilities of household A **and** B respectively, and α_A and α_B denote preferences of these households for the consumption of good 1. Similarly X_1^A and X_2^A, and X_1^B and X_2^B are consumptions of good 1 and good 2 by household A **and** B respectively. Only household A has an endowment of good 1 and it is $\omega_1^A=100$; and only household B has an endowment of good 2 and it is $\omega_2^B=200$; is α_A 0.4 and α_B is 0.6.

1. **Represent the initial endowment position of goods A and B of these two households using the Edgeworth box diagram with a number of indifference curves for each. [10]**
2. **Formulate the Lagrangian function for constrained optimisation for A and B. [9]**
3. **Provide the first order conditions necessary for optimisations by both households. [9]**
4. **Derive demand functions for both products by both households.[9]**
5. **State the market clearing conditions for both goods. [9]**
6. **Use good 1 as a numeraire. Find the relative price of good 2 that clears both markets and is consistent with maximization of utility (satisfaction) by both households given their budget constraints. [9]**
7. **Determine the income of each household. [9]**
8. **Evaluate optimal demands X_1^A and X_2^A, and X_1^B and X_2^B for those endowments and preferences. [9]**
9. **Check whether your solutions satisfy the market-clearing conditions required for a general equilibrium. [9]**
10. **What are the levels of utility for A and B at equilibrium? [9]**
11. **Represent the general equilibrium (optimal quantities, relative prices) in another Edgeworth box diagram. [9]**

32 Tutorials

32.1 Tutorial 1: Market and Demand, Taxes and welfare

Q1. Consider a market for a product A with linear demand (D) and supply (S) functions in terms of market price (P) as follows:

$$D = 150 - 3P \tag{1.1}$$

$$S = 30 + 2P \tag{1.2}$$

1. **What is the equilibrium price where demand equals supply ($D = S$) in this market? [15]**

2. **What amount of this product is bought and sold at equilibrium price? What is the consumer surplus? [15]**

3. **Now let there be a sales tax (t) on this commodity. Then suppliers get P^S but the consumers pay, $P^D = P^S + t$. Note that P^S is less than P, and P^D is above P. The difference between P^S and P^D is the amount of tax or the tax wedge (t) in market for this commodity after the imposition of this unit tax. Let $t = 2$. Find the prices, P^S and P^D and quantity bought and sold in tax distorted equilibrium. [20]**

4. **Draw a diagram to compare equilibrium before and after the imposition of tax rate t. Indicate the overall deadweight loss and and its components in terms consumer and producer surpluses because of this sales tax in this diagram. [20]**

5. **Using above solutions calculate the exact amount of consumer and producer surpluses lost due to this tax. [15]**

6. **Find price elasticities of demand and supply at the pre-tax equilibrium price P. Show that the burden of tax (deadweight loss) is higher for the less elastic part of the market. [15]**

Q2. Consider a consumer's utility (U) maximisation problem given by a Cobb-Douglas utility function as below:

$$\underset{x_1,x_2}{Max} U = x_1^{\alpha} x_2^{\beta},\ \alpha + \beta = 1; \text{s.t } E = p_1 x_1 + p_2 x_2 \tag{1.1}$$

here $p_1$and p_2 are prices of commodities $x_1$and x_2 and E denotes the total expenditure (amount of money) available.

1. **Derive Marshallian demand functions for both $x_1$and x_2 and associated indirect utility function. [20]**

2. **Prove that this demand function is homogenous of degree zero in price and income, increasing in income and decreasing in price. [20]**

3. **Formulate the dual of this problem and derive the expenditure function (E). [20]**

4. **By Shepherd's Lemma, $\frac{\partial E}{\partial p_i} = x_i(p_1, p_2, E)$. Evaluate this using the expenditure fuction derived above. Draw the compensated (Hicksian) and uncompensated (Marshallian) demand functions for x_1 in a well labeled diagram. [20]**

5. **Decompose the total price effect into income and substitution effects using the Slutskey equation. [20]**

Q3. Consider the utility maximisation problem of a consumer with the CES utility function as follows:

$$\max_{x_1, x2} u = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} \tag{7.1}$$

Subject to

$$M = p_1 x_1 + p_2 x_2 \tag{7.2}$$

1. **Write the Langrangian function for constrained optimisation and associated first order conditions**

2. **Derive demand function for x_1 and x_2.**

3. **Prove the these demand functions are homogenous of degree zero in prices p_1 and p_2 and income (M).**

4. **Show that the Marshallian demand functions for x_1 and x_2 could be derived from the indirect utility function following the Roy's identity, $\left[\frac{\partial V}{\partial p_i} = \frac{\partial L}{\partial p_i}\right]$.**

5. **Derive the compensated demand function for x_1 using the expenditure function.**

6. **What is Shephard's lemma? Prove that $\frac{\partial M}{\partial p_i} = \frac{\partial L}{\partial p_i} = x_i(p_x, p_y, M)$; where L stands for Langrangian function for unconstrained optimisation.**

32.2 Tutorial 2: Production, Cost and Supply Functions

Q1. Production function for a firm operating in the competitive market is given by

$$y = 2\sqrt{l} \tag{1.1}$$

where y is output and l is labour input. Product price is p and input price is w.

1. **Determine the cost function for this firm.**

2. **What is its profit function?**

3. **Determine its supply function.**

4. **What is its demand function for labour?**

5. **Discuss properties of the production, profit and cost functions.**

Q2. Passenger car market is operating under a monopolistic competition. A particular firm under this market will cut down it own prices if any another firm reduces its price but will not raise its price if another firm raises its price. For simplicity assume that there two firms in the market and their inverse demand and cost functions are as following:

Demand and cost function of firm I:

$$P_1 = 105 - 2q_1 - q_2; \quad C_1 = 5q_1^2 \tag{3.1}$$

Demand and cost function of firm II:

$$P_1 = 35 - q_1 - q_2; C_2 = q_2^2 \tag{3.2}$$

1. **Find the Cournot duopoly equilibrium as a base line for comparison. What are output, price and profit of each firm under this market conditions. [20]**

2. **Now firm I raises price of its own cars by 2 but the firm II does not change its own price. What will be the prices, output and profit of each firm? [15]**

3. **If the firm I reduces its price of cars by 2, how much firm II reduce its price to maintain its market share? What are the profits, level of output and prices for each firm? [15]**

4. **Put every result of the calculation in one table and explain the underlying factors behind these results.[15]**

5. **A firm under the monopolistic competition operates under less than full capacity setting price equal to its average cost in addition to $MR = MC$ rule. Explain. [15]**

6. **In what sense this is a monopoly? In what sense a competitive market? Illustrate discussion with some real world examples. [20]**

Optional problem

Q3. Consider profit function of a firm

$$\pi = py - rK - wL \tag{690}$$

Derive supply function and input demand function using Hotelling's Lemma when technology $y = K^{0.4}L^{0.4}$

$$y(p, w) = \frac{\partial \pi (p, w)}{\partial p} \tag{691}$$

$$x_i(w, p) = \frac{\partial \pi (p, w)}{\partial w} \tag{692}$$

32.3 Tutorial 3: Oligopoly and Monopolistic Competition

Q1. Market demand curve for mobile phones is given by

$$Q = 10 - \frac{1}{5}P \tag{2.1}$$

Two firms exist in the market to supply this product. Their cost functions are

$$C_1 = 4q_1 \text{ and } C_2 = 5q_2. \tag{2.2}$$

1. **Write the profit function for each of these two firms in this mobile phone market. [10]**
2. **Assume that these firms operate under the Cournot duopoly model. Each firm takes account of level of output supplied by another firm while determining its own quantity to be supplied to the market. Determine reaction functions of both firms and represent them in one diagram. [20]**
3. **How much will each produce? What will be the market price? How much profit will each make from selling mobile phones? [15]**
4. **What will be the consumer surplus from each of them? Show these consumer surpluses diagrams for each of these two firms.[15]**
5. **Assume that firm 1 is a leader and incorporates the output reaction function of its follower in its profit function. Modify it's profit function. Determine the level of output that firm 1 (the leader) will supply in the market? [15]**
6. **What will be the output of the firm 2, which is the follower firm in this mobile phone market? [5]**
7. **What will be the market price and levels of profit of firm 1 and firm 2? [10]**
8. **What would have been the level of output, market price and profit if they had formed a collusive cartel? [10].**

Q2. Consider a firm in a monopolistically competitive industry facing a demand for its products (Q) as a function of price (P) in the market as:

$$Q = A - B \cdot P \tag{4.1}$$

Here A is an intercept and B is the slope of the demand function.

9. **Prove that its marginal revenue (MR) is given by**

$$MR = P - \frac{Q}{B} \tag{4.2}$$

[15]

10. **If the cost function is $C = F + cQ$ with fixed cost F and variable cost c, then prove that the average cost (AC) declines because of the economy of scale. [15]**

11. **Given the size of the market (S) assume that the output sold by a firm (Q), number of firms (N), its own price (P) and average prices of firms $(\overline{P})$ are given by**

$$Q = S\left[\frac{1}{N} - b\left(P - \overline{P}\right)\right] \tag{4.3}$$

Note $Q = \frac{S}{N} + S.b.\overline{P} - S.b.P$; $\frac{S}{N} + S.b.\overline{P} = A$; $S.b. = B$ and $MR = MC \iff P - \frac{Q}{B} = c$. **Prove that price charged by a particular firm declines with the number of firms, $P = c + \frac{1}{b \cdot n}$. [15]**

12. **Show that the average cost rises to number of firms in the industry when all firms charge same price; $AC = \frac{n.F}{s} + c$. [15]**

13. **Determine the number of firms and price in equilibrium using $P = AC$. Explain entry and exit behavior and prices when number of firms are below or above this equilibrium point. How do collusive and strategic behaviors limit these conclusions. Discuss. [20]**

14. **Apply above model to explain interindustry and intra-industry international trade and its impact on prices and number of firms in a particular industry, e.g. passenger car or widescreen TV.[20]**

32.4 Tutorial 4: Bargaining and Cooperative Games

Q1. Find the Nash equilibrium in the prisoner's dilemma game given below.

Table 53: Prisonar's Dilemma Game

		Player A	
		Confess	Don't Confess
Player B	Confess	$(-7, -7)$	$(-1, -10)$
	Don't Confess	$(-10, -1)$	$(-2, -2)$

[Negative sign indicates bad payoff; -10 is worse than -7].
What would have been cooperative and the Pareto optimal solution?

Q2. One common example for a bargaining game is splitting a pie between two individuals, i and j. The total amount to be divided is 1. Their shares in this pie are given by θ_i and θ_j respectively and they should not claim more than what is on the table, i.e. $\theta_i + \theta_j \leq 1$. This implies a meaningful solution of the game requires $\theta_i \geq 0$ and $\theta_j \geq 0$. If the sum of claims is more than what is on the table each gets zero i.e. when $\theta_i + \theta_j > 1$ then $\theta_i = 0$ and $\theta_j = 0$.

Thus the Nash bargaining problem is given by

$$\max U = (\theta_i - 0)(\theta_j - 0) \tag{693}$$

subject to

$$\theta_i + \theta_j = 1 \tag{694}$$

Formulate the constrained optimisation of this problem. Find the optimal values of θ_i and θ_j that satisfy the Nash equilibrium.

Q3. In Spence's model of signalling, type 1 workers are less productive than type 2 workers. Workers signal their productivity type by choosing years of education to maximise their utility. As given below the utility of a worker is positively related to the wage rate (w) and negatively to the effort for education (e) but it is less costly for more productive workers to get education.

$$u_t(w_t, e) = 42\sqrt{w_t} - k_t e^{1.5} \text{ with } k_1 = 3;\ \ k_2 = 1\ \ w_1 = e;\ w_2 = 2e \tag{695}$$

Given the values of k_t and the above utility function find the optimal choice of e for each type of worker.

Q4. Consider a game in which player B has top and bottom strategies and player A has left and right strategies as following.

Table 54: Game of Mixed Strategy

		Player A	
		Left	Right
Player B	Top	$(30, -30)$	$(70, -70)$
	Bottom	$(80, -80)$	$(10, -10)$

Probability of playing Top by B is p and playing Bottom is $(1-p)$ if A plays Left . Similarly probability of B playing Top is p and playing Bottom $(1-p)$ if A plays Right. B likes to be equally well off no matter what A plays.

Find the optimal probability p of playing Top by player B solving this game by the mixed strategy.

Q5. **Nature left 1000 pounds on the table to be split between two players. What is the optimal solution from a symmetric bargaining game if the threat point is given by d(0,0)?**

Q6. Only two firms supply products in a certain market in which the market demand for the product is:

$$P = 150 - (q_1 + q_2) \tag{696}$$

Cost of production for each of the two firms is .

$$C_i = 11q_i \quad for \quad i = 1, 2 \tag{697}$$

a) **What is the total profit when these two firms collude?**
b) **What is the output in Cournot equilibrium? What kind of game is this?**

32.5 Tutorial 5: Signalling and Mechanism Design

Q1. Level of education signals quality of a worker. Spence (1973) model was among the first to illustrate how to analyse behavior of an employer and job applicants and role of signalling in job markets. Consider a situation where there are N individuals applying to work. In absence of education as the criteria of quality, employers cannot see who is a high quality worker and who is a low quality worker. Employers know that θ proportion of workers is of high quality and $(1-\theta)$ proportion is of bad quality. Therefore they pay each worker an average wage rate as:

$$\overline{w} = \theta w_h + (1-\theta) w_l \tag{6.1}$$

more productive worker is worth 40000 and less productive worker is worth 20000 and θ =0.5 then the average wage rate will be 30000. Let c the cost of being high quality worker be 15000.

1. **What are the wage rates in pooling and separating equilibrium? Is the separating equilibrium more efficient than the pooling equilibrium? Discuss why. [25]**
2. **Is it worth for high quality workers to signal their quality by the standard of their education if the cost of education is 5000? Why or why not discuss. [25]**
3. **Consider four different levels of cost of education, c, being at 5000, 10,000, 15,000 and 20,000. Determine in which of these cases is it appropriate for the good workers to signal their quality by taking higher education. [25]**
4. **Represent above results in a well labelled diagram appropriately. Explain. [25]**

Q2. Shareholders in a large corporation (such as the Royal Bank of Scotland) perceive that profit earned by the corporation depends largely on the level of effort put in by the Chief Executive Officer (CEO). Therefore they agree to provide a good amount of bonus to the CEO above his salary. The general public is skeptical of the effectiveness of a bonus scheme like this. In this context think of a project that would earn £800,000 profit if successful. The probability of success is 60 percent if the CEO puts in normal effort. This probability can rise to 90 percent if the CEO puts in extra effort. The CEO will put in extra effort only if an additional payment of £50,000 is made above the basic salary of £100,000. However, it is difficult for shareholders to monitor whether the CEO is putting in extra effort even if they pay an additional amount of £50,000.

1. **Is it profitable for owners to pay an extra £50,000 for the CEO? Why does such an extra payment not automatically guarantee a higher probability of profit? [20]**
2. **Design incentive compatibility and participation constraints in terms of the basic salary and a bonus so that the CEO puts in extra effort in return for an extra payment. [20]**
3. **Based on the above information, what is the minimum payment required by the CEO to put in extra effort? Do owners find it profitable to make such an extra payment as an incentive device? [20]**

4. **Consider now the case where the CEO can be of low or high productivity type. How can the level of education of a prospective CEO signal to the shareholders whether he or she is of high productivity type? [20]**

5. **How can shareholders signal that they cannot be fooled by a CEO who pretends (to the owners) to be putting in extra effort while actually putting in only normal effort? [20]**

Q2. Let there be two bidders bidding b_1 and b_2 but with true values v_1 and v_2. The highest bidder wins the auction at the price of the second-highest bid. The expected value for bidder 1 is then given by $prob\,(b_1 > b_2)\,(v_1 - b_2)$. **Prove that honesty is the best policy in this game.**

Q3. A22 is a taxi company in a certain city. There are two options for owners of the company. Option one is to lend all taxis to taxi drivers on a fixed fee (F) basis. Option two is to collide with the taxi drivers for maximisation of joint profit which could be divided between taxi drivers and owners according to their mutual agreement. The market demand and cost functions for this company are given as:

$$P = 24 - 0.5q; \qquad C = 12q \tag{698}$$

Prove the solutions of output, price, revenue, cost and profit are the same whether this taxi company operates under the fixed fee (F) contract or under the joint profit maximisation agreement.

[Hints: revenue: $R = P.q$ profits: $\Pi\,(q) = P.q - C$ vs. $\Pi\,(q) = P.q - F - C]$.

Q4. Productivity of a worker with the level of education e^* is a_2 and it is a_1 without education e^* ; i.e. productivity difference is $a_2 - a_1$ between educated and non-educated workers.

$$c_1 e^* \geq c_2 e^* \Longrightarrow c_1 \geq c_2 \tag{699}$$

Show how the cost of education relative to the productivity differences $a_2 - a_1$ is lower for the high quality worker than for the low quality worker.

32.6 Tutorial 6: Uncertainty and Insurance

Q1. **What are the measures of risk aversion for consumers with following utility functions? Which of these consumers is risk-averse, which one is risk neutral and which one is a risk lover?**

(a) Logarithmic utility in wealth: $U(W) = ln(W)$

(b) Cobb-Douglus type utility: $U(W) = W^{\frac{1}{2}}$

(c) Linear utility: $U(W) = aW$

(d) Exponential utility: $U(W) = \exp(aW)$

Q2. The amount of wealth in the good state is W. If a bad event occurs there will be a loss (L) and the probability of a loss is p.

The owner of the property can insure for amount (q) paying premium rate (m). The expected utility maximisation problem of the individual is implicitly written as:

$$\max_{q} EU = \ p.u\,(W - L - mq + q) + (1-p)\,u\,(W - mq) \tag{700}$$

The profit maximising condition of the insurance company with perfect competition in the insurance market is:

$$p\,(1-m)\,q - (1-p)\,mq = 0 \tag{701}$$

A risk averse consumer likes to get the same marginal utility whether in the good or bad state

$$u'\,(W - L - mq + q) = u'\,(W - mq) \tag{702}$$

Prove that the optimal premium rate equals the probability of loss (p), and that it is optimal for an individual facing uncertainty in this way to purchase full insurance.

32.7 Tutorial 7: Redistribution and welfare

Q4. An economy is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. They aim to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. Preferences for type 1 are given by $U_1 = \sqrt{Y_1}$ and for type 2 by $U_2 = \sqrt{Y_2}$. In a given year total output, Y, was 1000 billion pounds.

1. **What is the distribution of output between type 1 and type 2 that maximises the social welfare index? What is the maximum value of the social welfare index of this economy? [25]**
2. **What would have been the allocation if policy makers had given equal weight to the utility of both types of people in the economy such as $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$. By how much does the welfare index change in this case than compared to the social welfare in (a) above? [25]**
3. **How would the social welfare index change in sub-question (1) above if a tax rate of 20 percent is imposed in consumption and the tax receipts are not given back to any of these consumers? How much would the value of social welfare index be in this case? [25]**
4. **Assume that the policy makers still hold the welfare function to be $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$. How would the social welfare index change in (c) if all tax receipts are transferred to type 2 people? [**

Q2. Consider a global economy consisting to two countries, A and B. Each is endowed with labour and produces two goodsX_1 and X_2 . Country A is endowed with 200 units of labour and country B with 400 of it. The production functions and resource constraint in country A are $X_{1,A} = 5L_{1,A}$ and $X_2 = 2L_{2,A}$ and $L_{1,A} + L_{2,A} = 200$. Similarly the production functions and resource constraint in country B are $X_{1,B} = 2L_{1,B}$ and $X_2 = 5L_{2,B}$ and $L_{1,B} + L_{2,B} = 400$. Each country has a representative consumer, whose preferences are given by $U_A = X_{1,A}^{\alpha} X_{2,A}^{1-\alpha}$ and $U_B = X_{1,B}^{\alpha} X_{2,B}^{1-\alpha}$ respectively. For a specific year the value of α was 0.4 and β was 0.6.

a. Derive the production possibility frontier for each country and represent it in a diagram.

b. Indicate the optimal production of each country in autarky (with no global trade). What are the implicit price ratios between X_1 and X_2 in countries A and B?

c.Which country is more efficient in producing X_1 and which in producing X_2?

d.What would be the level of production of X_1 and X_2 in these two countries with complete specialisation? What would be the relative price ratio in the global market? How would consumption set change for each of these?

e.What is the gain from trade for each country from complete specialisation?

f. How do above results change if the country A prefers to consume more of X_1 and X_2 country B to consume more of X_2 i.e. if α was 0.6 and β was 0.4?

g.How can the WTO facilitate global trade to improve the global welfare?

Table 55: Parameters of the Autarky Model

	α	a_1	a_2	L
country 1	0.4	5	2	200
country 2	0.6	2	5	400

32.8 Tutorial 8: Taxes, welfare and general equilibrium

Q1. An individual gets utility (u) by consuming cashew nuts (x_1) and peanuts (x_2) as given by $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. This person has 200 to spend (m) between these two goods, thus the budget constraint is $m = p_1 x_1 + p_2 x_2$. Prices of cashew nuts and peanuts were $(p_1, p_2) = (2, 2)$ last month, giving the base line demand as $\left(x_1 = \frac{m}{2p_1}, x_2 = \frac{m}{2p_2}\right) = (50, 50)$ and the utility from this consumption was $u = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} = 50$.

Now an increase in VAT on cashew nuts raises its price to 4 but there is no change in the price of peanuts, $(p_1, p_2) = (4, 2)$. Income does not change and stays the same at 200. The new demand for cashew nuts and peanuts implied by their prices are $(x_1, x_2) = (25, 50)$. This person has become worse off because of higher prices due to increase in taxes. **Calculate the Hicksian compensating and equivalent variations of this price change.**

Q2. Consider a pure exchange general equilibrium model for an economy with individuals A and B with the set of parameters in the table given below.

Lagrangian for constrained optimisation for Household A :

$$\mathcal{L}^A = \left(X_1^A\right)^{\alpha_A} \left(X_2^A\right)^{1-\alpha_A} + \lambda \left(P_1 \omega_1^A + P_2 \omega_2^A - P_1 X_1^A - P_2 X_2^A\right) \tag{703}$$

Lagrangian for constrained optimisation for Household B :

$$\mathcal{L}^B = \left(X_1^B\right)^{\alpha_B} \left(X_2^B\right)^{1-\alpha_B} + \lambda \left(P_1 \omega_1^B + P_2 \omega_2^B - P_1 X_1^B - P_2 X_2^B\right) \tag{704}$$

Table 56: Parameters in Pure Exchange Model

	Household A	Household B
Endowments	$\left\{\omega_1^A, \omega_2^A\right\} = \{100, 0\}$	$\left\{\omega_1^B, \omega_2^B\right\} = \{0, 200\}$
Preference for X_1 (α)	0.6	0.4
Preference for X_2 $(1-\alpha)$	0.4	0.6

You may assume Walrasian numeraire: $P_1 = 1$ with this specification, and implied incomes for A and B are:

$$I^A = \omega_1^A \qquad I^B = P_2\omega_2^B \tag{705}$$

Derive the demand functions for both A and B individuals and find the relative price that clears the markets for both X_1 and X_2 .

Q3. Consider a general equilibrium model with taxes in which a representative household maximises utility subject to its budget constraint and the firm maximises profit subject to a technology constraint as given below.

$$\max \quad U = C \cdot L \tag{706}$$

Subject to

$$p\,(1+t)\,C + wL = w\overline{L} \tag{707}$$

The firm's profit maximisation problem is:

$$\max \quad \pi = p.Y - w.LS \tag{708}$$

Subject to

$$Y = LS \tag{709}$$

You may select $p = 1$ as a numeraire.

Find expressions for the wage rate, consumption, output, labour supply and demand for labour consistent with the general equilibrium.

32.9 Tutorial 9: Redistribution, Trade and welfare

Q4. An economy is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. They aim to maximise the social welfare function: $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$ where W is the index of the social welfare, U_1 represents the utility of type 1 people and U_2 is the utility of type 2 people. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies: $Y = Y_1 + Y_2$. Preferences for type 1 are given by $U_1 = \sqrt{Y_1}$ and for type 2 by $U_2 = \sqrt{Y_2}$. In a given year total output, Y, was 1000 billion pounds.

1. **What is the distribution of output between type 1 and type 2 that maximises the social welfare index? What is the maximum value of the social welfare index of this economy? [25]**

2. **What would have been the allocation if policy makers had given equal weight to the utility of both types of people in the economy such as $W = U_1^{\frac{1}{2}} U_2^{\frac{1}{2}}$. By how much does the welfare index change in this case than compared to the social welfare in (a) above? [25]**

3. **How would the social welfare index change in sub-question (1) above if a tax rate of 20 percent is imposed in consumption and the tax receipts are not given back to any of these consumers? How much would the value of social welfare index be in this case? [25]**

4. **Assume that the policy makers still hold the welfare function to be $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$. How would the social welfare index change in (c) if all tax receipts are transferred to type 2 people? [**

Q2. Consider a global economy consisting to two countries, A and B. Each is endowed with labour and produces two goodsX_1 and X_2 . Country A is endowed with 200 units of labour and country B with 400 of it. The production functions and resource constraint in country A are $X_{1,A} = 5L_{1,A}$ and $X_2 = 2L_{2,A}$ and $L_{1,A} + L_{2,A} = 200$. Similarly the production functions and resource constraint in country B are $X_{1,B} = 2L_{1,B}$ and $X_2 = 5L_{2,B}$ and $L_{1,B} + L_{2,B} = 400$. Each country has a representative consumer, whose preferences are given by $U_A = X_{1,A}^{\alpha} X_{2,A}^{1-\alpha}$ and $U_B = X_{1,B}^{\alpha} X_{2,B}^{1-\alpha}$ respectively. For a specific year the value of α was 0.4 and β was 0.6.

a. Derive the production possibility frontier for each country and represent it in a diagram.

b. Indicate the optimal production of each country in autarky (with no global trade). What are the implicit price ratios between X_1 and X_2 in countries A and B?

c.Which country is more efficient in producing X_1 and which in producing X_2?

d.What would be the level of production of X_1 and X_2 in these two countries with complete specialisation? What would be the relative price ratio in the global market? How would consumption set change for each of these?

e.What is the gain from trade for each country from complete specialisation?

f. How do above results change if the country A prefers to consume more of X_1 and X_2 country B to consume more of X_2 i.e. if α was 0.6 and β was 0.4?

g.How can the WTO facilitate global trade to improve the global welfare?

Table 57: Parameters of the Autarky Model

	α	a_1	a_2	L
country 1	0.4	5	2	200
country 2	0.6	2	5	400

32.10 Tutorial 10: Labour and capital markets

Q1. Let the demand curve for a competitive labour market be given by

$$L^D = 450 - 50\left(\frac{w}{p}\right) \tag{710}$$

and then the labour supply curve be:

$$L^S = 100\left(\frac{w}{p}\right) \tag{711}$$

a. What is the real wage $\left(\frac{w}{p}\right)$ and employment (L) in equlibrium?

b. Now suppose the government wants the real wage rate $\left(\frac{w}{p}\right)$ to be 4 adopting a special to a forlough scheme as in COVID-19 subsidising the workers. How much will be the demand for labour now? What will be the labour supply? How much should the government spend in wage subsidies?

c. Now suppose that the government sets the minimum real wage rate at 4, what will be the demand and supply of labour? How many people will be unemployed?

d. Illustrate above results in a diagram.

Q2. A car manufacturer sells each car at 8000 and pays 2000 for capital equipment per car. The nominal interest rate is 6%, appreciation of value of capital stock (capital gain) is 3% and the depreciation of capital stock is 3% per year.

The production function for this company is given by with .$Y = K^{\alpha}$ and $\alpha = 0.75$
What is the optimal capital stock for this manufacturer? (hint).

$$\Pi = \frac{F(K)}{(1+r)} - P_1^K K + \frac{(1-\delta)\,P_2^K K}{(1+r)} \tag{712}$$

$$\frac{\partial \Pi}{\partial K} = \frac{F'(K)}{(1+r)} - P_1^K + \frac{(1-\delta)\,P_2^K}{(1+r)} = 0 \tag{713}$$

$$MPK = (1+r)\,P_1^K - (1-\delta)\,P_2^K = 0 \tag{714}$$

$$MPK = \left[(1+r) - (1-\delta)\left(1+\pi^k\right)\right] P_1^K \tag{715}$$

$$MPK \simeq \left[r + \delta - \pi^k\right] P_1^K \tag{716}$$

$$MRPK = P.\alpha K^{\alpha-1} \simeq \left[r + \delta - \pi^k\right] P_1^K \tag{717}$$

$$8000.\left(0.75\right) K^{0.5-1} \simeq \left[0.06 + 0.03 - 0.03\right] \cdot 2000 \tag{718}$$

solve for K

$$6000.K^{-0.25} \simeq \left[0.06\right] \cdot 2000 \tag{719}$$

$$K = \left(\frac{3}{0.06}\right)^4 = 50^4 = 6,250,000 \tag{720}$$

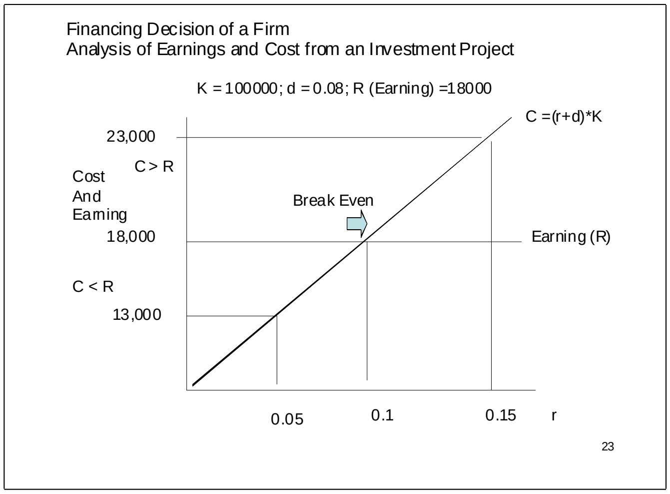
Show how the investment tax credit affect the optimal capital stock employed by a firm using an appropriate diagram.

Q3. A certain project costs 100,000. This project brings annual earning equal to 18000. Depreciation rate is 8% and the market interest vary as shown below? When is this investment profitable?

Table 58: Discount factor over time

	$Scen1$	$Scen2$	$Scen3$
r	0.05	0.1	0.15
δ	0.08	0.08	0.08
Cost	13000	18000	23000
Revenue	18000	18000	18000
	5000	0	-5000

Represent this result in a diagram with r in x-axix and cost and revenue in y-axis.



Q4. A person, who is 20 years of age now, is wondering whether (1) to start work immediately and earn £20000 salary per year that grows annually by 2 percent , or (2) to go to university for 3 years and start a job after graduation earning £30000 a year with annual growth rate of 4 percent. For each university year, the tuition is £3000 and subsistence cost is £6000. Retirement age is 65. Nominal interest rate equals the rate of inflation implying a zero real interest rate. What difference is made in the lifetime income of this person if he/she decides to go to university now? $\left[\text{hint: } LI = Y_0\left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right] - 3\times C \;\text{ or } LI = Y_0\left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right]\right]$

Q5. Project B earns more but is riskier than project A. Probability of success of projects A and B are given by η_a and η_b respectively.

a. Illustrate how the rate of interest rate should be lower in project A than in project B in equilibrium?

b. Probability of types A and B agents is given by p_a and p_b respectively. Prove under the asymmetric information a lender charging a pooling interest rate is unfair to the safe borrower A and more generous to the risky borrower B.

c. How can agent signal its worth? How can the lender ascertain the degree of moral hazard in B?

32.11 Tutorial 11: Externality

Consider a case where two friends share a public good $x = x_1 + x_2$ but consume private good y_i.

$$\max \; u_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} \tag{721}$$

subject to

$$10x_1 + y_1 = 300 \tag{722}$$

$$\mathcal{L}_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} - \lambda\left[300 - 10x_1 - y_1\right] \tag{723}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2}(x_1 + x_2)^{-\frac{1}{2}} y_1^{\frac{1}{2}} - 10\lambda = 0 \tag{724}$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = \frac{1}{2}(x_1 + x_2)^{\frac{1}{2}} y_1^{-\frac{1}{2}} - \lambda = 0 \tag{725}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 300 - 10x_1 - y_1 = 0 \tag{726}$$

From the first two FOC

$$\frac{y_1}{(x_1 + x_2)} = 10 \Longrightarrow y_1 = 10(x_1 + x_2) \tag{727}$$

Putting this y_1 back in the budget constraint
$300 - 10x_1 - y_1 = 300 - 10x_1 - 10(x_1 + x_2) = 0$

$$x_1 = 15 - \frac{x_2}{2} \tag{728}$$

As the problem is symmetric dimilar proces for individual 2 we get

$$\mathcal{L}_2 = (x_1 + x_2)^{\frac{1}{2}} y_2^{\frac{1}{2}} - \lambda\left[300 - 10x_2 - y_2\right] \tag{729}$$

$$x_2 = 15 - \frac{x_1}{2} \tag{730}$$

$$x_1 = 15 - \frac{x_2}{2} = 15 - \frac{1}{2}\left(15 - \frac{x_1}{2}\right) \Longrightarrow x_1 = \frac{4}{3}\frac{1}{2} \times 15 = 10 \tag{731}$$

$$x_1 = x_2 = 10 \Longrightarrow x_1 + x_2 = 20 \tag{732}$$

$$y_1 = 10(x_1 + x_2) = 10 \times 20 = 200 \tag{733}$$

Utility level in non-cooperative Nash scenario is:

$$u_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} = (20)^{\frac{1}{2}} 200^{\frac{1}{2}} = 63.2 = u_2 \tag{734}$$

Under the Samuelsonian rule

$$\frac{\frac{\partial \mathcal{L}}{\partial x_1}}{\frac{\partial \mathcal{L}}{\partial y_1}} + \frac{\frac{\partial \mathcal{L}}{\partial x_2}}{\frac{\partial \mathcal{L}}{\partial y_2}} = MRS_1 + MRS_2 = MRT \tag{735}$$

$$\frac{\frac{1}{2}(x_1 + x_2)^{-\frac{1}{2}} y_1^{\frac{1}{2}}}{\frac{1}{2}(x_1 + x_2)^{\frac{1}{2}} y_1^{-\frac{1}{2}}} + \frac{\frac{1}{2}(x_1 + x_2)^{-\frac{1}{2}} y_2^{\frac{1}{2}}}{\frac{1}{2}(x_1 + x_2)^{\frac{1}{2}} y_2^{-\frac{1}{2}}} = \frac{y_1}{x} + \frac{y_2}{x} = \frac{p_x}{p_y} = \frac{10}{1} = 10 \tag{736}$$

$$y_1 + y_2 = 10x \tag{737}$$

Combined budget constraint of both persons:

$$10x + y_1 + y_2 = 600 \tag{738}$$

$$10x + 10x = 600 \Longrightarrow x = 30 \tag{739}$$

$$y_1 + y_2 = 10x = 10 \times 30 = 300 \tag{740}$$

If the private good is equally devided each gets 150.

$$u_1 = (x_1 + x_2)^{\frac{1}{2}} y_1^{\frac{1}{2}} = (30)^{\frac{1}{2}} 150^{\frac{1}{2}} = 67.1 = u_2 \tag{741}$$

Table 59: Inefficienty of competitive equilibrium in case of positive externality

	Nash (CE)	Optimal (Samuelson)
x	20	30
y_1	200	150
y_2	200	150
u	63.2	67.1

Key questions: Pollution controls are less important in developing countries such as China and India. How does it affect the global environment?

32.12 Tutorial 12: Input-output model, linear programming and Trade and welfare

Q1. Consider a two sector economy with the material balance equations and technology coefficients defined as following:

$$X_1 = X_{11} + X_{12} + F_1 \tag{742}$$

$$X_2 = X_{21} + X_{22} + F_2 \tag{743}$$

$$a_{11} = \frac{X_{11}}{X_1};\ a_{12} = \frac{X_{12}}{X_2}; a_{21} = \frac{X_{21}}{X_1};\ a_{22} = \frac{X_{22}}{X_2}; \tag{744}$$

How could the changes in gross outputs of X_1 and X_2 after a 20 percent reduction in the final demands F_1 and F_2 be found using the input-output model for this economy? [Hint: express X_1 and X_2 in terms of technological coefficients $a_{i,j}$ and F_j]

Consider the Leontief technology matrix as given below

Table 60: Leontief Technology and Primary Input Coefficients

	X_1	X_2
X_1	0.1	0.2
X_2	0.2	0.3

Determine the gross outputs X_1 and X_2 if the final demands F_1 and F_2 were 70 and 100 respectively.

[Hints $X_1 = X_{11} + X_{12} + F_1; \quad X_2 = X_{21} + X_{22} + F_2$]

Q2. **What is the dual of the following linear programming problem? Write a Lagrangian function to represent this linear programming problem as a special case of non-linear constrained optimisation.**

$$\max \; R = 2X_1 + 4X_2 \tag{745}$$

Subject to

$$\begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \leq \begin{bmatrix} 200 \\ 300 \end{bmatrix}; \; X_1 \geq 0; X_2 \geq 0 \tag{746}$$

33 Texts:Recommended background texts:

References

[1] Krugman P. and R Wells (2018) Microeconomics, 5nd edition, Worth.

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Pre-requisite reading

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2022: Ben S. Bernanke, Douglas W. Diamond, Philip H. Dybvig
2021:David Card, Joshua D. Angrist and Guido W. Imbens
2020: Paul Milgram and Robert Wilson
2019: Abhijit Banerji,Esther Duflo and Michael Kremer
2018: Paul Romer and William Nordhaus
2017: Richard H. Thale
2016: Oliver Hart and Bernt Holmstrom
2015: Angus Deaton
2014: Jeane Tirole
2013: Hansen L P, E. Fama and R. Shiller
2012: Alvin E. Roth and Lloyd S. Shapley
2011: Thomas J. Sargent, Christopher A. Sims
2010: Peter A. Diamond, Dale T. Mortensen, Christopher A. Pissarides
2009: Elinor Ostrom, Oliver E. Williamson
2008: Paul Krugman
2007: Leonid Hurwicz, Eric S. Maskin, Roger B. Myerson
2006: Edmund S. Phelps
2005: Robert J. Aumann, Thomas C. Schelling
2004: Finn E. Kydland, Edward C. Prescott
2003: Robert F. Engle III, Clive W.J. Granger

2002: Daniel Kahneman, Vernon L. Smith
2001: George A. Akerlof, A. Michael Spence, Joseph E. Stiglitz
2000: James J. Heckman, Daniel L. McFadden
1999: Robert A. Mundell
1998: Amartya Sen
1997: Robert C. Merton, Myron S. Scholes
1996: James A. Mirrlees, William Vickrey
1995: Robert E. Lucas Jr.
1994: John C. Harsanyi, John F. Nash Jr., Reinhard Selten
1993: Robert W. Fogel, Douglass C. North
1992: Gary S. Becker
1991: Ronald H. Coase
1990: Harry M. Markowitz, Merton H. Miller, William F. Sharpe
1989: Trygve Haavelmo
1988: Maurice Allais
1987: Robert M. Solow
1986: James M. Buchanan Jr.
1985: Franco Modigliani
1984: Richard Stone
1983: Gerard Debreu
1982: George J. Stigler
1981: James Tobin
1980: Lawrence R. Klein
1979: Theodore W. Schultz, Sir Arthur Lewis
1978: Herbert A. Simon
1977: Bertil Ohlin, James E. Meade
1976: Milton Friedman
1975: Leonid Vitaliyevich Kantorovich, Tjalling C. Koopmans
1974: Gunnar Myrdal, Friedrich August von Hayek
1973: Wassily Leontief
1972: John R. Hicks, Kenneth J. Arrow
1971: Simon Kuznets
1970: Paul A. Samuelson
1969: Ragnar Frisch, Jan Tinbergen

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34.5 Model Codes and Computations

Economist use models in analysis. Lectures and Problems discussed ways of solving those models analytically. It is esier to compute ecenarios using software such as Excel or GAMS. Unzip these files in your directory in G: drive and do computations as necessary.

Name of the Model	Excel or analytical file	GAMS Programme file
Consumption-leisure tax	publicfinances_feb06.xls	Macrotax.gms
Financial sector	Finreg.xls	Stock.gms
Interest rate rule	Interest rate.xls	Intr_rule.gms
Bargaining	Bargain.xls	Bargain.gms
Demand supply in two goods	Analytical solution	Demand_supply_2.gms
Cartel	Duopoloy.xls	Cartel.gms

Pure Exchange	Purexchange.gms	Pexchange.gms
Price Leadership	Duopoly.xls	Stacklberg.gms
Cournot duopoly model	Duopoly.xls	Cournot.gms
Input output model	IO_3UK.xls	Input-output.gms
Model of credit crunch		Crdit_crunch.gms
Macro debt model	Debt.xls	Macro_debt.gms

Linear programming -max	Analytical solutions in book	Lp.gms
Linear Programming _min	Analytical solutions in book	Lp_min.gms
Ricardian Trade model	"	Tade_2.gms
Prototype tax model	Reported in the paper	Proto.gms
Labour supply with taxes	Analytical in book	Tax.gms; tcl.gms
Eneergy and Pollution model	Book chapter	combined.gms Pollute_twosector.gms
Overlapping generation	Olg.xls	Olg.gms
Intertemporal optimisation	reported in paper	Intemp1.gms
Open economy model with money	paper	twosector_hh.gms
Cob-Web Model	Analytical solutions	Cobweb.gms
Labour leisure	Reported in paper	Labourleisure.gms
Welfare calculations	Welfare.xls	
Model of Humber Side	Humber_GDP.xls	Humber.gms

This list includes only small models that can be solved easily. Some instructions on software is provided in the handbook. Demo version GAMS can be downloaded free from www.gams.com/download. Larger scale models require full license for appropriate solvers.

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See links
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https://share.hull.ac.uk/SitePages/Home.aspx
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https://www.res.org.uk/resource-library-page.html
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