THE UNIVERSITY OF HULL

Estimation and Inference with Nonstationary Panel Data

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by

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THESIS
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THESIS SUMMARY

Estimation and Inference with Nonstationary Panel Data

This PhD thesis applies the time-series concepts of unit-roots and cointegration to nonstationary panel data. The first three chapters set the scene for what follows and together are the first methodological core of the thesis, on nonstationary panel data estimation and testing.

In chapter 1 we consider the established panel unit root tests of Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) and also Pesaran (2005) for cross-sectional dependence, with a panel of 20 OECD inflation rates.

In chapter 2 we consider the established panel cointegration tests of Kao (1999), Pedroni (1999) and Larsson, Lyhagen and Lothgren (2001) with a panel of 25 OECD exchange rates to test for long run PPP, again including cross-sectional dependence.

In chapter 3 a more original contribution is given. We conduct an extensive empirical study of the long run determinants of consumption expenditure for a panel of 20 OECD countries. A panel data cointegrating regression is estimated using the panel DOLS and FMOLS estimators of Kao and Chiang (2000) and Pedroni (2000,2001). Using Bai and Kao (2005) we again consider cross-sectional dependence.

The second methodological core is the statistical inference of nonstationary panel data, in the last two chapters.

In chapter 4 is another original contribution using the bootstrap with nonstationary panel data. New bootstrap algorithms are presented for the panel DOLS estimators mentioned above and also the group-mean estimator of Pesaran and Smith (1995).

In our last original contribution, in chapter 5, we consider the asymptotic properties of nonstationary panel data estimators. The asymptotic normality and asymptotic consistency of our panel FMOLS, DOLS and OLS estimators are proved for the simple case of the panel cointegrating regression with a constant intercept and trend. The new sequential limit asymptotic theory of Phillips and Moon (1999) is highlighted.

INTRODUCTION

A longitudinal or panel data set is one that follows a given sample of individuals over time and thus provides multiple observations on each individual in the sample. One can obtain a panel dataset by carrying out a number of cross-section surveys at consecutive periods in time. A well known US panel dataset is the Panel Study of Income Dynamics (PSID) of the University of Michigan. The British Household Panel Study (BHPS) is a well known UK panel data set. The BHPS is a continuum of surveys that first started in 1991. Since then there have been 13-15 waves (or surveys). The panel consists of a sample of around 5,500 households with detailed information on opinions and socio-economic data. Other European panels exist, for example the German Socio-Economic Panel (GSOEP), see also Alessie, Kapteyn and Melenberg (1989) on the Intomart Dutch panel of households. Panel data possess many advantages over cross-section or time-series data. They give the researcher a much larger number of data points (or observations) thus increasing the degrees of freedom and reducing the collinearity amongst explanatory variables, therefore improving the efficiency of the estimates. They allow us to construct and test more complicated behavioural models than pure cross-section or time-series models. Finally panel data are better for studying the dynamics of adjustment, eg in labour studies a cross-section study can be used to show what proportion of the population is unemployed at a particular point in time, whilst repeated cross-sections, ie panel data, can show how this proportion changes over time. Some classic texts on the econometrics of panel data are Hsiao (1986), Matyas and Sevestre (1996) and Baltagi (2001).

More recently there has been much interest shown in large macro panels, with large N and large T. These are very different from the traditional, and hitherto very common, micro panels with small T and large N, customary for labour panels and consumer household panels (see Baltagi (2001) for a review). The new macro panels have originated from the new availability of large cross-country datasets such as the Penn World Tables (Summers and Heston (1991)). A feature of these new datasets is that contrary to the traditional panels which give rise to regressions with stationary regressors, these new cross-country datasets are to be used for regressions with nonstationary regressors. This is because as noticed by Nelson and Plosser (1982) the actual macroeconomic time-series contained in the panels have become identified as containing unit roots. This gives rise to the nonstationarity in the panel dataset to which we should apply the time-series methods of unit roots and cointegration. A good recent text here is Baltagi (2000) and see also Baltagi and Kao (2000) and Breitung and Pesaran (2005) for reviews.

As to be expected this new area of panel data econometrics has become equipped with its own tools of investigation. Consider the following three panel regressions:

Model 1

$y_{it} = \alpha + \delta t + \beta' x_{it} + \varepsilon_{it}$

Model 2

$y_{it} = \alpha_i + \lambda_t + \delta_i t + \beta' x_{it} + \varepsilon_{it}$

Model 3

$y_{it} = \alpha_i + \lambda_t + \delta_i t + \beta_i' x_{it} + \varepsilon_{it}$

Model 1 is the homogeneous panel data model with a constant intercept, trend and slope. Model 2 is the homogeneous panel data model with individual and time-specific effects, α_i and λ_t and individual-specific trends. Finally Model 3 is the heterogeneous panel data model with individual and time-specific effects and individual-specific trends. It is the use of the deterministic terms such as the individual-specific trend term and also the non-stationary variables, eg $\{y_{it}, x_{it}\} \sim$ I(1), taken from the parent time-series literature, that are the new additions to the formulation of the panel data regressions. Furthermore the usual division between estimating the α_i and λ_t as Fixed Effects or Random Effects is still possible. When fixed α_i and λ_t take the form of dummy variables to be treated like constants, whilst when random they have their own probability distribution, eg $E(\alpha_i) = E(\lambda_t) = 0$ with $Var(\alpha_i) = \sigma_\alpha^2$ and $Var(\lambda_t) = \sigma_\lambda^2$. It is these types of regressions, perhaps in their dynamic autoregressive form, that are to be investigated with nonstationary panel data.[1]

Thus this thesis is concerned with estimation and inference with nonstationary panel data. This very recent area of panel data study has already produced an eclectic mix of traditional time-series results as well as new and exciting results from panel data. Issues such as Panel Unit Roots and Panel Cointegration are important to ascertain information on the long run relationships between economic variables using panel data. Only once these notions have been identified can the issue of how best to extract estimates of the long run relationship, ie the cointegrating vector, be considered. Also in this thesis we deal only with the case of a single cointegrating vector in the panel data. Thus we focus our attention on the panel Fully Modified Ordinary Least Squares (FMOLS), panel Dynamic Ordinary Least Squares (DOLS) and panel Ordinary Least Squares (OLS) estimators that have recently been developed by Phillips (1999), Kao and Chiang (2000), Pedroni (2000,2001) and others. When more than one cointegrating vector exists in the panel, then the panel Vector Error Correction Methods (VECM) of

[1]More commonly a fixed effects specification is used, with perhaps a random factor structure to cope with cross-section dependence.

Groen and Kleibergen (2003), Larsson, Lyhagen and Lothgren (2001) and Breitung (2005) should be used. These methods are outside the scope of the present study for reasons of space. Finally we have taken care in the thesis to use relatively simple moderate to large panel datasets. This has enabled us to concentrate more on the statistical inferential issues associated with the use of nonstationary panel datasets.

In chapter 1 we discuss and apply some of the panel unit root tests that have emerged in the panel data literature, eg the Levin and Lin (1992,1993) tests, Levin, Lin and Chu (2002) test and the Im, Pesaran and Shin (2003) test. It is well known that the standard Dickey-Fuller type tests for unit roots lack power in distinguishing the unit root null hypothesis from stationary alternatives. One of the motivations for the panel unit root tests was to increase the power of these time-series unit root tests by adding the cross-section dimension to the dataset, giving a larger number of observations. This has been quite a success and in many empirical applications there has emerged the stark contrast whereby single country ADF tests conducted on such time-series as real exchange rates, inflation and investment, etc do not reject the null hypothesis of a unit root, whilst the panel unit root tests usually do. Also one of the main contributions of the thesis is to give a concise analysis and application of cross-section dependence in panels. The problem of cross-sectional dependence is unique to panel data and is a very serious one as in the existing nonstationary panel literature most of the studies assume cross-unit independence. In empirical applications this assumption was almost always violated. Hence recently new approaches to the problem of panel unit roots with cross-sectional dependence have come about which we discuss. Finally in chapter 1 we present an empirical application of some panel unit root tests and their extension to cater for cross-sectional dependence. This is done with a panel dataset of 20 OECD country inflation rates. In chapter 2 we discuss and apply some of the panel cointegration tests that have emerged in the literature, eg the Kao (1999) tests, Pedroni (1999) tests and Larsson, Lyhagen and Lothgren (2001) test. Again the aim has been to pool the cross-section and time-series dimension in order to benefit from the increased power of the panel cointegration tests. We illustrate these tests with an empirical application concerned with testing for long run PPP in a panel of 25 OECD countries. Here the panel unit root and panel cointegration tests are combined in a novel approach in order to gain a stronger overall consensus as to whether long run PPP exists. Also the issue of cross-sectional dependence is considered again.

In chapter 3 we continue with the issue of panel cointegration, this time with the aim of obtaining estimates of the long run economic relationships predicted by economic theory, that are contained in the panel. The setting for the study is the panel cointegration model with at most one single common cointegrating vector. As mentioned above this enables the panel analogues of the single-equation methodologies of the time-series literature to be applied. We discuss and apply the panel FMOLS and panel DOLS estimators of Kao and Chiang (2000) and Pedroni (2000,2001). These estimators are of recent origin and are designed specifically for panel regressions with I(1) variables, ie nonstationary panels. Our main contribution in this chapter is to present an extensive empirical application of these estimators in a panel data study of the determinants of consumption in 20 OECD countries and also to extend the model to cater for cross-sectional dependence. The recent modifications to the panel FMOLS estimation framework by Bai and Kao (2005), to cater for cross-sectional dependence, are highlighted in our applications and the latest contributions using the DSUR estimators are discussed, see Phillips and Sul (2003) for details.

In chapter 4 we consider some inferential issues related to the use of non-stationary panel data. Here we consider the Bootstrap. In the first part of the chapter we present a new method of constructing bootstrap confidence intervals for a panel cointegrating regression. This contribution has never before been seen in the panel data literature and it involves using the Pairs

Bootstrap method with a panel data cointegrating regression. A new modified bootstrap algorithm is presented for both the pooled Kao and Chiang (2000) DOLS panel estimator and the Pedroni (2001) group-mean DOLS panel estimator. Then using the exchange rate panel of chapter 2 bootstrap confidence intervals are computed and reported. In the second part of the chapter we show how the bootstrap can be used with other panel time-series models. A new method for constructing bootstrap quantiles is shown for a panel data AR(p) autoregression. This contribution also has never been presented before in the panel data literature and involves using the Block Bootstrap and Residual Bootstrap to construct the bootstrap samples. A new modified bootstrap algorithm is presented for the Pesaran and Smith (1995) group-mean panel estimator. Finally the inflation rate panel of chapter 1 is used to construct bootstrap quantiles.

In chapter 5, our final chapter, we continue in an inferential setting and discuss panel data asymptotic theory. The recent use of large N and large T macro panels necessitated the development of a new regression limit theory for nonstationary panel data by Phillips and Moon (1999). It was found that the asymptotic properties of the panel estimators such as panel DOLS and panel FMOLS were very different from their analogous time-series equivalents. Whereas the limiting distributions of the time-series FMOLS and

DOLS estimators converged to nonstandard functionals of Brownian motion, the panel FMOLS and DOLS estimators had limiting normal distributions which could be easily standardised after a suitable adjustment. This makes hypothesis testing and inference much simpler. The main contribution of this chapter is to present a detailed study of the new sequential limit theory of Phillips and Moon (1999). Nearly all the panel statistical tests and panel estimators discussed in the thesis can be based on an asymptotic theory which uses sequential limit probability theory arguments. We derive the asymptotic consistency and asymptotic normality properties of the panel FMOLS, DOLS and OLS estimators giving a much more detailed account than is usually given in the panel data literature.

List of Abbreviations

ACF=Autocorrelation Function
ADF=Augmented Dickey-Fuller
AIC=Akaike Information Criterion
AR(p)= Autoregression of Order p
BB=Block Bootstrap
BC=Bias Corrected
BCa=Bias Corrected and Accelerated
BHPS=British Household Panel Study
BM=Brownian Motion
Ch=Chapter
CADF=Cross-sectionally Augmented Dickey-Fuller
CCE=Correlated Common Effects Estimator
CD=Compact Disc
CDF=Cumulative Distribution Function
CFS=Coakley, Fuertes and Smith
CIPS= Cross-sectionally Augmented Im, Pesaran and Shin
CLT=Central Limit Theorem
CMT=Continuous Mapping Theorem
Cov=Covariance
CPI=Consumer Price Index
CSD=Cross-section Dependence
CUP-FMOLS=Continuously Updated Fully Modified Ordinary Least Squares
DGP=Data Generation Process
DF= Dickey-Fuller
DOLS=Dynamic Ordinary Least Squares
DSUR=Dynamic Seemingly Unrelated Regression
DW=Durbin Watson
E=Expectation
EDF=Empirical Distribution Function
ESDS=Economic and Social Data Service
FCLT=Functional Central Limit Theorem
FMOLS=Fully Modified Ordinary Least Squares
GDP=Gross Domestic Product
GSOEP=German Socio-Economic Panel
HAC=Heteroscedasticity And Autocorrelation Consistent
HEGY=Hylleberg, Engle, Granger and Yoo
I(0)=Integrated of Order 0
I(1)= Integrated of Order 1
I(2)= Integrated of Order 2
IC=Information Criterion
IFS=International Financial Statistics
IP=Invariance Principle
IID=Independently and Identically Distributed

IMF=International Monetary Fund
IPS = Im, Pesaran and Shin
IV=Instrumental Variable
KCC=Kao, Chiang and Chen
LL =Levin and Lin
LHS=Left Hand Side
LM=Lagrange Multiplier
LLL= Larsson, Lyhagen, and Lothgren
LLN=Law of Large Numbers
LR=Likelihood Ratio
MBB=Moving Block Bootstrap
MEI=Main Economic Indicators
MIMAS= Manchester Information & Associated Services
MPC=Marginal Propensity to Consume
NPT=Nonstationary Panel Time-Series
OECD=Organisation for Economic Cooperation and Development
OLS=Ordinary Least Squares
PSID=Panel Study of Income Dynamics
PFM=Panel Pooled Fully Modified
PPP=Purchasing Power Parity
PWT=Penn World Tables
QED=Quod Erat Demonstrandum (which was to be demonstrated)
RER=Real Exchange Rate
RHS=Right Hand Side
SE=Standard Error
SB=Stationary Bootstrap
SDOLS=System Dynamic Ordinary Least Squares
SDSUR=System Dynamic Seemingly Unrelated Regression
SIC=Schwartz Bayesian Information Criterion
SLLN=Strong Law of Large Numbers
SUR=Seemingly Unrelated Regression
UK=United Kingdom
US=United States (of America)
USA=United States of America
Var=Variance
VAR=Vector Autoregression
VECM=Vector Error-Correction Model
WLLN=Weak Law of Large Numbers

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Panel Data Unit Roots

1.1 Introduction

In this chapter we discuss unit root tests in panel data. Since the econometric methodology involved in testing for unit roots with panel data is similar to the time-series case we devote some of the chapter to the latter, to grasp the salient points.

Of the many panel unit root tests presented in the panel data literature the most popular have been the Levin and Lin (1992,1993) (see also Levin, Lin and Chu (2002)) and Im, Pesaran and Shin (2003) tests. These we discuss in detail. One of the weaknesses of many of the recent tests for unit roots in panel data was the reliance on the unrealistic assumption of cross-sectional independence. In empirical applications this assumption was often seen to be violated. Hence there has emerged a growing literature on panel unit root tests with cross-sectional dependence. One of our main contributions in this chapter is to bridge the gap between existing panel unit root techniques and the newly emerging literature on cross-sectional dependence. We do this with a thorough review and application. Thus we conclude the chapter with an empirical application of testing for unit roots in a panel dataset of inflation time-series accounting for cross-sectional dependence.

The sections are as follows. In section 1.2 we have unit root tests in time series, whilst in section 1.3 we have unit root tests with panel data. In section 1.4 are panel unit root tests with cross-sectional dependence. Finally in section 1.5 we have the empirical application.

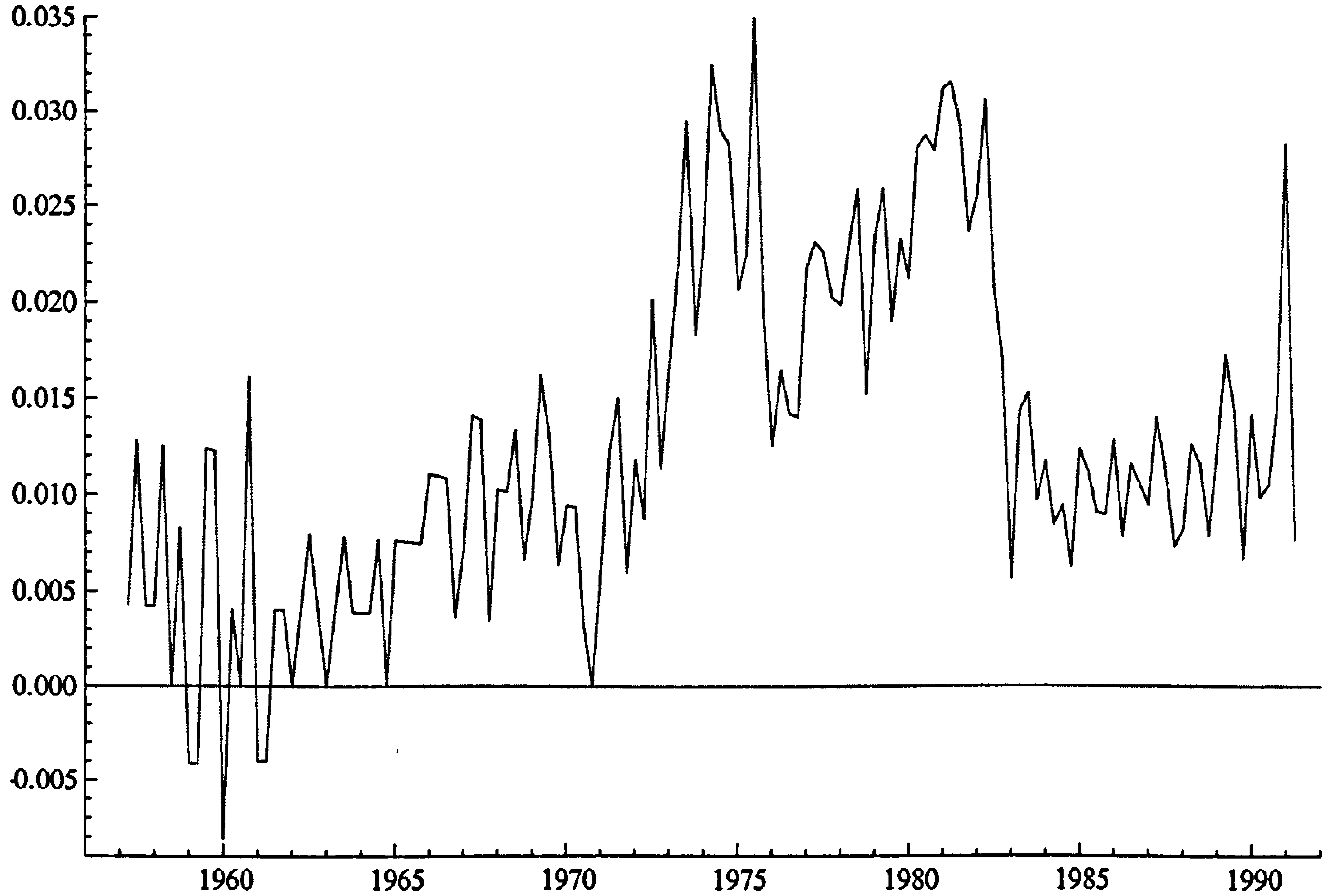
1.2 Unit Root Tests in Time Series

The concepts involved in testing for unit roots in panel data are very much analogous to the time-series case. From an inferential point of view treating a nonstationary regressor as if it were stationary will give very misleading and at worst nonsensical results. A variable is termed nonstationary if it contains a unit root. Such variables need to be differenced once or more to obtain a stationary variable.[1]

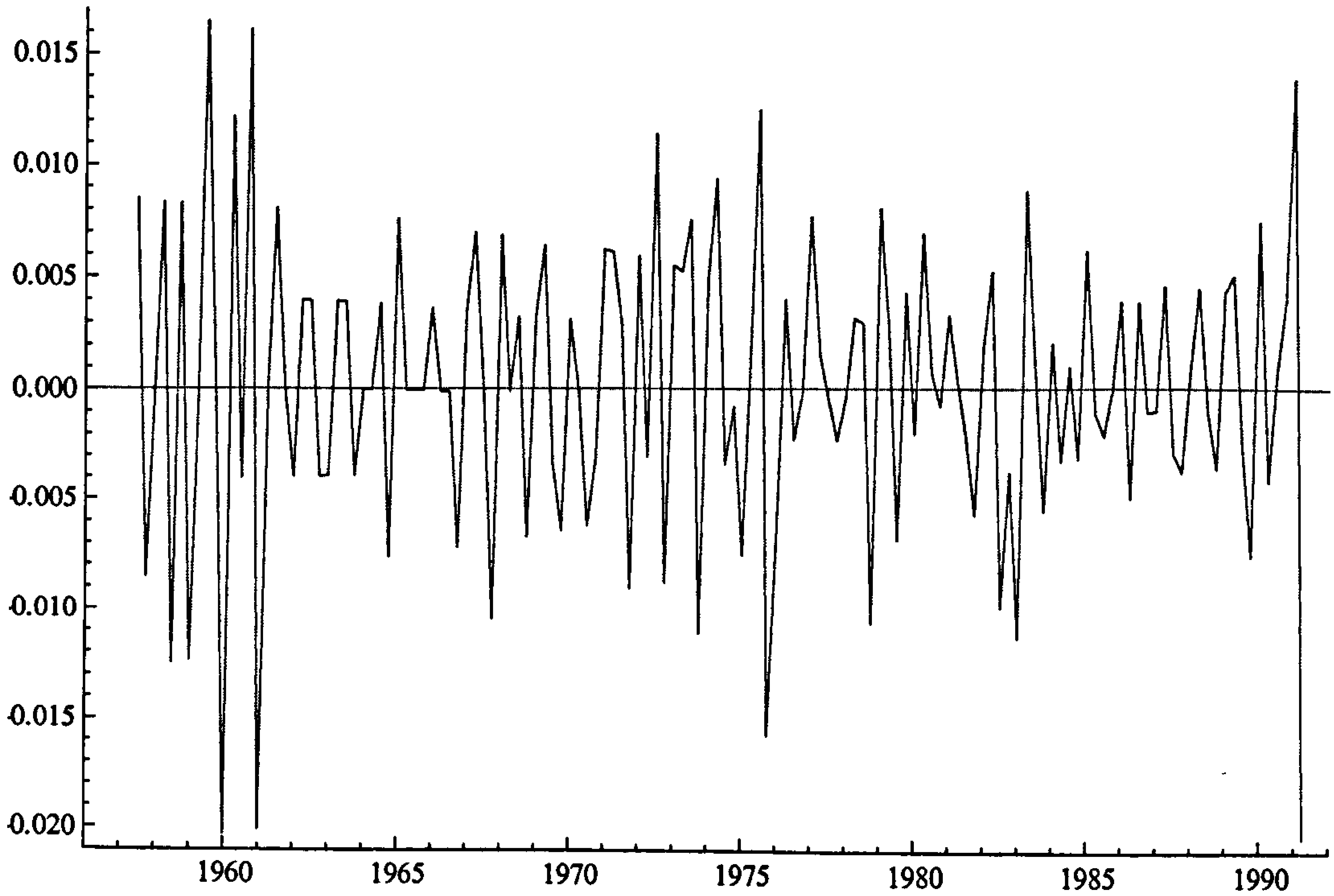
[1] A variable that has to be differenced once to acheive stationarity is termed an I(1) variable, ie integrated of order 1. Twice differencable variables are integrated of order 2, and so on. A stationary variable is I(0).

Figure 1.01

a) A Nonstationary Time-Series



b) A Stationary Time-Series



Here we may use the term stationary to mean covariance-stationary or weakly stationary. The time-series properties of a weakly stationary random variable X_t are

$$E(X_t) = \mu \quad \forall t \tag{1.1}$$

$$E(X_t - \mu)(X_{t-s} - \mu) = \gamma_s \quad \forall t, s. \tag{1.2}$$

Consider the AR(1) model

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{1.3}$$

where ε_t is $n.i.i.d(0, \sigma_e^2)$, ie a Gaussian white noise random disturbance term or shock. When $\rho = 1$ then we have a random walk model given by

$$Y_t = Y_{t-1} + \varepsilon_t. \tag{1.4}$$

By backward substitution write

$$Y_1 = Y_0 + \varepsilon_1 \tag{1.5}$$

$$Y_2 = Y_1 + \varepsilon_2 = Y_0 + \varepsilon_1 + \varepsilon_2 \tag{1.6}$$

$$\vdots \qquad \vdots \qquad \vdots \tag{1.7}$$

$$Y_t = Y_0 + \sum_{i=1}^{t} \varepsilon_i. \tag{1.8}$$

In the random walk model Y_t has infinite memory, ie shocks persist forever. Also given $Y_0 = 0$ then $E(Y_t) = E(\sum_{i=1}^{t} \varepsilon_i) = 0$ and $Var(Y_t) = Var(\sum_{i=1}^{t} \varepsilon_i) =$

$\sum_{i=1}^{t} Var(\varepsilon_i) = \sum_{i=1}^{t} \sigma_e^2 = t\sigma_e^2$. Then as $T \to \infty \quad Var(Y_t) \to \infty$.

Contrast this with the case where $|\rho| < 1$. Then given

$$Y_t = \rho Y_{t-1} + \varepsilon_t. \tag{1.9}$$

By backward substitution we have

$$Y_1 = \rho Y_0 + \varepsilon_1 \tag{1.10}$$

$$Y_2 = \rho Y_1 + \varepsilon_2 = \rho^2 Y_0 + \rho\varepsilon_1 + \varepsilon_2 \tag{1.11}$$

$$Y_3 = \rho Y_2 + \varepsilon_3 = \rho^3 Y_0 + \rho^2\varepsilon_1 + \rho\varepsilon_2 + \varepsilon_3 \tag{1.12}$$

$$\vdots \qquad \vdots \qquad \vdots \tag{1.13}$$

$$Y_t = \rho^t Y_0 + \sum_{i=0}^{t-1} \rho^i \varepsilon_{t-i}. \tag{1.14}$$

As $T \to \infty$ given $|\rho| < 1$ and $Y_0 = 0$, then $\sum_{i=0}^{t-1} \rho^i \varepsilon_{t-i} \to 0$ and so Y_t has finite memory. Hence the shocks die out. $E(Y_t) = E(\sum_{i=0}^{t-1} \rho^i \varepsilon_{t-i}) = 0$ and $Var(Y_t) = Var(\sum_{i=0}^{t-1} \rho^i \varepsilon_{t-i}) = \sigma_e^2 \sum_{i=0}^{t-1} \rho^{2i}$ and as $T \to \infty \quad Var(Y_t) = \frac{\sigma_e^2}{1-\rho^2} < \infty$. So when $|\rho| < 1$ it can be shown that Y_t is asymptotically stationary with means and covariances, etc independent of time. From an econometric modelling viewpoint then it is important to be aware of the problems involved when using nonstationary variables in regressions so as to choose an appropriate modelling strategy.

It was Nelson and Plosser (1982) who first commented on the nonstationarity of many U.S. macroeconomic time-series. This led to a large number of methodological and empirical studies on testing for unit roots in macroeconomic time-series data.

1.2.1 The Dickey-Fuller Tests for Unit Roots with Time Series Data

Fuller (1976) and Dickey and Fuller (1979,1981) were the first to develop tests for unit roots with time-series data. The model develops in a series of stages with constants and trends being added in turn to the regression. Consider the simple AR(1) model again

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{1.15}$$

where ε_t is $i.i.d(0, \sigma^2)$. Consider the test of the null hypothesis $H_0 : \rho = 1$. Under the null the true DGP is

$$Y_t = Y_{t-1} + \varepsilon_t. \tag{1.16}$$

However when $H_0 : |\rho| < 1$ the limiting distribution of $\hat{\rho}$ under the null is Gaussian and given by

$$\sqrt{T}(\hat{\rho} - \rho) \xrightarrow{d} N(0, 1 - \rho^2). \tag{1.17}$$

In[2] contrast when $H_0 : \rho = 1$ is true then the estimator $\hat{\rho}$ is termed, "super-consistent" converging at rate T instead of $\sqrt{T}$. Also the limiting distribution

[2]Here $\xrightarrow{d}$ means converges in distribution.

under the null is to a nonstandard ratio of functionals of Brownian motion using a functional central limit theorem (FCLT) as $T \to \infty$, ie

$$(1.18) \qquad T(\hat{\rho}-1) \Rightarrow \frac{(\frac{1}{2})\{[W(1)]^2-1\}}{\int_0^1 [W(r)]^2 dr}$$

where $W(1)$ and $W(r)$ are standard Brownian motion[3]. Critical values for this distribution are obtained by Monte Carlo simulation and can be found in Fuller (1976) p.371 case $\hat{\rho}$.

Similarly for the t-statistic, t_ρ, of ρ in equation (1.15) for the null hypothesis $H_0 : \rho = 1$, the limiting distribution is given, as $T \to \infty$, by

$$(1.19) \qquad t_\rho \Rightarrow \frac{(\frac{1}{2})\{[W(1)]^2-1\}^{\frac{1}{2}}}{\int_0^1 [W(r)]^2 dr}.$$

Critical values for this distribution are again obtained by Monte Carlo simulation and can be found in Fuller (1976) p.373 case $\hat{\tau}$.

The above AR(1) model could also be written as

$$(1.20) \qquad \Delta Y_t = \gamma Y_{t-1} + \varepsilon_t$$

where $\gamma = 1 - \rho$. Under the null hypothesis of $H_0 : \rho = 1$ then $\gamma = 0$. So we could use a t-test of $\gamma = 0$ to test for a unit root in Y_t. A summary of the Dickey-Fuller tests for unit roots is given in Hamilton (1994) p.502, Table 17.1.

[3] Here $\Rightarrow$ means converges weakly.

The unit root tests discussed make the assumption that ε_t is $i.i.d(0, \sigma^2)$. To cater for the case when ε_t is serially correlated the unit root tests evolved in two directions. First Dickey and Fuller (1981) used parametric corrections in the form of adding lagged differences of the dependent variable into the regression to, "whiten" the residuals.[4] As in Fuller (1976), p.374 consider the following AR(p) model

$$Y_t = \mu + \xi_1 Y_{t-1} + \xi_2 Y_{t-2} + \ldots + \xi_p Y_{t-p} + \varepsilon_t. \tag{1.21}$$

For this AR(p) model we have the equivalent ADF($p-1$) model given by

$$\Delta Y_t = \mu + \rho Y_{t-1} + \sum_{j=1}^{p-1} \alpha_j \Delta Y_{t-j} + \varepsilon_t \tag{1.22}$$

where $\rho = (\xi_1 + \xi_2 + \xi_3 + \ldots + \xi_{p-1} - 1)$, etc. When there is serial correlation of the residuals we use these "augmented" Dickey-Fuller (ADF) regressions for unit root tests.

The second method came from Phillips (1987) and his co-workers, in Phillips and Perron (1988) who used nonparametric corrections to the Dickey-Fuller model to cater for serial correlation. The two methods, ie the Augmented Dickey-Fuller or ADF method and the Phillips and Perron (1988) method are asymptotically equivalent as shown by the following distributions. For

[4]Give them white noise properties.

Z_α of[5] Phillips and Perron (1988) we have under the null of a unit root

$$Z_\alpha = T(\hat{\rho}-1) - \frac{1}{2}(T^2\hat{\sigma}_\rho^2 \div s^2)(\hat{\lambda}^2 - \hat{\gamma}_0) \Rightarrow \frac{(\frac{1}{2})\{[W(1)]^2 - 1\}}{\int_0^1 [W(r)]^2 dr} \tag{1.23}$$

where $\hat{\gamma}_j = \frac{1}{T}\sum_{t=j+1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_{t-j}$, $\hat{\varepsilon}_t$ is the OLS sample residual from the estimated regression, $\hat{\lambda}^2 = \hat{\gamma}_0 + 2\sum_{j=1}^{q}\left(1 - \frac{j}{(q+1)}\right)\hat{\gamma}_j$, $s^2 = \frac{1}{(T-k)}\sum_{t=1}^{T}\hat{\varepsilon}_t^2$, k is the number of parameters in the estimated regression and $\hat{\sigma}_\rho$ is the OLS standard error for $\hat{\rho}$. The same critical values as in the case without serial correlation are used. A summary of the Phillips and Perron (1988) tests for unit roots is given in Hamilton (1994) p.514, Table 17.2.

For the ADF tests we have the distribution under the null of a unit root of

$$\frac{T(\hat{\rho}-1)}{1-\hat{\alpha}_1-\hat{\alpha}_2-\ldots-\hat{\alpha}_{p-1}} \Rightarrow \frac{(\frac{1}{2})\{[W(1)]^2 - 1\}}{\int_0^1 [W(r)]^2 dr} \tag{1.24}$$

where $\alpha_1, \alpha_2, \ldots \alpha_{p-1}$ are as in the above ADF$(p-1)$ regression of equation (1.22). Again the same critical values as above are used. A summary of the Augmented Dickey-Fuller tests for unit roots is given in Hamilton (1994) p.528, Table 17.3.

1.2.2 A Unit Root Testing Strategy

In order to carry out unit root tests effectively one needs a formal testing strategy. If the researcher has knowledge of the DGP which generated the

[5]This is case 1 the model without a constant or trend.

data then this would dictate the choice of the test. If not one needs a reasonable testing strategy. It is wise if testing for one unit root to: (a) Graph the data in levels (b) Graph the data in differences (c) Graph the autocorrelations of the data in levels (d) Graph the autocorrelations of the data in differences. If testing for two unit roots then add to these (e) Graph the data in second differences (f) Graph the autocorrelations of the data in second differences. The formal testing strategy used in this thesis for ADF tests is the one followed by Perron (1988). Here a sequence of unit roots tests are carried out using t and F-tests in a certain order. This helps in identifying the model, ie one with a trend or not (see Perron (1988) for details).

1.2.3 Other Unit Root Tests and Extensions

The unit root tests just described have been extended in a number of directions. First the null hypothesis of stationarity as opposed to nonstationarity has been used by Kwaitkowski, Phillips and Schmidt (1992) and also by Leybourne and McCabe (1994). These models use structural time-series models for their testing framework. Tests for the presence of two unit roots in a time-series have been developed by Haldrup (1994) and Dickey and Pantula (1987). Whilst structural breaks have been incorporated into unit root tests by Perron (1989,1997) and Zivot and Andrews (1992). Finally it is prefer-able to use time-series data in its seasonally unadjusted form. This may necessitate seasonal differencing to obtain stationarity. Osborn (1990) has concluded several studies on seasonal unit roots in time-series models. Other contributions come from Hylleberg, Engle, Granger and Yoo (1990) (HEGY).

1.3 Unit Root Tests with Panel Data

Numerous unit root tests have been proposed for use with panel data, ie the Levin and Lin (1992,1993) and Levin, Lin and Chu (2002) tests, the Im, Pesaran and Shin (2003) tests, the Maddala and Wu (1999) and Hadri's (2000) test to name a few. We shall concentrate here on the first three.

1.3.1 The Levin and Lin Tests

In their paper, Levin and Lin (1992) (LL) develop unit root tests for the general model[6]:

$$\Delta y_{it} = \rho y_{it-1} + \alpha_0 + \delta t + \alpha_i + \theta_t + \varepsilon_{it} \tag{1.25}$$

for $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$.

Thus the autoregressive model incorporates a time trend and individual and time-specific effects. It is assumed in this paper that $\varepsilon_{it} \sim i.i.d.(0, \sigma^2)$. Levin and Lin consider several subcases of the above model. In all cases

[6]Here we write the general model and the subcases in first difference form.

the equation is estimated by OLS as a pooled regression and in all cases the limiting distributions are as $N \to \infty$ and $T \to \infty$. These submodels are

Model 1 $\Delta y_{it} = \rho y_{it-1} + \varepsilon_{it}, \quad H_0 : \rho = 0.$

Model 2 $\Delta y_{it} = \rho y_{it-1} + \alpha_0 + \varepsilon_{it}, \quad H_0 : \rho = 0.$

Model 3 $\Delta y_{it} = \rho y_{it-1} + \alpha_0 + \delta t + \varepsilon_{it}, \quad H_0 : \rho = 0, \delta = 0.$

Model 4 $\Delta y_{it} = \rho y_{it-1} + \theta_t + \varepsilon_{it}, \quad H_0 : \rho = 0.$

Model 5 $\Delta y_{it} = \rho y_{it-1} + \alpha_i + \varepsilon_{it}, \quad H_0 : \rho = 0, \alpha_i = 0, \forall i.$

Model 6 $\Delta y_{it} = \rho y_{it-1} + \alpha_i + \delta_i t + \varepsilon_{it}, \quad H_0 : \rho = 0, \delta_i = 0, \forall i.$

An important feature of the unit root test statistics is that in contrast to the nonstandard distributions of unit root test statistics for a single time series, the panel data test statistics have limiting normal distributions. Also convergence rates are faster with T (ie superconsistency as $T \to \infty$) than it is with N (ie $N \to \infty$). For models (1) to (4) we have under the null

(a) $T\sqrt{N}\hat{\rho} \Rightarrow N(0,2)$,[7]

(b) $t_\rho \Rightarrow N(0,1)$.[8]

For Model (5) if $\frac{\sqrt{N}}{T} \to 0$ then

[7] Where $\hat{\rho}$ is the OLS estimate of ρ and t_ρ is the t-statistic of $\hat{\rho}$.
[8] Again $\Rightarrow$ means weak convergence.

(a) $T\sqrt{N}\hat{\rho} + 3\sqrt{N} \Rightarrow N(0, 10.2)$,

(b) $\sqrt{1.25}t_\rho + \sqrt{1.875N} \Rightarrow N(0, 1)$.

Finally for Model (6) if $\frac{\sqrt{N}}{T} \to 0$ then

(a) $\sqrt{N}\{T\hat{\rho} + 7.5\} \Rightarrow N(0, \frac{645}{112})$,

(b) $\sqrt{\frac{448}{227}}\{t_\rho + \sqrt{3.75N}\} \Rightarrow N(0, 1)$.

In a later paper Levin and Lin (1993) (see also Levin, Lin and Chu (2002)) extend the model to incorporate error processes with heteroscedasticity and autocorrelation such as the following stationary invertible ARMA error process

$$\zeta_{it} = \sum_{j=0}^{\infty} \theta_{ij}\zeta_{it-j} + \varepsilon_{it} \qquad (1.26)$$

where $\zeta_{it}, \forall i, t$ has finite non-zero fourth moments and the variance of the innovation process ε_{it} is finite. In this model Levin and Lin prescribe the use of augmented Dickey-Fuller (ADF) tests to each individual series to test for unit roots. Using Model 5 then we have

$$\Delta y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \theta_{ij}\Delta y_{it-j} + \alpha_i + \varepsilon_{it}. \qquad (1.27)$$

In equation (1.27) the ρ_i is the autoregression coefficient for the ith equation and p_i is the order of the lag distribution function for the lags of the differenced dependent variable. The above regression is equivalent to performing two auxiliary regressions of Δy_{it} and y_{it-1} on the remaining variables in equation (1.27), ie

$$\Delta y_{it} = \sum_{j=1}^{p_i} \theta_{ij} \Delta y_{it-j} + \alpha_i + e_{it} \quad \text{and} \tag{1.28}$$

$$y_{it-1} = \sum_{j=1}^{p_i} \theta_{ij} \Delta y_{it-j} + \alpha_i + V_{it-1}.$$

Obtaining the residuals $\hat{e}_{it}$ and $\hat{V}_{it-1}$ regress $\hat{e}_{it}$ on $\hat{V}_{it-1}$,

$$\hat{e}_{it} = \rho_i \hat{V}_{it-1} + \varepsilon_{it} \tag{1.30}$$

to get ρ_i for the ith cross section.

The following expressions are next required to control for heteroscedasticity in ε_{it}

$$\hat{\sigma}^2_{e_i} = \frac{1}{T - p_i - 1} \sum_{t=p_i+2}^{T} (\hat{e}_{it} - \hat{\rho}_i \hat{V}_{it-1})^2 \quad \text{short-run variance} \tag{1.31}$$

$$\tilde{e}_{it} = \frac{\hat{e}_{it}}{\hat{\sigma}_{e_i}} \tag{1.32}$$

$$\tilde{V}_{it-1} = \frac{\hat{V}_{it-1}}{\hat{\sigma}_{e_i}} \tag{1.33}$$

$$\hat{\sigma}^2_{y_i} = \frac{1}{T-1} \sum_{t=2}^{T} \Delta y^2_{it} + 2 \sum_{L=1}^{\bar{K}} w_{\bar{K}L} \left(\frac{1}{T-1} \sum_{t=L+2}^{T} \Delta y_{it} \Delta y_{it-L} \right) \quad \text{long-run variance[9]} \tag{1.34}$$

$$\hat{S}_{NT} = \frac{1}{N} \sum_{i=1}^{N} \frac{\hat{\sigma}_{y_i}}{\hat{\sigma}_{e_i}} \quad \text{ratio of variances.} \tag{1.35}$$

[9] Here $\bar{K}$ is the lag truncation parameter and $w_{\bar{K}L}$ is the lag window, eg Bartlett or Parzen.

The final step is to estimate the panel regression (using all i and t) and the homoscedastic residuals

$$\tilde{e}_{it} = \rho \tilde{V}_{it-1} + \tilde{\varepsilon}_{it} \tag{1.36}$$

and compute the t-statistic

$$t_\rho = \frac{\hat{\rho}}{RSE(\hat{\rho})}, \tag{1.37}$$

where

$$RSE(\hat{\rho}) = \hat{\sigma}_\varepsilon \left[\sum_{i=1}^{N} \sum_{t=p_i+2}^{T} \tilde{V}_{it-1}^2 \right]^{-\frac{1}{2}} \tag{1.38}$$

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{N\tilde{T}} \sum_{i=1}^{N} \sum_{t=p_i+2}^{T} (\tilde{e}_{it} - \hat{\rho}\tilde{V}_{it-1})^2 \tag{1.39}$$

$$\bar{p} = \frac{1}{N} \sum_{i=1}^{N} p_i \quad \text{and} \quad \tilde{T} = (T - \bar{p} - 1). \tag{1.40}$$

The Levin and Lin statistic is an adjusted version of equation (1.37) and given by

$$t_\rho^* = \frac{t_\rho - N\tilde{T}\hat{S}_{NT}\hat{\sigma}_\varepsilon^{-2} RSE(\hat{\rho}) \mu_{\tilde{T}}^*}{\sigma_{\tilde{T}}^*} \tag{1.41}$$

where $\mu_{\tilde{T}}^*$ and $\sigma_{\tilde{T}}^*$ are the mean and standard deviation adjustment terms obtained by Monte Carlo and tabulated in their paper. Given the null hypothesis and the alternative hypothesis

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_N = \rho = 0 \tag{1.42}$$

$$H_1 : \rho_1 = \rho_2 = \ldots = \rho_N = \rho < 0 \tag{1.43}$$

the panel test statistic t_ρ^* has the property under the null

$$t_\rho^* \Rightarrow N(0,1) \quad \text{as} \quad T, N \to \infty \quad \text{and} \quad \frac{N}{T} \to 0. \tag{1.44}$$

1.3.2 The Im, Pesaran and Shin Test

Im, Pesaran and Shin (2003) (IPS) extend the Levin and Lin framework to allow for heterogeneity in the value of ρ_i under the alternative hypothesis. Let

$$\Delta y_{it} = \alpha_i + \rho_i y_{it-1} + \zeta_{it}, \tag{1.45}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ and where the errors ζ_{it} are serial correlated with different serial correlation properties across the units. The null and alternative hypotheses are

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_N = \rho = 0 \tag{1.46}$$

(1.47)$H_1 : \rho_i < 0, i = 1, 2, \ldots, N_1, \rho_i = 0, i = N_1 + 1, N_1 + 2, \ldots, N.$

Following the critique of Pesaran and Smith (1995) on pooled estimators in dynamic, heterogeneous panels, such as those used by Levin and Lin (1992) and (1993), Im, Pesaran and Shin (2003) propose a group-mean Lagrange multiplier (LM) statistic. The ADF regressions

$$\Delta y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \theta_{ij} \Delta y_{it-j} + \alpha_i + \varepsilon_{it}, \tag{1.48}$$

are estimated for each i and the LM-statistic for testing $\rho_i = 0$ is computed. Defining

$$(1.49) \qquad \bar{LM}_{NT} = \frac{1}{N}\sum_{i=1}^{N} LM_{iT}(p_i, \theta_i),$$

where $\theta_i = (\theta_{i1}, \theta_{i2}, \ldots, \theta_{ip_i})'$ and $LM_{iT}(p_i, \theta_i)$ is the individual LM-statistic for testing $\rho_i = 0$, the standardised LM-bar statistic is given by

$$(1.50) \qquad \Psi_{LM} = \frac{\sqrt{N}\left\{\bar{LM}_{NT} - N^{-1}\sum_{i=1}^{N} E[LM_{iT}(p_i, 0)|\rho_i = 0]\right\}}{\sqrt{N^{-1}\sum_{i=1}^{N} Var[LM_{iT}(p_i, 0)|\rho_i = 0]}}.$$

The values of $E[LM_{iT}(p_i, 0)|\rho_i = 0]$ and $Var[LM_{iT}(p_i, 0)|\rho_i = 0]$ are found by stochastic simulation and tabulated in their paper. It is shown that under $H_0 : \rho_i = 0$ for all i,

$$(1.51) \qquad \Psi_{\bar{LM}} \Rightarrow N(0, 1)$$

as $T, N \to \infty$ and $\frac{N}{T} \to k$ where k is some finite positive constant. Im, Pesaran and Shin (2003) also propose a group-mean t-bar statistic given by

$$(1.52) \qquad \Psi_{\bar{t}} = \frac{\sqrt{N}\left\{\bar{t}_{NT} - N^{-1}\sum_{i=1}^{N} E[t_{iT}(p_i, 0)|\rho_i = 0]\right\}}{\sqrt{N^{-1}\sum_{i=1}^{N} Var[t_{iT}(p_i, 0)|\rho_i = 0]}}$$

where

$$(1.53) \qquad \bar{t}_{NT} = N^{-1}\sum_{i=1}^{N} t_{iT}(p_i, \theta_i),$$

and $t_{iT}(p_i, \theta_i)$ is the individual t-statistic for testing $\rho_i = 0$ for all i. Again $E[t_{iT}(p_i, 0)|\rho_i = 0]$ and $Var[t_{iT}(p_i, 0)|\rho_i = 0]$ are found by stochastic simulation and tabulated in the paper. Also $\Psi_{\bar{t}} \Rightarrow N(0, 1)$ as above.

1.3.3 Other Panel Unit Root Tests and Extensions

There have emerged alongside the IPS and LL tests a number of other tests for unit roots in panel data. Harris and Tzavalis (1999) find in Monte Carlo simulations that the LL test has poor power properties when T is small. They propose a test for when T is small. Breitung (2000) develops a test to overcome the lack of power the LL and IPS tests suffer when fixed effects and trends are included in the DGP. Other important Monte Carlo simulation work on the power of the LL and IPS tests has been done by Karlsson and Lothgren (2000). Maddala and Wu (1999) have proposed a Fisher type test using p-values to test the null of a unit root. The advantages of this test are that it can handle unbalanced panels, it is easy to compute and can handle more general forms of cross-sectional dependence than LL and IPS. Hadri (2000) proposed a test based on the null of stationarity as opposed to nonstationarity. Structural breaks have been used in unit root tests by Culver and Papell (1997), Murray and Papell (2000) and Im, Lee and Tieslau (2005). Dreger and Reimers (2004) consider panel seasonal unit root tests. More recently new approaches have been proposed by Pedroni and Vogelsang (2005). Here they use kernel based estimators for panel unit root testing that are robust to incidental trends and cross-sectional dependence of unknown form. These are similar in spirit to the Phillips and Perron (1988) tests discussed in § 1.2. Jonsson (2005) studies the size distortions of cross-sectional dependence in panel unit root tests. The works of Bai (2003) and Bai and Ng (2002,2004) on factor models, initiated a new approach to the methods of panel unit root tests. These factor models are gaining ground with numerous applications such as Moon and Perron (2004), Phillips and Sul (2003), Pesaran (2003,2005) and Harris, Leybourne and McCabe (2005).

1.4 Panel Unit Root Tests with Cross-Sectional Dependence

Cross-sectional dependence occurs when the residuals in country i are correlated with the residuals in country j. One can detect certain dependencies by inspecting the cross-correlation matrix of the I(1) regressors. Cross-sectional dependence in panels can originate from a number of sources. One major source is global or common shocks, eg. the oil price shocks of the 1970's and their resulting inflation. Another example is in real exchange rates when using cross-country data, cross-sectional dependence is likely to arise due to the strong inter-economy linkages causing co-movement amongst the real exchange rates. Pesaran (2004) proposed a general diagnostic test for cross-sectional dependence.

The traditional way of dealing with cross-sectional dependencies has been to use time-effects in an error components model. However this assumes that the correlations are the same for each cross-unit, a very restrictive assumption. More recently, in the case of panel unit root tests, cross-sectional dependence has been modelled using a factor approach by Bai and Ng (2004), Phillips and Sul (2003), Moon and Perron (2003,2004) and others. Here one "defactors" the data using orthogonalisation procedures and then applies the standard panel unit root tests on the defactored data. The resulting new test statistics often have the meta-analysis form used in Maddala and Wu (1999) using p-values. Breitung and Das (2005) use a GLS SUR framework similar to the one that Phillips and Sul (2003) use for their dynamic panel data estimators for cross-sectionally dependent data. However for consistent estimates this requires that $T \geq N$. Chang (2002) use an IV approach for her panel unit root test with cross-sectional dependence using as instruments nonlinear transformations of the lagged levels in an augmented regression. Some Monte Carlo simulation studies of the finite sample performance of these tests incorporating cross-section dependence have come from Moon and Perron (2003) and Trapani (2004).

1.4.1 The Pesaran Panel Unit Root Test

In contrast to most other panel unit root tests that allow for cross-sectional dependence by defactoring the data, eg Bai and Ng (2004), Moon and Perron (2003,2004) and Phillips and Sul (2003), Pesaran (2003,2005) proposes a test where the standard DF or ADF regressions are augmented[10] with cross-section averages of lagged levels and first-differences of the individual series. Consider the model

$$y_{it} = (1-\theta_i)\mu + \theta_i y_{it-1} + u_{it} \tag{1.54}$$

for $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$.

This is a simple dynamic linear heterogeneous panel data model. To incorporate cross-sectional dependence into the model we assume the initial value $y_{i0} = 0$ and the error term u_{it} has the one-factor structure

$$u_{it} = \gamma_i f_t + \varepsilon_{it} \tag{1.55}$$

where f_t is the unobserved common effect and ε_{it} is the idiosyncratic error term. Write the above regression as

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \gamma_i f_t + \varepsilon_{it} \tag{1.56}$$

[10]Only Chang (2002) uses an augmented regression approach with her IV's.

where $\alpha_i = (1-\theta_i)\mu, \beta_i = -(1-\theta_i)$ and $\Delta y_{it} = y_{it} - y_{it-1}$.

The unit root hypothesis of $\theta_i = 1$ can be shown as

$$H_0 : \beta_i = 0 \quad \forall i \tag{1.57}$$

$$H_1 : \beta_i < 0, i = 1, 2, \ldots, N_1, \beta_i = 0, i = N_1 + 1, N_1 + 2, \ldots, N. \tag{1.58}$$

Given a number of assumptions on ε_{it}, f_t and γ_i it is shown in Pesaran (2002,2005b) that the common factor f_t can be proxied by the cross-section mean of y_{it}, ie $\bar{y}_t = \frac{1}{N}\sum_{j=1}^{N} y_{jt}$ and its lagged values $\bar{y}_{t-1}, \bar{y}_{t-2}, \ldots$, for N sufficiently large. In the case of no serial correlation in u_{it} then we base our test of the unit root hypothesis of equation (1.56) on the t-ratio of the OLS estimate of b_i in the following cross-sectionally augmented DF (CADF) regression

$$\Delta y_{it} = \alpha_i + b_i y_{it-1} + c_i \bar{y}_{t-1} + d_i \Delta \bar{y}_{t-1} + e_{it}. \tag{1.59}$$

Denote this t-ratio by $t_i(N,T)$. Pesaran shows that by sequential limit and joint limit probability theory these $t_i(N,T)$ statistics have limiting distributions called Cross-Sectionally Augmented Dickey-Fuller distributions. Pesaran (2003,2005) tabulates the critical values for the individual CADF statistics in the cases of no intercept, intercept and intercept and trend, included in the regression, for a range of values of N and T.

To generalise the CADF statistics to panel data Pesaran (2003,2005) proposes a version of the t-bar test of IPS (2003). This is the cross-sectionally augmented IPS test

$$CIPS(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T). \tag{1.60}$$

Pesaran (2003,2005) next extends the CIPS statistic to models with various forms of serial correlation. Although he discusses three such models he derives proofs only for the last, model 3, shown as follows

$$u_{it} = \rho_i u_{it-1} + \eta_{it} \tag{1.61}$$

$$\eta_{it} = \gamma_i f_t + \varepsilon_{it} \tag{1.62}$$

where $|\rho_i| < 1$ for $i = 1, 2, \ldots, N$. This yields

$$\Delta y_{it} = -\mu_i\beta_i(1-\rho_i) + \beta_i(1-\rho_i)y_{it-1} + \rho_i(1+\beta_i)\Delta y_{it-1} + \gamma_i f_t + \varepsilon_{it}. \tag{1.63}$$

All three model specifications yield the same ADF regressions but with different error specifications and parameter heterogeneity. The final estimation equation is given by the following cross-sectionally augmented DF regression. Extending the first-order autocorrelation error schemes to an AR(p) error process we get the following pth order cross-section time-series augmented regression

$$\Delta y_{it} = \alpha_i + b_i y_{it-1} + c_i \bar{y}_{t-1} + \sum_{j=0}^{P} d_{ij}\Delta\bar{y}_{t-j} + \sum_{j=0}^{P} \delta_{ij}\Delta y_{it-j} + e_{it}. \tag{1.64}$$

The relevant individual CADF statistic is given by the OLS t-ratio of b_i in the above regression for the ith individual. The critical values for the CIPS statistics used for the serially uncorrelated model apply equally to the serially correlated case. These are computed by Monte Carlo simulation and tabulated by Pesaran (2003,2005) in Tables 3a-3c.

1.5 An Empirical Application

1.5.1 Testing for Unit Roots in Inflation Panel Data

In this application we study whether there is a unit root in a panel of inflation time-series. We do this in two parts. In the following sections the methods of panel unit root testing discussed in § 1.3.1 and § 1.3.2 are presented for a panel of OECD inflation time-series. In the sections after we apply the Pesaran (2003,2005) panel data unit root test allowing for cross-sectional dependence of § 1.4.1 to see whether the panel data with cross-sectional dependence formulation sheds any further light on the issues of stationarity and nonstationarity.

1.5.2 The Dataset

The dataset is the Consumer Price Index (CPI) for twenty OECD countries obtained from the OECD Main Economic Indicators (MEI). We calculated the inflation rate by differencing the logarithm of the individual CPI's. The data was monthly time-series running from 1960:1 to 2000:8. So $N = 20$ and $T = 488$. Given monthly data for the countries it was expected that a lag of 11 or 12 periods (one year) would best fit the data. These were data determined in the ADF regressions rather than fixed a priori. Using the recursive t-statistic method suggested by Campbell and Perron (1991), we set an upper bound on k eg k_{max}, if the last lag included was significant we choose $k = k_{max}$, if not reduce k by one until the last lag becomes significant. If no lags are significant set $k = 0$. We set $k_{max} = 16$ for the monthly data in the tests. Also the 5% significance level of 1.96 of the asymptotic normal distribution is used as the critical value. The Akaike Information Criterion (AIC) was used also and gave similar results. These methods were used throughout the thesis when choosing ADF lag lengths.[11]

The ADF regressions contained individual-specific intercepts but did not contain a trend as this would have been consistent with ever accelerating inflation, (ie Model 5 of LL).

The issue of whether inflation is nonstationary or not has recently been studied using panel data by Culver and Papell (1997). They also applied the conventional time-series unit root tests to the individual inflation rates of 13 OECD countries. Their findings were that, contrary to the acceptance

[11]Ng and Perron (1995) discuss the advantages of the recursive t-statistic method.

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of the unit root hypothesis, found in most of the individual country time-series, the panel data unit root tests strongly reject it for the whole panel of 13 countries and various sub-panels. Culver and Papell (1997) also studied structural break models with the OECD time-series.

1.5.3 The Estimation Results

On a visual inspection of Figures 1.02-1.05 in Appendix 4 we see that in Figure 1.02 no trend is noticeable in any of the individual time-series, although many show a gradual incease up until 1980 and then a gradual decline. In Figure 1.03 we see the first differences of the time-series definitely exhibiting signs of stationarity and a zero mean. This is evidence of at least one unit root. Our suspicions are confirmed when we inspect the autocorrelation functions of Figures 1.04 and 1.05. The first shows the levels data dying away very slowly, indicative of a series with a long memory. That is, as discussed in § 1.2, the presence of a unit root means that shocks are persistent. In the second we see no persistence of the autocorrelations in the differenced data.

Table 1.01 **Individual Country Inflation ADF Regression Estimates**

Country	t-statistic	Lag
Austria	-3.095^b	11
Belgium	-2.366	12
Canada	-1.902	11
Denmark	-3.001^b	11
Finland	-3.155^b	12
France	-1.492	11
Germany	-2.323	11
Greece	-2.444	12
Iceland	-2.568	11
Ireland	-2.101	12
Italy	-2.120	12
Japan	-2.050	11
Luxembourg	-2.263	11
Norway	-2.436	11
Portugal	-2.886^b	12
Spain	-2.205	11
Sweden	-2.368	11
Switzerland	-2.799	12
U.K.	-2.306	12
U.S.	-2.067	12

In Table 1.01 we[12] have the results of the individual country ADF tests. It appears that the vast majority of the countries support the unit root hypothesis. Only Austria, Denmark, Finland and Portugal reject the null hypothesis at the 5% significance level. When considering the results of the IPS and LL panel unit root tests in Tables 1.02 and 1.03 we obtain seemingly conflicting results. IPS rejects the unit root null at the 1% significance level while LL accepts it. On closer inspection though we notice the following.

[12]Here b) means significant at the 5% level.

With the LL test the null hypothesis is that all members of the panel have a unit root, whilst the alternative is all members of the panel are stationary. This has been often criticised as unrealistic. The IPS test has the null that all members of the panel has a unit root, whilst the alternative is that at least one member of the panel is stationary. So we reject the null if any one member of the panel is stationary. This is a much more realistic hypothesis. In Monte Carlo simulations Maddala and Wu (1999) found the IPS test is the more powerful test. On these grounds and also since it does seem compatible with the results from the time-series unit root tests, that some countries reject the null, we give support to the IPS test and conclude that some but not all the members of the panel do not contain a unit root. To gain some further insights we consider the panel unit root tests with cross-sectional dependence.

Table 1.02 **IPS Panel Unit Root Tests**

$\Psi_{\bar{t}}$ stat
-4.88569[a]

Table 1.03 **LL Panel Unit Root Tests**

t_{ρ}^{*} stat
221.919

In Table 1.04 we have the results of the individual country CADF regressions and the picture changes dramatically. Now only five of the countries accept the null hypothesis these being France, Germany, Japan, Switzerland and the

US. Thus by accounting for cross-sectional dependence we have increased the power of the ADF tests to reject the unit root null. Finally in Table 1.05 we have the result of the Pesaran CIPS test and again the unit root null is rejected at the 1% significance level.

Table 1.04 **Individual Country Inflation CADF Regression Estimates**

Country	t-statistic	Lag
Austria	-4.31[a]	11
Belgium	-3.80[b]	12
Canada	-3.83[b]	11
Denmark	-3.93[a]	11
Finland	-5.40[a]	12
France	-2.34	11
Germany	-2.51	11
Greece	-3.38[b]	12
Iceland	-4.28[a]	11
Ireland	-3.44[b]	12
Italy	-5.25[a]	12
Japan	-1.81	11
Luxembourg	-4.26[a]	11
Norway	-4.44[a]	11
Portugal	-5.26[a]	12
Spain	-4.79[a]	11
Sweden	-4.87[a]	11
Switzerland	-2.56	12
U.K.	-3.83[b]	12
U.S.	-2.37	12

Table 1.05 **Pesaran Panel Unit Root Test**

CIPS(N,T) stat
-3.833[a]

The findings appear that by accounting for cross-sectional dependence one obtains stronger support for the stationary alternative hypotheses.

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Notes to the Tables

a) means significant at the 1% level, b) means significant at the 5% level. DF critical values $1\% = -3.44, 5\% = -2.87$. Pesaran CADF critical values $1\% = -3.84, 5\% = -3.23$. Pesaran CIPS critical values $1\% = -2.36, 5\% = -2.20$. N(0,1) one-sided critical values $1\% = -2.33, 5\% = -1.65$. Lag lengths chosen by Ng and Perron (1995) method.

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Chapter 2

Panel Data Cointegration

2.1 Introduction

In chapter 1 we tested for nonstationarity in panel data. In this chapter we build upon this framework and enquire, given the panel data in question has a unit root, whether or not there exists a long run equilibrium relationship amongst the variables of the panel (that is, enquire whether there is panel cointegration). To this end we discuss here the panel cointegration tests that have emerged in the panel data literature. As in chapter 1 much of their origin comes from the tests for cointegration of the time-series literature and so we devote some of the chapter to discussing these.

The tests for cointegration considered in this chapter are those proposed by Kao (1999), Pedroni (1999) and Larsson, Lyhagen and Lothgren (2001) and we apply these in an empirical application of testing for long run PPP with panel data.

The sections are as follows. In section 2.2 we have cointegration tests in time-series, whilst in section 2.3 we have residual based tests for cointegration. In section 2.4 is likelihood based tests for cointegration and in section 2.5 cointegration tests with panel data. Finally in section 2.6 we have the empirical application.

2.2 Cointegration Tests in Time Series

Similar to the case of unit roots in time-series and panel data, the concept of testing for cointegration in panel data is analogous to the time-series case. We have here in mind the Engle and Granger (1987) two-step method. In the first step one conducts a Dickey-Fuller type test for (non)cointegration. If there is cointegration then we may go a step further and estimate the cointegrating (or equilibrium) relationship.

To formally define cointegration consider an $(n \times 1)$ vector time-series Y_t. This vector Y_t is said to be cointegrated if each of the series taken individually is I(1), ie nonstationary with a unit root, while some linear combination of the series $\delta' Y_t$ is stationary or I(0) for some nonzero $(n \times 1)$ vector δ. We may note that this cointegrating vector may not be unique. For if $\delta' Y_t$ is stationary then so is $\alpha\delta' Y_t$, for any nonzero scalar α. Usually an arbitrary normalisation is made of the cointegrating vector such as that the first element is unity. Finally given Y_t if there are more than two variables contained in Y_t then there may be at least two cointegrating vectors δ_1 and δ_2 such that $\delta_1^{/}Y_t$ and $\delta_2^{/}Y_t$ are both stationary.

2.2.1 Cointegrating versus Spurious Regression

Before discussing the cointegrating regression we must note its opposite the spurious regression. A spurious regression is one where there is no relationship at all between Y_t a dependent variable and X_{1t} and X_{2t}, the candidate regressors, in their joint generation through the DGP, but we conclude wrongly from a regression analysis that such a relationship exists. The difference between a cointegrating regression and a spurious one is whether the linear combination of I(1) candidate variables and the dependent variable, is reduced to stationarity. Thus if U_t is the residual in the above regression, if it is I(0), given Y_t, X_{1t} and X_{2t} are I(1), then it is a cointegrating regression. If U_t is I(1) then it is a spurious regression. This property has been exploited for residual based tests of cointegration in both time-series and panel data. In general a spurious regression has the following characteristics: (a) estimates are not consistent (b) OLS t and F-statistics diverge (c) R^2 may not tend to 0. An alternative approach for testing for cointegration has come from Johansen (1988,1991,1995) in his multivariate maximum likelihood framework.

2.3 Residual Based Tests for Cointegration

2.3.1 The Phillips-Ouliaris-Hansen Test

Phillips and Ouliaris (1990) and Hansen (1992) build upon the approach of Engle and Granger (1987). The first step is to check the order of integration of each of our series and candidate regressors, so as to ensure they are all I(1). This is done usually by a Dickey-Fuller or ADF test of the type mentioned in chapter 1. Once this condition is satisfied the cointegrating regression is said to be "balanced" in its time-series properties, a necessary condition for cointegration. To obtain an estimate of the U_t the cointegrating regression is estimated by OLS called the "static" or "Engle-Granger" regression since the dynamics are ignored. Stock (1987) has shown that the OLS estimates of the cointegrating regression parameters are superconsistent converging faster than in the stationary case.

As in § 1.2.2, in the case of the Dickey-Fuller unit root test, we check whether the DGP is known to the researcher, ie whether economic theory has any a priori hypotheses about the coefficients in the cointegrating regression. If this is the case and the coefficients are known then one can proceed and conduct a Dickey-Fuller type unit root test on the residuals and use the Fuller (1976) and Dickey and Fuller (1981) critical values. If this is not the case and the coefficients are estimated by OLS then other critical values must be used with the Dickey-Fuller test for unit roots. These again are obtained by Monte Carlo simulation. If the U_t are serially correlated then Phillips and Perron (1988) or an ADF regression will be used for the unit root tests as discussed in chapter 1. A summary of the Phillips-Ouliaris-Hansen tests for cointegration is given in Hamilton (1994) Table 19.1.

An alternative method of computing critical values has come from MacKinnon (1991) who has provided response surfaces for calculating critical values appropriate for cointegration tests, which are applicable whatever the sample size. MacKinnon (1991) response surfaces have the general form

$$C(\alpha, T) = \kappa_\infty + \kappa_1/T + \kappa_2/T^2 \tag{2.1}$$

where $C(\alpha, T)$ is the one-sided $\alpha\%$ critical value for a sample of size T. κ_∞, κ_1 and κ_2 are given in a table of values by MacKinnon (1991) for various cases of constant and/or trend.

2.4 Likelihood based Tests for Cointegration

2.4.1 The Johansen Test

A second method for testing for cointegration comes from Johansen (1988,1991,1995) using a multivariate maximum likelihood approach. The cointegrating regression can be written

$$y_t = \alpha + \beta_1 X_{1t} +, \ldots, +\beta_k X_{kt} + u_t, \tag{2.2}$$

where $y_t, X_{1t}, \ldots, X_{kt}$ are I(1) variables and u_t is a stationary disturbance term. The framework of Johansen's (1991) cointegrating Vector Autoregression (VAR) is given by the following general Vector Error Correction model (VECM)

$$\Delta y_t = a_0 + a_1 t + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \psi w_t + \varepsilon_t, \tag{2.3}$$

for $t = 1, \ldots, T$.

Where y_t is an $(m \times 1)$ vector of jointly determined (endogenous) I(1) variables, w_t is a $(q \times 1)$ vector of exogenous/deterministic I(0) variables, excluding the intercepts and/or trends. The disturbance vector ε_t satisfies the following assumption

$$\varepsilon_t \sim i.i.d(0, \Sigma) \tag{2.4}$$

where Σ is a positive-definite matrix. The disturbances of the model ε_t are distributed independently of w_t, ie $E(\varepsilon_t | w_t) = 0$. The intercept and trend coefficients, a_0 and a_1 are $(m \times 1)$ vectors, Π is the long run multiplier matrix of order $(m \times m)$ $\Gamma_1, \Gamma_2, \ldots, \Gamma_{p-1}$ are $(m \times m)$ coefficient matrices capturing the short run dynamic effects and ψ is the $(m \times q)$ matrix of coefficients on the I(0) exogenous variables.

2.4.2 Cointegrating relations

The cointegrating VAR analysis is concerned with the estimation of equation (2.3) when the rank of the long run multiplier matrix Π could be at most equal to m. Therefore rank deficiency of Π can be represented as

$$H_0 : Rank(\Pi) = r < m. \tag{2.5}$$

Here we can write $\Pi = \alpha\beta'$ where α and β are $(m \times r)$ matrices each with full column rank r. In the case where Π is rank deficient we have $y_t \sim$ I(1), $\Delta y_t \sim$ I(0) and $\beta' y_t \sim$ I(0). The $(r \times 1)$ trend-stationary relations $\beta' y_t$ are referred to as the cointegrating relations and characterize the long run (steady state) of the VECM equation (2.3). This model can be used to examine the relationship between $y_t, X_{1t} \ldots X_{kt}$. An important feature of the Johansen model is that it is a multivariate systems framework. In the case of more than two regressors the single equation methodology of Engle and Granger breaks down since there can be more than one cointegrating relation between the variables. The framework of Johansen can accommodate up to

N cointegrating relations among $(N+1)$ variables.

2.5 Cointegration Tests with Panel Data

2.5.1 Cointegration Tests for Homogeneous Panels

We discuss here the residual based cointegration tests for homogeneous panels of Kao (1999) and Kao, Chiang and Chen (1999).

2.5.2 The Kao Tests

Consider the panel analogue of equation (2.2)

$$y_{it} = \alpha_i + \beta_1 X_{1it}, \ldots, +\beta_k X_{kit} + u_{it}, \qquad (2.6)$$

where $y_{it}, X_{1it}, \ldots, X_{kit}$ are assumed integrated processes of order one, $\forall i$, α_i an individual effect and u_{it} a stationary disturbance term. Thus equation (2.6) describes a system of cointegrated regressions with $y_{it}, X_{1it}, \ldots, X_{kit}$ assumed independent across cross-sectional units and with u_{it}.

Kao (1999), McCoskey and Kao (1999), Kao and Chiang (2000), Kao, Chiang and Chen (1999), Phillips and Moon (1999) and Pedroni (2004) analyse similar models for panel cointegration in homogeneous panels. Here we discuss the tests of Kao (1999) and Kao, Chiang and Chen (KCC) (1999) for whether a cointegrating relationship exists in the estimated equation. A DF type test is computed from the estimated residuals of equation (2.6) using

$$\hat{u}_{it} = \gamma \hat{u}_{it-1} + \pi_{it} \tag{2.7}$$

where $\hat{u}_{it}$ are the estimated residuals and π_{it} is assumed a white noise error term. To test the null hypothesis of no cointegration the null is written as $H_0 : \gamma = 1$. The OLS estimate, $\hat{\gamma}$, of γ can be given as

$$\hat{\gamma} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T} \hat{u}_{it}\hat{u}_{it-1}}{\sum_{i=1}^{N}\sum_{t=2}^{T} \hat{u}_{it-1}^2}. \tag{2.8}$$

Kao (1999) constructs five DF type tests using $\hat{\gamma}$, which we now show

(a) $DF_{\gamma} = \frac{T\sqrt{N}(\hat{\gamma}-1)+3\sqrt{N}}{\sqrt{10.2}}$.

(b) $DF_t = \sqrt{1.25}t_{\gamma} + \sqrt{1.875N}$.

(c) $DF_{\gamma}^{*} = \frac{\sqrt{N}T(\hat{\gamma}-1)+\left(3\sqrt{N}\hat{\sigma}_v^2/\hat{\sigma}_{0v}^2\right)}{\sqrt{3+\left(7.2\hat{\sigma}_v^4/\hat{\sigma}_{0v}^4\right)}}$.

(d) $DF_{t}^{*} = \frac{t_{\gamma}+\left(\sqrt{6N}\hat{\sigma}_v/2\hat{\sigma}_{0v}\right)}{\sqrt{\left(\hat{\sigma}_{0v}^2/2\hat{\sigma}_v^2\right)+\left(3\hat{\sigma}_v^2/10\hat{\sigma}_{0v}^2\right)}}$.

(e) $ADF = \frac{t_{ADF}+\left(\sqrt{6N}\hat{\sigma}_v/2\hat{\sigma}_{0v}\right)}{\sqrt{\left(\hat{\sigma}_{0v}^2/2\hat{\sigma}_v^2\right)+\left(3\hat{\sigma}_v^2/10\hat{\sigma}_{0v}^2\right)}}$.

Here t_{γ} is the t-statistic of $\hat{\gamma}$, $\hat{\sigma}_v^2 = \Sigma_u - \Sigma_{u\epsilon}\Sigma_{\epsilon}^{-1}$ and $\hat{\sigma}_{0v}^2 = \Omega_u - \Omega_{u\epsilon}\Omega_{\epsilon}^{-1}$ the contemporaneous covariance and long run covariance matrices[1] and where t_{ADF} is the t-statistic of $\hat{\gamma}$ in the ADF regression $\hat{u}_{it} = \gamma\hat{u}_{it-1} + \sum_{j=1}^{p}\theta_j\Delta\hat{u}_{it-j} + \pi_{it}$. The asymptotic distributions of $DF_{\gamma}, DF_t, DF_{\gamma}^{*}, DF_t^{*}$ and ADF converge to a standard normal distribution $N(0,1)$.

[1]See chapter 3 for a detailed discussion of these matrices. In the above these matrices are scalars.

2.5.3 Cointegration Tests for Heterogeneous Panels

We shall discuss here two types of cointegration tests for heterogeneous panels. The first is a residual based test proposed by Pedroni (1999), whilst the second a likelihood based test proposed by Larsson, Lyhagen and Lothgren (LLL) (2001).

2.5.4 The Pedroni Tests

Consider the following panel cointegrating regression

$$y_{it} = \alpha_i + \delta_i t + \beta_{1i} X_{1it}, \ldots, + \beta_{ki} X_{kit} + u_{it}, \tag{2.9}$$

where $y_{it}, X_{1it}, \ldots, X_{kit}$ are as before. However now the slope coefficients β_{1i} and β_{2i}, etc. are permitted to vary across the individual members of the panel. Pedroni (1999) focuses on reporting critical values for the null hypothesis of no cointegration versus cointegration in the panel cointegration regression of equation (2.9). Pedroni derives the asymptotic distribution and explores the small sample performance of seven different panel statistics for the above model. We report all of the panel statistics here:

(a) The Panel v-statistic

$$T^2 N^{\frac{3}{2}} Z_{\hat{v}NT} = T^2 N^{\frac{3}{2}} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \hat{L}_{11i}^{-2} \hat{u}_{it-1}^2 \right)^{-1}. \tag{2.10}$$

(b) The Panel ρ-statistic

$$(2.11) T\sqrt{N}Z_{\hat{\rho}NT-1} = T\sqrt{N}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{u}_{it-1}^{2}\right)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_{i}).$$

(c) The Panel t-statistic (nonparametric)

$$(2.12) \quad Z_{tNT} = \left(\tilde{\sigma}_{NT}^{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{u}_{it-1}^{2}\right)^{-\frac{1}{2}}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_{i}).$$

(d) The Group ρ-statistic

$$(2.13) TN^{-\frac{1}{2}}\tilde{Z}_{\hat{\rho}NT-1} = TN^{-\frac{1}{2}}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{u}_{it-1}^{2}\right)^{-1}\sum_{t=1}^{T}(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_{i}).$$

(e) The Group t-statistic (nonparametric)

$$(2.14) \quad N^{-\frac{1}{2}}\tilde{Z}_{tNT} = N^{-\frac{1}{2}}\sum_{i=1}^{N}\left(\hat{\sigma}_{i}^{2}\sum_{t=1}^{T}\hat{u}_{it-1}^{2}\right)^{-\frac{1}{2}}\sum_{t=1}^{T}(\hat{u}_{it-1}\Delta\hat{u}_{it}-\hat{\lambda}_{i}).$$

(f) The Panel t-statistic (parametric)

$$(2.15) \qquad Z_{tNT}^{*} = \left(\tilde{s}_{NT}^{*2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{u}_{it-1}^{*2}\right)^{-\frac{1}{2}}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}(\hat{u}_{it-1}^{*}\Delta\hat{u}_{it}^{*}).$$

(g) The Group t-statistic (parametric)

$$(2.16) \qquad N^{-\frac{1}{2}}\tilde{Z}_{tNT}^{*} = N^{-\frac{1}{2}}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{s}_{i}^{*2}\hat{u}_{it-1}^{*2}\right)^{-\frac{1}{2}}\sum_{t=1}^{T}(\hat{u}_{it-1}^{*}\Delta\hat{u}_{it}^{*}).$$

Where

$$(2.17) \qquad \hat{\lambda}_{i} = \frac{1}{T}\sum_{s=1}^{k_i}\left(1-\frac{s}{k_i+1}\right)\sum_{t=s+1}^{T}\hat{\mu}_{it}\hat{\mu}_{it-s}$$

and

$$(2.18) \qquad \hat{L}_{11i}^{2} = \frac{1}{T}\sum_{t=1}^{T}\hat{\eta}_{it}^{2} + \frac{2}{T}\sum_{s=1}^{k_i}\left(1-\frac{s}{k_i+1}\right)\sum_{t=s+1}^{T}\hat{\eta}_{it}\hat{\eta}_{it-s}.$$

Also $\hat{s}_i^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{\mu}_{it}^2$, $\hat{\sigma}_i^2 = \hat{s}_i^2 + 2\hat{\lambda}_i$, $\tilde{\sigma}_{NT}^2 = \frac{1}{N}\sum_{i=1}^{N}\hat{L}_{11i}^{-2}\hat{\sigma}_i^2$, $\hat{s}_i^{*2} = \frac{1}{T}\sum_{t=1}^{T}\hat{\mu}_{it}^{*2}$ and $\tilde{s}_{NT}^{*2} = \frac{1}{N}\sum_{i=1}^{N}\hat{s}_i^{*2}$, $\hat{\mu}_{it}, \hat{\mu}_{it}^*$ and $\hat{\eta}_{it}$ are obtained from the auxiliary regresions $\hat{u}_{it} = \hat{\gamma}_i\hat{u}_{it-1} + \hat{\mu}_{it}$, $\hat{u}_{it} = \hat{\gamma}_i\hat{u}_{it-1} + \sum_{k=1}^{K_i}\hat{\gamma}_{ik}\Delta\hat{u}_{it-k} + \hat{\mu}_{it}^*$ and $\Delta y_{it} = \hat{\beta}_{1i}\Delta X_{1it} + \ldots + \hat{\beta}_{ki}\Delta X_{kit} + \hat{\eta}_{it}$. For the Panel ρ-statistic the test for the null of no cointegration is

$$H_0 : \gamma_i = 1, \quad \forall i, \tag{2.19}$$

$$\text{against} \quad H_1 : \gamma_i = \gamma < 1, \quad \forall i. \tag{2.20}$$

Under certain assumptions Pedroni shows that following an appropriate standardisation each of the seven panel statistics are distributed as a standard normal distribution as $N \to \infty$ and $T \to \infty$. The particular standardisations required are computed by Monte Carlo simulation and tabulated in Pedroni's paper.

2.5.5 The Larsson, Lyhagen and Lothgren Test

LLL develop a panel cointegration test based on the multivariate maximum likelihood framework of Johansen (1988,1991,1995). The heterogeneous panel VECM is given by

$$\Delta y_{it} = \Pi_i y_{it-1} + \sum_{k=1}^{k_i-1}\Gamma_{ik}\Delta y_{it-k} + \varepsilon_{it} \tag{2.21}$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

Where Π_i is of order $(p \times p)$. If Π_i is of reduced rank we may let $\Pi_i = \alpha_i\beta_i'$

where α_i and β_i are of order $(p \times r_i)$ and of full column rank. Note that T must be large enough[2] so that equation (2.21) can be estimated separately for each group. We are interested in testing the hypothesis that all of the matrices $\Pi_i, \quad \forall i$, have rank $\leq r$. That is, we consider testing the hypothesis that all of the N groups in the panel have at most r cointegrating relations among the p variables. The following rank hypothesis is considered

$$H_0 : rank(\Pi_i) = r_i \leq r, \quad \forall i, \tag{2.22}$$

$$\text{against} \quad H_1 : rank(\Pi_i) = p, \quad \forall i. \tag{2.23}$$

Denote the trace statistic for group or country i, obtained from equation (2.3), as $LR_{iT}\{H(r)|H(p)\}$. Now define the LR-bar statistic as the average of the N individual trace statistics

$$\bar{LR}_{NT}\{H(r)|H(p)\} = \frac{1}{N}\sum_{i=1}^{N} LR_{iT}\{H(r)|H(p)\}. \tag{2.24}$$

LLL use a standardised LR-bar statistic for their panel cointegration rank test defined by

$$\Psi_{\bar{LR}}\{H(r)|H(p)\} = \frac{\sqrt{N}(\bar{LR}_{NT}\{H(r)|H(p)\} - E(Z_k))}{\sqrt{Var(Z_k)}}, \tag{2.25}$$

where $E(Z_k)$ and $Var(Z_k)$ are the mean and variance of the asymptotic trace statistic. These are computed by Monte Carlo simulations and are also given

[2]LLL states that the rule $T > Np+2$ should be used, where p is the number of variables.

in LLL. The $\Psi_{\bar{LR}}\{H(r)|H(p)\}$ statistic converges weakly in distribution to $N(0,1)$ as $T, N \to \infty$ and $\frac{N}{T} \to 0$.

2.5.6 Other tests for panel cointegration

A notable other test for cointegration in panels is the LM test of McCoskey and Kao (1998). This has the often more attractive null hypothesis of cointegration. Some finite sample Monte Carlo simulations have been performed by McCoskey and Kao (1998b) and Wu and Yin (1999). McCoskey and Kao (1998b) compare their LM statistic to two of Pedroni's (1999) statistics and find the LM test outperforms the other two.

More recent developments has seen the field of cross-sectional dependence extend to panel cointegration as in Pedroni and Vogelsang (2005) and also Chang (2005) who proposes residual based tests for cointegration in dependent panels. Banerjee and Carrion-i-Silvestre (2005) consider structural breaks and panel cointegration. Hu (2005) considers panel cointegration in panels of a mixture of I(0) and I(1) variables.

2.6 An Empirical Application

2.6.1 Testing for Long Run PPP with Panel Data

Though purchasing power parity (PPP) perfoms poorly in the short run, many economists still hold the view that over the long run, relative prices may move in proportion to the change in the nominal exchange rate, so that the real exchange rate will revert to its parity. This section aims to add support to this belief by using quite recent panel methods to test for PPP. Moreover we combine detailed evidence from both panel unit root tests and panel cointegration tests to study long run PPP, an approach that has so far not yet been undertaken with OECD datasets[3]. However many authors have commented one needs to be cautious when forming conclusions based on panel unit root and cointegration tests. There is always the possibility of a large range of outcomes due to the different hypotheses being tested, different power properties between tests and panels with different mixes of variables. Hence it may be impossible to get all tests to give the same result. Early empirical time-series studies for PPP are Adler and Lehman (1983), Frenkel (1981,1981b). More recent tests for the post Bretton Woods period are Meese and Rogoff (1988), Eddison and Pauls (1993). Empirical studies spanning longer time-series are Abauf and Jorion (1990), Lothian and Taylor (1996), Taylor (2002), Lee (1978) and Officer (1982). Most recently applied researchers have started to use panel methods to test for PPP, especially for the post Bretton Woods period, and with great success, ie. Oh (1996), Papell (1997), Wu (1996), MacDonald (1996) and Coakley and Fuertes (1997).

[3]Only one other study has used this approach-Cerrato and Sarantis (2003) for a panel of blackmarket exchange rates.

Recently PPP has been studied in panels with cross-sectional dependence, see O'Connell (1998) and Harris, Leybourne and McCabe (2005).

2.6.2 The Dataset

Our dataset is quarterly observations over the period 1957Q1-1991Q2, for 25 OECD countries obtained from the IMF International Financial Statistics (IFS). So $N = 25$ and $T = 138$. Thus E_t was taken as the market exchange rate per U.S. Dollar, P_t the domestic consumer price level (CPI) and P_t^* the foreign consumer price level (CPI), for each country.

2.6.3 The Econometric Methodology

One way to test for PPP is to test if the real exchange rate has a unit root. If PPP is to hold in the long run any shocks to the real exchange rate would be only transitory and the real rate should be mean reverting (or stationary). A strong form of PPP is as follows, let

$$q_t = e_t + p_t^* - p_t, \tag{2.26}$$

where q_t is the logarithm of the real exchange rate, e_t is the logarithm of the nominal exchange rate, p_t^* is the logarithm of the foreign price level and p_t is the logarithm of the domestic price level. We can use the augmented Dickey-Fuller (ADF) method to test for unit roots in q_t, utilising lags of differenced

q_t to counter any serial correlation in the time-series. The null hypothesis is $H_0 : \alpha = 0$ and $H_1 : \alpha < 0$.

Thus we have[4]

$$\Delta q_t = \mu + \alpha q_{t-1} + \sum_{j=1}^{k} \theta_j \Delta q_{t-j} + \varepsilon_t, \tag{2.27}$$

where μ is the intercept, α and θ_j the parameters of interest and ε_t a white noise disturbance term. Our choice of lag length k is again data determined rather than fixed a priori. We use the method suggested by Campbell and Perron (1991) discussed in chapter 1.

A second method for testing for long run PPP is by using the cointegration methods discussed above in § 2.3 and § 2.4 . The cointegrating regression can be written

$$e_t = \alpha + \beta_1 p_t + \beta_2 p_t^* + u_t, \tag{2.28}$$

where e_t, p_t and p_t^* are as before and u_t is a stationary disturbance term. We proceed by first testing that each of e_t, p_t and p_t^* is I(1) and then show that some linear combination of them, ie a cointegrating regression, is I(0). If long run PPP holds then e_t should be cointegrated with p_t and p_t^*. A strong PPP hypothesis requires the cointegrating vector to satisfy joint symmetry and proportionality conditions $\beta_1 = -\beta_2 = 1$. Whilst a weak PPP hypothesis might allow $0 < \beta_1 < 2$ and $-2 < \beta_2 < 0$.

[4]See the AR(1) model on p.7 for an example of the re-parameterisation of the autoregressive coefficient which is used here.

2.6.4 The Estimation Results

Visual inspection of the time-series graphs of nominal exchange rates in Figure 2.01 of Appendix 4 show them as stationary after first differencing. Also the autocorrelation functions in Figure 2.03 show a large degree of persistence which dissappears on taking first differences. In the graphs of Figure 2.01 the flat portions up to the 1970's reflect the fixed exchange rate system that existed prior to the breakdown of the Bretton Woods system in 1973. After this a floating exchange rate system existed in most countries. This change in exchange rate regime is a good example of a structural break which could be incorporated into our panel unit root and cointegration tests by the methods of Culver and Papell (1997), Im, Lee and Tieslau (2005) and Banerjee and Carrion-i-Silvestre (2005). One problem revealed here is that many countries exchange rates seem to move together, ie there are cross cointegrating relations between countries, eg the U.K. and Ireland have seemingly identical nominal exchange rates right up to the 1980's. It seems that some countries like the U.K. and Ireland may have their exchange rates pegged together. However current panel unit root and cointegration tests rule out the existence of such relationships,

$$E_{it} \neq E_{jt} \quad \text{and} \quad P_{it} \neq P_{jt} \quad \forall i, j. \tag{2.29}$$

See Banerjee, Marcellino and Osbat (2004,2005) and Pedroni (2001) for a discussion. This is one form of cross-sectional dependence which causes causes size distortions and severe loss of power (see O'Connell (1998)) in the panel unit root and cointegration tests if not catered for. Also on inspecting Figure 2.11 for stationarity in real exchange rates shows that most countries have real exchange rates with mean reversion occuring after shocks. See also A. Taylor (2002). This highlights the well known stylised fact of exchange rates being very sensitive to common global shocks across countries.

2.6.5 Panel Unit Root Tests

We use here the LL and IPS panel data unit root tests discussed in chapter 1 for the strong PPP model. To cater for serial correlation we have the following ADF test. Here the null is $H_0 : \rho_1 = \rho_2 =, \ldots, = \rho_N = \rho = \delta_i = 0$ and $H_1 : \rho_1 = \rho_2 =, \ldots, = \rho_N = \rho < 0, \delta_i \in \Re$

$$\Delta q_{it} = \rho_i q_{it-1} + \sum_{j=1}^{K_i} \theta_{ij} \Delta q_{it-j} + \alpha_i + \theta_t + \delta_i t + \varepsilon_{it}. \tag{2.30}$$

for $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$.

Thus the autoregressive model incorporates a time trend and individual and time-specific effects. Before conducting the panel unit root tests both LL and IPS also suggest an adjustment to account for any cross-sectional dependence if any is suspected in the panel. Assuming a single aggregate common factor having identical impact on all individuals in the panel, LL and IPS suggest eliminating the influence of the aggregate effects by subtracting cross-section averages from each variable. This removes the influence of the limited degree of cross-sectional dependence coming from the time-specific aggregate effects θ_t in equation (2.30). Thus use the demeaned series $q_{it} - \bar{q}_t$, where $\bar{q}_t = \frac{1}{N}\sum_{i=1}^{N} q_{it}$.

Table 2.01 **Individual Country ADF Regression Estimates**

COUNTRY	e_t t-stat	lag	p_t t-stat	lag	Δp_t t-stat	lag
Australia	-1.779	3	-2.483	4T	-2.048	3
Austria	-2.567	3T	-2.186	4T	-3.128[b]	3
Belgium	-2.097	3	-2.760	3T	-2.315	2
Canada	-1.445	3T	-2.911	4T	-1.637	3
Denmark	-2.281	3	-2.089	2T	-4.715[a]	1
Finland	-2.537	4T	-1.168	2	-1.719	1
France	-2.881	4T	-3.914[b]	4T	-3.444[c]	3T
Germany	-2.475	3T	-2.922	4T	-2.144	3
Greece	-1.139	4T	-1.620	5T	-3.526[b]	4T
Iceland	-1.449	0T	-2.166	4T	-6.372[a]	3T
Ireland	-3.123	4T	-1.988	1T	-1.281	3
Italy	-2.472	4T	-1.288	2	-1.578	1
Japan	-2.366	5T	-1.379	4T	-2.814[c]	3
Luxembourg	-2.097	3	-2.870	4T	-1.950	3
Mexico	-1.262	3T	-1.023	1	-3.898[a]	1
Netherland	-2.610	3T	-2.334	2T	-2.753[c]	3
New Zealand	-2.512	3T	-2.155	4T	-4.063[a]	2
Norway	-1.858	1T	-2.267	4	-3.146[b]	3
Portugal	-1.836	4T	-2.156	4T	-2.330	3
Spain	-2.436	3T	-1.749	2T	-3.519[a]	1
Sweden	-1.498	3	-2.483	4T	-3.091	3T
Switzerland	-2.618	4T	-3.491[b]	4T	-2.683[c]	4
Turkey	-0.033	3T	-2.285	1T	-4.319[a]	4T
U.K.	-3.089	3T	-2.640	4T	-2.482	4
U.S.	-	-	-2.053	3T	-2.053	2

In Table 2.01 we have the results of our individual country ADF tests. As indicated from our graphs all of the nominal exchange rate series accepts the unit root hypothesis and only two countries of the CPI time-series reject it at the 5% significance level, that is France and Switzerland. However a most disturbing characteristic of the panel is that a large number of the individual country CPI time-series exhibit two unit roots. This can be seen on re-inspection of Figures 2.05-2.10 with the CPI series requiring second differencing before showing the signs of a stationary series. Moreover our individual country ADF tests show 12 out of 25 accepting the unit root null in the differenced CPI time-series. This is at the 10% significance level. These are Australia, Belgium, Canada, Finland, Germany, Ireland, Italy Luxembourg, Portugal, Sweden, the UK and the US. This clearly indicates that these are $I(2)$ variables which has important implications for our panel unit root and cointegration methodology. For these variables can no longer form the balanced cointegrating regression in equation (2.28) neither can they seriously be considered to form the real exchange rate equation in equation (2.26). For this reason a sub-panel of 13 countries was used to test for PPP consisting of Austria, Denmark, France, Greece, Iceland, Japan, Mexico, the Netherlands, New Zealand, Norway, Spain, Switzerland and Turkey. Thus 13 countries which when using Japan as numeraire left us with a panel of 12 countries to test for PPP.

In Table 2.02 we have the results of our individual country sub-panel ADF tests on the real exchange rate (RER) together with the demeaned[5] series to cater for the cross-sectional dependence. The time-series graphs for these series are shown in Figures 2.11-2.18. In Table 2.02 only Iceland and Turkey reject the unit root null at the 1% significance level, whilst in the demeaned series France and Mexico also reject it at the same significance level. This indicates that the individual country ADF tests do not give support to long run PPP.

Table 2.02 **Individual Country RER ADF Regression Estimates**

COUNTRY	RER t-stat	lag	Demeaned RER t-stat	lag
Austria	-2.290	3T	-3.398	4T
Denmark	-2.486	3T	-2.574	2T
France	-2.818	4	-4.154[a]	4T
Greece	-2.684	4	-3.396	1T
Iceland	-3.642[a]	4	-4.653[a]	4
Mexico	-3.296	3T	-4.369[a]	3T
Netherland	-2.805	3T	-2.137	4
New Zealand	-2.095	3	-2.009	4
Norway	-2.325	1T	-1.646	3T
Spain	-2.757	3T	-4.061	4T
Switzerland	-2.812	4T	-2.123	5T
Turkey	-3.270[a]	0	-5.742[a]	3T

[5]Pedroni (2001) notes that care must be taken when using this procedure since it can be shown that the demeaned series can become stationary and cointegrating relations destroyed.

Table 2.03 **LL Panel Unit Root Tests**

RER t_ρ^* stat	Demeaned RER t_ρ^* stat
0.00028183	2.994

Table 2.04 **IPS Panel Unit Root Tests**

RER $\Psi_{\bar{t}}$ stat	Demeaned RER $\Psi_{\bar{t}}$ stat
-5.83133^a	-9.44733^a

In Tables 2.03 and 2.04 we have the results of our panel unit root tests. Both the RER and demeaned RER IPS tests reject the unit root null at the 1% significance level, whilst both the LL tests accept the null. Thus support for long run PPP is not clear cut by our panel unit root tests. However if we again take the position we did in chapter 1, in exactly the same situation with inflation time-series, we conclude that given the IPS test is the more powerful of the two tests and that its null and alternative hypothesis are more reasonable and its results compatible also with the evidence at the individual level, then we can give support to this panel test. In general the recent empirical evidence using panel data is in support of long run PPP. However, we often have, on the one hand, the inability of the individual country ADF tests to reject the null hypothesis of a unit root, and on the other, the overwhelming support for mean reversion, ie PPP, obtained by panel methods. One of the reasons for this stark contrast in results is attributed to the low power of the time-series unit root tests. Similar results are reported by MacDonald

(1996) using annual data for 20 OECD countries and Coakley and Fuertes (1997) using monthly data for 10 OECD countries, both for the post Bretton Woods period. Papell (1997) and Wu (1996) report similar results.

2.6.6 Panel Cointegration Tests

We can also use the panel cointegration tests discussed in § 2.5 to test for long run PPP. In Table 2.05 we briefly note the results of our Phillips-Ouliaris-Hansen individual country cointegration tests. In the regression with a constant only two countries are able to reject the null hypothesis of no cointegration. This again a symptom of the low power problem. McCoskey and Kao (1999) report similar problems.

Table 2.05 **Phillips-Ouliaris-Hansen Country Cointegration Tests**

COUNTRY	ADF t-stat (Const)
Austria	-3.0856
Denmark	-2.5821
France	-3.2365
Greece	-3.7800
Iceland	-3.9069[b]
Mexico	-3.3690
Netherland	-2.7879
New Zealand	-3.2012
Norway	-2.4018
Spain	-3.0537
Switzerland	-2.9494
Turkey	-4.3623[a]

The results of the panel cointegration tests are shown in Tables 2.06, 2.07 and 2.08. In Table 2.06 we see the null of no cointegration being rejected by all of the Kao statistics at the 1% level of significance. We can conclude that the Kao[6] test results strongly support the alternative hypothesis of cointegration. In Table 2.07 we have the results of the Pedroni[7] panel cointegration tests. The panel cointegration statistics are of two types. The first are within-dimension-based statistics termed, "Panel statistics". The second are between-dimension-based statistics termed, "Group statistics". Many of the test statistics are nonparametric tests that correct for serial correlation in analogous ways as the Phillips and Perron (1988) statistic. The issue of which test to use in a particular circumstance often arises. The nonparametric tests are robust to outliers but have poor size properties. The parametric tests have greater power when modelling processes with AR(p) errors. To compute the panel statistics an estimate of $\hat{L}_{11i}^2$, the long-run variance of $\hat{\eta}_{it}$, is needed. Pedroni (1999) recommends the Newey-West (1987) estimator for this. To calculate this type of estimator one needs an estimate of k_i for the lag window.[8] To estimate k_i the auto-correlation functions (ACF) of the residuals, $\hat{u}_{it}$,$\hat{\mu}_{it}$, $\hat{\eta}_{it}$ and $\hat{\mu}_{it}^*$ were inspected for the lag length of the decay in the residual auto-correlation. These ACF's are shown in Figures 2.19-2.22 in Appendix 4. In Figure 2.19 we see the auto-correlations of $\hat{u}_{it}$, just dying

[6]Kao test statistics computed using a pooled panel DOLS regression with fixed effects.
[7]The Pedroni tests computed using residuals obtained from equation by equation OLS regressions.
[8]Pedroni uses the Bartlett lag window in his paper.

out after 12 lags. Whilst in Figures 2.20-2.22, for the ACF's of $\hat{\mu}_{it}$, $\hat{\eta}_{it}$ and $\hat{\mu}^{*}_{it}$ respectively, we see the lag decay in the residuals not significantly different statistically from zero (indicated by the lags being contained within the confidence band of zero). For this reason k_i could be set to zero for both $\hat{\lambda}_i$ and $\hat{L}^2_{11i}$, however even when k_i is set to 10 the change in results is negligible. After computing the test statistics, for the model with a constant only, we found that the null of no cointegration was rejected by 4 out of 7 of the Pedroni panel cointegration test statistics, in three instances, at the 1% level of significance. For the model with a constant and trend, 2 out of 7 of the Pedroni panel cointegration test statistics rejected the null of no cointegration, but in 1 case, only at the 10% significance level. We find that with 6 out of 14 Pedroni panel cointegration tests rejecting the null of no cointegration the results are inconclusive.

Table 2.06 **Kao Panel Cointegration Tests**

	DF_t-stat	DF_γ-stat	DF^*_t-stat	DF^*_γ-stat	ADF-stat
Constant	125.86[a]	-10.535[a]	92.827[a]	-20.735[a]	84.2343[a]

Table 2.07 **Pedroni Panel Cointegration Tests**

	Panel-v	Panel-ρ	Panel-t	Group-ρ	Group-t	Panel-t^P	Group-t^P
Const	4.5367[a]	-1.8083[b]	2.5585	-1.1366	4.2209	-159.9640[a]	-4.3854[a]
Const+Tr	1.2813[c]	0.5622	5.2279	1.4144	6.7065	-153.2620[a]	-1.1464

We turn now to the likelihood based tests for cointegration which in certain circumstances will have more power than residual based tests. These results are given in Table 2.08. We show here the cointegration LR test trace statistics for each country, in columns five, six and seven. In 8 out of 12 countries the null hypothesis of $r = 1$ cointegrating vectors was chosen correctly. After normalising on p_t the cointegrating vector for strong PPP becomes $(1, -1, -1)$. This strong PPP hypothesis was tested by imposing the over-identifying restrictions $(1, -1, -1)$ on the cointegrating vector. The results shown in the third column of Table 2.08 show 9 out of 12 countries rejecting the strong PPP hypothesis. Also in Table 2.08 we find, using LLL panel cointegration tests, that there is evidence there exists a common cointegration rank in the panel, or at least a common largest rank of 2. This seemingly conflicts with the earlier results of the individual country Johansen LR tests, where 8 of the 12 cointegration vectors showed a rank of one. However due to the fact that three of the other cointegration rank estimates gave an $r = 2$ and one an $r = 0$ we conclude the LLL results are compatible with the Johansen results in that the LLL results only indicate a common cointegrating vector of maximum rank 2, exists for each country, and the possibility that this is of rank $r = 1$ is not ruled out. The more conclusive results of the likelihood based tests are comforting given the ambiguity of the Pedroni residual based tests. We can now conclude more surely from both panel unit root and cointegration tests there exists a long run relationship in the panel between exchange rates, domestic and foreign prices. These results do coincide with the findings of a number of empirical studies. Jacobson, Lyhagen, Larsson and Nessen (2002) use a multivariate VECM panel model for four OECD countries to test for PPP and find support for a weak form of PPP of the type discussed in this chapter. Their estimated cointegrating vector was $(1, -1.5, 0.9)$ instead of the strong form of $(1, -1, 1)$. A. Taylor (2002) finds evidence for the PPP hypothesis using the Johansen multivariate VECM and panels of up to six OECD and developing countries. Finally Pedroni (2001) decisively rejects the strong PPP hypothesis using his fully modified OLS and DOLS estimators and his panel cointegration tests for a panel of 20 OECD and developing countries.

Table 2.08 **LLL Panel Cointegration Tests**

COUNTRY	Lag	chi	rank	Cointegration LR Trace Statistics $LR_{iT}(H(r)\|H(p))$		
				r=0	r=1	r=2
Austria	4	23.39[a]	1	35.700	8.8566	1.4046
Denmark	3	19.56[a]	1	32.302	10.037	1.3893
France	2	21.99[a]	1	40.662	12.535	5.7219
Greece	4	23.31[a]	1	42.255	10.125	2.2571
Iceland	2	13.52[a]	2	42.139	17.855	3.0748
Mexico	2	5.059	1	33.047	12.227	2.4856
Netherland	2	17.04[a]	1	31.464	10.960	1.0352
New Zealand	2	25.57[a]	2	58.149	20.398	2.1970
Norway	3	20.42[a]	1	30.960	9.5516	0.8944
Spain	2	38.71[a]	1	55.921	14.121	2.9815
Switzerland	2	3.960	0	12.427	4.1593	0.0056
Turkey	4	8.325	2	43.963	18.307	2.1844
Aver. Trace				38.249	12.427	2.1359
$E(Z_k)$				14.955	6.086	1.137
$Var(Z_k)$				24.733	10.53	2.212
Statistic				16.225	6.769	2.326[b]

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Notes to the Tables

a) means significant at the 1% level, b) means significant at the 5% level, c) means significant at the 10% level. DF(constant only) critical values $1\% = -3.51, 5\% = -2.89, 10\% = -2.58$. DF(constant+trend) critical values $1\% = -4.04, 5\% = -3.45, 10\% = -3.15$. Phillips-Ouliaris-Hansen(constant only) critical values $1\% = -4.31, 5\% = -3.77$. Phillips-Ouliaris-Hansen(constant+trend) critical values $1\% = -4.36, 5\% = -3.80$. MacKinnon (constant only) critical values $1\% = -4.40, 5\% = -3.80$. N(0,1) one-sided (LHS) critical values $1\% = -2.33, 5\% = -1.65$. N(0,1) one-sided (RHS) critical values $1\% = 2.33, 5\% = 1.65, 10\% = 1.28$ N(0,1) two-sided critical values $1\% = \pm 2.58, 5\% = \pm 1.96$. T denotes model estimated with a constant and trend. Chi-squared statistic for test of over-identifying restrictions of (1,-1,-1) normalised on p_t. $\chi^2(2)$ critical values $1\% = 9.210, 5\% = 5.991$. Lag lengths chosen by Ng and Perron (1995) method.

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Chapter 3

Panel Data Cointegrating Regressions

3.1 Introduction

In chapters 1 and 2 we discussed panel unit root tests and panel cointegration tests respectively. In this chapter we complete the analysis by considering the estimation and inference of a panel data cointegrating regression. We highlight the FMOLS and DOLS panel estimators of Kao and Chiang (2000) and Pedroni (2000,2001).

Our contribution in this chapter is to provide an in depth study of consumption expenditure in a panel of 20 OECD countries. It's originality lies in the fact that a large number of panel cointegration estimators are used in the study. As well as comparing the panel DOLS and FMOLS estimators we also contrast the group-mean and pooled estimators in current use. We also note the important extension of the model to the estimation and inference of a panel data cointegrating regression with cross-sectional dependence. These modifications to the panel cointegration estimator have only recently been introduced and so the results could be seen as very illuminating.

The sections are as follows. In section 3.2 we have the efficient estimation of a panel data cointegrating regression, whilst in section 3.3 we have the estimation of a panel data cointegrating regression with cross-sectional dependence. In section 3.4 we present the empirical application.

3.2 The Efficient Estimation of a Panel Data Cointegrating Regression

With the increasing use of nonstationary panel data the focus of panel data econometrics has shifted towards the study of the asymptotics of macro panels, with large N (eg individuals) and large T (eg time-series), as opposed to the usual asymptotics of micro panels with large N and small T. This has necessitated the development of a new limit theory for nonstationary panel data, ie limit distributions for double indexed integrated processes, by Phillips and Moon (1999,2000). It was found that the statistical properties of the nonstationary panel data were very different from those of the nonstationary time-series data-the estimators of the former converging to Gaussian normal variates in the limit, whilst those of the latter had nonstandard limiting distributions that were composed of Brownian Motion functionals. The differences in the asymptotic statistical properties of the nonstationary panels have been highlighted by Kao and Chiang (1998,2000), Phillips and Moon (1999) and Pedroni (1996) in their works on the panel Fully Modified OLS (FMOLS), DOLS and OLS panel cointegration estimators. These works extend the field of panel cointegration to the estimation and inference of cointegrated regressions with panel data.

The FMOLS estimator of Phillips and Moon (1999) and Pedroni (2000) is the panel analogue of the Phillips and Hansen (1990) FMOLS estimator of the time-series literature. These FMOLS estimators use nonparametric corrections for bias and endogeneity problems in the OLS estimator. Similarly the DOLS estimator of Kao and Chiang (1998,2000) and Mark and Sul (1999), can be seen as the panel analogue of the Saikkonen (1991) and Stock and Watson (1993), DOLS estimators of the time-series literature. These DOLS estimators add leads and lags of the differenced regressors into the regression as parametric corrections for the bias and endogeniety problems. They are asymptotically equivalent to their FMOLS counterparts.

Since the introduction of these panel cointegration estimators a few Monte Carlo simulation studies of their finite sample properties, and some empirical applications, have appeared in the panel data literature. In a simulation study Kao and Chiang (2000) obtained mixed results for the FMOLS and OLS estimators and DOLS seemed more promising than both in estimating panel cointegration regressions. Kao, Chiang and Chen (1999) applied the panel cointegration methods developed in Kao and Chiang (2000) to study R&D spillovers. They found FMOLS and DOLS produced slightly different results but were unanimous on the main issues. Funk (1998) also studied the same R&D spillovers using the panel cointegration methods developed by Kao (1999), Kao and Chiang (2000) and Pesaran, Shin and Smith (1999). Pedroni (2001) developed group-mean DOLS and FMOLS estimators which are the average of the individual time-series DOLS and FMOLS estimators. He compared his DOLS estimator with the ones of Kao and Chiang (2000) and of Mark and Sul (1999). Sun (2004) proposed new panel cointegration estimators based on exponential kernel estimation. Other empirical applications have come from Ho (2002), Bac and Le Pen (2002), Dreger and Reimers (2003) and Westerland (2003).

3.2.1 The Kao and Chiang Pooled Panel Estimators

Consider the fixed effects panel regression

$$y_{it} = \alpha_i + x_{it}'\beta + u_{it} \tag{3.1}$$

$$\Delta x_{it} = \epsilon_{it} \tag{3.2}$$

where $\{y_{it}\} \sim I(1)$ is (1×1),$\{u_{it}\} \sim I(0)$ is (1×1) and $x_{it} = x_{it-1} + \epsilon_{it}$ so $x_{it} \sim I(1)$ is $(k \times 1)$. Independence is assumed of the $\{y_{it}\}$, $\{x_{it}\}$ and $\{u_{it}\}$ across i. Also the x_{it} are assumed not to be cointegrated.

Let $w_{it} = (u_{it}, \epsilon'_{it})'$.

The long run covariance matrix Ω of w_{it} (see Kao and Chiang (2000)) is

$$\Omega = \Sigma_{j=-\infty}^{\infty} E(w_{ij} w'_{i0}) \tag{3.3}$$

$$= \Sigma + \Gamma + \Gamma' \tag{3.4}$$

$$= \begin{bmatrix} \Omega_u & \Omega_{u\epsilon} \\ \Omega_{\epsilon u} & \Omega_\epsilon \end{bmatrix}. \tag{3.5}$$

The auto-covariance matrix of w_{it} is

$$\Gamma = \Sigma_{j=1}^{\infty} E(w_{ij} w'_{i0}) = \begin{bmatrix} \Gamma_u & \Gamma_{u\epsilon} \\ \Gamma_{\epsilon u} & \Gamma_\epsilon \end{bmatrix} \tag{3.6}$$

and Σ is the contemporaneous covariance matrix

$$\Sigma = E(w_{i0} w'_{i0}) = \begin{bmatrix} \Sigma_u & \Sigma_{u\epsilon} \\ \Sigma_{\epsilon u} & \Sigma_\epsilon \end{bmatrix}. \tag{3.7}$$

The one-sided long run covariance is

$$\Delta = \Sigma + \Gamma \tag{3.8}$$

$$= \Sigma_{j=0}^{\infty} E(w_{ij} w'_{i0}), \tag{3.9}$$

$$\text{with} \quad \Delta = \begin{bmatrix} \Delta_u & \Delta_{u\epsilon} \\ \Delta_{\epsilon u} & \Delta_\epsilon \end{bmatrix}. \tag{3.10}$$

Here we assume that the panels are homogeneous, ie the variances are constant across the cross-section units. When the panels are heterogeneous then,

$\Omega_i, \Gamma_i, \Delta_i$ and Σ_i are different for each i.

FMOLS estimators are formed by making the following corrections:

The endogeneity correction is obtained by

$$x^*_{it} = \hat{\Omega}_{i\epsilon}^{-\frac{1}{2}} x_{it}, \tag{3.11}$$

$$u^*_{it} = \hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}} \hat{u}^+_{it}, \tag{3.12}$$

$$\hat{u}^+_{it} = u_{it} - \hat{\Omega}_{iu\epsilon}\hat{\Omega}_{i\epsilon}^{-1}\epsilon_{it} \tag{3.13}$$

$$\hat{y}^+_{it} = y_{it} - \hat{\Omega}_{iu\epsilon}\hat{\Omega}_{i\epsilon}^{-1}\Delta x_{it} - \hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}}\left(\hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}} - \hat{\Omega}_{i\epsilon}^{-\frac{1}{2}}\right) x_{it}\hat{\beta} \tag{3.14}$$

$$y^*_{it} = \hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}} \hat{y}^+_{it} \tag{3.15}$$

and $\Omega_{iu.\epsilon} = \Omega_{iu} - \Omega_{iu\epsilon}\Omega_{i\epsilon}^{-1}\Omega_{i\epsilon u}$.

Also the correction term $\hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}}\left(\hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}} - \hat{\Omega}_{i\epsilon}^{-\frac{1}{2}}\right) x_{it}\hat{\beta}$ is needed[1].

The serial correlation correction is

$$\hat{\Delta}^+_{i\epsilon u} = (\hat{\Delta}_{i\epsilon u}, \hat{\Delta}_{i\epsilon})\begin{pmatrix} 1 \\ -\hat{\Omega}_{i\epsilon}^{-1}\hat{\Omega}_{i\epsilon u} \end{pmatrix} \tag{3.16}$$

$$= \hat{\Delta}_{i\epsilon u} - \hat{\Delta}_{i\epsilon}\hat{\Omega}_{i\epsilon}^{-1}\hat{\Omega}_{i\epsilon u}. \tag{3.17}$$

In heterogeneous panels then:

The pooled FMOLS estimator is given by

$$\hat{\beta}^*_{fm} = \left[\Sigma_{i=1}^{N}\Sigma_{t=1}^{T}(x^*_{it} - \bar{x}^*_i)(x^*_{it} - \bar{x}^*_i)'\right]^{-1}\left[\Sigma_{i=1}^{N}\left(\Sigma_{t=1}^{T}(x^*_{it} - \bar{x}^*_i)y^*_{it} - T\hat{\Delta}^*_{i\epsilon u}\right)\right] \tag{3.18}$$

[1]Here true β is replaced by a consistent estimate.

where $\hat{\Delta}^*_{i\epsilon u} = \hat{\Omega}_{i\epsilon}^{-\frac{1}{2}} \hat{\Delta}^+_{i\epsilon u} \hat{\Omega}_{iu.\epsilon}^{-\frac{1}{2}}$.

The t-statistic for an element of $\hat{\beta}^*_{fm}$ is

$$t_{j\hat{\beta}^*_{fm}} = \frac{\sqrt{N}T\left(\hat{\beta}^*_{jfm} - \beta\right)}{s_{j\hat{\beta}^*_{fm}}} \tag{3.19}$$

where $t_{j\hat{\beta}^*_{fm}}$ is the t-statistic of β^*_{jfm}, which is the jth element of β^*_{fm}. Also $s^2_{j\hat{\beta}^*_{fm}} = [6I_k]_{jj}$, the jth diagonal element of $[6I_k]$.

The pooled DOLS estimator, $\hat{\beta}^*_d$, is obtained from

$$y^*_{it} = \alpha_i + x^{*\prime}_{it}\beta + \Sigma_{j=-q_i}^{q_i} c_{ij}\Delta x^*_{it+j} + v^*_{it}. \tag{3.20}$$

The t-statistic for an element of $\hat{\beta}^*_d$ is

$$t_{j\hat{\beta}^*_d} = \frac{\sqrt{N}T\left(\hat{\beta}^*_{jd} - \beta\right)}{s_{j\hat{\beta}^*_d}} \tag{3.21}$$

where $t_{j\hat{\beta}^*_d}$ is the t-statistic of β^*_{jd}, which is the jth element of β^*_d. Also $s^2_{j\hat{\beta}^*_d} = [6I_k]_{jj}$, the jth diagonal element of $[6I_k]$.

We see here that the heterogeneous DOLS panel estimator of β has the same limiting distribution as the heterogeneous FMOLS panel estimator, this is proved in Kao and Chiang (2000).

In homogeneous panels then:

The pooled FMOLS estimator is given by

$$\hat{\beta}_{fm} = \left[\Sigma_{i=1}^{N}\Sigma_{t=1}^{T}(x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^{\prime}\right]^{-1}\left[\Sigma_{i=1}^{N}\left(\Sigma_{t=1}^{T}(x_{it} - \bar{x}_i)\hat{y}^+_{it} - T\hat{\Delta}^+_{\epsilon u}\right)\right]$$

(3.22)

where $\hat{\Delta}^{+}_{\epsilon u} = \hat{\Delta}_{\epsilon u} - \hat{\Delta}_{\epsilon}\hat{\Omega}_{\epsilon}^{-1}\hat{\Omega}_{\epsilon u}$ and $\hat{y}^{+}_{it} = y_{it} - \hat{\Omega}_{u\epsilon}\hat{\Omega}_{\epsilon}^{-1}\Delta x_{it}$.

The t-statistic for an element of $\hat{\beta}_{fm}$ is

$$t_{j\hat{\beta}_{fm}} = \frac{\sqrt{N}T\left(\hat{\beta}_{jfm} - \beta\right)}{s_{j\hat{\beta}_{fm}}} \tag{3.23}$$

where $t_{j\hat{\beta}_{fm}}$ is the t-statistic of β_{jfm}, which is the jth element of β_{fm}. Also $s^2_{j\hat{\beta}_{fm}=}\left[6\hat{\Omega}_{\epsilon}^{-1}\hat{\Omega}_{u.\epsilon}\right]_{jj}$, the jth diagonal element of $\left[6\hat{\Omega}_{\varepsilon}^{-1}\hat{\Omega}_{u.\varepsilon}\right]$, where $\Omega_{u.\epsilon} = \Omega_u - \Omega_{u\epsilon}\Omega_{\epsilon}^{-1}\Omega_{\epsilon u}$ and $\Omega = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\Omega_i$.

The pooled DOLS estimator, $\hat{\beta}_d$, is obtained from

$$y_{it} = \alpha_i + x'_{it}\beta + \Sigma_{j=-q_i}^{q_i} c_{ij}\Delta x_{it+j} + \bar{v}_{it}. \tag{3.24}$$

The t-statistic for an element of $\hat{\beta}_d$ is

$$t_{j\hat{\beta}_d} = \frac{\sqrt{N}T\left(\hat{\beta}_{jd} - \beta\right)}{s_{j\hat{\beta}_d}} \tag{3.25}$$

where $t_{j\hat{\beta}_d}$ is the t-statistic of $\hat{\beta}_{jd}$, which is the jth element of $\hat{\beta}_d$. Also $s^2_{j\hat{\beta}_d} = \left[6\hat{\Omega}_{\varepsilon}^{-1}\hat{\Omega}_{u.\varepsilon}\right]_{jj}$, the jth diagonal element of $\left[6\hat{\Omega}_{\varepsilon}^{-1}\hat{\Omega}_{u.\varepsilon}\right]$.

Again we see here the asymptotic equivalence of the homogeneous, DOLS and FMOLS, panel estimators of β. See again Kao and Chiang (2000) for a proof. Moreover an important departure from the usual panel literature is that the asymptotics are calculated using the sequential limit theorem of Phillips and Moon (1999).

We also use a Pedroni (2000) FMOLS t-statistic (see Pedroni (2000), Corollary 1.2, p.104) given by

$$(3.26) \qquad t_{\hat{\beta}_{NT}} = \left(\hat{\beta}_{NT} - \beta\right) \left(\sum_{i=1}^{N} \hat{\Omega}_{\varepsilon i}^{-1} \sum_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^{/}\right)^{0.5} .$$

To cater for the estimation of a constant intercept and trend in our regressions we use the modified Pedroni t-statistic

$$(3.27) \qquad t_{\hat{\beta}_{NT}} = \left(\hat{\beta}_{NT} - \beta\right) \left(\sum_{i=1}^{N} \Pi_i \sum_{t=1}^{T} x_{it}^{*} x_{it}^{*/}\right)^{0.5}$$

where Π_i is a $(k+1 \times k+1)$ block diagonal matrix with $\hat{\Omega}_{u.\varepsilon i}^{-1}$ and $\hat{\Omega}_{\varepsilon i}^{-1}$ along the diagonal and $x_{it}^{*} = (1, x_{it}^{/})^{/}$. This is the case for the model with an intercept only, in the model with an intercept and trend, Π_i is an analogous $(k+2 \times k+2)$ block diagonal matrix and $x_{it}^{*} = (1, t, x_{it}^{/})^{/}$.

The pooled OLS estimator of β is given by

$$(3.28) \hat{\beta}_{ols} = \left[\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^{/}\right]^{-1} \left[\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (x_{it} - \bar{x}_i)(y_{it} - \bar{y}_i)\right],$$

where $\bar{x}_i = \frac{\Sigma_{t=1}^{T}(x_{it})}{T}$ and $\bar{y}_i = \frac{\Sigma_{t=1}^{T}(y_{it})}{T}$.

The t-statistic for $\hat{\beta}_{ols}$ is

$$(3.29) \qquad t_{\hat{\beta}_{ols}} = (\hat{\beta}_{ols} - \beta)\left(\sigma^{-2} \Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)'\right)^{0.5},$$

where $\sigma^2 = \frac{(\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \hat{u}_{it}^2)}{NT-k}$.

3.2.2 The Pedroni Group-Mean Panel Estimators

Both the Pedroni (2000,2001) group-mean FMOLS and DOLS panel estimators are formed by averaging over the individual FMOLS and DOLS time-series estimators applied to the ith member of the panel. We show first how the panel DOLS estimator is formed. Using regression (3.24) Pedroni constructs his group-mean DOLS panel estimator as follows

$$\hat{\beta}^*_{GD} = \left[N^{-1}\Sigma_{i=1}^{N}\left(\Sigma_{t=1}^{T} z_{it} z'_{it}\right)^{-1}\left(\Sigma_{t=1}^{T} z_{it}\tilde{y}_{it}\right)\right]_1, \qquad (3.30)$$

where z_{it} is the $(K(2p+2) \times 1)$ vector of regressors

$$z_{it} = ((x_{1it} - \bar{x}_{1i}), \ldots, (x_{kit} - \bar{x}_{ki}), \Delta x_{1it-p}, \ldots, \Delta x_{kit-p}, \ldots, \Delta x_{kit+p})'. \qquad (3.31)$$

Here $\tilde{y}_{it} = (y_{it} - \bar{y}_i)$, $\bar{x}_{1i} = \frac{\Sigma_{t=1}^{T} x_{1it}}{T}$ and so on. The subscript 1 outside the square brackets indicate that we are considering only the first element of the vector for the pooled slope coefficient. The estimator can also be written simply as

$$\hat{\beta}^*_{GD} = N^{-1}\Sigma_{i=1}^{N}\hat{\beta}^*_{Di} \qquad (3.32)$$

where $\hat{\beta}^*_{Di}$ is the conventional DOLS time-series estimator applied to the ith member of the panel.

Let $\sigma_i^2 = \lim_{T\longrightarrow\infty} E\left[T^{-1}(\Sigma_{t=1}^{T}\hat{v}_{it})^2\right]$ be the long run variance of the residuals from the DOLS regression. This can be estimated using standard HAC methods, such as the Newey-West HAC estimator shown below. Then the t-statistic for the Pedroni estimator is written

$$t_{\hat{\beta}^*_{GD}} = N^{-0.5}\Sigma_{i=1}^{N} t_{\hat{\beta}^*_{Di}} \qquad (3.33)$$

where

$$t_{\hat{\beta}^*_{Di}} = \left(\hat{\beta}^*_{Di} - \beta\right)\left(\hat{\sigma}_i^{-2}\Sigma_{t=1}^T z_{it}z_{it}'\right)^{0.5}, \tag{3.34}$$

and z_{it} is as above. For estimation purposes the following HAC Newey and West (1987) standard error estimator, with the Bartlett kernel, was used for the above DOLS estimator

$$\hat{V} = \left(\frac{1}{T}\Sigma_{t=1}^T z_{it}z_{it}'\right)^{-1} \times \frac{1}{T}\left(\Sigma_{t=1}^T \hat{\hat{v}}_{it}^2 + \Sigma_{s=1}^q \left[1 - \frac{s}{q+1}\right]\Sigma_{t=1}^T\left(\hat{\hat{v}}_{it}\hat{\hat{v}}_{it-s} + \hat{\hat{v}}_{it-s}\hat{\hat{v}}_{it}\right)\right). \tag{3.35}$$

The Group-Mean FMOLS estimator is given by

$$\hat{\beta}^*_{GFM} = N^{-1}\Sigma_{i=1}^N \hat{\beta}^*_{FMi} \tag{3.36}$$

where $\hat{\beta}^*_{FMi}$ is the conventional FMOLS time-series estimator.

The associated t-statistic is

$$t_{\hat{\beta}^*_{GFM}} = N^{-0.5}\Sigma_{i=1}^N t_{\hat{\beta}^*_{FMi}}. \tag{3.37}$$

3.3 The Estimation of a Panel Data Cointegrating Regression with Cross-Sectional Dependence

In chapter 1 we dealt with panel unit root tests with cross-sectional dependence. When estimating a panel cointegrating regression the same problem of dependencies between cross-sectional units occurs violating the independence assumption of the panel. The problem of estimation and inference in panels with cross-sectional dependence has been considered recently by Pesaran (2002,2005b), Coakley, Fuertes and Smith (2002), Phillips and Sul (2003) and Bai and Kao (2005). However in all the above papers only Bai and Kao (2005) deal specifically with, and present results for, the cointegrated panel data regression model. That is the results and proofs of all the other papers are for the model with stationary regressors, although they state the models can be extended to I(1) regressors. Phillips and Sul (2003) deal with three main problems in their paper, which concerns dynamic panel data estimation, cross-section dependence, homogeneity restrictions and small sample bias. Pesaran (2002,2005b) uses a factor model to deal with the cross-section dependence problem. Similar to his panel unit root test discussed in chapter 1, Pesaran proposes eliminating the unobserved common factors by adding cross-section aggregates into the regression. Coakley, Fuertes and Smith (2002) (CFS) also use a factor model to cater for omitted global variables or common shocks (factors) correlated with the regressors proxying these by the principal components of the residuals from an auxiliary regression. Finally, for their panel cointegrating regression Bai and Kao (2005) propose a two-step FMOLS and continuous updated FMOLS (CUP-FM) estimator for the cross-sectional dependence, which they model by factors.

More recently Moon and Perron (2004), Mark, Ogaki and Sul (2005) and

Westerlund (2005), have used a Dynamic SUR estimator (DSUR), as in Phillips and Sul (2003), in some empirical applications (and Monte Carlo simulations) using panels with cross-sectional dependence. These panel cointegration estimators are useful in the case where N is small relative to T and so complement the panel estimators studied in this chapter which are for use with large N. Mark, Ogaki and Sul (2005) and Westerlund (2005) note, along the lines of Saikkonen (1993), that systems DOLS (SDOLS) estimation is more efficient than DOLS estimation. This results when one augments each equation's regression with leads and lags of regressors not only of the same equation but also of others. Similarly they distinguish between system DSUR (SDSUR) and DSUR noting the former is more efficient in the presence of cross-equation endogeniety. Both DSUR models cater for long run cross-equation correlation in the equilibrium errors. Thus Westerlund (2005) obtains the ranking $SDSUR \ll SDOLS \ll DSUR \ll DOLS$ where $\ll$ means "more efficient than".[2] Westerlund (2005) also considers new methods for the selection of lag lengths using data dependent information criteria such as the Schwartz Bayesian IC (SIC), the Akaike IC (AIC) and others.

[2]All three papers report similar efficiency rankings.

3.3.1 The Bai and Kao Panel FMOLS Estimator

Consider the fixed effects model of equations (3.1) and (3.2). To model cross-sectional dependencies, Bai and Kao (2005) use the following factor model for the error term

$$u_{it} = \lambda_i' F_t + \nu_{it} \tag{3.38}$$

where F_t is an $(r \times 1)$ vector of common factors, which we assume random, λ_i is an $(r \times 1)$ vector of factor loadings and ν_{it} the idiosyncratic error term. Now let $w_{it} = (F_t', \nu_{it}, \epsilon_{it}')'$.

The long run covariance matrix Ω_i of w_{it} (see Bai and Kao (2005)) is now

$$\Omega_i = \Sigma_{j=-\infty}^{\infty} E(w_{ij} w_{i0}') \tag{3.39}$$

$$= \Sigma_i + \Gamma_i + \Gamma_i' \tag{3.40}$$

$$= \begin{bmatrix} \Omega_{Fi} & \Omega_{Fui} & \Omega_{F\epsilon i} \\ \Omega_{uFi} & \Omega_{ui} & \Omega_{u\epsilon i} \\ \Omega_{\epsilon Fi} & \Omega_{\epsilon ui} & \Omega_{\epsilon i} \end{bmatrix}. \tag{3.41}$$

The auto-covariance matrix of w_{it} is now

$$\Gamma_i = \Sigma_{j=1}^{\infty} E(w_{ij} w_{i0}') = \begin{bmatrix} \Gamma_{Fi} & \Gamma_{Fui} & \Gamma_{F\epsilon i} \\ \Gamma_{uFi} & \Gamma_{ui} & \Gamma_{u\epsilon i} \\ \Gamma_{\epsilon Fi} & \Gamma_{\epsilon ui} & \Gamma_{\epsilon i} \end{bmatrix} \tag{3.42}$$

and Σ_i is now the contemporaneous covariance matrix given by

$$\Sigma_i = E(w_{i0} w_{i0}') = \begin{bmatrix} \Sigma_{Fi} & \Sigma_{Fui} & \Sigma_{F\epsilon i} \\ \Sigma_{uFi} & \Sigma_{ui} & \Sigma_{u\epsilon i} \\ \Sigma_{\epsilon Fi} & \Sigma_{\epsilon ui} & \Sigma_{\epsilon i} \end{bmatrix}. \tag{3.43}$$

The one-sided long run covariance is now

$$\Delta_i = \Sigma_i + \Gamma_i \tag{3.44}$$

$$(3.45) \qquad = \Sigma_{j=0}^{\infty} E(w_{ij} w_{i0}^{/}),$$

$$(3.46) \qquad \text{with} \quad \Delta_i = \begin{bmatrix} \Delta_{bi} & \Delta_{b\epsilon i} \\ \Delta_{\epsilon bi} & \Delta_{\epsilon i} \end{bmatrix}.$$

The FMOLS estimator is formed by making the following corrections:

The endogeniety correction is obtained by

$$(3.47) \qquad \hat{y}_{it}^{+} = y_{it} - \left(\hat{\lambda}_i^{/} \hat{\Omega}_{F\epsilon i} + \hat{\Omega}_{u\epsilon i}\right) \hat{\Omega}_{\epsilon i}^{-1} \Delta x_{it}.$$

The serial correlation correction has the form

$$(3.48) \qquad \hat{\Delta}_{b\epsilon i}^{+} = \hat{\Delta}_{b\epsilon i} - \hat{\Omega}_{b\epsilon i} \hat{\Omega}_{\epsilon i}^{-1} \Delta_{\epsilon i}.$$

Therefore the feasible two-step FMOLS estimator is

$$\hat{\beta}_{FM} = \left[\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^{/}\right]^{-1} \left[\Sigma_{i=1}^{N} \left(\Sigma_{t=1}^{T} y_{it}^{+} (x_{it} - \bar{x}_i)^{/} - T\left(\hat{\lambda}_i^{/} \hat{\Delta}_{F\epsilon i}^{+} + \hat{\Delta}_{u\epsilon i}^{+}\right)\right)\right].$$

(3.49)

The t-statistic for an element of $\hat{\beta}_{FM}$ is

$$(3.50) \qquad t_{j\hat{\beta}_{FM}} = \frac{\sqrt{N}T\left(\hat{\beta}_{jFM} - \beta_0\right)}{s_{j\hat{\beta}_{FM}}}$$

where $t_{j\hat{\beta}_{FM}}$ is the t-statistic of $\hat{\beta}_{jFM}$, which is the jth element of $\hat{\beta}_{FM}$. Also

$$(3.51) \quad s_{j\hat{\beta}_{FM}}^{2} = \left[6\hat{\Omega}_{\varepsilon}^{-1} \left\{\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\hat{\lambda}_i^{/} \hat{\Omega}_{F.\varepsilon i} \hat{\lambda}_i \hat{\Omega}_{\varepsilon i} + \hat{\Omega}_{u.\varepsilon i} \hat{\Omega}_{\varepsilon i}\right)\right\} \hat{\Omega}_{\varepsilon}^{-1}\right]_{jj},$$

the jth diagonal element of

$$(3.52) \qquad \left[6\hat{\Omega}_{\varepsilon}^{-1} \left\{\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\hat{\lambda}_i^{/} \hat{\Omega}_{F.\varepsilon i} \hat{\lambda}_i \hat{\Omega}_{\varepsilon i} + \hat{\Omega}_{u.\varepsilon i} \hat{\Omega}_{\varepsilon i}\right)\right\} \hat{\Omega}_{\varepsilon}^{-1}\right].$$

Here $\hat{\lambda}, \hat{F}, \hat{\Sigma}_i$ and $\hat{\Omega}_i$ are consistent estimates of λ, F, Σ_i and Ω_i obtained in the first estimation step. To estimate λ_i and F the principal components method of Bai and Ng (2002,2004) is used.

Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)'$ and $F = (F_1, F_2, \ldots, F_T)'$ and then write $Z = (\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_N)'$ a $(T \times N)$ matrix and $\hat{u}_i = (\hat{u}_{i1}, \hat{u}_{i2}, \ldots, \hat{u}_{iT})'$ where $\hat{u}_{it} = y_{it} - \hat{\alpha}_i - x_{it}'\hat{\beta}$, with a consistent estimator $\hat{\beta}$. The estimated $(T \times r)$ factor matrix denoted $\hat{F}$ is $\sqrt{T}$ times eigenvectors corresponding to the r largest eigenvalues of the $(T \times T)$ matrix ZZ' and $\hat{\lambda}' = \left(\hat{F}'\hat{F}\right)^{-1} \hat{F}'Z = \frac{\hat{F}'Z}{T}$ is the corresponding matrix of the estimated factor loadings.

While Σ and Ω can be estimated as follows

$$\hat{\Sigma} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{w}_{it}\hat{w}_{it}' \tag{3.53}$$

where $\hat{\nu}_{it} = \hat{u}_{it} - \hat{\lambda}_i'\hat{F}_t$ and $\hat{w}_{it} = \left(\hat{F}_t', \hat{v}_{it}, \Delta x_{it}'\right)'$. Also

$$\hat{\Omega} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{T} \sum_{t=1}^{T} \hat{w}_{it}\hat{w}_{it}' + \frac{1}{T} \sum_{\tau=1}^{\imath} \varpi_{\tau\imath} \sum_{t=\tau+1}^{T} \left(\hat{w}_{it}\hat{w}_{it-\tau}' + \hat{w}_{it-\tau}\hat{w}_{it}'\right) \right\} \tag{3.54}$$

where $\varpi_{\tau\imath}$[3] is some weight function or kernel.[4] Phillips and Moon (1999) show that $\hat{\Omega}$ and $\hat{\Sigma}$ are consistent for Ω and Σ. These estimators are also valid in the case without cross-sectional dependence in § 3.2. Also we have assumed here the number of factors r is known. However if this is not the case Bai and Ng (2002) have shown that the number of unknown factors k

[3] $\hat{u}_{it}$ residuals estimated from equation by equation OLS regressions.

[4] The Bartlett kernel with 4 lags was used in the computations.

can be found by minimising an information criterion (IC),

$$IC(k) = log(V(k)) + k\left(\frac{N+T}{NT}\right) log\left(\frac{NT}{N+T}\right) \tag{3.55}$$

where $V(k) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\hat{u}_{it} - \hat{\lambda}_i^{/}\hat{F}_t\right)^2$.

3.4 An Empirical Application

In our empirical multi-country consumption study we cover a much wider range of panel cointegration estimators and examine more candidate I(1) regressors than have previously been examined in such studies. Similar less detailed empirical studies have come from Pesaran, Shin and Smith (1999), Sarantis and Stewart (1999) and Larsson, Lyhagen and Lothgren (2001). In Pesaran, Shin and Smith (1999) alternative[5] estimators to existing panel estimators are developed called Pooled Mean-Group estimators. These can be used with stationary and nonstationary regressors. Pesaran, Shin and Smith use these to estimate aggregate consumption functions for 24 OECD economies over the period 1962-93. They find a long run equilibrium relationship in this panel cointegration model between the log of real consumption per capita and the log of real disposable income per capita and inflation. Sarantis and Stewart (1999) test for stationarity in the consumption-income ratio using a panel of 20 OECD countries and panel unit root tests. Their

[5] Termed as intermediate estimators between Kao's and Pedroni's estimators.

findings are that the ratio is generated by a nonstationary stochastic process and hence consumption and income do not form a long run equilibrium relationship. Larsson, Lyhagen and Lothgren (2001) estimate a panel consumption function using the panel error-correction methods developped in their paper. They use the same variables and definitions as Pesaran, Shin and Smith (1999) for a panel of 23 OECD countries over the period 1960-1994. They find that using individual country trace tests 17 countries in the panel select a rank of 1, whilst their panel test selects r=2 as the largest common rank. Thus showing good support for the cointegrated panel model.

3.4.1 The Data Set

We use a balanced panel of annual observations from 1961-1999 for 20 OECD countries obtained from the World Bank Development Indicators. So $N = 20$ and $T = 39$. The variables are:

1) Real Consumption Expenditure Per Capita (US$).

2) Real Gross Domestic Product Per Capita (US$)[6] (+).

3) Interest Rates (-): Subject to availability these were the short-term 24 hr Discount Rate (otherwise the 30 day Treasury Bill Rate or the Long-Term 10 year Government Bond Yield).

4) Inflation is the change in the logarithm of the (CPI) x 100 (-).

[6] The symbols in brackets indicate the expected sign of the coefficient.

5) The ratio of Real Liquid Assets to Real GDP (US$) (-).

Our estimation equation is[7]

Real $CON_{it}/N_{it} = \alpha_i + \delta_i t +$ Real $GDP_{it}/N_{it} +$ Real $LA_{it}/$Real $GDP_{it} + u_{it}$.

Here Real Consumption Expenditure Per Capita is final consumption expenditure deflated by the GDP deflator and divided by total population, similarly for the others, etc. Real Wealth or Real Liquid Assets consisted of Real National Savings plus Real Time, Savings and Demand Deposits at commercial banks and also when available the Real Stock of Bonds at commercial banks. The rationale behind the last variable is that there is assumed a desired ratio of real liquid assets to real GDP and that if the actual ratio falls short or exceeds the desired or equilibrium ratio, then consumers expenditure will either be constrained or expanded until equilibrium is reached again. Inspecting the time-series graphs of the ratio of real liquid assets to real GDP in Figure 3.09 of Appendix 4 we see the actual ratio falling in the majority of the countries in the panel (16 out of 20). This is expected to exert a negative influence on consumers expenditure as households constrain expenditure in order to boost savings in order to maintain a desired equilibrium level of real liquid assets to real GDP. In calculating the ratio of real liquid assets to real GDP we divided real liquid assets per capita by

[7] Two other regression formulations are used where Real LA/Real GDP is replaced by interest rates and inflation variables.

real GDP per capita. Since the population variable, N_{it}, appears in both the numerator and denominator it is cancelled out. To compute real national savings we multiplied real GDP by the national savings rate (% of GDP).

3.4.2 The Estimation Results[8]

Table 3.01 **Individual Country ADF Regression Estimates**

COUNTRY	$Int.Rate_t$ t-stat	lag	Inf_t t-stat	lag	LA_t/y_t t-stat	lag
Australia	-1.614	1	-1.260	4	-2.207	0T
Austria	-3.527[b]	3	-1.378	4	-1.971	0T
Belgium	-1.189	2	-3.141[b]	1	-1.386	0
Canada	-2.329	1	-1.962	1	-2.849	0T
Denmark	-1.368	3	-1.044	2	-1.758	1
Finland	-0.676	1	-2.041	1	-3.070	0T
France	-2.236	1	-1.526	1	-2.087	0T
Greece	-1.522	1	-1.474	2	-1.238	0T
Ireland	-0.980	4	-2.298	1	-1.672	0T
Italy	-1.340	3	-1.918	3	-2.884	1T
Japan	0.509	4	-1.324	2	-2.205	1T
Korea	-2.747	1	-2.871	1	-3.099	1T
Netherland	-3.415[b]	3	-1.220	4	-2.303	0
Norway	-1.583	3	-0.803	4	-3.118	0T
Portugal	-1.722	1	-1.165	4	-1.691	4
Spain	-1.317	2	-1.678	3	-2.392	0T
Sweden	-0.934	1	-0.970	2	-2.174	1
Switzerland	-2.327	1	-3.141[b]	1	-1.845	1
U.K.	-2.586	1	-1.290	3	-3.702[b]	1T
U.S.	-2.674	1	-3.084[b]	1	-4.121[b]	1T

Our consumption model then has similarities to a permanent income hy-

[8]This chapter makes extensive use of the Ox progamming language for the computation of the FMOLS, DOLS and OLS econometric estimators. Other software applications that are available include the Nonstationary Panel Time series (NPT) suite of programs written in Gauss by Professor Chihwa Kao and available for public use at http://www.maxwell.syr.edu/maxpages/faculty/cdkao/working/npt.html. See Appendix 2 for more details on econometric software packages.

pothesis model. In the initial stages a choice was made on the formulation of the consumption model. The linear form was opted for after a number of model selection tests (namely Bera and McAleer (1982) tests and the tests of loglinear vs linear specification of MacKinnon, White and Davidson (1983)), indicated the linear form better suited the data.

Table 3.02 **Individual Country ADF Regression Estimates**

COUNTRY	Con_t/N_t t-stat	lag	y_t/N_t t-stat	lag
Australia	-2.248	1T	-2.435	1
Austria	-2.807	1T	-3.245	4T
Belgium	-3.154	1T	-3.001	1T
Canada	-3.234	1T	-3.460	2T
Denmark	-2.773	1T	-2.996	4T
Finland	-3.734	4T	-4.233[a]	1T
France	-2.643	1T	-2.591	1T
Greece	-2.688	1T	-2.598	1T
Ireland	-2.120	1	-0.845	0T
Italy	-2.631	1T	-2.619	1T
Japan	-1.132	3T	-1.392	3T
Korea	-2.690	0	-2.582	0T
Netherland	-3.391	4T	-3.206	4T
Norway	-2.468	1T	-2.592	1T
Portugal	-2.732	1T	-2.502	1T
Spain	-2.833	1	-2.678	1
Sweden	-2.792	4	-2.714	4
Switzerland	-3.352	4T	-3.241	4T
U.K.	-3.002[b]	1	-3.333[b]	1
U.S.	-2.152	1T	-2.601	1T

All the initial time-series were pre-tested for nonstationarity using the ADF tests described in chapter 1. They were practically all found to be I(1) variables and so the Engle and Granger (1987) two-step modelling stategy could be applied to the panel data to undertake a panel cointegration consumption analysis. First test for panel unit roots and cointegration, if successful then estimate the panel cointegration vectors. The results of the country ADF tests are shown in Tables 3.01 and 3.02. Only one or two countries in each panel were found not to support the unit root hypothesis. These are Austria and the Netherlands in the interest rate panel, Belgium, Switzerland and the US in the inflation panel, the UK and the US in the ratio of real liquid assets to real GDP panel, the UK in the real consumption per capita panel and finally the UK and Finland in the real GDP per capita panel. All these except the Finland are significant at the 5% level. In order to increase the power of the univariate unit root tests panel unit root tests were undertaken. The individual country ADF tests were further supported by results from the Im, Pesaran and Shin (2003) panel unit root tests. All the variables in the panels were found nonstationary at the 1% level. The results of these tests are found in Table 3.03. In order to avoid the spurious regression problem we conduct panel cointegration tests. The panel cointegration tests of Kao (1999)[9] and Pedroni (1999)[10], described in chapter 2, were conducted. From the results of the Kao[11] tests in Table 3.04 we see the null hypothesis of no

[9]We use all 5 Kao test statistics in the tests.
[10]All seven statistics are reported here.
[11]We use the pooled fixed effect DOLS regressions for the Kao tests.

cointegration being rejected by all the test statistics for all the regressions.

Table 3.03 **IPS Panel Unit Root Tests**

	y_{it}/N_{it}	Con_{it}/N_{it}	$\text{L.A.}_{\cdot it}/y_{it}$	$\text{I.R.}_{\cdot it}$	Inf_{it}
$\Psi_{\bar{t}}$	-0.6479	-0.4928	-0.8321	-1.399435	-1.459278

Table 3.04 **Kao Panel Cointegration Tests**

Regression	DF_t-stat	DF_γ-stat	DF_t^*-stat	DF_γ^*-stat	ADF-stat
$\text{L.A.}_{\cdot it}/y_{it}$	63.260^a	-9.1948^a	38.214^a	-17.040^a	30.700^a
$\text{I.R.}_{\cdot it}$	66.612^a	-7.6548^a	41.215^a	-16.464^a	34.153^a
Inf_{it}	68.614^a	-6.8446^a	42.987^a	-15.636^a	36.342^a

Table 3.05 **Pedroni Panel Cointegration Tests**

Regr	Panel-v	Panel-ρ	Panel-t	Group-ρ	Group-t	Panel-t^P	Group-t^P
Con	1.4532^c	-2.8154^a	1.7879	0.0316	5.3425	4.2481	-1.7682^b
Con+Tr	−0.6340	−1.1884	3.6991	1.6821	7.3742	7.2856	-0.6232

The results of the Pedroni tests are shown in Table 3.05, to compute these statistics one needs an estimate of the lag truncation parameter in the Newey-West variance estimators. On inspection of the graph of $\hat{u}_{it}$, in Figure 3.21 of Appendix 4, a lag length of 4 was chosen for the kernel functions. However, as in chapter 2, the parameter k_i could equally have been set to zero as indicated by the autocorrelation functions for $\hat{\mu}_{it}$ and $\hat{\eta}_{it}$, in Figure 3.22 and Figure 3.23[12]. When constructing the FMOLS estimators one also needs to set the lag length for the Barltett scheme (or any lag window) in the Newey-

[12]Equation by equation OLS regressions were used to obtain the residuals for the Pedroni tests.

West (1987) HAC estimator. In this case one inspects the cross-correlation function of the residuals $\hat{u}_{it}$ and $\hat{\epsilon}_{it}$. This is unlike the case for the DOLS estimator[13] where one only needs to inspect the autocorrelation funtion of the residuals $\hat{v}_{it}$. A visual inspection of the graph of residual cross-correlations in Figure 3.25 in Appendix 4 indicated that a lag length of 4 periods was necessary in the FMOLS cases. As mentioned, in the analogous case for the DOLS estimator, the ACF gave us a lag length of 3-4 periods (4 again chosen), see Figure 3.26 in Appendix 4[14]. The results of the Pedroni tests in Table 3.05 are again less conclusive. In the model with a constant only, 3 of the test statistics rejected the null of no cointegration, one at the 1% level, one at the 5% level and one at the 10% level. In the model with a constant and trend no test statistics rejected the null of no cointegration. The poor perfomance of the Pedroni tests which was apparent in chapter 2 is attributed to the method by which the residuals are obtained, ie equation by equation OLS, and it is concluded perhaps alternative estimators may give better results. We can conclude on the strength of our Kao tests that there is a long run relationship between real consumption expenditure per capita and real GDP per capita and either of the other three candidate panel regressors.

[13] In all the DOLS regressions a uniform lag and lead of three periods was used with the differenced regressors.

[14] The residuals in Figure 3.26 are from the group-mean DOLS regressions, without a trend, for L.A.'s.

A number of regression diagnostic tests were carried out to check for specification errors. In the individual regressions for the mean-group estimators (and pooled regressions) a D.W. test was carried out for each regression. In each case the time-series showed strong residual autocorrelation, most D.W. statistics being below 1.000 (nearly all < 1.5). This indicates that the serial correlation and endogeneity problems in cointegration regressions are quite serious and should be tackled at all times using the HAC Newey-West or similar estimators. This method was used to compute standard errors and t-statistics for every cointegration estimator. Also an F-test on the explanatory power of the regressors was carried out for each regression. With F-statistics of $F_{(k-1,T-k)} = F_{(2,36)}$, in the individual regressions for the group-mean estimator and $F_{(k-1,N(T-k))} = F_{(2,720)}$ in the pooled regressions, we had critical values of $F_{(2,36)} = 3.26$ and 5.25 and $F_{(2,720)} = 3.00$ and 4.62 at the 5% and 1% levels of significance, respectively. These critical values are for the FMOLS regressions, the DOLS cases are analogous. These were greatly exceeded by computed F-statistics of 3 or 4 significant figures before the decimal point in most cases. These F-statistics, along with the D.W. statistics and R^2 statistic are shown together in the individual country regressions. The R^2 statistics were usually quite high (around 0.9 for the both the group-mean and pooled estimators). This indicated that the models were a very good fit of the data. The individual country regressions which go to make up the group-mean estimates for DOLS and FMOLS are reported in Tables 3.34-3.57 in Appendix 3.

A panel Chow test for the homogeneity of slope coefficients was also carried out using an F-statistic

$$F_{chow} = \frac{e'e - e_1'e_1 - e_2'e_2 \ldots - e_N'e_N/(N-1)(K+1)}{e_1'e_1 + e_2'e_2 \ldots + e_N'e_N/N(T-K-1)}. \tag{3.56}$$

Under $H_0 : \beta_i = \beta \quad \forall i \quad F \sim F_{((N-1)(K+1),N(T-K-1))}$. Where $e_i'e_i$ are the residual sum of squares from the individual country FMOLS Within estimates of the model, whilst $e'e$ are the residual sum of squares for the pooled FMOLS Within estimates of the fixed effects model. The $F_{(57,720)}$ critical values at the 1% and 5% significance levels were 1.54 and 1.36, respectively. Our computed F-statistics for the Real LA/Real GDP, Inflation and Interest Rates regressions, respectively, were 1.4540, 0.83709 and 0.76138. Hence we accept the hypothesis of a common slope for all countries, in all the regressions, at the 1% significance level. This result should be viewed with caution since the F-test is only applicable in the case of homoscedastic residuals and strictly exogenous regressors. Recently Phillips and Sul (2003) proposed homogeneity tests in dynamic panels with cross-sectional dependence using modified Hausman tests. Also Pesaran and Yamagata (2005) recently developed tests for slope homogeneity in large panels using a Swamy type test.

However both these tests are designed more for stationary dynamic autoregressions. Thus there does not seem to be a test for homogeneity that is easily applicable to our panel cointegration model.[15]

In Tables 3.06-3.33 we[16] have the results of our panel cointegration study. The analysis was conducted in three parts. In Part A, Tables 3.06-3.15 relate to the estimation of the model with a constant intercept and trend. In Part B, Tables 3.16-3.25 relate to the estimation of the fixed effects model with individual-specific intercepts and trends. Finally in Part C, in Tables 3.26-3.33, we have the results of the estimation of the fixed effects model with individual-specific intercepts and trends, allowing for cross-sectional dependence.

Unfortunately both the Kao pooled panel FMOLS estimators (heterogeneous and homogeneous) perfomed badly in the regressions. In the former, numerous problems were encountered in trying to compute the correction factor discussed in § 3.2.1. Also with the Kao homogeneous pooled panel estimator, not including individual-specific long run residual covariances led to a sharp deterioration in the results. These findings are not suprising as both Pedroni (2000) and Kao and Chiang (2000) strongly recommend not using the pooled FMOLS estimators in their papers based on their small sample performance

[15] A fully modified Wald test maybe the most appropriate.

[16] See the end of the chapter for Tables 3.06-3.33.

in Monte Carlo simulations. A slight modification to the Kao and Chiang pooled homogeneous estimators estimated with individual-specific long run residual covariance matrices had the effect of obtaining results comparable to the group-mean estimators. We also note that Kao and Chiang (2000) and Pedroni (2000) use two different methods to compute their t-statistics. Kao and Chiang (2000) use unweighted estimators that use the long run asymptotic innovation covariance matrix for their t-statistics. This is shown in § 3.2.1, equation (3.23). Pedroni (2000) uses weighted estimators that use weighted finite sample estimates of the regressor covariance matrix for their t-statistics, with weights that come from the long run asymptotic innovation covariance matrix. This is shown in § 3.2.1, equation (3.26). As mentioned in § 3.2.1, a modified Pedroni (2000) estimator is used in Part A, as shown in equation (3.27), to obtain t-statistics for the constant intercept and trend terms in the pooled models. In Part B and Part C the Kao and Chiang (2000) estimator is used for the t-statistics in the pooled models. In all the parts the R^2 figure is very high and also the F-statistics are strongly significant, indicating good fits of the data. However, considering the fact that all regressions exhibited substantial serial correlation, indicated by very low D.W. statistics, unreported in the main text, one can say that the OLS estimates have substantially biased standard errors and thus the t-statistics are of little use. Thus from an inferential point of view one should direct attention to the DOLS and FMOLS estimates only.

The results for Table 3.07-3.08 are similar and refer to the pooled (and modified) Kao panel estimators of the model without a trend. In all of these tables for the FMOLS and DOLS estimates, we have interest rates and inflation incorrectly signed. These are to be compared with Tables 3.09-3.10 of the group-mean estimators. Both the FMOLS and DOLS group-mean estimators performed well for this model with all coefficients correctly signed. This theme is familiar throughout the study. That is, for different groups of estimators, in different parts, the results tend to replicate themselves. In Tables 3.12-3.13 we have the model with the trend estimated. In most of these pooled estimates the trend is significant, however, it is usually estimated very close to zero. This is also the case for the group-mean estimates in Tables 3.14-3.15. Again the group-mean estimators have the correct signs on coefficients. The significance of most of the important estimates in Part A is good. The MPC ie, real GDP per capita coefficient, is always $0 < MPC < 1$, it is always very strongly significant and correctly signed for all estimators. For example for group-mean DOLS, in Table 3.09 we have MPC's of 0.73297, 0.73390 and 0.730221 as with a priori expectations, and t-statistics of (325.60),(274.91) and (214.51) respectively. Thus the group-mean estimators do better in terms of accuracy and are more efficient in the models with a constant only. In comparing the DOLS and FMOLS estimators in Part A there is often very little difference. For both groups both estimators deliver stable estimates of the regression parameters, of just below unity in absolute value, for all coefficients excluding the deterministic terms. However the group-mean estimates often push the real LA/real GDP coefficient to above 1. A slight defect is noted with the group-mean estimator in Table 3.09. With the group-mean estimators here the estimated coefficient and t-statistic have the opposite sign.[17]

The assumption of a fixed intercept and trend in the model of Part A is very restrictive. We we now relax this assumption and in Part B and Part C we estimate a fixed effects model with individual-specific intercepts and trends. To cater for the heterogeneous intercepts we transform the data into deviation-from-mean form. These FMOLS and DOLS estimates are termed "demeaned", in the results. To cater for the heterogeneous intercepts and trends we first transform the data into deviation-from-mean form and then we detrend the regressions. The following detrending procedure is used. First regress Real CON_{it}/N_{it} on an intercept and time-trend and obtain the residuals, say e_{1it}. Second regress Real GDP_{it}/N_{it} on an intercept and time-trend

[17] When averaging the individual country t-statistics when they are close to zero, positive and negative values tend to cancel, leading to perhaps the opposite sign to the coefficient.

and obtain the residuals, say e_{2it}. Third regress either of Real LA_{it}/Real GDP_{it}, $Inflation_{it}$ or Interest $Rates_{it}$ on an intercept and time-trend and obtain the residuals, say e_{3it}. Finally regress e_{1it} on e_{2it} and e_{3it}, which are free of the influence of the linear time-trend, to get the true slope coefficients required. These FMOLS and DOLS estimates are termed "demeaned and detrended", in the results[18]. In Tables 3.17-3.18 we see that allowing for differing intercepts improves the estimation of the inflation coefficients, in the pooled FMOLS estimates, which are now correctly signed and significant. Hoever it is still incorrectly signed in the pooled DOLS estimates as is interest rates for both estimators. Contrast this with the group-mean estimates in Tables 3.19-3.20 where all the coefficients are correctly signed except for interest rates in the DOLS regression, and all the coefficients significant except for interest rates in the FMOLS regression. For example, for group-mean DOLS, in Table 3.19, we have MPC's of 0.74035, 0.74491 and 0.75483 with t-statisics of (259.75),(190.34) and (207.21) respectively. A comparison of Tables 3.22-3.23 with Tables 3.24-3.25 in the model with a trend gives very similar findings as before. The group-mean estimators do much better in terms of accuracy, but now the pooled estimators are more efficient in the models with a trend. Finally comparing DOLS with FMOLS

[18]In some of the group-mean estimates the regressions were simply estimated with trend terms.

for both groups in Part B we see there is again very little difference between them, perhaps with FMOLS having slightly more significant estimates and more often the correct signs on coefficients and t-statistics. Thus they are slightly more efficient and accurate.

Finally Part C contains only the pooled FMOLS estimators of Bai and Kao (2005) for the fixed effects model with individual-specific intercepts and trends. These extend the basic framework to cater for cross-sectional dependence. The model is estimated with 5, 7, 9 and 12 factors. Here again being a pooled panel estimator one notices again the incorrect sign on interest rates for all regressions. The results in Tables 3.26-3.29 refer to the model with a constant only. We can see that in allowing for cross-sectional dependence we reduce the significance of the MPC coefficients, although all are still with a priori expectations and correctly signed. For example, for pooled FMOLS, in Table 3.26, we have MPC's of 0.73525, 0.73877 and 0.74491 with t-statistics of (62.682),(75.523) and (72.043) respectively. The coefficients of Real LA_{it}/Real GDP_{it} and $Inflation_{it}$ are also significant and correctly signed. In Tables 3.30-3.33, for the model with a constant and trend, we have similar results as before, except inflation is now insignificant in all the regressions. In both the models, with and without a trend, the estimated regression improves as we reduce the number of factors from 12 to 5. This is also supported by the IC criterion.

To conclude we can summarise the results of each part. It is clear that the pooled panel estimators are more unreliable estimators when compared to the group-mean estimators, but are equally efficient. This is generally the case for both models with and without a trend term. It is also apparent that most estimators improved in performance as we moved from the models of Part A to those of Part B indicating that the constant intercept and trend[19] assumption is too restrictive in this study. In ranking the estimators FMOLS appears to be more reliable at extracting strong signal-to-noise ratios throughout the samples and is to be preferred. But one must be careful to note that the FMOLS and DOLS results are very similar, in both the pooled and group-mean estimates. This raises the minor but important point of nonparametric versus parametric estimation. As in this study, Pedroni (2000) finds FMOLS estimators more robust and excelling in panels where there is considerable heterogeneity. He also prefers the group-mean estimators as in his Monte Carlo studies they exhibit much less size distortion compared to other estimators. DOLS estimators do well when there is not a lot of data available for estimation. However this is not a problem when using panel data. In most cases, as shown above, the far more stable

[19] In chapter 5 we derive asymptotic results for this simple model.

group-mean estimators should be preferred on the grounds that point estimates are much more accurate, and equally efficient. As a likely compromise between nonparametric and parametric estimators, the group-mean FMOLS estimator could be ranked as best, in this study, followed by the group-mean DOLS estimator with the pooled heterogeneous FMOLS and DOLS estimators following suit. Yet, when also considering cross-sectional dependence, (and given a well specified model) the performance of the Bai and Kao (2005) pooled FMOLS estimator is superior to the group-mean FMOLS and DOLS estimators. This may be the estimator's redeeming merit in large N panels to which the DSUR estimators cannot be applied.

Part A

Model with constant only[20]

Table 3.06 Pooled Panel OLS Estimates

Regression	(i)	(ii)	(iii)
Constant	10.757	0.54683	-3.4333
	(0.23181)	(0.016653)	(-0.092045)
Real GDP_{it}/N_{it}	0.74997	0.75216	0.75483
	(5.8210)	(5.5534)	(5.7722)
Real LA_{it}/Real GDP_{it}	-0.28553		
	(-0.24678)		
$Inflation_{it}$		0.19457	
		(0.069821)	
Interest $Rates_{it}$			0.60222
			(0.20615)
R^2	0.99696	0.99663	0.99685
F-statistic	127230	114770	123130

Table 3.07 Pooled Panel DOLS Estimates

Regression	(i)	(ii)	(iii)
Constant	8.7307	-1.3052	-5.4830
	(84.071)	(-8.2932)	(-28.397)
Real GDP_{it}/N_{it}	0.75452	0.75785	0.75891
	(145.05)	(136.23)	(133.88)
Real LA_{it}/Real GDP_{it}	-0.24600		
	(-10.752)		
$Inflation_{it}$		0.32595	
		(11.042)	
Interest $Rates_{it}$			0.77158
			(27.138)
R^2	0.99712	0.99692	0.99711
F-statistic	16531.0	15437.0	16479.0

[20]The dependent variable in Part A regressions is Real CON_{it}/N_{it}.

Table 3.08 **Pooled Panel FMOLS Estimates**

Regression	(i)	(ii)	(iii)
Constant	8.9684	1.8440	-3.2218
	(86.359)	(11.717)	(-16.689)
Real GDP_{it}/N_{it}	0.74970	0.75157	0.75593
	(144.13)	(135.10)	(133.35)
Real LA_{it}/Real GDP_{it}	-0.22860		
	(-9.9909)		
$Inflation_{it}$		0.029588	
		(1.0023)	
Interest $Rates_{it}$			0.58181
			(20.466)
R^2	0.99609	0.99600	0.99672
F-statistic	91770.0	89543.0	109510

Table 3.09 **Group Mean Panel DOLS Estimates**

Regression	(i)	(ii)	(iii)
Constant	31.737	4.3280	5.8599
	(29.424)	(3.7686)	(1.5210)
Real GDP_{it}/N_{it}	0.73297	0.73390	0.730221
	(325.60)	(274.91)	(214.51)
Real LA_{it}/Real GDP_{it}	-0.95989		
	(-29.720)		
$Inflation_{it}$		-0.42049	
		(-9.2602)	
Interest $Rates_{it}$			-0.12954
			(2.1487)
Average R^2	0.99978	0.99949	0.99952
F-statistic	-	-	-

Table 3.10 Group Mean Panel FMOLS Estimates

Regression	(i)	(ii)	(iii)
Constant	31.735	5.4750	1.9351
	(26.910)	(4.9986)	(0.19568)
Real GDP_{it}/N_{it}	0.74074	0.73296	0.74583
	(306.13)	(233.82)	(206.09)
Real LA_{it}/Real GDP_{it}	-1.0586		
	(-30.325)		
$Inflation_{it}$		-0.57345	
		(-11.637)	
Interest $Rates_{it}$			-0.16471
			(-0.84555)
Average R^2	0.99777	0.99738	0.99729
F-statistic	-	-	-

Model with constant and trend

Table 3.11 Pooled Panel OLS Estimates

Regression	(i)	(ii)	(iii)
Constant	10.103	-0.58641	-3.5571
	(0.20499)	(-0.014984)	(-0.089381)
Trend	0.045360	0.066262	0.010671
	(0.038975)	(0.052884)	(0.0089185)
Real GDP_{it}/N_{it}	0.74867	0.75015	0.75452
	(5.6340)	(5.3462)	(5.5680)
Real LA_{it}/Real GDP_{it}	-0.28562		
	(-0.24722)		
$Inflation_{it}$		0.22693	
		(0.079745)	
Interest $Rates_{it}$			0.59852
			(0.20287)
R^2	0.99696	0.99664	0.99686
F-statistic	84960.0	76830.0	81996.0

Table 3.12 Pooled Panel DOLS Estimates

Regression	(i)	(ii)	(iii)
Constant	7.7178	-1.8446	-5.1634
	(93.802)	(-13.034)	(-33.020)
Trend	0.057686	0.029612	-0.031010
	(16.090)	(4.8020)	(-4.5512)
Real GDP_{it}/N_{it}	0.75353	0.75716	0.75977
	(144.86)	(136.10)	(134.03)
Real LA_{it}/Real GDP_{it}	-0.24433		
	(-10.679)		
$Inflation_{it}$		0.33928	
		(11.493)	
Interest $Rates_{it}$			0.78946
			(27.771)
R^2	0.99714	0.99692	0.99712
F-statistic	15612.0	14522.0	15504.0

Table 3.13 Pooled Panel FMOLS Estimates

Regression	(i)	(ii)	(iii)
Constant	8.3540	0.78829	-2.5171
	(101.53)	(5.5701)	(-16.097)
Trend	0.057666	0.042487	-0.0032775
	(16.085)	(6.8898)	(-0.48102)
Real GDP_{it}/N_{it}	0.74761	0.75013	0.75432
	(143.73)	(134.84)	(133.07)
Real LA_{it}/Real GDP_{it}	-0.23127		
	(-10.108)		
$Inflation_{it}$		0.099043	
		(3.3551)	
Interest $Rates_{it}$			0.52417
			(18.439)
R^2	0.99620	0.99650	0.99685
F-statistic	61148.0	66469.0	73873.0

Table 3.14 **Group Mean Panel DOLS Estimates**

Regression	(i)	(ii)	(iii)
Constant	34.710	10.009	8.2637
	(28.911)	(4.1494)	(4.5482)
Trend	-0.015059	0.22578	0.035559
	(-0.23720)	(3.5248)	(4.2151)
Real GDP_{it}/N_{it}	0.73325	0.69601	0.73147
	(192.77)	(109.33)	(130.29)
Real LA_{it}/Real GDP_{it}	-1.0684		
	(-27.350)		
$Inflation_{it}$		-0.27920	
		(-7.2058)	
Interest $Rates_{it}$			-0.19944
			(-0.34646)
Average R^2	0.99981	0.99964	0.99969
F-statistic	-	-	-

Table 3.15 **Group Mean Panel FMOLS Estimates**

Regression	(i)	(ii)	(iii)
Constant	34.597	6.3529	5.5025
	(24.275)	(5.2481)	(3.1819)
Trend	0.014125	0.088841	0.090237
	(0.019343)	(4.6440)	(3.8999)
Real GDP_{it}/N_{it}	0.73757	0.72551	0.72646
	(237.04)	(129.90)	(133.64)
Real LA_{it}/Real GDP_{it}	-1.1442		
	(-23.970)		
$Inflation_{it}$		-0.24994	
		(-5.8020)	
Interest $Rates_{it}$			-0.13287
			(-1.6790)
Average R^2	0.99861	0.99904	0.99915
F-statistic	-	-	-

Part B

Fixed Effects Model with Individual Intercepts (Demeaned)[21]

Table 3.16 Pooled Panel OLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72839	0.73979	0.74255
	(7.1104)	(6.2580)	(6.2450)
Real LA_{it}/Real GDP_{it}	-0.52236		
	(-0.46113)		
$Inflation_{it}$		-0.055607	
		(-0.025880)	
Interest $Rates_{it}$			0.30029
			(0.12124)
R^2	0.99085	0.98708	0.98742
F-statistic	42135.0	29710.0	30533.0

Table 3.17 Pooled Panel DOLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72643	0.74055	0.74501
	(232.84)	(236.72)	(234.75)
Real LA_{it}/Real GDP_{it}	-0.50181		
	(-13.979)		
$Inflation_{it}$		0.033953	
		(1.5418)	
Interest $Rates_{it}$			0.51476
			(16.530)
R^2	0.99137	0.98821	0.98844
F-statistic	5484.2	4000.4	4082.3

[21] The dependent variable in Part B regressions is Real CON_{it}/N_{it}.

Table 3.18 **Pooled Panel FMOLS Estimates**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72627	0.73764	0.74321
	(232.79)	(235.80)	(234.19)
Real LA_{it}/Real GDP_{it}	-0.49518		
	(-13.794)		
$Inflation_{it}$		-0.24787	
		(-11.256)	
Interest $Rates_{it}$			0.27941
			(8.9726)
R^2	0.97387	0.96257	0.96415
F-statistic	13789.0	9516.3	9950.2

Table 3.19 **Group Mean Panel DOLS Estimates**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.74035	0.74491	0.75483
	(259.75)	(190.34)	(207.21)
Real LA_{it}/Real GDP_{it}	-0.97204		
	(-26.193)		
$Inflation_{it}$		-0.41214	
		(-5.3260)	
Interest $Rates_{it}$			0.039328
			(6.0353)
R^2	0.99973	0.99934	0.99950
F-statistic	-	-	-

Table 3.20 Group Mean Panel FMOLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.74074	0.73296	0.74583
	(306.13)	(233.82)	(206.09)
Real LA_{it}/Real GDP_{it}	-1.0586		
	(-30.325)		
$Inflation_{it}$		-0.57345	
		(-11.637)	
Interest $Rates_{it}$			-0.16471
			(-0.84555)
Average R^2	0.98167	0.96440	0.96229
F-statistic	-	-	-

Fixed Effects Model with Individual Intercepts and Individual Trends

(Demeaned and Detrended)

Table 3.21 Pooled Panel OLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72460	0.72796	0.72899
	(5.5082)	(4.4666)	(4.7624)
Real LA_{it}/Real GDP_{it}	-0.87334		
	(-0.43484)		
$Inflation_{it}$		0.020774	
		(0.010822)	
Interest $Rates_{it}$			0.14774
			(0.064128)
R^2	0.98373	0.97774	0.97791
F-statistic	23519.0	17082.0	17219.0

Table 3.22 Pooled Panel DOLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72226	0.72982	0.72931
	(253.69)	(302.74)	(308.99)
Real LA_{it}/Real GDP_{it}	-0.68916		
	(-21.037)		
$Inflation_{it}$		0.051108	
		(3.0116)	
Interest $Rates_{it}$			0.12957
			(5.5944)
R^2	0.98477	0.98084	0.98046
F-statistic	3085.7	2443.5	2395.6

Table 3.23 Pooled Panel FMOLS Estimates

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.72263	0.71585	0.72193
	(253.82)	(296.95)	(305.86)
Real LA_{it}/Real GDP_{it}	-0.75215		
	(-22.960)		
$Inflation_{it}$		-0.060837	
		(-3.5849)	
Interest $Rates_{it}$			0.033609
			(1.4512)
R^2	0.98255	0.97582	0.97630
F-statistic	20839.0	14933.0	15239.0

Table 3.24 **Group Mean Panel DOLS Estimates**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.74285	0.74761	0.74918
	(170.03)	(121.12)	(125.76)
Real LA_{it}/Real GDP_{it}	-1.0159		
	(-18.185)		
$Inflation_{it}$		-0.21834	
		(-3.6296)	
Interest $Rates_{it}$			0.0041037
			(2.7436)
Average R^2	0.99984	0.99970	0.99975
F-statistic	-	-	-

Table 3.25 **Group Mean Panel FMOLS Estimates**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73757	0.72551	0.72646
	(237.04)	(129.90)	(133.64)
Real LA_{it}/Real GDP_{it}	-1.1442		
	(-23.970)		
$Inflation_{it}$		-0.24994	
		(-5.8020)	
Interest $Rates_{it}$			-0.13287
			(-1.6790)
Average R^2	0.98261	0.98613	0.98677
F-statistic	-	-	-

Part C

The Fixed Effects Model with Individual Intercepts (Demeaned)[22]

Table 3.26 **Pooled Panel FMOLS Estimates (5 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73525	0.73877	0.74491
	(62.682)	(75.523)	(72.043)
Real LA_{it}/Real GDP_{it}	-0.35697		
	(-1.6383)		
$Inflation_{it}$		-0.28699	
		(-3.6899)	
Interest $Rates_{it}$			0.31773
			(7.4775)
$\bar{R}^2$	0.99045	0.98678	0.98741
F-statistic	38385.0	27621.0	29019.0
IC	4.1754	5.1701	5.1848

Table 3.27 **Pooled Panel FMOLS Estimates (7 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73624	0.73986	0.74511
	(56.106)	(74.028)	(69.468)
Real LA_{it}/Real GDP_{it}	-0.37007		
	(-1.5670)		
$Inflation_{it}$		-0.27485	
		(-3.4784)	
Interest $Rates_{it}$			0.33903
			(7.7094)
$\bar{R}^2$	0.99049	0.98681	0.98741
F-statistic	38548.0	27690.0	29008.0
IC	4.6302	5.6056	5.6268

[22]The dependent variable in Part C regressions is Real CON_{it}/N_{it}.

Table 3.28 **Pooled Panel FMOLS Estimates (9 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73590	0.74030	0.74542
	(57.610)	(72.697)	(68.285)
Real LA_{it}/Real GDP_{it}	-0.36702		
	(-1.5833)		
$Inflation_{it}$		-0.28083	
		(-3.5604)	
Interest $Rates_{it}$			0.35433
			(7.5395)
R^2	0.99049	0.98680	0.98740
F-statistic	38520.0	27659.0	28991.0
IC	5.0211	6.0162	6.0380

Table 3.29 **Pooled Panel FMOLS Estimates (12 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73614	0.73999	0.74504
	(56.941)	(71.541)	(68.437)
Real LA_{it}/Real GDP_{it}	-0.38031		
	(-1.6282)		
$Inflation_{it}$		-0.26758	
		(-3.3509)	
Interest $Rates_{it}$			0.33137
			(7.1729)
R^2	0.99053	0.98683	0.98741
F-statistic	38712.0	27727.0	29013.0
IC	5.6507	6.6325	6.6264

Fixed Effects Model with Individual Intercepts and Individual Trends

(Demeaned and Detrended)

Table 3.30 **Pooled Panel FMOLS Estimates (5 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73436	0.72186	0.73211
	(85.336)	(80.587)	(76.041)
Real LA_{it}/Real GDP_{it}	-0.45509		
	(-2.4439)		
$Inflation_{it}$		-0.081412	
		(-1.0869)	
Interest $Rates_{it}$			0.025979
			(1.0019)
R^2	0.98224	0.97746	0.97777
F-statistic	20464.0	16048.0	16271.0
IC	4.0335	5.0605	5.0575

Table 3.31 **Pooled Panel FMOLS Estimates (7 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73436	0.72315	0.73294
	(81.378)	(79.168)	(75.605)
Real LA_{it}/Real GDP_{it}	-0.40592		
	(-2.1228)		
$Inflation_{it}$		-0.080151	
		(-1.0683)	
Interest $Rates_{it}$			0.029831
			(1.0487)
R^2	0.98190	0.97751	0.97776
F-statistic	20077.0	16080.0	16268.0
IC	4.4557	5.4488	5.4623

Table 3.32 **Pooled Panel FMOLS Estimates (9 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73440	0.72396	0.73370
	(81.393)	(77.608)	(75.174)
Real LA_{it}/Real GDP_{it}	-0.39585		
	(-2.0675)		
$Inflation_{it}$		-0.076657	
		(-1.0139)	
Interest $Rates_{it}$			0.037720
			(1.2711)
R^2	0.98183	0.97754	0.97776
F-statistic	19993.0	16104.0	16269.0
IC	4.8509	5.8801	5.8904

Table 3.33 **Pooled Panel FMOLS Estimates (12 Factors)**

Regression	(i)	(ii)	(iii)
Real GDP_{it}/N_{it}	0.73461	0.72441	0.73421
	(80.555)	(77.272)	(74.675)
Real LA_{it}/Real GDP_{it}	-0.38798		
	(-2.0170)		
$Inflation_{it}$		-0.064364	
		(-0.84854)	
Interest $Rates_{it}$			0.042730
			(1.3602)
R^2	0.98176	0.97759	0.97776
F-statistic	19920.0	16137.0	16269.0
IC	5.4577	6.4789	6.4957

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Notes to the Tables

a) means significant at the 1% level, b) means significant at the 5% level. DF(constant only) critical values $1\% = -3.64, 5\% = -2.95$. DF(constant+trend) critical values $1\% = -4.22, 5\% = -3.55$. N(0,1) one-sided (LHS) critical values $1\% = -2.33, 5\% = -1.65$. N(0,1) one-sided (RHS) critical values $1\% = 2.33, 5\% = 1.65$. N(0,1) two-sided critical values $1\% = \pm 2.58, 5\% = \pm 1.96$. T denotes model estimated with a constant and trend. The figure shown in group-mean regressions is the Average R^2 of the individual countries. Lag lengths chosen by Ng and Perron (1995) method.

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Chapter 4

Nonstationary Panel Data and the Bootstrap

4.1 Introduction

In this chapter we illustrate how the bootstrap can be used with nonstationary panel data. There now exists a growing literature on the application of the bootstrap to time series models and a natural extension of this is to the area of stationary and nonstationary panel data.

Our contribution is to present a new and unique method to obtain bootstrap samples for constructing bootstrap confidence intervals for a panel data cointegrating regression, with the Kao and Chiang (2000) and Pedroni (2001) DOLS panel cointegration estimators being highlighted. Also using similar new and unique methods we show how the bootstrap can be used to compute the quantiles of a panel data AR(12) autoregression, using the Pesaran and Smith (1995) mean-group estimator.

The sections are as follows. In section 4.2 we describe the bootstrap and in section 4.3 the bootstrap confidence interval procedures are given. In section 4.4 the panel data estimators are discussed. In section 4.5 the bootstrap is used with time-series models and in section 4.6 the bootstrap panel data algorithms are presented. In section 4.7 are the bootstrap applications.

4.2 The Bootstrap

Since its introduction by Efron (1979) the bootstrap has been the focus of much research in statistics and econometrics. Numerous books have appeared on the topic, Davison and Hinkley (1997), Efron and Tibshirani (1993) and Hall (1992). Some bootstrap published research papers with an econometric orientation are Maddala and Jeong (1993), Hall (1994), Horowitz (1997) and Vinod (1993). The bootstrap is a method by which you can estimate the distribution of an estimator or test statistic by resampling your data. You actually treat the data as if it were the population for the purpose of evaluating the distribution of interest. In finite samples the bootstrap is often more accurate than first-order[1] asymptotic approximations. Thus it is a practical method of improving upon first-order asymptotic approximations. Such improvements are called asymptotic refinements and lead in general to more

[1]See § 4.7.3 on Efficient Estimation for a comment on the notion of first-order asymptotics.

efficient estimation. The bootstrap can provide asymptotic refinements in a number of situations, eg hypothesis testing and confidence interval estimation. The bootstrap can be used to obtain confidence intervals with reduced errors in coverage probabilities. That is the difference between the true and nominal coverage probabilities is often lower when the bootstrap is used than when first-order asymptotic approximations are used to obtain a confidence interval.[2]

A formal definition of Efron's (1979) nonparametric bootstrap is as follows. Consider a sample of I.I.D random variables $(y_1, y_2, \ldots, y_n)$ taken from a distribution characterised by the parameter θ. Let T be a statistic which is a function of the data, $T(\theta)$. T might be the sample mean of $(y_1, y_2, \ldots, y_n)$ for example. The bootstrap defines the empirical distribution function (EDF) of the data by assigning probability $\frac{1}{n}$ to each observed value of the random variables $y_i, \forall i$. Next the bootstrap draws repeated samples with replacement from the EDF, ie $(y_1, y_2, \ldots, y_n)$ to obtain a new bootstrap sample $(y_1^*, y_2^*, \ldots, y_n^*)$. We then use this sample to construct the bootstrap version of the statistic T, called T^*. We do this B times. The distribution of T^* is called the bootstrap distribution of T.

[2]Thus the real attraction of the bootstrap is that it offers a viable alternative in two important situations: (i) When calculation by mathematical analysis of the distribution of an estimator is too difficult or too tedious. (ii) When the asymptotic approximations of distributions used commonly for inference are inappropriate. This may occur when using small samples with estimators that have only asymptotic justification for their validity.

4.3 Bootstrap Confidence Interval Procedures

4.3.1 The standard or asymptotic confidence interval

The interval[3] estimate for a parameter β_k is just as useful as a point estimate. Together they tell us what the best estimate for β_k is and how much error we can expect. Texts such as Cramer (1946), Malinvaud (1980) and Casella and Berger (2002) provide general discussions of the interval estimate. Large sample theory is often used here with unknown confidence interval parameters substituted by their large sample plug-in estimates, which then provides asymptotic justification for the confidence interval.

Assume $Z = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \sim N(0,1)$ and let $z^{(\alpha)}$ be the 100αth percentile point of a $N(0,1)$ distribution as given by the standard normal table. Thus for $\alpha = 0.025$ and 0.05 then $z^{(0.025)} = -1.96$ and $z^{(0.05)} = -1.645$ and $z^{(1-\alpha)} = z^{(0.975)} = 1.96$ and $z^{(0.95)} = 1.645$, respectively etc. Thus we can write

$$Prob\left\{z^{(\alpha)} \leq \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \leq z^{(1-\alpha)}\right\} = 1 - 2\alpha \tag{4.1}$$

or

$$Prob\left\{\hat{\beta}_k - z^{(1-\alpha)} se(\hat{\beta}_k) \leq \beta_k \leq \hat{\beta}_k - z^{(\alpha)} se(\hat{\beta}_k)\right\} = 1 - 2\alpha \tag{4.2}$$

[3] When constructing confidence intervals for multiparameter vectors, eg $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_k)'$ we get k-dimensional confidence rectangles. To avoid notational difficulties we shall restrict our bootstrap confidence intervals to the single parameter case, ie the element $\hat{\beta}_k$ of $\hat{\beta}$. However, it should be noted that these single parameters belong to multiparameter vectors.

or

(4.3) $$Prob\left\{\beta_k \in \left[\hat{\beta}_k - z^{(1-\alpha)}se(\hat{\beta}_k), \hat{\beta}_k - z^{(\alpha)}se(\hat{\beta}_k)\right]\right\} = 1 - 2\alpha.$$

In general we can write

(4.4) $$\left[\hat{\beta}_k - z^{(1-\alpha)}se(\hat{\beta}_k), \hat{\beta}_k - z^{(\alpha)}se(\hat{\beta}_k)\right]$$

as the standard confidence interval for β_k with coverage probability $= 1-2\alpha$. We can also write the confidence interval as

(4.5) $$\left[\hat{\beta}_k \pm z^{(1-\alpha)}se(\hat{\beta}_k)\right].$$

The latter formula shows that $z^{(\alpha)} = -z^{(1-\alpha)}$, which when $\alpha = 0.05$ and $1 - 2\alpha = 0.90$ means that we get

(4.6) $$\left[\hat{\beta}_k \pm 1.645se(\hat{\beta}_k)\right].$$

We can also write equations (4.4),(4.5) and (4.6) in terms of the upper and lower confidence bounds, that is

(4.7) $$\hat{\theta}^s_{lo} = \left[\hat{\beta}_k - z^{(1-\alpha)}se(\hat{\beta}_k)\right] \quad = \text{Lower Bound}.$$

(4.8) $$\hat{\theta}^s_{up} = \left[\hat{\beta}_k - z^{(\alpha)}se(\hat{\beta}_k)\right] \quad = \text{Upper Bound}.$$

Hence the standard $(1 - 2\alpha)$ confidence interval becomes $\left[\hat{\theta}^s_{lo}, \hat{\theta}^s_{up}\right]$. The above confidence intervals are exact. However assuming $Z = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \sim N(0,1)$ holds only asymptotically, then these confidence intervals become approximations with large sample justification only.

4.3.2 The Percentile Method

This was developed by Efron (1981,1982) and uses the bootstrap estimates of β_k to construct a confidence interval. Given the bootstrap data set $\{X_{it}^{*b}, i = 1, \ldots, N, t = 1, \ldots, T\}$ for $b = 1, \ldots, B$, let the vector of bootstrap replications (ie $\hat{\beta}^*(b) = s(X_{it}^{*b})$, the estimate of β_k), be $\hat{\beta}^*$. Let $\hat{G}$ be the cumulative distribution function (CDF) of $\hat{\beta}^*$. Then the exact $(1-2\alpha)$ percentile confidence interval is defined by the α and $(1-\alpha)$ percentiles of $\hat{G}$

$$\left[\hat{\theta}_{lo}^{p}, \hat{\theta}_{up}^{p}\right] = \left[\hat{G}^{-1}(\alpha), \hat{G}^{-1}(1-\alpha)\right]. \tag{4.9}$$

Since $\hat{G}^{-1}(\alpha) = \hat{\beta}^{*(\alpha)} = 100\alpha$th percentile of the bootstrap distribution and $\hat{G}^{-1}(1-\alpha) = \hat{\beta}^{*(1-\alpha)} = 100(1-\alpha)$th percentile of the bootstrap distribution we have

$$\left[\hat{\theta}_{lo}^{p}, \hat{\theta}_{up}^{p}\right] = \left[\hat{\beta}^{*(\alpha)}, \hat{\beta}^{*(1-\alpha)}\right]. \tag{4.10}$$

To implement this in practice one uses a finite number of bootstrap replications. It is well known that the number of replications required to compute a confidence interval is around 1000 and is much greater than the number required to compute standard errors, ie around 100 (see Hall (1986) on the number of bootstrap replications needed to form a confidence interval). The percentile method does, however, have problems when used with small samples or with asymmetric distributions.

Percentile Bootstrap Algorithm

1. Generate B independent bootstrap data sets $X^{*1}, X^{*2}, \ldots, X^{*B}$, where $X^{*j} = \{X_{it}^{*j}, i = 1, \ldots, N, t = 1, \ldots, T\}$.

2. Compute the bootstrap replication $\hat{\beta}^*(b) = s(X_{it}^{*b})$ for $b = 1, \ldots, B$.

3. Let $\hat{\beta}_B^{*(\alpha)}$ be the 100αth empirical percentile of the $\hat{\beta}^*(b)$ values, ie the $B.\alpha$th value in the ordered list of B replications.

Hence if $B = 2000$ and $\alpha = 0.05$ then $\hat{\beta}_B^{*(\alpha)}$ is the 100th ordered value of the replications. Similarly $\hat{\beta}_B^{*(1-\alpha)}$ is the $100(1-\alpha)$th empirical percentile. Thus the approximate $(1-2\alpha)$ percentile interval is

$$\left[\hat{\theta}_{lo}^{p}, \hat{\theta}_{up}^{p}\right] = \left[\hat{\beta}_B^{*(\alpha)}, \hat{\beta}_B^{*(1-\alpha)}\right]. \tag{4.11}$$

B here denotes that the approximation is based on B replications. As $B \to \infty$ then $\hat{\beta}_B^{*(\alpha)} \to \hat{\beta}^{*(\alpha)}$ and $\hat{\beta}_B^{*(1-\alpha)} \to \hat{\beta}^{*(1-\alpha)}$.

4.3.3 Bias Corrected Method (BC)

Both the bias corrected method, and the bias corrected and accelerated method modify the percentile bootstrap method. Both were introduced by Efron (1987) and Efron and Tibsharani (1986). They depend on two parameters: (i) $\hat{a}$ called the acceleration and (ii) $\hat{z}_0$ called the bias correction. We now show how to calculate the bias corrected interval endpoints

$$\left[\hat{\theta}_{lo}^{bc}, \hat{\theta}_{up}^{bc}\right] = \left[\hat{\beta}^{*(\alpha_1)}, \hat{\beta}^{*(\alpha_2)}\right] \tag{4.12}$$

where $\alpha_1 = \Phi\left(2\hat{z}_0 + z^{(\alpha)}\right)$ and $\alpha_2 = \Phi\left(2\hat{z}_0 + z^{(1-\alpha)}\right)$.

Hence

$$\left[\hat{\theta}_{lo}^{bc}, \hat{\theta}_{up}^{bc}\right] = \left[\hat{G}^{-1}(\alpha_1), \hat{G}^{-1}(\alpha_2)\right], \tag{4.13}$$

$$= \left[\hat{G}^{-1}\left(\Phi\left(2\hat{z}_0 + z^{(\alpha)}\right)\right), \hat{G}^{-1}\left(\Phi\left(2\hat{z}_0 + z^{(1-\alpha)}\right)\right)\right]. \tag{4.14}$$

Where $\Phi(.)$ is the standard normal cumulative distribution function and $z^{(\alpha)}$ is the 100αth percentile point of a standard normal distribution, eg $z^{(0.95)} = 1.645$ and $\Phi(1.645) = 0.95$. When $\hat{z}_0 = 0$ the BC interval is the same as the percentile interval.

Computation of $\hat{z}_0$

We compute $\hat{z}_0$ directly from the proportion of bootstrap replications less than the original estimate of β_k, ie $\hat{\beta}_k$,[4]

$$\hat{z}_0 = \Phi^{-1}\left(\#\frac{\{\hat{\beta}^*(b) < \hat{\beta}_k\}}{B}\right). \tag{4.15}$$

Where Φ^{-1} is the inverse function of a standard normal cumulative distribution function, eg $\Phi^{-1}(0.95) = 1.645$.

[4] Here # means proportion.

4.3.4 Bias Corrected and Accelerated Method ($\mathbf{BC_a}$)

Here we calculate the BC_a interval endpoints as

$$\left[\hat{\theta}_{lo}^{bc_a}, \hat{\theta}_{up}^{bc_a}\right] = \left[\hat{\beta}^{*(\alpha_1)}, \hat{\beta}^{*(\alpha_2)}\right] \tag{4.16}$$

where

$$\alpha_1 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(\alpha)})}\right) \tag{4.17}$$

$$\alpha_2 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(1-\alpha)})}\right). \tag{4.18}$$

Hence

$$\left[\hat{\theta}_{lo}^{bc_a}, \hat{\theta}_{up}^{bc_a}\right] = \left[\hat{G}^{-1}(\alpha_1), \hat{G}^{-1}(\alpha_2)\right] \tag{4.19}$$

$$= \left[\hat{G}^{-1}\left(\Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(\alpha)})}\right)\right), \hat{G}^{-1}\left(\Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(1-\alpha)})}\right)\right)\right]. \tag{4.20}$$

Where $\Phi(.)$ is as before. Again when $\hat{a} = \hat{z}_0 = 0$ the BC_a interval is the same as the percentile interval.

Computation of $\hat{a}$

Of the number of parametric and nonparametric ways to compute $\hat{a}$, the simplest is probably the Jackknife estimate which we now explain. The jackknife method was developed by Quenouille (1949) and discussed in Efron (1982). See Wu (1986) for applications to regression analysis.

The Delete-One Jackknife in the Panel Data Regression Model

Given the linear panel data model

$$y_{it} = \alpha_i + x'_{it}\beta + e_{it} \qquad (4.21)$$

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$,

where $\{y_{it}\} \sim \mathrm{I}(1)$, $\{\mathrm{x}_{it}\} \sim \mathrm{I}(1)$, are random variables, $\{e_{it}\} \sim \mathrm{I}(0)$ a stationary disturbance term, and β and α_i are $((k-1)\times 1)$ and (1×1) parameters of interest, respectively. Then writing in matrix form

$$y = [I_N \otimes i_T]\beta_1 + X_s\beta_s + e \qquad (4.22)$$

$$= [I_N \otimes i_T \quad X_s] \begin{pmatrix} \beta_1 \\ \beta_s \end{pmatrix} + e \qquad (4.23)$$

$$= X\beta + e \qquad (4.24)$$

where $y = (y_{11}, \ldots, y_{NT})'$ is $(NT \times 1)$, $e = (e_{11}, \ldots, e_{NT})'$ is $(NT \times 1)$, $i_T = (1, 1, \ldots, 1)'$ is $(T \times 1)$, $\beta_s = (\beta_2, \ldots, \beta_k)'$ is $((k-1)\times 1)$, $\beta_1 = (\beta_{11}, \ldots, \beta_{1N})'$ is $(N \times 1)$, $X = [I_N \otimes i_T \quad X_s]$ is $(NT \times (k-1) + N)$ and $\beta = (\beta_1, \beta_s)'$ is $((K-1) + N \times 1)$. Note that $X_s = \begin{pmatrix} x_{21t}, x_{31t}, \ldots, x_{k1t} \\ x_{22t}, x_{32t}, \ldots, x_{k2t} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ x_{2nt}, x_{3nt}, \ldots, x_{knt} \end{pmatrix}$ and $var(e) = \Sigma$.

Given $X'X$ is non-singular and Σ a diagonal matrix with constant elements, the Ordinary Least Squares (OLS) estimator of β is

$$\hat{\beta} = (X'X)^{-1} X'y. \qquad (4.25)$$

Computation of the Delete-One Jackknife estimator

1. Let $\hat{\beta}_{(it)}$ be the itth jackknife (OLS) estimate of β obtained by recomputing $\hat{\beta}$ in equation (4.25) with the itth group $(y_{it}, 1, x_{2it}, \ldots, x_{kit})$ deleted from the sample.

2. Compute the jackknife estimator as

$$\hat{\beta}_{(.)} = \frac{\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \hat{\beta}_{(it)}}{NT}. \tag{4.26}$$

Since $\hat{\beta}_{(it)}$ is a $((k-1) + N \times 1)$ multiparameter vector we can choose our parameter of interest as $\hat{\beta}_{(kit)}$, ie the kth element of $\hat{\beta}_{(it)}$ (see above note on confidence intervals for multiparameter vectors). Then a simple expression for the single parameter accelerator constant is

$$\hat{a} = \frac{\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (\hat{\beta}_{(k.)} - \hat{\beta}_{(kit)})^3}{6 \left[\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} (\hat{\beta}_{(k.)} - \hat{\beta}_{(kit)})^2 \right]^{\frac{3}{2}}}. \tag{4.27}$$

where $\hat{\beta}_{(k.)}$ is the kth element of $\hat{\beta}_{(.)}$.

The confidence intervals mentioned so far, ie the standard or asymptotic, percentile, BC and BC_a work well if $\hat{\beta}_k$ or some transformation of it has an approximate Gaussian distribution and the other parameters of the model satisfy some relatively simple regularity assumptions. These are called transformation repecting properties of the interval and allow modifications and improvements to be made to the interval. See also Efron (1987).

4.3.5 The Bootstrap-t Method

This was introduced in Efron (1982) and detailed in Efron and Tibshirani (1993). The bootstrap-t improves upon the percentile method. It is less computer intensive than the double bootstrap[5] and easier to implement than the BC and BC_a methods, ie it involves no difficult computations. See DiCiccio and Romano (1988) for a review of bootstrap confidence intervals.

Consider the standard confidence interval derived in the previous section. Starting with $Z = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \sim N(0,1)$. This led to the exact confidence interval

$$\left[\hat{\beta}_k - z^{(1-\alpha)} se(\hat{\beta}_k), \hat{\beta}_k - z^{(\alpha)} se(\hat{\beta}_k)\right]. \tag{4.28}$$

We know that when we use plug-in estimates, when the variance of $\hat{\beta}_k$ is unknown, that this interval holds asymptotically only in large samples. In finite samples, then, we obtain only approximate confidence intervals. For small samples the approximation of Z was improved upon by W. Gosset in 1908 with his Student's t-distribution. Now for small samples of size n with plug-in estimates for $var(\hat{\beta}_k)$ we have $Z_t = \frac{\hat{\beta}_k - \beta_k}{se(\hat{\beta}_k)} \sim t_{(n-1)}$. Here $t_{(n-1)}$ means Student's t-distribution with $(n-1)$ degrees of freedom (d.f.). Also $t_{(n-1)} \to N(0,1)$ as $n \to \infty$. The percentiles of the t-distribution for varying degrees of freedom are tabulated in the Student's t-tables.

[5]The double bootstrap is sometimes called bootstrap iteration. It was developed in Hall and Martin (1988), Martin (1990) and Hall (1992) and is another way to improve on interval accuracy. It is a second-order accurate method.

Let $t^{(\alpha)}_{(n-1)}$ denote the αth percentile of the Student's t-distribution with $(n-1)$ d.f. Then our approximate $(1-2\alpha)$ confidence interval is

$$\left[\hat{\beta}_k - t^{(1-\alpha)}_{(n-1)} se(\hat{\beta}_k), \hat{\beta}_k - t^{(\alpha)}_{(n-1)} se(\hat{\beta}_k)\right]. \tag{4.29}$$

Our bootstrap-t interval is a generalisation of the above Student's t interval. The procedure estimates the distribution of Z_t directly from the data and tabulates percentiles that are appropriate for the data at hand.

The Bootstrap-t Algorithm

1. Generate B independent bootstrap data sets $X^{*1}, X^{*2}, \ldots, X^{*B}$, where $X^{*j} = \{X^{*j}_{it}, i = 1, \ldots, N, t = 1, \ldots, T\}$ and $j = 1, \ldots, B$.

2. Compute the bootstrap replication

$$Z^*(b) = \frac{\hat{\beta}^*(b) - \hat{\beta}_k}{\hat{se}^*(b)} \quad \text{for} \quad b = 1, \ldots, B. \tag{4.30}$$

   Where as above $\hat{\beta}^*(b)$ is the value of $\hat{\beta}_k$ for the bootstrap sample X^{*b} and $\hat{se}^*(b)$ is the estimated standard error of $\hat{\beta}^*(b)$ for the bootstrap sample X^{*b}. N.B. $\hat{se}^*(b)$ is estimated as the regression $se(\hat{\beta}_k)$ from each bootstrap sample X^{*b} regression.

3. Let Z^* be the ordered list of $Z^*(b)$ replications. Estimate the αth percentile of the sorted vector of $Z^*(b)$'s by the value $\hat{t}^{(\alpha)}$ such that

$$\left(\# \frac{\{Z^*(b) \leq \hat{t}^{(\alpha)}\}}{B}\right) = \alpha. \tag{4.31}$$

Thus if $B = 1000$ and $\alpha = 5\%$ then the 100αth empirical percentile is the $B.\alpha$th $= 1000.(0.05) = 50$th value of the ordered list of $Z^*(b)$ replications (or Z^*). Also for $\alpha = 95\%$, this gives the 950th value of the ordered list of $Z^*(b)$'s (or Z^*). Thus the bootstrap-t confidence interval is given by

$$\left[\hat{\beta}_k - \hat{t}^{(1-\alpha)} se(\hat{\beta}_k), \hat{\beta}_k - \hat{t}^{(\alpha)} se(\hat{\beta}_k)\right] \tag{4.32}$$

where $\hat{t}^{(\alpha)}$ is the αth percentile of the Z^* distribution.

We can also write equation (4.32) using endpoints as

$$\hat{\theta}^t_{lo} = \left[\hat{\beta}_k - \hat{t}^{(1-\alpha)} se(\hat{\beta}_k)\right] \quad = \text{Lower Bound.} \tag{4.33}$$

$$\hat{\theta}^t_{up} = \left[\hat{\beta}_k - \hat{t}^{(\alpha)} se(\hat{\beta}_k)\right] \quad = \text{Upper Bound.} \tag{4.34}$$

Hence the $(1 - 2\alpha)$ bootstrap-t confidence interval becomes $\left[\hat{\theta}^t_{lo}, \hat{\theta}^t_{up}\right]$.

4.4 The Panel Data Estimators

4.4.1 The Panel Cointegration Estimators

In this chapter we use some of the panel cointegration estimators discussed in detail in chapter 3, ie the Kao and Chiang (2000) pooled panel DOLS estimator and the Pedroni (2001) group-mean panel DOLS estimator. These panel cointegrating regression estimators utilise a pairs bootstrap technique for their application, discussed later. Finally the last panel estimator to be used here is also a group-mean estimator as follows.

4.4.2 The Pesaran and Smith Group-Mean Estimator

In their paper Pesaran and Smith (1995) propose the use of group-mean estimators for dynamic heterogeneuos panels. They show that the aggregation or pooling of dynamic heterogeneous panels can produce very misleading estimates. Here we use the group-mean estimator for a panel data autoregressive AR(p) model. Consider the heterogeneous panel data model

$$y_{it} = \alpha_i + \theta_{1i} y_{it-1} + \theta_{2i} y_{it-2} + \ldots + \theta_{pi} y_{it-p} + v_{it} \tag{4.35}$$

where $v_{it} \sim i.i.d.(0, \sigma^2)$ for $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

One alternative to using an ADF($p-1$) regression to form bootstrap samples and risk the bootstrap inconsistency problem when there is a unit root is to bootstrap the levels autoregression. Here a unit root in y_{it} coincides with $\theta_{1i} + \theta_{2i} + \ldots + \theta_{pi} = 1, \forall i$. Let us write the above model as

$$y_i = \alpha_i I + \theta_{1i} y_{i-1} + \theta_{2i} y_{i-2} + \ldots + \theta_{pi} y_{i-p} + v_i. \tag{4.36}$$

Or

$$y_i = X_i \beta_i + v_i \tag{4.37}$$

where $\beta_i = (\alpha_i, \theta_{1i}, \theta_{2i}, \ldots, \theta_{pi})'$ a $((p+1) \times 1)$ vector and $X_i = (I, y_{i-1}, y_{i-2}, \ldots, y_{i-p})$ a $(T \times (p+1))$ matrix also y_i and v_i are $(T \times 1)$ column vectors and I is a column vector of ones. Then the OLS estimator of β_i is

$$\hat{b}_i = \left(X_i^{\prime} X_i\right)^{-1} X_i^{\prime} y_i. \tag{4.38}$$

The group-mean estimator of Pesaran and Smith (1995) then becomes

$$\hat{b}_{GM} = \frac{1}{N}\sum_{i=1}^{N} \hat{b}_i. \tag{4.39}$$

Also the t-statistic of $\hat{b}_{GM}$, ie $t(\hat{b}_{GM})$, becomes the averaged t-statistics of $\hat{b}_i$, ie $t(\hat{b}_i)$,

$$t(\hat{b}_{GM}) = \frac{1}{N}\sum_{i=1}^{N} t(\hat{b}_i). \tag{4.40}$$

This estimator can now be bootstrapped in the same way as the group-mean DOLS estimator. However in this case we do not have N cointegrating regressions for which a pairs bootstrap is appropriate but N time-series regressions for which we may use a residual bootstrap or a block bootstrap scheme. See below for details.

4.5 The Bootstrap and Time Series Models

4.5.1 The Bootstrap and Cointegrating Regressions

The simple bootstrap method of Efron (1979) was originally designed for i.i.d. errors. When using time-series models, such as unit root and cointegration models, the bootstrap methodology needs to be modified to cope with errors that might not be i.i.d., eg the assumptions on u_{it} might range from white noise or weak stationarity to an m-dependent, strong mixing sequence. Li and Maddala (1996,1997) discuss a number of bootstrap methods that are applicable to time-series models. In particular cointegrating regressions are studied and the appropriate bootstrap method considered. Discussed are the recursive bootstrap, the moving blocks (MBB) bootstrap and the stationary (SB) bootstrap and it is explained which method suits a particular situation. Li and Maddala (1996,1997) also discuss the choice of procedure for the generation of the bootstrap samples when using cointegrating regressions and highlight the choice between the direct method of bootstrapping the data or the alternative of bootstrapping the residuals. They explain that for cointegrating regressions only the latter is appropriate. The basic argument is that all the information of the structure of the model should be used when generating the bootstrap samples. Only when the residual bootstrap method is used is this condition satisfied. They suggest the pairs bootstrap method for bootstrapping the residuals in the cointegrating regression. Thus estimate equation (3.24), in chapter 3, by DOLS and obtain residuals $\hat{v}_{it}$, noting that $\hat{\beta}_1 \ldots \hat{\beta}_k$ are superconsistent. Obtain also the residuals $\hat{w}_{1it} = \Delta x_{1it}, \ldots, \hat{w}_{kit} = \Delta x_{kit}$. Bootstrap the pairs $(\hat{v}_{it}, \hat{w}_{it}^{\prime})^{\prime}$ where $\hat{w}_{it}^{\prime} = (\hat{w}_{1it}, \ldots \hat{w}_{kit})$, perhaps after recentering the residuals. Next construct the bootstrap samples of $x^*_{1it}, \ldots, x^*_{kit}$ recursively and finally using $\hat{\beta}_1, \ldots \hat{\beta}_k, v^*_{it}, x^*_{1it} \ldots x^*_{kit}$, etc. $\forall i, t$, construct the sample $y^* = (y^*_{11}, y^*_{12}, \ldots, y^*_{it})'$ to be used for computing the bootstrap replications of $\beta^*_1, \ldots, \beta^*_k$ in equation (3.24). Finally Li and Maddala (1997) conduct a Monte Carlo experiment to compare the asymptotic FMOLS methods with MBB and SB methods using FMOLS, in a cointegrating regression with serial correlation in the errors and endogeneity of the regressors. They conclude that complications arise if there is serial correlation in the residuals of the cointegrating regression. Hence in the presence of serial correlation of the errors the pairs bootstrap should be modified to take this into account. If the auto-correlation structures of $\hat{v}_{it}$ are known then a recursive bootstrap can be applied to them, in addition to the pairs bootstrap. Otherwise for general unknown serial correlation one can use the moving block bootstrap, in addition to the pairs bootstrap. Li (1994) and Psaradakis (2001) also study the topic of bootstrapping cointegrating regressions as does Chang, Park and Song (2002) although they do so for the time-series case only. Chang, Park and Song (2002) employ the sieve bootstrap method coupled with the pairs bootstrap for generating bootstrap samples. They also conduct some Monte Carlo simulations. The sieve bootstrap (see Buhlmann (1997)) is used when the DGP can be represented as an infinite order autoregression.

The sieve bootstrap replaces this infinite order autoregression by an approximating finite order autoregression from which coeffients are estimated and residuals resampled. Balcombe (2004) and Fachin (2000) use the bootstrap with cointegrated systems, whilst Herwartz and Neumann (2005) use the wild bootstrap for systems of single-equation ECM's. In the wild bootstrap, introduced by Liu (1988), the error vector is resampled from a constructed distribution satisfying some conditions on the first three moments. A single observation is used to estimate the true distribution of the residual. Monte Carlo simulations of small sample properties are also carried out in the above papers. Other bootstrap applications in this area have come from Phillips (2001), Park (2003), Davidson (2002), Burridge and Taylor (2004). Park (2003b) and Chang (2004) have considered bootstrap unit root tests, the latter for panels with cross-sectional dependency. Other work on panels include Hahn and Newey (2004) who correct for incidental parameter biases in fixed effects models using the Jackknife. Although the use of the bootstrap in empirical and theoretical studies is quite widespread in the time-series literature, however the use of the bootstrap is very limited in the field of nonstationary (and stationary) panel data. This is due to the literature being in the very early stages of its development.

Our main contribution in this chapter then is to develop two different approaches to bootstrapping a panel cointegrating regression. In the first the pooled panel DOLS estimator of Kao and Chiang (2000) is studied using the pairs bootstrap method. The second is based on the Pedroni (2001) group-mean panel DOLS estimator. Here we apply the pairs bootstrap to each individual country in the panel and use the bootstrap methodology to compute group averages.

4.5.2 The Bootstrap and Time Series Regressions

One of the key characteristics of time-series data is the great deal of dependency that exists in the data. Thus when using the bootstrap with time-series data the bootstrap sampling must be carried out in a way that captures the dependence structure of the DGP. There are two main ways of doing this. The first is parametric. By fitting a parametric model the time dependent data is reduced to an i.i.d. structure suitable for niave bootstrap resampling. Such a parametric model is the stationary autoregressive AR(p) model. Efron and Tibshirani (1986) bootstrap an AR(1) and AR(2) model and Stine (1987) extends the analysis to an AR(p) model. Consider the general AR(p) model of the form

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_p y_{t-p} + \varepsilon_t. \tag{4.41}$$

where $\varepsilon_t \sim i.i.d(0, \sigma^2)$ and the roots of $(1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p)$ lie outside the unit circle. The recursive method, first introduced by Freedman and Peters (1984) and used by Efron and Tibshirani (1986), is used now to generate the bootstrap samples. Estimate equation (4.41) by OLS and obtain the residuals $\hat{\varepsilon}_t$. Rescale and/or recentre the residuals so $\bar{\varepsilon}_t = \left(\hat{\varepsilon}_t - \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\right)\left(\frac{T}{T-p}\right)^{\frac{1}{2}}$. Resample $\bar{\varepsilon}_t$ with replacement to get the bootstrap residuals ε_t^*. Then conditional on the initial conditions generate the bootstrap samples recursively using $y_t^* = \hat{\alpha}_1 y_{t-1}^* + \hat{\alpha}_2 y_{t-2}^* + \ldots + \hat{\alpha}_p y_{t-p}^* + \varepsilon_t^*$. Although conditioning on any particular initial conditions is asymptotically negligible, care must be taken. One solution is to set $y_t^* = y_t$ for $t = 1 - p, \ldots, 0$. Another is setting $(y_{1-p}, \ldots, y_0) = 0$, see Rayner (1990). Finally with the B bootstrap samples of y_t^* obtain B bootstrap replications of the parameters of interest $\alpha_1^{*(B)}, \alpha_2^{*(B)}, \ldots, \alpha_p^{*(B)}$, by OLS. Inoue and Kilian (2002) bootstrap an AR(p) model with possible unit roots this way.

The second way of bootstrapping dependent data is nonparametric and involves the resampling of blocks of data mentioned earlier, that is the MBB. Carlstein (1986) first discussed the idea of bootstrapping blocks (BB) of observations rather than individual observations. His blocks were non-overlapping. Later Kunsch (1989) and Liu and Singh (1992) introduced the moving block bootstrap (MBB) with overlapping blocks. Both block methods divide the data of n observations into blocks of length l and select b of these blocks by resampling with replacement from all the possible blocks. Assume $n = b \times l$, then in Carlstein's scheme there are just b blocks whilst in Kunsch's scheme there are $n - l + 1$ blocks. As an example let $n = 6$ and $l = 3$ and suppose the data are $X_t = \{4, 5, 8, 3, 1, 9\}$. The blocks according to Carlstein are $\{(4, 5, 8), (3, 1, 9)\} = 2$, whilst the blocks according to Kunsch are $\{(4, 5, 8,), (5, 8, 3), (8, 3, 1,), (3, 1, 9)\} = 4$. The MBB is reputedly more efficient than the BB but the available evidence indicates that the efficiency gain is small. A problem with both the block bootstrap methods is that the pseudo time-series generated by the block method is not stationary, even if the original series was. For this reason Politis and Romano (1994) introduced the stationary (block) bootstrap where the stationarity of the bootstrap is preserved. In their papers both Carlstein and Kunsch give some general rules for choice of the optimal block size. This is extended in Hall and Horowitz (1993). For the thesis we used a sensitivity analysis similar to the one discussed in Berkowitz and Kilian (2000). Consider a realisation of length T of a linear stationary time-series $\{y_t\}$.

1. Approximate the DGP by a parametric AR(p) .

2. Using a small pre-specified number of replications generate block boot-

strap data $\{y_t^*\}$ for many different block sizes k.

3. Calculate the statistics of interest for $\{y_t^*(k)\}$.

4. Select the block size k which on average produces the most accurate statistic or point estimates.

5. Use this optimal block size in the full replications block bootstrap model.

4.6 The Panel Data Bootstrap Algorithms

A very important part of the bootstrap methodology[6] is the generation of bootstrap samples. When the data is not i.i.d., as mentioned before, the bootstrap needs to be modified to cope with the new data structure. We describe now the new and unique bootstrap algorithms for generating the bootstrap samples for panel cointegrating regressions. These are original and developed by the author and hitherto unpresented in the panel literature. The methodologies for § 4.6.1 and 4.6.2 are similar. In both we apply the pairs bootstrap to the residuals of a cointegrating regression. In § 4.6.1 this for an NT observation regression, whereas in § 4.6.2 it is for a T observation

[6]This chapter makes extensive use of the Ox programming language for the implimentation of the bootstrap algorithms and computations of the bootstrap statistical models. A preprogrammed package available for bootstrap and simulation applications is the Bootstrap Ox Package written by Professor James Davidson available at http://www.oxmetrics.net/. See Appendix 2 for more details on the Ox software.

regression, before averaging over the cross-section dimension.

4.6.1 The Pairs Bootstrap and the Pooled DOLS estimator

The pairs bootstrap procedure is carried out by resampling the errors from the estimated equation (3.24), in chapter 3, and the stochastic error terms of the I(1) regressors.

1. Compute the predicted residuals using the estimates from equation (3.24), ie $\hat{\alpha}_i, \hat{\beta}$ and $\hat{c}_{ij}$ thus

   (4.42) $$\hat{\hat{v}}_{it} = y_{it} - \hat{\alpha}_i - x'_{it}\hat{\beta} - \Sigma_{j=-q}^{q}\hat{c}_{ij}\Delta x_{it+j}$$

   for $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

2. Obtain the residuals $\hat{w}_{1it} = \Delta x_{1it}, \ldots, \hat{w}_{kit} = \Delta x_{kit}$ and form the vector $\hat{w}_{it} = (\hat{w}_{1it}, \ldots, \hat{w}_{kit})'$ and $\hat{w} = (\hat{w}'_{i1}, \hat{w}'_{i2}, \ldots, \hat{w}'_{NT})'$. Recentre these using

   (4.43) $$\hat{w}_{(.)} = \frac{1}{NT}\Sigma_{t=1}^{T}\Sigma_{i=1}^{N}\hat{w}_{it}$$

   for $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

   Thus we have $\hat{w}^c = (\hat{w}^{c\prime}_{i1}, \hat{w}^{c\prime}_{i2}, \ldots, \hat{w}^{c\prime}_{NT})'$ where

   (4.44) $\hat{w}^c_{i1} = (\hat{w}_{i1} - \hat{w}_{(.)}), \hat{w}^c_{i2} = (\hat{w}_{i2} - \hat{w}_{(.)}), \ldots, \hat{w}^c_{NT} = (\hat{w}_{NT} - \hat{w}_{(.)})$.

3. If we assume that the residuals $\hat{\hat{v}}_{it}$ follow an AR(1) process then

$$\hat{\hat{v}}_{it} = \rho\hat{\hat{v}}_{it-1} + \varepsilon_{it} \tag{4.45}$$

where ε_{it} is a white noise error term.

Run the regression of equation (4.45) and obtain the residuals $\hat{\varepsilon}_{it}$ and also $\hat{\rho}$ and the vector of residuals $\hat{\varepsilon} = (\hat{\varepsilon}_{i1}, \hat{\varepsilon}_{i2}, \ldots, \hat{\varepsilon}_{NT})'$. Recentre these using

$$\hat{\varepsilon}_{(.)} = \frac{1}{NT}\Sigma_{t=1}^{T}\Sigma_{i=1}^{N}\hat{\varepsilon}_{it}. \tag{4.46}$$

Thus the vector of recentred residuals is $\hat{\varepsilon}^c = (\hat{\varepsilon}^c_{i1}, \hat{\varepsilon}^c_{i2}, \ldots, \hat{\varepsilon}^c_{NT})'$ where

$$\hat{\varepsilon}^c_{i1} = (\hat{\varepsilon}_{i1} - \hat{\varepsilon}_{(.)}), \hat{\varepsilon}^c_{i2} = (\hat{\varepsilon}_{i2} - \hat{\varepsilon}_{(.)}), \ldots, \hat{\varepsilon}^c_{NT} = (\hat{\varepsilon}_{NT} - \hat{\varepsilon}_{(.)}). \tag{4.47}$$

4. Resample with replacement from the paired vector of recentred residuals, $\hat{z}^c_{it} = (\hat{\varepsilon}^c_{it}, \hat{w}^{c/}_{it})'$ to get the bootstrap sample $z^*_{it} = (\varepsilon^*_{it}, w^{*/}_{it})'$. So that ε^* and w^* are the vectors of resampled residuals

$$\varepsilon^* = (\varepsilon^*_{i1}, \varepsilon^*_{i2}, \ldots, \varepsilon^*_{NT})' \quad \text{and} \quad w^* = (w^{*/}_{i1}, w^{*/}_{i2}, \ldots, w^{*/}_{NT})'. \tag{4.48}$$

5. Obtain the bootstrap samples of $x^*_{1it}, \ldots, x^*_{kit}$ by recursion using the initial conditions $x^*_{k00} = x_{k00}$. That is

$$x^*_{1it} = x^*_{1it-1} + w^*_{1it}, \ldots, x^*_{kit} = x^*_{kit-1} + w^*_{kit}. \tag{4.49}$$

Alternatively form x^*_{kit} from

(4.50) $$x^*_{kit} = x^*_{k00} + \Sigma_{t=1}^{T}\Sigma_{i=1}^{N} w^*_{kit}.$$

Also obtain the bootstrap samples of v^*_{it} by recursion using the estimated $\hat{\rho}$ of equation (4.45) and the initial conditions $v^*_{00} = \hat{v}_{00}$. Thus

(4.51) $$v^*_{it} = \hat{\rho} v^*_{it-1} + \varepsilon^*_{it}.$$

6. Construct the bootstrap samples of y^*_{it} from

(4.52) $$y^*_{it} = \hat{\alpha}_i + x^{*\prime}_{it}\hat{\beta} + \Sigma_{j=-q}^{q}\hat{c}_{ij}\Delta x^*_{it+j} + v^*_{it}.$$

7. Using the bootstrap samples y^*_{it}, α_i and x^*_{it} estimate $\beta^*(b)$ the bootstrap DOLS estimate of β.

8. Repeat steps (2) to (7) B times.

9. Construct the bootstrap distribution of β, ie $\beta^*(b)$ for $b = 1, ...5000$ and other bootstrap statistics.

4.6.2 The Pairs Bootstrap and the Group-Mean DOLS estimator

The pairs bootstrap procedure for the group-mean DOLS estimator is carried out by resampling the errors from the individual country estimates and I(1) regressor stochastic error terms. Form bootstrap samples and estimates for each country i and then average over the panel. This is done B times.

1. Using equation (3.24), in chapter 3, compute the DOLS estimates for the ith individual country and obtain the predicted residuals using $\hat{\alpha}, \hat{\beta}$ and $\hat{c}_j$ thus

(4.53) $$\hat{v}_t = y_t - \hat{\alpha} + x_t'\hat{\beta} + \Sigma_{j=-q}^{q}\hat{c}_j \Delta x_{t+j}$$

for $t = 1, \ldots, T$.

2. Obtain the residuals $\hat{w}_{1t} = \Delta x_{1t}, \ldots, \hat{w}_{kt} = \Delta x_{kt}$ and form the vector $\hat{w}_t = (\hat{w}_{1t}, \ldots, \hat{w}_{kt})$ and $\hat{w} = (\hat{w}_1', \hat{w}_2', \ldots, \hat{w}_T')'$. Recentre these using

(4.54) $$\hat{w}_{(.)} = \frac{1}{T}\Sigma_{t=1}^{T}\hat{w}_t$$

for $t = 1, \ldots, T$.

Thus we have $\hat{w}^c = (\hat{w}_1^{c\prime}, \hat{w}_2^{c\prime}, \ldots, \hat{w}_T^{c\prime})'$ where

(4.55) $$\hat{w}_1^c = (\hat{w}_1 - \hat{w}_{(.)}), \hat{w}_2^c = (\hat{w}_2 - \hat{w}_{(.)}), \ldots, \hat{w}_T^c = (\hat{w}_T - \hat{w}_{(.)}).$$

3. Similarly if we again assume that the residuals $\hat{v}_t$ follow an AR(1) process then

(4.56) $$\hat{v}_t = \rho \hat{v}_{t-1} + \varepsilon_t$$

where again ε_t is a white noise error term.

Run the regression of equation (4.56) and obtain the residuals $\hat{\varepsilon}_t$ and also $\hat{\rho}$ and the vector of residuals $\hat{\varepsilon} = (\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_T)'$. Recentre these using

$$(4.57) \qquad \hat{\varepsilon}_{(.)} = \frac{1}{T}\Sigma_{t=1}^{T}\hat{\varepsilon}_t.$$

Thus the vector of recentred residuals is $\hat{\varepsilon}^c = (\hat{\varepsilon}_1^c, \hat{\varepsilon}_2^c, \ldots, \hat{\varepsilon}_T^c)'$ where

$$(4.58) \qquad \hat{\varepsilon}_1^c = (\hat{\varepsilon}_1 - \hat{\varepsilon}_{(.)}), \hat{\varepsilon}_2^c = (\hat{\varepsilon}_2 - \hat{\varepsilon}_{(.)}), \ldots, \hat{\varepsilon}_T^c = (\hat{\varepsilon}_T - \hat{\varepsilon}_{(.)}).$$

4. Resample with replacement from the paired vector of recentred residuals, $\hat{z}_t^c = (\hat{\varepsilon}_t^c, \hat{w}_t^{c\prime})'$ to get the bootstrap sample $z_t^* = (\varepsilon_t^*, w_t^{*\prime})'$. So that ε^* and w^* are the vectors of resampled residuals

$$(4.59) \qquad \varepsilon^* = (\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_T^*)' \quad \text{and} \quad w^* = (w_1^*, w_2^*, \ldots, w_T^*)'.$$

5. Obtain the bootstrap samples of $x_{1t}^*, \ldots, x_{kt}^*$ by recursion using the initial conditions $x_{k0}^* = x_{k0}$. That is

$$(4.60) \qquad x_{1t}^* = x_{1t-1}^* + w_{1t}^*, \ldots, x_{kt}^* = x_{kt-1}^* + w_{kt}^*.$$

Alternatively form x_{kt}^* from

$$(4.61) \qquad x_{kt}^* = x_{k0}^* + \Sigma_{t=1}^{T} w_{kt}^*.$$

Also obtain the bootstrap samples of v_t^* by recursion using the estimated $\hat{\rho}$ of equation (4.56) and the initial conditions $v_0^* = \hat{v}_0$. Thus

$$(4.62) \qquad v_t^* = \hat{\rho} v_{t-1}^* + \varepsilon_t^*.$$

6. Construct the bootstrap samples of y_t^* from

$$(4.63) \qquad y_t^* = \hat{\alpha} + x_t^{*\prime}\hat{\beta} + \Sigma_{j=-q}^{q}\hat{c}_j \Delta x_{t+j}^* + v_t^*.$$

7. Using the bootstrap samples y_t^*, α and x_t^* estimate $\hat{\beta}^*(z)$ the bootstrap DOLS estimate of β for country z.

8. Repeat steps (2) to (7) for each country in the panel.

9. Compute the bootstrap group-mean $\beta^*(b)$ DOLS estimator as

$$(4.64) \qquad \beta^*(b) = \frac{\Sigma_{z=1}^{N}\hat{\beta}^*(z)}{N}.$$

10. Compute the bootstrap group-mean $\beta^*(b)$ DOLS t-statistic as[7]

$$(4.65) \qquad t_{(\beta^*(b))} = \frac{\Sigma_{z=1}^{N}\hat{t}^*(z)}{\sqrt{N}}.$$

where $\hat{t}^*(z)$ is the bootstrap DOLS estimate of the t-statistic of $\hat{\beta}^*(z)$ for country z.

11. For each panel of N countries construct B bootstrap samples using the initial regressions described above in step (1). That is repeat steps (2) to (10) B times.

12. Construct the bootstrap distribution of β and its t-statistic, ie $\beta^*(b)$ and $t_{(\beta^*(b))}$ for $b = 1, ...5000$.

[7]For the group-mean β DOLS bootstrap-t confidence interval the t-statistic $Z^*(b)$ of equation (4.30) was used in the above procedure.

4.6.3 The Pairs Bootstrap and the DOLS Asymptotic Covariance Matrix of the Residuals

The pairs bootstrap procedure for the DOLS asymptotic residual covariance matrix is carried out by resampling the errors from the individual country estimates and I(1) stochastic error terms. One forms bootstrap samples and estimates of the asymptotic covariance matrix for each country i and then averages over the panel. This is done B times.

1. Using an OLS regression for each individual country obtain the predicted residuals, ie $\hat{u}_t$ where

   (4.66) $$\hat{u}_t = y_t - \hat{\alpha} - x_t'\hat{\beta}$$

   for $t = 1, \ldots, T$.

2. Obtain the vector of residuals $\hat{u} = (\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_T)'$ and recentre the residuals using

   (4.67) $$\hat{u}_{(.)} = \frac{1}{T}\Sigma_{t=1}^{T}\hat{u}_t.$$

   Thus the vector of recentred residuals is $\hat{u}^c = (\hat{u}_1^c, \hat{u}_2^c, \ldots, \hat{u}_T^c)'$ where

   (4.68) $$\hat{u}_1^c = (\hat{u}_1 - \hat{u}_{(.)}), \hat{u}_2^c = (\hat{u}_2 - \hat{u}_{(.)}), \ldots, \hat{u}_T^c = (\hat{u}_T - \hat{u}_{(.)}).$$

3. Also obtain the residuals, $\hat{w}_{1t} = \Delta x_{1t}, \hat{w}_{2t} = \Delta x_{2t}, \ldots, \hat{w}_{kt} = \Delta x_{kt}$ and form the vector $\hat{w}_t = (\hat{w}_{1t}, \hat{w}_{2t}, \ldots, \hat{w}_{kT})'$ and $\hat{w} = (\hat{w}_1', \hat{w}_2', \ldots, \hat{w}_T')'$.

Recentre the residuals using

$$(4.69) \qquad \hat{w}_{(.)} = \frac{1}{T}\Sigma_{t=1}^{T}\hat{w}_t.$$

Thus the vector of recentred residuals is $\hat{w}^c = (\hat{w}_1^{c\prime}, \hat{w}_2^{c\prime}, \ldots, \hat{w}_T^{c\prime})'$ where

$$(4.70) \quad \hat{w}_1^c = (\hat{w}_1 - \hat{w}_{(.)}), \hat{w}_2^c = (\hat{w}_2 - \hat{w}_{(.)}), \ldots, \hat{w}_T^c = (\hat{w}_T - \hat{w}_{(.)}).$$

4. Resample with replacement from the paired vector of centred residuals $\hat{z}_t^c = (\hat{u}_t^c, \hat{w}_t^{c\prime})'$ to get the bootstrap sample $\hat{z}_t^* = (\hat{u}_t^*, \hat{w}_t^{*\prime})'$. So that u^* and w^* are the vectors of resampled residuals

$$(4.71) \quad u^* = (u_1^*, u_2^*, \ldots, u_T^*)' \quad \text{and} \quad w^* = (w_1^{*\prime}, w_2^{*\prime}, \ldots, w_T^{*\prime})'.$$

5. Using the bootstrap sample construct the bootstrap estimate of the long run asymptotic covariance matrix for country s, ie $\Omega^*(s)$ where

$$(4.72) \Omega^*(s) = \frac{1}{T}\Sigma_{t=1}^{T}\hat{z}_t^*\hat{z}_t^{*\prime} + \Sigma_{l=1}^{q}[1 - \frac{l}{q+1}]\frac{1}{T}\Sigma_{t=1}^{T}\left(\hat{z}_t^*\hat{z}_{t-l}^{*\prime} + \hat{z}_{t-l}^*\hat{z}_t^{*\prime}\right).$$

6. Repeat steps (2) to (5) for each country in the panel.

7. Compute the bootstrap average long run covariance matrix as

$$(4.73) \qquad \hat{\Omega}^*(b) = \frac{\Sigma_{s=1}^{N}\Omega^*(s)}{N}.$$

8. For each panel of N countries construct B bootstrap samples using the initial regressions described above in step (1). That is repeat steps (2) to (7) B times.

9. Construct the bootstrap distribution of Ω. That is obtain 5000 replications of $\hat{\Omega}^*(b)$, for $b = 1, \ldots, 5000$.

4.6.4 The Recursive Residual Bootstrap and the Group-Mean AR(p) estimator

The recursive residual bootstrap procedure for the group-mean AR(p) estimator is carried out by resampling the errors from the individual country estimates. Using the recursive bootstrap, form bootstrap samples and estimates for each country i and then average over the panel. This is done B times

1. Using equation (4.35) compute the AR(p) estimates for the ith individual country and obtain the predicted residuals using $\hat{\alpha}, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_p$ thus

   $$\hat{v}_t = y_t - \hat{\alpha} - \hat{\theta}_1 y_{t-1} - \hat{\theta}_2 y_{t-2} - \ldots - \hat{\theta}_p y_{t-p} \tag{4.74}$$

   for $t = 1, \ldots, T$.

2. After obtaining the residuals $\hat{v}_t$ form the vector of residuals $\hat{v} = (\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_T)'$.

Recentre these using

(4.75) $$\hat{v}_{(.)} = \frac{1}{T}\Sigma_{t=1}^{T}\hat{v}_t.$$

Thus the vector of recentred residuals is $\hat{v}^c = (\hat{v}_1^c, \hat{v}_2^c, \ldots, \hat{v}_T^c)'$

where

(4.76) $$\hat{v}_1^c = (\hat{v}_1 - \hat{v}_{(.)}), \hat{v}_2^c = (\hat{v}_2 - \hat{v}_{(.)}), \ldots, \hat{v}_T^c = (\hat{v}_T - \hat{v}_{(.)}).$$

3. Resample with replacement from the vector of recentred residuals, $\hat{v}^c = (\hat{v}_1^c, \hat{v}_2^c, \ldots, \hat{v}_T^c)'$ to get the bootstrap residual sample $v^* = (v_1^*, v_2^*, \ldots, v_T^*)'$.

4. Construct the bootstrap samples of y_t^* recursively from

(4.77) $$y_t^* = \hat{\alpha} + \hat{\theta}_1 y_{t-1}^* + \hat{\theta}_2 y_{t-2}^* + \ldots + \hat{\theta}_p y_{t-p}^* + v_t^*.$$

5. Using the bootstrap samples y_t^* estimate $\hat{\theta}^*(i) = (\alpha^*, \theta_1^*, \theta_2^*, \ldots, \theta_p^*)'$ the bootstrap AR(p) estimate of $\theta = (\alpha, \theta_1, \theta_2, \ldots, \theta_p)'$ for country i.

6. Repeat steps (2) to (5) for each country in the panel.

7. Compute the bootstrap group-mean AR(p) estimator $\theta^*(b)$ as

(4.78) $$\theta^*(b) = \frac{\Sigma_{i=1}^{N}\hat{\theta}^*(i)}{N}.$$

8. Compute the bootstrap group-mean AR(p) t-statistic as

(4.79) $$t_{(\theta^*(b))} = \frac{\Sigma_{i=1}^{N}\hat{t}^*(i)}{N}.$$

where $\hat{t}^*(i)$ is the bootstrap AR(p) estimate of the t-statistic of $\hat{\theta}^*(i)$ for country i.

9. For each panel of N countries construct B bootstrap samples and estimates using the initial regressions described above in step (1). That is repeat steps (2) to (8) B times.

10. Construct the bootstrap distribution of b_{GM} and its t-statistic, ie $\theta^*(b)$ and $t_{(\theta^*(b))}$ for $b = 1, ...5000$.

4.6.5 The Block Bootstrap and the Group-Mean AR(p) estimator

The block bootstrap procedure for the group-mean AR(p) estimator is carried out by resampling blocks from the individual country time-series of weakly dependent (stationary) data, forming bootstrap estimates from the new series and averaging these over the panel B times[8].

1. Consider a time-series of weakly dependent data for individual country i given by the sequence

$$\{X_1, X_2, X_3, \ldots, X_N\}. \tag{4.80}$$

[8]In the block bootstrap, one chooses a block length $L = N/k$, where N is the number of observations in the time-series and k is the number of blocks to resample. The idea is to choose a large enough block length L that observations more than L time units apart will be nearly independent and so mimic I.I.D. sampling.

2. Divide this into blocks of observations M_i, for $i = 1\ldots, k$, of equal length L. Thus the first block $M_1 = \{X_1, \ldots, X_L\}$, the second block $M_2 = \{X_{L+1}, \ldots, X_{2L}\}$ and so on to $M_k = \{X_{(k-1)L+1}, \ldots, X_{kL}\}$.

3. Resample k blocks randomly with replacement from the sequence

$$\{M_1, M_2, M_3, \ldots, M_k\}. \tag{4.81}$$

4. Denote the k resampled blocks as $M_1^*, M_2^*, M_3^*, \ldots, M_k^*$. Concatenate these blocks into one vector, that is lay the blocks end-to-end to form the vector

$$\{M_1^*, M_2^*, M_3^*, \ldots, M_k^*\} = \{X_1^*, X_2^*, X_3^*, \ldots, X_N^*\}. \tag{4.82}$$

5. Construct the N-vector y^* the bootstrap sample of the country time-series given by

$$y^* = \{X_1^*, X_2^*, X_3^*, \ldots, X_N^*\}. \tag{4.83}$$

7. Using the bootstrap sample y_t^*, for $t = 1, \ldots, N$, fit an AR(p) model to the bootstrap data

$$y_t^* = \alpha + \theta_1 y_{t-1}^* + \theta_2 y_{t-2}^* + \ldots + \theta_p y_{t-p}^* + \varepsilon_t. \tag{4.84}$$

8. Estimate $\hat{\theta}^*(i) = (\alpha^*, \theta_1^*, \theta_2^*, \ldots, \theta_p^*)'$ the bootstrap AR(p) estimate of $\theta = (\alpha, \theta_1, \theta_2, \ldots, \theta_p)'$ for country i.

9. Repeat steps (2) to (8) for each country in the panel.

10. Compute the bootstrap AR(p) group-mean $\theta^*(b)$ estimator as

$$\theta^*(b) = \frac{\Sigma_{i=1}^{N}\hat{\theta}^*(i)}{N}. \tag{4.85}$$

11. Compute the bootstrap AR(p) group-mean t-statistic as

$$t_{(\theta^*(b))} = \frac{\Sigma_{i=1}^{N}\hat{t}^*(i)}{N}. \tag{4.86}$$

    where $\hat{t}^*(i)$ is the bootstrap AR(p) estimate of the t-statistic of $\hat{\theta}^*(i)$ for country i.

12. For each panel of N countries construct B bootstrap samples and estimates using the initial time-series described above in step (1). That is repeat steps (2) to (10) B times.

13. Construct the bootstrap distribution of b_{GM} and its t-statistic, ie $\theta^*(b)$ and $t_{(\theta^*(b))}$ for $b = 1, ...5000$.

4.7 The Bootstrap Applications

4.7.1 Bootstrap Confidence Intervals for a panel data PPP Cointegrating Regression

In this application we use the pooled DOLS panel estimator of Kao and Chiang (2000) and the group-mean DOLS panel estimator of Pedroni (2001) to construct bootstrap confidence intervals for a purchasing power parity (PPP) panel cointegration regression.

In chapter 2 we showed that there existed a long-run equilibrium (cointegrating) relation between the nominal exchange rate and the prices of domestic and foreign goods in our panel data using our panel cointegration tests and the panel PPP cointegrating regression

$$e_{it} = \alpha_i + \beta_1 p_{it} + \beta_2 p^*_{it} + u_{it}, \tag{4.87}$$

where $\{e_{it}, p_{it}, p^*_{it}\} \sim \mathrm{I}(1)$ are the logarithms of the nominal exchange rate, domestic prices and foreign price level, respectively for country i at time t and similarly u_{it} is a stationary disturbance term. We now go one step further in the spirit of Engle and Granger (1987) and estimate the cointegration vectors using the panel data DOLS estimators described in § 4.4.1. The Kao pooled panel DOLS PPP estimator is obtained from the following regression

$$e_{it} = \alpha_i + \beta_1 p_{it} + \beta_2 p^*_{it} + \Sigma_{j=-q}^{q} d_{1ij} \Delta p_{it+j} + \Sigma_{j=-q}^{q} d_{2ij} \Delta p^*_{it+j} + \bar{u}_{it}. \tag{4.88}$$

The Pedroni group-mean panel DOLS PPP estimator is given by

$$\hat{\beta}^*_{GD} = \left[N^{-1} \Sigma_{i=1}^{N} \left(\Sigma_{t=1}^{T} z_{it} z'_{it} \right)^{-1} \left(\Sigma_{t=1}^{T} z_{it} e_{it} \right) \right]_{13}, \tag{4.89}$$

where z_{it} is the $(4(p+1)+N \times 1)$ vector of regressors

$$z_{it} = (1, 0, \ldots, 0, p_{it}, p^*_{it}, \Delta p_{it-p}, \ldots, \Delta p_{it+p}, \Delta p^*_{it-p}, \ldots, \Delta p^*_{it+p})'. \tag{4.90}$$

The subscript 13 outside the square brackets indicate that we are considering only the thirteenth element of the vector for the pooled slope coefficient.[9] In Kao, Chiang and Chen (1999) it was noted that there was not a coherent strategy to be applied for estimating the lengths of lags and leads in these panel cointegration models. Since this time Westerlund (2005) has developed data dependent methods for lag selection in panel cointegration models. The method used in this thesis was the general-to-specific method advocated by D.F. Hendry (1995). Here we start with an overparameterised model and use sequential test procedures to test down for a more parsimonious representation. In practice this meant setting leads and lags of 3 for both regressors and then testing for their significance. All insignificant regressors were subsequently dropped from the regression.

In our panel unit root and panel cointegration tests of chapter 2 some of the results were in favour of PPP using a strong form of the hypothesis which involves the joint symmetry and proportionality assumption $\beta_1 = -\beta_2 = 1$. Whilst other tests supported a weak form of PPP where the β_i coefficients fall within the range $(0 < \beta_1 < 2, -2 < \beta_2 < 0)$. Given the two main PPP hypotheses it is interesting to highlight the theories in a confidence interval framework. Thus given point estimates close to unity and a small variability

[9] We estimated both the Kao pooled and Pedroni group-mean regressions in levels form.

of the interval estimates would lend support to the strong PPP hypothesis. Whereas point estimates falling within the range $(0 < \beta_1 < 2, -2 < \beta_2 < 0)$ and interval estimates with large variability would lend support to the weak PPP hypothesis. One of the panel cointegration regressions is constructed by pooling the cross-section dimension and assuming heterogeneity in the intercepts, see Kao (1999), Kao, Chiang and Chen (1999) and Kao and Chiang (2000). Thus by assuming homogeneity of β across panels we in effect impose a strong PPP hypothesis, if it turns out that $\beta_1 = -\beta_2 = 1$ for all units. The other panel cointegration regression of Pedroni (2000,2001) differs in that he uses heterogeneous panels where the β coefficients are allowed to vary across individuals or countries. This is compatible with the weak PPP hypothesis given the country coefficients are allowed to vary within the range $(0 < \beta_1 < 2, -2 < \beta_2 < 0)$.

4.7.2 The Data Set

The data set was the sub-panel for the 12 OECD countries discussed in chapter 2. That is quarterly observations over the period 1957Q1-1991Q2, on E_{it} the nominal exchange rate, P_{it} the consumer price level (or CPI) and P_{it}^* the foreign (Japanese) consumer price level (or CPI).

In Table 4.01 we have the DOLS estimation results for the pooled panel cointegration model. A fixed effects model was estimated with individual-specific intercepts. Also three leads and lags were used with the differenced regressors to counter any serial correlation in the model. These are not reported here. The R^2 is very high with an $R^2 = 0.99368$ and also the F-statistic of the overall estimated regression is strongly significant at 14297.0. This indicates that the model is a very good fit of the data. However the regression D.W. statistic is very low D.W. = 0.18429 and so we should use the appropriate panel data tests for serial correlation. To test for first-order serial correlation in a fixed effects model we use the LM test developed in Baltagi and Li (1995). Consider the model of equation (4.88) with the errors described by the AR(1) process

$$\hat{\hat{u}}_{it} = \rho \hat{\hat{u}}_{it-1} + \varepsilon_{it}, \tag{4.91}$$

where ε_{it} is a Gaussian white noise process. Under the null hypothesis, $H_0 : \rho = 0$ and $H_1 : \rho \neq 0$

$$LM_1 = \left[\frac{NT^2}{(T-1)}\right] \left(\frac{\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \hat{\hat{u}}_{it} \hat{\hat{u}}_{it-1}}{\Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \hat{\hat{u}}_{it}^2}\right)^2 \tag{4.92}$$

As $T \longrightarrow \infty$ the statistic LM_1 is distributed as a χ_1^2 variable.

Our computed statistic $LM_1 = 1374.7$. With critical χ_1^2 values of 3.84 and 6.63 at the 5% and 1% significance levels, respectively, the null hypothesis is decisively rejected in favour of the hypothesis of first-order serial correlation in the errors.

Table 4.01 **Pooled Panel DOLS Regression Estimates**

Regression	p_{it}	p^*_{it}	DW	F-statistic	R^2
Pooled	1.0688 166.59 [0.000]	-1.1036 -176.94 [0.000]	0.18429 -	14297.0 [0.000]	0.99368 -

Our preliminary regression in Table 4.01 shows a quite well estimated panel PPP cointegrating regression. The expected signs and significance of the PPP regression parameters are satisfactory. Our estimates are $\beta_1 = 1.0688$ and $\beta_2 = -1.1036$, with t-statistics of (166.59) and (-176.94), respectively. These are shown underneath the point estimates, alongside the p-values. To test the joint proportionality and symmetry assumption of $\beta_1 = -\beta_2 = 1$ we conducted t-tests of the hypotheses $\beta_1 = 1$ and $\beta_2 = -1$. Our estimated t-statistics were 10.7202 and -16.6095. With critical values of ±1.96 we reject the strong PPP hypothesis at the 5% significance level.

In Table 4.02 we have the DOLS estimation results for the group-mean panel cointegration model. These are much more conservative than the pooled panel estimates. Both the regressor point estimates, and their associated HAC t-statistics and p-values (again shown under the point estimates), are consistently lower than those of their pooled panel counterparts. Eleven of twelve countries had domestic price estimates below unity and similarly for foreign prices. This indicates some underprediction in the model. The high R^2 values, in the final column, and significant F-statistics, in the fifth column, indicate that again we still have good fits of the data. However, again, all the individual country DW statistics are very much below unity indicating positive first-order serial correlation. This indicates that Newey-West (1987) HAC standard error estimators should be used at all times. Pedroni (2001) recommends these for his group-mean estimators due to endogeniety and serial correlation problems in the individual cointegrating regressions, see § 3.2.2 in chapter 3. These HAC Newey and West (1987) standard error estimators correctly account for the serial correlation structure in the errors by using the long run variance of $\hat{u}_{it}$, for each i. in their computations. The Barltett window was chosen to describe the lag structure of the Newey and West estimators with a truncation point of 10 (ie q=10). This was chosen after inspection of the sample autocorrelation function (ACF) of the residuals $\hat{u}_{it}$, for each i. The graph depicted lagged correlations persisting even after 10 lags, similar to the OLS residuals in Figure 2.19 of Appendix 4. Finally the leads and lags of the differenced regressors, shown under the R^2 figures in the final column, were chosen so that they were significant at around the $10 - 20\%$ significance level.

Table 4.02 **Individual Country DOLS Regression Estimates**

Regression	constant	p_t	p_t^*	DW/F-stat	R^2/le,la
Austria	4.5731	0.029890	-0.45177	0.14095	0.99805
	6.577 [0.000]	0.046 [0.962]	-0.904 [0.366]	11159.0	(0,1,3)
Denmark	2.4266	0.51624	-0.60395	0.13659	0.99525
	7.342 [0.000]	1.912 [0.055]	-1.856 [0.063]	9354.3	(1,0)
France	1.3761	0.44216	-0.32366	0.20458	0.99388
	5.288 [0.000]	1.889 [0.059]	-1.175 [0.239]	7252.0	(1,0)
Greece	3.1241	0.78002	-0.43285	0.15162	0.99934
	10.818 [0.000]	13.804 [0.000]	-3.712 [0.000]	39794.0	(1,1,1)
Iceland	1.1454	0.85646	-0.36978	0.48007	0.98846
	1.456 [0.145]	16.017 [0.000]	-1.694 [0.090]	2849.2	(1,1)
Mexico	4.6275	0.97118	-0.79893	0.40165	0.99919
	19.119 [0.000]	53.699 [0.000]	-11.324 [0.000]	55255.0	(1,0)
Netherl	1.7003	1.0962	-1.2788	0.20507	0.98686
	2.933 [0.003]	1.319 [0.187]	-1.803 [0.071]	1982.4	(1,2)
New Zeal	-0.83863	0.45388	-0.18369	0.24785	0.90869
	-2.847 [0.004]	4.076 [0.000]	-1.050 [0.293]	262.73	(3,0)
Norway	2.4977	0.46013	-0.59054	0.16370	0.99732
	14.549 [0.000]	2.946 [0.003]	-3.412 [0.000]	12374.0	(2,0)
Spain	4.1215	0.51075	-0.35914	0.14615	0.99868
	5.942 [0.000]	2.538 [0.011]	-1.020 [0.307]	33822.0	(1,0)
Switzerl	4.0434	-0.30050	-0.46362	0.22892	0.98260
	3.575 [0.000]	-0.428 [0.668]	-1.009 [0.312]	2522.5	(0,1)
Turkey	2.4982	0.97008	-0.17052	0.36323	0.99681
	4.388 [0.000]	20.577 [0.000]	-1.011 [0.312]	10383.0	(1,1)
Group-Mean	2.6079	0.56554	-0.50227	-	0.98709
	22.845[0.000]	34.177[0.000]	-8.416 [0.000]	-	-

Continuing the discussion of § 4.7.1, we now see that the pooled DOLS panel regression rejects the strong PPP hypothesis, whilst the group-mean DOLS panel regression gives good support to the weak PPP hypothesis. This indicates that our bootstrap confidence interval approach provides a quite general framework for making inferences on the long run PPP hypotheses.

Table 4.03 Bias Correction Constants $\hat{z}_0$

Model	p_t	p_t^*
Group-Mean		
Austria	-0.11	0.13
Denmark	0.18	-0.17
France	-0.18	0.20
Greece	-0.19	0.20
Iceland	-1.22*	-3.40*
Mexico	-0.36	0.09
Netherland	0.24	-0.24
New Zealand	-0.47	0.46
Norway	0.12	-0.10
Spain	0.07	0.03
Switzerland	0.86	0.07
Turkey	0.15	0.72
Average	0.0258333	0.1158333
Pooled	0.06	0.10

Table 4.04 Acceleration Constants $\hat{a}$

Model	p_t	p_t^*
Group-Mean		
Austria	-0.0082959	0.0133480
Denmark	-0.0153550	0.0168650
France	-0.0040728	0.0020222
Greece	0.0041270	0.0113320
Iceland	0.0066912	-0.0066642
Mexico	-0.0012256	0.0067793
Netherland	0.0136930	0.0122610
New Zealand	-0.0222160	0.0222100
Norway	-0.0244130	0.0246500
Spain	0.0058567	-0.0037768
Switzerland	-0.0173820	0.0221160
Turkey	0.0253340	-0.0225050
Average	-0.0031048	0.0082197
Pooled	0.0042951	-0.0079385

In[10] Tables 4.03 and 4.04 we have our bias and acceleration constant esti-

[10]Here * means outlier country omitted from average.

mates. The values for the group-mean estimator were obtained by applying the Jackknife method and bootstrap proportion method, described in § 4.3.3 and § 4.3.4, to each individual country and then taking the average.

4.7.3 Efficient Estimation

One of the purposes of this study is to see how well the bootstrap[11] works with nonstationary panel data. One would like to judge the performance and efficiency of the bootstrap in a general setting when compared with its counterparts from first-order asymptotic theory. We have already mentioned about the power of the bootstrap to deliver asymptotic refinements, these lead to more efficient estimates. There are methods of evaluating confidence intervals in order to make efficiency judgements. Two criteria are used to judge confidence intervals: (i) size and (ii) coverage probability. An optimal and hence efficient confidence interval is one with small size and large coverage, but these are difficult to obtain. We may measure coverage probability by the true coverage probability and size by the length of the interval.

We find that the difference between the true coverage probability and the nominal coverage probability of the asymptotic confidence interval is $O(n^{-1})$

[11] Horowitz (2000) notes that efficient estimation often results when one uses the parametric bootstrap, as opposed to the nonparametric bootstrap. If one knows the parametric distribution of the errors being sampled then this, when used, will be more accurate than using the empirical distribution function and leads to smaller errors. With the parametric bootstrap an assumption is made as to the form of the distribution being sampled (eg normal), whilst with the nonparametric bootstrap no assumptions are made.

that is

$$P(|\hat{\beta}_k| \leq z^{(\alpha)}) = 1 - \alpha + O(n^{-1}). \tag{4.93}$$

Whilst we have that for the bootstrap this error is $O(n^{-2})$ given by

$$P(|\hat{\beta}_k| \leq z^{*(\alpha)}) = 1 - \alpha + O(n^{-2}). \tag{4.94}$$

Note these confidence intervals are two-sided intervals. In general for one-sided and equal-tailed confidence intervals the difference between the true coverage probability and the nominal coverage probability for the asymptotic interval is $O(n^{-\frac{1}{2}})$. In this case the asymptotic confidence interval is said to be first-order accurate. However the analogous difference for the bootstrap confidence interval is $O(n^{-1})$. In which case we say it is second-order accurate. These notions of accuracy give us a good guide as to the expected performance of our confidence interval methodologies. We see that the errors are much smaller with the bootstrap than with the asymptotic confidence interval. The standard or asymptotic confidence interval and percentile intervals are first-order accurate, whilst the BC_a and bootstrap-t intervals are second-order accurate. See Efron (1987) for a good discussion. The asymptotic theory, for these accuracy, coverage probability and interval size concepts, have been rigourously proved by Hall (1992) using Edgeworth and Cornish Fisher expansions.

4.7.4 Pooled Panel DOLS Bootstrap Estimates[12]

Table 4.05 80% Nominal Confidence Interval for p_{it}

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	1.06059	1.07701	0.01642	0.2242
Percentile	1.03542	1.12571	0.09028	0.7998
BC	1.03889	1.13207	0.09317	0.7962
BCa	1.03914	1.13207	0.09266	0.7942
Bootstrap-t	1.01235	1.10157	0.08921	0.7714

Table 4.06 80% Nominal Confidence Interval for p^*_{it}

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	-1.11158	-1.09562	0.01595	0.0894
Percentile	-1.21073	-1.01533	0.19540	0.8000
BC	-1.19409	-0.99811	0.19597	0.7906
BCa	-1.19486	-0.99969	0.19516	0.7900
Bootstrap-t	-1.17862	-1.01241	0.16621	0.7198

Table 4.07 90% Nominal Confidence Interval for p_{it}

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	1.05821	1.07939	0.02117	0.2886
Percentile	1.02458	1.14430	0.11972	0.8996
BC	1.02767	1.15082	0.12314	0.8986
BCa	1.02828	1.15123	0.12294	0.8970
Bootstrap-t	0.99356	1.11301	0.11945	0.8434

Table 4.08 90% Nominal Confidence Interval for p^*_{it}

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	-1.11389	-1.09331	0.02058	0.1164
Percentile	-1.24673	-0.98258	0.26414	0.9002
BC	-1.22528	-0.96501	0.26026	0.8946
BCa	-1.22630	-0.96710	0.25924	0.8938
Bootstrap-t	-1.20963	-0.97853	0.23110	0.8504

[12] 5,000 Bootstrap replications used.

4.7.5 Group-Mean Panel DOLS Bootstrap Estimates[13]

Table 4.09 **80% Nominal Confidence Interval for p_{it}**

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	0.21157	0.91950	0.70793	0.7288
Percentile	0.19588	1.01833	0.82244	0.7998
BC	0.21722	1.02953	0.81231	0.7938
BCa	0.21159	1.03352	0.82192	0.7972
Bootstrap-t	0.62009	0.93756	0.31746	0.3216

Table 4.10 **80% Nominal Confidence Interval for p_{it}^***

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	-0.88010	-0.12443	0.75566	0.7714
Percentile	-0.82212	-0.04516	0.77700	0.8000
BC	-0.74471	0.02702	0.71770	0.7852
BCa	-0.74146	0.02974	0.71443	0.7842
Bootstrap-t	-0.96174	-0.61927	0.34247	0.2188

Table 4.11 **90% Nominal Confidence Interval for p_{it}**

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	0.10925	1.02182	0.91257	0.8422
Percentile	0.07750	1.14555	1.06804	0.9000
BC	0.09675	1.17140	1.07464	0.8998
BCa	0.09363	1.16971	1.07608	0.8994
Bootstrap-t	0.56869	0.98640	0.41771	0.4222

Table 4.12 **90% Nominal Confidence Interval for p_{it}^***

Method	Lower Bound	Upper Bound	Length of Conf. Int.	Cover. Prob%
Asymptotic	-0.98932	-0.01521	0.97410	0.8782
Percentile	-0.93765	0.06742	0.87022	0.8996
BC	-0.86895	0.14724	0.72171	0.8906
BCa	-0.86125	0.15501	1.01626	0.8890
Bootstrap-t	-1.01812	-0.57225	0.44586	0.2774

[13]5,000 Bootstrap replications used.

In Tables 4.05-4.08 we have the results of the pairs bootstrap Monte Carlo simulations for the pooled DOLS estimator. Here the percentile method performs best with the smallest coverage errors for both coefficients at both nominal levels. Here also the asymptotic method provides the shortest intervals for both coefficients at both nominal levels. It has intervals approximately one-tenth the size of those of the bootstrap-t method. However it does so at the cost of very large coverage errors. That is the confidence intervals are too narrow for 80% or 90% of all the bootstrap replications to fall in them. The very bad performance of the asymptotic method is due to the very small estimated standard errors of the model. However, the bootstrap-t method performs quite reasonably at both nominal levels. Here close to 80% and 90% of all the bootstrap replications fall in the bootstrap-t confidence interval at the respective nominal levels. The BC and BC_a methods are very similar with the BC_a method providing shorter intervals and the BC better coverage probabilities.

In Tables 4.09-4.12 we have the results of the pairs bootstrap Monte Carlo simulations for the group-mean DOLS estimator. With the group-mean DOLS estimates we have the percentile method again providing the smallest errors in coverage probabilities in both coefficients at both the 80% and 90% nominal levels. However now the bootstrap-t method provides the shortest confidence interval for both coefficients at both nominal levels. But again with very high cost in coverage probability. The asymptotic method performs very reasonably at both nominal levels as does the BC and BC_a, which again provide very similar results.

By modifying the bootstrap to correctly account for the serial correlation in the residuals of the cointegrating regression and using the pairs bootstrap, we see that the bootstrap confidence interval estimators perform, as well as, if not much better than the asymptotic confidence interval estimators. The asymptotic method is shown at times to be very unstable in this panel data cointegrating regression. With the pooled DOLS estimator it undercovers by over 60% at both nominal levels, whilst all the bootstrap methods deliver consistently small coverage errors of less than 5%f for both panel DOLS estimaors. The only exception being the bootstrap-t estimator in the group-mean estimates which also undercovers by as much as 60%. The pairs bootstrap method hence is shown to be remarkably accurate and efficient both with the group-mean and pooled DOLS estimators. Finally the percentile methods (including BC and BC_a) seem to be best in delivering optimal confidence intervals, in terms of shortest intervals and smallest coverage errors. Our results do coincide with some of the findings in the bootstrap literature. However, to our knowledge no applications exist, in the econometric literature, of bootstrap confidence interval studies with cointegrating panel regressions. Rilstone and Veall (1996) use the bootstrap to reduce the downward bias of SUR standard error estimates. However the panels they investigated gave rise to stationary regressors. They found the bootstrap-t estimator out performed the BCa estimator. Hansen (1999) proposed a Grid bootstrap for constructing confidence intervals in autoregressions when the auto-regressive root is close to unity, whilst Kilian (1999) conducted a Monte Carlo analysis of bootstrap confidence intervals for impulse response estimators. Also another econometric application of the bootstrap has come from Kazimi and Brownstone (1999) who provide bootstrap confidence bands for shrinkage OLS estimators. They find that of the numerous methods they consider that the BC_a methods perform best (and the double bootstrap poorly). In the general statistical literature DiCiccio and Romano (1988) provided examples of the percentile, BC, BC_a and percentile-t methods for the confidence intervals of a correlation coefficient and an exponential mean. They found that the BC and BC_a improve on the percentile method in terms of accuracy. Finally Efron (1987) introduced his corrections to the percentile method for the central $(1-\alpha)$ BC and BC_a intervals in a theoretical study. He showed the BC_a interval was nearly identical to the exact interval.

Figure 3.01 **Histograms of Pairs Bootstrap Replications of Pooled DOLS β₁, β₂**

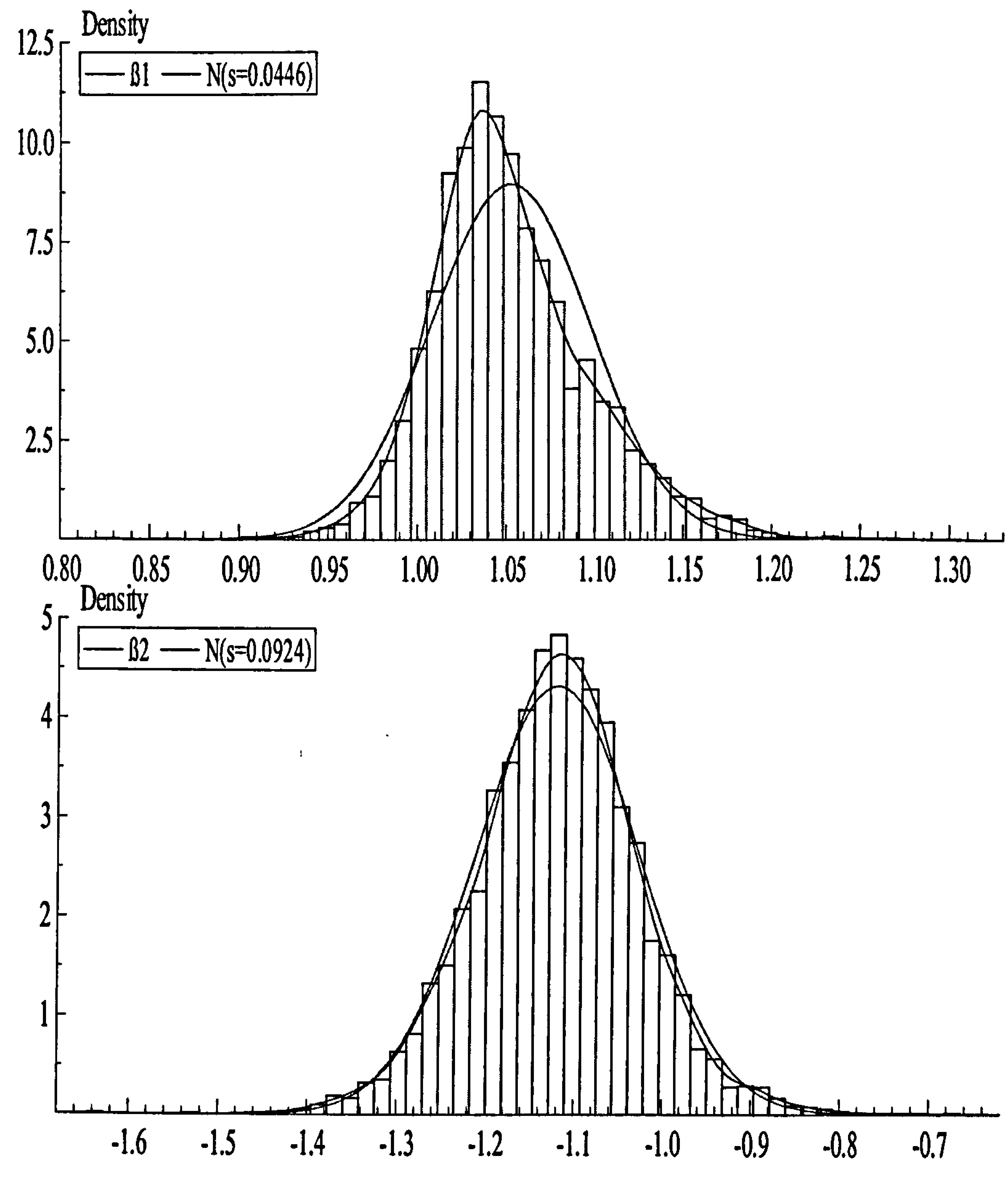

Figure 4.01 shows the histograms of the pooled DOLS bootstrap replications of β_1 and β_2, For β_2 we see the bootstrap distribution is a close approximation to the normal distribution (some slight mesokurtosis shown), whilst β_1 shows some positive skewness.

4.7.6 Bootstrap Quantiles for a panel data Inflation AR(12) Autoregression

In this application we use the panel group-mean estimator of Pesaran and Smith (1995) for bootstrap quantiles of an inflation AR(12) autoregression. In chapter 1 we showed that there may exist a unit root in our panel of inflation rates by our panel unit root tests. We now go further and show how these autregressions can be bootstrapped. The methodology follows Inoue and Kilian (2002) who also bootstrap autoregressions with possible unit roots.

4.7.7 The Data Set

The dataset is the same dataset for 20 OECD countries discussed in chapter 1, consisting of monthly observations from 1960q1-2000q8, on inflation rates. In Tables 4.18-4.22 of Appendix 3 we have the individual country AR(12) regression estimates. All show a reasonable fit of the data with R^2 above

0.5 in all cases except Japan, Finland and Portugal, and no evidence of serial correlation. All the F-statistics are significant. These individual country AR(12) regressions combine to produce the group-mean AR(12) regression shown in Table 4.13. The average $R^2 = 0.60768$ is quite high.

Table 4.13 **Group-Mean AR(12) Regression Estimates**

Cf	GROUP-MEAN
α_i	0.0010208
	9.4675 [0.000]
θ_{1i}	0.14814
	15.0554 [0.000]
θ_{2i}	0.024655
	2.3472 [0.019]
θ_{3i}	0.041889
	4.4179 [0.000]
θ_{4i}	0.025006
	2.1451 [0.032]
θ_{5i}	0.020952
	2.1136[0.034]
θ_{6i}	0.11465
	11.5756[0.000]
θ_{7i}	0.018694
	1.8502[0.064]
θ_{8i}	0.041117
	4.0175[0.000]
θ_{9i}	0.062330
	6.0664[0.000]
θ_{10i}	-0.00049967
	-0.0105[1.008]
θ_{11i}	0.028197
	3.0059[0.002]
θ_{12i}	0.28596
	29.6699 [0.000]
	Aver. $R^2 = 0.60768$

As mentioned we used the modelling strategy of Inoue and Kilian (2002) to bootstrap the AR(12) autoregressions with possible unit roots. This formulation avoids the well known bootstrap inconsistency problem of Basawa, Malik, McCormick, Reeves and Taylor (1991) and Datta (1996). Here the bootstrap is invalid in ADF(p) and AR(1) autoregressions when the autoregressive parameter is unity. This violates the consistency conditions of the Beran and Ducharne (1991) Theorem. The intuition behind their formulation is that only the bootstrap estimates of the slope parameters of the differenced regressors are valid when the bootstrap is used for an ADF(p) autoregression, when the time-series is I(1). The autoregressive parameter is not. However given that the slope parameters of the levels AR(p) autoregression can be shown to be linear combinations of the ADF(p) slope parameters of the differenced regressors, since the latter are valid for bootstrapping, then so are the former. Now our conditions for stability of the levels autoregression are that the roots of the polynomial in equation (4.41) lie outside or on the unit circle.

It is very important to note that the theoretical results for the BB and MBB have, up until recently, only been developed and proved for stationary weakly dependent processes. Many results for the nonstationary case remain unknown. Despite the lack of a theoretical basis for the BB and MBB with nonstationary data Li and Maddala (1996,1997) note that in Monte Carlo simulations these methods seem to improve significantly on asymptotic inference and thus recommend them for use by empirical researchers (see also Hinkley (1997)). More recent studies on the BB and MBB also consider this caveat, see Fachin (2000), p.3. Also Hidalgo (2003) and Paparoditis and Politis (2003) present new alternatives. Lahiri (1992) provided a proof for the MBB with nonstationary data, but his study was limited to the small case of a studentized sample mean. The first attempt to develop on the work of Li and Maddala (1996,1997) and Hinkley (1997) and also provide theoretical results for the MBB with dependent nonstationary data has come from Phillips (2001). He used the simple and block bootstrap in spurious regressions, ie with a random walk process and the I(1) residuals from a spurious regression. The results of Phillips (2001) showed that both bootstrap methods failed to reproduce the original properties of the regressions. Although Phillips (2001) found that the block bootstrap performed much better than the simple bootstrap in capturing data dependence in these models, he strongly advised against using the bootstrap for residual based tests for unit roots and cointegration. He paralleled his results from bootstrapping integrated data with the Basawa, Malik, McCormick, Reeves and Taylor (1991) problem of bootstrap inconsistency in unit root inference. However the results of Phillips (2001) can be criticised in that they are very case specific and that no Monte Carlo simulation study is carried out.

Our justification for using the BB with nonstationary data then is that in our simulations it adequately modelled the dependence structure in the non-stationary time-series and maintained the main characteristics of the data. Thus we do give evidence of a good performance of the BB, in our results below, where we see it performing similarly to the residual bootstrap of Inoue and Kilian (2002). We must however qualify these statements with a few observations. Firstly our bootstrap histograms, in Appendix 4, show the block bootstrap behaving very differently to the residual bootstrap, whereas the latter give rise to bootstrap distributions that have good normal properties, in Figures 4.04-4.05, the former have distributions, in Figures 4.06-4.07, that are very un-Gaussian and often bi-modal. This is caused by the bootstrap block length being too long and hints that the application may be rather crude here. It also must be noted that no theoretical results exist for the block bootstrap and the nonstationary AR(p) model and this is in direct contrast to the residual bootstrap. To implement the block bootstrap the sensitivity analysis[14] described in § 4.5.2 was conducted to ascertain the length of the blocks. This gave a length of 122 for both the block bootstrap methods, given an original time-series of 488 observations. This block size

[14]We show only the results for the block bootstrap as in this application the MBB produced slightly inferior results.

was consistent with the findings of Berkowitz and Kilian (2000) who found a block length of $k = 36$, for a time-series sample of $T = 80$, and $k = 120$, for a sample of $T = 480$ for the MBB.

Table 4.14 **Panel AR(12) Residual Bootstrap Regression Quantiles**

Level-%	α_i	θ_{1i}	θ_{4i}	θ_{8i}	θ_{12i}
0.99	0.0016751	0.16791	0.047878	0.060422	0.29855
0.975	0.0015967	0.16393	0.043109	0.057027	0.29509
0.95	0.0015420	0.16091	0.038682	0.053674	0.29172
0.9	0.0014886	0.15721	0.034408	0.049433	0.28772
0.8	0.0014142	0.15252	0.029627	0.044676	0.28308
0.5	0.0012744	0.14334	0.020353	0.035334	0.27419
0.2	0.0011448	0.13446	0.011102	0.026220	0.26554
0.1	0.0010867	0.12980	0.006297	0.021178	0.26085
0.05	0.0010344	0.12590	0.002472	0.017444	0.25687
0.025	0.0009873	0.12241	-0.000601	0.013717	0.25373
0.01	0.0009457	0.11892	-0.004981	0.009858	0.25045

Table 4.15 **Panel AR(12) Residual Bootstrap Regression Quantiles**

Level-%	t_{α_i}	$t_{\theta_{1i}}$	$t_{\theta_{4i}}$	$t_{\theta_{8i}}$	$t_{\theta_{12i}}$
0.99	2.8955	3.8100	0.98462	1.3244	6.9290
0.975	2.8408	3.7189	0.87508	1.2487	6.8553
0.95	2.8005	3.6478	0.78004	1.1733	6.7587
0.9	2.7389	3.5655	0.68463	1.0801	6.6594
0.8	2.6670	3.4573	0.57741	0.9737	6.5515
0.5	2.5294	3.2486	0.37421	0.7705	6.3171
0.2	2.3944	3.0457	0.17194	0.5665	6.1035
0.1	2.3225	2.9369	0.06794	0.4541	5.9888
0.05	2.2583	2.8514	-0.01423	0.3726	5.8948
0.025	2.2094	2.7772	-0.08746	0.2905	5.8175
0.01	2.1372	2.6929	-0.16954	0.2113	5.7341

In Tables 4.14-4.15 we have the AR(12) residual bootstrap regression quantiles for selected regression coefficient estimates and t-statistics, whilst in

Tables 4.16-4.17 we have the AR(12) block bootstrap regression quantiles for the same estimators. As with Phillips (2001) our results can be seen as being very case specific. No Monte Carlo simulation study is carried out nor any theoretical results given. Bearing this in mind, and our above discussion, however we can see that the results are similar for both methods and quite accurate. To get an idea of the bootstrap accuracy we can construct some nominal percentile confidence intervals for θ_{1i} and compare these with the group mean estimates in Table 4.13. At the $1 - 2\alpha = 90\%$ nominal level we have, for θ_{1i}, for the residual bootstrap method, an interval of 0.12590-0.16091 and for the block bootstrap, an interval of 0.10304-0.21270. Our group-mean estimate from Table 4.13 for θ_{1i} is 0.14814 which lies well inside both of the given intervals. The bootstrap t-statistics given in Tables 4.15 and 4.17 can also serve as exact finite sample critical values. Computing such critical values is often useful to avoid small sample bias.

Table 4.16 **Panel AR(12) Block Bootstrap Regression Quantiles**

Level-%	α_i	θ_{1i}	θ_{4i}	θ_{8i}	θ_{12i}
0.99	0.0031886	0.22593	0.037611	0.045742	0.30295
0.975	0.0027341	0.21937	0.037105	0.042864	0.29607
0.95	0.0023933	0.21270	0.036177	0.038635	0.28989
0.9	0.0020709	0.20491	0.034979	0.035155	0.28107
0.8	0.0017821	0.19569	0.032155	0.031137	0.26692
0.5	0.0013758	0.17815	0.025746	0.022638	0.24371
0.2	0.0010968	0.15189	0.020682	0.013177	0.21941
0.1	0.0009959	0.13658	0.016701	0.007366	0.20584
0.05	0.0009192	0.10304	-0.008874	0.003700	0.19653
0.025	0.0008370	0.08948	-0.011527	-0.003283	0.19093
0.01	0.0008034	0.08271	-0.017378	-0.006409	0.18372

Table 4.17 **Panel AR(12) Block Bootstrap Regression Quantiles**

Level-%	t_{α_i}	$t_{\theta_{1i}}$	$t_{\theta_{4i}}$	$t_{\theta_{8i}}$	$t_{\theta_{12i}}$
0.99	4.0944	5.0638	0.77147	0.92687	7.1328
0.975	3.8869	4.9518	0.74013	0.90040	6.9421
0.95	3.6217	4.7915	0.72622	0.82716	6.7793
0.9	3.3776	4.6129	0.68072	0.74957	6.5302
0.8	3.0786	4.4345	0.61760	0.67173	6.1596
0.5	2.7078	3.9869	0.49086	0.48367	5.5690
0.2	2.3717	3.4140	0.36109	0.26941	5.0087
0.1	2.2092	3.0672	0.24669	0.15069	4.6805
0.05	2.1234	2.3619	-0.25554	0.06869	4.4477
0.025	2.0151	2.0197	-0.32821	-0.09128	4.3302
0.01	1.9127	1.8464	-0.45394	-0.13686	4.1409

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Notes to the Tables

BC means Bias Corrected. BCa means Bias Corrected and Accelerated. 5,000 Bootstrap replications used. (*) means outlier country omitted from average. Lag lengths chosen by Ng and Perron (1995) method.

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Chapter 5

The Asymptotic Properties of Nonstationary Panel Data Estimators

5.1 Introduction

This chapter presents a study on the sequential limit asymptotic theory for nonstationary panel data estimators. It builds upon the recent pioneering work of Phillips and Moon (1999) concerning an asymptotic theory for double indexed integrated statistical processes. The homogeneous cointegrated panel data model is studied and estimators such as the panel FMOLS, panel DOLS and the panel OLS are derived. Asymptotic consistency and asymptotic normality is proved for the estimators with two specific cases for the panel data model being investigated. Firstly the case of the homogeneous panel data model with a constant intercept. Secondly the case of the homogeneous panel data model with a constant intercept and trend. The main contribution of this chapter is that it provides a more detailed analysis of the subject than is usually available in the panel data literature. Kao and Chiang (2000) study the model with varying intercepts and trends and also heterogeneous slopes, whilst Kauppi (2000) considers the model with near integrated regressors. The chapter evolves as follows. In section 5.2 an introduction is given to the new sequential limit panel data asymptotic theory. In section 5.3 some technical preliminaries are given. In section 5.4 we present the homogeneous cointegrated panel data model and the various estimators and in section 5.5 we have hypothesis tests. In the appendices, in Appendix 1, we have the proofs to the theorems discussed in the chapter.

5.2 The Panel Data Asymptotic Theory-Sequential Limit Probability Theory

Panel data limit theory concerns itself with double indexed processes X_{NT}, where both N and T tend to infinity. Recently Phillips and Moon (1999) have highlighted three different approaches to nonstationary panel data limit theory. The first, diagonal path asymptotics, allows N and T to pass to infinity along a diagonal path where one index is a monotonic increasing function of the other, of the type $T = T(N)$ as the index $N \to \infty$. The second approach discussed in detail by Phillips and Moon (1999) is called the sequential limit theory for double indexed integrated processes. Here one derives the limiting distribution in two steps. First one fixes (holds constant) one index, say N, the cross-section dimension of the panel, and allows the other, say T, the time-series dimension, to pass to infinity giving an intermediate limit. The final limit result is obtained by subsequently letting N tend to infinity. The final approach discussed by Phillips and Moon (1999) is joint limits where N and $T \to \infty$ simultaneously subject perhaps to some rate condition, say $\frac{N}{T} \to 0$. This last form of limit theory is the one most used and most well known in the panel data literature.

This chapter is solely concerned with the sequential limit theory and how it is applied. In general one may obtain different results when using sequential probability limits compared to joint probability limits. One may in certain situations prefer one to the other according to ease of use. In Phillips and Moon (1999) sufficient conditions are given under which joint probability limits and sequential probability limits give identical results.

It is now a well known fact that many macroeconomic time-series exhibit the characteristics of a nonstationary stochastic process. Nelson and Plosser (1982) provided convincing evidence that many macroeconomic time-series could be better described as integrated processes with drift. Subsequently there has emerged a growing body of literature concerned with the general theory of statistical inference for time-series regressions with integrated processes. Phillips and Durlauf (1986), Park and Phillips (1988) and Phillips and Hansen (1990) are just a few examples.

When dealing with nonstationary panel data the same need for a general theory of statistical inference for panel data regressions with I(1) processes arises, ie for a nonstationary panel data limit theory. The recent work of Phillips and Moon (1999), Kauppi (2000) and Kao and Chiang (2000), etc. have emerged to fill this gap.

One of the key characteristics of the limiting distributions of the integrated time-series processes is that the least squares estimators are not asymptotically normal when appropriately scaled and centred. This nonnormality results from the fact that suitably scaled sample moments converge weakly to random matrices rather than constant matrices. When dealing with panel data by using sequential probability limits this property of convergence to random matrices can be exploited to derive an intermediate limit by an application of an appropriate Law of Large Numbers (LLN) or FCLT across the time-series dimension as $T \to \infty$. The final limit being then obtained subsequently on using a second LLN and/or CLT for random matrices over the cross-section dimension, as $N \to \infty$. This then gives the Gaussian properties to the limiting distributions in the final limit and smooths out the nonnormality of the limiting distributions of the time-series dimension. This then is the appeal of the new panel data sequential limit theory. An important condition here is that the random matrices used in taking the second limit, using a LLN or CLT across the cross-section dimension, are all defined on the same probability space. This is so the sum of the limit of the random matrices is well defined on the same space. Phillips and Moon (1999) Appendix b, shows how one can accomodate this by enlargening the probability space if necessary.

5.3 Technical Preliminaries

5.3.1 A Functional Central Limit Theory (FCLT)

Consider the panel data discrete time stochastic process[1] $\{y_{it}\}_{it}^{\infty}$ where

$$y_{it} = \alpha y_{it-1} + u_{it}. \tag{5.1}$$

When $\alpha = 1$ this then describes a random walk process

$$y_{it} = y_{it-1} + u_{it}. \tag{5.2}$$

Under this representation by backward substitution we can write y_{it} in terms of a partial sum process of the innovation sequence $\{u_{it}\}_{it}^{\infty}$, ie

$$y_{it} = S_{it} + y_{i0}, \tag{5.3}$$

[1] Or dynamic panel data model or panel AR(1) model.

where $S_{it} = \sum_{j=1}^{t} u_{ij}$ and letting $S_{i0} = 0$, y_{i0} is the initial condition. We can choose many alternatives for y_{i0} (see Phillips (1987) for examples of different initial conditions in the time-series case), but here we let $y_{i0} = 0$.

As an introduction we distinguish two cases[2]:

(i) u_{it} is $i.i.d.(0, \sigma^2)$ a scalar. Hence $\{y_{it}\}$ is also a scalar sequence.

(ii) u_t is an $(n \times 1)$ vector $u_t = (u_{i1}, u_{i2}, \ldots, u_{in})'$, with $E(u_t) = \underline{0}$ an $(n \times 1)$ null vector, $E(u_t u_t') = \Sigma < \infty$ a positive definite symmetric matrix. Hence $\{y_t\}$ is an $(n \times 1)$ vector sequence. We shall be concerned with the limiting distributions of the standardised sums.

Case (i) Let u_{it} be a random scalar defined on probability space $(\Omega, \mathcal{B}, P)$ then

$$(5.4) \qquad X_T(r) = \frac{1}{\sqrt{T}\sigma} S_{i[Tr]} \qquad \frac{(j-1)}{T} \leq r < \frac{j}{T} \quad (j = 1, \ldots, T)$$

$$(5.5) \qquad X_T(1) = \frac{1}{\sqrt{T}\sigma} S_{iT}.$$

Here [ ] denotes the integer part of its argument.

We see the sample paths of

$$(5.6) \qquad X_T(r) \in D = D[0,1] \quad a.s.$$

the space of all real valued functions on $[0,1]$ that are right continuous at each point of $[0,1]$ and have finite left limits. Thus being in D space jump

[2]In what follows $\underline{0}$ will sometimes denote a null vector and othertimes a null matrix whilst 0 will always denote a null scalar.

discontinuities (or discontinuities of the first kind) are allowed.[3] We may endow D space with the modified metric $d(f,g)$ in Skorohod topology given by the norm

$$(5.7) \qquad \|(f,g)\| = d(f,g) = inf_{\varepsilon>0}\{\varepsilon : \|\lambda\| \leq \varepsilon, sup\,|f(t) - g(\lambda t)| \leq \varepsilon\},$$

where $\|\lambda\| = sup_{t \neq s} \left|\ln \frac{\lambda(t)-\lambda(s)}{t-s}\right|, \quad t,s \in [0,1]$. For a definition and the properties of this metric in Skorohod topology see Billingsley (1968) p.111-112. This renders $D[0,1]$ a separable and complete space.

Then for case (i) $X_T(r)$ is a random element in the function space D. Also under these conditions $X_T(r)$ can be shown to converge weakly to a limit process known as standard Brownian motion or the Wiener process. This result is called the functional central limit theorem (FCLT) (ie a CLT on a function space) or as an Invariance Principle (see Billingsley (1968) p.68 on Donsker's Theorem)[4]. The limit process called the Wiener process we denote by $W(r)$ has sample paths which lie in $C = C[0,1]$ the space of all real valued continuous functions on $[0,1]$ so $W(r) \in C[0,1]$ a.s. Moreover $W(r)$ is a Gaussian process, ie for fixed r $W(r) \sim N(0,r)$, and has independent increments, ie $W(u) - W(r)$ is independent of $W(s) - W(t)$ for all $0 \leq t \leq$

[3]Note that $\Omega = D[0,1]$ here then.

[4]The name Invariance Principle stems from the fact that if h is continuous on $C[0,1]$ then $X_n \Rightarrow W$ implies $h(X_n) \Rightarrow h(W)$. Thus the FCLT holds under very general conditions. There may be great dependence (and heterogeneity) among the innovations of the strong mixing form or they may be white noise or i.i.d. The FCLT is invariant to the conditions and holds in all cases.

$s \leq r \leq u$, also $W(s) - W(t) \sim N(0, s-t)$. We write

$$X_T(r) \Rightarrow W(r) \tag{5.8}$$

to denote the weak convergence of the process $X_T(r)$ to $W(r)$ ie " $\Rightarrow$ " signifies the weak convergence of the associated probability measures[5] as $T \to \infty$. See also Billingsley (1968) on convergence of probability measures. We will endow this space $C[0,1]$ with the norm $\|A\| = \left(\int_0^1 |A(x)|^2 dx\right)^{\frac{1}{2}}$. The FCLT or Invariance Principle (IP) discussed above are given a good treatment in Phillips and Durlauf (1986), Phillips (1987) and Phillips and Solo (1992).

Case (ii) When u_t is an $(n \times 1)$ random vector defined on some probability space $(\Omega, \mathcal{B}, P)$ then for $E(u_t) = \underline{0}, \forall t$ we have the vector partial sums $S_t = \sum_{j=1}^{t} u_j$ and the vector random function

$$X_T(r) = \frac{1}{\sqrt{T}} \Sigma^{-\frac{1}{2}} S_{[Tr]} \qquad \frac{(j-1)}{T} \leq r < \frac{j}{T} \quad (j = 1, \ldots, T) \tag{5.9}$$

$$X_T(1) = \frac{1}{\sqrt{T}} \Sigma^{-\frac{1}{2}} S_T. \tag{5.10}$$

Note that

$$X_T(r) \in D = D[0,1]^n \quad a.s. \tag{5.11}$$

where

$$D[0,1]^n = D[0,1] \times D[0,1] \times D[0,1] \times \ldots \times D[0,1] \tag{5.12}$$

[5] This is the analogue for function spaces of convergence in distribution for random variables see Hall and Heyde (1980) for a discussion.

the product metric space of n copies of the $D[0,1]$ space. We endow the $D[0,1]^n$ space with a suitable metric again in Skorohod topology. This metric renders $D[0,1]^n$ a complete and separable metric space. This metric being given now by the norm

$$(5.13) \qquad ||(x,y)|| = d(x,y) = max_i\{d(x_i, y_i) : x_i, y_i \in D[0,1]\}.$$

Then under these conditions

$$(5.14) \qquad X_T(r) \Rightarrow W(r).$$

Here $W(r)$ is a multivariate Wiener process with each element of $W(r)$ being a univariate Wiener process and independent of each other. Thus $W(r)$ is termed an n-dimensional Wiener process. Note again that

$$(5.15) \qquad W(r) \in C[0,1]^n \quad a.s.$$

where

$$(5.16) \qquad C[0,1]^n = C[0,1] \times C[0,1] \times C[0,1] \times \ldots \times C[0,1].$$

This is the product space of n copies of the $C[0,1]$ space defined above. We shall endow the $C[0,1]^n$ space with the following Euclidian norm $|| \quad ||$. Thus given any matrix A, $||A|| = \left(tr(A'A)\right)^{\frac{1}{2}}$.

The FCLT described operates under very restrictive conditions, which we may need to relax to cater for innovations that may be only weakly stationary and not *i.i.d.* The following proposition gives the necessary conditions which should suffice for our purposes, endowed with the above norms. We state the case for the vector sequence $\{u_t\}_t^\infty$ as in Phillips and Durlauf (1986). For scalar sequences see Phillips and Solo (1992).

Proposition 5.3.1 *Let $\{u_t\}_t^\infty$ be a weakly stationary sequence of $(n \times 1)$ random vectors. Given $S_T = \sum_{j=1}^T u_j$ if*

(i) $E(u_t) = \underline{0}$ *an* $(n \times 1)$ *null vector* $\forall t$.

(ii) $E\left|u_{i1}\right|^\delta < \infty \quad (i = 1, \ldots, n) \quad$ *for some* $2 \leq \delta < \infty$.

(iii) either $\sum_{m=1}^\infty \varphi_m^{1-\frac{1}{\delta}} < \infty$ *or* $\delta > 2$ *and* $\sum_{m=1}^\infty \alpha_m^{1-\frac{2}{\delta}} < \infty$ *then*

$$\Sigma = \lim_{T \to \infty} E(T^{-1} S_T S_T^{\prime}) \tag{5.17}$$

$$= E(u_1 u_1^{\prime}) + \sum_{k=2}^{\infty} \{E(u_1 u_k^{\prime}) + E(u_k u_1^{\prime})\}. \tag{5.18}$$

If Σ *is positive definite, then* $X_T(r) \Rightarrow W(r)$ *as* $T \to \infty$.

Again note that $X_T(r) \in D[0,1]^n$ a.s. and $W(r) \in C[0,1]^n$ a.s. as before.

Remark 5.3.2 *The conditions given allow a large degree of temporal dependence and heterogeneity among the innovation processes $\{u_t\}_t^\infty$. The strong and uniform mixing conditions given state how much dependence exists in the $\{u_t\}_t^\infty$ processes that are seperated by at least m periods. Events that are* *over m periods apart are independent. Hence allowing $m \to \infty$ we obtain asymptotic independence. For a good discussion on strong and uniform mixing conditions see White (1984). The summability condition in (iii) shows for $\sum_{n=1}^{\infty} \varphi_m^{1-\frac{1}{\delta}} < \infty$, φ_m is uniform mixing and for $\sum_{n=1}^{\infty} \alpha_m^{1-\frac{2}{\delta}} < \infty$, α_m is strong mixing. The conditions are satisfied, for say α_m, when the mixing decay rate is $\alpha_m = O(m^{-\lambda})$ for some $\lambda > \frac{\delta}{(\delta-2)}$. We can make precise statements about the memory of a sequence that we can relate to the moment conditions expressed in terms of δ. For sequences with longer memories δ is greater and hence our moment restrictions in (ii) increase accordingly (see White (1984) p.47)[6]. The case of strict stationarity follows as a special case.*

The FCLT is often used in conjunction with the Continuous Mapping Theorem (CMT) now given.

Lemma 5.3.3 *If as in Case (i) of equation (5.8) $X_T(r) \Rightarrow W(r)$ as $T \to \infty$ and h is any continuous functional[7] on $D[0,1]$, continuous that is except for at most a set of points $D_h \in D[0,1]$ for which $P(W(r) \in D_h) = 0$ then $h(X_T(r)) \Rightarrow h(W(r))$ as $T \to \infty$.*

Having established by FCLT the weak convergence of the partial sum processes to random matrices defined on Brownian motion the following theo-

[6]This way of showing the tradeoff between moment and mixing conditions was first developed by McLeish (1975).

[7]The extension to where h is an n-dimensional continuous functional is immediate.

rems concerning the Strong Law of Large Numbers (SLLN) and CLT will be useful. Also a formal definition of the sequential limit probability theory is given (see Phillips and Moon (1999) p.1065.).

Theorem 5.3.4 *(Komolgorov's Theorem) Let $\{Z_i\}$ be a sequence of i.i.d. random variables.*[8] *Then $\bar{Z}_n \xrightarrow{a.s.} \mu$ if and only if $E|Z_i| < \infty$ and $E(Z_i) = \mu$. Here $\bar{Z}_n = \frac{1}{N}\sum_{i=1}^{N} Z_i$.*

Theorem 5.3.5 *(Komolgorov's Theorem Multivariate Version) Let $\{Z_i\}$ be a sequence of i.i.d. random K-vectors with $E(Z_i) = \mu$. Then $\bar{Z}_n \xrightarrow{a.s.} \mu$ if and only if $E(Z_i)$ exists and $E(Z_i) < \infty$. Here $\bar{Z}_n = \frac{1}{N}\sum_{i=1}^{N} Z_i$.*

Theorem 5.3.6 *(Lindeberg-Levy CLT) Let $\{Z_i\}$ be a sequence of i.i.d. random variables. If $var(Z_i) = \sigma^2 < \infty, \quad \sigma^2 \neq 0$, then*

$$\frac{\sqrt{N}(\bar{Z}_n - \bar{\mu}_n)}{\bar{\sigma}_n} = \frac{\sqrt{N}(\bar{Z}_n - \mu)}{\sigma} \tag{5.19}$$

$$= \frac{1}{\sqrt{N}} \frac{\sum_{i=1}^{N}(Z_i - \mu)}{\sigma} \xrightarrow{d} N(0,1). \tag{5.20}$$

Theorem 5.3.7 *(Lindeberg-Levy CLT Multivariate Version) Let $\{Z_i\}$ be a sequence of i.i.d. random K-vectors with mean μ and covariance matrix $\Sigma < \infty$, then*

$$\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} Z_i - \mu\right) \xrightarrow{d} N(\underline{0}, \Sigma). \tag{5.21}$$

[8]Here $\xrightarrow{a.s.}$ means almost sure convergence.

Definition 5.3.8 *(a) A sequence of m-vectors $\{X_{NT}\}$ on a probability space $(\Omega, \mathcal{F}, P)$ is said to converge in probability to X sequentially written*[9] $X_{NT} \xrightarrow{p} X$ *in sequential limit as* $(T, N \to \infty)_{seq}$ *if*

$$\lim_{N\to\infty}\lim_{T\to\infty} P\{\|X_{NT} - X\| > \varepsilon\} = \underline{0} \quad \forall \varepsilon > 0. \tag{5.22}$$

(b) X_{NT} *converges in distribution sequentially to the m-vector X, written* $X_{NT} \Rightarrow X$*, in sequential limit as* $(T, N \to \infty)_{seq}$ *if*

$$\lim_{N\to\infty}\lim_{T\to\infty} |Ef(X_{NT}) - Ef(X)| = \underline{0} \quad \forall f \in C \tag{5.23}$$

where C is the class of all bounded, continuous, real functions on $\Re^m$ *and* $\underline{0}$ *is an* $(m \times 1)$ *null vector.*

Many LLN's and CLT's can be handled together using Slutsky's Theorem.

Proposition 5.3.9 *(Slutsky's Theorem) If* $X_n \xrightarrow{d} X$ *and* $Y_n \xrightarrow{d} \alpha$ *then*

(i) $X_n + Y_n \xrightarrow{d} X + \alpha$

(ii) $X_n Y_n \xrightarrow{d} \alpha X$

(iii) $\left(\frac{X_n}{Y_n}\right) \xrightarrow{d} \frac{X}{\alpha}$, *provided* $\alpha \neq 0$

To cater for the multivariate versions of Theorems (5.3.4) and (5.3.6), ie Theorems (5.3.5) and (5.3.7), we have the following Cramer-Wold device.

[9] Here $\xrightarrow{p}$ means converges in probability.

Proposition 5.3.10 *(Cramer-Wold device) Let $\{b_n\}$ be a sequence of random $(k \times 1)$ vectors and suppose that for any real $(k \times 1)$ vector λ such that $\lambda'\lambda = 1$ then $\lambda' b_n \xrightarrow{d} \lambda' Z$ where Z is a $(k \times 1)$ vector with joint distribution function $F(z)$. Then the limiting distribution function of b_n exists and equals $F(z)$.*

Finally in order to carry out block covariance matrix inversions we have the following partitioned matrix proposition.

Proposition 5.3.11 *Define the $(k \times k)$ nonsingular symmetric matrix*

$$A = \begin{bmatrix} B & C' \\ C & D \end{bmatrix}, \tag{5.24}$$

where B is $(k_1 \times k_1)$, C is $(k_2 \times k_1)$, D is $(k_2 \times k_2)$ and $k = (k_1 + k_2)$. Then defining $E = D - CB^{-1}C'$,

$$A^{-1} = \begin{bmatrix} B^{-1}(I + C'E^{-1}CB^{-1}) & -B^{-1}C'E^{-1} \\ -E^{-1}CB^{-1} & E^{-1} \end{bmatrix}. \tag{5.25}$$

Phillips and Moon (1999) and Kauppi (2000) have derived panel generalisations of LLN's and CLT's for double indexed processes. In Phillips and Moon (1999) Theorem 1 generalises a WLLN to double indexed processes that are independent across i, for all T. Theorem 2 generalises a CLT due to Eicker (1963) and Theorem 3 generalises the Lindeberg-Feller CLT to double indexed processes. Similarly in Kauppi (2000) Theorem 1 follows Phillips and

Moon's (1999) Theorem 1, Theorem 2 generalises Markov's LLN to double indexed processes whilst Theorem 3 follows Phillips and Moon's (1999) Theorem 3 again. Levin and Lin (1992,1993) use another form of limit theory for double indexed processes called triangular array asymptotics see Levin and Lin (1992,1993) Lemma 2.2 and 2.3 for details. Phillips (1987) also uses triangular arrays in his paper.[10]

5.4 The Homogeneous Cointegrated Panel Data Model

5.4.1 Case (1) The Model with a Constant

Consider the following homogeneous panel data model with a constant intercept

(5.26) $$y_{it} = \alpha + x_{it}^{\prime}\beta + u_{it} \qquad (1)$$

(5.27) $$x_{it} = x_{it-1} + \varepsilon_{it} \qquad (2)$$

where β is a $(k \times 1)$ vector of slope coefficients, $\{\alpha\}$ the intercept term, $\{u_{it}\}$ the stationary disturbance term, ie $\{u_{it}\} \sim$ I(0). Also $\{y_{it}\}$ which is (1×1) and $\{x_{it}\}$ which are $(k \times 1)$ are integrated processes of order one for all i. That is $\{\{y_{it}\}, \{x_{it}\}\} \sim$ I(1). This is the panel data version of Phillips triangular form (see Phillips (1991)) and thus describe a system of

[10]These triangular array methods form the basis of Phillips and Moon's (1999) diagonal path asymptotics.

cointegrated regressions.

The DOLS panel estimator

Following Kao and Chiang (2000) and Pedroni (2001) the Panel Data Dynamic Ordinary Least Squares (DOLS) estimator generalises the Saikkonen (1991) cointegrated regression estimator of the time-series literature to the panel setting. In contrast to the time-series fully modified FMOLS estimators of Phillips and Hansen (1990) and their panel analogues that use nonparametric corrections for serial correlation and endogeniety problems in the cointegrated OLS model, the DOLS estimator uses parametric corrections to account for serial correlation and endogeniety. One way to ensure that the $\{u_{it}\}$ (which we can assume is a strictly or covariance stationary sequence), is uncorrelated with $\{\varepsilon_{it}\}$ is to add past and future values of $\{\Delta x_{it}\}$ into the regression. Then we use the classical assumptions $E(\hat{u}_{it}\varepsilon_{it+k}) = \underline{0}$ a $(k \times 1)$ null vector, for $k = -j, -(j+1), \ldots, 0, 1, 2, \ldots (j-1), j$. Formally let $\{z_{it}\}$ be the residual from the linear projection of $\{u_{it}\}$ on $\{\varepsilon_{it-p}, \varepsilon_{it-p+1}, \ldots \varepsilon_{it-1}, \varepsilon_{it}, \varepsilon_{it+1}, \ldots \varepsilon_{it+p}\}$. So

$$u_{it} = \sum_{j=-p}^{p} c_{ij}\varepsilon_{it+j} + z_{it}. \tag{5.28}$$

Then z_{it} is uncorrelated with ε_{it-s} for $s = -p, -(p+1), \ldots, 0, 1, \ldots, p$.

Remark 5.4.1 *Two important assumptions made by Saikkonen (1991) (see* *also Kao and Chiang (2000)) is that the lags and leads of the* Δx_{it} *tend to infinity, across the time-series dimension, at a suitable rate and in the limit the coefficient matrices* c_{ij} *should be zero. These are stated formally as*

(i) $p \to \infty$ *as* $T \to \infty$ *such that* $\frac{p^3}{T} \to 0$

(ii) $T^{\frac{1}{2}} \sum_{|j|>p} ||c_{ij}|| \to 0.$

Substituting for u_{it} in (1) we get

$$y_{it} = \alpha + x_{it}^{\prime}\beta + \sum_{j=-p}^{p} c_{ij}\Delta x_{it-j} + z_{it} \qquad (1^i) \tag{5.29}$$

$$x_{it} = x_{it-1} + \varepsilon_{it}. \qquad (2^i) \tag{5.30}$$

Writing the above in matrix form we get

$$y_{it} = \alpha + X_{it}^{\prime}\Upsilon + z_{it} \tag{5.31}$$

where $X_{it} = [m_{it}^{\prime} \quad 1 \quad x_{it}^{\prime}]^{\prime}$ and $\Upsilon = \begin{bmatrix} \xi \\ \alpha \\ \beta \end{bmatrix}$,

where $m_{it} = (\varepsilon_{it-p}^{\prime}, \varepsilon_{it-p+1}^{\prime} \ldots, \varepsilon_{it-1}^{\prime}, \varepsilon_{it}^{\prime}, \varepsilon_{it+1}^{\prime}, \ldots, \varepsilon_{it+p}^{\prime})^{\prime}$

and $\xi = (c_{ip}^{\prime}, c_{ip-1}^{\prime} \ldots, c_{i-p}^{\prime})^{\prime}$.

Hence the DOLS estimator of Υ is

$$\hat{\Upsilon}_{DOLS1} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{\prime}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}y_{it} \tag{5.32}$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it} \\ 1 \\ x_{it} \end{bmatrix} [m_{it}^{\prime} \quad 1 \quad x_{it}^{\prime}]\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \end{bmatrix}\right) \tag{5.33}$$

$$(5.34)\quad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}^{/} & m_{it} & m_{it}x_{it}^{/} \\ m_{it}^{/} & 1 & x_{it}^{/} \\ x_{it}m_{it}^{/} & x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \end{bmatrix}\right).$$

Since $\varepsilon_{it-p} = \Delta x_{it-p}$ it is a $(k \times 1)$ column vector so $\varepsilon_{it-p}^{/}$ is a $(1 \times k)$ row vector. Hence m_{it} above is a $((2p+1)k \times 1)$ column vector and X_{it} is a $(((2p+2)k+1) \times 1)$ column vector.

Assumption 5.4.2 *Let* $w_{it} = \begin{bmatrix} z_{it} \\ \varepsilon_{it} \end{bmatrix}$ *be an* $(N^* \times 1)$[11] *vector generated by the linear process*

$$(5.35)\qquad w_{it} = \Psi(L)\epsilon_{it}$$

where $\Psi(L) = \sum_{j=0}^{\infty} \Psi_j L^j$ *so that* $w_{it} = \sum_{j=0}^{\infty} \Psi_j \epsilon_{it-j}$

(i) Ψ_j *is an* $(N^* \times N^*)$ *constant matrix with* $\Psi_0 = I_{N^*}$

(ii) $\sum_{j=0}^{\infty} j\|\Psi_j\| < \infty$ *so* $\{j\Psi_j\}_j^{\infty}$ *is an absolutely summable sequence*

(iii) $\Psi(1)$ *has full rank*

(iv) $\{\epsilon_{it}\}_{it}^{\infty}$ *is an i.i.d. sequence of* $(N^* \times 1)$ *vectors*

(v) $E(\epsilon_{it}) = \underline{0}$ *and* $E(\epsilon_{it}\epsilon_{it}^{/}) = \Sigma_i$ *a positive definite matrix such that* $P_iP_i^{/} = \Sigma_i$ *where* P_i *is the Cholesky factor of* Σ_i

(vi) $E\|\epsilon_{it}\|^4 < \infty$ *ie* ϵ_{it} *has finite fourth moments*[12].

[11] here $N^* = k+1$.

[12] Condition (ii) and condition (vi) ensure that the components of the long run covariance matrices are finite eg $\|\Omega_i\| = \|\Psi(1)\Sigma_i\Psi^{/}(1)\| < \infty$ since (ii) also implies $\|\Psi(1)\| < \infty$.

Remark 5.4.3 *Here the absolute summability of $\{j\Psi_j\}_j^\infty$ is stronger than the absolute summability condition $\sum_{j=0}^\infty \|\Psi_j\| < \infty$ and hence $\sum_{j=0}^\infty j\|\Psi_j\| < \infty$ implies $\sum_{j=0}^\infty \|\Psi_j\| < \infty$. The stronger condition is useful when using the Beveridge-Nelson Decomposition much used by Phillips and co-workers in his papers Phillips and Solo (1992) and Phillips and Moon (1999).*

Under the conditions of Proposition (5.3.1) and Assumption (5.4.2) we see that $\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it}$ satisfies the following multivariate invariance principle (again see Phillips and Durlauf (1986))

$$(5.36) \qquad \frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it} \Rightarrow W_i(r) = BM_i(\Omega_i) \quad \text{as} \quad T \to \infty \quad \forall i$$

where $W_i(r)$ is standard N^*-dimensional Brownian Motion (or Wiener Process) with covariance matrix $\Omega_i = \Psi(1)\Sigma_i\Psi'(1)$. Also $W_i(r) \in C[0,1]^{N^*}$ a.s. Since z_{it} is uncorrelated with $\varepsilon_{i\tau}$ for all t and τ we can partition $\Psi(L)$ and P_i accordingly and write

$$(5.37) \qquad P_i = \begin{bmatrix} P_{i11} & \underline{0}' \\ \underline{0} & P_{i22} \end{bmatrix} \qquad \Psi(L) = \begin{bmatrix} \Psi_{11}(L) & \underline{0}' \\ \underline{0} & \Psi_{22}(L) \end{bmatrix}$$

where P_{i22} and $\Psi_{22}(L)$ are $(k \times k)$ matrices and P_{i11} and $\Psi_{11}(L)$ (1×1) scalars. Hence P_i and $\Psi(L)$ are $(N^* \times N^*)$ matrices. Also $\underline{0}$ is the $(k \times 1)$ null vector. Then

$$(5.38) \qquad \Omega_i = \Psi(1)\Sigma_i\Psi'(1) = \Psi(1)P_iP_i'\Psi'(1)$$

$$(5.39) \qquad = \begin{bmatrix} [\Psi_{11}(1)P_{i11}]^2 & \underline{0}' \\ \underline{0} & \Psi_{22}(1)P_{i22}P_{i22}'\Psi_{22}'(1) \end{bmatrix}.$$

Assumption 5.4.4 *The* $(k \times 1)$ *vector* x_{it} *are not cointegrated (ie* Ω_{i22} *is nonsingular). Where* Ω_i *is also partitioned according to the above*

$$(5.40) \qquad = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} \\ \Omega_{i21} & \Omega_{i22} \end{bmatrix}$$

$$(5.41) \qquad = \begin{bmatrix} \Omega_{i11} & \underline{0}' \\ \underline{0} & \Omega_{i22} \end{bmatrix}.$$

Hence Ω_i is block diagonal here with the panel DOLS correction. Since Ω_i is usually unknown consistent estimates can be obtained of the components of Ω_i (shown below for the panel FMOLS estimator), which can be used to form feasible panel DOLS estimators. The following Lemma will simplify the DOLS computations to come for case (1)

Lemma 5.4.5 *By the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3)*

$$(5.42) \qquad \frac{1}{T^2}\sum_{t=1}^{T} x_{it}x_{it}' \Rightarrow \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\}\Lambda_{i22}' \qquad (a)$$

$$(5.43) \qquad \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} x_{it} \Rightarrow \Lambda_{i22}\int_0^1 [W_{i2}(r)]dr \qquad (b)$$

$$(5.44) \qquad \frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} z_{it} \Rightarrow \Lambda_{i11}W_{i1}(1) \qquad (c)$$

$$(5.45) \qquad \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it-p}x_{it}' \Rightarrow \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i2}(r)]'\right\}\Lambda_{i22}' + \sum_{v=1}^{\infty}\Gamma_{iv}' \qquad (d)$$

$$\frac{1}{T}\sum_{t=1}^{T} x_{it}z_{it} \Rightarrow \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11} \tag{5.46}$$ (e)

$$\frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}\varepsilon'_{it-s} \xrightarrow{p} \Gamma_{is} = E(\Delta x_{it})(\Delta x_{it-s})' \quad \textit{by LLN} \tag{5.47}$$ (f)

$$\frac{1}{T}\sum_{t=1}^{T} m_{it}m'_{it} \xrightarrow{p} V_i \quad \textit{by LLN} \tag{5.48}$$ (g)

$$\frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} m_{it}z_{it} \xrightarrow{d} N(\underline{0}, V_i\Lambda_{11}^2) \quad \textit{by CLT} \tag{5.49}$$ (h)

$$\frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} \varepsilon_{it} \Rightarrow \Lambda_{i22}W_{i2}(1) \tag{5.50}$$ (i)

where $\Lambda_{i11} = \Psi_{11}(1)P_{i11}$ *a* (1×1) *scalar,* $\Lambda_{i22} = \Psi_{22}(1)P_{i22}$ *a* $(k \times k)$ *matrix,*

$$m_{it} = \begin{bmatrix} \varepsilon_{it+p} \\ \varepsilon_{it+(p-1)} \\ \vdots \\ \varepsilon_{it} \\ \varepsilon_{it-1} \\ \varepsilon_{it-2} \\ \vdots \\ \varepsilon_{it-p} \end{bmatrix} \quad a\ ((2p+1)k \times 1)\ \textit{vector,}$$

$$V_i = E(m_{it}m'_{it}) = \begin{bmatrix} \Gamma_{i0} & \Gamma_{i1} & \dots & \Gamma_{ip} & \Gamma_{ip+1} & \dots & \Gamma_{i2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Gamma_{i-p} & & & & & & \Gamma_{ip} \\ \Gamma_{i-(p+1)} & \vdots & \vdots & \vdots & \vdots & \vdots & \Gamma_{ip+1} \\ \Gamma_{i-(p+2)} & & & & & & \Gamma_{i(p+2)} \\ \Gamma_{i-2p} & \dots & \dots & \dots & \dots & \dots & \Gamma_{i0} \end{bmatrix}$$

a $((2p+1)k \times (2p+1)k)$ *matrix.*

Note here $W_i(r)$ is partitioned as $W_i(r) = \left(W_{i1}(r), W'_{i2}(r)\right)'$ where $W_{i1}(r)$ is standard scalar Brownian motion and $W_{i2}(r)$ is k-dimensional standard Brownian motion. Also note the integrals above are understood to be taken with respect to Lesbesgue measure (that is $\int_0^1 W(r)dr$). See Pfieffer

(1990) for a good discussion of Lesbegue measure. The appendix of Levin and Lin (1992) gives a useful summary of some integrals of Brownian motion. See also Bispham (2002) for an introduction to Wiener processes [13] .

Theorem 5.4.6 *Suppose Proposition (5.3.1) and Assumptions (5.4.2) and (5.4.4) hold and the data are generated by* (1^i) *and* (2^i)*. Then by sequential limit probability theory*

(i) As $(N, T \to \infty)_{seq}$ $\quad \hat{\beta}_{DOLS1} \xrightarrow{p} \beta$

(ii) As $(N, T \to \infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{DOLS1} - \beta\right) \Rightarrow N\left(\underline{0}, 2\Omega_{22}^{-1}\Omega_{11}\right).$$

The FMOLS panel estimator

Consider again the model of (1) and (2).

Assumption 5.4.7 *Let* $\bar{w}_{it} = \begin{bmatrix} u_{it} \\ \varepsilon_{it} \end{bmatrix}$ *be an* $(N^* \times 1)$ *vector generated by the linear process*[14]

$$\bar{w}_{it} = \bar{\Psi}(L)\epsilon_{it} \tag{5.51}$$

where $\bar{\Psi}(L) = \sum_{j=0}^{\infty} \bar{\Psi}_j L^j$ *so that* $\bar{w}_{it} = \sum_{j=0}^{\infty} \bar{\Psi}_j \epsilon_{it-j}$

(i) $\bar{\Psi}_j$ *is an* $(N^* \times N^*)$ *constant matrix with* $\bar{\Psi}_0 = I_{N^*}$

(ii) $\sum_{j=0}^{\infty} j||\bar{\Psi}_j|| < \infty$ *so* $\{j\bar{\Psi}_j\}_j^{\infty}$ *is an absolutely summable sequence*

[13]Banerjee et al (1993) is useful and also the Mathematical Appendix in Pedroni (2000).
[14]Here $N^* = k + 1$ again.

(iii) $\bar{\Psi}(1)$ *has full rank*

(iv) $\{\epsilon_{it}\}_{it}^{\infty}$ *is an i.i.d. sequence of* $(N^* \times 1)$ *vectors*

(v) $E(\epsilon_{it}) = \underline{0}$ *and* $E(\epsilon_{it}\epsilon_{it}^{/}) = \Sigma_i$ *a positive definite matrix such that* $P_iP_i^{/} = \Sigma_i$ *where* P_i *is the Cholesky factor of* Σ_i

(vi) $E\left\|\epsilon_{it}\right\|^4 < \infty$ *ie* ϵ_{it} *has finite fourth moments.*

Again we see that under the conditions of Proposition (5.3.1) and Assumption (5.4.7) we have that $\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]}\bar{w}_{it}$ satisfies the following multivariate invariance principle (Phiilips and Durlauf (1986))

(5.52) $$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]}\bar{w}_{it} \Rightarrow \bar{W}_i(r) = BM_i(\bar{\Omega}_i) \quad \text{as} \quad T \to \infty \quad \forall i$$

where $\bar{\Omega}_i$ is the long run covariance matrix of $\{\bar{w}_{it}\}$ given by

(5.53) $$\bar{\Omega}_i = E(\bar{w}_{i0}\bar{w}_{i0}^{/}) + \sum_{k=1}^{\infty}E(\bar{w}_{i0}\bar{w}_{ik}^{/}) + \sum_{k=1}^{\infty}E(\bar{w}_{i0}\bar{w}_{ik}^{/})^{/}$$

(5.54) $$= \bar{\Phi}_i + \bar{\Pi}_i + \bar{\Pi}_i^{/}.$$

Here

(5.55) $$\bar{\Omega}_i = \begin{bmatrix} \bar{\Omega}_{i11} & \bar{\Omega}_{i12} \\ \bar{\Omega}_{i21} & \bar{\Omega}_{i22} \end{bmatrix}$$

where $\bar{\Omega}_{i11}$ is (1×1), $\bar{\Omega}_{i12}$ is $(1 \times k)$ and $\bar{\Omega}_{i22}$ is $(k \times k)$.

$$\bar{\Omega}_i = \begin{bmatrix} \sum_{k=0}^{\infty}E(u_{it}u_{it+k}) + \sum_{k=1}^{\infty}E(u_{it}u_{it+k}) & \sum_{k=0}^{\infty}E(u_{it}\varepsilon_{it+k}^{/}) + \sum_{k=1}^{\infty}E(\varepsilon_{it}^{/}u_{it+k}) \\ \sum_{k=0}^{\infty}E(\varepsilon_{it}u_{it+k}) + \sum_{k=1}^{\infty}E(u_{it}\varepsilon_{it+k}) & \sum_{k=0}^{\infty}E(\varepsilon_{it}\varepsilon_{it+k}^{/}) + \sum_{k=1}^{\infty}E(\varepsilon_{it}\varepsilon_{it+k}^{/}) \end{bmatrix}.$$

(5.56)

The one-sided long run covariance matrix is given by

$$\bar{\Gamma}_i = \sum_{k=0}^{\infty} E(\bar{w}_{i0}\bar{w}_{ik}^{/}) \tag{5.57}$$

$$= \bar{\Phi}_i + \bar{\Pi}_i \tag{5.58}$$

$$= \begin{bmatrix} \bar{\Gamma}_{i11} & \bar{\Gamma}_{i12} \\ \bar{\Gamma}_{i21} & \bar{\Gamma}_{i22} \end{bmatrix} \tag{5.59}$$

$$= \begin{bmatrix} \sum_{k=0}^{\infty} E(u_{it}u_{it+k}) & \sum_{k=0}^{\infty} E(u_{it}\varepsilon_{it+k}^{/}) \\ \sum_{k=0}^{\infty} E(\varepsilon_{it}u_{it+k}) & \sum_{k=0}^{\infty} E(\varepsilon_{it}\varepsilon_{it+k}^{/}) \end{bmatrix}. \tag{5.60}$$

Also $\bar{\Phi}_i = E(\bar{w}_{i0}\bar{w}_{i0}^{/})$ the contemporaneous covariance matrix and

$$\bar{\Pi}_i = \begin{bmatrix} \bar{\Pi}_{i11} & \bar{\Pi}_{i12} \\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22} \end{bmatrix}. \tag{5.61}$$

$$= \begin{bmatrix} \sum_{k=1}^{\infty} E(u_{it}u_{it+k}) & \sum_{k=1}^{\infty} E(u_{it}\varepsilon_{it+k}^{/}) \\ \sum_{k=1}^{\infty} E(\varepsilon_{it}u_{it+k}) & \sum_{k=1}^{\infty} E(\varepsilon_{it}\varepsilon_{it+k}^{/}) \end{bmatrix}. \tag{5.62}$$

Remark 5.4.8 *Note that the long run covariance matrix is an alternative way of writing* $\bar{\Omega}_i = \bar{\Psi}(1)\Sigma_i\bar{\Psi}^{/}(1) = \bar{\Omega}_i = \bar{\Psi}(1)P_iP_i^{/}\bar{\Psi}^{/}(1)$. *This is just the autocovariance-generating function* $G(z) = \bar{\Psi}(z)P_iP_i^{/}\bar{\Psi}^{/}(z^{-1})$ *evaluated at* $z = 1$ *and now conforms with the covariance matrix formulation given for the panel DOLS estimator.*

Assumption 5.4.9 *The* $(k \times 1)$ *vector* x_{it} *are not cointegrated (ie* $\bar{\Omega}_{i22}$ *is nonsingular).*

As mentioned earlier the FMOLS estimators of Phillips and Hansen (1990) and their panel analogues, eg Pedroni (2000), Kao and Chiang (2000) and

Phillips and Moon (1999) use nonparametric corrections for endogeneity and serial correlation problems in the model.

Consider first the effect of correlation between ε_{it} and u_{it}. This gives rise to a non-zero value for $\bar{\Omega}_{i21}$. This endogeneity problem is similar to the simultaneity bias in simultaneous equations. We can correct for this by subtracting $\bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\Delta x_{it}$ from y_{it} and u_{it} in (1). Define

$$u_{it}^{+} = u_{it} - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\varepsilon_{it}, \tag{5.63}$$

$$y_{it}^{+} = y_{it} - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\varepsilon_{it}. \tag{5.64}$$

So now

$$\begin{bmatrix} u_{it}^{+} \\ \varepsilon_{it} \end{bmatrix} = L_i' \begin{bmatrix} u_{it} \\ \varepsilon_{it} \end{bmatrix} = L_i' \bar{w}_{it} \tag{5.65}$$

where

$$L_i' = \begin{bmatrix} 1 & -\bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1} \\ \underline{0} & I_K \end{bmatrix} = \begin{bmatrix} L_{i1}' \\ L_{i2}' \end{bmatrix} \tag{5.66}$$

where L_i is $((k+1) \times (k+1))$,L_{i1}' is $(1 \times (k+1))$ and L_{i2}' is $(k \times (k+1))$.

Now

$$\begin{bmatrix} u_{it}^{+} \\ \varepsilon_{it} \end{bmatrix} = \begin{bmatrix} 1 & -\bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1} \\ \underline{0} & I_K \end{bmatrix} \begin{bmatrix} u_{it} \\ \varepsilon_{it} \end{bmatrix} \tag{5.67}$$

which has long run covariance matrix

$$L_i'\bar{\Omega}_i L_i = L_i' \begin{bmatrix} \bar{\Omega}_{i11} & \bar{\Omega}_{i12} \\ \bar{\Omega}_{i21} & \bar{\Omega}_{i22} \end{bmatrix} L_i \tag{5.68}$$

$$(5.69) \quad = \begin{bmatrix} 1 & -\bar{\Omega}_{i21}\bar{\Omega}_{i22}^{-1} \\ \underline{0} & I_K \end{bmatrix} \begin{bmatrix} \bar{\Omega}_{i11} & \bar{\Omega}_{i12} \\ \bar{\Omega}_{i21} & \bar{\Omega}_{i22} \end{bmatrix} \begin{bmatrix} 1 & \underline{0}' \\ -\bar{\Omega}_{i21}\bar{\Omega}_{i22}^{-1} & I_K \end{bmatrix}$$

$$(5.70) \quad = \begin{bmatrix} \bar{\Omega}_{i11}^{+} & \underline{0}' \\ \underline{0} & \bar{\Omega}_{i22} \end{bmatrix}.$$

where $\bar{\Omega}_{i11}^{+} = \bar{\Omega}_{i11} - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\bar{\Omega}_{i21}$. Thus the transformed long run covariance matrix is block diagonal as in the panel DOLS case.

The serial correlation problem arises from the constant term that appears in the OLS equation without correction, arising from the non-zero value for $\aleph^{+} = \sum_{k=0}^{\infty} E(\varepsilon_{it}u_{it+k}^{+}) = \bar{\Gamma}_{i21}^{+}$. Thus corrections are needed to remove what is termed the second-order bias in the time-series case[15] effects arising from the temporal correlation between ε_{it} and u_{it}^{+}. This is given by the off-diagonal elements of the one-sided long run covariance matrix $\bar{\Gamma}_i$. This is also called the bias correction as the estimator is knocked off centre and gives rise to the non-normality in the OLS time-series estimator. Write

$$(5.71) \quad \aleph^{+} = \sum_{k=0}^{\infty} E(\varepsilon_{it}u_{it+k}^{+})$$

$$(5.72) \quad = \sum_{k=0}^{\infty} E(\varepsilon_{it}[u_{it+k} - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\varepsilon_{it+k}])$$

$$(5.73) \quad = \sum_{k=0}^{\infty} E(\varepsilon_{it}u_{it+k}) - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1} \sum_{k=0}^{\infty} E(\varepsilon_{it}\varepsilon_{it+k}).$$

From above we see using our one-sided long run covariance matrix $\bar{\Gamma}_i$ that

$$(5.74) \quad \aleph^{+} = \bar{\Gamma}_{i21}^{+} = \bar{\Gamma}_{i21} - \bar{\Omega}_{i12}\bar{\Omega}_{i22}^{-1}\bar{\Gamma}_{i22}.$$

[15]In the time-series literature it is termed "second-order" because the consistency of time-series estimators is unaffected. In the panel case though they are. The bias does influence the centering of the time-series limiting distribution and normally indicates that the finite sample bias can be substantial.

Consistent estimators for the nuisance parameters $\bar{\Omega}_i, \bar{\Gamma}_i$ and $\bar{\Omega}_{i11}^{+}$ can be used to obtain feasible FMOLS estimators. Similar analogous consistent estimators are valid for use in the DOLS case. These estimators are the same as the time-series estimators of the long run covariance matrices as $T \to \infty$. Some are scalar as in Phillips and Perron (1988), p.340 and Phillips (1987), p.285 who discusses some of the conditions necessary on the lag window truncation parameter for consistent estimation. Further discussion of consistent estimation of covariance matrices can be found in White (1984) ch.6. For the matrix case Phillips and Durlauf (1986), p.479 give conditions for consistent estimates of long run covariance matrices.

Remark 5.4.10 *The conditions on the rate at which $l \to \infty$ as $T \to \infty$ necessary for consistent estimation of long run covariance matrices are*

(i) $l \to \infty$ *as* $T \to \infty$

(ii) $\frac{l^4}{T} \to 0$.

Newey and West (1987) Heteroscedasticity and Autocorrelation Consistent (HAC) estimators are also discussed by Phillips and Durlauf (1986) and can be used to ensure non-negative variances. Hence for the time-series estimates of $\bar{\Omega}_i, \bar{\Gamma}_i$ and $\bar{\Omega}_{i11}^{+}$ and the analogous DOLS nuisance parameters we can refer to Phillips and Durlauf (1986).

For the panel analogues of the long run covariance matrices we follow Kao and Chiang (2000). Note that under the assumption of a homogeneous panel we have the following simplifications $\bar{\Omega}_i = \bar{\Omega}$, $\bar{\Gamma}_i = \bar{\Gamma}$ and $\bar{\Omega}^+_{i11} = \bar{\Omega}^+_{11} \quad \forall i$ and similarly for the DOLS case. First obtain estimates of $\hat{u}_{it}$ and $\hat{\varepsilon}_{it}$ by the OLS regression of (1) and (2). For the DOLS $\hat{z}_{it}$ and $\hat{\varepsilon}_{it}$, use the OLS regression of (1^i) and (2^i). Thus we can form $\hat{w}_{it}$ and $\hat{\hat{w}}_{it}$.

Now $\hat{\hat{\Phi}}$ is estimated by

$$\hat{\hat{\Phi}} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\hat{w}}_{it}\hat{\hat{w}}_{it}' \tag{5.75}$$

and $\hat{\hat{\Omega}}$ is estimated by

$$\hat{\hat{\Omega}} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\sum_{t=1}^{T}\hat{\hat{w}}_{it}\hat{\hat{w}}_{it}' + \frac{1}{T}\sum_{\tau=1}^{l}\bar{\omega}_{\tau l}\sum_{t=\tau+1}^{T}(\hat{\hat{w}}_{it}\hat{\hat{w}}_{it-\tau}' + \hat{\hat{w}}_{it-\tau}\hat{\hat{w}}_{it}')\right) \tag{5.76}$$

where $\bar{\omega}_{\tau l}$ is some weight function or kernel. Popular choices are the Bartlett or Parzen kernels. By Phillips and Durlauf (1986) and a sequential limit theory $\hat{\hat{\Omega}}$ and $\hat{\hat{\Phi}}$ can be shown to be consistent estimators of $\bar{\Omega}$ and $\bar{\Phi}$. Phillips and Moon (1999) p.1084 also give detailed conditions for consistent estimates of the panel analogues of the time-series long run covariance matrices. Using a Parzen lag window Phillips and Moon use averages (as in $\hat{\hat{\Omega}}$ above) of the usual nonparametric, and consistent as $T \to \infty$, kernel estimates for each i. These methods are reproduced in Kauppi (2000) who uses the kernel estimation strategy of the panel pooled fully modified (PFM) estimator of

Phillips and Moon (1999).

Remark 5.4.11 *Comparing the DOLS and FMOLS long run covariance matrices we can see*

(5.77) $$\Omega_i = \begin{bmatrix} \Omega_{i11} & \underline{0}' \\ \underline{0} & \Omega_{i22} \end{bmatrix} = \begin{bmatrix} \Lambda_{i11}^2 & \underline{0}' \\ \underline{0} & \Lambda_{i22}\Lambda_{i22}' \end{bmatrix}$$

(5.78) $$= \begin{bmatrix} [P_{i11}\Psi_{11}(1)]^2 & \underline{0}' \\ \underline{0} & \Psi_{22}(1)P_{i22}P_{i22}'\Psi_{22}'(1) \end{bmatrix} \text{ for DOLS.}$$

(5.79) $$L_i'\bar{\Omega}_i L_i = \begin{bmatrix} \bar{\Omega}_{i11}^+ & \underline{0}' \\ \underline{0} & \bar{\Omega}_{i22} \end{bmatrix} \text{ for FMOLS.}$$

Now we still have $\bar{W}_i(r)$ defined as $\bar{W}_i(r) = \left(\bar{W}_{i1}(r), \bar{W}_{i2}'(r)\right)'$ where $\bar{W}_{i1}(r)$ and $\bar{W}_{i2}(r)$ are correlated since $\bar{W}_i(r)$ is standard Brownian motion with covariance matrix $\bar{\Omega}_i$ where the off-diagonal elements are nonzero as in equation (5.55). So we now define $\bar{W}_i^+(r) = BM_i(L_i'\bar{\Omega}_i L_i)$ where $\bar{W}_i^+(r)$ is standard N^*-dimensional Brownian Motion with covariance matrix

(5.80) $$L_i'\bar{\Omega}_i L_i = \begin{bmatrix} \bar{\Omega}_{i11}^+ & \underline{0}' \\ \underline{0} & \bar{\Omega}_{i22} \end{bmatrix} = \begin{bmatrix} \bar{\Omega}_{i11}^+ & \underline{0}' \\ \underline{0} & \bar{\Omega}_{i22}^+ \end{bmatrix} = \begin{bmatrix} \bar{\Lambda}_{i11}^{+2} & \underline{0}' \\ \underline{0} & \bar{\Lambda}_{i22}^+\bar{\Lambda}_{i22}'^+ \end{bmatrix}.$$

Partitioning $BM_i(L_i'\bar{\Omega}_i L_i)$ conformably with $\bar{w}_{it}$ then

$$BM_i(L_i'\bar{\Omega}_i L_i) = \begin{bmatrix} BM_i(L_i'\bar{\Omega}_i L_i) \\ BM_i(L_i'\bar{\Omega}_i L_i) \end{bmatrix} = \begin{bmatrix} \bar{\Lambda}_{i11}^{+2} & \underline{0}' \\ \underline{0} & \bar{\Lambda}_{i22}^+\bar{\Lambda}_{i22}'^+ \end{bmatrix} = \begin{bmatrix} \bar{W}_{i1}^+(r) \\ \bar{W}_{i2}^+(r) \end{bmatrix}.$$
(5.81)

Where $\bar{W}_i^+(r) = \left(\bar{W}_{i1}^+(r), \bar{W}_{i2}^{+\prime}(r)\right)'$ where $\bar{W}_{i1}^+(r)$ and $\bar{W}_{i2}^+(r)$ are independent. Note we cannot set $\bar{\Omega}_{i11}^+ = \Omega_{i11}$ except in special circumstances although it is easy to see that $\Omega_{i22} = \bar{\Omega}_{i22} = \bar{\Omega}_{i22}^+$

The feasible FMOLS panel estimator

On substitution of our estimators $\hat{\bar{\Omega}}_i$ and $\hat{\bar{\Gamma}}_i$ for $\bar{\Omega}_i$ and $\bar{\Gamma}_i$, etc we have the estimating regression

$$(5.82) \qquad y_{it} - \hat{\bar{\Omega}}_{i12}\hat{\bar{\Omega}}_{i22}^{-1}\varepsilon_{it} = \alpha + x_{it}^{/}\beta + u_{it} - \hat{\bar{\Omega}}_{i12}\hat{\bar{\Omega}}_{i22}^{-1}\varepsilon_{it} \qquad (1^{ii})$$

$$(5.83) \qquad x_{it} = x_{it-1} + \varepsilon_{it}. \qquad (2^{ii})$$

Writing the above in matrix form we get

$$(5.84) \qquad \hat{y}_{it}^{+} = X_{it}^{/}\Upsilon + \hat{u}_{it}^{+}$$

where $X_{it} = [1 \quad x_{it}^{/}]^{/}$ and $\Upsilon = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$.

Hence we have the FMOLS estimator of Υ as

$$(5.85) \qquad \hat{\Upsilon} = \begin{bmatrix} \hat{\alpha}_{FMOLS1} \\ \hat{\beta}_{FMOLS1} \end{bmatrix} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{/}\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T} X_{it}\hat{y}_{it}^{+}\right)$$

$$(5.86) \qquad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 \\ x_{it} \end{bmatrix}[1 \quad x_{it}^{/}]\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{y}_{it}^{+} \\ x_{it}\hat{y}_{it}^{+} - T\hat{\aleph}^{+} \end{bmatrix}\right)$$

$$(5.87) \qquad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{/} \\ x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{y}_{it}^{+} \\ x_{it}\hat{y}_{it}^{+} - T\hat{\aleph}^{+} \end{bmatrix}\right).$$

The following Lemma simplifies the FMOLS computations for case (1).

Lemma 5.4.12 *By the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3)*

$$(5.88) \qquad \frac{1}{T^2}\sum_{t=1}^{T} x_{it}x_{it}^{/} \Rightarrow \bar{\Lambda}_{i22}^{+}\left\{\int_0^1 [\bar{W}_{i2}^{+}(r)][\bar{W}_{i2}^{+}(r)]^{/}dr\right\}\bar{\Lambda}_{i22}^{+/} \qquad (a)$$

$$(5.89)\quad \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} x_{it} \Rightarrow \bar{\Lambda}^{+}_{i22}\int_0^1 [\bar{W}^{+}_{i2}(r)]dr \qquad (b)$$

$$(5.90)\quad \frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} \hat{u}^{+}_{it} \Rightarrow \bar{\Lambda}^{+}_{i11}\bar{W}^{+}_{i1}(1) \qquad (c)$$

$$(5.91)\quad \frac{1}{T}\sum_{t=1}^{T} x_{it}\hat{u}^{+}_{it} - \hat{\aleph}^{+} \Rightarrow \bar{\Lambda}^{+}_{i22}\left\{\int_0^1 [\bar{W}^{+}_{i2}(r)][d\bar{W}^{+}_{i1}(r)]\right\}\bar{\Lambda}^{+}_{i11} \qquad (d)$$

Theorem 5.4.13 *Suppose Proposition (5.3.1) and Assumptions (5.4.7) and (5.4.9) hold and the data are generated by* (1^{ii}) *and* (2^{ii}). *Then by sequential limit probability theory*

(i) *As* $(N, T \to \infty)_{seq}$ $\quad \hat{\beta}_{FMOLS1} \xrightarrow{p} \beta$

(ii) *As* $(N, T \to \infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{FMOLS1} - \beta\right) \Rightarrow N\left(\underline{0}, 2\Omega_{22}^{-1}\bar{\Omega}^{+}_{11}\right).$$

The OLS panel estimator

Consider again the model of (1) and (2) and the conditions of Assumptions (5.4.7) and (5.4.9). We can write for the untransformed model $\bar{\Omega}_i = \bar{\Psi}(1)\Sigma_i\bar{\Psi}'(1)$ as before. Using this framework for the OLS case write $\bar{\Psi}(1)\Sigma_i\bar{\Psi}'(1) = \Psi^*(1)\Sigma_i\Psi^{*\prime}(1)$. So now $\Lambda^*_i = \Psi^*(1)P_i$, $\Lambda^*_i = [\Lambda^*_{i11}, \Lambda^{*\prime}_{i22}]'$ and $\Lambda^*_i\Lambda^{*\prime}_i = \bar{\Omega}_i = \Omega^*_i$. Note that in this case $\Lambda^*_i\Lambda^{*\prime}_i$ is not block diagonal

$$(5.92)\qquad = \begin{bmatrix} [P_{i11}\Psi^*_{11}(1)]^2 & \Psi^*_{11}(1)P_{i11}P^{\prime}_{i22}\Psi^{*\prime}_{22}(1) \\ \Psi^*_{22}(1)P_{i22}P_{i11}\Psi^*_{11}(1) & \Psi^*_{22}(1)P_{i22}P^{\prime}_{i22}\Psi^{*\prime}_{22}(1) \end{bmatrix}$$

$$(5.93)\qquad = \begin{bmatrix} \Omega^*_{i11} & \Omega^*_{i12} \\ \Omega^*_{i21} & \Omega^*_{i22} \end{bmatrix}.$$

We can write the panel OLS estimator then as follows based on OLS estimation of (1) and (2) and the above assumptions. Writing (1) in matrix form we get

$$y_{it} = X_{it}^{/}\Upsilon + u_{it} \tag{5.94}$$

where $X_{it} = [1 \quad x_{it}^{/}]^{/}$ and $\Upsilon = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$.

Hence we have the OLS estimator of Υ as

$$\hat{\Upsilon} = \begin{bmatrix} \hat{\alpha}_{OLS1} \\ \hat{\beta}_{OLS1} \end{bmatrix} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{/}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}y_{it} \tag{5.95}$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 \\ x_{it} \end{bmatrix}[1 \quad x_{it}^{/}]\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \end{bmatrix}\right) \tag{5.96}$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{/} \\ x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \end{bmatrix}\right). \tag{5.97}$$

Theorem 5.4.14 *(Inconsistency) Suppose Proposition (5.3.1) and Assumptions (5.4.7) and (5.4.9) hold and the data are generated by* (1) *and* (2). *Then by sequential limit probability theory*

(i) As $(N, T \to \infty)_{seq}$ *OLS is inconsistent* $\hat{\beta}_{OLS1} \overset{p}{\nrightarrow} \beta$

(ii) As $(N, T \to \infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{OLS1} - \beta\right) \Rightarrow N\left(2\Gamma_{21}^{*}\Omega_{22}^{-1}, 2\Omega_{22}^{-1}\Omega_{21}^{*}\Omega_{21}^{*/}\Omega_{22}^{-1/}\right).$$

5.4.2 Case (2) The Model with a Constant and a Trend

Consider the following homogeneous panel data model with a constant intercept and trend

(5.98) $$y_{it} = \alpha + \delta t + x_{it}^{\prime}\beta + u_{it} \qquad (1^{iii})$$

(5.99) $$x_{it} = x_{it-1} + \varepsilon_{it}. \qquad (2^{iii})$$

This again is the panel data version of Phillips triangular form (Phillips (1991)) now with a constant intercept and deterministic trend added. The other parameters and variables are the same as before.

The DOLS panel estimator

The model develops exactly as before so we shall use the same notation as before with alterations explained where necessary. The estimating regression now becomes

(5.100) $$y_{it} = \alpha + \delta t + x_{it}^{\prime}\beta + \sum_{j=-p}^{p} c_{ij}\Delta x_{it-j} + z_{it} \qquad (1^{iv})$$

(5.101) $$x_{it} = x_{it-1} + \varepsilon_{it}. \qquad (2^{iv})$$

Writing the above in matrix form we get

(5.102) $$y_{it} = X_{it}^{\prime}\Upsilon + z_{it}$$

where $X_{it} = [m_{it}^{/} \quad 1 \quad x_{it}^{/} \quad t]^{/}$ and $\Upsilon = \begin{bmatrix} \xi \\ \alpha \\ \beta \\ \delta \end{bmatrix}$, with $m_{it}^{/}$ and ξ as before.

Hence the DOLS estimator of Υ is

$$(5.103) \qquad \hat{\Upsilon} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{/}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}y_{it}$$

$$(5.104) \qquad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it} \\ 1 \\ x_{it} \\ t \end{bmatrix} [m_{it}^{/} \quad 1 \quad x_{it}^{/} \quad t]\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right)$$

$$(5.105) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}^{/} & m_{it} & m_{it}x_{it}^{/} & m_{it}t \\ m_{it}^{/} & 1 & x_{it}^{/} & t \\ x_{it}m_{it}^{/} & x_{it} & x_{it}x_{it}^{/} & x_{it}t \\ tm_{it}^{/} & t & tx_{it}^{/} & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right).$$

The following Lemma will simplify the case (2) DOLS computations.

Lemma 5.4.15 *By the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3)*

$$(5.106) \qquad \frac{1}{T^2}\sum_{t=1}^{T} t \to \frac{1}{2} \qquad (a)$$

$$(5.107) \qquad \frac{1}{T^3}\sum_{t=1}^{T} t^2 \to \frac{1}{3} \qquad (b)$$

$$(5.108) \qquad \frac{1}{T^{v+1}}\sum_{t=1}^{T} t^v \to \frac{1}{(v+1)} \qquad (c)$$

$$(5.109) \qquad \frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} tx_{it}^{/} \Rightarrow \int_0^1 r[W_{i2}(r)]^{/} dr \Lambda_{i22}^{/} \qquad (d)$$

$$(5.110) \qquad \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} t\varepsilon_{it} \Rightarrow \Lambda_{i22}W_{i2}(1) - \Lambda_{i22}\int_0^1 [W_{i2}(r)]dr \qquad (e)$$

$$(5.111) \qquad \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tz_{it} \Rightarrow \Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 [W_{i1}(r)]dr \qquad (f)$$

Theorem 5.4.16 *Suppose Proposition (5.3.1) and Assumptions (5.4.2) and (5.4.4) hold and the data are generated by* (1^{iv}) *and* (2^{iv}). *Then by sequential limit probability theory*

(i) As $(N,T \to \infty)_{seq}$ $\quad \hat{\beta}_{DOLS2} \xrightarrow{p} \beta$

(ii) As $(N,T \to \infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{DOLS2} - \beta\right) \Rightarrow N\left(\underline{0}, 2\Omega_{22}^{-1}\Omega_{11}\right).$$

The FMOLS panel estimator

Again the model develops exactly as before so we shall use the same notation as before with alterations explained where necessary. Now our feasible FMOLS estimator can be obtained from the estimating regression

$$(5.112) \qquad y_{it} - \hat{\hat{\Omega}}_{i12}\hat{\hat{\Omega}}_{i22}^{-1}\varepsilon_{it} = \alpha + \delta t + x_{it}^{\prime}\beta + u_{it} - \hat{\hat{\Omega}}_{i12}\hat{\hat{\Omega}}_{i22}^{-1}\varepsilon_{it} \quad (1^{v})$$

$$(5.113) \qquad x_{it} = x_{it-1} + \varepsilon_{it}. \qquad (2^{v})$$

Writing the above in matrix form we get

$$(5.114) \qquad \hat{y}_{it}^{+} = X_{it}^{\prime}\Upsilon + \hat{u}_{it}^{+}$$

where $X_{it} = [1 \quad x_{it}^{\prime} \quad t]^{\prime}$ and $\Upsilon = \begin{bmatrix} \alpha \\ \beta \\ \delta \end{bmatrix}$.

Hence we have the FMOLS estimator of Υ as

$$(5.115) \qquad \hat{\Upsilon} = \begin{bmatrix} \hat{\alpha}_{FMOLS2} \\ \hat{\beta}_{FMOLS2} \\ \hat{\delta}_{FMOLS2} \end{bmatrix} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{\prime}\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T} X_{it}\hat{y}_{it}^{+}\right)$$

$$(5.116)\quad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix}1\\ x_{it}\\ t\end{bmatrix}\begin{bmatrix}1 & x_{it}^{/} & t\end{bmatrix}\right)^{-1}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix}\hat{y}_{it}^{+}\\ x_{it}\hat{y}_{it}^{+}-T\hat{\aleph}^{+}\\ t\hat{y}_{it}^{+}\end{bmatrix}\right)$$

$$(5.117)\quad = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix}1 & x_{it}^{/} & t\\ x_{it} & x_{it}x_{it}^{/} & x_{it}t\\ t & tx_{it}^{/} & t^2\end{bmatrix}\right)^{-1}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix}\hat{y}_{it}^{+}\\ x_{it}\hat{y}_{it}^{+}-T\hat{\aleph}^{+}\\ t\hat{y}_{it}^{+}\end{bmatrix}\right).$$

The following Lemma simplifies the FMOLS computations for case (2).

Lemma 5.4.17 *By the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3)*

$$(5.118)\quad \frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} tx_{it} \Rightarrow \bar{\Lambda}_{i22}^{+}\int_0^1 r[\bar{W}_{i2}^{+}(r)]dr \qquad (a)$$

$$(5.119)\quad \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} t\hat{u}_{it}^{+} \Rightarrow \bar{\Lambda}_{i11}^{+}\bar{W}_{i1}^{+}(1) - \bar{\Lambda}_{i11}^{+}\int_0^1 \bar{W}_{i1}^{+}(r)dr \qquad (b)$$

Theorem 5.4.18 *Suppose Proposition (5.3.1) and Assumptions (5.4.7) and (5.4.9) hold and the data are generated by* (1^v) *and* (2^v)*. Then by sequential limit probability theory*

(i) As $(N,T\to\infty)_{seq}$ $\quad\hat{\beta}_{FMOLS2}\xrightarrow{p}\beta$

(ii) As $(N,T\to\infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{FMOLS2}-\beta\right)\Rightarrow N\left(\underline{0},2\Omega_{22}^{-1}\bar{\Omega}_{11}^{+}\right).$$

The OLS panel estimator

Again the OLS case for the model of (1^{iii}) and (2^{iii}) and the conditions of Assumptions (5.4.7) and (5.4.9) are as follows. Let. $\bar{\Omega}_i = \Lambda_i^{*}\Lambda_i^{*/}$

$$(5.120)\quad = \begin{bmatrix}[P_{i11}\Psi_{11}^{*}(1)]^2 & \Psi_{11}^{*}(1)P_{i11}P_{i22}^{/}\Psi_{22}^{*/}(1)\\ \Psi_{22}^{*}(1)P_{i22}P_{i11}\Psi_{11}^{*}(1) & \Psi_{22}^{*}(1)P_{i22}P_{i22}^{/}\Psi_{22}^{*/}(1)\end{bmatrix}.$$

Writing (1^{iii}) in matrix form as before we get

$$y_{it} = X_{it}^{\prime}\Upsilon + u_{it} \tag{5.121}$$

where $X_{it} = [1 \quad x_{it}^{\prime} \quad t]^{\prime}$ and $\Upsilon = \begin{bmatrix} \alpha \\ \beta \\ \delta \end{bmatrix}$.

Hence we have the OLS estimator of Υ as

$$\hat{\Upsilon} = \begin{bmatrix} \hat{\alpha}_{OLS2} \\ \hat{\beta}_{OLS2} \\ \hat{\delta}_{OLS2} \end{bmatrix} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}X_{it}^{\prime}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T} X_{it}y_{it} \tag{5.122}$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 \\ x_{it} \\ t \end{bmatrix}[1 \quad x_{it}^{\prime} \quad t]\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right) \tag{5.123}$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{\prime} & t \\ x_{it} & x_{it}x_{it}^{\prime} & x_{it}t \\ t & tx_{it}^{\prime} & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right). \tag{5.124}$$

Theorem 5.4.19 *(Inconsistency) Suppose Proposition (5.3.1) and Assumptions (5.4.7) and (5.4.9) hold and the data are generated by* (1^{iii}) *and* (2^{iii}). *Then by sequential limit probability theory*

(i) As $(N, T \to \infty)_{seq}$ *OLS is inconsistent* $\hat{\beta}_{OLS2} \overset{p}{\nrightarrow} \beta$

(ii) As $(N, T \to \infty)_{seq}$

$$\sqrt{N}T\left(\hat{\beta}_{OLS2} - \beta\right) \Rightarrow N\left(2\Gamma_{21}^{*}\Omega_{22}^{-1}, 2\Omega_{22}^{-1}\Omega_{21}^{*}\Omega_{21}^{*\prime}\Omega_{22}^{-1\prime}\right).$$

5.5 Hypothesis Testing

We can use either of the panel FMOLS or DOLS estimators to give an example of how many hypotheses of interest can be tested in the homogeneous panel framework. Assume a set of linear restrictions given by the following linear combination of parameters

$$R\beta = r \qquad (5.125)$$

where R is $(q \times k)$, β is $(k \times 1)$ and r is $(q \times 1)$.

For example $R = [1, 1, \ldots, 1]$ and $r = 1$ gives $[1, 1, \ldots, 1] \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix} = 1$ and so $\beta_1 + \beta_2 + \ldots + \beta_k = 1$. This is the hypothesis that the elements of β sum to unity. As a preliminary note the following.

Lemma 5.5.1 *Let* $V_n^{-\frac{1}{2}} b_n \sim N(\underline{0}, I_k)$ *as* $T \to \infty$.
Then $b_n' V_n^{-1} b_n = b_n' V_n^{-\frac{1}{2}} V_n^{-\frac{1}{2}} b_n \sim \chi_k^2$.

Typically V_n will be unknown, but there will be a consistent estimator $\hat{V}_n$ such that $\hat{V}_n \xrightarrow{p} V_n$ or $\hat{V}_n - V_n \xrightarrow{p} \underline{0}$ a $(k \times k)$ null matrix.

Proposition 5.5.2 *Let* $V_n^{-\frac{1}{2}} b_n \sim N(\underline{0}, I_k)$ *as* $T \to \infty$ *and suppose there exists* V_n *positive semi-definite and symmetric such that* $\hat{V}_n - V_n \xrightarrow{p} \underline{0}$, *where* V_n *is O(1) and for all n sufficiently large, det*[16] $V_n > \delta > 0$. *Then* $b_n' \hat{V}_n^{-1} b_n \sim \chi_k^2$.

Now consider the main theorem of this section.

[16]det Q means determinant of Q here.

Theorem 5.5.3 *(Wald's Test) Let the conditions of Theorem (5.4.6) hold and let rank $R = q \leq k$. Then under $H_0 : R\beta = r$*

$$\Phi_n^{-\frac{1}{2}}\sqrt{N}T(R\hat{\beta}_{DOLS1} - r) \Rightarrow N\left(\underline{0}, I_k\right) \quad as \quad (N, T \to \infty)_{seq} \qquad (a) \tag{5.126}$$

where $\Phi_n = 2R\Omega_{22}^{-1}\Omega_{11}R'$.

The Wald Statistic

$$(W) = NT^2(R\hat{\beta}_{DOLS1} - r)\Phi_n^{-1}(R\hat{\beta}_{DOLS1} - r) \Rightarrow \chi_k^2 \quad as \quad (N, T \to \infty)_{seq}. \qquad (b)$$

(5.127)

APPENDIX 1

5.6 PROOFS TO THEOREMS

Proposition (5.3.1)

Proof

See Phillips and Durlauf (1986), Corollary 2.2.

Lemma (5.3.3)

Proof

See Billingsley (1968), Corollary 1, p.31.

Theorem (5.3.4)

Proof

See Rao (1973), p.115.

Theorem (5.3.5)

Proof

Let $Z_i = (Z_{i1}, \ldots, Z_{ik})'$ and $E(Z_i) = \mu = (\mu_1, \ldots, \mu_k)' < \infty, \forall i$. Consider now the real valued vector $\lambda = (\lambda_1, \ldots, \lambda_k)'$. Now write $Z_i'\lambda = (Z_{i1}\lambda_1 + \ldots + Z_{ik}\lambda_k)'$ and $E(Z_i'\lambda) = \mu'\lambda < \infty, \forall i$. Then by Komolgorov's Theorem (5.3.4) (see also Rao (1973), p.123, (xi)), we have

(5.128) $$\frac{1}{N}\sum_{i=1}^{N} Z_i'\lambda \overset{a.s.}{\rightarrow} \mu'\lambda \qquad \text{that is}$$

(5.129) $$\frac{1}{N}\sum_{i=1}^{N}(Z_{i1}\lambda_1 + \ldots + Z_{ik}\lambda_k)' \overset{a.s.}{\rightarrow} (\mu_1\lambda_1 + \ldots + \mu_k\lambda_k)'.$$

Given the above then by Proposition (5.3.10) we conclude

$$\frac{1}{N}\sum_{i=1}^{N} Z_i \xrightarrow{a.s.} \mu. \tag{5.130}$$

QED

Theorem (5.3.6)

Proof

See White (1984), p.108.

Theorem (5.3.7)

Proof

Let $Z_i = (Z_{i1}, \ldots, Z_{ik})'$ and $E(Z_i) = \mu = (\mu_1, \ldots, \mu_k)' < \infty, \forall i$ and $var(Z_i) = \Sigma < \infty$. Consider now the real valued vector $\lambda = (\lambda_1, \ldots, \lambda_k)'$. Now write $Z_i'\lambda = (Z_{i1}\lambda_1 + \ldots + Z_{ik}\lambda_k)'$ then $E(Z_i'\lambda) = \mu'\lambda < \infty, \forall i$ and $var(Z_i'\lambda) = \lambda'\Sigma\lambda < \infty, \forall i$. Then by the Lindeberg-Levy Theorem (5.3.6) (see also Rao (1973), p.123, (xi)), we have

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(Z_i'\lambda - \mu'\lambda) \xrightarrow{d} N(0, \lambda'\Sigma\lambda). \tag{5.131}$$

Given the above then by Proposition (5.3.10) we conclude

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(Z_i' - \mu') \xrightarrow{d} N(\underline{0}, \Sigma). \tag{5.132}$$

QED

Proposition (5.3.9)

Proof

See Rao (1973), p.122.

Proposition (5.3.10)

Proof

See Rao (1973), p.123.

Proposition (5.3.11)

Proof

See Goldberger (1964), p.27.

Lemma (5.4.5)

Proof

Consider the N^*-dimensional i.i.d vector process $\{v_{it}\}_{t=1}^{\infty}$, where $E(v_{it}) = \underline{0}$ and $E(v_{it}v_{it}^{/}) = I_{N^*}$. Write the vector partial sum process $\bar{G}_T(r)$ as

$$\bar{G}_T(r) = \frac{1}{T}(v_{i1} + v_{i2} + \ldots + v_{i[Tr]}).$$

Then by the multivariate FCLT and CMT as $T \to \infty$ we have

(5.134) $$\sqrt{T}\bar{G}_T(.) \Rightarrow W_i(.)$$

(5.135) $$\text{and} \quad \sqrt{T}\bar{G}_T(r) \Rightarrow W_i(r).$$

Here we assume $W_i(r)$ is an N^*-dimensional Wiener process with covariance matrix I_{N^*}. Now consider Assumption (5.4.2) where $w_{it} = \begin{bmatrix} z_{it} \\ \varepsilon_{it} \end{bmatrix}$. Here we could write

$$x_{it} = \varepsilon_{i1} + \varepsilon_{i2} + \ldots + \varepsilon_{it} \tag{5.136}$$

$$\text{and say} \quad \tilde{z}_{it} = z_{i1} + z_{i2} + \ldots + z_{it} \tag{5.137}$$

with both x_{i0} and z_{i0} equal to zero. Now let

$$\xi^*_{it} = \sum_{t=1}^{T} \begin{bmatrix} z_{it} \\ \varepsilon_{it} \end{bmatrix}. \tag{5.138}$$

Then we could write

$$\xi^*_{it} = w_{i1} + w_{i2} + \ldots + w_{it}. \tag{5.139}$$

Given $w_{it} = \Psi(L)\epsilon_{it}$ then let $\epsilon_{it} = P_i v_{it}$ so that $E(P_i v_{it} v_{it}' P_i') = P_i I_N P_i' = \Sigma_i$ again. Now write the partial sum process $G_T(r)$ as

$$G_T(r) = \frac{1}{T} \sum_{t=1}^{[Tr]} w_{it}. \tag{5.140}$$

Then

$$G_T(r) = \Psi(1) \frac{1}{T} \sum_{t=1}^{[Tr]} \epsilon_{it} = \Psi(1) P_i \bar{G}_T(r) \tag{5.141}$$

$$= \Psi(1) P_i \frac{1}{T} (v_{i1} + v_{i2} + \ldots + v_{i[Tr]}) \quad \text{so} \tag{5.142}$$

$$\sqrt{T} G_T(r) = \Psi(1) P_i \sqrt{T} \bar{G}_T(r). \tag{5.143}$$

But we know $\sqrt{T}\bar{G}_T(r) \Rightarrow W_i(r)$ so that

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} w_{it} \Rightarrow \Psi(1) P_i W_i(r) = \Lambda_i W_i(r) \quad \text{as} \quad T \to \infty. \tag{5.144}$$

Now consider

(a) Given the partial sum process $\sqrt{T}G_T(r) = \frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it}$ write the new function $S_T(r) = [\sqrt{T}G_T(r)][\sqrt{T}G_T(r)^{/}]$. But we know $\sqrt{T}G_T(r) \Rightarrow \Lambda_i W_i(r)$ as $T \to \infty$ so that $S_T(r) \Rightarrow [\Lambda_i W_i(r)][W_i^{/}(r)\Lambda_i^{/}]$ as $T \to \infty$. If

$$\xi_{it-1}^{*} = w_{i1} + w_{i2} + \ldots + w_{it-1} \tag{5.145}$$

then by the step function method

$$\frac{1}{T^2}\sum_{t=1}^{T}\xi_{it-1}^{*}\xi_{it-1}^{*/} = \frac{\xi_{i1}^{*}\xi_{i1}^{*/}}{T^2} + \frac{\xi_{i2}^{*}\xi_{i2}^{*/}}{T^2} + \ldots + \frac{\xi_{iT-1}^{*}\xi_{iT-1}^{*/}}{T^2}. \tag{5.146}$$

But this is just the integral of $S_T(r)$ using the step function method

$$\int_0^1 S_T(r)dr = \frac{1}{T^2}\sum_{t=1}^{T}\xi_{it-1}^{*}\xi_{it-1}^{*/}. \tag{5.147}$$

It follows then given $S_T(r) \Rightarrow [\Lambda_i W_i(r)][W_i^{/}(r)\Lambda_i^{/}]$ that

$$\frac{1}{T^2}\sum_{t=1}^{T}\xi_{it-1}^{*}\xi_{it-1}^{*/} \Rightarrow \Lambda_i\left\{\int_0^1 [W_i(r)][W_i(r)]^{/}dr\right\}\Lambda_i^{/} \quad \text{as} \quad T \to \infty. \tag{5.148}$$

This also implies

$$\frac{1}{T^2}\sum_{t=1}^{T}\xi_{it}^{*}\xi_{it}^{*/} \Rightarrow \Lambda_i\left\{\int_0^1 [W_i(r)][W_i(r)]^{/}dr\right\}\Lambda_i^{/} \quad \text{as} \quad T \to \infty. \tag{5.149}$$

Now for $\frac{1}{T^2}\sum_{t=1}^{T} x_{it}x_{it}^{/}$ we have

$$\frac{1}{T^2}\sum_{t=1}^{T} x_{it}x_{it}^{/} = [0 \quad I_k]\left[\frac{1}{T^2}\sum_{t=1}^{T}\xi_{it}^{*}\xi_{it}^{*/}\right]\begin{bmatrix} 0 \\ I_k \end{bmatrix} \Rightarrow \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]^{/}dr\right\}\Lambda_{i22}^{/} \quad T \to \infty. \tag{5.150}$$

QED.

(b) Given the partial sum process $\sqrt{T}G_T(r) = \frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it}$ and that

$$\xi^*_{it-1} = w_{i1} + w_{i2} + \ldots + w_{it-1} \tag{5.151}$$

then by the step function method

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\xi^*_{it-1} = \frac{\xi^*_{i1}}{T^{\frac{3}{2}}} + \frac{\xi^*_{i2}}{T^{\frac{3}{2}}} + \ldots + \frac{\xi^*_{iT-1}}{T^{\frac{3}{2}}}. \tag{5.152}$$

But this is just the integral of $\sqrt{T}G_T(r)$ again using the step function method

$$\int_0^1 \sqrt{T}G_T(r)dr = \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\xi^*_{it-1}. \tag{5.153}$$

But we know $\sqrt{T}G_T(r) \Rightarrow \Lambda_i W_i(r)$ hence

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\xi^*_{it-1} \Rightarrow \Lambda_i \int_0^1 W_i(r)dr \tag{5.154}$$

which implies

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\xi^*_{it} \Rightarrow \Lambda_i \int_0^1 W_i(r)dr. \tag{5.155}$$

Now for $\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} x_{it}$ we have

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} x_{it} = [0 \quad I_k]\left[\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\xi^*_{it}\right] \Rightarrow \Lambda_{i22}\int_0^1 W_{i2}(r)dr \quad \text{as} \quad T \to \infty. \tag{5.156}$$

QED

(c) Given $w_{it} = \Psi(L)\epsilon_{it}$ then let $\sqrt{T}G_T(r) = \frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it}$ but we know $\sqrt{T}G_T(r) = \Psi(1)P_i\sqrt{T}\bar{G}_T(r)$ and $\sqrt{T}\bar{G}_T(r) \Rightarrow W_i(r)$ so that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it} \Rightarrow \Lambda_i W_i(r) \quad \text{as} \quad T \to \infty. \tag{5.157}$$

Now evaluated at $r = 1$ the function gives us the result

(5.158) $$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it} \Rightarrow \Lambda_i W_i(1) \quad \text{as} \quad T \to \infty.$$

Now for $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_{it}$ we have

(5.159) $$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} z_{it} = [1 \quad \underline{0}]\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it}\right] \Rightarrow \Lambda_{i11} W_{i1}(1) \quad \text{as} \quad T \to \infty.$$

QED

(d) See Phillips (1988).

(e) By (d) we have

$$\frac{1}{T}\sum_{t=1}^{T} \xi_{it-1}^{*} w_{it}' \Rightarrow \Lambda_i \left\{\int_0^1 [W_i(r)][dW_i(r)]'\right\} \Lambda_i' + \sum_{v=1}^{\infty} E(w_{it} w_{it-v}') \quad \text{as} \quad T \to \infty.$$

(5.160)

This also holds for $\frac{1}{T}\sum_{t=1}^{T} \xi_{it}^{*} w_{it}'$. But we know that z_{it} is uncorrelated with $\varepsilon_{it-s}, \forall, s, t$. So for $\frac{1}{T}\sum_{t=1}^{T} x_{it} z_{it}$ we have

(5.161) $$\frac{1}{T}\sum_{t=1}^{T} x_{it} z_{it} = [0 \quad I_k]\frac{1}{T}\sum_{t=1}^{T} \xi_{it-1}^{*} w_{it}' \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

(5.162) $$\Rightarrow [0 \quad I_k]\Lambda_i \left\{\int_0^1 [W_i(r)][dW_i(r)]'\right\} \Lambda_i' \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

(5.163) $$+[0 \quad I_k]\sum_{v=1}^{\infty} E(w_{it} w_{it-v}') \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}.$$

The last term can be written

$$[0 \quad I_k]\sum_{v=1}^{\infty} E(w_{it} w_{it-v}') \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} = [0 \quad I_k]\sum_{v=1}^{\infty} E\begin{bmatrix} z_{it} \\ \varepsilon_{it} \end{bmatrix}[z_{it-v}\varepsilon_{it-v}]\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

(5.164)

(5.165) $$= [0 \quad I_k] \sum_{v=1}^{\infty} E \begin{bmatrix} z_{it} z_{it-v} & z_{it} \varepsilon_{it-v} \\ \varepsilon_{it} z_{it-v} & \varepsilon_{it} \varepsilon_{it-v}' \end{bmatrix} \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} = \underline{0}$$

by assumption. So we have by the first term

$$[0 \quad I_k] \Lambda_i \left\{ \int_0^1 [W_i(r)][dW_i(r)]' \right\} \Lambda_i' \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} \Rightarrow \Lambda_{i22} \left\{ \int_0^1 [W_{i2}(r)][dW_{i1}(r)]' \right\} \Lambda_{i11}'.$$
(5.166)

which gives us the desired result.

QED

(f) Given $E(w_{it} w_{it-s}') = \sum_{v=0}^{\infty} \Psi_{s+v} \Sigma_i \Psi_v < \infty$ for s=0,1,2,... Then by a LLN

(5.167) $$\frac{1}{T} \sum_{t=1}^{T} w_{it} w_{it-s}' \xrightarrow{p} E(w_{it} w_{it-s}') \quad \text{as} \quad T \to \infty.$$

Now for $\frac{1}{T} \sum_{t=1}^{T} \varepsilon_{it} \varepsilon_{it-s}'$ we have

$$\frac{1}{T} \sum_{t=1}^{T} \varepsilon_{it} \varepsilon_{it-s}' = [0 \quad I_k] \left[\frac{1}{T^2} \sum_{t=1}^{T} w_{it} w_{it-s}' \right] \begin{bmatrix} 0 \\ I_k \end{bmatrix} \xrightarrow{p} E(\Delta x_{it})(\Delta x_{it-s}') = \Gamma_{is} \quad \text{as} \quad T \to \infty.$$
(5.168)

See also Hamilton (1994) Proof of Proposition 10.2(d).

QED.

(g) Given $\mathrm{m}_{it} = \begin{bmatrix} \varepsilon_{it+p} \\ \varepsilon_{it+(p-1)} \\ \vdots \\ \varepsilon_{it} \\ \varepsilon_{it-1} \\ \varepsilon_{it-2} \\ \vdots \\ \varepsilon_{it-p} \end{bmatrix}$ a $((2p+1)k \times 1)$ vector.

Then by the LLN if $E(m_{it} m_{it}') < \infty$ if follows by (f) above that

(5.169) $$\frac{1}{T} \sum_{t=1}^{T} m_{it} m_{it}' \xrightarrow{p} V_i \quad \text{by LLN}$$

$$\text{where } V_i = E(m_{it}m_{it}') = \begin{bmatrix} \Gamma_{i0} & \Gamma_{i1} & \dots & \Gamma_{ip} & \Gamma_{i(p+1)} & \dots & \Gamma_{i2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Gamma_{i-p} & & & & & & \Gamma_{ip} \\ \Gamma_{i-(p+1)} & \vdots & \vdots & \vdots & \vdots & \vdots & \Gamma_{i(p+1)} \\ \Gamma_{i-(p+2)} & & & & & & \Gamma_{i(p+2)} \\ \Gamma_{i-2p} & \dots & \dots & \dots & \dots & \dots & \Gamma_{i0} \end{bmatrix}$$

a $((2p+1)k \times (2p+1)k)$ matrix.

QED

(h) Let

$$\frac{1}{T}\sum_{t=1}^{T} m_{it}z_{it} = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} \varepsilon_{it+p}z_{it} \\ \varepsilon_{it+(p-1)}z_{it} \\ \vdots \\ \varepsilon_{it}z_{it} \\ \varepsilon_{it-1}z_{it} \\ \varepsilon_{it-2}z_{it} \\ \vdots \\ \varepsilon_{it-p}z_{it} \end{bmatrix}. \tag{5.170}$$

By the LLN each element of the vector converges to zero as $T \to \infty$ so that

$$\frac{1}{T}\sum_{t=1}^{T} m_{it}z_{it} \xrightarrow{p} \underline{0} \tag{5.171}$$

and also

$$\frac{1}{T}\sum_{t=1}^{T} m_{it}m_{it}' \xrightarrow{p} V_i \quad \text{by the LLN and (g).} \tag{5.172}$$

Hence given $E(z_{it}^2) = \Lambda_{i11}^2$ then from (g) and applying the CLT we get the result

$$\frac{1}{T}\sum_{t=1}^{T} m_{it}z_{it} \xrightarrow{d} N(\underline{0}, \Lambda_{11}^2 V_i) \tag{5.173}$$

as $T \to \infty$. See also Hamilton (1994) equation 11.A.3.

QED

(i) From (c) we have that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} w_{it} \Rightarrow \Lambda_i W_i(r) \quad \text{as} \quad T \to \infty. \tag{5.174}$$

Now evaluated at $r = 1$ the function gives us the result

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it} \Rightarrow \Lambda_i W_i(1) \quad \text{as} \quad T \to \infty. \tag{5.175}$$

Now for $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\varepsilon_{it}$ we have

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\varepsilon_{it} = [0 \quad I_k]\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it}\right] \Rightarrow \Lambda_{i22} W_{i2}(1) \quad \text{as} \quad T \to \infty. \tag{5.176}$$

QED

Theorem (5.4.6)

Proof

We have from equation (5.34) the panel DOLS estimator

$$\hat{\Upsilon}_{DOLS1} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' \\ m_{it}' & 1 & x_{it}' \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \end{bmatrix}\right). \tag{5.177}$$

Substitute for y_{it} to obtain

$$(\hat{\Upsilon}_{DOLS1} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' \\ m_{it}' & 1 & x_{it}' \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}z_{it} \\ z_{it} \\ x_{it}z_{it} \end{bmatrix}\right). \tag{5.178}$$

To obtain the limiting distribution we must rescale the estimator with a scaling matrix. This is done because as discussed in chapter 1 different parameters have different convergence rates. The parameter β, for example, is termed "superconsistent" since it converges to its limiting distribution across the time-series dimension at rate T rather than $\sqrt{T}$ as in the usual stationary case. Define the scaling matrix

$$D_T = \begin{bmatrix} \sqrt{NT} I_{(2p+1)k} & \underline{0} & \underline{0} \\ \underline{0}' & \sqrt{NT} & \underline{0}' \\ \underline{0}' & \underline{0} & \sqrt{NT} I_k \end{bmatrix}. \tag{5.179}$$

So that

$$D_T(\hat{\Upsilon}_{DOLS1} - \Upsilon) = \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} I_{(2p+1)k} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1} I_k \end{bmatrix} \tag{5.180}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' \\ m_{it}' & 1 & x_{it}' \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' \end{bmatrix}\right)^{-1} \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} I_{(2p+1)k} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1} I_k \end{bmatrix}$$

$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} I_{(2p+1)k} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1} I_k \end{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T}\begin{bmatrix} m_{it}z_{it} \\ z_{it} \\ x_{it}z_{it} \end{bmatrix}.$$

Hence[17]

$$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS1} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \tag{5.181}$$

$$= \begin{bmatrix} N^{-1}T^{-1}\Sigma\Sigma m_{it}m_{it}' & N^{-1}T^{-1}\Sigma\Sigma m_{it} & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma m_{it}x_{it}' \\ N^{-1}T^{-1}\Sigma\Sigma m_{it}' & 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}' \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}m_{it}' & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}' \end{bmatrix}^{-1}$$

[17]See below for the dimensions of the null vectors and null matrices.

$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma m_{it}z_{it} \\ N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma z_{it} \\ N^{-\frac{1}{2}}T^{-1}\Sigma\Sigma x_{it}z_{it} \end{bmatrix}.$$

Now apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above more conveniently as

$$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS1} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \tag{5.182}$$

$$= \left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} T^{-1}\sum_{t=1}^{T} m_{it}m_{it}^{\prime} & T^{-1}\sum_{t=1}^{T} m_{it} & T^{-\frac{3}{2}}\sum_{t=1}^{T} m_{it}x_{it}^{\prime} \\ T^{-1}\sum_{t=1}^{T} m_{it}^{\prime} & 1 & T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it}^{\prime} \\ T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it}m_{it}^{\prime} & T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it} & T^{-2}\sum_{t=1}^{T} x_{it}x_{it}^{\prime} \end{bmatrix} \right)^{-1}$$

$$\times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} T^{-\frac{1}{2}}\sum_{t=1}^{T} m_{it}z_{it} \\ T^{-\frac{1}{2}}\sum_{t=1}^{T} z_{it} \\ T^{-1}\sum_{t=1}^{T} x_{it}z_{it} \end{bmatrix} \right).$$

By Lemma (5.4.5) and the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3) as $T \to \infty$, and N is held fixed we have by applying the FCLT to each element of the (3×3) block matrix and (3×1) block vector the following. Let

$$\frac{1}{T}\sum_{t=1}^{T} m_{it} = \frac{1}{T}\sum_{t=1} \left(\varepsilon_{it-p}^{\prime}, \varepsilon_{it-p+1}^{\prime} \ldots, \varepsilon_{it-1}^{\prime}, \varepsilon_{it}^{\prime}, \varepsilon_{it+1}^{\prime}, \ldots, \varepsilon_{it+p}^{\prime}\right)^{\prime} \tag{5.183}$$

$$= \left(\frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it-p}^{\prime}, \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it-p+1}^{\prime} \ldots, \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it+p}^{\prime}\right)^{\prime} \tag{5.184}$$

a $((2p+1)k \times 1)$ vector. But by Lemma (5.4.5) (i)

$$\frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} \varepsilon_{it} \Rightarrow \Lambda_{i22}W_{i2}(1) \quad . \tag{5.185}$$

Therefore

$$\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it} \Rightarrow \underline{0} \tag{5.186}$$

a $(k \times 1)$ null vector and so

$$\frac{1}{T}\sum_{t=1}^{T} m_{it} = \left(\frac{1}{T}\sum_{t=1}^{T}\varepsilon'_{it-p}, \frac{1}{T}\sum_{t=1}^{T}\varepsilon'_{it-p+1}\dots, \frac{1}{T}\sum_{t=1}^{T}\varepsilon'_{it+p}\right)' \tag{5.187}$$

$$\Rightarrow (0', 0', \dots, 0')' \quad \forall p = -1, -2, \dots, 0, +1, +2, \dots \tag{5.188}$$

a $((2p+1)k \times 1)$ null vector. So

$$\frac{1}{T}\sum_{t=1}^{T} m_{it} \Rightarrow \underline{0} \tag{5.189}$$

a $((2p+1)k \times 1)$ null vector. Similarly let

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} m_{it}x'_{it} = \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\left(\varepsilon'_{it-p}, \varepsilon'_{it-p+1}\dots, \varepsilon'_{it-1}, \varepsilon'_{it}, \varepsilon'_{it+1}, \dots, \varepsilon'_{it+p}\right)' x'_{it} \tag{5.190}$$

$$= \left(\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it-p}x'_{it}, \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it-p+1}x'_{it}\dots, \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it+p}x'_{it}\right)'. \tag{5.191}$$

But by Lemma (5.4.5) (d)

$$\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it-p}x'_{it} \Rightarrow \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i2}(r)]'\right\}\Lambda'_{i22} + \sum_{v=1}^{\infty}\Gamma'_{iv}. \tag{5.192}$$

Hence

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it-p}x'_{it} \Rightarrow \underline{0} \quad \forall p = -1, -2, \dots, 0, +1, +2, \dots \tag{5.193}$$

a $(k \times k)$ null matrix and so

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} m_{it}x'_{it} = \left(\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it-p}x'_{it}, \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it-p+1}x'_{it}\dots, \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\varepsilon_{it+p}x'_{it}\right)' \tag{5.194}$$

(5.195) $$\Rightarrow (\underline{0}, \underline{0}, \ldots, \underline{0})' \quad \forall p = -1, -2, \ldots, 0, +1, +2, \ldots.$$

Here some $(2p+1)k$ $(k \times k)$ null matrices. So

(5.196) $$\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} m_{it} x_{it}' \Rightarrow \underline{0}$$

a $((2p+1)k \times k)$ null matrix. By the rest of Lemma (5.4.5) (a)-(i)

(5.197) $$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS1} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \Rightarrow$$

$$\left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} V_i & \underline{0} & \underline{0} \\ \underline{0}' & 1 & \int_0^1 W_{i2}'(r) dr \Lambda_{i22}' \\ \underline{0}' & \Lambda_{i22} \int_0^1 W_{i2}(r) dr & \Lambda_{i22} \{ \int_0^1 [W_{i2}(r)][W_{i2}(r)]' dr \} \Lambda_{i22}' \end{bmatrix} \right)^{-1}$$
$$\times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} \Xi_i \\ \Lambda_{i11} W_{i1}(1) \\ \Lambda_{i22} \{ \int_0^1 [W_{i2}(r)][dW_{i1}(r)] \} \Lambda_{i11} \end{bmatrix} \right).$$

The first matrix is a (3×3) block diagonal matrix and we can see for the stationary Δx_{it-p} $\forall p = -1, -2, \ldots, 0, +1, +2, \ldots$ the coefficient $\hat{\xi}_{DOLS1}$ has a Gaussian distribution that is given by

(5.198) $$\sqrt{NT}(\hat{\xi}_{DOLS1} - \xi) \Rightarrow \left(\frac{1}{N} \sum_{i=1}^{N} V_i \right)^{-1} \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Xi_i \right)$$

where $\Xi_i \sim N(\underline{0}, V_i \Lambda_{11}^2)$ and V_i is the $((2p+1)k \times (2p+1)k)$ covariance matrix.

For the second stage of the sequential limit theory as $N \to \infty$ let us look at the (2×2) lower block diagonal matrix of the parameters of interest, ie

(5.199) $$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \Rightarrow$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W_{i2}'(r)dr\Lambda_{i22}' \\ \Lambda_{i22}\int_0^1 W_{i2}(r)dr & \Lambda_{i22}\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\}\Lambda_{i22}' \end{bmatrix}\right)^{-1}$$
$$\times\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \end{bmatrix}\right).$$

Now under the assumption of a homogeneous panel each element of the (2×2) block matrix and (2×1) block vector are independent and identically distributed random variables for all i. Hence we can apply the Lindeberg-Levy CLT to each element of the block vector and the Komolgorov SLLN to each element of both the block matrix and vector. Hence we can now show the asymptotic consistency and asymptotic normality of our parameters of interest $\hat{\alpha}_{DOLS1}$ and $\hat{\beta}_{DOLS1}$. For asymptotic consistency we use Theorem (5.3.5) Komolgorov's SLLN. We note that the limit of the inverse of a matrix is the inverse of the limit by the CMT. Then we shall apply the Komolgorov SLLN to each element of the (2×2) block matrix and (2×1) block vector before inverting as follows.

Asymptotic Consistency

(a) First take $\Lambda_{i22}\int_0^1 W_{i2}(r)dr$. We must first verify the conditions of Komolgorov's Multivariate SLLN Theorem (5.3.5) and write

$$Z_i = \Lambda_{i22}\int_0^1 W_{i2}(r)dr. \tag{5.200}$$

Then to show $E(Z_i) < \infty$ it suffices to show $E\left\|\Lambda_{i22}\int_0^1 W_{i2}(r)dr\right\| < \infty$ as in Phillips and Moon (1999), Lemma 4.

By the Cauchy-Schwartz Inequality

$$(5.201)\quad E\left\|\Lambda_{i22}\int_0^1 W_{i2}(r)dr\right\| \leq \left[E\left\|\Lambda_{i22}\right\|^2\right]^{\frac{1}{2}}\left[E\left\|\int_0^1 W_{i2}(r)dr\right\|^2\right]^{\frac{1}{2}}$$

$$(5.202)\quad \leq \left[E\left(\sqrt{tr(\Lambda_{i22}\Lambda_{i22}^{\prime})}\right)^2\right]^{\frac{1}{2}}$$

$$\times\left[E\left(\sqrt{tr\left(\int_0^1 W_{i2}(r)dr\right)\left(\int_0^1 W_{i2}(r)dr\right)^{\prime}}\right)^2\right]^{\frac{1}{2}}$$

$$(5.203)\quad \leq \left[E\left(tr(\Lambda_{i22}\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}}\left[E\left(tr\left(\int_0^1 W_{i2}(r)dr\right)\left(\int_0^1 W_{i2}(r)dr\right)^{\prime}\right)\right]^{\frac{1}{2}}.$$

Interchanging the expectation operator with the trace operator[18]

$$(5.204)\quad \leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}}\left[tr\left(E\left(\int_0^1 W_{i2}(r)dr\right)\left(\int_0^1 W_{i2}(r)dr\right)^{\prime}\right)\right]^{\frac{1}{2}}.$$

Now on evaluating the integrals in second term of the R.H.S.[19]

$$(5.205)\quad \leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}}\left[tr\left(\int_0^1\int_0^1 E[W_{i2}(s)][W_{i2}(t)]^{\prime}dsdt\right)\right]^{\frac{1}{2}}$$

$$(5.206)\quad \leq \left[tr\left(\Omega_{22}\right)\right]^{\frac{1}{2}}\left[tr\left(\frac{1}{3}I_k\right)\right]^{\frac{1}{2}} < \infty.$$

Denote the diagonal elements of the $(k \times k)$ finite symmetric positive definite matrix[20] Ω_{22} as $(\Omega_{1122}, \Omega_{22222}, \ldots, \Omega_{kk22})$. Then $tr(\Omega_{22}) = \Omega_{1122} + \Omega_{2222} +$

[18] As in the case of infinite sums we can interchange the order of the expectation operator and infinite sums if $\sum_{i=0}^{\infty} Z_i < \infty$. Hence $E\sum_{i=0}^{\infty} Z_i = \sum_{i=0}^{\infty} EZ_i$. Similarly with the trace operator because $tr(.)$ and $E(.)$ are both linear operators it is possible to write $tr(E[z]) = E[tr(z)]$ for any argument z. Hence above we have $E(tr(\Lambda_{i22}\Lambda_{i22}^{\prime})) = tr(E(\Lambda_{i22}\Lambda_{i22}^{\prime}))$.

[19] See Levin and Lin (1992) Appendix A2.2 and Pedroni (2000) Mathematical Appendix A17, for a scalar method of computation that can be applied to vector Brownian motion. Also note we can interchange the expectation operator and integral $\int_0^1$ since r is a number see Levin and Lin (1992) Appendices.

[20] We make use of the homogeneous panel assumption here in the proofs, so that $\Omega_{i22} = \Omega_{22}, \quad \forall i$, etc.

$\ldots + \Omega_{kk22} < \infty$ by Assumption (5.4.2) (ii) and (vi). Similarly $tr(I_k) = 1 + 1 + \ldots + 1 = k$ so $tr(\frac{1}{3}I_k) < \infty$. Hence

$$E\left\|\Lambda_{i22}\int_0^1 W_{i2}(r)dr\right\| \leq \left(\sqrt{\Omega_{1122} + \Omega_{2222} + \ldots + \Omega_{kk22}}\right)\left(\sqrt{\frac{k}{3}}\right) < \infty.$$

(5.207)

So we have verified the conditions of Theorem (5.3.5) and given $E(Z_i) = E\left(\Lambda_{i22}\int_0^1 W_{i2}(r)dr\right) = \underline{0}$ then by Komolgorov's SLLN

(5.208) $$\frac{1}{N}\sum_{i=1}^{N}\Lambda_{i22}\int_0^1 W_{i2}(r)dr \overset{a.s.}{\to} \underline{0} \quad \text{as} \quad N \to \infty.$$

(b) Now take $\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\}\Lambda_{i22}'$. Again to verify the conditions of Theorem (5.3.5) write

(5.209) $$Z_i = \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\}\Lambda_{i22}'.$$

As in Phillips and Moon (1999), Lemma 4 to show $E(Z_i) < \infty$ it suffices to show $E\left\|\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\}\Lambda_{i22}'\right\| < \infty$.

By the Cauchy-Schwartz Inequality

$$E\left\|\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\}\Lambda_{i22}'\right\| \leq \left[E\left\|\Lambda_{i22}\Lambda_{i22}'\right\|^2\right]^{\frac{1}{2}}\left[E\left\|\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\|^2\right]^{\frac{1}{2}}.$$

(5.210)

The above follows by the independence of Λ_{i22} and $\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr$

(5.211) $$\leq \left[E\left\|\Lambda_{i22}\right\|^4\right]^{\frac{1}{2}}\left[E\left\|\int_0^1 [W_{i2}(r)][W_{i2}(r)]'dr\right\|^2\right]^{\frac{1}{2}}$$

$$(5.212) \qquad \leq \left[E\left(\sqrt{tr(\Lambda_{i22}\Lambda_{i22}^{/})}\right)^4\right]^{\frac{1}{2}}$$

$$\times\left[E\left(\sqrt{tr\left(\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}(r)dr\right)\left(\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}dr\right)^{/}}\right)^2\right]^{\frac{1}{2}}.$$

Now write for some positive definite $(k\times k)$ matrix, $M_{i2}(r) = [W_{i2}(r)][W_{i2}(r)]^{/}$ for simplicity. Then interchanging the expectation operator and the trace operator again

$$(5.213) \leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{/})\right)^2\right]^{\frac{1}{2}}\left[tr\left(E\left(\int_0^1 M_{i2}(r)dr\right)\left(\int_0^1 M_{i2}(r)dr\right)^{/}\right)\right]^{\frac{1}{2}}.$$

As in (a) we have the reversal of E and $\int_0^1$

$$(5.214) \qquad \leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{/})^2\right)\right]^{\frac{1}{2}}\left[tr\left(\int_0^1\int_0^1 E[M_{i2}(s)M_{i2}(t)]^{/}dsdt\right)\right]^{\frac{1}{2}}.$$

On evaluating the integrals

$$(5.215) \qquad \leq \left[tr\left(\Omega_{22}\right)^2\right]^{\frac{1}{2}}\left[tr\left(\frac{1}{3}I_k\right)\right]^{\frac{1}{2}} < \infty.$$

Again as in (a) we have $tr(\Omega_{22})^2 = \Omega_{1122}^2 + \Omega_{2222}^2 + \ldots + \Omega_{kk22}^2 < \infty$ and $tr(I_k) = 1 + 1 + \ldots + 1 = k < \infty$ by Assumption (5.4.2) (ii) and (vi). Hence

$$(5.216) \qquad E\left\|\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}dr\right\}\Lambda_{i22}^{/}\right\|$$

$$(5.217) \qquad \leq \left(\sqrt{\Omega_{1122}^2 + \Omega_{2222}^2 + \ldots + \Omega_{kk22}^2}\right)\left(\sqrt{\frac{k}{3}}\right) < \infty.$$

Thus we have verified the conditions of Theorem (5.3.5) and since now

$$(5.218) \qquad E(Z_i) = E\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}dr\right\}\Lambda_{i22}^{/}\right)$$

$$(5.219) \qquad = E\left(\Lambda_{i22}\Lambda_{i22}^{/}\right)E\left(\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}dr\right).$$

The last line following by independence of Λ_{i22} and $\int_0^1 [W_{i2}(r)][W_{i2}(r)]' dr$. Thus we have on evaluating the integral in the second term of the R.H.S.

$$E(Z_i) = \left(\Omega_{22} \times \frac{1}{2} I_k\right) = \frac{1}{2}\Omega_{22}. \tag{5.220}$$

So that by Komolgorov's SLLN

$$\frac{1}{N}\sum_{i=1}^{N} \Lambda_{i22} \left\{\int_0^1 [W_{i2}(r)][W_{i2}(r)]' dr\right\} \Lambda'_{i22} \overset{a.s.}{\rightarrow} \frac{1}{2}\Omega_{22} \quad \text{as} \quad N \rightarrow \infty. \tag{5.221}$$

(c) Next $\Lambda_{i11} W_{i1}(1)$ and now we verify the conditions of Theorem (5.3.4) as follows. Write $Z_i = \Lambda_{i11} W_{i1}(1)$ a simple scalar composition so that $\mathrm{E}|Z_i| = E\left|\Lambda_{i11} W_{i1}(1)\right|$.

By the Cauchy-Schwartz Inequality write

$$E\left|\Lambda_{i11} W_{i1}(1)\right| \leq \left[E\left|\Lambda_{i11}\right|^2\right]^{\frac{1}{2}} \left[E\left|W_{i1}(1)\right|^2\right]^{\frac{1}{2}} \tag{5.222}$$

$$\leq (\sqrt{\Omega_{11}})(\sqrt{1}) < \infty \tag{5.223}$$

by Assumption (5.4.2) (ii) and (vi). Thus we have verified the conditions of Theorem (5.3.4) and since $E(\Lambda_{i11} W_{i1}(1)) = 0$ it follows

$$\frac{1}{N}\sum_{i=1}^{N} \Lambda_{i11} W_{i1}(1) \overset{a.s.}{\rightarrow} 0 \quad \text{as} \quad N \rightarrow \infty. \tag{5.224}$$

(d) Finally take $\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}$. Again to verify the conditions of Theorem (5.3.5) write

$$Z_i = \Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}. \tag{5.225}$$

To show $E(Z_i) < \infty$ we can write as in Phillips and Moon (1999), Lemma 4

$E\left\|\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right\| < \infty.$

By the Cauchy-Schwartz Inequality

$$E\left\|\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right\| \leq \left[E\left\|\Lambda_{i22}\Lambda_{i11}\right\|^2\right]^{\frac{1}{2}}\left[E\left\|\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\|^2\right]^{\frac{1}{2}} \tag{5.226}$$

$$\leq \left[E\left(\sqrt{tr(\Lambda_{i22}\Lambda_{i11}^2\Lambda_{i22}^{\prime})}\right)^2\right]^{\frac{1}{2}} \tag{5.227}$$

$$\times\left[E\left(\sqrt{tr\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)^{\prime}}\right)^2\right]^{\frac{1}{2}}.$$

Interchanging the expectation operator with the trace operator the first term on the R.H.S. is a quadratic form in Λ_{i11} and Λ_{i22}. Also the second term on R.H.S. is an Ito Stochastic Integral (see Phillips (1988))

$$\leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i11}^2\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}} \tag{5.228}$$

$$\times\left[tr\left(E\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)^{\prime}\right)\right]^{\frac{1}{2}}$$

$$\leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i11}^2\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}} \tag{5.229}$$

$$\times\left[tr\left(E\left(\int_0^1[W_{i2}(r)][W_{i2}(r)]^{\prime}dr\right)\right)\right]^{\frac{1}{2}}.$$

The second term on the R.H.S. follows by the properties of Ito Stochastic Integrals. Then we have using (a) for the first term on R.H.S. and (b) for the second term on R.H.S.

$$\leq \left[tr\left(\Omega_{11}\Omega_{22}\right)\right]^{\frac{1}{2}}\left[tr\left(\frac{1}{2}I_k\right)\right]^{\frac{1}{2}} < \infty. \tag{5.230}$$

So $tr(\Omega_{11}\Omega_{22}) = \Omega_{11}(\Omega_{1122} + \Omega_{2222} + \ldots + \Omega_{kk22}) < \infty$ and $\mathrm{tr}(\mathrm{I}_k) = 1 + 1 + \ldots + 1 = k$ so $tr\left(\frac{1}{2}I_k\right) < \infty$ by Assumption (5.4.2) (ii) and (vi). Hence

$$E\left\|\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right\| \leq \left(\sqrt{\Omega_{11}(\Omega_{1122} + \Omega_{2222} + \ldots + \Omega_{kk22})}\right)\left(\sqrt{\frac{k}{2}}\right) < \infty. \tag{5.231}$$

So we have verified the conditions of Theorem (5.3.5) and given $E(Z_i) = E\left(\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right) = \underline{0}$ then by Komolgorov's SLLN

$$\frac{1}{N}\sum_{i=1}^{N}\Lambda_{i22}\left\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11} \overset{a.s.}{\to} \underline{0} \quad \text{as} \quad N \to \infty. \tag{5.232}$$

Using Komolgorov's SLLN we have now shown that as $N \to \infty$ by (a)-(d)

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \underline{0}' \\ \underline{0} & \frac{1}{2}\Omega_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ \underline{0} \end{bmatrix}. \tag{5.233}$$

Thus

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ \underline{0} \end{bmatrix}. \tag{5.234}$$

Hence $\hat{\alpha}_{DOLS1} \xrightarrow{p} \alpha$ and $\hat{\beta}_{DOLS1} \xrightarrow{p} \beta$ as $(N,T \to \infty)_{seq}$ and so we have shown that $\hat{\alpha}_{DOLS1}$ and $\hat{\beta}_{DOLS1}$ are asymptotically consistent estimators as $(N,T \to \infty)_{seq}$.

Asymptotic Normality

For asymptotic normality we apply the Lindeberg-Levy CLT to each element of the (2×1) block vector. Since we already have the limiting distribution of the (2×2) block matrix the desired result follows after an application of

Proposition (5.3.9) (Slutsky's theorem).

$(a)/$ First take $\Lambda_{i11}W_{i1}(1)$. We must first verify the conditions of the Lindeberg-Levy CLT, that is given Z_i then $var(Z_i) = \sigma^2 < \infty$ and $\sigma^2 \neq 0$. Since $\Lambda_{i11}W_{i1}(1)$ is a scalar write $Z_i = \Lambda_{i11}W_{i1}(1)$. Then $var(Z_i) = var(\Lambda_{i11}W_{i1}(1)) = \Lambda_{i11}^2$, since W_{i1} is scalar Brownian motion $W_{i1} \sim N(0,1)$. Therefore

(5.235) $$var(Z_i) = var(\Lambda_{i11}W_{i1}(1)) = \Lambda_{i11}^2 = \Omega_{11} < \infty.$$

Hence we have satisfied the conditions of Theorem (5.3.6) and given $E(Z_i) = \mu$ then $E(\Lambda_{i11}W_{i1}(1)) = 0$ and by the Lindberg-Levy CLT

(5.236) $$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{(\Lambda_{i11}W_{i1}(1))}{\sqrt{\Omega_{11}}} \xrightarrow{d} N(0,1) \quad \text{as} \quad N \to \infty.$$

Hence we can write

(5.237) $$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Lambda_{i11}W_{i1}(1) \xrightarrow{d} N(0,\Omega_{11}) \quad \text{as} \quad N \to \infty.$$

$(b)/$ Now take $\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}$ and again to verify the conditions of the Lindeberg-Levy CLT write

(5.238) $$Z_i = \Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}.$$

Now $var(Z_i)$ is

(5.239) $$var\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)$$

and given

$$E\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)=\underline{0} \tag{5.240}$$

then

$$var\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)= \tag{5.241}$$

$$E\left[\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)^{/}\right] \tag{5.242}$$

$$= (\Lambda_{i22}\Lambda_{i11}^2\Lambda_{i22}^{/})E\left[\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)\left(\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right)^{/}\right] \tag{5.243}$$

$$= (\Omega_{22}\Omega_{11})\left[E\int_0^1[W_{i2}(r)][W_{i2}(r)]^{/}d(r)\right]. \tag{5.244}$$

The above bracketed term on the R.H.S. follows by the properties of Ito's Stochastic Integral. Then using (b) above we have

$$var(Z_i)=(\Omega_{22}\Omega_{11})\times\frac{1}{2}I_k=\frac{1}{2}\Omega_{22}\Omega_{11}<\infty\neq\underline{0}. \tag{5.245}$$

Hence we have satisfied the conditions of Theorem (5.3.6) and given $E(Z_i)=\mu$ we have $E(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11})=\underline{0}$ and so by the Lindberg-Levy CLT

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}}{\sqrt{\frac{1}{2}\Omega_{22}\Omega_{11}}}\right)\xrightarrow{d}N(\underline{0},I_k)\quad\text{as}\quad N\rightarrow\infty. \tag{5.246}$$

Hence we can also write

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda_{i22}\left\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\right\}\Lambda_{i11}\right)\xrightarrow{d}N\left(\underline{0},\frac{1}{2}\Omega_{22}\Omega_{11}\right)\quad\text{as}\quad N\rightarrow\infty. \tag{5.247}$$

Using the Lindeberg-Levy CLT we have now shown that as $N \to \infty$ by $(a)' - (b)'$

$$\left[\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left[\begin{array}{c}\Lambda_{i11}W_{i1}(1)\\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\end{array}\right]\right] \xrightarrow{d} N\left(\left[\begin{array}{c}0\\ \underline{0}\end{array}\right],\left[\begin{array}{cc}\Omega_{11} & \underline{0}'\\ \underline{0} & \frac{1}{2}\Omega_{22}\Omega_{11}\end{array}\right]\right).$$
(5.248)

Note that the off-diagonal zeros elements in the covariance matrix follow since[21]

$$cov\left(\Lambda_{i11}W_{i1}(1), \Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right) = \tag{5.249}$$

$$E\left(\Lambda_{i11}W_{i1}(1)\Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right) = \underline{0}. \tag{5.250}$$

Our final asymptotic normality result comes from using the Slutsky device Proposition (5.3.9) applied to our equation

$$\left[\begin{array}{c}\sqrt{N}T(\hat{\alpha}_{DOLS1} - \alpha)\\ \sqrt{N}T(\hat{\beta}_{DOLS1} - \beta)\end{array}\right] \Rightarrow \tag{5.251}$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\left[\begin{array}{cc}1 & \int_0^1 W_{i2}'(r)dr\Lambda_{i22}'\\ \Lambda_{i22}\int_0^1 W_{i2}(r)dr & \Lambda_{i22}\{\int_0^1[W_{i2}(r)][W_{i2}(r)]'dr\}\Lambda_{i22}'\end{array}\right]\right)^{-1}$$
$$\times\left(\frac{1}{N}\sum_{i=1}^{N}\left[\begin{array}{c}\Lambda_{i11}W_{i1}(1)\\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\end{array}\right]\right).$$

Which has the asymptotic distribution

$$\left[\begin{array}{c}\sqrt{N}T(\hat{\alpha}_{DOLS1} - \alpha)\\ \sqrt{N}T(\hat{\beta}_{DOLS1} - \beta)\end{array}\right] \Rightarrow N\left(\left[\begin{array}{c}0\\ \underline{0}\end{array}\right],\left[\begin{array}{cc}1 & \underline{0}'\\ \underline{0} & 2\Omega_{22}^{-1}\end{array}\right]\left[\begin{array}{cc}\Omega_{11} & \underline{0}'\\ \underline{0} & \frac{1}{2}\Omega_{22}\Omega_{11}\end{array}\right]\left[\begin{array}{cc}1 & \underline{0}'\\ \underline{0} & 2\Omega_{22}^{-1}\end{array}\right]\right).$$
(5.252)

[21] The usual formula for the covariance between random variables X and Y is $cov(X,Y) = E(X - E(X))(Y - E(Y)) = E(XY)$ when $E(X) = E(Y) = 0$ as above.

So

$$(5.253) \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS1} - \beta) \end{bmatrix} \Rightarrow N\left(\begin{bmatrix} 0 \\ \underline{0} \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \underline{0}' \\ \underline{0} & 2\Omega_{22}^{-1}\Omega_{11} \end{bmatrix} \right).$$

QED

Remark 5.6.1 *We can see now the Gaussian limiting distribution of our panel cointegration estimator. This also holds for FMOLS and OLS. The non-normality of the limiting distributions of the time-series estimators is smoothed out in their panel analogues by an application of an appropriate CLT across the cross-section dimension.*

Lemma (5.4.12)

Proof

(a) Follows Lemma (5.4.5) (a) with exactly the same derivations since x_{it} is identical in the DOLS case. We denote the analogous FMOLS case with a bar and plus, $\bar{W}_{i2}^{+}(r)$ over the Wiener processes and covariance matrices, $\bar{\Lambda}_{i11}^{+}$, etc.

(b) Again follows the same derivations as in Lemma (5.4.5) (b).

(c) This follows Lemma (5.4.5) (c) with z_{it} substituted by $\hat{u}_{it}^{+}$.

(d) We know $\hat{\aleph}^{+} = \sum_{v=0}^{\infty} E(\hat{\varepsilon}_{it}\hat{u}^{+}_{it+v}) = \hat{\Gamma}^{+}_{i21}$ so that by above

$$\frac{1}{N}\sum_{t=1}^{T}\xi^{*}_{it-1}L^{\prime}\hat{w}_{it} \Rightarrow \bar{\Lambda}^{+}_{i}\int_{0}^{1}[\bar{W}^{+}_{i}(r)][d\bar{W}^{+}_{i}(r)]\bar{\Lambda}^{+\prime}_{i} + \sum_{v=0}^{\infty}E[L^{\prime}\hat{w}_{it}\hat{w}^{\prime}_{it-v}L].$$

(5.254)

Then we have

(5.255) $$\frac{1}{T}\sum_{t=1}^{T}x_{it}\hat{u}^{+}_{it} = [0 \quad I_{K}]\frac{1}{N}\sum_{t=1}^{T}\xi^{*}_{it}L^{\prime}\hat{w}_{it}\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

$$\Rightarrow [0 \quad I_{K}]\bar{\Lambda}^{+}_{i}\int_{0}^{1}[\bar{W}^{+}_{i}(r)][d\bar{W}^{+}_{i}(r)]\bar{\Lambda}^{+\prime}_{i}\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} + [0 \quad I_{K}]\sum_{v=0}^{\infty}E\left(\begin{bmatrix} \hat{u}^{+}_{it} \\ \hat{\varepsilon}_{it} \end{bmatrix}[\hat{u}^{+}_{it-v}\hat{\varepsilon}_{it-v}]\right)^{\prime}\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

(5.256)

The last term can be written as

(5.257) $$= [0 \quad I_{K}]\sum_{v=0}^{\infty}E\left(\begin{bmatrix} \hat{u}^{+}_{it}\hat{u}^{+}_{it-v} & \hat{u}^{+}_{it}\hat{\varepsilon}_{it-v} \\ \hat{\varepsilon}_{it}\hat{u}^{+}_{it-v} & \hat{\varepsilon}_{it}\hat{\varepsilon}^{\prime}_{it-v} \end{bmatrix}\right)^{\prime}\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

(5.258) $$= \sum_{v=0}^{\infty}E[\hat{u}^{+}_{it}\hat{\varepsilon}_{it-v} \quad \hat{\varepsilon}_{it}\hat{\varepsilon}^{\prime}_{it-v}]\begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} = \sum_{v=0}^{\infty}E(\hat{u}^{+}_{it}\hat{\varepsilon}_{it-v}) = \sum_{v=0}^{\infty}E(\hat{\varepsilon}_{it}\hat{u}^{+}_{it+v}).$$

But this last term is just $\hat{\aleph}^{+}$ and so the result follows since the first term gives $\bar{\Lambda}^{+}_{i22}\left\{\int_{0}^{1}[\bar{W}^{+}_{i2}(r)][d\bar{W}^{+}_{i1}(r)]\bar{\Lambda}^{+\prime}_{i11}\right.$ so that

(5.259) $$\frac{1}{T}\sum_{t=1}^{T}x_{it}\hat{u}^{+}_{it} - \hat{\aleph}^{+} \Rightarrow \bar{\Lambda}^{+}_{i22}\left\{\int_{0}^{1}[\bar{W}^{+}_{i2}(r)][d\bar{W}^{+}_{i1}(r)]\bar{\Lambda}^{+\prime}_{i11} + \hat{\aleph}^{+} - \hat{\aleph}^{+}\right.$$

and the last two terms drop out, hence the result.

QED

Theorem (5.4.13)

Proof

We have from equation (5.87) the panel FMOLS estimator

$$(5.260)\ \hat{\Upsilon}_{FMOLS1} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{/} \\ x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{y}_{it}^{+} \\ x_{it}\hat{y}_{it}^{+} - T\hat{\aleph}^{+} \end{bmatrix}\right).$$

Substitute for $\hat{y}_{it}^{+}$ to obtain

$$(\hat{\Upsilon}_{FMOLS1} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{/} \\ x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{u}_{it}^{+} \\ x_{it}\hat{u}_{it}^{+} - T\hat{\aleph}^{+} \end{bmatrix}\right).$$

(5.261)

Again we rescale $(\hat{\Upsilon}_{FMOLS1} - \Upsilon)$ to obtain a non-degenerate limiting distribution. Where again $\hat{\beta}_{FMOLS1}$ is superconsistent converging across the time-series dimension at rate T. Define D_T now as

$$(5.262)\qquad D_T = \begin{bmatrix} \sqrt{NT} & \underline{0}^{/} \\ \underline{0} & \sqrt{N}TI_k \end{bmatrix}$$

then we have

$$(5.263)\qquad D_T(\hat{\Upsilon}_{FMOLS1} - \Upsilon) = \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}^{/} \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{/} \\ x_{it} & x_{it}x_{it}^{/} \end{bmatrix}\right)^{-1} \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}^{/} \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix}$$
$$\times\begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}^{/} \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{u}_{it}^{+} \\ x_{it}\hat{u}_{it}^{+} - T\hat{\aleph}^{+} \end{bmatrix}\right).$$

Hence

$$(5.264)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{FMOLS1} - \alpha) \\ \sqrt{N}T(\hat{\beta}_{FMOLS1} - \beta) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}^{/} \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}^{/} \end{bmatrix}^{-1}$$
$$\times\begin{bmatrix} N^{-\frac{1}{2}}\Sigma(T^{-\frac{1}{2}}\Sigma\hat{u}_{it}^{+}) \\ N^{-\frac{1}{2}}\Sigma(T^{-1}\Sigma x_{it}\hat{u}_{it}^{+} - \hat{\aleph}^{+}) \end{bmatrix}.$$

Now again apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above in the more convenient form as

$$\begin{bmatrix} \sqrt{N}T(\hat{\alpha}_{FMOLS1} - \alpha) \\ \sqrt{N}T(\hat{\beta}_{FMOLS1} - \beta) \end{bmatrix} \tag{5.265}$$

$$= \left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} 1 & T^{-\frac{3}{2}} \sum_{t=1}^{T} x_{it}' \\ T^{-\frac{3}{2}} \sum_{t=1}^{T} x_{it} & T^{-2} \sum_{t=1}^{T} x_{it} x_{it}' \end{bmatrix} \right)^{-1} \times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} T^{-\frac{1}{2}} \sum_{t=1}^{T} \hat{u}_{it}^{+} \\ T^{-1} \sum_{t=1}^{T} x_{it} \hat{u}_{it}^{+} - \hat{\aleph}^{+} \end{bmatrix} \right).$$

By Lemma (5.4.12) and the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3) as $T \to \infty$, and N is held fixed we have by applying the FCLT to each element of the (2×2) block matrix and (2×1) block vector the following.

By Lemma (5.4.12) (a)-(d)

$$\begin{bmatrix} \sqrt{N}T(\hat{\alpha}_{FMOLS1} - \alpha) \\ \sqrt{N}T(\hat{\beta}_{FMOLS1} - \beta) \end{bmatrix} \Rightarrow \tag{5.266}$$

$$\left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} 1 & \int_0^1 \bar{W}_{i2}^{+\prime}(r) dr \bar{\Lambda}_{i22}^{+\prime} \\ \bar{\Lambda}_{i22}^{+} \int_0^1 \bar{W}_{i2}^{+}(r) dr & \bar{\Lambda}_{i22}^{+} \{ \int_0^1 [\bar{W}_{i2}^{+}(r)][\bar{W}_{i2}^{+}(r)]' dr \} \bar{\Lambda}_{i22}^{+\prime} \end{bmatrix} \right)^{-1} \times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} \bar{\Lambda}_{i11}^{+} \bar{W}_{i1}^{+}(1) \\ \bar{\Lambda}_{i22}^{+} \{ \int_0^1 [\bar{W}_{i2}^{+}(r)][d\bar{W}_{i1}^{+}(r)] \} \bar{\Lambda}_{i11}^{+} \end{bmatrix} \right).$$

The rest of the proof of Theorem (5.4.13) follows along the same lines as the proof of Theorem (5.4.6). The only difference is to replace Ω_{11} and Ω_{22} by $\bar{\Omega}_{11}^{+}$ and $\bar{\Omega}_{22}^{+}$, respectively in the final computations. Note that the exactly analgous FMOLS case (to DOLS) is depicted with a bar and plus, $\bar{W}_{i2}^{+}(r)$

over the Wiener processes and covariance matrices, $\bar{\Lambda}_{i11}^{+}$, etc. As stated earlier $\Omega_{22} = \bar{\Omega}_{22}^{+}$ and so $\Lambda_{i22}\Lambda_{i22}^{\prime} = \bar{\Lambda}_{i22}^{+}\bar{\Lambda}_{i22}^{+\prime}$.

QED

Remark 5.6.2 *We noted earlier that in general $\Omega_{11} \neq \bar{\Omega}_{11}^{+}$. However in the special case it does then we have the well known result of the time-series literature of the asymptotic equivalence of the DOLS and FMOLS estimators this carries over to their panel data analogues if and only if $\Omega_{11} = \bar{\Omega}_{11}^{+}$. Conditions for this are stated generally in Banerjee et al (1993) and are as follows: If the Brownian motion $W_{i1}(r)$ is uncorrelated with $W_{i2}(r)$ at all frequencies, then the conditional process generating the residuals z_{it} is completely informative for the purpose of estimating β and the marginal process Δx_{it} generating ε_{it} in equation (2) can be ignored. In this case $\Omega_{11} = \bar{\Omega}_{11}^{+}$. Also the panel OLS estimator is biased and inefficient when compared to the asymptotically equivalent panel FMOLS and DOLS estimators.*

Theorem (5.4.14)

Proof

From equation (5.97) we have the following OLS estimator

$$\hat{\Upsilon}_{OLS1} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}^{\prime} \\ x_{it} & x_{it}x_{it}^{\prime} \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \end{bmatrix}\right). \quad (5.267)$$

Substituting for y_{it} we obtain

$$(5.268)\quad (\hat{\Upsilon}_{OLS1} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' \\ x_{it} & x_{it}x_{it}' \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} u_{it} \\ x_{it}u_{it} \end{bmatrix}\right).$$

Rescale the equation as before using

$$(5.269)\qquad D_T = \begin{bmatrix} \sqrt{NT} & \underline{0}' \\ \underline{0} & \sqrt{NT}I_k \end{bmatrix}$$

then we have

$$(5.270)\qquad D_T(\hat{\Upsilon}_{OLS1} - \Upsilon) = \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' \\ x_{it} & x_{it}x_{it}' \end{bmatrix}\right)^{-1} \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix}$$
$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k \end{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T}\begin{bmatrix} u_{it} \\ x_{it}u_{it} \end{bmatrix}.$$

Hence

$$(5.271)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}' \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}' \end{bmatrix}^{-1}$$
$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma u_{it} \\ N^{-\frac{1}{2}}T^{-1}\Sigma\Sigma x_{it}u_{it} \end{bmatrix}.$$

We now again apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above in the more convenient form as

$$(5.272)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix}$$

$$= \left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & T^{-\frac{3}{2}}\sum_{t=1}^{T}x_{it}' \\ T^{-\frac{3}{2}}\sum_{t=1}^{T}x_{it} & T^{-2}\sum_{t=1}^{T}x_{it}x_{it}' \end{bmatrix}\right)^{-1} \times \left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} T^{-\frac{1}{2}}\sum_{t=1}^{T}u_{it} \\ T^{-1}\sum_{t=1}^{T}x_{it}u_{it} \end{bmatrix}\right).$$

On substituting Λ_i^* for $\bar{\Lambda}_i^+$, etc we have an exactly analagous case for OLS to the proofs of Theorem (5.4.6) and Theorem (5.4.13) except for $T^{-1}\sum_{t=1}^{T}x_{it}u_{it}$. So

(5.273)
$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1}-\beta) \end{bmatrix} \Rightarrow$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W_{i2}^{*\prime}(r)dr\Lambda_{i22}^{*\prime} \\ \Lambda_{i22}^*\int_0^1 W_{i2}^*(r)dr & \Lambda_{i22}^*\{\int_0^1[W_{i2}^*(r)][W_{i2}^*(r)]'dr\}\Lambda_{i22}^{*\prime} \end{bmatrix}\right)^{-1} \times \left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda_{i11}^*W_{i1}^*(1) \\ \Lambda_{i22}^*\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\}\Lambda_{i11}^*+\Gamma_{i21}^* \end{bmatrix}\right)$$

where $\Gamma_{i21}^* = \sum_{v=0}^{\infty}E(\varepsilon_{it}u_{it+v})$ is the bias term which gives the distribution the non-zero mean. For a detailed derivation of the asymptotic distribution of the time-series OLS estimator in the cases with a constant and/or trend see Park and Phillips (1988).

Asymptotic Consistency

Using (c) and (d) above we have

(5.274)
$$\frac{1}{N}\sum_{i=1}^{N}\Lambda_{i11}^*W_{i1}^*(1) \overset{a.s.}{\to} 0 \quad \text{as} \quad N\to\infty.$$

and

(5.275)
$$\frac{1}{N}\sum_{i=1}^{N}\Lambda_{i22}^*\left\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\right\}\Lambda_{i11}^* \overset{a.s.}{\to} \underline{0} \quad \text{as} \quad N\to\infty.$$

However note that in the DOLS and FMOLS case $W_{i1}(r)$ is independent of $W_{i2}(r)$ and $\bar{W}_{i1}^{+}(r)$ is independent of $\bar{W}_{i2}^{+}(r)$. Also $\Lambda_{i11}^{2} = \Omega_{i11}$ and $\Lambda_{i22}\Lambda_{i22}^{/} = \Omega_{i22}$ and also $\bar{\Lambda}_{i11}^{+2} = \bar{\Omega}_{i11}^{+}$ and $\bar{\Lambda}_{i22}^{+}\bar{\Lambda}_{i22}^{+/} = \bar{\Omega}_{i22}^{+}$ only. This is due to the zero off diagonal elements in Ω_i. With OLS this is not the case and we must compute (d) under the assumption of correlation between $W_{i1}^{*}(r)$ and $W_{i2}^{*}(r)$ and where $\Lambda_{i11}^{*}\Lambda_{i22}^{*} = \Omega_{i21}^{*}$.

(d) Take $\Lambda_{i22}^{*}\left\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right\}\Lambda_{i11}^{*}$. To verify the conditions of Theorem (5.3.5) write

$$Z_i = \Lambda_{i22}^{*}\left\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right\}\Lambda_{i11}^{*}. \tag{5.276}$$

To show $E(Z_i) < \infty$ we can write as in Phillips and Moon (1999), Lemma 4 $E\left\|\Lambda_{i22}^{*}\left\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right\}\Lambda_{i11}^{*}\right\| < \infty$.

By the Cauchy-Schwartz Inequality

$$E\left\|\Lambda_{i22}^{*}\left\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right\}\Lambda_{i11}^{*}\right\| \leq \left[E\left\|\Lambda_{i22}^{*}\Lambda_{i11}^{*}\right\|^2\right]^{\frac{1}{2}}\left[E\left\|\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right\|^2\right]^{\frac{1}{2}}$$

(5.277)

$$\leq \left[E\left(\sqrt{tr(\Lambda_{i22}^{*}\Lambda_{i11}^{*2}\Lambda_{i22}^{*/})}\right)^2\right]^{\frac{1}{2}} \tag{5.278}$$

$$\times\left[E\left(\sqrt{tr\left(\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right)\left(\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\right)^{/}}\right)^2\right]^{\frac{1}{2}}.$$

Interchanging the expectation operator with the trace operator the first term on the R.H.S. is a quadratic form again in Λ_{i11}^{*} and Λ_{i22}^{*}. Also the second term on R.H.S. is an Ito Stochastic Integral (see Phillips (1988)).

$$\leq \left[tr\left(E(\Lambda^*_{i22}\Lambda^{*2}_{i11}\Lambda^{*/}_{i22})\right)\right]^{\frac{1}{2}} \tag{5.279}$$

$$\times \left[tr\left(E\left(\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right)\left(\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right)'\right)\right]^{\frac{1}{2}}$$

$$\leq \left[tr\left(E(\Lambda^*_{i22}\Lambda^{*2}_{i11}\Lambda^{*/}_{i22})\right)\right]^{\frac{1}{2}} \tag{5.280}$$

$$\times \left[tr\left(E\left(\int_0^1[W^*_{i2}(r)][W^*_{i2}(r)]'dr\right)\right)\right]^{\frac{1}{2}}.$$

The second term on the R.H.S. follows by the properties of Ito Stochastic Calculus. Now however $\Lambda^*_{i22}\Lambda^*_{i11} = \Omega^*_{i21}$ a $(k \times 1)$ vector so for the first term on R.H.S. $tr\left(E(\Lambda^*_{i22}\Lambda^{*2}_{i11}\Lambda^{*/}_{i22})\right) = tr\left(\Omega^*_{21}\Omega^{*/}_{21}\right)$. Denote the diagonal elements of the $(k \times k)$ finite symmetric positive definite matrix $(\Omega^*_{21}\Omega^{*/}_{21})$ as $(\Omega^*_{1121}, \Omega^*_{2221}, \ldots, \Omega^*_{kk21})$. Then $\sqrt{tr(\Omega^*_{21}\Omega^{*/}_{21})} = \sqrt{\Omega^*_{1121} + \Omega^*_{2221} + \ldots + \Omega^*_{kk21}} < \infty$ and $\text{tr}(\text{I}_k) = 1 + 1 + \ldots + 1 = k$ so $tr\left(\frac{1}{2}I_k\right) < \infty$ by Assumption (5.4.7) (ii) and (vi). Hence

$$E\left\|\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right\| \leq \left(\sqrt{(\Omega^*_{1121} + \Omega^*_{2221} + \ldots + \Omega^*_{kk21})}\right)\left(\sqrt{\frac{k}{2}}\right) < \infty. \tag{5.281}$$

So we have verified the conditions of Theorem (5.3.5). Now

$$E(Z_i) = E\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) = E(\Omega^*_{21})E\left(\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\right). \tag{5.282}$$

By the properties of Ito Stochastic Calculus the second term on R.H.S. is made up of two correlated Wiener processes $W^*_{i1}(r)$ and $W^*_{i2}(r)$. Thus

$$(5.283)\qquad \int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)] = \left(\frac{1}{2}I_k\right)\left(\chi^2(1) - 1\right).$$

However this has expectation zero so that $E\left(\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right) = \underline{0}$ and so $E(Z_i) = \Omega^*_{21}(\underline{0}) = \underline{0}$. Then by Komolgorov's SLLN

$$(5.284)\qquad \frac{1}{N}\sum_{i=1}^{N} \Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11} \stackrel{a.s.}{\to} \underline{0} \quad \text{as} \quad N \to \infty.$$

(e) Now take $\Lambda^*_{i22}\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21}$ again we verify the conditions of Theorem (5.3.5) as follows. From (d) above $E\left\|\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right\| < \infty$ so now we show only $E\left\|\Gamma^*_{i21}\right\|$. Write $Z_i = \Gamma^*_{i21}$. Since $\left\|\Gamma^*_{i21}\right\|$ is a constant then

$$(5.285)\qquad E\left\|Z_i\right\| = E\left\|\Gamma^*_{i21}\right\| = \left[tr\left(E(\Gamma^*_{i21}\Gamma^{*/}_{i21})\right)\right]^{\frac{1}{2}}.$$

Denote the diagonal elements of the $(k \times k)$ finite symmetric positive definite matrix $E(\Gamma^*_{i21}\Gamma^{*/}_{i21})$ as $(\Gamma^*_{1121}, \Gamma^*_{2221}, \ldots, \Gamma^*_{kk21})$. Then $\sqrt{trE(\Gamma^*_{i21}\Gamma^{*/}_{i21})} = \sqrt{(\Gamma^*_{1121} + \Gamma^*_{2221} + \ldots + \Gamma^*_{kk21})} < \infty$ since $(\Gamma^*_{21}\Gamma^{*/}_{21}) < \infty$ by Assumption (5.4.7) (ii) and (vi). Thus we have $E\left\|\Gamma^*_{i21}\right\| < \infty$ and hence

$$(5.286)\qquad E\left\|\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11} + \Gamma^*_{i21}\right\|$$

$$\leq E\left\|\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right\| + E\left\|\Gamma^*_{i21}\right\| < \infty$$

by the triangle inequality. Hence we have verified the conditions of Theorem

(5.3.5) and given

$E(Z_i) = E\left(\Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21}\right) = \Gamma^*_{i21}$

since we have $E(\Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11}) = \underline{0}$ by (d) above, then by Komolgorov's SLLN

$$(5.287)\quad \frac{1}{N}\sum_{i=1}^{N}\Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21} \overset{a.s.}{\to} \Gamma^*_{21} \quad \text{as} \quad N \to \infty.$$

Using Komolgorov's SLLN we have now shown that as $N \to \infty$ by (a)-(e)

$$(5.288)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \underline{0}' \\ \underline{0} & \frac{1}{2}\Omega_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ \Gamma^*_{21} \end{bmatrix}.$$

Thus

$$(5.289)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 2\Gamma^*_{21}\Omega^{-1}_{22} \end{bmatrix}.$$

Hence $\hat{\alpha}_{OLS1} \overset{p}{\to} \alpha$ but $\hat{\beta}_{OLS1} \overset{p}{\not\to} \beta$ as $(N, T \to \infty)_{seq}$ and so we have shown that $\hat{\alpha}_{OLS1}$ is asymptotically consistent but $\hat{\beta}_{OLS1}$ is not an asymptotically consistent estimator as $(N, T \to \infty)_{seq}$.

Remark 5.6.3 *It is of interest to compare our OLS bias term with the Kao and Chiang (2000) bias term for their OLS estimator in the same model*[22]*. Kao and Chiang (2000) compute their bias term as*[23]

$$\delta_{NT} = \left[\frac{1}{N}\Sigma_{i=1}^{N}\frac{1}{T^2}\Sigma_{i=1}^{T}(x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)'\right]^{-1}\left[\frac{1}{N}\Sigma_{i=1}^{N}\Omega_{\epsilon}^{\frac{1}{2}}\left(\int_0^1 \tilde{W}_i(r)dW_i'(r)\right)\Omega_{\epsilon}^{-\frac{1}{2}}\Omega_{\epsilon u} + \Delta_{\epsilon u}\right]$$

(5.290)

[22]Kao and Chiang (2000) give results for the fixed effects specification.

[23]In equating their notation with ours $\Omega_{\epsilon} = \Omega_{22}$, $\Omega_{\epsilon u} = \Omega^*_{21}$ and $\Delta_{\epsilon u} = \Gamma^*_{21}$.

where $\tilde{W}_i(r) = W_i(r) - \int_0^1 W_i(r)dr$ is demeaned Brownian motion. It is shown that $\delta_{NT} \xrightarrow{p} -3\Omega_\epsilon^{-1}\Omega_{\epsilon u} + 6\Omega_\epsilon^{-1}\Omega_{\epsilon u}$ as $(N, T \to \infty)_{seq}$.

The first term on R.H.S. of δ_{NT} converges as follows

$$(5.291)\ \frac{1}{N}\Sigma_{i=1}^{N}\frac{1}{T^2}\Sigma_{i=1}^{T}(x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^{/} \xrightarrow{p} \frac{1}{6}\Omega_\epsilon \quad as \quad (N, T \to \infty)_{seq}.$$

The denominator of 6 in the fraction on the R.H.S. above follows since the data has been demeaned. Thus we have demeaned Brownian motion.

The second term on R.H.S. converges as follows

$$\frac{1}{N}\Sigma_{i=1}^{N}\Omega_\epsilon^{\frac{1}{2}}\left(\int_0^1 \tilde{W}_i(r)dW_i^{/}(r)\right)\Omega_\epsilon^{-\frac{1}{2}}\Omega_{\epsilon u}+\Delta_{\epsilon u} \xrightarrow{p} -\frac{1}{2}\Omega_{\epsilon u}+\Delta_{\epsilon u} \quad as \quad N \to \infty.$$
(5.292)

The denominator of -2 in the fraction in the first term on the R.H.S. above follows since when using demeaned Brownian motion we get $E\left(\int_0^1 \tilde{W}_i(r)dW_i^{/}(r)\right) = -\frac{1}{2}I_N$. So

$$(5.293) \qquad \delta_{NT} \xrightarrow{p} \left(6\Omega_\epsilon^{-1}\right)\left(-\frac{1}{2}\Omega_{\epsilon u} + \Delta_{\epsilon u}\right) \quad as \quad (N, T \to \infty)_{seq}$$

and

$$(5.294) \qquad \delta_{NT} \xrightarrow{p} -3\Omega_\epsilon^{-1}\Omega_{\epsilon u} + 6\Omega_\epsilon^{-1}\Delta_{\epsilon u} \quad as \quad (N, T \to \infty)_{seq}.$$

In our model

$$\delta_{NT} = \left[\frac{1}{N}\Sigma_{i=1}^{N}\frac{1}{T^2}\Sigma_{i=1}^{T}x_{it}x_{it}^{/}\right]^{-1}\left[\frac{1}{N}\Sigma_{i=1}^{N}\Lambda_{i22}^{*}\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*} + \Gamma_{i21}^{*}\right].$$
(5.295)

Giving as before

$$(5.296) \qquad \frac{1}{N}\Sigma_{i=1}^{N}\frac{1}{T^2}\Sigma_{i=1}^{T}x_{it}x_{it}^{/} \xrightarrow{p} \frac{1}{2}\Omega_{22} \quad as \quad (N,T\to\infty)_{seq}$$

and

$$(5.297)\frac{1}{N}\Sigma_{i=1}^{N}\Lambda_{i22}^{*}\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*}+\Gamma_{i21}^{*} \xrightarrow{p} \Gamma_{21}^{*} \quad as \quad N\to\infty.$$

Note that in our case with the data not in deviation from mean form we get Brownian motion $E\left(\int_0^1 W_i(r)dW_i^{/}(r)\right)=\underline{0}$. *So the first term on R.H.S. above (involving* Ω_{21}^{*}*) drops out of the equation. Hence*

$$(5.298) \qquad \delta_{NT} \xrightarrow{p} 2\Omega_{22}^{-1}\Gamma_{21}^{*} \quad as \quad (N,T\to\infty)_{seq}.$$

Asymptotic Normality

For asymptotic normality we can apply the Lindeberg-Levy CLT to each element of the (2 × 1) block vector. Since we already have the limiting distribution of the (2 × 2) block matrix the desired result follows with an application of Slutsky's Theorem. Also we need only to apply the Lindeberg-Levy CLT to $\Lambda_{i22}^{*}\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*}+\Gamma_{i21}^{*}$ since we already have that of $\Lambda_{i11}^{*}W_{i1}^{*}(1)$ by $(a)^{/}$ with Λ_{i11} substituted by Λ_{i11}^{*}. This gives

$$(5.299) \qquad \frac{1}{\sqrt{N}}\sum_{i=1}^{N}(\Lambda_{i11}^{*}W_{i1}^{*}(1)) \xrightarrow{d} N(0,\Omega_{11}^{*}) \quad \text{as} \quad N\to\infty.$$

The Lindeberg-Levy CLT has already been applied to the first term in our bias equation above by $(b)^{/}$. However this was in the case of independence of $W_{i1}(r)$ and $W_{i2}(r)$ and $\Lambda_{i11}\Lambda_{i22} = \underline{0}$. With OLS these two are correlated so as above we write

$(b)'$ Take $\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}$ and again to verify the conditions of the Lindeberg-Levy CLT write

$$Z_i = \Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}. \tag{5.300}$$

Now $var(Z_i)$ is

$$var\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) \tag{5.301}$$

and given

$$E\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) = \underline{0} \tag{5.302}$$

by the properties of Ito's Stohastic Calculus and the new calculation of (d) above then

$$var\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) = \tag{5.303}$$

$$\mathrm{E}\left[\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right)\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right)'\right]$$

$$= (\Lambda^*_{i22}\Lambda^{*2}_{i11}\Lambda^{*/}_{i22})E\left[\left(\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right)\left(\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right)'\right]. \tag{5.304}$$

Since now $(\Lambda^*_{i22}\Lambda^{*2}_{i11}\Lambda^{*/}_{i22}) = \Omega^*_{i21}\Omega^{*/}_{i21}$ a $(k \times k)$ matrix as in (d) above, this gives the first term on R.H.S. For the second term on R.H.S. when $W^*_{i1}(r)$ and $W^*_{i2}(r)$ are correlated we have by the properties of Ito's Stochastic Integral

$$\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)] = \left(\frac{1}{2}I_k\right)\left(\chi^2(1) - 1\right) \tag{5.305}$$

but

(5.306) $$E\left(\int_0^1 [W_{i2}^*(r)][dW_{i1}^*(r)]\right) = \underline{0}$$

and

(5.307) $$var\left(\int_0^1 [W_{i2}^*(r)][dW_{i1}^*(r)]\right) = \frac{1}{2}I_k.$$

Hence we now get

(5.308) $$var(Z_i) = (\Omega_{21}^*\Omega_{21}^{*\prime}) \times \frac{1}{2}I_k = \frac{1}{2}\Omega_{21}^*\Omega_{21}^{*\prime} < \infty \neq \underline{0}.$$

Hence we have satisfied the conditions of Theorem (5.3.6) and given $E(Z_i) = \mu$ we have $E(\Lambda_{i22}^*\left\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\right\}\Lambda_{i11}^*) = \underline{0}$ and so by the Lindberg-Levy CLT

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\Lambda_{i22}^*\left\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\right\}\Lambda_{i11}^*}{\sqrt{\frac{1}{2}\Omega_{21}^*\Omega_{21}^{*\prime}}}\right) \xrightarrow{d} N(\underline{0}, I_k) \quad \text{as} \quad N \rightarrow \infty.$$

(5.309)

Hence we can also write

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda_{i22}^*\left\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\right\}\Lambda_{i11}^*\right) \xrightarrow{d} N\left(\underline{0}, \frac{1}{2}\Omega_{21}^*\Omega_{21}^{*\prime}\right) \quad \text{as} \quad N \rightarrow \infty.$$

(5.310)

(c)$'$ Now take $\Lambda_{i22}^*\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\}\Lambda_{i11}^* + \Gamma_{i21}^*$. We must verify the conditions of the Lindeberg-Levy CLT here. Let

(5.311) $$Z_i = \Lambda_{i22}^*\left\{\int_0^1[W_{i2}^*(r)][dW_{i1}^*(r)]\right\}\Lambda_{i11}^* + \Gamma_{i21}^*.$$

Then $var(Z_i)$ can be found from $var(Z_i) = E(Z_i - E(Z_i))(Z_i - E(Z_i))'$. Again note $E(Z_i) = \Gamma^*_{i21}$ because Γ^*_{i21} is a constant and

$$E\left(\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) = \underline{0} \tag{5.312}$$

from the new $(b)'$ above then

$$var\left(\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11} + \Gamma^*_{i21}\right) \tag{5.313}$$

$$= E\left[\left(\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right)\left(\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right)'\right]. \tag{5.314}$$

This has been found before in $(b)'$. Then using this information we have

$$var(Z_i) = (\Omega^*_{21}\Omega^{*\prime}_{21}) \times \frac{1}{2}I_k = \frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21} < \infty \neq \underline{0}. \tag{5.315}$$

Hence we have satisfied the conditions of Theorem (5.3.7) and given $E(Z_i) = \Gamma^*_{i21}$ we have by the Lindberg-Levy CLT

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}}{\sqrt{\frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21}}}\right) \xrightarrow{d} N(\underline{0}, I_k) \quad \text{as} \quad N \to \infty. \tag{5.316}$$

Hence we can also write

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda^*_{i22}\left\{\int_0^1 [W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) \xrightarrow{d} N\left(\underline{0}, \frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21}\right) \quad \text{as} \quad N \to \infty. \tag{5.317}$$

To find the off-diagonal elements in the asymptotic covariance matrix we have

$$(5.318) \qquad cov\left(\Lambda^*_{i11}W^*_{i1}(1), \Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21}\right)$$

$$(5.319) \qquad = E\left(\Lambda^*_{i11}W^*_{i1}(1)\left(\Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11}\right)\right)$$

$$(5.320) \qquad = E\left(\Lambda^*_{i11}W^*_{i1}(1)\Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11}\right) = \underline{0}.$$

The term on the R.H.S. equals zero by the properties of Ito's Stochastic Calculus when $W^*_{i1}(r)$ and $W^*_{i2}(r)$ are correlated.

Using the Lindeberg-Levy CLT we have now shown that as $N \to \infty$ by the above

$$(5.321) \qquad \left[\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda^*_{i11}W^*_{i1}(1) \\ \Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21} \end{bmatrix}\right]$$
$$\xrightarrow{d} N\left(\begin{bmatrix} 0 \\ \underline{0} \end{bmatrix}, \begin{bmatrix} \Omega^*_{11} & \underline{0}' \\ \underline{0} & \frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21} \end{bmatrix}\right).$$

Our final asymptotic normality result comes from using the Slutsky device of Proposition (5.3.10) applied to our equation

$$(5.322) \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix} \Rightarrow$$

$$\left[\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W^{*\prime}_{i2}(r)dr\Lambda^{*\prime}_{i22} \\ \Lambda^*_{i22}\int_0^1 W^*_{i2}(r)dr & \Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][W^*_{i2}(r)]'dr\}\Lambda^{*\prime}_{i22} \end{bmatrix}\right]^{-1}$$
$$\times\left[\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda^*_{i11}W^*_{i1}(1) \\ \Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21} \end{bmatrix}\right].$$

Which has asymptotic distribution (and using the result $\Omega^*_{22} = \Omega_{22}$)

$$(5.323) \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix}$$

$$\Rightarrow N\left(\begin{bmatrix} 0 \\ 2\Gamma^*_{21}\Omega^{-1}_{22} \end{bmatrix}, \begin{bmatrix} 1 & \underline{0}' \\ \underline{0} & 2\Omega^{-1}_{22} \end{bmatrix} \begin{bmatrix} \Omega^*_{11} & \underline{0}' \\ \underline{0} & \frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21} \end{bmatrix} \begin{bmatrix} 1 & \underline{0}' \\ \underline{0} & 2\Omega^{-1\prime}_{22} \end{bmatrix}\right).$$

So

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS1} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS1} - \beta) \end{bmatrix} \Rightarrow N\left(\begin{bmatrix} 0 \\ 2\Gamma^*_{21}\Omega^{-1}_{22} \end{bmatrix}, \begin{bmatrix} \Omega^*_{11} & \underline{0}' \\ \underline{0} & 2\Omega^{-1}_{22}\Omega^*_{21}\Omega^{*\prime}_{21}\Omega^{-1\prime}_{22} \end{bmatrix}\right).$$

(5.324)

QED

Lemma (5.4.15)

Proof

(a) Given

(5.325) $$\sum_{t=1}^{T} t = \frac{T(T+1)}{2} = \frac{T^2}{2} + \frac{T}{2}.$$

Then the leading term in $\sum_{t=1}^{T} t$ is $\frac{T^2}{2}$ thus

(5.326) $$\left(\frac{1}{T^2}\right) \sum_{t=1}^{T} t = \left(\frac{1}{T^2}\right) \left[\frac{T^2}{2} + \frac{T}{2}\right] = \frac{1}{2} + \frac{1}{2T} \to \frac{1}{2}$$

as $T \to \infty$.

QED

(b) Given

(5.327) $$\sum_{t=1}^{T} t^2 = \frac{T(T+1)(2T+1)}{6} = \frac{2T^3}{6} + \frac{3T^2}{6} + \frac{T}{6}.$$

Then the leading term in $\sum_{t=1}^{T} t^3$ is $\frac{T^3}{3}$ thus

(5.328) $$\left(\frac{1}{T^3}\right) \sum_{t=1}^{T} t^2 = \left(\frac{1}{T^3}\right) \left[\frac{2T^3}{6} + \frac{3T^2}{6} + \frac{T}{6}\right] = \frac{1}{3} + \frac{1}{2T} + \frac{1}{6T^2} \to \frac{1}{3}$$

as $T \to \infty$.

QED

(c) By induction we can see the general pattern. The leading term in $\sum_{t=1}^{T} t^v$ is $\frac{T^{v+1}}{(v+1)}$ thus

(5.329)
$$\left(\frac{1}{T^{(v+1)}}\right)\sum_{t=1}^{T} t^v \to \frac{1}{(v+1)}$$

as $T \to \infty$.

QED

(d) Using ξ^*_{it-1} and Lemma (5.4.5) (b) write

(5.330)
$$\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} t\xi^*_{it-1} = \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\left(\frac{t}{T}\right)\xi^*_{it-1} = \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} r\xi^*_{it-1}$$

where $r = \left(\frac{t}{T}\right)$. But we know from proof of Lemma (5.4.5) (b)

(5.331)
$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} r\xi^*_{it-1} \Rightarrow \Lambda_i \int_0^1 rW_i(r)dr.$$

So that

(5.332)
$$\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} t\xi^*_{it-1} \Rightarrow \Lambda_i \int_0^1 rW_i(r)dr$$

which implies

(5.333)
$$\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} t\xi^*_{it} \Rightarrow \Lambda_i \int_0^1 rW_i(r)dr.$$

Now for $\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} tx'_{it}$ we have

(5.334)
$$\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} tx'_{it} = [0 \quad I_k]\left[\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} t\xi^{*\prime}_{it}\right] \Rightarrow \int_0^1 r[W_{i2}(r)]'dr\Lambda'_{i22}$$

as $T \to \infty$.

QED

(e) We know that

$$\sum_{t=1}^{T} \xi_{it-1}^{*} = w_{i1}+(w_{i1}+w_{i2})+(w_{i1}+w_{i2}+w_{i3})+\ldots+(w_{i1}+w_{i2}+w_{i3}+\ldots+w_{iT-1})$$

(5.335)

$$= \sum_{t=1}^{T}(T-t)w_{it} = T\sum_{t=1}^{T} w_{it} - \sum_{t=1}^{T} tw_{it}. \tag{5.336}$$

So then

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} \xi_{it-1}^{*} = \frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} w_{it} - \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tw_{it}. \tag{5.337}$$

Hence

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tw_{it} = \frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} w_{it} - \frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} \xi_{it-1}^{*}. \tag{5.338}$$

But we know from (c) that

$$\frac{1}{T^{\frac{1}{2}}}\sum_{t=1}^{T} w_{it} = \sqrt{T}G_T(r) \tag{5.339}$$

evaluated at $r = 1$. Since $\sqrt{T}G_T(r) \Rightarrow P_i\Psi(1)W_i(r) = \Lambda_i W_i(r)$ then when evaluated at $r = 1$ $\sqrt{T}G_T(r) \Rightarrow \Lambda_i W_i(1)$. Similarly

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} \xi_{it-1}^{*} \Rightarrow \Lambda_i \int_0^1 W_i(r)dr \tag{5.340}$$

by proof of Lemma (5.4.5) (b) so that

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tw_{it} \Rightarrow \Lambda_i W_i(1) - \Lambda_i \int_0^1 W_i(r)dr. \tag{5.341}$$

This holds for all s=0,1,2,... Now for $\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} t\varepsilon_{it}$ we have

$$(5.342)\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} t\varepsilon_{it} = [0 \quad I_k]\left[\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tw_{it}\right] \Rightarrow \Lambda_{i22}W_{i2}(1) - \Lambda_{i22}\int_0^1 W_{i2}(r)dr$$

as $T \to \infty$.

QED

(f) This follows by (e) above by writing for $\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tz_{it}$

$$(5.343)\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tz_{it} = [1 \quad \underline{0}]\left[\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} tw_{it}\right] \Rightarrow \Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr.$$

as $T \to \infty$.

QED

Theorem (5.4.16)

Proof

We have from equation (5.105) the panel DOLS estimator

$$\hat{\Upsilon}_{DOLS2} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' & m_{it}t \\ m_{it}' & 1 & x_{it}' & t \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' & x_{it}t \\ tm_{it}' & t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}y_{it} \\ y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right).$$

(5.344)

Substitute for y_{it} to obtain

$$(\hat{\Upsilon}_{DOLS2} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' & m_{it}t \\ m_{it}' & 1 & x_{it}' & t \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' & x_{it}t \\ tm_{it}' & t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}z_{it} \\ z_{it} \\ x_{it}z_{it} \\ tz_{it} \end{bmatrix}\right).$$

(5.345)

Rescaling again by D_T to obtain a non-degenerate limiting distribution we note that not only is β "superconsistent" but also that $\hat{\delta}_{DOLS2}$ the time trend coefficient converges at rate $T^{\frac{3}{2}}$ so

$$D_T = \begin{bmatrix} \sqrt{NT}I_{(2p+1)k} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0}' & \sqrt{NT} & \underline{0}' & 0 \\ \underline{0}' & \underline{0} & \sqrt{NT}I_k & \underline{0} \\ \underline{0}' & 0 & \underline{0}' & \sqrt{NT^3} \end{bmatrix}. \tag{5.346}$$

Hence

$$D_T(\hat{\Upsilon}_{DOLS2} - \Upsilon) \tag{5.347}$$

$$= \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}I_{(2p+1)k} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ \underline{0}' & 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}m_{it}' & m_{it} & m_{it}x_{it}' & m_{it}t \\ m_{it}' & 1 & x_{it}' & t \\ x_{it}m_{it}' & x_{it} & x_{it}x_{it}' & x_{it}t \\ tm_{it}' & t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1}$$

$$\begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}I_{(2p+1)k} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ \underline{0}' & 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix}$$

$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}I_{(2p+1)k} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0}' & \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ \underline{0}' & 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} m_{it}z_{it} \\ z_{it} \\ x_{it}z_{it} \\ tz_{it} \end{bmatrix}\right).$$

So

$$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS2} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2} - \delta) \end{bmatrix} \tag{5.348}$$

$$= \begin{bmatrix} N^{-1}T^{-1}\Sigma\Sigma m_{it}m_{it}' & N^{-1}T^{-1}\Sigma\Sigma m_{it} & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma m_{it}x_{it}' & N^{-1}T^{-2}\Sigma\Sigma m_{it}t \\ N^{-1}T^{-1}\Sigma\Sigma m_{it}' & 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}' & N^{-1}T^{-2}\Sigma\Sigma t \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}m_{it}' & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}' & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma x_{it}t \\ N^{-1}T^{-2}\Sigma\Sigma m_{it}'t & N^{-1}T^{-2}\Sigma\Sigma t & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma x_{it}'t & N^{-1}T^{-3}\Sigma\Sigma t^2 \end{bmatrix}^{-1} \times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma m_{it}z_{it} \\ N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma z_{it} \\ N^{-\frac{1}{2}}T^{-1}\Sigma\Sigma x_{it}z_{it} \\ N^{-\frac{1}{2}}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}z_{it} \end{bmatrix}.$$

We now apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above more conveniently as

$$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS2} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2} - \delta) \end{bmatrix} \tag{5.349}$$

$$= \left(\frac{1}{N}\sum_{i=1}^{N} \begin{bmatrix} T^{-1}\sum_{t=1}^{T} m_{it}m_{it}' & T^{-1}\sum_{t=1}^{T} m_{it} & T^{-\frac{3}{2}}\sum_{t=1}^{T} m_{it}x_{it}' & T^{-2}\sum_{t=1}^{T} m_{it}t \\ T^{-1}\sum_{t=1}^{T} m_{it}' & 1 & T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it}' & T^{-2}\sum_{t=1}^{T} t \\ T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it}m_{it}' & T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it} & T^{-2}\sum_{t=1}^{T} x_{it}x_{it}' & T^{-\frac{5}{2}}\sum_{t=1}^{T} x_{it}t \\ T^{-2}\sum_{t=1}^{T} m_{it}'t & T^{-2}\sum_{t=1}^{T} t & T^{-\frac{5}{2}}\sum_{t=1}^{T} x_{it}'t & T^{-3}\sum_{t=1}^{T} t^2 \end{bmatrix} \right)^{-1} \times \left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \begin{bmatrix} T^{-\frac{1}{2}}\sum_{t=1}^{T} m_{it}z_{it} \\ T^{-\frac{1}{2}}\sum_{t=1}^{T} z_{it} \\ T^{-1}\sum_{t=1}^{T} x_{it}z_{it} \\ T^{-\frac{3}{2}}\sum_{t=1}^{T} tz_{it} \end{bmatrix} \right).$$

By Lemma (5.4.15) and the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3) as $T \to \infty$, and N is held fixed we have by applying the FCLT to each element of the (4×4) block matrix and (4×1) block vector the following.

Let

$$\frac{1}{T^2}\sum_{t=1}^{T} m_{it}t = \frac{1}{T^2}\sum_{t=1}\left(\varepsilon_{it-p}^{/}, \varepsilon_{it-p+1}^{/} \ldots, \varepsilon_{it-1}^{/}, \varepsilon_{it}^{/}, \varepsilon_{it+1}^{/}, \ldots, \varepsilon_{it+p}^{/}\right)^{/} t \tag{5.350}$$

$$= \left(\frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it-p}^{/}t, \frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it-p+1}^{/}t \ldots, \frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it+p}^{/}t\right)^{/} \tag{5.351}$$

a $((2p+1)k \times 1)$ vector. But by Lemma (5.4.15) (e)

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} t\varepsilon_{it} \Rightarrow \Lambda_{i22}W_{i2}(1) - \Lambda_{i22}\int_0^1 W_{i2}(r)dr. \tag{5.352}$$

Hence

$$\frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it-p}t \Rightarrow 0 \quad \forall p = -1, -2, \ldots, 0, +1, +2, \ldots \tag{5.353}$$

a $(k \times 1)$ null vector and so

$$\frac{1}{T^2}\sum_{t=1}^{T} m_{it}t = \left(\frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it-p}^{/}t, \frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it-p+1}^{/}t \ldots, \frac{1}{T^2}\sum_{t=1}^{T}\varepsilon_{it+p}^{/}t\right)^{/} \tag{5.354}$$

$$\Rightarrow (0', 0', \ldots, 0')' \quad \forall p = -1, -2, \ldots, 0, +1, +2, \ldots \tag{5.355}$$

So

$$\frac{1}{T^2}\sum_{t=1}^{T} m_{it}t \Rightarrow \underline{0} \tag{5.356}$$

a $((2p+1)k \times 1)$ null vector. By the rest of the Lemmas (5.4.5) and (5.4.15)

$$\begin{bmatrix} \sqrt{NT}(\hat{\xi}_{DOLS2} - \xi) \\ \sqrt{NT}(\hat{\alpha}_{DOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2} - \delta) \end{bmatrix} \Rightarrow \tag{5.357}$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} V_i & \underline{0} & \underline{0} & \underline{0} \\ \underline{0}' & 1 & \int_0^1 W_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{2} \\ \underline{0}' & \Lambda_{i22}\int_0^1 W_{i2}(r)dr & \Lambda_{i22}\{\int_0^1[W_{i2}(r)][W_{i2}(r)]'dr\}\Lambda_{i22}' & \Lambda_{i22}\int_0^1 rW_{i2}(r)dr \\ \underline{0}' & \frac{1}{2} & \int_0^1 rW_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{3}\end{bmatrix}\right)^{-1}$$

$$\times\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix}\Xi_i \\ \Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \\ \Lambda_{i11}W_{i1}(1)-\int_0^1\Lambda_{i11}W_{i1}(r)dr\end{bmatrix}\right).$$

The first (4×4) matrix is block diagonal and hence we can see for the stationary I(0) regressors Δx_{it-p} $\forall p=-1,-2,\ldots,0,+1,+2,\ldots.$ the coefficient $\hat{\xi}_{DOLS2}$ has a Gaussian distribution that is given again by

$$\sqrt{NT}(\hat{\xi}_{DOLS2}-\xi)\Rightarrow\left(\frac{1}{N}\sum_{i=1}^{N}V_i\right)^{-1}\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Xi_i\right) \tag{5.358}$$

where $\Xi_i\sim N(\underline{0},V_i\Lambda_{11}^2)$.

Again for the second stage of the sequential limit theory as $N\to\infty$ let us look at the (3×3) lower block diagonal matrix of the parameters of interest, ie

$$\begin{bmatrix}\sqrt{NT}(\hat{\alpha}_{DOLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2}-\delta)\end{bmatrix}\Rightarrow \tag{5.359}$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{2} \\ \Lambda_{i22}\int_0^1 W_{i2}(r)dr & \Lambda_{i22}\{\int_0^1[W_{i2}(r)][W_{i2}(r)]'dr\}\Lambda_{i22}' & \Lambda_{i22}\int_0^1 rW_{i2}(r)dr \\ \frac{1}{2} & \int_0^1 rW_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{3}\end{bmatrix}\right)^{-1}$$

$$\times\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix}\Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \\ \Lambda_{i11}W_{i1}(1)-\Lambda_{i11}\int_0^1 W_{i1}(r)dr\end{bmatrix}\right).$$

Again under the assumption of a homogeneous panel each element of the (3×3) block matrix and (3×1) block vector are independent and identically distributed random variables for all i. Hence we can apply the Lindeberg-Levy CLT to each element of the block vector and the Komolgorov SLLN to each element of both the block matrix and the block vector. We can now show the asymptotic consistency and asymptotic normality of our parameters of interest $\hat{\alpha}_{DOLS2}$,$\hat{\beta}_{DOLS2}$ and $\hat{\delta}_{DOLS2}$. For asymptotic consistency we again use Theorem (5.3.5) Komolgorov's SLLN. Taking note again that the limit of the inverse of a matrix is the inverse of the limit by the CMT. Then we shall again apply the Komolgorov SLLN to each element of the (3×3) block matrix and (3×1) block vector before inverting as follows.

Asymptotic Consistency

First note that elements (a)-(d) of the (3×3) block matrix have been considered in the proof of Theorem (5.4.6), as well as (e) so we shall consider only elements (f) and (g).

(f) Take $\Lambda_{i22} \int_0^1 rW_{i2}(r)dr$. We must first verify the conditions of Komolgorov's SLLN Theorem (5.3.5) so write

$$Z_i = \Lambda_{i22} \int_0^1 rW_{i2}(r)dr. \tag{5.360}$$

Then to show $E(Z_i) < \infty$ it suffices to show $E\left\|\Lambda_{i22} \int_0^1 rW_{i2}(r)dr\right\| < \infty$ as in Phillips and Moon (1999), Lemma 4.

By the Cauchy-Schwartz Inequality

$$(5.361)\quad E\left\|\Lambda_{i22}\int_0^1 rW_{i2}(r)dr\right\| \leq \left[E\left\|\Lambda_{i22}\right\|^2\right]^{\frac{1}{2}}\left[E\left\|\int_0^1 rW_{i2}(r)dr\right\|^2\right]^{\frac{1}{2}}$$

$$(5.362)\quad \leq \left[E\left(\sqrt{tr(\Lambda_{i22}\Lambda_{i22}^{\prime})}\right)^2\right]^{\frac{1}{2}}$$

$$\times\left[E\left(\sqrt{tr\left(\int_0^1 rW_{i2}(r)dr\right)\left(\int_0^1 rW_{i2}(r)dr\right)^{\prime}}\right)^2\right]^{\frac{1}{2}}$$

$$\leq \left[E\left(tr(\Lambda_{i22}\Lambda_{i22}^{\prime}\lambda)\right)\right]^{\frac{1}{2}}\left[E\left(tr\left(\int_0^1 rW_{i2}(r)dr\right)\left(\int_0^1 rW_{i2}(r)dr\right)^{\prime}\right)\right]^{\frac{1}{2}}.$$

(5.363)

Interchanging the expectation operator with the trace operator

$$\leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}}\left[tr\left(E\left(\int_0^1 rW_{i2}(r)dr\right)\left(\int_0^1 rW_{i2}(r)dr\right)^{\prime}\right)\right]^{\frac{1}{2}}.$$

(5.364)

Now on evaluating the integrals in second term of the R.H.S.

$$(5.365)\quad \leq \left[tr\left(E(\Lambda_{i22}\Lambda_{i22}^{\prime})\right)\right]^{\frac{1}{2}}\left[tr\left(\int_0^1\int_0^1 rsE[W_{i2}(s)][W_{i2}(t)]^{\prime}dsdt\right)\right]^{\frac{1}{2}}$$

$$(5.366)\quad \leq \left[tr\left(\Omega_{22}\right)\right]^{\frac{1}{2}}\left[tr\left(\frac{2}{15}I_k\right)\right]^{\frac{1}{2}} < \infty.$$

Again by the proof of Theorem (5.4.6) and Assumption (5.4.2) (ii) and (vi), $\mathrm{tr}(\Omega_{22}) = \Omega_{1122}+\Omega_{2222}+\ldots+\Omega_{kk22} < \infty$. Similarly $tr(I_k) = 1+1+\ldots+1 = k$ so $tr(\frac{2}{15}I_k) < \infty$. Hence

$$E\left\|\Lambda_{i22}\int_0^1 rW_{i2}(r)dr\right\| \leq \left(\sqrt{\Omega_{1122}+\Omega_{2222}+\ldots+\Omega_{kk22}}\right)\left(\sqrt{\frac{2k}{15}}\right) < \infty.$$

(5.367)

So we have verified the conditions of Theorem (5.3.5) and given $E(Z_i) = E\left(\Lambda_{i22}\int_0^1 rW_{i2}(r)dr\right) = \underline{0}$ then by Komolgorov's SLLN

$$\frac{1}{N}\sum_{i=1}^{N}\Lambda_{i22}\int_0^1 rW_{i2}(r)dr \overset{a.s.}{\to} \underline{0} \quad \text{as} \quad N \to \infty. \tag{5.368}$$

(g) Take now $\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr$.

This we can split into two parts since

$$\frac{1}{N}\sum_{i=1}^{N}(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr) \tag{5.369}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\Lambda_{i11}W_{i1}(1) - \frac{1}{N}\sum_{i=1}^{N}\Lambda_{i11}\int_0^1 W_{i1}(r)dr. \tag{5.370}$$

The first term on the R.H.S. is case (c) of the above so we need only to take $\Lambda_{i11}\int_0^1 W_{i1}(r)dr$ and thus verify the conditions of Komolgorov SLLN Theorem (5.3.4). Note the above is a scalar composition so

$$Z_i = \Lambda_{i11}\int_0^1 W_{i1}(r)dr \quad \text{and} \quad |Z_i| = \left|\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right|. \tag{5.371}$$

Now we verify $E|Z_i| = E\left|\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right| < \infty$. By the Cauchy-Schwartz Inequality

$$E\left|\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right| \leq \left[E\left|\Lambda_{i11}\right|^2\right]^{\frac{1}{2}}\left[E\left|\int_0^1 W_{i1}(r)dr\right|^2\right]^{\frac{1}{2}} \tag{5.372}$$

$$\leq (\sqrt{\Omega_{11}})(\sqrt{\frac{1}{3}}) < \infty. \tag{5.373}$$

By Assumption (5.4.2) (ii) and (vi) and evaluating the integral. Thus we have verified the conditions of Theorem (5.3.4) and since $E\left(\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) =$

$\mu = 0$ it follows,

$$(5.374)\qquad \frac{1}{N}\sum_{i=1}^{N}\Lambda_{i11}\int_0^1 W_{i1}(r)dr \xrightarrow{a.s.} 0 \quad \text{as} \quad N \to \infty.$$

Hence it follows

$$(5.375)\qquad \frac{1}{N}\sum_{i=1}^{N}\left(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \xrightarrow{a.s.} 0 \quad \text{as} \quad N \to \infty.$$

Using Komolgorov's SLLN we have now shown that as $N \to \infty$ by (a)-(g)[24]

$$(5.376)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2} - \delta) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \underline{0}' & \frac{1}{2} \\ \underline{0} & \frac{1}{2}\Omega_{22} & \underline{0} \\ \frac{1}{2} & \underline{0}' & \frac{1}{3} \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}.$$

Thus

$$(5.377)\qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2} - \delta) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}.$$

Hence $\hat{\alpha}_{DOLS2} \xrightarrow{p} \alpha$, $\hat{\beta}_{DOLS2} \xrightarrow{p} \beta$ and $\hat{\delta}_{DOLS2} \xrightarrow{p} \delta$ as $(N, T \to \infty)_{seq}$. Hence we have shown that $\hat{\alpha}_{DOLS2}, \hat{\beta}_{DOLS2}$ and $\hat{\delta}_{DOLS2}$ are asymptotically consistent estimators as $(N, T \to \infty)_{seq}$.

Asymptotic Normality

For asymptotic normality we apply the Lindeberg-Levy CLT to each element of the (3×1) block vector. Since we already have the limiting distribution of the block (3×3) matrix the desired result follows after an application of Proposition (5.3.9) Slutsky's device.

First note that elements $(a)' - (b)'$ of the (3×1) block vector have already

[24] Since $\frac{1}{2}$ and $\frac{1}{3}$ are constants there is no change on using the SLLN.

been considered in the proof of Theorem (5.4.6) so we shall consider only element $(d)'$ given by

$(d)'$ Take $\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr$, we must first verify the conditions of the Lindeberg-Levy CLT. That is given Z_i then show $var(Z_i) = \sigma^2 < \infty \neq 0$. Since $\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr$ is a scalar write

$$Z_i = \Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr. \tag{5.378}$$

Then

$$var(Z_i) = var\left(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \tag{5.379}$$

$$= var\left(\Lambda_{i11}W_{i1}(1)\right) - 2cov\left(\Lambda_{i11}W_{i1}(1), \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) + var\left(\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right)$$

(5.380)

$$= E\left(\Lambda_{i11}^2 W_{i1}(1)^2\right) - 2E\left(\Lambda_{i11}^2\int_0^1 W_{i1}(1)W_{i1}(r)dr\right) \tag{5.381}$$

$$+E\left(\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right)\left(\Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \tag{5.382}$$

$$= \Omega_{11} - 2\Omega_{11}\int_0^1 E[W_{i1}(1)][W_{i1}(r)]dr + \Omega_{11}\int_0^1\int_0^1 E[W_{i1}(r)][W_{i1}(r)]drds.$$

(5.383)

For the second term on the R.H.S. we have by the properties of Wiener processses the covariance $E[W_{i1}(1)][W_{i1}(r)] = min(r, 1)$ and since $r \leq 1$ it follows $E[W_{i1}(1)][W_{i1}(r)] = r$. The last term follows by (a) above. Thus

$$var(Z_i) = \Omega_{11} - 2\frac{1}{2}\Omega_{11} + \frac{1}{3}\Omega_{11} = \frac{1}{3}\Omega_{11} < \infty \neq 0. \tag{5.384}$$

Also

$$E\left(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \tag{5.385}$$

$$= \Lambda_{i11}E\left(W_{i1}(1)\right) - \Lambda_{i11}\int_0^1 E\left(W_{i1}(r)\right)dr = 0. \tag{5.386}$$

Hence we have satisfied the conditions of Theorem (5.3.4) and given $=$ $E\left(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) = \mu = 0$ then by the Lindeberg-Levy CLT

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr}{\sqrt{\frac{1}{3}\Omega_{11}}}\right) \xrightarrow{d} N(0,1) \quad \text{as} \quad N \to \infty. \tag{5.387}$$

Hence

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \xrightarrow{d} N\left(0, \frac{1}{3}\Omega_{11}\right) \quad \text{as} \quad N \to \infty. \tag{5.388}$$

Using Lindeberg-Levy's CLT we have now shown that as $N \to \infty$

by $(a)' - (d)'$

$$\left[\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \\ \Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr \end{bmatrix}\right] \xrightarrow{d} N\left(\begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \underline{0}' & \frac{1}{2}\Omega_{11} \\ \underline{0} & \frac{1}{2}\Omega_{22}\Omega_{11} & \underline{0} \\ \frac{1}{2}\Omega_{11} & \underline{0}' & \frac{1}{3}\Omega_{11} \end{bmatrix}\right). \tag{5.389}$$

Note again as in the proof of Theorem (5.4.6) the off-diagonal elements in the covariance matrix. These follow since

$$E\begin{bmatrix} \Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \\ \Lambda_{i11}W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr \end{bmatrix} \tag{5.390}$$

$$\times \left[\Lambda_{i11} W_{i1}(1), \quad \Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}, \quad \Lambda_{i11} W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right] =$$

$$\begin{bmatrix} A & B & C \\ D & E^* & F \\ G & H & I \end{bmatrix} \tag{5.391}$$

where $A - I$ are given by

$$A = E\left[\Lambda_{i11} W_{i1}(1) W_{i1}(1) \Lambda_{i11}\right] = \Omega_{11} \tag{5.392}$$

as in the proof of Theorem (5.4.6) $(a)'$.

$$B = E\left[\Lambda_{i11} W_{i1}(1)\Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right] = \underline{0}. \tag{5.393}$$

$$C = E\left[\Lambda_{i11} W_{i1}(1)\left(\Lambda_{i11} W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right)\right] = \tag{5.394}$$

$$= E\left(\Lambda_{i11}^2 W_{i1}(1)^2\right) - \Lambda_{i11}^2 E\left(\int_0^1 [W_{i1}(1)][W_{i1}(r)]dr\right) \tag{5.395}$$

$$= \Omega_{11} - \Omega_{11}\int_0^1 E[W_{i1}(1)][W_{i1}(r)]dr \tag{5.396}$$

As above $E[W_{i1}(1)][W_{i1}(r)] = min(r, 1) = r$ for $r \leq 1$ so on evaluating the integral

$$= \Omega_{11} - \Omega_{11}\int_0^1 r dr = \Omega_{11} - \frac{1}{2}\Omega_{11} = \frac{1}{2}\Omega_{11}. \tag{5.397}$$

$D = \underline{0}$ since it follows B.

$$E^* = E\left(\Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right)\left(\Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right)' = \frac{1}{2}\Omega_{22}\Omega_{11} \tag{5.398}$$

as in proof of Theorem (5.4.6) $(b)'$.

$$F = E\left(\Lambda_{i22}\{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11}\right)\left(\Lambda_{i11} W_{i1}(1) - \Lambda_{i11}\int_0^1 W_{i1}(r)dr\right) \tag{5.399}$$

$$= E\left[\Lambda_{i11} W_{i1}(1) \Lambda_{i22} \{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\} \Lambda_{i11}\right] \tag{5.400}$$

$$-E\left[\Lambda_{i11} \int_0^1 W_{i1}(r) dr \Lambda_{i22} \{\int_0^1 [W_{i2}(r)][dW_{i1}(r)]\} \Lambda_{i11}\right] = \underline{0} - \underline{0} = \underline{0}. \tag{5.401}$$

The first zero on R.H.S. follows from B and the second zero follows on evaluating the integrals

$$G = \frac{1}{2}\Omega_{11} \tag{5.402}$$

as it follows C.

$H = \underline{0}$ as it follows F.

$$I = E\left(\Lambda_{i11} W_{i1}(1) - \Lambda_{i11} \int_0^1 W_{i1}(r) dr\right)\left(\Lambda_{i11} W_{i1}(1) - \Lambda_{i11} \int_0^1 W_{i1}(r) dr\right)' = \frac{1}{3}\Omega_{11} \tag{5.403}$$

as in $(d)'$ above. Finally we need to calculate the inverse of our (3×3) block covariance matrix called P, say, using the partitioned matrix method of Proposition (5.3.11). Now for our $(k+2 \times k+2)$ matrix

$$\begin{bmatrix} 1 & \underline{0}' & \frac{1}{2} \\ \underline{0} & \frac{1}{2}\Omega_{22} & \underline{0} \\ \frac{1}{2} & \underline{0}' & \frac{1}{3} \end{bmatrix} \tag{5.404}$$

let

$B = 1$ a (1×1) scalar

$C' = [0, 0, \ldots, 0, \frac{1}{2}] = [\underline{0}', \frac{1}{2}]$ where $\underline{0}'$ is a $(1 \times k)$ vector of zeros. Hence C' is a $(1 \times k+1)$ vector.

$$C = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \frac{1}{2} \end{bmatrix}$$ where $\underline{0}$ is a $(k \times 1)$ vector of zeros.

$$D = \begin{bmatrix} \begin{bmatrix} \frac{1}{2}\Omega_{22} & \cdots \\ \vdots & \end{bmatrix} & \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \frac{1}{3} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega_{22} & \underline{0}' \\ \underline{0} & \frac{1}{3} \end{bmatrix}.$$

So our matrix $A = P$, say is

$$A = \begin{bmatrix} [1] & [0, 0, \dots, 0, \frac{1}{2}] \\ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} \frac{1}{2}\Omega_{22} & \cdots \\ \vdots & \end{bmatrix} & \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \frac{1}{3} \end{bmatrix} \end{bmatrix} \end{bmatrix}. \tag{5.405}$$

So by Proposition (5.3.11) we get

$B^{-1}(I - C'E^{-1}CB^{-1}) = 4.$

$$-B^{-1}C'E^{-1} = [0, 0. \dots, 0, 6] = [\underline{0}', 6]. \tag{5.406}$$

$$-E^{-1}CB^{-1} = \begin{bmatrix} 0. \\ 0 \\ \vdots \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} \underline{0} \\ 6 \end{bmatrix} \tag{5.407}$$

$$E^{-1} = \begin{bmatrix} \begin{bmatrix} \frac{1}{2}\Omega_{22} & \cdots \\ \vdots & \end{bmatrix} & \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \frac{1}{12} \end{bmatrix} \end{bmatrix}^{-1}. \tag{5.408}$$

Finally we get the matrix A^{-1} (or P^{-1} say), given by

$$= \begin{bmatrix} 4 & \underline{0}' & -6 \\ \underline{0} & 2\Omega_{22}^{-1} & \underline{0} \\ -6 & \underline{0}' & 12 \end{bmatrix}. \tag{5.409}$$

As in the proof of Theorem (5.4.6) our final asymptotic normality result comes from using the Slutsky Theorem applied to our equation

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2}-\delta) \end{bmatrix} \Rightarrow \tag{5.410}$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{2} \\ \Lambda_{i22}\int_0^1 W_{i2}(r)dr & \Lambda_{i22}\{\int_0^1[W_{i2}(r)][W_{i2}(r)]'dr\}\Lambda_{i22}' & \Lambda_{i22}\int_0^1 rW_{i2}(r)dr \\ \frac{1}{2} & \int_0^1 rW_{i2}'(r)dr\Lambda_{i22}' & \frac{1}{3} \end{bmatrix}\right)^{-1}$$
$$\times\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda_{i11}W_{i1}(1) \\ \Lambda_{i22}\{\int_0^1[W_{i2}(r)][dW_{i1}(r)]\}\Lambda_{i11} \\ \Lambda_{i11}W_{i1}(1)-\Lambda_{i11}\int_0^1 W_{i1}(r)dr \end{bmatrix}\right).$$

Which has asymptotic distribution

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2}-\delta) \end{bmatrix} \tag{5.411}$$

$$\Rightarrow N\left(\begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}, \begin{bmatrix} 4 & \underline{0}' & -6 \\ \underline{0} & 2\Omega_{22}^{-1} & \underline{0} \\ -6 & \underline{0}' & 12 \end{bmatrix}\begin{bmatrix} \Omega_{11} & \underline{0}' & \frac{1}{2}\Omega_{11} \\ \underline{0} & \frac{1}{2}\Omega_{22}\Omega_{11} & \underline{0} \\ \frac{1}{2}\Omega_{11} & \underline{0}' & \frac{1}{3}\Omega_{11} \end{bmatrix}\begin{bmatrix} 4 & \underline{0}' & -6 \\ \underline{0} & 2\Omega_{22}^{-1\prime} & \underline{0} \\ -6 & \underline{0}' & 12 \end{bmatrix}\right).$$

The asymptotic covariance matrix is

$$\begin{bmatrix} 4 & \underline{0}' & -6 \\ \underline{0} & 2\Omega_{22}^{-1} & \underline{0} \\ -6 & \underline{0}' & 12 \end{bmatrix}\begin{bmatrix} \Omega_{11} & \underline{0}' & \frac{1}{2}\Omega_{11} \\ \underline{0} & \frac{1}{2}\Omega_{22}\Omega_{11} & \underline{0} \\ \frac{1}{2}\Omega_{11} & \underline{0}' & \frac{1}{3}\Omega_{11} \end{bmatrix}\begin{bmatrix} 4 & \underline{0}' & -6 \\ \underline{0} & 2\Omega_{22}^{-1\prime} & \underline{0} \\ -6 & \underline{0}' & 12 \end{bmatrix} \tag{5.412}$$

$$= \begin{bmatrix} 4\Omega_{11} & \underline{0}' & -6\Omega_{11} \\ \underline{0} & 2\Omega_{22}^{-1}\Omega_{11} & \underline{0} \\ -6\Omega_{11} & \underline{0}' & 12\Omega_{11} \end{bmatrix}. \tag{5.413}$$

So

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{DOLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{DOLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{DOLS2}-\delta) \end{bmatrix} \Rightarrow N\left(\begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}, \begin{bmatrix} 4\Omega_{11} & \underline{0}' & -6\Omega_{11} \\ \underline{0} & 2\Omega_{22}^{-1}\Omega_{11} & \underline{0} \\ -6\Omega_{11} & \underline{0}' & 12\Omega_{11} \end{bmatrix}\right). \tag{5.414}$$

QED

Lemma (5.4.17)

Proof

(a) This follows from Lemma (5.4.15) (d) since it is the same for FMOLS except for the change in notation.

(b) This follows from Lemma (5.4.15) (f) with z_{it} substituted by $\hat{u}_{it}^{+}$.

Theorem (5.4.18)

Proof

We have from equation (5.117) the panel FMOLS estimator

$$\hat{\Upsilon}_{FMOLS2} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{y}_{it}^{+} \\ x_{it}\hat{y}_{it}^{+} - T\hat{\aleph}^{+} \\ t\hat{y}_{it}^{+} \end{bmatrix}\right).$$

(5.415)

Substituting for $\hat{y}_{it}^{+}$ we have

$$(\hat{\Upsilon}_{FMOLS2} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{u}_{it}^{+} \\ x_{it}\hat{u}_{it}^{+} - T\hat{\aleph}^{+} \\ t\hat{u}_{it}^{+} \end{bmatrix}\right).$$

(5.416)

Again rescale $(\hat{\Upsilon}_{FMOLS2} - \Upsilon)$ to obtain a non-degenerate limiting distribution. Here $\hat{\beta}_{FMOLS2}$ is superconsistent as before and converges at rate T

across the time-series dimension. Define

$$D_T = \begin{bmatrix} \sqrt{NT} & \underline{0}' & 0 \\ \underline{0} & \sqrt{NT} I_k & \underline{0} \\ 0 & \underline{0}' & \sqrt{NT^3} \end{bmatrix}. \tag{5.417}$$

So that

$$D_T(\hat{\Upsilon}_{FMOLS2} - \Upsilon) = \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix} \tag{5.418}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix}$$
$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix} \sum_{i=1}^{N}\left(\sum_{t=1}^{T}\begin{bmatrix} \hat{u}_{it}^{+} \\ x_{it}\hat{u}_{it}^{+} - T\hat{\aleph}^{+} \\ t\hat{u}_{it}^{+} \end{bmatrix}\right).$$

So

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{FMOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{FMOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{FMOLS2} - \delta) \end{bmatrix} \tag{5.419}$$

$$= \begin{bmatrix} 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}' & N^{-1}T^{-2}\Sigma\Sigma t \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}' & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma tx_{it} \\ N^{-1}T^{-2}\Sigma\Sigma t & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma tx_{it}' & N^{-1}T^{-3}\Sigma\Sigma t^2 \end{bmatrix}^{-1}$$
$$\times \begin{bmatrix} N^{-\frac{1}{2}}\Sigma(T^{-\frac{1}{2}}\Sigma\hat{u}_{it}^{+}) \\ N^{-\frac{1}{2}}\Sigma(T^{-1}\Sigma x_{it}\hat{u}_{it}^{+} - \hat{\aleph}^{+}) \\ N^{-\frac{1}{2}}\Sigma(T^{-\frac{3}{2}}\Sigma t\hat{u}_{it}^{+}) \end{bmatrix}.$$

We now apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above more conveniently as

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{FMOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{FMOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{FMOLS2} - \delta) \end{bmatrix} \tag{5.420}$$

$$= \left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} 1 & T^{-\frac{3}{2}} \sum_{t=1}^{T} x_{it}' & T^{-2} \sum_{t=1}^{T} t \\ T^{-\frac{3}{2}} \sum_{t=1}^{T} x_{it} & T^{-2} \sum_{t=1}^{T} x_{it} x_{it}' & T^{-\frac{5}{2}} \sum_{t=1}^{T} t x_{it} \\ T^{-2} \sum_{t=1}^{T} t & T^{-\frac{5}{2}} \sum_{t=1}^{T} t x_{it}' & T^{-3} \sum_{t=1}^{T} t^2 \end{bmatrix} \right)^{-1}$$

$$\times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} T^{-\frac{1}{2}} \sum_{t=1}^{T} \hat{u}_{it}^{+} \\ T^{-1} \sum_{t=1}^{T} x_{it} \hat{u}_{it}^{+} - \hat{\aleph}^{+} \\ T^{-\frac{3}{2}} \sum_{t=1}^{T} t \hat{u}_{it}^{+} \end{bmatrix} \right).$$

By Lemma (5.4.17) and the FCLT of Proposition (5.3.1) and the CMT of Lemma (5.3.3) as $T \to \infty$, and N is held fixed we have by applying the FCLT to each element of the (3×3) block matrix and (3×1) block vector the following.

By Lemma (5.4.12) and Lemma (5.4.17)

(5.421)
$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{FMOLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{FMOLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{FMOLS2} - \delta) \end{bmatrix} \Rightarrow$$

$$\left(\frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} 1 & \int_0^1 \bar{W}_{i2}^{+\prime}(r) dr \bar{\Lambda}_{i22}^{+\prime} & \frac{1}{2} \\ \bar{\Lambda}_{i22}^{+} \int_0^1 \bar{W}_{i2}^{+}(r) dr & \bar{\Lambda}_{i22}^{+} \{ \int_0^1 [\bar{W}_{i2}^{+}(r)][\bar{W}_{i2}^{+}(r)]' dr \} \bar{\Lambda}_{i22}^{+\prime} & \bar{\Lambda}_{i22}^{+} \int_0^1 r \bar{W}_{i2}^{+}(r) dr \\ \frac{1}{2} & \int_0^1 r \bar{W}_{i2}^{+\prime}(r) dr \bar{\Lambda}_{i22}^{+\prime} & \frac{1}{3} \end{bmatrix} \right)^{-1}$$

$$\times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} \bar{\Lambda}_{i11}^{+} \bar{W}_{i1}^{+}(1) \\ \bar{\Lambda}_{i22}^{+} \{ \int_0^1 [\bar{W}_{i2}^{+}(r)][d\bar{W}_{i1}^{+}(r)] \} \bar{\Lambda}_{i11}^{+} \\ \bar{\Lambda}_{i11}^{+} \bar{W}_{i1}^{+}(1) - \bar{\Lambda}_{i11}^{+} \int_0^1 \bar{W}_{i1}^{+}(r) dr \end{bmatrix} \right).$$

The rest of the proof of Theorem (5.4.18) follows along the same lines as the proof of Theorem (5.4.13). The only difference is to replace Ω_{11} and Ω_{22} by $\bar{\Omega}_{11}^{+}$ and $\bar{\Omega}_{22}^{+}$, respectively in the final computations. Again the exactly analagous FMOLS case (to DOLS) is depicted with a bar and plus, eg $\bar{W}_{i2}^{+}(r)$, over the Wiener processes and covariance matrices $\bar{\Lambda}_{i22}^{+}$. As stated earlier $\Omega_{22} = \bar{\Omega}_{22}^{+}$ and $\Lambda_{i22}\Lambda_{i22}' = \bar{\Lambda}_{i22}^{+}\bar{\Lambda}_{i22}^{+\prime}$.

QED

Theorem (5.4.19)

Proof

From equation (5.124) we have the OLS estimator of Υ as

$$\hat{\Upsilon}_{OLS2} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} y_{it} \\ x_{it}y_{it} \\ ty_{it} \end{bmatrix}\right). \tag{5.422}$$

Substituting for y_{it} we obtain

$$(\hat{\Upsilon}_{OLS2} - \Upsilon) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} u_{it} \\ x_{it}u_{it} \\ tu_{it} \end{bmatrix}\right). \tag{5.423}$$

Rescale the equation as before using

$$D_T = \begin{bmatrix} \sqrt{NT} & \underline{0}' & 0 \\ \underline{0} & \sqrt{NT}I_k & \underline{0} \\ 0 & \underline{0}' & \sqrt{NT^3} \end{bmatrix}. \tag{5.424}$$

So that

$$D_T(\hat{\Upsilon} - \Upsilon) = \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix} \tag{5.425}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\begin{bmatrix} 1 & x_{it}' & t \\ x_{it} & x_{it}x_{it}' & x_{it}t \\ t & tx_{it}' & t^2 \end{bmatrix}\right)^{-1} \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix}$$
$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}} & \underline{0}' & 0 \\ \underline{0} & N^{-\frac{1}{2}}T^{-1}I_k & \underline{0} \\ 0 & \underline{0}' & N^{-\frac{1}{2}}T^{-\frac{3}{2}} \end{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T}\begin{bmatrix} u_{it} \\ x_{it}u_{it} \\ tu_{it} \end{bmatrix}.$$

So

$$\text{(5.426)} \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it}^{/} & N^{-1}T^{-2}\Sigma\Sigma t \\ N^{-1}T^{-\frac{3}{2}}\Sigma\Sigma x_{it} & N^{-1}T^{-2}\Sigma\Sigma x_{it}x_{it}^{/} & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma t x_{it} \\ N^{-1}T^{-2}\Sigma\Sigma t & N^{-1}T^{-\frac{5}{2}}\Sigma\Sigma t x_{it}^{/} & N^{-1}T^{-3}\Sigma\Sigma t^2 \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} N^{-\frac{1}{2}}T^{-\frac{1}{2}}\Sigma\Sigma u_{it} \\ N^{-\frac{1}{2}}T^{-1}\Sigma\Sigma x_{it}u_{it} \\ N^{-\frac{1}{2}}T^{-\frac{3}{2}}\Sigma\Sigma t u_{it} \end{bmatrix}.$$

We now apply the sequential limit theory first holding N fixed and letting $T \to \infty$. To do this write the above in the more convenient form as

$$\text{(5.427)} \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix}$$

$$= \left(\frac{1}{N}\sum_{i=1}^{N} \begin{bmatrix} 1 & T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it}^{/} & T^{-2}\sum_{t=1}^{T} t \\ T^{-\frac{3}{2}}\sum_{t=1}^{T} x_{it} & T^{-2}\sum_{t=1}^{T} x_{it}x_{it}^{/} & T^{-\frac{5}{2}}\sum_{t=1}^{T} t x_{it} \\ T^{-2}\sum_{t=1}^{T} t & T^{-\frac{5}{2}}\sum_{t=1}^{T} t x_{it}^{/} & T^{-3}\sum_{t=1}^{T} t^2 \end{bmatrix} \right)^{-1}$$

$$\times \left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \begin{bmatrix} T^{-\frac{1}{2}}\sum_{t=1}^{T} u_{it} \\ T^{-1}\sum_{t=1}^{T} x_{it}u_{it} \\ T^{-\frac{3}{2}}\sum_{t=1}^{T} t u_{it} \end{bmatrix} \right).$$

On substituting Λ_i^* for $\bar{\Lambda}_i$, etc we have an exactly analagous case for OLS to the proofs of Theorems (5.4.6) and (5.4.13) except for $T^{-1}\sum_{t=1}^{T} x_{it}u_{it}$. So

$$\text{(5.428)} \qquad \begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix} \Rightarrow$$

$$= \left(\frac{1}{N}\sum_{i=1}^{N} \begin{bmatrix} 1 & \int_0^1 W_{i2}^{*/}(r)dr\Lambda_{i22}^{*/} & \frac{1}{2} \\ \Lambda_{i22}^{*}\int_0^1 W_{i2}^{*}(r)dr & \Lambda_{i22}^{*}\{\int_0^1 [W_{i2}^{*}(r)][W_{i2}^{*}(r)]^{/}dr\}\Lambda_{i22}^{*/} & \Lambda_{i22}^{*}\int_0^1 rW_{i2}^{*}(r)dr \\ \frac{1}{2} & \int_0^1 rW_{i2}^{*/}(r)dr\Lambda_{i22}^{*/} & \frac{1}{3} \end{bmatrix} \right)^{-1}$$

$$\times \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} \Lambda_{i11}^{*} W_{i1}^{*}(1) \\ \Lambda_{i22}^{*}\{\int_0^1 [W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*} + \Gamma_{i21}^{*} \\ \Lambda_{i11}^{*} W_{i1}^{*}(1) - \Lambda_{i11}^{*} \int_0^1 W_{i1}^{*}(r)dr \end{bmatrix} \right).$$

Asymptotic Consistency

Using (e) and also (c) and (g) above which are the same for OLS case since they only involve $W_{i1}^{*}(1)$ and $W_{i1}^{*}(r)$ then

(5.429) $$\frac{1}{N}\sum_{i=1}^{N} \Lambda_{i22}^{*}\{\int_0^1 [W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*} + \Gamma_{i21}^{*} \xrightarrow{a.s.} \Gamma_{21}^{*} \quad \text{as} \quad N \to \infty.$$

(5.430) $$\frac{1}{N}\sum_{i=1}^{N} \Lambda_{i11}^{*} W_{i1}^{*}(1) \xrightarrow{a.s.} 0 \quad \text{as} \quad N \to \infty.$$

(5.431) $$\frac{1}{N}\sum_{i=1}^{N} \left(\Lambda_{i11}^{*} W_{i1}^{*}(1) - \Lambda_{i11}^{*} \int_0^1 W_{i1}^{*}(r)dr \right) \xrightarrow{a.s.} 0 \quad \text{as} \quad N \to \infty.$$

Using Komolgorov's SLLN we have now shown that as $N \to \infty$ by (a)-(g)

(5.432) $$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2} - \delta) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \underline{0}' & \frac{1}{2} \\ \underline{0} & \frac{1}{2}\Omega_{22} & \underline{0} \\ \frac{1}{2} & \underline{0}' & \frac{1}{3} \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ \Gamma_{21}^{*} \\ 0 \end{bmatrix}.$$

Thus

(5.433) $$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2} - \alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2} - \beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2} - \delta) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 2\Gamma_{21}^{*}\Omega_{22}^{-1} \\ 0 \end{bmatrix}.$$

Hence $\hat{\alpha}_{OLS2} \xrightarrow{p} \alpha$ and $\hat{\delta}_{OLS2} \xrightarrow{p} \delta$ as $(N, T \to \infty)_{seq}$ but $\hat{\beta}_{OLS2} \not\xrightarrow{p} \beta$ as $(N, T \to \infty)_{seq}$. Hence we have shown that $\hat{\alpha}_{OLS2}$ and $\hat{\delta}_{OLS2}$ are asymptotically consistent estimators as $(N, T \to \infty)_{seq}$ but $\hat{\beta}_{OLS2}$ is not an asymptotically consistent estimator as $(N, T \to \infty)_{seq}$.

Asymptotic Normality

Again we apply the Lindeberg-Levy CLT to each element of the (3×1) vector and use the results of the proofs of Theorems (5.4.16) and (5.4.18) for the limiting distribution of the (3×3) block matrix. The desired result follows after an application of Slutsky's Theorem. Also from the proofs of Theorems (5.4.16) and (5.4.18) we have the following Lindeberg-Levy CLT results. From $(a)'$ and $(d)'$ on substituting for $\Lambda^*_{i11}, W^*_{i1}(r)$ and $W^*_{i1}(1)$ and also from $(c)'$ we have

$$\text{(5.434)} \qquad \frac{1}{\sqrt{N}}\sum_{i=1}^{N}(\Lambda^*_{i11}W^*_{i1}(1)) \xrightarrow{d} N(0,\Omega^*_{11}) \quad \text{as} \quad N \to \infty.$$

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda^*_{i22}\left\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\right\}\Lambda^*_{i11}\right) \xrightarrow{d} N\left(\underline{0},\frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21}\right) \quad \text{as} \quad N \to \infty.$$

(5.435)

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\Lambda^*_{i11}W^*_{i1}(1) - \Lambda^*_{i11}\int_0^1 W^*_{i1}(r)dr\right) \xrightarrow{d} N\left(0,\frac{1}{3}\Omega^*_{11}\right) \quad \text{as} \quad N \to \infty.$$

(5.436)

Now for the covariance matrix of the (3×1) block vector we have in the OLS case exactly the same result for the covariance matrix as in equation (5.391) in the proofs of Theorem (5.4.16) (with the slight change of notation for OLS) except for E^*. Hence using the Lindeberg-Levy CLT we have now shown that as $N \to \infty$ by $(a)' - (d)'$

$$\text{(5.437)} \qquad \left[\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda^*_{i11}W^*_{i1}(1) \\ \Lambda^*_{i22}\{\int_0^1[W^*_{i2}(r)][dW^*_{i1}(r)]\}\Lambda^*_{i11} + \Gamma^*_{i21} \\ \Lambda^*_{i11}W^*_{i1}(1) - \Lambda^*_{i11}\int_0^1 W^*_{i1}(r)dr \end{bmatrix}\right]$$

$$\xrightarrow{d} N\left(\begin{bmatrix} 0 \\ \underline{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega^*_{11} & \underline{0}' & \frac{1}{2}\Omega^*_{11} \\ \underline{0} & \frac{1}{2}\Omega^*_{21}\Omega^{*\prime}_{21} & \underline{0} \\ \frac{1}{2}\Omega^*_{11} & \underline{0}' & \frac{1}{3}\Omega^*_{11} \end{bmatrix}\right).$$

As in the proofs of Theorems (5.4.16) and (5.4.18) our final asymptotic normality result comes from using the Slutsky Theorem applied to our equation

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix} \Rightarrow \tag{5.438}$$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} 1 & \int_0^1 W_{i2}^{*\prime}(r)dr\Lambda_{i22}^{*\prime} & \frac{1}{2} \\ \Lambda_{i22}^{*}\int_0^1 W_{i2}^{*}(r)dr & \Lambda_{i22}^{*}\{\int_0^1[W_{i2}^{*}(r)][W_{i2}^{*}(r)]^{\prime}dr\}\Lambda_{i22}^{*\prime} & \Lambda_{i22}^{*}\int_0^1 rW_{i2}^{*}(r)dr \\ \frac{1}{2} & \int_0^1 rW_{i2}^{*\prime}(r)dr\Lambda_{i22}^{*\prime} & \frac{1}{3} \end{bmatrix}\right)^{-1}$$

$$\times\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\begin{bmatrix} \Lambda_{i11}^{*}W_{i1}^{*}(1) \\ \Lambda_{i22}^{*}\{\int_0^1[W_{i2}^{*}(r)][dW_{i1}^{*}(r)]\}\Lambda_{i11}^{*}+\Gamma_{i21}^{*} \\ \Lambda_{i11}^{*}W_{i1}^{*}(1)-\Lambda_{i11}^{*}\int_0^1 W_{i1}^{*}(r)dr \end{bmatrix}\right).$$

Which has asymptotic distribution

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix} \tag{5.439}$$

$$\Rightarrow N\left(\begin{bmatrix} 0 \\ 2\Gamma_{21}^{*}\Omega_{22}^{-1} \\ 0 \end{bmatrix}, \begin{bmatrix} 4 & \underline{0}^{\prime} & -6 \\ \underline{0} & 2\Omega_{22}^{-1} & \underline{0} \\ -6 & \underline{0}^{\prime} & 12 \end{bmatrix}\begin{bmatrix} \Omega_{11}^{*} & \underline{0}^{\prime} & \frac{1}{2}\Omega_{11}^{*} \\ \underline{0} & \frac{1}{2}\Omega_{21}^{*}\Omega_{21}^{*\prime} & \underline{0} \\ \frac{1}{2}\Omega_{11}^{*} & \underline{0}^{\prime} & \frac{1}{3}\Omega_{11}^{*} \end{bmatrix}\begin{bmatrix} 4 & \underline{0}^{\prime} & -6 \\ \underline{0} & 2\Omega_{22}^{-1\prime} & \underline{0} \\ -6 & \underline{0}^{\prime} & 12 \end{bmatrix}\right)$$

The asymptotic covariance matrix is

$$\begin{bmatrix} 4 & \underline{0}^{\prime} & -6 \\ \underline{0} & 2\Omega_{22}^{-1} & \underline{0} \\ -6 & \underline{0}^{\prime} & 12 \end{bmatrix}\begin{bmatrix} \Omega_{11}^{*} & \underline{0}^{\prime} & \frac{1}{2}\Omega_{11}^{*} \\ \underline{0} & \frac{1}{2}\Omega_{21}^{*}\Omega_{21}^{*\prime} & \underline{0} \\ \frac{1}{2}\Omega_{11}^{*} & \underline{0}^{\prime} & \frac{1}{3}\Omega_{11}^{*} \end{bmatrix}\begin{bmatrix} 4 & \underline{0}^{\prime} & -6 \\ \underline{0} & 2\Omega_{22}^{-1\prime} & \underline{0} \\ -6 & \underline{0}^{\prime} & 12 \end{bmatrix} \tag{5.440}$$

$$= \begin{bmatrix} 4\Omega_{11}^{*} & \underline{0}^{\prime} & -6\Omega_{11}^{*} \\ \underline{0} & 2\Omega_{22}^{-1}\Omega_{21}^{*}\Omega_{21}^{*\prime}\Omega_{22}^{-1\prime} & \underline{0} \\ -6\Omega_{11}^{*} & \underline{0}^{\prime} & 12\Omega_{11}^{*} \end{bmatrix}. \tag{5.441}$$

So

$$\begin{bmatrix} \sqrt{NT}(\hat{\alpha}_{OLS2}-\alpha) \\ \sqrt{NT}(\hat{\beta}_{OLS2}-\beta) \\ \sqrt{NT^3}(\hat{\delta}_{OLS2}-\delta) \end{bmatrix} \Rightarrow N\left(\begin{bmatrix} 0 \\ 2\Gamma_{21}^{*}\Omega_{22}^{-1} \\ 0 \end{bmatrix}, \begin{bmatrix} 4\Omega_{11}^{*} & \underline{0}^{\prime} & -6\Omega_{11}^{*} \\ \underline{0} & 2\Omega_{22}^{-1}\Omega_{21}^{*}\Omega_{21}^{*\prime}\Omega_{22}^{-1\prime} & \underline{0} \\ -6\Omega_{11}^{*} & \underline{0}^{\prime} & 12\Omega_{11}^{*} \end{bmatrix}\right). \tag{5.442}$$

QED

Lemma (5.5.1)

Proof

See White (1984), p.71.

Proposition (5.5.2)

Proof

See White (1984), p.71.

Theorem (5.5.3)

Proof

Under the null hypothesis $H_0 : R\beta = r$ then

$$R\hat{\beta}_{DOLS1} - r = R\left(\hat{\beta}_{DOLS1} - \beta\right). \tag{5.443}$$

So

$$\Phi_n^{-\frac{1}{2}}\sqrt{N}T\left(R\hat{\beta}_{DOLS} - r\right) = \Phi_n^{-\frac{1}{2}}\sqrt{N}TR\left(\hat{\beta}_{DOLS1} - \beta\right). \tag{5.444}$$

Now we know by Theorem (5.4.6) that

$$\sqrt{N}T\left(\hat{\beta}_{DOLS1} - \beta\right) \Rightarrow N(\underline{0}, 2\Omega_{22}^{-1}\Omega_{11}) \tag{5.445}$$

as $(N, T \to \infty)_{seq}$.

Let R be an O(1) sequence of nonstochastic $(q \times k)$ matrices with full rank

q. Then $\sqrt{N}TR\left(\hat{\beta}_{DOLS1} - \beta\right)$ is such that

$$\sqrt{N}TR\left(\hat{\beta}_{DOLS1} - \beta\right) \Rightarrow N(\underline{0}, 2R\Omega_{22}^{-1}\Omega_{11}R^{/}) \tag{5.446}$$

as $(N, T \rightarrow \infty)_{seq}$.

So

$$\Phi_n^{-\frac{1}{2}}\sqrt{N}T\left(R\hat{\beta}_{DOLS1} - r\right) \Rightarrow N(\underline{0}, I_k) \tag{5.447}$$

as $(N, T \rightarrow \infty)_{seq}$

where $\Phi_n = 2R\Omega_{22}^{-1}\Omega_{11}R^{/}$ and Φ_n and Φ_n^{-1} are O(1), which is the desired result for (a). Next, given the result in (a), (b) follows from applying Lemma (5.5.1).

QED

APPENDIX 2

DATABASE MANAGEMENT

This appendix is divided into three main sections:

1) A description of the panel datasets used in the thesis.

2) A brief introduction to the Ox and PcGive software programmes.

3) An explanation of the Library Information Systems used.

The Panel Datasets

We give here a brief description of the source and characteristics of the three panel datasets used in the thesis. The datasets are from the three largest worldwide economic organisations, the OECD, the IMF and the World Bank. The Organisation for Economic Co-operation and Development consists of 30 member countries mainly from the developed nations of Europe and North America who share a commitment to democratic government and the market economy. Information about the OECD can be obtained from http://www.oecd.org/. The International Monetary Fund is an international organisation of 184 member countries from all parts of the world both rich and poor. Its aim is to promote international monetary co-operation. Information about the IMF can be obtained from http://www.imf.org/. The World Bank consists of five closely linked international institutions owned by member countries. Its mission is to fight poverty. Information about the

World Bank can be obtained from http://www.worldbank.org/.

All three panel datasets consist of a balanced panel of time-series cross-section data. The data all from the post war period reflects a good variety of data observations at the monthly, quarterly and annual frequencies. Apart from the main data collecting agencies and publishers mentioned above the emergence of new data sources such as the Penn World Tables has made large panel datasets much more easily availabile.

Panel Dataset 1

Obtained from the OECD Main Economic Indicators dataset at the MIMAS Data Archive of Manchester University. 488 Monthly observations for 20 OECD countries on the Consumer Price Index. Details:

Countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Norway, Portugal, Spain, Sweden, Switzerland, U.K., U.S.

First Observation: 1960q1,

Last Observation: 2000q8,

CPI All Items: Index with Base Year 1995

All seasonally Unadjusted.

Panel Dataset 2

Obtained from the IMF International Financial Statistics dataset at the MI-

MAS Data Archive of Manchester University. 138 Quarterly observations for 25 OECD countries on the Nominal Exchange Rate and Consumer Price Index. Details:

Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, U.K., U.S.

First Observation: 1957q1,

Last Observation: 1991q2,

Exchange Rate: Market Rate F-AE

End of Period National Currency per U.S. Dollar.

CPI All Items: Consumer Prices F6-4

Index with Base Year 1985.

All seasonally Unadjusted.

Panel Dataset 3

Obtained from the World Bank World Development Indicators 2002 CD-Rom at Hull University. 39 Annual observations for 20 OECD countries on Consumption, Gross Domestic Product, Population, the GDP deflator and the CPI. Data on Interest Rates and Liquid Assets obtained from IMF IFS Yearbooks 1991 and 2000. Data on Savings Rates obtained from the Penn

World Tables. Details:

Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, U.K., U.S.

First Observation: 1961,

Last Observation: 1999,

Final Consumption Expenditure

Data are in current U.S. dollars (2002).

Gross Domestic Product

Data are in current U.S. dollars (2002).

Total Population: (Total population of country)

GDP deflator: (Base year varies by country)

Consumer Price Index

Index with base year 1995=100.

Savings Rates

Annual % of GDP.

Interest Rates: Annual % Rate of 3 types (24 hour Discount Rate, 30 Day Treasury Bill Rate or Long-Term 10 Year Government Bond Yield)

All seasonally adjusted.

The Datasets are provided on an accompanying CD-ROM. The CD-ROM contains only the transformed and estimated datasets for each panel. Note that the Liquid Assets variable in panel dataset 3, is a composite variable calculated as the Wealth of the Personal Sector comprising of Real National Savings plus Real Time, Savings and Demand Deposits at commercial banks and, when available, the Real Stock of Bonds at commercial banks. These data were obtained from the IMF IFS Yearbooks 1991 and 2000. These were in national currencies and had to be transformed into US$ and deflated by the GDP deflator.

An Outline of Ox

Ox is a high level programming language for use by econometricians, statisticians and other quantitative researchers. It is a programming language equivalent to Gauss. It exists within the PcGive suite of software programs and uses GiveWin software as a front end (ie for printing results to screen, etc). The PcGive Professional software is used for all the non-programmable regressions and other computations in the thesis. Ox and the PcGive software are distributed under the OxMetrics brandname by Timberlake Consultants. One way of using Ox is through its Object Orientated programming structure. The other alternative is the more flexible freestyle programming method used in this thesis for brevity. The OxMetrics development team has

its base at Oxford University. The OxMetrics development team host their own annual international conference and participate in numerous research and training activitives. The web address is http://www.oxmetrics.net/
All the important Ox programs used in the thesis and some of their datasets are stored on the accompanying CD-ROM with the three main panel datasets. A description of these is given in the list of Ox programs at the introduction. These have been written solely by the author.

The Library Information Systems

Most document examinations and searches have been carried out using the Athens Library Information System. The Athens system and Science Direct provided the basis of most online library information access. Information about these library infornation services can be found at http://www.athens.ac.uk/ and http://www.sciencedirect.com/. Other database archives such as the MIMAS archive of Manchester University, the Data Archive of Essex University and the new ESDS International, of the University of Manchester and Essex University, were the source of macroeconomic databases and econometric information. The web addresses of these are http://www.mimas.ac.uk and http://www.data-archive.ac.uk and http://www.esds.ac.uk .

APPENDIX 3

TABLES

Table 3.34 **Individual Country DOLS Regression Estimates**

CO	constant	y/N_t	LA/y_t	DW/F-stat	R^2
Ausl	28.396	0.77049	-1.2153	0.85683	0.99995
	5.3937[0.000]	38.684[0.000]	-9.1158[0.000]	26750.0	
Aus	-13.000	0.79677	0.23765	0.96391	0.99992
	-2.1589[0.042]	82.262[0.000]	1.5554[0.134]	17325.0	
Bel	35.213	0.73303	-1.0759	1.0614	0.99996
	15.353 [0.000]	183.89[0.000]	-14.927[0.000]	31064.0	
Can	-19.963	0.73155	1.1002	0.26516	0.99951
	-1.1606[0.258]	4.2325[0.000]	0.77062[0.449]	2831.5	
Den	57.886	0.70907	-1.6085	0.87062	0.99994
	10.094[0.000]	87.652[0.000]	-8.9733[0.000]	23159.0	
Fin	61.658	0.68145	-1.6511	1.0659	0.99981
	2.0805[0.049]	13.255[0.000]	-2.5919[0.016]	7155.3	
Fra	53.212	0.78421	-2.1154	0.83703	0.99991
	4.4273[0.000]	39.393[0.000]	-6.2496[0.000]	15533.0	
Gre	92.568	0.67723	-1.5569	0.95548	0.99934
	5.0363[0.000]	41.092[0.000]	-4.4249[0.000]	2091.1	
Ire	45.603	0.54975	-0.59585	0.69237	0.99951
	12.468[0.000]	10.299[0.000]	-2.4800[0.021]	2799.3	
Ita	27.714	0.74506	-0.89479	1.3751	0.99998
	19.798 [0.000]	115.44[0.000]	-13.366[0.000]	66230.0	
Jap	28.039	0.70662	-0.82793	0.86716	0.99993
	3.7549[0.001]	157.68[0.000]	-4.1117[0.000]	18406.0	
Kor	13.305	0.66891	-0.31390	0.86623	0.99877
	6.6312[0.000]	16.868[0.000]	-10.513[0.000]	1114.3	
Net	34.206	0.70205	-0.99784	0.81307	0.99993
	8.9719[0.000]	142.68[0.000]	-8.5826[0.000]	19670.0	
Nor	104.92	0.64818	-2.6346	1.4276	0.99992
	8.8052[0.000]	72.021[0.000]	-9.5058 [0.000]	17994.0	
Por	15.302	0.80298	-0.80638	1.5102	0.99955
	1.7184[0.099]	110.95[0.000]	-1.5645[0.131]	3024.0	
Spa	33.162	0.72808	-1.1032	0.67120	0.99994
	12.356[0.000]	46.826[0.000]	-8.3734[0.000]	24073.0	
Swe	85.772	0.72169	-2.8075	1.6634	0.99999
	13.960[0.000]	79.017[0.000]	-18.775[0.000]	127390.0	
Swi	13.411	0.73012	-0.24756	0.87447	0.99987
	2.3047[0.030]	84.394[0.000]	-2.2920[0.031]	10453.0	
UK	10.286	0.88654	-1.1641	1.0220	0.99997
	3.4942[0.002]	72.785[0.000]	-10.852[0.000]	47140.0	
US	-72.952	0.88556	1.0812	0.67149	0.99988
	-1.7354[0.096]	56.755[0.000]	1.4561[0.159]	11206.0	

Table 3.35 **Individual Country DOLS Regression Estimates**

CO	constant	trend	y/N_t	LA/y_t	DW/F-s	R^2
Ausl	20.370	0.062648	0.77485	-0.98240	0.93147	0.99995
	2.3336[0.029]	1.1076[0.279]	41.153[0.000]	-4.0313[0.000]	25667.0	
Aus	-16.852	0.17795	0.77743	0.33388	0.97156	0.99992
	-2.3790[0.026]	0.96088[0.347]	35.007[0.000]	1.8668[0.075]	16068.0	
Bel	34.186	-0.10763	0.75291	-1.0692	1.1268	0.99996
	14.076 [0.000]	-1.0502[0.305]	38.964[0.000]	-15.238[0.000]	29159.0	
Can	69.872	0.77103	0.61986	-2.2875	0.40810	0.99982
	3.2678[0.003]	4.6425[0.000]	6.5584[0.000]	-2.1786[0.040]	6959.2	
Den	50.631	-0.34624	0.78041	-1.6761	1.1888	0.99997
	12.961[0.000]	-4.6002[0.000]	47.858[0.000]	-14.875[0.000]	36379.0	
Fin	39.105	-0.32439	0.80031	-1.4730	1.2908	0.99984
	1.3289[0.197]	-1.9089[0.069]	10.278[0.000]	-2.5102[0.019]	7630.9	
Fra	49.202	0.33645	0.70331	-1.6461	1.4341	0.99997
	12.264[0.000]	9.6807[0.000]	66.009[0.000]	-13.449[0.000]	42938.0	
Gre	91.288	-0.62327	0.66274	-1.0035	1.0178	0.99935
	5.1831[0.000]	-0.58616[0.563]	22.638[0.000]	-1.0020[0.327]	1905.1	
Ire	44.826	-0.16524	0.55291	-0.38711	0.80596	0.99954
	13.214[0.000]	-1.2151[0.237]	11.356[0.000]	-1.3914[0.178]	2685.1	
Ita	24.750	0.030062	0.74475	-0.80613	1.4038	0.99998
	6.4652[0.000]	0.82383[0.418]	123.32[0.000]	-6.4770[0.000]	60569.0	
Jap	18.349	0.28412	0.68722	-0.62325	1.1143	0.99994
	2.8280[0.009]	2.9301[0.007]	92.598[0.000]	-3.7556[0.001]	21350.0	
Kor	12.755	-0.17664	0.68301	-0.23414	0.84920	0.99877
	4.1501[0.000]	-0.23716[0.814]	9.5429[0.000]	-0.69346[0.495]	1003.4	
Net	38.624	-0.53974	0.78930	-1.1923	1.2934	0.99997
	23.458[0.000]	-8.7773[0.000]	77.809[0.000]	-22.628[0.000]	46271.0	
Nor	124.31	-0.21745	0.66890	-3.1679	1.5996	0.99993
	6.5301[0.000]	-1.2553[0.222]	36.105[0.000]	-6.3672[0.000]	16965.0	
Por	5.7427	0.11163	0.81687	-0.53417	1.5669	0.99955
	0.42706[0.673]	0.89308[0.381]	48.312[0.000]	-0.95199[0.351]	2749.5	
Spa	48.924	-0.12030	0.72891	-1.6234	0.79577	0.99996
	8.3677[0.000]	-2.9121[0.008]	56.899[0.000]	-7.7666[0.000]	30878.0	
Swe	86.777	-0.0042110	0.72061	-2.8314	1.6643	0.99999
	12.180[0.000]	-0.28397[0.779]	72.428[0.000]	-16.427[0.000]	114680	
Swi	12.749	0.17062	0.71428	-0.22914	0.86202	0.99987
	2.1117[0.046]	0.40562[0.688]	17.866[0.000]	-1.9574[0.063]	9463.4	
UK	10.780	-0.0077880	0.88869	-1.2011	1.0168	0.99997
	2.2465[0.035]	-0.13083[0.897]	43.341[0.000]	-3.9747[0.000]	42384.0	
US	-72.191	0.38722	0.79772	1.2657	0.66455	0.99988
	-1.7137[0.100]	0.45233[0.655]	4.0943[0.000]	1.4926[0.149]	10212.0	

Table 3.36 Individual Country FMOLS Regression Estimates

CO	constant	y/N_t	LA/y_t	DW/F-stat	R^2
Ausl	41.060	0.74016	-1.4754	0.95974	0.99982
	8.9320[0.000]	50.611[0.000]	-11.688[0.000]	98956.0	
Aus	-61.485	0.85824	1.4670	0.99721	0.99852
	-5.9878 [0.000]	51.619[0.000]	5.7540[0.000]	12185.0	
Bel	22.926	0.74087	-0.66613	1.3149	0.99930
	6.7659[0.000]	128.22[0.000]	-6.6547[0.000]	25686.0	
Can	-23.338	0.72752	1.7002	1.8943	0.99079
	-1.4609[0.152]	12.816[0.000]	1.9534[0.058]	1937.0	
Den	20.190	0.74568	-0.52477	1.0973	0.99945
	2.0609 [0.046]	51.889[0.000]	-1.7938[0.081]	32613.	
Fin	149.53	0.58075	-3.8426	1.0035	0.99885
	10.803[0.000]	25.450[0.000]	-11.305[0.000]	15617.0	
Fra	56.991	0.77399	-2.1770	0.89948	0.99985
	7.5957 [0.000]	68.750 [0.000]	-9.2163[0.000]	122370	
Gre	75.613	0.69413	-1.2607	1.0532	0.99913
	5.0676 [0.000]	61.184[0.000]	-3.6577[0.000]	20670.0	
Ire	32.287	0.79234	-1.8316	2.0585	0.98510
	11.653[0.000]	27.895[0.000]	-15.470[0.000]	1190.1	
Ita	23.349	0.74155	-0.73589	0.67561	0.99986
	6.4422[0.000]	64.940 [0.000]	-6.0401[0.000]	126730	
Jap	34.407	0.70144	-1.0498	0.73500	0.99963
	3.7547[0.001]	142.76[0.000]	-4.1493[0.000]	48212.0	
Kor	17.551	0.63078	-0.22197	0.89415	0.98826
	5.1717[0.000]	12.048 [0.000]	-5.3941[0.000]	1515.4	
Net	24.758	0.70951	-0.70785	1.0511	0.99979
	5.2258[0.000]	105.96 [0.000]	-5.1435[0.000]	85201.0	
Nor	94.189	0.65403	-2.3800	1.5212	0.99988
	14.444[0.000]	121.22[0.000]	-15.066[0.000]	154750	
Por	33.647	0.79922	-1.8320	1.0802	0.99904
	7.3144[0.000]	80.680[0.000]	-8.2632[0.000]	18694.0	
Spa	34.222	0.73881	-1.2123	0.72570	0.99993
	13.138[0.000]	108.13[0.000]	-11.989[0.000]	245520	
Swe	107.52	0.70872	-3.5078	1.0315	0.99987
	19.349 [0.000]	84.567[0.000]	-23.085[0.000]	138820	
Swi	-15.200	0.77235	0.18723	1.1949	0.99910
	-2.3839[0.022]	71.256 [0.000]	1.7615[0.086]	20072.0	
UK	23.702	0.84586	-1.4832	1.0583	0.99981
	4.4017[0.000]	42.402[0.000]	-7.9041[0.000]	94106.0	
US	-57.221	0.85879	0.91174	1.3333	0.99943
	-1.9362 [0.060]	56.705 [0.000]	1.7295[0.092]	31633.0	

Table 3.37 **Individual Country FMOLS Regression Estimates**

CO	constant	trend	y/N_t	LA/y_t	DW/F-s	R^2
Ausl	48.192	-0.10633	0.74326	-1.6915	0.75449	0.99985
	6.1030[0.000]	-1.3363[0.190]	49.397[0.000]	-7.0838[0.000]	76133.0	
Aus	-112.36	1.0213	0.79534	2.7312	0.97808	0.99795
	-8.2314[0.000]	4.8149[0.000]	38.535[0.000]	8.0720[0.000]	5668.8	
Bel	20.582	-0.025371	0.74794	-0.60209	1.3121	0.99921
	5.7621[0.000]	-0.35278[0.727]	63.028[0.000]	-5.4998[0.000]	14784.0	
Can	88.066	1.1435	0.46274	-2.2177	2.0597	0.99215
	5.0394 [0.000]	8.4790[0.000]	9.2130[0.000]	-3.6373[0.000]	1474.4	
Den	24.392	-0.25760	0.79039	-0.86714	1.0963	0.99968
	4.6053 [0.000]	-3.6285[0.001]	65.549[0.000]	-5.0596[0.000]	36163.0	
Fin	181.09	-0.66091	0.67251	-5.0203	1.1939	0.99897
	12.198[0.197]	-5.1285[0.069]	27.188[0.000]	-12.236[0.019]	11261.0	
Fra	28.713	0.30359	0.72992	-1.0186	1.2457	0.99978
	5.3793[0.000]	6.0355[0.000]	80.836[0.000]	-5.4839[0.000]	54195.0	
Gre	77.802	0.072478	0.69459	-1.3704	1.0564	0.99914
	5.2488 [0.000]	0.22328[0.825]	52.971 [0.000]	-3.1149[0.005]	13528.0	
Ire	36.571	0.48489	0.72653	-2.0510	1.7243	0.99292
	11.853[0.000]	3.9899 [0.000]	23.095[0.000]	-11.476[0.000]	1635.9	
Ita	-7.8285	0.35049	0.74956	0.067266	0.76757	0.99978
	-0.67962[0.501]	2.8542[0.007]	63.697 [0.000]	0.21908[0.827]	52379.0	
Jap	36.198	0.19059	0.68373	-1.1270	0.79670	0.99964
	3.9799[0.000]	1.2831[0.207]	49.632 [0.000]	-4.4545[0.000]	32749.0	
Kor	6.2030	-1.9002	0.86497	0.55047	0.79233	0.99606
	2.5471[0.015]	-8.0177 [0.000]	20.765[0.000]	5.4650[0.000]	2948.8	
Net	28.857	-0.29992	0.75524	-0.86459	1.3975	0.99987
	12.058 [0.000]	-6.7077 [0.000]	109.61[0.000]	-12.156[0.000]	92713.0	
Nor	93.427	0.021505	0.65143	-2.3548	1.5187	0.99989
	14.550[0.000]	0.30951 [0.759]	61.016[0.000]	-14.968[0.000]	102190	
Por	37.317	-0.12822	0.79014	-1.7979	1.0813	0.99915
	4.9576[0.000]	-1.1182 [0.271]	60.557[0.000]	-7.1893[0.000]	13686.0	
Spa	22.678	0.037401	0.73290	-0.75340	1.0914	0.99984
	5.7618 [0.000]	1.3091[0.199]	117.75[0.000]	-5.4137[0.000]	71409.0	
Swe	117.65	-0.11051	0.70619	-3.7843	1.0747	0.99985
	17.470 [0.000]	-2.6591 [0.011]	83.731[0.000]	-20.560[0.000]	76151.0	
Swi	-16.415	0.14428	0.76016	0.21217	1.1731	0.99911
	-2.4132 [0.021]	0.58400 [0.565]	35.059 [0.000]	1.8416[0.079]	13036.0	
UK	28.033	-0.057755	0.85057	-1.6885	1.0153	0.99980
	3.9053 [0.000]	-0.95668[0.345]	41.649[0.000]	-5.8590[0.000]	57930.0	
US	-47.209	0.059340	0.84326	0.76344	1.2840	0.99949
	-1.5290 [0.135]	0.10934[0.913]	6.8208 [0.000]	1.3936[0.172]	22946.0	

Table 3.38 **Individual Country DOLS Regression Estimates**

CO	constant	y/N_t	Inf_t	DW/F-stat	R^2
Ausl	1.2068	0.77175	-0.58818	1.0950	0.99966
	0.18191[0.857]	21.203[0.000]	-1.7557[0.093]	4064.5	
Aus	2.4955	0.76296	-1.1715	1.0486	0.99995
	2.9432[0.007]	242.46[0.000]	-8.3265[0.000]	26669.0	
Bel	2.3034	0.76443	-0.27870	0.24338	0.99933
	0.35782 [0.723]	30.198[0.000]	-0.25304[0.802]	2043.9	
Can	-10.731	0.85408	-0.96987	0.98624	0.99979
	-2.0131[0.056]	30.231[0.000]	-5.2306[0.000]	6418.9	
Den	-5.3748	0.76872	1.0077	1.1616	0.99982
	-1.7944[0.086]	80.044[0.000]	5.2774[0.000]	7644.4	
Fin	5.4382	0.72950	-0.84686	0.83142	0.99952
	1.0140[0.321]	31.445[0.000]	-3.3967[0.002]	2850.3	
Fra	-3.8609	0.80697	-0.61141	1.2022	0.99989
	-2.3433[0.028]	112.14[0.000]	-5.7153[0.000]	12298.0	
Gre	27.655	0.72507	-0.85631	0.61496	0.99906
	1.8891[0.072]	26.823[0.000]	-0.84840[0.405]	1466.3	
Ire	43.085	0.45876	0.36993	0.48027	0.99871
	6.0728[0.000]	13.356[0.000]	0.87508[0.390]	1063.8	
Ita	6.6799	0.74420	-0.53677	0.47775	0.99974
	0.68911 [0.497]	15.985[0.000]	-1.7751[0.089]	5309.8	
Jap	4.0883	0.69842	-0.71923	0.89127	0.99988
	2.9745[0.006]	153.12[0.000]	-4.2931[0.000]	11912.0	
Kor	33.319	0.31046	-0.78052	0.80190	0.99635
	8.6560[0.000]	6.6372[0.000]	-4.3092[0.000]	375.85	
Net	2.8018	0.72080	-0.16498	0.92246	0.99983
	1.9321[0.066]	118.78[0.000]	-0.91424[0.370]	7876.5	
Nor	-3.5840	0.71676	-0.78586	1.4604	0.99965
	-1.2805[0.213]	75.658[0.000]	-3.7038[0.001]	3900.3	
Por	-1.3432	0.82485	-0.20590	1.2430	0.99945
	-0.34597[0.732]	32.645[0.000]	-0.78592[0.440]	2476.2	
Spa	1.9775	0.76665	-0.38422	1.2628	0.99987
	0.91750[0.368]	49.057[0.000]	-3.9599[0.000]	10485.0	
Swe	-6.7276	0.78005	0.64011	0.96968	0.99972
	-1.0852[0.289]	31.636[0.000]	1.5944[0.125]	4825.7	
Swi	-0.66310	0.75107	-0.55090	0.50125	0.99985
	-0.16956[0.866]	87.576[0.000]	-0.69874[0.492]	9017.4	
UK	-6.4419	0.86926	-0.43315	0.87875	0.99980
	-0.77253[0.448]	21.655[0.000]	-1.9093[0.069]	6839.5	
US	-5.7636	0.85318	-0.54307	0.52825	0.99985
	-0.96953[0.342]	48.840[0.000]	-1.2852[0.212]	9455.5	

Table 3.39 **Individual Country DOLS Regression Estimates**

CO	constant	trend	y/N_t	Inf_t	DW/F-s	R^2
Ausl	2.0221	0.23477	0.72147	0.11050	1.2557	0.99975
	0.38324[0.705]	2.8585[0.009]	21.320[0.000]	0.30586[0.762]	4914.9	
Aus	2.5403	0.035270	0.75780	-1.1802	1.0572	0.99995
	2.9319[0.007]	0.27554[0.785]	39.950[0.000]	-8.1558[0.000]	24025.0	
Bel	5.9415	1.0128	0.57821	0.98279	0.45629	0.99945
	1.0153[0.320]	1.6086[0.121]	4.9130[0.000]	0.81123[0.425]	2258.9	
Can	-24.292	-0.27232	0.96983	-1.4527	1.3214	0.99982
	-3.9695[0.000]	-2.8316[0.009]	21.272[0.000]	-6.7268[0.000]	6898.4	
Den	-3.8794	0.26904	0.72951	1.3364	1.1580	0.99983
	-1.3226[0.199]	1.5187[0.143]	26.730[0.000]	4.7890[0.000]	7339.1	
Fin	-6.5278	-0.55146	0.86416	-1.4153	1.0681	0.99961
	-0.93759[0.358]	-2.3003[0.031]	13.967[0.000]	-4.3198[0.000]	3176.5	
Fra	-0.50317	0.28947	0.74840	-0.27153	1.1514	0.99990
	-0.23750[0.814]	2.3558[0.027]	29.020[0.000]	-1.5379[0.138]	12981.0	
Gre	70.079	-1.3601	0.67727	-0.71232	1.1850	0.99961
	7.3326[0.000]	-6.1841[0.000]	46.657[0.000]	-1.5507[0.135]	3157.4	
Ire	37.126	-0.25439	0.53930	0.43812	0.49247	0.99877
	3.8392[0.000]	-0.88197[0.387]	5.5442[0.000]	1.0445[0.307]	999.80	
Ita	-16.417	0.35308	0.80016	-0.16404	0.86586	0.99988
	-2.4823[0.021]	5.1058[0.000]	31.216[0.000]	-0.98072[0.337]	10446.0	
Jap	3.5532	0.13697	0.68592	-0.75113	0.94858	0.99989
	2.6011[0.016]	1.1553[0.260]	58.981[0.000]	-4.7222[0.000]	11313.0	
Kor	19.192	-0.56615	0.66611	-0.38734	0.76792	0.99809
	4.8045[0.000]	-4.5762[0.000]	7.9678[0.000]	-2.6349[0.015]	644.16	
Net	2.7997	0.068610	0.70977	-0.12838	0.92918	0.99983
	1.9508[0.063]	0.30926[0.760]	19.616[0.000]	-0.59923[0.555]	7100.0	
Nor	-4.1500	-0.16575	0.73975	-0.97693	1.4469	0.99965
	-1.3546[0.189]	-0.46022[0.649]	14.546[0.000]	-2.0943[0.047]	3524.2	
Por	-8.2874	0.20013	0.83347	-0.077357	1.5054	0.99951
	-1.8772[0.073]	2.1985[0.038]	40.751[0.000]	-0.35783[0.723]	2531.5	
Spa	1.6046	0.10958	0.73835	-0.11275	1.2263	0.99990
	0.91158[0.371]	2.9544[0.007]	46.344[0.000]	-0.93022[0.362]	12224.0	
Swe	-2.9929	0.31392	0.72484	1.5420	1.0006	0.99977
	-0.52103[0.607]	2.3260[0.029]	22.425[0.000]	2.9251[0.007]	5315.7	
Swi	-0.055827	0.22841	0.72819	-0.63087	0.50944	0.99985
	-0.013682[0.989]	0.43511[0.667]	13.675[0.000]	-0.79400[0.435]	8223.7	
UK	0.63316	0.23072	0.79313	-0.17929	1.0859	0.99989
	0.10936[0.913]	4.1975[0.000]	24.596[0.000]	-1.1042[0.281]	11109.0	
US	121.80	4.2029	-0.085553	-1.5536	0.78930	0.99995
	5.3934[0.000]	5.6994[0.000]	-0.51868[0.609]	-5.5935[0.000]	23641.0	

Table 3.40 **Individual Country FMOLS Regression Estimates**

CO	constant	y/N_t	Inf_t	DW/F-stat	R^2
Ausl	4.9931	0.74617	-0.41896	1.0404	0.99879
	0.51445[0.610]	13.508 [0.000]	-0.92089[0.363]	14823.0	
Aus	2.5195	0.76295	-1.2292	1.3670	0.99988
	2.5814 [0.0140]	206.31 [0.000]	-8.2771[0.000]	146450	
Bel	2.4707	0.76201	-0.48279	0.48901	0.99909
	0.59119[0.558]	39.459 [0.000]	-0.86663[0.391]	19867.0	
Can	-11.097	0.85804	-1.0855	1.1501	0.99968
	-2.4125 [0.021]	35.441[0.000]	-6.0275[0.000]	56072.0	
Den	16.916	0.73717	-1.9166	1.5175	0.98889
	6.9891 [0.000]	85.757 [0.000]	-12.136[0.000]	1602.3	
Fin	5.2171	0.73514	-0.92698	0.91327	0.99929
	1.2326 [0.225]	38.294 [0.000]	-4.5405[0.000]	25292.0	
Fra	-2.7312	0.81052	-0.86491	0.38531	0.99925
	-0.56842 [0.573]	34.705 [0.000]	-2.8510[0.007]	24051.0	
Gre	27.512	0.71953	-0.52553	0.82275	0.99855
	3.7274 [0.001]	49.092[0.000]	-1.1769[0.246]	12405.0	
Ire	33.648	0.49877	0.89159	0.44252	0.99682
	4.7969 [0.000]	10.686[0.000]	2.9775[0.006]	5650.3	
Ita	9.1253	0.72853	-0.38319	1.0374	0.99944
	1.6962 [0.098]	28.016 [0.000]	-1.6118[0.115]	32321.0	
Jap	2.6219	0.69178	-0.71840	0.92611	0.99919
	1.1243[0.263]	81.636 [0.000]	-2.8041[0.008]	22204.0	
Kor	26.979	0.37228	-0.40674	0.58594	0.97450
	3.0805 [0.003]	3.3267 [0.002]	-1.2633[0.219]	688.02	
Net	0.93943	0.73445	-0.097199	0.43771	0.99953
	0.35748[0.722]	61.814 [0.000]	-0.31189[0.756]	38548.0	
Nor	-1.3378	0.70932	-0.59945	1.4102	0.99938
	-0.46397[0.645]	70.576[0.000]	-2.4462[0.019]	28809.0	
Por	0.87233	0.80607	-0.037987	1.0682	0.99784
	0.27285[0.786]	49.491[0.000]	-0.18102[0.857]	8313.4	
Spa	8.3900	0.72040	-0.27067	0.93052	0.99929
	2.3103 [0.026]	25.768[0.000]	-1.3529[0.184]	25285.0	
Swe	-8.3041	0.80633	-0.26235	1.2238	0.99898
	-0.94973[0.348]	22.008 [0.000]	-0.45893[0.649]	17681.0	
Swi	0.19999	0.75177	-1.1823	0.62459	0.99976
	0.072393[0.942]	97.473 [0.000]	-2.8621[0.006]	73770.0	
UK	-2.9592	0.84996	-0.44231	0.49495	0.99954
	-0.44070 [0.662]	23.292 [0.000]	-2.6065[0.013]	39004.0	
US	-6.4745	0.85810	-0.50954	0.66271	0.99984
	-2.1568[0.037]	69.052 [0.000]	-2.3262[0.025]	110570	

Table 3.41 **Individual Country FMOLS Regression Estimates**

CO	constant	trend	y/N_t	Inf_t	DW/F-s	R^2
Ausl	5.4936	0.34201	0.68824	0.26876	0.88171	0.99941
	0.91792 [0.364]	4.6860[0.000]	19.309 [0.000]	0.91897[0.364]	19843.0	
Aus	2.5183	0.091958	0.74976	-1.2169	1.5009	0.99988
	2.5643[0.014]	1.1092 [0.274]	65.215[0.000]	-8.1490[0.000]	93399.0	
Bel	2.7786	0.17949	0.73281	-0.31315	0.50747	0.99910
	0.68588[0.497]	0.70584[0.484]	17.116[0.000]	-0.53745[0.595]	12951.0	
Can	-7.1635	0.059235	0.82778	-0.98330	0.89029	0.99968
	-1.3924 [0.172]	0.93065[0.358]	26.596[0.000]	-5.0134[0.000]	36094.0	
Den	8.0041	0.64123	0.66675	0.30299	1.5414	0.99679
	2.3871 [0.022]	4.9163[0.000]	31.290 [0.000]	1.1142[0.272]	3627.7	
Fin	4.7478	-0.075026	0.74730	-1.0095	0.92029	0.99929
	1.0260[0.311]	-0.53471 [0.596]	22.631 [0.000]	-3.9271[0.000]	16368.0	
Fra	2.2411	0.58494	0.69665	-0.021414	0.62050	0.99963
	0.71124 [0.481]	5.0739 [0.000]	27.146[0.000]	-0.090520[0.928]	31361.0	
Gre	54.183	-0.96722	0.68280	0.047211	1.0447	0.99871
	4.3318 [0.000]	-2.4309[0.020]	34.513[0.000]	0.10276[0.919]	9053.5	
Ire	30.950	-0.46536	0.61071	0.57751	0.61954	0.99846
	6.9131[0.000]	-3.9789[0.000]	16.031 [0.000]	2.9517[0.005]	7558.2	
Ita	-5.0725	0.31307	0.75342	-0.12808	0.83166	0.99980
	-1.4473[0.156]	5.8866 [0.000]	52.764 [0.000]	-1.0385[0.306]	57544.0	
Jap	2.9023	-0.035691	0.69474	-0.72764	0.91142	0.99920
	0.99982[0.324]	-0.16619[0.868]	35.821[0.000]	-2.7925[0.008]	14547.0	
Kor	15.487	-0.63404	0.72383	-0.24798	0.51237	0.99596
	4.9289[0.000]	-8.1849 [0.000]	15.176 [0.000]	-2.0974[0.043]	2873.6	
Net	0.88552	0.027406	0.73058	-0.080310	0.45482	0.99953
	0.33234[0.742]	0.12941 [0.898]	21.842[0.000]	-0.23105[0.819]	24676.0	
Nor	-2.5328	-0.15903	0.73175	-0.70073	1.4278	0.99935
	-0.80636 [0.425]	-0.81773[0.419]	25.776 [0.000]	-2.4194[0.020]	17977.0	
Por	-2.1087	0.082496	0.81176	-0.013984	1.06876	0.99783
	-0.38141[0.706]	0.54933[0.588]	41.503[0.000]	-0.068270[0.946]	5373.8	
Spa	4.8635	0.17187	0.69970	0.079404	0.68169	0.99957
	1.6751[0.102]	3.0085 [0.004]	33.578[0.000]	0.49928[0.620]	27051.0	
Swe	-5.6056	0.39936	0.74011	0.99468	0.94803	0.99934
	-0.82284[0.416]	3.0492 [0.004]	22.097[0.000]	1.8718[0.069]	17548.0	
Swi	0.069797	0.26580	0.72700	-1.0820	0.68349	0.99976
	0.025907 [0.979]	1.0552 [0.298]	30.914 [0.000]	-2.6214[0.012]	47931.0	
UK	-1.9066	0.18678	0.81382	-0.19707	0.89450	0.99971
	-0.45234[0.653]	4.0014 [0.000]	34.775 [0.000]	-1.7930[0.081]	40701.0	
US	16.323	0.76753	0.68079	-0.54715	0.61238	0.99971
	1.2741 [0.211]	1.7808 [0.083]	6.8631 [0.000]	-2.6281[0.012]	81279.0	

Table 3.42 Individual Country DOLS Regression Estimates

CO	constant	y/N_t	$I.R._t$	DW/F-stat	R^2
Ausl	11.937	0.69173	0.27948	0.88427	0.99969
	1.5150[0.144]	21.099[0.000]	0.67387[0.507]	4449.8	
Aus	4.8392	0.77485	-2.0354	0.61860	0.99992
	2.4084[0.024]	153.56[0.000]	-4.5913[0.000]	17568.0	
Bel	-3.6686	0.74965	1.0032	0.68094	0.99968
	-1.2808[0.213]	43.618[0.000]	3.7248[0.001]	4240.2	
Can	-6.8568	0.82906	-0.46647	0.85975	0.99970
	-0.90954[0.372]	18.583[0.000]	-1.6362[0.116]	4580.9	
Den	-6.6423	0.76124	1.3430	0.92290	0.99976
	-1.3733[0.183]	42.960[0.000]	3.3738[0.002]	5730.7	
Fin	9.7500	0.71179	-0.850256	0.75744	0.99944
	1.1397[0.266]	15.543[0.000]	-1.40115[0.175]	2455.7	
Fra	1.1073	0.75351	0.32729	0.70462	0.99973
	0.25638[0.800]	33.421[0.000]	1.0596[0.300]	5037.9	
Gre	83.764	0.63812	-2.7446	1.0726	0.99924
	4.3253[0.000]	29.092[0.000]	-2.7503[0.011]	1816.9	
Ire	25.396	0.57164	0.49351	0.45923	0.99852
	2.2778[0.032]	11.077[0.000]	0.88177[0.387]	925.83	
Ita	-6.5151	0.76984	0.36413	1.3524	0.99990
	-2.6098 [0.015]	80.054[0.000]	4.6264[0.000]	14112.0	
Jap	15.984	0.66955	-2.5760	0.72553	0.99988
	3.4785[0.002]	55.182[0.000]	-3.8152[0.000]	11405.0	
Kor	4.9317	0.53944	0.67757	1.5441	0.99755
	1.1728[0.253]	12.050[0.000]	4.6247[0.000]	560.84	
Net	-5.5686	0.72657	1.3898	0.57040	0.99974
	-1.6226[0.118]	64.640[0.000]	2.2202[0.037]	5224.8	
Nor	-4.1103	0.70445	-0.00033863	1.0885	0.99979
	-1.9486[0.064]	47.278[0.000]	-0.00089796[0.999]	6396.7	
Por	5.2551	0.79041	-0.24178	1.5744	0.99878
	1.0986[0.283]	43.417[0.000]	-0.98836[0.333]	1128.7	
Spa	7.1879	0.69142	0.36459	1.2401	0.99984
	3.3970[0.002]	55.694[0.000]	3.7343[0.001]	8368.1	
Swe	-0.84014	0.75351	0.93255	0.88805	0.99967
	-0.12134[0.904]	27.971[0.000]	1.4610[0.158]	4153.5	
Swi	-3.1627	0.76285	-0.64197	0.47857	0.99985
	-1.2810[0.213]	88.098[0.000]	-0.69856[0.492]	9043.6	
UK	-10.599	0.86541	0.20665	0.99857	0.99984
	-1.4906[0.150]	27.311[0.000]	0.80990[0.426]	8729.5	
US	-4.9910	0.84936	-0.41591	1.1503	0.99991
	-1.6298[0.117]	88.699[0.000]	-1.6989[0.103]	15529.0	

Table 3.43 **Individual Country DOLS Regression Estimates**

CO	constant	trend	y/N_t	$I.R._t$	DW/F-s	R^2
Ausl	8.8133	0.38403	0.67505	0.029622	0.98332	0.99981
	1.4575[0.159]	3.3205[0.003]	26.623[0.000]	0.091630[0.927]	6442.7	
Aus	3.9280	-0.30777	0.82064	-1.8385	0.58610	0.99993
	1.8505[0.077]	-1.1422[0.265]	20.317[0.000]	-3.9371[0.000]	17244.0	
Bel	-1.3377	0.50379	0.66600	1.0200	1.0156	0.99975
	-0.53370[0.598]	2.2033[0.038]	16.508[0.000]	4.7704[0.000]	4902.5	
Can	-2.6053	0.086993	0.79309	-0.39141	0.79085	0.99971
	-0.26103[0.796]	0.65855[0.517]	11.202[0.000]	-1.2642[0.219]	4205.0	
Den	-5.3085	0.19293	0.73250	1.5193	0.99781	0.99978
	-1.1741[0.252]	1.2294[0.231]	25.820[0.000]	3.9092[0.000]	5493.3	
Fin	4.7968	-0.24270	0.78267	-1.4080	0.77061	0.99945
	0.41660[0.681]	-0.64588[0.525]	6.5784[0.000]	-1.3323[0.196]	2258.5	
Fra	4.7984	0.56063	0.67112	0.33054	1.3300	0.99991
	2.4846[0.021]	7.2640[0.000]	44.916[0.000]	2.4804[0.021]	13392.0	
Gre	86.639	-0.40248	0.63636	-2.2537	1.0944	0.99926
	4.5135[0.000]	-0.77178[0.448]	29.668[0.000]	-1.9425[0.065]	1667.3	
Ire	-19.0616	-1.7443	0.94275	3.7590	1.0551	0.99946
	-2.4159[0.024]	-7.2535[0.000]	16.807[0.000]	7.3038[0.000]	2265.4	
Ita	-10.142	0.14164	0.77420	0.31787	1.3428	0.99992
	-4.1887[0.000]	2.9492[0.007]	94.768[0.000]	4.7019[0.000]	14578.0	
Jap	19.521	0.46786	0.61437	-3.5555	1.2258	0.99994
	8.7549[0.000]	5.9058[0.000]	56.212[0.000]	-9.9727[0.000]	20688.0	
Kor	30.842	-1.2329	0.83563	-1.0771	0.94874	0.99918
	7.0432[0.000]	-7.2280[0.000]	17.071[0.000]	-4.1733[0.000]	1506.1	
Net	-4.5586	0.30219	0.67410	1.4167	0.69794	0.99976
	-1.4138[0.171]	1.0575[0.301]	13.312[0.000]	2.5195[0.019]	5054.4	
Nor	-3.2404	0.10015	0.69063	0.0053295	1.1160	0.99979
	-1.2922[0.209]	0.59820[0.555]	25.357[0.000]	0.014587[0.988]	5812.2	
Por	4.1125	0.056402	0.79104	-0.25559	1.6007	0.99879
	0.72373[0.476]	0.35459[0.726]	44.188[0.000]	-1.0540[0.303]	1016.7	
Spa	6.1389	0.083895	0.69216	0.26490	1.1661	0.99985
	2.8429[0.009]	1.5122[0.144]	57.643[0.000]	2.3013[0.031]	8203.4	
Swe	-5.5112	0.42499	0.75932	0.10662	1.0307	0.99974
	-0.94596[0.354]	2.6515[0.014]	34.96[0.000]	0.17792[0.860]	4837.6	
Swi	-3.4412	-0.49218	0.81001	-0.64892	0.47659	0.99986
	-1.4107[0.172]	-0.84389[0.407]	14.331[0.000]	-0.72133[0.478]	8612.6	
UK	6.3998	0.25278	0.77212	-0.42148	0.96887	0.99991
	0.88159[0.387]	3.6433[0.001]	21.667[0.000]	-1.5982[0.124]	13488.0	
US	44.491	1.5753	0.49572	-0.90836	1.1313	0.99994
	3.0079[0.006]	3.3884[0.002]	4.7380[0.000]	-3.8245[0.000]	20734.0	

Table 3.44 **Individual Country FMOLS Regression Estimates**

CO	constant	y/N_t	$I.R._t$	DW/F-stat	R^2
Ausl	0.69357	0.72033	0.73944	0.43322	0.99927
	0.089451[0.929]	19.334 [0.000]	2.0293[0.049]	24748.0	
Aus	2.8641	0.77481	-1.4600	0.89370	0.99983
	1.7224 [0.093]	153.40 [0.000]	-5.0017[0.000]	102920	
Bel	-5.5041	0.76834	0.75392	0.80215	0.99941
	-2.0021 [0.052]	61.231 [0.000]	2.8634[0.006]	30522.0	
Can	-13.678	0.87509	-0.74472	0.56297	0.99923
	-1.7185 [0.094]	20.158 [0.000]	-2.5090[0.016]	23378.0	
Den	-7.3114	0.77008	1.0361	0.97043	0.99946
	-1.2536 [0.218]	44.816 [0.000]	2.1399[0.039]	33437.0	
Fin	10.241	0.74611	-1.7088	0.73287	0.99885
	1.2766 [0.209]	26.598 [0.000]	-2.4346[0.019]	15601.0	
Fra	-8.9566	0.82278	-0.11043	0.31523	0.99906
	-1.3887 [0.173]	27.445 [0.000]	-0.25229[0.802]	19213.0	
Gre	25.589	0.71953	-0.23736	0.77059	0.99825
	1.9751 [0.055]	41.944[0.000]	-0.39867[0.692]	10279.0	
Ire	46.208	0.44062	0.22094	0.61412	0.99376
	4.2925 [0.000]	7.4422[0.000]	0.39554[0.696]	2867.2	
Ita	7.3428	0.71609	0.16435	0.76778	0.99944
	1.1956 [0.239]	30.453 [0.000]	0.74918[0.458]	32009.0	
Jap	15.501	0.66617	-2.6108	1.2046	0.99929
	3.7367[0.000]	63.962 [0.000]	-4.6229[0.000]	25183.0	
Kor	-8.1859	0.66892	1.1220	0.67877	0.96586
	-1.0485 [0.301]	7.4692 [0.000]	4.5010[0.000]	509.28	
Net	-2.7498	0.73212	0.65117	0.78042	0.99954
	-1.0553[0.298]	70.996 [0.000]	1.4430[0.157]	39520.0	
Nor	-5.7164	0.70836	0.18266	1.2054	0.99920
	-1.5409[0.132]	36.181[0.000]	0.39827[0.692]	22430.0	
Por	0.75668	0.80542	-0.013188	1.1168	0.99787
	0.17052[0.866]	49.796[0.000]	-0.062945[0.950]	8443.8	
Spa	6.4343	0.70619	0.19655	0.65881	0.99942
	1.6789 [0.101]	33.172[0.000]	1.1271[0.267]	30761.0	
Swe	-8.8781	0.79046	0.48647	1.0585	0.99914
	-1.1579[0.254]	27.767 [0.000]	0.81190[0.422]	20863.0	
Swi	-1.8190	0.76312	-1.3857	0.52478	0.99975
	-0.75051[0.457]	108.25 [0.000]	-2.6777[0.011]	71666.0	
UK	-6.4191	0.85599	-0.081353	0.60614	0.99938
	-0.78373 [0.438]	20.704 [0.000]	-0.27053[0.788]	28855.0	
US	-7.7100	0.86637	-0.49555	0.66169	0.99980
	-2.5629[0.014]	70.570 [0.000]	-2.0097[0.051]	90929.0	

Table 3.45 **Individual Country FMOLS Regression Estimates**

CO	constant	trend	y/N_t	$I.R._t$	DW /F-s	R^2
Ausl	-0.55998	0.27927	0.71759	0.31180	0.86262	0.99957
	-0.11355 [0.910]	3.7224[0.001]	30.263[0.000]	1.2022[0.237]	26857.0	
Aus	3.1356	-0.032925	0.77856	-1.4970	0.91581	0.99982
	1.7513[0.088]	-0.24476[0.808]	41.500[0.000]	-4.7517[0.000]	64824.0	
Bel	-3.9735	0.23362	0.72754	0.76236	0.86826	0.99943
	-1.4717[0.150]	1.5071 [0.140]	26.440 [0.000]	3.1063[0.003]	20439.0	
Can	-0.85904	0.23919	0.77639	-0.64265	0.52198	0.99937
	-0.11215 [0.911]	2.6427 [0.012]	16.979 [0.000]	-2.4317[0.020]	18361.0	
Den	-6.1595	0.11480	0.74986	1.1833	0.91693	0.99947
	-1.0416[0.304]	0.71753[0.477]	25.689[0.000]	2.3532[0.024]	22141.0	
Fin	12.126	0.14736	0.70337	-1.2327	0.66974	0.99889
	1.4894[0.145]	0.80106[0.428]	15.509[0.000]	-1.5568[0.128]	10499.0	
Fra	1.6218	0.56984	0.70016	0.016750	0.62577	0.99966
	0.46122[0.649]	6.0668 [0.000]	31.143 [0.000]	0.076897[0.939]	33836.0	
Gre	40.590	-1.1220	0.69518	0.94988	0.99284	0.99879
	3.5105[0.001]	-2.7894 [0.008]	44.260[0.000]	1.5148[0.138]	9622.8	
Ire	16.434	-0.79592	0.71437	1.2877	1.2798	0.99848
	2.9877 [0.005]	-7.2679[0.000]	18.367 [0.000]	4.6045[0.000]	7646.8	
Ita	-6.5894	0.32498	0.75144	0.076739	0.89356	0.99977
	-1.7747[0.084]	5.5808 [0.000]	56.778 [0.000]	0.61819[0.540]	50436.0	
Jap	15.286	0.071989	0.65936	-2.6411	1.2390	0.99927
	3.5867[0.001]	0.39358[0.697]	34.753 [0.000]	-4.7009[0.000]	15901.0	
Kor	18.415	-0.87663	0.78166	-0.32346	1.3118	0.99549
	4.8156[0.000]	-10.393 [0.000]	19.919 [0.000]	-2.3980[0.021]	2576.4	
Net	-2.6745	0.086865	0.71778	0.69210	0.84009	0.99953
	-1.0317 [0.309]	0.48847 [0.628]	23.390 [0.000]	1.5178[0.138]	25012.0	
Nor	-3.7072	0.15209	0.68895	0.074389	1.2825	0.99923
	-0.79438 [0.435]	0.59922 [0.555]	18.842 [0.000]	0.16364[0.871]	15238.0	
Por	1.1558	0.12648	0.80186	-0.24851	1.1160	0.99777
	0.19613[0.846]	0.81092[0.422]	44.797 [0.000]	-1.1745[0.248]	5223.6	
Spa	3.6277	0.13134	0.70986	0.14272	0.74250	0.99960
	1.1879 [0.242]	2.4054 [0.021]	42.774[0.000]	0.97370[0.336]	28816.0	
Swe	-12.703	0.24146	0.77626	0.86074	0.94533	0.99943
	-2.0455[0.048]	2.3293[0.025]	33.110[0.000]	1.7811[0.083]	20400.0	
Swi	-1.7124	0.30834	0.73285	-1.2563	0.52739	0.99977
	-0.73715 [0.465]	1.2931 [0.204]	32.117 [0.000]	-2.5404[0.015]	50300.0	
UK	2.6681	0.24396	0.79261	-0.38075	0.90767	0.99970
	0.60950 [0.546]	5.4114 [0.000]	35.095 [0.000]	-2.2850[0.028]	39229.0	
US	33.930	1.3607	0.55352	-0.79339	0.51828	0.99988
	2.7569[0.009]	3.3661 [0.001]	5.9631 [0.000]	-3.5827[0.001]	100260	

Table 3.46 **Individual Country DOLS Regressions (con)**

CO	y/N_t	LA/y_t	DW/F-stat	R^2
Ausl	0.76366	-1.0957	0.61773	0.99993
	27.730[0.000]	-8.1160[0.000]	20901.0	
Aus	0.79851	0.21941	0.58052	0.99987
	41.114[0.000]	0.73387[0.470	10790.0	
Bel	0.72943	-1.1306	0.86416	0.99995
	128.15[0.000]	-10.021[0.000]	26980.0	
Can	0.86435	0.94062	0.40129	0.99977
	8.9822[0.000]	1.0934[0.285]	6281.7	
Den	0.71490	-1.4901	1.0056	0.99996
	107.44[0.000]	-11.066[0.000]	31994.0	
Fin	0.67180	-1.7301	0.89835	0.99977
	9.8506[0.000]	-1.9336[0.065]	6198.1	
Fra	0.76720	-2.5590	1.1462	0.99997
	74.712[0.000]	-14.302[0.000]	43018.0	
Gre	0.68799	-1.1885	0.78076	0.99943
	41.483[0.000]	-2.1891[0.039]	2536.7	
Ire	0.56173	-0.75671	0.71276	0.99954
	13.852[0.000]	-4.1902[0.000]	3116.7	
Ita	0.72404	-0.73014	0.89862	0.99994
	52.243[0.000]	-5.9282[0.000]	24947.0	
Jap	0.69679	-1.3150	0.35167	0.99977
	60.522[0.002]	-1.6214[0.118]	6133.8	
Kor	0.65027	-0.19511	0.87767	0.99765
	11.300[0.000]	-4.2738[0.000]	609.56	
Net	0.70892	-0.82414	0.86948	0.99992
	103.98[0.000]	-6.0606[0.000]	18709.0	
Nor	0.64141	-2.9413	1.5097	0.99992
	82.583[0.000]	-10.336[0.000]	17621.0	
Por	0.81491	-2.0353	1.5131	0.99971
	109.90[0.000]	-3.8584[0.000]	5039.0	
Spa	0.73026	-1.1260	0.73483	0.99994
	42.959[0.000]	-7.2378[0.000]	22346.0	
Swe	0.73672	-2.4656	1.4925	0.99998
	64.840 [0.000]	-15.720[0.000]	76074.0	
Swi	0.74343	-0.25440	0.54976	0.99981
	54.923[0.000]	-1.5682[0.130]	7477.9	
UK	0.89970	-1.0862	1.2873	0.99997
	70.906[0.000]	-12.851[0.000]	57228.0	
US	0.90101	2.3231	0.64414	0.99990
	54.213[0.000]	2.3010[0.030]	13903.0	

Table 3.47 **Individual Country DOLS Regressions (con & trend)**

CO	y/N_t	LA/y_t	DW/F-s	R^2
Ausl	0.76029	-0.83393	0.62340	0.99993
	27.473[0.000]	-1.7540[0.093]	19307.0	
Aus	0.78673	0.33782	0.58669	0.99987
	30.921[0.000]	0.99705[0.329]	10032.0	
Bel	0.76389	-1.1887	1.0769	0.99996
	62.459[0.000]	-13.448[0.000]	34312.0	
Can	0.69456	-0.43786	0.70280	0.99987
	8.7296[0.000]	-0.63822[0.529]	9637.4	
Den	0.73320	-1.6155	0.91796	0.99996
	39.987[0.000]	-8.9346[0.000]	30282.0	
Fin	0.76780	-2.4303	1.3858	0.99986
	13.741[0.000]	-3.6581[0.001]	8932.1	
Fra	0.76153	-2.4669	1.1124	0.99997
	42.815[0.000]	-8.3270[0.000]	38988.0	
Gre	0.68800	-1.1897	0.78068	0.99943
	38.278[0.000]	-1.4629[0.157]	2283.7	
Ire	0.57198	-0.65805	0.69321	0.99957
	14.403[0.000]	-3.4165[0.002]	2995.4	
Ita	0.72400	-0.85891	0.89458	0.99994
	51.966[0.000]	-2.6452[0.014]	22705.0	
Jap	0.67575	-1.1863	0.35849	0.99981
	40.160[0.000]	-1.5566[0.133]	6739.7	
Kor	0.78562	0.38838	0.95924	0.99843
	25.268[0.000]	5.5886[0.000]	874.67	
Net	0.76039	-1.0684	0.94656	0.99997
	86.229[0.000]	-12.940[0.000]	39124.0	
Nor	0.65046	-3.0116	1.5008	0.99992
	41.074[0.000]	-9.8162[0.000]	16071.0	
Por	0.83502	-1.3608	1.5391	0.99973
	58.165[0.000]	-2.0856[0.048]	4781.2	
Spa	0.73350	-1.2397	0.72732	0.99994
	37.269[0.000]	-3.3000[0.003]	20231.0	
Swe	0.74039	-2.3701	1.5069	0.99998
	54.029[0.000]	-9.1714[0.000]	68838.0	
Swi	0.72389	-0.23889	0.54723	0.99981
	16.253[0.000]	-1.4356[0.165]	6832.8	
UK	0.90578	-1.2736	1.2780	0.99998
	44.140[0.000]	-5.6530[0.000]	53029.0	
US	0.79416	2.3848	0.54075	0.99991
	7.8840[0.000]	2.3285[0.029]	13667.0	

Table 3.48 **Individual Country FMOLS Regressions (con)**

CO	y/N_t	LA/y_t	DW/F-stat	R^2
Ausl	0.74016	-1.4754	0.61576	0.99356
	50.611[0.000]	-11.688[0.000]	2853.0	
Aus	0.85824	1.4670	0.54134	0.99531
	51.619[0.000]	5.7540[0.000]	3930.3	
Bel	0.74087	-0.66613	0.31154	0.99806
	128.22[0.000]	-6.6547[0.000]	9495.4	
Can	0.72752	1.17002	0.58461	0.91700
	12.816[0.000]	1.9534[0.058]	204.39	
Den	0.74568	-0.52477	0.27382	0.99291
	51.889[0.000]	-1.7938[0.081]	2591.7	
Fin	0.58075	-3.8426	0.96149	0.97174
	25.450[0.000]	-11.305[0.000]	636.23	
Fra	0.77399	-2.1770	0.76538	0.99785
	68.750 [0.000]	-9.2163[0.000]	8581.1	
Gre	0.69413	-1.2607	1.0514	0.99624
	61.184[0.000]	-3.6577[0.000]	4908.0	
Ire	0.79234	-1.8316	0.17487	0.90689
	27.895[0.000]	-15.470[0.000]	180.20	
Ita	0.74155	-0.73589	0.74544	0.99614
	64.940 [0.000]	-6.0401[0.000]	4776.2	
Jap	0.70144	-1.0498	0.26233	0.99921
	142.76[0.000]	-4.1493[0.000]	23265.0	
Kor	0.63078	-0.22197	0.43656	0.89679
	12.048 [0.000]	-5.3941[0.000]	160.75	
Net	0.70951	-0.70785	0.58605	0.99916
	105.96 [0.000]	-5.1435[0.000]	21925.0	
Nor	0.65403	-2.3800	1.5024	0.99889
	121.22[0.000]	-15.066[0.000]	16594.0	
Por	0.79922	-1.8320	0.90109	0.99602
	80.680[0.000]	-8.2632[0.000]	4632.8	
Spa	0.73881	-1.2123	0.66772	0.99830
	108.13[0.000]	-11.989[0.000]	10874.0	
Swe	0.70872	-3.5078	1.0880	0.99599
	84.567[0.000]	-23.085[0.000]	4595.4	
Swi	0.77235	0.18723	0.41571	0.99752
	71.256 [0.000]	1.7615[0.086]	7437.7	
UK	0.84586	-1.4832	0.44805	0.99088
	42.402[0.000]	-7.9041[0.000]	2009.2	
US	0.85879	0.91174	0.80542	0.99498
	56.705 [0.000]	1.7295[0.092]	3664.7	

Table 3.49 **Individual Country FMOLS Regressions (con & trend)**

CO	y/N_t	LA/y_t	DW/F-s	R^2
Ausl	0.74326	-1.6915	0.53287	0.99437
	49.397[0.000]	-7.0838[0.000]	3265.1	
Aus	0.79534	2.7312	0.58047	0.99429
	38.535[0.000]	8.0720[0.000]	3222.4	
Bel	0.74794	-0.60209	0.32626	0.99800
	63.028[0.000]	-5.4998[0.000]	9224.6	
Can	0.46274	-2.2177	0.69216	0.89869
	9.2130[0.000]	-3.6373[0.000]	164.11	
Den	0.79039	-0.86714	0.34769	0.99563
	65.549[0.000]	-5.0596[0.000]	4210.8	
Fin	0.67251	-5.0203	1.2171	0.97370
	27.188[0.000]	-12.236[0.000]	684.96	
Fra	0.72992	-1.0186	0.76411	0.99820
	80.836[0.000]	-5.4839[0.000]	10278.0	
Gre	0.69459	-1.3704	1.0526	0.99630
	52.971 [0.000]	-3.1149[0.003]	4980.8	
Ire	0.72653	-2.0510	0.24371	0.89820
	23.095 [0.000]	-11.476[0.000]	163.23	
Ita	0.74956	0.067266	0.81141	0.99432
	63.697[0.000]	0.21908[0.827]	3236.1	
Jap	0.68373	-1.1270	0.29522	0.99925
	49.632 [0.000]	-4.4545[0.000]	24652.0	
Kor	0.86497	0.55047	1.1042	0.93989
	20.765[0.000]	5.4650[0.061]	289.26	
Net	0.75524	-0.86459	0.76279	0.99958
	109.61[0.000]	-12.156[0.000]	44329.0	
Nor	0.65143	-2.3548	1.5041	0.99889
	61.016[0.000]	-14.968[0.000]	16694.0	
Por	0.79014	-1.7979	0.99067	0.99648
	60.557[0.000]	-7.1893[0.000]	5230.7	
Spa	0.73290	-0.75340	0.53759	0.99686
	117.75[0.000]	-5.4137[0.000]	5864.6	
Swe	0.70619	-3.7843	1.1293	0.99550
	83.731[0.000]	-20.560[0.000]	4096.8	
Swi	0.76016	0.21217	0.39302	0.99769
	35.059[0.000]	1.8416[0.073]	7989.9	
UK	0.85057	-1.6885	0.46851	0.99079
	41.649[0.000]	-5.8590[0.000]	1990.0	
US	0.84326	0.76344	0.83178	0.99558
	6.8208[0.000]	1.3936[0.171]	4163.6	

Table 3.50 **Individual Country DOLS Regressions (con)**

CO	y/N_t	Inf_t	DW/F-stat	R^2
Ausl	0.91022	-1.1665	0.93608	0.99967
	14.202[0.000]	-2.3294[0.028]	4329.3	
Aus	0.76826	-0.95551	1.0763	0.99994
	151.12[0.000]	-4.1424[0.000]	24971.0	
Bel	0.77973	0.16246	0.19686	0.99925
	17.712[0.000]	0.086945[0.931]	1921.6	
Can	0.88623	-0.98451	0.81939	0.99981
	23.373[0.000]	-4.6753[0.000]	7737.8	
Den	0.79689	1.3900	1.0682	0.99985
	68.955[0.000]	7.4575[0.000]	9849.5	
Fin	0.76242	-0.59338	0.87056	0.99952
	25.696[0.000]	-2.0691[0.049]	3017.2	
Fra	0.82776	-0.39594	0.79895	0.99990
	70.230[0.000]	-2.9408[0.007]	14239.0	
Gre	0.73728	-0.91272	0.62875	0.99922
	34.412[0.000]	-1.0482[0.305]	1850.2	
Ire	0.41819	0.060389	0.44026	0.99857
	7.9171[0.000]	0.15980[0.874]	1005.4	
Ita	0.63839	0.053013	0.75181	0.99969
	9.7269[0.000]	0.15195[0.880]	4562.9	
Jap	0.70367	-0.31190	0.38732	0.99968
	59.195[0.000]	-0.73800[0.467]	4455.5	
Kor	0.29865	-0.95095	0.94098	0.99333
	6.1396[0.001]	-3.7043[0.001]	214.23	
Net	0.72069	-0.16826	0.92848	0.99982
	83.440[0.000]	-0.71311[0.482]	7944.0	
Nor	0.69136	-1.1325	1.1744	0.99956
	51.115[0.000]	-3.8248[0.000]	3298.9	
Por	0.79089	0.094894	1.7479	0.99979
	69.989[0.000]	0.78833[0.438]	6981.2	
Spa	0.78203	-0.39320	0.71476	0.99986
	24.039[0.000]	-2.7775[0.010]	10403.0	
Swe	0.88818	0.000068268	0.89734	0.99981
	28.577[0.000]	0.00016969[0.999]	7456.9	
Swi	0.74724	-1.2777	0.52784	0.99979
	46.139[0.000]	-0.86143[0.397]	6715.0	
UK	0.88755	-0.41093	0.84612	0.99980
	17.723[0.000]	-1.7176[0.099]	7103.6	
US	0.86262	-0.34962	0.53711	0.99986
	43.958[0.000]	-0.92224[0.365]	10219.0	

Table 3.51 **Individual Country DOLS Regressions (con & trend)**

CO	y/N_t	Inf_t	DW/F-s	R^2
Ausl	0.81991	-0.42955	1.7555	0.99989
	34.331[0.000]	-2.2842[0.032]	11401.0	
Aus	0.74815	-0.98877	1.0925	0.99995
	66.154[0.000]	-4.6986[0.000]	25978.0	
Bel	0.68329	1.0204	0.27571	0.99937
	8.5241[0.000]	0.57617[0.570]	2059.3	
Can	0.97130	-1.1712	0.85002	0.99983
	11.657[0.000]	-4.4814[0.000]	7430.0	
Den	0.74679	1.5929	1.0676	0.99988
	34.840[0.000]	9.3201[0.000]	10508.0	
Fin	0.84823	-0.75126	1.0573	0.99957
	14.481[0.000]	-2.8753[0.008]	2974.4	
Fra	0.77395	-0.22927	0.93898	0.99993
	46.424[0.000]	-2.3366[0.028]	17962.0	
Gre	0.67981	-0.33970	0.83175	0.99946
	26.589[0.000]	-0.51038[0.614]	2375.1	
Ire	0.56119	0.39991	0.51577	0.99908
	9.2136[0.000]	1.3465[0.191]	1411.8	
Ita	0.76467	-0.033831	1.2671	0.99989
	23.529 [0.000]	-0.23086 [0.819]	11692.0	
Jap	0.68838	-0.30290	0.40807	0.99970
	32.931[0.000]	-0.72600[0.475]	4250.2	
Kor	0.80881	-0.073393	0.85165	0.99873
	14.350[0.000]	-0.57865[0.568]	1021.3	
Net	0.72873	-0.20547	0.94373	0.99982
	36.973[0.000]	-0.83187[0.414]	7203.9	
Nor	0.76446	-1.4928	1.1893	0.99962
	19.364[0.000]	-4.4016[0.000]	3382.5	
Por	0.81434	0.028689	1.6935	0.99981
	44.267[0.000]	0.22549[0.823]	6719.3	
Spa	0.75317	-0.16649	1.0132	0.99991
	34.996[0.000]	-1.5623[0.132]	14941.0	
Swe	0.80428	1.0454	1.1080	0.99989
	32.129[0.000]	3.2846[0.003]	11834.0	
Swi	0.70182	-0.93440	0.58160	0.99983
	24.912[0.000]	-0.72867[0.473]	7544.1	
UK	0.85373	-0.36461	0.96695	0.99987
	23.147[0.000]	-2.1459[0.043]	9672.4	
US	0.43717	-0.97053	0.61205	0.99991
	2.8483[0.009]	-2.5931[0.016]	14562.0	

Table 3.52 **Individual Country FMOLS Regressions (con)**

CO	y/N_t	Inf_t	DW/F-stat	R^2
Ausl	0.74617	-0.41896	0.36352	0.95531
	13.508 [0.000]	-0.92089[0.363]	395.42	
Aus	0.76295	-1.2292	1.2523	0.99940
	206.31 [0.000]	-8.2771[0.000]	30859.0	
Bel	0.76201	-0.48279	0.20155	0.99419
	39.459 [0.000]	-0.86663[0.391]	3167.8	
Can	0.85804	-1.0855	0.95935	0.97889
	35.441[0.000]	-6.0275[0.000]	857.83	
Den	0.73717	-1.9166	0.23005	0.95800
	85.757 [0.000]	-12.136[0.000]	421.98	
Fin	0.73514	-0.92698	0.96585	0.98082
	38.294 [0.000]	-4.5405[0.000]	945.89	
Fra	0.81052	-0.86491	0.18212	0.98886
	34.705 [0.000]	-2.8510[0.007]	1642.5	
Gre	0.71953	-0.52553	0.68208	0.99354
	49.092[0.000]	-1.1769[0.246]	2845.8	
Ire	0.49877	0.89159	0.43621	0.88928
	10.686[0.000]	2.9775[0.006]	148.58	
Ita	0.72853	-0.38319	0.50357	0.98362
	28.016 [0.000]	-1.6118[0.115]	1110.8	
Jap	0.69178	-0.71840	0.41715	0.99809
	81.636 [0.000]	-2.8041[0.008]	9685.5	
Kor	0.37228	-0.40674	0.51851	0.66267
	3.3267 [0.002]	-1.2633[0.219]	36.342	
Net	0.73445	-0.097199	0.41250	0.99747
	61.814 [0.000]	-0.31189[0.756]	7298.0	
Nor	0.70932	-0.59945	0.98548	0.99325
	70.576[0.000]	-2.4462[0.019]	2724.1	
Por	0.80607	-0.037987	0.85140	0.98755
	49.491[0.000]	-0.18102[0.857]	1467.1	
Spa	0.72040	-0.27067	0.32983	0.98313
	25.768[0.000]	-1.3529[0.184]	1077.8	
Swe	0.80633	-0.26235	0.32385	0.97603
	22.008 [0.000]	-0.45893[0.649]	753.26	
Swi	0.75177	-1.1823	0.55628	0.99882
	97.473 [0.000]	-2.8621[0.006]	15604.0	
UK	0.84996	-0.44231	0.61605	0.97206
	23.292 [0.000]	-2.6065[0.013]	643.60	
US	0.85810	-0.50954	0.65166	0.99705
	69.052 [0.000]	-2.3262[0.025]	6255.5	

Table 3.53 **Individual Country FMOLS Regressions (con & trend)**

CO	y/N_t	Inf_t	DW/F-s	R^2
Ausl	0.68824	0.26876	0.72944	0.97887
	19.309[0.000]	0.91897[0.364]	856.88	
Aus	0.74976	-1.2169	1.3208	0.99944
	65.215[0.000]	-8.1490[0.000]	32780.0	
Bel	0.73281	-0.31315	0.23742	0.99439
	17.116 [0.000]	-0.53745[0.594]	3277.4	
Can	0.82778	-0.98330	0.84984	0.98023
	26.596[0.000]	-5.0134[0.000]	917.05	
Den	0.66675	0.30299	0.31214	0.98596
	31.290[0.000]	1.1142[0.272]	1298.9	
Fin	0.74730	-1.0095	0.99033	0.98046
	22.631[0.000]	-3.9271[0.000]	928.30	
Fra	0.69665	-0.021414	0.54344	0.99463
	27.146[0.000]	-0.090520[0.928]	3428.1	
Gre	0.68280	0.047211	0.84293	0.99390
	34.513 [0.000]	0.102763[0.918]	3012.5	
Ire	0.61071	0.57751	0.57227	0.94670
	16.031 [0.000]	2.9517[0.005]	328.60	
Ita	0.75342	-0.12808	0.83928	0.99478
	52.764[0.000]	-1.0385[0.305]	3528.8	
Jap	0.69474	-0.72764	0.41809	0.99811
	35.821 [0.000]	-2.7925[0.008]	9748.5	
Kor	0.72383	-0.24798	0.53073	0.94158
	15.176[0.000]	-2.0974[0.042]	298.19	
Net	0.73058	-0.080310	0.41545	0.99746
	21.842[0.000]	-0.23105[0.818]	7271.9	
Nor	0.73175	-0.70073	1.0237	0.99337
	25.776[0.000]	-2.4194[0.020]	2773.9	
Por	0.81176	-0.013984	0.92264	0.98820
	41.503 [0.000]	-0.068270[0.945]	1548.9	
Spa	0.69970	0.079404	0.52244	0.99065
	33.578[0.000]	0.49928[0.620]	1960.3	
Swe	0.74011	0.99468	0.67123	0.98366
	22.097[0.000]	1.8718[0.069]	1113.8	
Swi	0.72700	-1.0820	0.57702	0.99891
	30.914 [0.000]	-2.6214[0.012]	16921.0	
UK	0.81382	-0.19707	0.77735	0.98388
	34.775 [0.000]	-1.7930[0.081]	1128.9	
US	0.68079	-0.54715	0.58152	0.99752
	6.8631 [0.000]	-2.6281[0.012]	7433.6	

Table 3.54 **Individual Country DOLS Regressions (con)**

CO	y/N_t	I.R.$_t$	DW/F-stat	R^2
Ausl	0.83214	0.78445	0.94550	0.99976
	18.990[0.000]	2.3922[0.025]	5894.8	
Aus	0.78099	-0.74526	0.70970	0.99993
	156.89[0.000]	-1.2356[0.229]	20918.0	
Bel	0.76922	1.4410	1.0229	0.99984
	77.104[0.000]	8.3324[0.002]	8769.9	
Can	0.92091	-0.58997	0.82552	0.99979
	20.829[0.000]	-2.2420[0.034]	6733.0	
Den	0.79230	2.5666	0.85614	0.99979
	41.886[0.000]	5.7039[0.000]	6802.2	
Fin	0.74117	0.22495	0.92105	0.99956
	20.696[0.000]	0.32568[0.747]	3237.3	
Fra	0.80596	0.67731	0.68459	0.99975
	29.539[0.000]	2.2052[0.037]	5719.7	
Gre	0.63614	-3.4168	0.79813	0.99908
	14.396[0.000]	-2.2295[0.035]	1562.6	
Ire	0.41885	-0.66083	0.41959	0.99837
	6.9601[0.000]	-1.3088[0.203]	877.88	
Ita	0.74919	0.48780	1.0043	0.99977
	20.817[0.000]	2.5336[0.018]	6158.2	
Jap	0.65455	-3.2786	0.64859	0.99982
	32.851[0.000]	-2.6853[0.013]	8056.8	
Kor	0.63296	0.91391	1.9153	0.99631
	10.598[0.000]	5.5311[0.000]	388.17	
Net	0.73101	2.8734	0.84559	0.99980
	83.903[0.000]	3.8955[0.000]	7191.5	
Nor	0.71193	-0.28033	1.0031	0.99977
	46.083[0.000]	-0.78337[0.441]	6145.8	
Por	0.80746	-0.0080864	1.5621	0.99951
	75.252[0.000]	-0.071393[0.943]	2904.0	
Spa	0.71201	0.50643	1.1983	0.99985
	41.127[0.000]	5.1547[0.000]	9339.7	
Swe	0.85487	1.5132	0.92901	0.99979
	33.715[0.000]	3.0271[0.005]	6696.8	
Swi	0.76822	-2.4459	0.56230	0.99981
	88.069[0.000]	-2.0286[0.054]	7577.6	
UK	0.92175	0.51987	1.1630	0.99984
	18.595[0.000]	1.8209[0.081]	8953.5	
US	0.85497	-0.29649	1.1741	0.99991
	88.395[0.000]	-1.3459[0.191]	16743.0	

Table 3.55 **Individual Country DOLS Regressions (con & trend)**

CO	y/N_t	$I.R._t$	DW/F-s	R^2
Ausl	0.76096	0.20341	1.3209	0.99989
	26.254[0.000]	0.91824[0.368]	11416.0	
Aus	0.78739	-0.85489	0.74442	0.99993
	46.677[0.000]	-1.3135[0.202]	19024.0	
Bel	0.73067	1.4019	1.0678	0.99986
	30.211 [0.000]	8.6811[0.000]	8990.2	
Can	0.98603	-0.68085	0.91900	0.99979
	9.4818[0.000]	-2.4015[0.025]	6207.5	
Den	0.74772	2.5271	0.86925	0.99982
	26.004[0.000]	6.2404[0.000]	7132.3	
Fin	0.77653	0.018296	0.97229	0.99956
	10.052[0.000]	0.023328[0.981]	2952.5	
Fra	0.70490	0.40867	0.86785	0.99990
	29.829[0.000]	2.3777[0.026]	13062.0	
Gre	0.60027	-2.1430	1.0031	0.99931
	16.274[0.000]	-1.6703[0.109]	1866.2	
Ire	0.74473	1.9369	0.56516	0.99919
	8.7005[0.000]	2.8070[0.010]	1590.4	
Ita	0.75467	0.25593	1.2551	0.99989
	42.273[0.000]	2.5273[0.019]	11944.0	
Jap	0.62787	-3.3207	0.96816	0.99988
	37.796[0.000]	-3.8441[0.000]	10382.0	
Kor	0.82296	-0.61855	1.2000	0.99949
	32.412[0.000]	-5.0708[0.000]	2541.9	
Net	0.71731	2.8346	0.86981	0.99980
	28.038[0.000]	3.9401[0.000]	6592.2	
Nor	0.73302	-0.27230	0.99847	0.99977
	24.660[0.000]	-0.75721[0.456]	5659.1	
Por	0.81458	-0.036137	1.5568	0.99951
	66.828[0.000]	-0.31356[0.756]	2656.9	
Spa	0.70603	0.35384	1.1035	0.99987
	42.295[0.000]	3.0645[0.005]	10167.0	
Swe	0.82842	0.78235	1.1378	0.99987
	43.514[0.000]	1.9717[0.061]	9646.2	
Swi	0.73384	-1.6674	0.51272	0.99982
	20.622[0.000]	-1.1746[0.252]	7352.4	
UK	0.80540	-0.24345	1.0024	0.99989
	13.739[0.000]	-0.67693[0.505]	11576.0	
US	0.60041	-0.80370	1.1301	0.99994
	6.7851[0.000]	-3.0585[0.005]	21271.0	

Table 3.56 **Individual Country FMOLS Regressions (con)**

CO	y/N_t	$I.R._t$	DW/F-stat	R^2
Ausl	0.72033 19.334 [0.000]	0.73944 2.0293[0.049]	0.41013 586.59	0.96943
Aus	0.77481 153.40 [0.000]	-1.4600 -5.0017[0.000]	0.95075 22982.0	0.99920
Bel	0.76834 61.231 [0.000]	0.75392 2.8634[0.006]	0.59582 4966.1	0.99629
Can	0.87509 20.158 [0.000]	-0.74472 -2.5090[0.016]	0.62096 448.7	0.96040
Den	0.77008 44.816 [0.000]	1.0361 2.1399[0.039]	0.45086 2223.4	0.99175
Fin	0.74611 26.598 [0.000]	-1.7088 -2.4346[0.019]	0.66404 581.89	0.96919
Fra	0.82278 27.445 [0.000]	-0.11043 -0.25229[0.802]	0.21956 1253.3	0.98545
Gre	0.71953 41.944[0.000]	-0.23736 -0.39867[0.692]	0.78839 2667.8	0.99311
Ire	0.44062 7.4422[0.000]	0.22094 0.39554[0.696]	0.46077 91.251	0.83144
Ita	0.71609 30.453 [0.000]	0.16435 0.74918[0.458]	0.55576 1312.3	0.98610
Jap	0.66617 63.962 [0.000]	-2.6108 -4.6229[0.000]	0.96132 10336.0	0.99821
Kor	0.66892 7.4692 [0.000]	1.1220 4.5010[0.000]	0.53181 35.545	0.65769
Net	0.73212 70.996 [0.000]	0.65117 1.4430[0.157]	0.70058 7560.4	0.99756
Nor	0.70836 36.181[0.000]	0.18266 0.39827[0.692]	0.68686 2145.7	0.99145
Por	0.80542 49.796[0.000]	-0.013188 -0.062945[0.950]	0.86954 1481.4	0.98767
Spa	0.70619 33.172[0.000]	0.19655 1.1271[0.267]	0.37261 1367.9	0.98666
Swe	0.79046 27.767 [0.000]	0.48647 0.81190[0.422]	0.38270 817.77	0.97788
Swi	0.76312 108.25 [0.000]	-1.3857 -2.6777[0.011]	0.66162 13342.0	0.99862
UK	0.85599 20.704 [0.000]	-0.081353 -0.27053[0.788]	0.39516 615.25	0.97081
US	0.86637 70.570 [0.000]	-0.49555 -2.0097[0.051]	0.64251 6082.4	0.99697

Table 3.57 **Individual Country FMOLS Regressions (con & trend)**

CO	y/N_t	$I.R._t$	DW /F-s	R^2
Ausl	0.71759	0.31180	0.80058	0.98260
	30.263[0.000]	1.2022[0.236]	1044.6	
Aus	0.77856	-1.4970	0.97454	0.99917
	41.500[0.000]	-4.7517[0.000]	22392.0	
Bel	0.72754	0.76236	0.70357	0.99652
	26.440 [0.000]	3.1063[0.003]	5292.6	
Can	0.77639	-0.64265	0.53179	0.96787
	16.979 [0.000]	-2.4317[0.019]	557.20	
Den	0.74986	1.1833	0.47339	0.99222
	25.689[0.000]	2.3532[0.024]	2359.8	
Fin	0.70337	-1.2327	0.65872	0.97264
	15.509[0.000]	-1.5568[0.128]	657.56	
Fra	0.70016	0.016750	0.54779	0.99482
	31.143 [0.000]	0.076897[0.939]	3551.0	
Gre	0.69518	0.94988	1.0218	0.99474
	44.260[0.000]	1.5148[0.138]	3496.5	
Ire	0.71437	1.2877	1.2152	0.94857
	18.367[0.000]	4.6045[0.000]	341.22	
Ita	0.75144	0.076739	0.83253	0.99461
	56.778[0.000]	0.61819[0.540]	3412.4	
Jap	0.65936	-2.6411	0.97975	0.99821
	34.753[0.000]	-4.7009[0.000]	10307.0	
Kor	0.78166	-0.32346	0.52955	0.95906
	19.919[0.000]	-2.3980[0.021]	433.40	
Net	0.71778	0.69210	0.75595	0.99753
	23.390[0.000]	1.5178[0.137]	7486.4	
Nor	0.68895	0.074389	0.74014	0.99173
	18.842[0.000]	0.16364[0.870]	2218.7	
Por	0.80186	-0.24851	0.84590	0.98725
	44.797[0.000]	-1.1745[0.247]	1432.8	
Spa	0.70986	0.14272	0.53146	0.99134
	42.774[0.000]	0.97370[0.336]	2117.9	
Swe	0.77626	0.86074	0.61645	0.98575
	33.110[0.000]	1.7811[0.083]	1279.3	
Swi	0.73285	-1.2563	0.58822	0.99886
	32.117[0.000]	-2.5404[0.015]	16143.0	
UK	0.79261	-0.38075	0.73495	0.98398
	35.095 [0.000]	-2.2850[0.028]	1136.2	
US	0.55352	-0.79339	0.58871	0.99788
	5.9631[0.000]	-3.5827[0.030]	8718.3	

Table 4.18 **Individual Country AR(12) Regression Estimates**

Cf	Austria	Belgium	Canada	Denmark
α	0.0013101	0.00042382	0.00047174	0.0012451
	2.9819 [0.003]	1.8894 [0.059]	1.8290 [0.068]	2.6299 [0.008]
θ_1	0.075730	0.28122	0.016208	0.082917
	1.9229 [0.055]	6.2829 [0.000]	0.37293 [0.709]	1.8303 [0.067]
θ_2	-0.056273	-0.065874	0.085250	0.045986
	-1.4313 [0.153]	-1.4143 [0.157]	1.9623 [0.050]	1.0123 [0.311]
θ_3	-0.0046351	-0.046925	0.10883	-0.016693
	-0.11750[0.906]	-1.0053 [0.315]	2.4928 [0.013]	-0.36710 [0.713]
θ_4	-0.12308	0.11733	0.13776	0.033316
	-3.1187 [0.001]	2.5641 [0.010]	3.1383[0.001]	0.73666 [0.461]
θ_5	0.056531	0.066050	0.032374	-0.040508
	1.4225 [0.155]	1.4336 [0.152]	0.73012 [0.465]	-0.89727 [0.370]
θ_6	-0.0082101	0.13057	0.029980	0.28552
	-0.20758 [0.835]	2.8283[0.004]	0.67439[0.500]	6.3197 [0.000]
θ_7	0.091635	-0.029829	0.0045686	-0.021244
	2.3138 [0.021]	-0.64613[0.518]	0.10270 [0.918]	-0.47019 [0.638]
θ_8	-0.067675	-0.004031	0.068106	0.066915
	-1.7005[0.089]	-0.087469 [0.930]	1.5307 [0.126]	1.4813 [0.139]
θ_9	0.0088047	0.20597	0.062862	0.10311
	0.22269 [0.823]	4.5014 [0.000]	1.4237 [0.155]	2.2789 [0.023]
θ_{10}	0.021247	0.0012787	-0.020056	-0.014176
	0.53715[0.591]	0.027395 [0.978]	-0.45552[0.648]	-0.31165 [0.755]
θ_{11}	0.087202	0.0079630	0.031533	0.057273
	2.2074 [0.027]	0.17091 [0.864]	0.71943 [0.472]	1.2607 [0.208]
θ_{12}	0.51295	0.21934	0.32448	0.15732
	12.982 [0.000]	4.9040[0.000]	7.4011[0.000]	3.4702 [0.000]
	$R^2 = 0.51324$	$R^2 = 0.67417$	$R^2 = 0.65737$	$R^2 = 0.50291$
	DW=2.0210	DW=1.9374	DW=2.0274	DW=1.9880
	F-stat=41.73[0.00]	F-stat=81.89[0.00]	F-stat=75.94[0.00]	F-stat=40.04[0.00]

Table 4.19 **Individual Country AR(12) Regression Estimates**

Cf	Finland	France	Germany	Greece
α	0.0016	0.00028595	0.0005481	0.0028995
	3.007 [0.002]	1.5054[0.132]	2.3314[0.020]	2.4032 [0.016]
θ_1	0.0333	0.28089	0.20648	-0.15713
	0.734 [0.463]	6.3052 [0.000]	4.8669 [0.000]	-3.6302 [0.000]
θ_2	0.0580	0.055225	-0.020167	-0.17466
	1.278 [0.201]	1.1890[0.235]	-0.46478[0.642]	-3.9779 [0.000]
θ_3	0.1520	0.19775	0.094449	-0.000946
	3.346 [0.000]	4.2520 [0.000]	2.1806 [0.029]	-0.021283 [0.983]
θ_4	-0.0522	-0.032288	-0.064920	0.037728
	-1.142 [0.253]	-0.68134[0.496]	-1.4902 [0.136]	0.84880 [0.396]
θ_5	0.0922	0.0052426	-0.027051	0.066585
	2.016 [0.044]	0.11082 [0.911]	-0.61964 [0.535]	1.5039 [0.133]
θ_6	0.0864	0.12674	0.020964	0.37970
	1.882 [0.060]	2.6747[0.007]	0.48004 [0.631]	8.6283[0.000]
θ_7	0.0202	-0.017319	-0.008019	0.13064
	0.440 [0.659]	-0.36544 [0.714]	-0.18356 [0.854]	2.9657[0.003]
θ_8	0.0223	0.076188	0.030215	0.092139
	0.487 [0.625]	1.6068 [0.108]	0.69186 [0.489]	2.0772 [0.038]
θ_9	0.1115	0.0092281	0.021490	0.039288
	2.440 [0.015]	0.19375[0.846]	0.49291 [0.622]	0.88265 [0.377]
θ_{10}	-0.0347	-0.016186	0.080007	-0.079357
	-0.763 [0.445]	-0.34546 [0.729]	1.8450 [0.065]	-1.7811 [0.075]
θ_{11}	0.0242	0.014143	0.077158	-0.000283
	0.534 [0.593]	0.30228 [0.762]	1.7723 [0.077]	-0.006446 [0.994]
θ_{12}	0.1420	0.23734	0.38476	0.32996
	3.129 [001]	5.2938 [0.000]	9.0218 [0.000]	7.5968 [0.000]
	$R^2 = 0.37640$	$R^2 = 0.82621$	$R^2 = 0.58803$	$R^2 = 0.51815$
	DW=2.0377	DW=1.9699	DW=2.0274	DW=2.0839
	F-stat=23.89[0.00]	F-stat=188.19[0.00]	F-stat=56.49[0.00]	F-stat=42.56[0.00]

Table 4.20 **Individual Country AR(12) Regression Estimates**

Cf	Iceland	Ireland	Italy	Japan
α	0.0024855	0.0003658	0.00042501	0.00070825
	1.9610 [0.050]	1.7395 [0.082]	1.6452 [0.100]	1.5645 [0.118]
θ_1	-0.068512	0.68713	0.45344	-0.010265
	-1.5172 [0.129]	15.369 [0.000]	10.110 [0.000]	-0.24502 [0.806]
θ_2	0.058245	0.085999	0.013848	-0.068337
	1.2873 [0.198]	1.5721 [0.116]	0.28019 [0.779]	-1.6405 [0.101]
θ_3	0.29251	-0.25854	0.039587	0.0010114
	6.4537[0.000]	-4.7112 [0.000]	0.80106 [0.423]	0.024214 [0.980]
θ_4	0.081822	0.15959	0.10939	-0.017453
	1.7313 [0.084]	2.8477 [0.004]	2.2116[0.027]	-0.42064 [0.674]
θ_5	0.12923	-0.015185	-0.050668	0.019134
	2.7264[0.006]	-0.26803 [0.788]	-1.0190 [0.308]	0.46281 [0.643]
θ_6	0.10079	-0.059700	0.12374	0.15196
	2.1098 [0.035]	-1.0551[0.291]	2.4878[0.013]	3.6819 [0.000]
θ_7	-0.0008847	0.053811	0.045142	0.057858
	-0.018519[0.985]	0.94815 [0.343]	0.90766[0.364]	1.4011 [0.161]
θ_8	0.019275	0.037804	0.026214	0.079171
	0.40664[0.684]	0.64674 [0.518]	0.52716 [0.598]	1.9134 [0.056]
θ_9	0.014619	0.10153	0.0059247	0.10492
	0.30932 [0.757]	1.7186[0.086]	0.11971 [0.904]	2.5258 [0.011]
θ_{10}	0.0028229	0.010692	-0.019719	-0.028464
	0.062303[0.950]	0.18518[0.853]	-0.39866[0.690]	-0.680346 [0.496]
θ_{11}	0.029713	-0.092571	-0.020772	0.099273
	0.65698[0.511]	-1.5962 [0.111]	-0.41982 [0.674]	2.3785 [0.017]
θ_{12}	0.17300	0.23200	0.20693	0.40659
	3.8324 [0.000]	4.9954[0.000]	4.6135[0.000]	9.6842[0.000]
	$R^2 = 0.52378$	$R^2 = 0.86929$	$R^2 = 0.83522$	$R^2 = 0.40739$
	DW=1.9901	DW=1.9202	DW=1.9387	DW=1.9708
	F-stat=43.53[0.00]	F-stat=263.25[0.00]	F-stat=200.63[0.00]	F-stat=27.21[0.00]

Table 4.21 **Individual Country AR(12) Regression Estimates**

Cf	Luxemb	Norway	Portugal	Spain
α	0.0005607	0.0010403	0.0025107	0.00098935
	2.0478 [0.041]	2.3743[0.017]	2.3959[0.016]	2.0602[0.039]
θ_1	0.021879	-0.010669	0.079310	0.21524
	0.49855 [0.618]	-0.25853 [0.796]	1.7558 0.079]	4.8213 [0.000]
θ_2	0.056954	0.064540	0.0030414	-0.037548
	1.2855 [0.199]	1.5645[0.118]	0.067297[0.946]	-0.82156 [0.411]
θ_3	0.077862	0.074305	0.089959	0.041336
	1.7543 [0.080]	1.7968 [0.073]	1.9927 [0.046]	0.90421[0.366]
θ_4	0.067771	0.0058013	0.011598	0.059245
	1.5287[0.127]	0.13973[0.888]	0.25645[0.797]	1.2973 [0.195]
θ_5	-0.032995	-0.045605	0.023484	0.0084617
	-0.74288 [0.457]	-1.1014 [0.271]	0.51928 [0.603]	0.18747[0.851]
θ_6	0.17618	0.13112	0.085748	0.14713
	3.9643[0.000]	3.1608[0.001]	1.8974 [0.058]	3.2605[0.001]
θ_7	-0.024667	0.010039	0.042689	-0.024915
	-0.54837 [0.583]	0.24240 [0.808]	0.94467 [0.345]	-0.55228[0.581]
θ_8	0.046794	0.061553	0.0072577	0.16119
	1.0335[0.301]	1.4875 [0.137]	0.16051 [0.872]	3.5721[0.000]
θ_9	0.083428	0.0096054	0.067851	0.060894
	1.8435 [0.065]	0.23154[0.817]	1.5007[0.134]	1.3343 [0.182]
θ_{10}	0.030126	-0.017746	0.044742	-0.030117
	0.66639[0.505]	-0.42943 [0.667]	0.99146 [0.321]	-0.65926 [0.510]
θ_{11}	0.017494	0.057277	0.072674	0.031912
	0.38710 [0.698]	1.3900 [0.165]	1.6088[0.108]	0.69838[0.485]
θ_{12}	0.31410	0.43775	0.16928	0.22620
	6.9656[0.000]	10.5788 [0.000]	3.7497 [0.000]	5.0705 [0.000]
	$R^2 = 0.54954$	$R^2 = 0.57288$	$R^2 = 0.29664$	$R^2 = 0.65528$
	DW=1.9750	DW=2.0120	DW=2.0397	DW=1.9819
	F-stat=48.29[0.00]	F-stat=53.09[0.00]	F-stat=16.69[0.00]	F-stat=75.24[0.00]

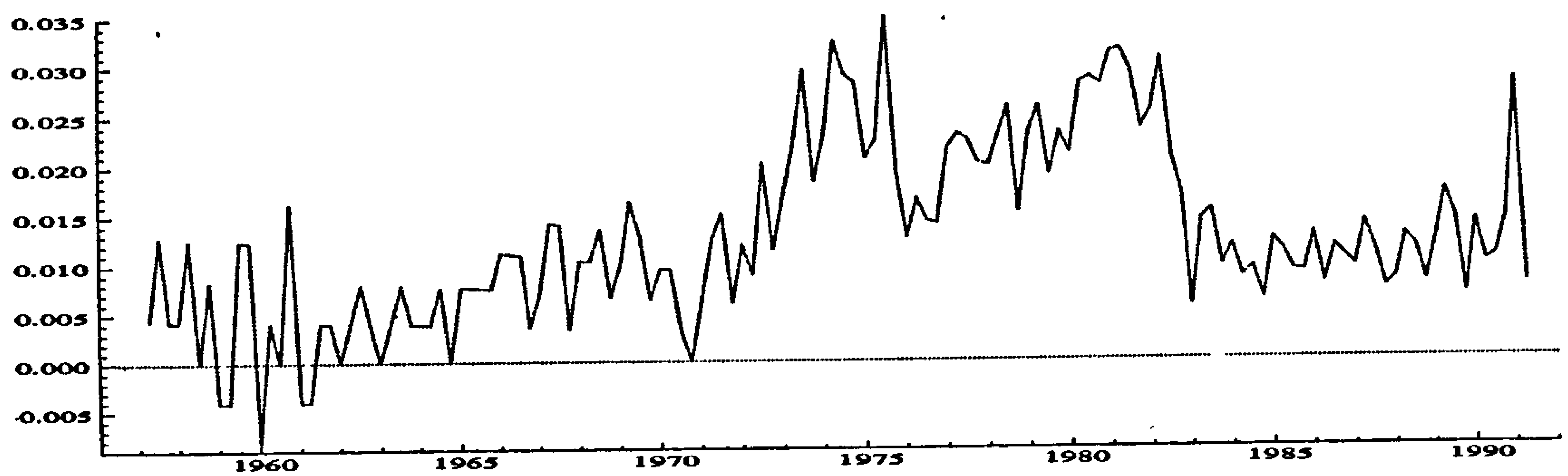
Table 4.22 **Individual Country AR(12) Regression Estimates**

Cf	Sweden	Switzerl	UK	US
α	0.00091974	0.00056031	0.00059700	0.00038584
	2.0993 [0.036]	2.1989 [0.028]	1.6917[0.091]	1.9852 [0.047]
θ_1	0.094583	0.17902	0.25342	0.24851
	2.1353 [0.033]	4.1961 [0.000]	6.3319 [0.000]	5.4487 [0.000]
θ_2	0.030643	0.11614	0.080573	0.16149
	0.69360 [0.488]	2.6790[0.007]	1.9348 [0.053]	3.4418 [0.000]
θ_3	-0.0054611	-0.029653	0.047072	-0.016108
	-0.12355 [0.901]	-0.68237 [0.495]	1.1260 [0.260]	-0.33849 [0.735]
θ_4	-0.0095873	-0.072427	-0.012858	0.063597
	-0.21709 [0.828]	-1.6737 [0.094]	-0.30735[0.758]	1.3447 [0.179]
θ_5	0.012714	0.034079	0.043619	0.041309
	0.29019[0.771]	0.78636 [0.432]	1.0408 [0.298]	0.86995 [0.384]
θ_6	0.061877	0.18149	0.14403	-0.0031660
	1.4178 [0.156]	4.1898[0.000]	3.4407[0.000]	-0.066620 [0.946]
θ_7	0.082968	-0.055473	-0.062547	0.079197
	1.9001[0.058]	-1.2802 [0.201]	-1.4931[0.136]	1.6654 [0.096]
θ_8	0.12397	-0.054824	-0.016052	0.045816
	2.8172[0.005]	-1.2638 [0.206]	-0.38208 [0.702]	0.96055[0.337]
θ_9	0.045143	0.093932	-0.033191	0.12962
	1.0173 [0.309]	2.1680[0.030]	-0.78980[0.430]	2.7152 [0.006]
θ_{10}	-0.018807	0.10161	0.011014	-0.034190
	-0.42308[0.672]	2.3340[0.020]	0.26236[0.793]	-0.71016 [0.477]
θ_{11}	0.12487	-0.064518	-0.050636	0.059973
	2.8103[0.005]	-1.48532[0.138]	-1.2105 [0.226]	1.2639[0.206]
θ_{12}	0.25863	0.37230	0.48978	0.12447
	5.7995 [0.000]	8.7173 [0.000]	12.185[0.000]	2.6990[0.007]
	$R^2 = 0.51706$	$R^2 = 0.55296$	$R^2 = 0.69757$	$R^2 = 0.76632$
	DW=2.0085	DW=1.8908	DW=1.7906	DW=1.9755
	F-stat=42.38[0.00]	F-stat=48.96[0.00]	F-stat=91.29[0.00]	F-stat=129.81[0.00]

APPENDIX 4

Figure 1.01

a) A Nonstationary Time-Series



b) A Stationary Time-Series

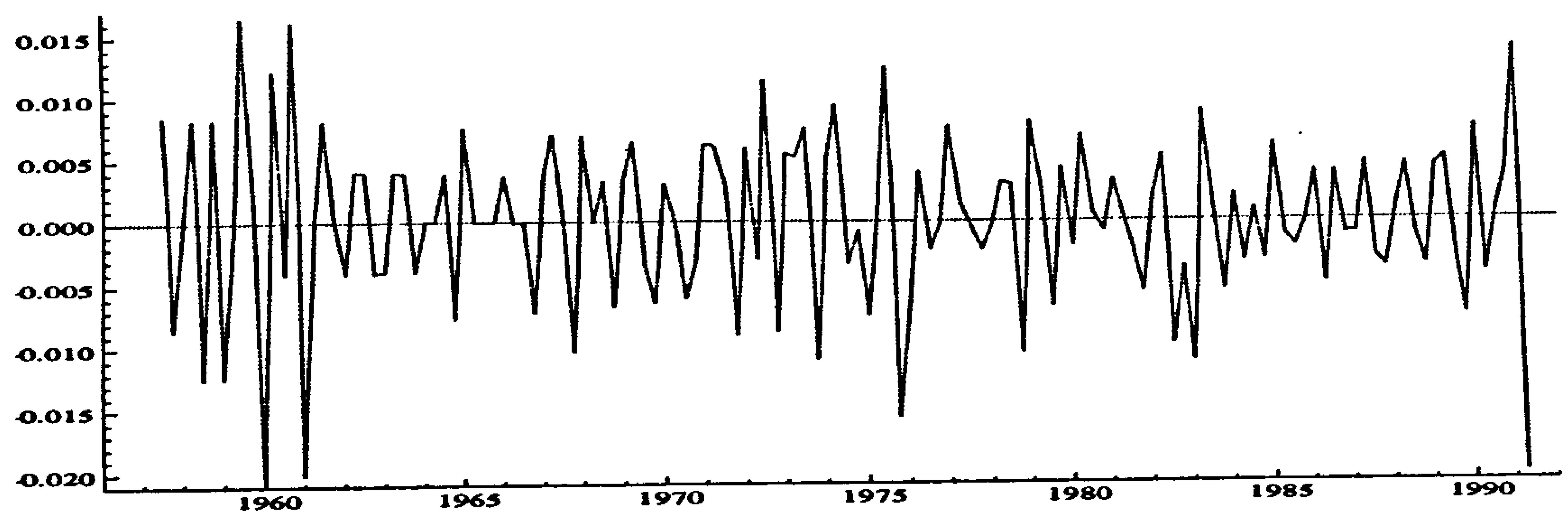


Figure 1.02 Inflation Time-Series

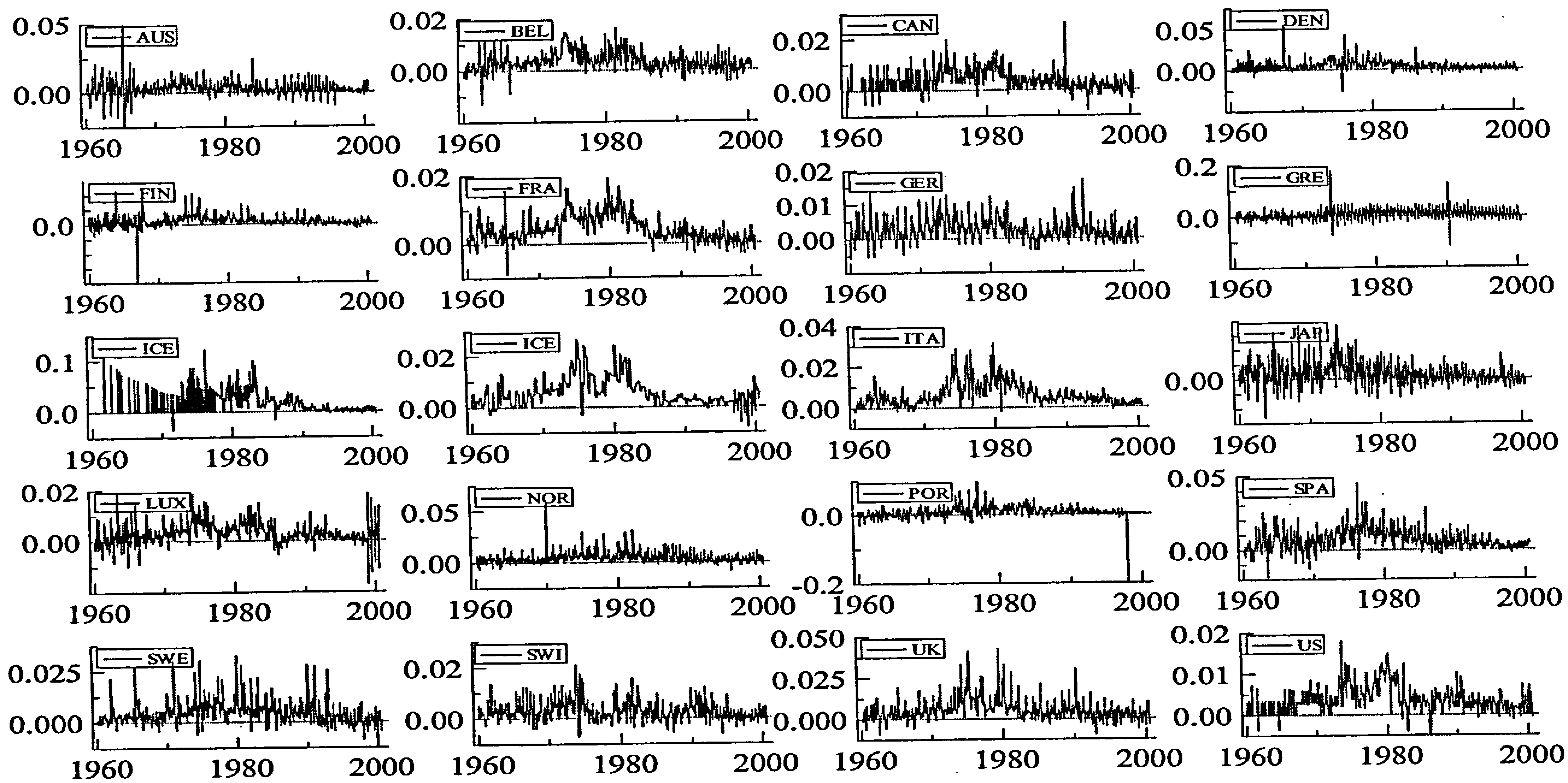


Figure 1.03 First Differences of Inflation Time-Series

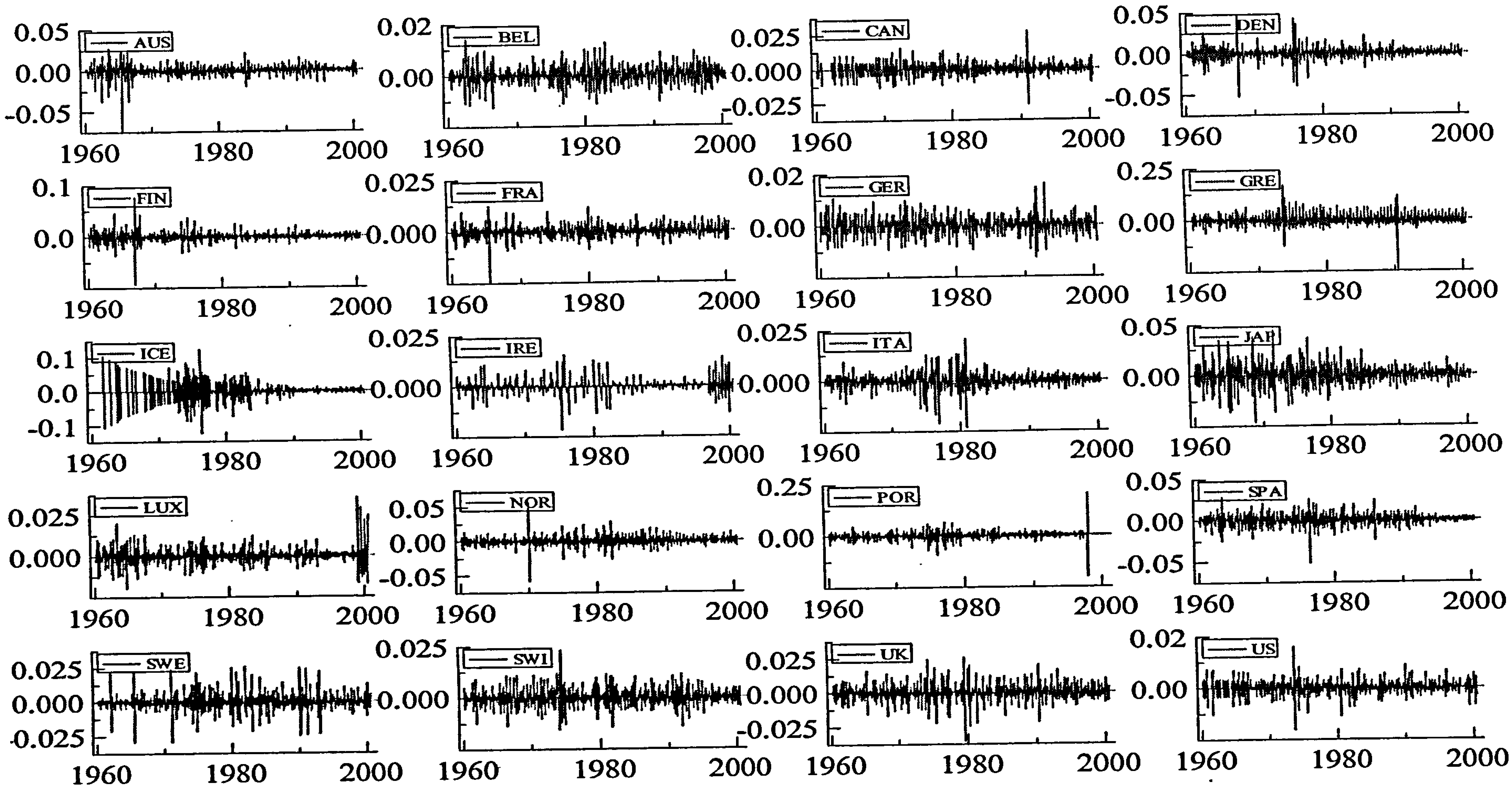


Figure 1.04 Autocorrelation Functions of Inflation Time-Series

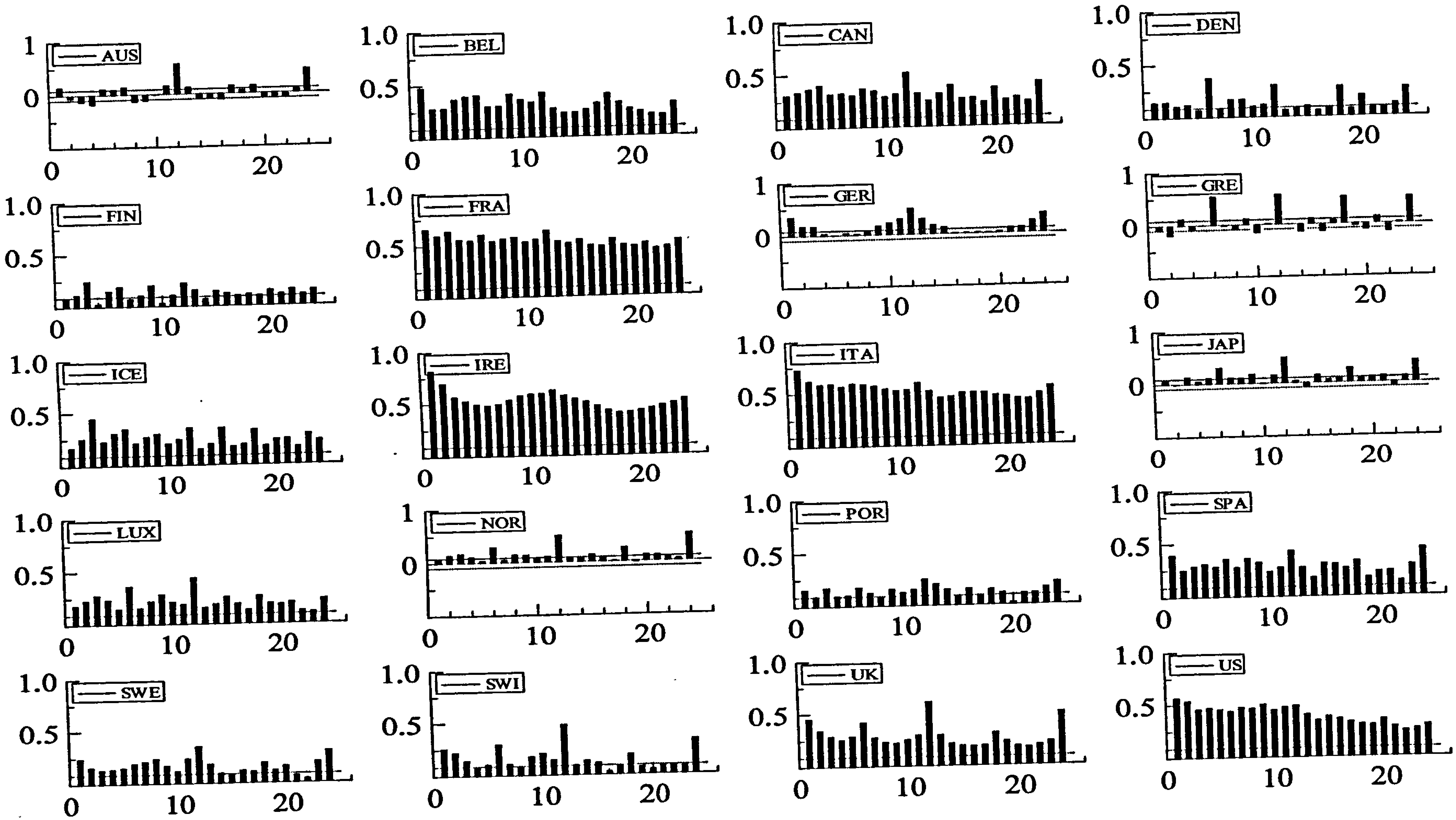


Figure 1.05 Autocorrelation Functions of First Differences of Inflation Time-Series

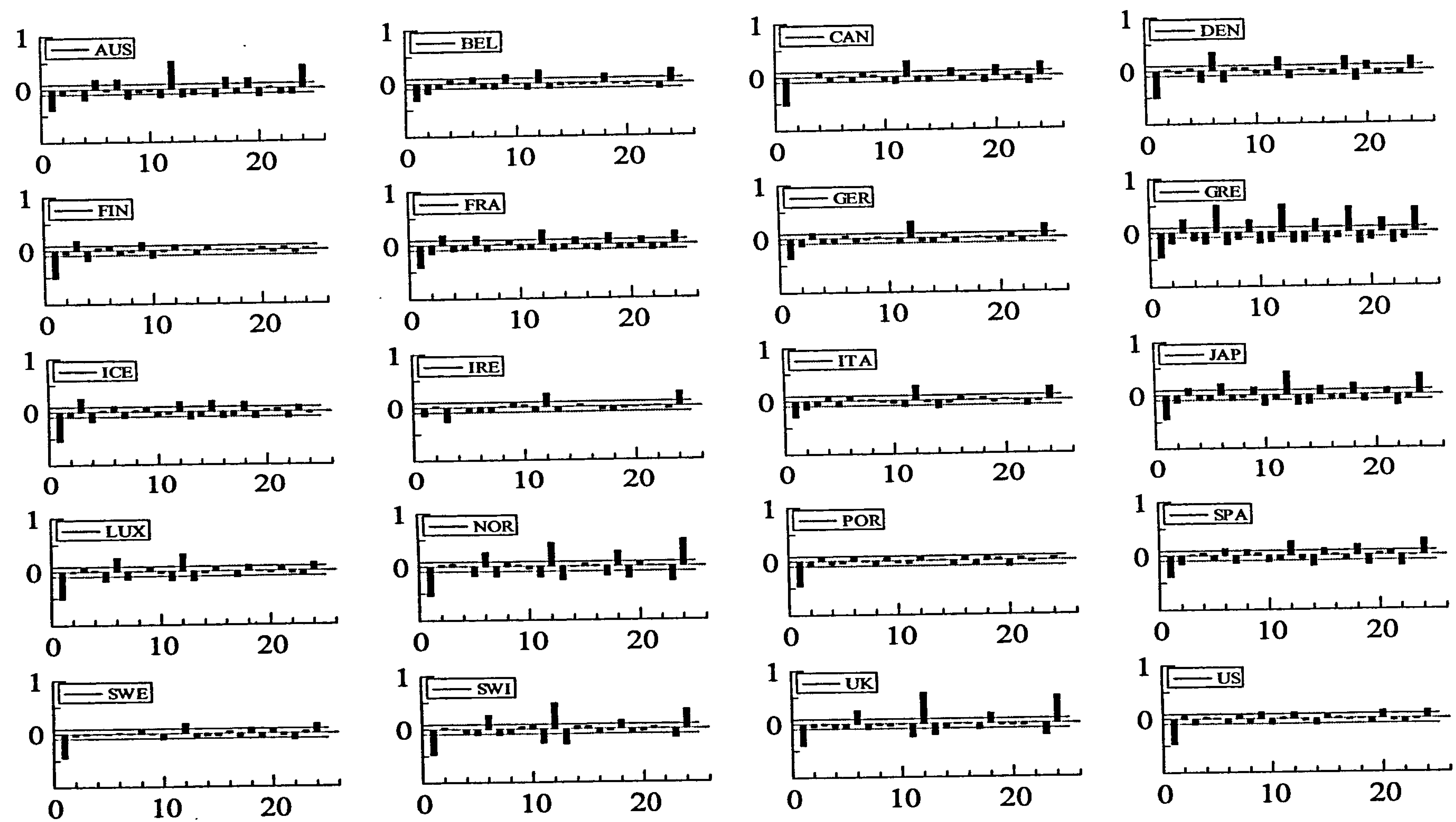


Figure 2.01 Nominal Exchange Rate Time-Series

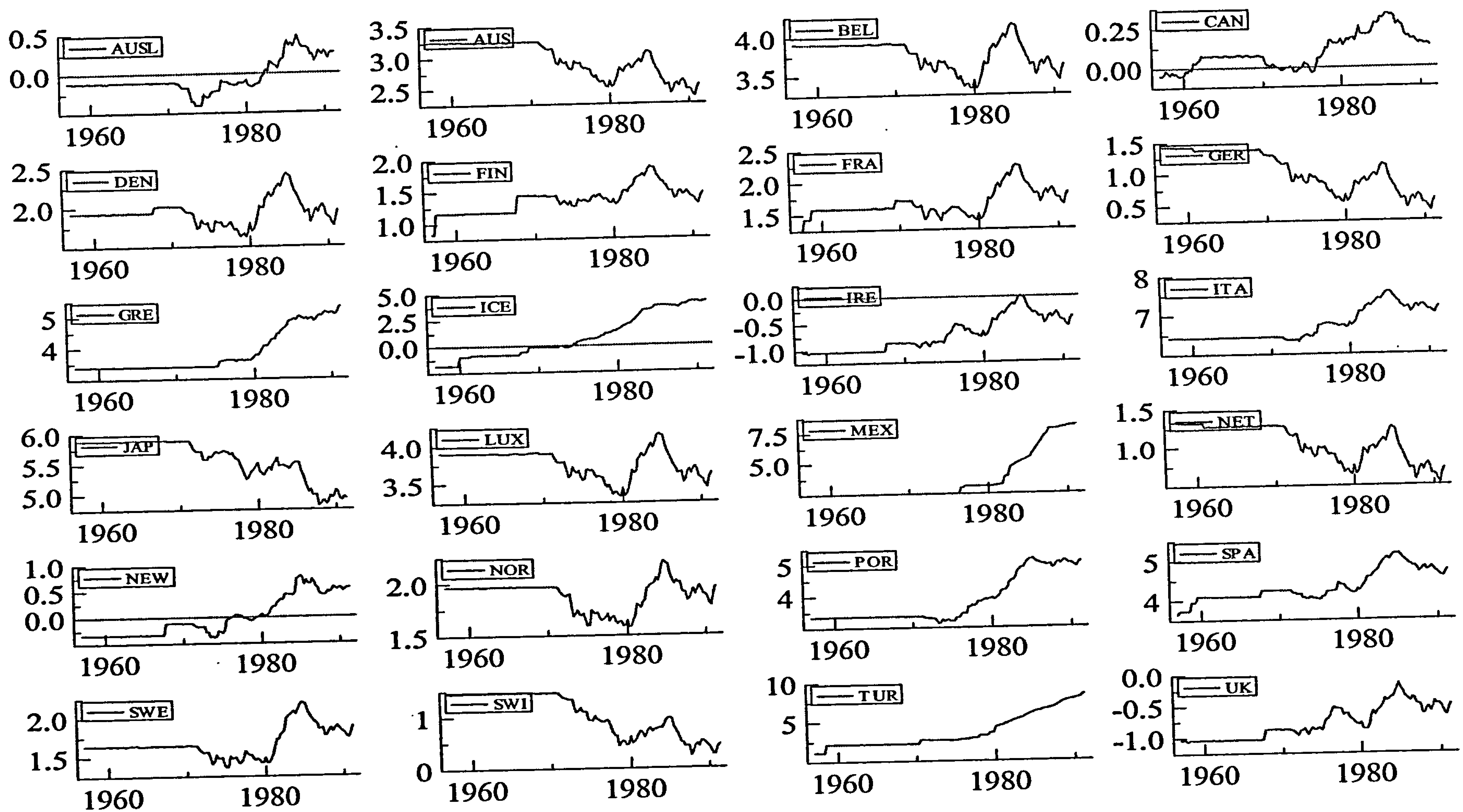


Figure 2.02 First Differences of Nominal Exchange Rate Time-Series

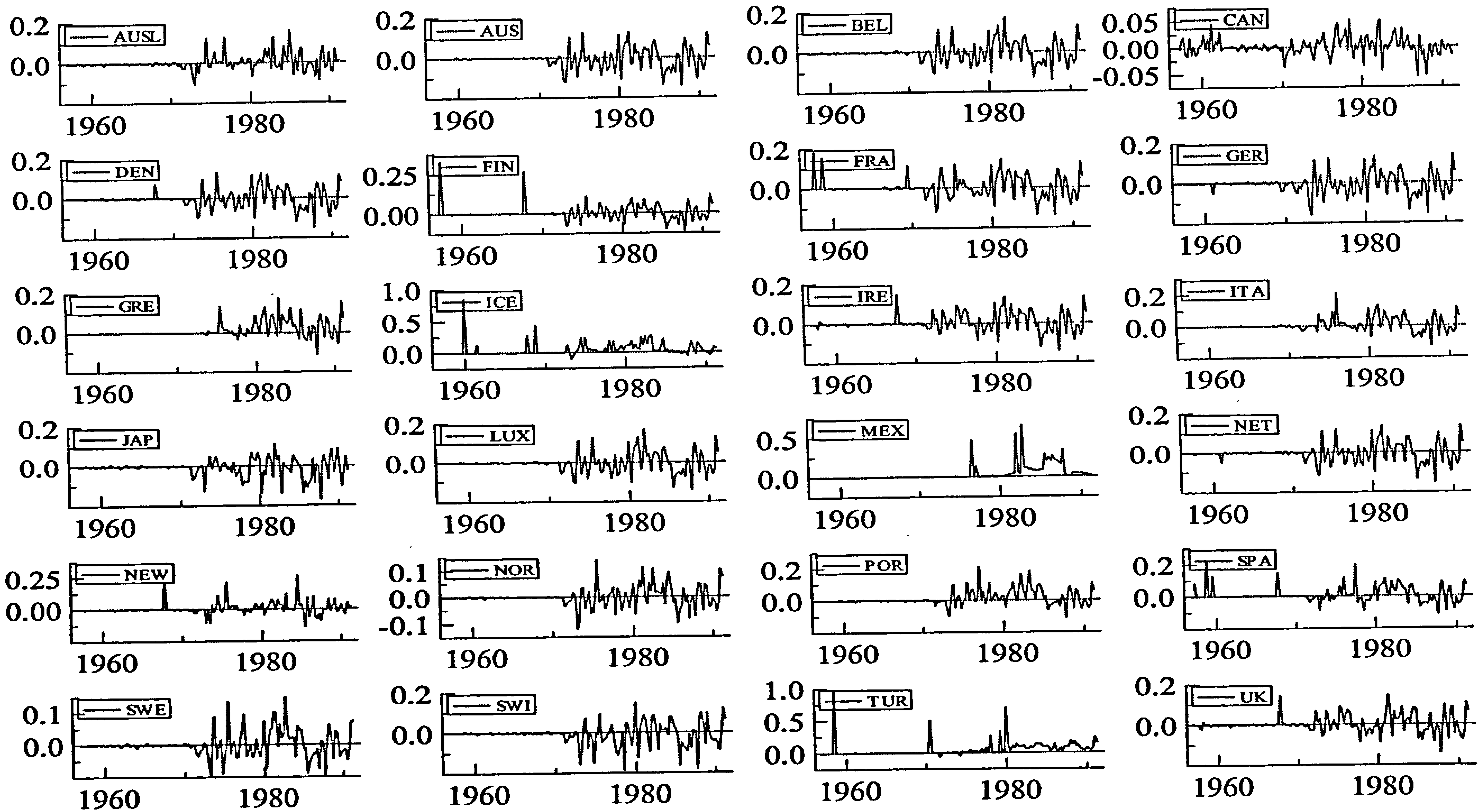


Figure 2.03 Autocorrelation Functions of Nominal Exchange Rate Time-Series

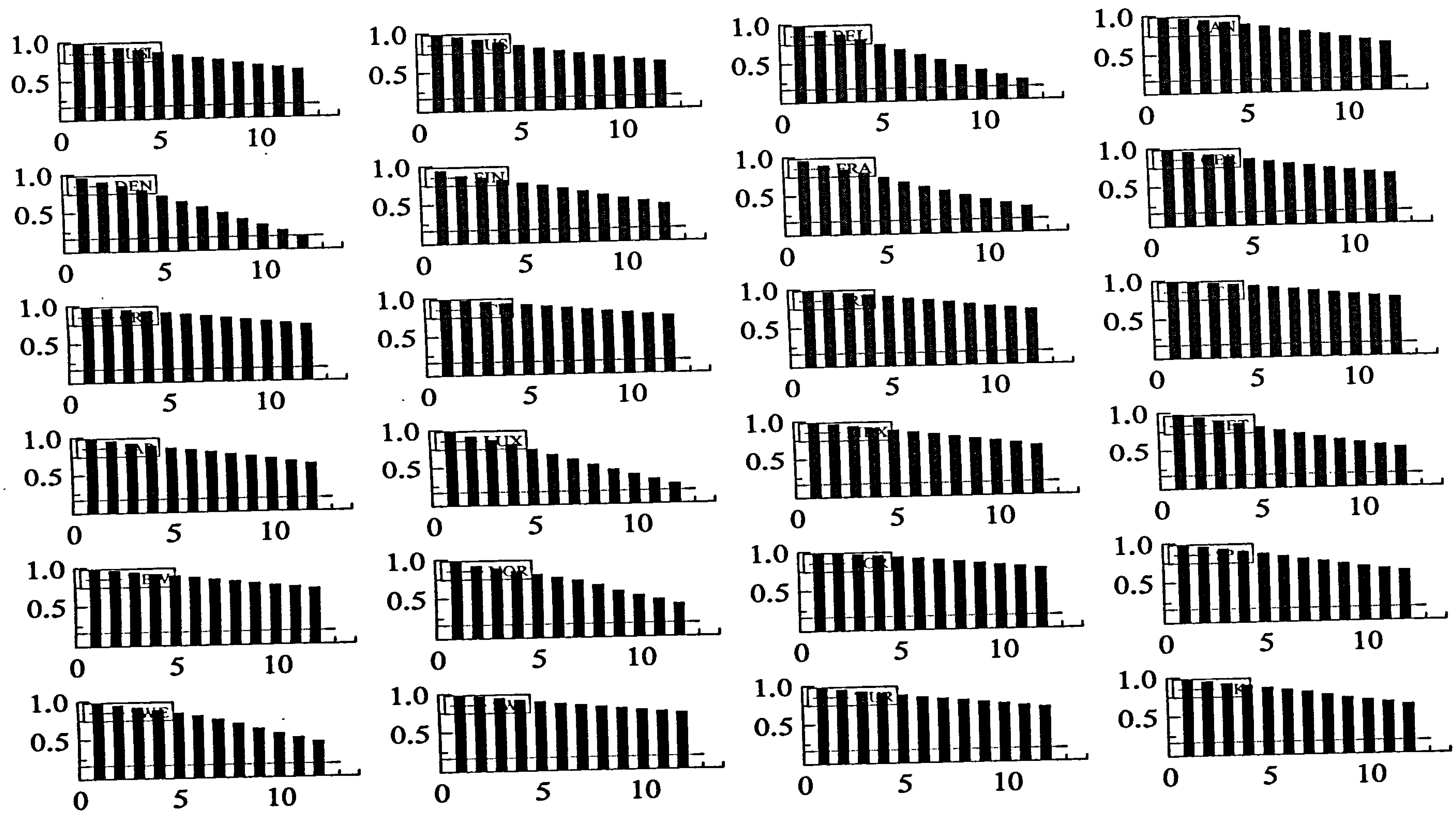


Figure 2.04 Autocorrelation Functions of First Differences of Nominal Exchange Rate Time-Series

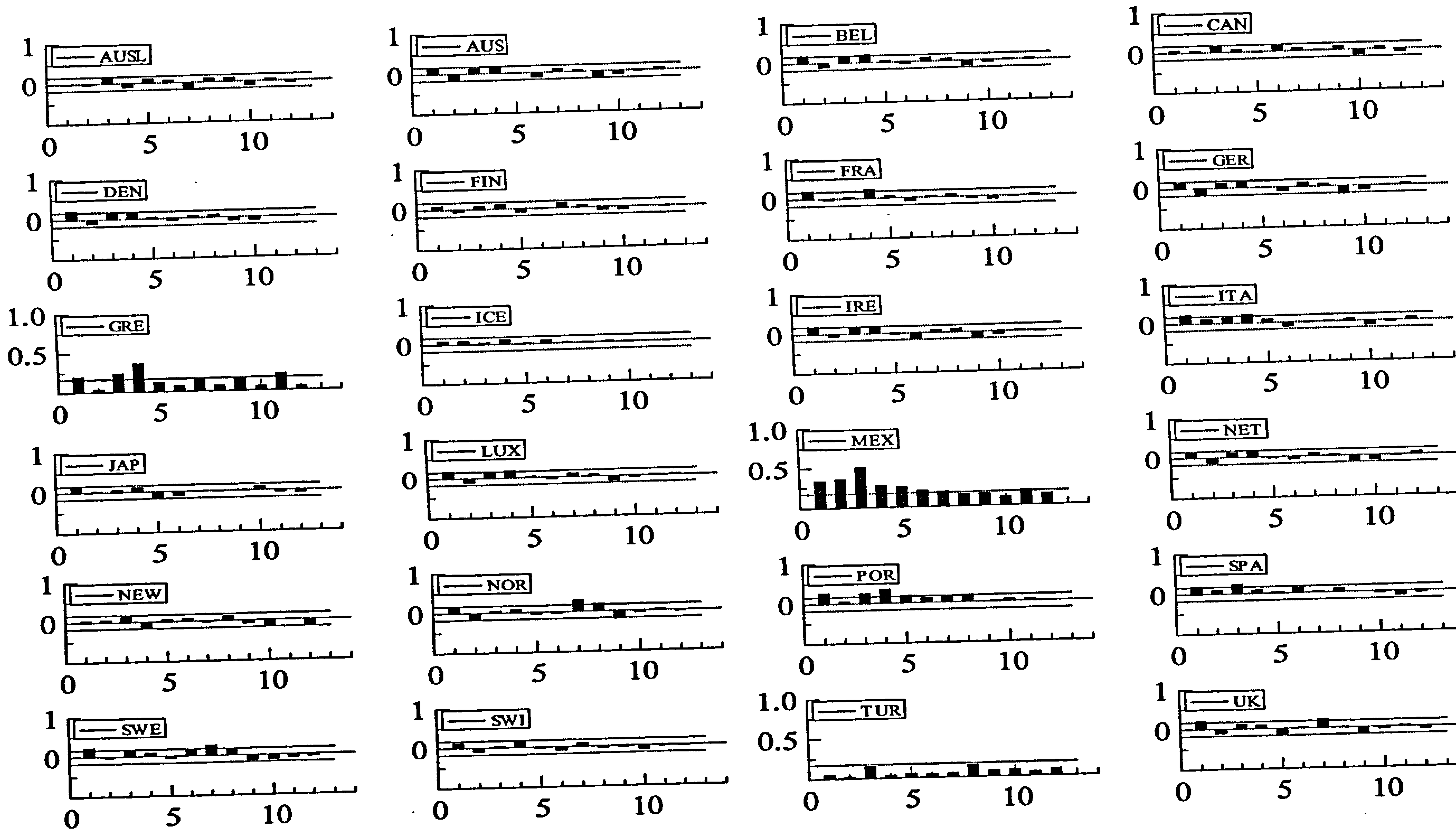


Figure 2.05 Consumer Price Index Time-Series

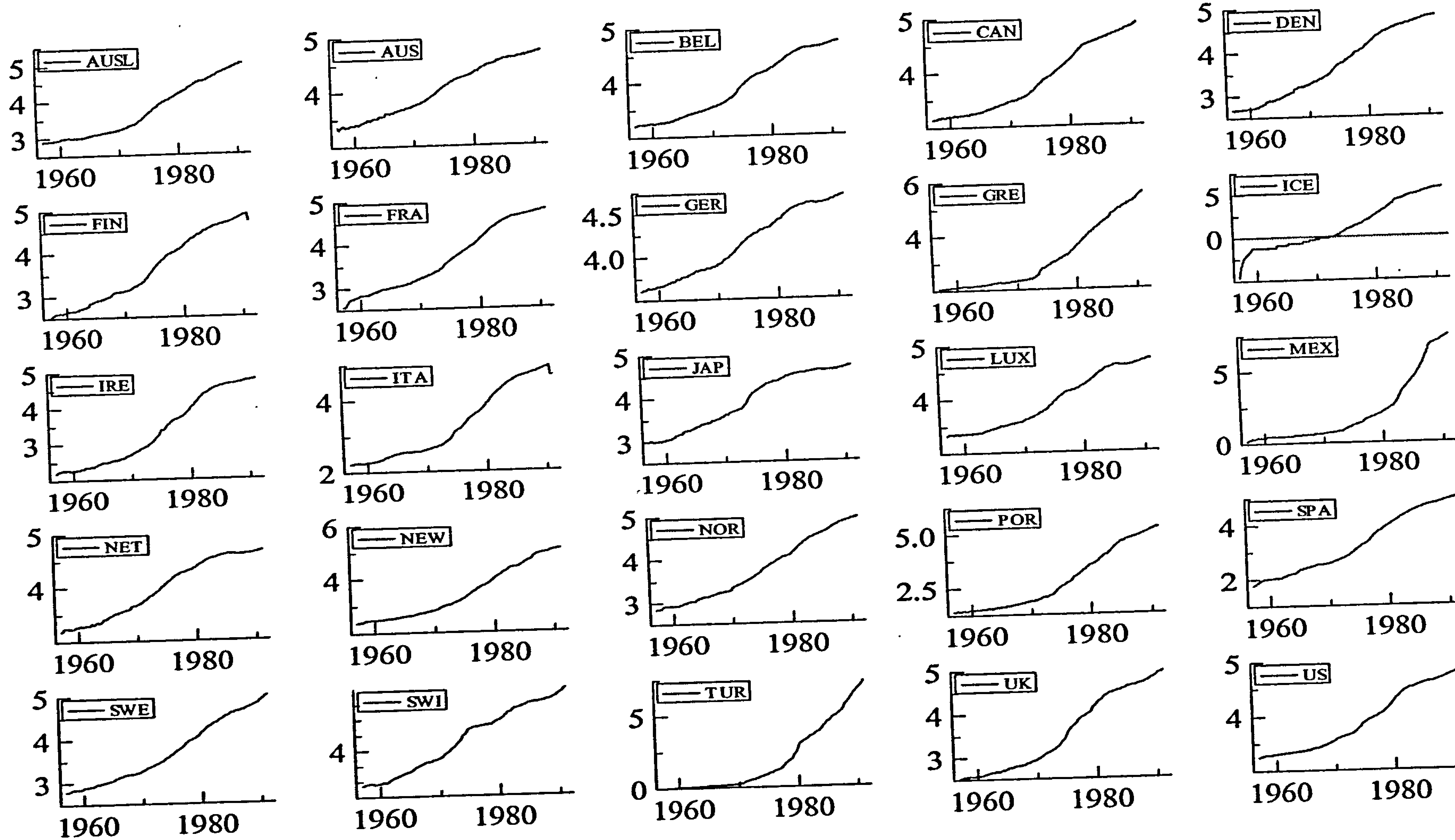


Figure 2.06 First Differences of Consumer Price Index Time-Series

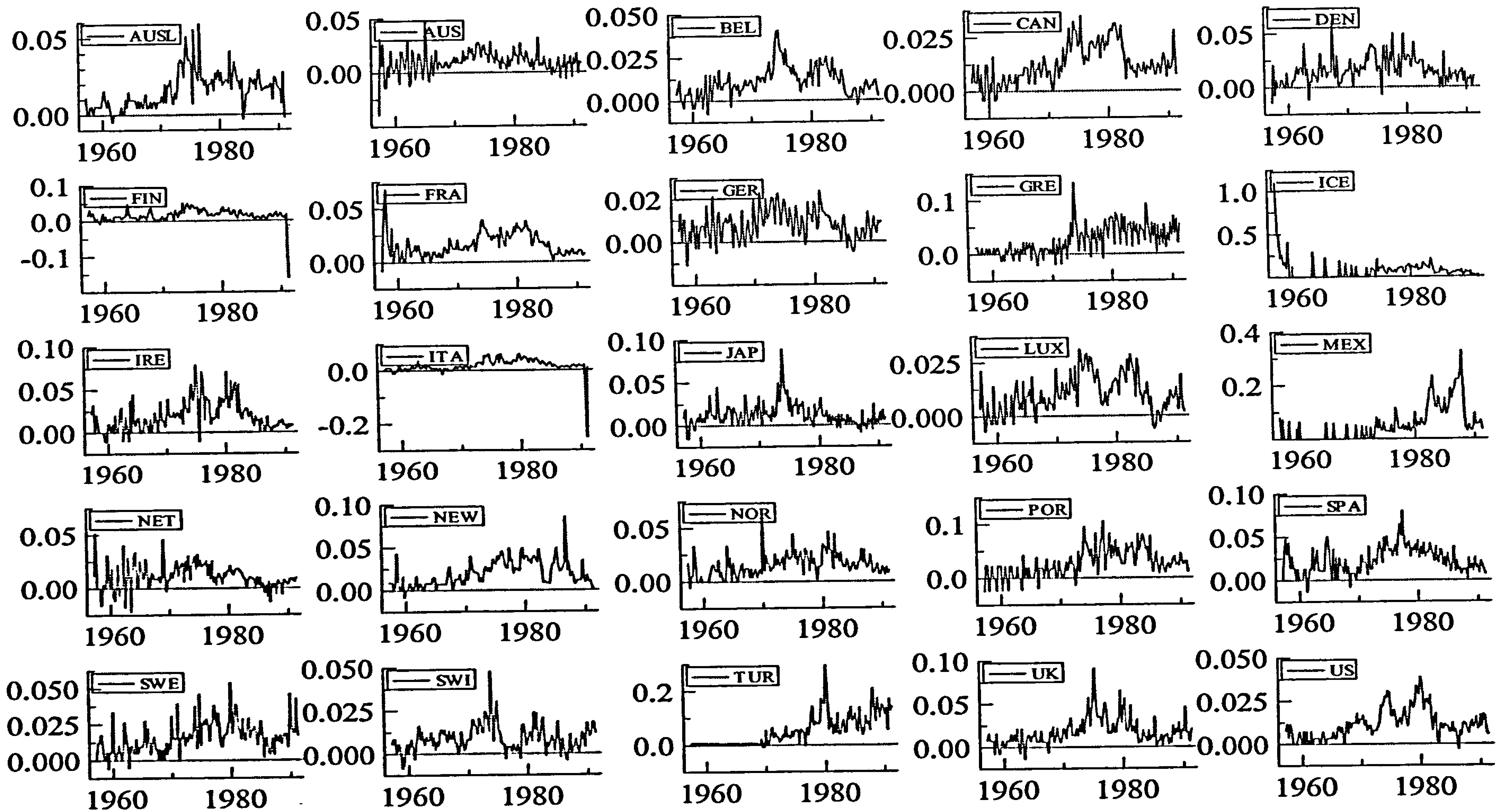


Figure 2.07 Second Differences of Consumer Price Index Time-Series

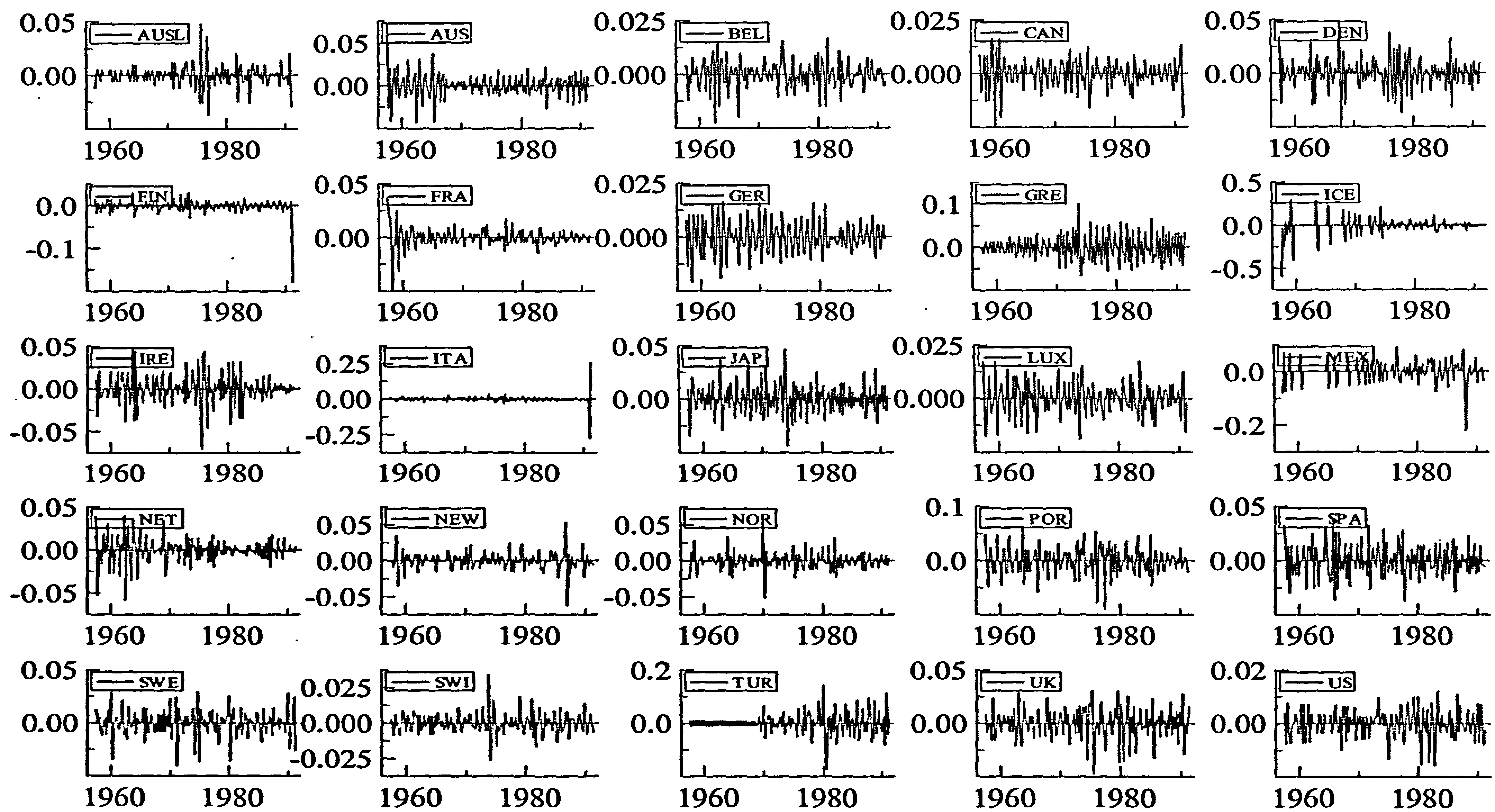


Figure 2.08 Autocorrelation Functions of Consumer Price Index Time-Series

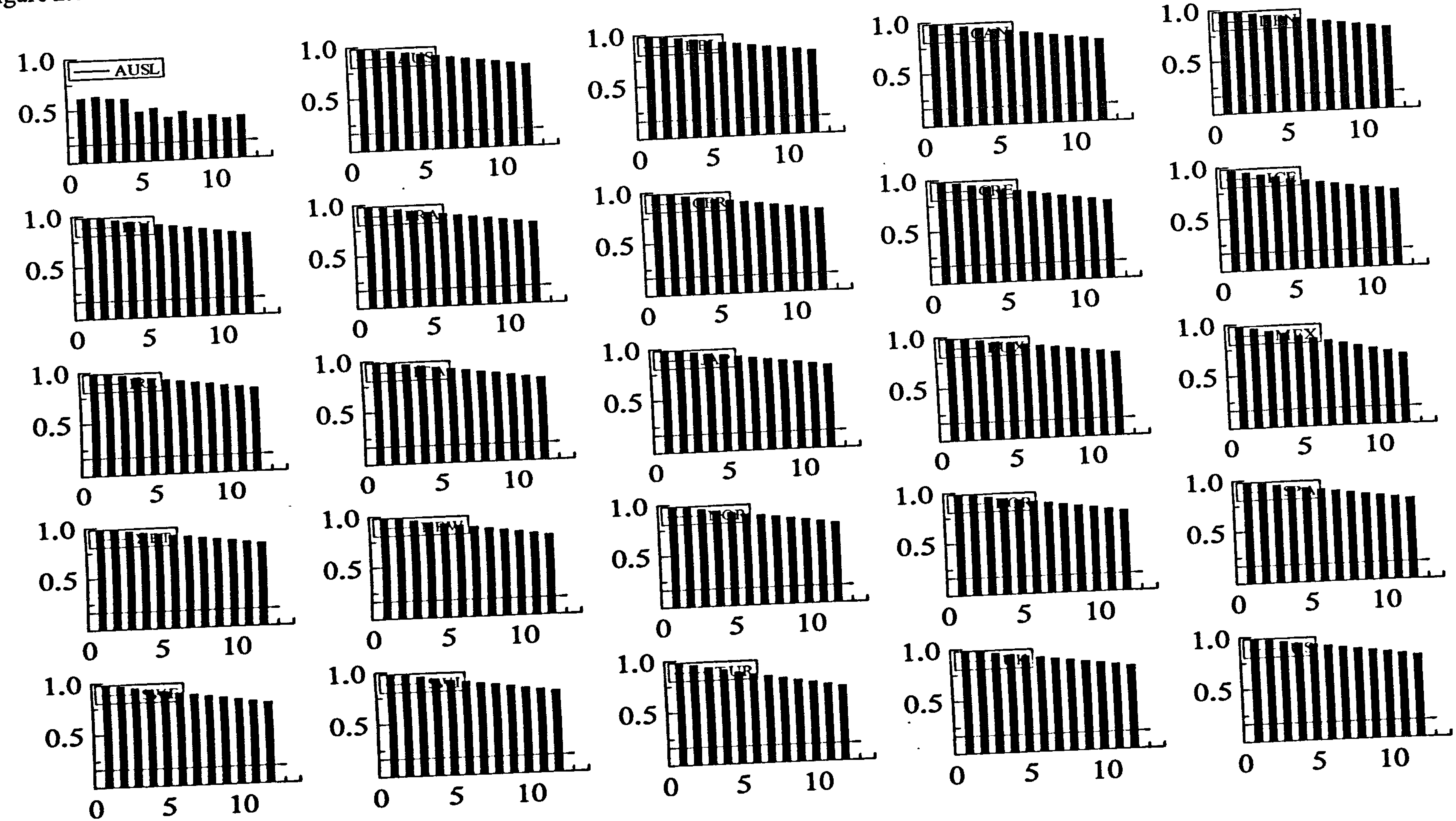


Figure 2.09 Autocorrelation Functions of First Differences of Consumer Price Index Time-Series

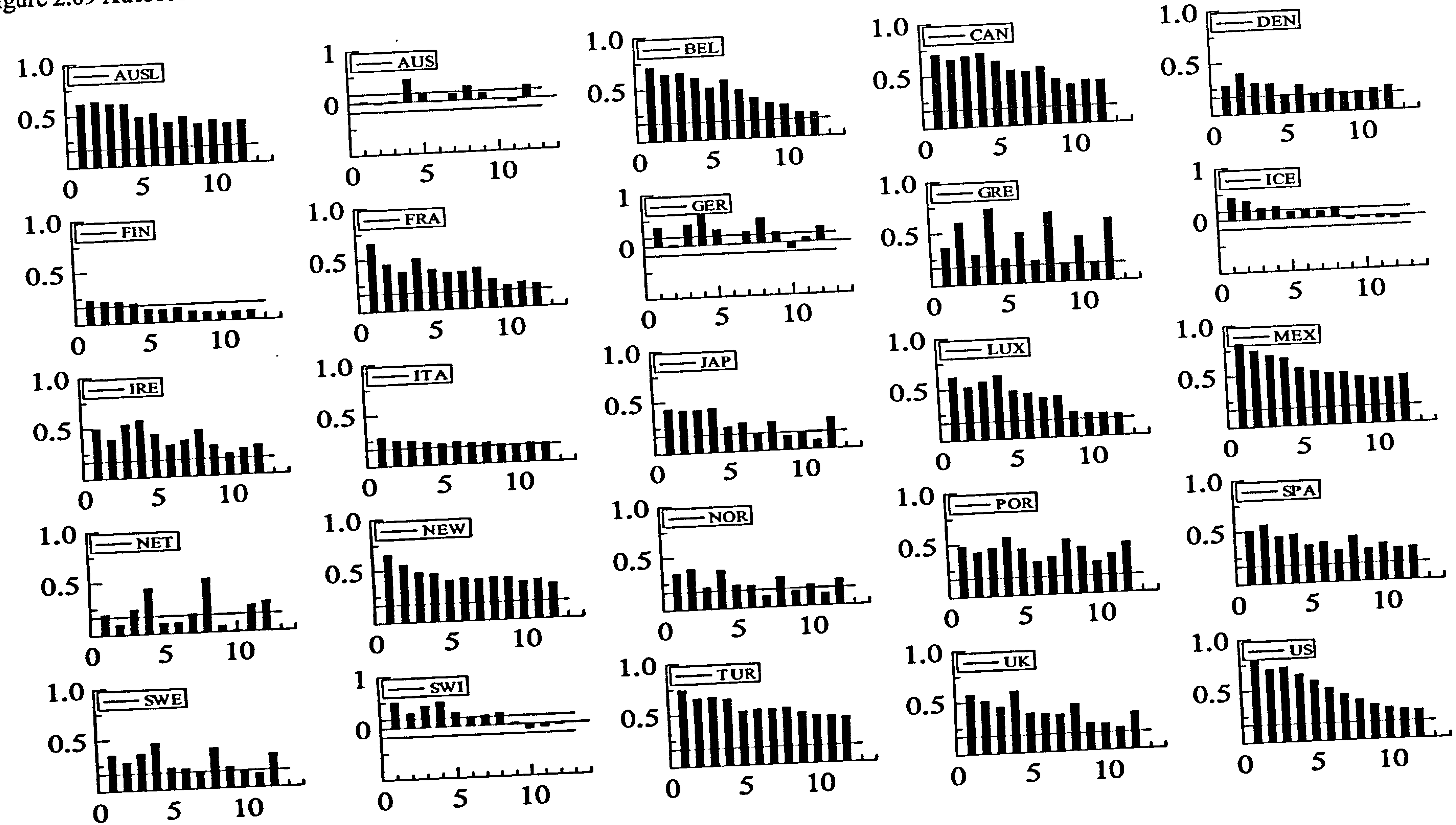


Figure 2.10 Autocorrelation Functions of Second Differences of Consumer Price Index Time-Series

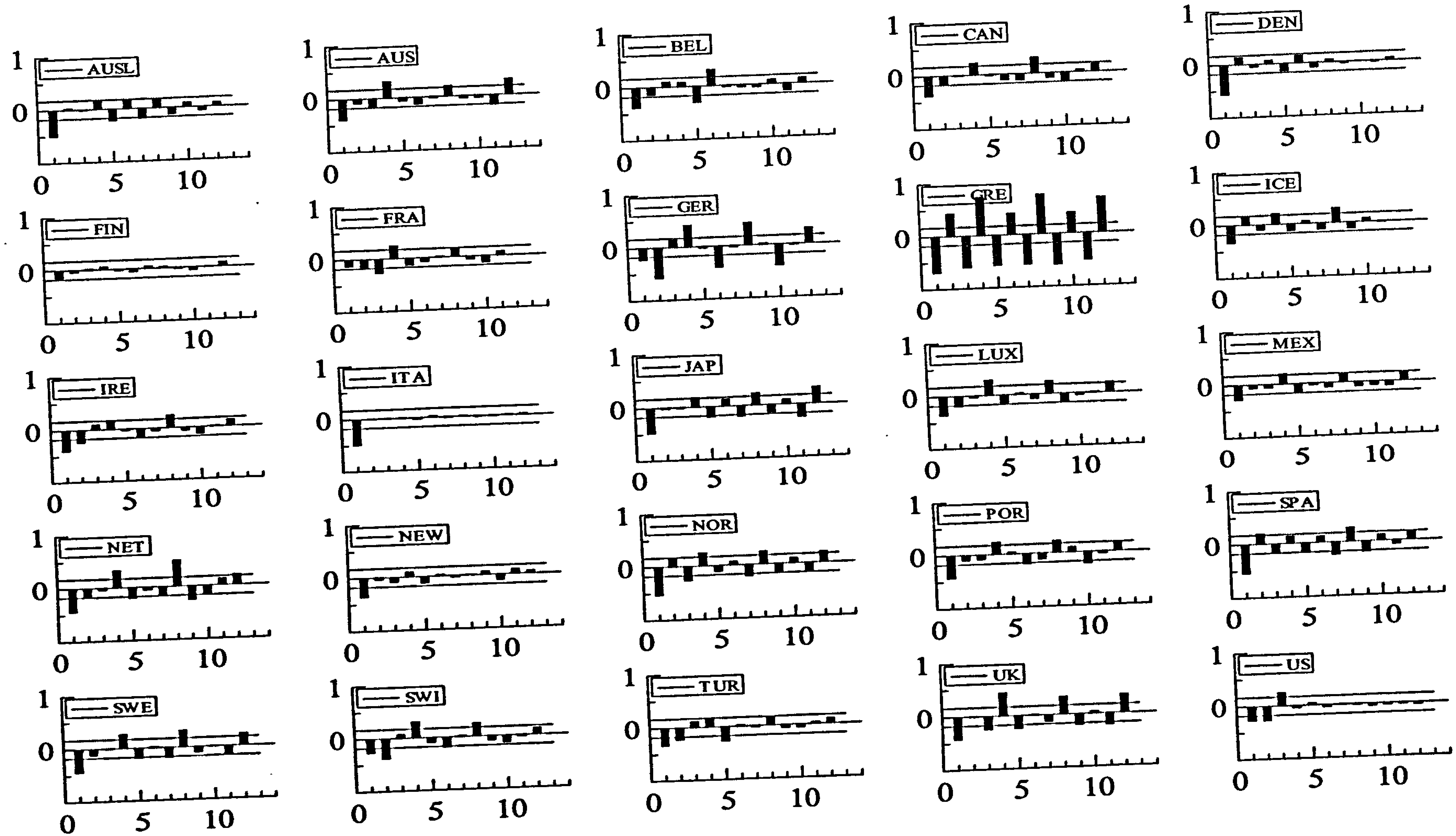


Figure 2.11 Real Exchange Rate Time-Series

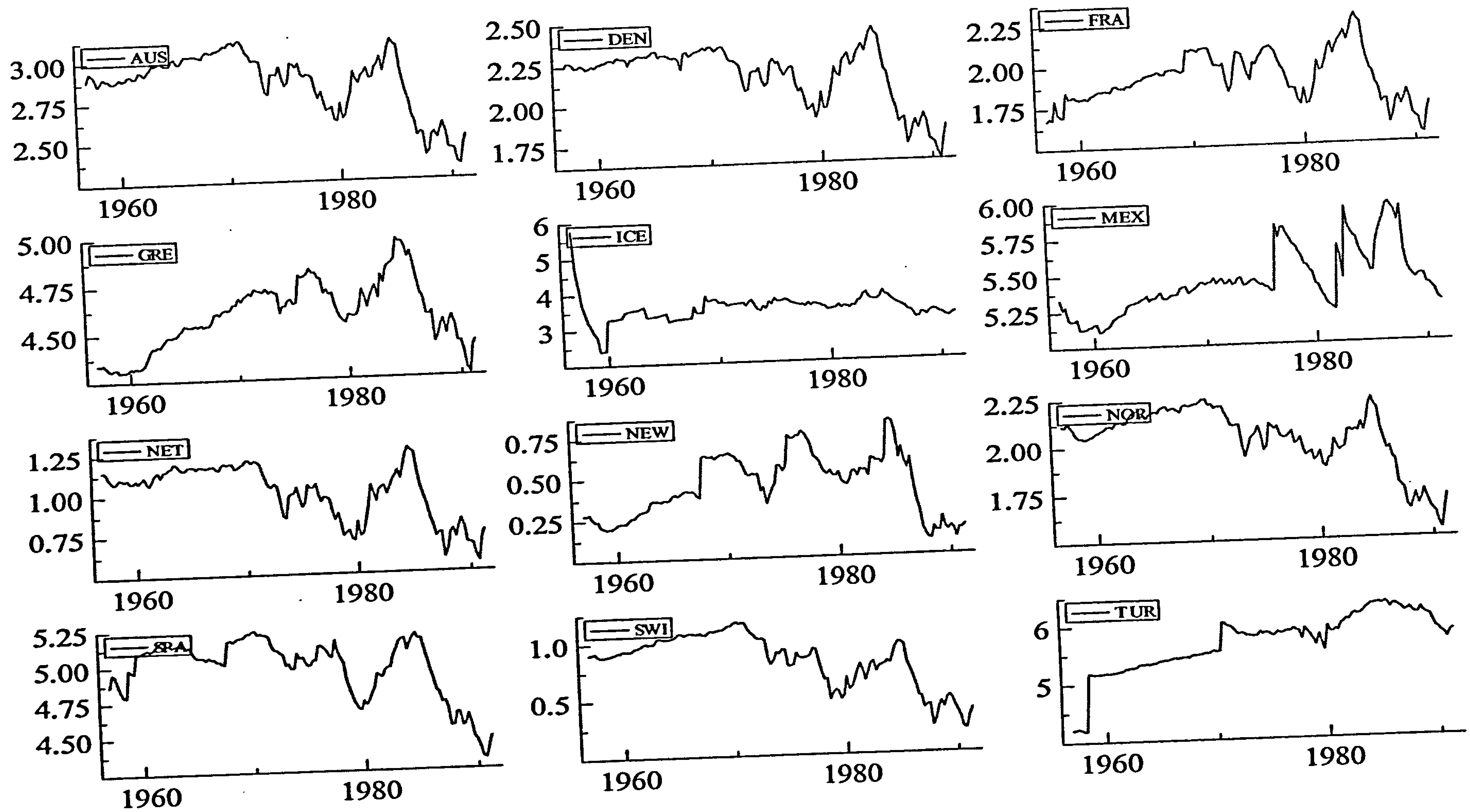


Figure 2.12 First Differences of Real Exchange Rate Time-Series

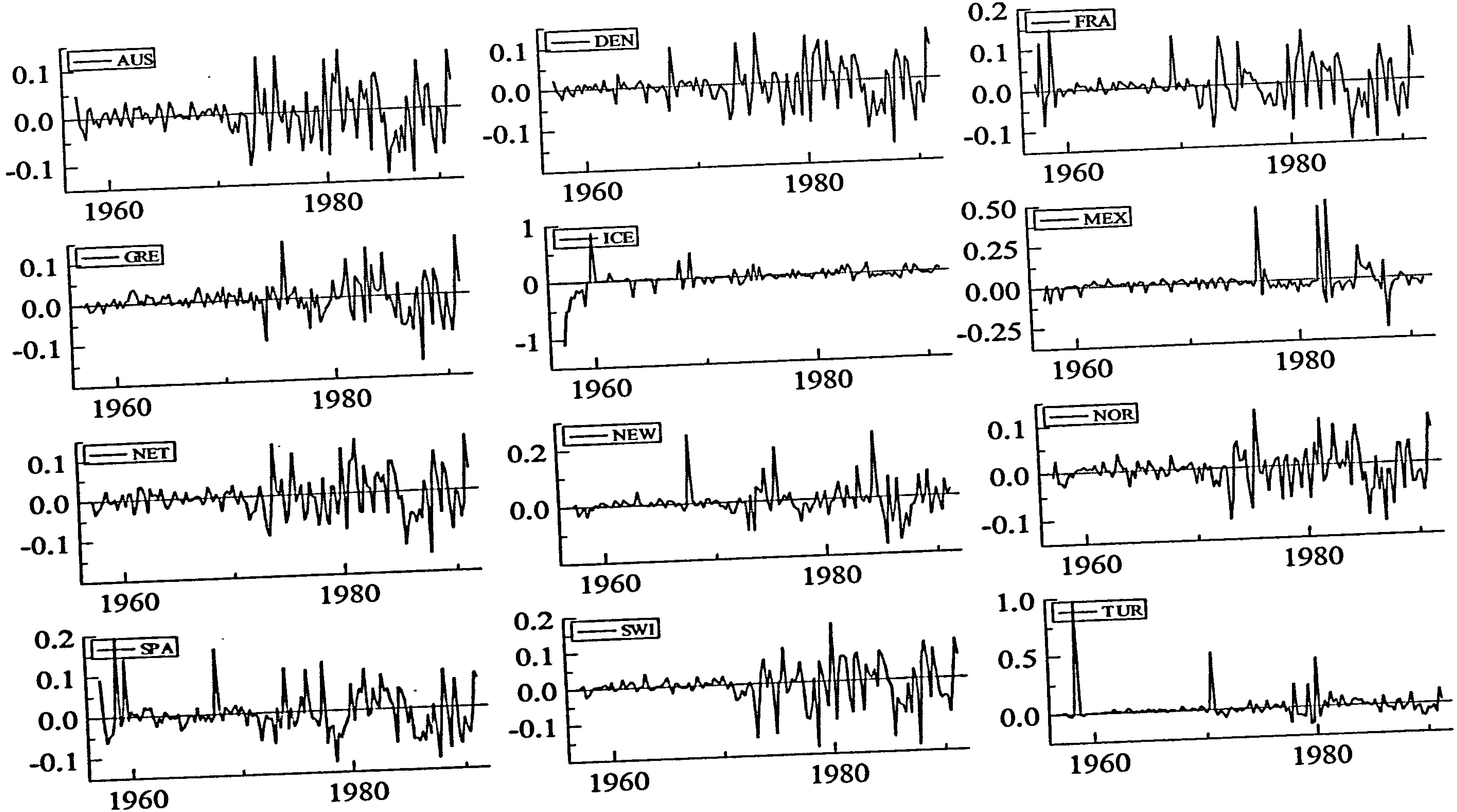


Figure 2.13 Autocorrelation Functions of Real Exchange Rate Time-Series

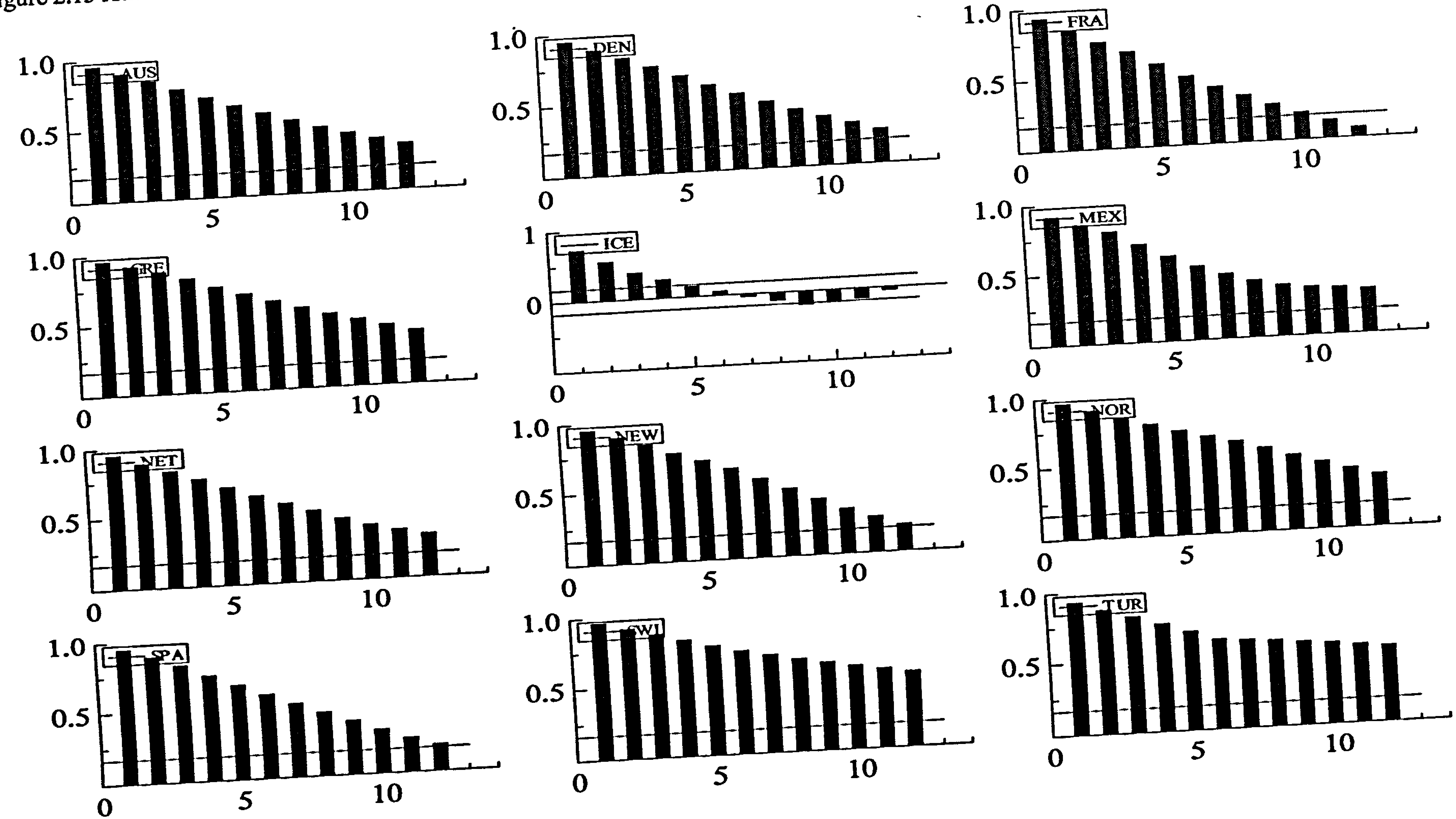


Figure 2.14 Autocorrelation Functions of First Differences of Real Exchange Rate Time-Series

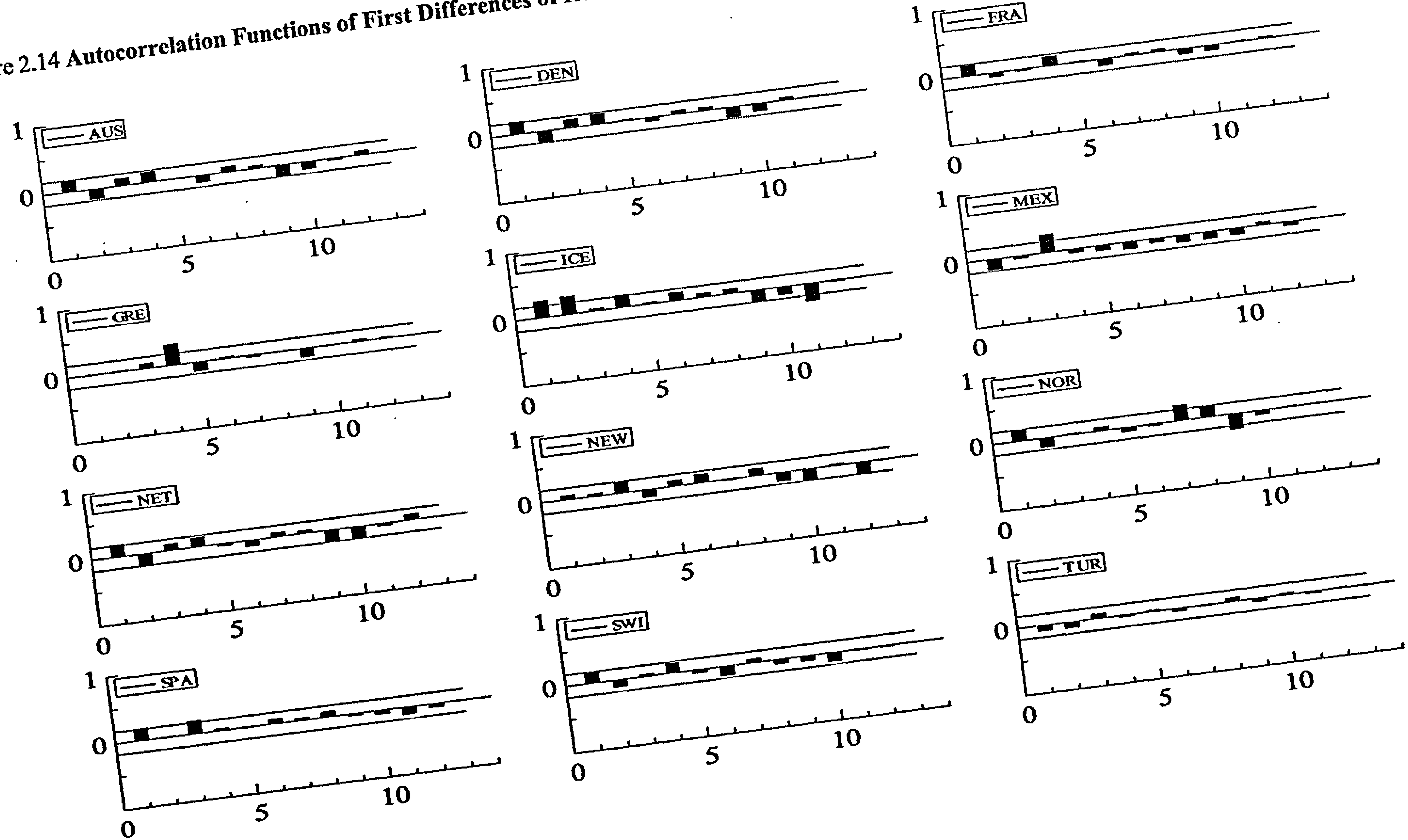


Figure 2.15 Demeaned Real Exchange Rate Time-Series

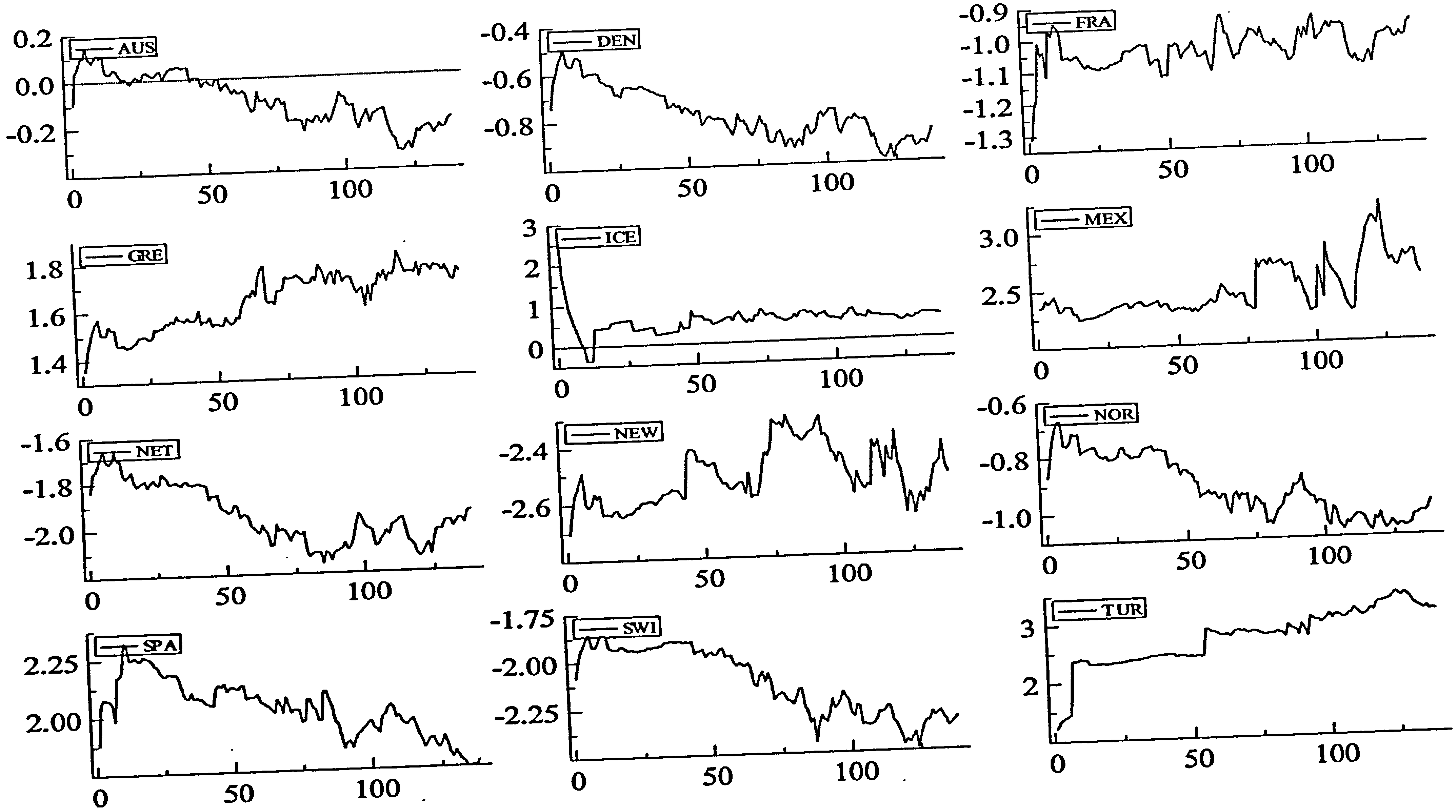


Figure 2.16 First Differences of Demeaned Real Exchange Rate Time-Series

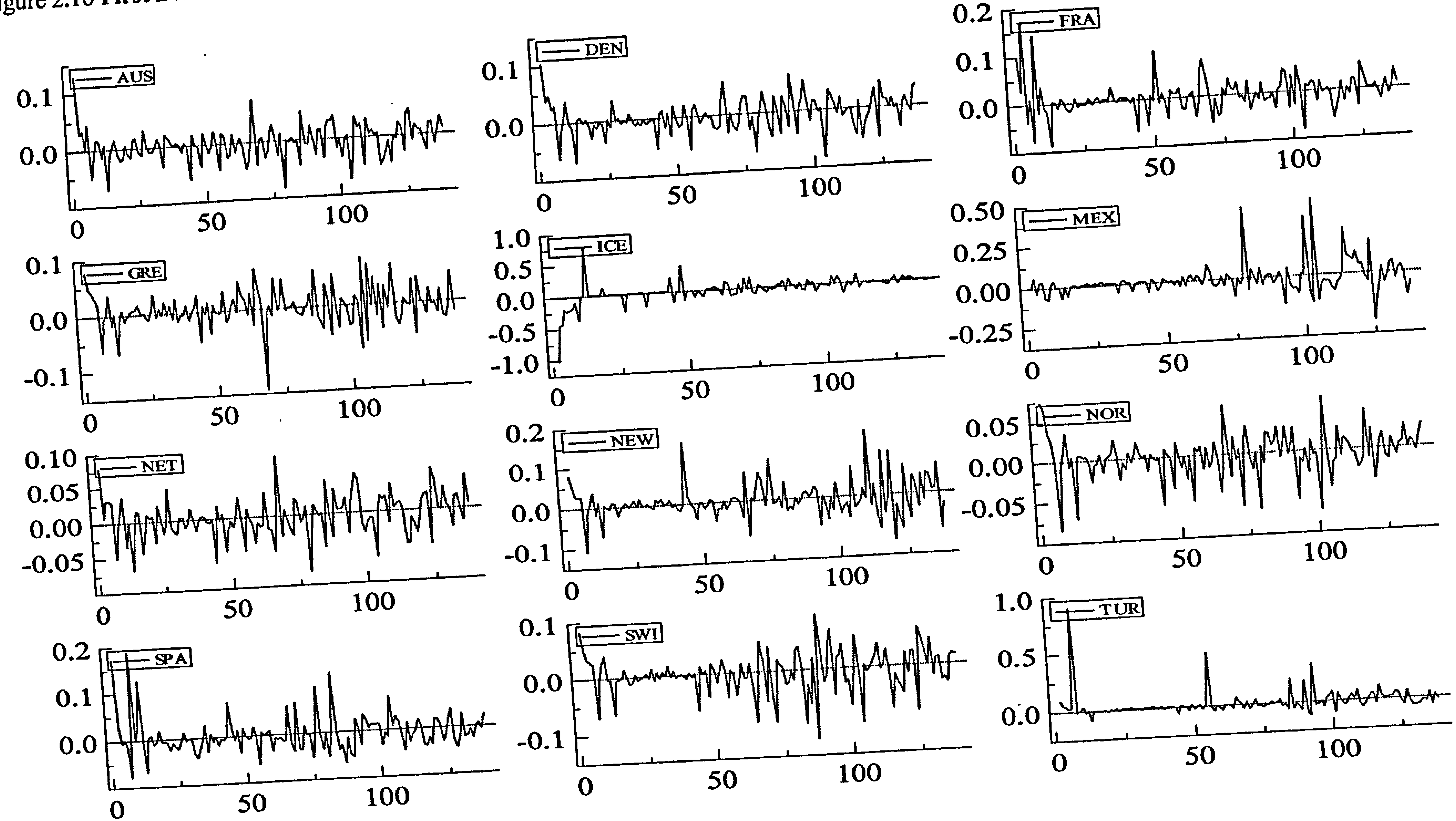


Figure 2.17 Autocorrelation Functions of Demeaned Real Exchange Rate Time-Series

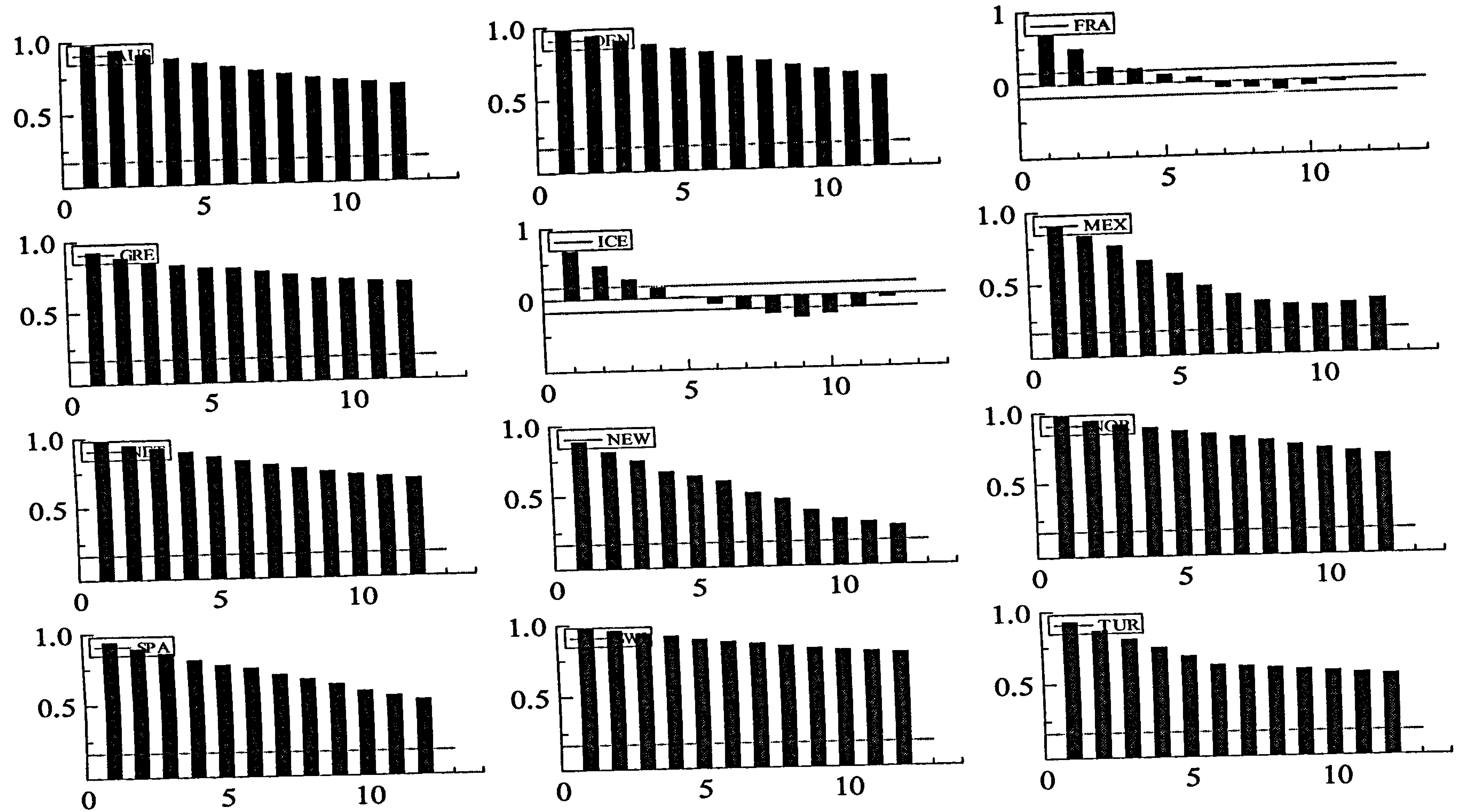


Figure 2.18 Autocorrelation Functions of First Differences of Demeaned Real Exchange Rate Time-Series

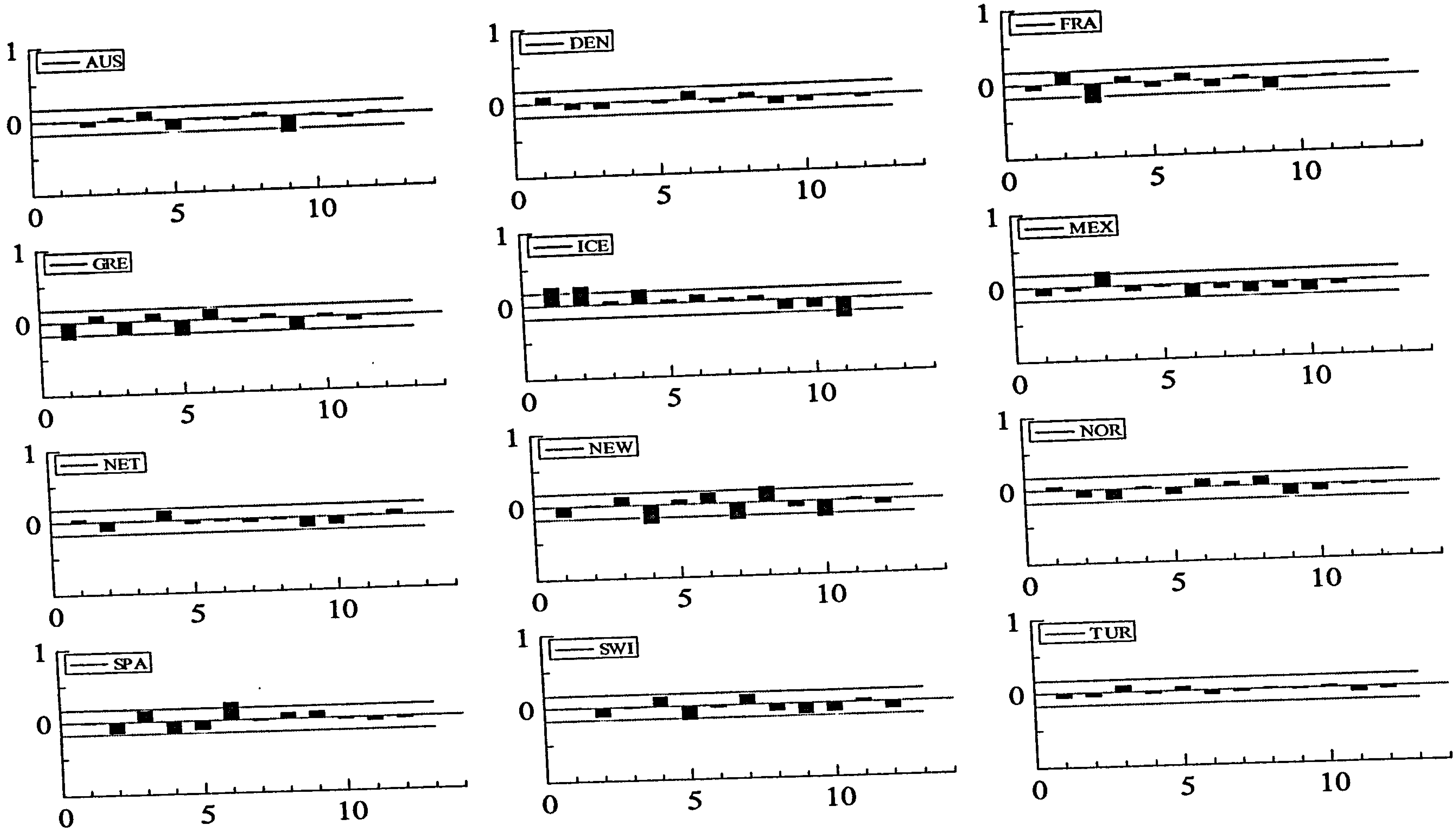


Figure 2.19 Autocorrelation Functions of the Residuals $\hat{u}_{it}$

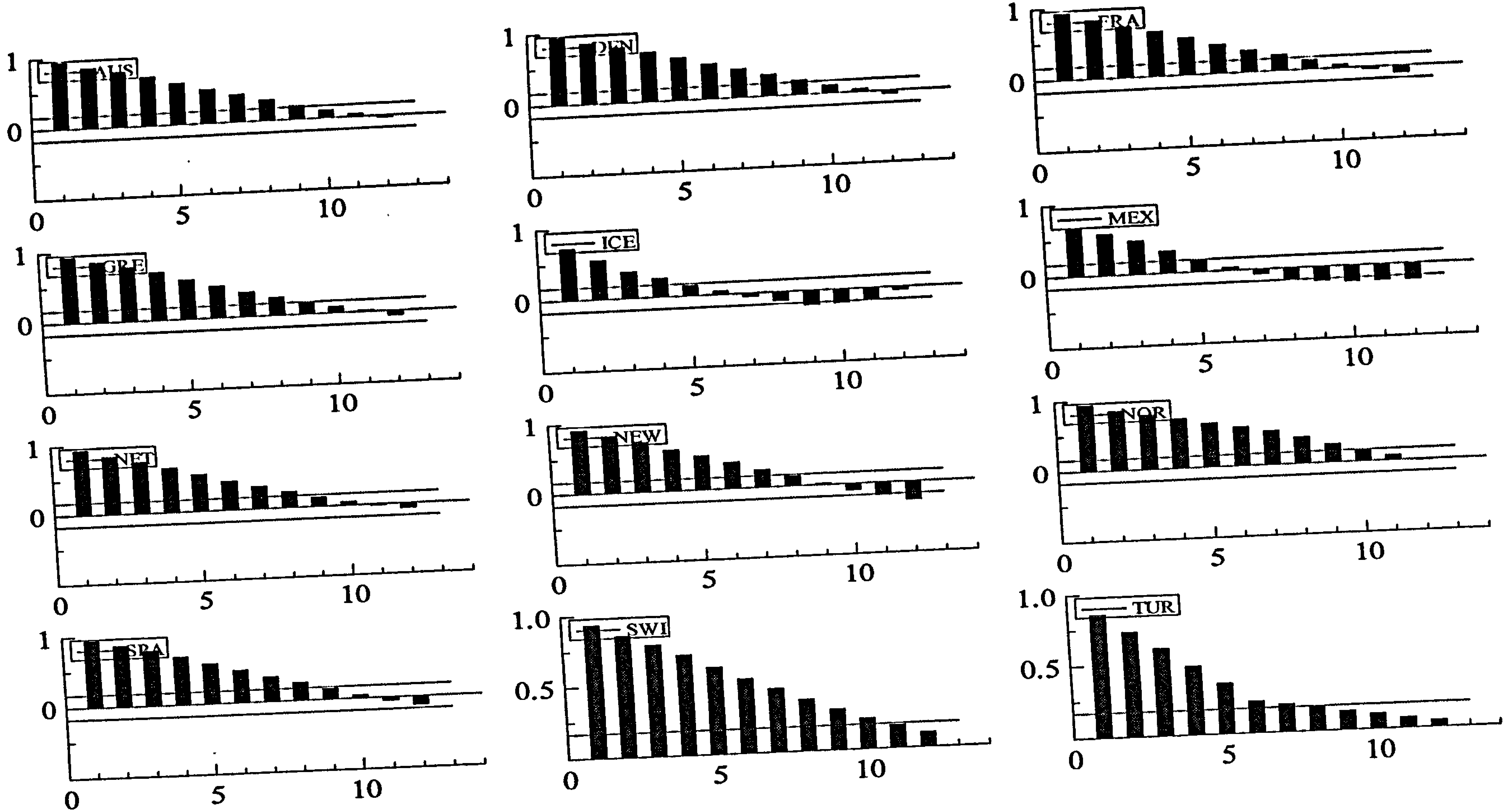


Figure 2.20 Autocorrelation Functions of the Residuals $\hat{\mu}_u$

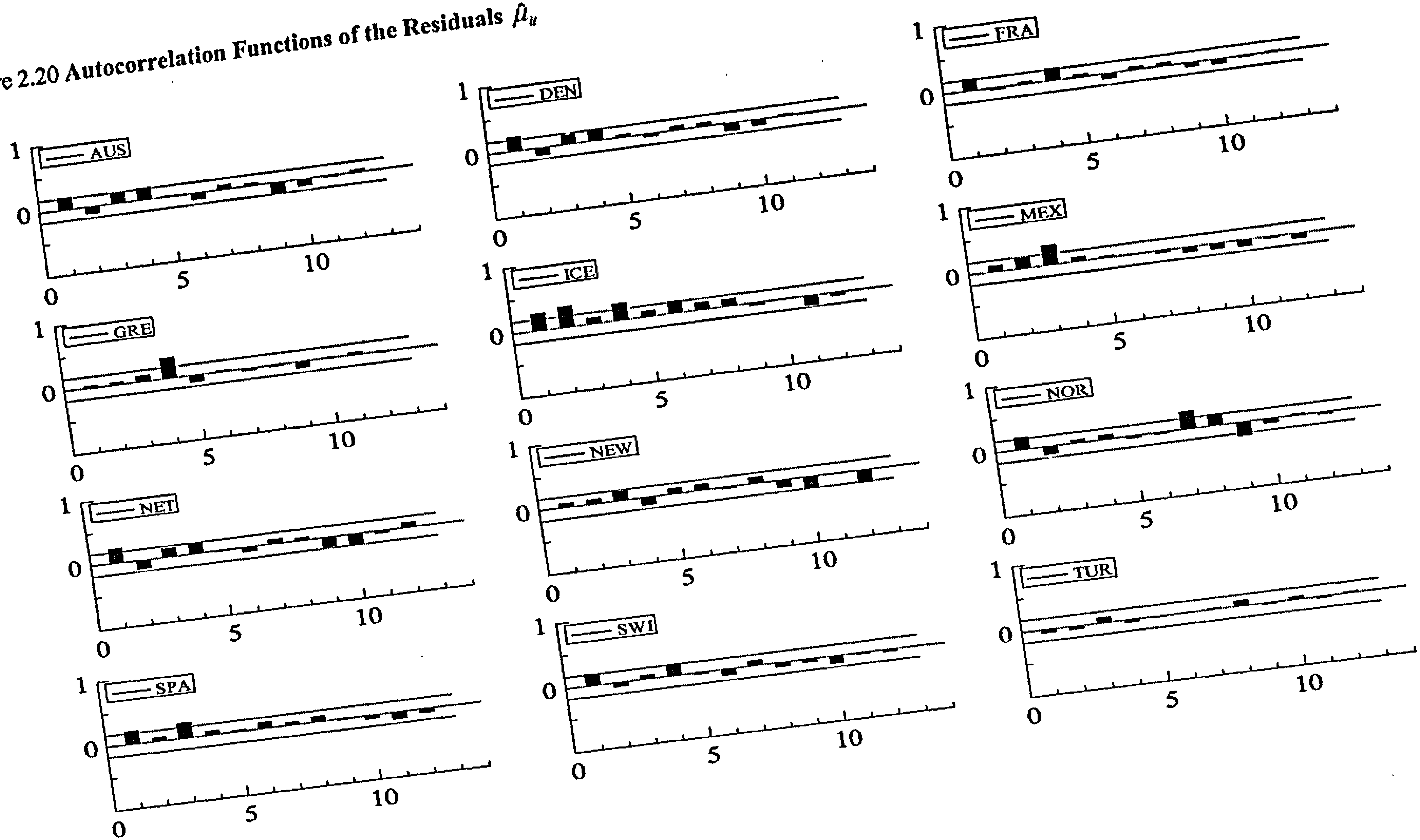


Figure 2.21 Autocorrelation Functions of the Residuals $\hat{\eta}_{it}$

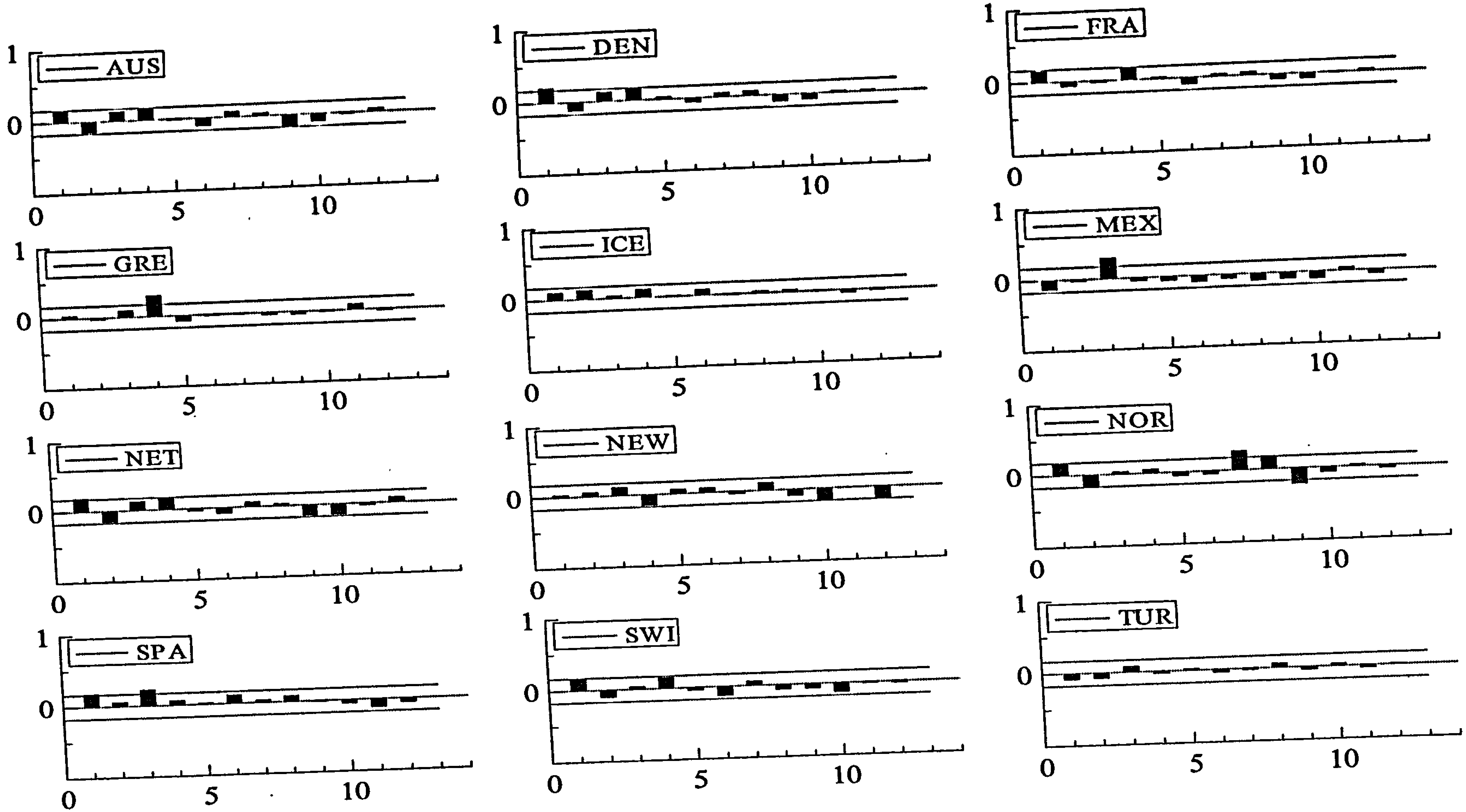


Figure 2.22 Autocorrelation Functions of the Residuals $\hat{\mu}^{*}_{it}$

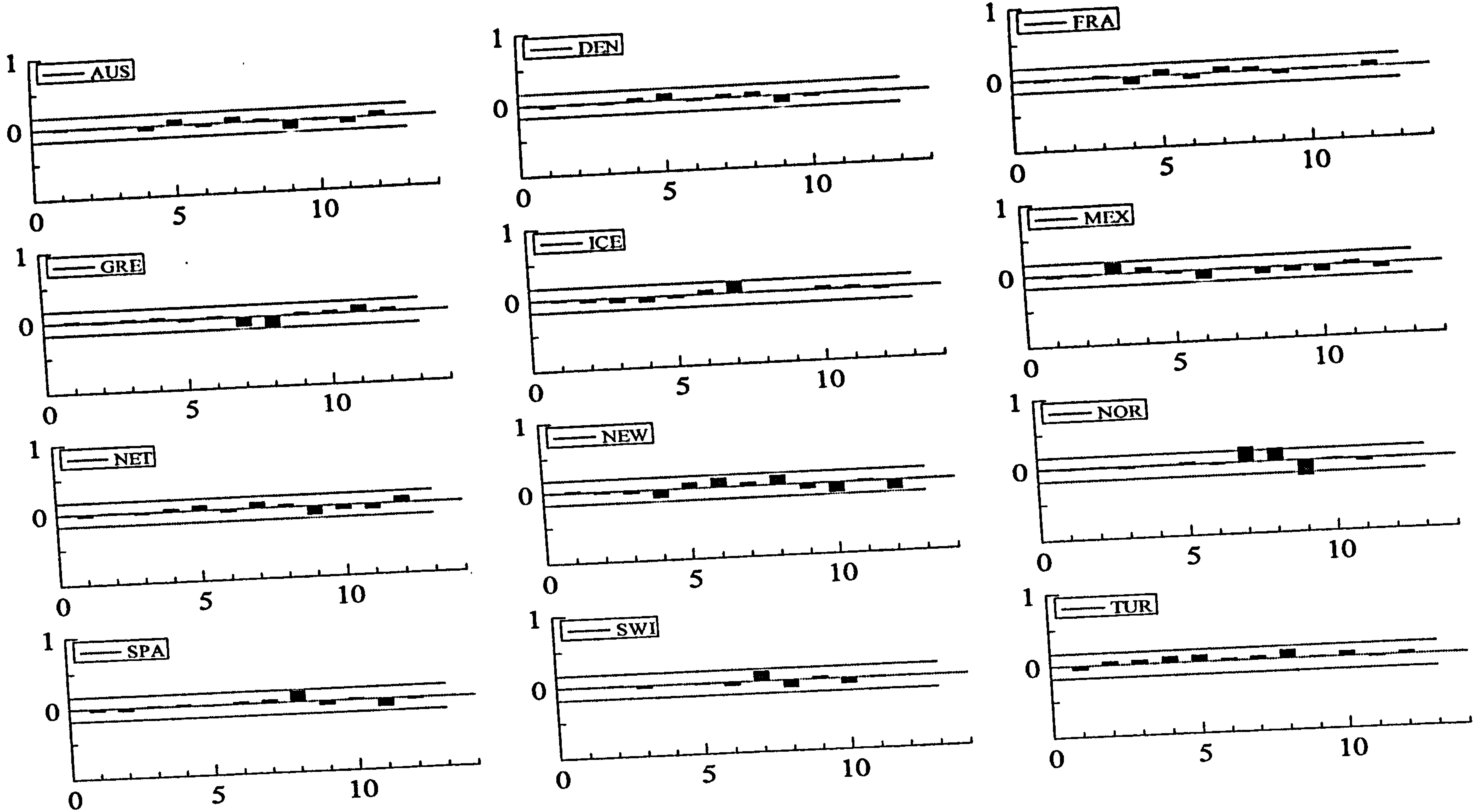


Figure 3.01 Real Consumption Per Capita Time-Series

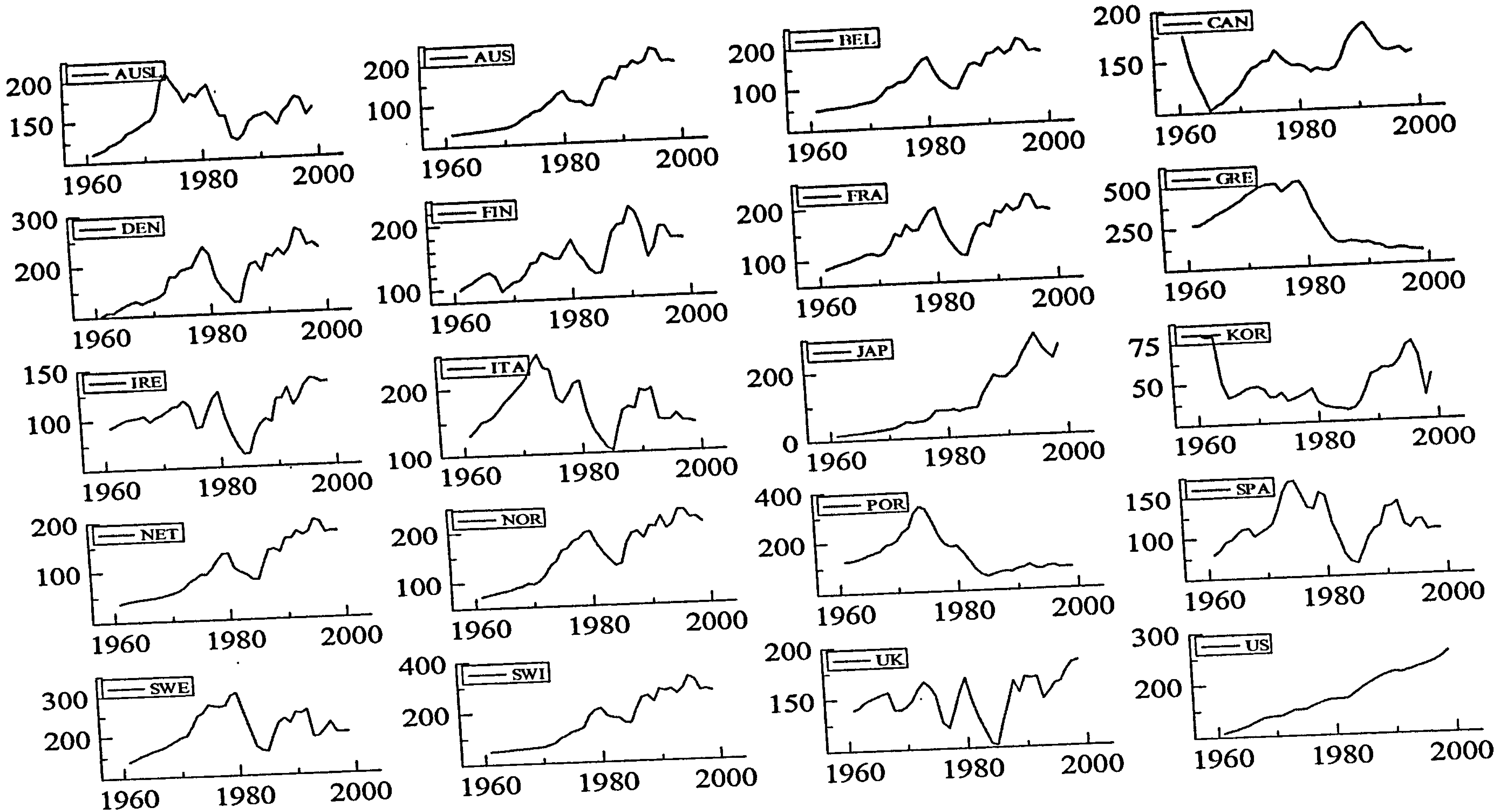


Figure 3.02 First Differences of Real Consumption Per Capita Time-Series

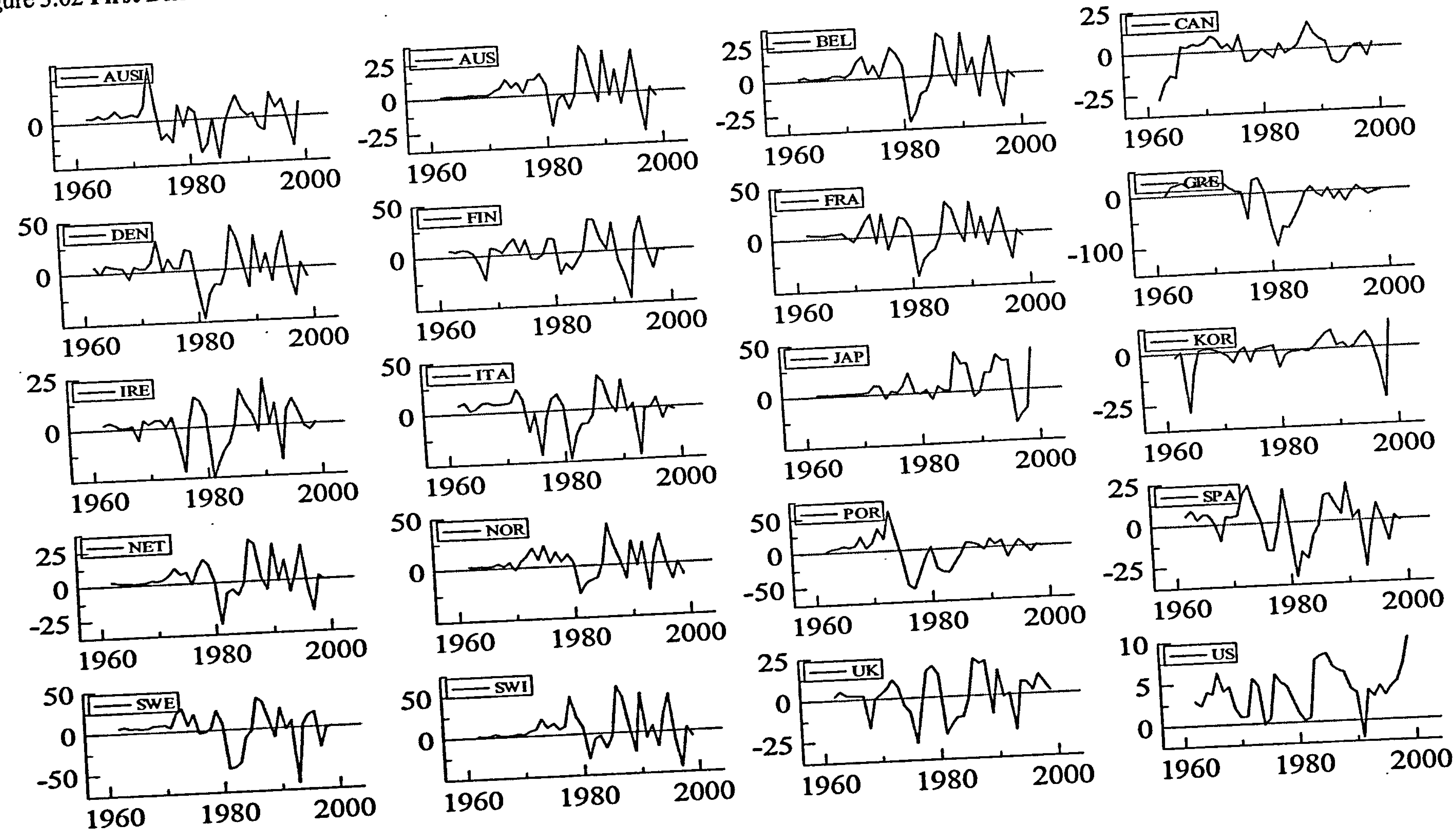


Figure 3.03 Autocorrelation Functions of Real Consumption Per Capita Time-Series

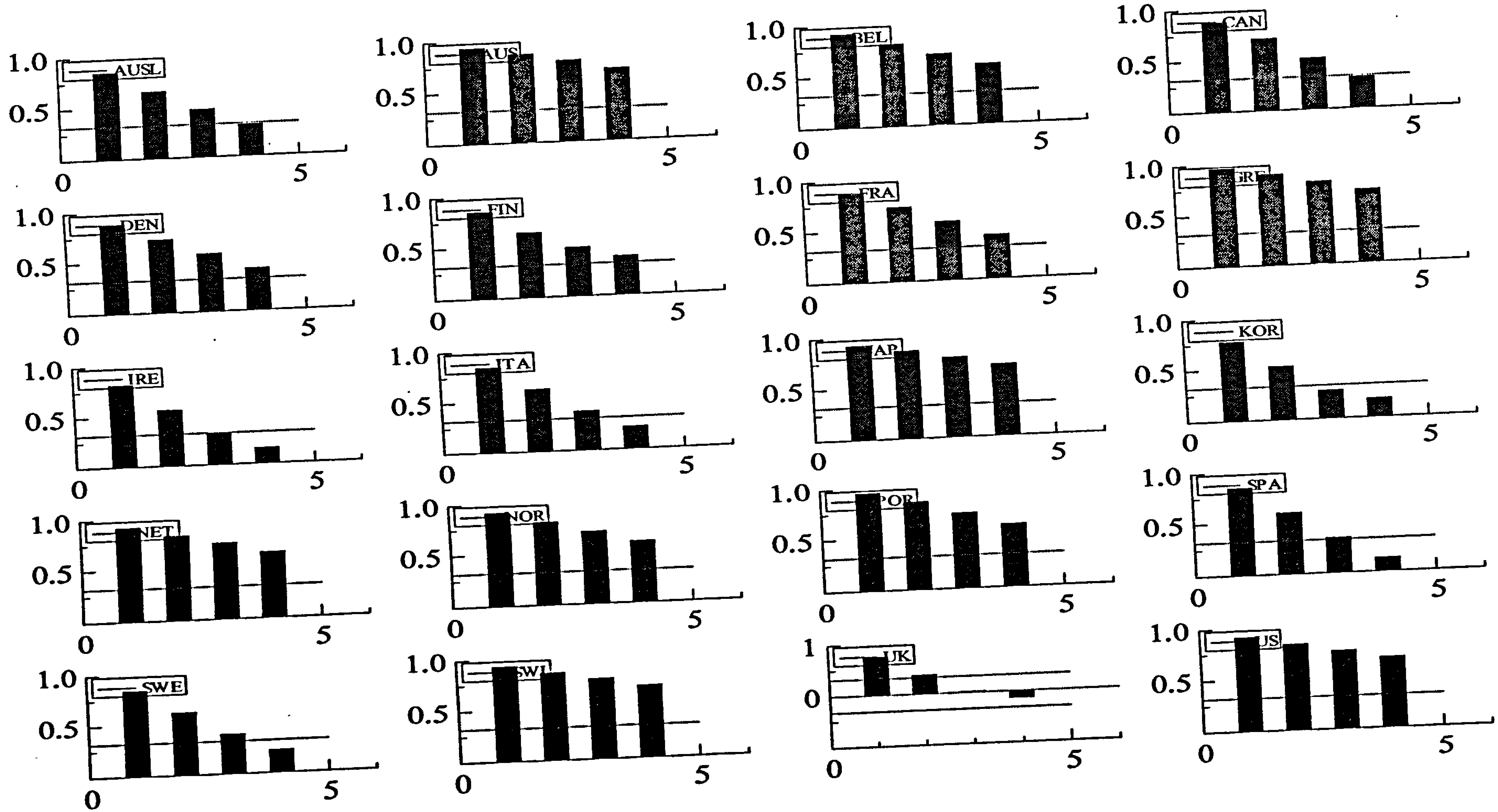


Figure 3.04 Autocorrelation Functions of First Differences of Real Consumption Per Capita Time-Series

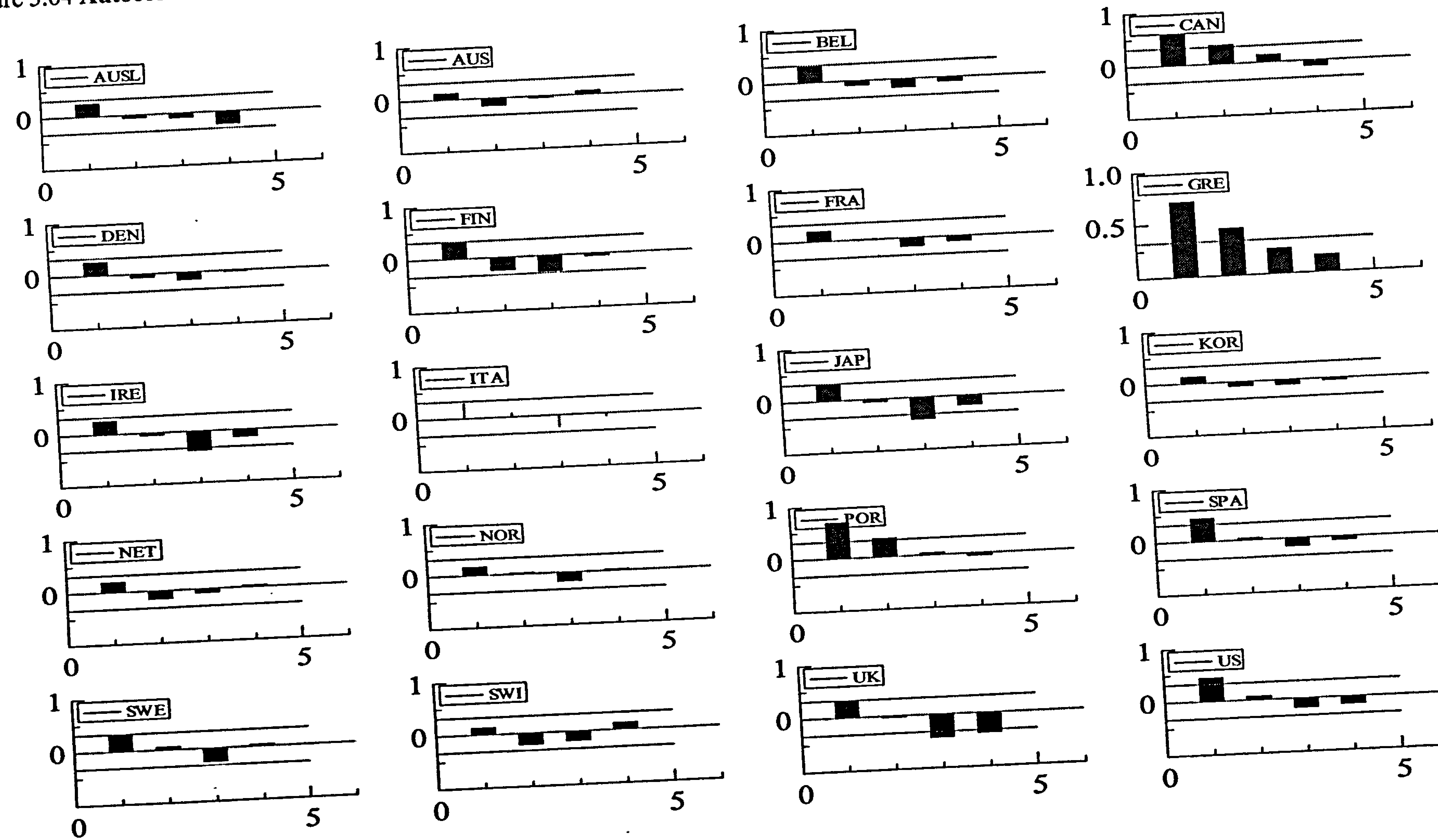


Figure 3.05 Real GDP Per Capita Time-Series

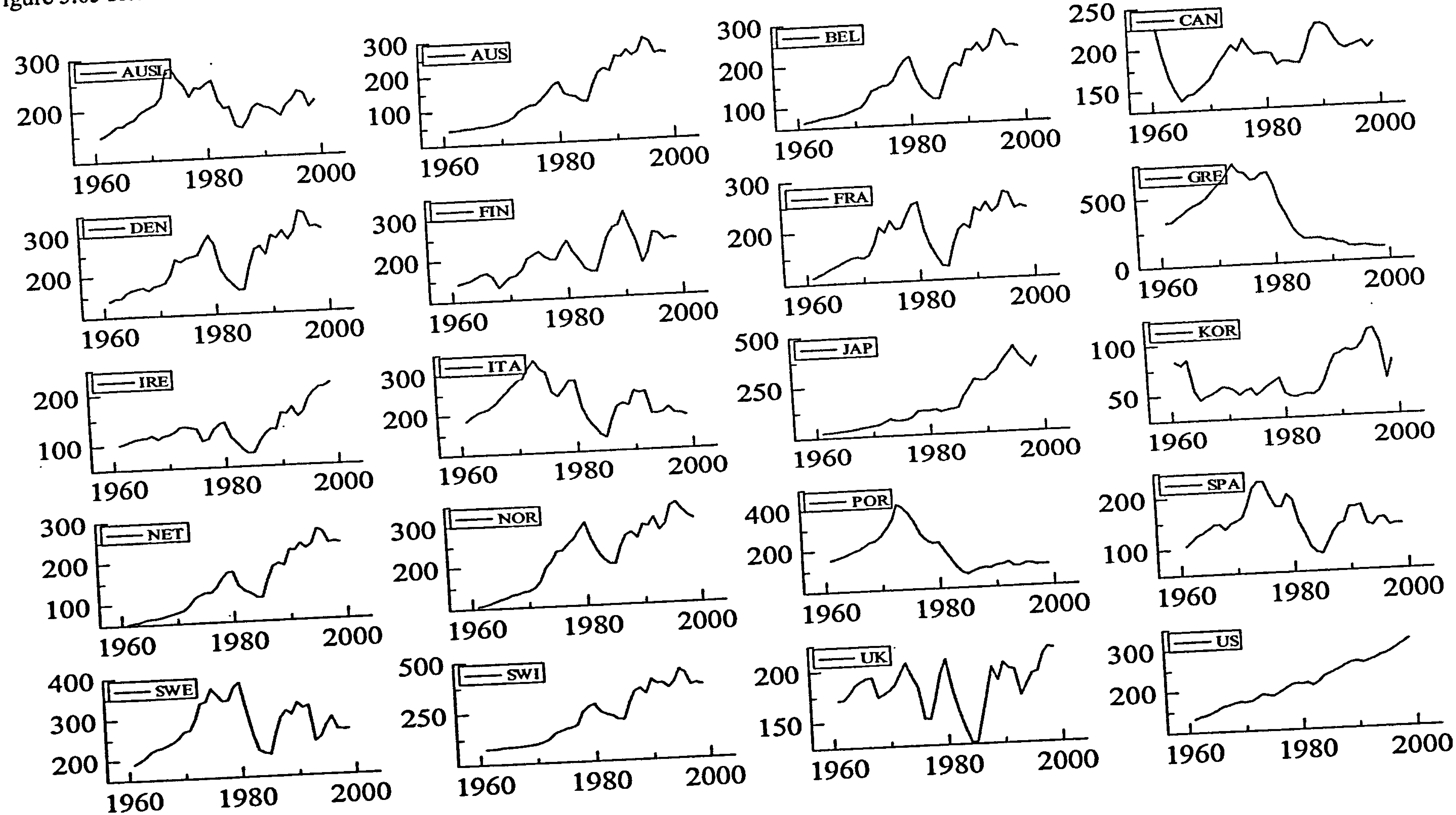


Figure 3.06 First Differences of Real GDP Per Capita Time-Series

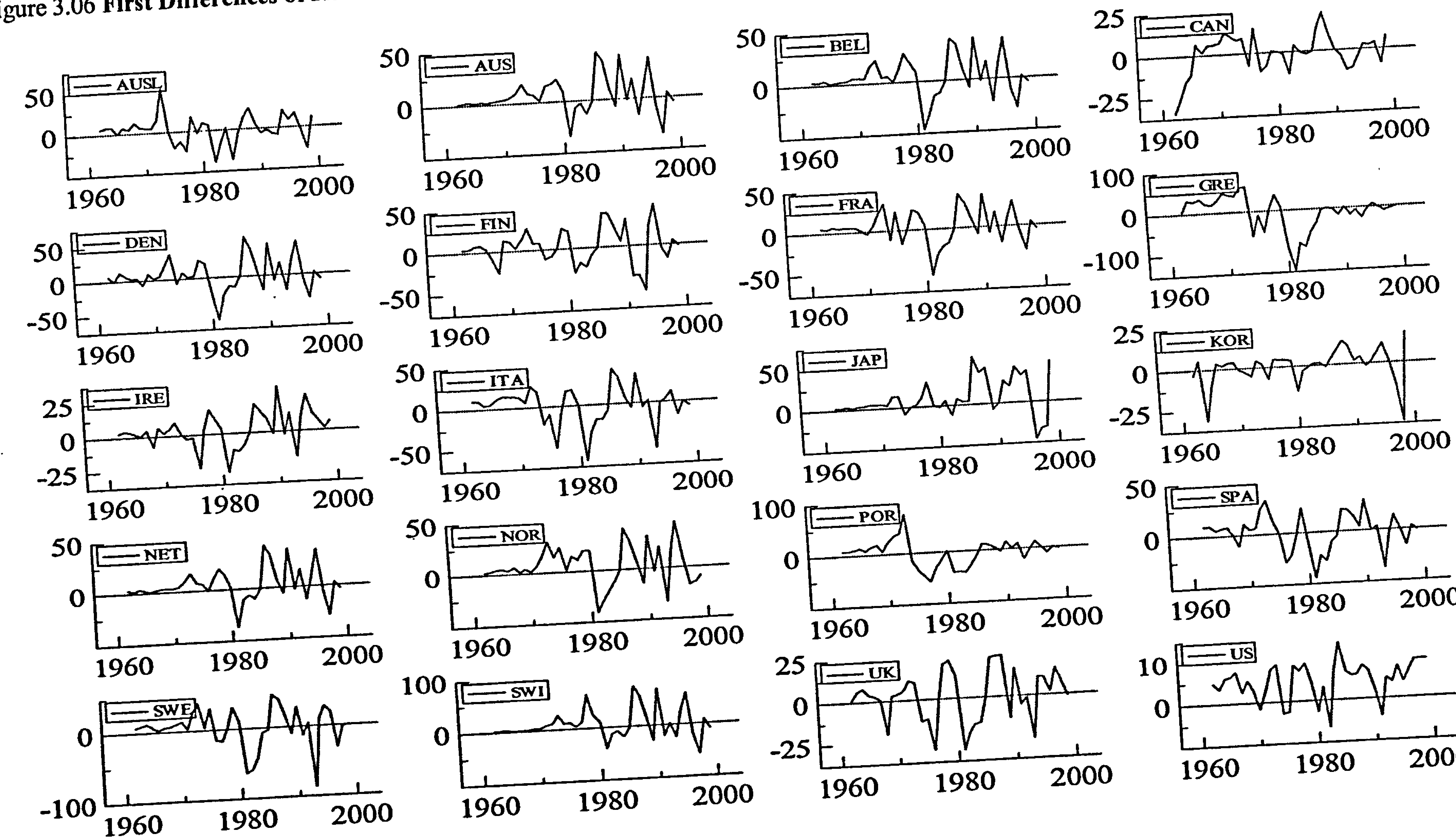


Figure 3.07 Autocorrelation Functions of Real GDP Per Capita Time-Series

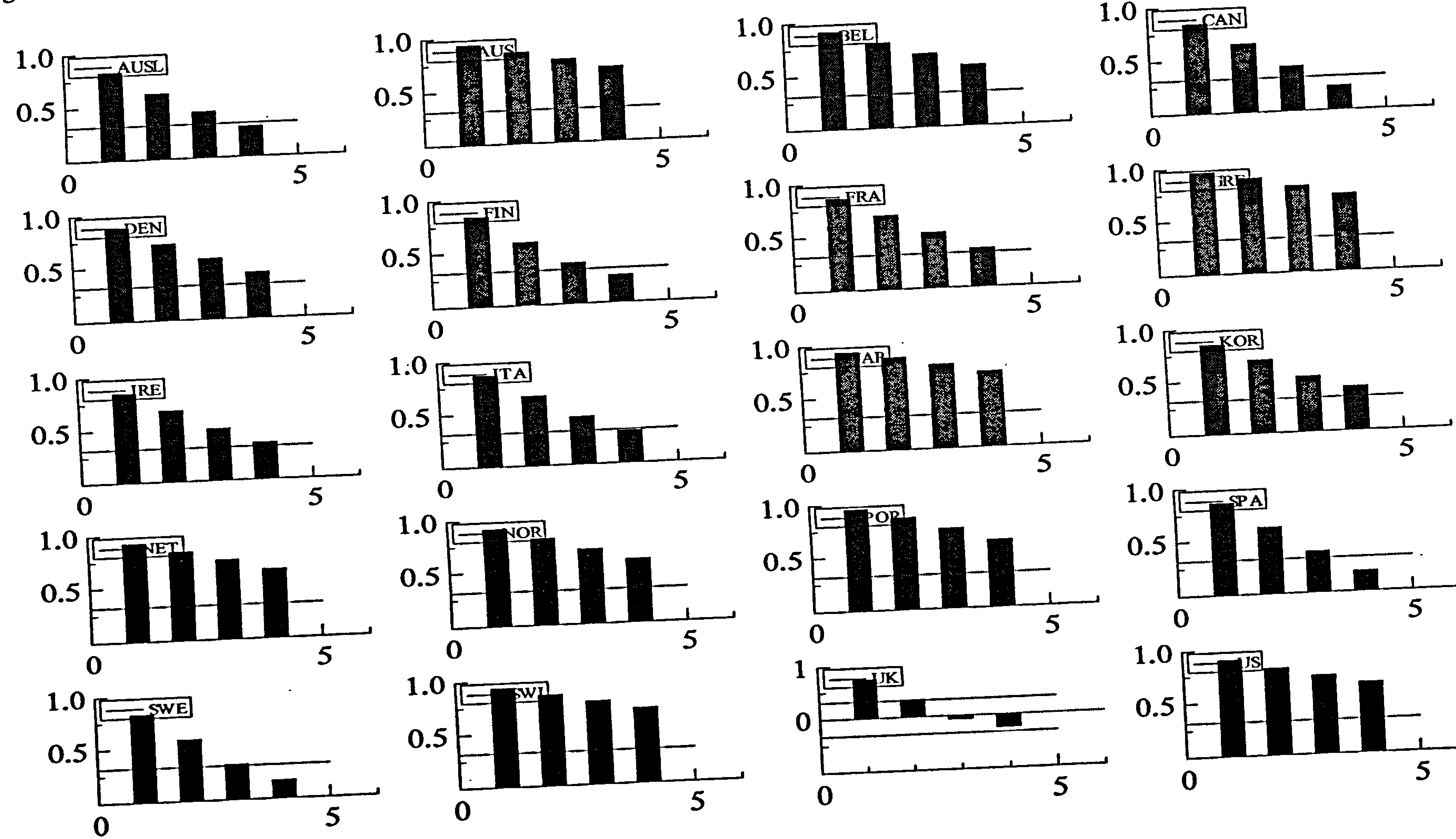


Figure 3.08 Autocorrelation Functions of First Differences of Real GDP Per Capita Time-Series

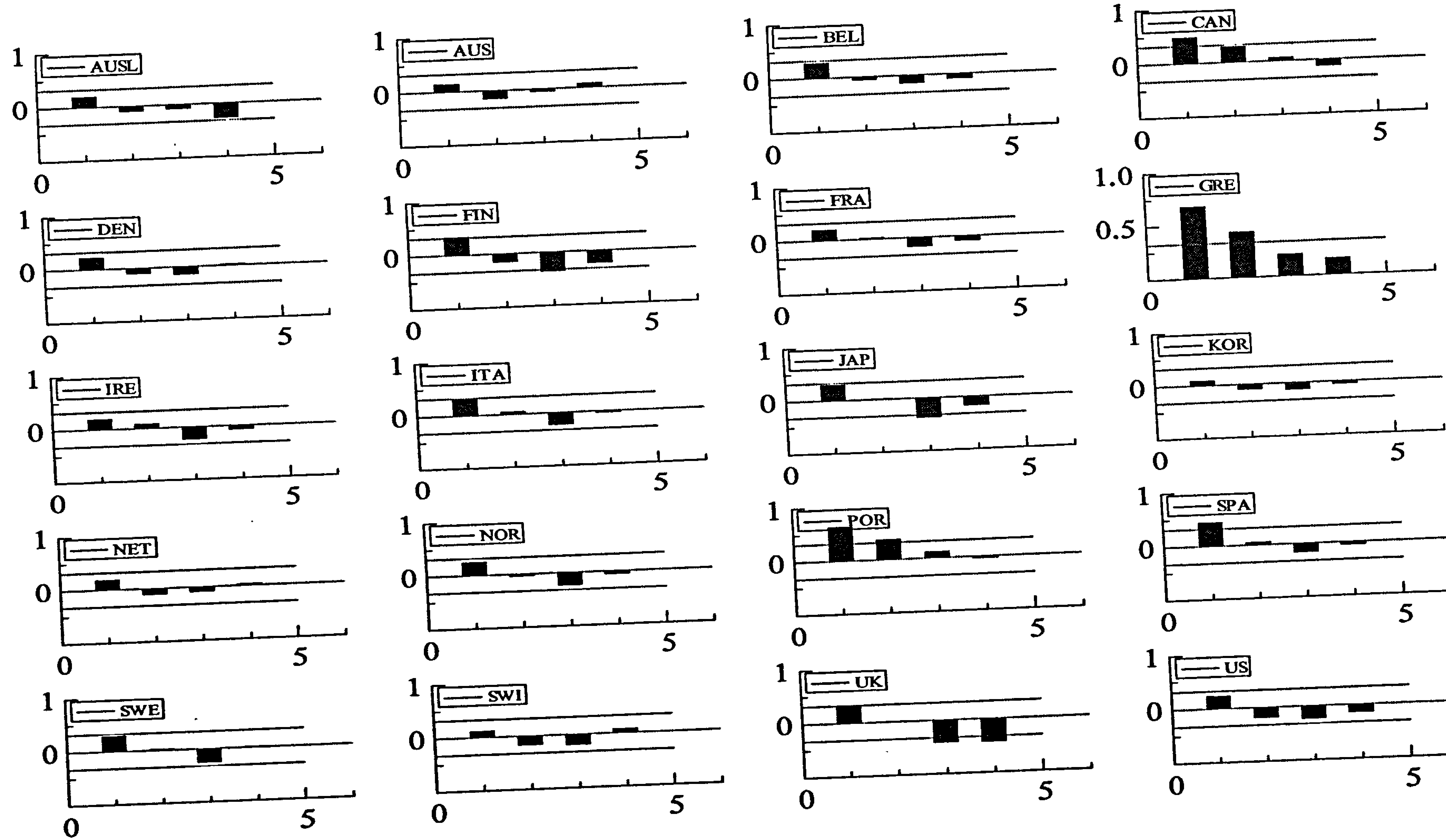


Figure 3.09 Real L.A. /Real G.D.P. Time-Series

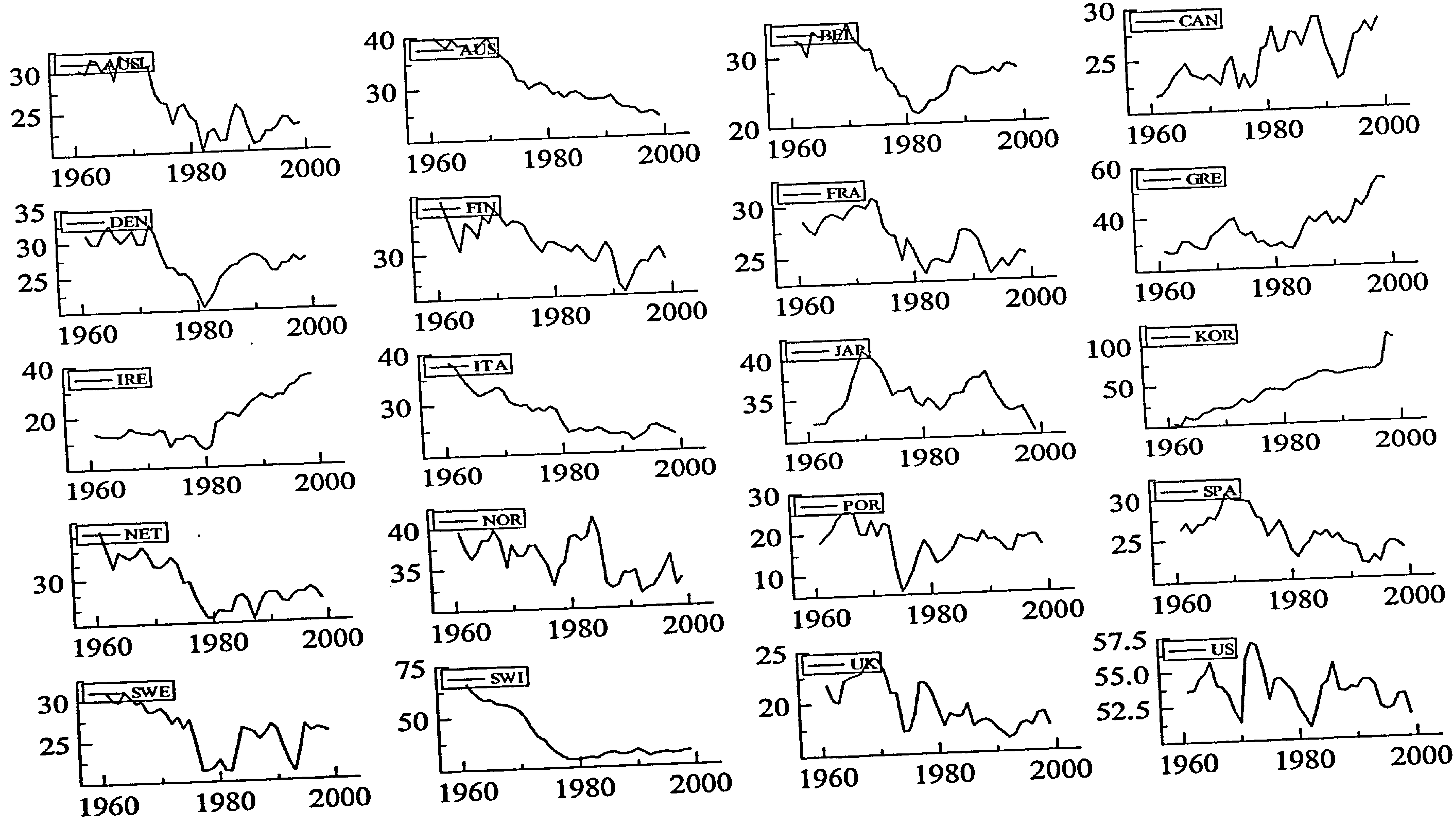


Figure 3.10 First Differences of Real L.A. /Real G.D.P. Time-Series

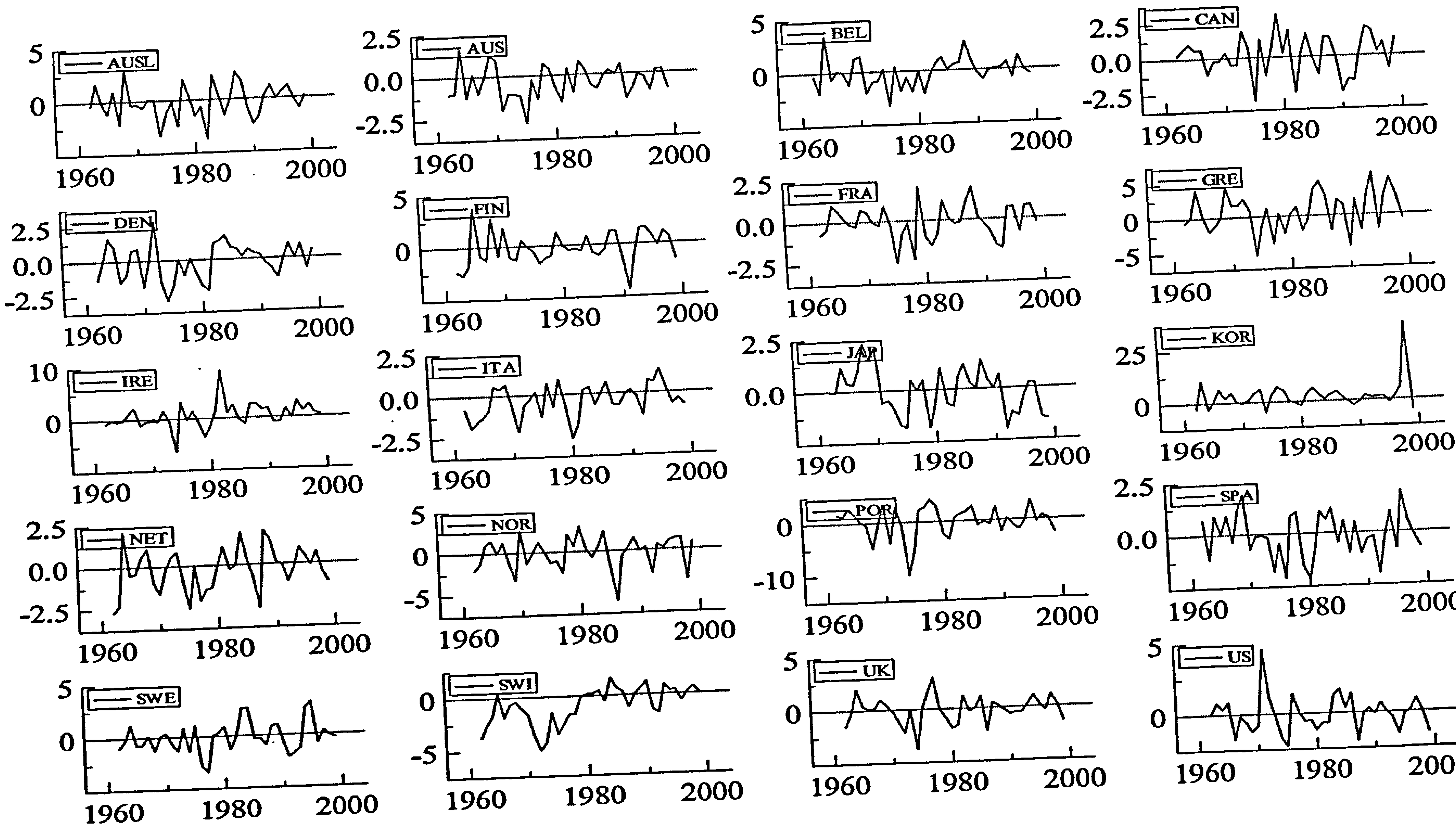


Figure 3.11 Autocorrelation Functions of Real L.A. /Real G.D.P. Time-Series

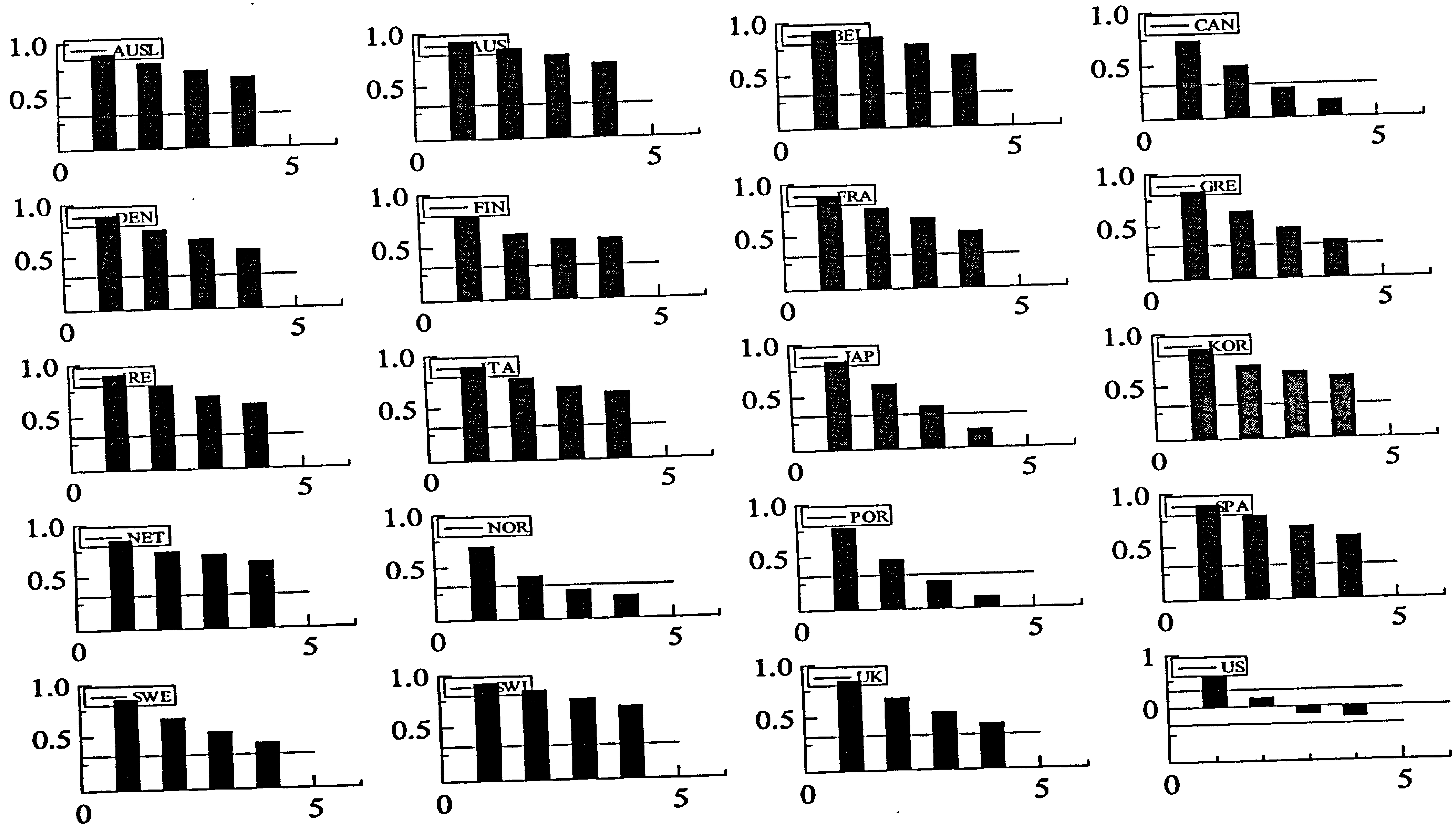


Figure 3.12 Autocorrelation Functions of First Differences of Real L.A. /Real G.D.P. Time-Series

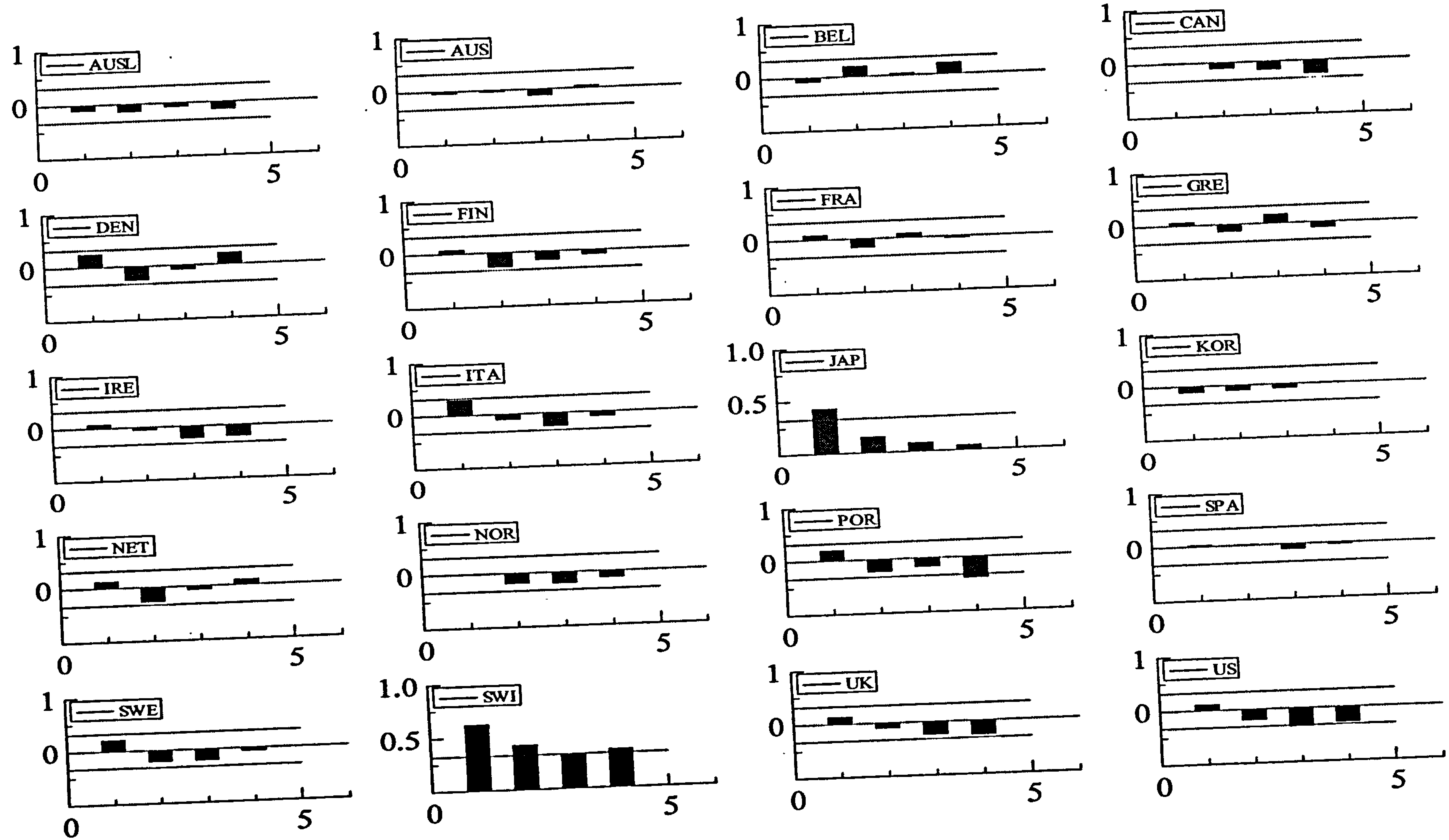


Figure 3.13 Interest Rate Time-Series

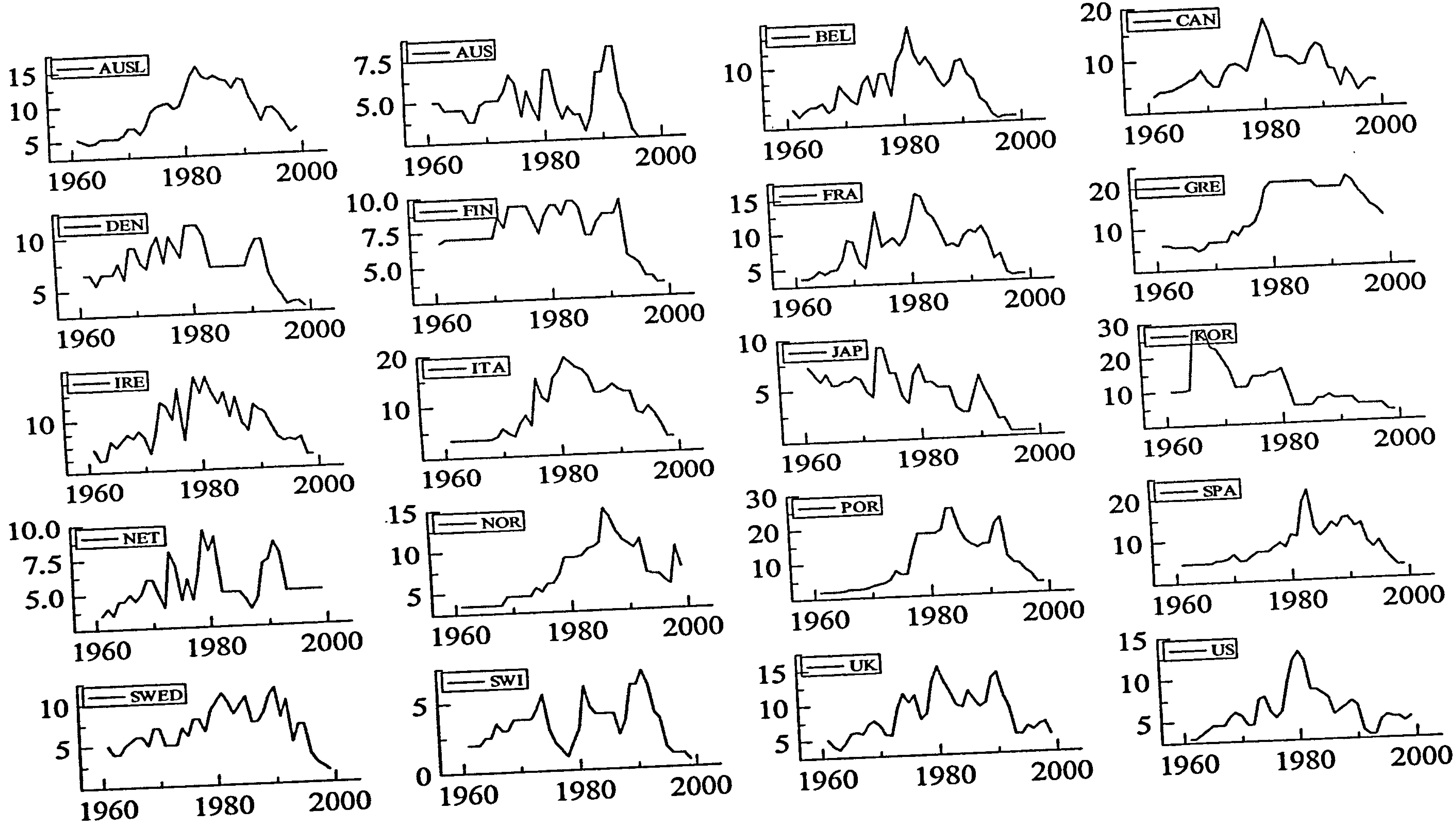


Figure 3.14 First Differences of Interest Rate Time-Series

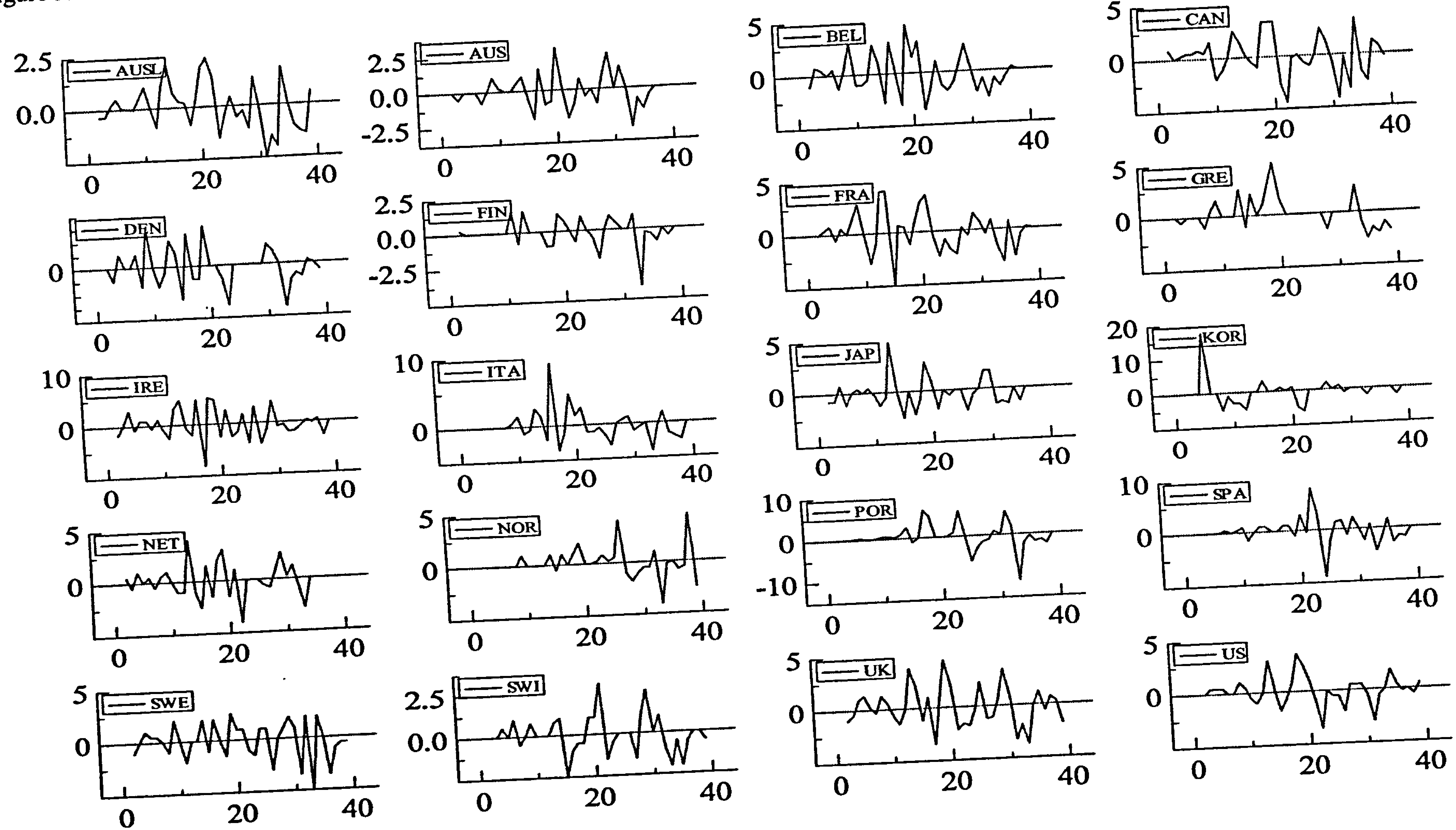


Figure 3.15 Autocorrelation Functions of Interest Rate Time-Series

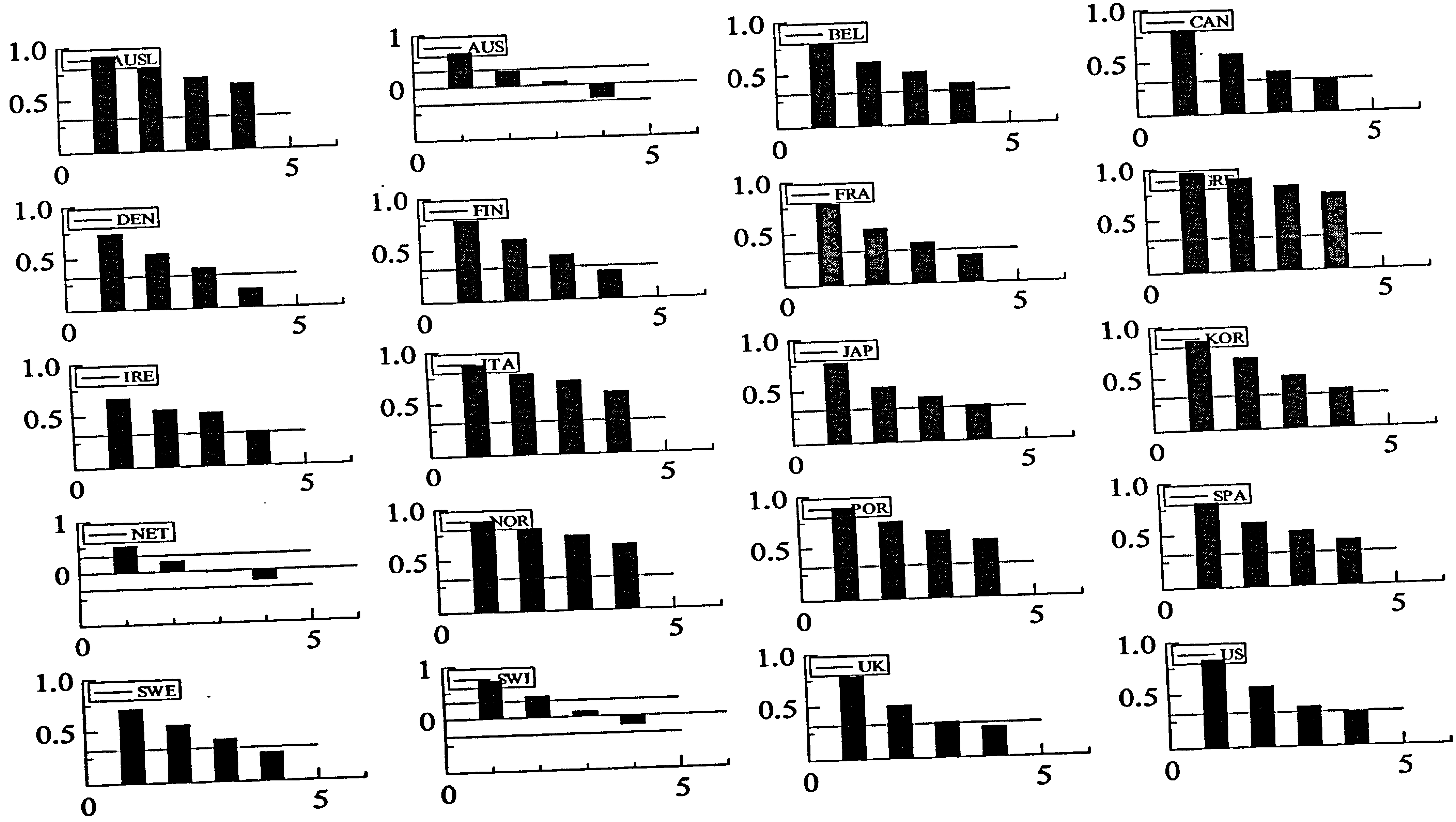


Figure 3.16 Autocorrelation Functions of First Differences of Interest Rate Time-Series

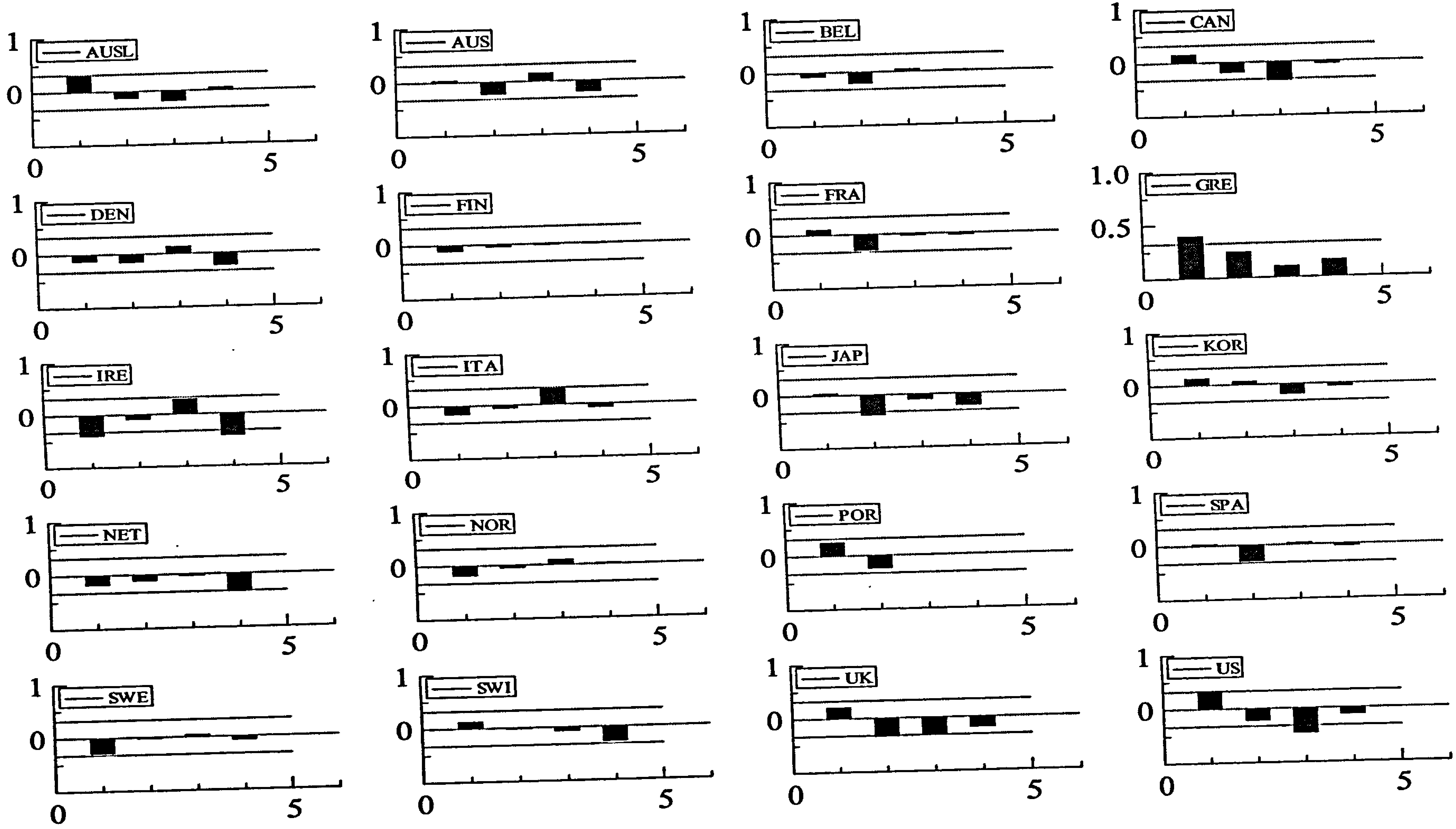


Figure 3.17 Inflation Time-Series

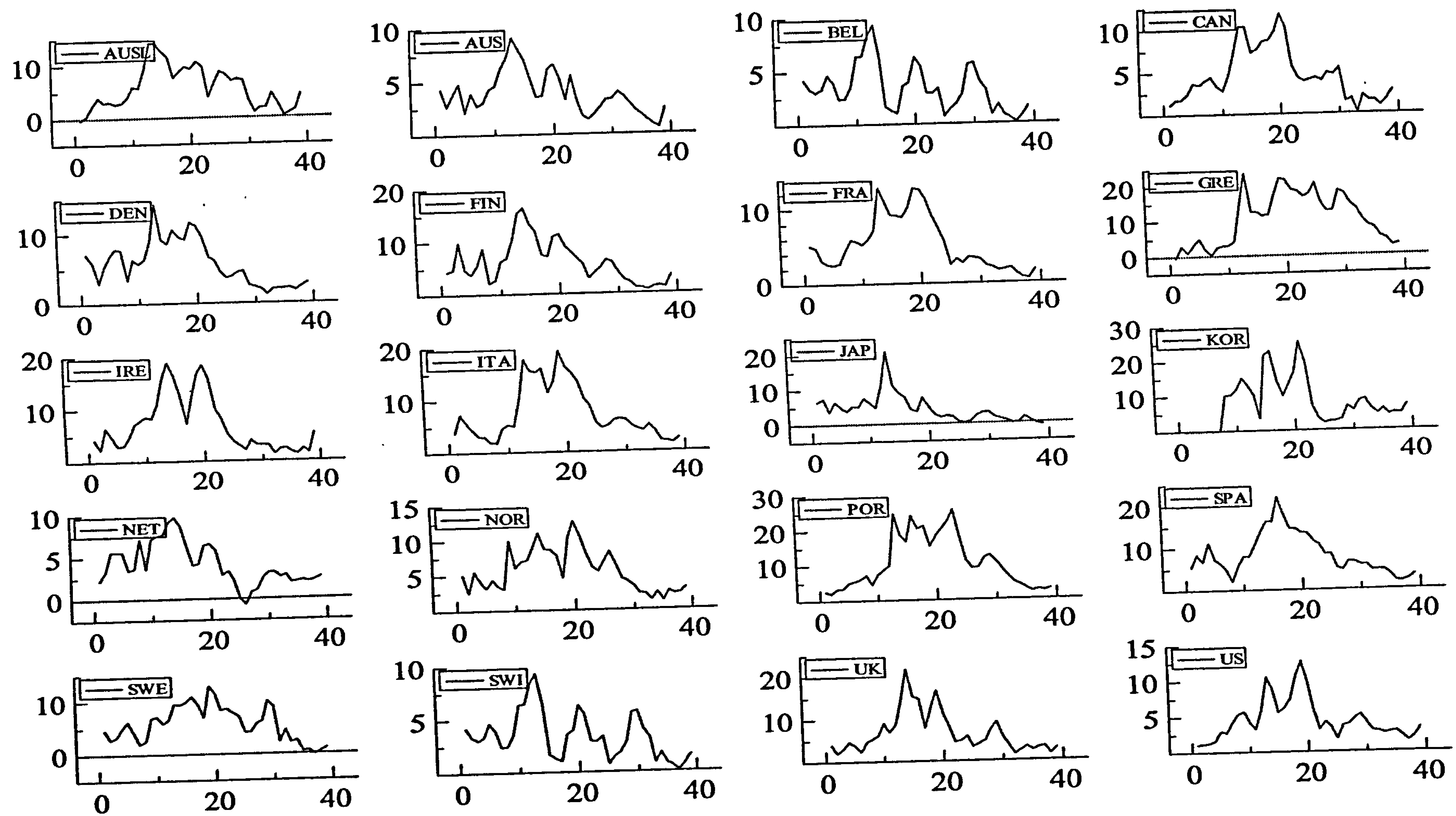


Figure 3.18 First Differences of Inflation Time-Series

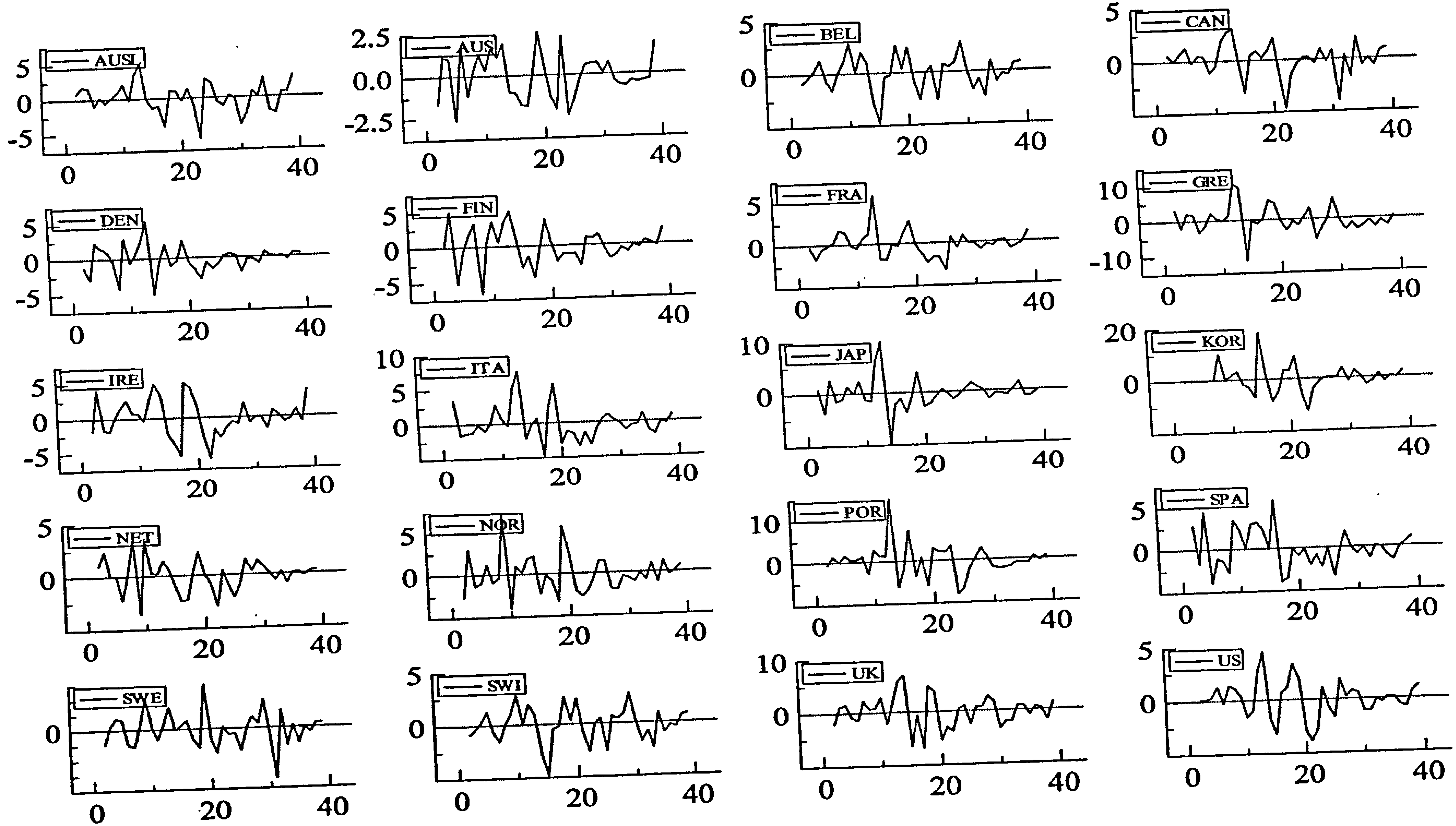


Figure 3.19 Autocorrelation Functions of Inflation Time-Series

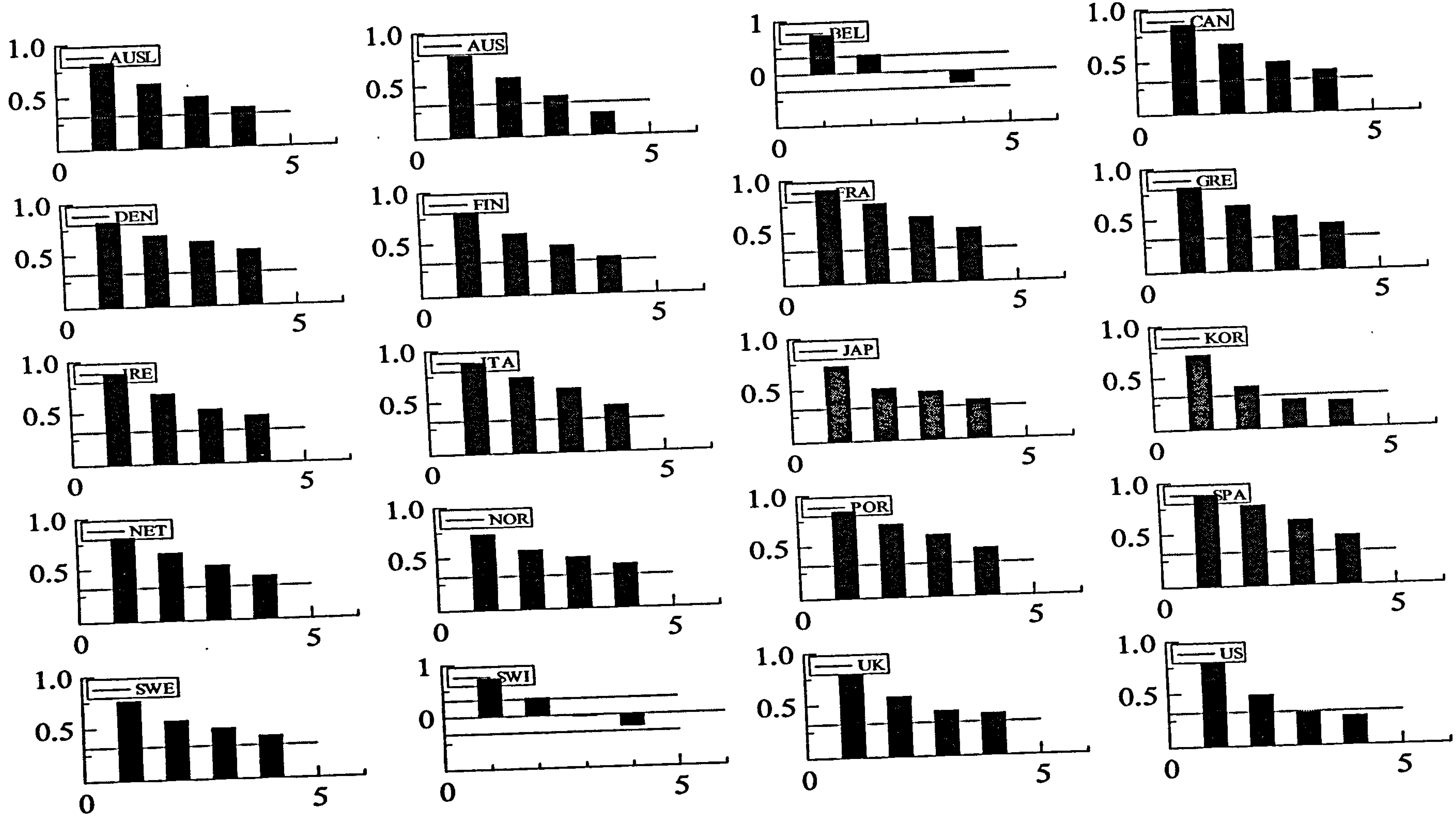


Figure 3.20 Autocorrelation Functions of First Differences of Inflation Time-Series

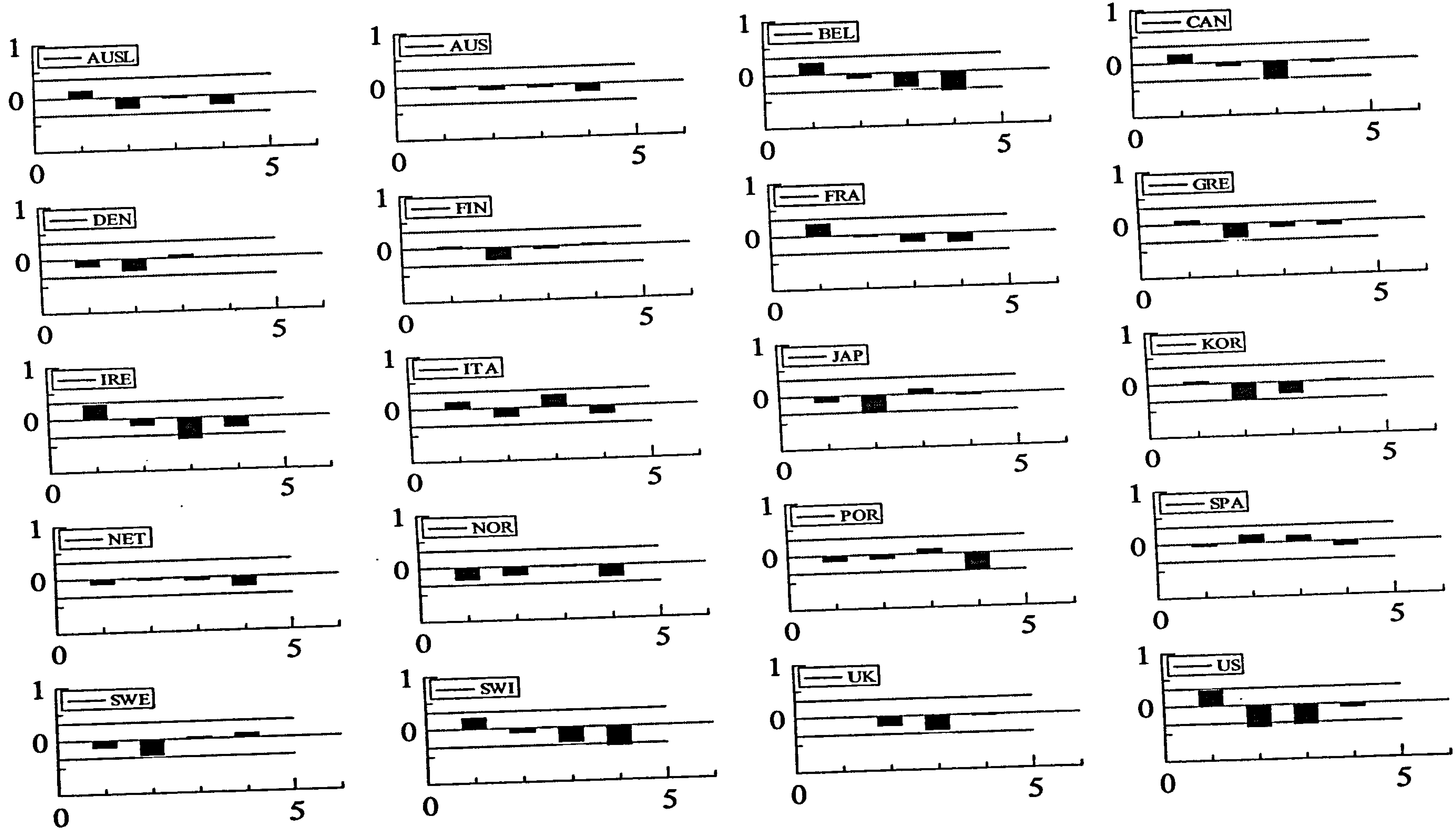


Figure 3.21 Autocorrelation Functions of the Residuals $\hat{u}_{it}$

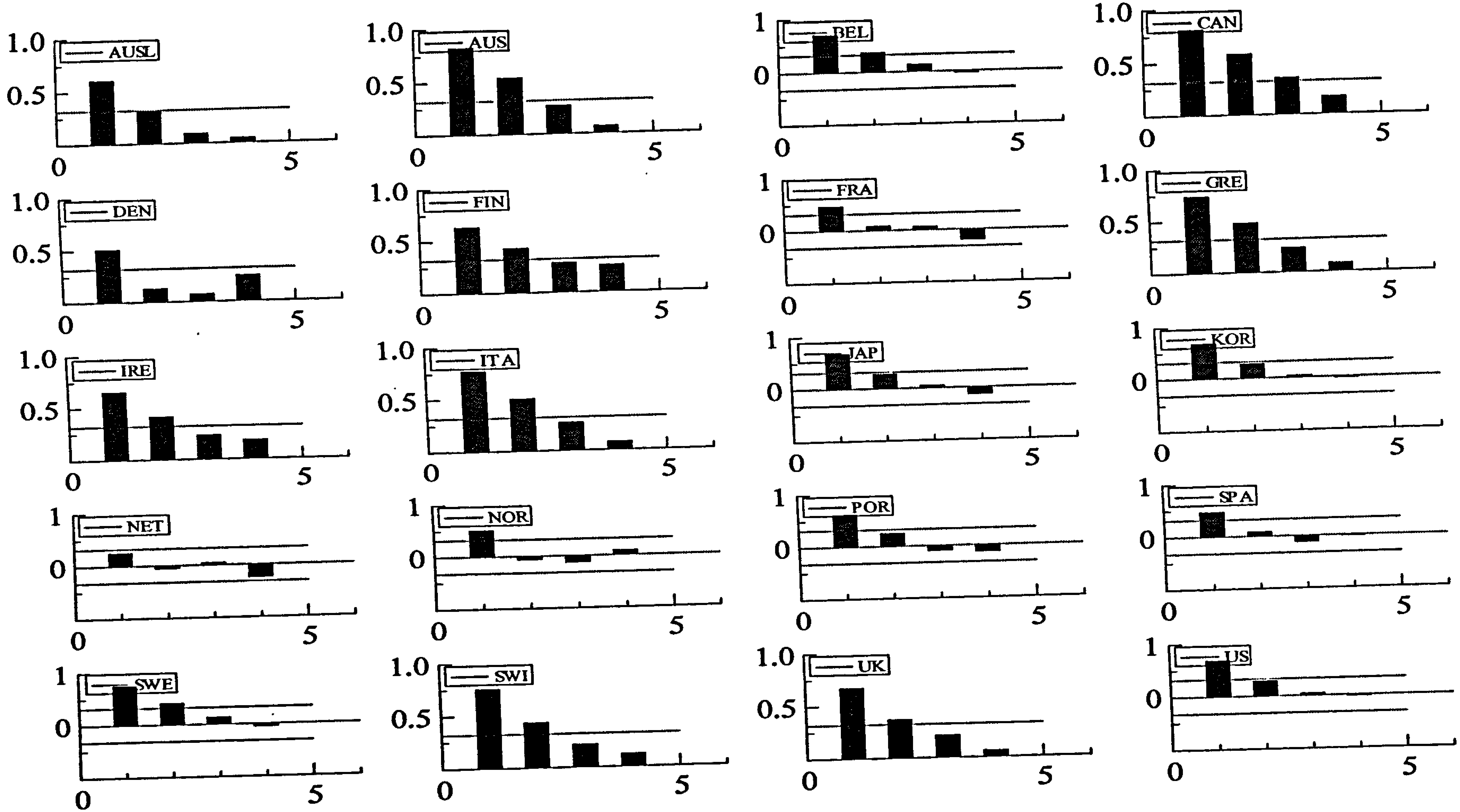


Figure 3.22 Autocorrelation Functions of the Residuals $\hat{\mu}_{it}$

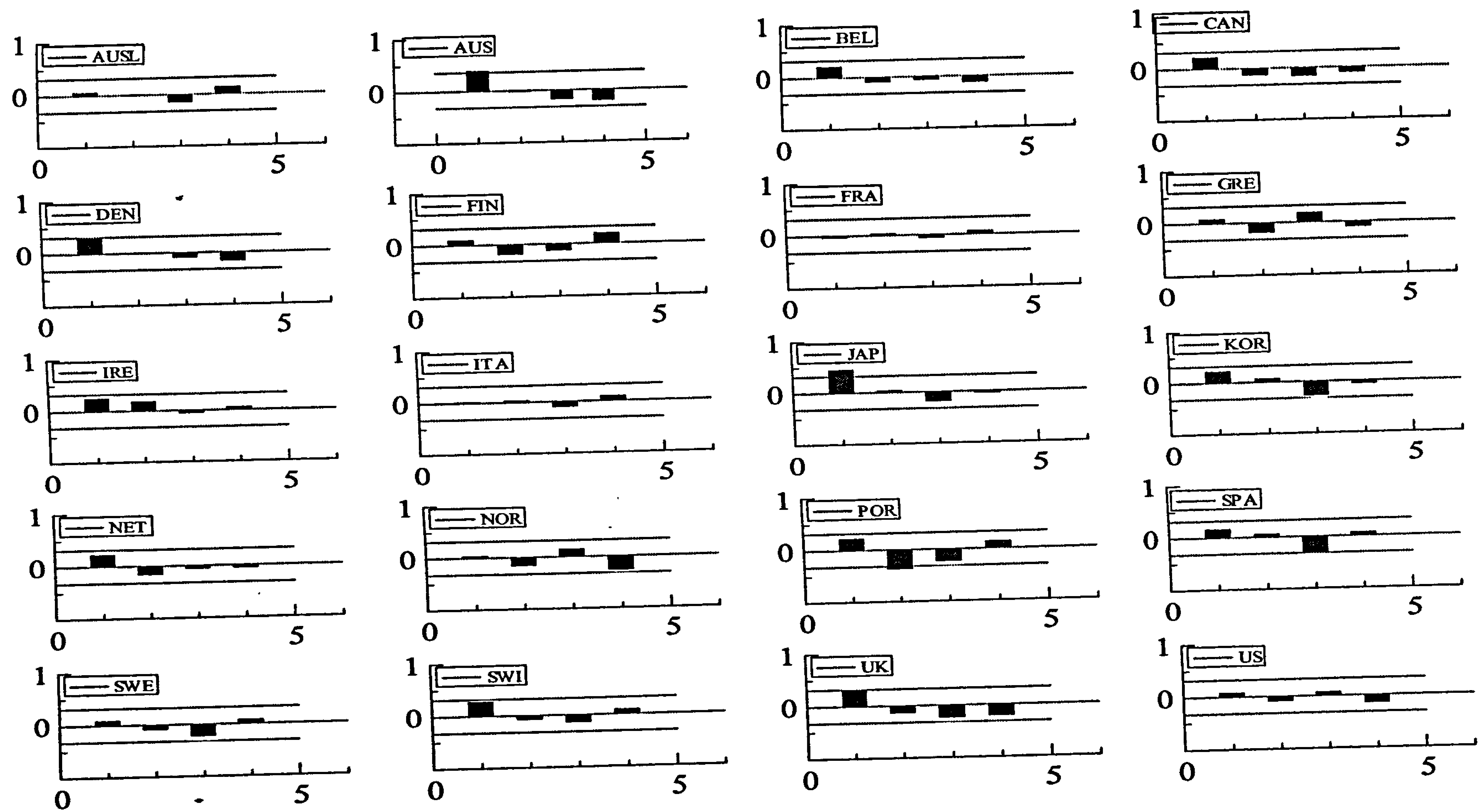


Figure 3.23 Autocorrelation Functions of the Residuals $\hat{\eta}_{it}$

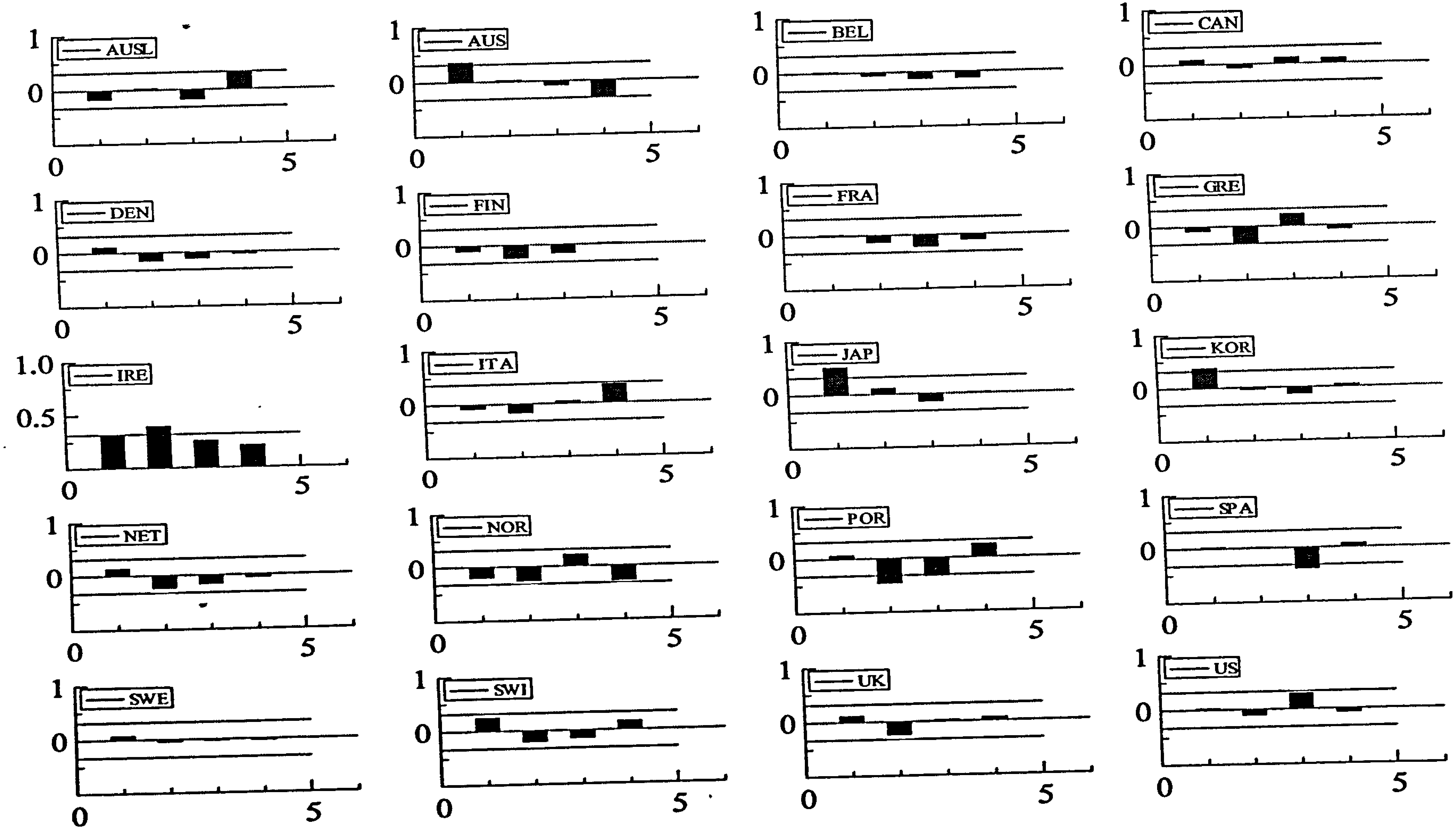


Figure 3.24 Autocorrelation Functions of the Residuals $\hat{\mu}^*_{it}$

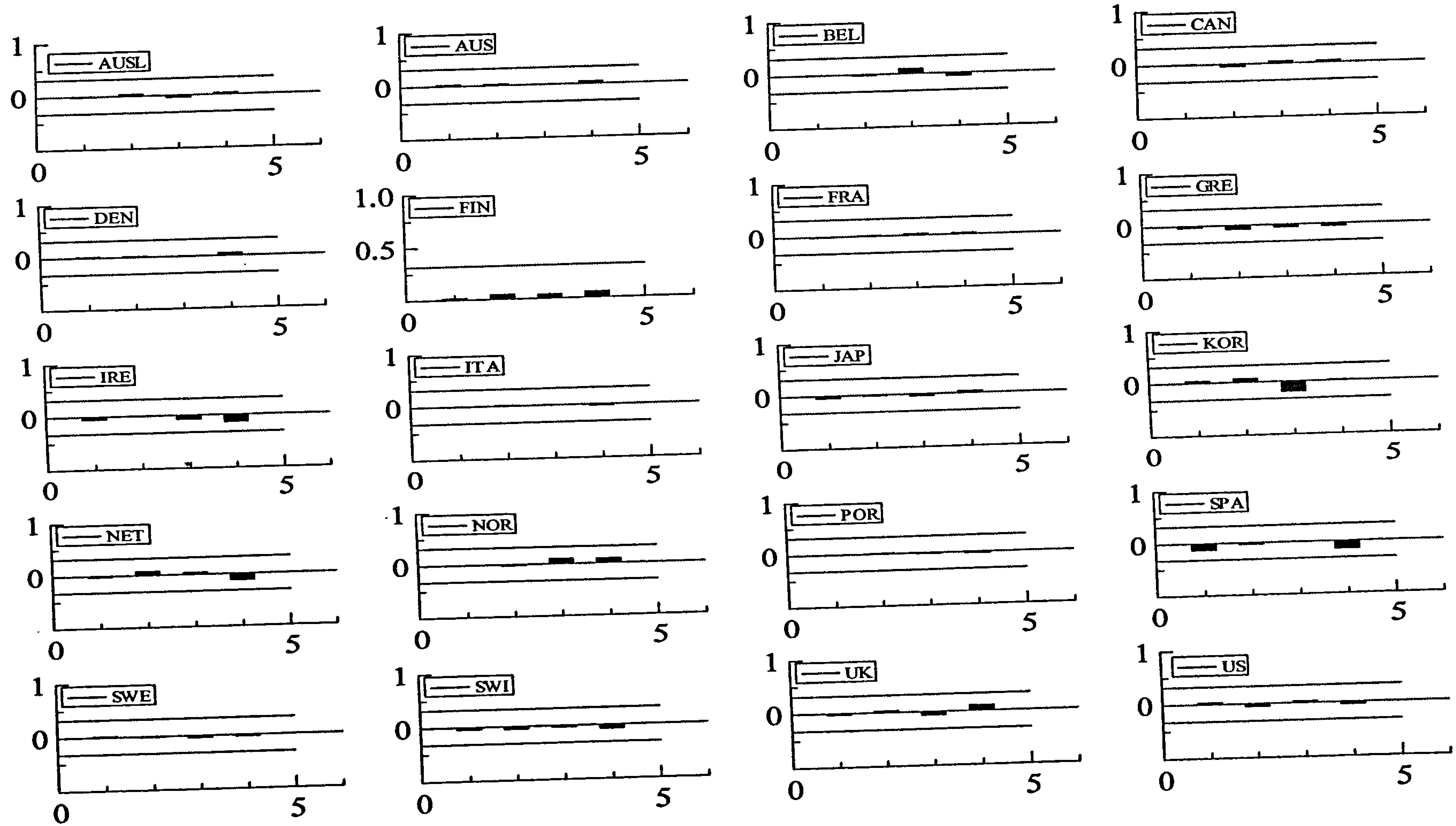


Figure 3.25 Cross-Correlation Function of the Residuals ($\hat{u}_{it}$ x REAGDPPCA)

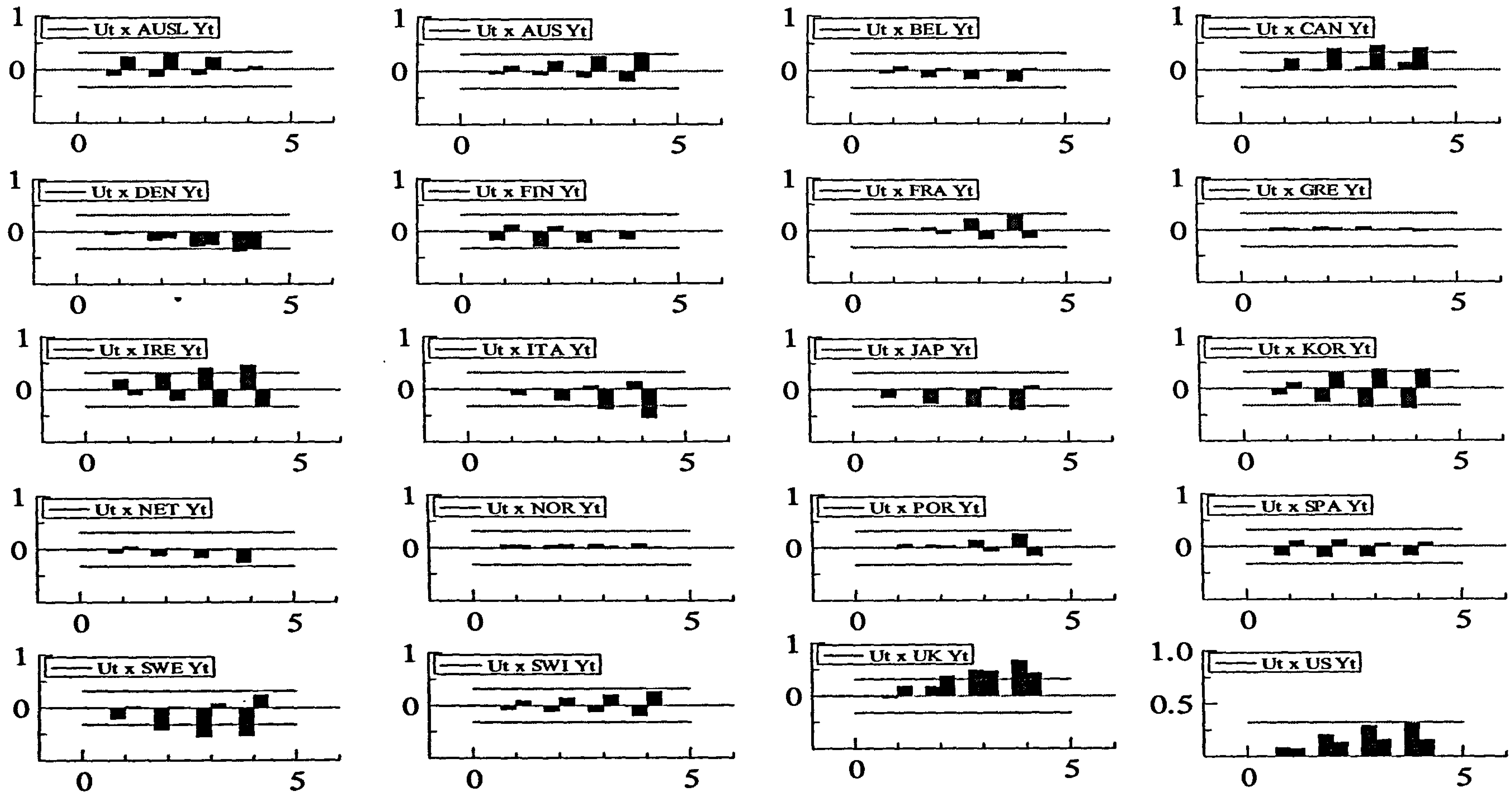


Figure 3.26 Autocorrelation Functions of the DOLS Residuals $\hat{v}_{it}$

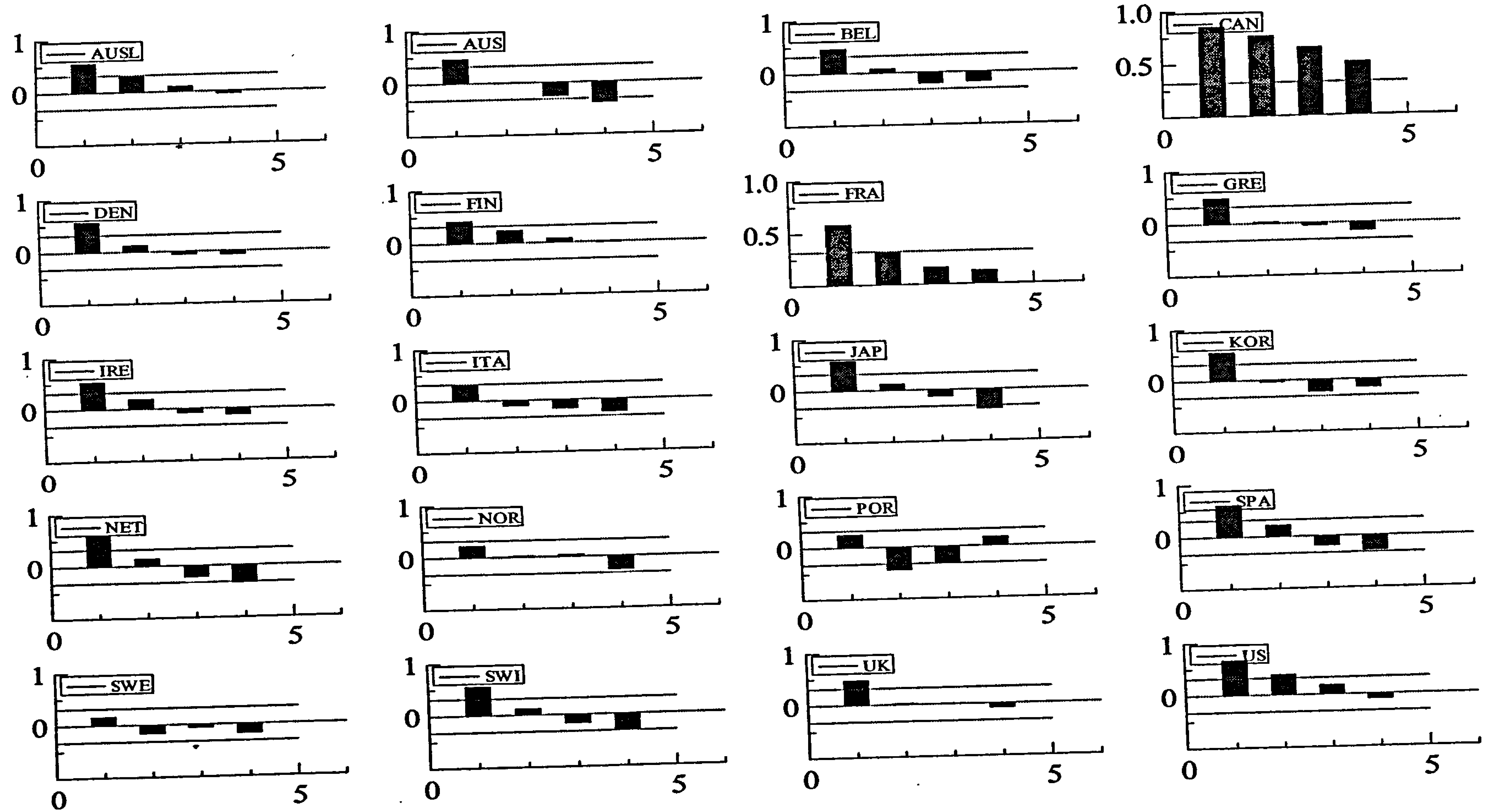


Figure 4.01 **Histograms of Pairs Bootstrap Replications of Pooled DOLS β_1 and β_2**

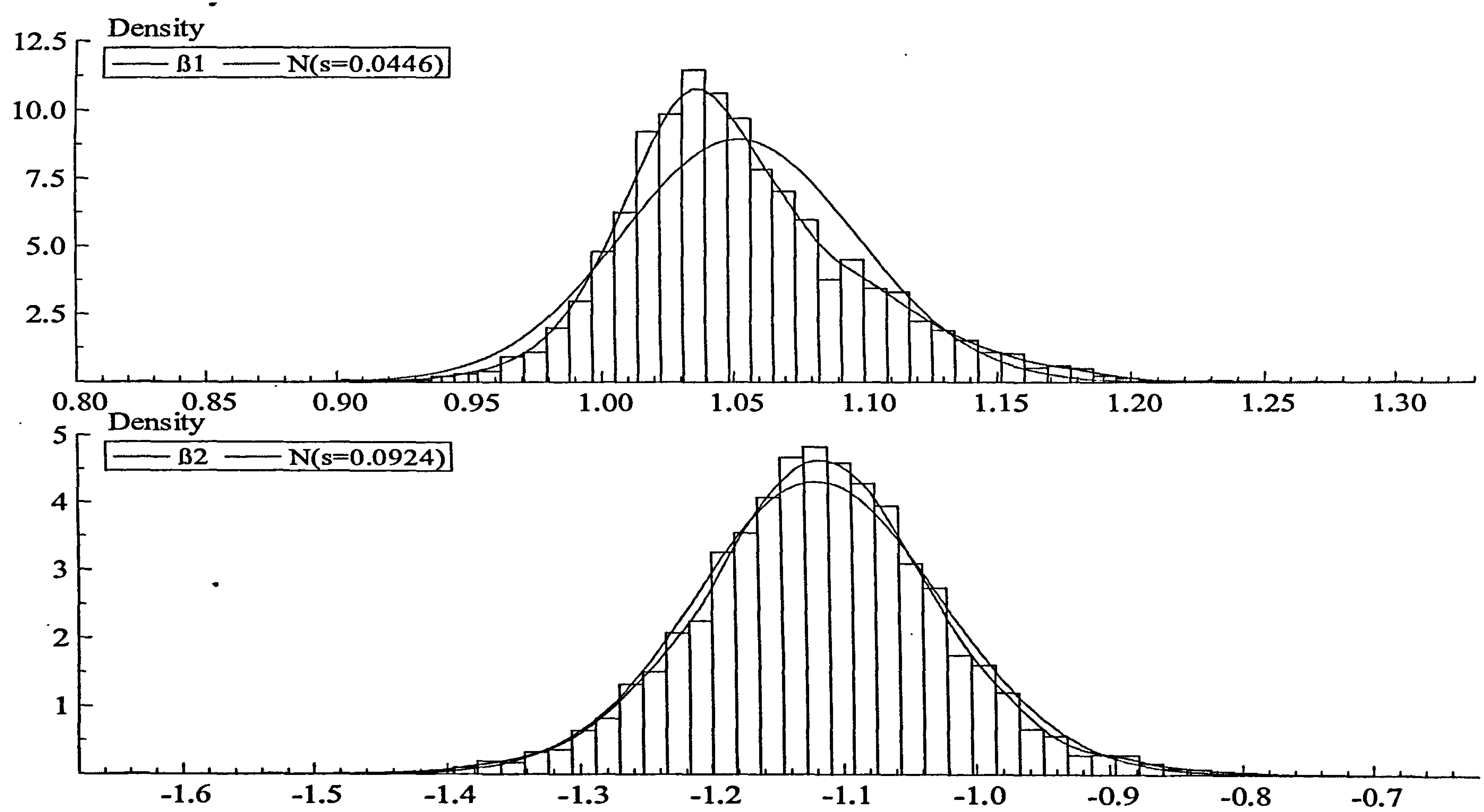


Figure 4.02 Histograms of Pairs Bootstrap Replications of Group-Mean DOLS β_1

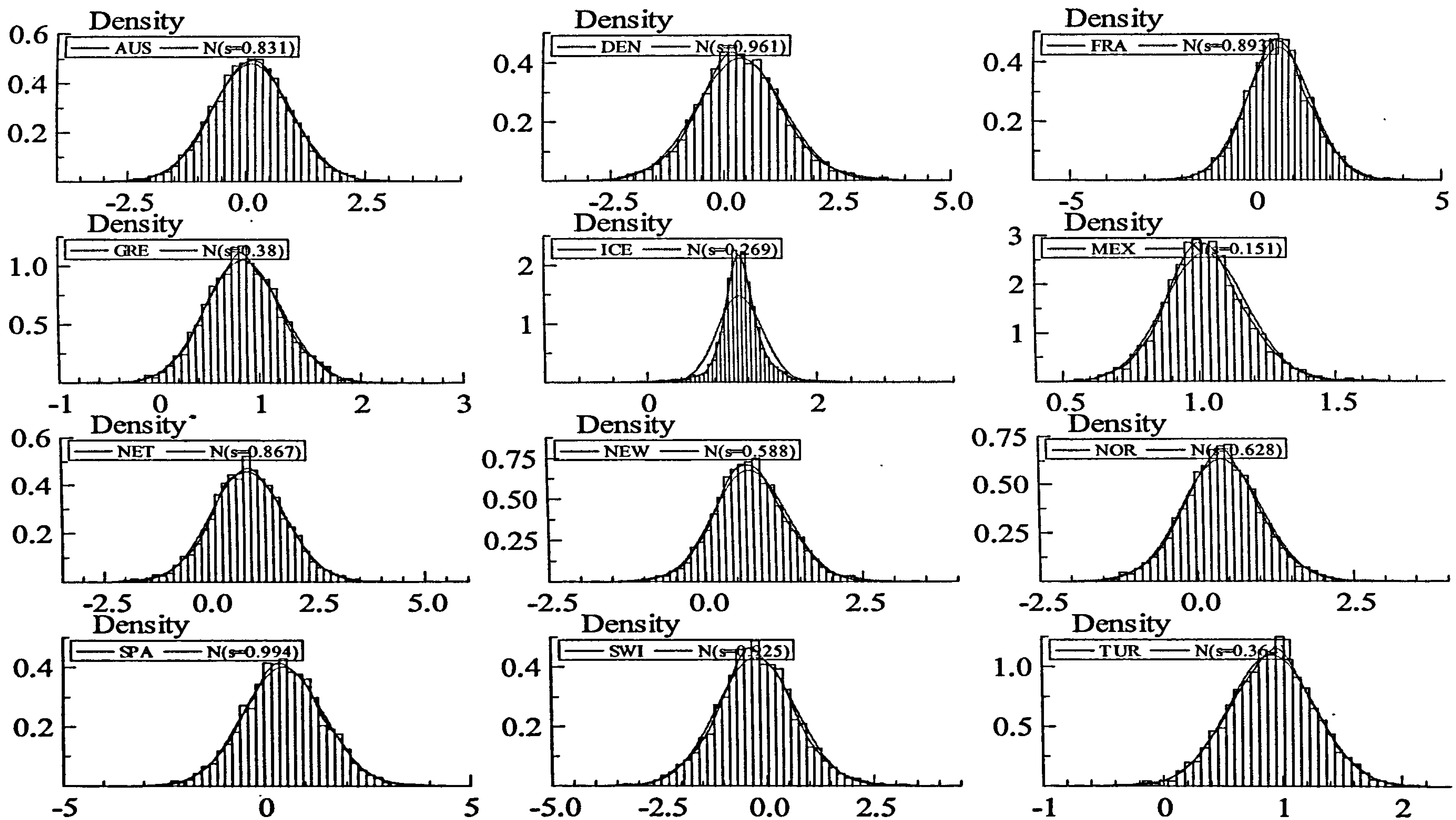


Figure 4.03 Histograms of Pairs Bootstrap Replications of Group-Mean DOLS β_2

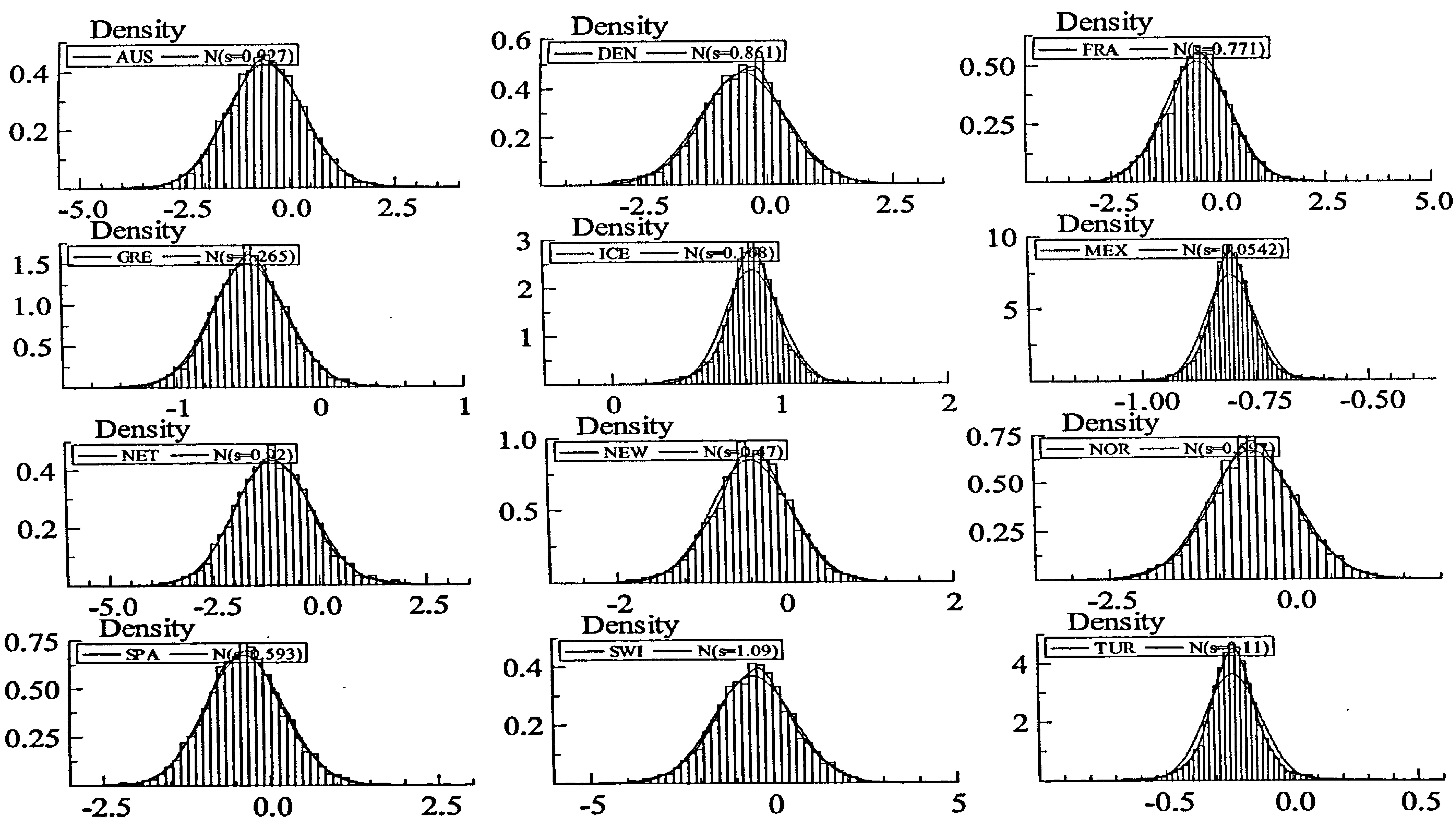


Figure 4.04 Histograms of Residual Bootstrap Replications of Group-Mean AR(12) θ_1

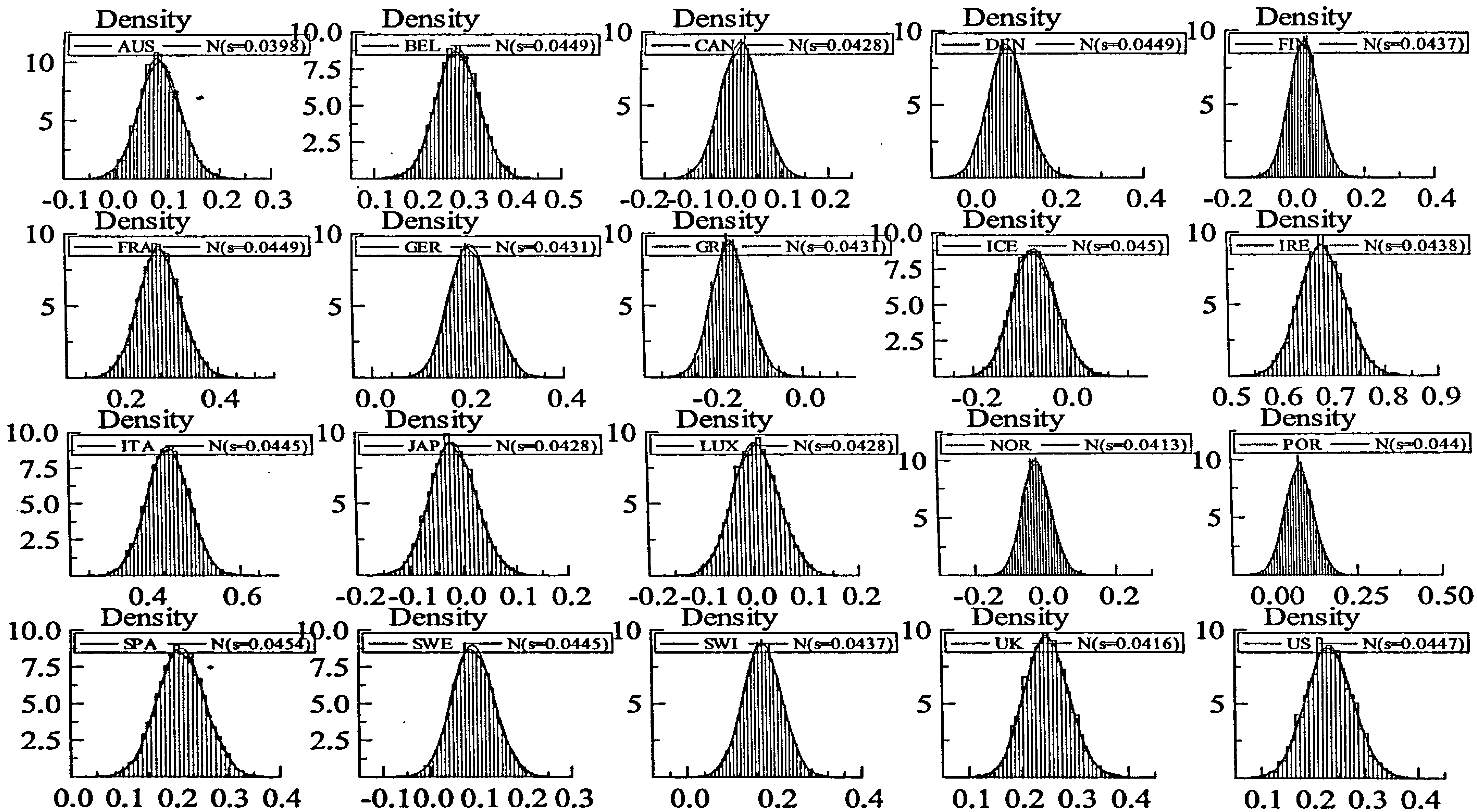


Figure 4.05 Histograms of Residual Bootstrap Replications of Group-Mean AR(12) θ_{12}

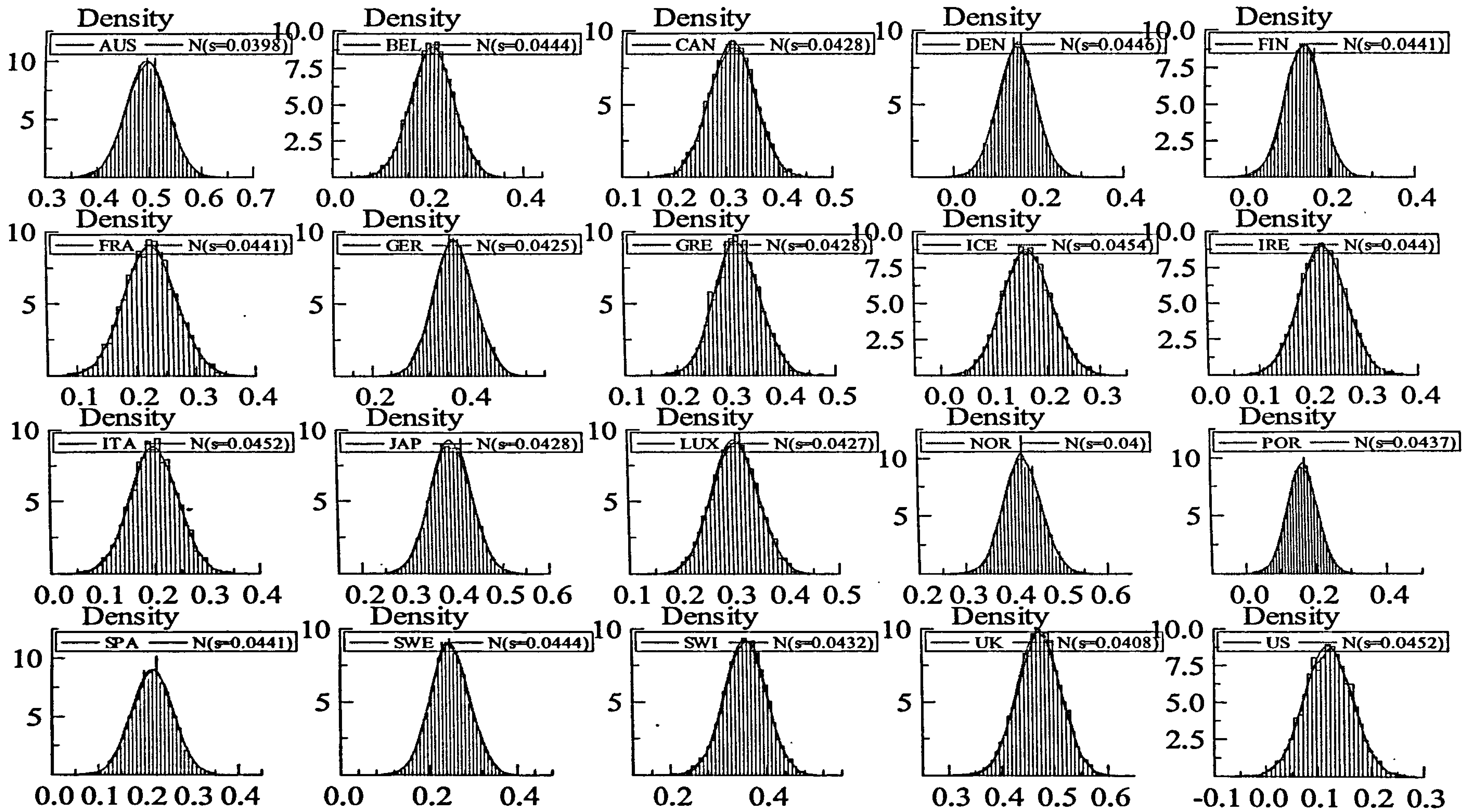


Figure 4.06 Histograms of Block Bootstrap Replications of Group-Mean AR(12) θ_1

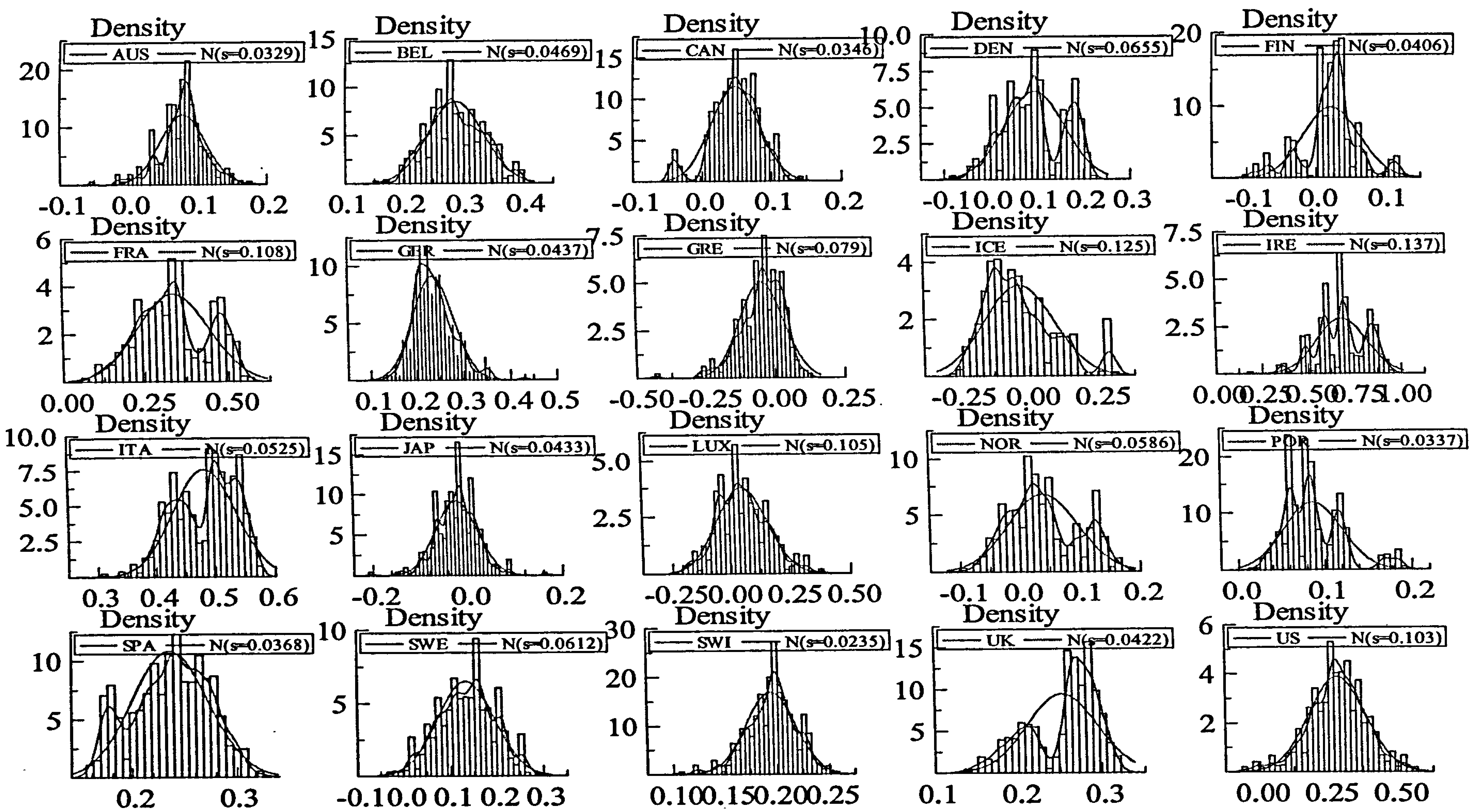
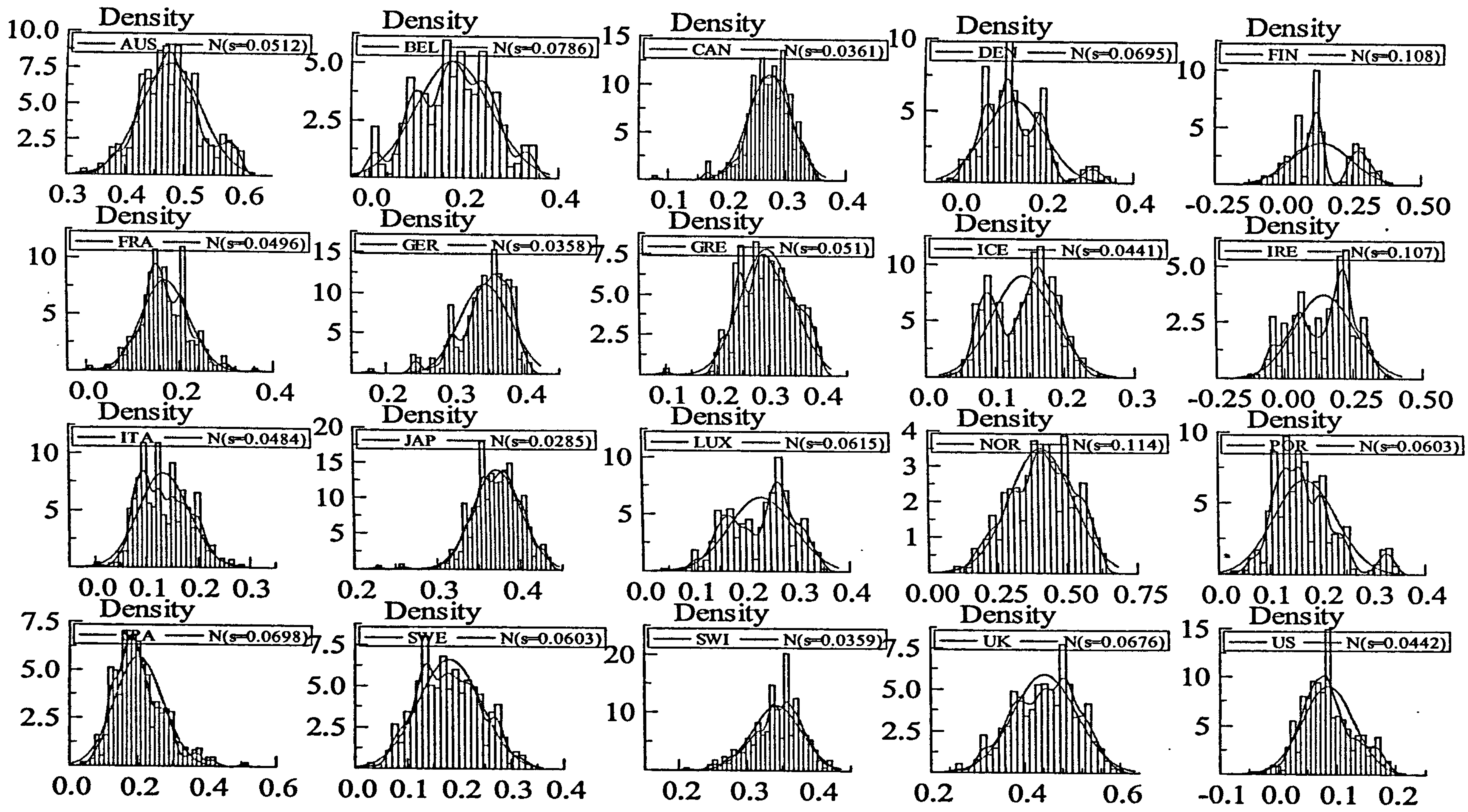


Figure 4.07 Histograms of Block Bootstrap Replications of Group-Mean AR(12) θ_{12}



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