DATA RETRIEVAL IN MIMO SYSTEMS AND THE EFFECTS OF CORRELATION ON THE CHANNEL CAPACITY

BEING A DISSERTATION SUBMITTED IN FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF MASTERS OF SCIENCE (MSc) IN WIRELESS SYSTEMS ENGINEERING

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BY

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DECLARATION

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ABSTRACT

Multiple-input multiple-output (MIMO) antenna design as used in communication is today, easily the most important field in the wireless field; as capacities of data are increased based on the inherent capability of the technology, without an increase in spectrum bandwidth.

This thesis analyses the ways through which data sent over some channel from a number of transmitters are recovered at the intended receivers; Maximum likelihood (ML) and Zero-forcing (ZF), are used for the data decoding; how effective these retrieval processes are and the imminent effects of correlation on the bit error rates as variants of signal to noise ratio, on the retrieved data capacities of the MIMO channels created, are all examined.

This is simply very important as wireless systems continue to impact on lives globally.

To drive the point home, the MIMO technology as it relates to this thesis is explicitly dissected to attempt a sound understanding of its modus operandi.

AKNOWLEDGMENT

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BACKGROUND INFORMATION

1.1 Introduction

Growth in human population and advancement in technology has come along with an increase in the need for effective, efficient and reliable communication. This has carved a large and demanding market for improvement in systems used worldwide.

The foundation for effectiveness in communication could be traced to the extensive work on information theory developed by Shannon, (1948); it provides all the possibilities for reliable communication involving information. In the classic paper "A mathematical theory of communication," he laid out the basic elements of communication:

- An information source that produces a message.
- A transmitter that operates on the message to create a signal which can be sent through a channel.
- A channel, which is the medium over which the signal, carrying the information that comprises the message is sent.
- A receiver which transforms the signal back into the message intended for delivery and,
- A destination which can be a person or a machine for whom or which the message is intended.

He significantly viewed that the fundamental problem of communication is that of reproducing at one point, either exactly or approximately, a message sent from another point.

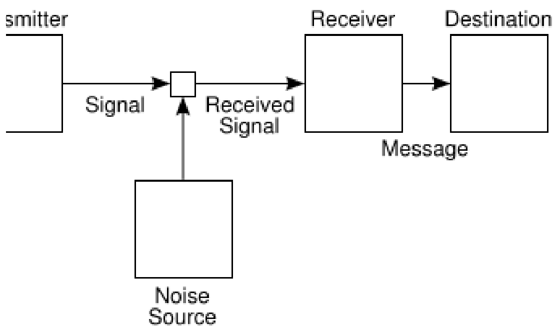


Fig.1 Shannon's diagram of a general communication system, Shannon (1948)

Hence, Shannon's law states that information cannot be transmitted at a rate greater than the capacity of the channel used, for an error free transmission.

1.2 History

Data transmission and reception in telecommunication started with what is cleverly termed "First generation networks," (1G); an analogue system of telecommunication introduced in the 1980s, which used frequency division multiplexing, (FDMA); a scheme in which numerous signals are combined for transmission on a single communication channel, each assigned a different frequency within the main channel. Bi et al (2001) and a host of other writers clearly note that the Second generation of networks, (2G), came along with digitisation of the systems and the use of (FDMA) and Time division multiple access (TDMA). TDMA; is a process of dividing up one communication channel into smaller time slots, in order to increase the amount of data that can be carried.

Code division multiple access (CDMA); a technique in which each channel transmits its bits as a coded channel with specific sequence of pulses, and a couple of other techniques like (EGPRS)- enhanced general packet radio services; a technology that allows improved data transmission rates are used in the third generation (3G) networks.

The fourth generation (4G) is based on better modulation schemes like orthogonal frequency division multiplexing (OFDM) – a large number of closely spaced orthogonal sub-carriers that divide data into parallel streams; one for each sub-carrier and multiple-input-multiple-output technology (MIMO). Sharony, (2006) rightly views the OFDM-MIMO combination as the cornerstone of future broadband wireless access.

These generational changes have ensured that not only voice but also data are communicated. In 1G, no data transmission was carried out. Peak data rates increased from 9.6kbps to 2Mbps for 2G and 3G respectively. It is envisioned that the 4G networks could have peak data rates of >20Mbps. (Wang, et al 2004). Hence, all the aforementioned multiple access schemes were used to allow users access resources, simultaneously, on some allocated radio spectrum.

It is viewed that the limitation of radio spectrum, the complexity of wireless propagation environment, coupled with the increasing demand for better quality of service and higher data transmission rates, it is imperative for better systems to evolve.

Since designers have wittingly divided frequency, time and a code, what may be left is space; Space-time division is the foundation concept of MIMO.

1.3 Wireless Background

All MIMO technology is wireless so a look into its background is to dive into some chronology of wireless communication, which is a modern branch of telecommunication in which information is transmitted from one point to another, regardless of distance and without resorting to the use of the known electric cables or wires.

In spite of its now proliferated use in communication vocabulary, the wireless field had been around since around the late 1800s; Marconi successfully demonstrated the art of wireless telegraphy in 1897. In a few years, 1901 precisely, transatlantic radio reception was achieved. So a gradual build-up had been on for almost a century. But the past decade has witnessed astronomical growth in wireless concepts for information communication, David et al (2005) reason that the success of 2G digitised standards and the large market of 3G networks, have provided concrete demonstration that good ideas from communication theory can have significant impact in practice; ultimately aiding the wireless revolution.

The Wireless concept arose as the world gradually became a global village; long inter-continental distance communication became necessary, making the deployment of wires, cables or other physical and tangible means of information carriage practically difficult and almost impossible. (Goldsmith, 1995)

In essence, wireless networks have evolved to be more expedient for both voice and data communication. Consequently, researchers and industry experts have concluded that the wireless channel capacity can be increased using multiple components in both transmit and receive ends, heralding MIMO.

Currently, wireless communications are heavily biased towards voice. However, recent studies indicate an exponential increase in the growth of wireless data traffic relative to voice. This is evident since the development of 802.11 data protocol standard by IEEE, as a distinct data technology that can work in a variety of radio spectrums. (Jankiraman, 2004)

The advent of multi-antenna systems, MIMO in particular, is borne out of the interest and strife to achieve high data rates at the receiver. This ideal has indeed been the case since the inception of wireless communication. Jankiraman, (2004), also views that a binding constraint in the evolution of the desired high data rates is the stringent limitation imposed on available spectrum; giving rise to more efficient signalling techniques like MIMO. Hence, the algorithms that achieve this, actually exploit the multipath structure by cleverly coding data in both time and space.

1.4 Constraints in Wireless Systems

What is wrong with the plain old wireless systems, with all its perceived advantages, why multi-channel-MIMO? Technical issues; as indeed no human system is perfect. A Wireless engineer's life is therefore saddled with the quest to make systems as closest to perfect as can be imagined!

- First, fading; the time variation of the channel strengths due to small scale effects like multipath and large scale effects like path loss via distance, attenuation and shadowing by obstacles. These fading effects are caused by Reflection; as signals impinge on smooth surfaces, Diffraction; as signals impinge on edge or corner of dense entity and Scatter; as signals impinge a rough surface.

- Second, unlike the wired world in which each transmitter-receiver pair are thought of as isolated point-to-point links, wireless users communicate over the air, significant interference existing between links and users, bringing to bear snags like signal to noise variations, (David et al 2005).

- Third, improved compression technologies have reduced the bandwidth needed for voice calls, but data traffic still needs more bandwidth as newer services come online. Hence, more bandwidth is desired especially for broadband services. But radio spectrum that provides bandwidths is a finite resource and multiple users must co-exist without causing interference to each other. (IEEE document on radio spectrum, 2002) So emerging technologies that improve wireless system efficiency are now very important. Presently, examples include coded multicarrier modulation, link-level retransmission, adaptive modulation and coding techniques and most importantly, smart antennas-particularly multiple-input-multiple-output (MIMO) technology.

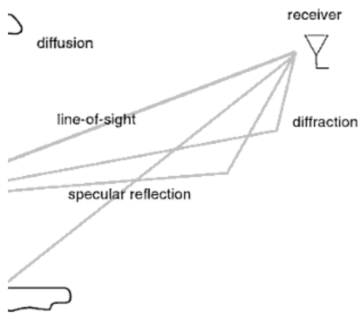


Fig. 2 A typical multipath scenarios, (Oestges et al, 2007)

These issues coupled with the growing consumer appetite for higher data rates and the endless demand for bandwidth and spectral availability has therefore made the resort to MIMO timely and credible.

Consequently, the MIMO technology is deployed providing interference cancellation, route and space diversity, synthesised arrays and space-time coding to combat the aforementioned bottle-necks, resolve issues related to quality and coverage, reduce capacity constraints in networks and ultimately enhance efficiency in wireless communication.

1.5 MIMO Evolution

Theoretical work asserted and developed by Teletar and Foschini form the basis of this technique; their pioneering work ignited much interest in this area-by predicting remarkable spectral efficiencies for systems with multiple antennas, when the channels exhibit rich scattering with variations that can be tracked accurately. (Goldsmith, 2004) This envisioned capability led to phenomenal research activity to characterise the aspects associated with MIMO and extend the concepts to multiuser systems; after developing the theories for MIMO, the direction was towards developing codes and schemes that will enable systems to approach the envisaged capacity limit.

It received momentum when Taroakh et al, (1998) introduced their trellis coding; a combined coding and modulation technique, for digital transmission over band-limited channels, and later

Alamouti introduced his space-time block coding techniques, to improve the link level techniques based on diversity, (Alamouti, 1998).

Then Bell laboratories introduced its Bell Laboratories Layered Space –Time coding technique (BLAST), developed by Gerald Foschini; the BLAST is a receiver which employs multiuser detection strategy and transmit diversity scheme which basically de-multiplexes the data stream to different sub-streams, each one sent to each antenna; demonstrating spectral efficiencies up to 42 bits/hertz. Thus, giving a phenomenal boost in spectral efficiency compared to the current 2-3 bits/hertz obtained in cellular mobiles and WLAN systems.

1.6 Motivation

As previously inferred, the wireless communication industry grew rapidly as newer technologies evolved to enhance greater quality of service. I was motivated by the fact that "smart antenna" technology is widely reputed and recognised as the promising technique to increase the spectrum efficiency of wireless networks.

Initially systems that exploit smart antennas had an array of multiple antennas only at one end of the communication link; e.g. single-input-multiple-output (SIMO) at the receive side, and multiple-input-single-output (MISO) at the transmit side.

MIMO is a more recent idea in which an antenna array is used at both the transmit and receive sides; thus, with an added spatial dimension, it has potentials far more than the conventional smart antenna systems leading to dramatic increase in the capacity of wireless links; increasing both the range of access and total performance of the system. It is evident that MIMO enhances these feats by ensuring that data streams, arriving from different paths and at different time combine to increase effectively, the receiver signal capturing power.

Paulraj et al, (2001), view that the data separation occurs in the spatial domain through different propagation paths in the rich scattering environment. Teletar (1999) says there are different conditions in the fixed line MIMO systems, if the channel changes are negligible and have channel state information at the transmitter; a water-filling analogy could be used, yielding to optimisation. If the channel state information (CSI), i.e. known channel properties of a communication link, is not perfect, the transmitter can rely on the average statistics from the receiver which can result in sub-optimal channel capacity.

In a nutshell, by suppressing the unwanted problems realised, MIMO system can increase the data capacity proportionally with the number of antennas.

1.7 Aims and Objectives

Comprehensibly, the numerous undesired problems realised in wireless communication are eliminated by MIMO concept; the following objectives as listed are determined to aid in meeting the main aim of the project, i.e. that of understanding the effects of correlation on the bit error rates in the retrieved data capacity and ultimately analysing effective data retrieval in MIMO systems.

- Firstly, to understand the basic concepts in MIMO, i.e. spatial multiplexing, diversity gains, etc as it relates to wireless communication in other to give informed perspective in providing solutions when required.
- To describe the physical environment with a view to understanding the scattering channels that exists between receivers and transmitters.
- To study some proposed channel models used to simulate data retrieval at the receivers, i.e. the Kronecker and Jake models with reference to their channel co-variance and limitations.
- To look at the channel models used in available for this scheme; the physical and analytical models and their derivatives like the one ring topology.
- To analyse some decoding algorithms like Zero-forcing, Nulling and Cancellation and Maximum Likelihood for the retrieval of data.
- To understand the effects of antenna correlation on channel performance, as it relates to the signal to noise ratios and corresponding bit error rates.
- To compare the performance of these algorithms with the intent of recommending the best.
- To learn how to make simulations of the desired results using Matlab.

1.8 Summary

This chapter is basically an introduction, starting from Shannon's views on information communication to the different aspects of modulation techniques that evolved and gradually led to wireless communication.

Then a background analysis of the wireless concept was viewed; from Marconi's invention of telegraphy to the IEEE 802.11 data protocol standard. Consequently, the limitations that abound in the wireless systems like multipath signal fading, etcetera; which necessitated better technological means to contain their effects and deliver higher data rates, was also glanced. So smart multi-antenna design, promising higher capacities, increase in through-put and range was researched and is in use as the single most important concept in wireless communication; MIMO.

Then an over view of the evolution of MIMO from theories asserted by (Teletar) and (Foschini) to the Trellis codes introduced by (Taroakh), and the tireless work by (Alamouti) in providing some fundamental wireless principles were given.

Finally, a statement about the development of the Blast techniques in the lucent laboratory that demonstrated higher spectral efficiencies compared to conventional techniques was made.

The motivation for the topic was stated and the basic aims and objectives also highlighted, all with the final goal of demonstrating the effects of correlation in the different decoding algorithms used for retrieval at the receiver, after data is sent using MIMO technology.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter tries to bring to the open, general aspects of the MIMO system and the review of what giants in the communication field had contributed in relation to this thesis.

The idea of using multiple antennas in both input and output ends is used in wireless systems, as the quest for higher data rates for systems that are power, bandwidth and complexity limited, continues. The large spectral efficiencies associated with MIMO channels are based on the premise that a rich scattering environment provides independent transmission paths from each transmit antenna to each receive antenna. Kahn (2005) states that the last decade had witnessed MIMO develop from purely theoretical analysis of its performance capacity to reality products for the ever expanding wireless market.

Data retrieval is done after symbols are effectively transmitted and received; the methods used are different in concept as they are different in results obtained.

Channel capacity in MIMO depends largely on the statistical properties and antenna element correlation of the channel; the channel varying drastically relative to the scattering, the distance between transmitter and receiver, antenna configuration and Doppler spread. (Kaveh et al, 2002) Czink (2005) believes that the radio propagation channel sorely determines the characteristics of all MIMO channels.

2.2 MIMO System

MIMO is one form of several smart antenna technologies, others being MISO- multiple input single outputs; SIMO-single input multiple outputs and the conventional single-input-single-output, SISO.

The MIMO system basically consists of the fundamental components of the technology as typically used in radio transmission environment; multiple receive and transmit antennas and a rich scattering environment between them, as depicted in fig (4).

When space time processing is used appropriately, the space time codes facilitates in achieving or at least approaching the MIMO capabilities in practical systems. (Molisch et al 2002)

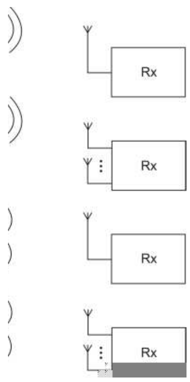


Fig. 3 Illustration of different antenna technologies, www.cst.com/UGM2009/MIMO August 2010

As seen in figure (3) above, conventional wireless communication technologies use single antennas, resulting in multipath leading to signal fading, cut-out and intermittent reception. This slows down data reception and increases error rates. The multipath is eliminated when two or more antennas are used; it even takes advantage of the multipath by using multiple smart antennas with an added spatial dimension and increases both the range of access and the total performance of the system. This has broadened the usefulness of wireless technologies for applications that increasingly call for greater performance.

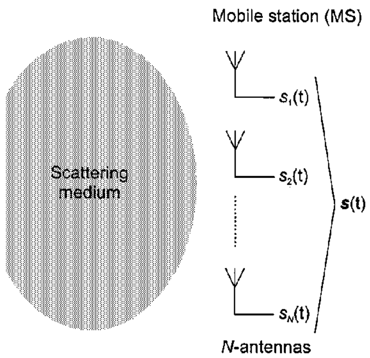


Fig.4 Multiple antenna arrays in a scattering environment. (Kermoal et al, 2002)

2.3 MIMO Functional Category

Robert, (2002) proposes that the MIMO system can easily be categorised into three basic functional processes; Pre-coding, Spatial multiplexes and Diversity coding.

2.3.1 Pre coding

Simply put, pre-coding is a multi-layer beam-forming method encompassing all spatial processing at the transmitter and requiring adequate knowledge of the channel state information.

Beam-forming is an important aspect in antenna usage; it is a signal processing technique used for directional signal transmission/reception in sensor arrays. In a single layer beam-forming employed in MIMO, the same signals with desired phases are emitted from each of the transmit antennas increasing the signal gain by constructive combination and reduces the multipath fading, adding resilience against channel ill-conditions; the Tomlinson-Haroshima linear model is an example.

2.3.2 Spatial Multiplexing

It is a common approach to harness the capacity of MIMO systems as it offers a linear increase in transmission rate for the same bandwidth. By it, independent information streams are modulated and transmitted. Having knowledge of the channel, the receiver separates the information stream using appropriate signal processing techniques. (Wang et al, 2007) Hence, it achieves optimal performance in highly scattered channels like the one depicted in fig.4, wherein the antenna elements are subjected to un-correlated fading.

Foschini (1996) views that if there is M number of antenna elements then the input data stream is first de-multiplexed into M number of sub streams. These sub streams are then modulated and transmitted all at the same time from the individual antenna elements. They are then demodulated to reproduce the M number sub streams and hence retrieve the original input data stream. Provided there is low correlation in the multipath propagation channels the transmitted signals will be received distinctly at the receive antenna elements. So parallel free channels at the receivers are created by the separation of the signal, i.e. if during transmission distortion make then arrive with dissimilar signatures.

2.3.3 Diversity coding

Oestges et al (2007), hint the principle of diversity to entail combating the impact of fading on the error rate by using special techniques to provide the receiver with multiple versions of the same transmitted signal, defining each of these versions as a diversity branch. So when these versions are affected by independent fading conditions, the likelihood that all the branches are in a fade simultaneously is dramatically reduced; helping to stabilise the link through channel hardening and leading to improved performance in terms of error rate.

This technique is used when CSI is not known at the transmitter; unlike in spatial multiplexing, here a single stream is transmitted and the signal coded using Space-time coding, diversity gain is then produced at the receiver with effective decoding of the signals.

Principles of full and sometimes near orthogonal coding- achieved by making the carriers orthogonal to one another to prevent interference between closely spaced carriers, are used to emit signals from each transmit antennas, i.e. in selecting possible signal channels; raising the chance of higher data rate, reducing the possibility of putting to use signal channels with high packet errors and improving the overall network throughput. Hence, this process uses the independent fading in the multiple antenna links to enhance signal diversity, (Paulraj et al, 1989).

The often mentioned fading, i.e. the degradation of signal strength or deviation of attenuation, can occur in time, frequency or space; diversity techniques may also be exploited in these domains, i.e. in time; coding and interleaving, in frequency; equalisation techniques or multi-carrier equalisation, space is what is left. Both time and frequency diversity techniques incur a loss in time and bandwidth and allow for the introduction of redundancy. (Oestges et al, 2007)

Spatial diversity in contrast sacrifices neither time nor bandwidth since it is provided by the use of multiple antennas. However, Chiau (2006) stresses that the space-time coding scheme does not increase capacity linearly like spatial multiplexing does, it increases the distance coverage of the system. So a combination of both schemes is required to meet the increase desired in both capacity and range in MIMO systems.

2.4 Space-Time Block Coding

Space-time block coding is a simple transmit diversity technique used in MIMO technology. It is based on using multiple antennas to transmit several copies of a data stream with the objective that some withstand the difficulty to be encountered in the physical path, in a good enough state to allow reliable decoding at the receivers. So the same data is coded and transmitted through different antennas, effectively improving the signal to noise ratio and doubling the channel power.

This includes aspects of delay diversity i.e. a method of not transmitting the same symbol simultaneously from both antennas, but with a delay between the transmissions. An example for a 2x2 MIMO channel says Jankiraman (2004) is to transmit a data signal from the first antenna and a delayed replica of the same signal from the second antenna after an interval. At the receiver, such a channel will look exactly like a two-path channel with independent path fading and equal average path power.

This approach has a negative effect of introducing interference between symbols and increases complexity in using detectors for data retrieval exponentially with the number of transmit antennas.

Consequently, there was a need to look for a better approach- this was fulfilled by Alamouti.

2.5 The Alamouti Scheme

Alamouti discovered the basics of using space-time transmit diversity, which is a symmetrical symbol mapping of parallel streams of data in time within the given space between transmitters and receivers. He modelled a simple and effective scheme for two antennas (Nt = 2), transmitting two symbols in two time intervals, achieving diversity gain of two.

The essential assumption of the Alamouti coding is that MIMO channel (*H*), stays construct for the two consecutive channel uses utilised by the space-time block code. Matsumoto et al (2003), infer that at the receiver, a space-time minimum mean square error restores the orthogonallity and estimates the sent chip vectors which are then dispread and scrambled to symbols that can be space-time decoded by means of maximum ratio combining.

The Alamouti code can be extended using the two transmit antenna codes as building blocks to design the extended Alamouti (EA)-STBC for four or higher transmit antennas.

But it is understood that the resulting transmission matrix loses its orthogonallity for N ≥ 2. Mecklenbrauker (2004) showed that this loss for the new schemes can be made less severe when gray-coded Quadrature pulse shift keying modulation is used (QPSK), i.e. a scheme that conveys data by changing the phase of the reference of the carrier wave using four points on the constellation diagram. Furthermore, he showed that starting with the four antenna scheme; linear receivers perform close to the theoretical bound for four-path diversity- ultimately offering significant gain over the two antenna case put forward by Alamouti, as shown in figure (5).

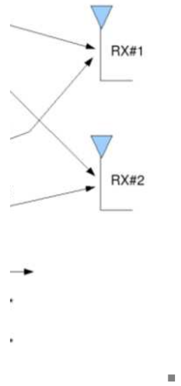


Fig.5 Two transmit, two receive Alamouti STBC, Barry et al (2004)

2.6 MIMO Channels

These are the paths created by the combined effects of transmitter-receiver elements; in conventional communication only one data stream is transmitted over a radio channel regardless of the number of antennas used. MIMO technology, which allows simultaneous transmission of multiple data, was deployed to increase the radio channel capacity-data capacity, needing no extra frequency spectrum. Increasing the transmission data in a given bandwidth using MIMO depends on parameters observed at the receiver; average power of desired signal, system related noise and co-channel interference.

These channels between the transmitters and receivers literarily link them and serve as the transportation medium for data; and are created by the utilization of space or antenna diversity at both transmitters and receivers. We need models for these channels to simply meet accuracy when making site specific decisions on antennas or to obtain same in theoretical analysis.

2.6.1 Channel Model Classification

Channel models can be classified in a number of ways; first, by considering the bandwidth of the system i.e. wideband and narrowband. According to Botonjic (2004), the wideband models treat the propagation channel as frequency selective; so different frequency sub channels have different channel response, hence needing additional modelling of multipath characteristics. While the narrowband models assume that the channel has frequency non-selective fading so the channel has the same response over the entire system bandwidth.

Example of the narrow band channel is the flat-fading channels, also known as amplitude varying channels, i.e. channels that have a constant gain and linear phase response over a bandwidth which is greater than that of the transmitted signal.

The wide band channels are frequency selective fading channels, i.e. channels that have constant gain and linear phase response over a bandwidth that is smaller than that of the transmitted signal.

Secondly, based on modelling approach, i.e. physical and non-physical (analytic model).

The non-physical model describes channels via statistical characteristics obtained from data, i.e. characterised based on the impulse response of the channel. Examples are:

- The correlation based models.
- Propagation motivated models
- The finite scattering model

The physical model is based on set-up parameters and theoretical results; choosing crucial parameters like angles of arrival and departure (AOA, AOD), carrier frequency and antenna spacing to describe MIMO propagation channels, it is subdivided into three:

- Deterministic models
- Geometry based stochastic models
- Non-geometric stochastic models.

There are three basic geometric channel models representation, i.e.

- One ring model

- Two ring model
- Elliptical model

One of these is analysed because of its significance, i.e. one ring model.

2.6.2 The one ring model

Hogstad et al (2004) consider this model as the starting point for channel capacity determination. In it, a transmitter is assumed to be elevated and the line of sight component is obstructed. The receiver is surrounded by an infinite number of local scatters forming a ring around it. The most effective scatterers are situated on the ring from where it got its name. Kahn (2005) says that the effective scatterers each has a random phase shift, uniformly distributed over (- п,п). Hence only waves that are reflected once by the scatterers are considered and we assume that all scattered waves that reach the receiver are equal in power.

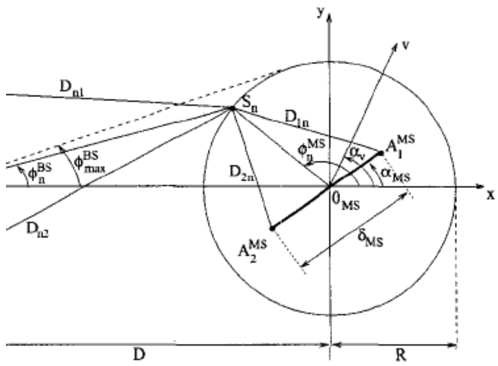


Fig.6 A geometric One-ring model for 2 x 2 channel with local scatterers around the mobile station, (MS). (Hogstad et al, 2004)

Other approaches to modelling and simulation include the “Clarke’s mathematical reference model” and its simplified simulation model due to Jake.

2.6.3 The Jake’s Model

This is a deterministic model, used often for the design of effective Rayleigh fading channel simulators that model the received complex low-pass envelop, of the stationary frequency non selective mobile fading channel, under isotropic condition. It assumes line of sight component is absent. Zheng et al (2002), infer that it is an appropriate analytical model for a zero-mean complex Gaussian noise process with uncorrelated impulse and quadrature components. This model allows an effective approximation of the desired analytic model by using finite number of low frequency oscillators. However, since it is deterministic it has difficulty to create multiple uncorrelated fading waveforms for MIMO channels. Hence, different modifications to the model are reported in many literatures.

Though widely accepted, it has some important limitations; it was shown by Pop et al (2002) that the Jake simulator is, in a wide-sense, non stationary when averaged across the physical ensemble of fading channels. So Pop and Beauliue, (2002), proposed an improved simulator to remove the problem by introducing random phase shifts in the low frequency oscillators. They also showed that higher order statistics of this improved simulator may not match the desired ones of Clarke's reference model.

To better simulate properties of the MIMO channel we can also use the Kronecker model.

2.6.4 The Kronecker Model

In this model, the transmitter and receiver correlations are assumed to be separable; which is a plus, since correlation effects on channel performance is a fundamental part of the thesis. It is also a well known stochastic narrowband MIMO radio channel model that creates channel realisation based on correlation. Its physical assumption means irrespective of which transmit weight vector is chosen, the scatterers surrounding the receiver are illuminated by one and the same power distribution.

However, Herdin et al (2002), show that the Kronecker assumption does not hold in general for realistic MIMO channels hence, lacking essential degree of freedom with respect to general conditions as it can also not generate diagonally dominated coupling matrices; leading to a systematic underestimation of channel capacity, and to a mismatch of the modelled and measured multipath structure.

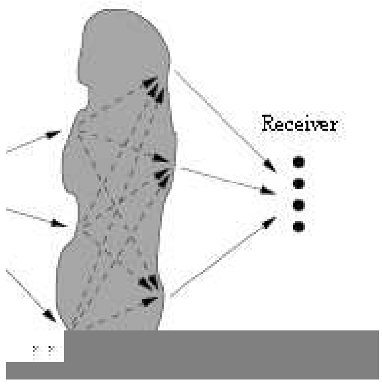


Fig.7 The Kronecker model enforces that all directions of departure are linked to all directions of arrival, the joint DOA-DOD spectrum of its synthesized channel is the product of the average DOA and the average DOD spectrum. (Bonek et al, 2007)

2.7 MIMO Channel Capacity

Based on Shannon’s views, a channel’s capacity is the amount of information that can be processed through it per unit time over a noisy channel. This is the maximum rate of information that can be transmitted via a channel; it is derived from the Shannon law to arrive at:

/s 2.1

Where τ the time slot, T is the time; n is the number of time slots.

So to increase the channel capacity, either n is increased or τ is decreased. Paulson (2009)

In a channel, uncertainty caused by noise is detrimental to information transfer, so the Information can be sent with a vanishing error rate while maintaining the information rate at the channel capacity, (Teletar et al, 1995). This is one of Shannon's most important deductions.

Nonetheless, he did not propose the exact encoding required for achieving this arbitrarily small error rate while maintaining the information rate at channel capacity. If the information rate from the source exceeds the channel capacity, then a message cannot be sent with an arbitrarily small error rate.

Channel capacity for SISO systems is given by the accompanying expression from Shannon – Hartley theorem:

CmaxB = log2 [1+SN] bps/herz 2.2

Where B= system bandwidth for a pulse with energy storage devices

S = total signal power

C_{max} = maximum capacity

N = total noise power

Paulson (2009) maintains that a similar expression for MIMO systems channel can be obtained.

MIMO channel capacity is realised for cases when the CSI is known and unknown to the transmitter. This gives information about maximum possibility transmission rate such that the probability of error is small.

When CSI is unknown to the transmitter the signal between are independent and power equally divided among the antennas at the transmitter; channels exists between each transmit and receive antenna, having a response that depends on the scatter environment i.e. the channel state information.

It is possible to learn the CSI at the transmitter using delay diversity scheme. When CSI is known to the transmitter each channel has an impulse response due to the entire multipath components of the channel. Optimal power allocation scheme is shown to be of a water filling algorithm and this system achieves higher capacity.

Generally, in systems where CSI knowledge is known to the transmitter, there is higher capacity than those in which only the receiver has knowledge.

Hence for MIMO systems the capacity can be written as:

$$C = \max_{Tr(R_{XX}) = M_T} \log_2 \left| I + \frac{\gamma}{M_T} H R_{XX} H^H \right| \text{ bps/hertz} \qquad 2.3$$

$H \in C^{n_t \times n_r}$ is the channel matrix,

$R_{XX} \in C^{n_t \times n_r}$ is the covariance of the signal vector,

M_T is the number of transmit antennas,

γ is the signal to noise ratio at the receiver,

$I \in R^{n_r \times n_t}$ is the identity matrix

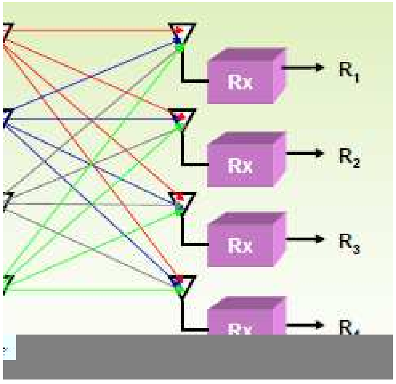


Fig.8 A 4x4 MIMO channel, Sharony (2006)

2.8 Layered Space-Time Architecture

Foschini (1996) designed this architecture for a Rayleigh fading environment for circumstances in which the transmitter does not have knowledge of the channel characteristics. This was seen as a newer method of presenting and processing higher dimensional signal (space) with the aim of leveraging the already developed one-dimensional codec technology, i.e. (SISO)

2.8.1 The BLAST

This stands for the Bell Laboratories Layered space-time trans-receiver which offers spatial multiplexing over multiple antenna systems; it was developed by Gerald Foschini at Lucent technologies Bell laboratories. In it, by carefully allocating to the transmitting antennas, data to be transmitted, multiple data stream are moved simultaneously within a single frequency band in order to attain the data capacity supported by MIMO; data capacity grows directly, in line with the number of antennas.

Two principal approaches to this are seen, i.e. the *D* and *V* Blasts architecture, (diagonal and vertical).

- **The V-Blast:** is a trans-receiver architecture in which independent data streams are multiplexed in an appropriate coordinate system, used over a deterministic time-invariant MIMO channel. It is a simplified version of the BLAST detection architecture well known for achieving high spectral efficiencies over the rich scattering environment.

The receiver transforms the received vector into another appropriate coordinate system to separately decode the different data streams.

At each symbol, it detects the strongest layer of the transmitted signal from each of the received signals; it cancels the effects of this strongest layer, i.e. using (MMSE-SIC); minimum mean square error- with successive interference cancellation, and then continues to detect the remaining layers and so on, (each stream is thought of as a layer). I infer from Foschini, that this architecture will always reconstruct transmitted signals by removing the fading effect of the channel in frequency domain.

Relatively, this architecture simply allocates equal power and rate to every transmit antenna, achieving the capacity of a fast fading channel and as a result, it becomes limited by the antenna with the smallest capacity as dedicated by the channel. Hence, it is strictly suboptimal for slow fading channels.

- **The D-Blast** is a modified architecture that codes across the transmit antennas, using multichannel arrays at both ends with diagonally layered coding structure in which code blocks are dispersed across diagonals in space-time. Since in the V-blast there is no coding across the sub-channels, outage occurs whenever one of them is in deep fade and cannot support the rate of the stream. However, Valenzuela (1998) remarks that this suffers from implementation complexities i.e. rate loss, because in the initialisation phase, some of the antennas have to be kept silent. It also suffers from error propagation i.e. if one layer is decoded incorrectly, subsequent layers are affected.

2.9 Equalization

Equalisation is the compensation of inter-symbol interference (ISI) created by multipath within time dispersive channels. Vitetta et al, (1998) say that the ISI occurs when the modulation bandwidth exceeds the coherence bandwidth of a radio channel causing modulation pulses to spread in time into adjacent symbols.

Equalisation, diversity and channel coding are techniques which can be used independently or in tandem to improve received signal quality, (Rappaport, 2002).

Diversity compensates for the fading channel impairments by using two or more antennas.

Channel coding improves the link performance by adding redundant data bits in the transmitted message so that if fading occurs in the channel, the data can still be recovered at the receiver.

So an equaliser within a receiver compensates for the average range of the expected channel amplitude and delay characteristics. Two types are observed; the linear and non-linear equalisers.

Therefore after all these processes comes a data decoding phase or the retrieval stage.

2.10 MIMO Decoding at the Receivers

Retrieving data in MIMO takes place at the receiver. I am looking at the software Matlab for its simulation. The modulation techniques could be Binary phase shift Keying (BPSK), Quadrature phase shift keying, (QPRS) and Quadrature amplitude modulation, (QAM); MIMO is not restricted to one technique.

Bottomley (2000), implemented a Rake receiver (a radio receiver originally designed to counter the effects of multipath), in hardware for a SISO channel in a CDMA system. It had the problem of in implementing equalisation for overloaded systems, i.e. in high scattering environments, its performance degrades appreciably. So, other receivers were figured out and designed to effectively decode signals sent by transmitters in a rich multipath environment.

Hence, from Paulson (2009), MIMO decoding can be done using least square algorithm (LSA), i.e. all information rewritten as least square problem or by using discrete means - since it is known that initially transmitted symbols come from a finite discrete set of constellation points, (a representation of a signal modulation scheme, displaying the signal as a two-dimensional scatter diagram in the complex plain at symbol sampling instants).

Or recursive least square algorithm (RLA), i.e. relying on error measures expressed in terms of time average of the actual received signal instead of the statistical approach of LSA. Examples of such decoding schemes include:

2.10.1 Zero-forcing

First proposed by Robert Lucky, it is a linear algorithm that first splits different data streams, decodes each by inverting the channel and eliminating multi stream interference. The MSI is an undesired signal distortion that arises when a sent symbol affects subsequently received ones.

To remove the MSI perfectly and invert the channel, infinite impulse response filtering is used. A ZF demodulator forces the interference between streams from different transmitted antennas to zero. The interference is then completely suppressed by multiplying the received signal with the Moore-penrose pseudo-inverse of the channel matrix.

$$X \cong H^*Y \qquad 2.4$$

Where * = Moore penrose pseudo-inverse.

H = channel matrix

Y = received signal vector.

A diagonal channel is obtained which is demodulated to get data.

Its advantage lie in the reduction of channel complexity, relatively it degrades channel performance.

2.10.2 Maximum Likelihood Receivers

Maximum likelihood is a decoding algorithm which results in the minimum probability of decoding to an incorrect code word when a priori probabilities of all the code words are equal.

This is considered the optimal method as it performs vector coding that searches through all the solution space of transmit symbol vectors for the one that maximises the probability of transmission, given a received symbol vector:

$$S = \arg[\min| Y- Hs|2] \qquad 2.5$$

Where S = estimated symbol vector;

H and Y remain the same as in ZF.

The ML receiver algorithm is difficult to implement but it achieves full diversity gain and zero power losses.

2.10.3 Null and Cancellation

This is an iterative system that uses an estimate to determine one of the entries of vectors i.e. ($s1$). The entry is assumed to be known so its effect is cancelled out to obtain a reduced order integer, least-square problem with m-1 unknowns, Paulson (2009). The decoder;

- Works out the continuous LS solution, s.
- Finds the symbol closest to the constellation point $s1$.
- Takes $s1$ to be that constellation point.
- Restates the problem with one less known.
- If there are more symbols to decode go to 1.

If the first estimated entry of the symbol is erroneous, estimation of the remaining entries could also be.

NC thus, attempts to progressively clean $\boldsymbol{x}$ from the interference corresponding to the layers already detected.

2.11 Summary

It is evident that the desire to meet higher data rate transmissions based on the inherent capability of utilising smart antenna technology, to obtain better performances over wireless channels, motivated much research about MIMO.

As a result, this chapter reviewed its system format with multiple antennas at both the receive and transmit sides, the advantageous use of the multi-path created, i.e. different signals arriving the receiver at various times and the Alamouti approach for using space-time diversity.

Hence, the three basic functional categories; firstly, pre-coding (space-time coding), in which antenna selection, spatial division multiplex, beam-forming and transmission diversity are done. Secondly, spatial multiplexing; i.e. the modulation and transmission of independent information streams with suppressed interference. And thirdly, diversity coding; encompassing diversity and array gains with effective decoding of the signals to reproduce transmitted data at the receiver, were all researched.

The channels for data transportation are analysed, its classification based firstly, on bandwidth of the channels, i.e. narrow and wide band widths. Secondly, the modelling approaches, i.e. physical and analytical. Then the One ring model was looked at, analysing its channel model basis, as it is viewed to be of much significance.

Then individual approaches to models were analysed, the Jake deterministic model; used often for the design of Rayleigh fading channel simulators. Kronecker model a stochastic narrowband model that creates channel realisation based on correlation.

The two different aspects involved in channel capacity realisation; when CSI is known and unknown to the transmitter were looked at. Then the layered space-time architecture (Blast) put forward by Foschini and their operating modes and basic limitations were analysed.

Finally, equalisation or decoding algorithms used at the receiver for data retrieval were looked at; discrete and least square approaches that enable Zero-forcing and Maximum-likelihood methods respectively and a different algorithm termed Nulling and cancellation.

CHAPTER THREE

MATHEMATICAL CONCEPTS AND DESIGN

3.1 Introduction

MIMO signalling operates by spreading available information in both space and time. After all the theories and literature concerning the enabling aspects of MIMO, here discussion of the mathematical elements of the models from radio propagation point of view is attempted.

It might not be needless to say that in-between the transmitter and receiver antennas used in MIMO a lot of mathematical computations are carried out accompanying the processes mentioned previously. These computations are what basically transform data from their binary form through encoding at the Tx, aid the modulation of data in the space-time in-between and finally helps in data retrieval at the Rx.

3.2 MIMO Mathematical Channel Model

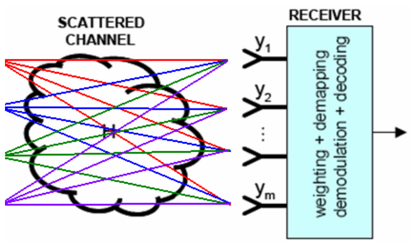


Fig. 9 MIMO model architecture, Paulson (2009).

In the figure above, every transmitter sends its data stream to all the receivers at such, each channel formed has an impulse response as a result of the multipath created by the components of the system. Bonek et al (2003) presume this model to be an enhanced stochastic model which is able to model the spatial properties of a realistic MIMO channel, and then one can restrict oneself to frequency- flat and stationary channels, which can be described as a single channel transfer matrix ***H***.

Hence, if a QAM symbol Sl is broadcast from Txl , then Rxm receives HlM Sl. When all deterministic components are removed from HlM, the stochastic part is said to be a Rayleigh random variable says Paulson, (2009).

The signal vector ***y*** at the receive antennas read as:

Y = Hx

Where ***x*** represents the transmit signal vector.

Usually noise ***n*** is included, i.e. $Y = Hx + n$ 3.1

Where ***n*** stands for the noise at the receiver, an example for a 3x3 system is:

= + Noise 3.2

This means that the symbol received at y1 is a mixture of all transmitted symbols:

$y1 = H11\ x1 + H21\ x2 + H31\ x3 + \ldots\ldots + Hn1\ xn$ 3.3

Paulson (2009), further reiterates that on its own the equation above has no way of unscrambling itself, so collecting all received symbols together yields:

$y1 = H11\ x1 + H21\ x2 + H31\ x3 + \ldots\ldots + Hn1\ xn$

$y2 = H12\ x1 + H22\ x2 + H32\ x3 + \ldots\ldots + Hn2\ xn$

.

.

.

$ym = H1m\ x1 + H2m\ x2 + H3m\ x3 + \ldots\ldots + Hnm\ xn$

Therefore our MIMO channel can be estimated if we send symbols from all the transmit antennas, i.e.

$y1 = Hx1$

$y2 = Hx2$

$yn = Hxn$

When the vectors are collected into matrices;

Y = (y1y2...yn) and X = (x1x2...xn), and

when ***X*** are all orthogonal;

$H = YX^{-1} \equiv H = YX^{T}$, 3.4

3.3 Space-Time Transmit Diversity - (ALAMOUTI CODE)

As stated earlier in chapter two and seen in fig (10), Alamouti discovered the basics for this scheme using two transmit antennas.

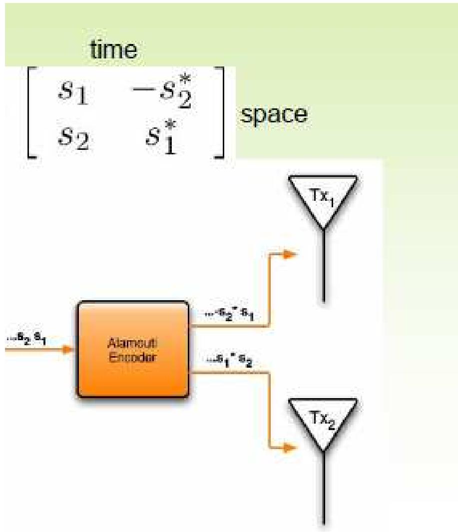


Fig. 10 Illustration of how the Alamouti code works, Sharony (2006).

Mecklenbrauker (2004) analysed that when data block (S1, S2*) is sent over the first antenna and block (S2, -S1*) over the second antenna, where * denotes complex conjugation. Assuming a flat fading channel with transmission coefficients h1, h2**,** the received vector ***r*** is formed by tracking two consecutive received data samples:

r = [r1,r2]T , in time, resulting in;

r = S12 h + V 3.5

Where h = [h1,h2]T is a complex channel vector and ***V*** is the noise vector at the receiver. The symbol block S12 is therefore defined as

S = 3.6

r1 = h1S1 + h2S2 + V1 3.7

r2* = -h2 S1* + h1*S2 + V2 3.8

(3.5) can be written as = + 3.9

Or $Y = H_V S + V$

Where vector $Y = [r_1, r_2]^T$ is introduced, the resulting virtual (2x2) channel matrix H_V is orthogonal, i.e. $H_V^H H_V = H_V H_V^H = h^2 I_2$ 3.10

3.4 One-ring Model

The geometric one ring model was viewed as the starting point for the derivation of a reference model for a MIMO channel, from which an efficient space-time simulation can be derived by applying the principle of deterministic modelling. So from fig (5); a transmitter, referred to as the base station (BS) is elevated and the line of sight component is obstructed. The receiver, represented by mobile station (MS) is surrounded by an infinite number of scatterers. Assuming $M_{BS} = M_{MS} = 2$ antennas at both MS and BS, the distance between them denoted by ***D*****,** radius of the ring on which the scatterers are located given as ***R***, and the angle spread is denoted as $\emptyset_{max}^{BS}$. In the same figure, we see that the MS moves with speed ***V*** in the direction determined by the angle $\propto_V$**.**

Hence, from Hogstad et al (2004), the time variant complex gains $h_{i,j}$ (t), (i, j = 1, 2) which connects the receiver and transmitter antenna elements are A_i^{MS} and A_j^{BS} respectively and given by:

$h_{11}(t) = \lim_{N \to \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n e^{j(2\omega f_n t + \theta_n)}$ 3.11

$h_{12}(t) = \lim_{N \to \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n^* b_n e^{j(2\omega f_n t + \theta_n)}$ 3.12

$h_{21}(t) = \lim_{N \to \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n b_n^* e^{j(2\omega f_n t + \theta_n)}$ 3.13

$h_{22}(t) = \lim_{N \to \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n^* b_n^* e^{j(2\omega f_n t + \theta_n)}$ 3.15

And $a_n = e^{j\frac{\pi\delta_{BS}}{\lambda}[\cos(\alpha_{BS}) + \emptyset_{max}^{BS} \sin \emptyset_n^{MS}]}$ 3.16

$b_n = e^{j\frac{\pi\delta_{MS}}{\lambda}\cos(\emptyset_n^{MS} - \alpha_V)}$ 3.17

$f_n = f_{max} \cos(\emptyset_n^{MS} - \propto_V)$ 3.18

(*) represents the complex conjugation function, λ is the wavelength and f_{max} is the maximum Doppler frequency.

Consequently, only $\emptyset_{maxBS}$ controls *D* and *R*, so $h_{i,j}$ (t) is a zero complex Gaussian process with a variance of unity, therefore the envelop of $h_{i,j}$ follows a Rayleigh distribution. Furthermore, since the capacity of this MIMO channel depends on the correlation between the channel gain $h_{i,j}$, i.e.

$$H = h11(t)h12(t)h21(t)h22(t) \qquad 3.19$$

So the channel capacity for the channel derived from this one-ring model, in bits/sec/hertz is given as:

$$C(t) = \log 2[\det(I1+ \ PBS,totalMBSPN \ \ HtHHt)] \qquad 3.20$$

Where I, is the 2x2 identity matrix, 'det' stands for the determinant matrix, P_N is the noise power.

3.5 MIMO Decoding

Rappaport (2002) is of the view that the function of a decoder is to estimate the encoded input information using a rule or method that results in the minimum possible errors, as there is a one-to-one correspondence between the information and code sequences.

For reasonable data decoding, we consider a system with equal numbers of transmit and receive antennas, also assuming a flat fading environment;

$$y = Hx + n \qquad 3.21$$

Where y is the received symbol; a $M_R \times 1$ vector, *H* is channel matrix of size $M_R \times M_T$, *x* the transmitted signal and *n* is the $M_R \times 1$ noise vector.

3.5.1 Zero-forcing

This removes all ISI by applying the inverse of the channel to the received signal, to restore the signal before the channel and implements matrix pseudo-inverse-ignoring noise. If we assume the channel matrix *H* is invertible, an estimate of the transmitted data is given as:

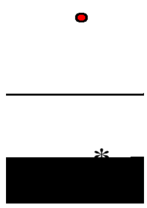
$$x = (H^{H}H)^{-1}Hx = H^{*}x \qquad 3.22$$

Where * is the pseudo-inverse. Since an inverse of *H* can only exist if the columns of *H* are independent, an independent identical distribution is assumed, i.e. H = $H\omega$. Therefore for a noiseless channel, the transmitted symbols can be calculated by;

$$x = H^{-1}y \qquad 3.23$$

Consequently, the term Zero-forcing was coined to correspond to, bringing down the inter symbol interference (ISI) to zero in a noise free case.

One could continuously work out an LS solution and then look for the nearest constellation point. i.e. using a QPSK as shown;



With $x = 1.1-0.9j-0.4-0.8j$

We can use ZF to decode to $1-1j-1-1j$

Since in reality in all MIMO receivers we need to contend with noise and multi symbol interference (MSI), it makes the ZF algorithm suboptimal.

3.5.2 Maximum likelihood

Maximum likelihood basically investigates SMT**,** i.e. all received signal sample combination, for the most probable transmitted signal vector. In this sense it searches for the most likely ***x*** given ***H*** and ***y***, making them difficult to implement.

So, when noise is factored in, the extra information added by increasing the number of antennas allows for significant noise reduction.

Therefore, using the least square algorithm, all the information can be used when the problem is re-written in a least square sense; so rather than solve:

$y = Hx$,

Solving for x, $x = \arg(\min\|y - Hx\|^2)$ 3.24

Based on the IEEE VTC 2000 document, its symbol error probability P_s depends on the distance, (Euclidean) between the different received vectors Hx. An upper bound on P_s can be obtained by assuming all possible code words have the same minimum distance (d_{min}). Hence the error probability for the ML decoder is given as:

$$P_s \leq \frac{C^m - 1}{2} \operatorname{erfc}\left(\sqrt{\frac{d_{min}^2}{4} \frac{E_s}{N_o}}\right) < C^m \exp\left(-\frac{d_{min}^2}{4} \frac{E_s}{N_o}\right) \quad 3.25$$

It is assumed in 3.25 that the all transmit antennas use the same constellation point ***C***, so total number of code words is C^m and d_{min}^2 is the squared minimum distance between two different received code words with unity normalised average power per receive antenna and $\frac{E_s}{N_o}$ is the average signal-to-noise ratio of the receive antennas.

Despite the difficulty in its implementation, ML receivers provide full M_R diversity and zero power loss.

3.5.3 Nulling and Cancellation

This is an iterative method, Seethaler et al, (2004) say a conventional Nulling and cancellation detection scheme for MIMO systems use layer-wise post detection minimum square errors (MSEs) as reliability measures for layer sorting; the MSEs being averaged measures that do not depend on the received vectors.

In addition, it contrasts the linear detection methods, i.e. ZF or ML and MMSE, in which all layers are jointly detected; NC uses a serial decision feedback approach to detect each layer separately. At each decoding step, a single layer is detected, corresponding to the received vector, it is then subtracted from ***x***, the other layers that have not yet been detected are "nulled out" using ZF or MMSE. Seethaler et al, 2004

Its basic approach is; for the ***ith*** layer ($i \in \{1,..., M_T\}$), the optimum decision on the data symbol $d_i \in \beta$ is given by the maximum a-posteriori map rule that maximises the approximate a-posteriori probability; App $P\{d_i = a \mid y_{ZF}\}$. The resulting maximum App $P\{d_i = \hat{d}_i \mid y_{ZF}$ is a measure reliability measure of the optimum symbol detection. The optimum symbol $\hat{d}_i$, for each layer *i,* is calculated, if the layer sorting approach is deemed. Then layer i is chosen, for which the reliability of the optimum decision is maximum, i.e.

$$i \triangleq \arg\max_{j\, i \in \{1...,M_T\}} P\{d_i = a \mid y_{ZF}\}. \qquad 3.26$$

And layer i is finally decoded in favour of $\hat{d}_i$, the result obtained used for interference cancellation.

3.6 Summary

This chapter bridges the gap between the theories introduced in chapter one and reviewed in two, with the simulation and results obtained in chapter four. This is important because, there exists underneath the different techniques that aid MIMO wireless technology, appreciable mathematical computations. These computational expressions, though mostly tedious and cumbersome, when understood give pragmatic meaning to the volumes of theories available on MIMO.

From the basic literature of transmit-receive symbols in a noisy channel, the mathematical expression Y = Hx + n is deduced which governs MIMO; from which the channel matrix could be estimated, and the received symbol could be determined knowing the transmitted ones.

The Alamouti code was analysed, mathematically, to give meaning to space-time transmit diversity.

The One-ring channel model is important in deriving the reference model for a MIMO channel, hence, the mathematics that support it was also analysed.

Then the all important MIMO decoding, i.e. the estimation of received symbols to predict and arrive at those transmitted were generated. Consequently, ZF with its characteristic channel inversion and the ML with its signature vector space of all possible combination, were generated. The NC was also generated but was not used for data retrieval because of time constraints.

While the generation of mathematical capacity expression would have fit here, it was given in chapter four since antenna correlation effects on retrieved data BER, hence the systems' overall performance were examined at length there.

CHAPTER FOUR

SIMULATION AND RESULTS

4.1 Introduction

The Oxford dictionary for modern English (4^{th} edition), defines simulation as a representation of a behaviour or characteristic of one system through the use of another system especially a computer program designed for the purpose.

Therefore, the data retrieval and the channel performance evaluation based on correlation effects, were done using Matlab program; a highly effective tool for such simulation. It stands for matrix laboratory; in representing the behaviour of data as they are transmitted and received in MIMO systems, some communication mathematics are used, these vary depending on the decoding algorithm employed or the channel model approach. The simulation will show the results and difference in the algorithms used. And the apparent effects of correlation between the antennas used in wireless communication, on the available capacities obtained is also viewed.

Details of the Matlab codes are given and the resulting plots of the determined results also shown. The script used was originally sourced online from http://www.dsplog.com, and modifications were made to suit the concepts and objectives of this project.

4.2 Zero-forcing Equaliser For a 2x2 MIMO (Using BPSK Modulation in a Rayleigh Channel)

Assuming that the channel is flat, i.e. meaning that the multi-path channel has only one tap-reducing the convolution operation to simply multiplication. The Matlab simulation model using ZF algorithm performs the following:

- Generate random binary sequence of +1's and -1's.
- Group them into pair of two symbols and send two symbols in one time slot.
- Multiply the symbols with the channel and then add white Gaussain noise.
- Equalise the received symbols.
- Perform hard decision decoding and count the bit errors.
- Repeat for multiple values of Eb/No; i.e. signal-to-noise ratio and plot the simulation and theoretical results.

The script is given in appendix (A); from the introduction of the variables to be used in the initial stages, the channel paths are built and the channel matrixes declared. At the transmitter, the symbols to be transmitted are then assigned, transmitted and are based in this case on BPSK. Noise addition to the channel is defined as shown in the input "`y = 10^(-Eb_N0_dB(ii)/20)*n;`".

Then the receiver structure is defined and the Moore Penrose pseudo-inverse is carried out on the unknown transmitted data i.e. on the matrix so formed by the combination of each transmitted symbol with the channel and noise. The accompanying inputs carry out the Moore penrose inverse on the channel **H**:

```
   % Forming the Zero Forcing equalization matrix W =
inv(H^H*H)*H^H
 % H^H*H is of dimension [nTx x nTx]. In this case [3 x 3]
 % Inverse of a [3x3] matrix [a b; c d] = 1/(ad-bc)[d -b;-c a]
 hCof = zeros(2,2,N/nTx)  ;
 hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
 hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
 hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
 hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
              hDen               =((hCof(1,1,:).*hCof(2,2,:))-
(hCof(1,2,:).*hCof(2,1,:)));% ad-bc term
                             hDen                              =
reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);%
formatting for division
 hInv = hCof./hDen; % inv(H^H*H)
 hMod =  reshape(conj(h),nRx,N); % H^H operation
```

The noise in the separated streams is correlated and consequently the SNRs are not independent. By so doing the received symbols are equalised. So, a continuous least square solution is worked, and the nearest constellation point is determined. Then a decision is taken, forcing the most unlikely transmitted symbols to zero; finally, retrieving the transmitted data. The accompanying graph indicates the results obtained.

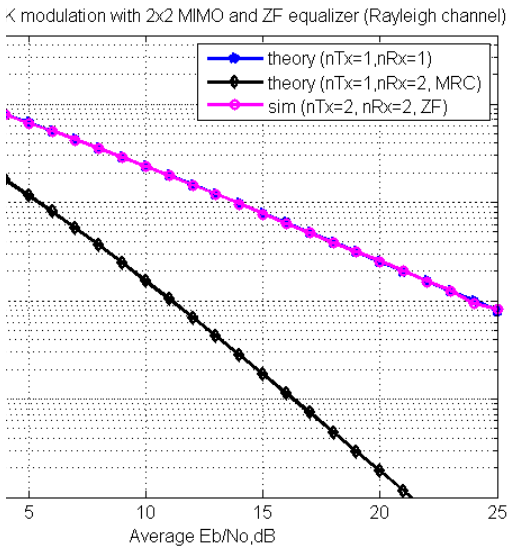


Fig11 Data retrieval illustration showing the relationship between SNR and BER to capacity for a 2x2 MIMO using ZF

4.3 Effects of Correlation on the BER and SNR of 2x2 MIMO Systems as it Relates to Capacity using ZF Decoding

To analyse the capacity changes in the above system, in relation to the BER and SNR of the retrieved data, a Kronecker model is employed and the correlation at both ends are varied to see their effects. Appendix (B) specifies the adjustments made to the original script in (A).

For starters, I varied the correlation at both ends using the same values, i.e. 0.5, 0.8, 1. Then I used different values at both ends to analyse the effects of total correlation at either receive or transmit ends and their subsequent contribution to the channel performance of the system under analysis. Appendix (B) is the script for a 0.5 correlation at both ends. Subsequent results of correlation were obtained by inserting the values in the script. Consequently, the following results were obtained:

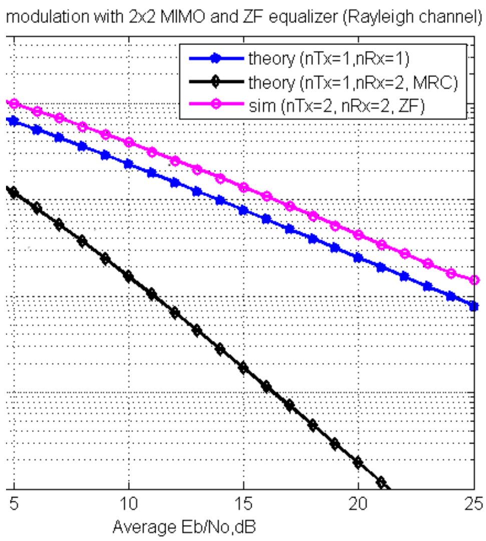


Fig.12 Zero-Forcing decoder with 0.5 correlations at both ends

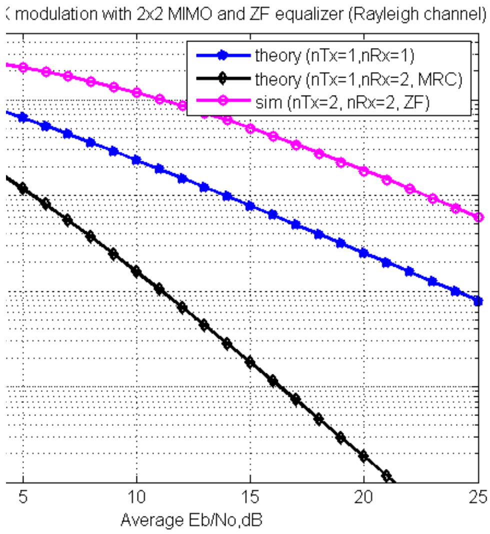


Fig.13 Zero-Forcing with 0.8 correlations at both ends

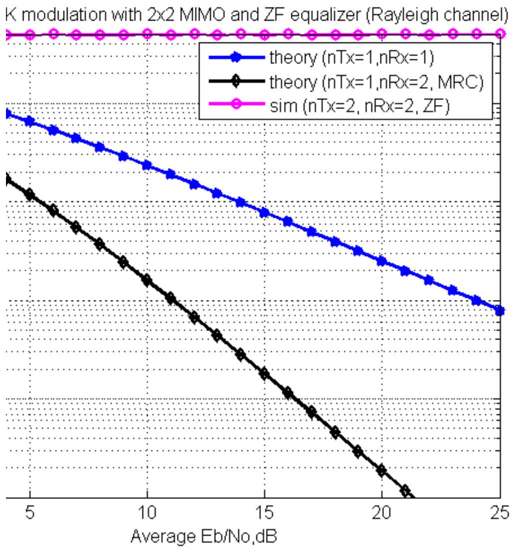


Fig.14 Zero-Forcing for unity (1) antenna correlation at both ends using the kronecker channel model

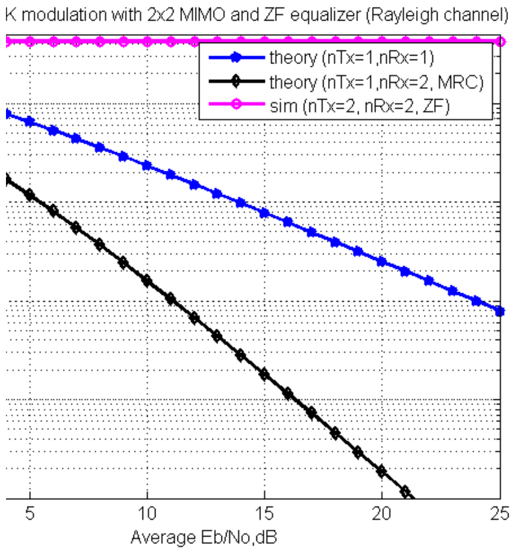


Fig.15 Zero-Forcing for null and unity correlation at either ends

4.4 Zero Forcing Equaliser for a 4x4 MIMO System

To analyse that capacity increments when more antennas are used at both the transmit and receive ends, a 4x4 multi-antenna system is used with all the previous assumptions for the 2x2 system maintained except that in this case not two symbols are sent in one time slot but four and Zero-forcing also used as the equaliser for data retrieval. Appendix C shows the algorithm as used on matlab and the accompanying figure, (16) shows the results plotted as obtained.

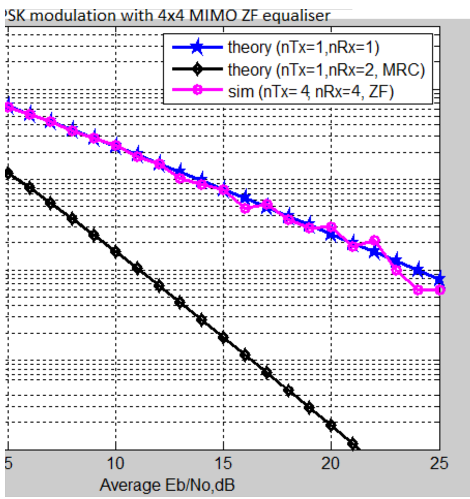


Fig16 Data retrieval illustration showing the relationship between SNR and BER to capacity for a 4x4 MIMO using ZF

4.5 Maximum Likelihood (ML) Equaliser For a 2x2 MIMO (using BPSK Modulation in a Rayleigh Channel)

The ML receiver tries to find x which minimizes, J = | y-Hx|2. Since the modulation scheme is BPSK, the possible values of x1 and x2 are either 1 or -1. To then find the Maximum Likelihood solution, the minimum from all four combination of x1 and x2 is needed. The transmit symbol has chosen estimates based on the minimum value of the four values, i.e.
If the minimum is J + 1, is +1 which translates to [1 1],
If the minimum is J + 1, is -1 which translates to [1 0],
If J – 1, is +1 it translates to [0 1],
If J-1, is -1 it translates to [0 0].

The Matlab simulation model using ML algorithm performs the following:

- Generate random binary sequence of +1's and -1's.
- Group them into pair of two symbols and send two symbols in one time slot.
- Multiply the symbol with the channel and then add white Gaussian noise.
- Find them minimum among the four possible transmit symbol combinations.
- Based on the minimum chose the estimate of the transmit symbol.
- Repeat for multiple values of EbN0 and plot the simulation and theoretical results.

The script is given in appendix D. As was previously done in the ZF script; the variables to be used are introduced initially. In fact, the same channel model and variables are used as in ZF. The only difference is in the method of decoding; a subroutine created by Paulson (2009) for Maximum likelihood decoding is called after the channel matrix is formed, and is given as appendix (E). In this script, after the transmitted symbols are multiplied with the Channel matrix and noise, a sample vector or space is formed of all the possible combinations. So, the

ML decoder investigates all the space of the received signal sample combination for the received symbol that maximises the likelihood or probability of being transmitted, or the received symbol that appears closest to what was transmitted given a known channel matrix **H** and known transmitted symbols **y**. This done by these lines in the script:

```
symbol_Vectors  =  [   kron(  ones(1,nSymbols)   ,   Symbols  )  ;
kron( Symbols , ones(1,nSymbols) ) ];
YMHS = kron(Y,ones(1,nSymbols^N)) - H*Symbol_Vectors;
Distance = sum( YMHS.*conj(YMHS) , 1 );
[ C , Index] = min(Distance);
Received_Symbols = Symbol_Vectors(:,Index);
```

Hence, the symbol is retrieved as the transmitted one, it is worthy of note that the sample for comparison increases with the number of antennas used, making this process complex. But the complexity can be reduced by a process called sphere decoding; i.e. finding a continuous LS solution and then search the transmit vectors close to it.

Consequently, fig (17) is the result for a 2x2mimo system after scripts (D) and (E) are ran. Then figs (18) and (19) are for a 3x3 and 4x4 systems respectively, obtained by making slight changes to the used scripts to increase the number of antennas used and direct the script to act accordingly.

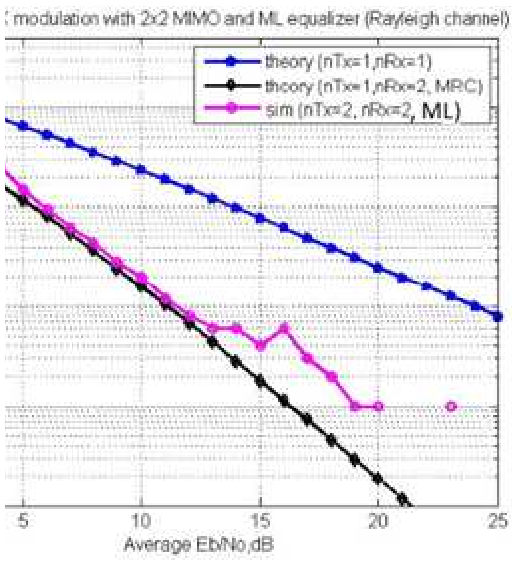


Fig 17 Data retrieval illustration showing the relationship between SNR and BER to capacity for a 2x2 MIMO using ML decoding

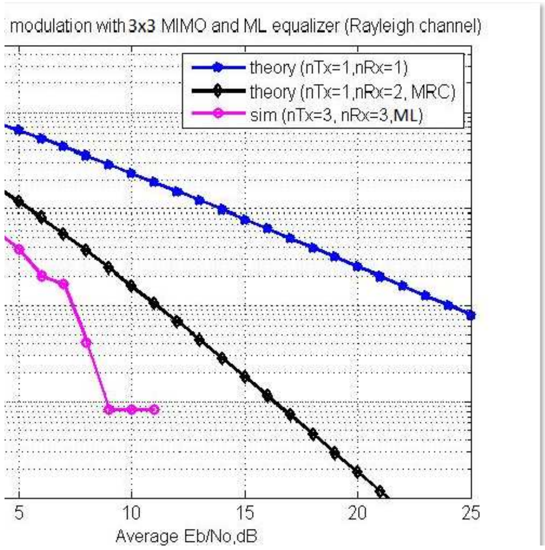


Fig 18 Data retrieval illustration showing the relationship between SNR and BER to capacity for a 3x3 MIMO using ML decoding

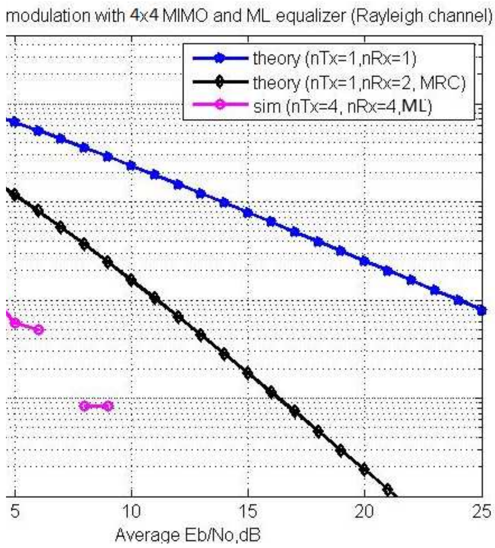


Fig 19 Data retrieval illustration showing the relationship between SNR and BER to capacity for a 4x4 MIMO using ML decoding

4.6 Effects of Correlation on the BER and SNR of MIMO Systems as it Relates to Capacity using ML Decoding

To investigate the effects of antenna correlation as BER and SNR change, on the performance of the channel capacity of MIMO systems, using the Maximum likelihood decoder, script D is modified such that correlation between transmit and received antennas can be varied. Script (F) is used for a 4x4 system; correlation is varied from 0.1 to 1. The results are given below:

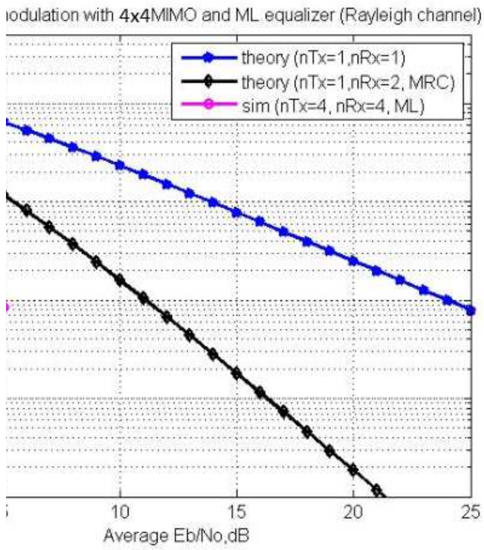


Fig. 20 Maximum Likelihood decoding with 0.1 correlations at both ends for a 4x4 system

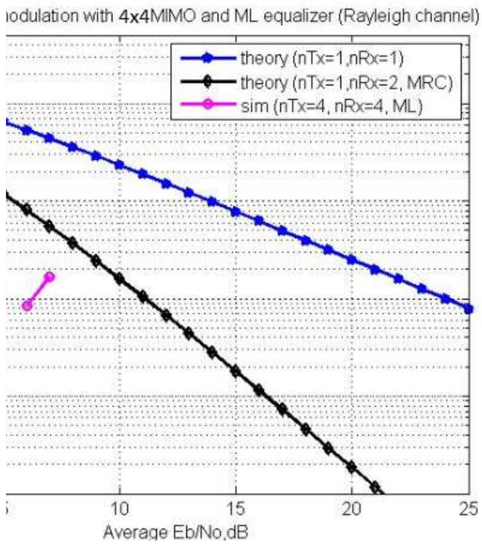


Fig. 21 Maximum Likelihood decoding with 0.5 correlations at both ends for a 4x4 system

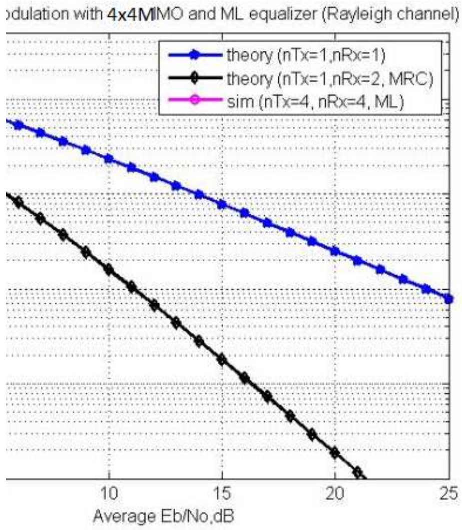


Fig. 22 Maximum Likelihood decoding with 0.9 correlations at both ends for a 4x4 system

The same script (F) is used to examine the effects of correlation on the 3x3 and 2x2 MIMO systems; the appropriate changes are made to the number of antennas and the channel matrix to give the accompanying results:

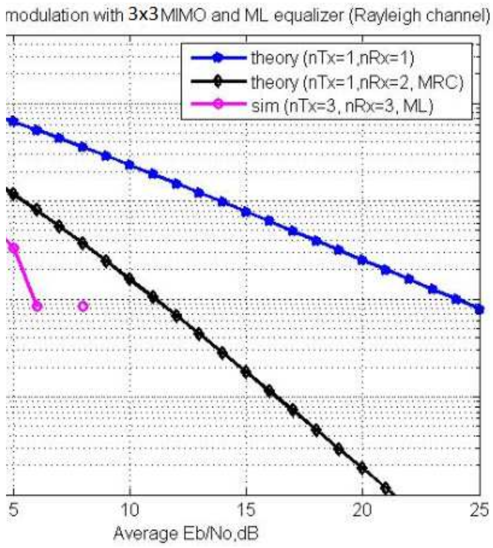


Fig. 23 Maximum Likelihood decoding with 0.1 correlations at both ends for a 3x3 system

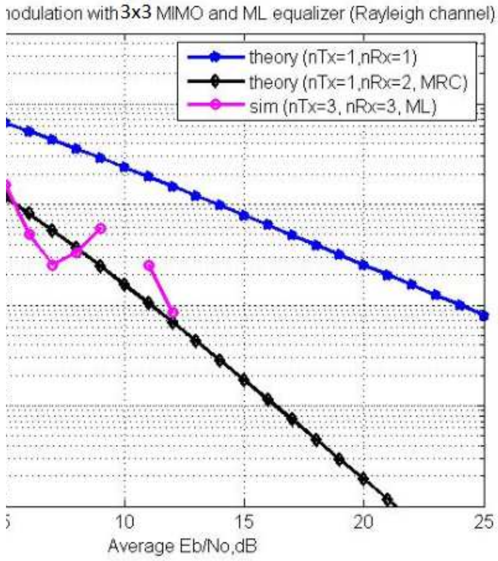


Fig. 24 Maximum Likelihood decoding with 0.5 correlations at both ends for a 3x3 system

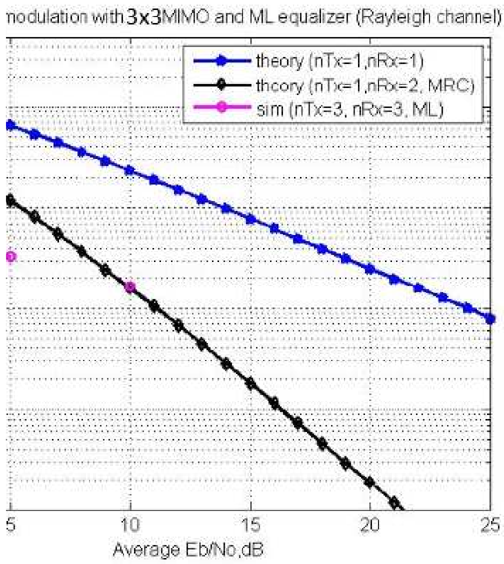


Fig. 25 Maximum Likelihood decoding with unity (1) correlations at both ends for a 3x3 system

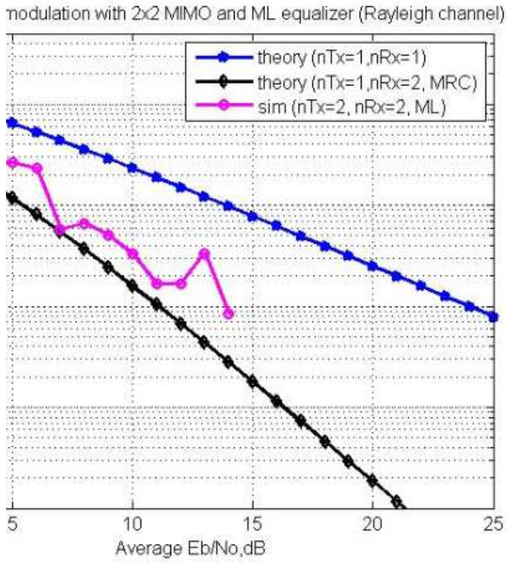


Fig. 26 Maximum Likelihood decoding with 0.5 correlations at both ends for a 2x2 system

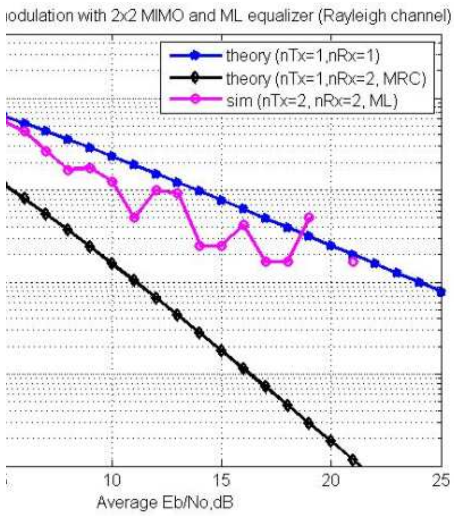


Fig. 27 Maximum Likelihood decoding with 0.8 correlations at both ends for a 2x2 system

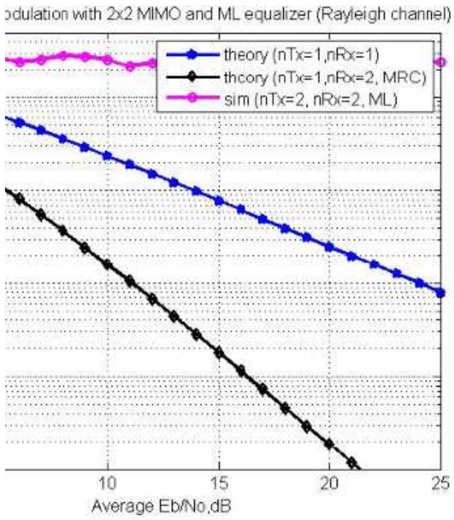


Fig. 28 Maximum Likelihood decoding with unity (1) correlations at both ends for a 2x2 system

4.7 Mean Capacity Simulation from Theoretical Equation for Unknown CSI

To analyse the capacities of our systems based on the BER/SNR values with respect to the equation derived in (4.4), i.e. that of theoretical capacity equation for MIMO systems when CSI is unknown at the transmitters, adjustments are done to the script in use. Appendix (G) shows that both ZF and ML decoding can be used to analyse the capacity. So (4.4) is used to calculate the capacity, using the channel matrix, receiver and transmit symbols that were employed in all previous analysis. The results obtained are:

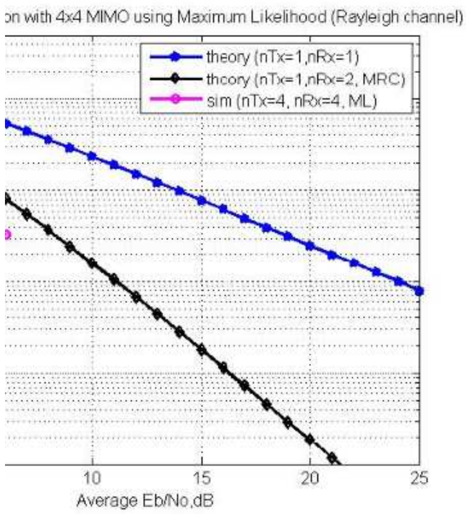


Fig. 29 MIMO capacity from theoretical computation for a 4x4 system using ML

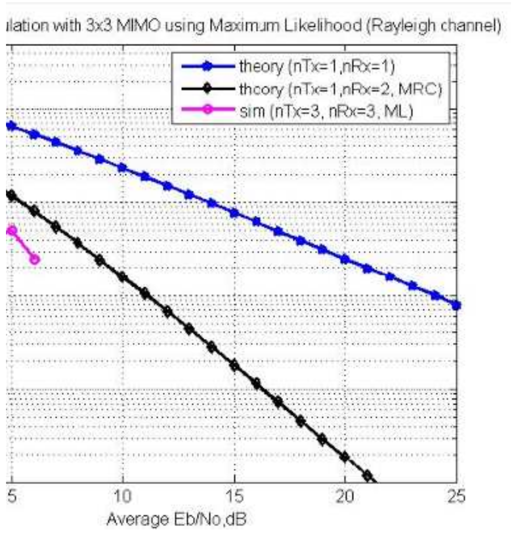


Fig. 30 MIMO capacity from theoretical computation for a 3x3 system using ML

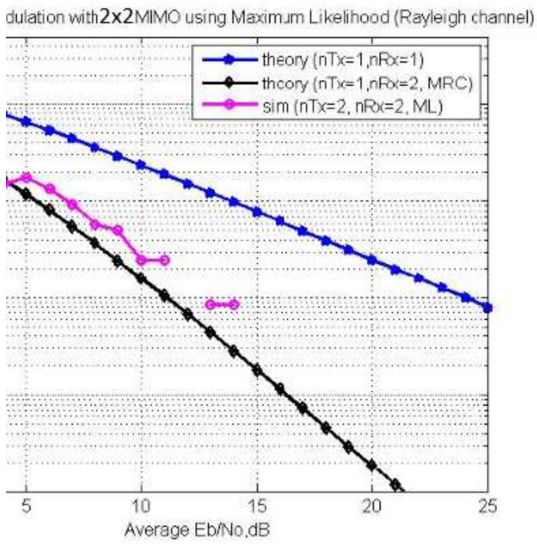


Fig. 31 MIMO capacity from theoretical computation for a 2x2 system using ML

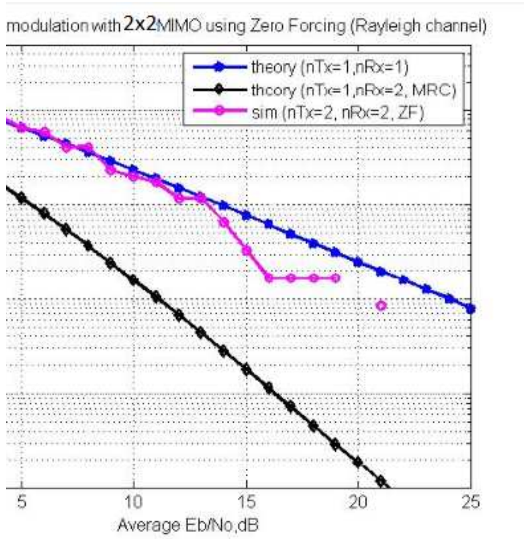


Fig. 32 MIMO capacity from theoretical computation for a 2x2 system using ZF

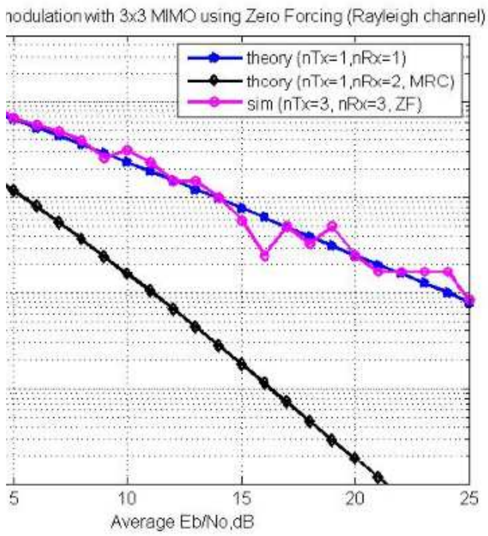


Fig. 33 MIMO capacity from theoretical computation for a 3x3 system using ZF

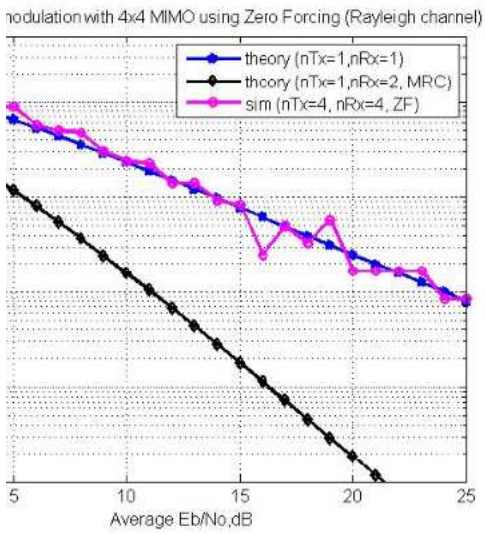


Fig 34 MIMO capacity from theoretical computation for a 4x4 system using ZF

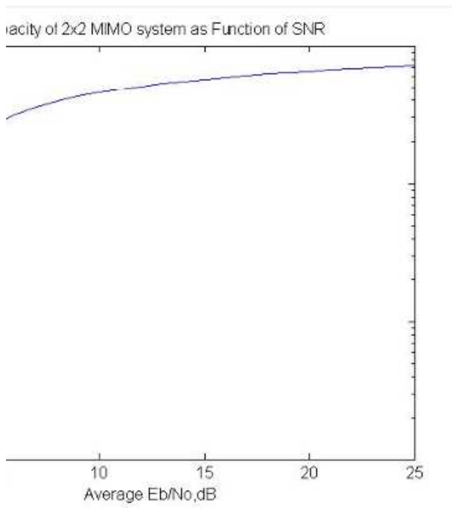


Fig. 35 MIMO capacity as a function of SNR from theoretical computation for a 2x2 system

4.8 Discussion of results

To meet the said aims of the project, of analysing data retrieval using different receiver algorithms, with a view to stating the most effective and also ascertain the effects of correlation no channels created in MIMO communication systems; the following discussion will try to give meaning to the results obtained.

In all the simulations done, the BER in retrieved data is plotted against the SNR. Generally, an *MxN* MIMO system has *M* transmit antennas and can transmit *M* symbols accordingly. When the number of antennas is increased and the BER in the retrieved data remains the same, it means the capacity is proportional to *M*, i.e. increases as *M* increases. However, if *M* increases and BER also increases, it signifies that the increase in symbols has made decoding more difficult; hence, meaning an inverse relation between capacity and *M.* So, an increase, decreases capacity. Consequently, I used the BER/SNR relation to help make deductions concerning data capacity available for retrieval at the receivers in MIMO systems.

Firstly, as expected, in fig 11 for a 2x2 MIMO system using maximum ratio combination and BPSK modulation in Rayleigh channel fading environment, the simulated results show a matching as obtained for a 1x1 system under the same conditions. Signifying that data was retrieved and the capacity increments that should be introduced by multiple antennas were seen, though it behaves like a SISO system probably because there is no CSI at the transmitters. Thus agreeing with Kumar et al (2007) and a host of other writers who uphold that the ZF is not the best possible way to equalise the received symbols as it helps to achieve data rate gains not diversity gains.

Thus, the figures show the relationship between the numbers of errors made for each bit of data retrieved from the channels formed between Rx and Tx against the SNR. Secondly, the plots try to show the number of bits that have errors relative to the total number of bits received and the capacities reached based on the SNR.

In theory, high values of SNR decreases channel estimation errors and improve capacity. This is seen in the result obtained in (11) as a SNR of 5db have higher bit error rates of 0.1 than SNR of 15db, with BER of 0.01; and the capacities as shown are all higher for higher SNRs. These mean that the errors in all data retrieved is lower for higher SNR, thus increasing channel capacity.

So, using Zero-forcing; after carrying out the Moore Penrose pseudo-inverse on the combination of transmitted data with the channel and noise, data was retrieved. And the performance of the capacity was seen to increase with increase in SNR and decreased bit error rates.

And the matching of the simulated and the theoretical plots in fig (11), shows that the systems fare well using the ZF algorithm.

Figs (12-14) show the effects of increased correlation on the capacity performance of the already retrieved data in fig 11(wherein there is 0 correlations).

For starters, as the antennas become more correlated from the case of no correlation in fig (11), to correlation values of 0.5, 0.8 and a totally correlated value of 1, the bit error rates increases, indicating possible capacity increments in the channel formed by the system.

In fig (12), for a 0.5 correlation, the BER gradually increases for increasing SNR, this is different from the result in fig (11) where correlation is 0.

Fig (13) also shows higher BER and SNR; much higher than that in fig (12) since the correlation is 0.8.

Fig (14) shows BER of about 1 which is very high but is realised for unity correlation.

Hence, the performance of the system gradually reduces, as indicated by the increase in the BER for higher SNR and viewed in the shift of the simulated plot up and away from the totally uncorrelated values; suggesting that, capacity is reduced and it is not the channels that matter in capacity deductions per se, but the Eigen values of the channel matrix formed.

Secondly, the kronecker model as discussed in chapter two is not a very accurate means of channel modelling; perhaps an accurate one must look at the CSI to model channels.

Figure (15) is an unlikely case since the correlation is varied differently at both ends, but it gives a result similar (14).i.e. the same as a totally correlated system, as far as BER for retrieved data is concerned but throughput is realised on either ends.

The above cases are for situations where the receiver has no of knowledge the channel state.

It is also observed that the capacity of the 4x4 system is larger than that of the 2x2 system as the multipath created by the combination of 4x4 elements is higher than that of a 2x2 elements. So, fig (16) shows the same matching as fig (11) even though more antennas are used; signifying that the receiver retrieves almost precisely the data sent by the transmitters in the noisy environment, and the 4x4 systems does almost as accurately as the 2x2 system for the same environment but with increased capacity.

Using the Maximum likelihood decoder shows similar trends as observed when using ZF; for a 2x2 system shown in fig (17), the BER decreases from about 0.01 for a 5db SNR to about 0.0001, for a 20db SNR. Though, similar to the behaviour noticed in using ZF, the ML performs better as observed in the BER and the increased capacities available for data transmission.

Increase in the number of antennas as seen in figs (18), and (19), for 3x3 and 4x4 MIMO systems respectively, shows lower BER in data retrieved for increasing SNR values; the 4x4 system giving the lowest BER of less than 0.0001, indicating it to have performed best and can have higher capacity as expected. And the overall BER for data retrieved using the ML decoding vis-à-vis that retrieved using the ZF decoding shows lower errors in data from ML for the same SNR and higher capacities as a result.

For the analysis of the effects of antenna correlation on the performance channel capacity of retrieved data in the ML decoding scenario, it is observed that BER gradually increases from about 0.001 for an uncorrelated, 4x4 system in fig (19), to about 0.01, for 0.1, 05 and 0.9 correlations in figs (20) and (21) and (22) respectively for increased SNR.

For a 3x3 system, using ML decoding, the same pattern of result is observed in Figs (23), (24) and (25); as correlation increases from 0.1 to 0.5 to 1, the BER increases, reducing the capacity of the system. Furthermore, for a 2x2 system the behaviour is also similar as seen in figs (26), (27) and (28); indicating that as more antennas are used, though the capacity increases, if correlation also increases, the errors in the retrieved data will increase, reducing the capacity of the system.

Generally, the reduction in capacity as correlation increases is analogous to the use of pipes in fluid mechanics. Jankiraman (2004) says that when two pipes are used between two reservoirs; the more the number of pipes the greater the quantum of flow of water from one to the other. As seen when a 4x4 MIMO system is analysed fig (16). This is similar to data pipes (channels) in communication. Hence for the 2x2 MIMO system, i.e. having two data pipes, there are two possible cases; either the data in the pipes are identical to each other or they are independent, and so completely different from each other. The same also happens in the 3x3 or 4x4 systems, but the means of exchange has increased since more water can flow through three and four pipes than through two, i.e. having four data pipes.

In the first case, the data goes as if through one data pipe; the pipes being replicas of the same contents. So the same signal is going through the pipes and no new information is getting transferred. This is the case of full correlation and because of this correlation the advantage of throughput is not obtained. Nonetheless a diversity of two, three or four is obtained as observed.

Secondly, when there is absolutely no correlation between the data carried in the pipes, the data streams are independent. Hence no diversity, but output in bits/sec is definitely higher than the first case.

Consequently, correlation is not good for communication and as seen in the MIMO system analysed, it reduces capacity.

Relatively, a point is reached in the systems under scrutiny, where increase in antennas will automatically entail increase in capacity. This is so because the increase come along with an increase in the complexity of the matrixes formed, though diversity will continue to increase with antenna increment as more symbols are available for detection, what is gained in diversity may be lost in throughput.

4.8.1 Capacity Evaluation Channel Unknown at the Transmitter

By and large from (2.3) in the literature review, an expression was given for the MIMO channel capacity, and it was made clear that there are different approaches to the capacity relation proposed by Shannon, hence varied capacity of data available for retrieval based on whether the channel state information is known to the transmitter or not.

(2.3) was given as $C = \max_{Tr(R_{XX}) = M_T} \log_2 \left| I_{MR} + \gamma M_T H R_{XX} H^H \right|$ bps/hertz 4.1

Jankiraman (2004) says the capacity ***C*** above is also called error-free spectral efficiency or data rate per unit bandwidth that can be sustained reliably over a MIMO link.

In the situation analysed in this thesis, where CSI is not known to the transmitter, the vector ***x*** is statistically independent i.e. $R_{xx} = I_{MT}$. Implying that the signals are independent and the power is equally divided among the transmit antennas.

In such a case then, the capacity is given as:

$$C = \log_2 \det \left(I_{MR} + E_s M_T N_o\, H H^H \right) \quad 4.2$$

Where Es/No is the SNR also represented by γ in (4.1).

Teletar (1995) warns that this is not Shannon capacity, since it is possible to out perform $R_{xx} = I_{M_T}$ if one has knowledge of CSI. Now the HH^H is an $M_R \times M_T$ positive semi-definite Hermitian Matrix. The Eigen decomposition of such a matrix is given by $Q\Lambda Q^H$ says Golub et al (1989), where ***Q*** the array matrix satisfying $Q^H Q = QQ^H = I_{M_R}$ and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots\ldots \lambda_{M_R}\}$ with $\lambda_i \geq 0$.

The MIMO channel capacity now becomes:

$$C = \log_2 \det\left(I_{M_R} + \frac{E_s}{M_T N_o} Q\Lambda Q^H\right) \qquad 4.3$$

Matrix simplification results in $C = \log_2 \det\left(I_{M_R} + \frac{E_s}{M_T N_o} \Lambda\right)$ 4.4

Or $C = \sum_{i=1}^{r} \log_2\left(1 + \frac{E_s}{M_T N_o} \lambda_i\right)$ 4.5

Where ***r*** is the rank of the channel and λ_i (i = 1, 2, 3...., r) are the positive Eigen values of HH^H. The rank is defined as the number of independent equations offered; (algebraic rank), and is always less than both the number of transmit and receive antennas, says Oesteges et al 2004.

So (4.5) expresses the capacity of a MIMO channel as the sum of the individual capacities of ***r*** SISO channels, each having a power gain of λ_i and transmit power of $\frac{E_s}{M_T}$.

Therefore, for the case presently under scrutiny, i.e. where CSI is unknown to the transmitter, we can relate the results obtained from our simulation to (4.5):

- The technique of multiple antennas as used in MIMO opens up multiple spatial channels $\mathbf{H}H^H$ between transmitters and receivers.

- The capacity of the channels created in such systems and hence the data available for subsequent retrieval depend on the order of diversity or rank ***r*** of the matrix of the channels formed, which is determined by the number of antennas used and positive Eigen values ***i*** of the channel.

- Since no knowledge of the channel state is available at the transmitters, equal transmit energy $\frac{E_s}{M_T}$ is allocated to each spatial channel.

The plots of MIMO and SISO as seen in fig (11) are the same, this could happen in special cases where a low correlation does not necessarily translate into higher capacity because of some special propagation scenarios, so the MIMO capacity can be low (sometime equal to a SISO capacity level). The effect was denoted as “key hole”, leading to a drop in capacity, Valenzuela et al, 2002. It is related to scenarios where rich scattering around the transmitters and receivers lead to correlation of the signals, while other propagation effects, like diffraction or wave guiding leads to rank reduction of the transfer function matrix; giving rise to significant local scattering around the Tx and Rx units, causing uncorrelated fading properties- hence low capacity.

4.8.2 Capacity Evaluation for Channel State Information known at Transmitter

When CSI is known at the transmitters, the capacity of MIMO can be increased by resorting to the so called “water filling principle” (Teletar, 1995). The water-filling principle can be derived by maximising the MIMO channel capacity under the rule that more power is allocated to the channel that is in good condition and less or none to the channel that is bad, i.e. assigning various levels of transmitted power pouring to various transmitting antennas, on the basis that the better a channel gets, the more the power assigned for data transmission through it, and vice-versa.

Hence, an analysis is taken from Paulraj (2003); consider a zero-mean circular symmetric complex Gaussian (ZMCSCG) signal vector, x of dimension $r \times 1$, where $\boldsymbol{r}$ is the rank of the channel $\boldsymbol{H}$ to be transmitted. The vector is multiplied by a matrix $\boldsymbol{V}$ prior to transmission (based on the fact that $H = U\Sigma V^H$ through singular value decomposition). At the receiver, the received signal vector $\boldsymbol{y}$ is multiplied by the matrix U^H , the input-output relationship of this operation is given by:

$$y = \sqrt{\frac{E_s}{M_T}} U^H H V x + U^H n \qquad 4.6$$

$$= \sqrt{\frac{E_s}{M_T}} \Sigma x + n$$

Where y is the transformed received signal vector and $\mathbf{n}$ is the ZMCSCG transformed noise vector of size $r \times 1$ with the covariance matrix $\epsilon\{nn^H\} = N_0 I_r$. The vector x satisfies $\epsilon\{xx^H\} = M_T$ to constrain the total transmit energy. Jankiraman (2004) infers that (4.6) shows that with channel knowledge at the transmitter H can be completely decomposed to r parallel SISO channels satisfying;

$$y_i = \sqrt{\frac{E_s}{M_T}}\sqrt{\lambda_i}\,x_i + n_i, \quad i = 1, 2, \ldots, r \qquad 4.7$$

So, the capacity of the MIMO channel is the sum of the individual parallel SISO channel capacities, given by;

$$C = \sum_{i}^{r}\log_2\left(1+ \frac{E_s\gamma_i}{M_T N_0}\lambda_i\right) \qquad 4.8$$

The above equation, (4.8), is the capacity relation for a channel in which CSI information is present at the transmitters.

Where $\gamma_i = \in\{|x_i|^2\}$ $(i = 1, 2, \ldots, r)$ is the transmit energy in the ith sub-channel such that $\sum_{i=1}^{r}\gamma_i = M_T$

Therefore to maximise mutual information, the transmitter can access the individual sub-channels and allocate variable power levels to them. So the mutual information maximisation problem becomes,

$$C = \max_{\sum_{i=1}^{r}\gamma_i} \sum_{i=1}^{r}\log_2\left(1+ \frac{E_s\gamma_i}{M_T N_0}\lambda_i\right) \qquad 4.9$$

Jankiraman (2004), concludes that using Lagrangian methods, the optimal energy allocation procedure can be written as:

$$\gamma_i^{opt} = \left(\mu - \frac{M_T N_0}{E_s\lambda_i}\right), \quad i = 1, 2, \ldots, r \text{ and} \qquad 4.10$$

$\sum_{i=1}^{r}\gamma_i^{opt} = M_T$ where μ is a constant.

The optimal energy allocation can then be determined iteratively through the water-pouring/filling algorithm. We set the iteration count p to 1 and calculate the constant μ in (4.9):

$$\mu = \frac{M_T}{(r-p+1)}\left[1+\frac{N_0}{E_s}\sum_{i=1}^{r-p+1}\frac{1}{\lambda_i}\right] \qquad 4.11$$

Using the value of μ the power allocated to the ith channel is calculated;

$$\gamma_i = \left(\mu - \frac{M_T N_0}{E_s\lambda_i}\right), \quad i = 1, 2, \ldots, r-p+1 \qquad 4.12$$

The optimal power allocating strategy, therefore allocates to those spatial sub-channels that are non negative. Consequently, from Jnkiraman's analysis, since this algorithm only concentrates on good-quality channels and rejects the bad ones during each channel realisation, it is expected that this method yields a capacity that is equal or better than the situation when the channel is unknown to the transmitter.

It can therefore be ultimately concluded that increased correlation reduces the CSI, as the conditioning of the channel matrix is degraded, since with more knowledge of the Channel; the capacity that can be realised at the receivers end could be greatly increased, increased correlation equally reduces capacity.

4.8.3 Relationship between the Theoretical and Simulated Channel Capacities

Initially, it was finalised that channel capacities are realised for two separate cases; when CSI is known and when it is unknown to the transmitter.

Since all the analysis in the project was made for cases where CSI is unknown, (4.4) gives the equation of MIMO channel capacity deduction for this thesis:

$$\mathbf{C} = \log 2 \det (IMR + EsMTNo \Lambda) \qquad 4.13$$

Hence, an attempt is made to calculate the channel capacities of the systems already analysed using ML and ZF decoding, based on (4.13) and compare with that obtained by the use of our previous scripts.

Fig (29) gives the result for a 4x4 system using ML decoding from the direct substitution of the used parameters into (4.13).

As seen from the result using the script in fig (19) the BER for a 4x4 system using ML decoding is about 10-3 for SNR of 5db. This is similar to the result obtained in (29); the BER for SNR of 5db is slightly above 10-3, but can be approximated to 10-3. Since their BER for the same SNR are almost the same, their performance show a similarity and the data capacities retrieved are almost equal, as seen in the simulated plots of both cases.

Fig (30) shows that the result for the calculated 3x3 system using (4.13) for ML decoder. If compared with fig (29), which is for a 4x4 system, for a 5db SNR, the BER increases in fig (30) and capacity reduces. However, if compared with fig (18) for the same number of antennas but using the script, the results are seen to be similar as the BER for SNR value of 5db is almost the same.

Fig (31), which is the result for the calculated 2x2 system, shows a much higher BER for the same SNR using the ML decoding, proving the fact that capacity is increased when antennas used are increased, the BER increasing with a reduction in the number of antennas.

When fig (31), i.e. result obtained by using the equation of capacity for unknown CSI (4.13), is further investigated and compared to fig (17) which has the same number of antennas but obtained using the script, the results obtained are also observed to be the same.

Hence, indicating that the results obtained from the script used agree with the result obtained by using the equations derived for the MIMO system when there is no CSI for the ML decoding algorithm.

Figures (32), (33) and (34) give the results obtained for ZF decoding in 2x2, 3x3 and 4x4 MIMO systems using the equation derived for the data capacity with no CSI at the receivers. As explained in the ML cases, the same observation are seen using the ZF decoding algorithm when compared to the corresponding results from the scripts.

Consequently, it can also be finalised that the equation that was derived gave the capacities that were realised from the use of the scripts, making it credible.

Figure (35) shows the capacity of a MIMO system as a function of SNR for a 2x2 system, the same result is obtained for the 3x3 and 4x4 systems using both ZF and ML decoding.

4.9 Comparisons between Zero-forcing and Maximum-Likelihood Decoders

From results obtained, it is apparent that the choice of a decoder has immense impact on the performance of wireless systems. This is more in MIMO systems as the effects of the multi-stream interference (MSI) resulting from the de-multiplexing of symbols over multiple antennas at the transmitter, creates different streams which interfere at the receivers.

The ML and ZF are both linear decoding schemes used for data retrieval; the methodology of using a linear filter is an aspect that makes these decoders distinct.

The ZF, is easy to implement as it just looks to eliminate interference between each independent stream originating from one of the multiple transmit antennas. It does this by inverting the known channel matrix, setting it equal to the linear filter and ignoring noise enhancement problems. Thus making it suboptimal as observed from the results obtained for data decoded using ZF from a 2x2 MIMO system in Fig (11) against that retrieved from the same system using ML in fig (17), also observed for the 4x4 systems in figs (16) and (19). Higher number of antenna systems result in lower BER and the result obtained can be said to be reliable since better error rates are observed for higher SNR.

The ML provides optimal performance for data retrieval. It is also the most appropriate for practical systems because it gives better BER which gives reliability in transmitted data. Though it is difficult to implement, its approach maximised the desired goals of MIMO. In this case, the identity matrix is equal to the linear filter; this inhibits an exhaustive search through the available symbols based on the modulation technique to find each estimated symbol. It gets complex and can be made less complex by using algorithms such as sphere decoding.

Other receivers are the minimum mean-squared estimate decoders which minimises MSI and noise to lower errors.

Then the non-linear receiver; the successive cancellation decoding, in which Nulling and cancellation falls (NC). The idea behind all successive cancellation receivers is, they treat each symbol as a layer and is stripped away to be decoded.

4.10 Summary

In a nutshell, this chapter covered the most important aspect of the thesis; the simulation of data retrieval, the analysis of available capacities as BER and SNR change at the receivers, when antenna correlation is factored.

Only two of the previously mentioned algorithms for data retrieval were used; the Zero-forcing and the Maximum likelihood.

The script used was originally sourced online, subsequent modification to meet desired results and understood concepts, were made.

Data retrieval for a 2x2 system using ZF decoding was first simulated; the capacity obtained as a function of the bit error rates and the signal to noise ratio were viewed. Furthermore, the effects of antenna correlation with varying values were also looked at. Then the 4x4 system was also analysed to show that as theoretically thought, capacity available for data communication increases with the number of antennas as the MIMO technology suggests, but also, only to a limit, since complexity grows.

Then the ML decoding was used for a 2x2, 3x3, and 4 x 4 MIMO systems, to retrieve unknown transmitted data. As previously done with the ZF, the data were retrieved as a function of the SNR and BER of the transmitted symbols. Again, the effects of antenna correlation on the performance capacity at the receiver for all the system were analysed.

All the results obtained were chronicled and detailed discussion was written based on observed behaviour in the simulations.

Considering the critical role of capacity in communication, the issue of channel state information and the capacities realised with or without it were further discussed. And mathematical relations based from the Shannon capacity evaluation were arrived at for both cases.

Finally, the capacity for the systems based on the equations were simulated and the relationship between the simulated results from the use of the modified script with those obtained by directly substituting the variables in the derived equation for a MIMO system with no CSI were analysed and seen to be almost accurate.

CHAPTER FIVE

CONCLUSION AND FURTHER STUDY

5.1 Introduction

From the stated objectives in chapter one, the following conclusions can be reached:

Firstly, that an understanding of the basic wireless concepts that govern MIMO technology as seen from the introduction and literature review, were achieved. Hence, the physical environment that abates the scattering of channels to give a rich multipath suited for the all important MIMO were also looked into and understood.

Secondly, the different channel model classification were studied, the resulting models thoroughly analysed, i.e. the Kronecker model (which was used in this thesis), the Jake and the one ring models. Literature about them was given in chapter two and their mathematical analysis, given in chapter three; so channel modelling, as it relates to this thesis was also achieved.

Thirdly, data decoding schemes and the algorithm that aid them were successfully analysed; the ZF, ML and NC, their literature and mathematical expressions also obtained. The real process of data decoding in different multi-antenna designs were simulated in chapter four, the results obtained were discussed, and an understanding was reached, that the ML decoder, though more complex to implement, performs better than the ZF decoder.

Then, correlation between the multi-antennas was seen to affect the Eigen values of the matrices formed in the decoding schemes. Thus, the BER was seen to be affected by the correlation and as a result, the capacity was reasoned to be affected.

Lastly, the Matlab simulation using the script from www.dsplog.com was understood and modifications were made to suit the thesis requirements.

In a nutshell, these emphasise in no small measure that not only were the set objectives met but the results obtained in this thesis concur with the agreed standards in the wireless world.

Consequently, I can now make meaningful and sound contribution, based on a knowledgeable and informed perspective, about MIMO generally and specifically its data retrieval schemes, the correlation effects on BER as SNR increases on channel capacity and the capacities available when CSI is known or unknown.

So, all that were done in this thesis are summarised below, from all that was understood, as a way of concluding the thesis.

5.2 Conclusion

This project was originally aimed at retrieving transmitted data at the receiving end using discussed algorithms like Maximum likelihood and Zero-forcing and checking the inescapable effects of correlation of the antennas used in the data capacities realised, as seen from the error rates and SNR.

To achieve this, the basic understanding of what MIMO is and how it works was attempted; MIMO antenna design was analysed to be more beneficial in capacity realisation as against the previously used technologies of SIMO or MISO. This was understood against the back drop that more channels are created by the use of more antennas and the rich scattering that results when utilised, provides a superb foundation for increased capacities. So, spatial multiplexing and other related ideas that aid this technique were analysed.

But unlike Gaussian channels, wireless channels suffer from attenuation caused by copies of the transmitted signal reaching the receivers at different times. It will be difficult to retrieve the transmitted signal without the use of diversity techniques and array gain. Array gain for example provide the validity in that premise; as it is the average increase in the SNR at the receiver that arises from the coherent combining effects of multiple antennas at the receiver or transmitter or both.

Another method is to use the multiple antenna elements in the spatial multiplexing or BLAST approach. So the vertical and horizontal Bell layered space time techniques were reviewed.

Prior to MIMO development, wireless communication systems analysts viewed multipath propagation as a problem to be mitigated towards achieving reliable data/voice transfers. MIMO is the first technology that treats multipath propagation as an opportunity to increase link capacity.

Analysing the performance of MIMO system, need proper modelling of the system to realise reasonable results. MIMO channels are categorised based on the nature of the impulse response of the channels; flat, frequency selective and slow fading channels. While the first results from multipath fading and was used in all the analysis of this project, the other two are a direct result of delay spread.

A channel matrix characterises a MIMO system with a number of transmit and receive antennas. The entries of the matrix are the channel impulse response for all transmit-receive antenna pair.

Signals from transmitters are spatially multiplexed and expected to mix with and always be corrupted by noise. Upon receiving such signals, receivers separate the mixed signals always filter out the noise. This separation is achieved if the equations formed by the combination of the matrixes are linear and independent, so each antenna uses a statistically independent channel.

In addition, wireless channels are also affected by distortions of transmitted data symbols. The symbols being typical forms of amplitude and phase modulation pulses; so neighbouring data symbols interfere with each other, causing inter symbol interference, which in turn make data detection for retrieval a herculean task.

But the algorithms used were designed with all these problems at heart, though different, they all aim to retrieve data based on the channel matrix and transmitted symbols presented.

While Zero-forcing basically tries to remove all ISI by applying the inverse of the channel to the received signal; thereby restoring the signal before the channel and then implementing a matrix pseudo-inverse on the signal. It does this while ignoring the effects of noise, i.e. bringing down the ISI to zero.

The Maximum Likelihood on the other hand, tries to retrieve data by investigating all the sample of the received signal combination, for the transmitted signal vector that maximises the probability of being transmitted.

While the former is easier to simulate and run, it is also observed from the results to be less effective or sub optimal. The later is more effective as lower BER is obtained in retrieved data for the same SNR, but more complex to implement.

All simulations were done using the Matlab program; the results were obtained when the increasing SNRs are matched against BER for all data retrieved.

Other decoding schemes like Nulling and cancellation and Minimum mean successive cancellation (MMSE) were not analysed because of time constrains.

The capacity available for retrieved data is central to this thesis hence, the effects of correlation among the multiple antennas used at either ends to the capacity of the system was analysed in detail. Different multi-antenna design were looked at and different correlation values also used. It is often assumed that the elements of the channel matrix are independent, this is practise is never the case. So correlation affects the capacity realised; if there is correlation the capacity of the MIMO systems are decreased. And when the correlation is increased as was observed from the results obtained, BER continues to increase and the capacity continues to decrease appreciably. Paulson (2009), says, channels when correlated, result in similar paths with reflections off the same objects and effect adjacent channels, i.e. from adjacent antennas. Hence, more correlation means fewer effective channels and so lower capacity.

From the theories and simulation, it can then be said that though capacity increases linearly with increase in the number of antennas used, the capacity can be reduced when correlation is increased. In practise, antennas are usually separated by about a wavelength to reduce correlation.

The capacity of a system was first analysed by Shannon in 1948, however, data capacity available for retrieval at the receiver can now be viewed in two separate perspectives; when there is channel state information and when it is absent.

This thesis was analysed from the former perspective and the results obtained agree with those obtained by using the sourced, remodelled and implemented script.

But more capacity is expected when there is CSI, since the transmitter makes an informed decision while allocating data to channels to be transmitted; reducing errors and increasing capacity.

5.3 Further Study

So further work on this thesis could be done in looking at firstly, a better channel model than the kronecker model incorporated into the script. The Jake model or a model known as the Monte Carlo model can each be attempted.

Secondly, a case where there is CSI at the receivers could be analysed and the implementation of other decoding algorithms, i.e. the Nulling and cancellation, e.t.c.

Thirdly, negative values of correlation could be attempted to view its impact on the capacity. And since the ZF algorithm is easier to implement but less optimal and the ML, optimal but difficult to implement, a situation where both are used at the receiver could also be analysed, and depending on what quality is desired at a specific time, the system could be designed to choose which decoding algorithm to use.

CHAPTER SIX

APPENDIXES

6.1 APPENDIX A

```
% Script for computing the BER for BPSK modulation in a
% Rayleigh fading channel with 2 Tx, 2Rx MIMO channel
% Zero Forcing equalization
 clear
N = 10^6; % number of bits or symbols
Eb_N0_dB = [ 0:25] ; % multiple Eb/N0 values
nTx = 2;
nRx = 2;
% build CSI and noise arrays
h_iid = 1/sqrt(2)*[ randn(nRx,nTx,N/nTx) + 1i*randn(nRx,nTx,N/nTx)] ;
Rayleigh channel
h = h_iid;                                   % declare CSI array
tx_antenna_Correlation = 0; % correlation at transmitter
rx_antenna_Correlation = 0; % correlation at receiver
R_tx = [  1 tx_antenna_Correlation ; tx_antenna_Correlation 1] ;
sqrt_R_tx = sqrtm(R_tx)';
R_rx = [  1 rx_antenna_Correlation ; rx_antenna_Correlation 1] ;
sqrt_R_rx = sqrtm(R_rx);
for i = 1:N/nTx
h(:,:,i)   =   sqrt_R_rx   *   squeeze(h_iid(:,:,i))   *   sqrt_R_tx;
%Kronecker model
end
n  =  1/sqrt(2)*[ randn(nRx,N/nTx)  +  1i*randn(nRx,N/nTx)] ;  %  white
gaussian noise, 0dB variance
 for ii = 1:length(Eb_N0_dB)
% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 0
sMod = kron(s,ones(nRx,1)); %
sMod = reshape(sMod,[ nRx,nTx,N/nTx] ); % grouping in [ nRx,nTx,N/NTx ]
matrix
% Channel and noise Noise addition
y = squeeze(sum(h.*sMod,2)) + 10^(-Eb_N0_dB(ii)/20)*n;
% Receiver
% Forming the Zero Forcing equalization matrix W = inv(H^H*H)*H^H
% H^H*H is of dimension [ nTx x nTx] . In this case [ 2 x 2]
% Inverse of a [ 2x2] matrix [ a b; c d] = 1/(ad-bc)[ d -b;-c a]
hCof = zeros(2,2,N/nTx)  ;
hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
hDen = ((hCof(1,1,:).*hCof(2,2,:)) -(hCof(1,2,:).*hCof(2,1,:)));%ad-
bc term
hDen = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);
% formatting for division
hInv = hCof./hDen; % inv(H^H*H)
hMod =  reshape(conj(h),nRx,N); % H^H operation
yMod  =  kron(y,ones(1,2));  %  formatting  the  received  symbol  for
equalization
yMod = sum(hMod.*yMod,1); % H^H * y
```

```
yMod =  kron(reshape(yMod,2,N/nTx),ones(1,2)); % formatting
yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^H*H)*H^H*y
% receiver - hard decision decoding
ipHat = real(yHat)>0;
% counting the errors
nErr(ii) = size(find([ ip- ipHat] ),2);
 end
simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));
p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));

close all
figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([ 0 25 10^-5 0.5] )
grid on
legend('theory  (nTx=1,nRx=1)',  'theory  (nTx=1,nRx=2,  MRC)',  'sim
(nTx=2, nRx=2, ZF)');
xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');
title('BER  for  BPSK  modulation  with  2x2  MIMO  and  ML  equalizer
(Rayleigh channel)');
```

6.2 APPENDIX B

```
% build CSI and noise arrays
```

```
h_iid = 1/sqrt(2)*[ randn(nRx,nTx,N/nTx) + 1i*randn(nRx,nTx,N/nTx)];
Rayleigh channel
h = h_iid;                                  % declare CSI array
tx_antenna_Correlation = 0; % correlation at transmitter
rx_antenna_Correlation = 0; % correlation at receiver
R_tx = [ 1 tx_antenna_Correlation ; tx_antenna_Correlation 1];
sqrt_R_tx = sqrtm(R_tx)';
R_rx = [ 1 rx_antenna_Correlation ; rx_antenna_Correlation 1];
sqrt_R_rx = sqrtm(R_rx);
for i = 1:N/nTx
h(:,:,i) = sqrt_R_rx * squeeze(h_iid(:,:,i)) * sqrt_R_tx;
%Kronecker model
end
n = 1/sqrt(2)*[ randn(nRx,N/nTx) + 1i*randn(nRx,N/nTx)]; % white
gaussian noise, 0dB variance
for ii = 1:length(Eb_N0_dB)
```

6.3 APPENDIX C

```
% Script for computing the BER for BPSK modulation in a

% Rayleigh fading channel with 4Tx, 4Rx MIMO channel

% Zero Forcing equalization
clear
N = 10^4;                 % number of bits or symbols
Eb_N0_dB = [ 0:25] ;      % multiple Eb/N0 values
nTx = 4;
nRx = 4;
nSymbolPeriodsToSimulate = N/nTx;
% build CSI and noise arrays
h_iid = 1/sqrt(2)*[ randn(nRx,nTx,N/nTx) + 1i*randn(nRx,nTx,N/nTx)] ;
% Rayleigh channel
h = h_iid;                                    % declare CSI array
tx_antenna_Correlation  =  0;                           %  correlation  at
transmitter
rx_antenna_Correlation  =  0;                           %  correlation  at
reciever
%R_tx   =   [   1   tx_antenna_Correlation   tx_antenna_Correlation;
tx_antenna_Correlation                                                1
tx_antenna_Correlation:tx_antenna_Correlation tx_antenna_Correlation
1] ;
R_tx = eye(nTx);
sqrt_R_tx = sqrtm(R_tx)';

%R_rx   =   [   1   rx_antenna_Correlation   rx_antenna_Correlation;
rx_antenna_Correlation                                                1
rx_antenna_Correlation;rx_antenna_Correlation rx_antenna_Correlation
1] ;
R_rx = eye(nRx);
sqrt_R_rx = sqrtm(R_rx);
for i = 1:nSymbolPeriodsToSimulate
h(:,:,i)   =   sqrt_R_rx   *   squeeze(h_iid(:,:,i))   *   sqrt_R_tx;   %
Kronecker model
end
n       =       1/sqrt(2)*[ randn(nRx,nSymbolPeriodsToSimulate)       +
1i*randn(nRx,nSymbolPeriodsToSimulate)] ; % white gaussian noise, 0dB
variance
for ii = 1:length(Eb_N0_dB)
% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 1
sMod   =   reshape(s,nTx,nSymbolPeriodsToSimulate);   %   grouping   in
[ nTx,N/NTx ] matrix
%Channel and noise Noise addition
y = 10^(-Eb_N0_dB(ii)/20)*n;
for i = 1:nSymbolPeriodsToSimulate
y(:,i) = y(:,i) + squeeze(h(:,:,i))*squeeze(sMod(:,i));
end
% Receiver
if false
% Forming the Zero Forcing equalization matrix W = inv(H^H*H)*H^H
% H^H*H is of dimension [ nTx x nTx] . In this case [ 3 x 3]
% Inverse of a [ 3x3] matrix [ a b; c d] = 1/(ad-bc)[ d -b;-c a]
hCof = zeros(2,2,N/nTx)  ;
```

```
hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
hDen  =  ((hCof(1,1,:).*hCof(2,2,:))-(hCof(1,2,:).*hCof(2,1,:)));%ad-
bc term
hDen  = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);   %
formatting for division
hInv = hCof./hDen; % inv(H^H*H)
hMod =  reshape(conj(h),nRx,N); % H^H operation
yMod  =  kron(y,ones(1,2));  %  formatting  the  received  symbol  for
equalization
yMod = sum(hMod.*yMod,1); % H^H * y
yMod =  kron(reshape(yMod,2,N/nTx),ones(1,2)); % formatting
yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^H*H)*H^H*y
%receiver - hard decision decoding
ipHat = real(yHat)>0;
 else
% use Kevin's subroutine
for i = 1:nSymbolPeriodsToSimulate
ipHat(:,i)  = Zero_Forcing(y(:,i), h(:,:,i) , [ -1,1] );             %
use zero forcing
end
end
%counting the errors
nErr(ii) = length(find( ipHat ~= sMod ));
end
simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));
p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));
close all
figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([ 0 25 10^-5 0.5])
grid on
legend('theory  (nTx=1,nRx=1)',  'theory  (nTx=1,nRx=2,  MRC)',  'sim
(nTx=4,  nRx=4,  ZF)');
xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');
title('BER  for  BPSK  modulation  with  2x2  MIMO  and  ML  equalizer
(Rayleigh channel)');
```

6.4 APPENDIX D

```
% Script for computing the BER for BPSK modulation in a
% Rayleigh fading channel with 2Tx, 2Rx MIMO channel
% Maximum likelihood

clear
N = 12000;                % number of bits or symbols
Eb_N0_dB = [0:25];       % multiple Eb/N0 values
nTx = 2;
```

```
nRx = 2;

nSymbolPeriodsToSimulate = N/nTx;
% build CSI and noise arrays
h_iid = 1/sqrt(2)*[ randn(nRx,nTx,N/nTx) + 1i*randn(nRx,nTx,N/nTx)] ;
% Rayleigh channel
h = h_iid;                                   % declare CSI array
tx_antenna_Correlation = 0;                            % correlation at
transmitter
rx_antenna_Correlation = 0;                            % correlation at
reciever
%R_tx = [ 1 tx_antenna_Correlation tx_antenna_Correlation;
tx_antenna_Correlation 1
tx_antenna_Correlation:tx_antenna_Correlation tx_antenna_Correlation
1] ;
R_tx = eye(nTx);
sqrt_R_tx = sqrtm(R_tx)';
%R_rx = [ 1 rx_antenna_Correlation rx_antenna_Correlation;
rx_antenna_Correlation 1
rx_antenna_Correlation;rx_antenna_Correlation rx_antenna_Correlation
1] ;
R_rx = eye(nRx);
sqrt_R_rx = sqrtm(R_rx);
for i = 1:nSymbolPeriodsToSimulate
       h(:,:,i) = sqrt_R_rx * squeeze(h_iid(:,:,i)) * sqrt_R_tx;
% Kronecker model
end
n = 1/sqrt(2)*[ randn(nRx,nSymbolPeriodsToSimulate) +
1i*randn(nRx,nSymbolPeriodsToSimulate)] ; % white gaussian noise, 0dB
variance
for ii = 1:length(Eb_N0_dB)

% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 1
sMod = reshape(s,nTx,nSymbolPeriodsToSimulate); % grouping in
[ nTx,N/NTx ] matrix
% Channel and noise Noise addition
y = 10^(-Eb_N0_dB(ii)/20)*n;
for i = 1:nSymbolPeriodsToSimulate
y(:,i) = y(:,i) + squeeze(h(:,:,i))*squeeze(sMod(:,i));
end

% Receiver

 if false

             % Forming the Zero Forcing equalization matrix W =
inv(H^H*H)*H^H
        % H^H*H is of dimension [ nTx x nTx] . In this case [ 3 x 3]
         % Inverse of a [ 3x3] matrix [ a b; c d] = 1/(ad-bc)[ d -b;-c
a]
        hCof = zeros(2,2,N/nTx)  ;
        hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
        hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
        hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
```

```
        hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
                          hDen    =    ((hCof(1,1,:).*hCof(2,2,:))    -
(hCof(1,2,:).*hCof(2,1,:))); % ad-bc term
                                                          hDen        =
reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);             %
formatting for division
        hInv = hCof./hDen; % inv(H^H*H)

        hMod =  reshape(conj(h),nRx,N); % H^H operation

         yMod = kron(y,ones(1,2)); % formatting the received symbol
for equalization
        yMod = sum(hMod.*yMod,1); % H^H * y
        yMod =  kron(reshape(yMod,2,N/nTx),ones(1,2)); % formatting
        yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^H*H)*H^H*y

        % receiver - hard decision decoding
        ipHat = real(yHat)>0;

    else

        % use Kevin's subroutine
        for i = 1:nSymbolPeriodsToSimulate
%             ipHat(:,i) = Zero_Forcing(y(:,i), h(:,:,i) , [ -1,1] );
% use zero forcing
            ipHat(:,i) = ML_Decode_2_2(y(:,i), h(:,:,i) , [ -1,1] );
% use ML decoding
  end
  end

    % counting the errors
    nErr(ii) = length(find( ipHat ~= sMod ));

end

simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));
p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));

close all
figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([ 0 25 10^-5 0.5] )
grid on
legend('theory  (nTx=1,nRx=1)',  'theory  (nTx=1,nRx=2,  MRC)',  'sim
(nTx=2, nRx=2, ML)');
xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');
title('BER  for  BPSK  modulation  with  2x2  MIMO  and  ML  equalizer
(Rayleigh channel)');
```

6.5 Appendix E

```
% Maximum Liklihood decoding for N=2
function [ Received_Symbols] = ML_Decode_2_2(Y,H,Symbols)
N=2;
nSymbols = length(Symbols);
Symbol_Vectors = [ kron( ones(1,nSymbols) , Symbols ) ;
kron( Symbols , ones(1,nSymbols) ) ];
YMHS = kron(Y,ones(1,nSymbols^N)) - H*Symbol_Vectors;
Distance = sum( YMHS.*conj(YMHS) , 1 );
[ C , Index] = min(Distance);
Received_Symbols = Symbol_Vectors(:,Index);
```

6.6 Appendix F

```
% Script for computing the BER for BPSK modulation in a
% Rayleigh fading channel with 4Tx, 4Rx MIMO channel
% Maximum likelihood equalization
clear
N = 1200;                % number of bits or symbols
Eb_N0_dB = [ 0:25] ;     % multiple Eb/N0 values
nTx = 4;
nRx = 4;
nSymbolPeriodsToSimulate = N/nTx;

% build CSI and noise arrays
h_iid    =1/sqrt(2)*[ randn(nRx,nTx,N/nTx)+1i*randn(nRx,nTx,N/nTx)] ;%
Rayleigh channel
```

```
h = h_iid;                                % declare CSI array
tx_antenna_Correlation =0;                         % correlation at
transmitter
rx_antenna_Correlation =0;                % correlation at receiver
%T_tx = [ 1 tx_antenna_Correlation tx_antenna_Correlation 1];
R_tx = [ 1 tx_antenna_Correlation 0 0; tx_antenna_Correlation 1
tx_antenna_Correlation 0;0 tx_antenna_Correlation 1
tx_antenna_Correlation;0 0 tx_antenna_Correlation 1];
sqrt_R_tx = sqrtm(R_tx)';
%R_rx = [ 1 rx_antenna_Correlation rx_antenna_Correlation 1];
R_rx =[1 rx_antenna_Correlation 0 0; rx_antenna_Correlation 1
rx_antenna_Correlation 0;0 rx_antenna_Correlation 1
rx_antenna_Correlation;0 0 rx_antenna_Correlation 1];
sqrt_R_rx = sqrtm(R_rx);
for i = 1:nSymbolPeriodsToSimulate
h(:,:,i) = sqrt_R_rx * squeeze(h_iid(:,:,i)) * sqrt_R_tx; %
Kronecker model
end
n = 1/sqrt(2)*[ randn(nRx,nSymbolPeriodsToSimulate)+
1i*randn(nRx,nSymbolPeriodsToSimulate)]; % white gaussian noise,0dB
variance
for ii = 1:length(Eb_N0_dB)

% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 1
sMod = reshape(s,nTx,nSymbolPeriodsToSimulate); % grouping in
[ nTx,N/NTx ] matrix

%Channel and noise Noise addition
y = 10^(-Eb_N0_dB(ii)/20)*n;
for i = 1:nSymbolPeriodsToSimulate
y(:,i) = y(:,i) + squeeze(h(:,:,i))*squeeze(sMod(:,i));
end

% Receiver

if false
% Forming the Zero Forcing equalization matrix W = inv(H^H*H)*H^H
% H^H*H is of dimension [ nTx x nTx] . In this case [ 3 x 3]
% Inverse of a [ 3x3] matrix [ a b; c d] = 1/(ad-bc)[ d -b;-c a]
hCof = zeros(2,2,N/nTx)  ;
hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
hDen = ((hCof(1,1,:).*hCof(2,2,:)) - (hCof(1,2,:).*hCof(2,1,:))); %
ad-bc term
hDen = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);  %
formatting for division
hInv = hCof./hDen; % inv(H^H*H)
hMod =  reshape(conj(h),nRx,N); % H^H operation
yMod = kron(y,ones(1,2)); % formatting the received symbol for
equalization
yMod = sum(hMod.*yMod,1); % H^H * y
yMod =  kron(reshape(yMod,2,N/nTx),ones(1,2)); % formatting
```

```
yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^H*H)*H^H*y

%receiver - hard decision decoding
ipHat = real(yHat)>0;
else
% use Kevin's subroutine
for i = 1:nSymbolPeriodsToSimulate
% ipHat(:,i) = Zero_Forcing(y(:,i), h(:,:,i) , [ -1,1] );% use zero
forcing
ipHat(:,i) = ML_Decode_4_4(y(:,i), h(:,:,i) , [ -1,1] ); % use ML
decoding
end
end
% counting the errors
nErr(ii) = length(find( ipHat ~= sMod ));
end
simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));
p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));

close all
figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([ 0 25 10^-5 0.5])
grid on
legend('theory (nTx=1,nRx=1)', 'theory (nTx=1,nRx=2, MRC)', 'sim
(nTx=4, nRx=4, ML)');
xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');
title('BER for BPSK modulation with 2x2 MIMO and ML equalizer
(Rayleigh channel)');
```

6.7 APPENDIX G

```
% Script for computing the BER for BPSK modulation in a
% Rayleigh fading channel with 4Tx, 4Rx MIMO channel
% Maximum likelihood equalization
 clear
DecodeMethod = 'Maximum Likelihood';
%DecodeMethod = 'Zero Forcing';
N = 1200;              % number of bits or symbols
Eb_N0_dB = [0:25];     % multiple Eb/N0 values
nTx = 4;
nRx = 4;
nSymbolPeriodsToSimulate = N/nTx;
% build CSI and noise arrays
h_iid = 1/sqrt(2)*[randn(nRx,nTx,N/nTx) + 1i*randn(nRx,nTx,N/nTx)];
% Rayleigh channel
h = h_iid;                                 % declare CSI array
tx_antenna_Correlation =0;                          % correlation at
transmitter
rx_antenna_Correlation =0;                % correlation at reciever
```

```
%T_tx = [  1 tx_antenna_Correlation tx_antenna_Correlation 1];
R_tx  =  [   1  tx_antenna_Correlation  0  0;  tx_antenna_Correlation  1
tx_antenna_Correlation       0;0       tx_antenna_Correlation       1
tx_antenna_Correlation;0 0 tx_antenna_Correlation 1];
sqrt_R_tx = sqrtm(R_tx)';
%R_rx = [  1 rx_antenna_Correlation rx_antenna_Correlation 1];
R_rx   =[ 1   rx_antenna_Correlation   0   0;   rx_antenna_Correlation  1
rx_antenna_Correlation       0;0       rx_antenna_Correlation       1
rx_antenna_Correlation;0 0 rx_antenna_Correlation 1];
sqrt_R_rx = sqrtm(R_rx);
for i = 1:nSymbolPeriodsToSimulate
h(:,:,i)   =   sqrt_R_rx   *   squeeze(h_iid(:,:,i))   *   sqrt_R_tx;   %
Kronecker model
end
n=1/sqrt(2)*[ randn(nRx,nSymbolPeriodsToSimulate)+
1i*randn(nRx,nSymbolPeriodsToSimulate)] ; % white gaussian noise, 0dB
variance
for ii = 1:length(Eb_N0_dB)

% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
 s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 1
sMod   =   reshape(s,nTx,nSymbolPeriodsToSimulate);   %   grouping   in
[ nTx,N/NTx ] matrix

% Channel and noise Noise addition
y = 10^(-Eb_N0_dB(ii)/20)*n;
Capacity = zeros( nSymbolPeriodsToSimulate,1 );
for i = 1:nSymbolPeriodsToSimulate
ChannelMatrix = squeeze(h(:,:,i));
y(:,i) = y(:,i) + ChannelMatrix*squeeze(sMod(:,i));
Lambda = eig( ChannelMatrix );
Capacity(i)        =        log2(        det(        eye(nRx)        +
Eb_N0_dB(ii)/nTx*diag(Lambda) ) );
end
MeanCapacity(ii) = mean(Capacity);

% Receiver
if false
% Forming the Zero Forcing equalization matrix W = inv(H^H*H)*H^H
% H^H*H is of dimension [ nTx x nTx] . In this case [ 3 x 3]
% Inverse of a [ 3x3] matrix [ a b; c d] = 1/(ad-bc)[ d -b;-c a]
hCof = zeros(2,2,N/nTx)  ;
hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);  % d term
hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term
hDen  =  ((hCof(1,1,:).*hCof(2,2,:))  -  (hCof(1,2,:).*hCof(2,1,:)));%
ad-bc term
hDen  =  reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);  %
formatting for division
hInv = hCof./hDen; % inv(H^H*H)
hMod =  reshape(conj(h),nRx,N); % H^H operation
yMod  =  kron(y,ones(1,2));  %  formatting  the  received  symbol  for
equalization
yMod = sum(hMod.*yMod,1); % H^H * y
```

```
yMod =  kron(reshape(yMod,2,N/nTx),ones(1,2)); % formatting
yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^*H)*H^H*y
% receiver - hard decision decoding
ipHat = real(yHat)>0;
else
% use Kevin's subroutine
switch DecodeMethod
case 'Maximum Likelihood'
for i = 1:nSymbolPeriodsToSimulate
ipHat(:,i)  = ML_Decode_4_4(y(:,i), h(:,:,i) , [ -1,1] );% use ML
decoding
 end
 case 'Zero Forcing'
 for i = 1:nSymbolPeriodsToSimulate
ipHat(:,i)  = Zero_Forcing(y(:,i), h(:,:,i) , [ -1,1] );% use zero
forcing
 end
 otherwise
 disp(' ERROR unknown decoding ethod');
 end
 end

% counting the errors
nErr(ii) = length(find( ipHat ~= sMod ));
end
simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));
p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));

close all
figure(1)
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([ 0 25 10^-5 0.5])
grid on
legend('theory  (nTx=1,nRx=1)',  'theory  (nTx=1,nRx=2,  MRC)',  'sim
(nTx=4, nRx=4, ML)');
xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');
title([ 'BER for BPSK modulation with 4x4 MIMO using ' DecodeMethod '
(Rayleigh channel)'] );

figure(2)
semilogy(Eb_N0_dB,MeanCapacity);
xlabel('Average Eb/No,dB');
ylabel('Capacity (b/s/Hz)');
title('Capacity of 4x4 MIMO system as Function of SNR');
```

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