

The University of Hull

**Observer-Based Robust Fault  
Estimation For  
Fault-Tolerant Control**

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by

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# Abstract

A control system is fault-tolerant if it possesses the capability of optimizing the system stability and admissible performance subject to bounded faults, complexity and modeling uncertainty. Based on this definition this thesis is concerned with the theoretical developments of the combination of robust fault estimation (FE) and robust active fault tolerant control (AFTC) for systems with both faults and uncertainties.

This thesis develops robust strategies for AFTC involving a joint problem of on-line robust FE and robust adaptive control. The disturbances and modeling uncertainty affect the FE and FTC performance. Hence, the proposed robust observer-based fault estimator schemes are combined with several control methods to achieve the desired system performance and robust active fault tolerance. The controller approaches involve concepts of output feedback control, adaptive control, robust observer-based state feedback control. A new robust FE method has been developed initially to take into account the joint effect of both fault and disturbance signals, thereby rejecting the disturbances and enhancing the accuracy of the fault estimation. This is then extended to encompass the robustness with respect to modeling uncertainty.

As an extension to the robust FE and FTC scheme a further development is made for direct application to smooth non-linear systems via the use of linear parameter-varying systems (LPV) modeling.

The main contributions of the research are thus:

- The development of a robust observer-based FE method and integration design for the FE and AFTC systems with the bounded time derivative fault magnitudes, providing the solution based on linear matrix inequality (LMI) methodology. A stability proof for the integrated design of the robust FE within the FTC system.
- An improvement is given to the proposed robust observer-based FE method and integrated design for FE and AFTC systems under the existence of different disturbance structures.
- New guidance for the choice of learning rate of the robust FE algorithm.
- Some improvement compared with the recent literature by considering the FTC problem in a more general way, for example by using LPV modeling.

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# Chapter 1.

## Introduction

### 1.1 Introduction

Since the milestone Watt used the steam pressure of a steam engine to regulate the engine speed via fly-ball governor (1760), and with the effort of many other pioneers, the theory and practice of control system design advanced rapidly. In 1868 James Clerk Maxwell analysed the non-linear dynamics of the fly-ball governor and the control began to encompass complex dynamical system (Smithers, 1994). Important new concepts and tools were developed in connection with telephone and radio communications in the 1920s and 1930s, and further accelerated the development in World War II. Later, automation became a household word as industry began to depend more and more upon automatically controlled machinery (Bennett, 1996).

Today, control systems are everywhere in our lives, constantly making our lives more comfortable and more pleasant until the system ‘loses its life’ and failure occurs. Control systems are in our kitchens, in our DVD-players and computers. They are driving the elevators, motors and we have them in our cars, ships, aircraft and spacecraft. Control systems are present in every modern industry, used to control robots, nuclear power plants, solar energy generators, chemical reactors and so on. (Kanev, 2010).

As modern technological systems increase in complexity, the corresponding control systems become more and more sophisticated. A conventional feedback control design for a complex system may result in an unsatisfactory performance, or even instability (Patton, 1997). The requirement for high standard control performance and highly unstable systems which optimize the cost and the control effort challenge the development of control system methodologies. Hence, the subject of control has evolved from simple mechanical feedback structures into sophisticated and advanced electronic devices. During the last 60 years, it has seen the emergence of multivariable

and robust control ideas (Skogestad and Postlethwaite, 1996; Morari and Zafiriou, 1989; Stefani, et.al, 2002, Tan, Chen and Marquez, 2002) to increase the practical performance capabilities and at the same time ensure stability in the face of modelling uncertainty and robustness to noise and disturbances. Many control methods have attracted many researchers in the last 20 years such as predictive control (Pachter, Chandler and Mears, 1995, Monaco *et al.* 1997; Huzmezan and Maciejowski, 1998; Kale and Chipperfield, 2005), robust control (Morari and Zafiriou, 1989; Zhou, Doyle and Glover, 1995) and adaptive control (Isermann, Lachmann and Matko, 1992; Åström and Wittenmark, 1989).

Although there are fruitful results in theory, very few of them have been well applied to industry problems except in the field of model-based predictive control.

Some unexpected scenarios or unusual system events mean that the performance and even the stability of the designed closed-loop system can be degraded. These unexpected scenarios may be faults, failures or system damage, which are usually not considered in the controller design process. The need to account for faults in a closed-loop system has been the main motivation for this research, providing some new concepts in *fault-tolerant control* (FTC) of uncertain systems.

## 1.2 What is the need for Fault tolerant control?

Faults in control systems are events that occur abnormally, often as unexpected scenarios at unexpected times. Isermann and Ballé (1997) make the definition for a fault is:

<p>A <b>Fault</b> is an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition</p>
---

Faults are almost impossible to predict and prevent. As control system faults develop and become more severe they may lead to total system “*failure*” depending on the precise conditions, and the criticality of faults, and if appropriate and prompt action is not taken [Definitions established by the Technical Committee for IFAC (International

Federation of Automatic Control) Symposium SAFEPROCESS (Fault Detection Supervision and Safety for Technical Processes), Isermann and Ballé, 1997].

On the other hand, a ‘*Failure*’ describes the condition when the system is no longer performing the required function. It is then necessary to explain the difference between the terms “*Faults*” and “*Failure*”. The term failure suggests complete breakdown of a system component or function, whilst the term fault can be used to indicate that a malfunction may be tolerable at its present stage. A fault should be diagnosed as early as possible even if it is tolerable at its early stage, to prevent any serious consequences (Chen and Patton, 1999). A failure (i.e. a system function involving the faulty components may lead to a failure), ranges from failure of simple components (actuators or sensors) that can be replaced by redundancy to very significant dramatic incidents as a result of failures. Some samples are:

1. The high-speed train crash in WenZhou, China, on 23rd July 2011 (see Figure 1-1).



Figure 1-1: Train crash in Wen Zhou China on 23rd July 2011.

40 people were killed, at least 192 people were injured, 12 of whom with severe injuries. Because of a design flaw, the railway signaling systems indicated the wrong signals guiding the train D301 to run at high speed onto a track occupied by another train D3115 which resulted in the collision with the second-deadliest high-speed rail accident in history following the 1998 Eschede train disaster in Germany. (Wikipedia, 2012).

2. ELAL Flight 1862 Bijlmermeer Incident, on 4 October 1992, a Boeing 747, ELAL Flight 1862, cargo plane of the Israeli airline ELAL, crashed into an apartment building in Bijlmermeer, Amsterdam, Netherlands. The damage to the right wing, resulting in reduced lift with leading to the following crash, which

caused 43 people were killed, plus 39 persons on the ground. Many more were injured, see Figure 1-2. (Wikipedia, 2012).



Figure 1-2: ELAL flight 1862 crash accident caused by mechanical failure

3. 271 persons on board and 2 on the ground were killed in the crash of the American Airlines flight 191, a McDonnell-Douglas DC-10 aircraft, at Chicago O'Hare international Airport on 25 May 1979, which because of the engine on the left wing was separated and flipped over the top of the wing, see Figure 1-3. (Patton, 1997; Kanev,2010)



Figure 1-3: Crash accident of American Airlines flight 191 in 1979

4. A catastrophic nuclear accident occurred on 26 April 1986 at the Chernobyl Nuclear Power Plant in Ukraine. The deficiency of the reactor design and the operator's abnormal operation result in the reactor explosion and fire released large quantities of radioactive contamination into the atmosphere, this disaster ultimately involved over 500,000 people exposed to radiation and cost an estimated 18 billion rubles. (Wikipedia, 2012).

An interesting question arises naturally, "Could these disasters be prevented if something had been done in time?" The answer is hard to give, as it depends on many other resistible and irresistible factors but not only control signals (e.g. motor burned, engine failed). However, in most situations the occurrences of faults in the systems

cannot be prevented, subsequent analysis often reveals that the consequences of faults could be avoided or, at least, their seriousness could be minimized. If faults can be detected and diagnosed in a timely way then it may be possible to reconfigure the control system to ensure safe operation until the time when the system can be shut down for maintenance. In order to minimize the chances of disaster, safety-critical systems must possess the properties of increased reliability, safety and with all this also fault-tolerance.

A way to achieve these attributes is FTC system design. From the investigations and the research results for the respective accidents listed above, all could have been avoided. The Chinese State Administration of Work Safety investigation group showed that the accident could have been totally prevented (Luo, 2011). An FTC system could have been designed to lead to a safe end to the Chernobyl reactor before it exploded (Mahmoud *et al.*, 2003). Maciejowski and Jones (2003) demonstrated in simulation, using a model-based predictive control approach that the ELAL 1862 disaster could have been avoided and it may have been possible to control the crippled aircraft using a form of FTC in the flight control system to maintain the required controllability for the purpose of a quick landing back at Schiphol airport, Amsterdam.

In another accident case involving DELTA flight 1080 a fire destroyed some of the control rod/pulley mechanisms of the actuators on one wing. The pilot was able to reconfigure the remaining lateral control sections and successfully land the aircraft safely. The elevator jammed at 19 degrees up and the pilot had been given an alarm that this fault had occurred (Patton, 1997). However, it is now apparent that the pilot's action "reconfiguring" action could have been automated by a suitable FTC scheme.

All these cases have shown clearly the demand and importance of increasing the fault tolerance of a system in order to improve the extend to which the safety and reliability are available of modern and complex controlled systems.

### **1.3 Fault classification**

As defined in Section 1.2, a fault is an event that may occur in different parts of the controlled system. In general, according to different locations of fault occurrence within a control system, faults are classified as: (1) actuator faults, (2) sensors faults, (3) components faults (see Figure 1-4). These are defined in more detail as follows:

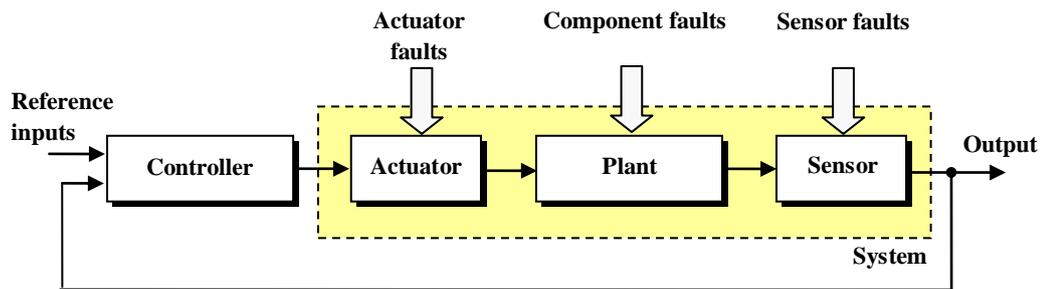


Figure 1-4: Types of faults in a control system

**Actuator faults** occur in equipment such as motors, valves, solenoids, relays, pneumatic device. An actuator fault is normally represented in the literature as a partial or total loss of the actuator’s control action effectiveness. It may be the result of a jam, become ‘stuck’, due to damage to bearings or gears, caused by changes from the design characteristics or complete failure. An example of a complete loss of an actuator is an actuator that produces no action in spite of the control input applied to it.

**Component faults** exist in the components of the plant itself, i.e. all faults that cannot be categorized as sensor or actuator faults will normally be considered as component faults. These faults represent changes in the physical parameters of the system, e.g. mass change, aerodynamic coefficients, damping constant, mismatching model, etc., that are often due to structural damages. They change the dynamical I/O characteristics of the system.

**Sensor faults** represent incorrect readings from the system measurement sensors. Sensor faults are always due to poor calibration or bias, scaling errors or a change in the sensors dynamic characteristics which cause errors on the sensors outputs, but not on the plant dynamics. Sensor faults can also be further divided into partial or total sensor faults, where a total sensor fault is a sensor failure.

From a modelling point of view, the faults are classified as *additive* or *multiplicative* faults (see Figure 1-5). An additive fault is considered as an additional external signal, i.e. unknown input, whilst a multiplicative fault is considered as a parameter deviation.



Figure 1-5: Additive fault and Multiplicative fault

On the other hand, faults are also classified on the basis of their time characteristics (see Figure 1-6) as abrupt, incipient and intermittent. Hardware damage is the normal reason for occurrence of the abrupt faults. Abrupt faults refer to changes that occur at time scales much faster than the nominal dynamics of the system. They have very important roles through their effect on the performance and stability of the controlled system. As a result, they have to be detected before the system becomes safety-critical, i.e. before their effect on the system leads to a crash. Incipient faults are small and slowly developing faults, sometimes called soft faults. Compared with abrupt faults, incipient faults are much more difficult to detect, because they have slow time characteristics, but their further development may cause very serious consequences. Finally, intermittent faults are faults that appear and disappear repeatedly.

Here, all the types of faults with their characteristics and the motivation for FTC are illustrated in Figure 1-6. In Section 1.4 some basic fault principles leading to the FTC concept are described.

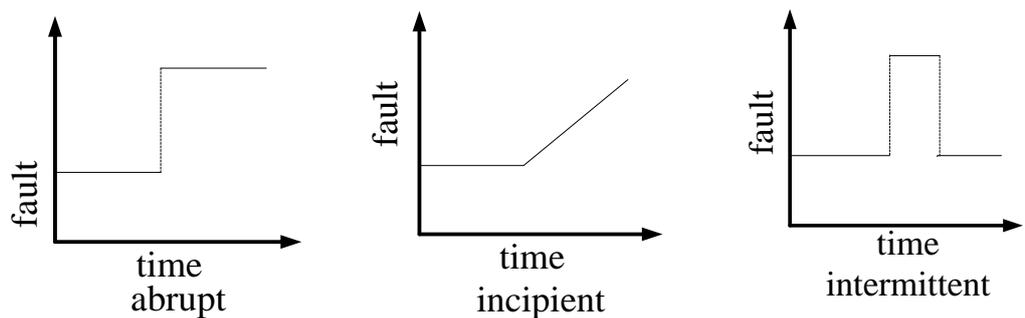


Figure 1-6: Abrupt faults, incipient faults and intermittent faults

## 1.4 Fault tolerant control architecture

Sections 1.4.1 and 1.4.2 provide a simple description to illustrate the FTC concept, including the architecture and some terminology for fault diagnosis, which is just a precursor for fault estimation (FE). More details will be given in Chapter 2.

### 1.4.1 Fault tolerant control architecture

Normally, the architecture of FTC comprises two parts which are “diagnosis” and “controller redesign” (see Figure 1-7). These two parts (or blocks) act together to carry out the FTC function.

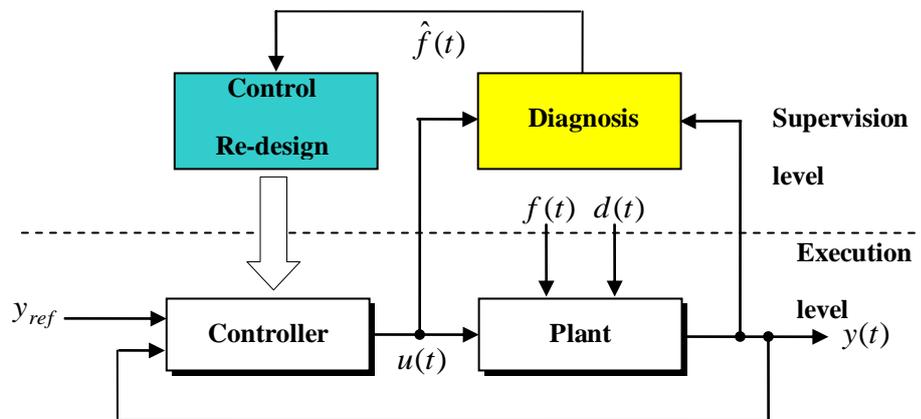


Figure 1-7: The architecture of FTC (Blanke *et al.*, 2003)

1. *The diagnosis block* uses the measured inputs and outputs and tests their consistency with the plant model. Its result is a characterisation of the fault with sufficient accuracy for the controller re-designs.
2. *The re-design block* uses the fault information and adjusts the controller to the faulty situation.

Figure 1-7 illustrates that FTC extends the usual feedback controller by a *supervisor*, which includes the diagnostic function and the controller re-design blocks. In the fault-free case (no fault happens), the system works as before, on the *execution level*. (Note: it is more suitable for active fault tolerant control, see Section 2.2). The nominal controller (sometimes referred to as the “baseline” controller or nominal controller, see Patton, 1997), which is designed for the healthy system, attenuates the disturbance  $d(t)$  and guarantees the closed-loop system’s good reference following quality and other requirements. In this situation, the diagnostic block recognizes that the closed-loop

system is faultless or healthy (fault-free) and there is no necessity to redesign the control law.

When a fault  $f(t)$  occurs, the *supervision level* makes the control loop fault-tolerant. The diagnostic block identifies the fault and “orders” the controller re-design block to change the control law to satisfy the stability and performance requirement for the faulty system. However, it can also achieve fault tolerance without using the structure given in Figure 1-7 via other established control methods, which will be introduced in Section 2.2 in the following. (Blanke *et al.*, 2003)

### 1.4.2 Fault diagnosis

A supervision level which is used to detect faults and diagnose their location and significance in a system is called “fault diagnosis system”. Such a system normally consists of the following tasks:

**Fault detection:** To make a binary decision – either that something has gone wrong or that everything is fine.

**Fault isolation:** To determine the location of the fault, e.g. sensor or actuator has become faulty.

**Fault identification:** To estimate the magnitude and type or nature of the fault.

The relative importance of these three tasks is clearly subjective. However the fault detection is absolutely necessary for any practical system and fault isolation is of the same importance. Fault identification, on the other hand, whilst undoubtedly helpful, may not be essential if no controller designing action is involved. As a result, in most literature, fault diagnosis is very often considered as fault detection and isolation or FDI (Chen and Patton, 1999). However, in AFTC, the fault feature is one of the most critical items of information of the process of controller redesign, therefore fault identification or FE plays a very important role in AFTC process, the FD block needs to provide not only the fault alarm and location of the faults, but also the feature of the faults. To summarise, the significant difference between FDI and FD is whether the function of fault identification or FE is included or not. In this thesis, FDI is considered to only consist of the tasks of fault detection and fault isolation.

## 1.5 The Robust Fault Estimation Approach to FTC

This research focuses on developing methods for estimating the fault rather than to detect the presence of a fault via the use of residual signal under the influence of uncertain in systems. As summarized above, FE plays as important a role as fault detection and isolation (FDI) in FTC, which provides important information related to the faults for the controller re-design process, compared with FDI, FE is a direct way to provide fault information such as the magnitude and the severity of the fault, from this point of view, the FE is the same as Fault identification. Furthermore, FE can also be used to isolate the fault in the same system by comparison with other FE signals, i.e. a multiple observer-based FE approach like multiple observer based FDI approach (Berec, 1998; Menke and Maybeck, 1998).

This thesis is concerned with the active approach to FTC and in particular the use of FE embedded within an adaptive control problem. In this approach the fault isolation decision process is obviated, as the accommodation to the fault(s) is automatic within the adaptive scheme. Hence, in this work the residual generation problem of fault detection is replaced by one of FE which is called “fault estimator” and it is observer-based (see Figure 1-8). A residual is a fault indicator or an accentuating signal which reflects the faulty situation of the monitored system, in this research the residual means the different situation between the faulty system and the observer. Chen and Patton (1999) pointed out that an ideal residual signal for FDI, even in an uncertain system application, can be defined as a robust estimator of the fault to be detected. If this ideal residual generator remains insensitive to uncertainty and modelling errors it can be further defined to be robust.

However, for FTC, the role of FE is as important as that of FDI, the requirement has been increased to not only know if the faults occur or not but also for the specific property of the faults, meanwhile the robustness is a challenge applied to the FE problem as well.

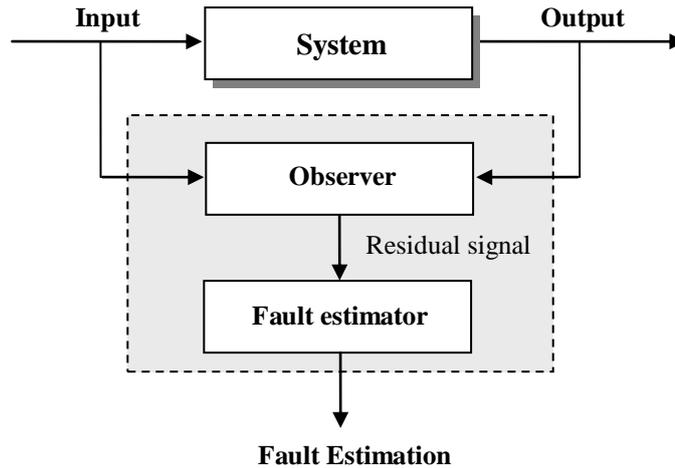


Figure 1-8: Observer-based fault estimator in controlled system (modified from Chen and Patton, 1999)

As a matter of fact, only a mathematical nominal model of the system is usually available, not necessarily considering the presence of exogenous disturbance inputs and noise, and also time-varying parameters and non-modeled plant dynamics (Nobrega, Abdalla and Grigoriadis, 2008). The robustness problem for FDI was first defined by Patton, Frank and Clark (1989) and studied in more detail by Chen and Patton (1999). The robustness of FE can be stated as the degree to which the sensitivity of the fault estimates (to the real faults) can remain invariant (relatively constant) in the presence of model-reality differences (e.g. parameter variations, disturbances, and noise).

The robustness challenge is one of understanding how to reconstruct the fault information precisely in the presence of uncertainty and how to build a robust FTC scheme that is based on the robust fault estimates to satisfy the overall system performance and stability requirements. This is a multi and joint robustness problem which is actually a multi-objective challenge for FTC and as such is probably the most difficult challenge in modern and advanced control science.

As summarized above the main challenges to be faced for robust FE for FTC systems are:

- (1) The difficulty in achieving accurate or robust FE in the presence of other exogenous disturbance inputs and noise, as well as time-varying parameters and non-modeled plant dynamics.

- (2) The stability of the FTC system is difficult to maintain after the controller redesign based on the use of the FE. The occurrence of a fault can make the system deviate far from its nominal operation and can lead to a severe change in system behaviour. Even bounded faults can cause the closed-loop system to deviate rapidly from its required operation and hence the fault accommodation time is a critical parameter. The requirement for rapid reaction to faults can mean that the FDI or fault diagnosis (FD) procedure, if used, may slow down the accommodation process. The accommodation ability of a control system depends on several factors, for instance, the magnitude of the fault, the robustness of the system, etc. Therefore, to overcome such problems, new controllers must be developed with accommodation capabilities and tolerance to faults.
- (3) The need to consider robustness in the FTC design. The FTC design challenge is not only to maintain the stability of the closed loop system, but also to achieve robustness in terms of a suitable performance requirement.
- (4) The integration of the whole FTC system involving the observer, fault estimator, and fault compensation mechanism is a huge and complex problem.

## 1.6 Thesis Structure and Contributions

The thesis is arranged in the following manner:

**Chapter 2** is concerned with a review of the main literature of the combined fields of FDI and FTC field. The Chapter introduces concepts of FTC and FDI from the beginning and following that, some general classifications on the different FTC and FDI strategies are presented. The main concepts and strategies behind some of the FTC and FDI schemes in the literature, as well as their advantages and disadvantages are also discussed. Then, a description of the residual generator structure in model-based FDI is presented and an example mathematical model of a general faulty system is also given. Attention then turns to an emphasis on the use of robust FDI methods that can be achieved using disturbance-decoupling techniques via the unknown input observer (UIO). Finally, some review of mature methods using model-based FE is given which can be seen as an extension of model-based FDI.

**Chapter 3** introduces a new approach to fault compensation for FTC using Model Reference Control (MRC) combined with Fast Adaptive FE (FAFE) (Zhang, Jiang and Cocquempot). The proposed FTC scheme is composed of two parts; (1) FAFE produces the FE and (2) model reference control design. The observer gain is calculated by using Linear Matrix Inequality (LMI) approach, based on knowledge of the fault bounds. The contribution of this Chapter is to use MRC to design a fault estimator based only on the reference model and not on the plant dynamics, which is helpful when dealing with some nonlinear systems or systems that are difficult to linearise, e.g. for robot manipulators or for aerospace systems. This leads to the use of a simpler parameterization of the fault estimator LMI computation compared with the estimator approach developed by Zhang, Jiang and Cocquempot (2008). The Chapter also illustrates the benefits of this new method when applied in FTC. The basic design process and the technical analysis are undertaken based on a Two-Link Manipulator example to introduce the main features of the design.

**Chapter 4** presents a new robust FE scheme and its application for FTC. A switching function which is a nonlinear part in the sliding mode observer design (Edwards, Spurgeon and Patton, 2000) of sliding mode theory is imported into the observer-based fault estimator design to improve the accuracy of the estimation for additive actuator and sensor faults. The robustness improvement is shown via the switching function term which rejects the effect from the unmatched uncertainties (Edwards and Spurgeon, 1998) or exogenous disturbances or noise. The main contribution of this Chapter is that by using the proposed estimation scheme one can obtain the state estimation and FE at the same time. Beside these, this novel observer can also be used in an FDI problem. The FTC controller design uses a control law based upon state estimate feedback control designed using  $H_\infty$  optimization. After pursuing the existence conditions for the observer-based fault estimator, the convergence of the observer and the stability of the system are proved by using individual Lyapunov functions. The proof for satisfying the stability and the performance index of the whole system is also discussed. Finally, a simple nonlinear inverted pendulum with additive actuator and sensor fault examples is used to illustrate the whole design process, respectively.

**Chapter 5** presents a novel adaptive robust FE method for actuator failure, two different situations are considered --- “the stuck” situation and the “loss of effectiveness” situation and adaptive controller design for their accommodation. Both matched and unmatched uncertainties are considered either in a fault supervising scenario or in the

case of controller redesign. Matched uncertainty is difficult to deal with when doing the robust FE, but the concept is more easily handled in FTC, whilst unmatched uncertainty is difficult to deal with when doing the FTC, but is more straightforward in FE. The main contributions are listed as: (1) the development of the method -- robust adaptive FE; (2) improvement of the work (Jin and Yang, 2009) by using outputs and state estimate feedback to design the control law instead of state feedback control. The gains are calculated by an LMI method using the MatLab LMI tool box. Finally, the proposed method is tested via two simple mechanical examples (1) linear rocket fairing structural-acoustic model and (2) nonlinear single link manipulator system.

**Chapter 6** addresses the robust FE problem of linear parameter-varying (LPV) systems where the state space equation depends on the time-varying system parameters as an alternative to robust residual generation for FD as discussed in Chapter 2. On the other hand, it can be deemed as an extension of the method developed in Chapter 4 for nonlinear systems. Faulty system performance is affected by an additive actuator fault, however, because of the use of a LPV system framework, the additive actuator fault is transformed into a multiplicative fault, and this is a big challenge for the proposed method. The problem is solved by using some subtle mathematical transformation as used in Chapter 3. The fault gains of the estimator, observer and controller are characterized via a set of LMIs with the robustness property to exogenous disturbance. The integration design process is similar to the designs described in Chapters 4 and 5. Finally, to demonstrate the proposed method, an illustrative example of a two-link manipulator is provided and the polytopic LPV model of this system is also presented. The main contributions of this Chapter are (1) There is an extension of the method developed in Chapter 4 for nonlinear systems, (2) The work gives a guidance for the choice of the learning rate of the fault estimator, and (3) compared with the previous work (Patton, Chen and Klinkhieo, 2012), an improved way for considering the system robustness when designing the FTC scheme is considered.

**Chapter 7** summarises and provides a general overall conclusion for the research described in the thesis. Suggestions and recommendations as to how the research can be further developed through future projects are also presented.

# **Chapter 2.**

## **Overview on Fault tolerant control, FDI and FD**

### **2.1 Introduction**

This Chapter provides a literature review of the main research topics and published work on FTC, FDI and FD. Section 2.2 provides a general overview of the subject of FTC, including the concept of FTC and some general classifications on different FTC strategies. Section 2.3 discusses the topics of FDI and FD in some detail, and reviews some previous research in this field, which mainly focus on the Model-Based Approach. In Section 2.4, a specific literature review is given concerned with the main concepts related to the use of FE for different strategies of AFTC. Section 2.4 also discusses a potential correspondence between residual generation and the FE concept and their combined structure, stating the importance of these issues for FTC theory.

### **2.2 Fault tolerant control**

#### **2.2.1 Fault tolerant control general overview**

When considering the increased safety and performance requirements of modern control systems, it is now widely understood that conventional (classical) feedback control is generally inadequate. New controllers are always being developed which are capable of tolerating component malfunctions whilst still maintaining desirable and robust performance and stability properties. A control system that possesses such a capability is often called as an FTC system (Patton, 1997; Blanke *et al.*, 2003).

In the literature, most application areas where FTC has been used are some safety critical systems such as aircraft, spacecraft, chemical processing plants, and nuclear power plants. This is traditionally an important field of research and application for aircraft flight control system design (Steinberg, 2005; Alwi, 2008; Edwards, Tombaerts and Hafid, 2010).

Patton (1997) stated in his survey that, ‘. . . *Research into fault tolerant control is largely motivated by the control problems encountered in aircraft system design. The goal is to provide a self-repairing capability to enable the pilots to land the aircraft safely in the event of serious fault . . .*’.

Even today it is still true that FTC comprises *four* major research areas (Patton, 1997), which are FDI, Robust Control, Reconfigurable Control, and Supervision (see Figure 2-1). The function of FDI is to detect if any fault occurs and determine the location of the fault. Information is then passed to the reconfiguration block for controller redesign to adapt to the fault, therefore recovering a suitable level of stability and acceptable performance.

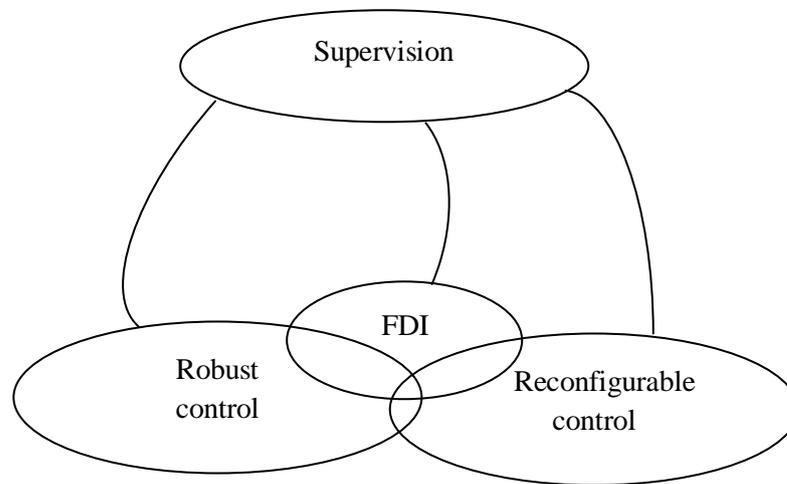


Figure 2-1: The scattered areas of fault- tolerant control research (Patton, 1997)

In the context of FTC, robust control relates to a fixed controller designed to tolerate changes of the plant dynamics. The ideal FTC system satisfies its goals under all faulty conditions. Fault tolerance is achieved without changing the controller parameters (Blanke, 2003). It is important to note that even without the FDI function the parameter-fixed controller is still able to provide a limited capability for overcoming the effect of a fault. This is the subject of Passive fault tolerant control (PFTC) described further in Section 2.2.2.

## 2.2.2 Classification of Fault tolerant control

In the literature (Patton, 1997; Zhang and Jiang, 2008), FTC systems are classified into *two* major groups: Passive fault tolerant control (PFTC) and Active Fault Tolerant Control (AFTC) (see Figure 2-2), as follows:

**PFTC:** A closed-loop system can have limited fault-tolerance by means of a carefully chosen feedback design, taking care of the effects of both faults and system uncertainties. Such a system is sometimes called a PFTC system (Patton, 1997). In other words, it means that when a fault occurs, the control law does not need to be adjusted to adapt to the fault, but still maintains stability and some acceptable level of degradation of performance. During the last two decades, the PFTC is more connected with reliable control, for which various approaches have been proposed (Keating *et al.*, 1995; Tyler and Morari, 1994; Veillette, 1995; Niemann *et al.*, 1997; Stoustrup *et al.*, 1997; Zhou and Ren, 2001; Chen and Patton, 2001; Niemann and Stoustrup, 2002, 2005; Niemann, 2005; Wang, Tan and Zhang, 2010). However, the main disadvantage of this approach is the very limited fault-tolerance, because of the use of a “non-intelligent” controller. It is hard to reject all the faulty situations, since neither diagnostic information is used nor the knowledge of fault occurrence (where and how serious the fault is) (Patton, 1997).

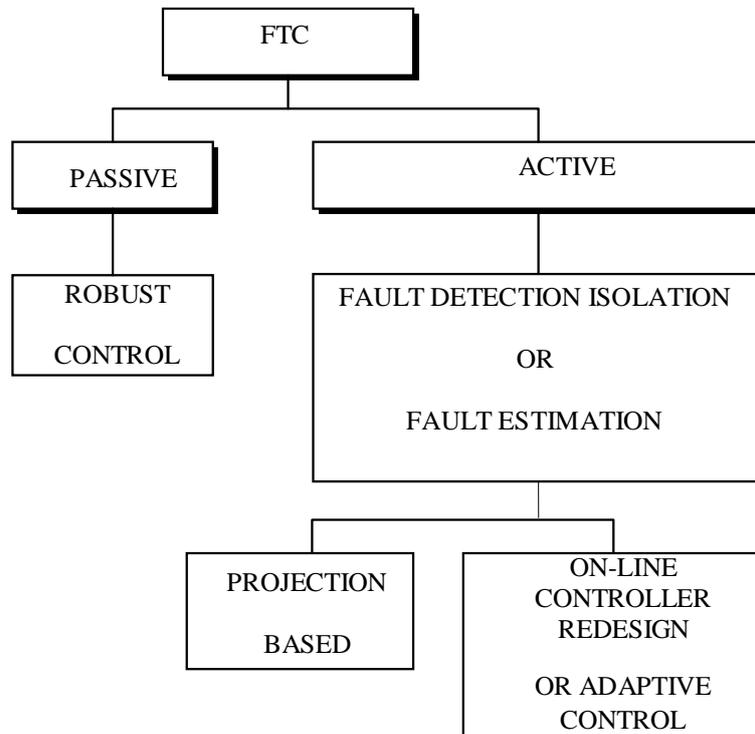


Figure 2-2: Fault-tolerant control methods (Patton, 1997)

**AFTC:** Active fault tolerance has this title because on-line fault accommodation is used. AFTC systems actively deal with faults or failures of system components by reconfiguring control actions in order to maintain the stability and acceptable performance of the entire system, subject to faults or failures. However, in such circumstances, degraded performance must be accepted (Patton, 1997; Blanke *et al.*,

2001; Zhang and Jiang, 2008). Active approaches are further divided into *two* main types of methods based on the way the post-fault controller is formed: (a) projection-based methods and (b) on-line automatic controller redesign methods. The projection based approach includes the use of a new control law that is selected from a set of off-line predesigned controllers. Normally, each controller from the set is designed for a particular type of malfunction and is switched on whenever the corresponding fault has been detected and isolated by FDI scheme. The on-line redesign methods involve on-line re-computation of the controller gain, which is often referred to as *reconfigurable control* (Patton, 1997), or by a recalculation of both the structure and the parameters of the controller, called *restructurable control* (Patton, 1997). As the projection based method can only deal with a restricted finite class of fault problems the on-line redesign method is more advanced and widely used in the FTC field. However, it is computationally the most expensive method as it often boils down to on-line optimization (Kanev, 2006).

In contrast to studies on PFTC, more and more researchers have been attracted to focus on the development of AFTC due to their improved performance and their ability to deal with a wider class of faults. During the last two decades, there have been fruitful results in this field. Most of these are considered with the following categories: Pseudo-inverse modelling (Gao and Antsaklis, 1991; Staroswiecki, 2005); Adaptive control (Tao, Chen and Joshi, 2002; Jiang, Staroswiecki and Cocquempot, 2006; Zhang and Chen, 2008; Jin and Yang, 2009; Zhang and Qin, 2009); Model predictive control (Maciejowski and Jones, 2003); Model following control (Gao and Antsaklis, 1992; Tao, Ma and Joshi, 2001; Zhang and Jiang, 2002; Mirkin and Gutman, 2005); Multiple-model methods (Zhang and Jiang, 2002; Yen and Ho, 2003; Theilliol, Sauter, and Ponsart, 2003); Eigenstructure assignment (Jiang, 1994; Zhang and Jiang, 2001; Zhang, 2000, Wang, Liang and Duan, 2005); Sliding mode and variable structure control (Alwi and Edwards, 2008, Hess and Wells, 2003; Shin, Moon and Kim, 2005; Demirci and Kerestecioğlu, 2005); Linear parameter varying (LPV) methods (Weng, Patton and Cui, 2007, 2008; Patton, Chen and Klinkhieo, 2012;). Some discussion on different AFTC strategies is presented in the following.

### **FTC via pseudo-inverse modelling:**

The pseudo-inverse method (PIM) (Gao and Antsaklis, 1991; Staroswiecki, 2005) is one of the most famous active methods to AFTC, due to its simple computation requirement

and ability to deal with a large range of system faults. Consider a nominal linear state-space system:

$$\left. \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \right\} \quad (2-1)$$

where  $x \in \mathfrak{R}^n, u \in \mathfrak{R}^m, y \in \mathfrak{R}^p$ , and  $A, B$  and  $C$  are appropriately dimensioned matrices. The state-feedback control law  $u = Kx$  is designed under the assumption that the entire state vector is measurable. A very general faulty system model can then be constructed as:

$$\left. \begin{aligned} \dot{x}_f &= A_f x_f + B_f u \\ y_f &= C_f x_f \end{aligned} \right\} \quad (2-2)$$

where  $A_f$  is the faulty system matrix,  $B_f$  and  $C_f$  are faulty input weighting matrix and faulty output matrix, respectively. The problem is to determine the new feedback control law  $u = K_f x_f$  by computing the new state-feedback gain matrix  $K_f$  so that the closed-loop matrix in (2-2) is “closest” to the one in (2-1), according to (Gao and Antsaklis, 1991),  $J = \|(A + BK) - (A_f + B_f K_f)\|_F$  is minimised, when  $K_f = B_f^{++}(A + BK - A_f)$ , where  $\|\cdot\|_F$  stands for the Frobenius norm and  $B_f^{++}$  represents the pseudo-inverse of the matrix  $B_f$ . The advantages of this method is that it is very simple both in design and in computation which make it is easy for on-line application. Furthermore, it is able to handle those faults that affect the changes in all state-space matrices of the system. The disadvantage is that, the stability of the closed-loop system cannot be guaranteed by the optimal gain computation. Meanwhile, this method is based on state-feedback control, which means it asks for all the knowledge of the state variables. However, in general, it is difficult to measure all the state variables as well as the existence of the sensor faults. More recently, Staroswiecki has considered this approach to FTC as a “model-matching” problem, providing some interesting insight into new developments.

### **FTC via adaptive control:**

The motivation for using adaptive control was invented during the 1950s for high performance aircraft. Adaptive control is used in order to automatically adjust the controller gains in real time to force the plant to follow the trajectories of a desired performance (Patton, 1997). Due to their ability to adapt to changes in system

parameters, these methods can be referred to as “self-reconfigurable”. For example, the “FD” block is not essential in the FTC design. More specifically, adaptive control is categorized into two sub problems, which are *direct adaptation* and *indirect adaptation* (Åström and Wittenmark, 1989). For direct adaptive control approach, the controller is designed directly without estimating the system parameters. Whilst for the indirect adaptive control method, there are two steps in designing the controller. Firstly, by estimating the system parameters, i.e. the system matrix pair  $(A, B)$  needs to be estimated according to the changes caused by faults. Secondly, using the estimated information to design the controller (Alwi, 2008). The adaptive control approach can be typically combined with other control methods to achieve fault tolerance e.g. by using Model Reference Adaptive Control (Morse, 1990; Nie and Patton, 2011), Multiple-Model Adaptive Control (Rauch, 1995).

### FTC via multiple model switching:

The multiple-model switching (MMS) approach is another active approach to achieve fault tolerance, which belongs to the projection based-methods category (see Figure 2-3).

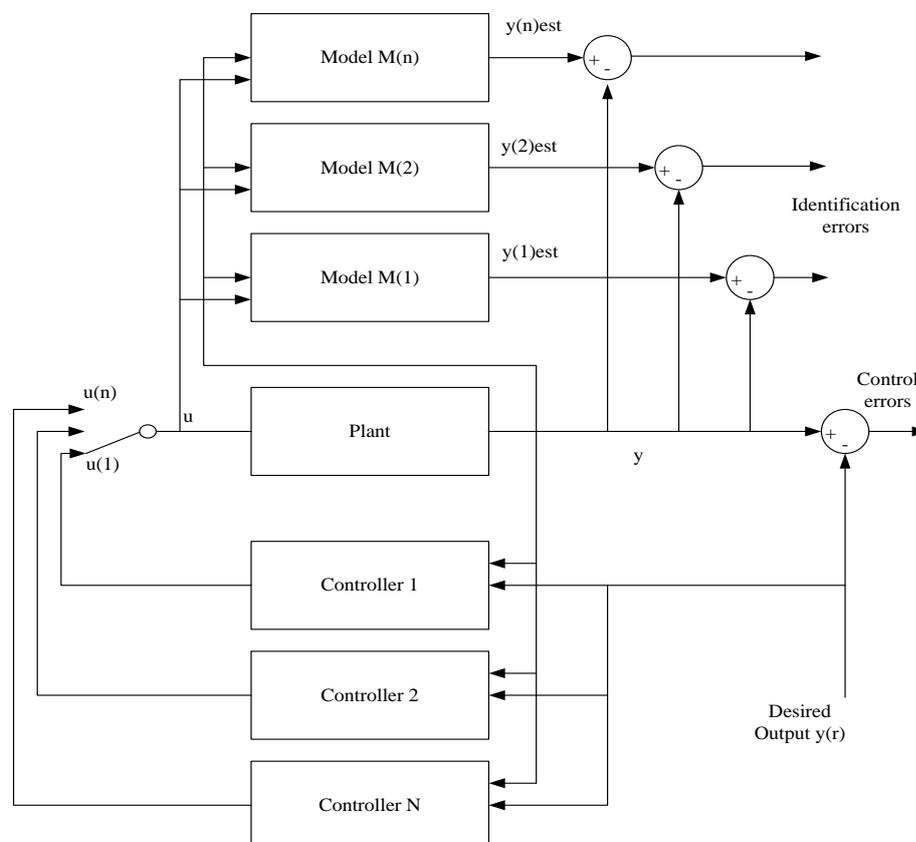


Figure 2-3: Architecture of Multiple-model switching method FTC method (adopted from Narendra and Balakrishnan, 1997)

Multiple-model schemes are based on a finite set of linear models that describe the system in different faulty conditions. For each model a controller is designed off-line. From an FTC point of view, during the process, the predesigned controllers are considered as a backup and only “wake up” when faults occur. Clearly, the core part for the MMS approach is the FDI block, which FDI must provide the fault information in a suitably correct format (i.e. type and location) to make sure that the appropriate controller is activated. Meanwhile, the predesigned controllers have to be able to handle all the possible faulty models. Consequently, the MM approach calls for FDI with the property of high robustness. Many robust FDI methods can be found in (Saif and Guan, 1993; Patton and Chen, 1998; Chen and Patton, 1999; Xiong and Saif, 2000). The disadvantage of this method is that it can only handle a limited set of anticipated faults. The advantage is that the model uncertainty can be easily considered by designing a local robust controller.

**FTC via eigenstructure assignment:**

Another approach to achieve fault tolerance is to use the eigenstructure assignment (EA) approach (Zhang and Jiang, 1999). The main idea is to assign the most dominant eigenvalues while at the same time minimizing the 2-norm of the difference between the corresponding eigenvectors. The difference from PIM is that, EA focuses on matching the eigenvalues and eigenvectors of the matrices of the nominal and faulty closed-loop systems (Kanev, 2006). Recalling the nominal system Eq. (2-1) with a state-feedback gain  $u = Kx$ , the EA method calculates the state-feedback gain  $K$  for the faulty model Eq. (2-2) and this is equivalent to finding a solution for the question summarised below (Zhang and Jiang, 1999):

$$\left\{ \begin{array}{l} \text{find} \quad K_f \\ \text{such that} \quad (A_f + B_f K_f) v_j^f = \lambda_j v_j^f, j = 1, \dots, n. \\ \text{and} \quad v_j^f = \arg \min_{v_j^f} (v_j - v_j^f)^T G_j (v_j - v_j^f) \end{array} \right\} \quad (2-3)$$

where  $(\lambda_j, v_j), j = 1, 2, \dots, n$  are the eigenvalues and corresponding eigenvectors of  $A$  matrix of the nominal closed-loop system,  $G_j$  is the weighting matrix. In a more general way, the process of finding the post-fault system control scheme  $K_f$  is actually to place the eigenvalues of the faulty system at the same locations as the ones of the nominal closed-loop system. Meanwhile, the eigenvector directions are kept as close as possible. Also, because both eigenvalues and eigenvectors determine the time response of the

closed-loop system, for preserving the time response of the nominal closed-loop in the event of fault occurrence, EA is one of the best choices. The disadvantage is illustrated by Kanev (2006) that the model uncertainties cannot be easily incorporated into the optimization problem, and that only static controllers are considered. However work by Liu and Patton (1998) showed earlier that dynamic feedback controllers can be designed using EA, and hence there actually is a way to improve this approach. This literature has not been followed further by other investigators.

### **Integrated FD and FTC:**

There are also many research studies considering the problems of FTC and FD in an integrated way. Zhang and Jiang (1999) proposed an integrated design method for FD and FTC based on multiple-model (MM) methods, in which the FD part uses a two-level adaptive Kalman filter (Wu, Zhang and Zhou, 1998), and FD with adaptive methods (Bošković and Mehra 2003), and FD with PID controller (Zhou and Frank, 1998). However, these methods do not consider uncertainty which is a drawback. More recently, another integrated FD and FTC design based on  $H_\infty$  polytopic MM design dealing with linear parameter varying (LPV) systems was proposed by Weng, Patton and Cui (2007) (see Figure 2-4). Chen, Patton and Klinkhieo (2012) further developed this method by combining it with state-feedback pole-placement. Many other methods can be found in review papers (Zhang and Jiang, 2008; Hwang *et al.*, 2010).

## **2.3 Overview of relationships between FDI and FE in FTC**

### **2.3.1 Classification of FDI**

There are many classifications of FDI in the literature and one of the most well known classifications for FDI is in terms of whether or not a model-based approach is used. Since the emphasis of this thesis is on the observer-based FE, the introduction will mainly focus on the main concepts of model-based FDI, providing a review of the important literature. When considering the requirements for FTC systems, model-based FDI schemes are grouped based on their properties into two major categories which are FDI using residual schemes and FDI with ability of estimating the faults (see Figure2-4).

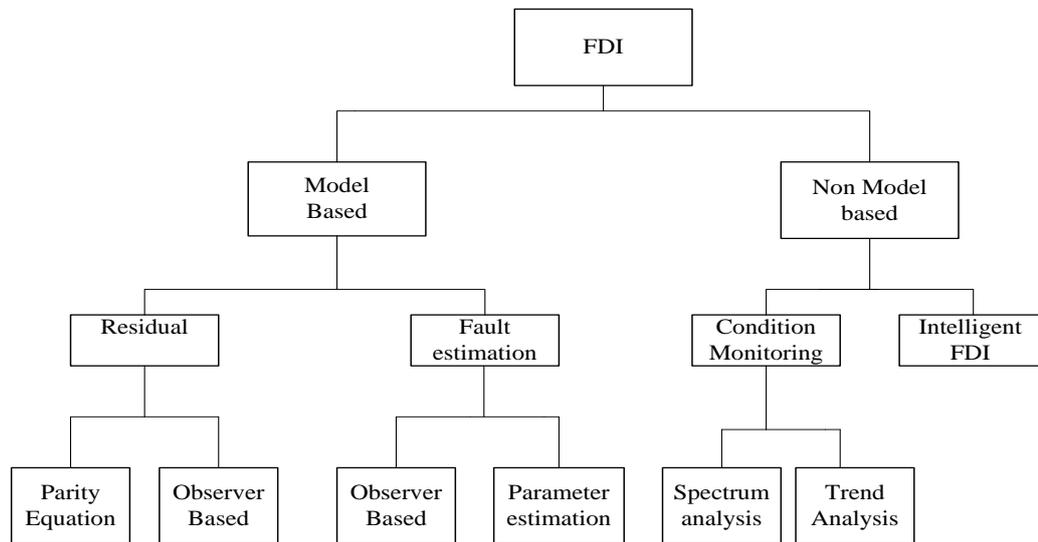


Figure 2-4: FDI classification (obtained from Chen and Patton, 1999)

### 2.3.2 Model-based FDI

Traditional non model-based FDI methods monitor the level of a particular signal, and take action when the signal reaches a given threshold. Meanwhile, a traditional approach to FDI, in the wider application context, is based on “*hardware redundancy*” methods which use multiple lanes of sensors, actuators, computers and software to measure and/or control a particular variable. A voting scheme is usually applied to the hardware redundant system to decide if and when a fault has occurred and its likely location amongst redundant system components. However, there are several drawbacks associated with non model-based approaches to FDI, for example:

- (1) The possibility of false alarms in the event of noise, the input variations and the change of operating point. Sometimes this means that it is difficult to identify the faults from operation related disturbance.
- (2) A single fault could cause many system signals to exceed their limits and appear as multiple faults, which makes fault isolation hard to achieve.
- (3) Demand for extra equipment and high maintenance cost.

According to these disadvantages and the common restriction that no additional hardware is required, model-based FDI offers a powerful way of achieving the roles of both detection and isolation of faults. The idea is that analytical relationships (“*analytical redundancy*”) among several model variables (or their estimates) can be

used to develop “indicators” or “residual” signals that when tested (usually with a logical threshold) can indicate if a fault is present or not (Patton, Frank and Clarke, 1989; Gertler, 1998; Chen and Patton, 1999; Patton, Frank and Clarke, 2000; Isermann, 2005). This is the *fault detection* part of the FDI process. By further using the analytical redundancy, it is also possible to develop a logical process for determining the location of the fault from information about the effect that the fault has on the dynamical system structure - this is the *fault isolation* part of the FDI process. Fault isolation is often referred to as FDI “decision-making”, as it is the stage during which fault decisions are really made.

Hence, the conceptual structure of a model based FDI system is shown in Figure 2-5. This two-stage structure was first suggested by Chow and Willsky (1980) and is now widely accepted by the FDI/FD community.

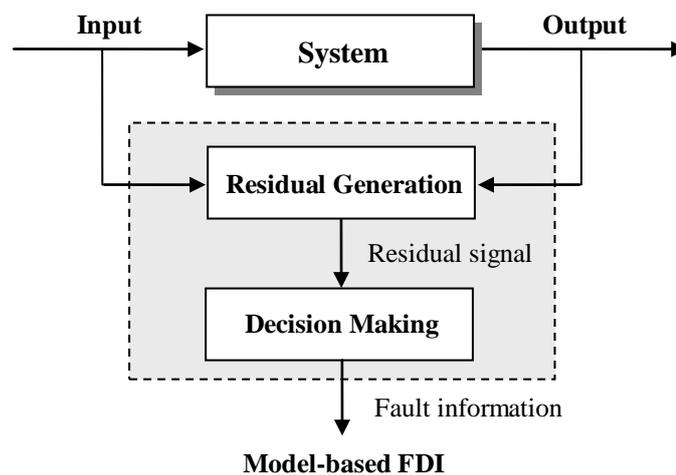


Figure 2-5: Conceptual structure of model-based fault diagnosis

(Adapted from Chen and Patton, 1999)

There are many advantages of using the model-based approach. For example, there are no additional hardware components needed in order to realize an FDI algorithm. A model-based FDI approach can be applied in software on the process computer. Furthermore, the measurements necessary to control the process are sufficient for the FDI algorithm so that no additional sensors have to be adapted (Chen and Patton, 1999).

Figure 2-5 shows clearly that the generation of the residual signal is the main issue in this two-stage structure model-based FDI. In the nominal or fault-free condition, the residuals should be zero, and nonzero when faults occur. This residual signal is

sometimes applied with a threshold to avoid false alarms disturbances or uncertainty (Chen and Patton, 1999). The monitor indicates the occurrence of the faults, when the residual signal exceeds the threshold.

### 2.3.3 Residual generation approaches

There are a variety of methods available for residual generation both for continuous and discrete system models which are described in the books (Patton, Frank and Clarke, 1989; Chen and Patton, 1999; Patton, Frank and Clarke, 2000; Isermann, 2005). This Section introduces the most commonly used model-based residual generation techniques.

#### Parity relation method

The Parity Relation Method is a popular method for residual generation. The basic idea is illustrated in (Chen and Patton, 1999) “*The basic idea of the parity relation approach is to provide a proper check of the parity (consistency) of the measurements of the monitored system*”. To describe this method, consider a system where the output equations is:

$$y(k) = Cx(k) + f(k) + \xi(k) \quad (2-4)$$

where  $y(k) \in R^q$  is measurement vector,  $x(k) \in R^n$  is the state vector, and  $f(k)$  is the vector of sensor faults,  $\xi(k)$  is a noise vector and  $C$  is an  $q \times n$  measurement matrix. If the consistency is based on hardware redundancy, which means there is more than the minimum number of sensors, hence the following dimension condition  $q > n$ ; and  $rank(C) = n$  should be satisfied. Then the parity vector (residual) signal can be generated by

$$r(k) = Vy(k) \quad (2-5)$$

$$s. t. VC = 0$$

When this condition is true, the parity vector (residual) can be written as:

$$r(k) = v_1[f_1(k) + \xi_1(k)] + \dots + v_m[f_m(k) + \xi_m(k)] \quad (2-6)$$

where  $v_i$  is the  $i$ th column of  $V$ ,  $f_i(k)$  is the  $i$ th element of  $f(k)$  which denotes the fault in the  $i$ th sensor (Chen and Patton, 1999). The benefit of this method for residual

generation is simple in the design and implementation and is useful in applications based on hardware redundancy such as strap-down inertial guidance systems (Chen and Patton, 1999). Other theoretical developments of this approach can be found in (Patton and Chen, 1994; Isermann, 2005; Zhang and Jiang, 2008).

### The observer-based Approach

The observer-based approach can be used to generate residuals via the difference between the estimated and actual system outputs. The main advantage of this approach over the use of parity equations is that it is more suitable for tolerating some degree of nonlinearity and modelling uncertainty. For this reason the observer approach is given more attention in the literature (Patton and Chen, 1997).

The main concept of the observer approach is to estimate the outputs of the system from the measurements (or a subset of measurements) by using either a Luenberger observer in a deterministic setting or by using a Kalman filter in the stochastic setting (Patton and Chen, 1997). The (weighted) output estimation error is used as a residual (Patton and Chen, 1999). Before summarising the observer design process, consider a general case of a dynamical system with faults that can be represented by the following state space model:

$$\dot{x}(t) = Ax(t) + Bu(t) + F_1f(t) \quad (2-7)$$

$$y(t) = Cx(t) + F_2f(t) \quad (2-8)$$

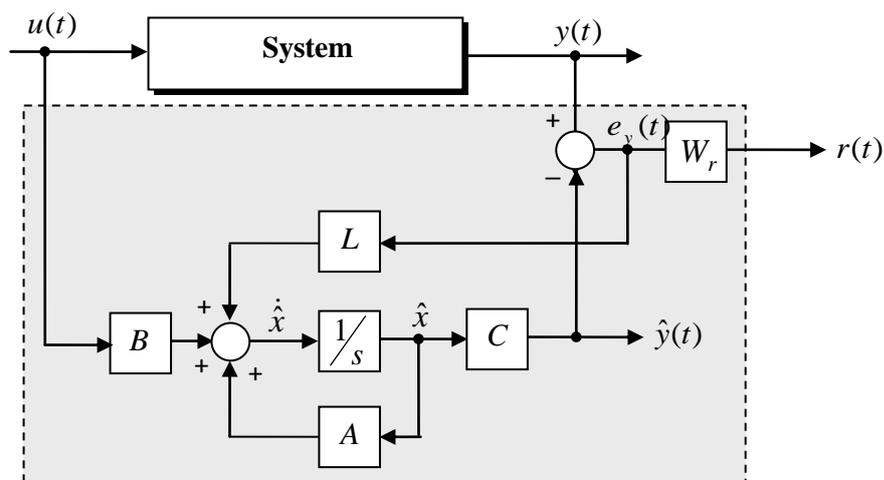


Figure 2-6: State observer for FDI (from Chen and Patton, 1999)

The observer shown in Figure 2-6 can be described by the following equations:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + Le_y(t)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (2-9)$$

Where  $e_y(t) = y(t) - C\hat{x}(t)$  and  $L \in \mathfrak{R}^{n \times q}$  is a designed observer gain,  $\hat{x}(t)$  is the estimated state,  $\hat{y}(t)$  is the estimated output and  $e_y(t)$  is the output estimation error.

The state estimation error can be expressed as  $e(t) = x(t) - \hat{x}(t)$ , thus:

$$\begin{aligned} \dot{e}(t) &= A[x(t) - \hat{x}(t)] - Le_y(t) \\ &= Ae(t) - LCe(t) \\ &= (A - LC)e(t) \end{aligned} \quad (2-10)$$

Applying this state observer to the system of Eq. (2-7) and (2-8) with actuator, component and sensor faults, the output estimation error  $e_y(t)$  can then be expressed as:

$$\begin{aligned} e_y(t) &= y(t) - \hat{y}(t) \\ &= Ce(t) + Ff(t) \end{aligned} \quad (2-11)$$

The state estimation error can be written as:

$$\begin{aligned} \dot{e}(t) &= Ax(t) + Bu(t) + F_1f(t) - Ax(t) - Bu(t) - Le_y(t) \\ &= (A - LC)e(t) + F_1f(t) - LF_2f(t) \end{aligned} \quad (2-12)$$

The weighted residual can then be generated as:

$$r(t) = W_r e_y(t) \quad (2-13)$$

The matrices  $W_r \in \mathfrak{R}^{p \times q}$  can be designed to generate residuals with desired characteristics, for example, a time response performance requirement can be achieved using eigenstructure assignment (Patton and Chen, 1991b; Patton and Chen, 2000). Since  $e_y(t)$  depends on  $y(t)$ ,  $\hat{y}(t)$  and hence also on  $F_1f(t)$ ,  $F_2f(t)$ ,  $r(t)$  can be shown to be a residual signal that can be designed to be specially sensitive to the fault signals.

However, since it is impossible to avoid the modelling errors and disturbances when considering non-linear, uncertain and complex engineering systems, there is a need to

include robust design methods that can minimise the effects of the disturbance and uncertainty whilst maximising the sensitivity to faults, in the presence of model – reality differences. Normally, parameter variations and disturbances act on a real process in an uncertain way, so that it may be difficult to design a fault diagnosis system which is highly sensitive to faults, whilst insensitive to uncertainty and un-modelled disturbances. This is a very big challenge. One way to solve this problem is to use the unknown input observer (UIO) (Patton and Chen, 1997), which ingeniously decouples the disturbances and faults into two different input channels. Many other robust observer-based methods for residual generation can be found in the book (Chen and Patton, 1999) and in (Gertler, 1997; Isermann, 2005, 2011; Zhang and Jiang, 2008).

### **Parameter estimation approach**

In this approach, the parameters of the model of the system are estimated using the input-output measurements of the system (Isermann, 1984, 2006, 2011), which is very useful when dealing with the components faults. The main idea is that residuals are generated by detecting a change in the system parameters. These residuals can then be used to detect and isolate faults. The main drawback of this approach is that the model parameters should have a physical meaning and they should correspond to the actual physical parameters of the system. If this condition is not true it is difficult to distinguish fault effects on the residual from causal effects of parametric variation, uncertainty or other time-varying system properties (e.g. changing disturbance or even system structure changes). Moreover, if the model structure is nonlinear in its parameters, non-linear modelling methods or non-linear feedback structures should be applied and these may cause serious difficulties in the case of complex (difficult to model) systems. Robust parameter estimation techniques may be applied to account for system-model mismatch.

There are still several other methods for residual generation, e.g. the Factorization method (Viswanadham, Taylor and Luce, 1987) which is not introduced here. For more details please refer to (Ding and Frank, 1990, Chen and Patton, 1999; Isermann, 2011).

## 2.4 Fault estimation/identification

For the FDI process, faults are detected and located, but no other useful information about the faults is provided. However, for some FTC schemes, only detecting and isolating the faults are not enough, further information about the nature and behaviour of the faults are usually required.

Fault estimation/identification is one step further than FDI, the basic idea is to estimate or reconstruct the real faults in systems, which play the same important role as FDI does in some FTC systems. For example, with regard to sensor fault FTC, if the sensor fault can be estimated, this information can be used directly to correct the corrupted sensor measurements before they are used by the controller.

As for FDI, the FE process also comprises two stages, for which the first one is still the residual generation step, whilst the second step is fault estimation/identification (sometimes called fault reconstruction) by using the generated residual signals in the first step. The structure for FE is shown in Figure 1-8

Similarly, the core of the FE problem also consists of the residual generation so that it is natural to consider if the methods of residual generation for FDI can be transferred to FE. Anyway, according to the literature, in the past three decades, some methods of residual generation for FDI have been successfully used for FE, such as parity-relations (Nguang, Zhang and Ding, 2007) but more attention has been put in the field of the observer-based residual generation for fault estimation (Zhang, Jiang and Shi, 2010).

An important contribution on this topic is the thread of research initiated by (Zhang, Jiang and Cocquempot 2008) in which they combine an observer-based residual (as a first stage) with a proportional - plus integral fault estimator – the so called Fast Adaptive Fault Estimation (FAFE). The FAFE approach is actually used in Chapter 3 of this thesis in connection with Model-reference control of a Two-link manipulator system.

Several other FE approaches have been adopted from observer-based schemes. For example, sliding mode observers can be designed for the FE role satisfying excellent robustness conditions (Edwards, Spurgeon, and Patton 2000; Floquet, Edwards and Spurgeon, 2007; Tan, Edwards and Kuang, 2005). Further observer-based FE methods are Iterative learning observers (Chen and Saif, 2006), FE based-on filters (Zhang and

Jiang, 1999; Wu, Zhang and Zhou, 2000; Nobrega, Abdalla and Grigoriadis, 2008; Weng, Patton and Cui, 2007), Descriptor observers (Gao and Ding, 2007), Direct reconstruction approach (Liu and Duan, 2012; Corless and Tu, 1998; Zhu and Cen, 2010), and Adaptive observers (Wang and Daley, 1996; Xu and Zhang, 2004; Jiang, Staroswiecki, and Cocquempot, 2002, 2006; Wang, Jiang and Shi, 2008; Patton and Klinkhieo, 2010) and Unknown input observer (Saif and Guan, 1993; Wang and Lum, 2007; Wang, 2010). These combined FDI and FE methods comprise the whole Fault Diagnosis block (Figure 1-7), which acts a critical role in fault tolerant control or reconfigurable control process. This thesis is based on the use of observer-based FE strategies for FTC. Hence the remainder of this Chapter focuses on a selection of observer-based FE methods to form a background for Chapters 3, 4 and 5 of the thesis.

### **Adaptive observer fault estimation in FTC design**

The basic idea for this method is to estimate the actuator efficiency by setting up an augmented observer-based fault estimator. Different FE methods depend on different kinds of faults considered, e.g. additive faults are always reconstructed/estimated directly using residual-based information (errors between measurement and output estimates) directly (Jing, Staroswiecki and Cocquempot, 2006). To illustrate the adaptive observer FE methods design process, consider the following linear system with additive fault as follows:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Ef(t) \\ y(t) &= Cx(t)\end{aligned}\tag{2-14}$$

where  $x(t) \in \mathfrak{R}^n$  is the state vector,  $u(t) \in \mathfrak{R}^m$  is the input vector,  $y(t) \in \mathfrak{R}^p$  is the output vector and  $f(t) \in \mathfrak{R}^r$  represents the additive fault.  $A, B, E$  and  $C$  are known constant real matrices of appropriate dimensions, the matrix  $E$  is of full column rank and the pair  $(A, C)$  is observable. Then the adaptive fault estimator is constructed as:

$$\begin{aligned}\hat{\dot{x}}(t) &= A\hat{x}(t) + Bu(t) + E\hat{f}(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t)\end{aligned}\tag{2-15}$$

where  $\hat{x}(t) \in \mathfrak{R}^n$  is the observer state vector,  $\hat{y}(t) \in \mathfrak{R}^p$  is the observer output vector and  $\hat{f}(t) \in \mathfrak{R}^r$  is the estimate of the fault  $f(t)$ . By denoting  $e(t) = x(t) - \hat{x}(t)$ ,

$e_y(t) = y(t) - \hat{y}(t)$ ,  $e_f(t) = f(t) - \hat{f}(t)$  and subtracting Eq. (2-15) from Eq. (2-14) the error dynamics is acquired as:

$$\begin{aligned}\dot{e}(t) &= (A - LC)e(t) + Ee_f(t) \\ e_y(t) &= Ce(t)\end{aligned}\tag{2-16}$$

As there is no way to know the dynamic information of the real faults, in the earlier literature the faults are commonly considered as constant, i.e.  $\dot{f}(t) = 0$  hence the fault estimator for this case has the simplified form:

$$\dot{\hat{f}}(t) = \Gamma F e_y(t)\tag{2-17}$$

where  $\Gamma$  is the learning rate. As a result the derivative of the error of the fault with respect to time can be written as:

$$\dot{e}_f(t) = -\dot{\hat{f}}(t) = -\Gamma F e_y(t)\tag{2-18}$$

As the pair  $(A, C)$  is observable, the design objective is changed to one of solving the problem of finding S.P.D. matrices  $P, Q \in \mathfrak{R}^{n \times n}$  to satisfy the following conditions:

$$\left\{ \begin{array}{l} P(A - LC) + (A - LC)^T P = -Q \\ E^T P = FC \end{array} \right\}\tag{2-19}$$

If the above equation has suitable solutions for  $P, Q$ , then the observer-based estimator can realise asymptotic convergence to zero for both  $e(t)$  and  $e_y(t)$ .

Some researchers used the modified adaptive observer for other faults, like actuator faults and sensor faults (Wang and Daley, 1996, 1997; Patton and Klinkhieo, 2010). The main disadvantage of this method is that the fault may have unpredictable behaviour and in most of the time is not constant, which means the derivative of it is not zero and may further cause high gains (Patton and Klinkhieo, 2010). Meanwhile since there is disturbance and other uncertainty in the system, the precise value of the estimate may be degraded and as a result, some researchers adopted the unknown input observer (UIO) instead of the conventional observer to improve the robustness (Saif and Guan, 1993; Wang and Lum, 2007). However the UIO requires two structural conditions associated with the channels of the unknown inputs to meet the complete rejection of the uncertainty. However, there are many results in observer-based FE algorithm, but only

a few (Gao and Ding, 2007; Zhang, Jiang and Shi, 2009) are used in FTC (Zhang and Jiang, 2008).

### **Sliding mode observer for fault reconstruction**

In the work of Yang and Saif (1995) the sliding mode observer was applied to an FDI residual problem. The idea was to ensure that the sliding motion was broken when faults occurred in the system and a residual was generated containing information about the fault. In more recent work using sliding mode observers the faults are reconstructed or estimated (Edwards, Spurgeon and Patton, 2000; Tan and Edwards, 2003; Jiang, Staroswiecki and Cocquempot, 2004). In these special sliding mode-observer based approaches, not only the faults can be detected and isolated, the estimated faults can be further used in the reconfigurable control design process. The difference between these methods form the idea behind the paper (Jiang, Staroswiecki and Cocquempot, 2004), which shows that under certain geometric conditions, the original nonlinear system is transformed into two different subsystems with uncertainty. The first subsystem is in generalised observer canonical form, which is not affected by faults, whilst the second is affected by faults. A sliding mode observer is then constructed for the first subsystem to enable the estimation of the faults to be achieved from the second subsystem. However, Edwards *et al* (2000) provided an alternative way for faults reconstruction by proposing so-called *equivalent output injection* concept which represents the average behaviour of the switching function and represents the effort necessary to maintain the motion on the sliding surface (Edwards, Spurgeon and Patton, 2000). The drawback of this method is that, when dealing with the sensor fault, it has to be assumed that the fault is slowly changing, which is a very strict condition.

There are a number of important issues when designing AFTC. The most significant one is probably the integration between the FD and the FTC functions. Most research studies on FD and reconfigurable control/FTC have been carried out as two separate entities. These two subjects are investigated mostly by separate fields or groups of researchers with seemingly little interaction between them. To be more specific, many FD algorithms do not consider the closed-loop operation of the system on the one hand, and many FTC methods assume the availability of perfect fault estimation from the FD scheme on the other hand. Furthermore, most of the FDI/FD techniques are developed as a diagnostic or a supervising tool, but not as an aspect of FTC. The result is that some existing FD methods may not satisfy the requirement for controller reconfiguration or

may not guarantee that a satisfactory post-fault performance or even stability can be maintained by such a scheme under FTC conditions. Hence, it is very important that designs of the FD and FTC, when carried out separately, are each performed taking the presence and imperfection of the other into account. For example, from the viewpoint of reconfigurable controls design (Zhang and Jiang, 2006, 2008); “... (a) *What are the needs and requirements for FD?* (b) *What information can be provided by the existing FD techniques for overall FTC designs?* (c) *How to analyze systematically the interaction between FD and the reconfigurable control system?* (d) *How to design the FD and reconfigurable controls in an integrated manner for on-line and real-time applications? ...*”

A common situation in practice is that there are disturbances/uncertainty existing in any systems, which can both affect the performance of the FE from an FD aspect. From this an imprecise FE signal may further threaten the stability of the closed-loop system. For this reason, the FTC should necessarily be capable of dealing with disturbance/uncertainty in the FD estimates and should perform satisfactorily, at least for the stability, during the transition period that the FD scheme is required to diagnose the fault.

Another issue in practice is that the dynamics of a real system cannot be represented accurately enough by linear dynamical models so that nonlinear models or linear model with nonlinearity terms (i.e. time-varying parts) have to be used, such as the dynamics of the manipulator/robot, and aerospace systems (flight control, satellite attitude control, etc). This necessitates the development of techniques for FTC design that can explicitly deal with nonlinearities in the mathematical representation of the system.

Because of these, requirements for further research and developments for integration of FD and FTC methods makes this an open field for research. Some of these challenges are taken up in this thesis with several novel robust solutions.

## **2.5 Conclusion**

This Chapter has presented a brief introduction and literature review on the field of FD and FTC. These include some definitions and terminology regularly used in FDI, FE and FTC. For some historical reasons, some terminologies have not been unified in FTC,

as a result, the introduction and literature review for FD is divided into two parts of FDI and FE. Different methods of FTC were discussed based on the given classification. Then, this Chapter has also briefly discussed the residual generation issue, and its importance to FDI and FE, further to FTC, as well as different methods for residual generation, in which the observer-based method is specially mentioned.

Some discussion and literature review on FE methods is carried out, and the adaptive observer-based and sliding mode observer-based methods for FE, are outlined.

The Chapter also lists the challenges involved in the integration of the designs of FD and FTC. In the Chapter 3, a new FTC scheme with a combined Model-reference control and FAFE are given, with application to a model example of a Two-link manipulator non-linear system.

# Chapter 3.

## Fault estimation and Model Reference

## Control-based active FTC\*

### 3.1 Introduction

The use of model reference tracking is a well known way in the literature of achieving control reconfiguration or adaptation (Landau, 1979). Actually, MRC is quite synonymous with FTC and this is explained below. Several research studies have used model reference schemes for active FTC (Tao, Joshi and Ma, 2001; Taware and Tao, 2003; Jiang and Zhang, 2006; Yang, Qi and Shan, 2009; Mirkin and Gutman, 2007). Several advantageous features of MRC make it a popular approach for FTC. Performance specifications are given in the time domain, such as rise time, damping ratio, decoupling effects etc, and these characteristics can be easily represented in terms of an ideal system response, which become the reference signals that the closed-loop system must follow for tracking purposes. Another advantage of using model reference control for FTC is that it allows the reference model to be changed online to cope with changes in the operational conditions especially during faults or failures (Duan, Wang and Huang, 2004; Qu and Dawson, 1994). In fact the so-called blending model reference adaptive concept is deemed a meaningful approach for fault accommodation within FTC.

On the other hand, linear models are traditionally used for both estimation and control within the framework of robustness analysis and design. In a classical way the joint performance of the FTC estimation and control compensation of such systems may only be acceptable in a region of operation close to the defined equilibria and numerous studies have emerged focused on robust fault detection and isolation (FDI), robust fault estimation and robust FTC, based on this limitation.

However, many real system applications have no unique linearization equilibria e.g. advanced aircraft and various forms of robotic systems, which present a significant challenge to the use of linear modelling methods. Within the wider field of control the problem of feedback design for such systems has been of considerable interest in the literature, see for example (Marino and Tomei, 1997); (Astolfi *et al.*, 2007).

This Chapter highlights the benefit of combining an adaptive fault estimation law and a model reference framework for achieving on-line fault estimation and FTC for time-varying affine systems. The main contribution of this Chapter is the design of an on-line active fault estimator and fault compensator to achieve FTC objective based only on the reference model and not on the plant dynamics. This leads to the use of a simpler parameterization in the fault estimator in terms of LMI computation compared with the estimator approach developed by Zhang, Jiang and Cocquempot (2008).

This Chapter is constructed as follows. Section 3.2 presents the faulty time-varying affine system, and Section 3.3 summarises briefly the MRC approach to feedback design. Section 3.4 outlines the structure of the combined fault estimation and control compensation scheme, based on the MRC approach. Section 3.5 describes a non-linear system example of a two-link manipulator (TLM), illustrating the MRC strategy for a scenario when two actuator faults act independently on each joint. Concluding comments are given in Section 3.6.

## 3.2 Preliminaries

Consider a faulty time-varying affine system of the form:

$$\dot{x}(t) = A[x(t)]x(t) + B[x(t)]u(t) + Ef(t) \quad (3-1)$$

$E \in \mathfrak{R}^{n \times q}$  is a full column rank fault weighting matrix and  $f(t) \in \mathfrak{R}^q$  denotes the bounded  $q$  system faults. Note that  $Ef(t)$  could represent either an additive or a multiplicative fault.  $A[x(t)] \in \mathfrak{R}^{n \times n}$ ,  $B[x(t)] \in \mathfrak{R}^{n \times m}$  are smooth, continuous and controllable functions of the state vector  $x(t) \in \mathfrak{R}^n$  (supposed all can be measured) whilst  $u(t) \in \mathfrak{R}^m$  is a stabilising state feedback control vector. It is further assumed that  $B[x(t)]$  is full column rank for all states  $x(t)$  defined by Eq. (3-1).

### 3.3 Model Reference Control System

Consider the problem of developing an associated linear time-invariant (LTI) “open-loop” MRC reference model for a linear time-varying multivariable plant in Eq. (3-1), with  $f(t) = 0$  as follows:

$$\dot{w}(t) = A_m w(t) + B_m \text{ref}(t) \quad (3-2)$$

$w(t) \in \mathfrak{R}^n$  is the reference model state vector and  $\text{ref}(t) \in \mathfrak{R}^r$  is a time-varying input signal designed to achieve a required reference trajectory for the states  $w(t)$ .  $(A_m, B_m)$  is a compatibly dimensioned controllable pair with stable  $A_m$ .

The error state vector,  $e(t) \in \mathfrak{R}^n$  is defined as:

$$e(t) = x(t) - w(t) \quad (3-3)$$

The error system dynamics are determined from Eq. (3-1), Eq. (3-2) as:

$$\dot{e}(t) = A_m e(t) + (A[x(t)] - A_m)x(t) + B[x(t)]u(t) - B_m \text{ref}(t) \quad (3-4)$$

The restriction arising from use of a MRC design strategy is given in terms of the well-known *perfect model matching conditions* of Erzberger (1968) and Chen (1968) as:

$$\text{rank}[B: A_m - A] = \text{rank}(B) = \text{rank}[B: B_m] \quad (3-5)$$

For real applications, these conditions are easy to satisfy as the reference model can be chosen by the designer. The plant control signal can be designed as:

$$u = B^{++}[x(t)][-K_1[x(t)]x(t) + K_2 \text{ref}(t)] \quad (3-6)$$

$B^{++}[x(t)] \in \mathfrak{R}^{m \times n}$  is a suitable pseudo-inverse matrix of the full column rank matrix  $B[x(t)]$  and  $K_1[x(t)] \in \mathfrak{R}^{n \times n}$ ,  $K_2 \in \mathfrak{R}^{n \times r}$  are feedback matrices given by:

$$K_1[x(t)] = A[x(t)] - A_m \quad (3-7)$$

$$K_2 = B_m \quad (3-8)$$

Hence, Eq. (3-4) can be reduced to:

$$\dot{e}(t) = A_m e(t) \quad (3-9)$$

Assuming the matching condition above, the error  $e(t)$  tends asymptotically to zero at a rate determined by the placement of the eigenvalues of  $A_m$  in the open left plane.

### 3.4 Model Reference FTC Strategy

An FTC design strategy is required to compensate for the effects of the faults acting in Eq. (3-1). The current study is based on the estimator proposed by (Zhang, Jiang and Cocquempot, 2008). However, the estimation is applied within an FTC fault compensation mechanism that makes use of a MRC structure.

Now consider the system Eq. (3-1) for the case  $f(t) \neq 0$ . In order to proceed to the fault estimator design the following *two* Assumptions A1 and A2 must be satisfied:

A1.  $\text{Rank}(E) = q$ .

A2. The invariant zeros of  $(A_m, I, E)$  lie in the open left half-plane (LHP) (Kudva, Viswanadham and Ramakrishna, 1980).

Assume that the fault estimator has the following dynamics:

$$\dot{\hat{f}}(t) = WL[\dot{e}(t) + e(t)] \quad (3-10)$$

where  $W \in \mathfrak{R}^{q \times q}$  is the learning rate, and  $\dot{e}(t)$  can be achieved via Eq. (3-9),  $\hat{f}(t) \in \mathfrak{R}^q$  is the fault estimate and  $L \in \mathfrak{R}^{q \times n}$  is a suitable estimator gain determined using a suitable LMI calculation as described below.

A similar estimation structure is the augmented state observer (ASO) by (Patton and Klinkhieo, 2009). However, Eq. (3-10) has a more general proportional-plus-integral (P-I) structure (compared with the proportional only ASO approach). The estimation error is  $e_f(t) = f(t) - \hat{f}(t)$ , and the error dynamics are given by:

$$\dot{e}_f(t) = \dot{f}(t) - \dot{\hat{f}}(t) = \dot{f}(t) - WL(\dot{e}(t) + e(t)) \quad (3-11)$$

The proportional  $L\dot{e}(t)$  plus integral  $Le(t)$  action on the error system in Eq. (3-11) provides degrees of design freedom to shape the estimator tracking performance.

Here it is proposed that the fault estimate signal  $\hat{f}(t)$  can be added to the control signal in Eq. (3-6) to compensate the fault signal  $f(t)$ , according to the MRC structure of Figure 3-1.

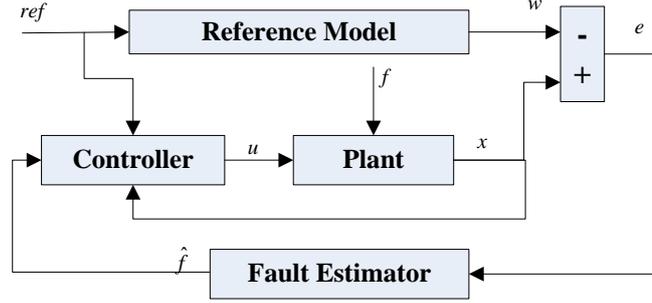


Figure 3-1: Model Reference FTC scheme

The fault-tolerant performance of this system depends on the robustness of the fault estimation applied to the control input:

$$u(t) = B^{++}[x(t)][-K_1[x(t)]x(t) + K_2ref(t) - E\hat{f}(t)] \quad (3-12)$$

It is further assumed that  $B(x)^{++}E \neq 0$ . From Eq. (3-1)-(3-12), it can be shown that:

$$\dot{e}(t) = A_m e(t) + E e_f(t) \quad (3-13)$$

The existence of suitable symmetric positive definite S.P.D. Lyapunov matrices to guarantee the stability of the error system Eq. (3-13) is determined as follows (Zhang, Jiang and Cocquempot, 2008):

**Theorem 3.1:** Under the Assumptions A1 and A2, if there exist S.P.D. matrices  $P \in \mathfrak{R}^{n \times n}$ ,  $G \in \mathfrak{R}^{q \times q}$ , and matrix  $L \in \mathfrak{R}^{q \times p}$  to make the following two conditions hold:

$$E^T P = L ; \quad (3-14)$$

$$\begin{bmatrix} PA_m + A_m^T P & -A_m^T P E \\ -E^T P A_m & -2LE + G \end{bmatrix} < 0, \quad (3-15)$$

then the fault estimation error can be guaranteed to remain in a bounded region on the  $q$ -dimension fault space. The proof is taken from (Zhang, Jiang and Cocquempot, 2008) but is given here for completeness as this approach is modified for the MRC design.

**Proof:**

The Lyapunov function can be considered as:

$$V(t) = e^T(t)Pe(t) + e_f^T(t)W^{-1}e_f(t) \quad (3-16)$$

According to Eq. (3-10) after differentiation of  $V(t)$  with respect to time (3-16) becomes:

$$\begin{aligned} \dot{V}(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + 2e_f^T(t)W^{-1}\dot{e}_f(t) \\ &= [A_m e(t) + E e_f(t)]^T Pe(t) + e^T(t)P[A_m e(t) + E e_f(t)] \\ &\quad + 2e_f^T(t)W^{-1}[\dot{f}(t) - WL(\dot{e}(t) + e(t))] \\ &= e^T(t)A_m^T Pe(t) + e_f^T(t)E^T Pe(t) + e^T(t)PA_m e(t) \\ &\quad + e^T(t)PE e_f(t) + 2e_f^T(t)W^{-1}\dot{f}(t) - 2e_f^T(t)L(\dot{e}(t) + e(t)) \\ &= e^T(t)(A_m^T P + PA_m)e(t) + 2e_f^T(t)E^T Pe(t) \\ &\quad - 2e_f^T(t)L[A_m e(t) + E e_f(t) + e(t)] + 2e_f^T(t)W^{-1}\dot{f}(t) \end{aligned} \quad (3-17)$$

From Eq. (3-17), Eq. (3-16) can be re-written in the form:

$$\begin{aligned} \dot{V}(t) &= e^T(t)(A_m^T P + PA_m)e(t) - 2e^T(t)A_m^T PE e_f(t) \\ &\quad - 2e_f^T(t)LE e_f(t) + 2e_f^T(t)W^{-1}\dot{f}(t) \end{aligned} \quad (3-18)$$

**Lemma 3.1:** (Jiang, Wang and Soh, 2002) given a scalar  $a > 0$  and a S.P.D. matrix  $Q \in \Re^{n \times n}$ , for which the following inequality holds:

$$2x^T y \leq \frac{1}{a}x^T Qx + ay^T Q^{-1}y \quad x, y \in \Re^n. \quad (3-19)$$

Then Eq. (3-18) satisfies:

$$\begin{aligned}
\dot{V}(t) &\leq e^T(t)(A_m^T P + P A_m)e(t) - 2e^T(t)A_m^T P E e_f(t) \\
&\quad - 2e_f^T(t) L E e_f(t) + e_f^T(t) G e_f(t) \\
&\quad + \dot{f}^T(t) W^{-1} G^{-1} W^{-1} \dot{f}(t) \\
&\leq e^T(t)(A_m^T P + P A_m)e(t) - 2e^T(t)A_m^T P E e_f(t) \\
&\quad - 2e_f^T(t) L E e_f(t) + e_f^T(t) G e_f(t) \\
&\quad + \|\dot{f}(t)\|^2 \lambda_{\max}(W^{-1} G^{-1} W^{-1})
\end{aligned} \tag{3-20}$$

By defining a vector  $\theta(t)$  as  $\theta(t) = \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix}$ , Eq. (3-20) can now be re-written more succinctly as:

$$\dot{V}(t) \leq \theta^T(t) \Psi \theta(t) + \mathfrak{Q}(t) \tag{3-21}$$

where:

$$\Psi = \begin{bmatrix} P A_m + A_m^T P & -A_m^T P E \\ -E^T P A_m & -2L E + G \end{bmatrix}, \text{ and } \mathfrak{Q}(t) = \|\dot{f}(t)\|^2 \lambda_{\max}(G^{-1})$$

Then Eq. (3-21) can be rewritten as:

$$\dot{V}(t) < -\lambda_{\min}(-\Psi) \|\theta(t)\|^2 + \mathfrak{Q}(t) \tag{3-22}$$

where  $\lambda_{\max}(\ast)$  and  $\lambda_{\min}(\ast)$  are the largest and smallest eigenvalues of the matrix  $(\ast)$ , respectively. Eq. (3-22) shows that when  $-\lambda_{\min}(-\Psi) \|\theta(t)\|^2 > \mathfrak{Q}(t)$ ,  $\dot{V}(t) < 0$ , which is satisfied for  $\|\theta(t)\|^2 < \frac{\mathfrak{Q}(t)}{-\lambda_{\min}(-\Psi)}$ , which implies that  $\begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix}$  is bounded within a small finite range determined by the derivative of the fault  $\dot{f}(t)$ . Furthermore, the lower the value of the scalar  $\|\dot{f}(t)\|^2$  the faster will be the fault estimation speed, i.e. when  $\|\dot{f}(t)\|^2 = 0$  (the fault is constant with the error vector  $\theta(t) = 0$ ) and the fault estimator achieves perfect tracking. Q.E.D.

In order to solve the LMI Eq. (3-15) subject to Eq. (3-14) a well known procedure of (Corless and Tu, 1998) can be used to transfer Eq. (3-14) into a (convex optimization)

LMI problem. For this case and based on the selected reference model of Eq. (3-13), a new LMI must be solved, as follows (Zhang, Jiang and Cocquempot, 2008):

$$\begin{bmatrix} -JI & L - E^T P \\ PE - L^T & -JI \end{bmatrix} < 0 \quad (3-23)$$

The LMIs Eq. (3-15) and Eq. (3-23) are solved simultaneously to determine the matrices  $P, G$  and hence  $L$  so that the model-reference estimator can be determined.

However, the fault estimator Eq. (3-10) can be equivalently re-expressed as  $\hat{f}(t) = WL[e(t) + \int_{t_f}^t e(t)]$  where  $t_f$  denotes the instant when the faults occurs.

### 3.5 Two-Link Manipulator Case Study

To illustrate the mathematical discussion above, a tutorial example of the actuator fault compensation problem is considered using a nonlinear simulation of the two-link manipulator/robot. The field of robotics is concerned with the principles, design, manufacture, and application of robots, and is a broad application area involving many areas such as Physics, mechanical design, motion analysis and planning, actuators and drivers, control design, sensors, signal and image processing, computer algorithms, and study of behaviour of machines, animals, and even human beings (McKerrow, 1991; Slotine and Li, 1991; Hassen, *et al.*, 2000).

Robot manipulators are familiar examples of position-controllable mechanical systems (Hassen, *et al.*, 2000). However, their nonlinear dynamics present challenging control problems, since traditional linear control approaches do not easily apply. The objective of this Section is to model the complete nonlinear dynamics of an example of a two-joint manipulator, so that the movement control, e.g. from one point to another in two-dimensional space, is facilitated.

#### 3.5.1 Two-Link Manipulator Dynamics

Basically, there are three types of dynamic torques that arise from the motion of the manipulator: Inertial, Centripetal, and Coriolis torques (McKerrow, 1991; Slotine and

Li, 1991; Hassen, *et al.*, 2000). Inertial torques are proportional to acceleration of each joint in accordance with Newton's second law. Centripetal torques arise from the centripetal forces which constrain a body to rotate about a point. They are directed towards the centre of the uniform circular motion, and are proportional to the square of the velocity. Coriolis torques result from vertical forces derived from the interaction of two rotating links and are proportional to the product of the joint velocities of those links.

For simplicity, the two-link robotic manipulator is considered to rotate in the vertical plane, whose position can be described by a 2-vector  $\varphi(t) = [\varphi_1(t), \varphi_2(t)]^T$  of joint angles, and whose actuator inputs consist of a 2-vector  $u(t) = [u_1(t), u_2(t)]^T$  of torques applied at the manipulator joints as shown in Figure 3-2.

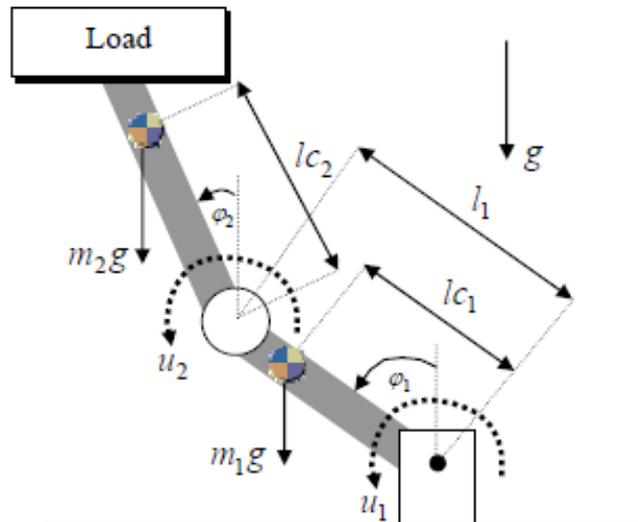


Figure 3-2: Two link planar manipulator structure

Using the vectors  $\dot{\varphi}(t)$  and  $\ddot{\varphi}(t)$  to denote the joint velocities and accelerations, respectively the dynamics of this simple manipulator can be written in the more general form (McKerrow, 1991; Slotine and Li, 1991; Hassen, *et al.*, 2000) as:

$$E[\varphi(t)]\ddot{\varphi}(t) + O[\varphi(t), \dot{\varphi}(t)]\dot{\varphi}(t) + G[\varphi(t)] = u(t) \quad (3-24)$$

where:  $\mathcal{E}[\varphi(t)] \in \mathfrak{R}^{2 \times 2}$  is the manipulator inertia tensor matrix (which is S.P.D.),  $O[\varphi(t), \dot{\varphi}(t)]\dot{\varphi}(t) \in \mathfrak{R}^2$  is the vector function containing the Centripetal and Coriolis torques, i.e.  $O[\varphi(t), \dot{\varphi}(t)] \in \mathfrak{R}^2$  and  $G[\varphi(t)] \in \mathfrak{R}^2$  are the gravitational torques.

Consider the following numerical example taken from (Hassen, *et al*, 2000, Klinkhieo, 2009) and modified here as a demonstration for the proposed design strategy in this Section.

The Euler-Lagrange dynamic model of the TLM is given in state space notation as:

$$\begin{aligned} & (m_1 l c_1^2 + m_2 l_1^2 + J_1) \ddot{\varphi}_1(t) \\ & + m_2 l_1 l c_2 \cos[\varphi_1(t) - \varphi_2(t)] \ddot{\varphi}_2(t) + m_2 l_1 l c_2 \sin[\varphi_1(t) - \varphi_2(t)] \dot{\varphi}_2^2(t) \\ & - (m_1 l c_1 + m_2 l_1) g \sin \varphi_1(t) = u_1(t) \end{aligned} \quad (3-25)$$

$$\begin{aligned} & m_2 l_1 l c_2 \cos[\varphi_1(t) - \varphi_2(t)] \ddot{\varphi}_1(t) \\ & + (m_2 l c_2^2 + J_2) \ddot{\varphi}_2(t) - m_2 l_1 l c_2 \sin[\varphi_1(t) - \varphi_2(t)] \dot{\varphi}_1^2(t) \\ & - m_2 g l c_2 \sin \varphi_2(t) = u_2(t) \end{aligned} \quad (3-26)$$

where:

$J_1$ : Inertia of arm-1 and load

$J_2$ : Inertia of arm-2

$l_1$ : Distance between joint-1 and joint-2

$l c_1$ : Distance of joint-1 from centre of mass arm-1

$l c_2$ : Distance of joint-2 from centre of mass arm-2

$m_1$ : Mass of arm-1 and load

$m_2$ : Mass of arm-2

$\varphi_1(t), \dot{\varphi}_1(t)$  and  $\varphi_2(t), \dot{\varphi}_2(t)$  are state variables representing the angle and angular velocity of Link-1 and Link-2, respectively.  $u_1(t)$  and  $u_2(t)$  are the control signals. The associated parameters are given in Table.1.

**Table 3-1:** Parameters of TLM

Parameters	$J_1$	$J_2$	$l_1$	$lc_1$	$lc_2$	$m_1$	$m_2$	$g$
Values	0.833	0.417	1.0	0.5	0.5	10.0	5.0	9.80
Units	$Kg * m^2$	$Kg * m^2$	$m$	$m$	$m$	$Kg$	$Kg$	$m/s^2$

### 3.5.2 Simplification of the TLM system

To facilitate the development of the MRC design the controlled non-linear TLM dynamics can be simplified using the following notation:

$$a_1 = m_1 lc_1^2 + m_2 l_1^2 + J_1, \quad a_2 = m_2 l_1 lc_2,$$

$$a_3 = (m_1 lc_1 + m_2 l_1)g, \quad a_4 = m_2 lc_2^2 + J_2,$$

$$a_5 = m_2 g lc_2$$

The notation can be simplified as follows:

$$M_{(32)} = a_2 \sin[\varphi_1(t) - \varphi_2(t)] \dot{\varphi}_2(t), \quad M_{(33)} = a_1,$$

$$M_{(34)} = a_2 \cos[\varphi_1(t) - \varphi_2(t)], \quad M_{(41)} = -a_2 \sin[\varphi_1(t) - \varphi_2(t)] \dot{\varphi}_1(t),$$

$$M_{(43)} = M_{(34)}, \quad M_{(44)} = a_4.$$

Now define:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & M_{(32)} & M_{(33)} & M_{(34)} \\ M_{(41)} & 0 & M_{(43)} & M_{(44)} \end{bmatrix}, x(t) = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \\ \dot{\varphi}_1(t) \\ \dot{\varphi}_2(t) \end{bmatrix},$$

Where  $M_{(i,p)}, i, p \in N$  are the elements of the new matrix  $M$ . Then the TLM dynamical description Eq. (3-25) and (3-26) can be written as:

$$M\dot{x}(t) = \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ I_2 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ a_3 \sin \varphi_1(t) \\ a_5 \sin \varphi_2(t) \end{bmatrix}, \quad (3-27)$$

where  $I_2$  is the identity matrix on  $\mathfrak{R}^2$ . For all multi-link manipulator systems, including the TLM system, the matrix  $M$  is full rank, so that  $M^{-1}$  exists. Hence, Eq. (3-27) can be transformed into:

$$\dot{x}(t) = M^{-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} x(t) + M^{-1} \left( \begin{bmatrix} 0 \\ I_2 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ a_3 \sin \varphi_1(t) \\ a_5 \sin \varphi_2(t) \end{bmatrix} \right) \quad (3-28)$$

As  $\varphi_1(t), \varphi_2(t)$  are measured angles, the MRC design strategy can be simplified by decoupling the gravity terms  $a_3 \sin \varphi_1(t)$  and  $a_5 \sin \varphi_2(t)$  in Eq. (3-28), using as basic form of feedback linearization, with:

$$u(t) = u_s(t) - u_g(t) \quad (3-29)$$

where:

$$u_s(t) = \begin{bmatrix} u_{s1}(t) \\ u_{s2}(t) \end{bmatrix} \text{ and } u_g(t) = \begin{bmatrix} a_3 \sin \varphi_1(t) \\ a_5 \sin \varphi_2(t) \end{bmatrix}$$

It follows that Eq. (3-27) is simplified to the structure:

$$\dot{x}(t) = A[x(t)]x(t) + B[x(t)]u_s(t) \quad (3-30)$$

where:

$$A[x(t)] = M^{-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix}, B[x(t)] = M^{-1} \begin{bmatrix} 0 \\ I_2 \end{bmatrix},$$

This completes the TLM model system simplification procedure.

### 3.5.3 MRC-based FTC of TLM system

For this example, the matrices  $A[x(t)]$  and  $B[x(t)]$  of the plant model have the following structure:

$$A[x(t)] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A_{31}[x(t)] & A_{32}[x(t)] & A_{33}[x(t)] & A_{34}[x(t)] \\ A_{41}[x(t)] & A_{42}[x(t)] & A_{43}[x(t)] & A_{44}[x(t)] \end{bmatrix},$$

$$B[x(t)] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31}[x(t)] & B_{32}[x(t)] \\ B_{41}[x(t)] & B_{42}[x(t)] \end{bmatrix}$$

$A_{31}[x(t)], \dots, A_{44}[x(t)]$  and  $B_{31}[x(t)], \dots, B_{42}[x(t)]$  represent the elements in matrices  $A$  and  $B$  as functions of the plant states. The controllable reference model is obtained from the structure of the matrices  $A[x(t)]$  and  $B[x(t)]$  in Eq. (3-30). Once, a candidate set of model parameters is selected (e.g. as a single point in the linearization). A suitable reference model for this system is assumed to retain the structure of  $A[x(t)]$  and  $B[x(t)]$ . A suitable pair  $A_m B_m$  has been chosen with spectrum of  $\rho(A_m) = (-4, -6, -8, -10)$  as follows:

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -44.1098 & 14.2476 & -13.7422 & 2.0281 \\ 13.5021 & -47.8890 & 1.9389 & -14.2578 \end{bmatrix},$$

$$B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$$

For this example, the control problem is that of moving the two links to constant reference angles (corresponding to  $\varphi_1(t), \varphi_2(t)$ ) of 10 deg and 5 deg, respectively. As a special case (regulator tracking) problem for this example the solution for  $ref(t)$  is set to  $\dot{w}(t) = 0$  in (2). The initial values  $w(0)$  of the states  $w$  in Eq. (3-2) are also set to:

$\begin{bmatrix} w_1(0) \\ w_2(0) \\ w_3(0) \\ w_4(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ , which are also the equilibrium values  $w_e$  of  $w(t)$ . The required

reference signal  $ref(t)$  is thus given by

$$ref(t) = -B_m^{++} A_m w_e$$

with  $B_m^{++} = \begin{bmatrix} 0 & 0 & -4 & 3 \\ 0 & 0 & 3 & -2 \end{bmatrix}$

The solution for  $ref(t)$  is expressed in terms of the reference angles  $w_1$  and  $w_2$  as:

$$ref(t) = [Q] \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} -217 & 201 \\ 159.334 & -138.521 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \quad (3-31)$$

This reference model system is applied to the MRC-FTC design Eq. (3-10)-(3-13), and the control signal is given in terms of the pseudo-inverse  $B^{++}[x(t)]$  of  $B[x(t)]$  as:

$$u(t) = B^{++}[x(t)][-K_1(x(t))x(t) + K_2 ref(t)] - \hat{f}(t) - u_g(t) \quad (3-32)$$

Now consider a vector of actuator faults  $f(t)$  acting on the TLM system joints according to:

$$\dot{x}(t) = A[x(t)]x(t) + B[x(t)]u_s(t) + B[x(t)]f(t) \quad (3-33)$$

where  $f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$ , and:

$$f_1(t) = \begin{cases} 0 & 0s \leq t \leq 5s \\ 2sint & 5s \leq t \end{cases}, f_2(t) = \begin{cases} 0 & 0s \leq t \leq 10s \\ 2 & 10s \leq t \end{cases}$$

Note that  $B[x(t)]f(t)$  has the structure:

$$B[x(t)]f(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31}[x(t)] & B_{32}[x(t)] \\ B_{41}[x(t)] & B_{42}[x(t)] \end{bmatrix} f(t) \quad (3-34)$$

Hence, the term  $Ef(t)$  of Eq. (3-1) can be re-written, for this case, as  $Ef_{new}(t)$

$$\text{where } E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, f_{new}(t) = \begin{bmatrix} f_{new1}(t) \\ f_{new2}(t) \end{bmatrix} = B[x(t)]f(t),$$

This shows that although  $B[x(t)]$  is time-varying according to  $(t) \in \mathfrak{R}^4$ , the fault distribution can still be represented via a constant distribution matrix  $E$  operating on a transformed (but bounded) fault  $f_{new}(t)$ .  $f_{new}(t)$  is bounded since both  $B[x(t)]$  and  $f(t)$  are bounded and  $\hat{f}(t) = B^{++}[x(t)]$ . Giving the learning rate of the fault estimator  $W = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$  and by using the Matlab LMI toolbox a solution to Eq. (3-15) and Eq. (3-23) can be obtained as:

$$P = \begin{bmatrix} 64.6210 & 0.1039 & 4.5897 & 1.0857 \\ 0.1039 & 65.5793 & 1.0596 & 4.3703 \\ 4.5897 & 1.0596 & 0.4053 & 0.1678 \\ 1.0857 & 4.3703 & 0.1678 & 0.3676 \end{bmatrix}$$

$$L = \begin{bmatrix} 4.5897 & 1.0596 & 0.4053 & 0.1678 \\ 1.0857 & 4.3703 & 0.1678 & 0.3676 \end{bmatrix}, G = \begin{bmatrix} 0.0623 & 0.0471 \\ 0.0471 & 0.0504 \end{bmatrix}$$

Figure 3-3 shows the two faults estimates  $f_{e1}(t)$  and  $f_{e2}(t)$  soon converge to the real faults  $f_1(t)$  and  $f_2(t)$  after an oscillatory transient period.

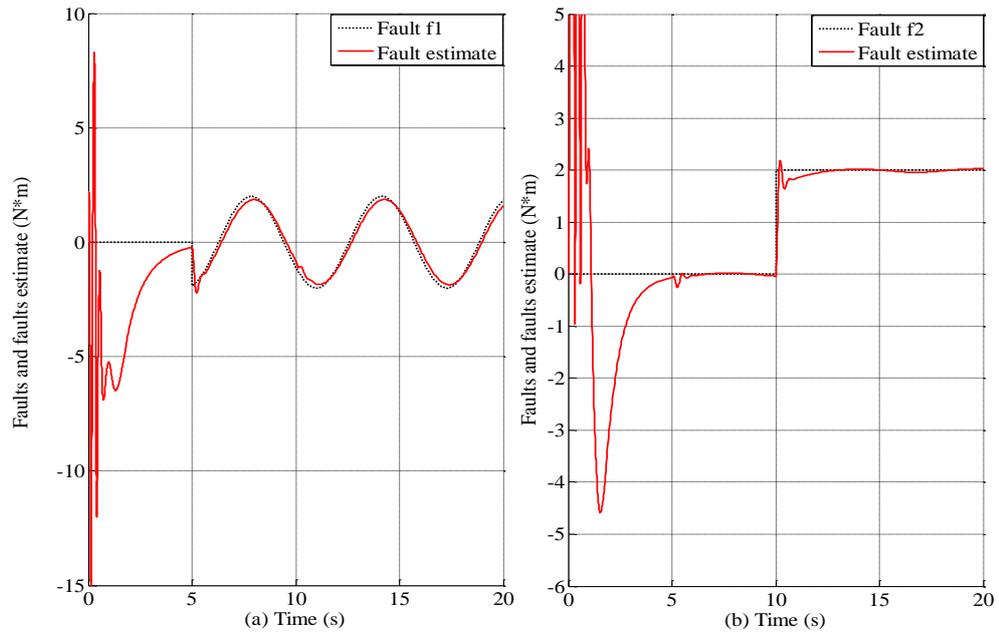


Figure 3-3: Faults  $f_1(t)$ ,  $f_2(t)$  and their estimates  $f_{e1}(t)$ ,  $f_{e2}(t)$

According to the inequality (3-22), the error between the actual fault and its estimate is bounded in a small region. Also the fault time derivative is bounded from the above mathematical analysis. From the Figure 3-3, the adopted fault derivative goes to infinity at  $t = 5s$  and  $t = 10s$ , and even if the bound requirement is a potential limitation of this method, the result still shows that good fault estimation is achieved as the fault is accurately estimated on-line. Some oscillations appear at the commencement of the simulation run, which are due to the initial errors between the reference model and the TLM state variables. However, these oscillations can be minimised by choosing the same initial conditions for the reference model and the TLM plant.

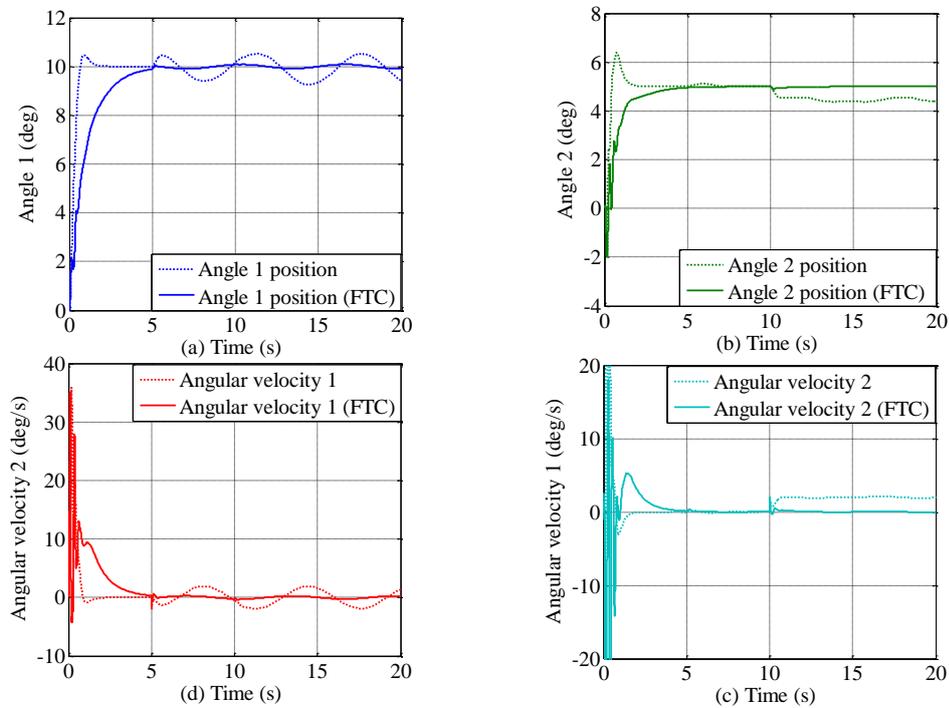


Figure 3-4: Fault TLM responses (initial conditions: 0; 0; 0; 0) with (solid) and without FTC action (dotted).

Figure 3-4 gives the TLM comparisons for the system state response of two cases (a) with FTC action applied and (b) without FTC action. Without the FTC the system state variables are strongly affected by the occurrence of faults. The angle and angular velocity for Joint-1 oscillate around the reference points, whilst for Joint-2, the angle and angular velocity follow their reference levels with steady-state following errors. It is clear that after compensation the angle state responses become almost independent of

the fault effects. From this study it is understood that the proportional term  $\dot{e}(t)$  (the estimation equivalent of proportional control) in  $\hat{f}(t) = WL(\dot{e}(t) + e(t))$  as introduced by (Zhang, Jiang and Cocquempot, 2008) provides good estimator design freedom for minimising this effect.

### 3.6 Conclusion

This Chapter proposes a strategy of active (direct) FTC for systems that have no unique equilibria, such as nonlinear systems and LPV systems, making use of fault estimation and compensation via MRC design. When all the required assumptions are satisfied the aim of the combined on-line fault estimation and compensation is described in terms of a pre-designed reference model. The reference model is used to derive the fault estimator parameters as well as the controller structure. The controller stability guarantee is provided by the stability of the reference model, whilst the estimator stability arises from the solution of an appropriate LMI-based Lyapunov condition. The fact that the estimator parameters are based on a reference model, rather than on the plant itself is an important improvement over existing methods. Furthermore, the approach is important for systems that have no unique equilibria i.e. that cannot be uniquely linearised.

This Chapter also describes an example of FTC for a non-linear TLM dynamical system with independently acting joint faults. The fault estimation errors are very small and good control compensation performance is demonstrated. Further research on this approach will inevitably involve a deeper understanding of robustness issues that may be applied to further enhance the performance of the method, which is equivalent to finding a suitable reference model with enough robustness against the disturbances and other system uncertainty.

In fact, the disturbance/uncertainty heavily affects the FE and FTC performances. Imprecise estimation results may risk the stability of the entire control system. As a result, the improvement of the robustness of the fault estimator is an important challenge. Based on this, in Chapter 4, a novel robust FE method is proposed, as well as the integration design to solve the joint robust FE and robust AFTC problem.

# **Chapter 4.**

## **Robust Fault Estimation and Fault Tolerant Control based on Observer-based Fault Estimator Method**

### **4.1 Introduction**

With the reference of the classification of FTC system given in Chapter 1, this Chapter is concerned with the active robust approach to FTC, involving robust fault estimation and robust fault compensation. This approach to achieve FTC obviates the need for reconfiguring or reconstructing the controller to tolerate the faults.

As stated in Chapter 2, there have been a significant amount of studies on FTC based on FE methods (Patton, 1997) (Wang and Daley, 1996) (Wang, Huang and Daley, 1997) (Edwards, Spurgeon and Patton, 2000) (Zhang, Jiang and Cocquempot, 2008) (Zhang and Jiang, 1999) (Rodrigues *et al*, 2005) (Patton and Klinkhieo, 2010) (Zhang, Jiang and Shi, 2009) (Gao and Ding, 2007) (Nie and Patton, 2011). Some of these deal with the tolerance to sensor faults and others deal with the FTC problem for actuator faults (Jiang and Staroswiecki, 2002) (Zhang, Jiang and Cocquempot, 2008) (Zhang, Jiang and Shi, 2009) (Nie and Patton, 2011).

In this Chapter, a novel FE method is proposed - the Robust Fault Estimation Method and FTC action based on it, using fault accommodation. This method is motivated by (Patton, Putra and Klinkhieo, 2010; Edwards, Spurgeon and Patton, 2000; Zhang, Jiang and Cocquempot, 2002; Zhu and Cen, 2010). Patton, Putra and Klinkhieo (2010) describe a novel ASO (augmented state observer) which considers the fault as an augmented state of the new system, but with the disadvantage of high gain and normally the actuator fault is not often a constant. Zhang, Jiang and Cocquempot (2008) proposed

the so-called fast adaptive fault estimator (FAFE) method which is outlined in Chapter 3. The FAFE approach does take into account the possibility of time-varying faults. However, the original work by Zhang, Jiang and Cocquempot (2008) does not prove the asymptotic convergence of the fault estimation error system. Edwards Supergon and Patton (2000) proposed the well known sliding mode observer-based fault estimator but this is only suited to cases in which the faults change slowly and this sliding mode estimation was not applied to the FTC problem. Zhu and Cen (2010) produced an interesting adaptive observer but without any fault estimation and FTC work. With this background in mind, the main contribution of this Chapter is to investigate the properties of this Robust Fault Estimation approach for FE and FTC.

The work of this Chapter considers two kinds of faults (1) actuator faults and (2) sensor faults. Based on this novel method, both faults can be estimated to improve the performance and stability of the system in control under the influence of the exogenous disturbance. Meanwhile, this novel approach to FE can also be used in the FDI problem, if required.

It is also very important to note that the FTC schemes proposed in this Chapter are adaptive systems as the on-line fault estimates are updated continuously and the estimates are used to compensate the faults acting within the control channels. The compensation is achieved within the observer estimation error system with the consequence that the control signal has a time-varying component, the adaptive part of the control. However, using on-line compensation means that the fault isolation task of FDI is not strictly required, although this function can be useful, and also can achieve the robustness to the disturbance for the simultaneous state and fault estimation.

This Chapter consists of 6 sections, Section 2 summarises the robust state and actuator fault estimate observer design. Section 3 considers the sensor fault based on the theorem set up in Section 2. Section 4 outlines the actuator and sensor FTC design based on the observer set up in Section 2. Section 5 describes a linear inverted pendulum tutorial example to illustrate the actuator and sensor fault estimation and FTC. Conclusions are given in the last Section. Moreover this Chapter also produces the foundation for the work described in Chapters 6 and 7.

## 4.2 Robust state and Fault Estimation observer

Consider a state space representation of a fault-free and disturbance-free (nominal) linear time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4-1)$$

$$y(t) = Cx(t)$$

Where  $x(t) \in \mathfrak{R}^n$  is the state vector,  $y(t) \in \mathfrak{R}^p$  is the output vector,  $u(t) \in \mathfrak{R}^m$  is the input control signal vector,  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times m}$ ,  $C \in \mathfrak{R}^{p \times n}$ , are all known appropriate dimension matrices, the matrix pair  $(A, B)$  is assumed controllable and the pair  $(A, C)$  is assumed to be observable.

When the nominal system Eq. (4-1) is affected by actuator faults and external disturbance simultaneously, the original system is now described as:

$$\dot{x}(t) = Ax(t) + Bu(t) + B_f f_a(t) + Ed(t) \quad (4-2)$$

$$y(t) = Cx(t)$$

where  $f_a(t) \in \mathfrak{R}^q$  and  $d(t) \in \mathfrak{R}^r$  are the actuator faults and exogenous disturbance vectors, respectively.  $B_f \in \mathfrak{R}^{n \times q}$ ,  $E \in \mathfrak{R}^{n \times r}$  are known real constant matrices. It is very important to know that for solving robust FDI problems, a mathematical representation for expression of modelling uncertainty is required. Patton and Chen (Patton and Chen, 1992, 1993; Patton, Chen and Zhang, 1992; Chen, 1995) provide several methodologies to represent modelling uncertainties in structured format from various sources, as additive disturbances with an estimated distribution matrix (Chen and Patton, 1999). It can be concluded that from a mathematical point of view, the expression of the modelling uncertainty has the same effect in the system as the disturbance. As a result, a general description for system uncertainty and disturbance is expressed in a form of one distribution matrix  $E$  multiplying the disturbance or uncertainty vector, i.e.  $Ed(t)$ , such as in Eq. (4-2) is no loss of generality. This Section is concerned with the robust fault estimator design. Hence, the following Assumptions can be made:

**Assumption 4.1:**  $Rank(B_f) = q$ ,  $Rank(E) = r$  and  $rank(B_f \quad E) = q + r \leq p$ .

**Assumption 4.2:** The norms of  $d(t)$ ,  $f_a(t)$  and its first derivative of  $\dot{f}_a(t)$  are bounded such that:

$\|d(t)\| \leq d_1$ ,  $\|f_a(t)\| \leq d_2$ ,  $\|\dot{f}_a(t)\| \leq d_3$ , for all  $t \geq 0$ .  $d_1, d_2, d_3$  are known positive constants, namely,  $d_1 \geq 0, d_2 \geq 0, d_3 \geq 0$   $d_1, d_2, d_3 \in \mathfrak{R}$ .

A full-order state observer for system Eq. (4-2) using output information can be designed as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_f \hat{f}_a(t) + \eta[y(t), \hat{x}(t)] + L[y(t) - \hat{y}(t)] \quad (4-3)$$

$$\hat{y}(t) = Cx(t) \quad (4-4)$$

where  $\eta[y(t), \hat{x}(t)]$  is a nonlinear design function whose design is given under Theorem 4.1. The error dynamic system between the plant and the observer is then represented as:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + B_f e_f(t) + E(d) - \eta[y(t), \hat{x}(t)] - L[y(t) - \hat{y}(t)] \\ &= (A - LC)e(t) + B_f e_f(t) + Ed(t) - \eta[y(t), \hat{x}(t)] \end{aligned} \quad (4-5)$$

$$e_y(t) = Ce(t) \quad (4-6)$$

where  $e(t) = x(t) - \hat{x}(t)$  and  $e_f(t) = f_a(t) - \hat{f}_a(t)$ . The fault estimator system can be stated mathematically as:

$$\dot{\hat{f}}_a(t) = WF_1 e_y(t) \quad (4-7)$$

Where  $e_y(t)$  is calculated by  $y(t) - \hat{y}(t)$ ,  $W$  is the learning rate,  $F_1$  is the fault estimator gain matrix to be designed. Meanwhile, it should be noted that an estimate of the actuator fault  $\hat{f}_a(t)$  can easily be derived by taking the integral of both sides of Eq. (4-7), so that  $\hat{f}_a(t) = \int_0^t WF_1 e_y(t) dt$ .

The idea behind Eq. (4-5) is easy to grasp, if  $\eta[y(t), \hat{x}(t)]$  has an effect on reducing the influence from the exogenous disturbance for states and fault estimates, then the error dynamics of Eq. (4-5) becomes robust. Theorem 4.1 is established following a motivation by the observer designed by Zhu and Cen (2010). It is noted that in the work

of (Zhu and Cen, 2010) an adaptive and robust full-order observer was constructed, the robustness is shown by the special design for the term  $\eta[y(t), \hat{x}(t)]$ , similar to the discontinuous control component used in sliding mode control law (Edwards and Spurgeon, 1998). However, Zhu and Cen design the observer for the fault-free case which is not the concept described here since the current work is based on the use of the term  $\eta[y(t), \hat{x}(t)]$ .

**Theorem 4.1:** Under Assumptions 4.1-4.2, if there exist symmetric positive definite matrices  $P \in \mathfrak{R}^{n \times n}$ ,  $R \in \mathfrak{R}^{n \times n}$  and matrices  $Y \in \mathfrak{R}^{n \times p}$ ,  $F_1 \in \mathfrak{R}^{q \times p}$  and  $F_2 \in \mathfrak{R}^{r \times p}$ , such that the following conditions hold:

$$PA - YC + A^T P - C^T Y^T = -R < 0, \quad (4-8)$$

$$B_f^T P = F_1 C \quad (4-9)$$

$$E^T P = F_2 C \quad (4-10)$$

where  $Y = PL$ . The robust full-order observer determined by Eq. (4-3), (4-4), (4-7) has a non-linear function the term  $\eta[y(t), \hat{x}(t)]$  of the state estimates and output measurements, is as follows:

$$\begin{aligned} \eta[y(t), \hat{x}(t)] = & \frac{d_1 E F_2 e_y(t)}{\|F_2 e_y(t)\|} + \frac{E F_2 e_y(t)}{2 \|F_2 e_y(t)\|^2} [ad_3^2 \lambda_{\max}(W^{-1} G^{-1} W^{-1}) \\ & + \frac{1}{\alpha} \lambda_{\max}(G + \tau I) (d_2 + \|\int_0^t W F_1 e_y(t) dt\|)^2] \end{aligned} \quad (4-11)$$

where  $G \in \mathfrak{R}^{q \times q}$  is a design matrix,  $G^{-1}$  is the inverse of the matrix  $G$ ,  $\tau$  is a small positive constant to ensure when the time  $t$  goes to infinity the state estimation  $\hat{x}(t)$  and fault estimation  $\hat{f}_a(t)$  converge asymptotically to the actual state  $x(t)$  and the actuator fault  $f_a(t)$ , respectively. The term  $\|\int_0^t W F_1 e_y(t) dt\|$  is the  $L_2$  norm of the integral  $\int_0^t W F_1 e_y(t) dt$ , namely the norm of the fault estimate  $\hat{f}_a(t)$ .

**Proof:**

Consider a Lyapunov function candidate:

$$V(t) = e^T(t) P e(t) + e_f^T(t) W^{-1} e_f(t) \quad (4-12)$$

The derivative of  $V$  along with the error dynamic system (4-5), (4-6) is:

$$\begin{aligned}
\dot{V}(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + 2e_f^T(t)W^{-1}\dot{e}_f(t) \\
&= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PB_f e_f(t) \\
&\quad + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\
&\quad + 2e_f^T(t)W^{-1}\dot{f}_a(t) - 2e_f^T(t)W^{-1}\dot{\hat{f}}_a(t)
\end{aligned} \tag{4-13}$$

By **Lemma 3.1** it is easy to show that:

$$\begin{aligned}
2e_f^T(t)W^{-1}\dot{f}_a(t) &\leq \frac{1}{a}e_f^T(t)Ge_f(t) + a\dot{f}_a^T(t)W^{-1}G^{-1}W^{-1}\dot{f}_a(t) \\
&\leq \frac{1}{a}e_f^T(t)Ge_f(t) + a\|\dot{f}_a(t)\|^2\lambda_{\max}(W^{-1}G^{-1}W^{-1})
\end{aligned} \tag{4-14}$$

where  $G$  is a symmetric positive matrix and  $a$  is a positive constant chosen appropriately by the designer. According to Eq. (4-14) and substituting Eq. (4-7) into Eq. (4-13), then Eq. (4-13) becomes:

$$\begin{aligned}
\dot{V}(t) &\leq e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PB_f e_f(t) \\
&\quad + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\
&\quad + \frac{1}{a}e_f^T(t)Ge_f(t) + a\|\dot{f}_a(t)\|^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) \\
&\quad - 2e_f^T(t)F_1Ce(t)
\end{aligned} \tag{4-15}$$

where  $\lambda_{\max}(\ast)$  denotes the largest eigenvalue of the matrix defined in the space  $(\ast)$ . In order to obtain the appropriate quadratic form to prove the Lyapunov stability of the system of Eqs. (4-5) and (4-6), a subtle mathematical transformation can be made by adding a positive term  $\frac{1}{a}e_f^T(t)\tau Ie_f(t)$  to the inequality (4-15) to change its structure, then the inequality (4-15) becomes:

$$\begin{aligned}
\dot{V}(t) &\leq e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PB_f e_f(t) \\
&\quad + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] + \frac{1}{a}e_f^T(t)(G + \tau I)e_f(t)
\end{aligned}$$

$$+a\|\dot{f}_a(t)\|^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) - 2e_f^T(t)F_1Ce(t) \quad (4-16)$$

To maintain the same evaluation, on subtracting the term  $\frac{1}{a}e_f^T(t)\tau Ie_f(t)$  from inequality (4-16), then inequality (4-16) becomes:

$$\begin{aligned} \dot{V}(t) &\leq e^T(t)[(A-LC)^TP + P(A-LC)]e(t) + 2e^T(t)PB_f e_f(t) \\ &\quad + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + \frac{1}{a}e_f^T(t)(G + \tau I)e_f(t) - \frac{1}{a}e_f^T(t)\tau Ie_f(t) \\ &\quad + a\|\dot{f}_a(t)\|^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) - 2e_f^T(t)F_1Ce(t) \end{aligned} \quad (4-17)$$

Since  $B_f^TP = F_1C$  from Eq. (4-9), then the terms  $2e^T(t)PB_f e_f(t)$  and  $-2e_f^T(t)F_1Ce(t)$  sum to zero, hence:

$$\begin{aligned} \dot{V}(t) &\leq e^T(t)[(A-LC)^TP + P(A-LC)]e(t) + 2e^T(t)PEd(t) \\ &\quad - 2e^T(t)P\eta[y(t), \hat{x}(t)] + \frac{1}{a}e_f^T(t)(G + \tau I)e_f(t) \\ &\quad - \frac{1}{a}e_f^T(t)\tau Ie_f(t) + a\|\dot{f}_a(t)\|^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) \end{aligned} \quad (4-18)$$

As  $\|\dot{f}_a(t)\| \leq d_3$  (**Assumption 4.2**), and by defining a vector  $\theta$  as  $\theta(t) = \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix}$ ,

Eq.(4-18) can now be rewritten as:

$$\begin{aligned} \dot{V}(t) &\leq \theta^T(t)\Psi\theta(t) + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + ad_3^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) + \frac{1}{a}e_f^T(t)(G + \tau I)e_f(t) \end{aligned} \quad (4-19)$$

where  $\Psi = \begin{bmatrix} P(A-LC) + (A-LC)^TP & 0 \\ 0 & -\frac{1}{a}\tau I \end{bmatrix}$ .

Also noting that  $\|e_f(t)\| = \|f_a(t) - \hat{f}_a(t)\| \leq \|f_a(t)\| + \|\hat{f}_a(t)\| = d_2 + \|\hat{f}_a(t)\|$  which further indicates that:

$$\begin{aligned}
\frac{1}{a} e_f^T(t)(G + \tau I)e_f(t) &\leq \frac{1}{a} \lambda_{\max}(G + \tau I) \|e_f(t)\|^2 \\
&\leq \frac{1}{a} \lambda_{\max}(G + \tau I) (d_2 + \|\hat{f}_a(t)\|)^2
\end{aligned} \tag{4-20}$$

Then Eq. (4-20) is transformed to:

$$\begin{aligned}
\dot{V}(t) &\leq \theta^T(t)\Psi\theta(t) + 2d_1\|E^T P e(t)\| - 2e^T(t)PE\eta[y(t), \hat{x}(t)] \\
&\quad + ad_3^2\lambda_{\max}(W^{-1}G^{-1}W^{-1}) + \frac{1}{a}\lambda_{\max}(G + \tau I)(d_2 + \|\hat{f}_a(t)\|)^2
\end{aligned} \tag{4-21}$$

Replace  $\eta[y(t), \hat{x}(t)]$  by the right hand side of Eq. (4-11) into Eq. (4-21), and the following result is obtained:

$$\dot{V}(t) \leq \theta^T(t)\Psi\theta(t) < 0 \tag{4-22}$$

That is because, on the basis of Eq. (4-8),  $P(A - LC) + (A - LC)^T P = PA - YC + A^T P - C^T Y^T < 0$  where  $Y = PL$ , and  $-\frac{1}{a}\tau I < 0$ . So that  $\Psi < 0$  which means that

$\theta(t) = \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix}$  converges asymptotically to zero. On the other hand, the state and the fault estimates track the trajectories of the plant states and actuator faults, respectively.

Q.E.D.

**Remark 4.1:** For a single input system  $B_f = B$ , the matrix  $B$  replaces  $B_f$  during the observer design. For multi-input systems, faults may occur in several actuators, at this time the matrix  $B_f$  is a linear subspace of the matrix  $B$ .

**Remark 4.2:** When no fault occurs, i.e.  $f_a = 0$ , the proposed robust observer by Eq. (4-3), (4-4), (4-7) and (4-11) with removed fault estimation term  $\hat{f}_a(t)$ , is robust in the sense of rejecting exogenous disturbances. This is consistent with the work of Zhu and Cen (2010). When a fault occurs, as the matrix  $B_f$  is different from the matrix  $E$ , and their columns are linearly independent respectively, the term  $\eta[y(t), \hat{x}(t)]$  only has the effect of rejecting the disturbances rather than rejecting the actuator fault signal. The consequence of this is that the system output  $y$  is only perturbed by the actuator fault  $f_a$ . Hence, the output error  $e_y$  does not approach zero asymptotically. The proposed observer Eqs. (4-3), (4-4), (4-7) and (4-11) can thus serve as a *fault detection observer*.

The logical fault/no-fault classification is given as follows by checking the system output residual  $e_y(t)$ :

$$\begin{cases} \|e_y(t)\| \leq \rho & \text{no actuator fault occurs} \\ \|e_y(t)\| > \rho & \text{actuator fault occurs} \end{cases}$$

where  $\rho$  is a threshold designed for fault detection, i.e. to avoid false alarms. Meanwhile, when the actuator fault weighting matrix  $B_f$  is unknown, the weighting matrix  $B_f$  in the observer structure can be taken by directly using the input matrix  $B$ . As the fault estimate shows the fault magnitude and identifies the location of the fault, it can also be considered as a *robust fault isolation observer*.

**Remark 4.3:** From a theoretical point of view, when  $e_y(t) = 0$ ,  $\eta[y(t), \hat{x}(t)]$  has no significance where its denominator is equal to zero. For this case the observer dynamics are the same as the plant dynamics. Under this situation, no further action is required. To take care of this condition the nonlinear law  $\eta[y(t), \hat{x}(t)]$  is always chosen as:

$$\eta[y(t), \hat{x}(t)] =$$

$$\left\{ \begin{array}{ll} \frac{(d_1 + \partial)EF_2e_y(t)}{\|F_2e_y(t)\|} + \frac{EF_2e_y(t)}{2\|F_2e_y(t)\|^2} [ad_3^2\lambda_{\max}(W^{-1}G^{-1}W^{-1})] & \text{if } e_y(t) \neq 0 \\ + \frac{1}{a}\lambda_{\max}(G + \tau I)(d_2 + \|\int_0^t WF_1e_y(t)dt\|^2) & \\ 0 & \text{if } e_y(t) = 0 \end{array} \right\} \quad (4-23)$$

where  $\partial$  is a small positive constant. Hence Eq. (4-23) can be used instead of Eq. (4-11) when  $e_y(t) = 0$ . It is also important to point out that the inequality  $P(A - LC) + (A - LC)^T P < 0$  is nonlinear, and it is very difficult to find solutions satisfying the nonlinear inequality (due to lack of convexity). However, by setting  $Y = PL$ , this inequality can be transformed into the form of Eq. (4-8), which then leads to an LMI problem.

Although inequality (4-8) can be solved efficiently using the MatLab LMI toolbox, difficulties arise when solving the inequality (4-8), Eqs. (4-9) and (4-10) simultaneously. It turns out that a simultaneous solution of inequality (4-8), Eq. (4-9) and (4-10) cannot be guaranteed. However, this problem can be converted into the following optimization

problem (Zhang, Jiang and Cocquempot, 2008; Corless and Tu, 1998), for which the solution is more straightforward.

This procedure is summarised as follows. Solve the following two LMIs, and thereby minimize  $J$  subject to inequality (4-8):

$$\begin{bmatrix} -I & F_1 C - B^T P \\ (F_1 C - B^T P)^T & -I \end{bmatrix} < 0 \quad (4-24)$$

$$\begin{bmatrix} -I & F_2 C - E^T P \\ (F_2 C - E^T P)^T & -I \end{bmatrix} < 0 \quad (4-25)$$

The matrices  $F_1, F_2, P, Y$  can be determined by solving inequalities (4-8), (4-24) and (4-25) simultaneously. The final observer gain matrix is then given by  $L = P^{-1}Y$ , where  $P^{-1}$  is the inverse of the S.P.D. matrix  $P$ .

### 4.3 Robust state and sensor fault estimation

All the results obtained in Section 4.2 apply to the actuator fault case. In this Section the proposed observer Eqs. (4-3), (4-4), (4-7) and robust design strategy involving Eq. (4-11) are extended to deal with the sensor fault case. In this part, consider the system with additive sensor faults described as:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4-26)$$

$$y(t) = Cx(t) + Df_s(t) + Ed(t) \quad (4-27)$$

Where  $f_s(t) \in \mathfrak{R}^s$  represents the sensor fault vector. The matrices  $A, B$  and  $C$  defined as for Eq. (4-1) and Eq. (4-2).  $D \in \mathfrak{R}^{p \times s}$  is a full column rank known constant real weighting matrix.

By constructing an augmented system (Edwards, 2004) (Edwards and Tan, 2006), the previous results can be extended to the sensor fault estimation problem. Consider a new state  $z_s(t) \in \mathfrak{R}^{l \times l}$  that is a filtered version of  $y(t)$  as:

$$\dot{z}_s(t) = -A_s z_s(t) + A_s [y(t)]$$

$$= -A_s z_s(t) + A_s C x(t) + A_s D f_s(t) + A_s E d(t) \quad (4-28)$$

$$y_s(t) = C_s z_s(t) \quad (4-29)$$

where  $-A_s \in \mathfrak{R}^{p \times p}$  is a stable matrix and  $C_s = I_p$ . Then the augmented system can be written as:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}_s(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -A_s C & -A_s \end{bmatrix} \begin{bmatrix} x(t) \\ z_s(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ A_s E \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ A_s D \end{bmatrix} f_s(t) \quad (4-30)$$

$$\begin{bmatrix} y(t) \\ y_s(t) \end{bmatrix} = \begin{bmatrix} 0 & C_s \end{bmatrix} \begin{bmatrix} x(t) \\ z_s(t) \end{bmatrix} \quad (4-31)$$

Also denote:

$$\bar{x}(t) = [x(t)^T \quad z_s(t)^T]^T$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -A_s C & -A_s \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} 0 \\ A_s E \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} 0 \\ A_s D \end{bmatrix}, \bar{C} = [0 \quad I_p];$$

$$\bar{y}(t) = [y(t)^T \quad y_s(t)^T]^T$$

Then Eq. (4-26) and (4-27) can be rewritten in compact notation as:

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u + \bar{D}f_s(t) + \bar{E}d(t) \quad (4-32)$$

$$\bar{y}(t) = \bar{C}\bar{x}(t) \quad (4-33)$$

Then the sensor fault of the original system appears as an ‘actuator fault’ and so the proposed actuator fault estimation approach described earlier can be adopted.

**Remark 4.4:** In the augmented system, the control pair  $(\bar{A}, \bar{B})$  is controllable as  $(A, B)$  is controllable, and the observable pair  $(\bar{A}, \bar{C})$  is observable as  $(A, C)$  is observable, these properties are important in the FTC problem. The proof is omitted here.

Before giving **Theorem 4.2**, it is necessary to make the following Assumptions:

**Assumption 4.3:**  $\text{Rank}(\bar{D}) = s$ ,  $\text{Rank}(\bar{E}) = r$  and  $\text{Rank}(\bar{D} \quad \bar{E}) = s + r \leq p$ .

**Assumption 4.4:** The norms of  $d(t)$ , the sensor fault  $f_s(t)$  and its first derivative of  $\dot{f}_s(t)$  are bounded such that:

$\|d(t)\| \leq d_1$ ,  $\|f_s(t)\| \leq d_2$ ,  $\|\dot{f}_s(t)\| \leq d_3$  for all  $t \geq 0$ , which  $d_1 \geq 0$ ,  $d_2 \geq 0$ , and  $d_3 \geq 0$ , where  $d_1, d_2, d_3 \in \mathfrak{R}$ .

**Theorem 4.2:** Under Assumptions 4.3 and 4.4, if there exist S.P.D. matrices  $\bar{P} \in \mathfrak{R}^{(n+p) \times (n+p)}$ ,  $\bar{R} \in \mathfrak{R}^{(n+p) \times (n+p)}$  and matrices  $\bar{Y} \in \mathfrak{R}^{(n+p) \times p}$ ,  $\bar{F}_1 \in \mathfrak{R}^{s \times p}$  and  $\bar{F}_2 \in \mathfrak{R}^{s \times p}$ , such that the following conditions hold:

$$\bar{P}\bar{A} - \bar{Y}\bar{C} + \bar{A}^T\bar{P} - \bar{C}^T\bar{Y}^T = -\bar{R} < 0 \quad (4-34)$$

$$\bar{B}^T\bar{P} = \bar{F}_1\bar{C} \quad (4-35)$$

$$\bar{E}^T\bar{P} = \bar{F}_2\bar{C} \quad (4-36)$$

where  $\bar{Y} = \bar{P}\bar{L}$ , then the augmented robust state and fault estimator (the notations are given in Section 4.2) is:

$$\dot{\hat{x}}(t) = \bar{A}\hat{x}(t) + \bar{B}u + \bar{D}\hat{f}_s(t) + \bar{\eta}[\bar{y}(t), \hat{x}(t)] + \bar{L}[\bar{y}(t) - \hat{y}(t)] \quad (4-37)$$

$$\hat{y}(t) = \bar{C}\hat{x}(t) \quad (4-38)$$

$$\hat{f}_s(t) = \bar{W}\bar{F}_1\bar{e}_y(t) \quad (4-39)$$

$$\begin{aligned} \bar{\eta}[\bar{y}(t), \hat{x}(t)] = & \frac{d_1\bar{E}\bar{F}_2\bar{e}_y(t)}{\|\bar{F}_2\bar{e}_y(t)\|} + \frac{\bar{E}\bar{F}_2\bar{e}_y(t)}{2\|\bar{F}_2\bar{e}_y(t)\|^2} [ad_3^2\lambda_{\max}(\bar{W}^{-1}\bar{G}^{-1}\bar{W}^{-1}) \\ & + \frac{1}{a}\lambda_{\max}(\bar{G} + \bar{\tau}I)(d_2 + \|\int_0^t \bar{W}\bar{F}_1\bar{e}_y(t)dt\|)^2] \end{aligned} \quad (4-40)$$

where  $\bar{e}_y(t) = \bar{y}(t) - \hat{y}(t)$ , then this estimator will guarantee that the estimates of the states and actuator faults converge asymptotically to the real states and the actuator faults of the augmented system Eq. (4-32) and (4-33), providing a robust realization of the states and sensor faults estimates of the original system Eq. (4-26) and (4-27). The proof is similar to that given in **Theorem 4.1** and is omitted here.

**Remark 4.5:** In a similar manner to what is described in **Remark 4.2-4.3**. This novel robust state and fault observer design can also be using in sensor fault detection and also in sensor fault compensation. In practice, the sliding control law is chosen as ‘larger than the bound’ to achieve the robustness. As a matter of fact the magnitude of  $\partial$  chosen may affect the accuracy of the estimated results. It should be chosen as small as possible depending on the design requirements.

## 4.4 Robust fault tolerant control

### 4.4.1 Robust actuator fault tolerant control

According to the state and fault estimates achieved in Section 4.2, it is natural to consider the FTC design by using this information. In this Section an integrated robust actuator FTC law design is given based on the proposed robust actuator fault estimator principle. Figure 4-1 shows the proposed FTC strategy.

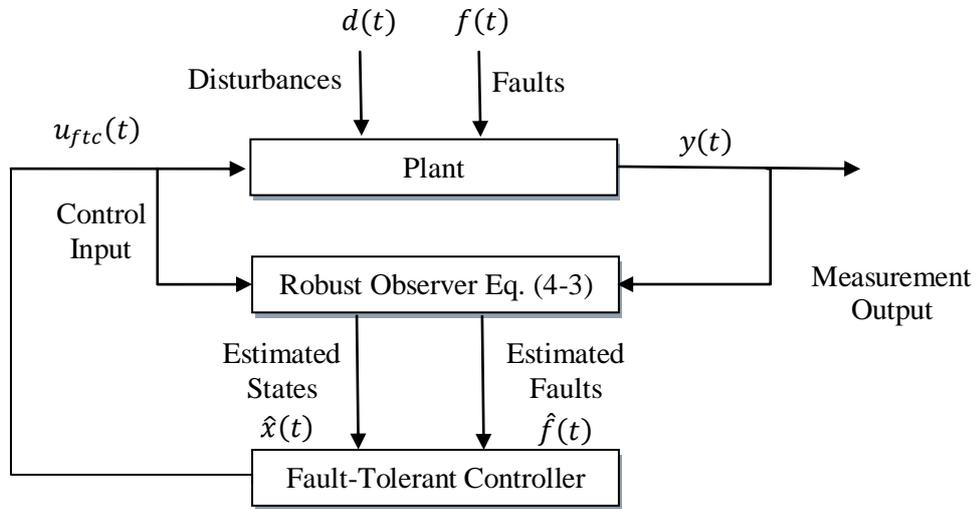


Figure 4-1: Fault estimation and fault-tolerant control

Now consider the system Eq. (4-2). Once again, the full state-feedback control law is given as:

$$u_n(t) = -Kx(t) + r(t) \quad (4-41)$$

where  $K \in \mathfrak{R}^{m \times n}$  is the feedback gain matrix and  $r(t) \in \mathfrak{R}^m$  is the reference input.

Consider the fault-free situation ( $f_a(t) = 0$ ), (assuming that  $r(t) = 0$ ). Substituting Eq. (4-41) into the Eq. (4-2) the closed-loop system description is obtained as:

$$\dot{x}(t) = (A - BK)x(t) + Ed(t) \quad (4-42)$$

Then Theorem 4.3 can be stated as:

**Theorem 4.3:** The closed-loop system Eq. (4-42) is asymptotically stable, and  $\|T_{dy}\|_\infty < \gamma$ , if there exist a symmetric positive definite matrix  $Q \in \mathfrak{R}^{n \times n}$ , and a matrix  $K \in \mathfrak{R}^{m \times n}$  such that the following condition hold:

$$\begin{bmatrix} Q(A + BK) + (A + BK)^T Q & QE & I_n \\ E^T Q & -\gamma I_r & 0 \\ I_n & 0 & -\gamma I_n \end{bmatrix} < 0 \quad (4-43)$$

As the above inequality is nonlinear, by pre-multiplying and post-multiplying  $\text{diag}[\gamma^{1/2}Q^{-1} \ \gamma^{1/2}I_r \ \gamma^{1/2}I_n]$  in inequality Eq. (4-43) and letting  $X = \gamma Q^{-1}$ , Theorem 4.4 can then be stated as:

**Theorem 4.4:** The closed-loop system Eq. (4-42) is asymptotically stable, and  $\|T_{dy}\|_\infty < \gamma$ , if there exist a S.P.D. matrix  $X \in \mathfrak{R}^{n \times n}$ , and a matrix  $Z \in \mathfrak{R}^{m \times n}$  such that the following condition holds:

$$\begin{bmatrix} AX + BZ + (AX + BZ)^T & E & X^T \\ E^T & -\gamma^2 I_r & 0 \\ X & 0 & -I_n \end{bmatrix} < 0 \quad (4-44)$$

where  $\|T_{dy}\|_\infty$  is the  $L_2$  gain of the transfer function from the exogenous disturbance  $d$  to the system output  $y$ , and  $\gamma > 0$  is the prescribed  $H_\infty$  performance. Furthermore, if a feasible solution  $(X, Z)$  exists in the above LMIs, the state-feedback gain can be computed as  $K = ZX^{-1}$ . This is the well known robust  $H_\infty$  State Feedback Control Theorem (Yu, 2002).

However, the state vector  $x(t)$  is not always available, and the estimated value  $\hat{x}(t)$  is substituted for  $x(t)$ . So the observer-based state estimated feedback controller is:

$$u_{no}(t) = -K\hat{x}(t) \quad (4-45)$$

Motivated by the observer system of Eqs. (4-3), (4-4), (4-7) and (4-11), the observer-based robust FTC system is constructed as:

$$\left. \begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \eta(y, \hat{x}) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= Cx(t) \\ \dot{\hat{f}}_a(t) &= WF_1 e_y(t) \\ \eta[y(t), \hat{x}(t)] &= \frac{d_1 EF_2 e_y(t)}{\|F_2 e_y(t)\|} + \frac{EF_2 e_y(t)}{2\|F_2 e_y(t)\|^2} [ad_3^2 \lambda_{\max}(W^{-1}G^{-1}W^{-1})] \\ &\quad + \frac{1}{a} \lambda_{\max}(G + \tau I) (d_2 + \|\int_0^t WF_1 e_y(t) dt\|)^2 \\ u_{FTC}(t) &= u_{no}(t) - B^{++} B_f \hat{f}_a(t) \end{aligned} \right\} \quad (4-46)$$

where  $B^{++} = B^T(BB^T)^{-1}$  is the right pseudo inverse of the matrix of  $B$ . Substituting the FTC law  $u_{FTC}(t)$  of Eq. (4-46) into Eq. (4-2), one obtains:

$$\begin{aligned} \dot{x}(t) &= Ax(t) - BK\hat{x}(t) + B_f e_f(t) + Ed(t) \\ &= (A - BK)x(t) + BKe(t) + B_f e_f(t) + Ed(t) \end{aligned} \quad (4-47)$$

It is natural to question whether the controller and observer can still be designed independently, because of the existence of the fault and uncertainty. It is clear that the well known Separation Principle is no longer satisfied. However, the controller and observer designs can still be made separately (Gao and Ding, 2007) (Zhang, Jiang and Shi, 2009) to preserve:

(1) The asymptotically stability of the closed-loop system Eq. (4-47).

(2)  $\|T_{dy}\|_{\infty} < \gamma$ ,

Theorem 4.5 can then be stated as:

**Theorem 4.5:** Under the Assumptions 4.1 and 4.2, the closed-loop system Eq. (4-47) is asymptotically stable or fault tolerant and  $\|T_{dy}\|_{\infty} < \gamma$ , if Theorems 4.1 and 4.4 hold.

**Stability proof:**

Consider the following Lyapunov function

$$V_c(t) = x^T(t)Qx(t) \quad (4-48)$$

where  $Q = X^{-1} \in \mathfrak{R}^{n \times n}$ .

Then the derivative of  $V_c(t)$  with respect to time is obtained as:

$$\begin{aligned}
\dot{V}_c(t) &= x^T(t)(Q(A - BK) + (A - BK)^T Q)x(t) + 2x^T(t)QB e_f(t) \\
&\quad + 2x^T(t)QBKe(t) + 2x^T(t)QE d(t) \\
&= x^T(t)\Phi x(t) + 2x^T(t)QB e_f(t) + 2x^T(t)QBKe(t) \\
&\quad + 2x^T(t)QE d(t)
\end{aligned} \tag{4-49}$$

where  $\Phi = Q(A - BK) + (A - BK)^T Q$ .

By choosing  $\beta = \lambda_{\min}(-\Phi)$  Eq. (4-49) becomes:

$$\begin{aligned}
\dot{V}_c(t) &\leq -\beta \|x(t)\|^2 + 2x^T(t)QB e_f(t) + 2x^T(t)QBKe(t) \\
&\quad + 2x^T(t)QE d(t)
\end{aligned} \tag{4-50}$$

Consider a new Lyapunov function as:

$$V_w(t) = V_c(t) + \mu V_o(t) \tag{4-51}$$

where  $\mu$  is a positive scalar and  $V_o(t) = e^T(t)Pe(t) + e_f^T(t)W^{-1}e_f(t)$ , according to the proof of Theorem 4.1 and Inequality. (4-50), the derivative of  $V_w(t)$  can be described by:

$$\begin{aligned}
\dot{V}_w(t) &\leq -\beta \|x(t)\|^2 + \mu \theta^T(t)\Psi \theta(t) + 2x^T(t)QB e_f(t) + 2x^T(t)QBKe(t) \\
&\quad + 2x^T(t)QE d(t)
\end{aligned} \tag{4-52}$$

Setting

$$\alpha = \lambda_{\min}(-\Psi),$$

$$\varepsilon = \|[QBK \quad QB]\|,$$

Thus, for the zero disturbance case,  $\dot{V}_w(t)$  can be rewritten as:

$$\begin{aligned}
\dot{V}_w(t) &\leq -\mu\alpha\|\theta(t)\|^2 - \beta\|x(t)\|^2 + 2\varepsilon\|x(t)\| \cdot \|\theta(t)\| \\
&= -\mu\alpha\left(\|\theta(t)\|^2 - \frac{2\varepsilon}{\mu\alpha}\|x(t)\| \cdot \|\theta(t)\| + \frac{\varepsilon^2}{\mu^2\alpha^2}\|x(t)\|^2\right) \\
&\quad + \frac{\varepsilon^2}{\mu^2\alpha^2}\|x(t)\|^2 - \beta\|x(t)\|^2 \\
&\leq -\frac{1}{\mu\alpha}(\mu\alpha\beta - \varepsilon^2)\|x(t)\|^2
\end{aligned} \tag{4-53}$$

and choosing  $\mu > \frac{\varepsilon^2}{\alpha\beta}$ , one obtains  $\dot{V}_w(t) < 0$  such that the system Eq. (4-47) is asymptotically stable.

**Remark 4.6:** With the situation of the disturbance absent from the system, the state and fault estimation error  $e(t)$  and  $e_f(t)$  affect the Separation Principle. Furthermore, imprecise estimation may cause the error to become large, which may induce a potential stability problem (especially during a transient). Under the situation of the absent disturbance, the switching function term in the observer can be removed, and the proposed observer-based estimator is changed to a conventional one (See Section 2.4).

Furthermore, when the disturbances exists, the  $H_\infty$  performance index proof is given below.

The proof of the guaranteed performance index follows by considering:

$$H = \dot{V}_w(t) + x(t)^T x(t) - \gamma^2 d(t)^T d(t) \tag{4-54}$$

The following inequality is obtained here by using Eq. (4-49):

$$\begin{aligned}
H &\leq x^T(t)\Phi x(t) + 2\varepsilon\|x(t)\| \cdot \|\theta(t)\| + 2x^T(t)QE d(t) \\
&\quad - \mu\alpha\|\theta(t)\|^2 + x(t)^T x(t) - \gamma^2 d(t)^T d(t)
\end{aligned} \tag{4-55}$$

According to Theorem 4.3, the following inequality can be obtained:

$$\begin{bmatrix} x(t)^T & d(t)^T & x(t)^T \end{bmatrix} \begin{bmatrix} \Phi & QE & I_n \\ E^T Q & -\gamma^2 I_r & 0 \\ I_n & 0 & -I_n \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \\ x(t) \end{bmatrix} < 0 \tag{4-56}$$

This leads to:

$$x^T(t)\Phi x(t) + 2x^T(t)QE d(t) - \gamma^2 d(t)^T d(t) + x(t)^T x(t) < 0 \quad (4-57)$$

By defining  $x_d^T(t) = [x^T(t) \quad d^T(t)]^T$ ,  $\bar{\Phi} = \begin{bmatrix} \Phi + I_n & QE \\ E^T Q & -\gamma^2 I_r \end{bmatrix}$ , then inequality (4-57) leads to  $x_d^T(t)\bar{\Phi}x_d(t) < 0$ . Denote  $\beta_d = \lambda_{\min}(-\bar{\Phi})$ , then the following result can be achieved:

$$H \leq -\beta_d \|x_d(t)\|^2 + 2\varepsilon \|x(t)\| \cdot \|\theta(t)\| - \mu\alpha \|\theta(t)\|^2 \quad (4-58)$$

In a similar manner to Eq. (4-53), by choosing  $\mu > \frac{\varepsilon^2}{\alpha\beta_{dm}}$  with  $\beta_{dm} = \min\{\beta \quad \beta_d\}$ , Eq. (4-58) finally becomes,

$$H \leq -\frac{1}{\mu\alpha} (\mu\alpha\beta_{dm} - \varepsilon^2) \|x_d(t)\|^2 < 0 \quad (4-59)$$

Consider zero initial conditions and integrating both sides of Eq. (4-54) leads to the inequality:

$$x^T(t)\Phi x(t) + \int_0^T x(t)^T x(t) d\tau < \int_0^T \gamma^2 d(t)^T d(t) d\tau \quad (4-60)$$

when the time  $T$  goes to infinity the following result is obtained:

$$\|x(t)\|_2^2 < \gamma^2 \|d(t)\|_2^2 \quad (4-61)$$

Q.E.D.

**Remark 4.7:** Theorem 4.4 shows that robust fault estimator and the related observer-based FTC system can be designed independently and the system Eq. (4-45) is global asymptotically stable. Meanwhile, by using the proposed robust FTC design, one can achieve the state and fault estimates and the fault compensation simultaneously.

#### 4.4.2 Robust sensor fault tolerant control

Section 4.4.1 describes the robust actuator FTC design strategy, it is important to note that it is an integrated design for achieving FTC which involves the estimator, observer and controller design. On the other hand, for the sensor FTC problem, there are two different methods that can be applied. (1) *Sensor fault accommodation*: when the system

output is involved in the controller design, sensor fault accommodation is needed, because the sensor fault further affects the system performance. This case can be transformed into an actuator FTC problem in a similar manner to the sensor fault estimation concept described in Section 4.3 (Figure 4-2). The stability is then easy to obtain based on the approach developed in this Section. Hence there is no requirement to give a proof here.

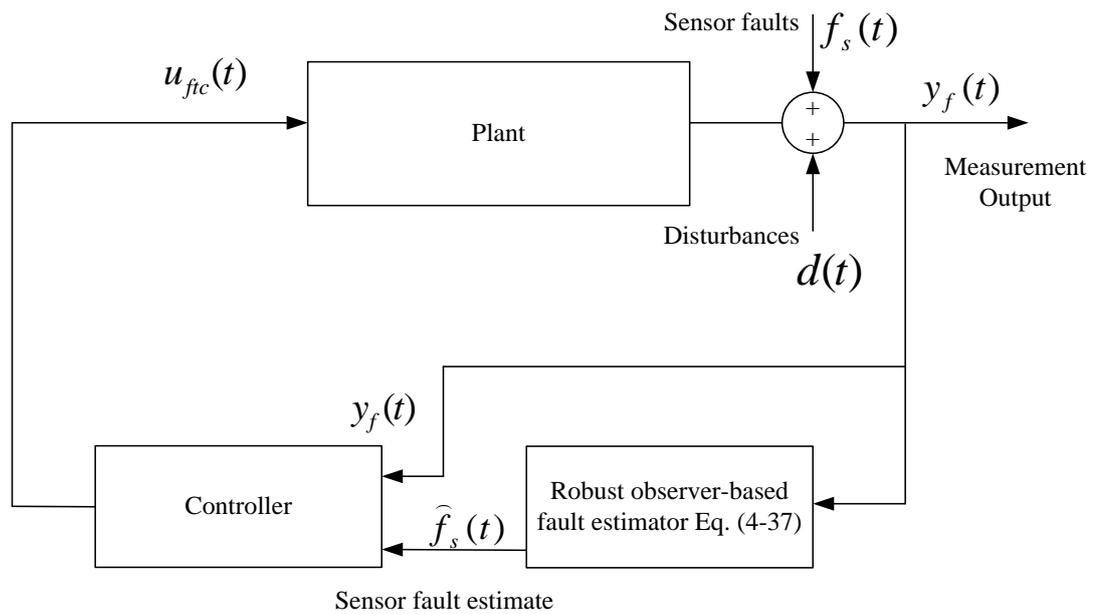


Figure 4-2: Sensor fault accommodation

(2) *Sensor fault compensation*: when the controller design does not involve the system output, under this situation, the state estimates of Eq. (4-37) provides the fault-free system output response, then the healthy system output response can be obtained by subtracting the fault estimate  $\hat{f}_s(t)$  directly from the faulty measurement  $y_f(t)$  as illustrated in Figure 4-3. To obtain an accurate estimate of the “ideal” output signal  $y(t)$  the fault estimate  $\hat{f}_s(t)$  must be robust to the disturbance, uncertainty.

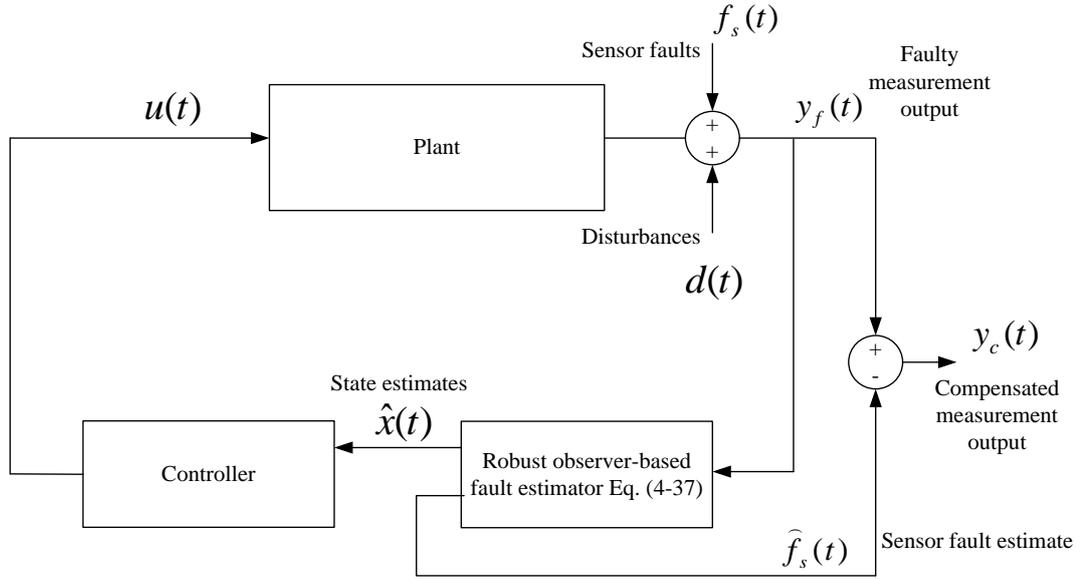


Figure 4-3: Sensor fault compensation

However, in this thesis, the FTC scheme is based on the observer feedback control. As a result, in the following simulation, the sensor fault compensation is adopted.

## 4.5 Simulation and Results

To illustrate the above discussion a tutorial example of the inverted pendulum (Patton and Klinkhieo, 2010) with a cart is used here to illustrate a redesigned example of a simplified nonlinear actuator fault compensation problem in the presence of disturbance (Figure 4-4). The cart is linked by a transmission belt to a drive wheel which is driven by a DC motor to rotate the pendulum into vertical position in the vertical plane by force control  $u(t)$  on the cart. The nonlinear equations of motion including actuator fault and external disturbance  $d(t)$  on the cart are:

$$\dot{x}_p(t) = \dot{x}_p(t) + d(t)$$

$$\dot{\theta}_p(t) = \dot{\theta}_p(t)$$

$$\ddot{x}_p(t) = \frac{1}{M_0} [-m^2 l^2 g \sin(\theta_p(t)) \cos(\theta_p(t)) + J m l \sin(\theta_p(t)) \dot{\theta}_p(t)]$$

$$\begin{aligned}
& +F_{\theta}ml\cos(\theta_p(t))\dot{\theta}_p(t) - JF_x\dot{x}_p(t) \\
& +J(u(t) + f_a(t))] \\
\ddot{\theta}_p(t) = & \frac{1}{M_0} [(M + m)mlg\sin(\theta_p(t)) - m^2l^2 \sin(\theta_p(t)) \cos(\theta_p(t))\dot{\theta}_p^2(t) \\
& +mlF_x \cos(\theta_p(t)) \dot{x}_p(t) - (M + m)F_{\theta}\dot{\theta}_p(t) \\
& -ml\cos(\theta_p(t))(u(t) + f_a(t))] - d(t) \tag{4-62}
\end{aligned}$$

Where  $x_p(t)$ ,  $\theta_p(t)$  are the cart position and the pendulum angle, respectively. The system parameters are given in Table 4-1.

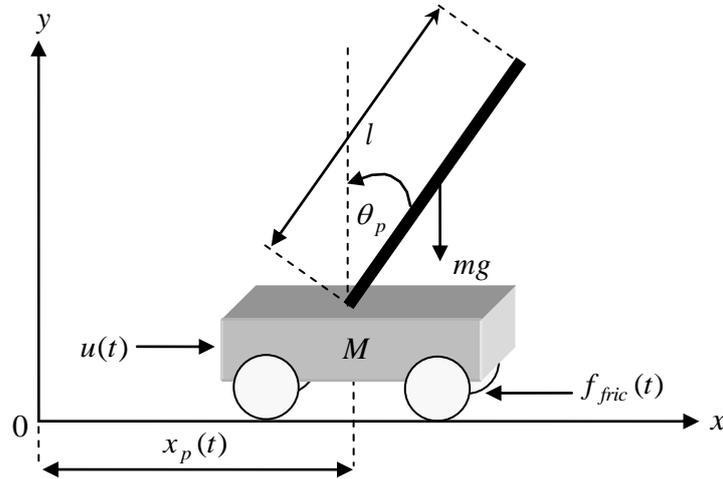


Figure 4-4: Inverted pendulum system

**Table 4-1:** The inverted pendulum parameters

Constants	$M$	$m$	$J$	$l$	$F_x$	$F_{\theta}$	$g$
Values	3.2	0.535	0.062	0.365	6.2	0.009	9.807
Units	Kg	Kg	Kg*m <sup>2</sup>	m	Kg/s	Kg*m <sup>2</sup>	m/s <sup>2</sup>

A linearization of the left hand side of Eq. (4-62) has been made at the equilibrium point:  $\dot{x}_p(t) = \dot{\theta}_p(t) = \theta_p(t) = 0$ . These results in the system triple corresponding to single input  $u(t)$  and measurements  $(t) \in \mathfrak{R}^3$ . The three measurements (cart position, pendulum angular position and cart velocity) replicate the measurements of the laboratory system.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.9333 & -1.9872 & 0.0091 \\ 0 & 36.9771 & 6.2589 & -0.1738 \end{bmatrix}, B = B_f = \begin{bmatrix} 0 \\ 0 \\ 0.3205 \\ -1.0095 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

#### 4.5.1 Robust actuator fault estimation and fault tolerant control

The actuator fault is created by:

$$f_a(t) = \begin{cases} 2 & 0 \leq t \leq 2s \\ t & 2s < t \leq 6s \\ 8 & 6s < t \leq 8s \\ 6 & 8s < t \leq 10s \\ 8\sin(2\pi t) + 1 & 12s < t \leq 15.78s \end{cases}$$

Setting  $\gamma = 1.1332$ , then according to Theorem 4.5, and solving Eq. (4-41), leads to:

$$X = \begin{bmatrix} 0.5963 & 0.0644 & -1.2164 & 0.0217 \\ 0.0644 & 0.5668 & -1.1444 & -1.1672 \\ -1.2164 & -1.1444 & 6.8789 & -3.8911 \\ 0.0217 & -1.1672 & -3.8911 & 17.3458 \end{bmatrix},$$

$$Z = [-6.1385 \quad 18.3548 \quad -43.4119 \quad 53.0528]$$

$$K = [138.0660 \quad 280.7071 \quad 88.3307 \quad 41.5903]$$

When doing the simultaneous observer design, by including a learning rate  $W = 10$ , and constant  $a = 1$ ,  $\vartheta = 0.005$ ,  $\bar{G} = 1$ ,  $\tau = 0.01$   $d_2 = 20$ . Then according to Theorem 4.1, inequalities (4-8), (4-24) and (4-25) are solved to give:

$$Y = 1.0e + 003^* \begin{bmatrix} 1.0212 & 1.1899 & -1.8639 \\ -0.6472 & 0.6471 & -0.7682 \\ 1.2601 & 0.3260 & 0.3587 \\ 0.1085 & 0.1312 & -0.0857 \end{bmatrix},$$

$$P = \begin{bmatrix} 75.2457 & 0.4470 & -25.3015 & 1.3109 \\ 0.4470 & 119.0912 & -54.4671 & -6.3340 \\ -25.3015 & -54.4671 & 113.5291 & 4.1288 \\ 1.3109 & -6.3340 & 4.1288 & 1.3109 \end{bmatrix},$$

$$L = \begin{bmatrix} 18.2576 & 16.6843 & -25.1024 \\ 3.3027 & 17.0498 & -13.7017 \\ 15.6157 & 9.8709 & -5.7987 \\ 31.2772 & 134.6995 & -88.2495 \end{bmatrix},$$

$$F_1 = [-9.4329 \quad -11.0635 \quad 32.2199],$$

$$F_2 = [73.9348 \quad 6.7810 \quad -29.4303],$$

These matrices are implemented in Eqs. (4-46), together with the non-linear pendulum simulation to achieve the robust FE and FTC.

The simulation results for the system output response estimation are shown below:

Figure 4-5 shows the external disturbance  $d(t)$  considered here is a Gaussian zero-mean white noise signal with variance 0.01.

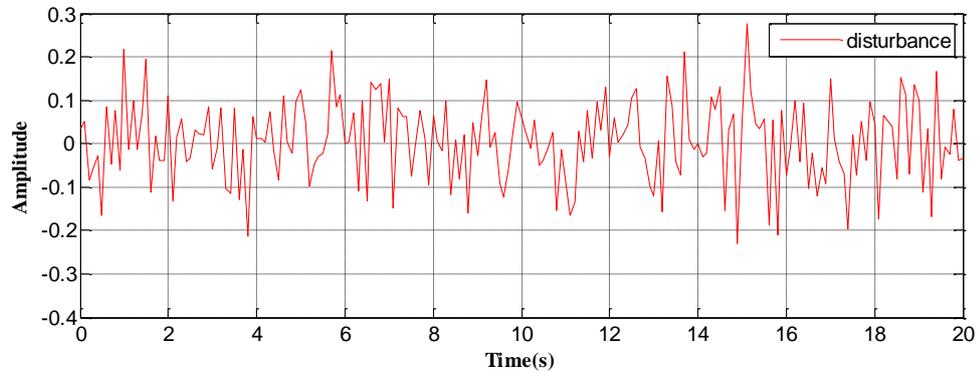


Figure 4-5: The external disturbance

Figure 4-6 indicates the comparison of the inverted pendulum system output responses without and with the FTC action.

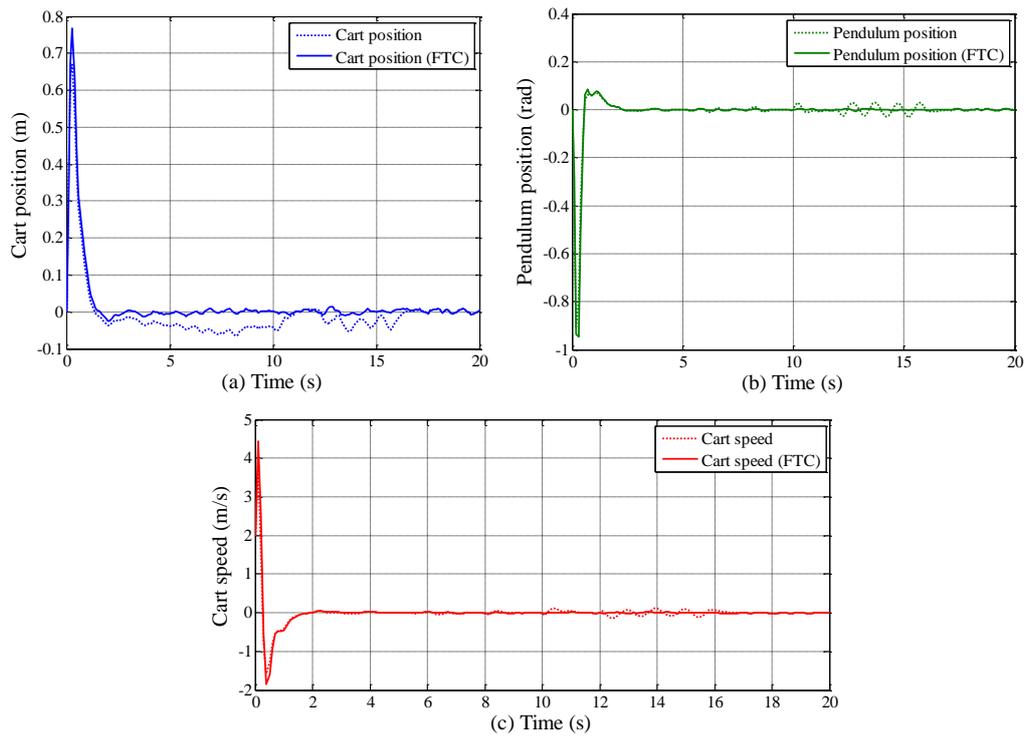


Figure 4-6: Comparison of the inverted pendulum system output responses with FTC (solid line) and without (dashed line)

Figures 4-6 (a), (b) and (c) show the system outputs of cart position, pendulum position and cart velocity responses with and without the FTC action, respectively. They clearly show that without the FTC applied, the inverted pendulum system exhibits a faulty scenario, with limit cycle oscillation around the vertical equilibrium point (the origin), which is affected by the actuator fault. In contrast, when the FTC scheme is applied, the limit cycle oscillation is significantly reduced, this is because the on-line updated actuator fault estimate compensates the influence caused by the real actuator fault.

Figure 4-7 and 4-8 show the result of the errors of the inverted pendulum system state responses and their estimates with and without the FTC action.

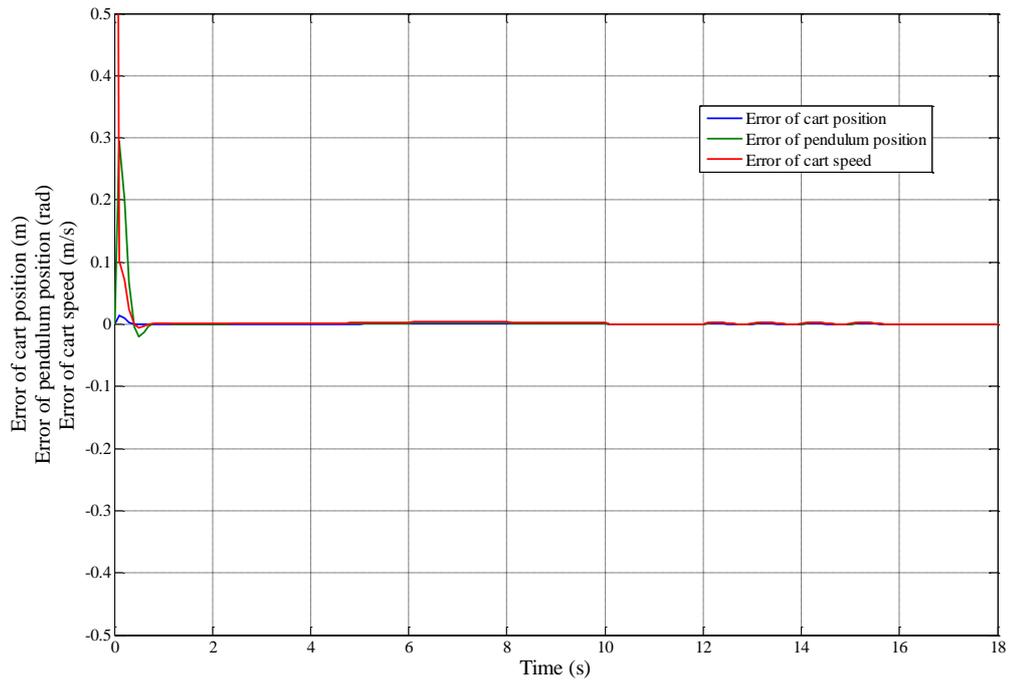


Figure 4-7: Errors of the system state responses and their estimates with FTC

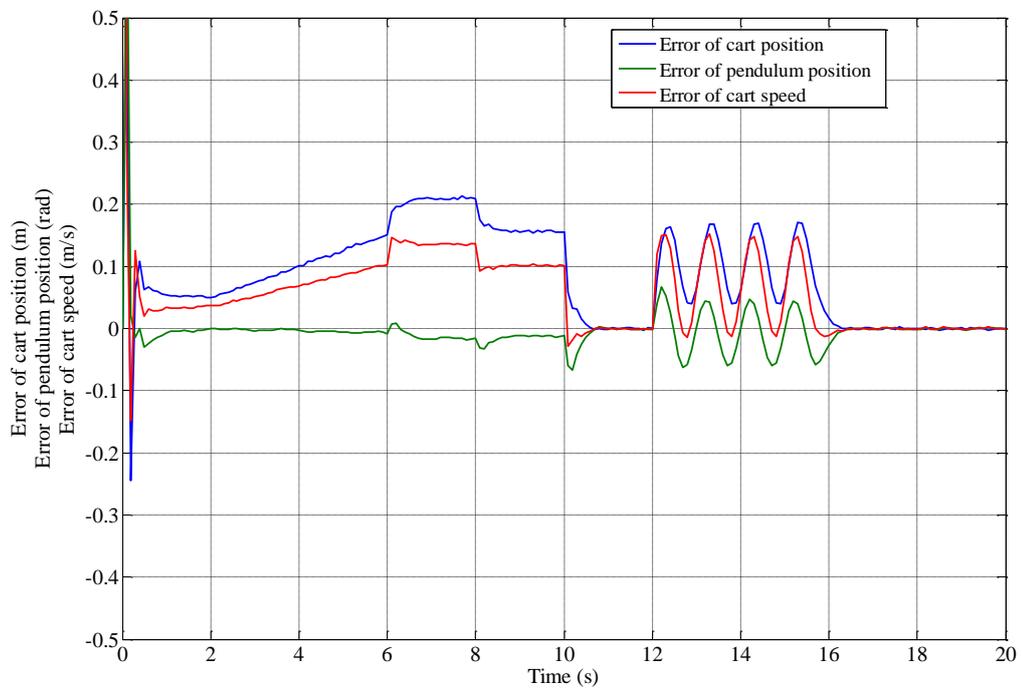


Figure 4-8: Errors of the system state responses and their estimates without FTC

Figures 4-5 and 4-6 show the influence on the errors of the system state and their estimates by excluding or including the FTC compensation. Without the FTC action applied, the errors of the inverted pendulum system state responses are severely affected by the actuator fault, which means the estimates and the system states are ‘far away’ from each other due to the fault. However, when the FTC is applied, the fault is compensated by its estimate. As a result, the errors between the inverted pendulum system state responses and their estimates converge to zero, which means that the system state estimates track the plant state with good performance.

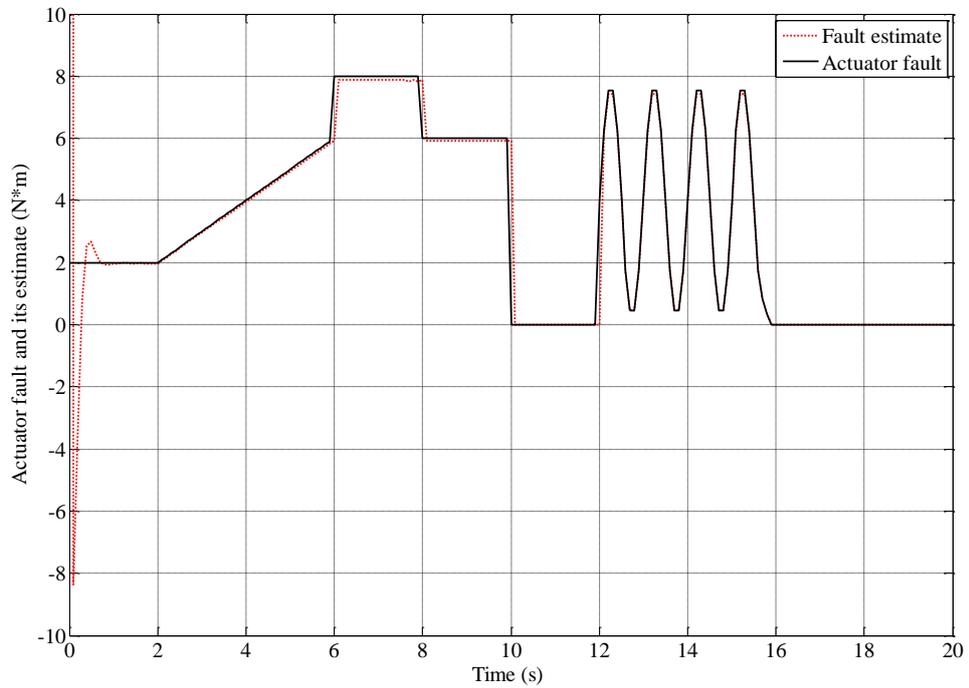


Figure 4-9: Actuator fault and its estimate

Figure 4-9 shows a satisfactory result of the estimation of the actuator fault. Though the exogenous disturbance exists, the estimator generates a robust actuator fault estimate  $\hat{f}_a(t)$ .

It should be emphasised that in the Section 4.5.2 the robust control, robust state and fault estimation, and robust fault compensation are implemented simultaneously, via the proposed robust observer design (Section 4.2). One can achieve the system states and fault estimates on-line to update the FTC law  $u_{FTC}(t) = u_{no}(t) - B^{++}B_f\hat{f}_a(t)$ . Expressed another way, even for the FTC scheme the existence of the exogenous

disturbance not only affects the system output stability or performance but also affects the accuracy of the estimates of the system states and faults, enhancing the difficulties in both controller and observer, even in the FTC design. Therefore, according to the above results and analysis, it can be concluded that the proposed robust FTC algorithm in Section 4.4.1 can provide satisfactory plant state and fault estimates, with good FTC action. Meanwhile, a good robustness to bounded exogenous disturbance is also exhibited.

## 4.5.2 Robust sensor fault estimation and fault compensation

In this Section, the simulation results for the robust sensor fault estimation are given. It is necessary to state that, the sensor fault and disturbance affect the output measurement, but without the actuator fault and system disturbance. In the simulation, the time variation of the sensor fault and its weighting matrix are given as:

$$f_s(t) = \begin{cases} 0 & 0 \leq t \leq 2s \\ t/8 & 2s < t \leq 6s \\ 1 & 6s < t \leq 8s \\ 0.75 & 8s < t \leq 10s \\ 5.5 - t/2 & 10s < t \leq 11s \\ 0 & 11s < t \leq 12s \\ 0.8\sin(3t + \pi/6) & 12s < t \leq 15.78s \end{cases}, D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

which means the fault occurs in the angle measurement of the pendulum, and the disturbance affects the first output measurement of the cart position. Following the procedure outlined in Section 4.3 the sensor fault is transformed to an equivalent actuator fault and the resulting augmented system is then constructed as:

$$A_s = \begin{bmatrix} 5.0 & 0 & 0 \\ 0 & 5.2 & 0 \\ 0 & 0 & 5.4 \end{bmatrix}, C_s = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, a = 1$$

By taking a learning rate  $W = 30$ , and constant  $a = 1$ ,  $\partial = 0.005$ ,  $\bar{G} = 1$ ,  $\tau = 0.01$ ,  $d_2 = 20$  and solving the Eqs. (4-35) - (4-36) and the inequality (4-34) in Theorem 4.2, one can obtain that:

$$\bar{P} = \begin{bmatrix} 6.0376 & 10.9640 & 6.0376 & 3.1838 & 0 & 2.0565 & -2.1352 \\ 10.9640 & 32.7282 & 10.9640 & 6.2937 & 0 & -4.8597 & -4.2886 \\ 6.0376 & 10.9640 & 6.0376 & 3.1838 & 0 & 2.0565 & -2.1352 \\ 3.1838 & 6.2937 & 3.1838 & 1.9344 & 0 & 1.6495 & -1.3311 \\ 0 & 0 & 0 & 0 & 17.9563 & 0.0401 & 10.7425 \\ -0.0453 & -4.8597 & -2.0565 & 1.6495 & 0.0401 & 49.3349 & -1.3305 \\ -0.1083 & -4.2886 & 2.1352 & -1.3311 & 10.7425 & -1.3305 & 21.5571 \end{bmatrix},$$

$$\bar{Y} = 10^3 * \begin{bmatrix} -0.0332 & -0.2205 & 0.2194 \\ -0.2980 & -0.1034 & 0.1824 \\ -0.0332 & -0.2205 & 0.2194 \\ -0.30379 & -0.1750 & 0.0490 \\ 0.7010 & 0.0995 & 0.5858 \\ -0.0075 & 0.9316 & -0.0129 \\ 0.3379 & 0.3203 & 1.0813 \end{bmatrix},$$

$$\bar{G} = 1, \tau = 0.01$$

$$\bar{L} = \begin{bmatrix} 116.4850 & -0.8009 & 10.4722 \\ -19.1262 & 66.1835 & -6.9753 \\ -67.1743 & 66.5788 & 165.1906 \\ -41.3485 & -437.5924 & -204.5673 \\ 43.6807 & -0.5252 & 0.8481 \\ -2.9544 & 37.6040 & 0.0012 \\ -7.7480 & 10.0028 & 53.1153 \end{bmatrix},$$

$$\bar{F}_1 = [17.9563 \quad 0.0401 \quad 10.7425],$$

$$\bar{F}_2 = [0 \quad -0.0034 \quad 0.1521].$$

Figure 4-10 shows the external disturbance  $d(t)$  considered here is a Gaussian zero-mean white noise with variance 0.01.

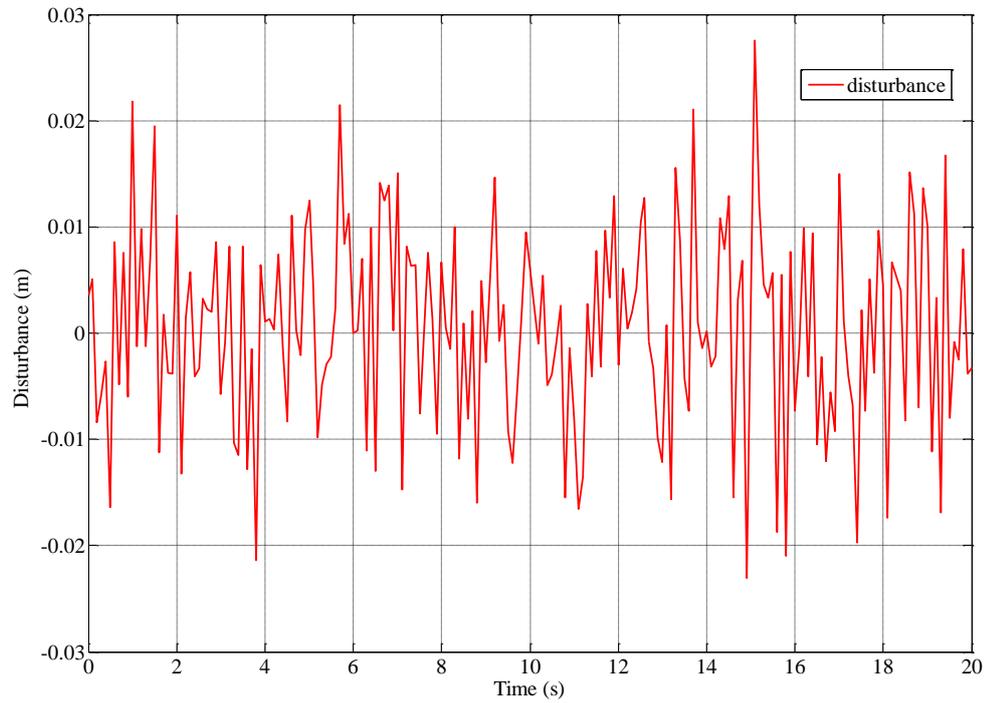


Figure 4-10: The external disturbance

Figure 4-11 shows the sensor fault and its estimate. The plot shows that the system sensor fault estimate tracks the real system sensor fault almost exactly. The result can be further used in a sensor fault compensation problem, based on compensating the sensor fault according to the scheme depicted in Figure 4-3, following results are obtained.

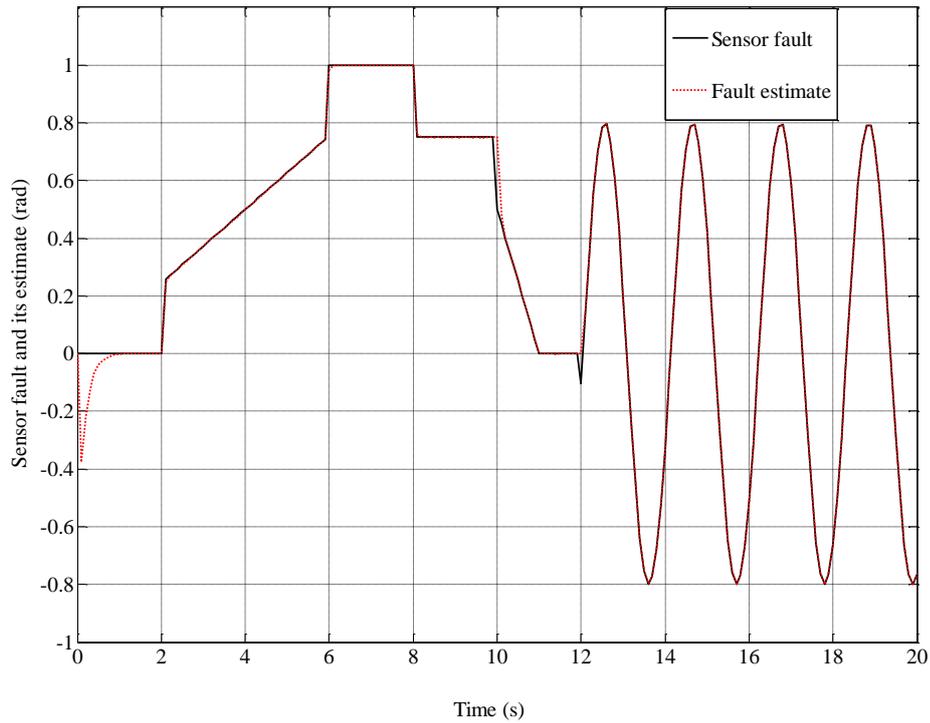


Figure 4-11: Pendulum system sensor fault (solid line) and its estimate (dashed line)

A fault occurs at  $t = 2s$ , and following this, the fault estimate immediately tracks the real fault signal, and the good tracking performance continues to the end of the simulation. The fault signal is constructed by a combination of ramp, pulse and continuous sine wave signals. This complex fault profile is chosen to evaluate the applicability of the estimator to various fault types (fast faults, slow drift faults, sinusoidal faults).

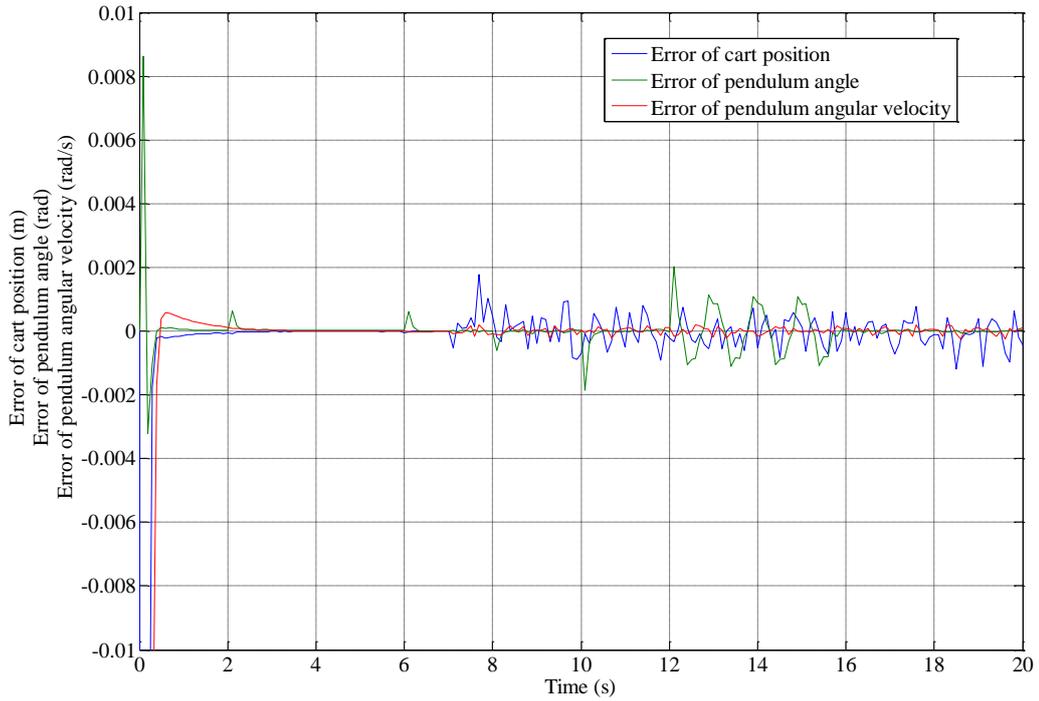


Figure 4-12: Errors of the pendulum system output responses and their estimates

Figure 4-12 indicates that the errors of the system output responses and their estimates go to zero quickly from the beginning of the simulation. The observer resumes approximately the same state estimate error performance as the nominal and fault-free system. Whilst when switching off the nonlinear term  $\eta[y(t), \hat{x}(t)]$  at  $t = 7s$ , the robustness to the disturbance and uncertainty is lost and the estimation errors oscillate around the time axis due to the disturbance  $d(t)$ . This shows evidence of good estimation robustness provided through the term  $\eta[y(t), \hat{x}(t)]$ .

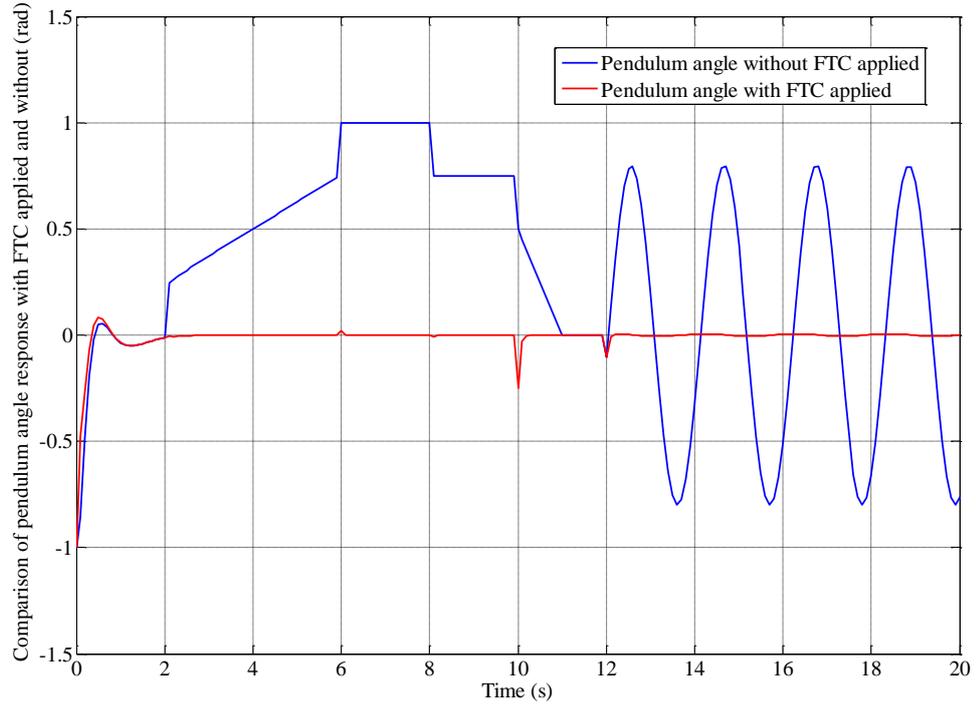


Figure 4-13: Comparisons of pendulum angle responses with and without fault compensation.

Figure 4-13 shows the comparisons of the system output response, comprising the pendulum angle  $\theta(t)$ , (initial condition sets at  $\theta_0 = -1 \text{ rad}$ ) without and with fault compensation applied, respectively. It is clear that without the fault compensation, the output response of  $\theta(t)$  is dominated by the fault completely, whilst with the action of the fault compensation, the output response is very close to the nominal case (fault-free and disturbance-free) as shown in Figure 4-14. Some spikes can be seen corresponding to the discontinuities at the fault signal transitions. It is necessary to state that the proposed robust observer-based estimator in Section 4.2 is under the assumption that the derivative of the fault signal is bounded. The derivative of the transition for the tested fault signal is infinity, and hence some transition spikes occur in the fault accommodation process. Meanwhile, the estimation performance also depends on the estimator's learning rate  $W$  and the assigned observer eigenvalues.

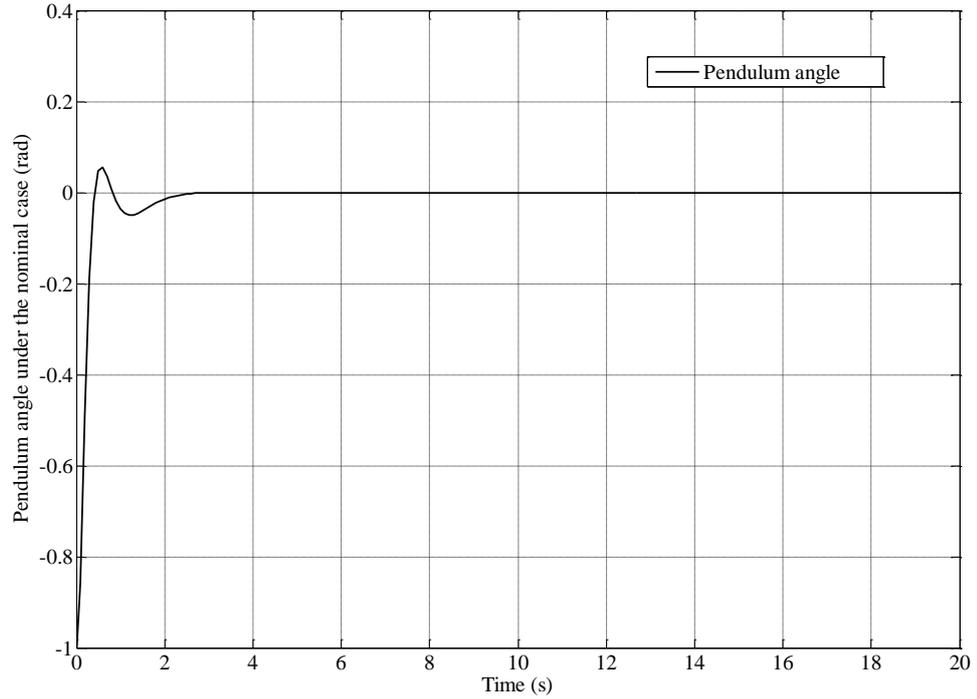


Figure 4-14: Pendulum angle response with fault-free and disturbance-free case

Sections 4.5.1 and 4.5.2 give the actuator FTC and sensor fault compensation on the nonlinear inverted pendulum examples, respectively, to show the power of the proposed observer-based estimator. By using this observer the actuator fault and sensor fault can both be estimated and their estimates can be further used in the FTC design. It also gives a special design approach with the sensor fault FE and FTC problems transferred to the actuator fault FE and FTC problem by importing an output filter for the faulty output involved in the control law design. On the other hand, when the FTC design is based on the observer, the sensor fault compensation can be directly achieved by subtracting the fault from the faulty output measurement. From the above results and analysis, the proposed observer-based estimator can deal with a general fault estimation problem, meanwhile, a good robustness to the external disturbance of the proposed observer is also illustrated.

## 4.6 Conclusion

In this Chapter a new approach to on-line robust state and fault estimation is presented which is based on a conventional observer involving a nonlinear switching function. The nonlinear switching function comprises two parts, one part is based on information from the disturbance and the second contains the fault information. The nonlinear switching function is not only successful in reducing the influence from disturbance to the estimation performance but also keeps the errors between the real states and their estimates and the fault and fault estimate from diverging. Two types of faults (actuator faults and sensor faults) are tested on the proposed observer and are in agreement with the theory. The computation of the observer and fault estimator gains are solved via the LMI MatLab Tool box, which provides the optimal solution.

Later in this Chapter, a robust FTC approach is proposed by combining the robust observer with robust  $H_\infty$  optimization. For the FTC system the controller and robust observer are designed separately. The FTC control law is up-dated on-line by using the plant output error and the fault estimation is acquired from the integration of the plant and observer. By using this FTC-observer based method, the robust plant state and fault estimation, and robust FTC can be achieved simultaneously.

A tutorial example of a nonlinear inverted pendulum is used as a demonstration study. This is an important example as the work has shown clearly the strong robustness of the proposed methods in the system states and fault estimation and FTC control design. Meanwhile, because of the non-restrictive assumptions, the proposed observer and FTC scheme can be widely used in other application systems.

In Chapter 5, a more general fault model is considered rather than the additive actuator fault, and accordingly a robust adaptive FE and AFTC are described.

# Chapter 5.

## Robust Adaptive Actuator Fault

## Estimation and Fault Tolerant Control

### 5.1 Introduction

This Chapter is concerned with the active approach to FTC, involving fault estimation and fault compensation. The work of this Chapter only considers actuator faults since if the sensor fault can be estimated, this faulty information can be used directly to correct the fault from the sensor measurements prior to the implementation of the further FTC design, meanwhile the FTC scheme on sensor fault stated in Chapter 4 can also be adopted. The motivation is to develop a top-down integrated design scheme for robust adaptive FTC based on fault estimation and compensation. There are many previous works that can form in some case a background to its research (Jin and Yang, 2009; Fan and Song, 2010; Gao and Ding, 2005, 2007; Zhang, Jiang and Shi, 2007, 2010; Ye and Yang, 2006; Boskovic and Mehra, 2001; Gayaka and Yao, 2011; Zhang, Parisini and Polycarpou, 2004). Compared with their work, a new robust adaptive observer-based actuator fault estimator design strategy is described, which is a development of the work of Chapter 4.

The actuator fault considered in this Chapter is illustrated in Section 5.2, where a more general description for actuator faults comprising actuator loss of effectiveness and fault stuck situations.

Section 5.3 describes the concept of robust adaptive FTC based on the estimation acquired from the observer system proposed in Section 5.2. Different structural disturbances are introduced and appropriately robust FTC design strategies are developed. For matched disturbances, the work is based on (Jin and Yang 2009; Fan and Song, 2010). However, in their work, the assumption is made that the system states are all measurable, and this assumption may not be valid in real applications.

Furthermore, for the unmatched disturbance case, Zhang, Jiang and Shi (2007) give the estimation design but have not considered the disturbances when developing their FTC scheme.

Section 5.4 provides two case studies to illustrate the applicability of the proposed FE and FTC design approach.

## 5.2 Robust adaptive actuator fault estimation

### 5.2.1 Preliminaries and problem statement

Consider a state space representation of a linear continuous time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5-1)$$

$$y(t) = Cx(t)$$

Where  $x(t) \in \mathfrak{R}^n$  is the state vector,  $y(t) \in \mathfrak{R}^p$  is the output vector,  $u(t) \in \mathfrak{R}^m$  is the input control signal vector, and with  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times m}$ ,  $C \in \mathfrak{R}^{p \times n}$ . The matrix pair  $(A, B)$  is controllable and  $(A, C)$  is observable. Thus, the system Eq. (5-1) can be referred to as a “nominal system”.

The nominal system Eq. (5-1) with external input disturbances is described as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (5-2)$$

$$y(t) = Cx(t)$$

where  $E \in \mathfrak{R}^{n \times r}$ . As described in Chapter 1, there are several types of faults which may occur during system operation. In this chapter the actuator fault is considered to fall into one of the following three categories, which including outage, loss of effectiveness and stuck faults (Wang and Lum, 2007; Boskovic and Mehra, 2002; Chen and Patton, 1999). The definitions of these three category actuator faults are given below:

#### (1) Outage fault scenario

The actuator produces zero force and moment, i.e. it becomes ineffective.

## (2) Loss of effectiveness fault scenario

A loss of effectiveness corresponds to a decrease in actuator gain resulting in a deflection that is smaller than the commanded position.

## (3) Stuck fault scenario

For this scenario, the actuator is stuck in a certain position at an unknown time and does not respond to subsequent commands.

As a result, a general actuator fault model is considered in this Chapter. This is adopted from the work of Jin and Yang (2009). Let  $u_{ij}^F(t)$  represent the signal from the  $i_{th}$  actuator that has failed in the  $j_{th}$  faulty mode. Then the general actuator fault model as described before is denoted as:

$$u_{ij}^F(t) = \rho_i^j(t)u_i(t) + \sigma_i^j u_{si}(t) \quad (5-3)$$

$$i = 1, \dots, m, j = 1, \dots, N$$

where  $\rho_i^j(t)$  is the unknown time-varying actuator efficiency factor, the index  $j$  denotes the  $j_{th}$  faulty mode.  $N$  is the number of total faulty modes which contains a maximum of  $2^m$  elements, and  $\underline{\rho}_i^j$  and  $\bar{\rho}_i^j$  represents the known lower and upper bounds of  $\rho_i^j(t)$ , respectively.  $u_{si}(t)$  is the unparameterizable bounded time-varying stuck-actuator fault in the  $i_{th}$  actuator (Tao, Joshi and Ma, 2001). Note the practical case for which  $0 \leq \underline{\rho}_i^j \leq \rho_i^j(t) \leq \bar{\rho}_i^j$ , and  $\sigma_i^j$  is an unknown constant defined as:

$$\sigma_i^j = \begin{cases} 0, & \rho_i^j(t) > 0 \\ 0 \text{ or } 1, & \rho_i^j(t) = 0 \end{cases} \quad (5-4)$$

The model parameters described above are summarised in Table 5-1

**Table 5-1:** Fault model

<b>Fault model</b>	$\underline{\rho}_i^j$	$\bar{\rho}_i^j$	$\sigma_i^j$
<b>Normal</b>	1	1	0
<b>Outage</b>	0	0	0
<b>Loss of effectiveness</b>	$> 0$	$< 1$	0
<b>Stuck</b>	0	0	1

Denote:

$$u_j^F(t) = [u_{1j}^F(t), u_{2j}^F(t), \dots, u_{mj}^F(t)]^T = \rho^j(t)u(t) + \sigma_i^j u_s(t) \quad (5-5)$$

where  $\rho^j(t) = \text{diag}_i\{\rho_i^j(t)\}$ ,  $\rho_i^j(t) \in [\underline{\rho}_i^j, \bar{\rho}_i^j]$ ,  $\sigma^j = \text{diag}_i\{\sigma_i^j\}$ ,  $i = 1, 2, \dots, m, j = 1, 2, \dots, N$ .

Consider the above parameterization description of the actuator faults for all possible faulty modes. A general actuator faults uniform can be characterized as:

$$u^F(t) = \rho(t)u(t) + \sigma u_s(t)$$

where  $\rho(t) = \text{diag}\{\rho_1(t), \dots, \rho_m(t)\} \in \{\rho^1(t), \dots, \rho^N(t)\}$  and

$$\sigma = \text{diag}\{\sigma_1, \dots, \sigma_m\} \in \{\sigma^1, \dots, \sigma^N\}.$$

Hence, the dynamics of the system Eq. (5-2) with actuator faults Eq. (5-5) are described by:

$$\dot{x}(t) = Ax(t) + B\rho(t)u(t) + B\sigma u_s(t) + Ed(t) \quad (5-6)$$

$$y(t) = Cx(t)$$

To ensure that the robust adaptive fault estimation objective can be achieved, the following Assumptions in estimator design are assumed to be valid.

**Assumption 5.1:** All pairs  $(A, B\rho(t))$  are uniformly completely controllable for any actuator failure mode  $\rho(t) \in \{\rho^1(t), \dots, \rho^N(t)\}$  under consideration.

**Assumption 5.2:**  $\text{Rank}(B) = m$ ,  $\text{Rank}(E) = r$  and  $\text{rank}(B \quad E) = m + r \leq p$

**Assumption 5.3:** The norm of the vector  $d(t)$  is bounded such that  $\|d(t)\| \leq d_1$ , for all  $t \geq 0$ , and for which  $d_1$ , is a known positive constants namely,  $d_1 \geq 0$ ,  $d_1 \in \mathfrak{R}$ .

**Assumption 5.4:** The stuck-actuator fault is a piece-wise continuous bounded function. That is, there exists an unknown positive constant  $d_2$ , such that,  $\|u_s(t)\| \leq d_2$ .

**Remark 5.1:** As far Assumption 5.1, Assumption 5.2 makes sure that the input weighting matrix  $B$  and disturbance weighting matrix  $E$  are linear independent, and the output can be used to provide enough information for the fault estimation and disturbance rejection. Assumption 5.3 leads to a determination of the disturbance bound, respectively.

## 5.2.2 Robust adaptive fault estimator strategy

According to the plant described in Eq. (5-6), the term  $\sigma u_s(t)$  can be considered as an actuator fault occurring during the nominal working process, which can be denoted as  $f(t) = \sigma u_s(t)$ . As a result Eq.(5-6) can be rewritten as:

$$\dot{x}(t) = Ax(t) + B\rho(t)u(t) + Bf(t) + Ed(t) \quad (5-7)$$

$$y(t) = Cx(t)$$

Then an observer-based robust adaptive fault estimator model is constructed as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{\rho}(t)u(t) + B\hat{f}(t) + \eta[y(t), \hat{x}(t)] + L[y(t) - \hat{y}(t)] \quad (5-8)$$

$$\eta[y(t), \hat{x}(t)] = \frac{d_1 E F_2 e_y(t)}{\|F_2 e_y(t)\|}$$

$$\dot{\hat{\rho}}_i(t) = \text{Proj}_{[\underline{\rho}_i^j, \bar{\rho}_i^j]} \{-\alpha_i F_{1i} e_y(t) u_i^T(t)\}$$

$$\hat{y}(t) = C\hat{x}(t)$$

$$\dot{\hat{f}}(t) = WF_1 e_y(t) \quad (5-9)$$

where  $W$  is the learning rate,  $F_{1i}$  is the  $i$ th row of the fault estimator gain  $F_1$  to be designed.  $\eta[y(t), \hat{x}(t)]$  is a nonlinear switching function used to reduce the influence from exogenous disturbance (see Section 4.2 for description of this term).  $\hat{\rho}(t)$  is the estimate of the stuck fault index, i.e.

$\hat{\rho}(t) = \text{diag}\{\hat{\rho}_1(t), \dots, \hat{\rho}_m(t)\} \in \mathfrak{R}^{m \times m}$ .  $\alpha_i$  is the adaptation rate which can be adjusted and may affect the convergence rate of the adaptive estimation.

The error dynamic system is obtained by subtracting Eq. (5-8) from Eq. (5-7):

$$\begin{aligned} \dot{e}(t) &= Ae(t) + B[\Delta\rho(t)]u(t) + Be_f(t) + Ed(t) - \eta[y(t), \hat{x}(t)] - L[y(t) - \hat{y}(t)] \\ &= (A - LC)e(t) + B[\Delta\rho(t)]u(t) + Be_f(t) + Ed(t) - \eta[y(t), \hat{x}(t)] \end{aligned} \quad (5-10)$$

$$e_y(t) = Ce(t)$$

where  $e(t) = x(t) - \hat{x}(t)$  is the state error,  $e_f(t) = f(t) - \hat{f}(t)$  and  $\Delta\rho(t) = \rho(t) - \hat{\rho}(t)$  represent the stuck actuator fault error and the error of the index of the actuator loss of effectiveness, respectively. Then the following Theorem is established.

**Theorem 5.1:** Under Assumptions 5.1-5.4, if there exist symmetric positive definite matrices  $P, R \in \mathfrak{R}^{n \times n}$ , and matrices  $Y \in \mathfrak{R}^{n \times p}$ ,  $F_1 \in \mathfrak{R}^{m \times p}$  and  $F_2 \in \mathfrak{R}^{r \times p}$ , such that the following conditions hold:

$$PA - YC + A^T P - C^T Y^T = -R < 0, \quad (5-11)$$

$$B^T P = F_1 C \quad (5-12)$$

$$E^T P = F_2 C \quad (5-13)$$

where  $F_1 \in \mathfrak{R}^{m \times p}$  and  $F_2 \in \mathfrak{R}^{r \times p}$  are design matrices, then the state estimate  $\hat{x}(t)$ , fault estimate  $\hat{f}(t)$  and the estimate of the index of the actuator loss of effectiveness, from the robust full-order observer determined by Eq. (5-11) converge to the actual state  $x(t)$ , the actuator fault  $f(t)$  and the real index of the actuator effectiveness, respectively.

Before proving Theorem 5.1, the Lemma 5.1 must be introduced.

**Lemma 5.1** (Barbalat Lemma (solutine and Li, 1991)). If  $w: \mathfrak{R} \rightarrow \mathfrak{R}^+$  is a uniformly continuous function for  $t \geq 0$  and if the limit of the integral:

$$\lim_{t \rightarrow \infty} \int_0^t |w(\tau)| d\tau \quad (5-14)$$

Exists and is finite, then:

$$\lim_{t \rightarrow \infty} w(t) = 0 \quad (5-15)$$

**Proof:**

One Lyapunov function candidate is chosen as:

$$V_o(t) = e^T(t)Pe(t) + e_f^T(t)W^{-1}e_f(t) + \text{trace}(\Delta\rho(t)\Gamma^{-1}\Delta\rho(t)) \quad (5-16)$$

where  $\Gamma = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_m)$ .

Then the derivative of  $V(t)$  with respect to time is:

$$\begin{aligned} \dot{V}_o(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + 2e^T(t)PB[\Delta\rho(t)]u(t) + 2e_f^T(t)W^{-1}\dot{e}_f(t) \\ &\quad + 2\text{trace}[\Delta\rho(t)\Gamma^{-1}\Delta\dot{\rho}(t)] \\ &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PB[\Delta\rho(t)]u(t) \\ &\quad + 2e^T(t)PBe_f(t) + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + 2e_f^T(t)W^{-1}\dot{f}(t) - 2e_f^T(t)W^{-1}\dot{\hat{f}}(t) \\ &\quad - 2\text{trace}[\Delta\rho(t)F_1e_y(t)u^T(t)] \end{aligned} \quad (5-17)$$

Note that when the actuator is stuck the derivative of the fault  $\dot{f}(t) = 0$ , therefore the term  $2e_f^T(t)W^{-1}\dot{f}(t)$  in Eq. (5-17) is removed. Furthermore, as  $\rho(t)$  is a constant diagonal matrix,  $\Delta\dot{\rho}(t) = -\dot{\hat{\rho}}(t)$ . Using Eq. (5-12) and the expression of  $\dot{\hat{f}}(t)$ , Eq. (5-17) can be rewritten as:

$$\begin{aligned} \dot{V}_o(t) &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PB[\Delta\rho(t)]u(t) \\ &\quad + 2e^T(t)PEd(t) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \end{aligned}$$

$$-2\text{trace}[\Delta\rho(t)F_1e_y(t)u^T(t)] \quad (5-18)$$

Also note that:

$$\begin{aligned} -2\text{trace}[\Delta\rho(t)F_1e_y(t)u^T(t)] &= -2\text{trace}[u^T(t)\Delta\rho(t)F_1Ce(t)] \\ &= -2\text{trace}[u^T(t)\Delta\rho(t)B^TPe(t)] \\ &= -2e^T(t)PB[\Delta\rho(t)]u(t) \end{aligned} \quad (5-19)$$

According to Eqs. (5-18) and (5-19), Eq. (5-17) follows that:

$$\begin{aligned} \dot{V}_o(t) &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PEd(t) \\ &\quad - 2e^T(t)P\eta[y(t), \hat{x}(t)] \end{aligned} \quad (5-20)$$

It can be shown that:

$$\dot{V}_o(t) \leq -e^T(t)Re(t) + 2d_1\|E^TPe(t)\| - 2e^T(t)PE\eta[y(t), \hat{x}(t)] \quad (5-21)$$

Now on substitution  $\eta[y(t), \hat{x}(t)] = \frac{d_1EF_2e_y(t)}{\|F_2e_y(t)\|}$  into Eq. (5-21):

$$\dot{V}_o(t) \leq -e^T(t)Re(t) \leq -\delta(t) < 0 \quad (5-22)$$

where  $\delta(t) = \lambda_{\min}(R)\|e(t)\|$ ,  $\lambda_{\min}(\cdot)$  is the minimum eigenvalue of  $(\cdot)$ . Integrating the above equation from zero to  $t$  yields:

$$0 < \int_0^t \delta(\tau)d\tau \leq \int_0^t \delta(\tau)d\tau + V_o(t) \leq V_o(0) \quad (5-23)$$

Because of  $(t) > 0, V_o(t) > 0$ , when  $t \rightarrow \infty$ , the above integral is always less than or equal to  $V_o(0)$ , so there exists a positive constant  $p$  for the limit of the integral  $\lim_{t \rightarrow \infty} \int_0^t \delta(\tau)d\tau = p$ . Hence, by Lemma 5.1 the limit of  $\delta(t)$ ,  $\lim_{t \rightarrow \infty} \delta(t) = 0$ , which implies that  $\lim_{t \rightarrow \infty} e(t) = 0$ .

**Remark 5.2:** For all these cases of actuator fault scenarios considered in this Chapter the proposed observer-based fault estimator is always suitable as long as the relationship Eq. (5-12) holds.

**Remark 5.3:** However, because the existence of the time-variant item  $\Delta\rho(t)$  in the dynamic error system in Eq. (5-10), Eq. (5-22) only guarantees the stability of the dynamic error system but no description of the convergence of the error  $e(t)$  to zero is provided. Based on the Barbalat's Lemma, the asymptotic estimation of the states can be guaranteed.

**Remark 5.4:** Since the directions corresponding to the columns of the weighting matrix of actuator fault ( $B$ ) are assumed to be different from those of the unknown disturbance ( $E$ ), i.e. their columns are linearly independent, the nonlinear switching function  $\eta[y(t), \hat{x}(t)]$  cannot affect the actuator fault. As a result, when the dynamic system is fault free ( $f_a(t) = 0$ ), the output error  $e_y(t)$  approaches zero asymptotically, for all three actuator fault scenarios, the output error  $e_y(t)$  is affected only by the corresponding actuator fault  $f_a(t)$ , and is unaffected by whether or not the existence of the disturbance. For this reason, the proposed observer-based estimator can also be treated as a robust fault detection observer. Hence similar as the discussion in Chapter 4, thus, by checking the residual  $e_y(t)$  as following logical fault classification can be made:

$$D(e_y(t)) = \left. \begin{array}{l} \|e_y(t)\| \leq \beta \quad \text{no actuator fault occurs} \\ \|e_y(t)\| > \beta \quad \text{at least one actuator fault has occurred} \end{array} \right\}$$

where  $\beta$  is a threshold designed for fault detection.

**Remark 5.5:** It is important that the actuator effectiveness estimator and stuck fault estimator are designed separately. Hence, when faults are detected, the actuator effectiveness estimator and stuck fault estimator can distinguish where the corresponding fault occurred and which type of fault is present. Hence, the proposed observer-based estimator can also serve as an FDI observer.

However, sometimes it is not an easy task to obtain the exact value of the disturbance upper bound in practice. In other words, the smallest value of the parameter  $d_1$  in Assumption 5.3 is difficult to determine *a priori*. Therefore an adaptive gain  $\hat{d}_1(t)$  is employed to adapt this unknown constant  $d_1$ . Then the form of the proposed observer-based estimator Eq. (5-8) and Eq. (5-9) can be modified to:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{\rho}(t)u(t) + B\hat{f}(t) + \eta[y(t), \hat{x}(t)] + L[y(t) - \hat{y}(t)] \quad (5-24)$$

$$\hat{f}(t) = WF_1e_y(t)$$

$$\eta[y(t), \hat{x}(t)] = \frac{\hat{d}_1 EF_2e_y(t)}{\|F_2e_y(t)\|}$$

$$\dot{\hat{\rho}}_i(t) = Proj_{[\underline{\rho}_i^j, \bar{\rho}_i^j]} \{-\alpha_i F_{1i}e_y(t)u_i^T(t)\}$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5-25)$$

then Theorem 5.2 can be derived:

**Theorem 5.2:** Under Assumption 5.1 - 5.4 but  $d_1$  is an unknown positive constant, if Theorem 5.1 holds, then the adaptive law:

$$\dot{\hat{d}}_1(t) = \nu \|F_2e_y(t)\| \quad (5-26)$$

where  $\nu > 0$  is the learning rate, and the robust full-order observer determined by Eq. (5-8) and (5-9), can realise the state estimation  $\hat{x}(t)$ , fault estimation  $\hat{f}(t)$  and the estimation of the index of the actuator lost effectiveness converge to the actual state  $x(t)$ , actuator fault  $f(t)$  and the real index of the actuator effectiveness, respectively.

**Proof:**

A new Lyapunov function is chosen as:

$$V_a(t) = V_o(t) + \nu^{-1}e_{d_1}^2(t) \quad (5-27)$$

where  $e_{d_1}(t) = d_1 - \hat{d}_1(t)$ , after differentiate  $V_a(t)$  with respect to time, Eq. (5-20) can be modified to:

$$\begin{aligned} \dot{V}_a(t) = & e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PEd(t) \\ & - 2e^T(t)P\eta[y(t), \hat{x}(t)] + 2\nu^{-1}e_{d_1}(t)\dot{e}_{d_1}(t) \end{aligned} \quad (5-28)$$

because of  $\dot{e}_{d_1}(t) = -\dot{\hat{d}}_1(t)$ , and substituting  $\eta[y(t), \hat{x}(t)] = \frac{\hat{d}_1 E F_2 e_y(t)}{\|F_2 e_y(t)\|}$  into Eq. (5-28), it follows that:

$$\begin{aligned} \dot{V}_a(t) &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PEd(t) \\ &\quad - 2e^T(t)P \frac{\hat{d}_1 E F_2 e_y(t)}{\|F_2 e_y(t)\|} - 2e_{d_1} \dot{\hat{d}}_1(t) \end{aligned} \quad (5-29)$$

According to Eq. (5-13) and the adaptive law Eq. (5-26), Eq. (5-29) is rewritten as:

$$\begin{aligned} \dot{V}_a(t) &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) + 2e^T(t)PEd(t) \\ &\quad - 2e^T(t)PE\hat{d}(t) - 2e_{d_1}(t)\|E^T P e(t)\| \\ &= e^T(t)[(A - LC)^T P + P(A - LC)]e(t) \\ &\quad + 2e^T(t)PEe_{d_1}(t) - 2e_{d_1}(t)\|E^T P e(t)\| \end{aligned} \quad (5-30)$$

Thus Eq. (5-30) can be simplified as:

$$\dot{V}_a(t) \leq e^T(t)[(A - LC)^T P + P(A - LC)]e(t)$$

Using inequality (5-11), it then follows that:

$$\dot{V}_a(t) \leq -\lambda_{\min}(R)\|e(t)\| \quad (5-31)$$

From Theorem 5.1, and by importing Lemma 5.1, it is easily to guarantee that  $e(t)$  converges asymptotically to zero. Q.E.D.

**Remark 5.6:** For sensor FE, an augmented stable system can also be used. This concept has been described in Section 4.3.

### 5.3 Robust Adaptive Actuator Fault Tolerant Control

In this Section, two different types of uncertainty are considered, which are *matched disturbance/uncertainty* and *unmatched disturbance/uncertainty*. Any disturbance/uncertainty which does not lie within the range space of the input

distribution matrix is described as unmatched modelling disturbance/uncertainty, and vice versa. The relationship between the weighting matrix  $E$  of the matched uncertainty and the input weighting matrix  $B$  can be expressed as  $\mathcal{R}(E) \subset \mathcal{R}(B)$ , where  $\mathcal{R}(\ast)$  represents the range of the  $\ast$  (Edwards and Spurgeon, 1998).

For the case of matched uncertainty, there is more difficulty in actuator FE process. Since it affects the input channel directly, this results in the poor performance not only in dealing with FE but also for the FTC process. Consequently, to achieve fault tolerance with robustness to matched disturbances/uncertainty, some researchers consider the matched uncertainty with actuator faults together when doing robust FTC designs (Edwards and Spurgeon, 1998; Cunha *et al.*, 2003; Jin and Yang, 2009). Whilst for the unmatched uncertainty case, the main challenge lies in the robust FTC process. It is also easy to decouple the disturbances/uncertainty from the input signal and many mature FE methods can be applied (Chen and Patton, 1998). In this Section, both these two types of uncertainty are considered and regarding the analysis are given.

### 5.3.1 Robust adaptive actuator fault tolerant control against matched uncertainty

To consider the matched uncertainty, a new Assumption must to be made as follows:

**Assumption 5.5:** For the system Eq. (5-6), there exists a matrix function  $M \in \mathfrak{R}^{m \times r}$  such that  $E = BM$ .

And then the system description Eq. (5-6) is rewritten as:

$$\dot{x}(t) = Ax(t) + B\rho(t)u(t) + B\sigma u_s(t) + BMd(t) \quad (5-32)$$

$$y(t) = Cx(t)$$

And the corresponding observer is designed as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\rho(t)u(t) + \eta[y(t), \hat{x}(t)] + L[y(t) - \hat{y}(t)] \quad (5-33)$$

$$\hat{y}(t) = C\hat{x}(t)$$

Then the error dynamics are obtained as:

$$\dot{e}(t) = (A - LC)e(t) + B\sigma u_s(t) + BMd(t) - \eta[y(t), \hat{x}(t)] \quad (5-34)$$

It is noted that the observer input signal is the same as the plant dynamics as the object of the observer Eq. (5-33) here is only to acquire the state estimate but not the fault estimate, which is further using in the direct reconfigurable adaptive control law. To achieve asymptotic convergence of the error dynamics of Eq. (5-34), the design of the nonlinear term  $\eta(y, \hat{x}) = \frac{d_1 E F_2 e_y(t)}{\|F_2 e_y(t)\|}$  is kept the same as the one in Eq. (5-24). Meanwhile Theorem 5.2 is applied with the corresponding Assumptions 5.1-5.4 (it depends on different designs for  $\eta[y(t), \hat{x}(t)]$ , if the adaptive algorithm is adopted, then the Assumption 5.3 reduces the strictness on the known upper bound of the disturbance). The proof is similar to that of Theorem 5.2, and is omitted here. Now the observer-based FTC adaptive control scheme is given below as:

$$u_f(t) = \hat{\rho}^{-1}(t)[-K_1 \hat{x}(t) + \eta_1(y, t)] \quad (5-35)$$

where  $K_1 \in \mathfrak{R}^{m \times n}$  is the feedback gain matrix,  $\eta_1(y, t) = [\eta_{11}(y, t), \eta_{12}(y, t), \dots, \eta_{1m}(y, t)]^T \in \mathfrak{R}^m$  is the adaptive FTC law will define later, and  $\hat{\rho}^{-1}(t)$  is the inverse of the estimate of the actuator effectiveness which is modified from Eq. (5-8), and given as:

$$\hat{\rho}_i(t) = Proj_{[\underline{\rho}_i^j, \bar{\rho}_i^j]} \{-\alpha_i F_{3i} y(t) u_{fi}^T(t)\} \quad (5-36)$$

where  $F_{3i}$  is the  $i$ th row of  $F_3$ ,  $u_{fi}^T$  is the  $i$ th column of the FTC control law,  $Proj\{\}$  and  $\alpha_i$  are as same as defined in Theorem 5.1, and  $\eta_1(y, t)$  is given as:

$$\eta_1(y, t) = -\frac{\hat{K}_2(t) \|F_3 y(t)\|}{\|F_3 y(t)\|} \quad (5-37)$$

Furthermore,  $\hat{K}_2$  is updated by the following law:

$$\dot{\hat{K}}_2(t) = \gamma \|F_3 y(t)\| \quad (5-38)$$

where  $\gamma$  is design positive constant and  $\hat{K}_2(t)$  is finite. From Eq. (5-38) it can be clearly seen that  $\hat{K}_2(t) \geq 0$ , as long as the initial value  $\hat{K}_2(t_0) \geq 0$ .  $\hat{K}_2(t)$  here is adaptive law to cover the positive unknown constant  $K_2 = \|d_2\| + \|Md\| \geq 0$ , based on

Assumptions 5.3 and 5.4, but with  $d_1$  unknown. By denoting  $\tilde{K}_2(t) = K_2 - \hat{K}_2(t)$ , it is clear that  $\dot{\tilde{K}}_2(t) = -\dot{\hat{K}}_2(t)$ .

Thus, returning to the FTC law Eq. (5-35) and referring to Eqs. (5-36) – (5-38) for according to the design of the parameter in Eq. (5-32), the closed-loop FTC system model is rewritten as:

$$\dot{x}(t) = Ax(t) + B\rho(t)\hat{\rho}^{-1}(t)(-K_1\hat{x}(t) + \eta_1(y, t)) + B\sigma u_s(t) + BMd(t) \quad (5-39)$$

$$y(t) = Cx(t) \quad (5-40)$$

Eq. (5-39) can be further re-arranged as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\rho(t)\hat{\rho}^{-1}(t)[-K_1\hat{x}(t) + \eta_1(y, t)] + B\sigma u_s(t) + BMd(t) \\ &= Ax(t) + B\hat{\rho}(t)\hat{\rho}^{-1}(t)[-K_1\hat{x}(t) + \eta_1(y, t)] + B\sigma u_s(t) + BMd(t) \\ &\quad + B\rho(t)u_f(t) - B\hat{\rho}(t)u_f(t) \\ &= Ax(t) - BK_1\hat{x}(t) + B\eta_1(y, t) + B\sigma u_s(t) + BMd(t) \\ &\quad + B\Delta\rho(t)u_f(t) \\ &= (A - BK_1)x(t) + BK_1e(t) + B\eta_1(y, t) + B\sigma u_s(t) + BMd(t) \\ &\quad + B\Delta\rho(t)u_f(t) \end{aligned} \quad (5-41)$$

Then the asymptotic stability of the closed-loop system Eq. (5-41) and the error dynamics (5-34) are guaranteed by the following Theorem:

**Theorem 5.3:** under the Assumptions 5.1-5.5, for the whole systems Eq. (5-41) and (5-34), if there exists a symmetric positive definite matrices  $Q, G \in \mathfrak{R}^{n \times n}$ , and matrix  $F_3 \in \mathfrak{R}^{m \times p}$ , such that the following conditions:

$$(A - BK_1)^T Q + Q(A - BK_1) = -G < 0 \quad (5-42)$$

$$B^T Q = F_3 C \quad (5-43)$$

and Theorem 5.2 hold, then the observer-based FTC of Eq. (5-35) with its parameters designed according to Eqs. (5-36) - (5-38) can realise the whole system of Eq.(5-41) and (5-34). This system is asymptotically stable, where the observer design can refer to Theorem 5.1 and Theorem 5.2.

**Proof:**

First, consider a Lyapunov function for the nominal system of Eq. (5-1) described as follows:

$$V_c(t) = x^T(t)Qx(t) \quad (5-44)$$

The derivative of  $V_c(t)$  with respect to time is obtained as:

$$\begin{aligned} \dot{V}_c(t) &= x^T(t)(Q(A - BK) + (A - BK)^T Q)x(t) \\ &= -x^T(t)Gx(t) \leq -\beta\|x(t)\|^2 \end{aligned} \quad (5-45)$$

where  $G = (A - BK_1)^T Q + Q(A - BK_1)$  and  $\beta = \lambda_{\min}(G)$ .  $\beta$  is an unknown positive scalar. If  $G > 0$ , then  $\dot{V}_c(t) < 0$ . Therefore the nominal system Eq. (5-1) is asymptotically stable.

Secondly, for the whole system Eqs. (5-41) and (5-34), a new candidate Lyapunov function is set up as:

$$V_w(t) = V_c(t) + \mu V_o(t) + \text{trace}(\Delta\rho(t)\Gamma^{-1}\Delta\rho(t)) + \gamma^{-1}\tilde{K}_2^2(t) \quad (5-46)$$

where  $V_o(t)$  is different from the Lyapunov function chosen for Theorem 5.1 and 5.2. Here  $V_o(t)$  is chosen as  $V_o(t) = e^T(t)Pe(t)$ .

According to the Theorem 5.1, the derivative of  $V_o(t)$  can be described by Eq. (5-20), and based on Eq. (5-22) the following result can be obtained:

$$\dot{V}_o(t) \leq -\lambda_{\min}(R)\|e(t)\|^2 \quad (5-47)$$

Denote  $\alpha = \lambda_{\min}(R)$ , where  $\alpha$  is a positive scalar. By bringing Eqs. (5-47) and (5-45) into the finite derivative of Eq. (5-46) with respect to time, the following result is obtained:

$$\begin{aligned}
\dot{V}_w(t) &= \dot{V}_c(t) + \mu\dot{V}_o(t) + 2\text{trace}(\Delta\rho(t)\Gamma^{-1}\Delta\dot{\rho}(t)) - 2\gamma^{-1}\tilde{K}_2(t)\dot{\hat{K}}_2(t) \\
&\leq -\beta\|x(t)\|^2 - \alpha\|e(t)\|^2 + 2x^T(t)QBK_1e(t) \\
&\quad + 2x^T(t)QB\eta_1(y,t) + 2x^T(t)QB\sigma u_s(t) \\
&\quad + 2x^T(t)QBMd(t) + 2x^T(t)QB\Delta\rho(t)u_f(t) \\
&\quad - 2\gamma^{-1}\tilde{K}_2(t)\dot{\hat{K}}_2(t) \\
&\leq -\beta\|x(t)\|^2 - \mu\alpha\|e(t)\|^2 + 2x^T(t)QBK_1e(t) \\
&\quad + 2x^T(t)QB\eta_1(y,t) + 2\|x^T(t)QB\|(\|\sigma u_s(t)\| + \|Md(t)\|) \\
&\quad + 2x^T(t)QB\Delta\rho(t)u_f(t) - 2\gamma^{-1}\tilde{K}_2(t)\dot{\hat{K}}_2(t) \\
&\quad + 2\text{trace}(\Delta\rho(t)\Gamma^{-1}\Delta\dot{\rho}(t)) \tag{5-48}
\end{aligned}$$

In a similar manner to Theorem 5.1, the terms  $2\text{trace}(\Delta\rho(t)\Gamma^{-1}\Delta\dot{\rho}(t))$  and  $+2x^T(t)QB\Delta\rho(t)u_f(t)$  are cancelled, and based on Eqs. (5-38) and (5-43), Eq. (5-48) becomes:

$$\begin{aligned}
\dot{V}_w(t) &\leq -\beta\|x(t)\|^2 - \mu\alpha\|e(t)\|^2 + 2x^T(t)QBK_1e(t) \\
&\quad + 2\|F_3y(t)\|\tilde{K}_2 + 2\|F_3y(t)\|(\|\sigma u_s(t)\| + \|Md(t)\|) \\
&\quad - 2\gamma^{-1}\tilde{K}_2(t)\dot{\hat{K}}_2(t) \\
&= -\beta\|x(t)\|^2 - \mu\alpha\|e(t)\|^2 + 2x^T(t)QBK_1e(t) \\
&\quad + 2\|F_3y(t)\|\tilde{K}_2 - 2\|F_3y(t)\|\tilde{K}_2(t) \tag{5-49}
\end{aligned}$$

Finally, Eq. (5-49) can be rewritten in compact form as:

$$\dot{V}_w(t) \leq -\beta\|x(t)\|^2 - \mu\alpha\|e(t)\|^2 + 2x^T(t)QBK_1e(t) \tag{5-50}$$

Denote  $\varepsilon = \|QBK_1\|$ , then  $\dot{V}_w(t)$  can be rewritten as:

$$\dot{V}_w(t) \leq -\beta\|x(t)\|^2 - \mu\alpha\|e(t)\|^2 + 2\varepsilon\|x(t)\| \cdot \|e(t)\|$$

$$\begin{aligned}
&= -\mu\alpha \left( \|e(t)\|^2 - \frac{2\varepsilon}{\mu\alpha} \|x(t)\| \cdot \|e(t)\| + \frac{\varepsilon^2}{\mu^2\alpha^2} \|x(t)\|^2 \right) \\
&\quad + \frac{\varepsilon^2}{\mu^2\alpha^2} \|x(t)\|^2 - \beta \|x(t)\|^2 \\
&\leq -\frac{1}{\mu\alpha} (\mu\alpha\beta - \varepsilon^2) \|x(t)\|^2 \tag{5-51}
\end{aligned}$$

By choosing  $\mu > \frac{\varepsilon^2}{\alpha\beta}$ , it is easy to show that  $\dot{V}_w(t) < 0$ , for any  $x(t) \neq 0$ . Thus the global observer-based adaptive fault-tolerant compensation control problem with disturbance rejection for the whole systems Eqs. (5-41) and (5-34) is solvable, and the state  $x(t)$  and error  $e(t)$  converges asymptotically to zero. Q.E.D.

The proposed robust observer-based adaptive FTC method can deal with systems with matched uncertainty/disturbance. Compared with the work in (Jin and Yang, 2009; Ye and Yang, 2006; Zhang, Jiang and Shi, 2007), the contribution lies in the following aspects. Jin and Yang (2009) consider that all the states of the system are available at every instant. However this is not always possible in practice. In this Section, an observer-based FTC design has solved this problem. Zhang, Jiang and Shi (2007) designed the reconfigurable control scheme based on the so-called Fast Fault Estimation method, but without considering any disturbance or uncertainty when applying the FTC. Ye and Yang (2006) proposed the Indirect Adaptive Method for FTC. However, they did not consider the uncertainty/disturbances. Meanwhile, the method proposed in this Section can solve the more general actuator fault problem corresponding to a time-varying fault effect factor  $\rho(t)$ , which cannot be solved by the Indirect Adaptive Method.

### 5.3.2 Robust adaptive FTC for unmatched uncertainty

In practice, not all the uncertainty is matched, which means that the Assumption 5.5 is not always satisfied. In this Section, the unmatched uncertainty/disturbance is considered when designing the FTC scheme. As described in Section 5.3.1, for unmatched uncertainty the challenge lies in choice of method to solve the robustness of uncertainty/disturbance problem of the fault accommodation. The corresponding fault estimation problem is dealt with well.

Consider the faulty system Eq. (5-6) again, but this time Assumption 5.5 is not satisfied.

Then in keeping with the robust fault estimation method described in Section 5.2, the actuator effectiveness factor  $\rho(t)$  can be estimated with property of robustness to the disturbance  $d(t)$ . On the basis of that, the adaptive FTC scheme is given as for Eq. (5-35). However, for the design of the feedback gain  $K_1$  in Eq. (5-35) the reader should refer to Chapter 4 Eq. (4-41), which is based on  $H_\infty$  optimization to satisfy the system performance  $\|T_{dy}\|_\infty < \gamma$ . On the other hand, the second term in the adaptive control law Eq. (5-37) is designed as:

$$\eta_1(y, t) = -\frac{\hat{d}_2(t)\|F_3y(t)\|}{\|F_3y(t)\|} \quad (5-52)$$

where  $\hat{d}_2(t)$  is the estimate of the positive parameter  $d_2$  with Assumption 5.4  $\|f(t)\| = \|u_s(t)\| \leq d_2$ . Meanwhile  $\hat{d}_2(t)$  is updated by the same law as  $\hat{K}_2(t)$  in Eq. (5-37).

On the basis of the above description, the whole observer-based FTC dynamics are modified from Eq. (5-41):

$$\begin{aligned} \dot{x}(t) = & (A - BK_1)x(t) + BK_1e(t) + B\eta_1(y, t) + B\sigma u_s(t) + Ed(t) \\ & + B\Delta\rho(t)u_f(t) \end{aligned} \quad (5-53)$$

Then the asymptotic stability of the closed-loop system Eq. (5-53) and the error dynamics (5-24) are guaranteed.

**Theorem 5.4:** Under the Assumptions 5.1-5.4, the whole closed - loop system of Eqs. (5-53) and (5-10) is asymptotic stable and fault tolerant and, furthermore  $\|T_{dy}\|_\infty < \gamma$ , if Theorems 4.4, 5.1 and 5.3 hold.

As the proof of Theorem 5.4 constitutes the combination of the Theorems 4.3, 4.5 and Theorem 5.3, it is thus omitted here.

### 5.3.3 Robust adaptive fault tolerant control for mixed-uncertainty

Section 5.3.1 and Section 5.3.2 discuss the robust adaptive FTC schemes for matched uncertainty and unmatched uncertainty respectively. However, in practice these two

different forms of uncertainties can exist simultaneously. In this Section, the main object is to discuss the robust FTC for the two cases, (1) the single disturbance works not only in the input channel but outside the input channel, (2) the multiple disturbance works on the structure of the overall system dynamics.

For the first situation, the state-space model of the faulty system with mixed-uncertainty is described as:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B\rho(t)u(t) + Bf(t) + E_1d(t) + E_2d(t) \\ y(t) &= Cx(t)\end{aligned}\tag{5-54}$$

This implies that the weighting matrix  $E$  in Eq. (5-7) can be further divided into  $E = E_1 + E_2$ , in which  $E_1$  represents the distribution matrix of the matched uncertainty which satisfies the Assumption 5.5 with  $E_1 = BM$  and  $E_2$  represents the weighting or distribution matrix of one of the unmatched uncertainties. Under this situation, the estimation of the actuator effectiveness factor  $\hat{\rho}(t)$  can be achieved according to the design by Theorem 5.1 and Theorem 5.2. For the case of the estimate of the stuck fault  $f(t)$ , Eq. (5-54) becomes:

$$\dot{x}(t) = Ax(t) + B\rho(t)u(t) + B \underbrace{(f(t) + Md(t))}_{f_{new}(t)} + E_2d(t)\tag{5-55}$$

where  $f_{new}(t) = f(t) + Md(t)$ , thus the new defined actuator fault  $f_{new}(t)$  and disturbance  $d(t)$  distributed by matrix  $E_2$  can be estimated separately (similar as the fault estimation process), and the original actuator fault estimate  $\hat{f}(t)$  can be achieved by using  $\hat{f}(t) = f_{new}(t) - M\hat{d}(t)$ , where  $\hat{d}(t)$  is the estimate of the disturbance. The method for disturbance estimation is as the same as the fault estimation Eq. (4-41).

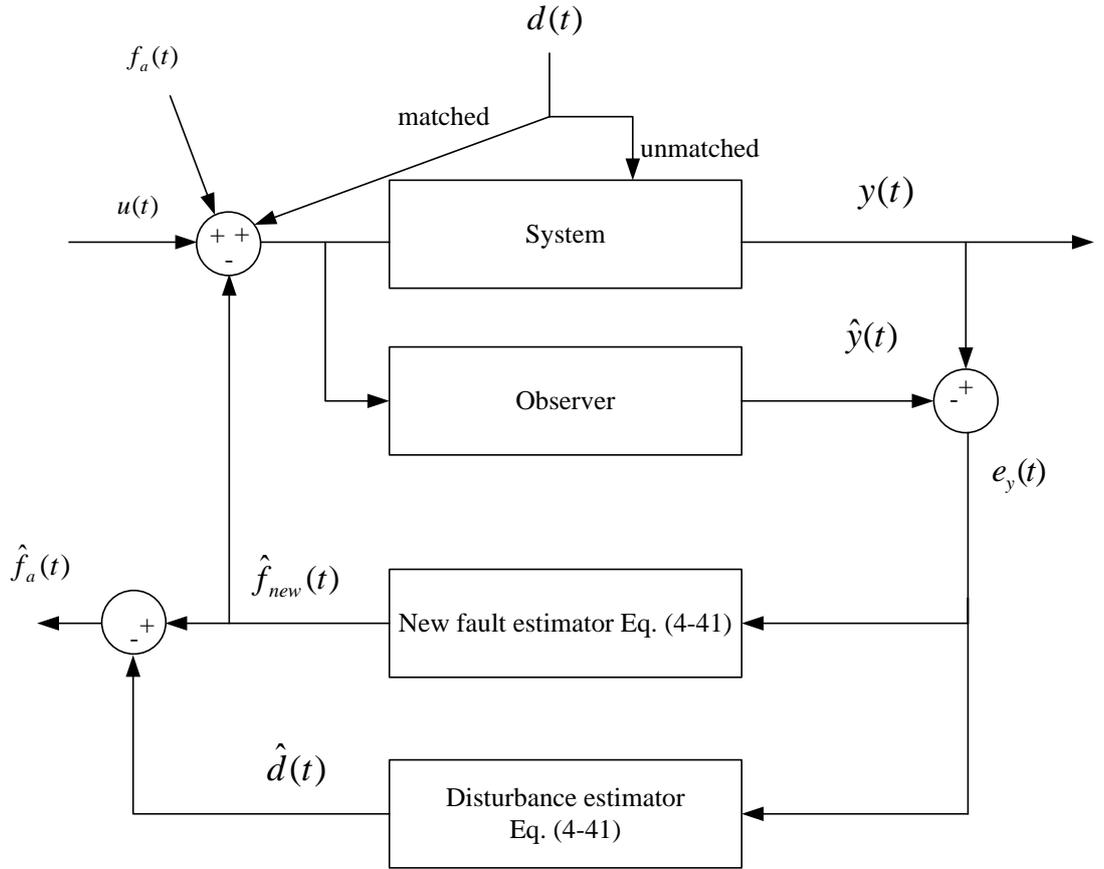


Figure 5-2: Structure of the FE and FTC with separable single disturbance

Whilst for FTC scheme, the method proposed in Section 5.3.2 can be applied to achieve the FTC, and the structure of the FE and FTC is shown in Figure 5-2.

For the second situation, the system dynamics are written as:

$$\dot{x}(t) = Ax(t) + B\rho(t)u(t) + Bf(t) + E_1d_1(t) + E_2d_2(t) \quad (5-56)$$

$$y(t) = Cx(t)$$

where  $d_1(t)$  is the matched disturbance/uncertainty and  $d_2(t)$  is the unmatched disturbance/uncertainty. Under this situation, the proposed robust adaptive fault estimation method is no longer suitable for the stuck fault  $f(t)$  estimation. This can be considered an interesting topic for future research. However, it must be stated that the fault tolerance is still achieved via combine the method proposed in Sections 5.3.1 and 5.3.2.

## 5.4 Simulation and results

In this Section, two tutorial examples of a linear rocket fairing structural-acoustic model and a non-linear flexible joint robot link problem are used to illustrate the FTC design strategy of this Chapter.

### 5.4.1 Case study for Rocket fairing structural-acoustic model

This linear model is obtained from (Tang, *et al.*, 2006) with external disturbance input added, which is used to illustrate the method in Section 5.3.1. The whole model comprises two sub-systems, which are structural modelling and rigid-wall acoustic cavity modelling. The structural model for the fairing is formulated as:

$$\dot{w}(t) = A_s w(t) + B_s u(t) + H_s d(t) \quad (5-57)$$

$$y_s(t) = C_s w(t)$$

where  $w(t) = [z^T(t), \dot{z}^T(t)]^T \in \mathfrak{R}^{2n_1}$  is the structural state vector with  $z(t) \in \mathfrak{R}^{n_1}$  and  $\dot{z}(t) \in \mathfrak{R}^{n_1}$  being structural displacement (cm) and velocity vectors (cm/s) for  $n_1$  modes, respectively.  $u(t) \in \mathfrak{R}^m$  is a vector of  $m$  control inputs at the structural nodes, i.e. the structural actuator outputs whose components may fail during system operation,  $y_s(t) \in \mathfrak{R}^{n_1}$  is a vector of structural displacements,  $d(t) \in \mathfrak{R}^q$  is a disturbance vector,  $A_s \in \mathfrak{R}^{2n_1 \times 2n_1}$ ,  $B_s \in \mathfrak{R}^{2n_1 \times m}$ ,  $C_s \in \mathfrak{R}^{n_1 \times 2n_1}$ , and  $H_s \in \mathfrak{R}^{n_1 \times q}$  are structure-related matrices associated with each corresponding mode.

The model for the air cavity enclosed by the fairing structure can be expressed as:

$$\dot{r}(t) = A_a r(t) + B_a w(t) \quad (5-58)$$

$$y_a(t) = C_a r(t)$$

where  $r(t) \in \mathfrak{R}^{2n_2}$ ,  $y_a(t) \in \mathfrak{R}^l$  is the vector of pressures within the cavity,  $A_a \in \mathfrak{R}^{2n_2 \times 2n_2}$ ,  $B_a \in \mathfrak{R}^{2n_2 \times 2n_1}$ , and  $C_a \in \mathfrak{R}^{l \times 2n_2}$  are acoustic-related matrices for  $n_2$  acoustic modes. And the overall structural-acoustic fairing model combines the structural and acoustic models as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (5-59)$$

$$y(t) = Cx(t)$$

where  $A = \begin{bmatrix} A_s & B_{sa} \\ B_a & A_a \end{bmatrix}$ ,  $B = \begin{bmatrix} B_s \\ 0 \end{bmatrix}$ ,  $E = \begin{bmatrix} H_s \\ 0 \end{bmatrix}$ ,  $C = [C_s \quad 0]$  and  $x(t) = [w^T(t), r^T(t)]^T \in \mathfrak{R}^n$ ,  $n = 2n_1 + 2n_2$ , is the state vector of the fully coupled fairing system, and  $B_{sa} \in \mathfrak{R}^{2n_1 \times 2n_2}$  is the matrix associated with vibroacoustical-related pressure acting at the fairing structure. For a single mode fairing model the parameter are given as:

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.1150 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M = \begin{bmatrix} 1.5 \\ -2 \\ -1 \end{bmatrix}, E = BM = \begin{bmatrix} -2.3500 \\ -0.9875 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

In the simulation the following faulty case is considered, the first actuator has totally failed. This is a special stuck actuator situation, and the second actuator loss of effectiveness has become  $\rho_1(t) = 0.6$ . The external disturbance is a Gaussian zero-mean white noise with variance 0.01 as shown in Figure 5-3.

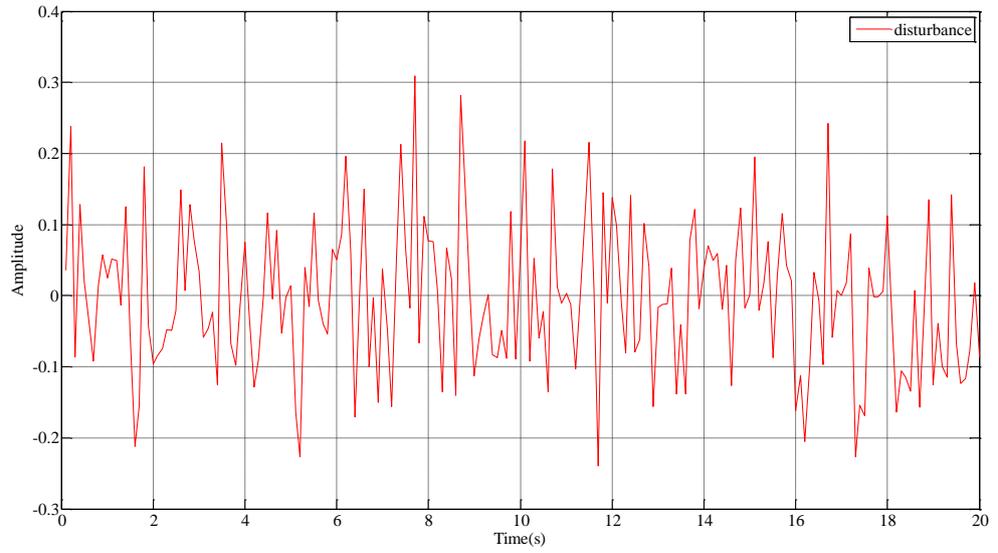


Figure 5-3: The external disturbance

According to Theorem 5.4, the following results can be obtained:

$$L = \begin{bmatrix} 17.5585 & 3.9909 \\ 2.3853 & 5.0891 \\ 99.5645 & 22.0599 \\ 57.7250 & 13.8069 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.1223 & -0.0360 & -0.0011 & -0.0008 \\ -0.0360 & 0.2821 & 0.0017 & 0.0005 \\ -0.0011 & 0.0017 & 0.0015 & -0.0006 \\ -0.0008 & 0.0005 & -0.0006 & 0.0017 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 2.5956 & -0.2730 & -1.0617 & -0.8154 \\ 2.4081 & -0.4127 & -1.2002 & -0.9280 \\ 2.0027 & -0.0839 & -0.7777 & -0.5500 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.2126 & -0.2491 & 0.0547 & -0.1612 \\ -0.2491 & 0.3364 & -0.1383 & 0.1747 \\ 0.0547 & -0.1383 & 0.2511 & -0.0006 \\ -0.1612 & 0.1747 & 0.0395 & 0.3195 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0.0872 & 0.2390 \\ 0.1608 & 0.1698 \\ 0.0611 & 0.2128 \end{bmatrix}, F_3 = \begin{bmatrix} -0.0302 & 0.0789 \\ 0.1304 & -0.1169 \\ -0.0522 & 0.0991 \end{bmatrix}$$

Figure 5-4 illustrates the comparison of the real actuator effectiveness factor and its estimate.

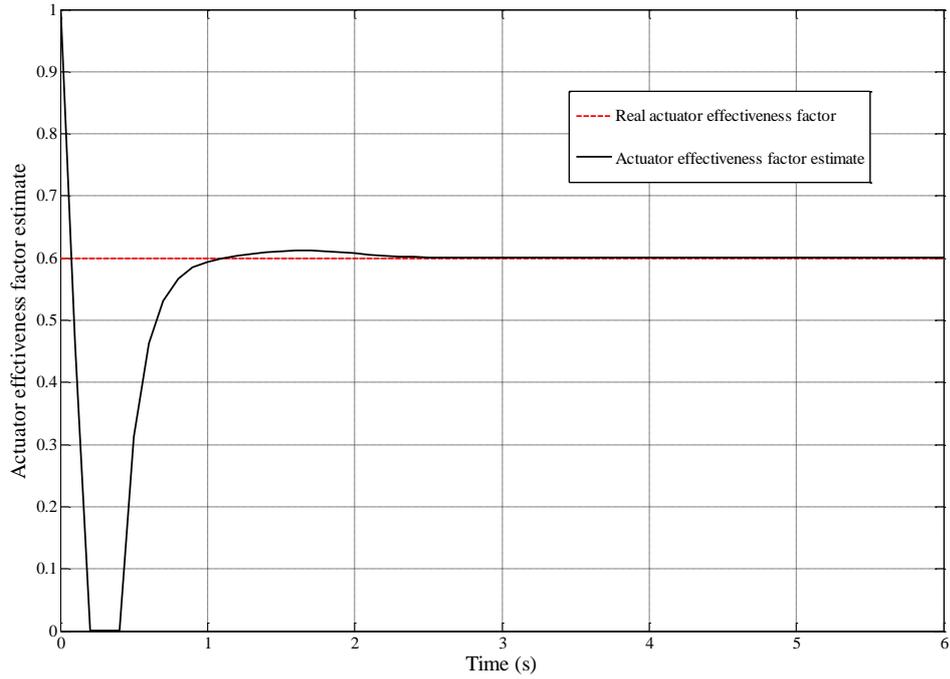


Figure 5-4: Comparisons of the real actuator effectiveness factor and its estimate

Figure 5-4 shows the actuator effectiveness factor estimation computed using Eq. (5-36) with setting  $\alpha = 10$ , and the initial value  $\hat{\rho}_1(t_0) = 1$ . The actuator effectiveness factor estimate signal is dropped from its initial value and goes out of the limited projected range  $[0,1]$ , then it returns and oscillates around the real actuator effectiveness factor  $\rho_1(t) = 0.6$ . Finally, the estimate remains almost exactly at the level of  $\rho_1(t) = 0.6$  with a small offset. The phenomenon can be explained as the factor estimate signal is based on the error of the system output and the observer output. At the beginning of the response, the error is very large, which causes the effectiveness factor estimate signal to exceed the limit. As the error is reduced and converges to zero with respect to as time, the changing rate of the factor estimate decreases as well, until it reaches the true effectiveness factor value.

Figure 5-5 shows the comparison of the output error with and without the nonlinear term  $\eta(y, \hat{x})$ .

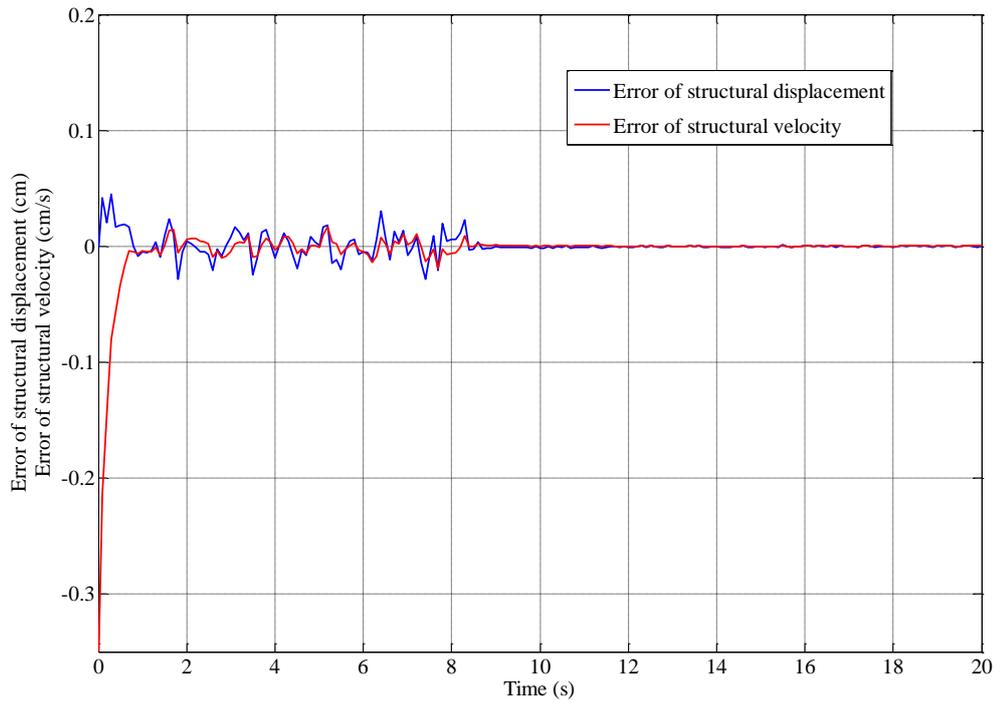


Figure 5-5: Comparison of the output error without and with nonlinear term  $\eta[y(t), \hat{x}(t)]$

Figure 5-5 shows that the errors of the system output responses and their estimates go to zero quickly when switching on the nonlinear term  $\eta[y(t), \hat{x}(t)]$  at 8s. Before that, the errors are heavily affected by the disturbance  $d(t)$  which means the proposed observer has strong robustness against the disturbance.

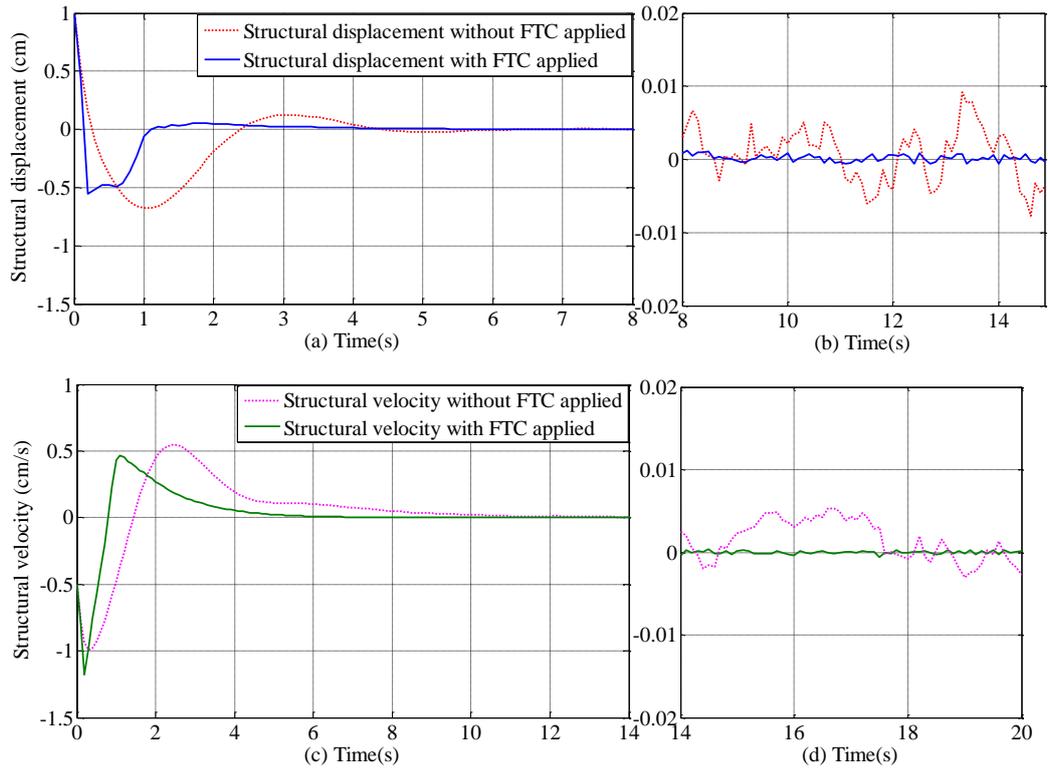


Figure 5-6: Comparisons of the system output responses with FTC and without

Figure 5-6 (a) and (c) show the comparisons of the system output response curves with and without application of the proposed FTC strategy applied and without for the above mentioned faulty case. It is clear to see that in the presence of faulty actuators and external matched disturbances with the initial values  $z(0) = 1$  cm and  $\dot{z}(0) = -0.5$  cm/s for the structural displacement and structural velocity, respectively. The system with FTC applied is stabilized more quickly than the situation without the FTC action applied.

Figures 5-6 (b) and (d) show after system is stabilized, the matched disturbance affects the system output responses heavily when the FTC action is not applied. Whilst with the application of the FTC the system is lightly affected by the matched disturbance.

Figure 5-7 shows the response of the estimated positive constant  $\hat{K}_2(t)$ . As the design for  $\hat{K}_2(t)$  Eq. (5-38) with the  $\gamma = 1$ , it is easy to see from the initial value of  $\hat{K}_2(t_0) = 0$ , the response curve increases quickly in the beginning, because of the large state errors in the start. However, it increases slowly along the variation of the state errors as shown in Figure 5-5, and it increases all the time, and is always positive.

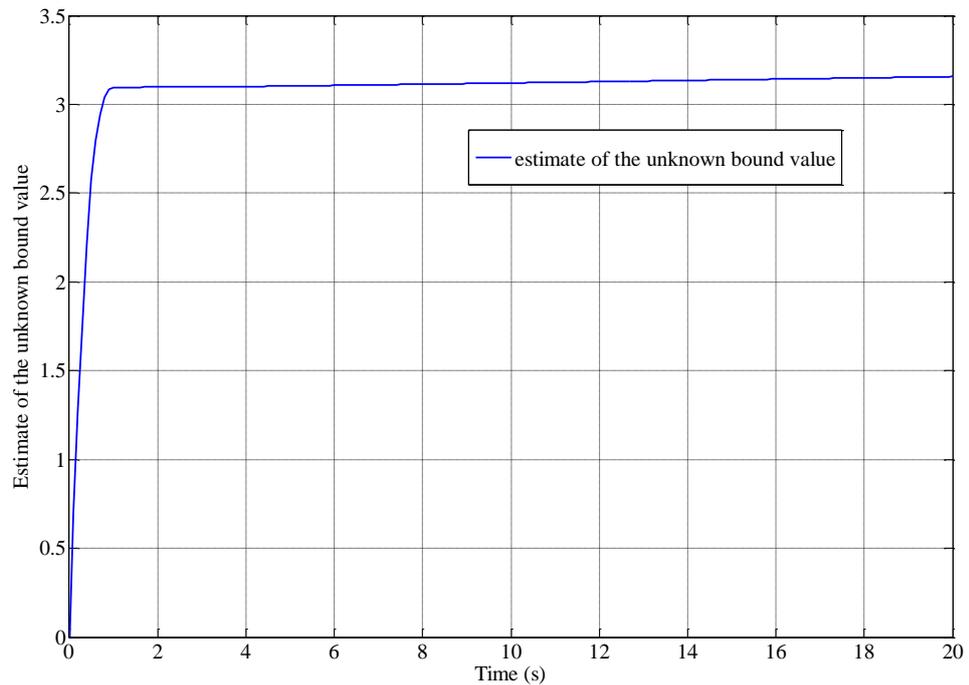


Figure 5-7: Estimate of the unknown bound value  $K_2$

#### 5.4.2 Case study for nonlinear single flexible joint robot link

In this Section, a nonlinear single flexible joint robot link problem is used to illustrate the observer-based FTC design described in Section 5.3.2. This case study corresponds to a single input model. Hence, no consideration is given to the stuck actuator fault case and only actuator loss of effectiveness is considered. The model is obtained from (Spong, 1987; Raghavan, 1992). A one-link manipulator with revolute joints actuated by a DC motor is shown in Figure 5-8.

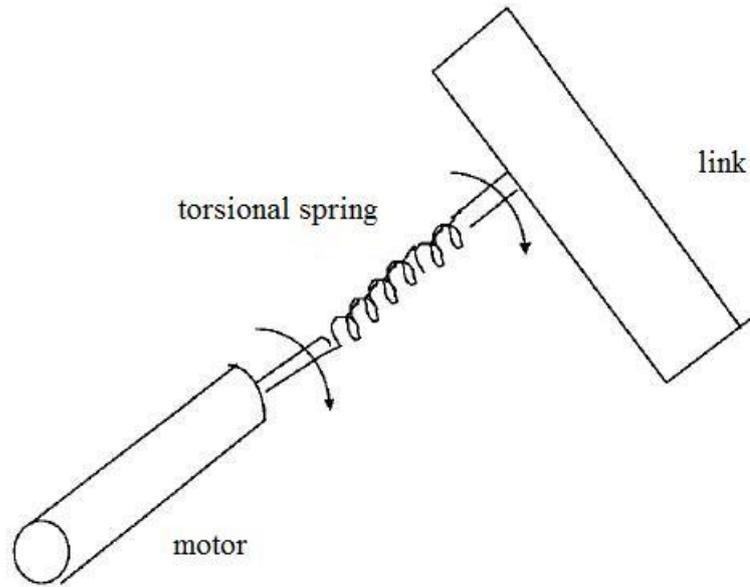


Figure 5-8: Robotic systems with flexible joint.

The corresponding state-space model is:

$$\dot{\theta}_m(t) = \omega_m(t)$$

$$\dot{\omega}_m(t) = \frac{k}{J_m}(\theta_1(t) - \theta_m(t)) - \frac{b}{J_m}\omega_m(t) + \frac{K_r}{J_m}u(t)$$

$$\dot{\theta}_1(t) = \omega_1(t)$$

$$\dot{\omega}_1(t) = -\frac{k}{J_1}(\theta_1(t) - \theta_m(t)) - \frac{mgh}{J_1}\sin(\theta_1(t))$$

where  $\theta_m(t)$ ,  $\omega_m(t)$  are the angular rotation and angular velocity of the motor respectively, and  $\theta_1(t)$ ,  $\omega_1(t)$  are the angular position and angular velocity of the link, respectively.  $J_m$  is the inertia of the motor;  $J_1$  is the inertia of the link;  $b$  is viscous friction coefficient;  $m$  is the pointer mass;  $k$  is the torsional spring constant;  $K_r$  is the amplifier gain;  $h$  is the link length; and  $g$  is the gravity constant.

The corresponding parameters are shown in Table 5-2

**Table 5-2:** Parameters and values of the single link robotic systems

Constants	$J_m$	$J_1$	$b$	$m$	$k$	$K_\tau$	$h$	$g$
Values	0.037	0.093	0.046	0.21	1.8	0.8	0.15	9.831
Units	Kg*m <sup>2</sup>	Kg*m <sup>2</sup>	Nm/V	Kg	Nm/rad	Nm/V	M	m/ s <sup>2</sup>

The system dynamics are clearly nonlinear with the non-linear terms in the model being due to joint flexibility. The systems dynamic model can be written as:

$$\dot{x}(t) = Ax(t) + \varphi(x, t) + Bu(t) \quad (5-60)$$

$$y(t) = Cx(t)$$

where  $x(t) = [\theta_m(t), \omega_m(t), \theta_1(t), \omega_1(t)]$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \varphi(x, t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33\sin(\theta_1(t)) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

The nonlinearity  $\varphi(x, t)$  in Eq. (5-60) can be further divided in to a multiplication of a weighting matrix and a disturbance signal vector, as:

$$\varphi(x, t) = Ed(x, t) \quad (5-61)$$

In practice, many forms of uncertainty/disturbance/nonlinearities can be described in the format of Eq. (5-61) (Patton and Chen, 1992, 1993; Patton, Chen and Zhang, 1992; Chen, 1995). On the other hand, for this robotic problem, the system nonlinearity takes the form of a kind of unmatched uncertainty/disturbance, so that the approaches proposed in Section 5.3.2 can be applied for the purpose of achieving robust fault estimation and FTC.

In the simulation, the faulty actuator has loss of effectiveness at  $\rho(t) = 0.6$  the FTC gain is taken from the work of Jiang, Staroswiecki and Cocquempot (2006). The following results are thus obtained:

$$K_1 = [0.7603 \quad 0.5054 \quad -0.0518 \quad 0.9547]$$

$$L = \begin{bmatrix} 0.6378 & -0.9368 & -0.0320 \\ -0.7581 & 33.6777 & 12.9531 \\ 2.0343 & 6.8107 & 7.6651 \\ -16.2669 & -61.5948 & 465.2538 \end{bmatrix}$$

$$P = \begin{bmatrix} 34.7299 & 0.9561 & -0.1829 & 0 \\ 0.9561 & 1.2356 & -0.6166 & 0 \\ -0.1829 & -0.6166 & 4.6731 & -0.0003 \\ 0 & 0 & -0.0003 & 0.1000 \end{bmatrix}$$

$$F_1 = 1.0e - 3 * [-0.0211 \quad 0 \quad -0.2558]$$

$$F_2 = [20.6517 \quad 26.6887 \quad -13.3192]$$

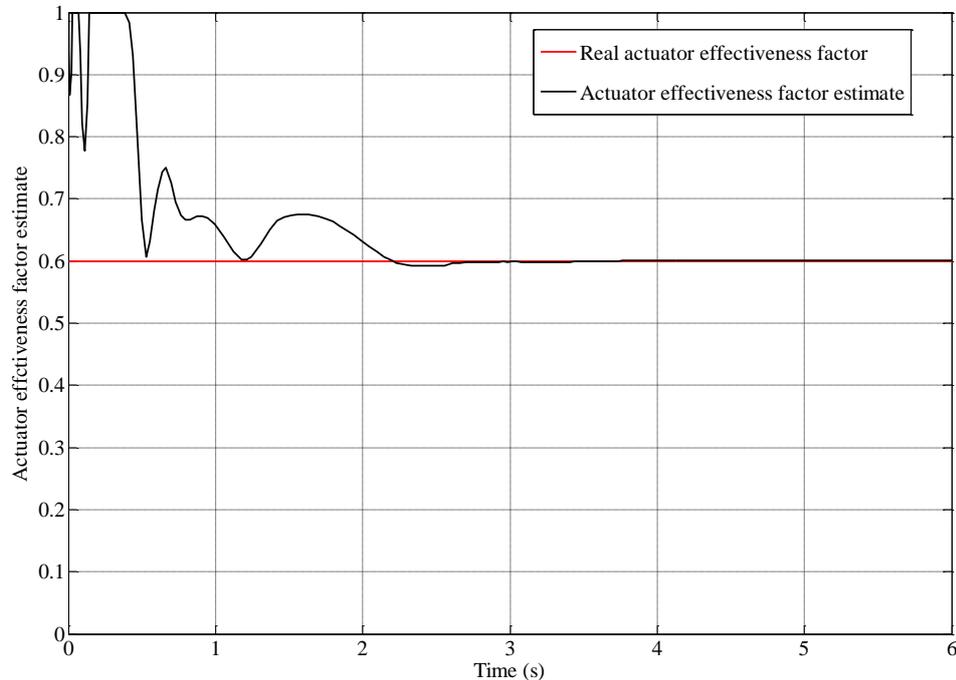


Figure 5-9: Comparison of the real actuator effectiveness factor and its estimate

Figure 5-9 shows the comparison of the real actuator effectiveness factor and its estimate. The actuator effectiveness factor estimate signal tracks the real actuator

effectiveness factor after 3 seconds, though the nonlinearity causes some oscillations at the beginning.

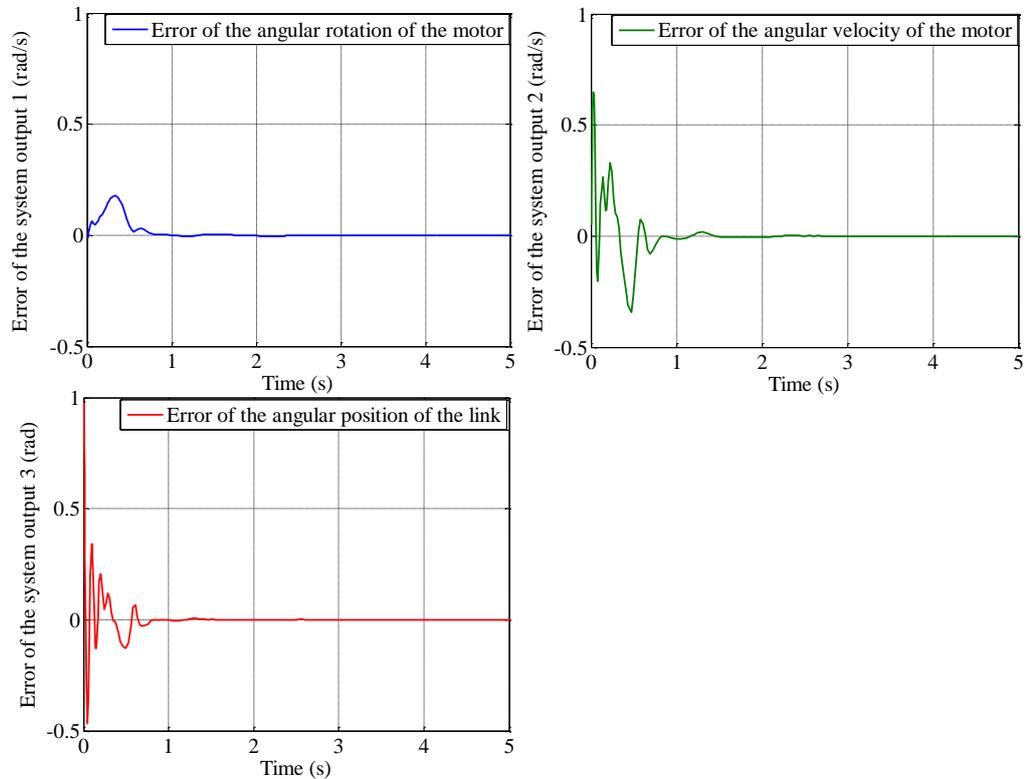


Figure 5-10: System output errors

Figure 5-10 illustrates the errors of the system output  $e_y(t)$ , based on the proposed integrated robust observer and fault estimator. It can be clearly seen that the errors converge to zero after a few of seconds, which means that the proposed observer is able to track the real system dynamics. There is, however, some oscillation at the beginning of the simulation.

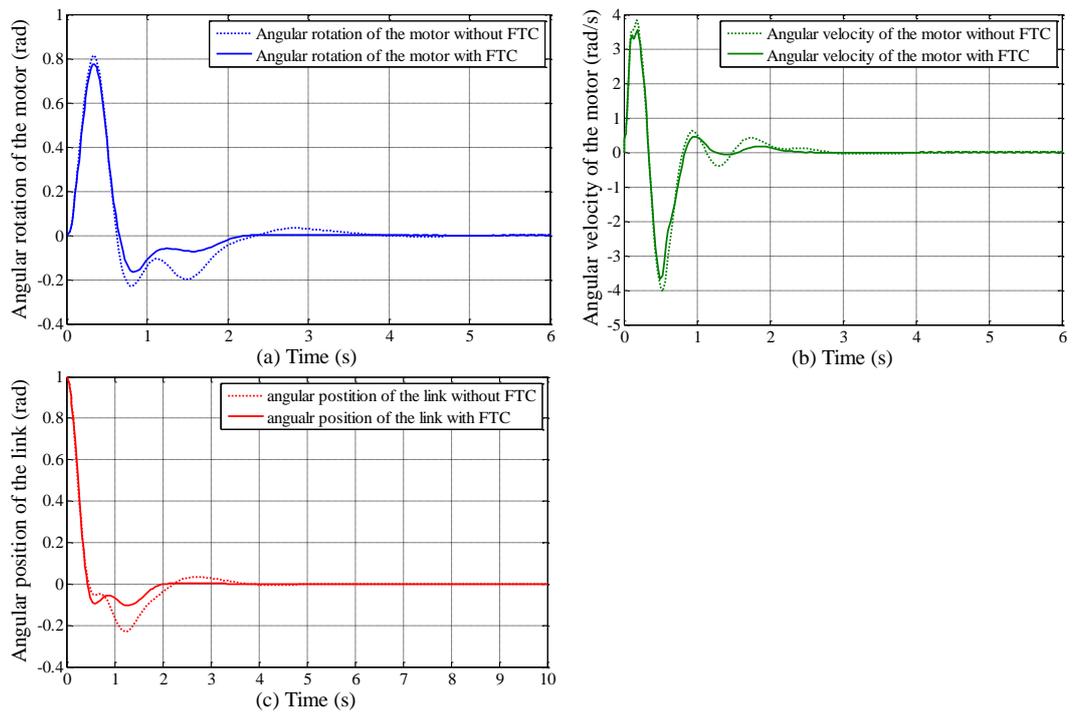


Figure 5-11: Response curves of the system output with FTC applied and without

The system output performance is clearly improved after the FTC applied. Under the described faulty case, the fault accommodation successfully improves the system performance in terms of the settling response time compared with the faulty situation. It suggests that the integration of the proposed observer and the controller designs can achieve the FTC objective. Meanwhile, it also suggests that some types of nonlinearity/uncertainty problems can be transferred to a system representation in which the external disturbances and unknown inputs can be handled.

Based on the results and analysis of these two cases studies, one can conclude that under certain assumptions, the proposed design strategy in this Chapter for robust fault estimation and further extension to robust FTC is applicable for real FTC system problems. Meanwhile, the approach can be further extended to systems with bounded non-linearities or combined with other control methods.

## 5.5 Conclusion

The main content of this Chapter is the provision of a strategy of robust adaptive actuator FE and compensation on the basis of the robust observer-based estimator proposed in Chapter 4. However compared with Chapter 4, this Chapter gives a more general fault description, which including *three* types of actuator faults.

In Section 5.2, a robust adaptive FE strategy is described and the validation of its applicability for cases of different actuator faults is given. By adding an adaptive updating law the requirement for the upper bound on the disturbances/uncertainties is recovered, and hence the applicability of the FE approach is enhanced. This is a significant contribution to this subject.

Section 5.3 discusses the robust adaptive FTC design based on the estimation information acquired in Section 5.2. More specifically, different forms of robust adaptive FTC strategies dealing with both matched and unmatched disturbances are proposed. A brief discussion of the mixed-structure (matched/unmatched) disturbance situation is given. Concerning the matched disturbance case, the on-line FE and combined FTC is capable of providing a high degree of fault-tolerance for actuator faults for which both the fault signal  $f_a(t)$  and its derivative signal  $\dot{f}_a(t)$  are bounded. With this, an excellent recovery of the system closed-loop performance is demonstrated.

It should also be emphasised that in all of the work of this Chapter (and for the whole thesis) the state estimates rather than the system states are used for the FTC. The recent FTC work (Jin and Yang, 2009) on fault compensation is limited to the use of full state rather than state estimate information.

Section 5.4 gives two examples to show the applicability of the proposed methods. The linear rocket fairing structural-acoustic example and the nonlinear single flexible joint manipulator example are used to support the proposal for combining FE and FTC under the matched disturbance and unmatched disturbance, respectively. The results show the power and potential of this integrated design, which successfully connects the observer, estimator and controller together to achieve the robust FTC goals.

When considering sensor faults, the approach can make use of the augmented system model as described in Section 4.3. In this Chapter, both additive and multiplicative faults are considered. All the above methods are based on linear time invariant systems, although there is a nonlinear application. However, real-life engineering systems are always nonlinear or are linear system with some nonlinear terms. Hence the application of the purposed FE approach to FTC for nonlinear systems is discussed in Chapter 6.

# Chapter 6.

## Fault Estimation and Compensation for LPV system

### 6.1 Introduction

There has been significant interest in the control of time-varying systems over many years (Leith and Leithead, 2000; Balas, 2002). In recent years, the development of fault estimation and compensation for Linear Parameter Varying (LPV) systems via polytopic modelling methodology have gained a great deal of interest, especially for applications related to robots, vehicle systems, and aerospace control (Wu, 2001; Ganguli, Marcos and Balas, 2002; Balas, 2002; Weng, Patton and Cui, 2008; Balas, 2012). The LPV description preserves the linear time invariant (LTI) structure, the only difference occurs when computing the coefficients. The parameter vector is a continuously time-dependent known function facilitating the evaluation of the transformed nonlinear system at every sample instant.

For control applications the LPV modelling approach facilitates the direct application of classical control structures directly on the time-varying and non-linear system with robust results. FDI schemes based on the LPV system formulation have also been developed (Bokor and Balas, 2004; Bokor and Kulcsar, 2004; Henry and Zolghadri, 2005; Rodrigues, Theilliol and Sauter, 2005; Casavola, *et al.*, 2007, 2008; Weng, Patton and Cui, 2008; Zhang, Jiang and Chen, 2009; Armeni, Casavola and Mosca, 2009; Kulcsar, Bokor and Shinar, 2010; Chen and Patton, 2011; Hamdi, *et al.*, 2011; Astorga-Zaragoza, *et al.*, 2011 ; Alwi, Edwards and Marcos, 2012).

For FTC, Chen *et al* (1999) tackled an FTC flight control design study using a *Linear Fractional Transformation* (LFT) approach via the LMI framework. AFTC controllers are either based on on-line FE (fault compensation) or FDI/FDD and control system reconfiguration. The FE approaches require the generation of estimates of possible

faults to allow the FTC controller to tolerate the faults. Generally speaking the FDI problem is not relevant within this framework unless the more general FDD problem is considered as this includes FE. As discussed in Chapters 4 and 5, the AFTC Strategy that includes FE is a powerful approach to on-line controller reconfiguration. Many researchers have extended the integrated design of the FE and FTC successfully within an LPV field (Ganguli, Marcos, and Balas, 2002; Rodrigues, Theilliol and Sauter, 2005, 2007; Patton, Chen and Klinkhieo, 2012). However, in these studies, no one proved the control stability for the integrated design of the FE and FTC and in every case the actuator loss of effectiveness is considered without additive faults. In this chapter, the contribution concerns the additive actuator faults, with the provision of control stability for the integrated design. Also, there are still many other passive LPV approaches to achieve fault tolerance (Weng, Patton and Cui, 2007; Song and Yang, 2011; Alwi, Edwards and Marcos, 2012), by combining the AFTC literature, which comprises the background to the work of this Chapter.

This Chapter proposes a new design approach for AFTC including a polytopic LPV fault estimator for systems which can be characterized via a set of Linear Matrix Inequalities (LMIs) based on efficient interior-point algorithms (Apkarian, Gahinet and Becker, 1995). From another point, the work in this Chapter is an extension to the work of Chapter 4. A robust polytopic LPV estimator is synthesized for providing actuator fault estimation which is used in an AFTC scheme to schedule the state feedback gain. This gain is also calculated using LMIs in the fault-free case in order to maintain the system performance over a wide operating range within a proposed polytopic model. The applicability of the proposed method is demonstrated through a nonlinear two-link manipulator system with a fault in the torque input at one manipulator joint. This is a nonlinear system that can be represented well by a polytopic model and this is also proposed in Section 6.5.

## **6.2 General overview of the LPV approach**

An LPV system is a mathematical realization/description of the linear parameter-varying nature of a dynamical system. LPV systems depend on a set of time-varying scheduling parameters over time. These systems are represented in state space

(continuous or discrete) form. The LPV model has the structure of a time-varying linear system with the parameter-dependent matrix quadruple  $[A(\theta), B(\theta), C(\theta), D(\theta)]$ .

where:  $A(\theta) \in \mathfrak{R}^{n \times n}$ ,  $B(\theta) \in \mathfrak{R}^{n \times m}$ ,  $C(\theta) \in \mathfrak{R}^{p \times n}$  and  $D(\theta) \in \mathfrak{R}^{p \times m}$  as follows:

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t) + D(\theta)u(t) \end{aligned} \quad (6-1)$$

where:  $\theta$  is a vector of smoothly changing system parameters. In this Chapter the distribution matrix of the input signal working on each output channel is set to be zero, which means that  $D(\theta) = 0$ .

An LPV system can also be reduced to a *Linear Time-Varying* (LTV) system with a given parameter trajectory and it can be reformulated as a *Linear Time-Invariant* (LTI) system with a given constant trajectory [i.e.  $\theta$  is a constant]. From a control point of view, the LPV control design is closely related to the gain-scheduling problem (Apkarian *et al.*, 1995; Leith and Leithead, 2000). The LPV approach is motivated by the problem of obtaining and designing multiple models and controllers and the lack of performance and stability proofs for classical gain-scheduling (Balas, 2002; Ganguli *et al.*, 2002). The advantage of the LPV approach to nonlinear systems, compared with the multiple model switching and tuning (MMST) and interactive multiple-model (IMM) methods is that the LPV controllers do not need to be designed for all linearization points (Leith and Leithead, 2000; Wu, 2001; Klinkhieo, 2009).

## 6.3 Robust Actuator Fault Estimation for LPV System

### 6.3.1 Robust actuator fault estimation for LPV system

Consider the LPV system with actuator and uncertainty described by the state-space equation as follows:

$$\dot{x}_f(t) = A(\theta)x_f(t) + B(\theta)u(t) + B_f(\theta)f_a(t) + E(\theta)\xi(t, y, u) \quad (6-2)$$

$$y_f(t) = C(\theta)x_f(t) + D(\theta)u(t) \quad (6-3)$$

where  $x(t) \in \mathfrak{R}^n$ ,  $u(t) \in \mathfrak{R}^m$ ,  $y(t) \in \mathfrak{R}^p$  are the states, control inputs and outputs, respectively. The  $\xi(t, y, u) \in \mathfrak{R}^r$  encapsulate the modelling uncertainty in the system.  $f_a(t) \in \mathfrak{R}^q$  represents the actuator fault vector.  $\theta \in \mathfrak{R}^s$  is a time-varying parameter vector, and  $A(\theta)$ ,  $B(\theta)$ ,  $E(\theta)$ ,  $C(\theta)$   $D(\theta)$  are the appropriate matrices with appropriate dimensions. The pairs  $(A(\theta), B(\theta))$  are controllable and pairs  $(A(\theta), C(\theta))$  are observable, with  $\mathcal{R}(B_f(\theta)) \in \mathcal{R}(B(\theta))$ .

Some Assumptions apply to the system of Eqs. (6-2) - (6-3) (Apkarian, Gahinet and Becker, 1995) for the actuator FE problem as follows:

**Assumption 6.1:** The vector  $\theta(t)$  varies in a polytope  $\Omega$  with vertices:

$\theta_1, \theta_2, \dots, \theta_N$  ( $N = 2^s$ ), i.e.:

$$\theta(t) \in \Omega := Co\{\theta_1, \theta_2, \dots, \theta_N\} = \left\{ \sum_{i=1}^N \alpha_i \theta_i : \alpha_i \geq 0, \sum_{i=1}^N \alpha_i = 1 \right\}$$

**Assumption 6.2:** The state-space matrices depend affinely on the vector  $\theta(t)$ , and the system of Eqs. (6-2) - (6-3) is assumed to be polytopic with  $N$  vertices, i.e.:

$$[A(\theta), B(\theta), E(\theta), C(\theta), D(\theta)] \in Co \left\{ \begin{array}{l} (A(\theta_i), B(\theta_i), M(\theta_i), C(\theta_i), D(\theta_i)) \\ i = 1, \dots, N \end{array} \right\}$$

**Assumption 6.3:**  $E(\theta), C(\theta)$  are parameter independent, i.e.  $E(\theta_i) = E$ ,  $C(\theta_i) = C$ ,  $i = 1, \dots, N$ .

**Assumption 6.4:** It is assumed that the parameter varying matrix  $B_f(\theta)$  can be factorized into:

$$B_f(\theta) = B_f H(\theta)$$

where  $B \in \mathfrak{R}^{n \times q}$  is a fixed constant matrix, and  $H(\theta) \in \mathfrak{R}^{q \times q}$  is a parameter-dependent matrix which is assumed to be inevitable and bounded by  $\|H(\theta)\| \leq h_1$ . Furthermore, its derivative with respect to time is bounded by  $\left\| \frac{d[H(\theta)]}{dt} \right\| \leq h_2$ .

**Assumption 6.5:**  $Rank(B_f) = q$ ,  $Rank(E) = r$  and  $rank(B_f, E) = q + r \leq p$ .

**Assumption 6.6:** The system of Eqs. (6-2) - (6-3) is stable under the affect of the actuator fault  $f_a(t)$  and unknown uncertainty  $\xi(t, y, u)$ . Meanwhile  $\xi(t, y, u)$ ,  $f_a(t)$  and its time derivative  $\dot{f}_a(t)$  are bounded by  $\|\xi(t, y, u)\| \leq d_1$ ,  $\|f_a(t)\| \leq d_2$ ,  $\|\dot{f}_a(t)\| \leq d_3$ , where  $d_1, d_2, d_3$  and are known positive constants.

By Assumptions 6.3 and 6.4, the system dynamics of Eq. (6-2) are written as:

$$\dot{x}_f(t) = A(\theta)x_f(t) + B(\theta)u(t) + B_f \underbrace{H(\theta)f_a(t)}_{f_{new}(\theta, t)} + E\xi(t, y, u) \quad (6-4)$$

Now it is useful to simplify the notation by defining  $f_{new}(t) = H(\theta)f_a(t)$ , (note, here for simple notation, the  $f_{new}(\theta, t)$  is replaced by  $f_{new}(t)$ ) and on the basis of Assumptions 6.2 and 6.6 the newly defined fault vector  $f_{new}(t)$  is also bounded and it is assumed that the bound is  $d_4$ ,  $\|f_{new}(t)\| \leq h_1 d_2 = d_4$ . Furthermore, the time derivate  $\dot{f}_{new}(t)$  is also assumed to be bounded by  $\|\dot{f}_{new}(t)\| \leq h_2 d_3 = d_5$ , and then the objective is changed to a new FE problem. In order to achieve that, the following observer-based fault estimator is constructed:

$$\begin{aligned} \dot{\hat{x}}_f(t) &= A(\theta)\hat{x}_f(t) + B(\theta)u(t) + B_f \hat{f}_{new}(t) \\ &\quad + L(\theta)[y_f(t) - \hat{y}_f(t)] + \eta(t, y, \hat{y}_f) \end{aligned} \quad (6-5)$$

$$\hat{y}_f(t) = C\hat{x}_f(t) + D(\theta)u(t) \quad (6-6)$$

where  $\hat{x}_f(t) \in \mathfrak{R}^n$  is the observer state vector,  $\hat{y}_f(t) \in \mathfrak{R}^p$  is the observer output vector,  $\hat{f}_{new}(t) \in \mathfrak{R}^q$  is the estimate of the newly defined fault  $f_{new}(t)$ .  $L(\theta) \in \mathfrak{R}^{n \times p}$  is the designed gain matrix, and  $L(\theta) := C_o \left\{ \begin{array}{c} L(\theta_i) \\ i = 1, \dots, N \end{array} \right\}$ .

Denoting:

$$e(t) = x_f(t) - \hat{x}_f(t), e_y(t) = y_f(t) - \hat{y}_f(t),$$

$$e_{f_{new}}(t) = f_{new}(t) - \hat{f}_{new}(t).$$

Then the error dynamics are obtained by subtracting the observer dynamics of Eq. (6-5)-(6-6) from the system dynamic description Eq. (6-1)-(6-2), given by:

$$\dot{e}(t) = (A(\theta) - L(\theta)C)e(t) + B_f e_f(t) + E\xi(t, y, u) - \eta[y(t), \hat{x}_f(t)] \quad (6-7)$$

$$e_y(t) = Ce(t) \quad (6-8)$$

The nonlinear uncertainty rejection term  $\eta(t, y, \hat{y}_f)$  and the FE algorithm are given by:

$$\begin{aligned} \eta[y(t), \hat{x}_f(t)] = & \frac{d_1 M F_2 e_y(t)}{\|F_2 e_y(t)\|} + \frac{M F_2 e_y(t)}{2\|F_2 e_y(t)\|^2} [ad_4^2 \lambda_{\max}(W^{-1}G^{-1}W^{-1}) \\ & + \frac{1}{a} \lambda_{\max}(G + \tau I_q)(d_2 + \|\int_0^t W F_1 e_y(t) dt\|)^2] \end{aligned} \quad (6-9)$$

$$\dot{\hat{f}}_{new}(t) = W F_1 e_y(t) \quad (6-10)$$

where  $F_1 \in \mathfrak{R}^{q \times p}$ ,  $F_2 \in \mathfrak{R}^{r \times p}$  are fixed constant gain matrices defined later and  $W \in \mathfrak{R}^{r \times r}$  is a S.P.D. matrix that represents the learning rate. Then the following Theorem 6.1 is obtained:

**Theorem 6.1:** Under the Assumptions 6.1-6.6, if there exist a S.P.D. matrix  $P \in \mathfrak{R}^{n \times n}$ , and matrices  $Y(\theta) \in \mathfrak{R}^{n \times p}$ ,  $F_1 \in \mathfrak{R}^{q \times p}$ ,  $F_2 \in \mathfrak{R}^{r \times p}$  such that:

$$PA(\theta) + A^T(\theta)P - Y(\theta)C - C^T Y^T(\theta) < 0 \quad (6-11)$$

$$B_f^T P = F_1 C \quad (6-12)$$

$$E^T P = F_2 C \quad (6-13)$$

hold, where  $Y(\theta) = PL(\theta)$ . Then the state observer-based on Eqs. (6-7) - (6-9) and the FE algorithm Eq. (6-10) can realize the asymptotic convergence to zero of  $e(t)$  and  $e_{f_{new}}(t)$ . This means that the estimate of the system state vector  $\hat{x}_f(t)$  and the estimate of the newly defined fault vector  $\hat{f}_{new}(t)$  from the proposed robust full-order observer converge to the actual state  $x_f(t)$  and the actuator fault  $f_{new}(t)$ .

**Proof:**

Consider the following candidate Lyapunov function:

$$V_o(t) = e^T(t)Pe(t) + e_{f_{new}}^T(t)W^{-1}e_{f_{new}}(t) \quad (6-14)$$

The derivative of  $V$  along a trajectory of the error dynamic system of Eqs. (4-6) and (4-7) is given by:

$$\begin{aligned}
\dot{V}_o(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + 2e_{fnew}^T(t)W^{-1}\dot{e}_{fnew}(t) \\
&= e^T(t)[(A(\theta) - L(\theta)C)^TP + P(A(\theta) - L(\theta)C)]e(t) \\
&\quad + 2e^T(t)PB_f e_{fnew}(t) + 2e^T(t)PE\xi(t, y, u) \\
&\quad - 2e^T(t)P\eta[y(t), \hat{x}(t)] + 2e_{fnew}^T(t)W^{-1}\dot{f}_{new}(t) \\
&\quad - 2e_{fnew}^T(t)W^{-1}\dot{f}_{new}(\theta, t)
\end{aligned} \tag{6-15}$$

By **Lemma 3.1**, it is straightforward to show that:

$$\begin{aligned}
&2e_{fnew}^T(t)W^{-1}\dot{f}_{new}(t) \\
&\leq \frac{1}{a}e_{fnew}^T(t)Ge_{fnew}(t) + a\dot{f}_{new}^T(t)W^{-1}G^{-1}W^{-1}\dot{f}_{new}(t) \\
&\leq \frac{1}{a}e_{fnew}^T(t)Ge_{fnew}(t) + a\|\dot{f}_{new}(t)\|^2\lambda_{max}(W^{-1}G^{-1}W^{-1})
\end{aligned} \tag{6-16}$$

where  $a$  is a positive constant and  $G \in \mathfrak{R}^q$  is a symmetric positive vector, both to be chosen by the designer. By Eq. (6-16) and substituting Eq. (6-10) into Eq. (6-15) then Eq. (6-15) becomes:

$$\begin{aligned}
\dot{V}_o(t) &\leq e^T(t)[(A(\theta) - L(\theta)C)^TP + P(A(\theta) - L(\theta)C)]e(t) \\
&\quad + 2e^T(t)PB_f e_{fnew}(t) + 2e^T(t)PE\xi(t, y, u) \\
&\quad - 2e^T(t)P\eta[y(t), \hat{x}(t)] + \frac{1}{a}e_{fnew}^T(t)(G + \tau I_q)e_{fnew}(t) \\
&\quad - \frac{1}{a}e_f^T(t)\tau I_q e_f(t) + a\|\dot{f}_{new}(t)\|^2\lambda_{max}(W^{-1}G^{-1}W^{-1}) \\
&\quad - 2e_{fnew}^T(t)F_1Ce(t)
\end{aligned} \tag{6-17}$$

where  $\lambda_{max}(\cdot)$  denotes the largest eigenvalue of the matrix defined in (\*).

Now denoting  $(A(\theta) - L(\theta)C)^T P + P(A(\theta) - L(\theta)C) = \Psi_{11}$ , and by using Eq. (6-12) in Theorem 6.1, Eq. (6-17) is changed to:

$$\begin{aligned} \dot{V}_o(t) &\leq e^T(t)\Psi_{11}e(t) + 2e^T(t)PE\xi(t, y, u) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + \frac{1}{a}e_{f_{new}}^T(t)(G + \tau I)e_{f_{new}}(t) - \frac{1}{a}e_{f_{new}}^T(t)\tau I e_{f_{new}}(t) \\ &\quad + a\|\dot{f}_{new}(t)\|^2 \lambda_{max}(W^{-1}G^{-1}W^{-1}) \end{aligned} \quad (6-18)$$

According to Assumptions 6.4 and 6.6, and by defining a vector  $\theta$  as  $\theta(t) = \begin{bmatrix} e(t) \\ e_{f_{new}}(t) \end{bmatrix}$ , Eq. (6-17) can be rewritten as:

$$\begin{aligned} \dot{V}_o(t) &\leq \theta^T(t)\Psi\theta(t) + 2e^T(t)PE\xi(t, y, u) - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + ad_5^2 \lambda_{max}(W^{-1}G^{-1}W^{-1}) + \frac{1}{a}e_{f_{new}}^T(t)(G + \tau I_q)e_{f_{new}}(t) \end{aligned} \quad (6-19)$$

where  $\Psi = \begin{bmatrix} \Psi_{11} & 0 \\ 0 & -\frac{1}{a}\tau I_q \end{bmatrix}$ . Meanwhile, by noting that:

$$\|e_{f_{new}}(t)\| \leq \|f_{new}(t) - \hat{f}_{new}(t)\| \leq \|f_{new}(t)\| + \|\hat{f}_{new}(t)\| = d_4 + \|\hat{f}_{new}(t)\|$$

which further indicates that:

$$\begin{aligned} \frac{1}{a}e_{f_{new}}^T(t)(G + \tau I)e_{f_{new}}(t) &\leq \lambda_{max}(G + \tau I_q)\|e_{f_{new}}(t)\|^2 \\ &\leq \lambda_{max}(G + \tau I_q)(d_4 + \|\hat{f}_{new}(t)\|)^2 \end{aligned} \quad (6-20)$$

By using  $\|\xi(t, y, u)\| \leq d_1$ , then the Eq. (6-19) is transformed to:

$$\begin{aligned} \dot{V}_o(t) &\leq \theta^T(t)\Psi\theta(t) + 2d_1\|E^T P e(t)\| - 2e^T(t)P\eta[y(t), \hat{x}(t)] \\ &\quad + ad_5^2 \lambda_{max}(W^{-1}G^{-1}W^{-1}) + \lambda_{max}(G + \tau I_q)(d_4 + \|\hat{f}_{new}(t)\|)^2 \end{aligned} \quad (6-21)$$

Substituting  $\eta[y(t), \hat{x}(t)]$  given by Eq. (6-8) into Eq. (6-21), the following result is obtained:

$$\dot{V}_o(t) \leq \theta^T(t)\Psi\theta(t) < 0 \quad (6-22)$$

That is because  $\Psi_{11} < 0$  and  $-\frac{1}{a}\tau I_q < 0$ , hence  $\Psi < 0$  which means that  $\theta(t) = \begin{bmatrix} e(t) \\ e_{f_{new}}(t) \end{bmatrix}$  converges asymptotically to zero. In other words, the state and the fault estimates track the trajectory of the plant states and the actuator faults, respectively.

Q.E.D.

**Remark 6.1:** However, difficulties result from attempts to solve the *three* conditions described in Theorem 6.1. The latter two conditions are not LMI conditions but they can be transferred to an optimization problem as shown by Eqs. (4-22) and (4-23) of Chapter 4. On the other hand the first condition must be solved even though the varying parameter is measurable during the whole system dynamic process. For this a heavy on-line computation is needed which asks for powerful support from hardware, i.e. memory capacity and central processing unit. For solving this problem, an alternative approach for solving a convex set has been given by Apkarian, Gahinet and Becker (1995) and from their work the following corollary can be obtained.

**Corollary 6.1:** If there exist a S.P.D. matrix  $P \in \mathfrak{R}^{n \times n}$ , matrices  $Y_i \in \mathfrak{R}^{n \times p}$ ,  $F_1 \in \mathfrak{R}^{q \times p}$ ,  $F_2 \in \mathfrak{R}^{r \times p}$  such that:

$$PA(\theta_i) + A^T(\theta_i)P - Y(\theta_i)C - C^T Y^T(\theta_i) < 0, (i = 1, \dots, N) \quad (6-23)$$

and condition Eq. (6-12) and (6-13) hold, where  $Y_i = PL_i$ , then the observer-based Eq. (6-7)-(6-9) and FE algorithm Eq. (6-10) can ensure that  $e(t)$  and  $e_f(t)$  converge to zero asymptotically. This means that the estimate of the system state vector  $\hat{x}_f(t)$  and the estimate of the newly defined fault vector  $\hat{f}_{new}(\theta, t)$  as [defined in Eq. (6-3)] from the proposed robust full-order observer converges to the actual state  $x_f(t)$  and actuator fault  $f_{new}(\theta, t)$ , respectively.

**Proof:** The reader can refer to the proof of Theorem 6.1 and the proof is thus omitted here.

### 6.3.2 The Relevance of Exponential Stability

The observer or observer-based FDI/ FE block plays a very important role in the integrated FTC design process, and its performance such as a slow decay rate may result

in imprecise estimates, which further affect the whole design performance, i.e. the stability of the closed-loop system. Hence, a high-performance observer is preferable. One way to ensure a rapid convergence of the estimation error is to change the asymptotic stability of the error dynamic system to *exponential convergence* ( $\beta$ -stability of the observer).

To illustrate the concept of exponential convergence, consider the Lyapunov function described as below (Hamdi, *et al.*, 2011)

$$V_{o1}(t) = e^T(t)Pe(t) \quad (6-24)$$

The exponential convergence of the estimation error is guaranteed if:

$$\exists P = P^T > 0: \quad \dot{V}_{o1}(t) + 2\beta V_{o1}(t) < 0 \quad (6-25)$$

where  $\beta$  is the decay rate. Indeed the solution of Eq. (6-25) is given as follows:

$$V_{o1}(t) \leq V(0) \exp(-2\beta t), \forall t \geq 0 \quad (6-26)$$

Due to  $\lambda_{min}(P)\|e(t)\|^2 \leq V_{o1}(t) \leq \lambda_{max}(P)\exp(-2\beta t)\|e(0)\|^2$ , the norm of the estimation error is bounded as follows:

$$\|e(t)\| \leq \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} \exp(-\beta t) \|e(0)\|, \forall t \geq 0 \quad (6-27)$$

Therefore, for the system of Eqs. (6-7) and (6-8), the following Theorem is obtained:

**Theorem 6.2:** The exponential convergence ( $\beta$ -stability) of the error dynamic system of Eqs. (6-7) and (6-8) is achieved if there exists a S.P.D. matrix  $P \in \mathfrak{R}^n$ , matrices  $Y(\theta_i) = PL(\theta_i)$  and a positive scalar  $\beta$  such that:

$$PA(\theta_i) + A^T(\theta_i)P - Y(\theta_i)C - C^T Y^T(\theta_i) + 2\beta P < 0, (i = 1, \dots, N) \quad (6-28)$$

$$\beta < \frac{\tau \lambda_{min}(W)}{2\alpha} \quad (6-29)$$

**Proof:**

Consider the Lyapunov function described in Eq. (6-14) and by Eq. (6-25), if the  $\beta$ -stability is achieved, then:

$$\dot{V}_o(t) + 2\beta V_o(t) < 0 \quad (6-30)$$

Substituting Eq. (6-22) into Eq. (6-30) it thus follows that:

$$\begin{bmatrix} \Psi_{11} & 0 \\ 0 & -\frac{1}{a}\tau I_q \end{bmatrix} + 2\beta \begin{bmatrix} P & 0 \\ 0 & W^{-1} \end{bmatrix} < 0 \quad (6-31)$$

So that:

$$\Psi_{11} + 2\beta P < 0 \quad (6-32)$$

With:

$$-\frac{1}{a}\tau I_q + 2\beta W^{-1} < 0 \quad (6-33)$$

Meanwhile, Eq. (6-33) shows that:

$$\beta I_q < \frac{\tau I_q W}{2a} \quad (6-34)$$

From which:

$$\beta < \frac{\tau \lambda_{\min}(W)}{2a} \quad (6-35)$$

Q.E.D.

**Remark 6.2:** From Eq. (6-35) it can be concluded that, the decay rates of the fault estimation error dynamics are mainly decided by small positive numbers  $\tau$ ,  $a$  and the fault estimator learning rate  $W$ , and it is not hard to investigate that there is a trade-off relationship between the  $\beta$  and the nonlinear switching function  $\eta[y(t), \hat{x}_f(t)]$  in Eq. (6-9). This causes the values of  $\tau$  and  $W$  to increase with a small value of  $a$  to guarantee the performance of the estimator. This increases the decay rates for the fault estimator and the term  $\lambda_{\max}(W^{-1}G^{-1}W^{-1})$  in  $\eta[y(t), \hat{x}_f(t)]$  becomes small. However, the term  $\frac{1}{a}\lambda_{\max}(G + \tau I_q)$  becomes large, and vice versa. As a result, when the value  $a$  is fixed, under achievable practical actuator fault estimation limits the actuator fault estimator learning rate  $W$  is as larger as possible, and  $\tau$  can be very small.

Exponential convergence is a strong form of convergence, and it also implies asymptotic convergence as a special case (Orjuela, *et al.*, 2008). Indeed, for a decay rate equal to zero,  $\beta = 0$ , the asymptotic convergence of the error dynamics is obtained.

### 6.3.3 LMI Region Pole Placement

From a LMI region pole placement point of view, when estimation errors achieve the exponential convergence, all the eigenvalues of the matrix of the matrix  $\mathcal{B} = \begin{bmatrix} \Psi_{11} + 2\beta P & 0 \\ 0 & -\frac{1}{a}\tau I_q + 2\beta W^{-1} \end{bmatrix}$  lies in the left complex plane, with  $Re(\lambda_i) < -\beta$ , demonstrated by the shaded region of Figure 6-1.

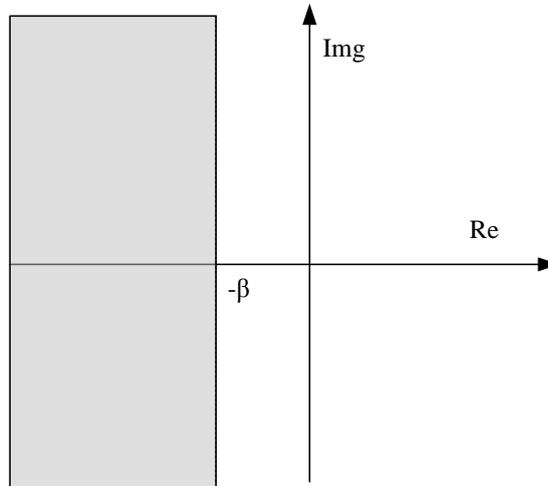


Figure 6-1: Eigenvalues locations for  $\beta$ -stability

As shown in Figure 6-1, Theorem 6.2 only ensures the position of the real parts of the eigenvalues of  $\mathcal{B}$ .

However, the imaginary part of the eigenvalues of  $\mathcal{B}$  must also be considered as they cause the error trajectory to oscillate and hence must be limited. Hence, it is clear that the problem of obtaining satisfactory observer performance is one of suitable eigenvalue placement. To solve this problem, the LMI region pole placement constraint is applied.

According to (Arzelier, Bernussou and Garcia, 1993; Chilali, Gahinet and Apkarian, 1999), the Definition of the LMI region is given as follows:

**Definition 6.1:** (LMI region) A subset  $\mathcal{D}$  of the complex plane is called an LMI region if there exist a symmetric matrix  $V = [V_{kl}] \in \Re^{m \times m}$  and a matrix  $T = [T_{kl}] \in \Re^{m \times m}$  such that:

$$\mathcal{D} = \{z \in \mathbb{C} : f_{\mathcal{D}}(z) < 0\} \quad (6-36)$$

with:

$$f_{\mathcal{D}}(z) = V + zT + \bar{z}T^T = [V_{kl} + T_{kl}z + T_{lk}\bar{z}]_{1 \leq k, l \leq m} \quad (6-37)$$

$f_{\mathcal{D}}(z)$  is called the *characteristic function of  $\mathcal{D}$* . Then according to Chilali, Gahinet and Apkarian (1999), the  $\mathcal{D}$ - stability of a matrix  $S$  is guaranteed as follows.

**Theorem 6.3:** The matrix  $S$  is  $\mathcal{D}$ - stable if and only if there exists a symmetric matrix  $U$  such that:

$$\begin{aligned} O_{\mathcal{D}}(S, U) &:= V \otimes U + T \otimes (SU) + T^T \otimes (US^T) \\ &= [V_{kl}U + T_{kl}(A(\theta) - B(\theta)K) + T_{lk}(A(\theta) - B(\theta)K)^T] < 0, \quad U > 0 \end{aligned} \quad (6-38)$$

By using an LMI eigenvalue placement constraint, the expected system performance can be achieved.

Recall the conditions for estimation of the error dynamics Eq. (6-7) and (6-8) to achieve the  $\beta$ -stability, Eqs. (6-32) and (6-33), in Eq. (6-33), as the value are all on the negative real axis. Hence, there is no need to change the eigenvalue assignment for the lower

right hand block of the matrix  $\mathfrak{B} = \begin{bmatrix} \Psi_{11} + 2\beta P & 0 \\ 0 & -\frac{1}{a}\tau I_q + 2\beta W^{-1} \end{bmatrix}$ , whilst, its top left

hand block defines the observer's decay rate. This means that under modelling uncertainty conditions or with other disturbances present, the unwanted observer eigenvalues may further affect the stability of the overall system which may even become unstable. Hence, whilst guaranteeing the  $\beta$ -stability of the observer and fault estimator the observer eigenvalues locations should be more constrained compared with the fault estimator eigenvalues.

On the basis of the above discussion, combining Theorem 6.2 and Theorem 6.3, leads to the following corollary.

**Corollary 6.2:** The closed-loop LPV system Eq. (6-7) and (6-8) is  $\beta$ -stable, and the eigenvalues of the matrix  $\Psi_{11}$  lie in the LMI region  $\mathcal{D}$ , if Theorem 6.2 and:

$$\begin{aligned} & O_{\mathcal{D}}[(A(\theta_i) - L(\theta)C), P] \\ & = V_{kl}P + T_{kl}[PA(\theta_i) - Y(\theta_i)C] + T_{lk}[A^T(\theta_i)P - C^TY^T(\theta_i)] < 0, \\ & \qquad \qquad \qquad (i = 1, \dots, N) \end{aligned} \quad (6-39)$$

hold, where the notations are given in Theorems 6.2 and 6.3.

**Remark 6.3:** Normally, the real parts of the eigenvalues of the matrix  $\Psi_{11}$  lie on the left of the fault estimator eigenvalues. This is not only guarantees the decay rates of responses of the whole system comprising the observer and fault estimator, but also provides quick decay speeds for the observer error only. This is very important to ensure that the system state estimate tracks the real system dynamics quickly enough to stabilize the feedback system.

After the above discussion, the solutions for the observer gain, the polytopic coordinates and the actuator fault estimates can be obtained by the following steps:

- (1) A set of solutions for  $Y(\theta_i)$ , ( $i = 1, \dots, N$ ) arise from the application of Corollary 6.1. It is then straightforward to obtain accordingly  $L(\theta_i)$ , ( $i = 1, \dots, N$ ) by pre-multiplying the inverse of  $P$ . The function matrix  $L(\theta)$  is then computed by:

$$L(\theta) = \sum_{i=1}^N \alpha_i L(\theta_i) \quad (6-40)$$

- (2) The weighting parameters  $\alpha_i$  are any solutions of the convex decomposition problem:

$$\theta = \sum_{i=1}^N \alpha_i \theta_i \quad (6-41)$$

- (3) According to Corollary 6.1, the actuator fault estimation signal  $\hat{f}_{new}(\theta, t)$  is obtained. Also, as the matrix  $H(\theta)$  is invertible, the real actuator fault signal  $f_a(t)$  can be achieved by the following reverse transformation:

$$\hat{f}_a(t) = H^{-1}(\theta)\hat{f}_{new}(t) \quad (6-42)$$

where  $H^{-1}(\theta)$  is the inverse of the matrix  $H(\theta)$ .

**Remark 6.4:** It is reasonable assume that the newly defined actuator fault  $f_{new}(t) = H(\theta)f_a(t)$  is bounded. However, there may be some problem with this assumption when applied to its derivative  $\|\dot{f}_{new}(t)\|$ . If this assumption is not true, another approach will be required to solve this problem by changing the condition Eq. (6-12) to:

$$B_f(\theta_i)^T P = F_1(\theta_i)C \quad (6-43)$$

Meanwhile, considering the estimation of the original actuator fault  $f_a(t)$ , the FE algorithm is changed to:

$$\hat{f}_a(t) = WF_1(\theta)e_y(t) \quad (6-44)$$

In a similar manner to Eq. (6-23) the gain  $F_1(\theta)$  can be achieved by:

$$F_1(\theta) = \sum_{i=1}^N \alpha_i F_1(\theta_i) \quad (6-45)$$

## 6.4 Robust actuator fault tolerant control

As for previous chapters, in this Section the estimated system state variable  $\hat{x}_f(t)$  and actuator fault  $\hat{f}_a(t)$  are applied in an FTC scheme against uncertainty/disturbance. The robust  $H_\infty$  feedback control law is first developed for the fault-free system with robust performance against uncertainty/disturbance, and then the fault tolerance is achieved by improving the robust  $H_\infty$  feedback control law with fault accommodation property.

### 6.4.1 Robust $H_\infty$ control law design for LPV system in fault-free case

First, under the Assumption 6.1-6.6, consider the following fault-free system dynamics with uncertainty:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + E\xi(t, y, u) \quad (6-46)$$

$$y(t) = C(\theta)x(t) \quad (6-47)$$

where all the notations are provided in Eqs. (6-2) and (6-3). Consider a feedback control law with fixed feedback gain as:

$$u(t) = -K_1 x(t) \quad (6-48)$$

where  $K_1 \in \mathfrak{R}^{m \times n}$  and then the closed-loop system dynamics are obtained by substituting Eq. (6-48) into Eq. (6-46), as follows:

$$\dot{x}(t) = [A(\theta) - B(\theta)K_1]x(t) + E\xi(t, y, u) \quad (6-49)$$

By Apkarian *et al.*, (1995) the following Lemma 6.1 is introduced:

**Lemma 6.1:** For the LPV system Eq. (6-49), the following statements are equivalent:

- (1) The  $L_2$ - induced norm of the operator mapping  $\xi(t, y, u)$  into  $x(t)$  is bounded by a scalar number  $\gamma$  for all parameter trajectories  $\theta$  in the polytope  $\Omega$
- (2) For the parameter trajectories  $\theta$  in the polytope  $\Omega$ , there exists  $Q = Q^T > 0$  satisfying the system of LMIs:

$$\begin{bmatrix} Q(A(\theta) - B(\theta)K_1) + (A(\theta) - B(\theta)K_1)^T Q & QE & I_n \\ E^T Q & -\gamma I_r & 0 \\ I_n & 0 & -\gamma I_n \end{bmatrix} < 0 \quad (6-50)$$

where  $\gamma$  is defined as:

$$\|T_{\xi y}\|_{\infty} = \sup \frac{\|x\|_2}{\|\xi(t, y, u)\|_2} < \gamma, \quad 0 < \|\xi(t, y, u)\|_2 < d_1 \quad (6-51)$$

As the top left-hand term in Eq. (6-50) is a nonlinear parameter-varying matrix inequality, by pre-multiplying and post-multiplying  $diag[\gamma^{1/2}Q^{-1} \quad \gamma^{1/2}I_r \quad \gamma^{1/2}I_n]$  with the inequality Eq. (6-50), the following alternative matrix inequality for Eq. (6-50) is obtained:

$$\begin{bmatrix} A(\theta)X - B(\theta)Z + (A(\theta)X - B(\theta)Z)^T & E & X^T \\ E^T & -\gamma^2 I_r & 0 \\ X & 0 & -I_n \end{bmatrix} < 0 \quad (6-52)$$

where  $X = \gamma Q^{-1}$  and  $Z = K_1 X$  and then on the basis of the convex optimization the following Theorem is obtained.

**Theorem 6.4:** The closed-loop LPV system Eq. (6-49) is asymptotically stable, and  $\|T_{\xi y}\|_{\infty} < \gamma$ , if there exist a S.P.D. matrix  $X \in \mathfrak{R}^{n \times n}$ , and a matrix  $Z \in \mathfrak{R}^{m \times n}$  such that the following set of LMIs are satisfied:

$$\begin{bmatrix} A(\theta_i)X - B(\theta_i)Z + XA^T(\theta_i) - Z^T B^T(\theta_i) & E & X^T \\ E^T & -\gamma^2 I_r & 0 \\ X & 0 & -I_n \end{bmatrix} < 0, \quad (6-53)$$

$(i = 1, \dots, N)$

where  $X = \gamma Q^{-1}$  and  $Z = K_1 X$ . Furthermore, if a feasible solution  $(X, Z)$  exists in the above set of LMIs, the state-feedback gain can be computed as  $K_1 = ZX^{-1}$ . For more details the reader is referred to Chapter 4 and (Apkarian, Gahinet and Becker, 1995).

**Remark 6.5:** Theorem 6.4 also implies that the closed-loop nominal LPV dynamic system Eq. (6-1) with  $D(\theta) = 0$ , under the state feedback control law  $u(t) = -K_1 x(t)$  is asymptotically stable. Since for a Lyapunov candidate function:

$$V_c(t) = x^T(t)Qx(t) \quad (6-54)$$

The derivative of  $V_c(t)$  with respect to time is:

$$\begin{aligned} \dot{V}_c(t) &= \dot{x}^T(t)Qx(t) + x^T(t)Q\dot{x}(t) \\ &= x^T(t)[(A(\theta) - B(\theta)K_1)^T Q + Q(A(\theta) - B(\theta)K_1)]x(t) \end{aligned} \quad (6-55)$$

Also, since  $(A(\theta) - B(\theta)K_1)^T Q + Q(A(\theta) - B(\theta)K_1) = \Phi < 0$ , it follows that:

$$\dot{V}_c(t) < x^T(t)\Phi x(t) \quad (6-56)$$

By duality with the LPV observer design, to achieve good controller performance the eigenvalues of the matrix  $A(\theta) - B(\theta)K_1$  must also be constrained to lie in a prescribed complex region. However, the closed-loop observer eigenvalue constraints should be stricter than the closed-loop control system, which means the convergence speed of the error must be faster than that of the control system to preserve the overall system stability.

**Remark 6.6:** In a similar manner to the observer gain design, the state feedback control law  $K_1$  can also be designed based on a  $\theta$  parameter-dependent form, i.e.  $K_1(\theta)$ . In this Chapter the  $K_1(\theta)$  is chosen as a fixed constant matrix.

## 6.4.2 Robust $H_\infty$ active FTC law design for LPV system

After designing the state feedback control law by assigning the eigenvalues of the  $A(\theta) - B(\theta)K_1$  in the desired LMI region  $\mathcal{D}$ , the active robust FTC control law becomes the next main objective to achieve the property of fault tolerance as well as robustness against uncertainty. However, as described in Section 6.2, the full order observer-based fault estimator is not only able to provide the estimate of the actuator fault signal but also the estimate of the state vector. Based on that, to be directed against the faulty system Eq. (6-2), the following active FTC law is considered:

$$u_{ftc}(t) = u(t) + u_f(t) \quad (6-57)$$

where  $u(t) = -K_1\hat{x}(t)$ , is defined in Eq. (6-48), and the pseudo-inverse control law  $u_f(t)$  is described as:

$$u_f(t) = -B^{++}(\theta)B_f(\theta)\hat{f}_a(t) \quad (6-58)$$

in which  $B^{++}(\theta)$  is the right pseudo-inverse matrix of  $B(\theta)$ , and  $B^{++}(\theta) = B^T(\theta)[B(\theta)B^T(\theta)]^{-1}$ . Substituting the FTC law  $u_{ftc}(t)$  into Eq. (6-2) and (6-3), the closed-loop faulty system is obtained in Eqs. (6-59) and (6-60):

$$\dot{x}_f(t) = [A(\theta) - B(\theta)K_1]x_f(t) + B_f e_{fnew}(t) + E\xi(t, y, u) \quad (6-59)$$

$$y_f(t) = Cx_f(t) \quad (6-60)$$

According to the analysis of Section 4.4.1, the observer-based fault estimator combined with the  $H_\infty$  robust control design is able to achieve asymptotic stability, fault tolerance to the actuator fault  $f_a(t)$  and robustness with system performance  $\|T_{\xi y}\|_\infty < \gamma$  for the closed-loop system Eq. (6-59) and (6-60). The proof is similar to that give in Section 4.4.1 and is omitted here.

## 6.5 Simulations on Two Link Manipulator System

To illustrate the mathematical discussion above, a tutorial example of the actuator fault compensation problem is considered using a nonlinear simulation of the two-link manipulator. More information on the introduction of this mechanical model can be found in Section 3.5. However, in this Section a polytope representation of this model is given.

### 6.5.1 Polytopic model of two-link manipulator

It should be noted that this work differs from the work by Adams *et al* (1996). In their work, the terms  $O(\varphi, \dot{\varphi})$  are taken into account in the design of the robust control approaches for a two-link flexible manipulator. However, in this study the terms  $O(\varphi, \dot{\varphi})$  are not considered because they are not bounded. However, it is a quite normal way to solve this problem, (Patton, Chen and Klinkhieo, 2012; Kajiwara, Apkarian and Gahinet, 1999). Taking this limitation into account Eq. (3-24) becomes:

$$M(\varphi)\ddot{\varphi} - G(\varphi) = u \quad (6-61)$$

where the vector  $\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$ , and the ranges of both the angles  $\varphi_1, \varphi_2$  are  $[-\pi, \pi]$ , and accordingly the following matrices are obtained:

$$M(\varphi) = \begin{bmatrix} a_1 & a_2 \cos(\varphi_1 - \varphi_2) \\ a_2 \cos(\varphi_1 - \varphi_2) & a_4 \end{bmatrix}, \quad G(\varphi) = \begin{bmatrix} a_3 \sin \varphi_1 \\ a_5 \sin \varphi_2 \end{bmatrix} \quad (6-62)$$

The nonlinear term  $\phi_1(\varphi) = \cos(\varphi_1 - \varphi_2)$  in  $M(\varphi)$  is clearly a bounded function, where  $-1 \leq \phi_1(\varphi) \leq 1$  [see Figure 6-2 (b)]. Hence,  $M(\varphi)$  can be represented by a polytope whose vertices are defined by:

$$M(\varphi) \in Co\{M_1, M_2\} \quad (6-63)$$

where

$$M_1 = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} a_1 & -a_2 \\ -a_2 & a_4 \end{bmatrix} \quad (6-64)$$

To facilitate a state-space formulation, the vector field  $G(\varphi)$  with  $\varphi \in \mathfrak{R}^2$  is rearranged in the form of  $G^g(\varphi)\varphi$  and function  $\varphi_2(\varphi)$  in the following function can now be

defined which is bounded. The bound of  $\varphi_2(\varphi)$  is  $-0.2 \leq \varphi_2(\varphi) \leq 1$  as shown in Figure 6-2 (a).

$$\varphi_2(\varphi)\varphi_1 = \left(\frac{\sin(\varphi_1)}{\varphi_1}\right)\varphi_1 \quad (6-65)$$

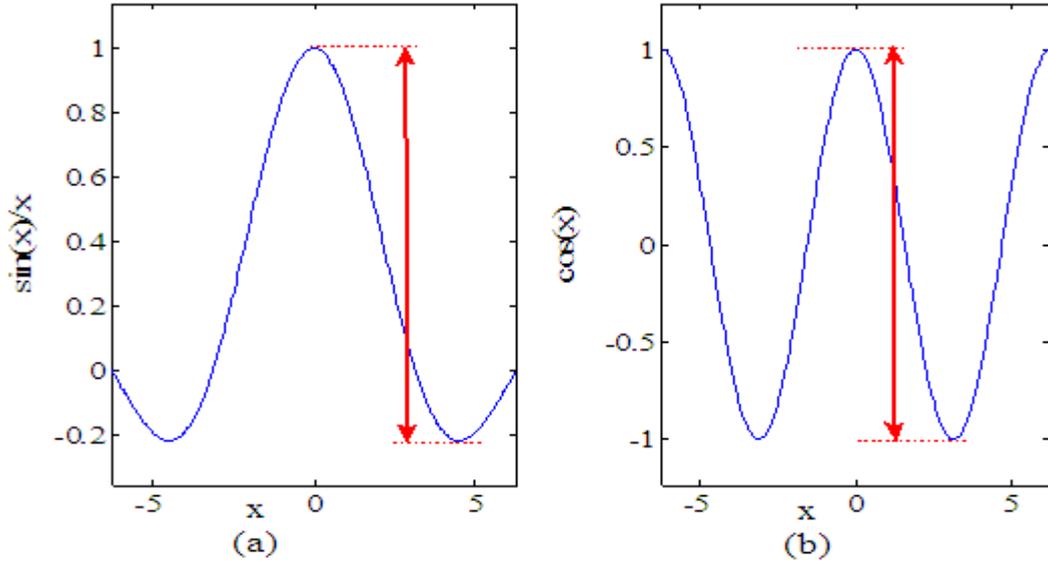


Figure 6-2: Variation of parameters used for the simulation (Balakrishnan, 1997)

From the boundedness of functions  $\varphi_2(\varphi)$  in terms of the angle  $\varphi$ ,  $G^g(\varphi)$  is considered as a polytope as follows:

$$G^g(\varphi) \in Co\{G_1^g(\varphi), G_2^g(\varphi), G_3^g(\varphi), G_4^g(\varphi)\} \quad (6-66)$$

where

$$G_1^g(\varphi) = \begin{bmatrix} -0.2a_3 & 0 \\ 0 & -0.2a_5 \end{bmatrix}, \quad G_2^g(\varphi) = \begin{bmatrix} a_3 & 0 \\ 0 & -0.2a_5 \end{bmatrix}$$

$$G_3^g(\varphi) = \begin{bmatrix} -0.2a_3 & 0 \\ 0 & a_5 \end{bmatrix}, \quad G_4^g(\varphi) = \begin{bmatrix} a_3 & 0 \\ 0 & a_5 \end{bmatrix}$$

To define the state space representation of the two-link manipulator system, let  $x(t) =$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \\ \dot{\varphi}_1(t) \\ \dot{\varphi}_2(t) \end{bmatrix} \text{ and } W_b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and then the nonlinear dynamics of the two link}$$

manipulator system is described by the following descriptor system:

$$\begin{bmatrix} I_2 & 0 \\ 0 & M(\varphi) \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & I_2 \\ -G^g(\varphi) & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + W_b u(t) \quad (6-67)$$

which can be further transferred to:

$$\dot{x}(t) = A(\varphi)x(t) + B(\varphi)u(t) \quad (6-68)$$

Let the matrix  $\Pi$  be a non-singular matrix given by:

$$\Pi = \begin{bmatrix} I_2 & 0 \\ 0 & M(\varphi) \end{bmatrix} \quad (6-69)$$

It is important to note that  $\Pi$  is a non-singular block diagonal matrix, because in the manipulator system example, as shown in Eq. (6-69) the eigenvalues of  $\Pi$  totally depend on the lower right-hand block  $M(\varphi)$ . With further investigation on  $M(\varphi)$  as described in Eq. (6-64), its determinant (which is also the determinant of the matrix  $\Pi$ ) is equal to  $a_1 a_4$  and is a fixed value only determined by the mechanical parameters such as the link length and link mass and so on. Hence it thus follows that:

$$A(\varphi) = \Pi^{-1} \begin{bmatrix} 0 & I_2 \\ -G^g(\varphi) & 0 \end{bmatrix}$$

$$B(\varphi) = \Pi^{-1} W_b$$

## 6.5.2 Actuator fault estimation

Consider that an additive actuator fault  $f_a(t)$  occurs on the first torque in the nominal time-varying model of the nonlinear two link manipulator dynamical system, then the faulty system description is given below:

$$\dot{x}(t) = A(\varphi)x(t) + B(\varphi)u(t) + B_f(\varphi)f_a(t) + E\xi(t, y, u) \quad (6-70)$$

$$y(t) = Cx(t) \quad (6-71)$$

And according to the previous mathematical analysis in this chapter, Eq. (6-70) is further changed to:

$$\dot{x}(t) = A(\varphi_i)x(t) + B(\varphi_i)u(t) + B_f f_{new} + E\xi(t, y, u),$$

$$(i = 1, \dots, 4) \quad (6-72)$$

The corresponding matrices for the polytopic model are given as an 8-vertex linear system as follows [where elements in  $B(\varphi)$  change sign due to sign changes in the diagonal elements of  $M(\varphi)$ ]:

**Vertex system 1:**

$$A_{11} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & -8.0159 & 0 & 0 \\ -29.3861 & 26.7184 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & -0.3272 \\ -0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 2:**

$$A_{12} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & 1.6032 & 0 & 0 \\ -29.3861 & -5.3437 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & -0.3272 \\ -0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 3:**

$$A_{13} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.9189 & -8.0159 & 0 & 0 \\ 5.8772 & 26.7184 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & -0.3272 \\ -0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 4:**

$$A_{14} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.9189 & 1.6032 & 0 & 0 \\ 5.8772 & -5.3437 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & -0.3272 \\ -0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 5:**

$$A_{21} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & -8.0159 & 0 & 0 \\ -29.3861 & 26.7184 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & 0.3272 \\ 0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 6:**

$$A_{22} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & -8.0159 & 0 & 0 \\ -29.3861 & 26.7184 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & 0.3272 \\ 0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 7:**

$$A_{23} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & -8.0159 & 0 & 0 \\ -29.3861 & 26.7184 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & 0.3272 \\ 0.3272 & 1.0905 \end{bmatrix}$$

**Vertex system 8:**

$$A_{24} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5947 & -8.0159 & 0 & 0 \\ -29.3861 & 26.7184 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.2182 & 0.3272 \\ 0.3272 & 1.0905 \end{bmatrix}$$

And the other matrices related to the system such as the fixed actuator fault distribution

matrix are given as  $B_f = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ , disturbance/uncertainty matrix  $E = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ , and the output

matrix  $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ . Thus the actuator fault estimate  $\hat{f}_{new}(t)$  and further  $\hat{f}_a(t)$

can be implemented by on the basis of Theorem 6.2 with placement of the eigenvalues in the region  $Re(\lambda) < -30$ . The solutions are listed below by using the MatLab LMI toolbox.

$$P = \begin{bmatrix} 48.0769 & 0 & 0 & 0 \\ 0 & 35.1613 & -22.7447 & -0.4219 \\ 0 & -22.7447 & 15.1046 & 0.2730 \\ 0 & -0.4219 & 0.2730 & 0.0141 \end{bmatrix};$$

$$Y_1 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ -0.4333 & 1.2259 & -0.6825 \\ 0.2879 & -0.7959 & 0.4573 \\ 0.0049 & 0.0080 & -0.0064 \end{bmatrix},$$

$$L_1 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ 0.0196 & -0.0080 & 0.0406 \\ -0.0294 & 2.5267 & -1.6173 \end{bmatrix}$$

$$Y_2 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ -0.4333 & 1.0206 & -0.6825 \\ 0.2879 & -0.6594 & 0.4573 \\ 0.0049 & 0.0102 & -0.0064 \end{bmatrix},$$

$$L_2 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ 0.0196 & 0.0016 & 0.0406 \\ -0.0294 & 2.4946 & -1.6173 \end{bmatrix}$$

$$Y_3 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ 0.0867 & 1.2259 & -0.6825 \\ -0.0576 & -0.7959 & 0.4573 \\ 0.0010 & 0.0080 & -0.0064 \end{bmatrix},$$

$$L_3 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ -0.0039 & -0.0080 & 0.0406 \\ 0.0059 & 2.5267 & -1.6173 \end{bmatrix}$$

$$Y_4 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ 0.0867 & 1.0206 & -0.6825 \\ 0.2879 & -0.6594 & 0.4573 \\ -0.0010 & 0.0102 & -0.0064 \end{bmatrix},$$

$$L_4 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ -0.0039 & 0.0016 & 0.0406 \\ 0.0059 & 2.4946 & -1.6173 \end{bmatrix}$$

$$Y_5 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ -0.4581 & 0.8612 & -0.6825 \\ 0.3040 & -0.5538 & 0.4573 \\ 0.0058 & 0.0124 & -0.0064 \end{bmatrix},$$

$$L_5 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ 0.0196 & 0.0080 & 0.0406 \\ 0.0294 & 2.5267 & -1.6173 \end{bmatrix}$$

$$Y_6 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ -0.4581 & 1.0936 & -0.6825 \\ 0.3040 & -0.7078 & 0.4573 \\ 0.0058 & 0.0093 & -0.0064 \end{bmatrix},$$

$$L_6 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ 0.0196 & -0.0016 & 0.0406 \\ 0.0294 & 2.4946 & -1.6173 \end{bmatrix}$$

$$Y_7 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ 0.0916 & 0.8612 & -0.6825 \\ -0.0608 & -0.5538 & 0.4573 \\ -0.0012 & 0.0124 & -0.0064 \end{bmatrix},$$

$$L_2 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ -0.0039 & 0.0080 & 0.0406 \\ -0.0059 & 2.5267 & -1.6173 \end{bmatrix}$$

$$Y_8 = 10^3 * \begin{bmatrix} 1.9495 & 0 & 0.0481 \\ 0.0916 & 1.0936 & -0.6825 \\ -0.0608 & -0.7078 & 0.4573 \\ -0.0012 & 0.0093 & -0.0064 \end{bmatrix},$$

$$L_8 = 10^3 * \begin{bmatrix} 0.0406 & 0 & 0.0010 \\ 0 & 0.0600 & -0.0126 \\ -0.0039 & -0.0016 & 0.0406 \\ -0.0059 & 2.4946 & -1.6173 \end{bmatrix}$$

The fault estimator gains are  $F_1 = \begin{bmatrix} 0 & 0 & 96.1539 \\ 0.4219 & -0.4219 & 0 \end{bmatrix}$  and  $F_2 = [44.0243 \quad 0.0090 \quad 0]$ . The actuator fault is simulated using:

$$f_a(t) = \begin{cases} 5 & 0 \leq t \leq 2s \\ 2.5t & 2s < t \leq 6s \\ 20 & 6s < t \leq 8s \\ 15 & 8s < t \leq 10s \\ 20\sin(2\pi t) + 1 & 12s < t \leq 15.78s \end{cases}$$

Figure 6-3 shows the comparison of the actuator fault and its estimate. The learning rate of the fault estimator is chosen as  $W = 30$ , and a Gaussian random disturbance  $d(t)$  of zero-mean and variance is 0.05 included.

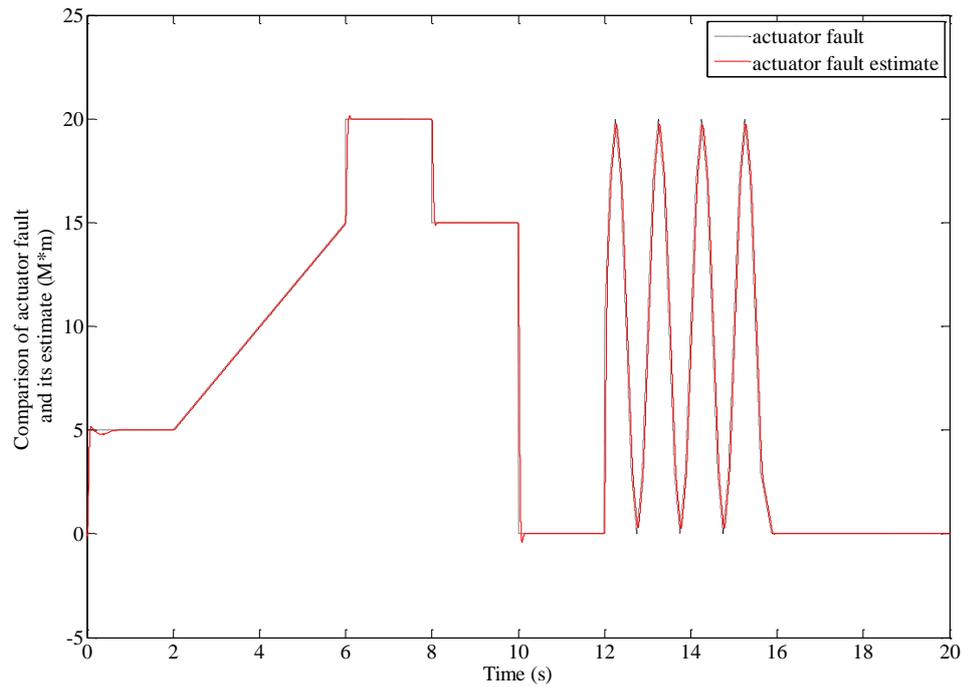


Figure 6-3: Comparison of the actuator fault and its estimate.

Figure 6-4 shows the actuator fault estimate by using a conventional fault estimator (Wang and Daley, 1996). By comparison of the results of Figure 6-3 and 6-4, it is clear that though both estimators provide good online FE the robust one proposed in this Chapter is more robust against the disturbance than the conventional one.

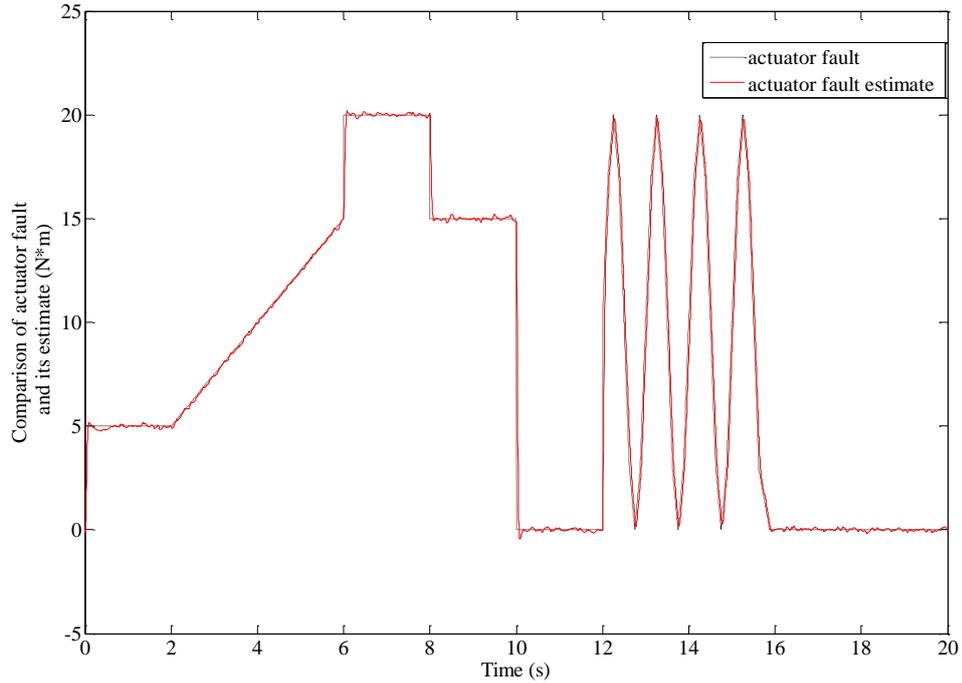


Figure 6-4: Comparison of actuator fault and its estimate (conventional estimator)

In this Section the design and performance of the fault estimator for the manipulator system have been given. After verifying the FE performance it can then be used further to design the AFTC scheme. As discussed in Chapter 5, the disturbance considered here is an un-matched disturbance. However, in the real single or multi-link manipulator system, the disturbance/uncertainty usually comes from the system itself. This means the disturbance acts in the same channel as the actuator fault signal. Under this situation, the disturbance and fault can be treated together as a new fault to apply the combined FE and FTC functions.

### 6.5.3 Active Fault tolerant control for two link manipulator system

As described the design steps in Section 6.4 the first object is to design a constant gain matrix nominal state feedback control gain matrix  $K_1$  to stabilise the fault-free open-loop system on each vertex, which can be obtained by implementing Theorem 6.4 and solving by MatLab Tool LMI box. However, here the robust feedback control gain is fallen from (Patton, Chen and Klinkhieo, 2012) as:

$$K_1 = \begin{bmatrix} -109.3065 & 0 & -32.8321 & 0 \\ 0 & -53.0095 & -53.0095 & -22.1276 \end{bmatrix}$$

After applying the FTC law of Eq. (6-57), the following result is obtained.

Figure 6-5 shows the comparison of the two link manipulator system output response with and without the FTC action.

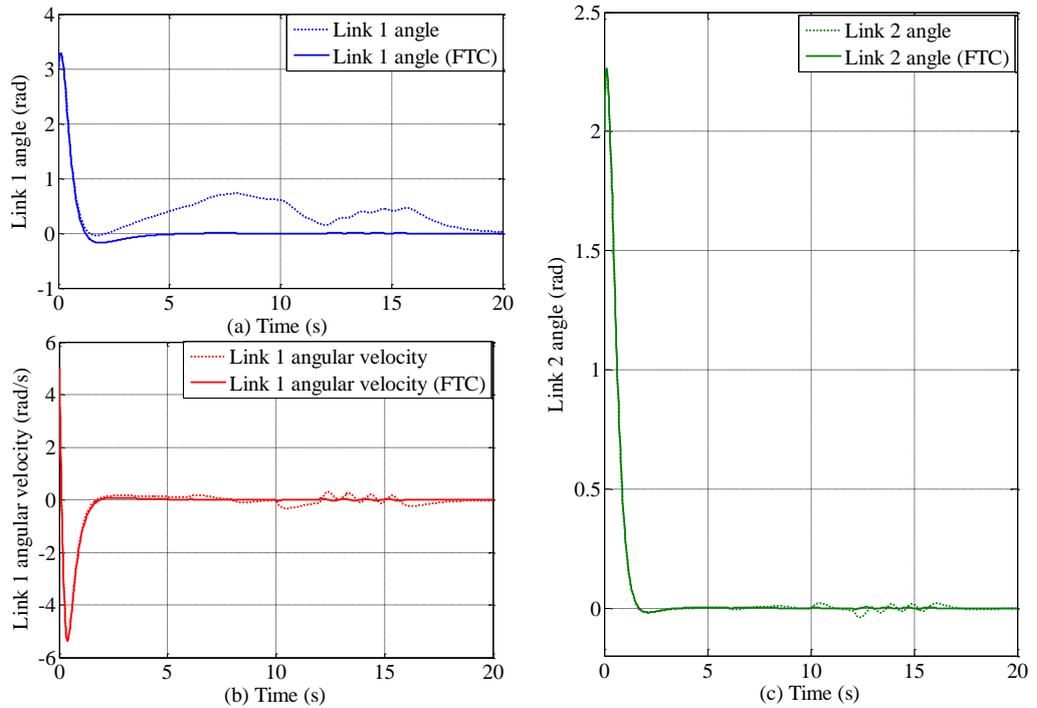


Figure 6-5: Comparison of the two link manipulator system output responses with FTC (solid line) and without (dashed line)

The result shows that the system actuator fault has been successfully compensated resulting in a good performance on the system output. Without the FTC strategy applied, the system output trajectory is moved away from the true (fault-free) trajectory, which means the system is heavily affected by the actuator fault. On the other hand, this highlights the power of the proposed strategy of the integrated design of LPV fault estimator and LPV compensation.

## 6.6 Conclusion

This Chapter proposes a new robust strategy of an active FTC and polytopic LPV estimator for nonlinear systems which can be implemented via a set of parameterized LMIs using efficient interior point algorithms.

In the work of this Chapter, an on-line observer-based robust polytopic LPV fault estimator is synthesized for providing the estimate of actuator fault which can be further combined with a scheduled predefined state feedback controller for the nominal system in an active FTC strategy. The multiplicative actuator fault considered is transformed into a new fault with a fixed distribution matrix. Then the real faults are calculated by the estimate of the newly defined fault, which means this estimation method can be further applied within an FTC scheme with both system and multiplicative faults.

The time-varying gain of the LPV estimator is based on a set of predefined gains in each vertex, and combined by a set of weighting scalars. By using the eigenvalue assignment approach, the eigenvalues of the observer are assigned in the desired LMI region to improve the performance of the LPV fault estimator. The same procedure is also applied in the controller design. The solutions of these gains are calculated using LMIs and implemented via the MaTlab LMI toolbox.

This Chapter also proves the principle of the observer-based estimator gain (learning rate) chosen to suitably increase the estimator gain and achieve a fault estimate signal with fast tracking of the real fault. From another point of view, the polytopic robust fault estimator and FTC strategy for LPV systems is an extension to the one for LTI systems.

The proposed active robust FTC scheme is investigated using the two-link manipulator with an actuator fault acting on the torque input of the first manipulator joint under the case of un-matched disturbance. Simulation results show that the design of the polytopic LPV estimator can follow the fault rapidly and effectively with robustness to the disturbance signal. This ensures that the system will continue to operate safely and with satisfactory performance via the on-line AFTC controller.

# Chapter 7.

## Conclusions and Future work

In this thesis some novel approaches are presented in the domain to active fault tolerant control with the main focus on achieving robust fault estimate under the effect of the disturbances/uncertainty. This Chapter discusses on the results presented in this thesis and summaries the contributions and provides suggestions for future research.

### 7.1 Conclusions

There are two main research streams in the field of FTC systems design. The first is FD which deals with the detection and diagnosis of faults that occur in the controlled system. The second is FTC investigating the problem of achieving fault-tolerance via designs of passive and/or active FTC schemes. Although there are major publications in this field, the interaction between these two research lines is still rather weak. The FTC methods that rely on FD assume that perfectly robust fault estimates are always available. The FD approaches, on the other hand, do not consider the presence and the needs of the FTC function. As a result of these issues it is difficult in practice to proceed with the integration of the two functions within an overall FTC scheme. This challenge becomes even greater when the process system is non-linear and when disturbances and modeling uncertainties are present.

Hence, this thesis considers the FD and FTC problems in the presence of disturbances and modeling uncertainties as a new integration design problem, focused on robustness in the post-fault situation. The goal has been to attempt to reduce the gap between real-world engineering systems and the models used in the analysis and design stages.

For the FD scheme, there are two main rank assumptions that must be imposed for robust fault estimation. The first rank condition requires that the rank of the output matrix be larger than the sum of the ranks for the fault distribution matrix and the disturbance/modeling uncertainty weighting matrix, guaranteeing sufficient information

to ensure that a robust fault estimate can be computed from the system outputs. The second assumption requires the disturbance/uncertainty weighting matrix lies in the null space of the input (control) weighting matrix, i.e.  $\mathcal{R}(E) \cap \mathcal{R}(B) = \emptyset$ .

This thesis deals with the active approach to FTC based on robust on-line estimation, nominal robust control and fault compensation integrated together to achieve robust system stability and fault-tolerance. The design solutions are achieved using the MatLab LMI Tool Box.

The definitions and significance of faults, failures and different types of faults have been presented briefly, along with the industry drivers and practical requirements. Chapters 1 and 2 provide an introduction and overview of the model-based FD and FTC reconfiguration approaches. The research is motivated by issues of increased demand of reliable, safe and available and sustainable control systems. Some conventional methods for FDI, FTC and FE have been given with discussion on their advantages and disadvantages and these have been compared with the methods proposed.

In Chapter 3, an FTC strategy using an adaptive fault estimator based on model reference control is proposed to deal with systems that have no unique linearization equilibria and for which the classical “direct” approach to FTC via fault estimation/compensation cannot easily be achieved via a linear time invariant systems approach. The fault estimator design is based only on the reference model and not the plant dynamics; this makes it possible to deal with nonlinear systems. By using the proposed design, the faulty system tracks the output responses of the predefined reference model. A tutorial application study of adaptive estimation and compensation in a nonlinear two link manipulator system is used to illustrate the applicability of the proposed method.

Since the closed-loop system has the property of tracking the reference model, there is no need to consider the system stability after the fault is accommodated by one control signal component which is a function of the fault estimate. Although the estimator is adopted from earlier research, the computation is reduced and merged into the whole FTC design.

However, the main purpose of Chapter 3 is to focus on the integration of the FE strategy of FAFE within a model-reference control framework. Chapter 3 does not focus on a robustness problem since it is assumed that all the states of the manipulator system are measurable, offering an avenue of further work in this area.

The challenge of robustness is then taken up in Chapter 4 via the proposal of a novel observer-based fault estimator design. The main motivation is to improve the robustness for the conventional observer-based adaptive fault estimator. This achieved by importing a nonlinear switching function typically used in the discontinuous gain of a sliding mode observer system. Based on this, the switching function is able to decrease the effect of the fault estimates on the disturbance/uncertainty.

In keeping with the main theme of the thesis, the integration of the robustness for the fault estimate and the FTC robustness is achieved in Chapter 4. This Chapter also gives the stability proof for the integrated system. It is important to note that when the disturbance/uncertainty and faults exist in the system, the well-known Separation Principle breaks down and the stability proof shows the observer and controller can still be separately designed whilst still yielding a robust result when combined. This is in essence a way of recovery for the Separation Principle. To illustrate this novel method, a nonlinear inverted pendulum system example which considers the effects of additive actuator and sensor faults. The results show the power of the proposed method.

Chapter 5 considers a more general actuator fault model, which includes three different forms of faults: actuator loses effectiveness, stuck faults and actuator outage. Accordingly, a more general adaptive robust observer-based fault estimator is established. By importing the adaptive scheme, the restriction condition for the fault estimator design is relaxed compared with the approach taken in Chapter 4. Chapter 5 also gives two different AFTC design strategies focused on both matched and unmatched disturbance/uncertainty highlighting the challenges that arise from the cases of matched and unmatched disturbance/uncertainty acting in either the FE and/or the controller function of the FTC scheme.

For the case of matched uncertainty the difficulty lies in the robust FE process but in the FTC process this is easily handled. For the unmatched case the opposite is true, so that for this case the robust FE scheme easily handles the disturbance/uncertainty, whilst the

opposite is true for the FTC scheme. For the situation in which both matched and unmatched disturbances/uncertainty are considered together the combination of the robust FE and robust FTC schemes work well to enhance their joint robustness.

Two case studies, both with actuator faults that are: (i) a linear rocket fairing structural-acoustic and (ii) a nonlinear single link manipulator model are applied to illustrate the strength of this integrated active FTC method for the two design examples. A description and discussion of the mathematical issues concerned with this mixed-disturbance/uncertainty problem is also given. Hence, the main contributions in Chapter 5 are: (1) the removal of the requirement for the disturbance bound imposed in the approach used in Chapter 4. (2) Three different integration approaches to robust AFTC are described according to individual disturbance/uncertainty cases. (3) Compared with the conventional adaptive FE design (Zhang, Jiang and Cui, 2007) the simulation results show that the proposed method exhibits enhanced robustness. (4) Comparing with the work of (Jin and Yang, 2009), this work uses state estimate feedback instead of full state feedback control. Hence, the approach taken in Chapter 5 is considered of more general application to systems where subsets of states are not measurable.

The work in Chapter 4 is based on linear invariant systems and this approach is not realistic when considering application to non-linear systems. A large class of nonlinear systems can be reduced to LPV representations based on linearization along trajectories of the parameters. It is thus appropriate in Chapter 6 to develop a polytopic LPV strategy as an extension to the methodology of Chapter 4 that can be used for a certain class of affine non-linear systems. In this Chapter a polytopic LPV observer-based fault estimator is synthesized for robust FE and an AFTC scheme is designed to schedule some predefined state feedback controllers to each parameter vertex. Chapter 6 also gives evidence for the dependence of the FE convergence on the algorithm learning rate. By restricting the eigenvalues of the observer dynamics and closed-loop system to lie in pre-designed regions in the complex plane (formulated for each case as an LMI problem) a suitable compromise between estimation performance and robustness to disturbance/uncertainty can be achieved.

The nonlinear two link manipulator system simulation includes a joint actuator fault and the disturbance model is chosen to reflect the availability for the proposed design. The resulting simulation results demonstrate the power of this method. In summary, the contributions for Chapter 6 are: (1) The proposal of an LPV polytopic observer-based robust fault estimator for application to time-varying or affine non-linear systems; (2) Guidance is provided for the choice of the robust FE system learning rate; (3) Compared with the work of (Patton, Chen and Klinkhieo, 2012) which only considered the disturbance in the FE, this work takes the robustness of both the FE and AFTC problems into account.

## **7.2 Future Work**

This thesis describes the past four years of research work. However, there are interesting aspects of this work that are worth pursuing further.

(1) Although the combination of model reference control and adaptive fault estimation produces good results in Chapter 3, it is developed subject to the perfect matching condition, which is a heavy restriction for real applications. Imperfect model matching could cause system uncertainty and affect the system performance.

(2) All the work in this thesis only considers actuator faults and sensor faults but component faults should be investigated within an FTC framework. Meanwhile, how to deal with the simultaneous occurrence of the three types of faults would be another valuable and challenging topic.

(3) As described in Chapter 5, mixed disturbance/uncertainty is another problem that need to be considered carefully for the combined FE and FTC strategy.

(4) There are no considerations of the system time delay problem in this thesis. From an FE point of view, some delay always exists between the real fault and the fault estimate. As a consequence, when dealing the AFTC process, the inclusion of various forms of system delays becomes an interesting challenge in terms of stability and performance.

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