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Robust De-centralized Control and Estimation for Inter-connected Systems

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by

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To my parents and my wife

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Abstract

The thesis is concerned with the theoretical development of the control of inter-connected systems to achieve the whole overall stability and specific performance. A special included feature is the Fault-Tolerant Control (FTC) problem for the inter-connected system in terms of local subsystem actuator fault estimation. Hence, the thesis describes the main FTC challenges of distributed control of uncertain non-linear inter-connected systems. The basic principle adopted throughout the work is that the controller has two components, one involving the nominal control with unmatched components including uncertainties and disturbances. The second controller dealing with matched components including uncertainties and actuator faults.

The main contributions of the thesis are summarised as follows:

- The non-linear inter-connected systems are controlled by two controllers. The linear part via a linear matrix inequality (LMI) technique and the discontinuous part by using Integral Sliding Mode Control (ISMC) based on state feedback control.
- The development of a new observer-based state estimate control strategy for non-linear inter-connected systems. The technique is applied either to every individual subsystem or to the whole as one shot system.
- A new proposal of Adaptive Output Integral Sliding Mode Control (AOISMC) based only on output information plus static output feedback control is designed via an LMI formulation to control non-linear inter-connected systems. The new method is verified by application to a mathematical example representing an electrical power generator.
- The development of a new method to design a dynamic control based on an LMI framework with Output Integral Sliding Mode Control (OISMC) to improve the stability and performance.
- Using the above framework, making use of LMI tools and ISMC, a method of on-line actuator fault estimation has been proposed using the Proportional Multiple Integral Observer (PMIO) for fault estimation applicable to non-linear inter-connected systems.

List of Abbreviations

Symbols

$\|\cdot\|$	Euclidean norm (vectors) or induced spectral norm (matrices)
$\|a\|$	The absolute value of the real number a
$\lambda_{\max}$, $\lambda_{\min}$	Largest and smallest eigenvalues of a square matrix
$\mathbb{R}$	Field of real numbers
$\mathsf{N}(A)$	Null space of the matrix A
$\mathsf{R}(A)$	Range space of the matrix A

Abbreviations

ACSS	Autonomous Coordination and Supervision Scheme
AFTC	Active fault tolerant control
AOISMC	Adaptive output integral sliding mode control
FAC	Fault accommodation
FDD	Fault detection and diagnosis
FDI	Fault detection and isolation
FES	Fault estimation
FRC	Fault reconfiguration
FTC	Fault tolerant control

ISMC	Integral sliding mode control
LMI	Linear matrix inequality
LQR	Linear Quadratic Regulator
OISMC	Output integral sliding mode control
PFTC	Passive fault tolerant control
PIM	Pseudo inverse methods
PMI	Proportional multiple integral
PMIO	Proportional multiple integral observer
SMC	Sliding mode control
SMO	Sliding mode observer
s.p.d.	Symmetric positive definite

List of Publications

Within the period of this thesis the following papers were accepted and submitted for publication:

- Larbah E. and Patton R.J. 2013. Static output feedback adaptive integral sliding control for inter-connected non-linear systems, *ASCC 2013*, Istanbul, Turkey, 26-29th June.

- Larbah E. and Patton R.J. 2012. Robust De-centralized control design using integral sliding mode control, Proc.of the 2012 UKACC International Conference on Control, 81- 86 Cardiff, UK, Sept. 3th-6th.

- Larbah E. and Patton R.J. 2010. Fault tolerant "plug and play" vibration control in building structures, 49th IEEE Conference on Decision and Control, *CDC 2010*, Atlanta,. 2462-2467. Georgia, USA.

- Larbah E. and Patton R.J. 2010. Fault tolerant control in high building structures, Conference on Control and Fault-Tolerant Systems, *SysTol'10*, 855-860. Nice, France, Oct.

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Chapter 1 : Introduction

1.1 Introduction

As a result of the industrial and technological progress in modern day life, systems have become increasingly more complex as they contain a number of subsystems which in turn interact with each other. This has led to the creation of inter-connected systems or large-scale systems such as electrical grids, airports, manufacturing plants, ecological systems, infrastructure and computer communication systems (Boubour *et al.*, 1997, Al-Abdullah, 1984, Brittain,Otaduy,Rovere and Perez, 1988, Dimirovski,Jing,Yuan and Zhang, 1998, Chou and Cheng, 2000, Yan, Edwards and Spurgeon, 2004, Hua,Yuanwei,Siying and Lina, 2006, Kalsi,Jianming and Zak, 2008, Batool,Horacio and Tongwen, 2009, Dhbaibi,Tlili,Elloumi and Benhadj Braiek, 2009, Challouf *et al.*, 2010, Changqing,Patton and Zong, 2010). As a result of these interactions, the control of these systems has become more complicated in order to ensure stability and to achieve the required specifications (Vidyasagar, 1981, Travé,Titli and Tarras, 1989, Chen and Patton, 1999).

According to (Zecevic and Šiljak, 2010) an inter-connected system is a system that consists of small inter-connected systems; these interconnections may either be physical or geographical or be based on a mathematical concept. In order to effectively study the structure of large-scale systems, it is practical to divide them into several smaller systems; some of which are inter-connected and thus become inter-connected systems. This approach facilitates the development of individual subsystem controllers with desired stability and performance (Al-Abdullah, 1984, Changqing,Patton and Zong, 2010).

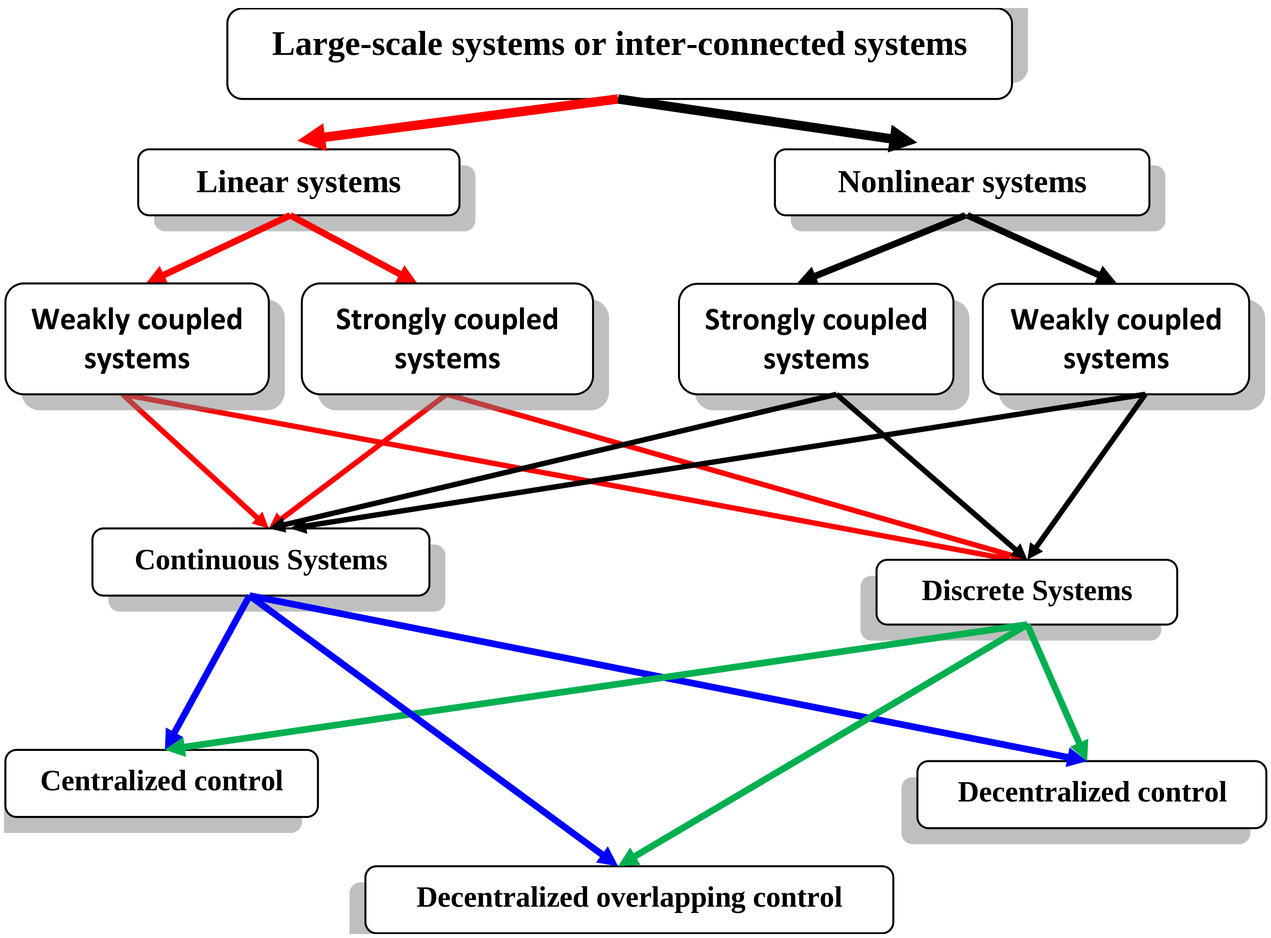

Figure 1-1: Classification of inter-connected systems and types of control

Inter-connected systems consist of several subsystems which are connected to each other. Each subsystem is dedicated to the performance of a specific task and all systems are directed towards the same goal (Šiljak, 1991, Tlili and Braiek, 2011). Figure 1-1 illustrates the methods of designing a control strategy and the different types of inter-connected systems which can be represented by a mathematical model. It can be observed, from Figure 1-1 that some subsystems may be strongly or weakly coupled.

In the case of weakly coupled subsystems, it is easy to design a controller for each subsystem as the value of interactions between them can be ignored and each subsystem is isolated from the other subsystems (Jamshidi, 1997). Whereas in the case of strongly coupled subsystems the interactions cannot be neglected and must be taken into consideration when designing the control (Jamshidi, 1997, Vidyasagar, 1981). The approach to design depends on whether or not the interactions are known or unknown.

Some researchers agree that the difficulty and complexity of inter-connected systems come from the combined complexity of each subsystem and the complexity of all the subsystems taken as a whole. For the local subsystem the main challenges to control design arise from the nature of the uncertainty, time delays, subsystem interactions as well as actuator, sensor or component faults. When all the subsystems are considered together the system dimension and the fault propagation effects are considered as the system complexity (Šiljak, 1991,Jamshidi, 1997,Zecevic and Šiljak,2010, Zhang and Zhang, 2012).

1.2 Methods of controlling inter-connected systems

It is generally agreed that an appropriate control design method should take into account the interactions between the subsystems at the analysis stage (Vidyasagar, 1981).

The traditional method of controlling inter-connected systems is achieved through the use of a centralized controller as shown in Figure 1-2 where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ are the control inputs, $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs and $d_i(t)$ are the bounded unknown disturbances.

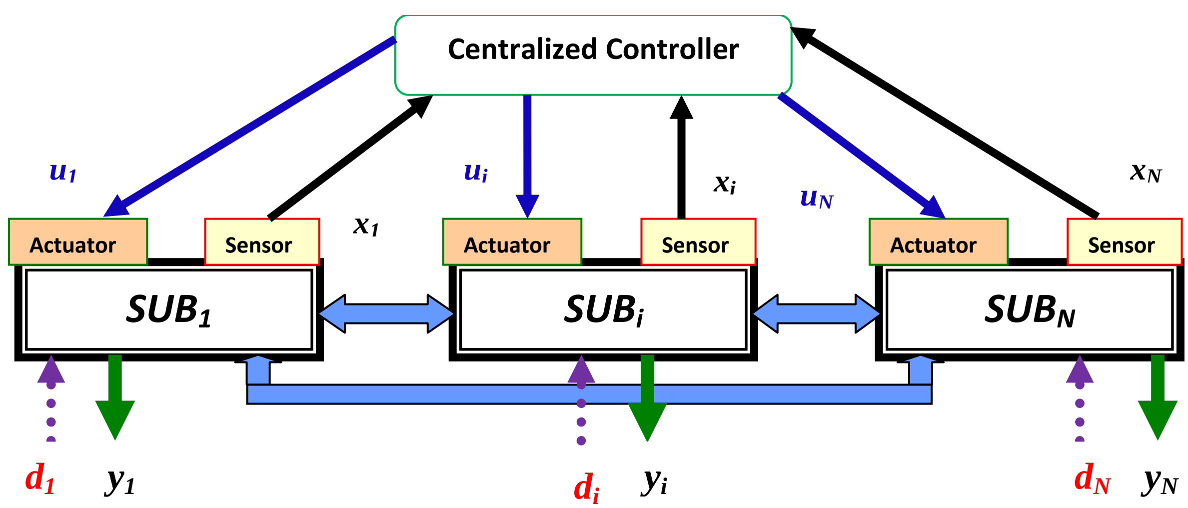

Figure 1-2: Centralized control of inter-connected systems

However, this method requires a large exchange of information between subsystems, thereby increasing both the cost and complexity (Boubour *et al.*, 1997,Šiljak, 1996, Patton *et al.*, 2007). Also, in the event of a malfunction in the centralized controller, a loss of control of all subsystems may cause the system to become unstable.

Consequently, de-centralized control is used in inter-connected systems, in order for each subsystem to have its own controller. In this strategy signals are taken individually from each subsystem to develop the appropriate control action involving the one subsystem alone. Figure 1-3 shows the architecture of the decentralized control system and its distribution (Sandell,Varaiya,Athans and Safonov, 1978,Hassan and Singh, 1980,Šiljak and Stipanovic,2001, Kalsi,Jianming and Zak, 2008).

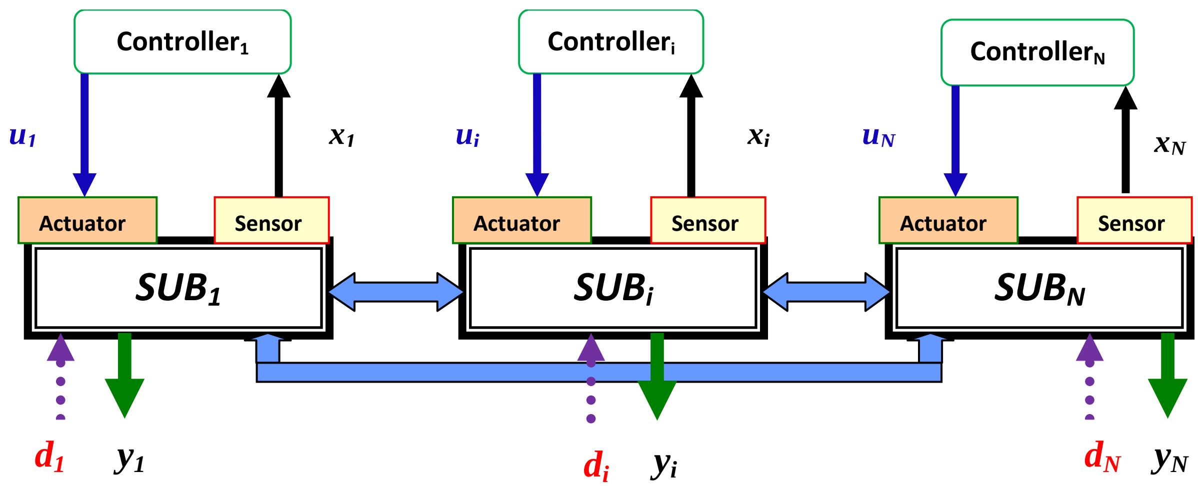

Figure 1-3: De-centralized control of inter-connected systems

In general the centralized control system requires a more complex discrete-time computer representation than would be required for each of the individual de-centralized control subsystems. The complexity arises from system order as well as numerical conditioning arising from widely separated discrete-time eigenvalues (Sandell,Varaiya,Athans and Safonov, 1978). These issues have an impact on the complexity of the control design. There is also another type of control design called the de-centralised overlapping control as shown in Figure 1-4. This type is a combination of centralized control and de-centralized control, in which all local controllers are connected to each other (Özgüner,Khorrami and İftar, 1988, Stanković,Stipanović and Šiljak, 2007). This type of control has several disadvantages, similar to the centralized control. However, it provides stability to all the subsystems in the event of malfunctions that may occur in any one subsystem.

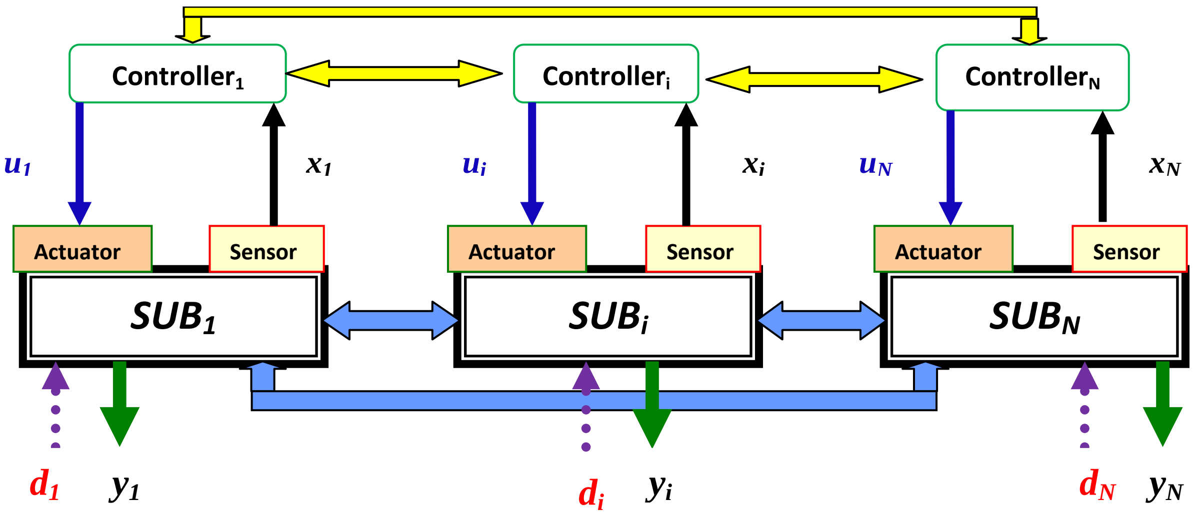

Figure 1-4: De-centralized overlapping control of inter-connected systems

The last method uses the so-called multi-level control which depends on the local control design of every subsystem and a coordinator. The research about multilevel control started in the 1960s and has attracted significantly more attention since the 1970s when the starting point was that each local controller should guarantee stability and performance for its subsystem (Singh,Hassan and Titli, 1976, Hassan and Boukas, 2007). The role of the coordinator is to deal with the interconnections between the subsystems to ensure global stability, as shown in Figure 1-5 (Singh and Titli, 1978,Brittain,Otaduy,Rovere and Perez, 1988, Changqing,Patton and Zong, 2010).

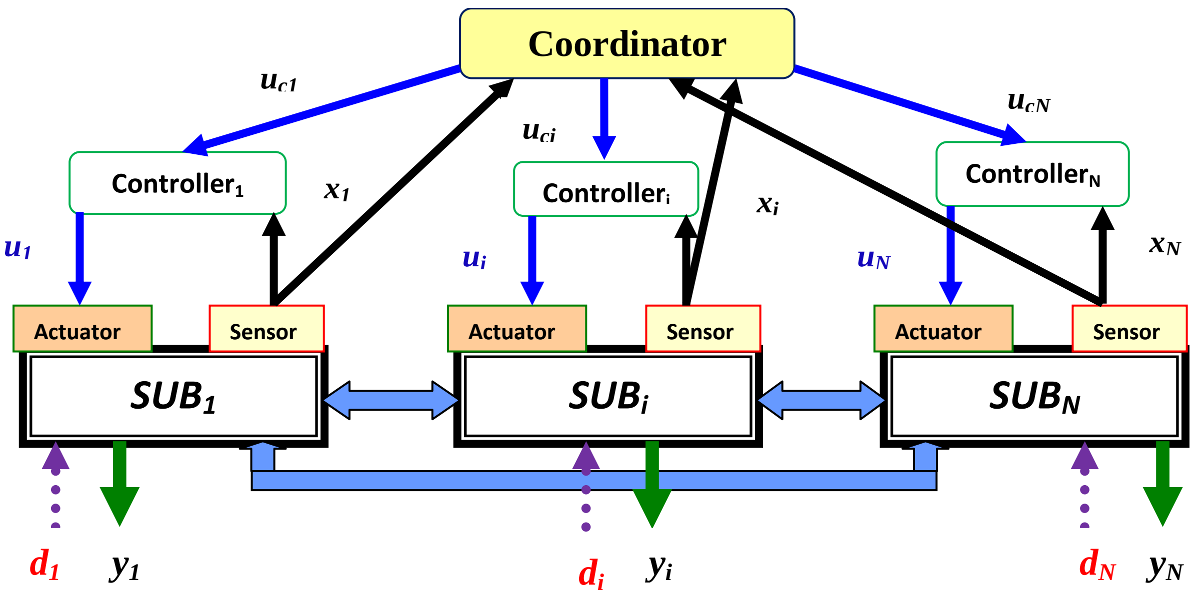

Figure 1-5: Multi-level control of inter-connected systems

1.2.1 Control design and faults in inter-connected systems

Classical control methods do not provide satisfactory solutions for the study and design of the control of inter-connected systems, since these approaches were developed before the introduction of robust control theory. Robust strategies for dealing with modelling uncertainty as well as for achieving specific performance and stability, especially where systems are non-linear can be added (Trave,Titli and Tarras, 1989).Two classical techniques have generally been used for the study of inter-connected systems in terms of stability and control design; these are Lyapunov stability and input-output stability, though there is a considerable amount of overlap between them. The Lyapunov stability method has received attention from some researchers such as Miller and Michel and Šiljak (Vidyasagar, 1981). Consequently, it is relatively easy to obtain documents, papers or books for studies on this subject and the development of control using this approach (Vidyasagar, 1981).

One approach to the design of controllers for inter-connected systems can be made via optimal control theory applied to each subsystem and also to the global system. The popularity of optimal control in recent years has meant that few researchers now follow this subject. Nevertheless, the approach to subsystem control and fault tolerant control is interesting when viewed from an optimal control standpoint (Patton *et al.*, 2007).

One approach to fault-tolerant control is to develop a suitable control action to compensate for the faults within the control system within pre-defined limits. However, it is important to know the existence of these faults and how to avoid them, as they may cause increased malfunction, instability, or even prevent the closed-loop system from functioning.

A fault prevents a system from achieving the required specifications and can arise within the system itself or may result from changes in the system's parameters or working environments which then have a direct impact on the functions of the system (Chen and Patton, 1999, Halim,Edwards and Chee, 2011) . A fault can be defined as '… a non-permitted deviation of a characteristic property (feature), of the system from the acceptable, usual, standard condition…' (Isermann, 2011).

Faults can develop into "failures" which are extreme cases of faults giving failed system function or operation (Chen and Patton, 1999) and consequently incorrect human operator decisions as in the case of the "3-mile island" accident in USA (Patton,Frank and Clark, 1989). Faults are malfunctions that cause the system to perform incorrectly. However, failures (Be definition) are a special class of faults that cause the system operation to fail.

Faults often occur in systems over time. It is difficult to prevent their occurrence and, if not treated quickly, can result in severe damage depending on their severity. Examples of systems where the faults and failures led to disastrous consequences are as follows:

- **Train wreck in the city of Buenos Aires, Argentina on February 22,2012**
  This incident was the derailing of a train as it departed from one of the stations in Buenos Aires. Investigations revealed that the cause of the accident was a malfunction in the brakes, as shown in Figure 1-6. This incident resulted in the death of at least 49 people and an additional 600 injuries (BBC, 2012).
- **Plane crash near the city of Parañaque in the Philippines on December 10,2011**
  A mechanical failure in mechanical parts of a plane caused it to crash near a school. Nearby houses were set on fire, which resulted in the death of 14 people and injuring 10. Figure 1-7 shows part of this tragedy (CBN, 2012).

Figure 1-6: Buenos Aires train wreck

Figure 1-7: Plane crash in Philippines

- **Explosion of the famous Chernobyl reactor on April 28,1986**

  An explosion caused by human dereliction or disregard and inexperience caused the loss of control of the reactor resulting in the release of nuclear radiation. 31 workers were killed and 100,000 people were displaced to neighbouring cities. It led to wide spread injury and disease, including cancer, and the contamination of more than 1.4 million hectares of farmland. The material losses were calculated to be $6.7 billion (World Nuclear Association, 2012).

The study of faults and malfunctions or failures in systems, especially in the inter-connected systems is a new subject established during the past few years. Understanding the different types of faults and where they occur makes it easier to control or to reduce their impact.

1.2.1.1 The classification of faults in inter-connected systems

(Chen and Patton, 1999) classifies clearly different faults in systems by the location of these faults (where they act in the subsystem). According to this classification, the faults can be distinguished as follows:

Actuator faults: Any faults that may occur in the actuator system will hinder its performance of its duties. The fault may be transmitted and cause malfunction or failure, for example, asymmetry due to breakdown of a winding of the stator in an electrical machine that is used in elevators.

Sensor fault: This type of fault will cause reduction in the accuracy of measurements, resulting in incorrect readings or to a situation where readings are not even taken such as e.g. potentiometers, accelerometers, tachometers, strain gauges, etc, as mentioned in (Isermann, 2011).

System Components fault: This will cause some changes in the components of the system, e.g. parametric changes or faults in sub-component like a pump. Hence, during the modelling of the system the faults may appear as changes in the system parameters, for example in a chemical process changes in concentration if not normal changes may be considered as component faults.

However, in inter-connected systems, from a subsystem point of view, the interactions between this subsystem and other subsystems are considered as unusual signals or interaction faults.

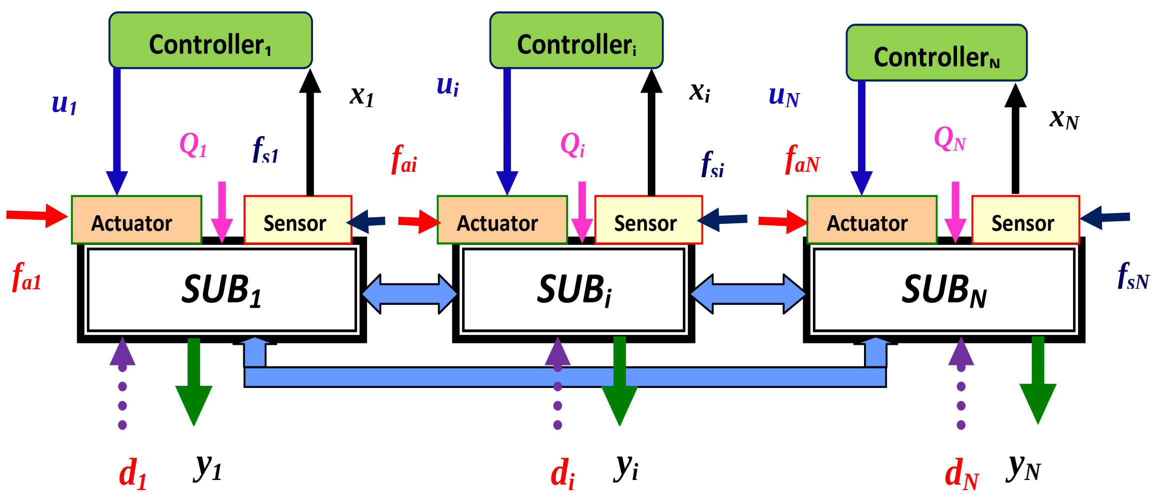

Figure 1-8: Types of faults in inter-connected systems

Figure 1-8 shows these types of systems and their components and the faults that may occur. The types of faults are; sensor fault (*fs*), actuator fault (*fa*) and system fault (*Q*). The systems are also affected by disturbances and interactions, which are considered to be important factors in inter-connected systems, because if a fault or malfunction happens in one subsystem then all of the other subsystems will be affected by this fault. Consequently, it could lead to the instability of some of them or of the whole system.

From a literature view point the faults are classified according to the way that the faults are modelled, e.g. in state space. In this way additive or multiplicative faults can be considered by the way they act in the system model (Patton,Frank and Clark, 1989, Chen and Patton, 1999). The fault model can then be used as a representation of real physical faults that may occur at any part of the modelled system.

1.2.1.2 Fault diagnosis and fault-tolerant control

The history of fault diagnosis and fault-tolerant control have received a lot of attention in the literature from the 1970s from both theoretical and application-based perspectives {for fault diagnosis}(Patton,Frank and Clark, 1989, Gertler, 1998, Patton,Frank and Clark, 2000, Isermann, 2011, Zhang and Zhang, 2012) and {for fault-tolerant control} (Patton *et al.*, 2007, Blanke,Kinnaert,Lunze and Staroswiecki, 2006, Zhang and Jiang, 2008).

1.2.1.2.1 Fault diagnosis

Faults and failures can occur at any time and in any part of the whole system. The effects of these faults or failures must be minimized and their proliferation reduced, otherwise they may worsen and cause disasters. Time is an important factor in the discovery of faults or failures in inter-connected systems and determining their location makes them easier to deal with as well ensuring more reliable control of the systems, e.g. by using fault-tolerant control. A fault must be reliably detected very quickly after its onset, i.e. the fault must be detected “promptly”. As shown in Figure 1-9 the longer it takes for the fault to be detected the lower the possibility of taking remedial action to the fault, e.g. by using redundancy in hardware or analytical forms.

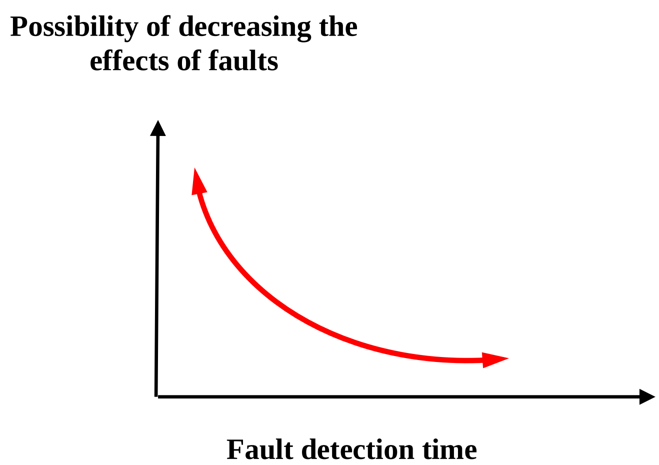

Figure 1-9: Relationship between fault detection time and possibility of dealing with faults

The importance of having prompt information about control system faults is thus important. Some systems have a bigger margin for allowable fault detection. For example, when considering the functions of a mobile phone, if there is noise on the line, although an error has occurred it is still possible to understand the other person's speech. This would allow for a greater margin for faults. However, when this margin is very narrow any problems may cause damage that would lead to a disaster, and hence critical behaviour of the system is lowered. For example in a nuclear reactor any faults could result in a nuclear radiation leak. Conclusively, the margin of possible faults is inversely proportional to the system integrity, as shown in Figure 1-10.

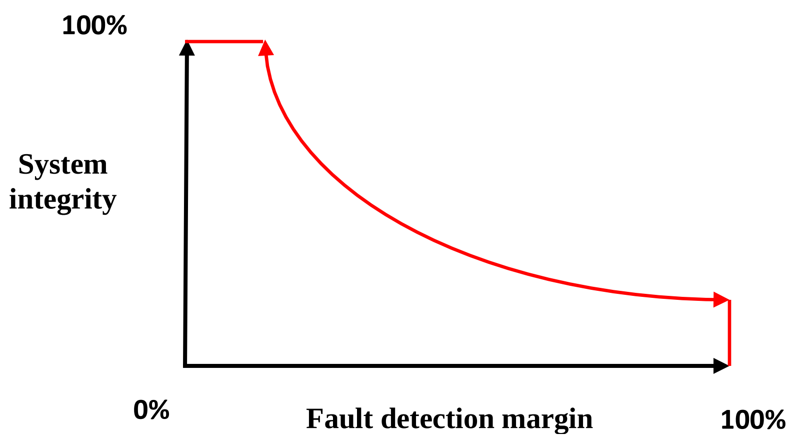

Figure 1-10: Relationship between importance of the system and margin of faults

In general it is required to design systems to minimize their energy consumption and design control methods to prevent them from faults and failures. These faults may be expensive in terms of both economics and human resources. This is due to the high cost of energy and the lack of natural resources. Faults and failures have a strong relationship with the economy. For example, faults in the electricity supply network or energy affect all interests and could cripple the daily handling of all community facilities and result in economic loss (Blanke,Kinnaert,Lunze and Staroswiecki, 2006).

According to (Patton,Frank and Clark, 1989) the most widely used method for diagnosing faults is the model-based fault diagnosis which is based on the analysis of a specific signal and depends on the predetermined level to make the right decision. An important benefit of using the model-based method is that it is implemented purely using software and does not require any hardware (Patton,Frank and Clark, 1989, Chen and Patton, 1999, Patton,Frank and Clark, 2000). This is the so-called "analytical redundancy" which is also used for the fault diagnosis function.

The majority of the methods used in fault detection are based on the so-called model-based fault detection which uses a mathematical model equivalent to the system itself to see faults (Patton,Frank and Clark, 1989, Gertler, 1995).An easy and simple method of studying faults is to analyse the output signal of the system (Zhang and Jiang, 2008). The detection and isolation of faults should not be impaired by changes in system parameters, in other words the FDI function should be robust against modelling uncertainty and some parametric variations. The FDI function should also be insensitive to disturbances in the system that can affect the detection performance (Chen and Patton, 1999).

According to Isermann (2011) a failure is a permanent disability in a system meaning that a certain role of a system ceases to be varied, i.e. part of the system fails, possibly a permanent disability of one or more functions of the system, imparting on system performance during a defined period. A system may fail with single or multiple failures, often these failures evolve from faults. On the other hand malfunction is a temporary breakdown in the system which causes it to stop working and which may cause the system to enter into a failed state. If the system continues to work even when there are certain faults, the system is said to be tolerant of those faults (or even failures).

The relationship between faults failures and malfunction is shown in Figure 1-11. If a failure continues to enter the system then full failure will occur, or if the failure changes to become a malfunction. Hence, the system will continue to function in the malfunctioned state as the faulty system. This process can be vice-versa; switching between failure and malfunction.

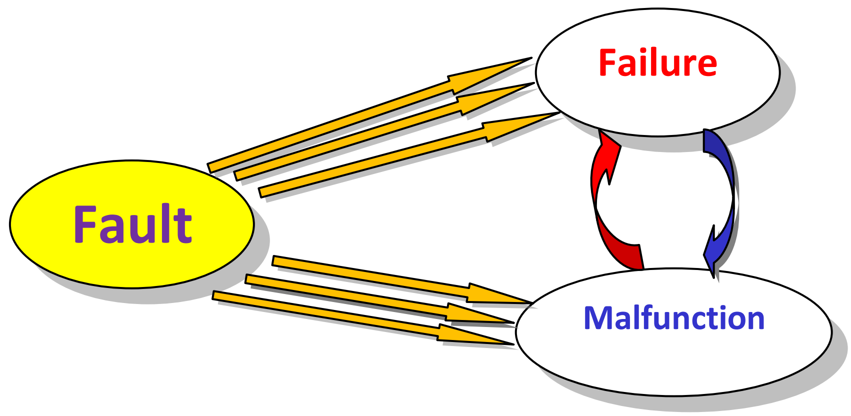

Figure 1-11: Relationship between fault, failure and malfunction in systems

This leads to the actual subject of Fault Detection and Isolation (FDI). The fault detection part is defined as the procedure for detecting that a fault has occurred. The fault isolation procedure is used to determine the location of the fault, i.e. where the fault acts in the system.

However, as mentioned in (Patton,Frank and Clark, 1989, Chen and Patton, 1999, Isermann, 2011), fault diagnosis can be divided into several sub-parts:

- **Fault detection**: detection presents two cases of logic or which indicates whether there is a fault or not.
- **Fault isolation**: determine the place where the fault or faults occurred, for example in the actuator or sensor.
- **Fault identification and fault estimation**: determine the type, properties and the nature of the specific fault; one method is the estimation of this fault.

A set of general steps for achieving good FDI in control systems can be given as:

1- Treat the fault using any method of FDI, regardless of its size, because if left untreated, small faults may turn into malfunction or failure.
2- Locate the fault in any part of the system accurately; this will make it easier and quicker to deal with.
3- Know the specifications of this fault, its shape, its size and its type by FDI.
4- Take appropriate action to deal with the fault accurately and quickly as determined by the operator.

1.2.1.2.2 Fault-tolerant control

According to Patton (1997) Fault tolerant Control (FTC) is a very important concept in control systems theory "...the main task to be tackled in achieving fault tolerance is the design of a controller with suitable structure to guarantee stability and satisfactory performance, not only when all control components are operational, but also in the case when sensors, actuators (or other components e.g. the control computer hardware or software) malfunction. This has sometimes been referred to as a control system which possesses integrity or which has control loops which possess loop integrity."

In the field of FTC, there are two main principles which determine how to study and control faults. In the first principle, for each element where a fault may occur there is an alternative; this is for reasons such as security, economy and human safety, for example, in aircrafts and spacecrafts. The second principle is based on changes in the controller or through the estimation of some signals, such as state estimation in the case of sensor faults. This principle is less costly and involves lower maintenance costs than the first one. However, it cannot be used in all cases, for example, in the event of complete failure in the actuator (Blanke,Kinnaert,Lunze and Staroswiecki, 2006, Gertler, 1998).

According to Patton (1997) FTC methods are classified into Passive fault-tolerant control (PFTC) and Active fault-tolerant control (AFTC).Whereas the main difference between these two methods is whether or not a reconfiguration or adaption of the control design is used. In the passive FTC, from limited to minor fault effects on the system can be tolerated by designing robust control with a best choosing design method. On the other hand, active

FTC methods depend on obtaining online fault information to reconfigure or reconstruct the structure of the controller according to this fault.

According to the method which is used to obtain the fault information Active FTC methods are classified to two branches whether or not there is a dependence on the use of FDI residual signals or a dependence on using fault estimation to overcome the effect of bounded faults. FTC methods can certainly deal with practical faults arising in real systems (Patton, 1997).

Figure 1-12 shows a general overview of the main classification of FTC and the used methods to achieve PFTC or AFTC.

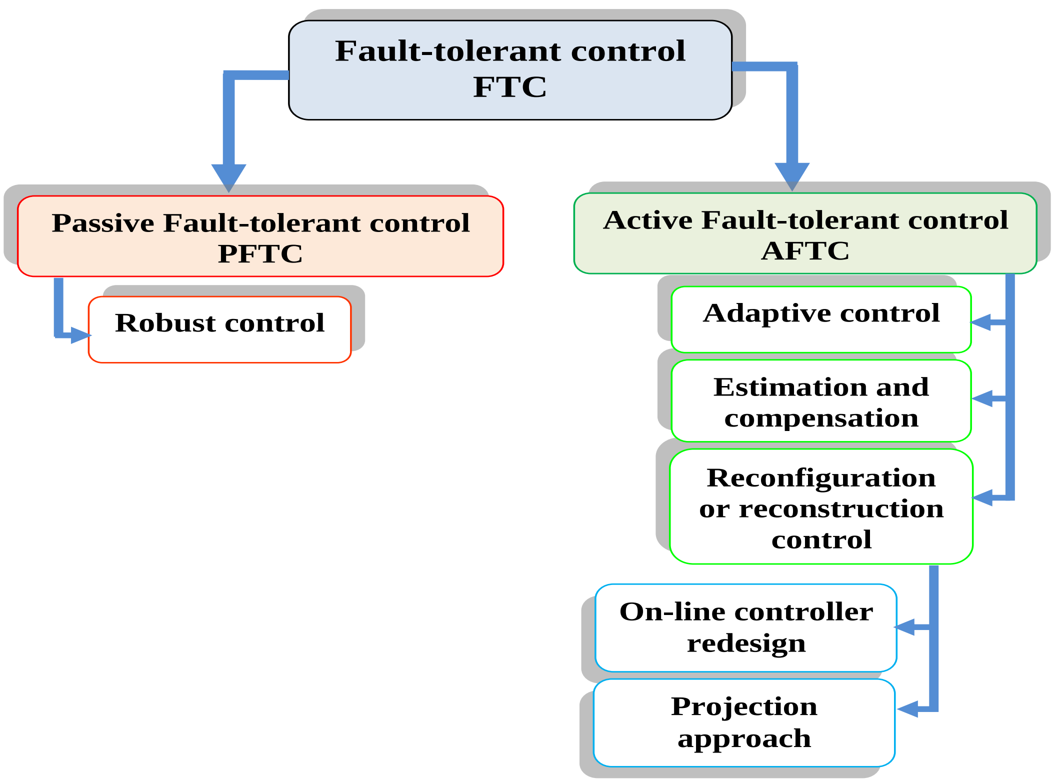

Figure 1-12 : General classification of FTC methods (adapted from Patton, 1997)

1.3 Thesis structure and contributions

This thesis is primarily concerned with the challenges of designing various approaches to robust de-centralized control systems to handle the joint problem of robust local controller design, for uncertain inter-connected systems. However, as an extension to the robustness concepts some new ideas about the FTC problem for inter-connected systems are also developed. The main idea is to isolate each subsystem from other subsystems of an inter-connected system, taking account of actuator faults, uncertainties and exogenous disturbances as aspects of local control system design.

The remaining Chapters of the thesis are arranged as follows:

Chapter 2 focuses on the study and review of the most important methods and techniques in controlling linear inter-connected systems. The first method of control is the Hierarchical optimisation method with known interconnections, which includes (the Goal coordination approach, the three-level method of Tamura for discrete dynamical systems, the interaction prediction approach and the three level prediction principle controllers). Comparisons are made between these methods, all of which use the same concept of controlling in multi-levels.

The second method is Hierarchical feedback control with unknown interconnections. The Chapter ends by discussing the design of a hierarchical feedback control, and robust decentralised control using model following and robust decentralised control using parameter perturbation. In this Chapter all the types of faults and failures that can occur are discussed; included in the study are: Actuator faults (*malfunction in actuator*), Process faults (*change in the parameters of dynamic systems*) and Sensor faults (*malfunction in sensor*).Also how to build an observer to estimate faults .An example of vibration control of a three storey building excited by seismic data is discussed, including the approach to developing the inter-connected subsystems of the hierarchical distributed system structure.

Chapter 3 presents a novel technique to control Lipschitz non-linear inter-connected systems with matched and unmatched uncertainties, unknown interactions and exogenous disturbances. This method depends on utilizing a state feedback distributed controller that depends on two components; one which decreases the effect of matching elements and actuator faults by using the ISMC, and the other which can be designed based on LMI state feedback to reduce the effect of unmatched elements and to achieve predetermined specifications. This part of the controller can be achieved in several ways, depending on whether the interactions between the subsystems are known or not. For unknown and known interactions there are two methods to design a controller: (a) An LMI formulation for each subsystem separately and (b) using an LMI approach applied to the overall system as a single (one shot) system. Comparisons are made between the relative advantages and disadvantages of all of the above methods.

Chapter 4 focuses on a new approach to estimate the state of Lipschitz non-linear inter-connected systems with matched uncertainties and unknown interactions. Measuring the state is not always possible, due to several reasons, such as the cost of measurements and maintenance. This Chapter describes a new approach to observer-based control design for inter-connected systems. There are two steps in this method: (i) One part of the controller gain and the observer gain can be achieved by using an appropriate LMI, (ii) An ISMC is designed to cancel the effect of any matched components in the subsystem or bounded actuator faults. This method can be designed in two ways. One way is to design all subsystems as a compact system and the second way is to design each subsystem individually; if the total number of systems is not too big, then it is possible to tune every subsystem separately, but if the number is too large, it is easier to use the compact system (One shot) to design the observers and the control gains in one step.

Chapter 5 investigates and presents a new proposal of an LMI-based design based on static output feedback with Adaptive Output Integral Sliding Mode Control (AOISMC) of non-linear inter-connected systems. The AOISMC is used to control unknown matched components with unknown bounds. This Chapter offers a new technique to deal with the non-linear terms arising from the application of the LMI formulation using the output signals to design the controller gain. An example of a single machine connected to an infinite power bus system is used as an example to verify this approach and compare all simulations results achieved by using OISMC and AOISMC. This study also includes the effects of actuator faults in the power system.

Chapter 6 considers a new design of an output dynamic controller that depends on LMI design to verify the systems stability and achieve specific performance. Because the output signals are the only information available OISMC is used to deal with the transient effects of any matched components. Finally, an example of two coupled inverted pendula is used to illustrate the new approach. Simulated responses of the subsystems are provided for specific actuator fault cases.

Chapter 7 presents an approach to estimate the actuator faults of uncertain Lipschitz non-linear inter-connected systems via state observation applied to each subsystem, in turn.

It is assumed that the individual subsystem observers can estimate the state variables and actuator faults on-line and at the same time instants. This is achieved by designing an appropriate controller with a given performance and robustness specification. It is possible to add an observer to every subsystem, called the de-centralized observer, to estimate actuator faults, which depends on the nature of the required fault detection process.

Chapter 8 summarizes all the work accomplished in the thesis, highlighting the most important points and offering recommendations that can benefit other future researchers interested in this subject.

Chapter 2 :Outline review of linear inter-connected systems

2.1 Introduction

As summarized in Figure 1-1 of Chapter 1 inter-connected dynamical systems can be re-presented in either linear of non-linear model forms. It is interesting initially to consider the concepts from a linear systems standpoint. Hence, this Chapter focuses on the linear systems approach to provide a background for the basic concepts involved, in terms of the system behaviour corresponding to small changes around an operating point (Dutton,Thompson and Barraclough, 1997). The de-centralized system strategy involves an optimisation framework to stabilize the linear subsystems of the inter-connected system according to the linear dynamics (Al-Abdullah, 1984,Brittain,Otaduy,Rovere and Perez, 1988, Trave,Titli and Tarras, 1989).

This Chapter first outlines the multi-level control approach to de-centralised systems using a state variable system description. Multi-level control strategies incorporating either two or three levels are investigated along with the use of hierarchal optimization methods (Singh,Hassan and Titli, 1976,Hassan and Boukas, 2007, Changqing,Patton and Zong, 2010) .

Each level of the multi-level system has a special role for dealing with specific performance of all the inter-connected subsystems. This Chapter investigates the basic concepts of several architectures that may be used to achieve stability and for each subsystem and as well as the stability of the overall system (Brittain,Otaduy,Rovere and Perez, 1988, Larbah and Patton, 2010a).

For example, one level can deal with the stability of each local subsystem whilst in a second level the effects of the subsystem interactions can be minimized and conditions for achieving stability of the overall stability of the system can be determined.

The work outlines the concept of multi-levelled (or *hierarchical*) control used to achieve optimum “local” and “global” stability, disturbances minimization as well as optimal feedback control performance, with carefully defined optimality criteria (Singh,Hassan and Titli, 1976, Sandell,Varaiya,Athans and Safonov, 1978, Hassan and Singh, 1980).

The system must also have a degree of tolerance to system faults occurring either at subsystem or at higher system levels. To handle the faults each fault is considered as signal acting within the system dynamics and a study of possible faults that may occur must be completed (Larbah and Patton, 2010a). To decide on the severity of a particular fault acting in the inter-connected system each of the fault signals is estimated by a state observer.

An example of a tall building structure is described which has a *two-level* hierarchical control approach to ensure good performance in the presence of seismic disturbance excitation acting during an earthquake. For this problem an optimal control approach is compared with the use of H_∞ disturbance minimization method. Both methods are compared subject to the criterion of minimum deviation of each building floor to a central datum line. Each floor of the building is considered to have a semi-active or passive damping system in place. The semi-active damping is achieved through the use of a special actuator the magneto-rheological (*MR)* damper (Shayeghi,Kalasar and Shayeghi, 2009, Larbah and Patton, 2010b).

The effects of actuator faults have also been studied in some detail, identifying whether an actuator fault occurs only on one floor or whether there are several actuator faults each affecting different floors of the building structure. The main contributions of this Chapter are to apply optimal hierarchical feedback control to a three-floor high building, and study how the controller is affected by the actuator faults[1]. The approached used is also compared with the use of H_∞ control to illustrate the robustness of the plug and play concept.

The structure of this Chapter is as follows. Firstly, Section 2.2 reviews the methods of controlling linear inter-connected systems focussed on well-known approaches. Section 2.3 describes the concept of hierarchical feedback control used to illustrate the optimal control

[1] Part of the work presented in this chapter has been published in:

- Eshag Larbah and Ron J. Patton, Fault tolerant control in high building structures, Conference on Control and Fault-Tolerant Systems (SysTol'10), October 6-8, 2010, Nice, France
- Eshag Larbah and R.J. Patton, Fault tolerant "plug and play" vibration control in building structures, 49th IEEE Conference on.Decision and Control, December 15-17, 2010, Atlanta, Georgia USA

formulation for the derivation of the hierarchical feedback control. Section 2.4 illustrates the development of a robust decentralised control system using model-following control. Section 2.5 describes an approach to fault-tolerant control in linear inter-connected systems, illustrating how the faults can affect the overall system performance. Section 2.6 describes the example of a model system for the vibration control of a three storey building excited by seismic data. This includes the approach to developing the inter-connected subsystems of the hierarchical distributed system structure. Finally, the Chapter conclusions are stated in Section 2.7.

2.2 Problem formulation

The linear model of the inter-connected system and corresponding controller designs are discussed in this Chapter. Chapters 3, 4, 5, 6 & 7 are concerned with the application of various de-centralized control and estimation methods for systems with non-linear model dynamics. The linear model is described by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i z_i(t) + E_i d_i \tag{2-1}$$

$$z_i(t) = \sum_{j=1}^{N} L_{ij} x_j(t) \tag{2-2}$$

$$y_i(t) = T_i x_i(t) \qquad i = 1, \dots \dots, N \tag{2-3}$$

where $x_i(t) \in R^{n\times 1}$ is a state vector, $u_i(t) \in R^{m\times 1}$ is a control vector , $y_i(t) \in R^{q\times 1}$ is the output signal , $z_i(t) \in R^{q\times 1}$ is a vector of interconnections between individual subsystems and d_i is an unknown bounded disturbance. A_i ,B_i, C_i , E_i and T_i are known matrices of appropriate dimensions and L_{ij} is an interconnection matrix between subsystems i and j, and $x(0) = x_{io}$ is an initial state vector.

2.3 Methods of controlling linear inter-connected systems

To achieve the required objectives and to improve the performance of a distributed hierarchical structure, coordination functions are required to deal with the interactions between the subsystems (Patton *et al.*, 2007) . Figure 1-5 illustrates how to use a multi-level system to achieve multi-objectives of acceptable stability, robust performance and good fault tolerance.

(Jamshidi, 1997) stated that some of these methods are applicable to hierarchical structures with two or more levels; the lower level (*local controller*) is responsible for local variables to complete local optimisation or local requirements, whereas in highest level (*coordinator*) deals only with global variables. The role of the coordinator is thus to force the local controllers to achieve global stability and global optimisation. In the classical work on *large-scale systems* optimal control is used at both local and global levels of the system (Singh and Titli, 1978, Jamshidi, 1997, Stanislaw, 2003).

Several alternative methods can be used to develop optimal strategies for multi-level control of de-centralised but inter-connected systems. This Chapter presents a general overview of the main techniques reported in the literature.

2.3.1 Hierarchical optimization and control for distributed linear inter-connected systems with quadratic cost function

In optimal control, a linear quadratic cost function is used to achieve the optimal trajectory and this approach is applicable for inter-connected systems. However, if the system is large-scale and inter-connected the optimization problem can become very computationally expensive. However, if the optimisation problem is divided into N sub-optimisation problems according to the number of subsystems, then the process of the whole optimisation problem is made more efficient and faster (Krumov, 2008).

The optimisation problem of inter-connected systems comprising n linear inter-connected dynamical subsystems provides the subsystem control variables $u_i(t)$ by minimizing the performance function (Singh,Hassan and Titli, 1976, Singh and Titli, 1978, Hassan and Singh, 1980, Jamshidi, 1997):

$$J = \sum_{i=1}^{N} \left(\frac{1}{2} x_i^T(T) P_i x_i(T) + \int_0^T \frac{1}{2} (x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t)) dt\right) \qquad i = 1, \dots\dots, N \tag{2-4}$$

Subject to Eqs. (2-1) & (2-3)

where Q_i and P_i are positive semi definite matrices, R_i is a positive definite matrix .

The principle of optimal optimization is to form the *Hamiltonian* function for the *i*[th] subsystem as:

$$H_i = \frac{1}{2} x_i^T Q_i x_i + \frac{1}{2} u_i^T R_i u_i + \lambda_i^T \left(z_i - \sum_{j=1}^{N} L_{ij} x_j \right) + P_i^T (A_i x_i + B_i u_i + C_i z_i + E_i d_i) \quad (2\text{-}5)$$

The *Lagrangian* of the overall system is:

$$L = \sum_{i=1}^{N} [\frac{1}{2} x_i^T(T) P_i x_i(T) + \int_0^T \frac{1}{2} (x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t) + z_i^T(t) R_i z_i(t) + \lambda_i^T (z_i(t) - \sum_{j=1}^{N} L_{ij} x_j (t)))dt] \quad (2\text{-}6)$$

where λ_i are *Lagranage* multipliers and P_i are adjoint variables.

The necessary conditions to satisfy the optimisation criteria are:

$$\frac{\partial H_i}{\partial u_i} = 0 \ or \ u_i = -R_i^{-1} B_i^T P_i \quad (2\text{-}7)$$

$$\frac{\partial H_i}{\partial P_i} = \dot{x}_i = A_i x_i + B_i u_i + C_i z_i + E_i d_i \quad (2\text{-}8)$$

$$\frac{\partial H_i}{\partial x_i} = - \dot{P}_i = Q_i x_i - \sum_{j=1}^{N} L^T{}_{ij} \ \lambda_i \ + A_i^T P_i \quad (2\text{-}9)$$

$$\frac{\partial H_i}{\partial \lambda_i} = 0 \ or \ z_i = \sum_{j=1}^{N} L_{ij} x_j \quad (2\text{-}10)$$

$$\frac{\partial H_i}{\partial z_i} = 0 \ or \ \lambda_i \ = -C_i^T P_i \quad (2\text{-}11)$$

$$\lambda_i(T) = 0 \quad (2\text{-}12)$$

The optimal feedback control $u_i(t)$ is formed as:

$$u_i(t) = -R_i^{-1} B_i^T P_i \quad (2\text{-}13)$$

This is the general formula to find the optimal feedback control, which leads to several approaches for calculating the optimal trajectory depending on the system as well as on the number of levels of hierarchy in the multi-level distributed control problem.

2.3.1.1 The Goal coordination approach

The *Goal Coordination* approach has *two* levels of hierarchy. Independently, the subsystem minimisation is subject to subsystem constraints by the initially given *Lagrange* multipliers $\lambda_i = \lambda^*{}_i$ (Singh,Hassan and Titli, 1976, Jamshidi, 1997). The crucial part of this technique is the first step in which the higher level sends the initial value of the λ_i to the lower level to compute the subsystem *Lagrangian* L_i, where the L_i are minimised subject to their constraints. After the minimisation process, x_i, u_i and z_i are determined and are sent back to the higher level to calculate the interconnection error (e_i) that is calculated by:

$$e_i = z_i - \sum_{j=1}^{N} L_{ij} x_j \qquad (2\text{-}14)$$

This process continues until a specified error is reached, at which point the global optimisation has been achieved. The global *Lagrangian* L is obtained via the summation of the individual L_i so that L has an *additively separable* property; if one subsystem is removed the global Lagrangian L takes account of this via the so-called “plug and play” concept. This means that it is quite easy to add new subsystems or to remove any old subsystems (Shayeghi,Kalasar and Shayeghi, 2009) whilst the global system adjusts the optimal performance. The principle of this approach is shown in Figure 2-1.

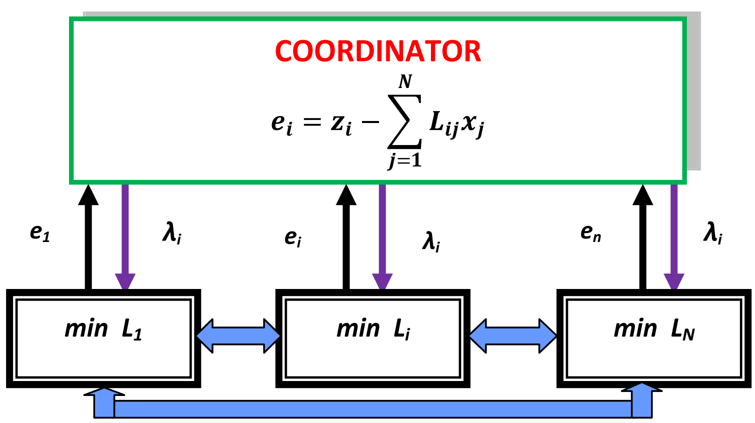

Figure 2-1: Goal coordination approach

2.3.1.2 The three-level method of Tamura for discrete-time systems

The formulation of this method is made in discrete-time and it is a modification of the Goal Coordination approach where three levels are used. The difference between this method and the Goal coordination approach is that the optimisation process is divided into k optimisation problems, where k is the sampling instant. That means decomposing the *Lagrangian* into *sub-Largrangians* for each subsystem (Singh,Hassan and Titli, 1976,Singh and Titli, 1978, Jamshidi, 1997). Figure 2-2 illustrates the general idea of this method.

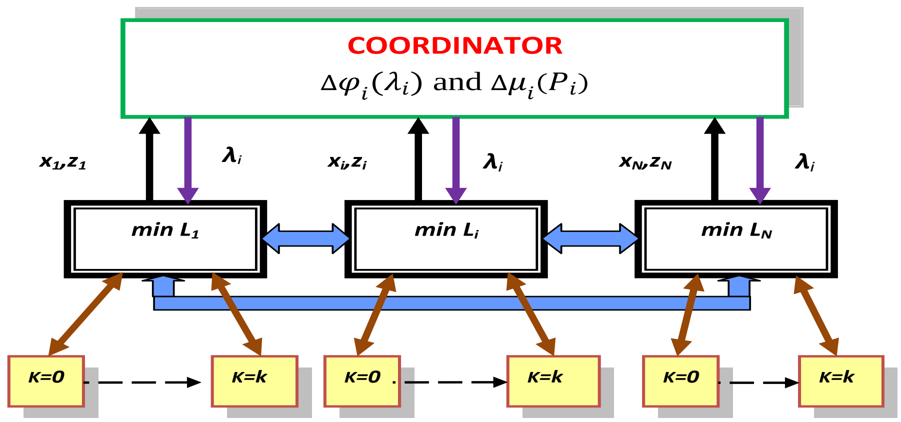

Figure 2-2: Three level method of Tamura

The lower level receives new λ_i values from the higher level to solve the optimisation problem k times, and each time the higher level receives updated values of $x_i(k)$ and $z_i(k)$ obtained from the second level. These are then used to reduce the computation errors in both $\varphi_i(\lambda_i)$ and $\mu_i(P_i)$, where the calculations of these error are given by:

$$\Delta\varphi_i(\lambda_i)|_{\lambda_i=\lambda_i^*}=e_i = z_i - \sum_{j=1}^{N} L_{ij}\, x_j \tag{2-15}$$

$$\Delta\mu_i(P_i)|_{p_i=p_i^*} = -x_i(k+1) + A_i x_i(k) + B_i u_i(k) + C_i z_i(k) \tag{2-16}$$

The overall optimisation is achieved when both $\Delta\varphi_i(\lambda_i)$ and $\Delta\mu_i(P_i)$ become equal to zero or specified value. This method is used in the case of a open-loop system and the gain is calculated *on-line*, however, there is a time-delay in the subsystem which increases the complexity of the calculation and may lead to an unstable model (Singh and Titli, 1978).

2.3.1.3 The Interaction Prediction approach

This approach is formulated in continuous-time and the principle depends on using *two* levels, as in the Goal Coordination approach (Singh and Titli, 1978, Hassan and Singh, 1980). However, the main difference is that the updated values of λ_i and z_i are calculated as coordination vectors at the higher level. After the coordination vectors have been obtained, they are used to minimise every L_i in the lower level to obtain the updated values of x_i, u_i and P_i . These values are then sent back to the higher level to calculate the new values of the coordination vectors. The optimisation process is stopped if the calculations of the errors e_i given by Eq. (2-14) reach an acceptable value, then the last values of x_i and u_i are used as the optimal values. The steps of this approach are shown in Figure 2-3 (Singh,Hassan and Titli, 1976, Jamshidi, 1997).

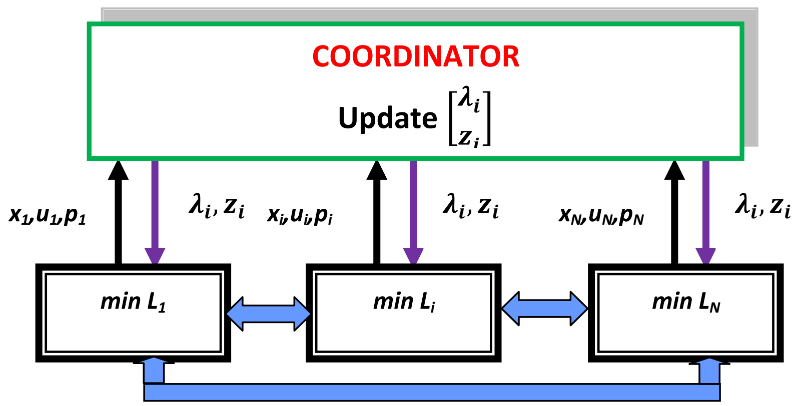

Figure 2-3: Interaction prediction approach

2.3.1.4 The three-level prediction principle controller

This method is an upgraded version of the Interaction Prediction approach (Singh,Hassan and Titli, 1976, Singh and Titli, 1978), where the coordination is divided to two parts; 1st coordinator and 2nd coordinator (Singh and Titli, 1978). The role of the 1st coordinator is to send the initial value of $z_i{}^1$ and $\pi_i{}^1$ to the 2nd coordinator to update $\lambda_i{}^1$ where π is a variable . The 2nd coordinator sends $\lambda_i{}^1$ to all subsystems to minimise sub-Largrangian, following the 2nd coordinator receives the updated $\lambda_i{}^{1+i}$ to calculate an error $e^k{}_{\lambda i}$ using the formula:

$$e^k{}_{\lambda i} = \lambda_i{}^{1+i} - \lambda_i{}^1 \qquad (2\text{-}17)$$

This procedure continues until the error reaches a suitable value. Then the last values of $z_i{}^{1+i}$ and $\pi_i{}^{1+i}$ are sent back to the 1st coordinator to calculate errors $e^k{}_{zi}$ and $e^k{}_{\pi i}$ using Eqs.(2-18) & (2-19):

$$e^k{}_{\pi i} = \pi_i{}^{1+i} - \pi_i{}^1 \tag{2-18}$$

$$e^k{}_{zi} = z_i{}^{1+i} - z_i{}^1 \quad 2\text{-}19)$$

The optimisation process is stopped if the calculation of all errors $e^k{}_{\lambda i}$, $e^k{}_{\pi i}$ and $e^k{}_{zi}$ reaches an acceptable value. The steps of the algorithm describing how to enter data until the algorithm reaches the optimal control and trajectory are illustrated in Figure 2-4 (Singh and Titli, 1978). Although this method gives accurate optimal values, it requires complex calculations.

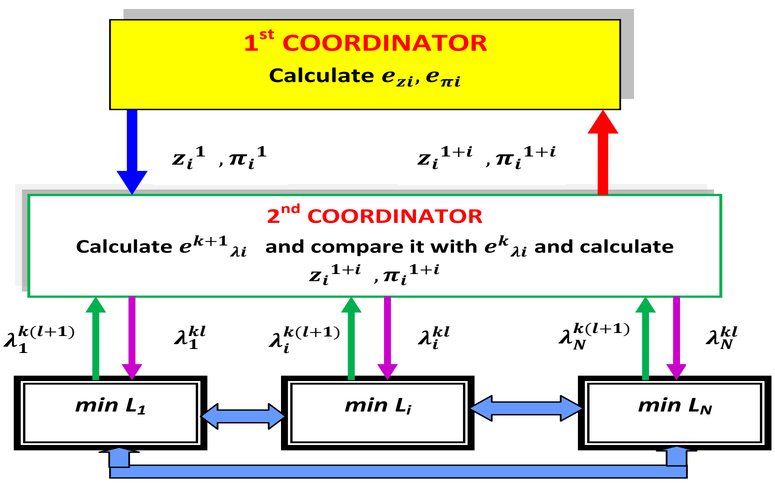

Figure 2-4: Three-level prediction principle controller

2.3.1.5 Comparison between hierarchical distributed control systems methods

Table 2-1 below illustrates the comparison between the different methods of control in hierarchical distributed control systems with quadratic cost functions.

Method	Number of levels	Type of system	Notes
The Goal coordination approach	Two	Continuous time	Cost function includes the interaction term $z_i(t)$does not correspond to a realistic (the state $x_i(t)$ and the control signal $u_i(t)$)
The three level method of Tamura	Three	Discrete time	Complicated and gives attractive results for open-loop control design only.
The interaction prediction approach	Two	Continuous time	Easy and fast but the minimisation only takes the interaction constraints but it requires fewer calculations.
The three level prediction principle controller	Three	Continuous time	Fast to calculate the optimal controller and the trajectory but it is complicated.

Table 2-1: Comparison of hierarchical distributed control systems methods

2.3.2 De-centralized iterative learning control for inter-connected systems with unknown interconnections

This method has been proposed by (Hansheng,Kawabata and Kawabata, 2003). The advantage of this method is that it does not depend on the availability of information about the interactions between the subsystems. Therefore this method requires less computation to obtain the gains to satisfy a desired output. The principle of this approach is shown in Figure 2-1.

Consider the inter-connected system has a form:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i z_i(t) + E_i d_i(t) \quad (2\text{-}20)$$

$$z_i = \sum_{j=1}^{N} L_{ij} x_j \quad (2\text{-}21)$$

$$y_i(t) = T_i x_i(t) \qquad i = 1, \dots\dots, N \quad (2\text{-}22)$$

with $x_i(0) = x_{io}$, for each subsystem the error between the desired local output and the actual local output is:

$$e_i(t) = y_i^d(t) - y_i(t) \quad (2\text{-}23)$$

where $d_i(t)$ is $\in \mathbb{R}^h$ unknown bounded process disturbance , $y_i^d(t)$ is desired local output and $y_i(t)$ is actual local output .

Through the iterative learning process the limited error is:

$$\lim_{k\to\infty} \left|\left|e_i^k(t)\right|\right| = \lim_{k\to\infty} \left|\left|y_i^d(t) - y_i^k(t)\right|\right| = 0 \qquad i = 1, \dots\dots N \quad (2\text{-}24)$$

Where k is an iteration index and the de-centralized iterative learning control algorithm is:

$$u_i^{k+1}(t) = u_i^k(t) + \Gamma_i^k e_i^k(t) \quad (2\text{-}25)$$

The initial state of the learning control algorithm is given by:

$$x_i^{k+1}(t) = x_i^k(t_o) + B_i \Gamma_i^k e_i^k(t_o) \quad (2\text{-}26)$$

where Γ_i^k is an iterative learning control gain matrix and can be calculated from:

$||I_i - C_i B_i \Gamma_i|| < 1$

when $I_i \in R$ is an identity matrix, $x_i^k(t_o)$ and $e_i^k(t_o)$ are initial values of the state and error to begin the iteration.

This kind of control is executed online; the steps of the algorithm describing how to execute the data until reaching the optimal control and trajectory are illustrated in Figure 2-6.

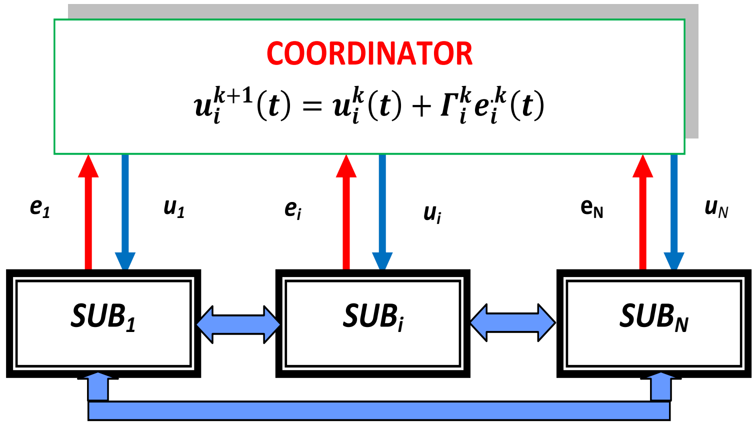

Figure 2-5: De-centralized iterative learning control for inter-connected systems with unknown interconnections

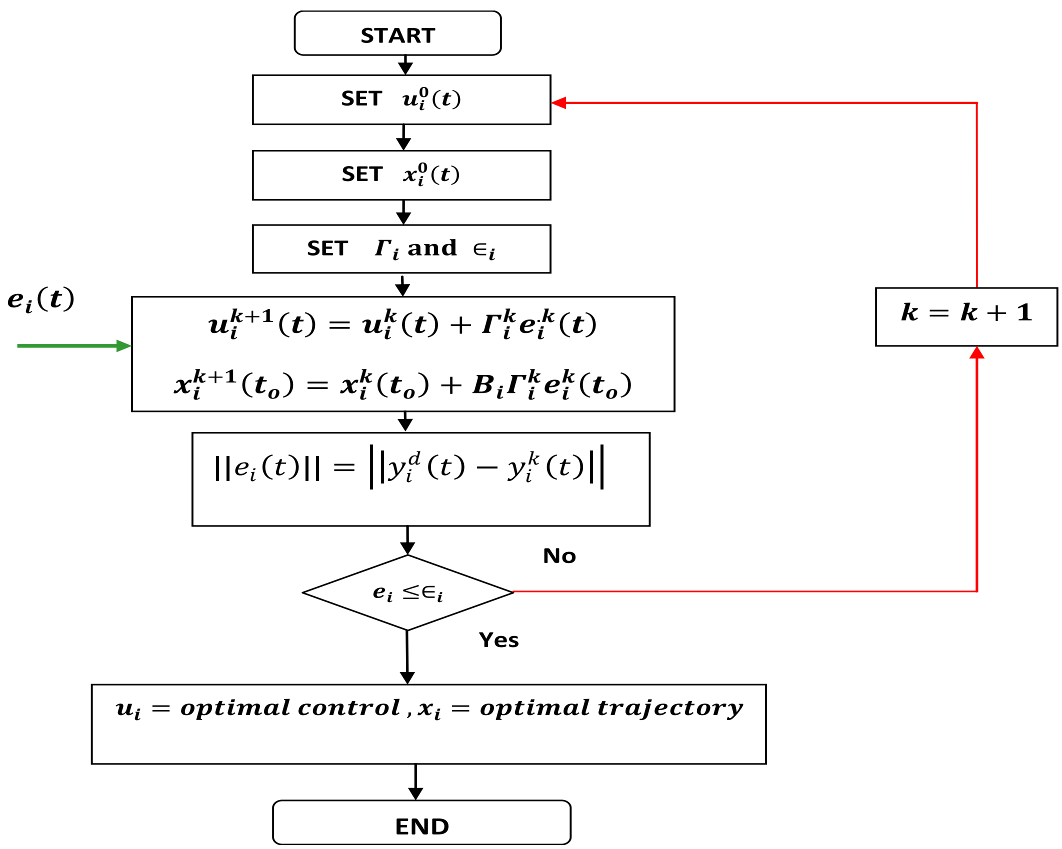

Figure 2-6: Flowchart of the algorithm of de-centralized iterative learning control for inter-connected systems with unknown interconnections

2.4 Hierarchical feedback control design

To implement the controller in hierarchical distributed inter-connected systems most of the previous strategies propose an optimal feedback approach to satisfy the stability and desired performance (Sandell,Varaiya,Athans and Safonov, 1978, Kalsi,Jianming and Zak, 2008). Although all the methods can be used to control inter-connected systems, every method has a different calculation to obtain the controller gains. The general formula to obtain the optimal hierarchical feedback control is described as:

From Eq. (2-12) the control signal $u_i(t) = -R_i^{-1} B_i^T P_i$

Let the P_i be the adjoint variables given by the solution of a Matrix Ricatti equation as follows:

$$P_i(t) = K_i x_i(t) + S_i(t) \tag{2-27}$$

$$u_i(t) = -R_i^{-1} B_i^T K_i x_i(t) - R_i^{-1} B_i^T K_i \, S_i(t) \tag{2-28}$$

where $-R_i^{-1} B_i^T K_i x_i(t)$ is the local control , $-R_i^{-1} B_i^T S_i(t)$ is the effect of the global control (coordinator) and K_i is the local optimal gain and S_i is an open-loop interaction compensation control vector.

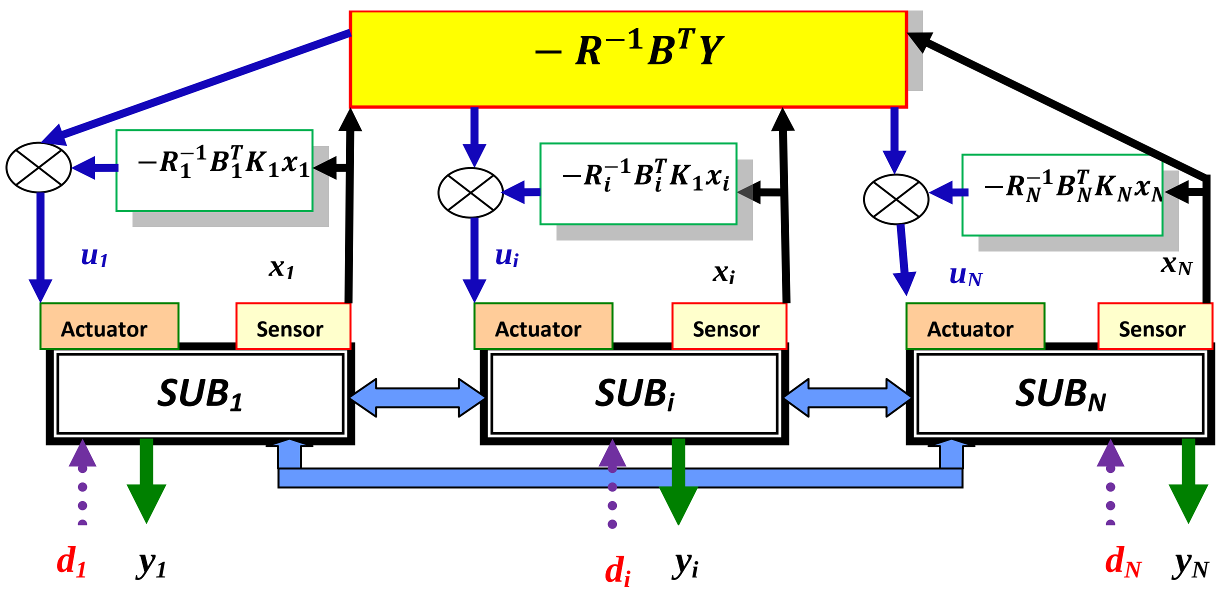

Figure 2-7: Hierarchical feedback control

Where the $-R^{-1}B^TY$ compensates for the interactions, $R = diag(R_i)$, $B = diag(B_i)$ and $Y = diag(S_i)$ .The local control depends on its state and the coordinator depends on all the states of the overall system as shown in Figure 2-7 (Singh,Hassan and Titli, 1976, Singh and Titli, 1978, Patton *et al.*, 2007)

2.5 Robust de-centralized control

The problems of robust stability in inter-connected systems with complex interconnections become more important in closed-loop systems (Chen and Wong, 1987, Larbah and Patton, 2012). Several problems may occur such as the change in the parameters of the system as well as in the inaccuracy of the model (uncertainties), which can lead to system instability. Consequently, there are many methods to solve this problem, including the following model.

2.5.1 Robust de-centralized control using model following

The basic concept is to construct a dynamic model of the interactions for every subsystem and use the states of this model as the interaction inputs(Singh,Hassan and Titli, 1976, Singh and Titli, 1978). The idea of this technique is to combine the original system model with an interaction model into a single system. As a consequence of the inherent modelling uncertainty arising from the use of the interaction model an additional optimization must be used to modify the interaction model parameters based on the online system measurements (Hassan and Singh, 1980). The following additional features of this design approach should be noted (Singh and Titli, 1978):

- The de-centralised control gains are independent of the initial conditions.
- The controller is easy to calculate.

2.6 Fault-tolerant control in inter-connected linear systems

As mentioned in Chapter 1, the idea of using fault-tolerant control is motivated by (a) the detectability of the various faults that occur in the inter-connected system, (b) how these faults affect the overall system stability and performance, and (c) how to compensate for the faults and reduce their effects on the subsystems. The faults that can occur in the systems can be classified, according to (Witczak, 2007) into:

1- Actuator faults (actuator malfunctions)
2- Process faults (parametric changes in the system dynamics)
3- Sensor faults (sensor malfunctions)

Subsystem information about these types of faults can be used to detect faults on-line by using appropriately chosen observers. The observers are used to estimate these faults and give enough information about them.

2.6.1 Estimating faults in linear inter-connected systems

Fault detection can be achieved by estimating certain measured or unmeasured signals from the system (Witczak, 2007) . The model without any faults as described in Eqs. (2-1) & (2-3).

The effects of faults may enter the system via $u_i(t)$, $y_i(t)$ and $z_i(t)$ or the faults may affect the subsystem parameters directly.

The model of the linear system with faults can be expressed as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i z_i(t) + E_{ci} f_{ci}(t) + E_{zi} f_{zi}(t) \tag{2-29}$$

$$y_i(t) = W_i x_i(t) + E_{si} f_{si}(t) \qquad i = 1, \dots \dots, N \tag{2-30}$$

Where the $f_{ci}(t)$ are the actuator faults, the $f_{zi}(t)$ are faults that come from the interactions, and the $f_{si}(t)$ are the subsystem sensor faults. The E_{ci}, E_{zi} and E_{si} are known inputs matrices with appropriate dimensions.

2.6.2 Full observer to estimate faults in inter-connected systems

Model-based fault diagnosis is needed to detect faults by residual generation to indicate whether or not the faults have occurred. Therefore, suitable decisions can be made to either compensate for the effects of the faults in the system or to use hardware redundancy to replace a faulty component (sensor, actuator, etc) (Chen and Patton, 1999, Menighed,Aubrun and Yamé, 2009).

$$\dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i u_i(t) + C_i z_i(t) + L_i[y_i(t) - W_i \hat{x}_i(t)] \tag{2-31}$$

The structure of the full observer used in an inter-connected system is shown in Figure 2-8.

For this system ,the state error $e_{xi}(t) = x_i(t) - \hat{x}_i(t)$ (2-32)

where: $\dot{e}_{xi}(t) = (A_i - L_i W_i)e_{xi}(t) + E_{ci} f_{ci}(t) + E_{zi} f_{zi}(t) - L_i E_{si} f_{si}(t)$ (2-33)

If there is no fault in the subsystem then $\dot{e}_{xi}(t)$ tends to zero, but if the fault has occurred then $\dot{e}_{xi}(t) \neq 0$

where $E_{ci} f_{ci}(t) + E_{zi} f_{zi}(t) - L_i E_{si} f_{si}(t) \neq 0$

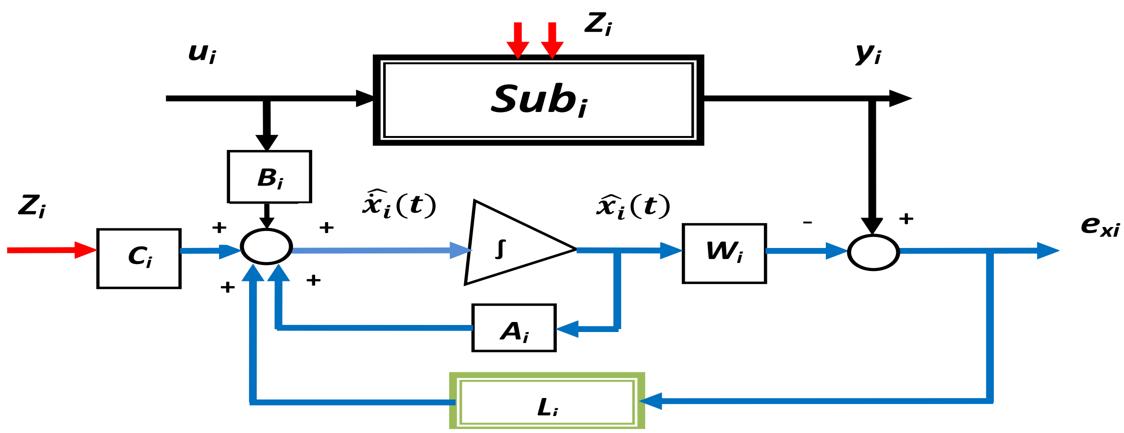

Figure 2-8: Full observer with i^{th} subsystem

Before examining the faults in the subsystems it is necessary to understand the subsystem well, especially in complex subsystems with interconnections. In these subsystems it is necessary to distinguish between signals coming from other subsystems to the faults. Then their challenge is made to design a robust and adaptive controller to achieve stability and specific performance. It is possible to include observers in an inter-connected systems framework to give an indication of the presence of faults; Figure 2-9 illustrates the idea of including observers for this *fault detection* role in an inter-connected system.

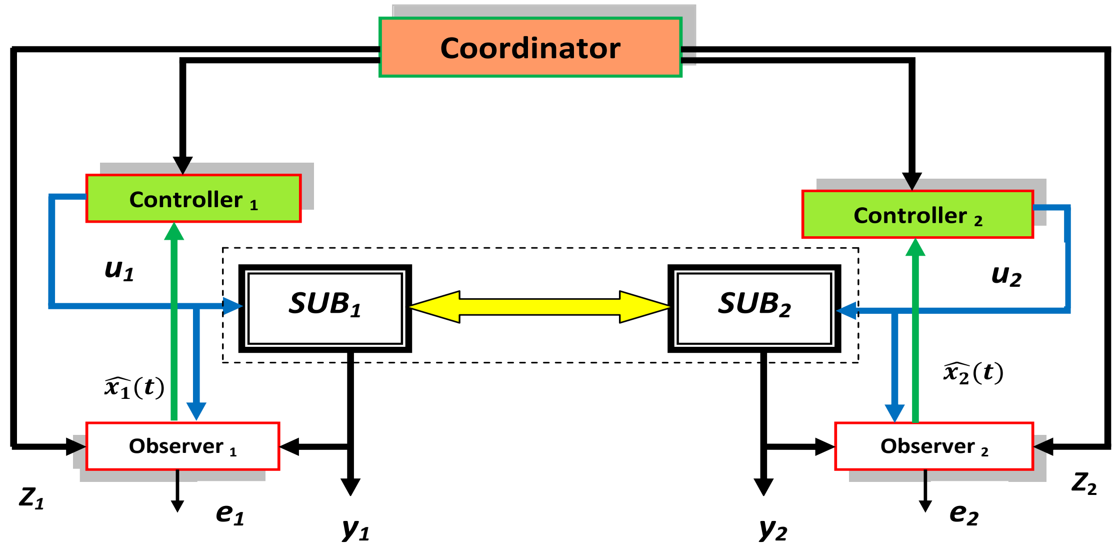

Figure 2-9: Coordinate all the subsystems with its controller and observer

2.7 Fault-tolerant vibration control in building structures

According to a report by the US Geological Survey National Earthquake Information Centre, the number of earthquakes occurring worldwide during 1990 to 2008 has doubled (USGS, 2010). This study focuses on a control example of a multi-floor building structure, with aim to minimize floor displacements under seismic vibration. The seismic data corresponds to the El Centro Richter 7.1 earthquake of July 1940 (NISEE, 2010).

The control of the structural integrity of large buildings subjected to severe external forces from natural phenomena such as earthquakes and wind storms has become one of the biggest challenges in civil and structural engineering (Spencer,Christenson and Dyke, 1999, Sk and Ramaswamy, 2009). During an earthquake, large buildings are subjected to very significant structural stresses and bending moments that can be minimized using control methods (Spencer,Dyke and Deoskar, 1997).

It is normal practice in earthquake zones for the various floors of multi-storey buildings to be constructed to include actuator devices to either provide strong vibration isolation or actuate the structure to compensate for high bending loads. There are three types of actuators in use in buildings; these are passive, active or semi-active (Jerome and Kincho, 2004).The passive actuator does not use sensors and control devices but provides shock absorption or damping to lower the effect of the external forces and moments acting on the structure. Although passive devices require no external energy their parameters are tuned at design to give vibration isolation for a limited range of externally acting forces and moments. On the other hand active devices require special sensors and control design and cannot operate in power failure outages as they consume large amounts of energy and are typically not used on every floor of a building (Sk and Ramaswamy, 2009).

The semi-active actuator devices for vibration control combine the advantages of passive and active methods, providing adaption to high exogenous load levels, through a mechanism to tune their physical properties to facilitate good control performance (Yalla,Kareem and Kantor, 2001).

The requirement for robust adaption to variable external forces and moments acting on a building leads to interesting control challenges. The controlled structure *must have fault-tolerance* in the sense that a suitable level of control performance should be maintained, even when power outages occur. Furthermore, if an actuator should become faulty or even fail the cooperative structure control through the building should be tolerant to this type of malfunction.

The literature on this application subject does discuss the use of various control methods including centralized and de-centralized control techniques (Lynch,Law and Blume, 2002). However, the published studies do not consider fault-tolerance properties in the sense of improving the integrity of the vibration control system subject to actuator and/or sensor faults or failures.

This Chapter proposes that a de-centralized and hierarchical approach to the structural control problem provides good FTC properties in the presence of individual floor actuator fault or failures. The system is a two-level control autonomous coordination and supervision scheme (ACSS) scheme with an autonomous coordinator (at the highest level). The second level comprises the subsystems representing the individual floor structures with their semi-active actuator systems. The subsystems are assumed to be inter-connected as a consequence of the building structure.

The main goal is to demonstrate a flexible control "plug and play" property of the FTC system (Kambhampati,Patton and Uppal, 2006, Patton *et al.*, 2007).Wherein a good structural control performance is maintained throughout the building, while there are for example malfunctions or failures of semi-active actuators in one or more floors of the structure. When a semi-active actuator device fails the distributed two-level system takes account of the loss of one subsystem and autonomously reconfigures/redistributes the control action among the remaining floor actuation systems. The mathematical structure of this two-level scheme shows that if a subsystem fails, i.e. its function is “unplugged” a suitable performance can be maintained using the additive separable control performance of the distributed system receding horizon control structure. In a sense the hierarchical control system utilizes a form of redundancy that exists between the distributed subsystems by balancing the system subsequent to faults and failures.

The two-level distributed control design is compared against a single level (aggregated) H-∞ control method with control applied to each floor actuation system.

2.7.1 Building structure modeling and control challenges

The concept of an *N*-floor building structure is shown in Figure 2-10. The physical problem is one of continuous bending of the vertical structure in response to seismic vibration excitation. An assumption usually made is that the structural bending can be further represented as a simple linear displacement of a rigid structure. Hence, the horizontal displacements due to seismic vibrations are taken from a datum line.

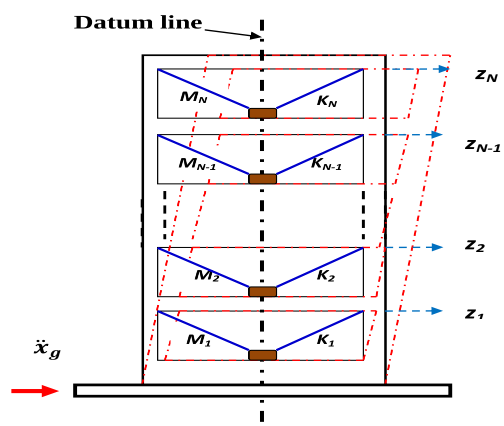

Figure 2-10: *N*- Floor building structure

The deflection from the datum is a continuum resulting from the bending of the structure. However, a rigid body structure can be assumed and hence a simplified representation of the horizontal displacements $Z_i\,, i = 1, \dots\dots, N$ of each floor can be achieved using linear second order system dynamics. Here we assume that each floor is actuated as shown in

Figure 2-10. The aggregate lumped *2N* order system representation (corresponding to the *N*-floors) is N degrees of freedom lumped mass vector representation as follows (Nguyen,Dalvand,Yu and Ha, 2008):

$$M\ddot{Z}(t) + C\dot{Z}(t) + KZ(t) = L_u u(t) + L_w w(t) \tag{2-34}$$

where $Z(t) = [Z_1, Z_2, \ldots . Z_N]^T \in R^{N\times 1}$ is vector of linear displacements, with respect to the datum. $M \in R^{NxN}, C \in R^{NxN}$ and $K \in R^{NxN}$ contain the masses, viscous damping coefficients and stiffness coefficients of each floor. $u(t) \in R^{N\times 1}$ is the control and $w(t)$ is the scalar external seismic excitation. $L_u(t) \in R^{N\times N}$ and $L_w(t) \in R^{N\times 1}$ are the control and excitation matrices within the framework of a single displacement control system acting on each floor (Swartz and Lynch, 2009).

The linear state-space model incorporating the lumped mass models of each floor is:

$$\dot{x}_z(t) = A_z x_z(t) + B_z u(t) + E_z \ddot{x}_g \tag{2-35}$$

The states are defined as: $x_z = [Z \quad \dot{Z}]^T \in R^{2N\times 1}$, and

$$A_z = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \in R^{2N\times 2N}, B_z = \begin{bmatrix} 0 \\ M^{-1}L_u \end{bmatrix} \in R^{2N\times N}, E_z = \begin{bmatrix} -l \\ 0 \end{bmatrix} \in R^{2N\times 1} \tag{2-36}$$

x_g is the horizontal ground displacement and $L_w = -Ml$

The displacement and velocity variables in $x_z(t)$ relate to the datum. To take into account the inter-floor displacements the following transformation must be used (Yang,Jerome and Kincho, 2009):

$$x = \left[Z_1, \dot{Z}_1, (Z_2 - Z_1), (\dot{Z}_2 - \dot{Z}_1), \ldots, (Z_N - Z_{N-1}), (\dot{Z}_N - \dot{Z}_{N-1})\right]^T \tag{2-37}$$

The transformation matrix is $\Lambda \in R^{2N\times 2N}$, so that: $x = \Lambda x_z \Rightarrow x_z = \Lambda^{-1} x$

The transformed state space representation is:

$$\dot{x}(t) = Ax(t) + Bu(t) + E\ddot{x}_g \tag{2-38}$$

where $x(t) \in R^{2N\times 1}$, $u(t) \in R^{N\times 1}$, $A \in R^{2N\times 2N}$, $B \in R^{2N\times N}$ and $E \in R^{2N\times 1}$

$$A = \Lambda A_z \Lambda^{-1} , \; B = \Lambda B_z \; \text{ and } \; E = \Lambda E_z \tag{2-39}$$

The output of this model system is:

$$y(t) = C_x x(t) + D_x u(t) \tag{2-40}$$

where $C_x \in R^{P\times 2N}$ and $D_x \in R^{P\times M}$

2.7.2 Vibration control strategies

The greatest challenge for the control of vibration in building structures in the presence of an earthquake is to maintaining structural integrity. The main control objective is thus to enhance the degree to which the structure is isolated from the vibration. This control enhancement must be achieved at each principal stage of the building structure, i.e. at each floor of the building. A number of methods have been proposed to address the required enhancements ranging from LQR and LQG to H_2 / H_∞ , sliding mode and pole-placement (Shayeghi,Kalasar and Shayeghi, 2009). For this application problem it is very important to maintain good control function, not only under severe earthquake conditions but also to ensure that if faults or failures occur the control system is sufficiently tolerant to these malfunctions. Example of faults may be actuator or sensor malfunctions or the fact that power outages can cause system failure. When faults occur the system should either be reconfigured or make use of inherent redundancy to maintain the required control action. Hence, this Chapter focuses on the requirement for good FTC performance.

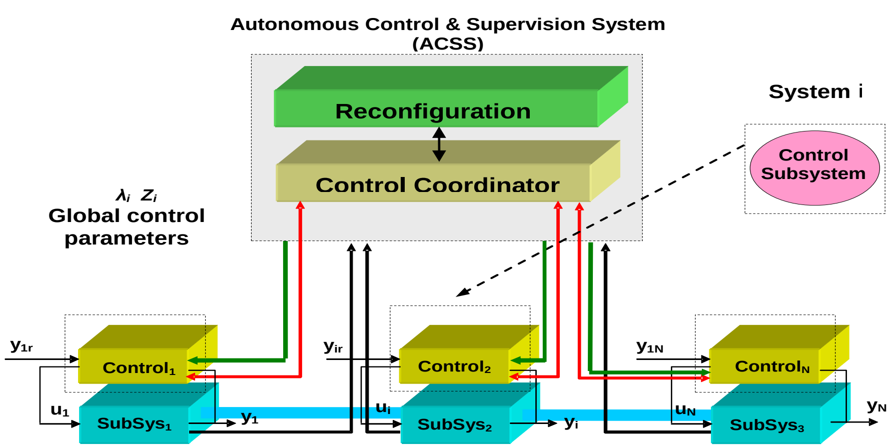

Figure 2-11: Autonomous Coordination & Supervision Scheme (ACSS)

Figure 2-11 illustrates a suitable architecture which facilitates this requirement, based on a two-level decomposition of the overall system into a higher level autonomous coordination of local control systems at a subsystem level. This is the autonomous control and supervision system (ACSS) proposed in (Patton *et al.*, 2007).

The coordinator role includes an element of management of system reconfiguration in the event of control failure of one subsystem. The coordinator minimizes the interaction imbalance among the local controllers as a result of a controller malfunction. Hence, the coordination can be viewed as a global control function comprising the highest level of the two-level structure. It will be shown that by using this two-level structure with LQR and local and global constraints a “plug and play” feature enables the system to reconfigure easily, subsequent to a local control malfunction.

In the context of building vibration control it is important to be able to maintain good control performance and fault tolerance when the control actuation fails to function correctly on one or more floors of the building.

2.7.2.1 H-infinity (H_∞) control

In this study the performance of the two-level distributed control scheme to seismic excitation is compared with that of a single level centralized (or aggregate) output feedback using a robust control design. The H_∞ approach is considered to be a suitable approach (in terms of robustness and disturbance rejection) controller, for a system such as Eqs.(2-38)-(2-40), that has vibration disturbance (Burns, 2001). As the H_∞ approach to robust control is very well known it is only necessary here to outline the main concept. The H_∞ output feedback design (for actuation at multiple floors) is made using the MATLAB© LMI toolbox, based on the concept of disturbance minimization as follows:

The closed-loop system transfer function from the seismic disturbance to the outputs is considered as T_{yw}(s), where:

$$||T_{y\ddot{x}_g}(s)||_\infty = sup_\omega \bar{\sigma}[T_{y\ddot{x}_g}(j\omega)] \quad (2\text{-}41)$$

$\bar{\sigma}$ denotes the largest singular value of the matrix $T_{yw}(s)$ and $\sup_\omega$ denotes the least upper bound of a set of real numbers, covering the angular frequency range of interest (Ashish, 2002) . This centralized control system is used here only as a basis for control performance comparison with the two-level distributed scheme. It will be shown that this single level structure cannot be used to satisfy the FTC requirements for a multi-floor building system.

2.7.2.2 Two-Level hierarchical approach to fault-tolerant vibration control of building structures

The concept of interest is that of achieving FTC if a subsystem should malfunction. For this study and to correspond with Figure 2-10 an individual subsystem is defined as the local dynamics of a single floor with its actuation. To provide an explanation for the way in which a two-level hierarchical control system can achieve the required fault-tolerance it is first necessary to examine the properties of the two-level scheme initially proposed by (Singh,Hassan and Titli, 1976).

In order to implement a two-level distributed control system a method is required to decompose the system into appropriate subsystems (Singh,Hassan and Titli, 1976, Jamshidi, 1997).

A suitable decomposition is essential to construct the use of more than one level of control in the hierarchical scheme of Figure 2-11. The decomposition of the system Eqs.(2-38)& (2-40) can be made in terms of n subsystems, as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i z_i(t) \qquad (2\text{-}42)$$

$$z_i(t) = \sum_{j=1}^{N} L_{ij}\, x_j(t) \qquad i = 1, \dots \dots, N \qquad (2\text{-}43)$$

where $x_i(t) \in R^{n\times 1}$, $u_i(t) \in R^{m\times 1}$ are the subsystem states and controls. $z_i(t) \in R^{q\times 1}$ represents the interconnection states between the i^{th} and $N-1$ remaining subsystems.

Following the procedure of (Singh,Hassan and Titli, 1976), based on constrained LQR the global optimization problem for the control of an inter-connected system is:

$\min J$, where J is a cost function defined as:

$$J = \sum_{i=1}^{N} \frac{1}{2} \int_{to}^{T} (x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t))dt \qquad (2\text{-}44)$$

where Q_i is a positive semi definite matrix and R_i is a positive definite matrix, t_o is the initial time and T is the final time. It can be seen that this cost function is “additively separable”, since the individual subsystem cost components are added together to give the global cost (Singh and Titli, 1978). This leads to the important concept of “plug and play” through which the global controller (coordinator) can effectively reschedule the control action, for example if a subsystem malfunctions of fails. The following theoretical derivation from (Jamshidi, 1997) maintains the plug and play concept noted in (Kambhampati,Patton and Uppal, 2006). As the main contribution of this work is to use this concept for FTC it is necessary to outline here the mathematical structure of the two-level control system, showing the concept of interaction compensation that is important to achieve fault-tolerance. When local control of a subsystem fails the interaction compensation is an important function of the global control and can lead to good FTC performance. From optimization theory the *Lagrangian* of the global system is:

$$L = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \frac{1}{2} \int_{to}^{T} \{ (x_i^T Q_i x_i + u_i^T R_i u_i) + \lambda_i^T (z_i - \sum_{j=1}^{N} L_{ij} x_j(t)) + P_i^T (-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + C_i z_i(t)) \} dt \tag{2-45}$$

where λ_i are Lagrange multipliers and P_i is the adjoint vector.

It can be seen that L is *additively separable* for any given z_i and λ_i , preserving the plug and play property.

The Hamiltonian for each subsystem can be written as in (Stefani *et al.*, 2002):

$$H_i = \frac{1}{2} x_i^T(t) Q_i x_i(t) + \frac{1}{2} u_i^T(t) R_i u_i(t) + \lambda_i^T (z_i(t) - \sum_{j=1}^{N} L_{ij} x_j(t)) + P_i^T \big(A_i x_i(t) + B_i u_i(t) + C_i z_i(t) \big) \tag{2-46}$$

From Eq. (2-46), the necessary optimality conditions are:

$$\frac{\partial H_i}{\partial x_i} = -\dot{P}_i = Q_i x_i(t) + A_i^T P_i(t) - \sum_{j=1}^{N} [L_{ji}^T \lambda_j] \tag{2-47}$$

with $P(T) = 0$

$$\frac{\partial H_i}{\partial u_i} = 0 \implies u_i(t) = -R_i^{-1} B_i^T P_i(t) \tag{2-48}$$

$$\frac{\partial H_i}{\partial z_i} = 0 \implies \lambda_i = -C_i^T P_i(t) \tag{2-49}$$

$$\frac{\partial H_i}{\partial \lambda_i} = 0 \;\; \Longrightarrow z_i(t) = \sum_{j=1}^{N} L_{ij}\, x_j\,(t) \tag{2-50}$$

Using the procedure of (Singh and Titli, 1978) the compensation of the interaction is computed via partial feedback control as follows:

Let $P_i(t) = K_i x_i(t) + S_i(t)$ where K_i is the local optimal gain determined from the solution of the standard LQR matrix Riccati equation, and S_i is an open-loop interaction compensation control vector.

It then follows that:

$$\dot{P}_i(t) = K_i \dot{x}_i(t) + \dot{K}_i x_i(t) + \dot{S}_i(t) \tag{2-51}$$

Substituting Eq.(2-48) into Eq.(2-42) gives the closed-loop i^{th} subsystem state equation as:

$$\dot{x}_i(t) = A_i x_i(t) - B_i R_i^{-1} B_i^T K_i x_i(t) - B_i R_i^{-1} B_i^T S_i(t) + C_i z_i(t) \tag{2-52}$$

Substituting Eq.(2-47) and Eq.(2-52) into Eq.(2-51) yields:

$$\begin{aligned} -Q_i x_i(t) - A_i^T P_i(t) + \sum_{j=1}^{N} [L_{ji}^T \lambda_j] \\ = K_i \left(A_i x_i(t) - B_i R_i^{-1} B_i^T K_i x_i(t) - B_i R_i^{-1} B_i^T S_i(t) + C_i z_i(t) \right) \\ + \dot{K}_i x_i(t) + \dot{S}_i(t) \end{aligned} \tag{2-53}$$

On re-arranging Eq. (2-53) the local control K_i is then given from the solution to the Riccati equation:

$$\dot{K}_i + A_i^T K_i + K_i A_i - K_i B_i R_i^{-1} B_i^T K_i + Q_i = 0 \qquad \text{with } K_i(T) = 0 \tag{2-54}$$

And the compensation vector S_i can be determined from the solution of:

$$\dot{S}_i = -A_i^T S_i + K_i B_i R_i^{-1} B_i^T S_i - K_i C_i z_i + \textstyle\sum_{j=1}^{N} [L_{ji}^T \lambda_j] \qquad \text{with } S_i(T) = 0 \tag{2-55}$$

Substituting Eq.(2-49) & Eq.(2-50)into Eq. (2-53) leads to:

$$\dot{S}_i = -A_i^T S_i + K_i B_i R_i^{-1} B_i^T S_i - K_i C_i \left(\sum_{j=1}^{N} L_{ij}\, x_j\,(t) \right) - \sum_{j=1}^{N} [L_{ji}^T C_j^T P_j\,(t)] \tag{2-56}$$

and substituting $P_i(t) = K_i x_i(t) + S_i(t)$ into Eq. (3-56) we get:

$$\dot{S}_i = -A_i^T S_i + K_i B_i R_i^{-1} B_i^T S_i - K_i C_i \left(\sum_{j=1}^{N} L_{ij} x_j \right) - \sum_{j=1}^{N} [L_{ji}^T C_j^T (K_j x_j + S_j)] \qquad (2\text{-}57)$$

The solution to Eq. (2-57) gives the updated compensation matrix S_i. S_i can thus be updated and the process repeated. The total subsystem control is thus:

$$u_i(t) = -R_i^{-1} B_i^T K_i x_i(t) - R_i^{-1} B_i^T S_i(t) \qquad (2\text{-}58)$$

where $-R_i^{-1} B_i^T K_i x_i(t)$ is the local control and $-R_i^{-1} B_i^T S_i(t)$ is the "global" control action of the interaction compensation provided by the coordination function (Singh,Hassan and Titli, 1976).

This formulation shows clearly that if the i^{th} subsystem control malfunctions or suffers an unusual disturbance then the interaction state will reflect the change and the global control will provide some compensation through the update of the interaction compensation vector S_i. This ability of the two-level distributed system to compensate for local control faults (or failures) corresponds to the plug and play feature in which a local actuator could be completely removed from the overall system structure in a fault-tolerant way. The building example of Sections 2.6.3 illustrates this fault-tolerance principle with results that include the removal of one or more floor actuation systems.

2.7.3 Three-floor building example

Consider the structure of three-story building model as shown in Figure 2-12, which has one semi-active actuator installed at each floor, this example has been adopted from (Yang,Jerome and Kincho, 2009) where the mass, stiffness coefficient, and damping coefficient matrices of the building are:

$$M = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \times 10^3 \text{kg} \quad , K = \begin{bmatrix} 3.4 & -1.8 & 0 \\ -1.8 & 3.4 & -1.6 \\ 0 & -1.6 & 1.6 \end{bmatrix} \times 10^6\ N\ m^{-1}$$

$$C = \begin{bmatrix} 12.4 & -5.16 & 0 \\ -5.16 & 12.4 & -4.59 \\ 0 & -4.59 & 7.2 \end{bmatrix} \times 10^3 N\ m\ s^{-1}$$

From Eq. (2-36) and Eq.(2-37) , the state-space model of the building is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -266.7 & -1.2 & 300 & 0.8603 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 266.7 & 0.7647 & -600 & -2.156 & 266.7 & 0.7647 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 300 & 0.8603 & -533.3 & -1.965 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 1.667 & 0 \\ 0 & 0 & 0 \\ 1.667 & -3.333 & 1.667 \\ 0 & 0 & 0 \\ 0 & 1.667 & -3.333 \end{bmatrix} \times 10^{-4}, E = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2\text{-}59)$$

2.7.3.1 Linear system decomposition

A procedure of (Jamshidi, 1997) can be used to decompose the linear building structure dynamic system of Eqs. (2-38)-(2-40) into k inter-connected linear subsystems. First this system is re-written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ . \\ . \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} A_1 & L_{12} & . & . & . & . & . & L_{1k} \\ L_{21} & A_2 & L_{23} & . & . & . & & L_{2k} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ L_{k1} & . & . & . & . & . & . & A_k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_k \end{bmatrix} + \begin{bmatrix} B_1 & B_{12} & . & . & . & . & . & B_{1k} \\ B_{21} & B_2 & B_{23} & . & . & . & & B_{2k} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ B_{k1} & . & . & . & . & . & . & B_k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ . \\ u_k \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_k \end{bmatrix} W \quad (2\text{-}60)$$

From this the subsystems are described as:

$$\dot{x}_1 = A_1 x_1 + L_{12} x_2 + \ ... + L_{1k} x_k + B_1 u_1 + B_{12} u_2 + \ ... + B_{1k} u_k + E_1 W \quad (2\text{-}61)$$

$$\dot{x}_2 = L_{21} x_1 + A_2 x_2 + \ ... + L_{1k} x_k + B_{21} u_1 + B_2 u_2 + \ ... + B_{2k} u_k + E_2 W \quad (2\text{-}62)$$

$$\dot{x}_k = L_{k1} x_1 + L_{k2} x_2 + \ ... + A_k x_k + B_{k1} u_1 + B_{k2} u_2 \ ... + B_k u_k + E_k W \quad (2\text{-}63)$$

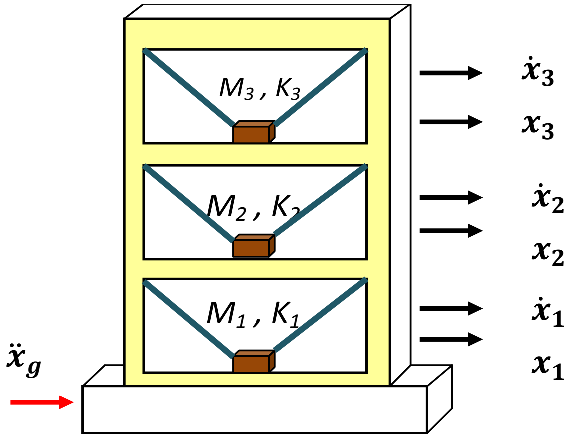

Figure 2-12: Three-story building

In this case the system is described by Eq.(2-59) can be divided into three subsystems defined as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -266.7 & -1.2 & 300 & 0.8603 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 266.7 & 0.7647 & -600 & -2.156 & 266.7 & 0.7647 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 300 & 0.8603 & -533.3 & -1.965 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 1.667 & 0 \\ 0 & 0 & 0 \\ 1.667 & -3.333 & 1.667 \\ 0 & 0 & 0 \\ 0 & 1.667 & -3.333 \end{bmatrix} \times 10^{-4}$$

The three subsystem models are as follows:

1st Subsystem

$$A_1 = \begin{bmatrix} 0 & 1 \\ -266.7 & -1.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ -1.667 \end{bmatrix} \times 10^{-4},\ B_{12} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4},$$
$$B_{13} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times 10^{-4} \text{ and } E_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad (2\text{-}64)$$

The interconnection matrices between 2nd subsystem and the 1st as well as between 3rd subsystem and the 1st respectively are:

$$L_{12} = \begin{bmatrix} 0 & 0 \\ 300 & 0.8603 \end{bmatrix} \text{ and } L_{13} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2-65}$$

2nd Subsystem:

$$A_2 = \begin{bmatrix} 0 & 1 \\ -600 & -2.156 \end{bmatrix}, \; B_{21} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4}, B_2 = \begin{bmatrix} 0 \\ -3.333 \end{bmatrix} \times 10^{-4}, \\ B_{23} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4} \text{ and } E_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2-66}$$

The interconnection matrices between 1st subsystem and the 2nd as well as between 3rd subsystem and the 2nd respectively are:

$$L_{21} = \begin{bmatrix} 0 & 0 \\ 266.7 & 0.7647 \end{bmatrix} \text{ and } L_{23} = \begin{bmatrix} 0 & 0 \\ 266.7 & 0.7647 \end{bmatrix} \tag{2-67}$$

3rd Subsystem:

$$A_3 = \begin{bmatrix} 0 & 1 \\ -533.3 & -1.965 \end{bmatrix}, \; B_{31} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times 10^{-4}, \; B_{32} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4}, \\ B_3 = \begin{bmatrix} 0 \\ -3.333 \end{bmatrix} \times 10^{-4} \text{ and } E_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2-68}$$

The interconnection matrices between 1st subsystem and the 3rd as well as between 2nd subsystem and the 3rd respectively are:

$$L_{31} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } L_{32} = \begin{bmatrix} 0 & 0 \\ 300 & 0.8603 \end{bmatrix} \tag{2-69}$$

Figure 2-13 shows the validation of the decomposed (distributed) model system by comparing the outputs with the aggregated system outputs. The 1940 El Centro earthquake record is used as a single seismic disturbance input (at ground level) to evaluate the effectiveness of both the two-level hierarchical and the aggregated systems with H_∞ control (NISEE, 2010).

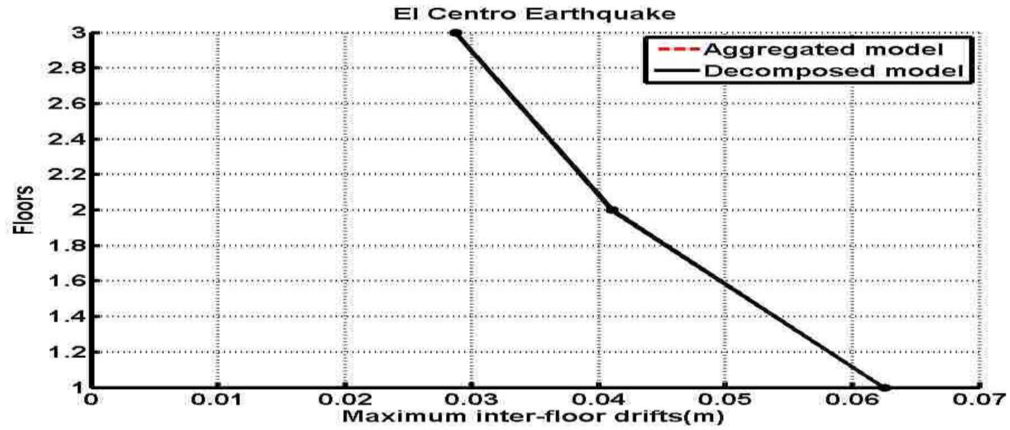

Figure 2-13: Displacements for both aggregated and distributed systems

2.7.3.2 Plug and play FTC results

Figure 2-14, 2-15 & 2-16 show the displacement response of all three building floors when excited by the El Centro earthquake seismic data for cases of: (i) no control, (ii) H_∞ output feedback control and (iii) two-level hierarchical control.

Figure 2-17 illustrates the maximum inter-floor displacements of all three floors (i) without control, (ii) the H_∞ control and (iii) with the two-level control. With no faults this result shows that the centralized H_∞ solution is very similar to the two-level control result. For the 1st and 2nd floors the H_∞ control is slightly better, but for the 3rd floor the two-level control gives the best result.

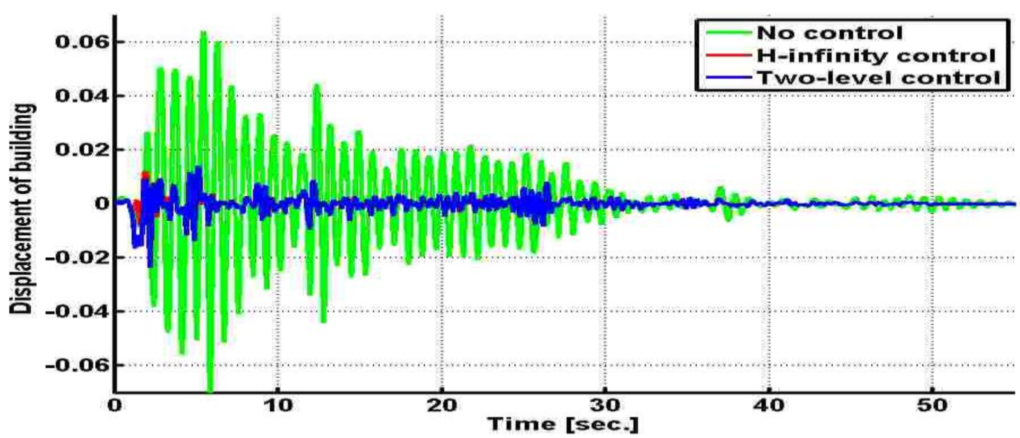

Figure 2-14: 1st floor displacements

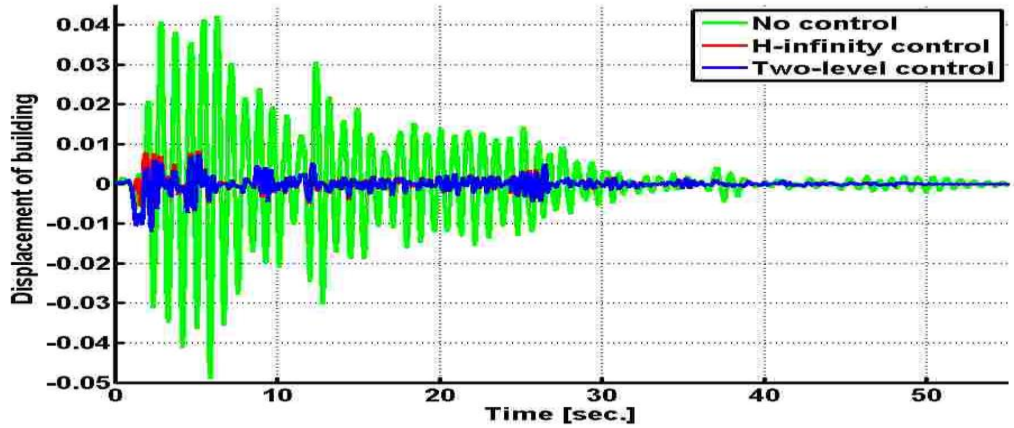

Figure 2-15: 2nd floor displacements

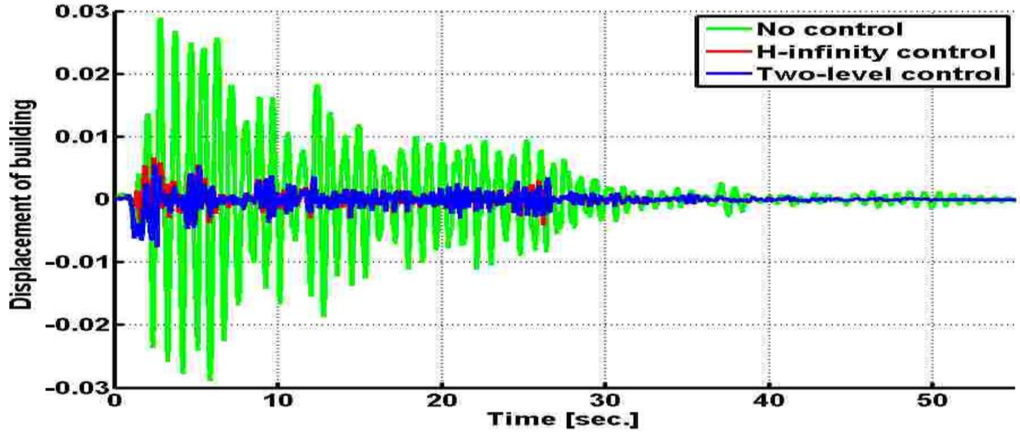

Figure 2-16: 3rd floor displacements

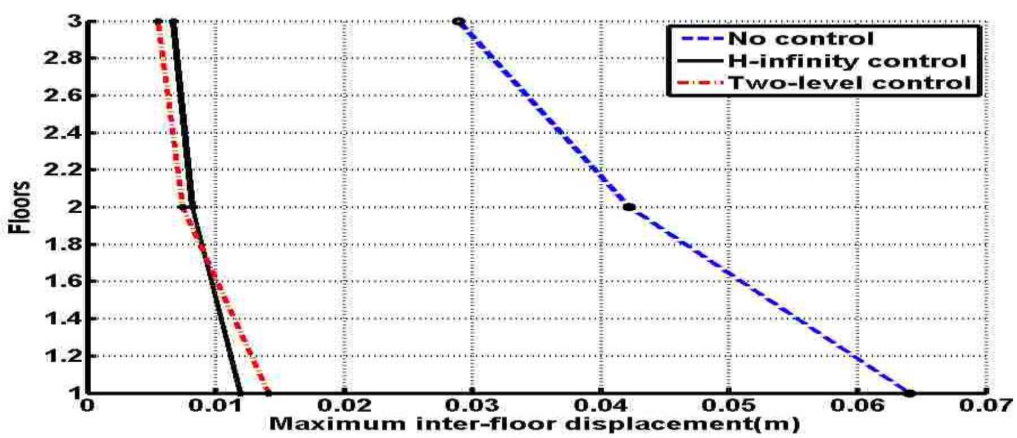

Figure 2-17: Maximum inter-floor displacements

Figure 2-18 shows the normal force output of the 1st floor actuator subject to the seismic input excitation, under two-level control operation.

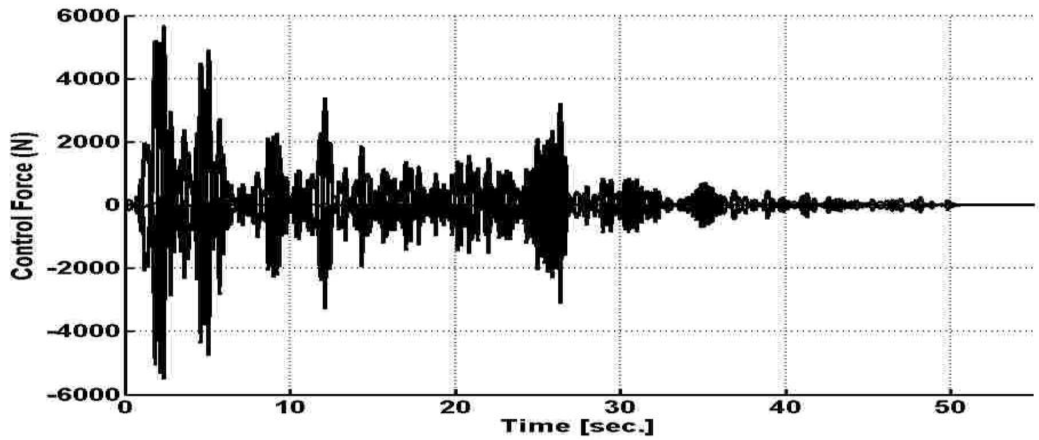

Figure 2-18: 1st floor actuator force with no faults

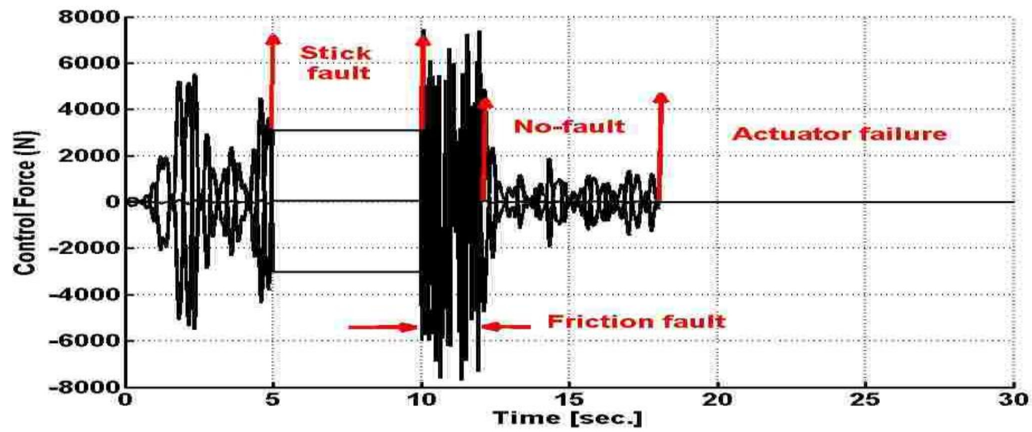

Figure 2-19: 1st floor actuator malfunction and failure

Figure 2-19 illustrates the simulation of a realistic fault/failure scenario for this actuator with the two-level control. After normal operation (0 to 5 s) the actuator is considered to be sticking (5 to 10 s). To simulate the effect of a limit cycle caused by friction a random noise uniformly distributed [0, 8000] is applied to the actuator input (10 to 12 s). After a normal operation period of (12 to 18 s) the simulation includes total actuator failure, meaning that the local system (1st floor) no longer has vibration control action.

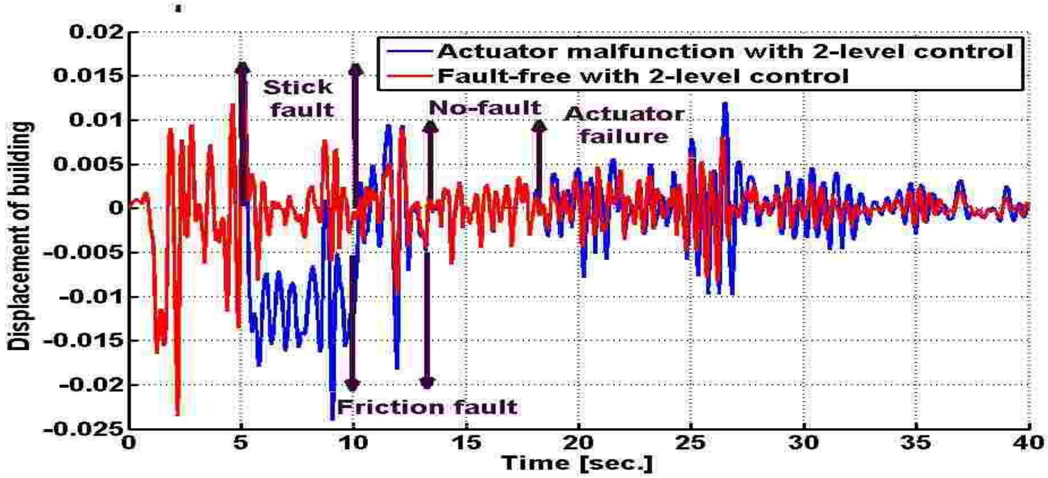

Figure 2-20: 1st floor displacement (two-level control and actuator faults)

The corresponding 1st floor displacement Figure 2-20 illustrates the comparison between the normal and faulty operations.

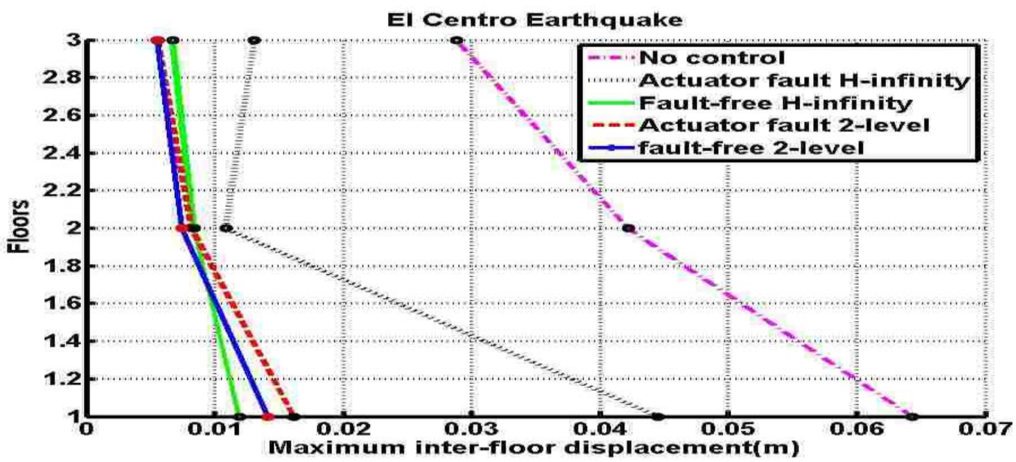

Figure 2-21: Maximum inter-floor displacements

Using the same 1st floor actuator fault/failure scenario depicted in Figure 2-19 & 2-20 the maximum inter-floor displacements for the cases of (i) no control, (ii) H_∞ control and (iii) the two-level hierarchical control are shown in Figure 2-21, 2-22 & 2-23. A comparison of the floor displacement results for both the two-level and H_∞ control designs shows that when there is a fault/failure in one actuator (1st, 2nd or 3rd floors) the two-level scheme gives the least displacement. This is the case as the two-level system is able to react to the control malfunction via the global control compensation function of the term $R_i^{-1}B_i^T S_i(t)$ in Eq. (2-58).

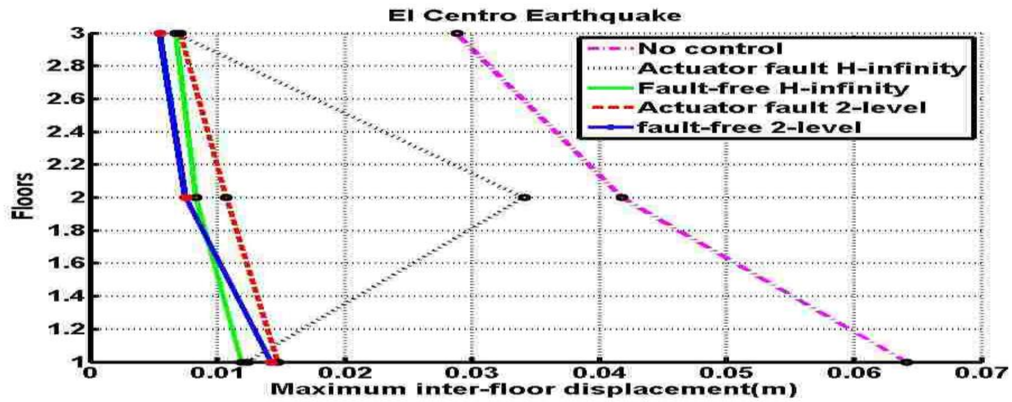

Figure 2-22: Maximum inter-floor displacement: 2nd floor actuator fault

The malfunction of the subsystem control is compensated by re-scheduling the control actions to the remaining subsystems. This is the case, even when the subsystem control fails completely (i.e. the control has no effect). Thus is the essence of the *plug and play* concept of the two-level distributed control that can be seen via the additively separable nature of the global cost function of Eq.(2-44). There is, however upper bounds to the permissible control signals that have not been determined in this work.

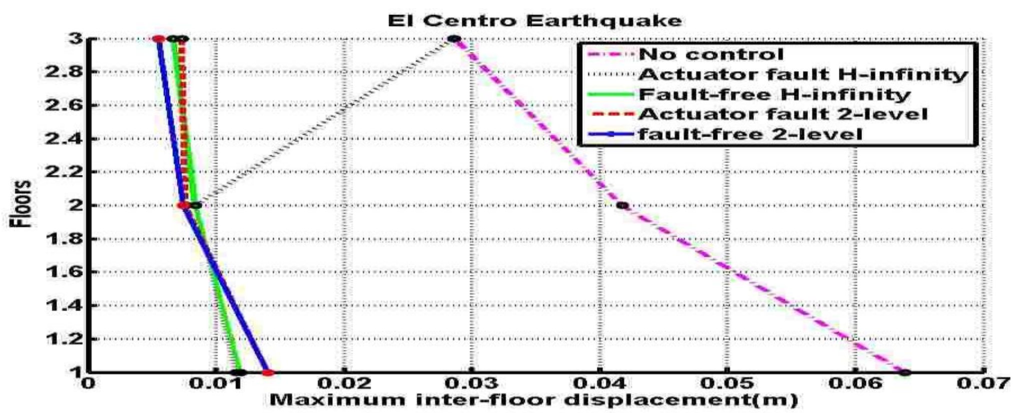

Figure 2-23: Maximum inter-floor displacement: 3rd floor actuator fault

Figure 2-24 shows the comparison between the H_∞ and two-level control systems for the case when the coordinator itself malfunctions. Each subsystem operates independently through three local control systems (based on LQR design).

Figure 2-25 shows the case when the model parameters of Eq.(2-59) are perturbed by -10%, for which the floor displacements for the two-level and H_∞ control systems cases converge above the 2nd floor level. The H_∞ solution shows better robustness between the 1st and 2nd floor levels.

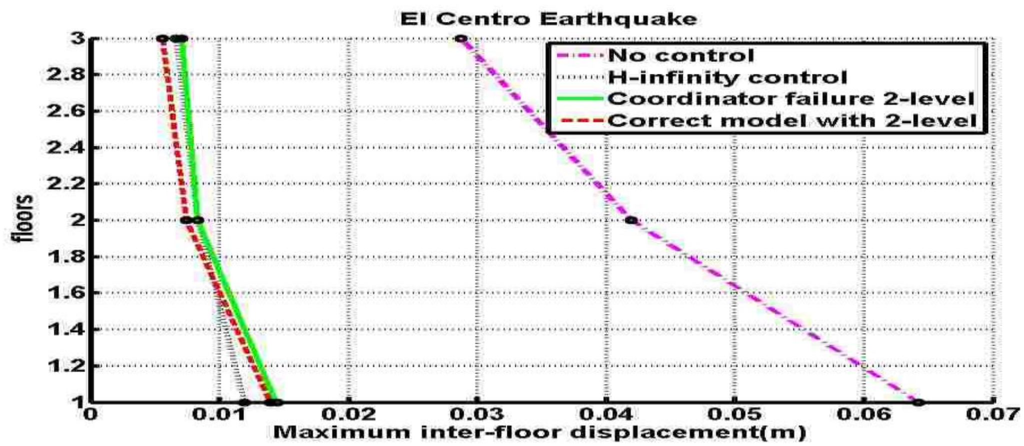

Figure 2-24: Maximum inter-floor displacement: Coordinator failure

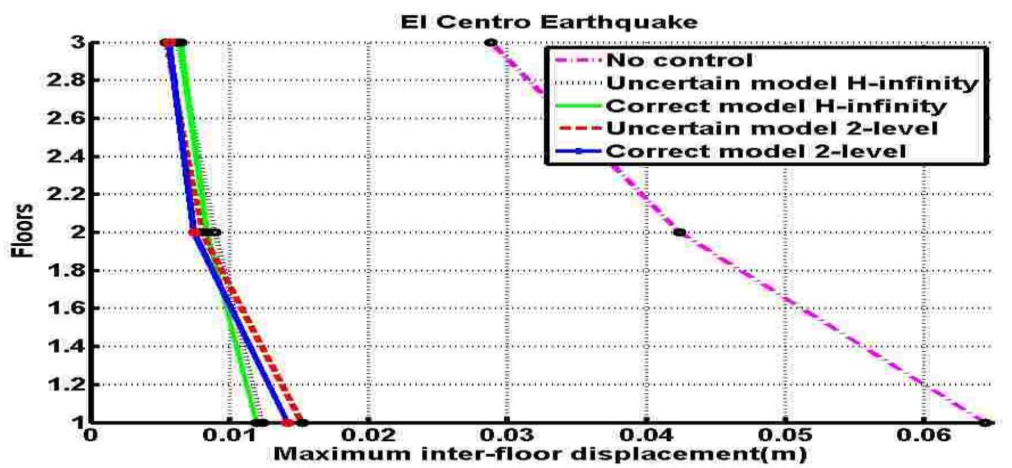

Figure 2-25: Maximum displacements: 10% mismatched & correct models with H_∞ & two-level control

2.8 Conclusion

This Chapter outlines the linear inter-connected systems methods and makes a comparison of the hieratical distributed control methods, as well as their FTC properties. Particular attention is paid to the interaction prediction approach with the inclusion of fault-tolerant control. The fault estimation is used to detect and isolate the presence of faults so that a system reconfiguration can be carried out to replace the faulty subsystem component by a healthy one.

An example of controlling tall buildings in the event of an earthquake is given using the interaction prediction approach. An interesting result observed in this example is that systems that are controlled via two-level hierarchical control exhibit a special form of fault-tolerance. If subsystems malfunction or fail, the global control has the capability of compensating for the individual controller malfunctions, to maintain the required control performance. This is the so-called “plug and play” feature of two-level distributed systems, based on constrained *LQR* control with additive separable global cost. This concept is illustrated using an example of displacement control in multi-floor buildings subject to seismic excitation.

The global performance of the two-level scheme, both for floor actuator malfunctions and model uncertainties have been compared with a centralized (aggregated) robust control design using H_∞ optimization with respect to the exogenous seismic inputs. When the coordinator of the two-level systems fails completely, the control performance (in terms of minimal floor displacement) is similar to that of the H_∞ design. The H_∞ design deteriorates if any of the floor actuators malfunctions or fails. However, the two-level system shows a strong ability to maintain good displacement control in the presence of actuator faults and even in the presence of a completely failed floor actuator, illustrating the significance of the plug and play feature.

In reality, all systems have non-linear dynamics. It is therefore of value to examine the performance of de-centralised control and FTC when applied to non-linear inter-connected systems. Consequently, the remaining Chapters of the thesis focus on the development of inter-connected de-centralised control schemes that take into account the system non-linearity and provide robust performance in the presence of modelling uncertainty. The aim is to develop robust schemes that also have good FTC properties. The thesis describes several approaches to robust de-centralised control and these are compared using various non-linear system examples and the approach to FTC is based on robust fault estimation, together with system reconfiguration if a fault is detected and located.

Chapter 3 : Control of inter-connected systems via LMI combined with ISMC

3.1 Introduction

Chapter 2 discussed the concept of inter-connected systems, starting from the more general non-linear system description and focusing on the development of linear systems concepts.

As a further development this chapter is based entirely on the consideration of a non-linear inter-connected system with bounded uncertainty, non-linear interconnections as well as bounded disturbances. In this work the Lipschitz non-linear model has been considered as a tutorial example to which the appropriate analysis non-linear inter-connected system design is made. This fits with the subject of de-centralized control of inter-connected systems which has received a very significant amount of research attention during more than three decades (Gertler, 1995, Šiljak and Stipanovic, 2001, Zecevic and Šiljak, 2005, Castaños,Xu and Fridman, 2006, Batool,Horacio and Tongwen, 2009, Dhbaibi,Tlili,Elloumi and Benhadj Braiek, 2009, Zecevic and Šiljak, 2010).The literature is rich with the development of powerful strategies for handling the combined problem of minimizing the effect of non-linear interactions whilst designing suitable robust de-centralized (or local) control systems. The de-centralized control concept is naturally extendable to large-scale systems, comprising significant complexity in terms of many inter-connected systems. Well-known design methods include H_∞ optimization and multi-objective designs based on linear matrix inequalities (LMI).

This thesis is based on developments of sliding mode control (SMC) applicable to inter-connected systems. The SMC theory provides a powerful approach to inherent robustness to unknown matched uncertainties, actuator faults and disturbances (Utkin, 1992, Edwards and Spurgeon, 1998, Pisano and Usai, 2011, Mondal and Mahanta, 2012) and has attractive properties for ensuring the robustness of some inter-connected systems. When the system is on the sliding surface, the system is robust to unknown matched uncertainties, bounded actuator fault and disturbances but it is still sensitive to unmatched ones (Edwards and Spurgeon, 1998, Poznyak,Fridman and Bejarano, 2004, Hamayun,Edwards and Alwi, 2010).

The concept of sliding and its inherent robustness to matched uncertainties has been applied to the de-centralized control problem by a number of investigators during the last decade (Yan,Spurgeon and Edwards, 2003,Yan,Edwards and Spurgeon, 2004, Castaños and Fridman, 2005).

Following this background this study concentrates on a further development of SMC, the so-called integral sliding mode control (ISMC) that ensures the system state starts functioning on the sliding surfaces from initial time, without the requirement of a sliding surface "reaching phase" (Castaños and Fridman, 2005, Hamayun,Edwards and Alwi, 2010). ISMC offers some robustness advantages over classical SMC because it compensates unknown matched uncertainties, actuator faults and disturbances from initial time (Edwards,Spurgeon and Patton, 2000,Chang, 2009, Changqing,Patton and Zong, 2010). However, one disadvantage of the standard approach to ISMC is that a comprehensive knowledge of the state vector is required as well as the initial state vector during implementation. A valuable concept in this current study is the use of a combination of a sensitivity minimization approach with the ISMC design to enhance the overall robustness from initial time, thereby ensuring sliding but also reducing the effect of the unmatched uncertainties on the closed-loop system performance. This can be achieved, for example by using an H_∞ optimization approach with an LMI framework (Bejarano,Fridman and Poznyak, 2007,Jeang-Lin and Huan-Chan, 2009, Mondal and Mahanta, 2012). Thus, the control action is the sum of two signals; the first rejects the unknown matched uncertainties and the actuator fault and disturbances, whilst the second is responsible for the system stability and works to ensure that the required level of robustness performance is achieved by using the LMI strategy. An alternative control design procedure to make the subsystem controllers insensitive to any bounded matched components after applying sliding mode control is described by (Šiljak and Stipanovic, 2001)

Using these concepts this Chapter focuses on the design of state feedback control assuming a Lipschitz non-linear state space system model, where each subsystem is dependent only on its own subsystem states, within the required de-centralized control.

The main contributions of this Chapter are as follows:

1- The suggestion of developing a new ISMC with an LMI-based control design of non-linear inter-connected systems covering two approaches. The first is to design independent control systems for each subsystem, whilst the second approaches the design of the whole system as a "*one shot*" approach involving a single overall simultaneous design of all the subsystems.
2- The applications of all these techniques are applied to a three-floored building structure involving a study of the faults that may occur in the actuators, in addition to an investigation of the ability of the overall system to compensate for the effects these actuator faults.

The remaining Sections are planned as follows: Section 3.2 presents the main problem formulation of non-linear inter-connected systems as well as stating the necessary assumptions. Section 3.3 describes the control design methods that use the combined ISMC and LMI-based design strategy. These design methods focus on the subsystem description comprising both known and unknown interactions applicable to the two possible approaches to controller design, namely the *one shot* and individual subsystems (taken one at a time). Section 3.4 gives an example of a three storey building assumed to have a seismic excitation considered as a disturbance input. This non-linear system building example with floor actuator faults is used as a benchmark problem to compare the properties of each of the one shot and individual subsystem control design approaches. Finally, the conclusions are presented in Section 3.5.

3.2 Problem description and basic assumptions

Consider an inter-connected system that consists of many subsystems, and every subsystem is a continuous-time system which can be described as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t)$$
$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots \dots, N \tag{3-1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ are the control inputs and $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs. A_i, B_i, C_i and E_i are known matrices of appropriate dimensions. $Z_i(t) \in \mathbb{R}^n$ represents the unknown time-varying interactions between the subsystems, containing matched and unmatched components.

Hence, $Z_i = Z_{mi} + Z_{ui}$ where Z_{mi} is a matched component of Z_i and Z_{ui} are the unmatched components (Castaños,Xu and Fridman, 2006).

Dropping the subscripts in $Z_i(t)$ and using the Bezout identity $I_n = BB^+ + B^\perp B^{\perp +}$, where $B^+ = (B^T B)^{-1} B^T$, $B^\perp$ is a null space of B^T, $Z_i = B_i {B_i}^+ Z_i + {B_i}^\perp {B_i}^{\perp +} Z_i$ and $B^T B^\perp = 0$, then $Z_i = B_i {B_i}^+ Z_i + \zeta_i$ where $\zeta_i = {B_i}^\perp {B_i}^{\perp +} Z_i$ contains the unmatched uncertainty components.

$W_i(x_i, t)$ represents the subsystem unknown modelling uncertainties that satisfy the matching condition $W_i(x_i, t) = B_i Q_i(x_i, t)$, $d_i(t)$ is an unknown bounded disturbance, $f_i(t) \in \mathbb{R}^k$ denotes the actuator faults, where $f_i(t) = -K(t) u_i$ and $K(t) = diag(K_i)$ with $0 \le K_i \le 1$, $K_i = 0$ that means the actuator is working perfectly and if $K_i = 1$ the i^{th} actuator has failed completely therefore the subsystem does not respond to any control signal otherwise for $0 \le K_i \le 1$ a fault (without failure) is present.

Assumptions:

A1-The pair (A_i, B_i) is controllable and (C_i, A_i) is an observable.

A2-The matrices B_i has full rank m_i.

A3-The initial state $x_i(t_o)$ is bounded.

A4- $Z_i(t)$ Euclidean bounded norms as:

$\|Z_i(t)\| \le \beta_i(x_i, t)$ where $\beta_i(x_i, t)$ is known non-linear function and it vanishes when $x_i(t)$ tends to zereo (Šiljak and Stipanovic, 2001).

A5- $Q_i(x_i, t))$ are bounded as:

$\|Q_i(x_i)\| \le \kappa_i \|x_i\|$,where $\kappa_i > 0$ are known Lipschitz constants (Changqing,Patton and Zong, 2010).

A6- $d_i(t)$ bounded norms as:

$\|d_i(t)\| \le \gamma_i \ \|x_i\|$,where $\gamma_i > 0$ are known constants.

A7- $f_i(t)$ bounded norms as:

$\|f_i(t)\| \le \eta_i \ \|x_i\|$,where $\eta_i > 0$ are known Lipschitz constants and $K_i < 1$.

Following the Assumptions above Eq. (3-1) becomes:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + B_i B_i^{+} Z_i(t) + \zeta_i(t) + B_i Q_i(x_i, t) + E_i d_i(t) + B_i f_i(t)$$
$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N \tag{3-2}$$

The control signal contains *two* components as:

$$u_i(t) = u_i^{LMI}(t) + u_i^{ISM}(t) \tag{3-3}$$

Where u_i^{LMI} is responsible for stabilizing the system, obtaining the desired performance and decreasing the effects of unmatched components, u_i^{ISM} is a discontinuous control responsible for rejecting the effects of matched components (uncertainties, disturbances and actuator faults).

Substituting Eq. (3-3) into Eq. (3-2) yields the *ith* subsystem including the ISMC:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + B_i u_i^{ISM}(t) + B_i B_i^{+} Z_i(t) + \zeta_i(t) + B_i Q_i(x_i, t) + E_i d_i(t) + B_i f_i(t) \tag{3-4}$$
$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N$$

3.3 Control design techniques

The design of a control system based on the inter-connected systems depends on knowledge of the type and extent of the subsystem interconnections. Consequently, the control design is classified as follows:

1- All the interconnections are unknown.
2- All the interconnections are known.

Control designs based on these classifications are considered as follows.

3.3.1 All the interconnections are unknown

If it is assumed that all the subsystems are connected to each other and the i^{th} subsystem interconnections Z_i are unknown, then this implies that the distribution matrices of the interconnections are also unknown. The de-centralized subsystem control system corresponding to this scenario is now developed using the classical ISMC approach (without minimizing the effects of the unmatched uncertainty):

3.3.1.1 Integral sliding mode control (ISMC) design

The ISMC design to deal with any matched components can be achieved through the following two steps:

1- Design a sliding surface to satisfy a chosen linear system performance specification when the system is on the sliding surface.

2- Design an appropriate discontinuous control to maintain the chosen sliding motion.

The integral sliding switching surface is proposed as:

$$\sigma_i(x_i,t) = G_i[x_i(t) - x_i(t_o) - \int_{t_o}^{t} (A_i x_i(t) + B_i u_i^{LMI}(t))dt] \tag{3-5}$$

where G_i is an appropriate design matrix that must satisfy the condition that G_iB_i is invertible if the actuator has not failed completely. The integral term provides the freedom to add any linear controller that satisfies the prescribed time response performance specification. The structure of Eq.(3-5) implies that there is independent freedom to choose any control system design method of the linear component of the feedback. This is an important characteristic of the ISMC design problem.

The so-called *equivalent control* $u_{eqi}(t)$ can maintain the state motion on the sliding surface if the actuator fault is bounded by letting the time derivative of $\sigma_i(x_i,t)$ be identically zero, i.e. $\dot{\sigma}_i(x_i,t) = 0$ (Zinober, 1990,Utkin and Jingxin, 1996, Cao and Xu, 2001).

$$\dot{\sigma}_i(x_i,t) = G_i\dot{x}_i(t) - G_iA_ix_i(t) - G_iB_iu_i^{LMI}(t) = 0 \tag{3-6}$$

Substituting Eq. (3-4) into Eq. (3-6) yields:

$$G_i A_i x_i + G_i B_i u_i^{LMI} + G_i B_i u_i^{ISM} + G_i B_i B_i^{+} Z_i(t) + G_i \zeta_i(t) + G_i B_i Q_i(x_i, t) + G_i E_i d_i(t) + G_i B_i f_i(t) - G_i A_i x_i - G_i B_i u_i^{LMI} = 0 \quad (3\text{-}7)$$

This leads to the equivalent control signal if $f_i(t)$ is not failure ($K_i \neq 0$):

$$u_{eqi}(t) = -(G_i B_i)^{-1}[G_i B_i B_i^{+} Z_i(t) + G_i \zeta_i(t) + G_i B_i Q_i(x_i, t) + G_i E_i d_i(t) + GiBifit \quad (3\text{-}8)$$

Rearranging Eq. (3-8):

$$u_{eqi}(t) = -B_i^{+} Z_i(t) - (G_i B_i)^{-1} G_i \zeta_i(t) - Q_i(x_i, t) - (G_i B_i)^{-1} G_i E_i d_i(t) - f_i(t) \quad (3\text{-}9)$$

Substituting (3-9) into (3-4) gives the equivalent dynamic equation of the i^{th} subsystem in sliding mode as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + [I_i - B_i (G_i B_i)^{-1} G_i] \zeta_i(t) + [I_i - B_i (G_i B_i)^{-1} G_i] E_i d_i(t) \quad (3\text{-}10)$$

From Eq. (3-10) the unknown matched uncertainties $(G_i B_i)^{-1} G_i \zeta_i(t)$ and actuator faults $f_i(t)$ are completely nullified. However, the dynamics of the subsystem on the sliding surface still contain the unknown unmatched uncertainties $[I_i - B_i (G_i B_i)^{-1} G_i] \zeta_i(t)$ and disturbances $[I_i - B_i (G_i B_i)^{-1} G_i] E_i d_i(t)$.

The proposed discontinuous control takes the form:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i, t)}{\|\sigma_i(x_i, t)\|} \quad (3\text{-}11)$$

It is assumed that μ_i is a positive scalar function and a possible choice (according to the stability of the subsystem) is:

$$\mu_i > \|(G_i B_i)^{-1} G_i\| \beta_i(x_i, t) + \kappa_i \|x_i\| + \gamma_i \|(G_i B_i)^{-1} G_i E_i\| \, \|x_i\| + \eta_i \, \|x_i\| \quad (3\text{-}12)$$

To maintain the subsystem state on the sliding surface, let $\sigma_i(x_i, t) = 0$. Then consider the subsystem stability by choosing the following subsystem Lyapunov functions: $\sum_{i=1}^{N} V_i\,(\sigma_i(x_i, t)) = \sum_{i=1}^{N} \|\sigma_i(x_i, t)\| > 0$

For stability of each subsystem the time derivatives of each of the $V_i(\sigma_i(x_i,t))$ must be negative, i.e. $\dot{V}_i\big(\sigma_i(x_i,t)\big) < 0$. this can be verified as follows:

$$\dot{V}_i(\sigma_i(x_i,t)) = \frac{\sigma_i^T(x_i,t)\dot{\sigma}_i(x_i,t)}{\|\sigma_i(x_i,t)\|} \tag{3-13}$$

where:

$$\dot{\sigma}_i(x_i,t) = G_i B_i u_i^{ISM} + G_i Z_i(t) + G_i B_i Q_i(x_i,t) + G_i E_i d_i(t) + G_i B_i f_i(t) \tag{3-14}$$

Substituting the proposed discontinuous control as in Eq.(3-11) into Eq.(3-14) and substituting the result into Eq.(3-13) yields:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) = \sum_{i=1}^{N}[-G_iB_i\mu_i + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}\,G_iZ_i(t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}\,G_iB_iQ_i(x_i,t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}\,G_iE_id_i(t) + \frac{\sigma_i^T(x_i,t)}{\|\sigma_i(x_i,t)\|}\,G_iB_if_i(t)] \tag{3-15}$$

Re-arranging Eq.(3-15) as:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \leq \sum_{i=1}^{n}[-(G_iB)[\mu_i - (G_iB_i)^{-1}G_i\|Z_i\| - \|Q_i\| - (G_iB_i)^{-1}G_iE_i\|d_i\| - \|f_i\|] \tag{3-16}$$

According to Assumptions (4, 5, 6 and 7):

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \leq \sum_{i=1}^{N}[-(G_iB_i)\big[\mu_i - \|(G_iB_i)^{-1}G_i\|\beta_i(x_i,t) - \kappa_i\|x_i\| - \gamma_i\|(G_iB_i)^{-1}G_iE_i\|\|x_i\| - \eta_i\,\|x_i\|\big] \tag{3-17}$$

Furthermore according to the choice of in Eq. (3-12) that leads to $\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i,t)\right) \leq 0$. This implies that the choices of the μ_i according to the inequality Eq.(3-12) indeed guarantee the stability of the sliding surface by using the proposed discontinuous control.

The matrices G_i must be chosen in order to reduce the norm of $\Psi_i\zeta_i(t)$ and $\Psi_i E_i d_i(t)$, as well as reducing the amplification of Ψ_i to the unknown unmatched uncertainties and disturbances (Castaños,Xu and Fridman, 2006).

By using the classical projection theorem, there is a unique vector $m_o \in M$ such that $\|x - m_o\| \leq \|x - m\|$ for all $m \in M$ as shown in Figure 3-1. To minimize the vector m_o and $(x - m_o)$ they must be orthogonal to each other on M (Luen, 1997).

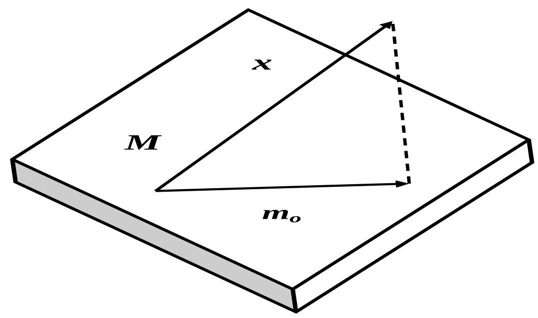

Figure 3-1: The projection theorem

In the case of unmatched uncertainties where $[\zeta_i - B_i(G_iB_i)^{-1}G_i\zeta_i] = \zeta_i - B_i\varphi_i$ and $\varphi_i = (G_iB_i)^{-1}G_i\zeta_i$. Then as in the classical projection theorem, a search for φ_i which would make $(\zeta_i - B_i\varphi_i)$ orthogonal to span (B_i) is as shown in Figure 3-2.

The sufficient condition is:

$B_i^T\zeta_i - B_i^T B_i\varphi_i = 0$.

This leads to $\varphi_i = (B_i^T B_i)^{-1}B_i^T\zeta_i = B_i^+\zeta_i$

Then $\varphi_i = (G_iB_i)^{-1}G_i\zeta_i = B_i^+\zeta_i$

From the last equation $(G_iB_i)^{-1}G_i = B_i^+$ where $B_i^+ = (B_i^T B_i)^{-1}B_i^T$

As a result the best chose of G_i is to take $G_i = B_i^T$

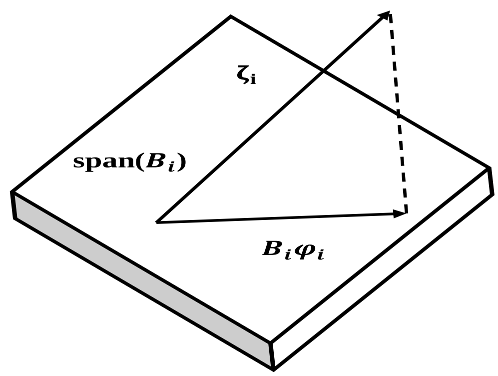

Figure 3-2: The projection theorem of choosing the matrix G_i

The matrices which minimize the norms Ψ_{i} are B_i^T or B_i^+ which are the same as:

$$\Psi_i = [I_i - B_i(B_i^T B_i)^{-1} B_i^T] = [I_i - B_i B_i^+] \tag{3-18}$$

In addition, the choice of G_i must not amplify the unknown unmatched uncertainties and disturbances where the following identity holds $\|\Psi_i\| = 1$ (Castaños,Xu and Fridman, 2006).

$$\begin{aligned}\|\Psi_i\|^2 = \Psi_i^T \Psi_i = [I_i - B_i B_i^+]^T [I_i - B_i B_i^+] = I_i - B_i^{T+} B_i^T - B_i B_i^+ + B_i^{T+} B_i^T B_i B_i^+ \\ = [I_i - B_i B_i^+]\end{aligned} \tag{3-19}$$

This leads to:

$$\Psi_i^T \Psi_i = \Psi_i \tag{3-20}$$

From (3-20) it can be seen that the matrices Ψ_i are symmetric which implies that all the eigenvalues of $\Psi_i \in \mathbb{R}^n$ are real. Suppose that λ_{ik} with $k = 1,2, .. n$ is an eigenvalue of Ψ_i and v_{ik} is the corresponding eigenvector (Castaños and Fridman, 2005), then:

$$\Psi_i v_{ik} = \lambda_{ik} v_{ik} \Longrightarrow v_{ik}^T \Psi_i^T \Psi_i v_{ik} = \lambda_{ik}^2 \|v_{ik}\|^2 \tag{3-21}$$

According to Eq. (3-20):

$$v_{ik}^T \Psi_i^T \Psi_i v_{ik} = v_{ik}^T \Psi_i v_{ik} = \lambda_{ik} \|v_{ik}\|^2 \tag{3-22}$$

That leads to:

$v_{ik}^T \Psi_i^T \Psi_i v_{ik} = v_{ik}^T \Psi_i v_{ik}$ then $\lambda_{ik}^2 \|v_{ik}\|^2 = \lambda_{ik} \|v_{ik}\|^2$

This means that $\lambda_{ik} = \lambda_{ik}^2$, the solutions of this equation are $\lambda_{ik1} = 0$ and $\lambda_{ik2} = 1$. The rank of $[I_i - B_i B_i^+] \neq 0$. Therefore, Ψ_i must have λ_{ik} different from zero and as a result the maximum eigenvalue =1, as a result $\|\Psi_i\| = 1$ this means that the unknown unmatched uncertainties and disturbances are not amplified.

By substituting the matrices B_i^+ instead of G_i in Eq. (3-10), then:

1- ***The terms:***

$$[I_i - B_i(G_i B_i)^{-1} G_i]\zeta_i(t) = [I_i - B_i(B_i^+ B_i)^{-1} B_i^+] B_i^\perp B_i^{\perp +} Z_i(t)$$

Where the $\zeta_i(t) = B_i^\perp B_i^{\perp +} Z_i(t)$ since the $B_i^T B_i^\perp = 0$, these terms can be re-written as:

$$[I_i - B_i(B_i^+ B_i)^{-1} B_i^+] B_i^\perp B_i^{\perp +} Z_i(t) = B_i^\perp B_i^{\perp +} Z_i(t) \tag{3-23}$$

2- ***The terms:***

$$[I_i - B_i(G_i B_i)^{-1} G_i] E_i d_i(t) = [I_i - B_i(B_i^+ B_i)^{-1} B_i^+] E_i d_i(t) = [I_i - B_i B_i^+] E_i d_i(t) \tag{3-24}$$

Substituting Eq. (3-23) and Eq. (3-24) into Eq. (3-10) yields:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + B_i^\perp B_i^{\perp +} Z_i(t) + [I_i - B_i B_i^+] E_i d_i(t) \tag{3-25}$$

The dynamics of the subsystems on the sliding surfaces can thus be described as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + T_i Z_i(t) + M_i d_i(t) \tag{3-26}$$

Where $T_i = B_i^\perp B_i^{\perp +}$and $M_i = [I_i - B_i B_i^+] E_i$

From Eq. (3-26) it can be observed that the unknown unmatched uncertainties and disturbances have not been completely eliminated. As a result another method must be found to enhance the properties of the control design to reduce of the influences of the unknown unmatched uncertainties and disturbances.

Note:

When applying the $u_i^{ISM}(t)$ to the subsystems, so-called chattering motion takes place, i.e. repeated discontinuous motion in a small vicinity of each sliding surface is present. The chattering motion can be reduced by the addition of a small constant $\mathfrak{z}_i > 0$ if all uncertainties disappear and the subsystem is stable ,where the control will be as (Changqing,Patton and Zong, 2010).

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$$

3.3.1.2 Continuous control design via LMI framework

The Linear matrix inequality (LMI) has been used for over 100 years to study and analyse systems and specifically the first use of LMI theory to study the stability of systems was introduced by Lyapunov. The first use of LMI theory in control engineering appeared in the Soviet Union in 1940 and was later used in convex optimization problems by Boyd (Stephen,Laurent,Eric and Venkataraman, 1994). (Šiljak and Stipanovic, 2001) first proposed the LMI framework to study the design of the control of large-scale systems.

The possible design of LMI methods can be classified as outlined in Section 3-1 by either:

1- Designing a controller for each subsystem individually, using LMI framework.

2- Designing a one shot controller for the overall system structure using LMI framework.

Both procedures use the same ISMC technique, and the difference between these two subsystem controller design strategies lies only in the approach taken. Some systems consist of several subsystems with each subsystem having its own characteristics with some interconnections, but some inter-connected systems are considered as large-scale systems. Consequently, the subsystem design approach depends on the subsystem requirements.

3.3.1.2.1 Continuous control design via LMI for each subsystem individually where none of the interconnections are known

The LMI framework is used as a tool to find the appropriate gains that can guarantee the stability and decrease the effect of any bounded unmatched compounds and disturbances. The object here is to design closed-loop gains based on LMI-based design formulation is through a one-step solution to the set of LMIs to satisfy performances design.

After designing the ISMC the closed-loop subsystem takes the form:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + \Gamma_i J_i(t) \qquad (3\text{-}27)$$

where $\Gamma_i = [T_i \quad M_i]$ and $J_i(t) = \begin{bmatrix} Z_i(t) \\ d_i(t) \end{bmatrix}$

Suppose $J_i(t)$ is an unknown input, but confirms the condition of quadratic inequality.

$$J_i^T(t) J_i(t) \leq \alpha_i^2 x_i^T(t) x_i(t) \tag{3-28}$$

where the $\alpha_i > 0$ are bounding constants. (Šiljak and Stipanovic, 2001).

To develop a robust control law, insert the feedback control into the formula:

$$u_i^{LMI}(t) = k_i x_i(t) \tag{3-29}$$

Where the k_i are the gains that stabilise the subsystem under a specific performance. The objective of the control design is to choose suitable values for the k_i to minimize the effect of the $J_i(t)$ on the system of Eq.(3-27). It is required to attenuate the $J_i(t)$ to the appropriate level ϵ_i. The Lyapunov function candidate, $V_i(x_i, t) = x_i^T(t) P_i x_i(t)$, can be used to check the stability of the closed-loop system, where the $P_i > 0$ are s.p.d. matrices.

The time derivatives of the $V_i(x_i, t)$ are given by:

$$\dot{V}_i(x_i, t) = \dot{x}_i^T(t) P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t) \tag{3-30}$$

Substituting Eq. (3-29) into Eq. (3-27) and then substituting the result into Eq.(3-30) yields:

$$\begin{aligned} \dot{V}_i(x_i, t) = {} & [A_i x_i(t) + B_i u_i^{LMI}(t) + \Gamma_i J_i(t)]^T P_i x_i(t) + x_i^T(t) P_i [A_i x_i(t) \\ & + B_i u_i^{LMI}(t) + \Gamma_i J_i(t)] \end{aligned} \tag{3-31}$$

Rearranging Eq. (3-29) gives:

$$\begin{aligned} \dot{V}_i(x_i, t) = {} & x_i^T(t) A_i^T P_i x_i(t) + x_i^T k_i^T B_i^T P_i x_i(t) + J_i^T(t) \Gamma_i^T P_i x_i(t) + x_i^T P_i A_i x_i(t) \\ & + x_i^T(t) P_i B_i k_i x_i(t) + x_i^T(t) P_i \Gamma_i J_i(t) \end{aligned} \tag{3-32}$$

The stability of the system Eq. (3-27) requires that $\dot{V}_i(x_i, t) < 0$ for all $x_i(t) \neq 0$.

The equation Eq. (3-31) could now be rewritten as:

$$\mathcal{Z}_i^T \mathcal{D}_i \mathcal{Z}_i < 0 \tag{3-33}$$

Where: $\mathcal{Z}_i = \begin{bmatrix} x_i(t) \\ J_i(t) \end{bmatrix}$ and $\mathcal{D}_i = \begin{bmatrix} A_i^T P_i + P_i A_i + k_i^T B_i^T P_i + P_i B_i k_i & P_i \Gamma_i \\ \Gamma_i^T P_i & 0 \end{bmatrix}$

In order to check the stability of the condition, matrices $\mathcal{D}_i$ must be negative-definite, i.e. $\mathcal{D}_i < 0$.

The equation Eq. (3-28) could be rewritten as:

$$\mathcal{Z}_i^T \mathcal{O}_i \mathcal{Z}_i \leq 0 \tag{3-34}$$

Where: $\mathcal{Z}_i = \begin{bmatrix} x_i(t) \\ J_i(t) \end{bmatrix}$ and $\mathcal{O}_i = \begin{bmatrix} -\alpha_i^2 I_i & 0 \\ 0 & I_i \end{bmatrix}$

Eqs. (3-33)&(3-34) can be combined into one single equation by using the S-procedure (Šiljak and Stipanovic, 2001). If $\mathcal{D}_i$ and $\mathcal{O}_i$ are symmetric matrices then $\mathcal{Z}_\mathrm{i}^\mathrm{T} \mathcal{D}_\mathrm{i} \mathcal{Z}_\mathrm{i} < 0$ and $\mathcal{Z}_\mathrm{i}^\mathrm{T} \mathcal{O}_\mathrm{i} \mathcal{Z}_\mathrm{i} \leq 0$, where there is a number $\tau_i > 0$ that satisfies the relation $\mathcal{D}_\mathrm{i} - \tau_\mathrm{i} \mathcal{O}_\mathrm{i} < 0$. Therefore the combination of the two equations is:

$$\mathcal{D}_i - \tau_i \mathcal{O}_i = \begin{bmatrix} A_i^T P_i + P_i A_i + k_i^T B_i^T P_i + P_i B_i k_i & P_i \Gamma_i \\ \Gamma_i^T P_i & 0 \end{bmatrix} - \tau_i \begin{bmatrix} -\alpha_i^2 I_i & 0 \\ 0 & I_i \end{bmatrix} < 0 \tag{3-35}$$

Put $\mathcal{Y}_i = \frac{P_i}{\tau_i}$ in Eq. (3-35) this yields:

$$\begin{bmatrix} A_i^T \mathcal{Y}_i + \mathcal{Y}_i A_i + k_i^T B_i^T \mathcal{Y}_i + \mathcal{Y}_i B_i k_i + \alpha_i^2 I_i & \mathcal{Y}_i \Gamma_i \\ \Gamma_i^T \mathcal{Y}_i & -I_i \end{bmatrix} < 0 \tag{3-36}$$

Eq. (3-36) cannot be solved by LMI because of the bilinear terms $\mathcal{Y}_i B_i k_i$. To overcome this limitation the non-convex problem must be converted into a convex problem. To achieve this, both sides of Eq.(3-36) must be multiplied by the matrices $\begin{bmatrix} \mathcal{Y}_i^{-1} & 0 \\ 0 & I_i \end{bmatrix}$ and $\mathcal{P}_i = \mathcal{Y}_i^{-1}$.

$$\begin{bmatrix} \mathcal{P}_i A_i^T + A_i \mathcal{P}_i + \mathcal{P}_i k_i^T B_i^T + B_i k_i \mathcal{P}_i + \alpha_i^2 \mathcal{P}_i \mathcal{P}_i & \Gamma_i \\ \Gamma_i^T & -I_i \end{bmatrix} < 0 \tag{3-37}$$

By putting $N_i = k_i \mathcal{P}_i$, $\epsilon_i = \frac{1}{\alpha_i^2}$ and by further using the Schur complement the inequality (3-37) and can be re-formulated as:

$$\begin{bmatrix} \mathcal{P}_i A_i^T + A_i \mathcal{P}_i + N_i^T B_i^T + B_i N_i + \Upsilon_i^T \Upsilon_i & \Gamma_i & \mathcal{P}_i \\ \Gamma_i^T & -I_i & 0 \\ \mathcal{P}_i & 0 & -\epsilon_i I_i \end{bmatrix} < 0 \tag{3-38}$$

where Υ_i is a tuning matrix that can be used to obtain specific subsystem responses. The de-centralized control gain matrix k_i is obtained by $k_i = N_i \mathcal{P}_i^{-1}$

Algorithm 3-1:

1- Calculate $\sigma_i(x_i, t)$ from the Eq.(3-5)

2- Get the discontinuous from $u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\|+\mathfrak{z}_i}$

3- Minimizing ϵ_i subject to $\mathcal{P}_\mathrm{i} > 0$ and the Eq.(3-38), by solving the LMI this leads to the calculation of the controller gain from $k_i = N_i \mathcal{P}_i^{-1}$

For real system implementation it is also essential to ensure that the numerical conditioning of $k_i = N_i \mathcal{P}_i^{-1}$must satisfy a maximum condition number, giving an acceptable Euclidean norm $\|K\|_2$ of the gain. If this norm is too large, the conditioning can be improved by adding extra inequality constraints (for $\mathcal{P}_\mathrm{i}$ and N_i) to the LMI of Eq.(3-38) In fact the Euclidean norms of both $\mathcal{P}_i^{-1}$ and N_i must be jointly minimized via additional LMI constraints. This can be done as follows (Zecevic and Šiljak, 2005) as:

The first condition is $\|N_i\|^2 < k_{Ni} I$, where k_{Ni} is a scalar variable, and by using Schur complement the appropriate LMI condition becomes:

$$\begin{bmatrix} -k_{Ni} I_i & N_i^T \\ N_i & -I_i \end{bmatrix} < 0 \qquad (3\text{-}39)$$

The algorithm that now seeks to compute the gain $k_i = N_i \mathcal{P}_i^{-1}$ satisfying these conditioning constraints is stated (Šiljak and Stipanovic, 2001) where $\mathcal{P}_i > k_{Pi} I_i$ and by using Schur complement the appropriate LMI condition becomes:

$$\begin{bmatrix} \mathcal{P}_i & I_i \\ I_i & k_{Pi} I_i \end{bmatrix} > 0 \qquad (3\text{-}40)$$

where the k_{Pi} are scalar variables.

Algorithm 3-2:

1- Calculate $\sigma_i(x_i, t)$ from the Eq.(3-5)

2- Get the discontinuous from $u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\|+\mathfrak{z}_i}$

3- Minimize $(\epsilon_i + k_{Ni} + k_{Pi})$ subject to $\mathcal{P}_i > 0$, the Eqs. (3-38) , (3-39) & (3-40)

4- The controller gain can be calculated from $k_i = N_i \, \mathcal{P}_i^{-1}$

The principle of this approach when it is applied to inter-connected systems is shown in Figure 3-3.

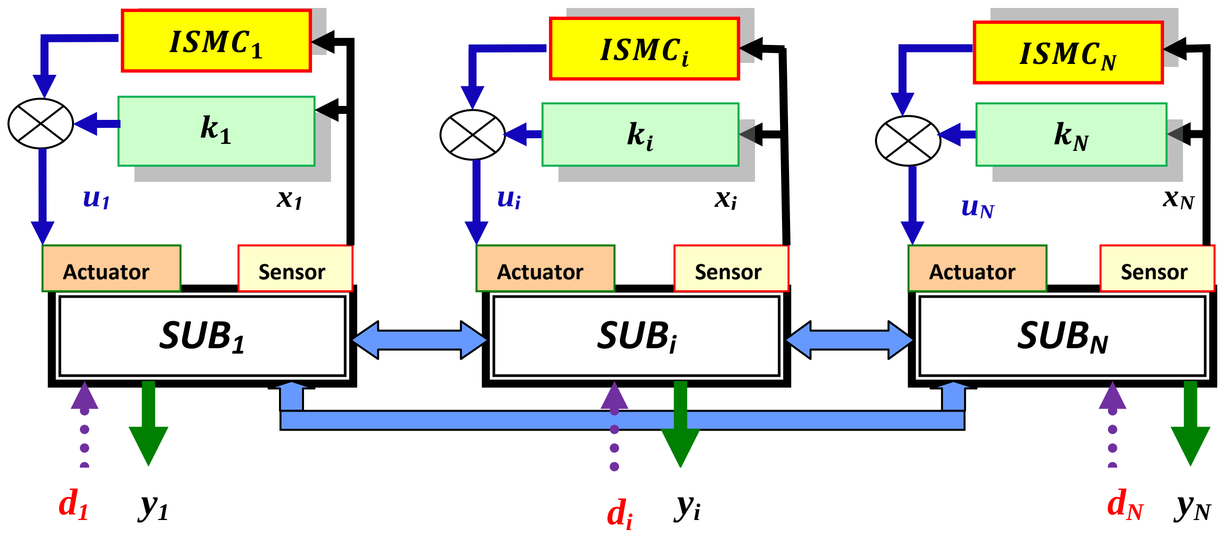

Figure 3-3: Control of inter-connected systems via LMI + ISMC subsystem by subsystem

3.3.1.2.2 Continuous control design via LMI for all subsystems (one shot)

The objective of this procedure is to design a de-centralized control that robustly regulates the state of the overall system without any information exchange between the controllers. On other hand each de-centralized control uses only available local information.

After the ISMC is designed to the i^{th} subsystem, the dynamic i^{th} subsystem is described by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + \Gamma_i J_i(t) \tag{3-41}$$

where $\Gamma_i = [T_i \quad M_i]$ and $J_i(t) = \begin{bmatrix} Z_i(t) \\ d_i(t) \end{bmatrix}$

Suppose $J_i(t)$ is an unknown input but conforms to the quadratic inequality condition.

$$J_i{}^T(t) J_i(t) \le \alpha_i^2 x_i{}^T(t) x_i(t) \tag{3-42}$$

Where the $\alpha_i > 0$ are scalar parameters. Then the overall (one shot) system can be written as:

$$\dot{X}(t) = A_d X(t) + B_d U(t) + \Gamma_d J(t) \tag{3-43}$$

where $(t) = [x_1, x_2, \dots \dots, x_n]$, $U(t) = [u_1^{LMI}, u_2^{LMI}, \dots \dots, u_n^{LMI}]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $\Gamma_d = diag(\Gamma_i)$ and $J(t) = [J_1, J_2, \dots \dots, J_n]$, where *diag* is a block diagonal matrix.

To develop a robust control law, let the feedback have the following form:

$$U(t) = KX(t) \tag{3-44}$$

where K is the controller gain. The purpose of the choice of the value of K is to minimize the effect of $J(t)$ on the one shot system.

The unknown input disturbance $J(t)$ satisfies the condition of the quadratic inequality:

$$J^T(t)J(t) \leq \alpha^2 X^T(t)X(t) \tag{3-45}$$

where $\alpha > 0$ is a scalar parameter.

To check the stability of this one shot closed-loop system choose a candidate Lyapunov function candidate $V(X,t) = X^T(t)PX(t)$, where $P = diag(P_i)$ and $P_i > 0$ are s.p.d. matrices. The choice of control method depends on the designer and can be either de-centralized control by choosing P as a diagonal matrix or by de-centralized overlapping control by choosing P as a non-diagonal matrix. The stability of the one shot system can be checked as described in Section 3.3.1.2.1. This leads to an inequality which can be written as:

After using the S-procedure (Šiljak and Stipanovic, 2001), combining Eqs (3-43) & (3-45), and following and application of the Schur complement then the inequality is:

$$\begin{bmatrix} \mathcal{P}A_d^T + A_d\mathcal{P} + N^T B_d^T + B_d N + \Upsilon^T \Upsilon & \Gamma_d & \mathcal{P} \\ \Gamma_d^T & -I & 0 \\ \mathcal{P} & 0 & -\epsilon I \end{bmatrix} < 0 \tag{3-46}$$

Where Υ is a *tuning matrix*, $N = K\mathcal{P}$ and $\epsilon = \frac{1}{\alpha^2}$

Algorithm 3-3:

1- Calculate $\sigma_i(x_i, t)$ from the Eq.(3-5)
2- Get the discontinuous from $u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$
3- Calculate the aggregate system from Eq.(3-43)
4- Minimize ϵ subject to $\mathcal{P} > 0$ and the Eq.(3-46)
5- Get the controller gain $K = N\mathcal{P}^{-1}$

As in *Algorithm 3-1*, limit the gain so that it is not too high by adding other conditions to the LMI algorithm. In addition, a condition can be added to the matrix N as:

$$\begin{bmatrix} -k_N I & N^T \\ N & -I \end{bmatrix} < 0 \tag{3-47}$$

As well as adding another condition to the matrix .

$$\begin{bmatrix} \mathcal{P} & I \\ I & k_P \mathrm{I} \end{bmatrix} > 0 \tag{3-48}$$

Where k_N and k_P are scalar variables.

Algorithm 3-4:

The same procedure as in *Algorithm 3-3* is used by replacing step 3 by:

Minimize $(\epsilon + k_N + k_P)$ subject to $\mathcal{P} > 0$, the Eqs. (3-46) , (3-47) & (3-48)

The control of the overall or one shot system is achieved by using the ISMC and LMI gains. According to the choice of form of the matrix P there are two design possibilities: (1) P is a diagonal matrix then the control of the one shot system will be as shown in Figure 3-3. But if P is a non-diagonal matrix the control of the one shot system is attained by using the combined ISMC with LMI minimization of the unmatched uncertainty, as shown in Figure 3-4.

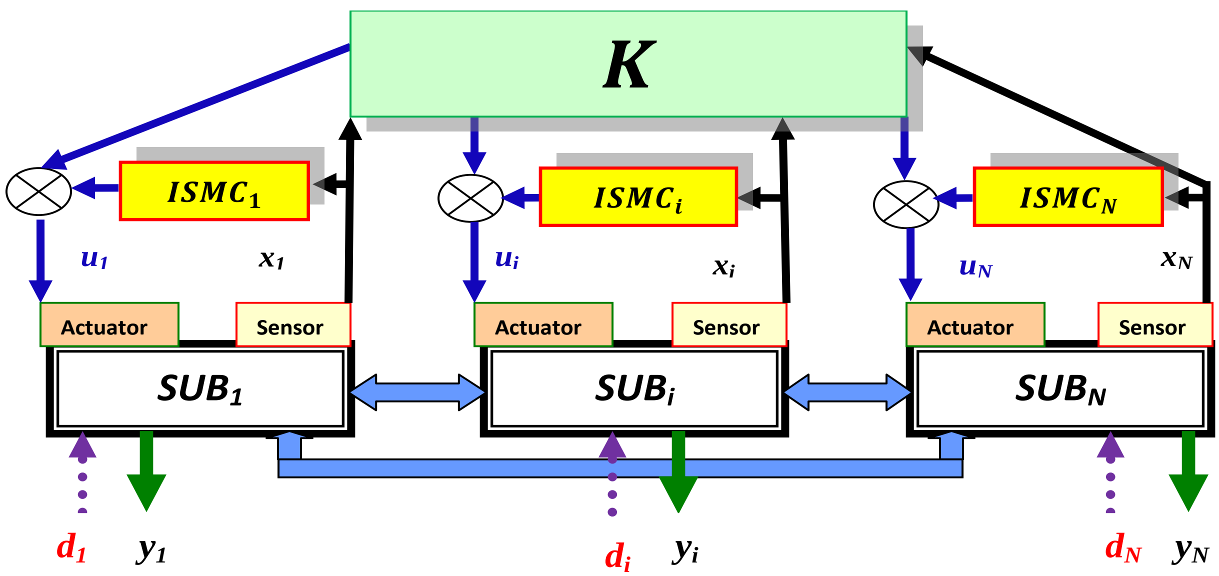

Figure 3-4: Control of inter-connected systems via LMI + ISMC (One shot)

3.3.2 All the interconnections are known

It can now be assumed that all the subsystems are connected to each other and that all the interconnections between them are known. For this scenario the dynamics of each subsystem can be described by:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + B_i u_i^{ISM}(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t) \tag{3-49}$$

$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N$$

Assume further that the known interconnections can be described as:

$$Z_i(t) = \sum_{j=1}^{N} (L_{ij} x_j(t) + V_{ij} u_j(t)) \tag{3-50}$$

where $x_j(t)$ and $u_j(t)$ are the state and control of other inter-connected systems, and L_{ij} and V_{ij} are the appropriate interconnection matrices between the i^{th} and j^{th} subsystems.

3.3.2.1 Integral sliding mode control (ISMC) design

Based on the inter-connected system of Eqs.(3-49) & (3-50) with known interactions, an integral sliding surface can be developed that is different from the one outlined in Section 3.3.1.1. The new sliding surface formulation includes the effects of the interaction terms in an ISMC design as follows:

$$\sigma_i(x_i, t) = G_i[x_i(t) - x_i(t_o) - \int_{t_o}^{t} \left(A_i x_i(t) + B_i u_i^{LMI}(t) + Z_i(t)\right) dt] \tag{3-51}$$

As stated in Section 3.3.1.1 although G_i are design matrices they must satisfy the condition that the $G_i B_i$ are invertible.

The so-called *equivalent control* $u_{eqi}(t)$ can maintain the sliding surface by forcing the time derivative of $\sigma_i(x_i, t)$ to be identically zero, i.e. $\dot{\sigma}_i(x_i, t) = 0$ (Cao and Xu, 2001).

$$\dot{\sigma}_i(x_i, t) = G_i \dot{x}_i(t) - G_i A_i x_i(t) - G_i B_i u_i^{LMI}(t) - G_i Z_i(t) = 0 \tag{3-52}$$

Substituting Eq. (3-49) into Eq. (3-52) yields:

$$G_iA_ix_i + G_iB_iu_i^{LMI} + G_iB_iu_i^{ISM} + G_iZ_i(t) + G_iB_iQ_i(x_i,t) + G_iE_id_i(t) + G_iB_if_i(t) - G_iA_ix_i - G_iB_iu_i^{LMI} - G_iZ_i(x_i,t) = 0 \quad (3\text{-}53)$$

Then the equivalent control is:

$$u_{eqi}(t) = u_i^{ISM} = -(G_iB_i)^{-1}[G_iB_iQ_i(x_i,t) + G_iE_id_i(t) + G_iB_if_i(t)] \quad (3\text{-}54)$$

Eq. (3-54) can be re-written as:

$$u_{eqi}(t) = u_i^{ISM} = -Q_i(x_i,t) - (G_iB_i)^{-1}G_iE_id_i(t) - f_i(t) \quad (3\text{-}55)$$

Substituting Eq. (3-53) into Eq. (3-49) gives the i^{th} subsystem state equation after applying ISMC as follows:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{LMI}(t) + Z_i(t) + [I_i - B_i(G_iB_i)^{-1}G_i]E_id_i(t) \quad (3\text{-}56)$$

From Eq. (3-56) the unknown matched uncertainties and actuator faults are completely deleted by the sliding action, but during sliding the sliding surfaces still contain the unknown disturbance components that are multiplied by $\Psi_i = [I_i - B_i(G_iB_i)^{-1}G_i]$.

The proposed discontinuous sliding control is:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\|} \quad (3\text{-}57)$$

where the μ_i are positive scalar functions with suitable choice:

$$\mu_i > (G_iB_i)^{-1}G_i\beta_i(x_i,t) + \kappa_i\|x_i\| + (G_iB_i)^{-1}G_iE_i\gamma_i\|x_i\| + \eta_i\|x_i\| \quad (3\text{-}58)$$

The proof of how to obtain suitable functions μ_i to ensure the stability of each subsystem with $u_i^{ISM}(t)$ is the same as described in Section 3.3.1.1.

The same procedure is applied when choosing the matrix G_i to reduce the norm of $\Psi_iE_id_i(t)$ or reducing the amplification of Ψ_i to the unknown disturbance (Castaños,Xu and Fridman, 2006).

A suitable choice of the G_i to guarantee that the G_iB_i are full rank is $G_i = B_i^+$ and this can be used to compensate in Eq. (3-56) where the term:

$$[I_i - B_i(G_iB_i)^{-1}G_i]E_id_i(t) = [I_i - B_i(B_i^+B_i)^{-1}B_i^+]E_id_i(t) = [I_i - B_iB_i^+]E_id_i(t)$$

After applying ISMC and compensating $G_i = B_i^+$, the subsystem dynamics can then be expressed as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + Z_i(t) + [I_i - B_i B_i^{+}]E_i d_i(t) \quad (3\text{-}59)$$

The dynamics of the subsystem on the sliding surface:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + Z_i(t) + M_i d_i(t) \quad (3\text{-}60)$$

where $M_i = [I_i - B_i B_i^{+}]E_i$

From Eq. (3-60) it can be observed that the unknown and unmatched disturbances have not been completely eliminated. Hence, another method must be used to remove their influences. The final form of discontinuous control after adding a small constant $\mathfrak{z}_i > 0$ is:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i} \quad (3\text{-}61)$$

Where the constants $\mathfrak{z}_i$ are selected, as described in Section 3.3.1.1 to keep the subsystem state motion close to the sliding boundary and thereby avoid excessive chattering motion.

3.3.2.2 Continuous control design via LMI for overall systems (one shot)

After designing the ISMC, each subsystem still contains interactions $Z_i(t)$ and disturbances. The equation of the subsystem is:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{LMI}(t) + Z_i(t) + M_i d_i(t) \quad (3\text{-}62)$$

Where $Z_i(t) = \sum_{j=1}^{N}(L_{ij}x_j(t) + V_{ij}u_j(t))$ and $M_i = [I_i - B_i B_i^{+}]E_i$

The overall or one shot system form of Eq. (3-62) can be written as:

$$\dot{X}(t) = A_d X(t) + B_d U(t) + LX(t) + VU(t) + M_d d_d(t) \quad (3\text{-}63)$$

where $(t) = [x_1, x_2, \dots\dots., x_n]$, $U(t) = [u_1^{LMI}, u_2^{LMI}, \dots\dots., u_n^{LMI}]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $M_d = diag(M_i)$, $L = nondiag(L_{ij})$, $V = nondiag(V_{ij})$ and $d_d(t) = [d_1, d_2, \dots\dots., d_n]$, where *diag* denotes a block diagonal matrix and *nondiag* denotes a block non-diagonal matrix, therefore Eq. (3-63) could be rewritten as :

$$\dot{X}(t) = \mathcal{A}X(t) + \mathfrak{B}U(t) + M_d d_d(t) \quad (3\text{-}64)$$

where $\mathcal{A} = A_d + L$ and $\mathfrak{B} = B_d + V$

To develop a robust state feedback control, let the control signal $U(t)$ be given by:

$$U(t) = KX(t) \tag{3-65}$$

The objective of K is to minimize the influence of the unknown input disturbances $d_d(t)$ on the one shot system. Suppose that $d_d(t)$ is an unknown input disturbance and satisfies the quadratic inequality condition:

$$d_d{}^T(t)d_d(t) \leq \alpha^2 X^T(t)X(t) \tag{3-66}$$

where $\alpha > 0$ is a scalar parameter. The Lyapunov candidate function $V(X,t) = X^T(t)PX(t)$ can be used to check the stability of the closed-loop system. Where $P = diag(P_i)$ and $P_i > 0$ is a s.p.d. matrix. The reason for choosing P as a diagonal matrix is to obtain the de-centralized control. However, if P is not a diagonal matrix, the de-centralized overlapping approach can be obtained, as stated in Section 3.3.1.2.2.

The stability of the one shot system is derived according the procedure outlined in Section 3.3.1.2.1. This then leads to the following inequality:

$$\begin{bmatrix} \mathcal{P}\mathcal{A}^T + \mathcal{A}\mathcal{P} + N^T\mathfrak{B}^T + \mathfrak{B}N + \Upsilon^T\,\Upsilon & M_d & \mathcal{P} \\ M_d^T & -I & 0 \\ \mathcal{P} & 0 & -\epsilon I \end{bmatrix} < 0 \tag{3-67}$$

where Υ is a tuning matrix. Feasible solutions for $\mathcal{P}$ and N in the linear matrix inequality (3-67) can be found using interior point methods provided by the Matlab LMI toolbox, according to the following *Algorithm*:

Algorithm 3-5:

1- Calculate $\sigma_i(x_i, t)$ from the Eq.(3-51)
2- Get the discontinuous control signal from the Eq.(3-61)
3- Calculate the overall system from the Eq.(3-64)
4- Minimize the ϵ subject to $\mathcal{P} > 0$ and the Eq. (3-67). After solving this LMI problem, the gains can be calculated from $K = N\mathcal{P}^{-1}$

It is acceptable practice to limit the magnitudes of the gain elements, by adding further inequalities to the LMI algorithm.

The condition $\|N\|^2 < k_N I$ can be added to the matrix N where the Euclidean norms of the N is bounded as in(Zecevic and Šiljak, 2005).

where k_N is a scalar variable, and by using Schur complement the appropriate LMI condition becomes:

$$\begin{bmatrix} -k_N I & N^T \\ N & -I \end{bmatrix} < 0 \tag{3-68}$$

Another condition ($\mathcal{P} > k_P \mathrm{I}$) can be added to the matrices $\mathcal{P}$. Once again invoking the Schur complement this maximization is achieved via the following LMI:

$$\begin{bmatrix} \mathcal{P} & I \\ I & k_P \mathrm{I} \end{bmatrix} > 0 \tag{3-69}$$

where k_P is a scalar variable.

The algorithm that now seeks to compute the gain $K = N\mathcal{P}^{-1}$ satisfying these conditioning constraints is stated as:

Algorithm 3-6:

The procedure is as given under *Algorithm 3-5* but step 4 must be changed to:

Minimize $(\epsilon + k_N + k_P)$ subject to $\mathcal{P} > 0$, the Eqs. (3-67) , (3-68) & (3-69).

Following this the scheme for the control of the one shot system with known interconnections is illustrated in Figure 3-5.

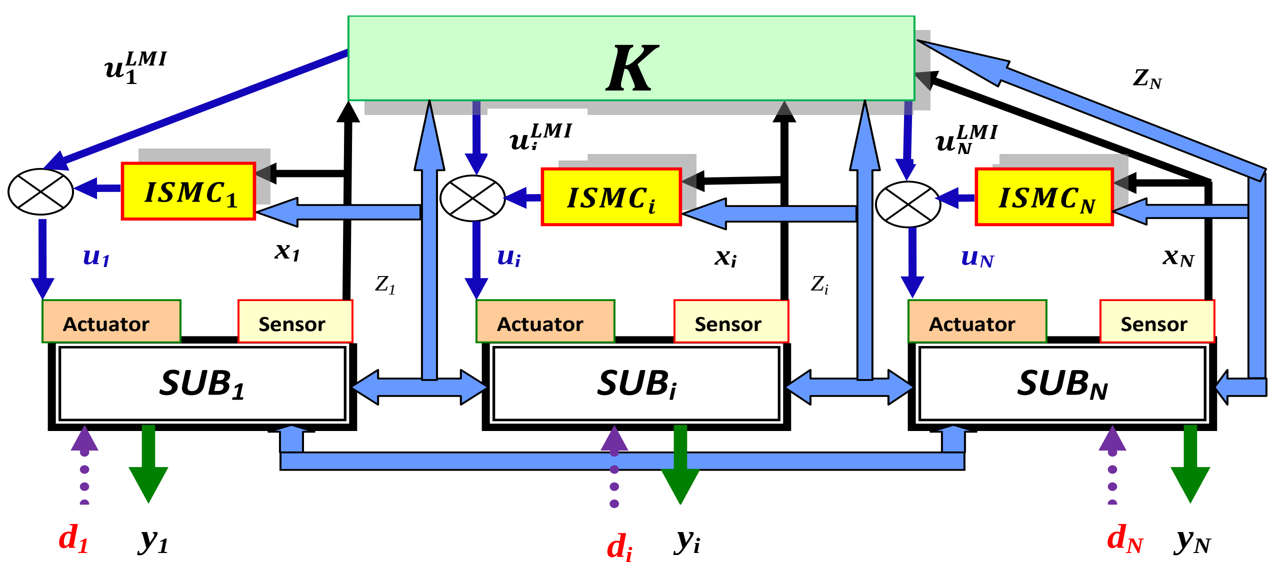

Figure 3-5: Control of inter-connected systems (known interactions) via LMI + ISMC one shot system

3.4 Control of dynamic non-linear model of a multi-floor building structure

It is appropriate here to provide an example of a Lipschitz non-linear system which has practical application value whilst at the same time enables all the concepts of Sections 3.1, 3.2 and 3.3 to be applied. For this purpose the linear example of a multi-floor building control problem described in Section 2.7.3 can be extended to a non-linear representation incorporating non-linear damping elements.

Recall from Section 2.7.3 that the equations that describe the movement of the building with respect to a datum line are obtained by using Newton's second law according to the following assumptions:

1- The mass of each floor acts as a point mass acting on the central datum line, neglecting the effect of the walls and other structures within the floor.
2- All the floors are affected by the movement of a horizontal seismic force due to an earthquake.

The equations of motion that describe the structure are given by:

1- First-floor equation*:*

$$m_1\ddot{x}_1(t) + c_1\dot{x}_1(t) + k_1x_1(t) - c_2\big(\dot{x}_2(t) - \dot{x}_1(t)\big) - k_2\big(x_2(t) - x_1(t)\big) = -m_1\ddot{x}_g(t) - f_1(t) + f_2(t) \quad (3\text{-}70)$$

2- Second-floor equation:

$$m_2\ddot{x}_2(t) + c_2(\dot{x}_2(t) - \dot{x}_1(t)) + k_2\big(x_2(t) - x_1(t)\big) - c_3\big(\dot{x}_3(t) - \dot{x}_2(t)\big) - k_3\big(x_3(t) - x_2(t)\big) = -m_2\ddot{x}_g(t) - f_2(t) + f_1(t) \quad (3\text{-}71)$$

3- The $(n-1)$-Floor equation:

$$m_{n-1}\ddot{x}_{n-1}(t) + c_{n-1}(\dot{x}_{n-1}(t) - \dot{x}_{n-2}(t)) + k_{n-1}\big(x_{n-1}(t) - x_{n-2}(t)\big) - c_n\big(\dot{x}_n(t) - \dot{x}_{n-1}(t)\big) - k_n\big(x_n(t) - x_{n-1}(t)\big) = -m_{n-1}\ddot{x}_g(t) - f_{n-1}(t) + f_{n-2}(t) \quad (3\text{-}72)$$

4- The (n)-Floor equation:

$$m_n\ddot{x}_n(t) + c_n(\dot{x}_n(t) - \dot{x}_{n-1}(t)) + k_n\big(x_n(t) - x_{n-1}(t)\big) = -m_n\ddot{x}_g(t) - f_n(t) \qquad (3\text{-}73)$$

where $x_i(t)$ is the displacement away from the vertical datum line of the i^{th} floor and m_i, c_i and k_i are mass, damping and stiffness of the floor i^{th} respectively, f_i and $\ddot{x}_g(t)$ are the control force and one dimensional horizontal ground acceleration, respectively. The response of a *n*-floor building to an earthquake is illustrated in Figure 3-6.

Consider an example of a three-storey building and rewrite the equations by putting $\dot{x}_i(t) = v_i(t)$ and $\dot{v}_i(t) = \ddot{x}_i(t)$, where $v_i(t)$ is the horizontal velocity of the i^{th} floor.

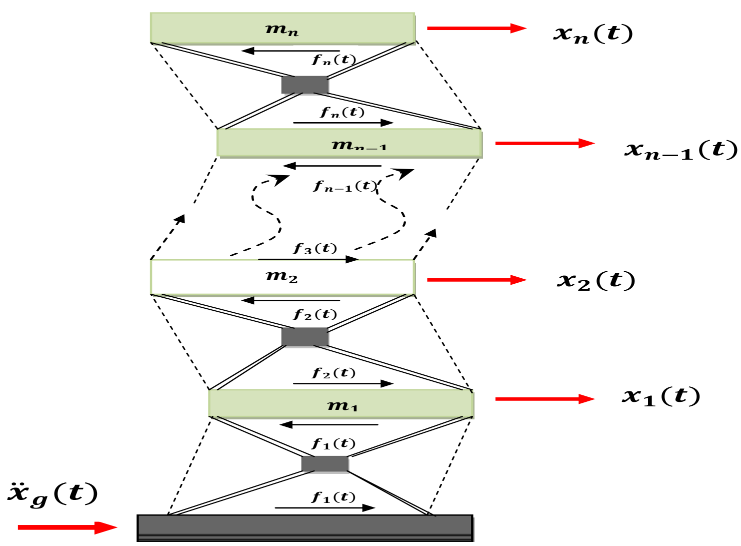

Figure 3-6: The effects of one dimensional earthquake on building of *n*-floor

The Newtonian equations representing the motion of each floor of the building can now be expressed as:

1- First-floor equation:

$$\begin{aligned}\dot{v}_1(t) = &-m_1^{-1}(c_1 + c_2)v_1(t) - m_1^{-1}(k_1 + k_2)x_1(t) \\ &+ m_1^{-1}c_2v_2(t) + m_1^{-1}k_2x_2(t) - \ddot{x}_g(t) - m_1^{-1}f_1(t) + m_1^{-1}f_2(t)\end{aligned} \qquad (3\text{-}74)$$

2- Second-floor equation:

$$\dot{v}_2(t) = -m_2^{-1}(c_2 + c_3)v_2(t) - m_2^{-1}(k_2 + k_3)x_2(t) + m_2^{-1}c_2v_1(t) + m_2^{-1}k_2x_1(t) + m_2^{-1}c_3v_3(t) + m_2^{-1}k_3x_3(t) - \ddot{x}_g(t) - m_2^{-1}f_2(t) + m_2^{-1}f_3(t) \tag{3-75}$$

3- Third-floor equation:

$$\dot{v}_3(t) = -m_3^{-1}c_3v_3(t) - m_3^{-1}k_3x_3(t) + m_3^{-1}c_3v_2(t) + m_3^{-1}k_3x_2(t) - \ddot{x}_g(t) - m_3^{-1}f_3(t) \tag{3-76}$$

Putting all these equations into state space form it follows that:

$$\begin{bmatrix} \dot{x}_1(t) \\ v_1(t) \\ \dot{x}_2(t) \\ v_2(t) \\ \dot{x}_3(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -m_1^{-1}(k_1+k_2) & -m_1^{-1}(c_1+c_2) & m_1^{-1}k_2 & m_1^{-1}c_2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ m_2^{-1}k_2 & m_2^{-1}c_2 & -m_2^{-1}(k_2+k_3) & -m_2^{-1}(c_2+c_3) & m_2^{-1}c_3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & m_3^{-1}k_3 & m_3^{-1}c_3 & -m_3^{-1}c_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ v_1(t) \\ x_2(t) \\ v_2(t) \\ x_3(t) \\ v_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -m_1^{-1} & m_1^{-1} & 0 \\ 0 & 0 & 0 \\ 0 & -m_2^{-1} & m_2^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & -m_3^{-1} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \ddot{x}_g(t) \tag{3-77}$$

A horizontal force is applied at each floor using a magneto-rheological (*MR damper*) to provide suitable damping of the floor structure to the seismic excitation, i.e to invoke a form of semi-active damping by varying the damping force. The semi-active MR damper is a control device that can adjust the structural damping to reduce the vibrations caused by the seismic excitation (Dyke and Spencer, 1997).

The non-linear equations that describe the MR damper force are adopted from (Kwok *et al.*, 2006) as follows:

$$f_i(t) = c_{0i}v_i(t) + k_{0i}x_i(t) + \propto_{0i} \mathcal{S}_i(t) + h_{0i}u_i(t) + c_{1i}u_i(t)v_i(t) + k_{1i}u_i(t)x_i(t) \tag{3-78}$$

where $u_i(t) = G_i x_i(t)$ is a control signal and G_i is a gain. The k_{0i} , c_{0i} are coefficients of the MR damper and $\mathcal{S}_i(t)$ is an evolutionary variable that describes the hysteresis according to:

$$\mathcal{S}_i(t) = \tanh(\beta_i v_i(t) + \varrho_i sign(x_i(t))) \tag{3-79}$$

where $\varrho_i = \varrho_{0i} + \varrho_{1i}u_i(t)$, β_i is a constant against the control signal and the parameters $\propto_{0i}$, h_{0i} , c_{1i} , k_{1i} , ϱ_{0i} and ϱ_{1i} are constants .

Rewriting the Eq. (3-77) by adding Eq. (3-78) and using three MR dampers with the same specifications and $Q_i(t) = c_{1i}u_i(t)v_i(t) + k_{1i}u_i(t)x_i(t)$, this yields the state-space model of the building as:

$$\begin{bmatrix} \dot{x}_1(t) \\ v_1(t) \\ \dot{x}_2(t) \\ v_2(t) \\ \dot{x}_3(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -m_1^{-1}(k_1+k_2+k_{01}) & -m_1^{-1}(c_1+c_2+c_{01}) & m_1^{-1}(k_2+k_{01}) & m_1^{-1}(c_2+c_{01}) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ m_2^{-1}k_2 & m_2^{-1}c_2 & -m_2^{-1}(k_2+k_3+k_{01}) & -m_2^{-1}(c_2+c_3+c_{01}) & m_2^{-1}(k_3+k_{01}) & m_2^{-1}(c_3+c_{01}) \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & m_3^{-1}k_3 & m_3^{-1}c_3 & -m_3^{-1}(k_3+k_{01}) & -m_3^{-1}(c_3+c_{01}) \end{bmatrix} \begin{bmatrix} x_1(t) \\ v_1(t) \\ x_2(t) \\ v_2(t) \\ x_3(t) \\ v_3(t) \end{bmatrix} \tag{3-80}$$

$$
+\begin{bmatrix} 0 & 0 & 0 \\ -m_1^{-1}h_{01} & m_1^{-1}h_{01} & 0 \\ 0 & 0 & 0 \\ 0 & -m_2^{-1}h_{01} & m_2^{-1}h_{01} \\ 0 & 0 & 0 \\ 0 & 0 & -m_3^{-1}h_{01} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}
$$

$$
+\begin{bmatrix} 0 & 0 & 0 \\ -m_1^{-1} \propto_{01} & m_1^{-1} \propto_{01} & 0 \\ 0 & 0 & 0 \\ 0 & -m_2^{-1} \propto_{01} & m_2^{-1} \propto_{01} \\ 0 & 0 & 0 \\ 0 & 0 & -m_3^{-1} \propto_{01} \end{bmatrix} \begin{bmatrix} \mathcal{S}_1(t) \\ \mathcal{S}_2(t) \\ \mathcal{S}_3(t) \end{bmatrix}
$$

$$
+\begin{bmatrix} 0 & 0 & 0 \\ -m_1^{-1} & m_1^{-1} & 0 \\ 0 & 0 & 0 \\ 0 & -m_2^{-1} & m_2^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & -m_3^{-1} \end{bmatrix} \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \ddot{x}_g(t)
$$

Consider an example of a building structure with three-storey building model where one installed semi-active actuator MR is added at each floor.

The mass of each floor $m_1 = m_2 = m_3 = 6 * 10^3\ kg$, the stiffness coefficients of each floor are $k_1 = 1.6 * 10^6\ N/m$, $k_2 = 1.8 * 10^6\ N/m$ and $k_3 = 1.6 * 10^6\ N/m$.

The damping coefficients of each floor are:

$c_1 = 7.2 * 10^3\ N/(m/s)$, $c_2 = 5.16 * 10^3\ N/(m/s)$ and $c_3 = 7.2 * 10^3\ N/(m/s)$.

The other MR damper constants are:

$k_{01} = 18 * 10^4\ N/m$, $c_{01} = 7 * 10^3\ N/(m/s)$, $\propto_{01} = -30.86$, $h_{01} = 1$,

$c_{11} = 53\ N/(m/s)$, $k_{11} = -3.4 * 10^2\ N/m$, $\beta_1 = 0.08$, $\varrho_{01} = 0.43$ and $\varrho_{11} = 0.54$.

Substituting all these parameters into Eq. (3-80) yields the final state-space model for the velocities and positions of the three floors as:

$$
\begin{bmatrix} \dot{x}_1(t) \\ v_1(t) \\ \dot{x}_2(t) \\ v_2(t) \\ \dot{x}_3(t) \\ v_3(t) \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -596.6667 & -3.2267 & 330.00 & 2.0267 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 266.6667 & 1.20 & -596.6667 & -3.2267 & 296.6667 & 2.3667 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 266.6667 & 1.20 & -296.6667 & -2.3667 \end{bmatrix} \begin{bmatrix} x_1(t) \\ v_1(t) \\ x_2(t) \\ v_2(t) \\ x_3(t) \\ v_3(t) \end{bmatrix} \tag{3-81}
$$

$$
+ \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 1.667 & 0 \\ 0 & 0 & 0 \\ 0 & -1.667 & 1.667 \\ 0 & 0 & 0 \\ 0 & 0 & -1.667 \end{bmatrix} \times 10^{-4} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 & 0 & 0 \\ 0.0051 & -0.0051 & 0 \\ 0 & 0 & 0 \\ 0 & 0.0051 & -0.0051 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0051 \end{bmatrix} \begin{bmatrix} \mathcal{S}_1(t) \\ \mathcal{S}_2(t) \\ \mathcal{S}_3(t) \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 1.667 & 0 \\ 0 & 0 & 0 \\ 0 & -1.667 & 1.667 \\ 0 & 0 & 0 \\ 0 & 0 & -1.667 \end{bmatrix} \times 10^{-4} \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \ddot{x}_g(t)
$$

System decomposition:

A procedure of (Jamshidi, 1997) is used to decompose the building structure dynamic system into n inter-connected subsystems. The compact system is re-written as:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ . \\ . \\ . \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} A_1 & L_{12} & . & . & . & . & . & L_{1n} \\ L_{21} & A_2 & L_{23} & . & . & . & & L_{2n} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ L_{n1} & . & . & . & . & . & . & A_n \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ . \\ x_k(t) \end{bmatrix}$$

$$+ \begin{bmatrix} B_1 & B_{12} & . & . & . & . & . & B_{1n} \\ B_{21} & B_2 & B_{23} & . & . & . & & B_{2n} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ B_{n1} & . & . & . & . & . & . & B_n \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ . \\ u_n(t) \end{bmatrix} \tag{3-82}$$

$$+ \begin{bmatrix} T_1 & T_{12} & . & . & . & . & . & T_{1n} \\ T_{21} & T_2 & T_{23} & . & . & . & & T_{2n} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ T_{n1} & . & . & . & . & . & . & T_n \end{bmatrix} \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ . \\ . \\ . \\ Z_n(t) \end{bmatrix} + \begin{bmatrix} B_1 & B_{12} & . & . & . & . & . & B_{1n} \\ B_{21} & B_2 & B_{23} & . & . & . & & B_{2n} \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ B_{n1} & . & . & . & . & . & . & B_n \end{bmatrix} \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ . \\ . \\ . \\ Q_n(t) \end{bmatrix}$$

$$+ \begin{bmatrix} E_1 & O & . & . & . & . & . & 0 \\ 0 & E_2 & 0 & . & . & . & . & 0 \\ & & & . & & & & \\ & & & . & & & & \\ & & & . & & & & \\ 0 & . & . & . & . & . & . & E_n \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \\ . \\ . \\ . \\ d_n(t) \end{bmatrix}$$

From Eq. (3-82) the subsystems are described as:

***1st Subsystem*:**

$$\begin{aligned} \dot{x}_1(t) = A_1 x_1(t) + L_{12} x_2(t) + \ ... + L_n x_n(t) + B_1 u_1(t) + B_{12} u_2(t) + \ ... \\ + \ B_{1n} u_n(t) + T_1 Z_1(t) + T_{12} Z_2(t) + \ ... + \ T_{1n} Z_n(t) + B_1 Q_1(t) \\ + B_{12} Q_2(t) + \ ... + \ B_{1n} Q_n(t) + E_1 d_1(t) \end{aligned} \tag{3-83}$$

2nd Subsystem:

$$\begin{aligned} \dot{x}_2(t) = L_{21} x_1(t) + A_2 x_2(t) + \ ... + L_{1n} x_n(t) + B_{21} u_1(t) + B_2 u_2(t) + \ ... \\ + B_{2n} u_n(t) + T_{21} Z_1(t) + T_2 Z_2(t) + \ ... + T_{2n} Z_n(t) + B_{21} Q_1(t) \\ + B_2 Q_2(t) + \ ... + B_{2n} Q_n(t) + E_2 d_2(t) \end{aligned} \tag{3-84}$$

3rd Subsystem:

$$\dot{x}_3(t) = L_{31}x_1(t) + L_{32}x_2(t) + \ \dots + A_3x_3(t) + B_{31}u_1(t) + B_{32}u_2(t) \ \dots$$
$$+ \ B_3u_3(t) + +T_{31}Z_1(t) + T_{32}Z_2(t) \ \dots + \ T_3Z_3(t) + B_{31}Q_1(t) \qquad (3\text{-}85)$$
$$+ B_{32}Q_2(t) \ \dots + \ B_3Q_3(t) + E_3d_3(t)$$

In this case the system described by Eq. (3-81) can be divided into three subsystems defined as follows:

1st Subsystem: $A_1 = \begin{bmatrix} 0 & 1 \\ -596.6667 & -3.2267 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ -1.667 \end{bmatrix} \times 10^{-4}$,

$T_1 = \begin{bmatrix} 0 \\ 0.0051 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

The interconnection matrices between the 1st and 2nd subsystems (floors) as well as between the 1st and 3rd subsystems are:

$L_{12} = \begin{bmatrix} 0 & 0 \\ 330 & 2.0267 \end{bmatrix}$, $L_{13} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4}$, $B_{13} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$,

$T_{12} = \begin{bmatrix} 0 \\ -0.0051 \end{bmatrix}$ and $T_{13} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2nd Subsystem: $A_2 = \begin{bmatrix} 0 & 1 \\ -596.6667 & -3.2267 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ -1.667 \end{bmatrix} \times 10^{-4}$,

$T_2 = \begin{bmatrix} 0 \\ 0.0051 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

The interconnection matrices between the 1st and 2nd subsystems as well as between the 2nd and 3rd subsystems are:

$L_{21} = \begin{bmatrix} 0 & 0 \\ 266.6667 & 1.20 \end{bmatrix}$, $L_{23} = \begin{bmatrix} 0 & 0 \\ 296.6667 & 2.3667 \end{bmatrix}$, $B_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$,

$B_{23} = \begin{bmatrix} 0 \\ 1.667 \end{bmatrix} \times 10^{-4}$, $T_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $T_{23} = \begin{bmatrix} 0 \\ -0.0051 \end{bmatrix}$

3rd Subsystem: $A_3 = \begin{bmatrix} 0 & 1 \\ -296.6667 & -2.3667 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ -1.667 \end{bmatrix} \times 10^{-4}$,

$T_3 = \begin{bmatrix} 0 \\ 0.0051 \end{bmatrix}$ and $E_3 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

The interconnection matrices between the 1st and 3rd subsystems as well as between the 2nd and 3rd subsystems are:

$L_{31} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $L_{32} = \begin{bmatrix} 0 & 0 \\ 266.6667 & 1.20 \end{bmatrix}$, $B_{31} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $B_{32} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $T_{31} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

and $T_{32} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The 1940 El Centro earthquake record is used as a single seismic disturbance input (acting at ground level) and MATLAB is used to simulate the response of the building where all supposed interactions are unknown (NISEE, 2010).

3.4.1 Simulations and results of using algorithm 3-2

The continuous control $u_i^{LMI}(t)$ is designed by the LMI tool where the solution of *algorithm 3-2* yields the gains:

$k_1 = [1.0493 \quad 0.0031] \times 10^3$, $k_2 = [1.0493 \quad 0.0031] \times 10^3$

and $k_3 = [-3.3462 \quad 0.2394] \times 10^3$

Where $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.2$ and $\Upsilon_1 = \Upsilon_2 = \Upsilon_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

The control signal will be as: $u_i(t) = k_i x_i(t) - \mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$

Figures 3-7, 3-8 & 3-9 illustrate the displacement response of all three floors of the building when stimulated by the El Centro earthquake. The earthquake seismic data records correspond to cases of: (i) no control, (ii) with passive actuator only and (iii) with semi-active actuator with control $\{u_i^{LMI}(t) + u_i^{ISM}(t)\}$. Figure 3-10 shows the maximum inter-floor displacements of all three floors (i) without control, (ii) with passive actuator only and (iii) with actuator + $(u_i^{LMI}(t) + u_i^{ISM}(t))$. The results show that the actuator with the control signals $(u_i^{LMI}(t) + u_i^{ISM}(t))$ is slightly better than the one using the passive actuator (without additional control), especially on the 3rd floor.

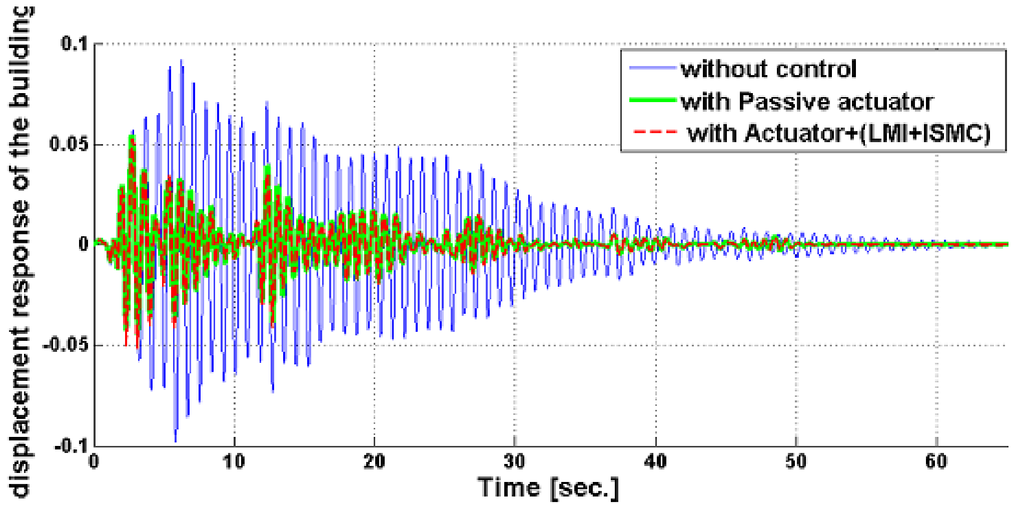

Figure 3-7: 1st floor displacements

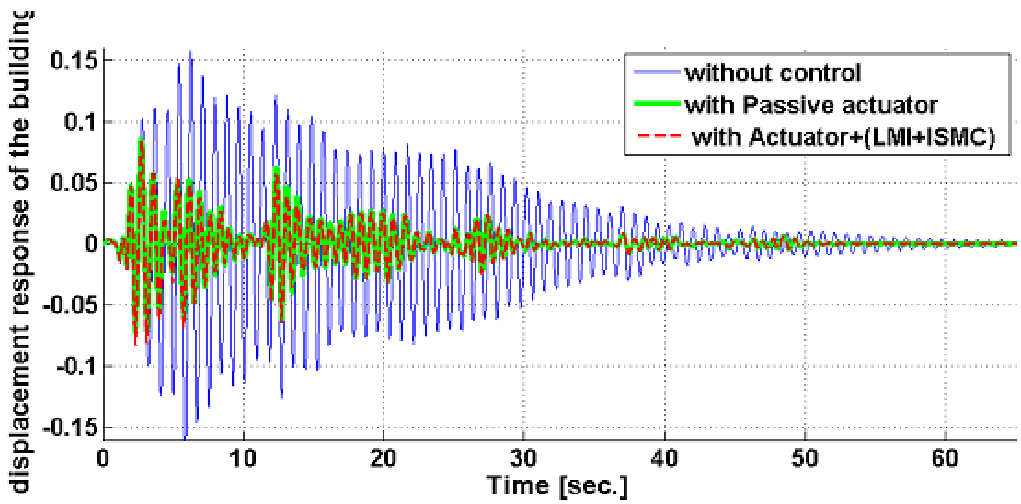

Figure 3-8: 2nd floor displacements

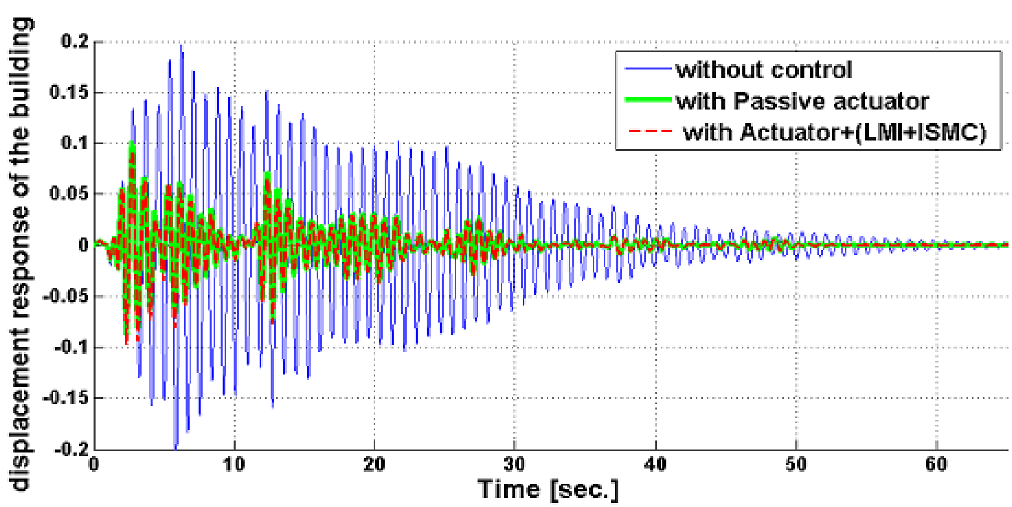

Figure 3-9: 3rd floor displacements

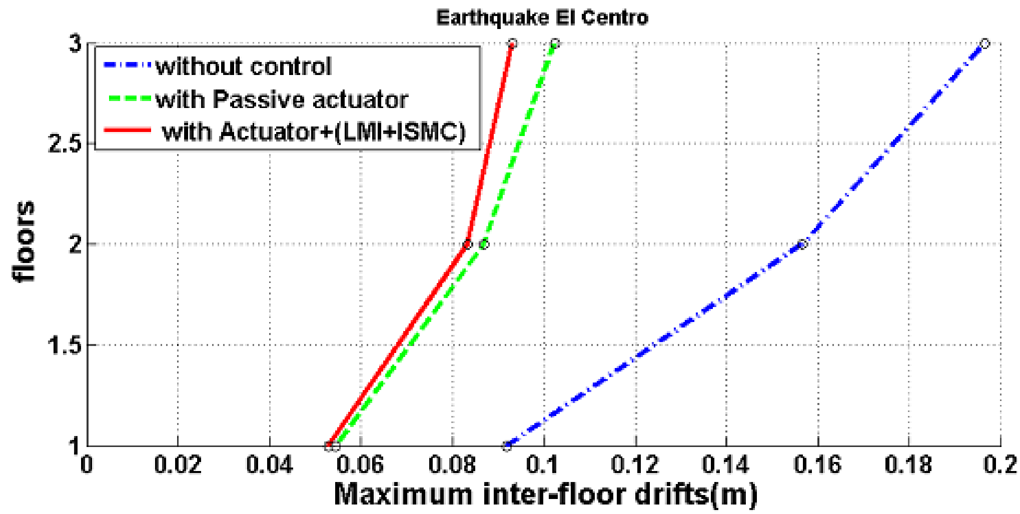

Figure 3-10: Maximum inter-floor displacements without fault

Figure 3-11 shows the normal control signal of the 1st floor actuator subject to the seismic input excitation, with the semi-active control applied via $u_i^{LMI}(t)+u_i^{ISM}(t)$.

3.4.2 Building simulation with actuator fault included

The above results show the effect of introducing a semi-active control action to the building system in which it is assumed that none of the actuators malfunction. Figure 3-11 shows an example of the total control force acting on the 1st floor mass. It is assumed that there are no faults acting, i.e. the semi-active actuators are working normally.

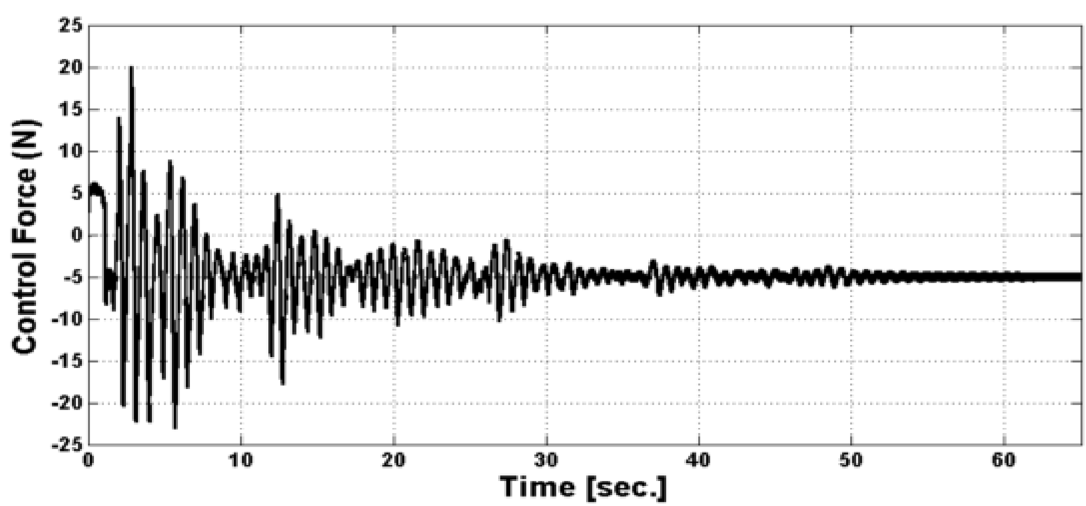

Figure 3-11: 1st floor control force with no faults

It is interesting to repeat the simulated building system example by including the effects of faults in the semi-active actuators as follows.

Figure 3-12 illustrates the simulation of a realistic fault scenario for this actuator with 70% of actuator fault. Using the same 1st floor actuator fault with 70% of actuator fault and 100% actuator failure, the maximum inter-floor displacements for the cases of (i) no control, (ii) with passive actuator only and (iii) with semi-active actuator with control $\{u_i^{LMI}(t)+u_i^{ISM}(t)\}$ are shown in Figures 3-13, 3-14 , 3-15 & 3-16. A comparison of the floor displacement results indicate that the designed control $(u_i^{LMI}(t)+u_i^{ISM}(t))$ gives better results in the case of faults and failures in terms of integrity of the building (i.e. with respect to the low level of floor displacements from the datum line.

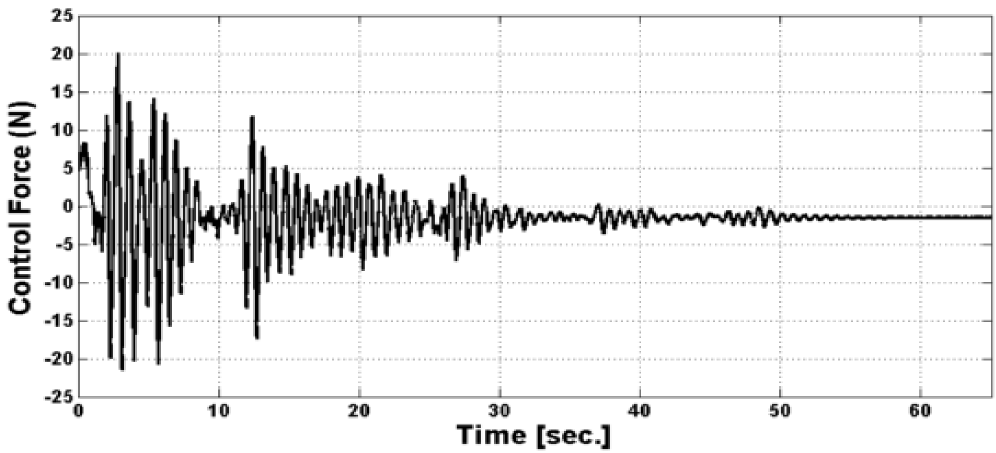

Figure 3-12: 1st floor actuator force with 70% actuator fault

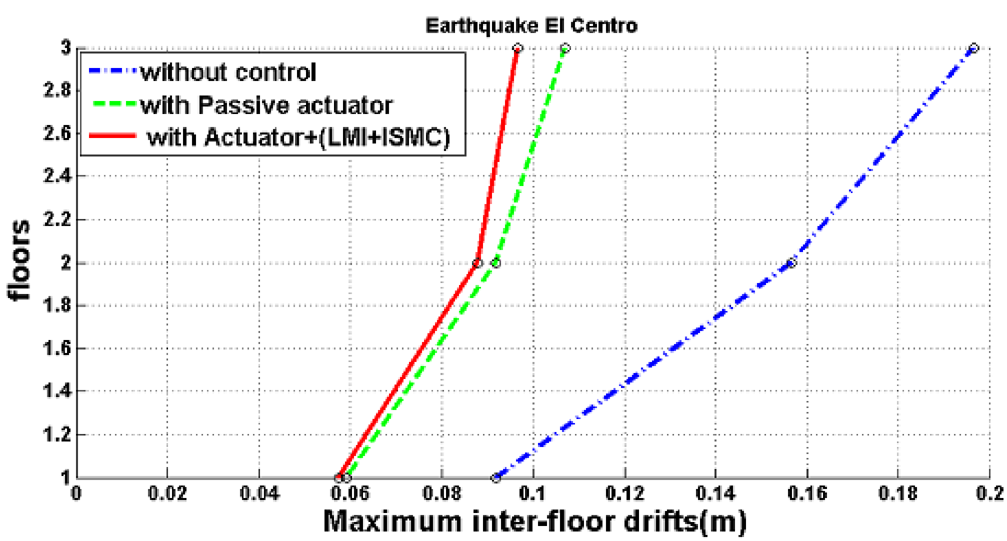

Figure 3-13: Maximum inter-floor displacement: 1st floor 70% actuator fault

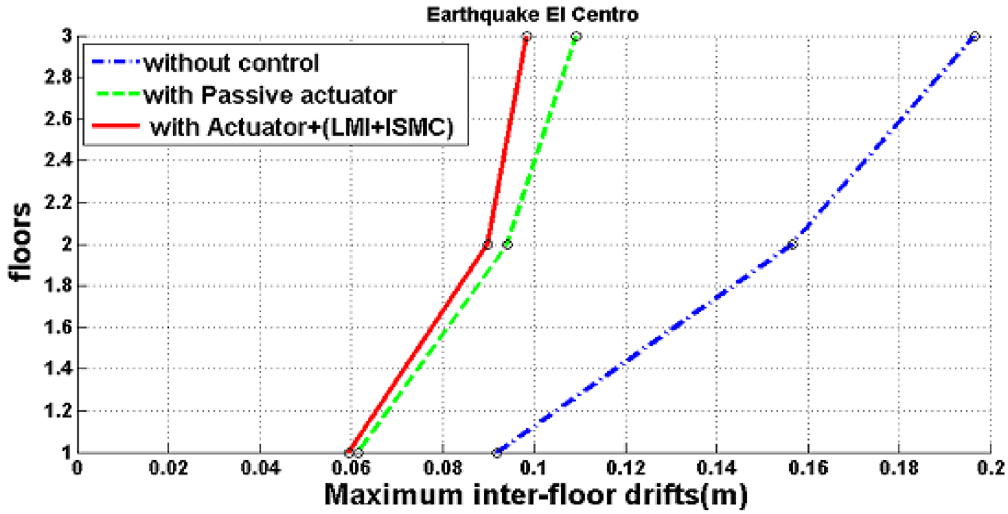

Figure 3-14: Maximum inter-floor displacement: 1st floor 100% actuator failure

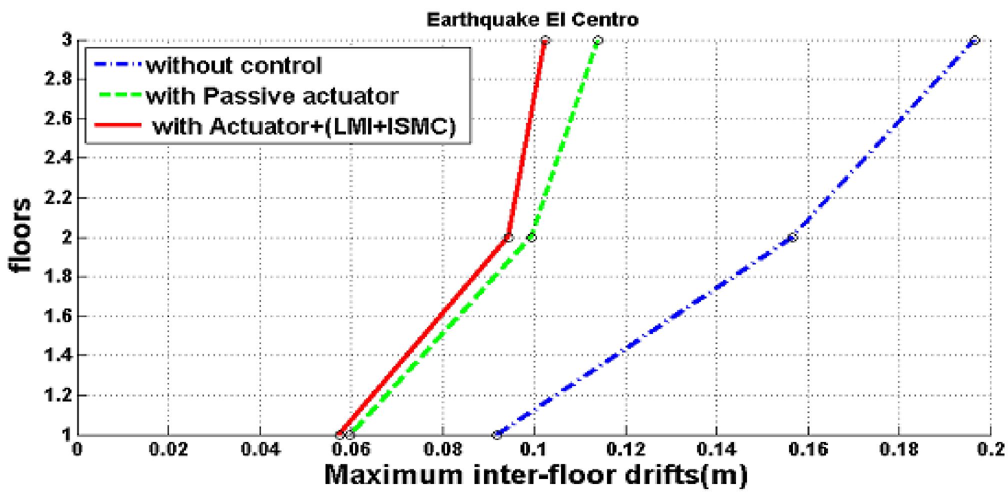

Figure 3-15: Maximum inter-floor displacement: 1st and 2nd floor 70% actuator fault

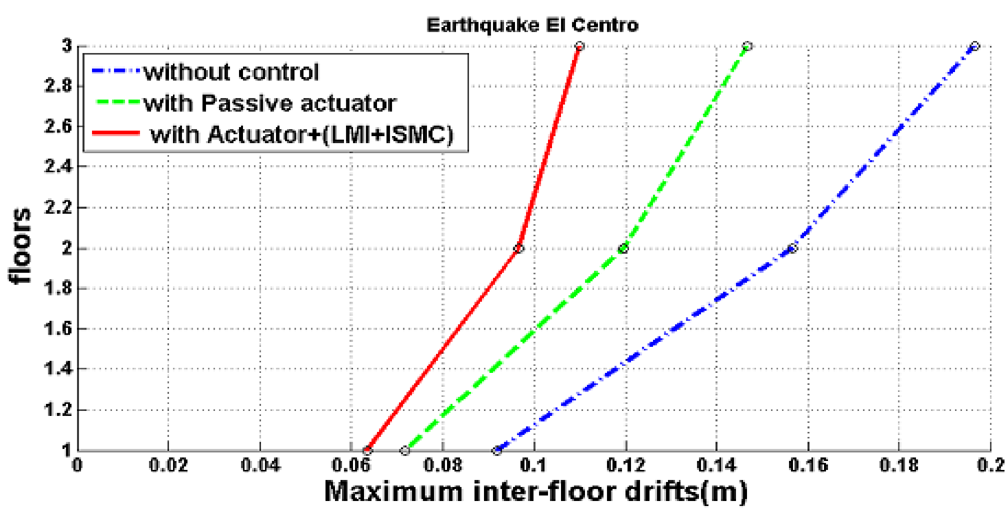

Figure 3-16: Maximum inter-floor displacement: 1st, 2nd and 3rd floor 70% actuator fault

3.4.3 Simulations and results of using Algorithm 3-4 (one shot)

When using the same example of the building structure with three floors, as illustrated in Eq. (3-81), it can be seen there are three subsystems. After putting all the three subsystems together into the one shot system the resulting position and velocity linear state space part of the non-linear Lipchitz system of Eq. (3-86) has the following parameters:

$$A_d = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -596.6667 & -3.2267 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -596.6667 & -3.2267 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -296.6667 & -2.3667 \end{bmatrix}$$

and

$$B_d = \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1.667 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1.667 \end{bmatrix} \times 10^{-4}$$

Algorithm 3-4 (see Section 3.3.1.2.2) is now used to examine the response of this building model to the seismic disturbance signal from the 1940 El Centro earthquake record. The gains are calculated using the MATLAB LMI toolbox and MATLAB is also used to simulate the response of the building where all the interactions are supposed unknown.

The continuous control $u_i^{LMI}(t)$ is designed according to the one shot LMI of *Algorithm 3-4*.The solution gained by using this algorithm, after choosing P as a diagonal matrix, yields the gain as:

$$K = \begin{bmatrix} -8.5493 & 0.1444 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8.5493 & 0.1444 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.3235 & 0.0792 \end{bmatrix} \times 10^3$$

where $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.2$ and $\Upsilon = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

The overall control is given by: $u_i(t) = k_i x_i(t) - \mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$

The maximum inter-floor displacements of all three floors in the case of (i) no control, (ii) with passive actuator only and (iii) with semi-active actuator with control $\{u_i^{LMI}(t)+u_i^{ISM}(t)\}$ are shown in Figure 3-17, where there is no any fault in any actuator floor. From the Figure 3-17 it is clear that the semi-active actuators operating in all the three floors with the added control signals perform better than the passive actuators do.

The control force that is applied to the 1st floor actuator stabilizes the building and a decrease the effects of earthquake is shown in Figure 3-18. The control force is applied to the 1st floor actuator which has a 70% actuator fault. The simulation of this control force is illustrated in Figure 3-19 .

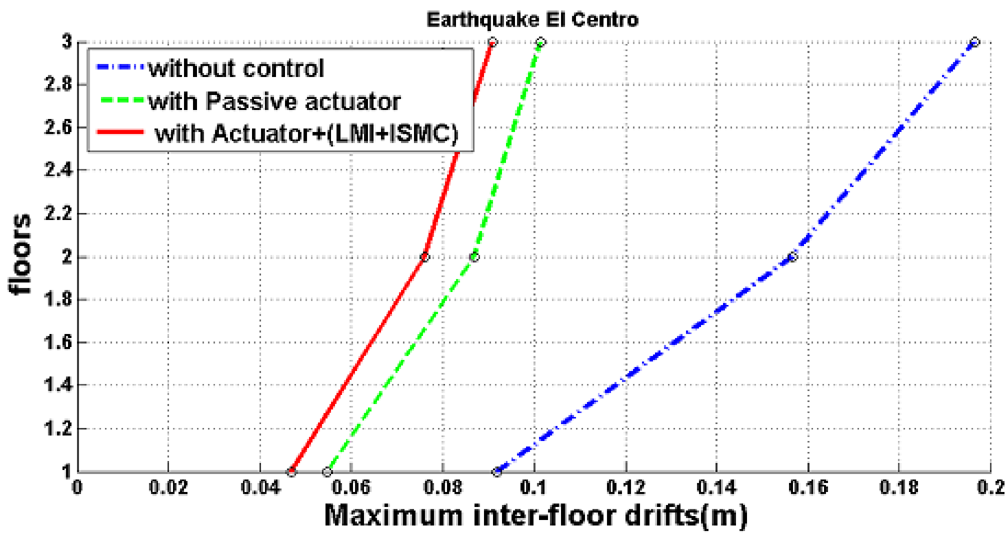

Figure 3-17 : Maximum inter-floor displacements without fault

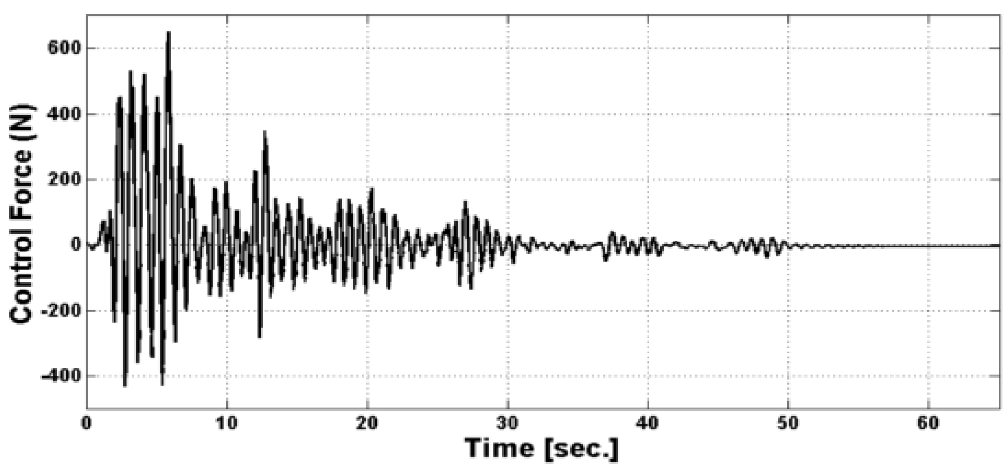

Figure 3-18: 1st floor control force with no faults

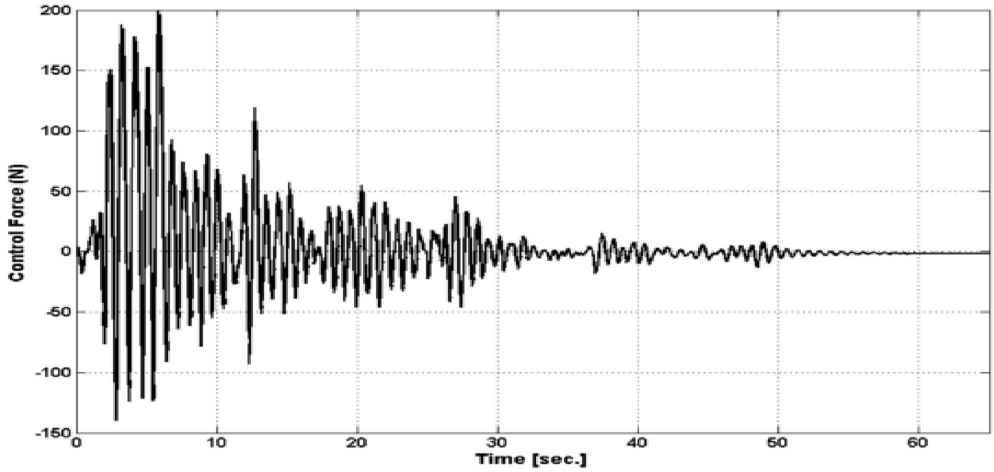

Figure 3-19 : 1st floor actuator force with 70% actuator fault

As a consequence of the 70% actuator fault in the 1st floor actuator, the maximum inter-floor displacements for the cases of (i) no control, (ii) with passive actuator only and (iii) with semi-active actuator with control $\{u_i^{LMI}(t)+u_i^{ISM}(t)\}$ is shown in Figure 3-20 . The maximum inter-floor displacements, 100% actuator failure, in the 1st floor actuator are shown in Figure 3-21.

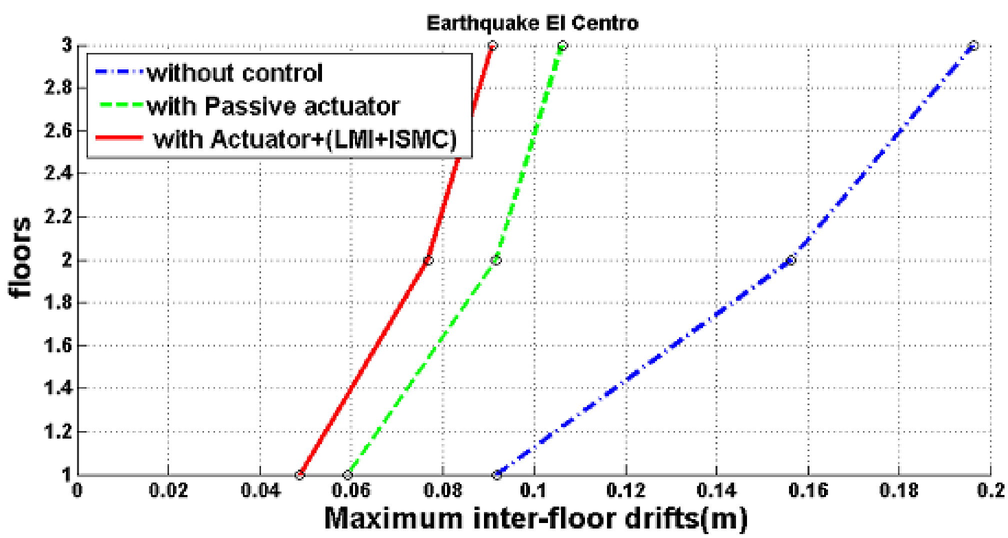

Figure 3-20 : Maximum inter-floor displacement: 1st floor 70% actuator fault

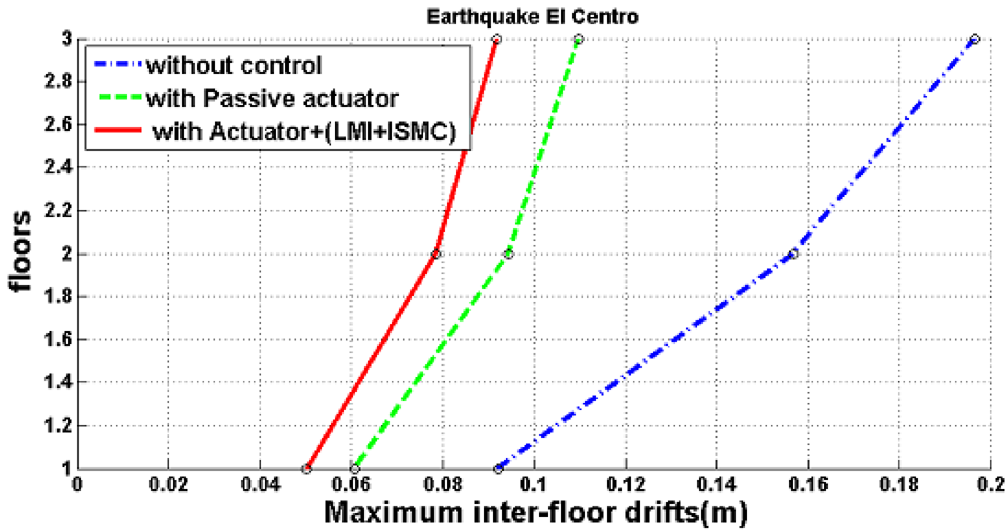

Figure 3-21 : Maximum inter-floor displacement: 1st floor 100% actuator failure

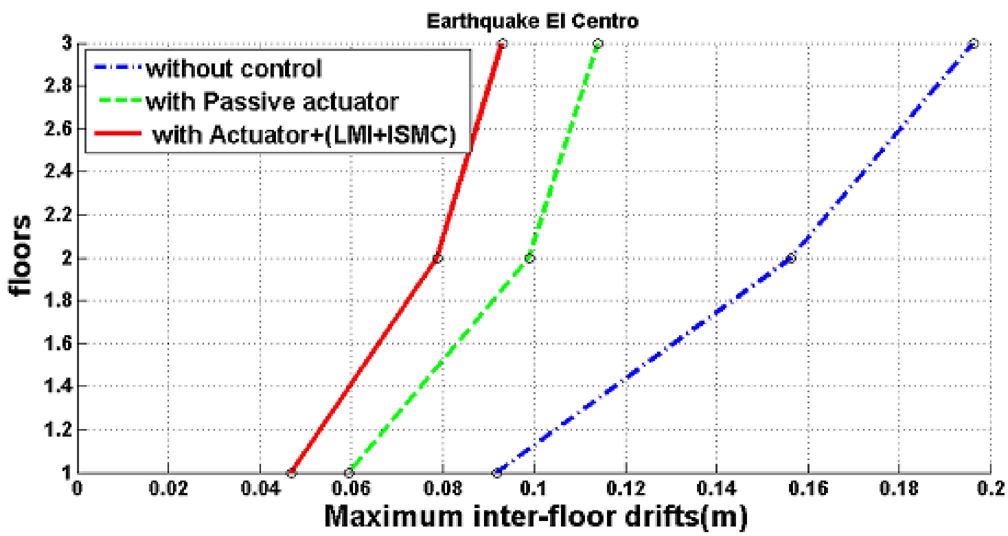

Figure 3-22 : Maximum inter-floor displacement: 1st and 2nd floor 70% actuator fault

Figure 3-22 shows the maximum inter-floor displacements of the three floors of the building with 70% actuator faults in the 1st and 2nd floors after applying the ISMC with the gains that are obtained using algorithm 3-4. Figure 3-23 shows the results for the 70% actuator faults in 1st, 2nd and 3rd actuator floors. The simulated building system is seen to have better integrity with the semi-active actuator control than for the passive actuator cases.

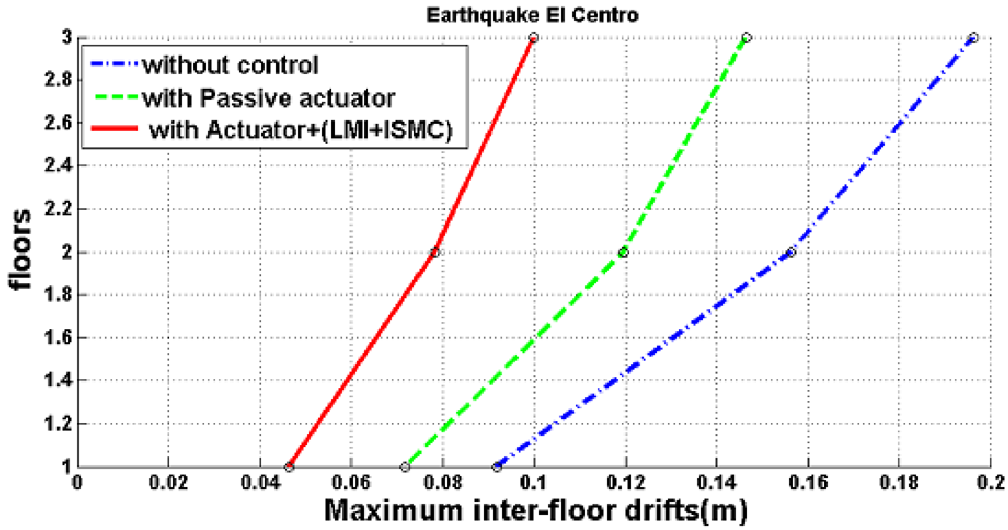

Figure 3-23 : Maximum inter-floor displacement: 1st, 2nd and 3rd floor 70% actuator fault

3.4.4 Simulations and results of using algorithm 3-6 (overlapping)

This new technique is applied to the same building example as illustrated in Eq. (3-81) , based on the 1940 El Centro earthquake seismic disturbance.

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -596.6667 & -3.2267 & 330.000 & 2.0267 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 300.0000 & 0.8600 & -596.6667 & -3.2267 & 296.6667 & 2.3667 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 266.6667 & 1.2000 & -296.6667 & -2.3667 \end{bmatrix}$$

and $$\mathfrak{B} = \begin{bmatrix} 0 & 0 & 0 \\ -1.667 & 1.667 & 0 \\ 0 & 0 & 0 \\ 0 & -1.667 & 1.667 \\ 0 & 0 & 0 \\ 0 & 0 & -1.667 \end{bmatrix} \times 10^{-4}$$

The MATLAB LMI toolbox is used to calculate the gain from Section 3.3.2.2, using *Algorithm 3-6*. In addition, MATLAB is used to simulate the response of the three floor building where all the supposed interactions are known.

The continuous control $u_i^{LMI}(t)$ for the one shot system is designed by the LMI, where P is a non-diagonal matrix with gain:

The gain:

$$K = \begin{bmatrix} -1.1197 & 0.0023 & 1.0963 & 0.0013 & -0.4752 & -0.0015 \\ 0.5523 & -0.0003 & -1.7142 & 0.0042 & 1.1562 & 0.0077 \\ 1.5672 & 0.0023 & -0.0123 & 0.0004 & -1.1863 & 0.0051 \end{bmatrix} \times 10^4$$

Where $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.2$ and $\Upsilon = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

The overall control will be as: $u_i(t) = k_i x_i(t) - \mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$

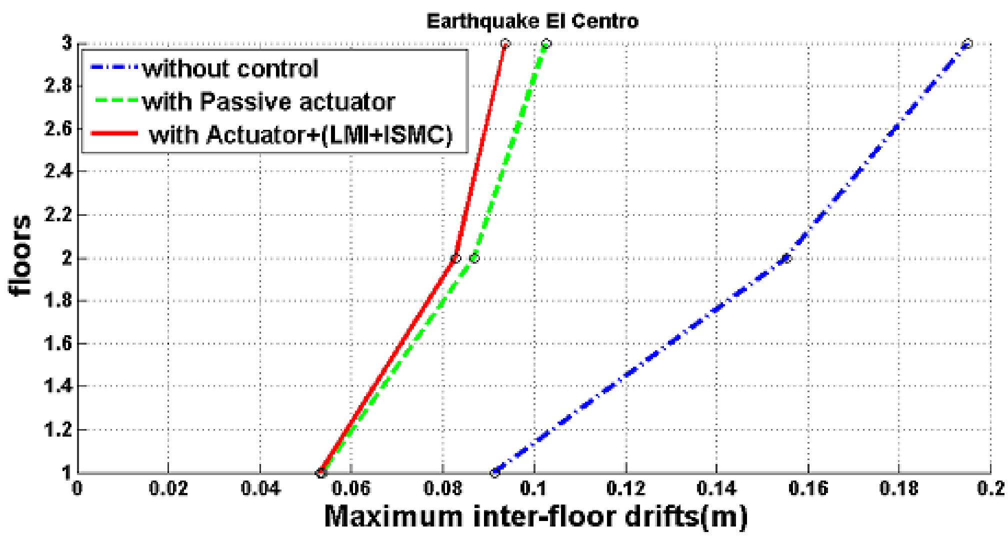

Figure 3-24 : Maximum inter-floor displacements without fault

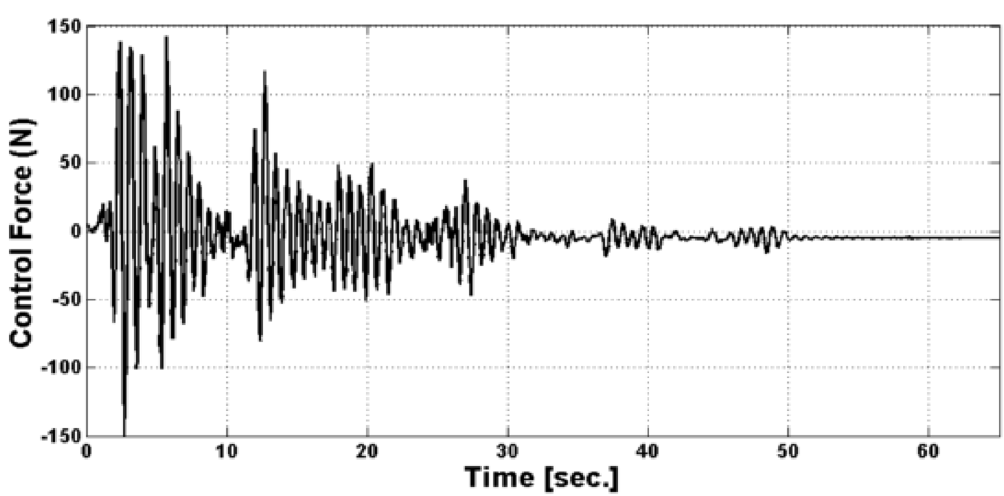

Figure 3-25 : 1st floor control force with no faults

Figure 3-24 shows the maximum inter-floor displacements that may take place in all three floors without any actuator faults for the cases where there is no control, there is only the passive actuator (with no controller) and where the semi-active actuators are controlled using the LMI and ISMC designs. The simulation results show that the semi-active actuator with control in the third floor performs a little better compared with the case when only passive actuation is applied. However, the results for the first floor are almost the same.

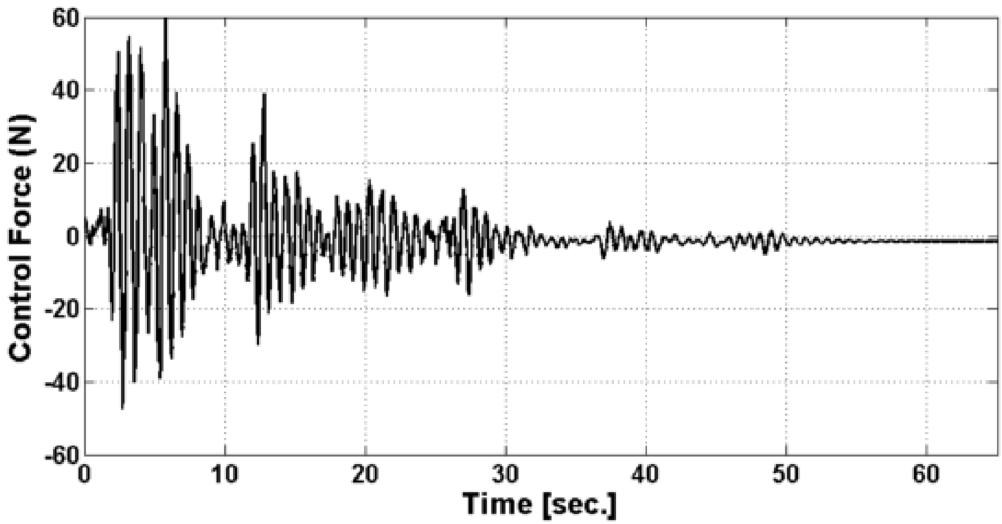

Figure 3-26 : 1st floor actuator force with 70% actuator fault

The control signal for the 1st floor fault-free semi-active actuator, subject to the 1940 El Centro earthquake seismic excitation is shown in Figure 3-25 . Figure 3-26 illustrates the same control signal, but with a 70% actuator fault in the 1st floor semi-active actuator.

Figure 3-27 shows the maximum inter-floor displacements of the three floors where the 1st floor semi-active actuator has a 70% fault. It can be seen clearly that the controlled actuator is well able to handle the fault. Figure 3-28 shows that for the 1st floor semi-active actuator with 100% failure, there is very little difference of the maximum inter-floor displacements with and without control.

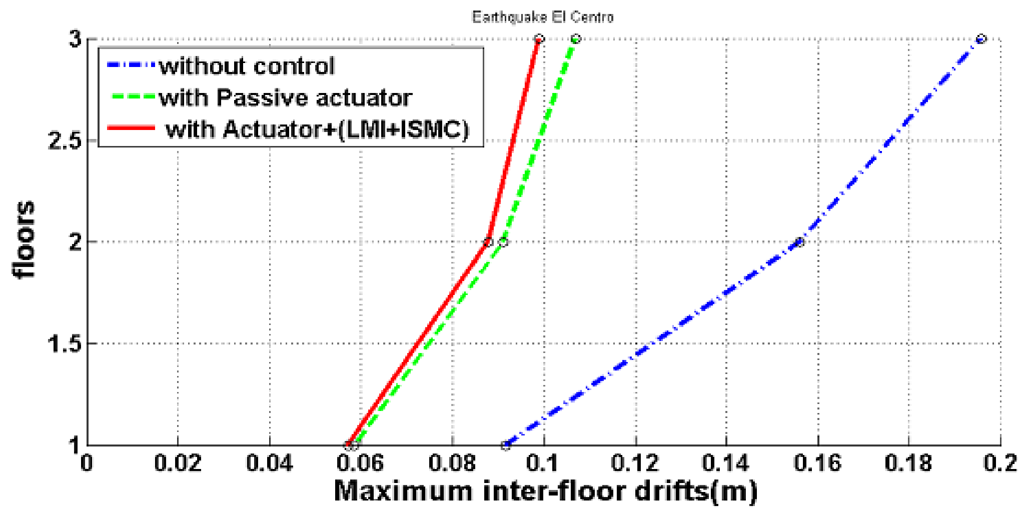

Figure 3-27 : Maximum inter-floor displacement: 1st floor 70% actuator fault

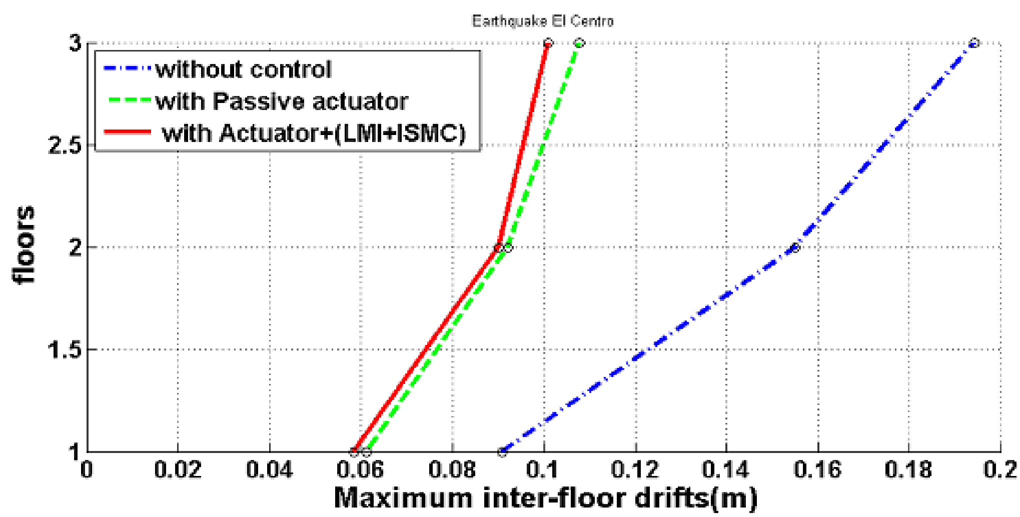

Figure 3-28 : Maximum inter-floor displacement: 1st floor 100% actuator failure

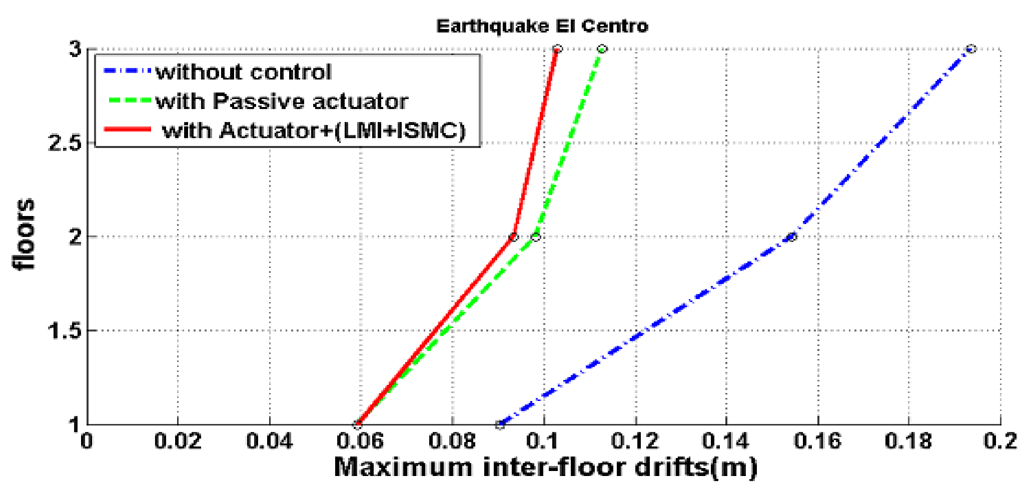

Figure 3-29 : Maximum inter-floor displacement: 1st and 2nd floor 70% actuator fault

Figure 3-29 shows that for the case when the 1st and 2nd floor semi-active actuators have 70% faults, the semi-active actuator with the control is slightly better than the system operation with the passive actuator. However, if all the three of the floor actuators have 70% faults the semi-active actuators all perform better than their passive counterparts, demonstrating the effectiveness of the floor controllers in dealing with these bounded actuator faults as shown in Figure 3-30.

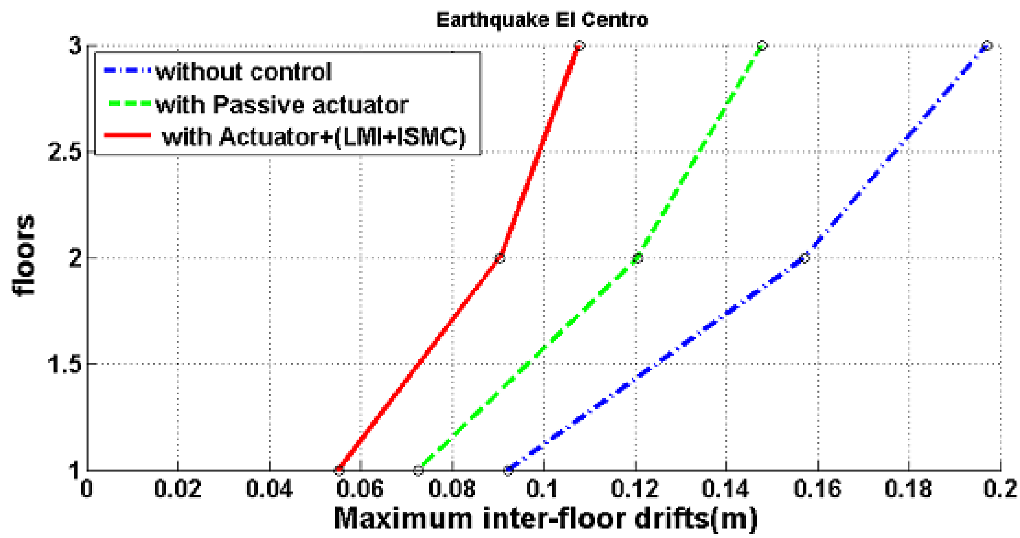

Figure 3-30 : Maximum inter-floor displacement: 1st, 2nd and 3rd floor 70% actuator fault

3.4.5 Comparison of all methods

Table 3-1 shows a comparison between the applications of the three ISMC design methods. From the table, it can be observed that a slightly higher gain can be obtained from using the method based on Algorithm 3-6. The control force resulting from the design using Algorithm 3-2 shows the most suitable range of variation between -20 to 20 N. On the other hand, designs based on algorithm 3-4 use the highest power, as the force variations are between -400 to 600 N. However, this design method produces a better power performance for the case of 70% actuator faults applied in all three subsystems (floors) and also the case of 100% failure in the 1st floor. The Algorithms 3-2 and 3-4 have de-centralized gains with unknown interactions but the Algorithm 3-2 has de-centralized overlapping gain with known interactions.

Contents / Method	Interconnections	Gain	Control force with no fault (N)	Response to 70% actuator fault in all (three floors) (m)	Response to 100% failure in (1st floor) (m)
(ISMC+LMI) for each subsystem	Unknown	De-centralized control	-20 to 20	1st floor 0.06 2nd floor 0.095 3rd floor 0.15	1st floor 0.06 2nd floor 0.09 3rd floor 0.098
(ISMC+LMI) for overall system (one shot)	Unknown	De-centralized control	-400 to 600	1st floor 0.045 2nd floor 0.078 3rd floor 0.1	1st floor 0.05 2nd floor 0.08 3rd floor 0.09
(ISMC+LMI) for overall system (one shot)	Known	De-centralized overlapping control	-150 to 150	1st floor 0.055 2nd floor 0.09 3rd floor 0.13	1st floor 0.06 2nd floor 0.09 3rd floor 0.1

Table 3-1 : Comparison of control inter-connected systems methods

3.5 Conclusion

In this Chapter, the ISMC with an LMI-based design taking into account the non-linear interaction between the inter-connected building floor systems has been presented for three different methods. The methods depend on the configuration of the controller where the first method gives de-centralized control for the case of unknown interactions. The second method is the same as the first method but the design considers all the subsystems together as a one shot system. Finally, the third method gives de-centralized overlapping control where all the interactions are known.

The main common challenge to the application of all the three design methods is the requirement for decreasing the effects of the disturbances and interactions between the subsystems. There are additional challenges of isolating each floor subsystem from the propagation of the effects of faults that may arise within the other floor subsystems. This effectively involves the application of the “plug and play” concept.

The controllers all contain two control signals; the first signal designed by using ISMC to deal with any matched components including uncertainties, disturbances and bounded actuator faults, and the second control signal is designed via the LMI formulation. This second control signal is responsible for dealing with any unmatched components to stabilise the system and to achieve a required performance in terms of minimum floor displacement.

These methods are dependent on the availability of all states, however, when it is not easy to obtain all the states then the best solution is to estimate them. Chapter 4 focuses on the design method dependent on the use of observer-based control, leading to the de-centralized observer approach.

Chapter 4 : De-centralized observed-based control design with ISMC for inter-connected systems

4.1 Introduction

Some research on control of non-linear inter-connected systems focuses on inter-connected systems with uncertainties, e.g. unknown non-linear interconnections and disturbances, presenting robustness design challenges involving control specifications for each subsystem. These systems are particularly difficult to design when faced with limitations arising from uncertainty matching conditions and lack of available state information (Shafai,Ghadami and Saif, 2011).

In most cases the design of robust de-centralized systems focuses on state feedback problems. However, in reality only output information is available and this adds a further challenge to the robust design problem. It is often the case that the controller designs must depend to a degree on estimated states, and hence it is common practice in the literature to investigate the observer based feedback control approach with state estimates based on local information (Pagilla and Zhu, 2005, Dhbaibi,Tlili,Elloumi and Benhadj, 2009). The derivation of robust output feedback for de-centralized control systems with uncertain interconnection remains a difficult challenge in the literature (Stanković,Stipanović and Šiljak, 2007, Huan,Jeang and Yon, 2012).

Observer-based strategies represent a commonly used way of dealing with output feedback design and there are two observer-based control paradigms for de-centralized systems. Firstly, a separate "decentralized" observer is designed for each subsystem, taking account of local information (Aldeen,Lau and Marsh, 1998, Trinh and Aldeen, 1998). The second approach involves the use of "inter-connected observers" in which each observer measurement and input information is shared with observers from other local subsystems (Dhbaibi,Tlili and Benhadj, 2008).

In many branches of control systems there is a need to compensate robustly for effects of either system uncertainties or input disturbances or even faults, to maintain required closed-loop performance and stability. One such approach is the use of sliding mode control (SMC) in which the system dynamic behaviour can be forced to be independent of inputs, and certain disturbances and modelling uncertainties, once the so-called sliding regime has been reached (Changqing,Patton and Zong, 2010, Larbah and Patton, 2012). Several studies of inter-connected de-centralized systems have focused on the use of SMC as a basis for solving robustness (Yan,Spurgeon and Edwards, 2003, Ghadami and Shafai, 2011).

However, the classical approach to SMC requires (i) a reachability condition to guarantee that the SMC sliding or switching motion in state space can be reached from arbitrary initial conditions, and (ii) that two control components, one linear and one discontinuous are designed to achieve reachability and satisfy the sliding mode design objectives (Poznyak,Fridman and Bejarano, 2004).

In the case of output integral sliding mode (OISMC) the requirements for both (i) & (ii) above are obviated, making the use of OISMC very attractive for robust control of de-centralized systems (Poznyak,Fridman and Bejarano, 2004, Castaños,Xu and Fridman, 2006, Bejarano,Fridman and Poznyak, 2007).

This Chapter focuses on the use of OISMC for de-centralized control, based on estimated state feedback. It is assumed that the local system states are not measurable and hence the *de-centralized observer* approach outlined above is used as a part of a state-estimate feedback design problem. De-centralized observers are used as a part of the strategy to de-couple the effects of interconnections between subsystems. Although, each observer has linear feedback structure the observer-based control is formulated using a single LMI procedure to satisfy both Lyapunov *stability* and performance of the augmented state space form of the observer-based controller state-space system. This relates to the classical Separation Principle only in the sense that objective for each subsystem is to provide effective recovery of the Separation Principle and hence also effective decentralization. This is achieved through the use of the single LMI approach involving the feedback designs for each observer and controller (Zhu and Pagilla, 2007). The system description involves both matched and unmatched uncertainty components (arising from interconnections and external disturbances) and the Chapter deals with both forms of uncertainty.

The main contributions in this Chapter can be summarised as follows:

1- The new proposal of an observer-based control design in LMI framework combined with OISMC to non-linear inter-connected system[2].
2- The impacts of actuator faults on the other subsystems and the role of the proposed control approach to deal with these faults.

The Chapter is structured as follows. Section 4.2 describes the problem formulation. Then Section 4.3 considered the proposed control approach that includes OISMC in the first part and LMI observer–based control design one for a compact system (one shot) and the other for every subsystem individually. Section 4.4 describes a numerical example with three non-linear inter-connected systems to illustrate the design approach and simulation performance Section 4.5 gives some conclusions and the further work.

4.2 Model description and problem statement

As described in Section 3.2 non-linear inter-connected system comprising *n* subsystems can be described by:

$$\begin{gathered}\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t) \\ y_i(t) = C_i x_i(t) \qquad i = 1, \dots \dots, N\end{gathered} \tag{4-1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ are the control inputs and $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs. A_i, B_i, C_i and E_i are known matrices of appropriate dimensions. $Z_i(t) \in \mathbb{R}^n$ represents the unknown time-varying interactions between the subsystems, containing matched and unmatched components. Hence, $Z_i = Z_{mi} + Z_{ui}$ where Z_{mi} is a matched component of Z_i and Z_{ui} are the unmatched components (Castaños,Xu and Fridman, 2006).

Dropping the subscripts in $Z_i(t)$ and using the Bezout identity $I_n = BB^+ + B^\perp B^{\perp +}$

where $B^+ = (B^T B)^{-1} B^T$, $Z_i = B_i {B_i}^+ Z_i + {B_i}^\perp {B_i}^{\perp +} Z_i$ and ${B_i}^T {B_i}^\perp = 0$, then $Z_i = B_i {B_i}^+ Z_i + \zeta_i$ where $\zeta_i = {B_i}^\perp {B_i}^{\perp +} Z_i$ contains the unmatched uncertainty components.

[2] Part of the work presented in this chapter has been published in:

Larbah, E. and Patton, R.J. 2012. Robust decentralized control design using integral sliding mode control, The 2012 UKACC International Conference on Control, Cardiff, UK. 81 - 86.

$W_i(x_i,t)$ represent the subsystem unknown modelling uncertainties that satisfies the matching condition $W_i(x_i,t) = B_i Q_i(x_i,t)$.

The $d_i(t)$ are unknown bounded disturbances, $f_i(t) \in \mathbb{R}^k$ denote the actuator faults, where $f_i(t) = -K(t)u_i$. $K(t) = diag(K_i)$ with $0 \leq K_i \leq 1$, $K_i = 0$ means that the i^{th} actuator is working perfectly and if $K_i = 1$ the actuator has failed completely, otherwise the fault is present.

***Assumptions*:**

As described in Chapter 3 in Section 3.2, the same assumptions from (**A1** to **A7**) are considered in this Section.

Applying (**A1** to **A7)** to Eq.(4-1), it then follows that:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + B_i B_i^{+} Z_i(t) + \zeta_i(t) + B_i Q_i(x_i,t) + E_i d_i(t) + B_i f_i(t)$$
$$y_i(t) = C_i x_i(t) \qquad i = 1, \dots, N \tag{4-2}$$

The control signal includes *two* components:

$$u_i(t) = u_i^{OBC}(t) + u_i^{ISM}(t) \tag{4-3}$$

where u_i^{OBC} is responsible for stabilizing the system and affects the desired performance and decreases the effect of unmatched components where the state is not available. u_i^{ISM} is a discontinuous control responsible for rejecting the effects of matched components (uncertainties and actuator faults) where the state is not available.

4.3 Control design methods

As described in Eq. (4-3) the subsystem control signal includes *two* parts with each part designed using a different method where (i) $u_i^{ISM}(t)$ is designed by output integral sliding mode control OISMC where the state is not available and only the estimated state is obtainable, and (ii) $u_i^{OBC}(t)$ feedback control depends on the estimated state as shown in Figure 4-1.

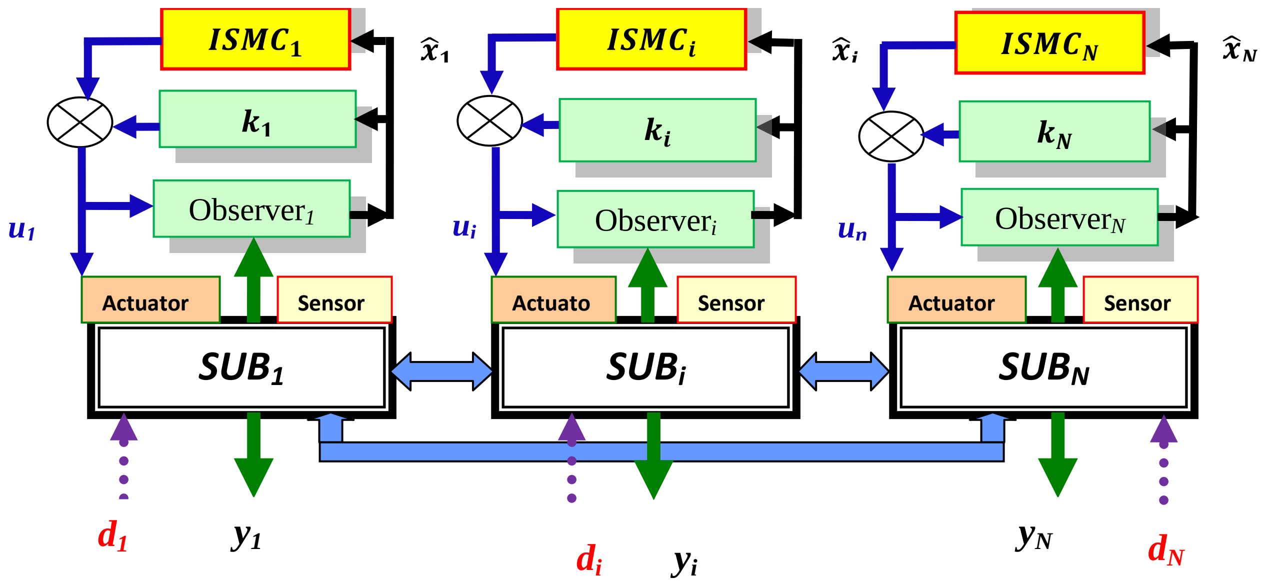

Figure 4-1: Output control of inter-connected systems via LMI+ISMC

4.3.1 Output integral sliding mode control (OISMC)

As outlined in Section 4.2 the OISMC can be used to obviate the requirement for a reachability condition. The output feedback case of the integral sliding mode control can then be developed by defining the following output integral sliding switching surface for i^{th} subsystem:

$$\sigma_i(y_i, \hat{x}_i, t) = G_i[y_i(t) - y_i(t_o) - \int_{t_o}^{t} \left(C_i A_i \hat{x}_i(t) + C_i B_i u_i^{OBC}(t)\right) dt] \qquad (4\text{-}4)$$

where $G_i \in \mathbb{R}^{mxp}$ is a design freedom matrix that must satisfy the invertibility of $G_i C_i B_i$.

The two steps involved in the design of an OISMC are as follows:

1- Based on the estimated state, design a sliding surface that satisfies the system performance and guarantees design specification from the initial moment when the system is on the sliding surface.

2- Propose a suitable discontinuous control based on the estimated state to keep the system trajectory close to or on the sliding surface.

In the OISMC the design freedom of the integral action can be used to design a control law that satisfies the prescribed closed-loop performance.

The equivalent control $u_{eqi}(t)$ can maintain the subsystem i^{th} on the sliding surface by forcing the time derivative of $\sigma_i(y_i, \hat{x}_i, t)$ in Eq.(4-4) to be zero-valued (Cao and Xu, 2001) i.e.:

$$\dot{\sigma}_i(y_i, \hat{x}_i, t) = G_i \dot{y}_i(t) - G_i C_i A_i \hat{x}_i(t) - G_i C_i B_i u_i^{OBC}(t) = 0 \tag{4-5}$$

Then substituting Eq. (4-2) and Eq. (4-3) into Eq. (4-5) yields:

$$\begin{aligned} G_i C_i A_i x_i(t) &+ G_i C_i B_i u_i^{OBC} + G_i C_i B_i u_i^{ISM} + G_i C_i B_i B_i^{+} Z_i(t) + G_i C_i \zeta_i(t) \\ &+ G_i C_i B_i Q_i(x_i, t) + G_i C_i E_i d_i(t) + G_i C_i B_i f_i(t) - G_i C_i A_i \hat{x}_i(t) \\ &- G_i C_i B_i u_i^{OBC} = 0 \end{aligned} \tag{4-6}$$

Hence, the so-called *equivalent control* for the output feedback case is:

$$\begin{aligned} u_{eqi}(t) &= u_i^{ISM} \\ &= -(G_i C_i B_i)^{-1} G_i C_i A_i (x_i(t) - \hat{x}_i(t)) - B_i^{+} Z_i(t) \\ &- (G_i C_i B_i)^{-1} G_i C_i \zeta_i(t) - Q_i(x_i, t) - (G_i C_i B_i)^{-1} G_i C_i E_i d_i(t) - f_i(t) \end{aligned} \tag{4-7}$$

Substituting Eq. (4-7) into Eq. (4-2) gives the i^{th} subsystem state equation as:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i^{OBC}(t) + [I_i - B_i (G_i C_i B_i)^{-1} G_i C_i] \zeta_i(t) \\ &+ [I_i - B_i (G_i C_i B_i)^{-1} G_i C_i] E_i d_i(t) - (G_i C_i B_i)^{-1} G_i C_i A_i (x_i(t) \\ &- \hat{x}_i(t)) \end{aligned} \tag{4-8}$$

From Eq. (4-8) the unknown matched uncertainties and the bounded actuator faults (not the complete failure) are completely nulled but the dynamics on the sliding surface contain the unknown unmatched uncertainties, disturbance and the state error. The terms in Eq.(4-8) involving unknown unmatched uncertainties and disturbances are multiplied by a matrix:

$$\Psi_i = [I_i - B_i (G_i C_i B_i)^{-1} G_i C_i]$$

To simplify the notation Eq. (4-8) can now be re-written as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{OBC}(t) + \Psi_i \zeta_i(t) + \Psi_i E_i d_i(t) - M_i e_i(t) \tag{4-9}$$

where $M_i = (G_iC_iB_i)^{-1}G_iC_iA_i$ and $e_i(t) = x_i(t) - \hat{x}_i(t) \in \mathbb{R}^n$ is the estimation error. The proposed discontinuous control is:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\| + \mathfrak{z}_i} \tag{4-10}$$

The parameters $\mathfrak{z}_i > 0$ are chosen to reduce the amount of "chattering" of the motion around the sliding surface (Changqing,Patton and Zong, 2010).To satisfy subsystem stability the positive scalar μ_i is chosen according to the following derivation:

$$\begin{aligned}\mu_i > &\|(G_iC_iB_i)^{-1}G_iC_i\beta_i\|(x_i,t) + \kappa_i\|x_i\| + \gamma_i\|(G_iC_iB_i)^{-1}G_iC_iE_i\|\|x_i\| + \eta_i\ \|x_i\| \\ &+ \|(G_iC_iB_i)^{-1}G_iC_iA_i\|\|e_i(t)\|\end{aligned} \tag{4-11}$$

To ensure sliding motion let $\sigma_i(y_i, \hat{x}_i, t) = 0$. Furthermore, the stability of the inter-connected system Eq. (4-1) on the sliding surface is considered in terms of a positive definite summation of individual Lyapunov subsystems components as:

$\sum_{i=1}^{N} V_i\ (\sigma_i(y_i, \hat{x}_i, t)) = \sum_{i=1}^{N}\|\sigma_i(y_i, \hat{x}_i, t)\| > 0$

The derivative of the subsystem Lyapunov functions is:

$$\dot{V}_i(\sigma_i(y_i, \hat{x}_i, t)) = \frac{\sigma_i^T(y_i, \hat{x}_i, t)\dot{\sigma}_i(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|} \tag{4-12}$$

Hence, from Eqs. (4-4) , (4-5) & (4-12) it can be shown that:

$$\begin{aligned}&\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(y_i, \hat{x}_i, t)\right) \\ &= \sum_{i=1}^{N}[-G_iC_iB_i\mu_i + \frac{\sigma_i^T(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|}\ G_iC_iZ_i(t) \\ &+ \frac{\sigma_i^T(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|}\ G_iC_iB_iQ_i(x_i, t) + \frac{\sigma_i^T(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|} G_iC_iE_id_i(t) \\ &+ \frac{\sigma_i^T(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|}\ G_iC_iB_if_i(t) + \frac{\sigma_i^T(y_i, \hat{x}_i, t)}{\|\sigma_i(y_i, \hat{x}_i, t)\|} G_iC_iA_ie_i(t)]\end{aligned} \tag{4-13}$$

which Eq. (4-13) can be re-written as:

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(y_i, \hat{x}_i, t)\right)$$

$$\leq \sum_{i=1}^{N}[-(G_iC_iB_i)[\mu_i - (G_iC_iB_i)^{-1}G_iC_i\|Z_i\| - \|Q_i\| - (G_iC_iB_i)^{-1}G_iC_iE_i\|d_i\| - \|f_i\| + (G_iC_iB_i)^{-1}G_iC_iA_i\|e_i(t)\|] \tag{4-14}$$

Then, according to **A4, A5**, **A6** & **A7** from Chapter 3 Section 3.2 Eq.(4-14):

$$\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(y_i, \hat{x}_i, t)\right)$$

$$\leq \sum_{i=1}^{N}[-(G_iC_iB_i)[\mu_i - (G_iC_iB_i)^{-1}G_iC_i\beta_i(x_i, t) - \kappa_i\|x_i\| - (G_iC_iB_i)^{-1}G_iC_iE_i\gamma_i\,\|x_i\| - \eta_i\,\|x_i\| + (G_iC_iB_i)^{-1}G_iC_iA_i\|e_i(t)\|] \tag{4-15}$$

By suitable choice of μ_i in Eq. (4-11) then $\sum_{i=1}^{N} \dot{V}_i\left(\sigma_i(x_i, t)\right) \leq 0$

To minimize the norms $\|\Psi_i\zeta_i(t)\|$ and $\|\Psi_iE_id_i(t)\|$ corresponding to the unmatched uncertainty and disturbances, respectively, the matrix G_i must be carefully chosen (Castaños,Xu and Fridman, 2006). One choice is $G_i = B_i^TC_i^+$ which if substituted into Eq. (4-8) leads to the following:

(i) **The term:** $[I_i - B_i(B_i^+B_i)^{-1}B_i^+]B_i^{\perp}B_i^{\perp+}Z_i(t)$, with $B_i^TB_i^{\perp} = 0$, i.e.:

$$[I_i - B_i(B_i^+B_i)^{-1}B_i^+]B_i^{\perp}B_i^{\perp+}Z_i(t) = B_i^{\perp}B_i^{\perp+}Z_i(t) \tag{4-16}$$

(ii) **The term:**

$$[I_i - B_i(B_i^+B_i)^{-1}B_i^+]E_id_i(t) = [I_i - B_iB_i^+]E_id_i(t) \tag{4-17}$$

Substituting Eqs. (4-16) & (4-17) into Eq. (4-8) yields the subsystem dynamics during sliding:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{OBC}(t) + T_iZ_i(t) + H_id_i(t) - M_ie_i(t) \tag{4-18}$$

where $T_i = B_i^{\perp}B_i^{\perp+}$ and $H_i = [I_i - B_iB_i^+]E_i$

From Eq. (4-18) it can be observed that the unknown unmatched uncertainties $T_i Z_i(t)$ and disturbances $H_i d_i(t)$ have not been minimized. Hence, another method must be found to minimize these terms and to limit their influence on the subsystem dynamics.

4.3.2 Observer-based control design via LMI framework

Control design is depended on the topology of the connections between subsystems. Therefore LMIs formulation is used according to two procedures:

1- LMIs for overall subsystem one system as (one shot).
2- LMIs for each subsystem individually.

However, there are two procedures to design the continuous control, but they both use the same discontinuous control (OISMC), enabling the designer to choose the best procedure to control inter-connected systems depends on the number of subsystems in the compact (or centralised) system.

4.3.2.1 LMI observer–based control design of a compact system (one shot)

After designing the OISMC, the subsystem sliding dynamics are:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{OBC}(t) + \Gamma_i J_i(t) - M_i e_i(t) \tag{4-19}$$

where $\Gamma_i = [T_i \quad H_i]$ and $J_i(t) = \begin{bmatrix} Z_i(t) \\ d_i(t) \end{bmatrix}$

The aggregated system dynamics are given by:

$$\dot{X}(t) = A_d X(t) + B_d U^{OBC}(t) + \Gamma_d J(t) - M_d e(t) \tag{4-20}$$

Where $(t) = [x_1, x_2, \dots \dots, x_n]$, $U^{OBC}(t) = [u_1^{OBC}, u_2^{OBC}, \dots \dots, u_n^{OBC}]$,

$e(t) = [e_1, e_2, \dots \dots, e_n]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $\Gamma_d = diag(\Gamma_i)$ and

$J(t) = [J_1, J_2, \dots \dots, J_n]$, where "$diag$" represents the block diagonal matrix.

To develop a robust control law for the aggregate system (one shot) consider a state estimate feedback of the form:

$$U^{OBC}(t) = K\hat{X}(t) = KX(t) - Ke(t) \tag{4-21}$$

where $K = diag(k_i)$ is the de-centralized control system gain that stabilizes the system under a specific performance objective. The design objective is to choose the gain K to minimize the effect of $J(t)$ on all the subsystems. Suppose further that $J(t)$ is the unknown input disturbance which satisfies the quadratic inequality ((Zecevic and Šiljak, 2010):

$$J^T(t)J(t) \leq \alpha^2 X^T(t)X(t) \tag{4-22}$$

where $\alpha > 0$ a positive constant. Any suitable observer can be used to estimate the aggregate system state $\hat{X}(t)$. However, the observer subsystems are given by:

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i^{OBC}(t) + L_i(y_i(t) - C_i\hat{x}_i(t)) \tag{4-23}$$

where L_i is the subsystem observer gain. The aggregate observer dynamics are thus:

$$\dot{\hat{X}}(t) = A_d\hat{X}(t) + B_d U^{OBC}(t) + L_d(Y(t) - C_d\hat{X}(t)) \tag{4-24}$$

where $(t) = [y_1, y_2, \dots \dots., y_n]$, $L_d = diag(L_i)$ and $C_d = diag(C_i)$

Subtracting Eq. (4-20) from Eq. (4-24) yields the state estimation error:

$$\dot{e}(t) = (A_d - L_d C_d)e(t) + \Gamma_d J(t) - M_d e(t) \tag{4-25}$$

To check the stability of the observer-based closed-loop system the following candidate Lyapunov function is used:

$V(X,t) = X^T(t)PX(t) + e^T(t)Fe(t)$ where $P > 0$ and $F > 0$.

The time derivative of $V(X,t)$is thus:

$$\dot{V}(X,t) = \dot{X}^T(t)PX(t) + X^T(t)P\dot{X}(t) + \dot{e}^T(t)Fe(t) + e^T(t)F\dot{e}(t) \tag{4-26}$$

Substituting Eqs. (4-21) & (4-20) into Eq. (4-26) and substituting Eq. (4-25) into Eq. (4-26):

$$\begin{aligned} \dot{V}(X,t) = {} & X^T(t)[{A_d}^T P + K^T {B_d}^T P + PB_d K]X(t) - e^T[K^T {B_d}^T P + {M_d}^T P]X(t) \\ & + J^T(t){\Gamma_d}^T PX(t) - X^T(t)[PB_d K + PM_d]e(t) + X^T(t)P\Gamma_d J(t) \\ & + e^T(t)[{A_d}^T F + {C_d}^T {L_d}^T F - {M_d}^T F + FA_d + FL_d C_d - FM_d]e(t) \\ & + J^T(t){\Gamma_d}^T Fe(t) + e^T(t)F\Gamma_d J(t) \end{aligned} \tag{4-27}$$

The stability of the subsystem Eq. (4-27) requires that condition $\dot{V}(X,t) < 0 \; \forall \; X(t) \neq 0$ so that the Lyapunov stability theory is satisfied. Equation Eq. (4-27) can then be re-written as:

$$\mathcal{Z}^T\mathcal{D}\mathcal{Z} < 0 \tag{4-28}$$

where: $\mathcal{Z} = \begin{bmatrix} X(t) \\ e(t) \\ J(t) \end{bmatrix}$ and :

$$\mathcal{D} = \begin{bmatrix} {A_d}^T P + PA_d + K^T {B_d}^T P + PB_d K & -PB_d K - PM_d & P\Gamma_d \\ -K^T {B_d}^T P - {M_d}^T P & {A_d}^T F + FA_d + {C_d}^T {L_d}^T F + FL_d C_d - FM_d - {M_d}^T F & F\Gamma_d \\ {\Gamma_d}^T P & {\Gamma_d}^T F & 0 \end{bmatrix} < 0 \tag{4-29}$$

To guarantee stability of the system Eq. (4-28) the matrix $\mathcal{D}$ must be negative-definite.

Furthermore, Eq. (4-22) can be rewritten as:

$$\mathcal{Z}^{\mathrm{T}}\mathcal{O}\mathcal{Z} \leq 0 \tag{4-30}$$

where: $\mathcal{Z}_i = \begin{bmatrix} X(t) \\ e(t) \\ J(t) \end{bmatrix}$ and $\mathcal{O} = \begin{bmatrix} -\alpha^2 I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$

To combine Eqs. (4-28) & (4-30) into a single inequality matrix ,the so-called S-procedure is now used (Šiljak and Stipanovic, 2001).

If $\mathcal{D}$ and $\mathcal{O}$ can be considered as symmetric matrices then $\mathcal{Z}^T\mathcal{D}\mathcal{Z} < 0$ and $\mathcal{Z}^T\mathcal{O}\mathcal{Z} \leq 0$. Hence, there is a number $\tau > 0$ where $-\tau\mathcal{O} < 0$ and it follows that:

$$\mathcal{D} - \tau\mathcal{O} = \begin{bmatrix} {A_d}^T P + PA_d + K^T {B_d}^T P + PB_d K + \tau\alpha^2 I & -PB_d K - PM_d & P\Gamma_d \\ -K^T {B_d}^T P - {M_d}^T P & {A_d}^T F + FA_d + {C_d}^T {L_d}^T F + FL_d C_d - FM_d - {M_d}^T F & F\Gamma_d \\ {\Gamma_d}^T P & {\Gamma_d}^T F & -\tau I \end{bmatrix} < 0 \tag{4-31}$$

Substituting $\mathcal{Y} = \frac{P}{\tau}$ and Ⅎ $= \frac{F}{\tau}$ into Eq. (4-31) yields:

$$\Pi = \begin{bmatrix} {A_d}^T\mathcal{Y} + \mathcal{Y}A_d + K^T{B_d}^T\mathcal{Y} + \mathcal{Y}B_dK + \alpha^2 I & -\mathcal{Y}B_dK - \mathcal{Y}M_d & \mathcal{Y}\Gamma_d \\ -K^T{B_d}^T\mathcal{Y} - {M_d}^T\mathcal{Y} & {A_d}^T\Finv + \Finv A_d + {C_d}^T{L_d}^T\Finv + \Finv L_dC_d - \Finv M_d - {M_d}^T\Finv & \Finv\Gamma_d \\ {\Gamma_d}^T\mathcal{Y} & {\Gamma_d}^T\Finv & -I \end{bmatrix} < 0 \tag{4-32}$$

The inequality Eq. (4-32) cannot be solved via an LMI tool since it includes the non-linear term $\mathcal{Y}B_dK$ to overcome this problem both sides of Eq. (4-32) must be multiplied by the matrix $\mathcal{W}$:

$\mathcal{W} = \begin{bmatrix} \mathcal{Y}^{-1} & 0 \\ 0 & \mathcal{T} \end{bmatrix}$ where $\mathcal{T} = \begin{bmatrix} \mathcal{Y}^{-1} & 0 \\ 0 & S \end{bmatrix}$ where S is a design parameter.

Hence:

$$\mathcal{W}\Pi\mathcal{W}^T = \begin{bmatrix} \mathcal{Y}^{-1} & 0 \\ 0 & \mathcal{T} \end{bmatrix} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^{-1} & 0 \\ 0 & \mathcal{T} \end{bmatrix} = \begin{bmatrix} \mathcal{Y}^{-1}\Pi_{11}\,\mathcal{Y}^{-1} & \mathcal{Y}^{-1}\Pi_{12}\mathcal{T} \\ \mathcal{T}\Pi_{21}\mathcal{Y}^{-1} & \mathcal{T}\Pi_{22}\mathcal{T} \end{bmatrix} \tag{4-33}$$

Also, by making a partition of Π_i into four parts:

$$\Pi_{11} = {A_d}^T\mathcal{Y} + \mathcal{Y}A_d + K^T{B_d}^T\mathcal{Y} + \mathcal{Y}B_dK + \alpha^2 I \tag{4-34}$$

$$\Pi_{12} = [-\mathcal{Y}B_dK - \mathcal{Y}M_d \quad \mathcal{Y}\Gamma_d] \tag{4-35}$$

$$\Pi_{21} = \begin{bmatrix} -K^T{B_d}^T\mathcal{Y} - {M_d}^T\mathcal{Y} \\ {\Gamma_d}^T\mathcal{Y} \end{bmatrix} \tag{4-36}$$

$$\Pi_{22} = \begin{bmatrix} {A_d}^T\Finv + \Finv A_d + {C_d}^T{L_d}^T\Finv + \Finv L_dC_d - \Finv M_d - {M_d}^T\Finv & \Finv\Gamma_d \\ {\Gamma_d}^T\Finv & -I \end{bmatrix} \tag{4-37}$$

The term $\mathcal{T}\Pi_{22}\mathcal{T}$ can then be described using (Ichalal,Marx,Ragot and Maquin, 2010) as:

$$\mathcal{T}\Pi_{22}\mathcal{T} \le -\lambda(\mathcal{T} + \mathcal{T}^T) - \lambda^2\Pi_{22}^{-1} \tag{4-38}$$

where $\lambda > 0$ is used for tuning to get an acceptable response.

$$\mathcal{W}\Pi\mathcal{W}^T = \begin{bmatrix} \mathcal{Y}^{-1}\Pi_{11}\,\mathcal{Y}^{-1} & \mathcal{Y}^{-1}\Pi_{12}\mathcal{T} \\ \mathcal{T}\Pi_{21}\mathcal{Y}^{-1} & -\lambda(\mathcal{T} + \mathcal{T}^T) - \lambda^2\Pi_{22}^{-1} \end{bmatrix} < 0 \tag{4-39}$$

By using the Schur complement Eq. (4-39) could rewritten as:

$$\begin{bmatrix} \mathcal{Y}^{-1}\Pi_{11}\,\mathcal{Y}^{-1} & \mathcal{Y}^{-1}\Pi_{12}\mathcal{T} & 0 \\ \mathcal{T}\Pi_{21}\mathcal{Y}^{-1} & -2\lambda\mathcal{T} & \lambda I \\ 0 & \lambda I & \Pi_{22} \end{bmatrix} < 0 \qquad (4\text{-}40)$$

Substituting $\mathcal{P} = \mathcal{Y}^{-1}$ in Eq.(4-40) and also substituting Eqs. (4-34) , (4-35) , (4-36) & (4-37) into Eq. (4-40) yields:

$$\begin{bmatrix} \mathbb{W} & -B_dK\mathcal{P} - M_d\mathcal{P} & \Gamma_d S & 0 & 0 \\ -\mathcal{P}K^TB_d{}^T - \mathcal{P}M_d{}^T & -2\lambda\mathcal{P} & 0 & \lambda I & 0 \\ S\Gamma_d{}^T & 0 & -2\lambda S & 0 & \lambda I \\ 0 & \lambda I & 0 & \mathbb{L} & F\Gamma_d \\ 0 & 0 & \lambda I & \Gamma_d{}^T\Finv & -I \end{bmatrix} < 0 \qquad (4\text{-}41)$$

where $\mathbb{W} = \mathcal{P}A_d{}^T + A_d\mathcal{P} + \mathcal{P}K^TB_d{}^T + B_dK\mathcal{P} + \alpha^2\mathcal{P}\mathcal{P}$ and

$\mathbb{L} = A_d{}^T\Finv + \Finv A_d + C_d{}^TL_d{}^T\Finv + \Finv L_dC_d - \Finv M_d - M_d{}^T\Finv$

Choose $= I$, and substituting $N = K\mathcal{P}$, $R = \Finv L_d$ and $\epsilon = \frac{1}{\alpha^2}$, by using the Schur complement Eq. (4-41) is re-written as:

$$\begin{bmatrix} \overline{\overline{\mathbb{W}}} & -B_dN - M_d\mathcal{P} & \Gamma_d & 0 & 0 & \mathcal{P} \\ -N^TB_d{}^T - \mathcal{P}M_d{}^T & -2\lambda\mathcal{P} & 0 & \lambda I & 0 & 0 \\ \Gamma_d{}^T & 0 & -2\lambda & 0 & \lambda I & 0 \\ 0 & \lambda I & 0 & \overline{\overline{\mathbb{L}}} & F\Gamma_d & 0 \\ 0 & 0 & \lambda I & \Gamma_d{}^T\Finv & -I & 0 \\ \mathcal{P} & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix} < 0 \qquad (4\text{-}42)$$

where $\overline{\overline{\mathbb{W}}} = \mathcal{P}A_d{}^T + A_d\mathcal{P} + N^TB_d{}^T + B_dN$ and

$$\overline{\overline{\mathbb{L}}} = A_d{}^T\Finv + \Finv A_d + C_d{}^TR_d^T + RC_d - \Finv M_d - M_d{}^T\Finv$$

The augmented system is globally stable in the Lyapunov sense if the matrices N, , R and L_d can be found to satisfy the LMI of Eq.(4-42):

There are two approaches to solving this LMI, as follows:

Algorithm 4-1:

1- Calculate the $\sigma_i(y_i, \hat{x}_i, t)$ from the Eq.(4-4)
2- Use the $\sigma_i(y_i, \hat{x}_i, t)$ to get the discontinuous controller from the Eq.(4-10)
3- Calculate the aggregate system from the Eq.(4-20)
4- Calculate the aggregate observer from Eq(4-23)
5- Minimize ϵ subject to $\mathcal{P} > 0$, $\Finv > 0$ and the Eq. (4-42)

6- The aggregate system control gain can be calculated from $K = N\mathcal{P}^{-1}$

7- The aggregate system observer gain can be calculated from $L_d = \Finv^{-1}R$

To minimize the gain magnitude the conditioning of the matrices N and R are given in terms of norm bounds $\|N\|^2 < K_N I$ and $\|R\|^2 < K_R I$ (Zecevic and Šiljak, 2005):

where K_N and K_R are scalar variables, and by using the Schur complement inequalities Eqs. (4-43) & (4-44) can be added to Eq. (4-42)as follows:

$$\begin{bmatrix} -K_N I & N^T \\ N & -I \end{bmatrix} < 0 \quad \text{and} \quad \begin{bmatrix} -K_R I & R^T \\ R & -I \end{bmatrix} < 0 \tag{4-43}$$

Additional inequalities can be added to the matrices $\mathcal{P}$ and $\Finv$ (Zecevic and Šiljak, 2005).

$$\begin{bmatrix} \mathcal{P} & I \\ I & K_P I \end{bmatrix} > 0 \quad \text{and} \quad \begin{bmatrix} \Finv & I \\ I & K_{\Finv} I \end{bmatrix} > 0 \tag{4-44}$$

where K_P and $K_{\Finv}$ are scalar variables.

Algorithm 4-2:

The same procedure as in *algorithm 4-1* is used by replacing step 5 by:

Minimize $(\epsilon + K_N + K_P + K_R + K_{\Finv})$ subject to $\mathcal{P} > 0$, $\Finv > 0$, the Eqs. (4-42) , (4-43) & (4-44).

4.3.2.2 LMI observer–based control design for every subsystem individually

After design the OISMC as in Section 4.3.1 the subsystem on sliding mode will be as in Eq. (4-19):

To develop a robust control law let the feedback control has the form:

$$u_i^{OBC}(t) = k_i \hat{x}_i(t) = k_i x_i(t) - k_i e_i(t) \tag{4-45}$$

where k_i is the constant gain. It is assumed that this gain is determined via an LMI tool to stabile the subsystem i^{th} under specific performance as well as decrease the influence of $J_i(t)$ on the subsystem.

The estimated state $\hat{x}_i(t)$ can be obtained by using a Luenberger observer based on *n* inter-connected system. This de-centralized observer is designed depending on local information (input and output) and no connections between the other observers or subsystems. The *i*^th^ subsystem observers can be expressed as:

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i^{OBC}(t) + L_i(y_i(t) - C_i\hat{x}_i(t)) \tag{4-46}$$

where L_i is the constant gain that stabiles observer i^{th}.

By subtracting Eq. (4-46) from Eq. (4-45) the resulting equation gives the subsystem state estimation errors as follows:

$$\dot{e}_i(t) = (A_i - L_iC_i)e_i(t) + \Gamma_i J_i(t) - M_i e_i(t) \tag{4-47}$$

The same Lyapunov procedure as described in Section 4.3.2.1 is used to check the stability of each closed-loop subsystem comprising local subsystem controllers and observers, using the Lyapunov candidate functions $V_i(x_i, t) = x_i^T(t)P_i x_i(t) + e_i^T(t)F_i e_i(t)$, with $P_i > 0$ and $F_i > 0$.

The results that are obtained from this procedure give the flowing LMI:

$$\begin{bmatrix} \overline{\overline{\mathbb{W}}}_i & -B_iN_i - M_i\mathcal{P}_i & \Gamma_i & 0 & 0 & \mathcal{P}_i \\ -N_i^T B_i^T - \mathcal{P}_i M_i^T & -2\lambda_i\mathcal{P}_i & 0 & \lambda_i I_i & 0 & 0 \\ \Gamma_i^T & 0 & -2\lambda_i & 0 & \lambda_i I_i & 0 \\ 0 & \lambda_i I_i & 0 & \overline{\overline{\mathbb{L}}}_i & F_i\Gamma_i & 0 \\ 0 & 0 & \lambda_i I_i & \Gamma_i^T \Finv_i & -I_i & 0 \\ \mathcal{P}_i & 0 & 0 & 0 & 0 & -\epsilon_i I_i \end{bmatrix} < 0 \tag{4-48}$$

where $\overline{\overline{\mathbb{W}}}_i = \mathcal{P}_i A_i^T + A_i\mathcal{P}_i + N_i^T B_i^T + B_i N_i$

$\overline{\overline{\mathbb{L}}} = A_i^T \Finv_i + \Finv_i A_i + C_i^T R_i^T + R_i C_i - \Finv_i M_i - M_i^T \Finv_i$

The other variables are:

$\Finv_i > 0$, $\mathcal{P}_i > 0$, $N_i = k_i\mathcal{P}_i$, $R_i = \Finv_i L_i$, $\epsilon_i = \frac{1}{\alpha_i^2}$ and λ_i is a tuning matrix for every subsystem.

Algorithm 4-3:

1- Calculate the $\sigma_i(y_i, \hat{x}_i, t)$ from the Eq.(4-4)
2- Use the $\sigma_i(y_i, \hat{x}_i, t)$ to get the discontinuous controllers from Eq.(4-10)
3- Minimize the ϵ_i subject to $\mathcal{P}_i > 0$, $\Finv_i > 0$ & Eq. (4-48)

4- The controller gains can be calculated from the $k_i = N_i \, \mathcal{P}_i^{-1}$

5- The observer gain can be calculated from the $L_i = Ⅎ_i^{-1} R_i$

This algorithm follows *Algorithm 3-2*, with the addition of conditions to reduce the norms of the control and observer gains in the LMI framework.

To limit the conditioning of the controller designs the Euclidean norms $\|N_i\|$ and $\|R_i\|$ of the gains N_i and R_i are bounded according to the following:

$$\begin{bmatrix} -k_{Ni} I_i & N_i^T \\ N_i & -I_i \end{bmatrix} < 0 \quad \text{and} \quad \begin{bmatrix} -k_{Ri} I_i & R_i^T \\ R_i & -I_i \end{bmatrix} < 0 \qquad (4\text{-}49)$$

where k_{Ni} and k_{Ri} are scalar variables.

To limit the conditioning of the observer designs the Euclidean norms $\|\mathcal{P}_i\|$ and $\|Ⅎ_i\|$ are bounded as follows:

$$\begin{bmatrix} \mathcal{P}_i & I_i \\ I_i & k_{Pi} I_i \end{bmatrix} > 0 \quad \text{and} \quad \begin{bmatrix} Ⅎ_i & I_i \\ I_i & k_{Ⅎ_i} I_i \end{bmatrix} > 0 \qquad (4\text{-}50)$$

where k_{Pi} and $k_{Ⅎ_i}$ are scalar variables.

Algorithm 4-4:

The same procedure as in *algorithm 4-3* is used by replacing step 3 by:

Minimize $(\epsilon_i + k_{Ni} + k_{Pi} + k_{Ri} + k_{Ⅎ_i})$ subject to $\mathcal{P}_i > 0$, $Ⅎ_i > 0$, the Eqs. (4-48) , (4-49) & (4-50).

4.4 Numerical example

Consider the following numerical example consisting of three non-linear inter-connected systems adapted from (Castaños,Xu and Fridman, 2006) :

1st Subsystem:

$$A_1 = \begin{bmatrix} 0 & -6 \\ 6 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, z_1 = \left(\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} \right)$$

$$W_1(x_1, t) = \begin{bmatrix} 0 \\ 4\cos(2t) x_{11} - 2\sin(t) x_{12} \end{bmatrix}, x_1(0) = \begin{bmatrix} 0.4 \\ -0.1 \end{bmatrix} \text{ and } x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}$$

2nd Subsystem:

$$A_2 = \begin{bmatrix} 0 & -1 \\ -2 & -7 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},\ z_2 = \left(\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix}\right)$$

$$W_2(x_2,t) = \begin{bmatrix} 0 \\ 2\sin(t)x_{21} + 4\cos(2t)x_{22} \end{bmatrix},\ x_2(0) = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix} \text{ and } x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

3rd Subsystem:

$$A_3 = \begin{bmatrix} 0 & -1 \\ -4 & -5 \end{bmatrix},B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_3 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},\ z_3 = \left(\begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}\right)$$

$$W_3(x_3,t) = \begin{bmatrix} 0 \\ 6\sin(t)x_{31} + 2\cos(2t)x_{32} \end{bmatrix},\ x_3(0) = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix} \text{ and } x_3(t) = \begin{bmatrix} x_{31}(t) \\ x_{32}(t) \end{bmatrix}$$

$$d_1(t) = d_2(t) = d_3(t) = 0.3 + 0.01\sin(100t)$$

It can be seen that the subsystem systems without controls are unstable.

4.4.1 Simulations and results

This Section illustrates results and simulations of applying the above algorithms to the numerical example. The same OISMC design is used for each subsystem but a different choice of LMI algorithm is used for each of the controller and observer gains.

4.4.1.1 Observer-based control design of compact system (one shot)

The continuous subsystem control $u_i^{OBC}(t)$ is obtained via the LMI tool corresponding to *Algorithm 4-2* yielding the following controller and observer gains:

Controller gains:

$k_1 = [-4.9522 \quad -4.4306]$, $k_2 = [5.6854 \quad 3.6946]$ and $k_3 = [7.6854 \quad 1.6946]$

Observer gains:

$L_1 = \begin{bmatrix} 5.2947 & -0.1504 \\ 0 & 0 \end{bmatrix}$, $L_2 = \begin{bmatrix} 7.7790 & 0 \\ 0 & 0 \end{bmatrix}$ and $L_3 = \begin{bmatrix} 7.7790 & 0 \\ 0 & 0 \end{bmatrix}$

The tuning design parameter of the aggregate systems is chosen to be $\lambda = 1.86$

Furthermore, the gains of the discontinuous controllers are: $\mu_1 = \mu_2 = \mu_3 = 5$ and the constants chosen to eliminate the chattering motion are $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.2$

All three subsystems without controls are unstable, one reason being the interactions between them and the disturbances. Figure 4-2 shows the response of these systems without control.

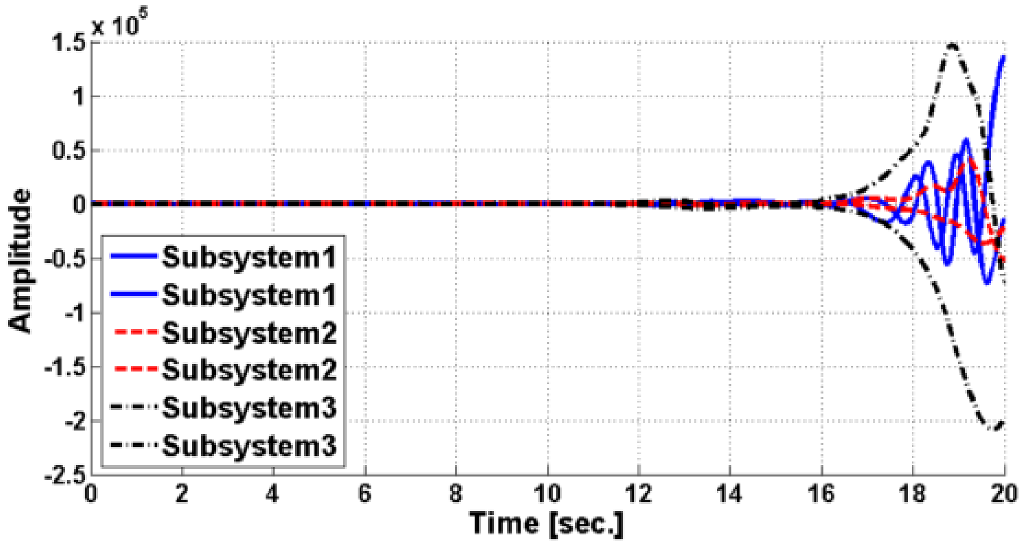

Figure 4-2 : Three subsystems without control

Figure 4-3 shows the response of all three subsystems under using output de-centralized (LMI + OISMC) with no faults. These results illustrate that the controllers give an acceptable and stable response for all of the subsystems.

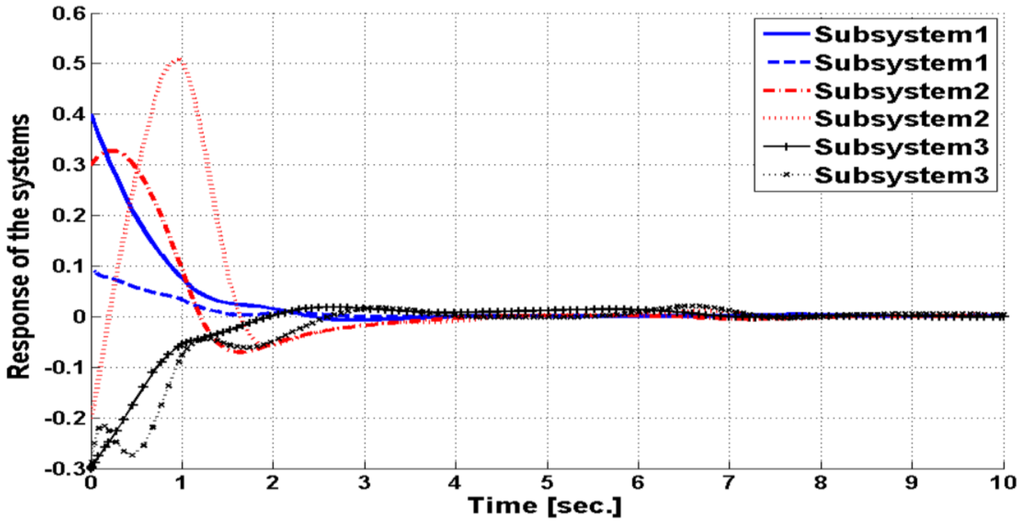

Figure 4-3: Three subsystems with controls and without faults

Figure 4-4 ,Figure **4-5** &Figure **4-6** illustrate the simulation results of the states of every subsystem and their estimates when there are no faults in all subsystems (actuators and sensors). From these results, the estimated and true state values are almost identical, with a little difference at the start of the simulated responses.

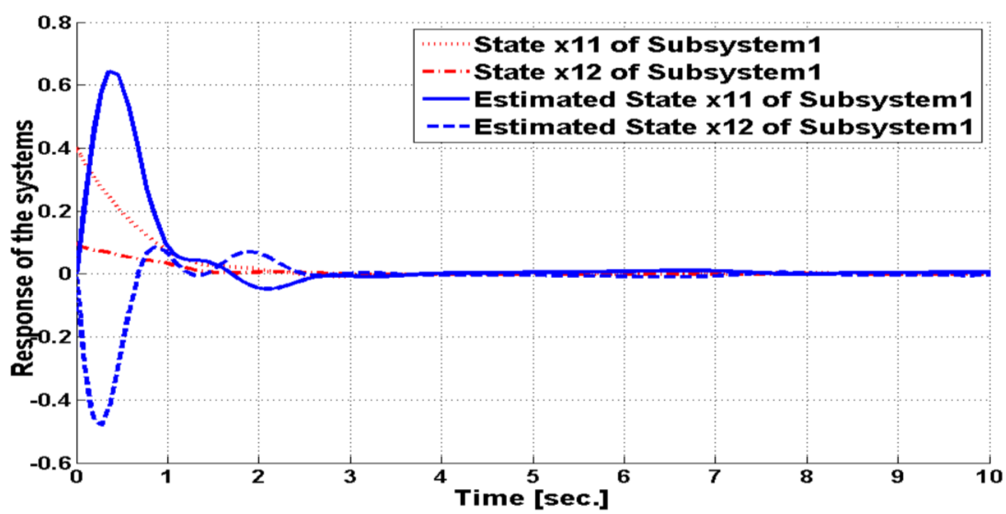

Figure 4-4: States of 1st subsystem and its estimated

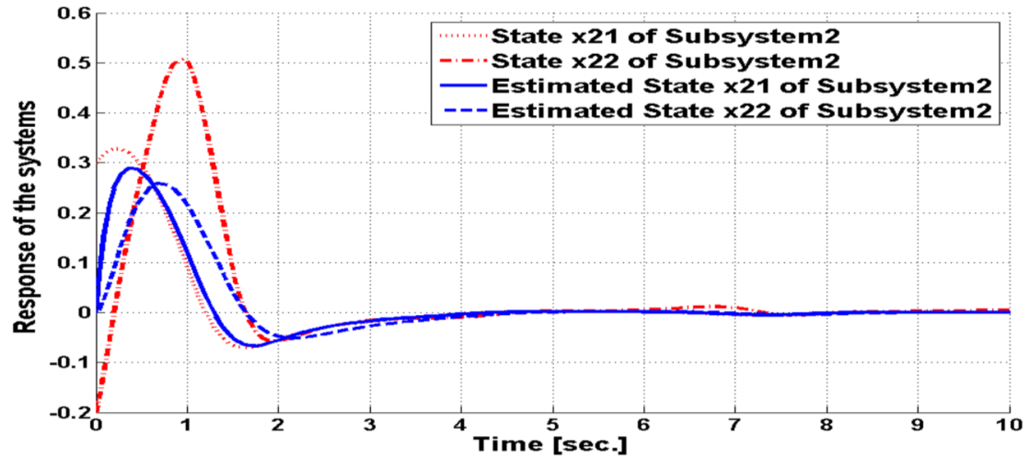

Figure 4-5 : States of 2nd subsystem and it's estimated

These estimated states are used as feedback signals to control the subsystems. Figure 4-7 illustrates the simulation response of a 50% fault scenario in the 1st subsystem actuator, with the remaining two actuators considered as fault-free. The effects of 1st subsystem actuator fault on the remaining two subsystems are also shown. From the Figure, it is clear that the controller compensates the faults and attenuates any disturbance effects.

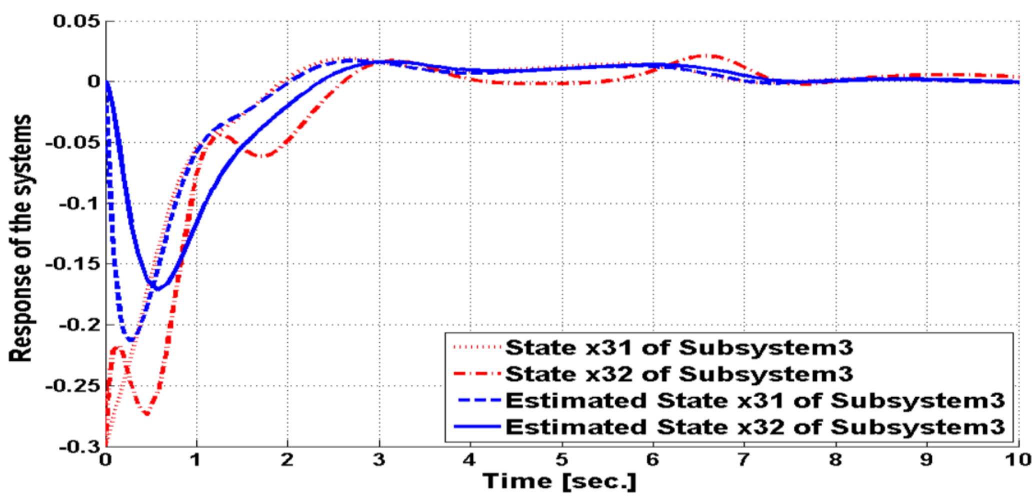

Figure 4-6 : States of 3rd subsystem and its estimates

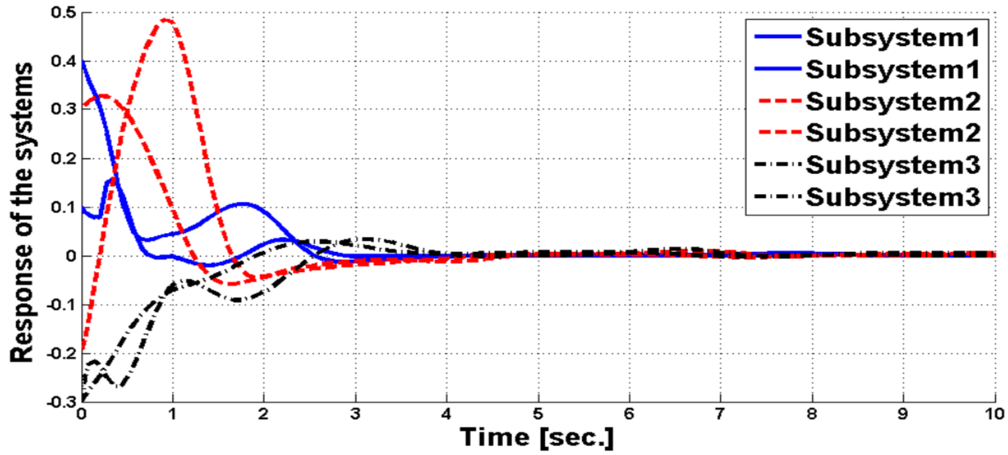

Figure 4-7: Three subsystems with controls and with a 50% actuator fault in 1st subsystem

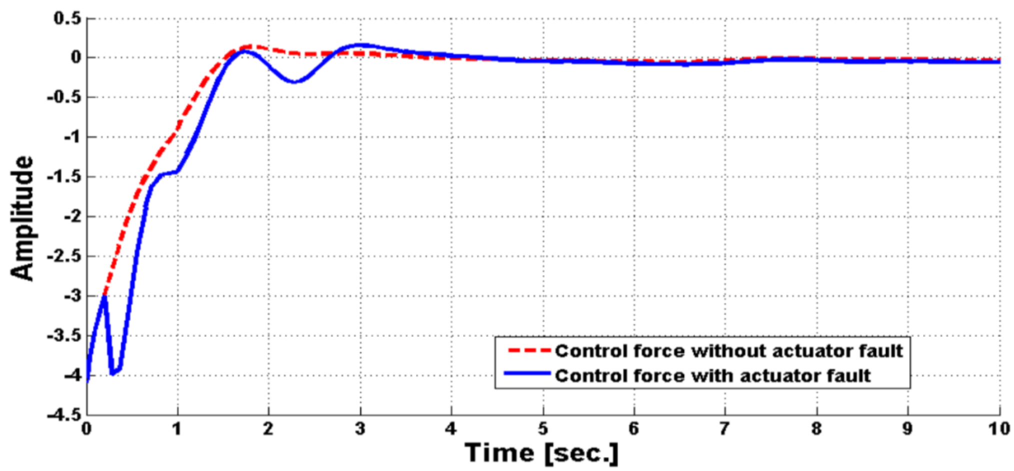

Figure 4-8: Control signal of the 1st subsystem with 50% actuator fault

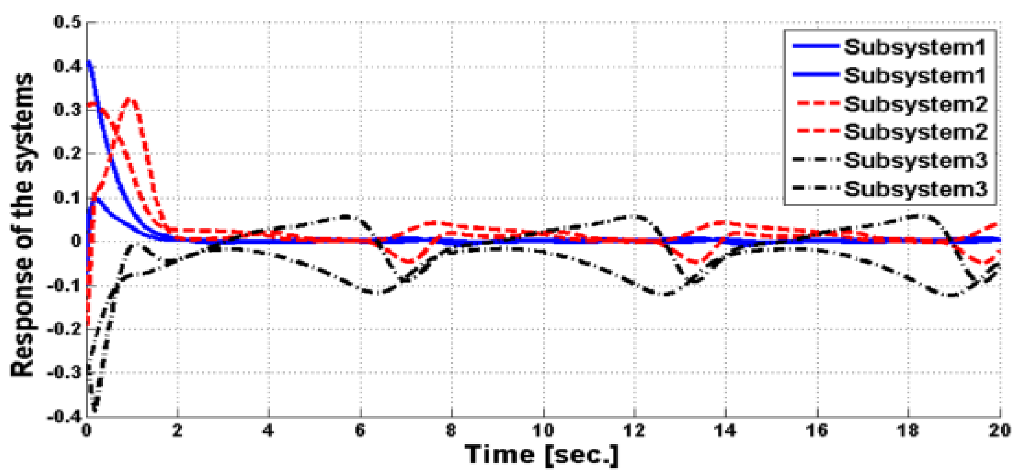

Figure 4-9: Three subsystems with controls and with 50% actuator fault in 1st, 2nd and 3rd subsystems

Figure 4-8 shows the control signal of the 1st subsystem for (a) no fault and (b) for a 50% actuator fault. Therefore the control signal compensates the actuator fault.

Figure **4-9** shows the simulation of a fault scenario for all three subsystems with 50% actuator faults. From the Figure it can be seen that all the subsystems are affected by the faults and disturbance. The faults cause a loss of desired response. However, all subsystems will be unstable, as shown in Figure 4-10 when all the subsystems have 70% faults in their actuators.

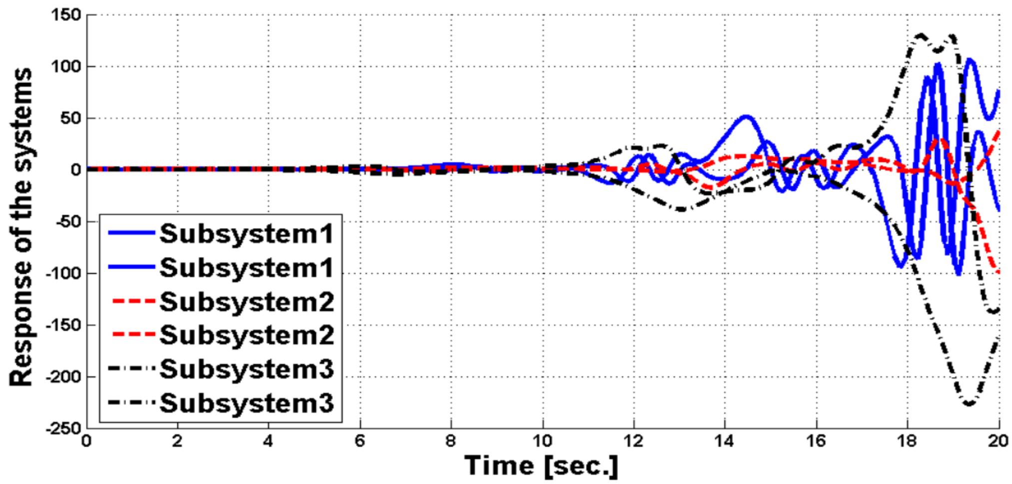

Figure 4-10 : Three subsystems with controls and with 70% actuator fault of 1st, 2nd and 3rd subsystems

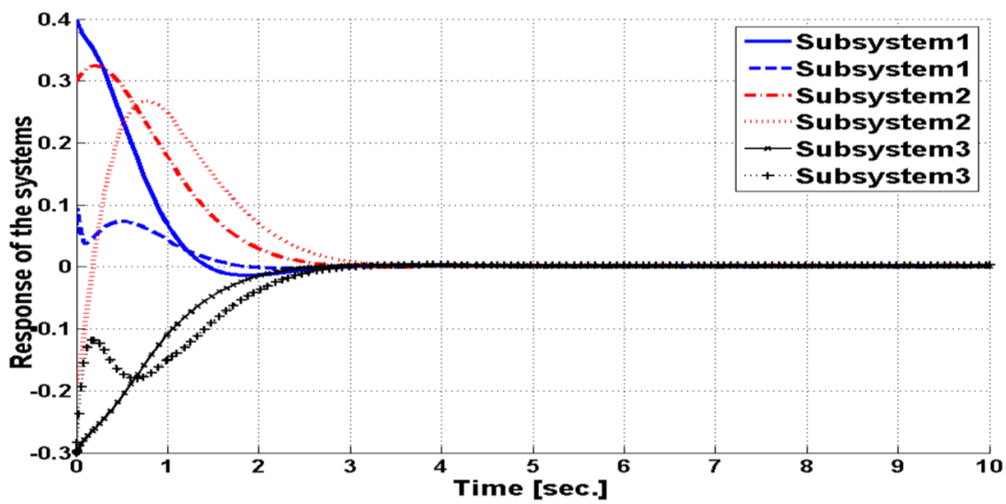

Figure 4-11: Three linear subsystems with controls but no faults and no interconnections or uncertainties

Figure 4-11 shows the response of all three linear subsystems resulting from the use of output de-centralized LMI + OISMC with no faults, no interconnections and no uncertainties.

4.4.1.2 Observer-based control design of subsystem by subsystem

The continuous control $u_i^{OBC}(t)$ designed by the LMI tool of *algorithm 4-4* yields:

Controller's gains:

k_1 = [-5.0191 -4.3222] , k_2 = [6.9282 3.0775] and k_3 = [8.9282 1.0775]

Algorithm 4-4 also yields:

Observer's gains:

$L_1 = \begin{bmatrix} 8.3427 & 0 \\ 0 & 0 \end{bmatrix}$, $L_2 = \begin{bmatrix} 52.9197 & -0.0065 \\ 0 & 0 \end{bmatrix}$ and $L_3 = \begin{bmatrix} 52.9197 & -0.0065 \\ 0 & 0 \end{bmatrix}$

where the design parameters of the subsystems are:

λ_1= 1.91, λ_2= 1.99 and λ_3= 1.99

The resulting subsystem control signals are $u_i(t) = k_i x_i(t) - \mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$

where $\mu_1 = \mu_2 = \mu_3 = 5$ and $\mathfrak{z}_1 = \mathfrak{z}_2 = \mathfrak{z}_3 = 0.2$

Figure 4-12 shows the response of all three subsystems by using output de-centralized (LMI + OISMC) control with no faults. From the results the proposed method gives controllers that can stabilize the inter-connected systems based on feedback from the estimated state. It can be seen that the controllers produce acceptable responses.

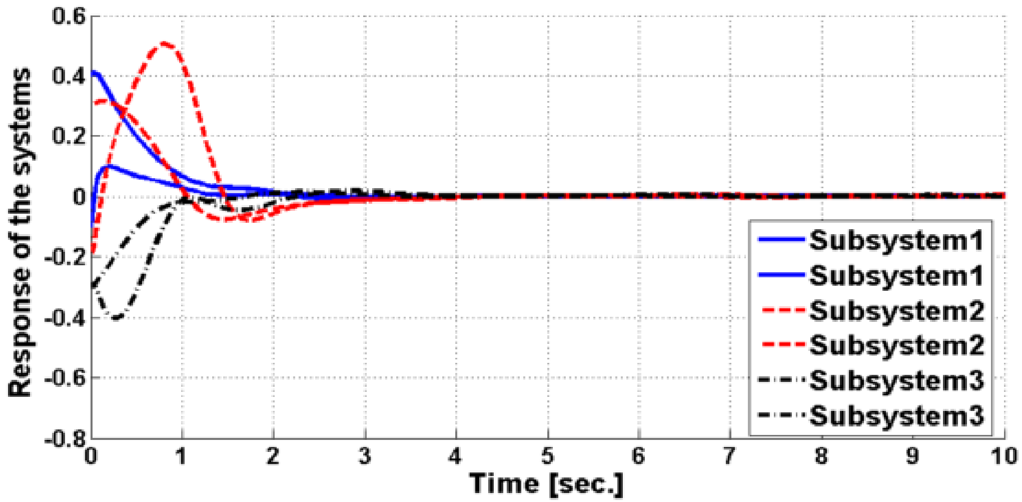

Figure 4-12 : Three subsystems with controls and without faults

The states and their estimates for all the inter-connected subsystems are shown in Figure 4-13 , Figure **4-14** & Figure **4-15** . All the simulations of the states correspond to the fault-free case. Moreover, the Figures show that the estimated values nearly match the true subsystem states, apart from small transient differences.

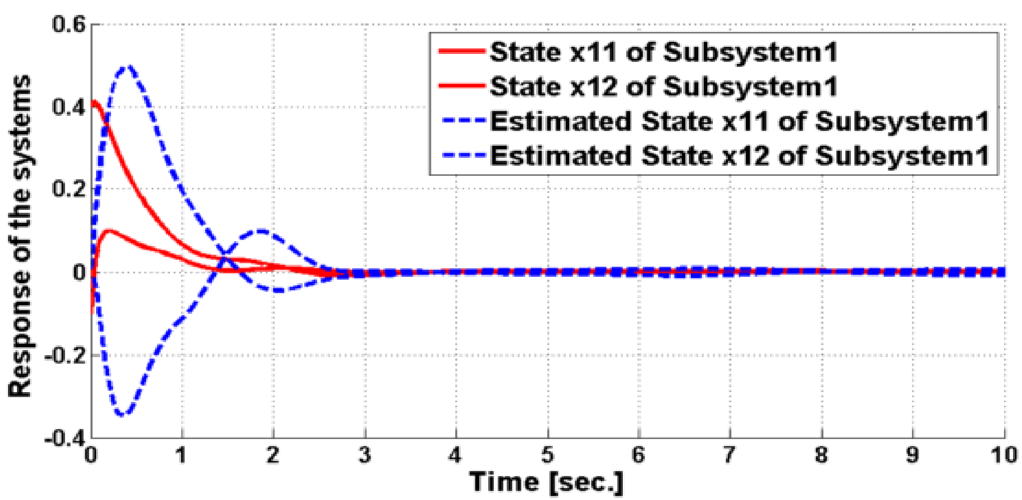

Figure 4-13 : States of 1st subsystem and its estimated

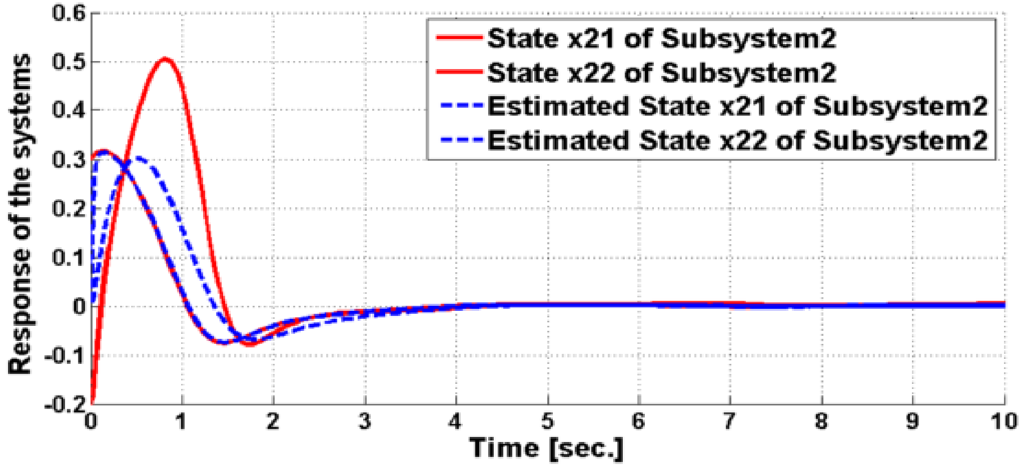

Figure 4-14 : States of 2nd subsystem and it's estimated

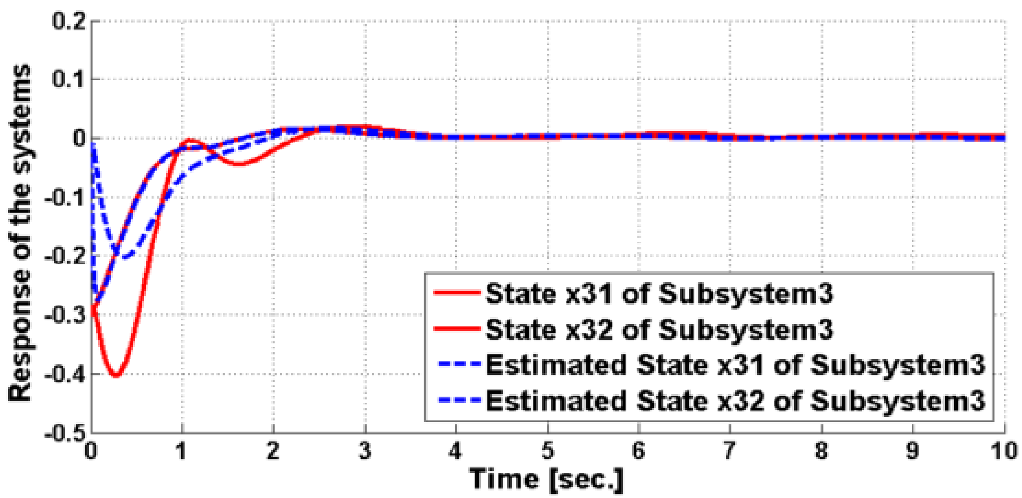

Figure 4-15 : States of 3rd subsystem and its estimated

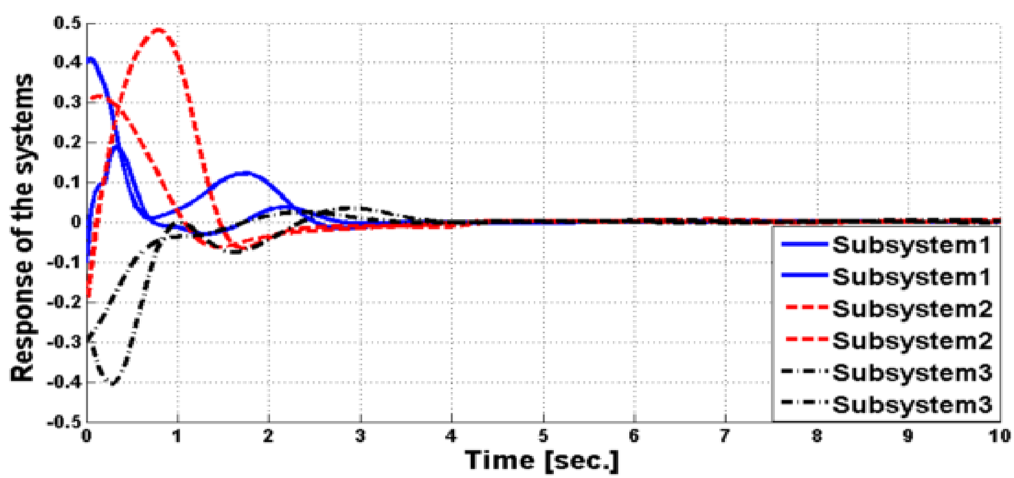

Figure 4-16 : Three subsystems with controls and with 50% actuator fault in 1st subsystem

The effects of a 50% actuator fault of the 1st subsystem on the 1st subsystem itself and the other subsystems is shown in Figure 4-16, where it is assumed that the remaining subsystems are fault-free,. The Figure also shows that all subsystems can compensates the faults and the controller of the 1st subsystem is still capable of stabilizing the 1st subsystem very effectively. The control signal of the 1st subsystem with a 50% actuator fault can be seen in Figure 4-17.

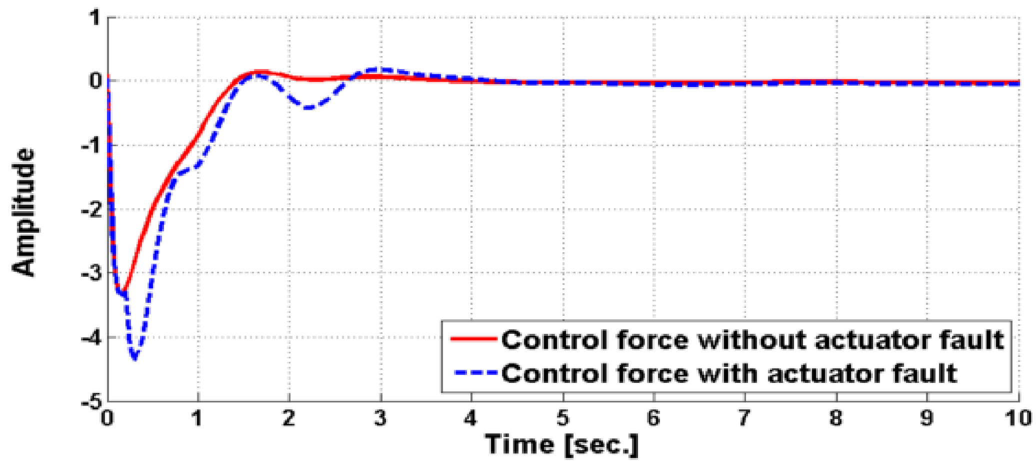

Figure 4-17 : The control signal of the 1st subsystem with a 50% actuator fault

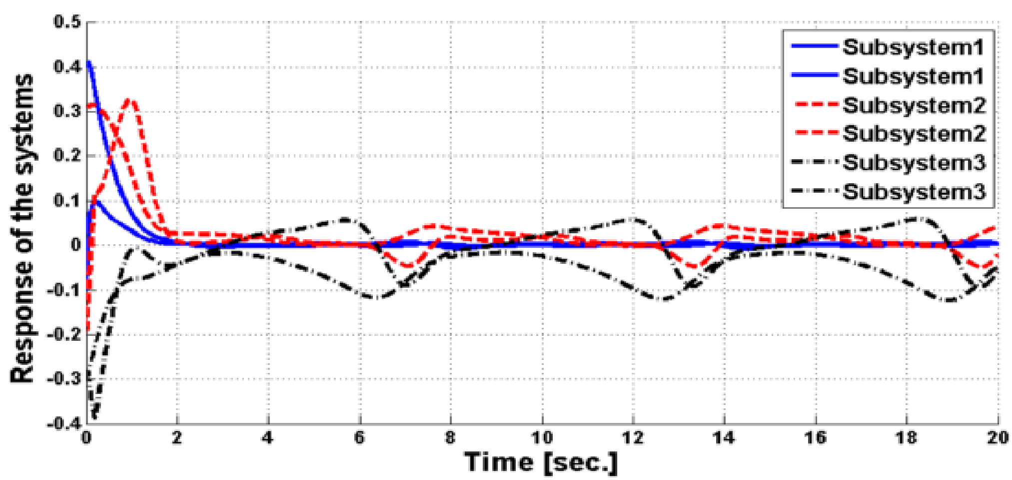

Figure 4-18 : Three subsystems with controls and with 50% actuator faults in the 1st, 2nd and 3rd subsystems

All three subsystems have 50% actuator faults with the responses shown in Figure 4-18. The Figure shows that all the subsystems are affected by each other but they are still stable.

However, if all the faults are increased, all subsystems are unstable, as shown in Figure 4-19 where all the subsystems have 70% faults in their actuators.

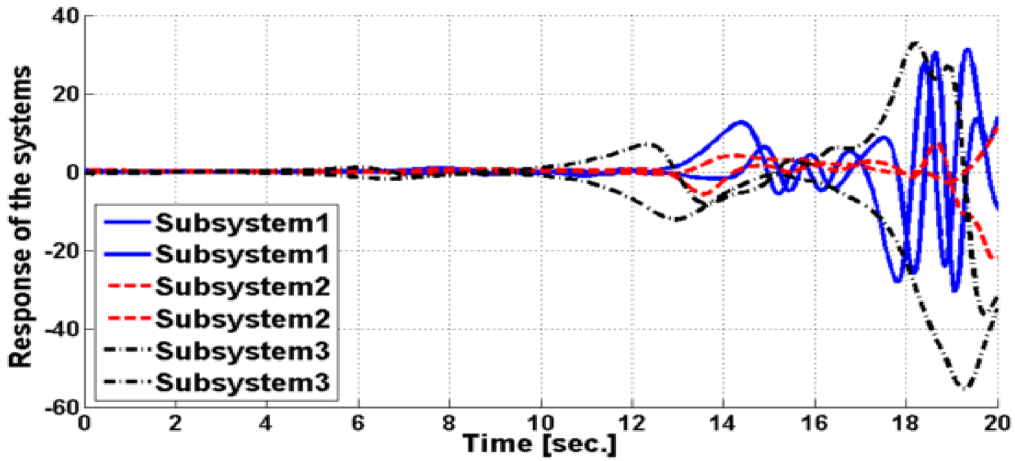

Figure 4-19 : Three subsystems with controls and 70% actuator fault of 1st, 2nd and 3rd subsystems

4.4.1.3 Comparison of the two methods

The comparison of the proposed methods to control non-linear inter-connected systems is illustrated in Table 4-1. To summarize it is assumed that the estimated states replace the true states in a state estimate feedback structure to stabilize the subsystems.

However, the two methods have some similarity but there are some differences in the LMI calculations. The aggregate system only requires the solution of one LMI but in other methods each LMI requires separate solutions according to the number of subsystems. Hence, the LMI solution strategy for the aggregate system is straightforward but the LMI designs for each subsystem give flexibility to tune every subsystem individually to achieve specific subsystem responses and behaviour in large-scale inter-connected systems.

From the Table 4-1, each design method can be used in specific applications. The LMI approach for the aggregate system is useful for large-scale inter-connected systems. In the case of smaller inter-connected systems the approach based on the use of individual subsystem LMI designs becomes more useful. This also shows up the concept of "plug and play" in which individual subsystems can be added or removed if the corresponding subsystems fails or malfunctions (Kragelund,Leth and Wisniewski, 2010, Michelsen and Stoustrup, 2010).

Method / Contents	Observer-based control via LMI for each subsystem separately	Observer-based control via LMI for overall system (one shot)
ISMC	The same discontinuous control	The same discontinuous control
Calculations of gains by LMI	Solve the LMI n times	Solve the LMI only once.
Choosing tuning parameter λ	Every subsystem has its own λ_i	One λ for the aggregate system
Gains of controllers	Assumed identical in all subsystems	Assumed identical in all subsystems
Gains of observers	Slightly higher norms	Lower norm values
Response	Adjust individual subsystems	Adjust only the aggregate system
The best use	Good for small inter-connected systems	Good for large-scale inter connected systems

Table 4-1: Comparison of two proposed methods

4.5 Conclusion

A major challenge of the control of uncertain inter-connected systems is to remove or compensate for the effects of uncertainly and disturbances acting in the subsystems so that an ideal decentralization can be achieved. In the ideal case, the resulting hitherto inter-connected system now becomes a truly de-centralized structure in which the subsystems can be designed independently. This approach to the control of complex systems has important consequences for security and fault-tolerance, e.g. if one subsystem fails then this failure does not influence the integrity of the remaining subsystems.

It is assumed in this work that the subsystem states are not available for control and hence the outputs are used together with the classical notion of the state estimate feedback to develop a strategy for de-centralization. Hence, the output de-centralized control is achieved via OISMC together with linear observer design to give robust performance for both matched and unmatched uncertainty and disturbances. The combination of the OISMC and an LMI design framework can be used to give suitably robust control and the parameters λ_i used to achieve desired subsystem response or parameters λ of the aggregate system, according to the method chosen.

Depending on whether or not the subsystems are connected as a large-scale system (according to the designer's consideration of the size of a large-scale system) it is possible to design observer-based control for all the subsystem as an aggregate (one-shot) system. The aggregate design uses a single LMI to achieve stability as well as the minimization of matched/unmatched uncertainties and interactions and control performance specification. But if they are connected in a small-scale system it is possible to design observer-based control for each subsystem and perform tuning to achieve a desired response.

Whilst the design procedure is considerably complex the system implementation is nothing more than the OISMC and linear observers are applied locally to each subsystem.

Suitably robust control of non-linear inter-connected systems via estimated states can be a very challenging design problem since the approach requires a satisfactory implementation of a suitable Separation Principle recovery procedure.

An alternative solution would make use of the output signal directly as a feedback signal in the so-called static or dynamic output feedback control problem as described in Chapters 5 and 6.

Chapter 5 :Static output feedback adaptive ISMC for inter-connected non-linear systems

5.1 Introduction

One of the challenges of feedback control of inter-connected systems is how best to use static feedback design in the presence of modelling uncertainty and interconnections (Yu,Jun-e,Lin and Juan, 2010). During the last few years several researchers have studied the stability of inter-connected systems using static output control, developing different static feedback control strategies (Yan,Edwards and Spurgeon, 2004, Zecevic and Šiljak, 2004). Despite the excellent research more effort is required for further investigations (Cao,Lam and Sun, 1998). Most of the methods used to create static feedback are based on linear systems and little effort and attention has been given to studying non-linear systems. This is especially true for large-scale or inter-connected systems, due to challenges of stability, robustness and complexity of computation (Zecevic and Šiljak, 2004, Yuanwei and Dimirovski, 2006).

For practical applications static output control has several advantages; it may lead to a reduction in implementation cost and a decrease the cost of maintenance (Trofino-Neto and Kucera, 1993). However, the design conditions for controllable and observable static output systems are well known even if the design freedom for such systems is restricted. This is the case for systems with no interconnections as the design complexity increases dramatically when inter-connections are considered (Stephen,Laurent,Eric and Venkataraman, 1994).

A large-scale system can be considered as a combination of inter-connected dynamic subsystems, e.g. infrastructure systems (electrical grids, telecommunications networks and building structure) and some industries such as production lines and oil industries that cannot easily have a single control structure (Poznyak,Fridman and Bejarano, 2004). This Chapter focuses on inter-connected systems that are Lipschitz, i.e. with bounded non-linearity. The non-linearities may come from one or more subsystems or may reflect some non-linear dynamic interconnections between subsystems (Dimirovski,Jing,Yuan and Zhang, 1998). The main challenges for the design of control systems for inter-connected non-linear systems are to design de-centralized controllers that ensure the global stability when the interactions are unknown (Yuanwei and Dimirovski, 2006).

There are many techniques to design static output feedback control; one of them is based on linear matrix inequalities (LMI) which are used to handle the multi-objective control problem arising from the need to solve a joint stability, uncertainty robustness and performance design problem. This is actually an appropriate approach for the design of linear robust control for system that is actually non-linear with bounded non-linearity (Benton and Smith, 1999,Veselý, 2001,Daniel, 2003, Gadewadikar *et al.*, 2007). However, the complexity in this design approach arises from the difficulty in directly determining the feedback gains because of the need to convert the non-convex optimization to a convex one.

Most researchers who use LMI tools rely on the iterative algorithm approach to compute the appropriate gains after determining suitable convergence criteria (Juntao, 2009). The use of iteration is difficult because it depends on choosing the best initial state feedback matrix, which may lead to a non-LMI solution (Wenlong,Yaqiu and Liping, 2006). Also some methods are based on iterative calculation, substitutive LMI and a min/max algorithm (Poznyak,Fridman and Bejarano, 2004). At every iteration cycle the LMIs must be resolved, but these methods do not necessarily guarantee that a solution can be determined (Cao,Lam and Sun, 1998).

An approach to output feedback control of de-centralized systems which does not require the use of iterative LMI calculations can be based on the use of ISMC as an extension of the classical state feedback work of (Castaños and Fridman, 2005, Castaños,Xu and Fridman, 2006) which considers problems in which the disturbances satisfy a matching condition. In the work described in this Chapter a more general ISMC problem is considered in which (a) the control is based on static output feedback, (b) the disturbances considered can be both matched and unmatched. It is shown in this work that the key to achieve both (a) and (b) in a de-centralized control problem is to use adaptive control as a tool that takes into account changes in system parameters, to ensure that the response of the system is automatically controlled according to required closed-loop and stability performance specifications (Bejarano,Fridman and Poznyak, 2007).

This new approach to ISMC is used to enable robust control to be achieved at the local levels of the de-centralized inter-connected system. It will be shown that the ISMC is capable of rejecting matched uncertainties/disturbances arising from a subsystem or the interconnections between subsystems (Mondal and Mahanta, 2012). The idea relies on splitting the control signal into *two* parts; continuous and discontinuous control. The first part is designed via LMI condition and the discontinuous gain is designed by an adaptive output feedback integral sliding control (AOISMC) in which the gain is a function of the output feedback, taking into account known upper bounds for both disturbances and matched components.

The main contributions in this Chapter can be summarized as:

3- The new proposal of an LMI-based design on static output feedback with AOISMC of non-linear inter-connected systems in two cases, one to design a control for every subsystem individually and the second to design the overall centralised system in a "one shot" design[3].
4- Application of this new method to an example of the model of a single electrical machine connected to an infinite electrical bus system, as well as to study the scenario when faults occur in actuators and assess the impact of the faults on the stability of whole system.

The Chapter is organized as follows. Section 5.2 describes the problem formulation. Then Section 5.3 considers the proposed control approach that includes the proposal for a new AOISMC in the first part and an LMI-based output feedback control design in the second part. Section 5.4 describes the electrical power systems example comprising two inter-connected systems to illustrate the design approach. The simulated performance of this example is given and a comparison is made between the OISMC and adaptive OISMC design approaches. Finally, a concluding discussion is presented in Section 5.5.

[3] Part of the work presented in this chapter has been published in:

Larbah, E. and Patton, R.J. 2013. Static output feedback adaptive integral sliding control for interconnected nonlinear systems. The 9th Asian Control Conference (ASCC) June 23th -26th .

5.2 System description

Consider a non-linear inter-connected state-space system comprising n subsystems as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t)$$
$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N \tag{5-1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ are the control inputs and $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs. $A_i \in \mathbb{R}^{n\times n}$ is a subsystem characteristic matrix, $B_i \in \mathbb{R}^{n\times m}$ is the subsystem control input matrix, $C_i \in \mathbb{R}^{p\times n}$ is the subsystem output matrix and $E_i \in \mathbb{R}^{n\times q}$ is the subsystem external disturbance matrix. It is assumed that all of the matrices A_i B_i, C_i & E_i are known and that the $Z_i(t)$ denote the interactions between the subsystems. The $W_i(x_i, t)$ denote the unknown uncertainties that satisfy the matching conditions $W_i(x_i, t) = B_i Q_i(x_i, t)$. $d_i(t)$ represent an unknown bounded disturbance.

$f_i(t) \in \mathbb{R}^k$ denote the actuator faults where $f_i = -K(t)u_i$ and for which $K(t) = diag(K_i)$ and $0 \le K_i \le 1$. $K_i = 0$. This means that the actuator is working perfectly and if $K_i = 1$ the actuator has failed completely, otherwise the fault is present.

Some of the assumptions are taken into account as mentioned in Section 3.2 of Chapter 3 from (**A1** to **A7**).

Suppose that: $\Gamma = I_n - BB^+$ where B^+ is pseudo-inverse of the matrix B , so that $B^+ = (B^T B)^{-1} B^T$ and I_n is the $n \times n$ identity matrix.

It is also assumed that the interactions between the subsystems contain two matched and unmatched components, so that $Z_i = Z_{mi} + Z_{ui}$ where $Z_{mi} = B_i B_i^+ Z_i$ is a matched component of Z_i and $Z_{ui} = \Gamma_i Z_i$ is an unmatched component of Z_i (Ghadami and Shafai, 2011) in which $\Gamma_i = I_{ni} - B_i B_i^+$.

The same procedure is applied to the components $E_i d_i$ which divide into two further matched and unmatched components $E_i d_i = E_i d_{mi} + E_i d_{ui}$, with $E_i d_{mi} = B_i B_i^+ E_i d_i$ and $E_i d_{ui} = \Gamma_i E_i d_i$.

After substituting all assumptions the subsystem of Eq. (5-1) becomes:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + B_i \Phi_{mi}(t) + \Phi_{ui}(t) \tag{5-2}$$
$$y_i(t) = C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N$$

where Φ_{mi} is a matched component, $\Phi_{mi} = B_i^{+}Z_i(t) + Q_i(x_i, t) + B_i^{+}E_i d_i(t) + f_i(t)$ and Φ_{ui} is an unmatched component, $\Phi_{ui} = Z_{ui}(t) + E_i d_{ui} = [\Gamma_i \quad \Gamma_i E_i]\begin{bmatrix} Z_i \\ d_i \end{bmatrix} = r_i w_i$.

The matched and unmatched components are bounded by known positive constants ϵ_{0i} , ϵ_{1i} , γ_{0i} and γ_{1i} such as:

$$\|\Phi_{mi}\| \leq \epsilon_{0i} + \epsilon_{1i}\|y_i\| \tag{5-3}$$

and

$$\|\Phi_{ui}\| \leq \gamma_{0i} + \gamma_{1i}\|y_i\| \tag{5-4}$$

The control signal contains *two* components:

$$u_i(t) = u_i^{Otp}(t) + u_i^{ISM}(t) \tag{5-5}$$

where u_i^{Otp} is responsible for stabilizing the system and achieving the desired performance whilst also decreasing the effect of unmatched components where the state is not available. u_i^{ISM} is a discontinuous control responsible for eliminating the effects of matched components (uncertainties and actuator faults).

Substituting Eq.(5-5) into Eq.(5-2) yields:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i^{Otp}(t) + B_i u_i^{ISM}(t) + B_i \Phi_{mi}(t) + \Phi_{ui}(t) \\ y_i(t) &= C_i x_i(t) \qquad\qquad i = 1, \dots\dots, N \end{aligned} \tag{5-6}$$

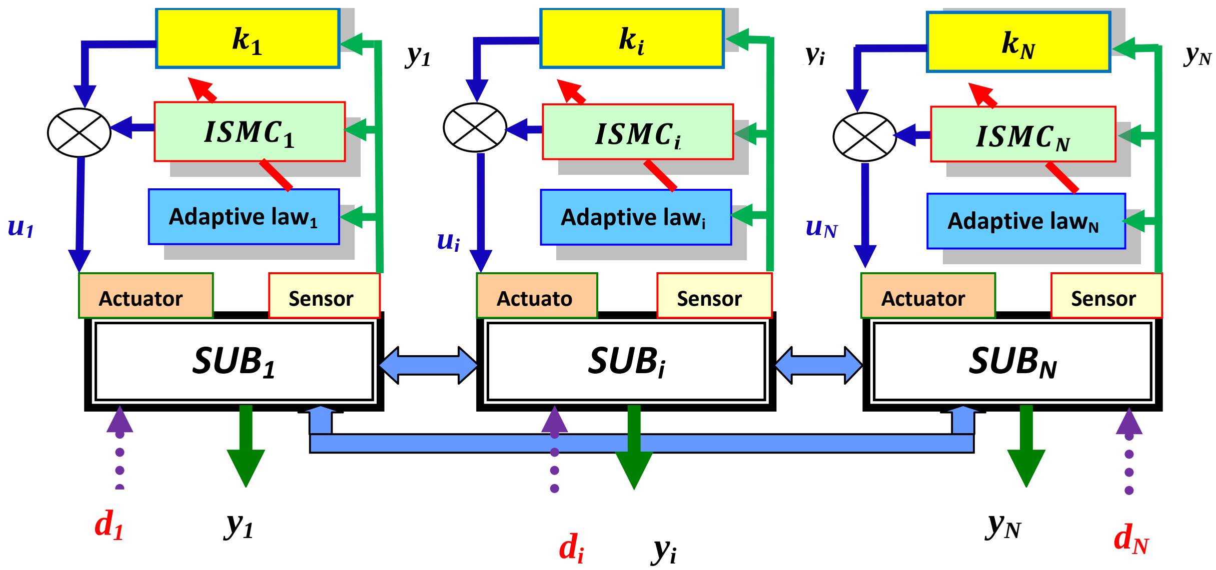

Figure 5-1: Static output control of inter-connected systems via LMI+AOISMC

5.3 Control design methods

In this Section two design methods are described for the control of inter-connected systems which depend on the type and extent of knowledge of the interactions between the subsystems. Suppose that all subsystems are connected to each other and all the interactions are not known, so that the input or "distribution" matrices of the interactions are not known. It is assumed that the control input matrices and the disturbance distributions are known. As summarised in Section 5.2 the control signal contains *two* parts one is designed via an LMI framework and the second one is designed by a proposed AOISMC approach as follows.

5.3.1 Adaptive output integral sliding mode control (AOISMC)

An application based on the use of adaptive control is able to improve the behaviour of the system if compared with the use of a fixed gain controller (Nguyen, 2012). Adaptive control has great advantages when applied to non-linear subsystems containing uncertainties, interactions, disturbances and faults. Based on the behavioural changes that may occur in the system, the control tunes its parameters online to change the system response to ensure stability and achieve a pre-specified performance, in the presence of system parameter changes, uncertain perturbations or disturbances (Wai, 2000, Yu and Fei, 2011). Therefore the control is able to adapt the system parameters to events that may change the system behaviour. Accordingly, the controller is changed depending on internal and external system changes to reach the best level of stability. Despite several advantages of adaptive control, it nevertheless can suffer from a reliability problem when implemented in safety critical systems (Nguyen, 2012).

The output integral sliding switching surface is defined as:

$$\sigma_i(y_i,t) = G_i[y_i(t) - y_i(t_o)] - \int_{t_o}^{t} \left(u_i^{Otp}(t) + G_i C_i A_i \mathbb{G}_i y_i(t)\right) dt \tag{5-7}$$

where the G_i are design freedom matrices satisfying the conditions $G_i = C_i B_i$ where the G_i are invertible and the $\mathbb{G}_i$ are design matrices to be chosen.

The steps of the proposed AOISMC design are:

1- Design sliding surface based on outputs that insure the performance of the system.
2- Design an appropriate discontinuous control to force the system to be maintained in the sliding surface.
3- Design the required adaptive gains for the AOISMC discontinuous control.

For this problem the equivalent controls (for each subsystem) $u_{eqi}(t)$ can maintain the sliding surfaces by ensuring that the time derivatives of $\sigma_i(y_i,t) = 0$ are given by:

$$\begin{gathered} \dot{\sigma}_i(y_i,t) = G_i \dot{y}_i(t) - u_i^{Otp}(t) - G_i C_i A_i \mathbb{G}_i y_i(t) \\ \dot{\sigma}_i(y_i,t) = G_i C_i \dot{x}_i(t) - u_i^{Otp}(t) - G_i C_i A_i \mathbb{G}_i C_i x_i(t) = 0 \end{gathered} \tag{5-8}$$

Substituting Eq. (5-6) into Eq. (5-8) yields:

$$\begin{aligned} G_i C_i A_i x_i(t) + G_i C_i B_i u_i^{Otp}(t) + G_i C_i B_i u_i^{ISM}(t) + G_i C_i B_i \Phi_{mi}(t) + G_i C_i \Phi_{ui}(t) \\ - u_i^{Otp}(t) - G_i C_i A_i \mathbb{G}_i C_i x_i(t) = 0 \end{aligned} \tag{5-9}$$

Selecting the G_i as $G_i = (C_i B_i)^+$

Then:

$$u_{eqi}(t) = -[(C_iB_i)^+C_iA_ix_i(t) + \Phi_{mi}(t) + (C_iB_i)^+C_i\Phi_{ui}(t) - (C_iB_i)^+C_iA_i\mathbb{G}_iC_ix_i(t)] \tag{5-10}$$

Now, substituting Eq. (5-10) into Eq. (5-6) gives:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{Otp}(t) + [I_i - B_i(C_iB_i)^+C_i]\Phi_{ui}(t) \tag{5-11}$$

where: $-B_i(C_iB_i)^+C_iA_ix_i(t) + B_i(C_iB_i)^+C_iA_i\mathbb{G}_iC_ix_i(t) = 0$

Leading to the selection $\mathbb{G}_i = (C_i)^+$

Then the *i*[th] subsystem on the sliding surface will be:

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i^{Otp}(t) + T_i\Phi_{ui}(t) \tag{5-12}$$

where $T_i = [I_i - B_i(C_iB_i)^+C_i]$

From Eq. (5-12) the *i*[th] subsystem still has unmatched components.

The proposed adaptive discontinuous controls for each subsystem are:

$$u_i^{ISM}(t) = -\bar{\mu}_i\frac{\sigma_i(y_i,t)}{\|\sigma_i(y_i,t)\|} \tag{5-13}$$

where $\bar{\mu}_i = \bar{\mu}_{i0} + \bar{\mu}_{i1}\|y_i(t)\|$, $\bar{\mu}_{i0}$ and $\bar{\mu}_{i1}$ are adaptive values to adapt the unknown constant parameters of μ_{i0} and μ_{i1} .

The parameter adaptive errors are defined as:

$\tilde{\mu}_{i0} = \bar{\mu}_{i0} - \mu_{i0}$ and $\tilde{\mu}_{i1} = \bar{\mu}_{i1} - \mu_{i1}$.The two adaptive gains are specified as:

$$\dot{\tilde{\mu}}_{i0} \cong q_{i0}\|\sigma_i(y_i,t)\| \tag{5-14}$$

and

$$\dot{\tilde{\mu}}_{i1} \cong q_{i1}\|\sigma_i(y_i,t)\|\|y_i(t)\| \tag{5-15}$$

where q_{i0} and q_{i1} are constants defined by the designer. To study the stability of the system Eq. (5-2) with the proposed discontinuous control and the two adaptive gains, one must check if the subsystem is globally stable on its sliding surface. To check the global stability, consider a candidate Lyapunov function as:

$$V_i(\sigma_i,y_i,t) = \frac{1}{2}[\sigma_i^T(y_i,t)\sigma_i(y_i,t) + q_{i0}^{-1}\tilde{\mu}_{i0}^2 + q_{i1}^{-1}\tilde{\mu}_{i1}^2] \tag{5-16}$$

To maintain the motion close to the sliding surface consider the time derivative of V_i yielding:

$$\dot{V}_i(\sigma_i, y_i, t) = {\sigma_i}^T(y_i, t)\dot{\sigma}_i(y_i, t) + {q_{i0}}^{-1}\tilde{\mu}_{i0}\dot{\tilde{\mu}}_{i0} + {q_{i1}}^{-1}\tilde{\mu}_{i1}\dot{\tilde{\mu}}_{i1} \tag{5-17}$$

Substituting Eq. (5-13) into Eq. (5-9) gives:

$$\dot{\sigma}_i(y_i, t) = -\bar{\mu}_i \frac{\sigma_i(y_i, t)}{\|\sigma_i(y_i, t)\|} + \Phi_{mi}(t) + (C_i B_i)^+ C_i \Phi_{ui}(t) \tag{5-18}$$

Substituting Eq. (5-14), Eq. (5-15) and Eq. (5-18) into Eq.(5-17) gives:

$$\begin{aligned}\dot{V}_i(\sigma_i, y_i, t) = {\sigma_i}^T(y_i, t)\left[-\bar{\mu}_i \frac{\sigma_i(y_i, t)}{\|\sigma_i(y_i, t)\|} + \Phi_{mi}(t) + (C_i B_i)^+ C_i \Phi_{ui}(t)\right] \\ + (\bar{\mu}_{i0} - \mu_{i0})\|\sigma_i\| + (\bar{\mu}_{i1} - \mu_{i1})\|\sigma_i\|\|y_i(t)\|\end{aligned} \tag{5-19}$$

Substituting Eq. (5-3) and Eq. (5-4) into Eq. (5-19) leads to:

$$\begin{aligned}\dot{V}_i(\sigma_i, y_i, t) \le -\|\sigma_i\|\bar{\mu}_i + \epsilon_{0i}\|\sigma_i\| + \epsilon_{1i}\|\sigma_i\|\|y_i\| + \gamma_{0i}\|\sigma_i\|\|(C_i B_i)^+ C_i\| \\ + \gamma_{1i}\|\sigma_i\|\|(C_i B_i)^+ C_i\|\|y_i\| + (\bar{\mu}_{i0} - \mu_{i0})\|\sigma_i\| + (\bar{\mu}_{i1} \\ - \mu_{i1})\|\sigma_i\|\|y_i(t)\|\end{aligned} \tag{5-20}$$

Substituting $\overline{\mu}_i$ by $\bar{\mu}_{i0} + \bar{\mu}_{i1}\|y_i(t)\|$, $\bar{\mu}_{i0} > (\epsilon_{0i} + \gamma_{0i})$, $\overline{\mu}_{i1} > (\epsilon_{1i} + \gamma_{1i})$ and rearranging Eq. (5-20) as:

$$\begin{aligned}\dot{V}_i(\sigma_i, y_i, t) \le -\bar{\mu}_{i0}\|\sigma_i\| - \bar{\mu}_{i1}\|\sigma_i\|\|y_i(t)\| + \mu_{i0}\|\sigma_i\| + \mu_{i1}\|\sigma_i\|\|y_i(t)\| \\ + (\bar{\mu}_{i0} - \mu_{i0})\|\sigma_i\| + (\bar{\mu}_{i1} - \mu_{i1})\|\sigma_i\|\|y_i(t)\|\end{aligned} \tag{5-21}$$

It can be shown that:

$$\dot{V}_i(\sigma_i, y_i, t) \le 0 \tag{5-22}$$

From Eq. (5-22) the discontinuous controllers and the *two* adaptive gains for each subsystem will guarantee stability of the sliding surface and ensure that the dynamic subsystem will remain on the sliding surface.

Note:

When applying the $u_i^{ISM}(t)$ on the system chattering will occur and the defect can be eliminated by the addition of a small constant $\mathfrak{z}_i > 0$ (Changqing,Patton and Zong, 2010).

The final version of the discontinuous control is:

$u_i^{ISM}(t) = -\bar{\mu}_i \frac{\sigma_i(y_i,t)}{\|\sigma_i(y_i,t)\| + \mathfrak{z}_i}$, where $\bar{\mu}_i = \bar{\mu}_{i0} + \bar{\mu}_{i1}\|y_i(t)\|$ and

From a practical view threshold must added to the adaptive gains:

$$\bar{\mu}_{i0} \begin{cases} \bar{\mu}_{i0inl} + q_{i0}\int\|\sigma_i(y_i,t)\|dt & \text{if Threshold < Thr} \\ \bar{\mu}_{i0inl} & \text{if threshold} \geq \text{Thr} \end{cases} \tag{5-23}$$

and

$$\bar{\mu}_{i1} \begin{cases} \bar{\mu}_{i1inl} + q_{i1}\int\|\sigma_i(y_i,t)\|\|y_i(t)\|dt & \text{if Threshold < Thr} \\ \bar{\mu}_{i1inl} & \text{if threshold} \geq \text{Thr} \end{cases} \tag{5-24}$$

Where the $\bar{\mu}_{i0inl}$ and $\bar{\mu}_{i1inl}$ are initial values of $\bar{\mu}_{i0}$ and $\bar{\mu}_{i1}$ respectively, Thr is a specific threshold.

5.3.2 Output integral sliding mode control (OISMC)

Using the same integral sliding switching surfaces for each subsystem as defined in Eq. (5-7). The i^{th} subsystem on the sliding surface as in Eq. (5-12).

The difference between this controller and the AOISMC is the gain of the discontinuous controller is fixed on specific value. The proposed discontinuous subsystem control signals are:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(y_i,t)}{\|\sigma_i(y_i,t)\| + \mathfrak{z}_i} \tag{5-25}$$

where the μ_i are positive scalar .

5.3.3 Continuous control design via LMI formulation

After the design of the AOISMC the dynamic subsystem will be as in Eq. (5-12). As mentioned above the system still has unmatched components to solve this problem and hence the LMI strategy for solving this problem cannot be used since output feedback rather than a state feedback control is being developed. This leads to a non-convex optimization problem which must be converted to an appropriate convex solution problem.

It is important to note here (see Section 3.3.1.2) that achievable designs making use of LMI tools can be classified into *two* main procedures:

1- Design controller via LMI for each subsystem individually.
2- Design controller via LMI for overall subsystem (one shot).

Hence, the designer must choose the more appropriate of these two strategies according to the number of interconnections. For systems with a low number of interconnections procedure (1) can be used. On the other hand for a larger-scale system with a significant number of connected subsystems procedure (2) is essential, i.e. as a one shot system design.

5.3.3.1 Control design via LMI framework for each subsystem individually

The required static output feedback control has the form:

$$u_i^{Otp}(t) = K_i y_i(t) \tag{5-26}$$

Where the K_i are controllers required to stabilize the dynamic system and reach a specific performance.

Substituting Eq. (5-26) into Eq. (5-12) yields:

$$\dot{x}_i(t) = A_i x_i(t) + B_i K_i y_i(t) + T_i \Phi_{ui}(t) = A_i x_i(t) + B_i K_i C_i x_i(t) + T_i \Phi_{ui}(t) \tag{5-27}$$

Eq. (5-27) can be re-arranged as:

$$\dot{x}_i(t) = A_i x_i(t) + B_i [\, K_i \quad 0_i \,] \mathcal{T}_i x_i(t) + T_i \Phi_{ui}(t) \tag{5-28}$$

Where the $\mathcal{T}_i = \begin{bmatrix} C_i \\ Oc_i \end{bmatrix}$ are square and non-singular and the Oc_i are the orthogonal bases of the null space of C_i (Prempain and Postlethwaite, 2001).

Making a transformation of the $x_i(t)$ to $\mathcal{T}_i^{-1}\tilde{x}_i(t)$ and substituting into Eq. (5-28) yields:

$$\mathcal{T}_i^{-1}\dot{\tilde{x}}_i(t) = A_i \mathcal{T}_i^{-1}\tilde{x}_i(t) + B_i [\, K_i \quad 0_i \,] \mathcal{T}_i \mathcal{T}_i^{-1}\tilde{x}_i(t) + T_i \Phi_{ui}(t) \tag{5-29}$$

Rearranging Eq. (5-29) as:

$$\dot{\tilde{x}}_i(t) = A_{ci}\tilde{x}_i(t) + B_{ci} K_i C_{ci}\tilde{x}_i(t) + T_{ci}\Phi_{ui}(t) \tag{5-30}$$

$A_{ci} = \mathcal{T}_i A_i \mathcal{T}_i^{-1}$, $B_{ci} = \mathcal{T}_i B_i$, $C_{ci} = [\, I_{(p\times p)i} \quad 0_i \,]$ and $T_{ci} = \mathcal{T}_i T_i$

Suppose that the $\Phi_{ui}(t)$ are so-called "unknown inputs" satisfying the condition of quadratic inequality for each subsystem (Šiljak and Stipanovic, 2001), as follows:

$$\Phi_{ui}{}^{T}(t)\Phi_{ui}(t) \le \alpha_i^2 \tilde{x}_i{}^{T}(t)\tilde{x}_i(t) \tag{5-31}$$

where the $\alpha_i > 0$ are scalar parameters.

The Lyapunov function candidates $V_i(\tilde{x}_i, t) = \tilde{x}_i^T(t)P_i\tilde{x}_i(t)$ are used to check the stability of the closed-loop system, where $P_i > 0$.

The time derivative of the $V_i(\tilde{x}_i, t)$ are:

$$\dot{V}_i(\tilde{x}_i, t) = \dot{\tilde{x}}_i{}^{T}(t)P_i\tilde{x}_i(t) + \tilde{x}_i^T(t)P_i\dot{\tilde{x}}_i(t) \tag{5-32}$$

Substituting Eq. (5-30) into Eq. (5-32) yields:

$$\begin{aligned}\dot{V}_i(\tilde{x}_i, t) = {} & [A_{ci}\tilde{x}_i(t) + B_{ci}K_iC_{ci}\tilde{x}_i(t) + T_{ci}\Phi_{ui}(t)]^T P_i\tilde{x}_i(t) + \tilde{x}_i^T(t)P_i[A_{ci}\tilde{x}_i(t) \\ & + B_{ci}K_iC_{ci}\tilde{x}_i(t) + T_{ci}\Phi_{ui}(t)]\end{aligned} \tag{5-33}$$

Rearranging Eq. (5-33) as:

$$\begin{aligned}\dot{V}_i(\tilde{x}_i, t) = {} & x_i^T(t)A_{ci}^T P_i\tilde{x}_i(t) + \tilde{x}_i^T C_{ci}^T K_i^T B_{ci}^T P_i\tilde{x}_i(t) + \Phi_{ui}{}^{T}(t)T_{ci}^T P_i\tilde{x}_i(t) \\ & + \tilde{x}_i^T(t)P_iA_{ci}\tilde{x}_i(t) + \tilde{x}_i^T(t)P_iB_{ci}C_{ci}K_i\tilde{x}_i(t) + \tilde{x}_i^T(t)P_iT_{ci}\Phi_{ui}(t)\end{aligned} \tag{5-34}$$

The stability of the system Eq. (5-30) requires that $\dot{V}_i(\tilde{x}_i, t) < 0$ for all $\tilde{x}_i(t) \neq 0$.

Equation (5-34) can then be rewritten as:

$$\mathcal{Z}_i^T \mathcal{D}_i \mathcal{Z}_i < 0 \tag{5-35}$$

$\mathcal{Z}_i = \begin{bmatrix} \tilde{x}_i(t) \\ \Phi_{ui}(t) \end{bmatrix}$ and $\mathcal{D}_i = \begin{bmatrix} A_{ci}^T P_i + P_iA_{ci} + C_{ci}^T K_i^T B_{ci}^T P_i + P_iB_{ci}K_iC_{ci} & P_iT_{ci} \\ T_{ci}^T P_i & 0 \end{bmatrix}$

In order to check the stability condition matrices $\mathcal{D}_i$ must be negative-definite, i.e. $\mathcal{D}_i < 0$ and also Eq. (5-31) can be rewritten as:

$$\mathcal{Z}_i^T \mathcal{O}_i \mathcal{Z}_i \le 0 \tag{5-36}$$

where: $\mathcal{Z}_i = \begin{bmatrix} \tilde{x}_i(t) \\ \Phi_{ui}(t) \end{bmatrix}$ and $\mathcal{O}_i = \begin{bmatrix} -\alpha_i^2 I_i & 0 \\ 0 & I_i \end{bmatrix}$

To organize the equations into a single equation the S-procedure can be used (Šiljak and Stipanovic, 2001).

If $\mathcal{D}_i$ and $\mathcal{O}_i$ are symmetric matrices then $\mathcal{Z}_i^T\mathcal{D}_i\mathcal{Z}_i < 0$ and $\mathcal{Z}_i^T\mathcal{O}_i\mathcal{Z}_i \leq 0$ there is a number $\tau_i > 0$ where $\mathcal{D}_i - \tau_i\mathcal{O}_i < 0$. In order to put the equations in a single set of equations (one for each subsystem) to study the stability, it can be shown that:

$$\mathcal{D}_i - \tau_i\mathcal{O}_i = \begin{bmatrix} A_{ci}^T P_i + P_i A_{ci} + C_{ci}^T K_i^T B_{ci}^T P_i + P_i B_{ci} K_i C_{ci} & P_i T_{ci} \\ T_{ci}^T P_i & 0 \end{bmatrix} - \tau_i \begin{bmatrix} -\alpha_i^2 I_i & 0 \\ 0 & I_i \end{bmatrix} < 0 \quad (5\text{-}37)$$

Putting $\mathcal{Y}_i = \frac{P_i}{\tau_i}$ into Eq. (5-37) yields:

$$\begin{bmatrix} A_{ci}^T\mathcal{Y}_i + \mathcal{Y}_i A_{ci} + C_{ci}^T K_i^T B_{ci}^T \mathcal{Y}_i + \mathcal{Y}_i B_{ci} K_i C_{ci} + \alpha_i^2 I_i & \mathcal{Y}_i T_{ci} \\ T_{ci}^T \mathcal{Y}_i & -I_i \end{bmatrix} < 0 \quad (5\text{-}38)$$

Eq. (5-38) is not an LMI since it contains the non-linear term $\mathcal{Y}_i B_{ci} K_i C_{ci}$. To overcome this problem, both sides of Eq. (5-38) must be multiplied by the matrices $\begin{bmatrix} \mathcal{Y}_i^{-1} & 0 \\ 0 & I_i \end{bmatrix}$ setting the $\mathcal{P}_i = \mathcal{Y}_i^{-1}$, as:

$$\begin{bmatrix} \mathcal{P}_i A_{ci}^T + A_{ci}\mathcal{P}_i + \mathcal{P}_i C_{ci}^T K_i^T B_{ci}^T + B_{ci} K_i C_{ci} \mathcal{P}_i + \alpha_i^2 \mathcal{P}_i \mathcal{P}_i & T_{ci} \\ T_{ci}^T & -I_i \end{bmatrix} < 0 \quad (5\text{-}39)$$

Rearranging Eq. (5-39) and using the Schur complement Eq. (5-39) can be rewritten as:

$$\begin{bmatrix} \mathcal{P}_i A_{ci}^T + A_{ci}\mathcal{P}_i + \mathcal{P}_i C_{ci}^T K_i^T B_{ci}^T + B_{ci} K_i C_{ci} \mathcal{P}_i & T_{ci} & \mathcal{P}_i \\ T_{ci}^T & -I_i & 0 \\ \mathcal{P}_i & 0 & -\epsilon_i I_i \end{bmatrix} < 0 \quad (5\text{-}40)$$

To ensure that the $\mathcal{P}_i$ are s.p.d matrices whilst satisfying the inequality Eq. (5-40), the $\mathcal{P}_i$ can be chosen as (Prempain and Postlethwaite, 2001):

$$\mathcal{P}_i = \mathcal{P}_i^T = \begin{bmatrix} P_{1i} & P_{1i}N_i \\ N_i^T P_{1i} & P_{2i} + N_i^T P_{1i} N_i \end{bmatrix} \quad (5\text{-}41)$$

$P_{1i} = P_{1i}^T \in \mathbb{R}^p$, $P_{2i} = P_{2i}^T \in \mathbb{R}^{(n-p)\times(n-p)}$ and $N_i \in \mathbb{R}^{p\times(n-p)}$. P_i can be rearranged as:

$$P_i = P_i^T = T_{Ni} P_{di} T_{Ni}^T \quad (5\text{-}42)$$

$P_{di} = \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix}$ and $T_{Ni} = \begin{bmatrix} I_{(p\times p)i} & 0 \\ N_i^T & I_{((n-p)\times(n-p))i} \end{bmatrix}$ with $det(N_i) \neq 0$

Substituting Eq.(5-42) into Eq. (5-40) yields:

$$\begin{bmatrix} T_{Ni}P_{di}T_{Ni}^{T}A_{ci}^{T} + A_{ci}\,T_{Ni}P_{di}T_{Ni}^{T} + T_{Ni}P_{di}T_{Ni}^{T}C_{ci}^{T}K_{i}^{T}B_{ci}^{T} + B_{ci}K_{i}C_{ci}\,T_{Ni}P_{di}T_{Ni}^{T} & T_{ci} & T_{Ni}P_{di}T_{Ni}^{T} \\ T_{ci}^{T} & -I_{i} & 0 \\ T_{Ni}P_{di}T_{Ni}^{T} & 0 & -\,\epsilon_{i}I_{i} \end{bmatrix} < 0 \tag{5-43}$$

Pre- and post-multiplying Eq. (5-43) by $diag(T_{Ni}^{-T}, I_i, I_i) = \Pi_i$ yields:

$$\begin{bmatrix} P_{di}T_{Ni}^{T}A_{ci}^{T}T_{Ni}^{-T} + T_{Ni}^{-T}A_{ci}\,T_{Ni}P_{di} + P_{di}T_{Ni}^{T}C_{ci}^{T}K_{i}^{T}B_{ci}^{T}T_{Ni}^{-T} + T_{Ni}^{-T}B_{ci}K_{i}C_{ci}\,T_{Ni}P_{di} & T_{Ni}^{-T}T_{ci} & P_{di}T_{Ni}^{T} \\ T_{ci}^{T}T_{Ni}^{-T} & -I_{i} & 0 \\ T_{Ni}P_{di} & 0 & -\,\epsilon_{i}I_{i} \end{bmatrix} < 0 \tag{5-44}$$

It then follows that:

$$T_{Ni}^{-T}B_{ci}K_{i}C_{ci}\,T_{Ni}P_{di} = T_{Ni}^{-T}B_{ci}K_{i}C_{ci}P_{di} = T_{Ni}^{-T}B_{ci}K_{i}\;P_{1i}C_{ci} \tag{5-45}$$

Since: $C_{ci}\,T_{Ni} = \begin{bmatrix} I_{i(p\times p)} & 0_i \end{bmatrix} \begin{bmatrix} I_{i(p\times p)} & 0_i \\ N_i^T & I_{i((n-p)\times(n-p))} \end{bmatrix} = \begin{bmatrix} I_{i(p\times p)} & 0_i \end{bmatrix} = C_{ci}$

So that, $C_{ci}P_{di} = \begin{bmatrix} I_{i(p\times p)} & 0_i \end{bmatrix} \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i2} \end{bmatrix} = P_{1i}\begin{bmatrix} I_{i(p\times p)} & 0_i \end{bmatrix} = P_{1i}C_{ci}$

Hence:

$$T_{Ni}^{-T}B_{ci}K_{i}\;P_{1i}C_{ci} = T_{Ni}^{-T}B_{ci}\mathcal{H}_{i}C_{ci} \tag{5-46}$$

Where the $\mathcal{H}_i = K_i\;P_{1i}$. It follows that the subsystem gain matrices are given by:

$$K_i = \mathcal{H}_i P_{1i}^{-1} \tag{5-47}$$

Substituting Eq. (5-46) into Eq. (5-44) and rearranging yields:

$$\begin{bmatrix} P_{di}T_{Ni}^{T}A_{ci}^{T}T_{Ni}^{-T} + T_{Ni}^{-T}A_{ci}\,T_{Ni}P_{di} + C_{ci}^{T}\mathcal{H}_{i}^{T}B_{ci}^{T}T_{Ni}^{-T} + T_{Ni}^{-T}B_{ci}\mathcal{H}_{i}C_{ci} & T_{Ni}^{-T}T_{ci} & P_{di}T_{Ni}^{T} \\ T_{ci}^{T}T_{Ni}^{-T} & -I_{i} & 0 \\ T_{Ni}P_{di} & 0 & -\,\epsilon_{i}I_{i} \end{bmatrix} < 0 \tag{5-48}$$

The N_i can then be chosen as tuning matrices to achieve the specific subsystem performances.

Algorithm 5-1:

1- Calculate the $\sigma_i(y_i, t)$ from the Eq.(5-7)
2- Design the OISMC from the Eqs. (5-13), (5-23) &(5-24)
3- Calculate the Oc_i and then find $\mathcal{T}_i$ from the Eq. (5-28)
4- Transform the subsystem by the Eq. (5-29)
5- Choose the tuning parameters N_i and calculate the T_{Ni} from the Eq.(5-42)
6- Minimize ϵ_i subject to $P_{di} > 0$, $P_{1i} > 0$, $P_{2i} > 0$ and the Eq. (5-48)
7- Calculate the controller gains from $K_i = \mathcal{H}_i P_{1i}^{-1}$

The Euclidean norms of the gains K_i can be limited in two steps. (i) The Euclidean norms of $\mathcal{H}_i$ can be constrained as $\|\mathcal{H}_i\|^2 < k_{\mathcal{H}i} I_i$, where the $k_{\mathcal{H}i}$ are scalar variables. This is achieved by adding another LMI conditions (Zecevic and Šiljak, 2005). Then using the Schur complement it follows that:

$$\begin{bmatrix} -k_{\mathcal{H}i} I_i & {\mathcal{H}_i}^T \\ \mathcal{H}_i & -I_i \end{bmatrix} < 0 \tag{5-49}$$

(ii) A second condition must be added to the matrices $P_{di} > k_{Pi} I_i$ (Zecevic and Šiljak, 2005) , as:

$$\begin{bmatrix} P_{di} & I_i \\ I_i & k_{Pi} I_i \end{bmatrix} > 0 \tag{5-50}$$

where k_{Pi} are scalar variables.

Algorithm 5-2:

The same procedure as in *Algorithm 5-1* is used by replacing step 6 by:

Minimize $(\epsilon_i + k_{\mathcal{H}i} + k_{Pi})$ subject to $P_{di} > 0$, $P_{1i} > 0$, $P_{2i} > 0$, the Eqs.(5-48) , (5-49) & (5-50).

5.3.3.2 One shot Control design via LMI

As mentioned in the Section 5.3.1 the subsystem dynamics during ideal sliding are described by Eq. (5-12) .

The dynamics of the centralized or aggregated system that contains all the inter-connected subsystems are:

$$\dot{X}(t) = A_d X(t) + B_d U^{OTP}(t) + T_d J(t) \tag{5-51}$$

where: $X(t) = [x_1, x_2, \dots \dots, x_n]$, $U^{OTP}(t) = \left[u_1^{Otp}, u_2^{Otp}, \dots \dots, u_n^{Otp}\right]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $\Gamma_d = diag(\Gamma_i)$, $Y(t) = [y_1, y_2, \dots \dots, y_n]$, $C_d = diag(C_i)$ and $J(t) = [J_1, J_2, \dots \dots, J_n]$ where "$diag$" represents the block diagonal matrix.

The control signal to achieve a specific performance can be constructed as:

$$U^{OTP}(t) = KY(t) = KCX(t) \tag{5-52}$$

where $K = diag(k_i)$. The gain K is to be designed via LMI to minimize the effect of $J(t)$ on the aggregated system. Suppose that $J(t)$ is the unknown input disturbance which satisfies the quadratic inequality:

$$J^T(t)J(t) \leq \alpha^2 X^T(t)X(t) \tag{5-53}$$

Making a transformation of $X(t)$ to $\mathcal{T}^{-1}X(t)$ and substituting it in Eq. (5-51) yields:

$$\mathcal{T}^{-1}\dot{X}(t) = A_d\mathcal{T}^{-1}X(t) + B_d[\,K \quad 0\,]\mathcal{T}\mathcal{T}^{-1}X(t) + T_d J(t) \tag{5-54}$$

Rearranging Eq. (5-54) as:

$$\dot{X}(t) = A_{cd}X(t) + B_{cd}KC_{cd}X(t) + T_{cd}J(t) \tag{5-55}$$

$A_{cd} = \mathcal{T}A_d\mathcal{T}^{-1}$, $B_{cd} = \mathcal{T}^{-1}B_d$, $C_{cd} = [\,I_{(p\times p)} \quad 0\,]$ and $T_{cd} = \mathcal{T}^{-1}T_d$

The procedure of Section 5.3.2.1 is used to check the stability conditions and derive the gains K in a one-step solution of the following LMI:

$$\begin{bmatrix} P_d T_N^T A_{cd}^T T_N^{-T} + T_N^{-T} A_{cd}\, T_N P_d + C_{cd}^T \mathcal{H}_d^T B_{cd}^T T_N^{-T} + T_N^{-T} B_{cd}\mathcal{H}_d C_{cd} & T_N^{-T} T_c & P_d T_N^T \\ T_c^T T_N^{-T} & -I & 0 \\ T_N P_d & 0 & -\epsilon I \end{bmatrix} < 0 \tag{5-56}$$

N can be chosen as a tuning design matrix to achieve specific performances in the compact system from :

$$T_N = \begin{bmatrix} I_{(p\times p)} & 0 \\ N^T & I_{((n-p)\times(n-p))} \end{bmatrix}$$

The following *Algorithms* 5-3 and 5-4 are alternative ways to solve Eq. (5-56), as follows:

Algorithm 5-3:

1- Calculate $\sigma_i(y_i, t)$ from the Eq.(5-7)
2- Design OISMC from the Eqs. (5-13), (5-23) &(5-24)
3- Calculate the aggregate system from the Eq.(5-51)
4- Calculate Oc and then find $\mathcal{T}$
5- Transform the subsystem by the Eq. (5-54)
6- Choose the tuning parameter N and calculate T_N
7- Minimize ϵ subject to $P_d > 0$, $P_1 > 0$, $P_2 > 0$ and the Eq. (5-56)
8- Calculate the controller gain from $K = \mathcal{H}_d P_1^{-1}$

In some application the *Algorithm 5-3* can give a high gain norm.

Algorithm 5-4 is used to provide an approach to bounding this norm via two additional LMIs as follows:

$$\begin{bmatrix} -k_{\mathcal{H}} I & {\mathcal{H}_d}^T \\ \mathcal{H}_d & -I \end{bmatrix} < 0 \tag{5-57}$$

and

$$\begin{bmatrix} P_d & I \\ I & k_P I \end{bmatrix} > 0 \tag{5-58}$$

where k_P and $k_{\mathcal{H}}$ are scalar variables.

Algorithm 5-4:

This procedure is as described as in A*lgorithm 5-3* but step 6 must be changed to:

Minimize $(\epsilon + k_{\mathcal{H}} + k_P)$ subject to $P_d > 0$, $P_1 > 0$, $P_2 > 0$, the Eqs. (5-56), (5-57) & (5-58).

5.4 Application example (Power system)

This technique is applied to an example problem of a single power generation system requiring both electrical (excitation) control and mechanical (steam valve) control. The generation system is assumed to be connected to an infinite bus so that the bus itself does not provide additional loading variations to the generator. This machine consists of a turbine connected to an electrical generator that is in turn producing power on the infinite bus. The control objectives are to improve the transient stability and control the output of the generator during the transient, as shown in Figure 5-2.

Power systems are more vulnerable to faults or failures because they can be considered dynamically as several overlapping systems with some mechanical and electrical components. The task of the controller is to keep the rotor angle fixed at a specific value even in the case of the emergence of some faults that can occur in the generator itself, the transformer, the turbine or in several other components (Saha,Aldeen and Tan, 2011).

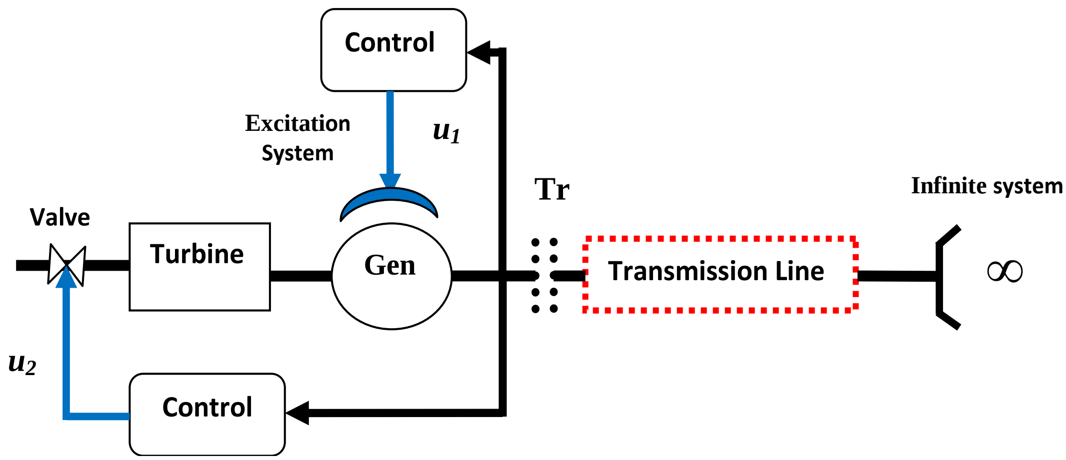

Figure 5-2: Single machine connected to infinite bus system

5.4.1 Power system model

It is conventional to model the single machine infinite bus problem using a set of so-called *d-q* axis Parks equations in terms of equations describing the electrical flux and voltage and mechanical torque. The model comprises a set of six well-known non-linear equations (Subramaniam and Malik, 1973, Brittain,Otaduy,Rovere and Perez, 1988) as follows:

$$\dot{x}_1 = \dot{\delta} = x_2$$
$$\dot{x}_2 = \dot{S} = a_{21} sin2x_1 + a_{22}x_2 + a_{23}x_3 sinx_1 + c_1 x_5$$
$$\dot{x}_3 = \dot{\lambda_{fd}} = a_{31} cosx_1 + a_{32}x_3 + x_4 \quad (5\text{-}59)$$
$$\dot{x}_4 = \dot{E_{fq}} = a_{41} cosx_1 + a_{43}x_3 + a_{44}x_4 + k_1 u_1$$
$$\dot{x}_5 = \dot{T_m} = a_{52}x_2 + a_{55}x_5 + k_2 u_2$$
$$\dot{x}_6 = \dot{\lambda_{fq}} = a_{61} sinx_1 + a_{65}x_5 + a_{66}x_6$$

where δ is the rotor angle (rad), S is the speed deviation around a nominal value (rad s^{-1}), λ_{fd} and λ_{fq} are the field flux linkages. E_{fq} is an excitation bus voltage and T_m is the per unit (p.u.) mechanical power normalised according to the maximum Torque.

All the parameters of the system are described in

Table **5-1** .

$a_{21} = -\frac{e_o^2(X_d' - X_q')}{4M(X_e + X_q')(X_e + X_d')}$	$a_{22} = -\frac{K_d}{M}$	$a_{23} = -\frac{e_o}{2MT_{do}'(X_e + X_d')}$
$c_1 = \frac{1}{M}$	$a_{31} = \frac{e_o(X_d + X_d')}{(X_e + X_d')}$	$a_{32} = -\frac{(X_e + X_d)}{T_{do}'(X_e + X_d')}$
$a_{41} = -\frac{k_v e_o X_d'}{\tau_v(X_e + X_d')}$	$a_{43} = -\frac{k_v X_e}{\tau_v T_{do}'(X_e + X_d')}$	$a_{44} = -\frac{1}{\tau_v}$
$k_1 = \frac{k_v}{\tau_v}$	$a_{52} = -\frac{k_m}{\tau_m}$	$a_{55} = -\frac{1}{\tau_m}$
$k_2 = \frac{1}{\tau_m}$	$a_{61} = -\frac{e_o(X_q + X_q')}{(X_e + X_q')}$	$a_{65} = -\frac{(X_e + X_q)}{T_{qo}'(X_e + X_q')}$
$a_{66} = -\frac{(X_e + X_q)}{T_{qo}'(X_e + X_q')}$		

Table 5-1 : Parameters of the single machine infinite bus power

System parameters

e_o is an infinite bus voltage (p.u) , $X_d^{'}, X_q^{'}$ are d-q axis transient reactance, M is the inertia constant , X_e is a transmission line and transformer reactance, K_d is a damping coefficient, $T_{do}^{'}, T_{qo}^{'}$ are d-q-axis open circuit time constants (s), X_d, X_q are d-q-axis synchronous reactances, k_v is a voltage regulator gain, τ_v is a voltage regulator time constant, k_m is a governor and turbine loop gain and τ_m is a governor and turbine loop time constant .

The system has been decomposed into two subsystems as:

1^st^ Subsystem:

$$\dot{x}_{11} = x_{12}$$
$$\dot{x}_{12} = a_{21}sin2x_{11} + a_{22}x_{12} + a_{23}x_3 sinx_{11} + c_1 x_{15} \qquad (5\text{-}60)$$
$$\dot{x}_{13} = a_{31}cosx_{11} + a_{32}x_{13} + x_{14}$$
$$\dot{x}_{14} = a_{41}cosx_{11} + a_{43}x_{13} + a_{44}x_{14} + k_1 u_1$$

2^nd^ Subsystem:

$$\dot{x}_{25} = \dot{T_m} = a_{52}x_{12} + a_{55}x_{25} + k_2 u_2 \qquad (5\text{-}61)$$
$$\dot{x}_{26} = \dot{\lambda_{fq}} = a_{61}sinx_{11} + a_{65}x_{25} + a_{66}x_{26}$$

System parameters of the single power example

All reactances are in p.u values and time constants are in seconds as in

Table 5-2:

$e_o = 1$	$X_d^{'}$ =-0.176	$X_q^{'} = 0$	M =0.0338	X_e =0.00186
K_d =0.0732	$T_{do}^{'}$ =5.41588	$T_{qo}^{'}$ =-0.4901	X_d =-0.96719	$X_q = 0$
$k_v = 3$	$\tau_v = 2$	k_m =0.204	τ_m =0.2	

Table 5-2 : System parameters of the single power generator

1st Subsystem:

$$A_1=\begin{bmatrix}0 & 1 & 0 & 0\\ a_{22} & 0 & 0 & 0\\ 0 & 0 & a_{32} & 1\\ 0 & 0 & a_{43} & a_{44}\end{bmatrix},\ B_1=\begin{bmatrix}0\\0\\0\\k_1\end{bmatrix},\ C_1=\begin{bmatrix}0.8 & 0.1 & 0 & 0\\ 0 & 1 & 0 & 0.1\end{bmatrix},$$

$$E_1=\begin{bmatrix}0.1\\0.1\\0.1\\0.1\end{bmatrix},\ z_1=\begin{bmatrix}0 & 0\\ c_1 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_{25}\\ x_{26}\end{bmatrix},\ W_1(x_1,t)=\begin{bmatrix}0\\ a_{21}sin2x_{11}+a_{23}x_3sinx_{11}\\ a_{31}cosx_{11}\\ a_{41}cosx_{11}\end{bmatrix} \quad (5\text{-}62)$$

$$,\ x_1(0)=\begin{bmatrix}0.7105\\0\\5.604\\0.8\end{bmatrix} \text{ and } x_1(t)=\begin{bmatrix}x_{11}(t)\\x_{12}(t)\\x_{13}(t)\\x_{14}(t)\end{bmatrix}$$

2nd Subsystem:

$$A_2=\begin{bmatrix}a_{55} & 0\\ a_{65} & a_{66}\end{bmatrix},\ B_2=\begin{bmatrix}k_2\\0\end{bmatrix},\ C_2=\begin{bmatrix}0 & 1\end{bmatrix},\ E_2=\begin{bmatrix}0.1\\0.1\end{bmatrix},$$

$$z_2=\begin{bmatrix}a_{52} & 0\\ 0 & a_{61}sin\end{bmatrix}\begin{bmatrix}x_{11}\\x_{12}\end{bmatrix},\ x_2(0)=\begin{bmatrix}0.8\\2.645\end{bmatrix} \text{ and } x_2(t)=\begin{bmatrix}x_{25}(t)\\x_{26}(t)\end{bmatrix} \quad (5\text{-}63)$$

5.4.2 Simulation results

The continuous control $u_i^{Otp}(t)$ is designed via the LMI described in *Algorithm 5-2,* leading to the gains:

$K_1=[0.1611 \quad -0.2981]$ and $K_2=-3.9167$

The subsystem parameter design or tuning matrices are:

$$N_1=\begin{bmatrix}0.08 & 0 & 0.08 & 0\\ 0 & 0.08 & 0 & 0.08\\ 0.08 & 0 & 0 & 0\\ 0 & 0.08 & 0 & 0\end{bmatrix} \text{ and } N_2=\begin{bmatrix}0.6 & 0\\ 0 & 0.6\end{bmatrix}$$

The adaptive OISMC parameters (defined in Section 5.2.1) are as follows:

***1st Subsystem**:*

$\bar{\mu}_{10inl} = 0.1$, $q_{10} = -3$, $\bar{\mu}_{11inl} = 0.1$, $q_{11} = -2$ and $\mathfrak{z}_1 = 0.2$

2nd Subsystem:

$\bar{\mu}_{20inl} = -2$, $q_{20} = -4$, $\bar{\mu}_{21inl} = -0.1$, $q_{21} = -2$ and $\mathfrak{z}_2 = 0.2$

For the case of the OISMC the parameters are:

The discontinuous control is $u_i^{ISM}(t) = \mu_i \frac{\sigma_i(x_i,t)}{\|\sigma_i(x_i,t)\| + \mathfrak{z}_i}$

where $\mu_1 = \mu_2 = 5$ and $\mathfrak{z}_1 = \mathfrak{z}_2 = 0.2$

The outputs of the two subsystems without controls are shown in Figure 5-3 & Figure **5-4**. Figure 5-5 shows the response of the rotor angle of the single machine system using output de-centralized control applied to the continuous control component $u_i^{Otp}(t)$. The discontinuous control has been designed for the two cases of (i) OISMC and (ii) AOISMC, both with no faults.

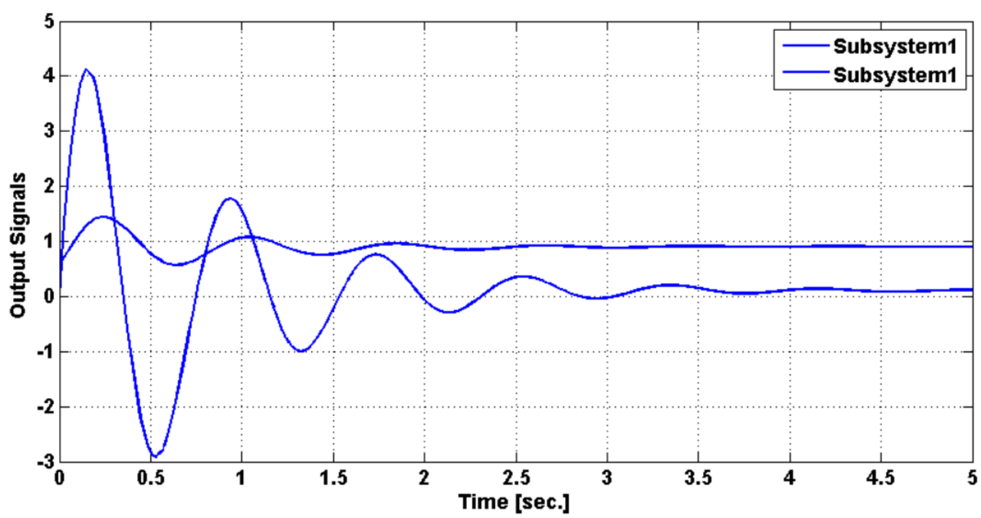

Figure 5-3 : Response of 1st subsystem without control (rotor angle)

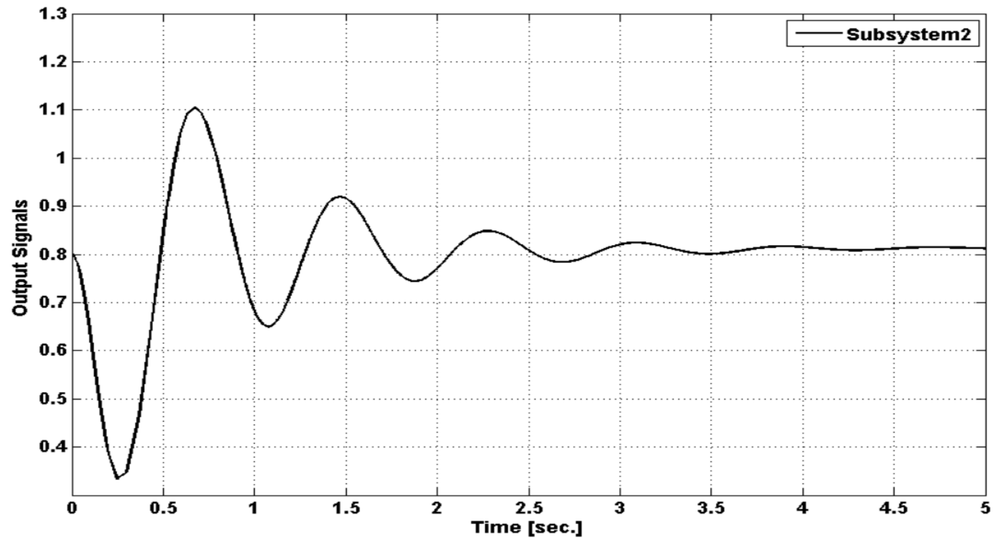

Figure 5-4: Response of 2nd subsystem without control

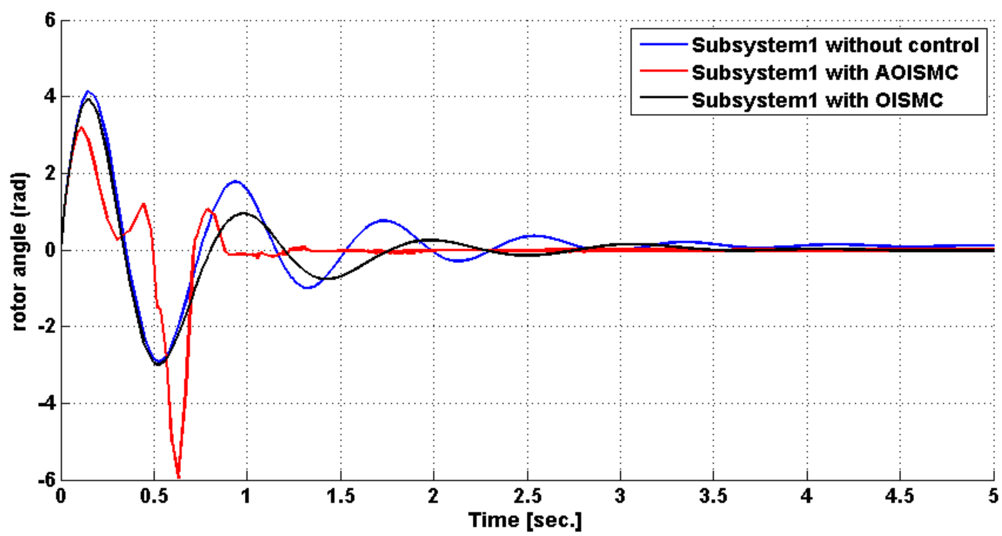

Figure 5-5 : The rotor angle with OISMC and AOISMC with no fault

From Figure 5-5 it can be seen that the AOISMC gives better results compared with the OISMC. The AOISMC has some higher transient phase oscillation but it is more stable than the OISMC. Figure 5-6 shows the simulation with a 50% actuator fault in the 1st subsystem with AOISMC. If all the actuator faults are increased to 70%; the 1st subsystem is still stable, as shown in Figure 5-7 .

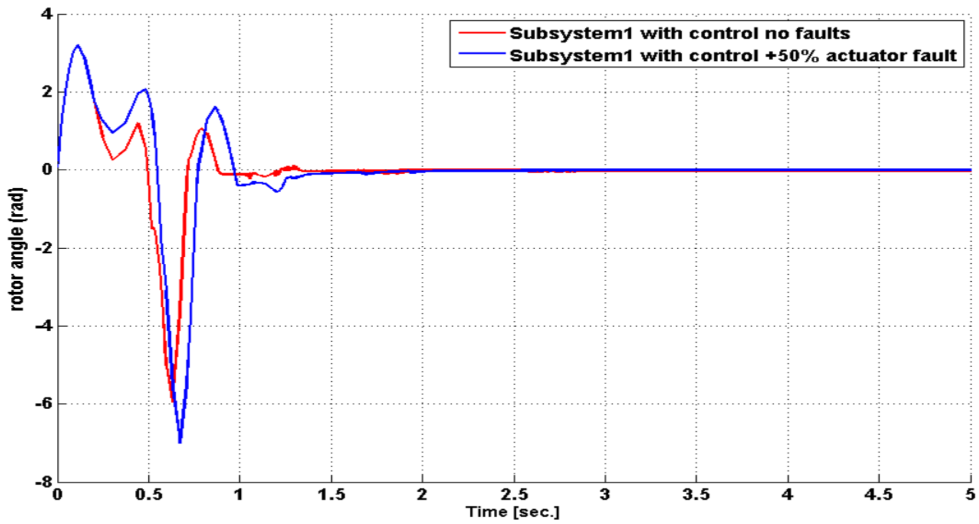

Figure 5-6 : 1st subsystem with AOISMC and 50% actuator faults in only in 1st subsystem

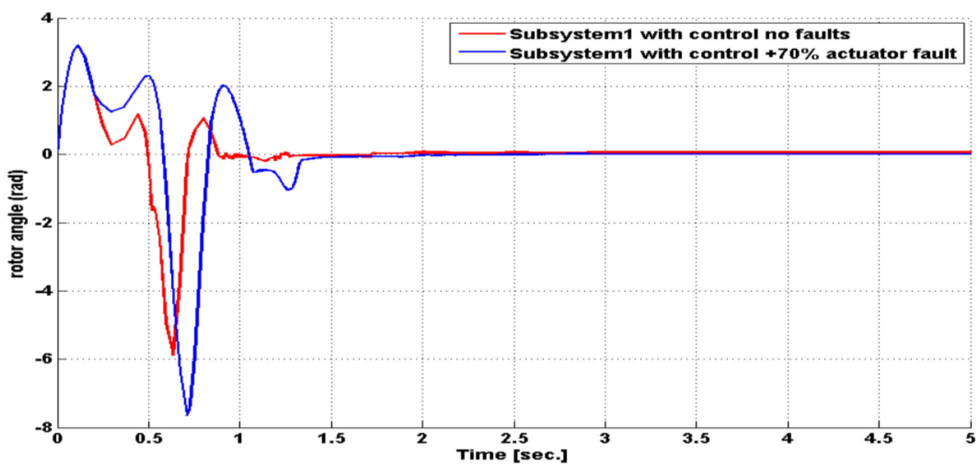

Figure 5-7 : 1st subsystem with AOISMC and 70% actuator faults in only in 1st subsystem

Figure 5-8 illustrates the simulation of a 70% fault of the 2nd subsystem actuator and its effects on other subsystems when there is no fault in the 1st subsystem, by applying the AOISMC. From Figure 5-8 , the controller compensates the faults and decreases the effects of the disturbances. Furthermore, the AOISMC gives even better control action, in terms of the low rotor angle deviation.

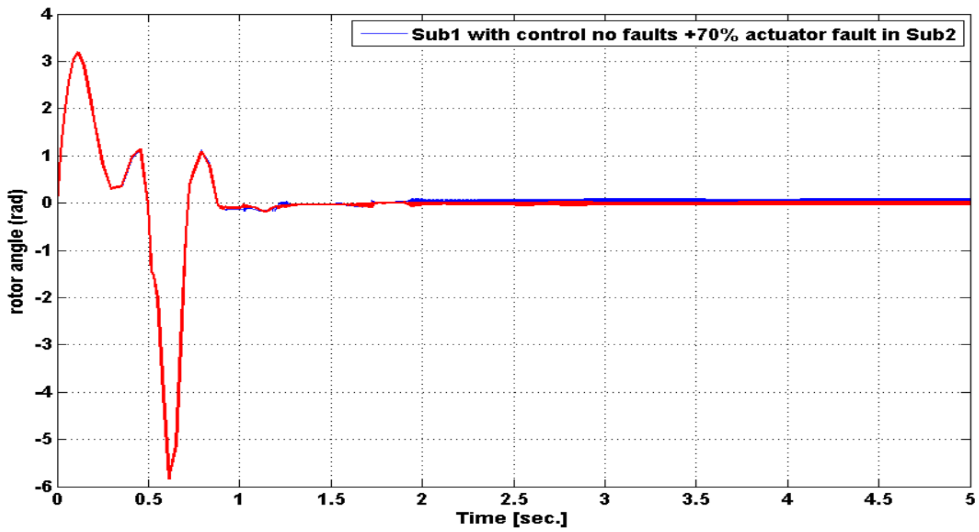

Figure 5-8 : 2nd subsystem with AOISMC and 70% actuator faults in only in 2nd subsystem

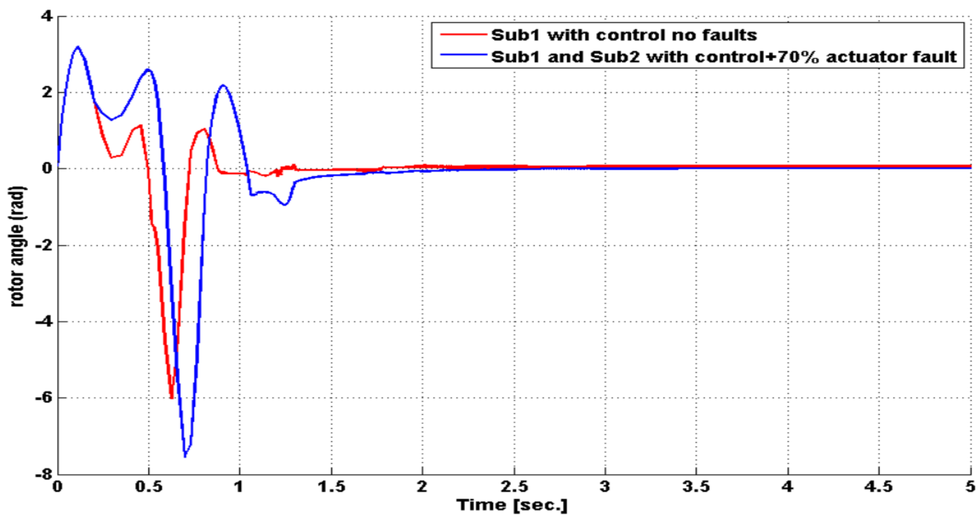

Figure 5-9 : Two subsystems with AOISMC with 70% actuator faults in all subsystems

Figure 5-9 shows the AOISMC simulation of the 70% actuator fault scenario for two subsystems. From Figure 5-9 all the subsystems are affected by the faults but all are stable but are still subjected to some rotor angle deviations in the transient phase. However, if the actuator fault is increased in the 2nd subsystem until the actuator completely fails whilst the 1st subsystem has no actuator faults. That leads to a big transient change in the rotor angle after 3 sec, as shown in Figure 5-10 .

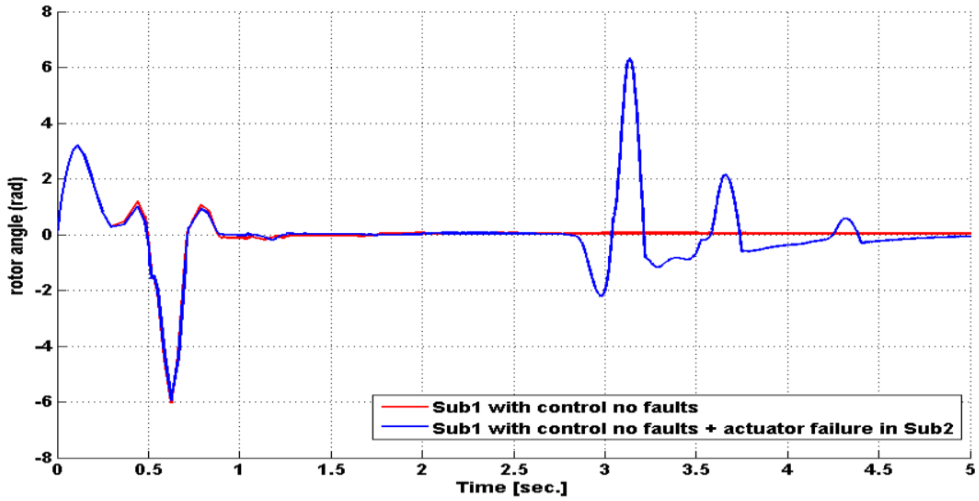

Figure 5-10 : AOISMC with 1st subsystem fault-free and 2nd subsystem with actuator failure

5.4.3 Comparison of OISMC and AOISMC

The Table 5-3 illustrates some points of comparison between the OISMC and AOISMC. From the table the LMI design with OISMC method only requires the choice of μ_i and $\mathfrak{z}_i$ but the LMI design with AOISMC requires a suitable selection of the initial values of $\bar{\mu}_{i0inl}$ and $\bar{\mu}_{i1inl}$. Suitable values of q_{i0} , q_{i1} and $\mathfrak{z}_i$ must also be chosen. Although, the time response of the AOISMC and the OISMC are slightly different, the AOISMC does give better performance in the case of 70% actuator fault in subsystems.

Method / **Contents**	**LMI with OISMC**	**LMI with AOISMC**
OISMC	OISMC	OISMC + two adaptive laws
Response	Only choosing μ_i and $\mathfrak{z}_i$	Choosing initial values of $\bar{\mu}_{i0inl}$ and $\bar{\mu}_{i1inl}$ as well as q_{i0} , q_{i1} and $\mathfrak{z}_i$
Stability	Less stable if the actuator faults or disturbances are increased	More stable if the faults or disturbances are increased

Table 5-3: Comparison of control inter-connected systems methods

5.5 Conclusion

A major challenge of the control of uncertain inter-connected systems is to remove or compensate for the effects of uncertainties and disturbances acting in the subsystems so that an ideal decentralization can be achieved. In the ideal case, the resulting hitherto inter-connected system now becomes a truly de-centralized structure in which the subsystems can be designed independently. This approach to the control of complex systems is an important contribution to the subject of fault-tolerant control for inter-connected systems.

An example of this fault tolerance can be seen if one subsystem fails then this failure does not influence the integrity of the remaining subsystems. Using static output de-centralized AOISMC in the inter-connected systems can give rise to robust performance in the elimination of the faults as well as any matched inputs. The controller is designed by the LMI method to achieve certain specifications while minimizing any mismatched inputs by using only the output signal. However, the combined AOISMC and LMI controllers may have limited ability to reduce the impact of faults and external inputs. However, these controllers are designed specifically to give good robust control action and by tuning the matrices N_i a desired response of every subsystem can also be achieved. When comparing the AOISMC with OISMC, the AOISMC gives the best results in the event of faults or disturbance in the actuators.

In some applications, for example with lack of measured states, the designer needs more freedom in the control design. The appropriate solution then is to consider dynamic output feedback design. Chapter 6 considers the design of de-centralized non-linear inter-connected systems via dynamic output feedback control, offering significant advantages over the static output feedback approaches described in this Chapter.

Chapter 6 : Dynamic output feedback ISMC with LMI for inter-connected non-linear systems

6.1 Introduction

As mentioned in Chapter 1 modern systems have become large and inter-connected to each other with increasing complexity. This stimulates the need for the design of *'de-centralized control'* that makes use of local states or output information (Park,Choi and Kong, 2007, Sung and Jin, 2012). The design of controllers for inter-connected systems requires knowledge of the dynamics of the individual subsystems. Since it is difficult to know the interactions between the subsystems, it is important to limit the effects of these interactions. Hence, a suitable de-centralized controller should take care to reduce the neglected interaction impacts on the subsystems, leading to improved stability and performance in the subsystems and also in the overall inter-connected system (Stanković and Šiljak, 2009).

The work required for non-linear inter-connected systems is to find a method to design a local control where each subsystem uses only local information, ensuring local (subsystem) stability and achieving some performance requirements (Pagilla and Zhu, 2005, Yongliang and Prabhakar, 2005, Batool,Horacio and Tongwen, 2009). So the designed control should ensure stability if there is any change in the interaction signals between these subsystems or under the action of external disturbances or internal uncertainties. Although robust stability and performance are the most important design objectives for the majority of control problems, these objectives can be hard to meet in practice because of uncertainty in the form of unmatched components and/or disturbances. Methods that seek to eliminate or reduce the impact of unmatched components and disturbances on robust stability and performance are complex, especially if the controller is designed on the basis of the nominal linear model (Ning and Wei-hua, 2007).

In the case of not being able to get access to some of the system states, the design freedom available to ensure stability and achieve the desired goals becomes limited. Hence, another approach must be used to recover the design freedom so that the performance and stability goals can be reached. The available signals that can be measured are output signals. For this reason, observer-based control and output feedback control have received considerable attention in the literature (Vidyasagar, 1981, Aldeen,Lau and Marsh, 1998, Trinh and Aldeen, 1998, Ghadami and Shafai, 2001, Pagilla and Zhu, 2005, Dhbaibi,Tlili,Elloumi and Benhadj Braiek, 2009, Tognetti,Oliveira and Peres, 2012). Specifically, within the framework of non-linear inter-connected systems the aforementioned output feedback methods have been proposed in the literature (Huan,Jeang and Yon, 2012, Park,Choi and Kong, 2007).

It is difficult and sometimes impossible to implement a full-order dynamic controller on a large-scale system (Tognetti,Oliveira and Peres, 2012). Therefore, a suitable way to deal with these systems is to decompose them into smaller subsystems with some interactions between them. For that reason each subsystem has a controller that can be designed alone by appropriate methods. One of these methods is an H_2/H_∞ based on dynamic output control for non-linear systems where the uncertainties are considered to be in polytope bounded form (Ning and Wei-hua, 2007, Zhao *et al.*, 2012). This approach has also been used to control systems with bounded uncertainties (Ning and Wei-hua, 2007). Normally the dynamic controller has a dynamic order that is the same as the system order, which gives the possibility of converting the design problem to an LMI optimization problem (Tognetti,Oliveira and Peres, 2012). One of the advantages of using dynamic output control is that it is an approach to attempt to recover the original state variable freedom by augmenting the system order. The increased design freedom means that there is increased potential for improving stability and enhancing system performance.

Sliding mode control (SMC) is a control system design method used to tackle some robust control problems. However, an SMC system has some limitations. For example, before reaching the sliding surface the system becomes sensitive to so-called unmatched components or unmatched exogenous disturbances (Edwards and Spurgeon, 1998,Pisano and Usai, 2011, Mondal and Mahanta, 2012). One strong limitation of the SMC approach is that it is often assumed that all the states are available for control. The output feedback approaches to SMC are more challenging, either requiring the use of dynamic compensation design (dynamic output feedback) or multi-objective design tools such as linear matrix inequalities (LMIs) in the static output feedback case. The challenge of output feedback and its practical use has been the cause of a steady increase in interest in output feedback based SMC (Edwards and Spurgeon, 1998, Ning and Wei-hua, 2007, Jeang-Lin and Huan-Chan, 2009).

When ISMC is used in control design, matched uncertainties do not resemble unmatched uncertainties or any disturbance for the reason that the latter affects the system behaviour and performance, even if the system is in the sliding surface (Poznyak,Fridman and Bejarano, 2004, Castaños,Xu and Fridman, 2006, Larbah and Patton, 2012). As a result unmatched compounds cannot be completely removed when the system is in the sliding surface, and so a technique must be found to reduce their influence on the system stability (Huan,Jeang and Yon, 2012).

The method proposed here consists of the combined use of output integral sliding mode control and a dynamic controller based on LMI design for a subsystem containing matched and unmatched uncertainties and with unmatched exogenous disturbances. To guarantee the robust stability in the closed-loop system and minimize the effect of disturbances on the sliding surface, the proposed approach ensures stability, and minimizes the impact of any disturbances. The method itself is simple in design, so the LMI is solved for each subsystem to find a linear feedback gain prior to designing the discontinuous component of the controller. As a result the controller consists of both linear and non-linear components. The non-linear part is responsible for dealing with matched uncertainties designed by using ISMC, whilst the linear part is responsible for reducing the impact of any unmatched uncertainties or external disturbances designed via an LMI formulation.

This Chapter investigates the methodology and benefit of utilizing available output signals to design an LMI-based dynamic output feedback controller within an OISMC framework applicable to de-centralized control, problems.

The main contribution of this Chapter is a new LMI-based design method for dynamic output feedback having the structure of an OISMC for non-linear inter-connected systems. Consequently, to verify this method an example of two inter-connected inverted pendulum systems is studied.

The Chapter is structured as follows. Section 6.2 describes the problem formulation. Then Section 6.3 shows the proposed control method that includes OISMC in the first part and an LMI-based output feedback dynamic control design in the second part. Section 6.4 describes the two inter-connected pendula application example used to illustrate the new approach and the simulation response of the subsystems. Finally, Section 6.5 presents a conclusion and further discussion.

6.2 Definition and problem formulation

Consider a non-linear inter-connected system comprising *n* subsystems described by:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t) \\ y_i(t) &= C_i x_i(t) \qquad i = 1, \ldots \ldots, N \end{aligned} \tag{6-1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ is are the control inputs and $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs. $A_i \in \mathbb{R}^{n\times n}$ is a known subsystem characteristic matrix, $B_i \in \mathbb{R}^{n\times m}$ is the known subsystem control input matrix, $C_i \in \mathbb{R}^{p\times n}$ is the known subsystem output matrix and $E_i \in \mathbb{R}^{n\times q}$ is the known subsystem external disturbance matrix. $Z_i(t)$ denotes the interactions between subsystems. $W_i(x_i, t)$ denotes the unknown uncertainties that satisfy the matching condition $W_i(x_i, t) = B_i Q_i(x_i, t)$.

The $d_i(t)$ represents an unknown bounded disturbance, $f_i(t) \in \mathbb{R}^k$ denotes the actuator faults where $f_i = -K(t)u_i$ and for which $K(t) = diag(K_i)$ and $0 \le K_i \le 1$. $K_i = 0$. As described in Section 3.2 that $0 \le K_i \le 1$ means that the actuator is working correctly and if $K_i = 1$ the actuator has failed completely, otherwise the fault is present.

Suppose that: $\Gamma = I_n - BB^+$ where B^+ is pseudo-inverse of a matrix B, $B^+ = (B^T B)^{-1} B^T$ and I_n is the $n \times n$ identity matrix.

It is assumed that the interactions between subsystems contain *two* components matched and unmatched components, respectively. so $Z_i = Z_{mi} + Z_{ui}$ where Z_{mi} is a matched component of Z_i and Z_{ui} is an unmatched component of Z_i (Shafai,Ghadami and Saif, 2011).

where $Z_{mi} = B_i B_i^+ Z_i$ and $Z_{ui} = \Gamma_i Z_i$

The same procedure is applied for the disturbance component $E_i d_i$, which is decomposed into matched and unmatched components via $E_i d_i = d_{mi} + d_{ui}$, with $d_{mi} = B_i B_i^+ E_i d_i$ and $d_{ui} = \Gamma_i E_i d_i$.

After substituting all of the subsystem assumptions the subsystem dynamics of Eq.(6-1) now becomes:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + B_i \Phi_{mi}(t) + \Phi_{ui}(t) \\ y_i(t) &= C_i x_i(t) \qquad i = 1, \dots\dots, N \end{aligned} \tag{6-2}$$

where Φ_{mi} is a matched component, $\Phi_{mi} = B_i^{+} Z_i(t) + Q_i(x_i, t) + B_i^{+} E_i d_i(t) + f_i(t)$ and Φ_{ui} is an unmatched component, $\Phi_{ui} = Z_{ui}(t) + d_{ui} = [\,\Gamma_i \quad \Gamma_i E_i\,] \begin{bmatrix} Z_i \\ d_i \end{bmatrix} = r_i w_i$

From Section 6.1 the control signal contains *two* components:

$$u_i(t) = u_i^{Dyn}(t) + u_i^{ISM}(t) \tag{6-3}$$

where u_i^{Dyn} is responsible for stabilizing the system and achieving the desired performance. On the other hand u_i^{ISM} is a discontinuous control responsible for eliminating the effects of matched components.

Substituting Eq.(6-3) into Eq.(5-2) yields:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i^{Dyn}(t) + B_i u_i^{ISM}(t) + B_i \Phi_{mi}(t) + \Phi_{ui}(t) \\ y_i(t) &= C_i x_i(t) \qquad i = 1, \dots\dots, N \end{aligned} \tag{6-4}$$

6.3 De-centralized dynamic output feedback control design

For inter-connected systems the approach to control design depends on the topology of the connections of the inter-connected systems (Šiljak and Stipanovic, 2001). These topologies are either based on physical interpretation or on logical meaning. According to the available topology the control system design uses either a one shot strategy or requires the individual control of each of the subsystems.

The control signal contains *two* parts one is designed by solving LMI formulation of the control problem and the second part is designed by OISLMC as following.

6.3.1 Output integral sliding mode control (OISLMC)

It is possible to use any of the commonly used or adaptive output integral sliding mode control methods to design the discontinuous control as is described in Chapter 5.

The integral sliding switching surfaces for each subsystem are defined as:

$$\sigma_i(y_i,t) = G_i[y_i(t) - y_i(t_o)] - \int_{t_o}^{t} \left(u_i^{Dyn}(t) + G_i C_i A_i \mathbb{G}_i y_i(t)\right) dt \tag{6-5}$$

where the G_i are design matrices that must satisfy the condition that the $G_i = C_i B_i$ are invertible and the $\mathbb{G}_i$ are matrices chosen so that $\mathbb{G}_i = (C_i)^+$

Then the i^{th} subsystem on the sliding surface will be:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^{Dyn}(t) + T_i \Phi_{ui}(t) \tag{6-6}$$

where $T_i = [I_i - B_i(C_i B_i)^+ C_i]$

From Eq. (5-12) the i^{th} subsystem still has unmatched components.

The proposed discontinuous subsystem control signals are:

$$u_i^{ISM}(t) = -\mu_i \frac{\sigma_i(y_i,t)}{\|\sigma_i(y_i,t)\| + \mathfrak{z}_i} \tag{6-7}$$

where the μ_i are positive scalar .

6.3.2 Dynamic output-feedback control design via LMI framework

After design of the OISMC the dynamic subsystem will be as in Eq. (6-6). Design of the LMI technique depends on the interactions between the subsystems, which can be classified into two procedures as mentioned in Chapters 3,4 and 5 .One procedure is based on individual subsystem design, whilst the second design is made on the overall system, the so-called one shot procedure.

6.3.2.1 LMI-based approach to local dynamic control design

The dynamic output feedback control will be as:

$$\dot{x}_{ci}(t) = A_{ci}x_{ci}(t) + B_{ci}y_i(t) \qquad (6\text{-}8)$$
$$u_i^{Dyn}(t) = C_{ci}x_{ci}(t) + D_{ci}y_i(t)$$

where $x_{ci}(t)$ is the state of the dynamic controller, with A_{ci}, B_{ci} , C_{ci} and D_{ci} the constant controller gain matrices with appropriate dimensions. And $u_i^{Dyn}(t)$ is a linear part responsible for stabilizing the dynamic system.

Augmenting the subsystem Eq. (6-6) and the equations of the controller Eq. (6-8) after substituting the control signal $u_i^{Dyn}(t)$ into the subsystem dynamics of Eq.(6-6) yields:

$$\dot{x}_{agi}(t) = A_{agi}x_{agi}(t) + B_{agi}K_{dyi}C_{agi}x_{agi}(t) + T_{agi}\Phi_{ui}(t) \qquad (6\text{-}9)$$

where

$$x_{agi}(t) = \begin{bmatrix} x_i(t) \\ x_{ci}(t) \end{bmatrix}, \quad A_{agi} = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{agi} = \begin{bmatrix} B_i & 0 \\ 0 & I_i \end{bmatrix}, \quad K_{dyi} = \begin{bmatrix} D_{ci} & C_{ci} \\ B_{ci} & A_{ci} \end{bmatrix},$$

$C_{agi} = \begin{bmatrix} C_i & 0 \\ 0 & I_i \end{bmatrix}$ and $T_{agi} = \begin{bmatrix} T_i \\ 0 \end{bmatrix}$

Here the same LMI design procedure as described in Chapter 5 is used to find a static output controller to control the non-linear inter-connected system. Following this, Eq.(6-9) can describe the system with the dynamical controller in one equation. As a result the LMI formulation is as follows:

By transforming x_{agi} to $\tilde{x}_{agi}$, after using the S-procedure (Šiljak and Stipanovic, 2001) and combining the two Eqs (6-9) & $\Phi_{ui}{}^T(t)\Phi_{ui}(t) \leq \alpha_i^2 \tilde{x}_{agi}{}^T(t)\tilde{x}_{agi}(t)$.

Then by using the Schur complement then the inequality becomes:

$$\begin{bmatrix} P_{di}T_{Ni}^T A_{agci}^T T_{Ni}^{-T} + T_{Ni}^{-T}A_{agci}\, T_{Ni}P_{di} + C_{agci}^T \mathcal{H}_i^T B_{agci}^T T_{Ni}^{-T} + T_{Ni}^{-T}B_{agci}\mathcal{H}_i C_{aci} & T_{Ni}^{-T}T_{agci} & P_{di}T_{Ni}^T \\ T_{agci}^T T_{Ni}^{-T} & -I_i & 0 \\ T_{Ni}P_{di} & 0 & -\epsilon_i I_i \end{bmatrix} < 0 \qquad (6\text{-}10)$$

where the $P_{di} = \begin{bmatrix} P_{1i} & 0_i \\ 0_i & P_{2i} \end{bmatrix}$, $T_{Ni} = \begin{bmatrix} I_{i(p\times p)} & 0_i \\ N_i^T & I_{i((n-p)\times(n-p))} \end{bmatrix}$ and N_i can be chosen as *tuning matrices* to achieve specific performances in each subsystem.

The algorithms to solve this problem are:

Algorithm 6-1:

1- Design OISMC from the Eqs. (6-5) & (6-7)
2- Calculate the augmenting matrices A_{agi} , B_{agi} , C_{agi} and T_{agi} from the Eq.(6-9)
3- Calculate Oc_i and then find $\mathcal{T}_i$ from the Eq. (5-28)
4- Transform the subsystem by the Eq. (5-29)
5- Choose the tuning parameter N_i and calculate T_{Ni} from the Eq.(5-42)
6- Minimize the ϵ_i subject to $P_{di} > 0$, $P_{1i} > 0$, $P_{2i} > 0$ and the Eq.(6-10)
7- Calculate the controller gain from $K_{dyi} = \mathcal{H}_i P_{1i}^{-1}$
8- Find the dynamic control parameters from $K_{dyi} = \begin{bmatrix} D_{ci} & C_{ci} \\ B_{ci} & A_{ci} \end{bmatrix}$
9- Build the dynamic output feedback control from the Eq.(6-8)

As described in the Chapter 5 in *Algorithm 5-2,* two other LMI conditions are added to decrease the magnitude of the Euclidean norm of the subsystem controller gains $\|K_{dyi}\|_2$ (improved the numerical conditioning of the feedback design) as follows:

$$\begin{bmatrix} -k_{\mathcal{H}i} I_i & \mathcal{H}_i^T \\ \mathcal{H}_i & -I_i \end{bmatrix} < 0 \qquad (6\text{-}11)$$

and

$$\begin{bmatrix} P_{di} & I_i \\ I_i & k_{Pi} I_i \end{bmatrix} > 0 \qquad (6\text{-}12)$$

where $k_{\mathcal{H}i}$ and k_{Pi} are a scalar variables.

Algorithm 6-2:

The procedure is as given under A*lgorithm 6-1* but step 6 must be changed to:

Minimize $(\epsilon_i + k_{\mathcal{H}i} + k_{Pi})$ subject to $P_{di} > 0$, $P_{1i} > 0$, $P_{2i} > 0$, the Eqs. (6-10) , (6-11) & (6-12) .

6.3.2.2 One shot LMI–based dynamic control design

As mentioned in Section 6.3.2.1 the dynamics of each subsystem in the sliding mode is described by Eq. (5-12) .

A one shot system dynamic that contains all inter-connected subsystems is given by:

$$\dot{X}(t) = A_d X(t) + B_d U^{Dyn}(t) + T_d J(t) \tag{6-13}$$

where: $X(t) = [x_1, x_2, \dots \dots, x_n]$, $U^{Dyn}(t) = [u_1^{Dyn}, u_2^{Dyn}, \dots \dots, u_n^{Dyn}]$, $A_d = diag(A_i)$, $B_d = diag(B_i)$, $\Gamma_d = diag(\Gamma_i)$, $Y(t) = [y_1, y_2, \dots \dots, y_n]$, $C_d = diag(C_i)$ and $J(t) = [J_1, J_2, \dots \dots, J_n]$. where "$diag$" represents the block diagonal matrix.

The one shot dynamic control can be constructed as:

$$\begin{aligned} \dot{X}_c(t) &= A_{cd} X_c(t) + B_{cd} Y(t) \\ U^{Dyn}(t) &= C_{cd} X_c(t) + D_{cd} Y(t) \end{aligned} \tag{6-14}$$

where $X_c(t) = [x_{C1}, x_{C2}, \dots \dots, x_{Cn}]$ is the controller's state , $A_{cd} = diag(A_{ci})$, $B_{cd} = diag(B_{ci})$, $C_{cd} = diag(C_{ci})$ and $D_{cd} = diag(D_{ci})$ are constant controller gain matrices with appropriate dimensions.

After substituting the control signal $U^{Dyn}(t)$ in the one shot system, the augmentation of the one shot system Eq. (6-13) and the one shot controller Eq. (6-14) are:

$$\dot{X}_{ag}(t) = A_{ag} X_{ag}(t) + B_{ag} K_{dy} C_{ag} X_{ag}(t) + T_{ag} \Phi_u(t) \tag{6-15}$$

where

$X_{ag}(t) = \begin{bmatrix} X(t) \\ X_c(t) \end{bmatrix}$, $A_{ag} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}$, $B_{ag} = \begin{bmatrix} B_d & 0 \\ 0 & I \end{bmatrix}$, $K_{dy} = \begin{bmatrix} D_{cd} & C_{cd} \\ B_{cd} & A_{cd} \end{bmatrix}$, $C_{ag} = \begin{bmatrix} C_d & 0 \\ 0 & I \end{bmatrix}$ and $T_{ag} = \begin{bmatrix} T_d \\ 0 \end{bmatrix}$

As described above, the formulation of the LMI based control is:

$$\begin{bmatrix} P_d T_N^T A_{agc}^T T_N^{-T} + T_N^{-T} A_{agc} \; T_N P_d + C_{agc}^T \mathcal{H}^T B_{agc}^T T_N^{-T} + T_N^{-T} B_{agc} \mathcal{H} C_{ac} & T_N^{-T} T_{agc} & P_d T_N^T \\ T_{agc}^T T_N^{-T} & -I & 0 \\ T_N P_d & 0 & -\epsilon I \end{bmatrix} < 0 \quad (6\text{-}16)$$

where $P_d = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, $T_N = \begin{bmatrix} I_{(p\times p)} & 0 \\ N^T & I_{((n-p)\times(n-p))} \end{bmatrix}$ and N can be chosen as tuning matrices to achieve specific performances in each subsystem.

The algorithms to solve this problem are:

Algorithm 6-3:

1- Design OISMC from the Eqs. (6-5) & (6-7)
2- Calculate the augmenting matrices A_{ag} , B_{ag} , C_{ag} and T_{ag} from the Eq.(6-15)
3- Transform the subsystem.
4- Choose the tuning parameter N and calculate T_N from $T_N = \begin{bmatrix} I_{(p\times p)} & 0 \\ N^T & I_{((n-p)\times(n-p))} \end{bmatrix}$
5- Minimize ϵ subject to $P_d > 0$, $P_1 > 0$, $P_2 > 0$ and the Eq.(6-16)
6- Calculate the controller gain from $K_{dy} = \mathcal{H} P_1^{-1}$
7- Find the dynamic control parameters from $K_{dy} = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix}$
8- Build the dynamic output feedback control from the Eq.(6-14)

As described in Chapter 5 in *Algorithm 5-3,* two other LMI conditions are added to decrease the magnitude of the Euclidean norm of the subsystem controller gains $\| K_{dy} \|_2$ (improved the numerical conditioning of the feedback design) as follows:

$$\begin{bmatrix} -k_{\mathcal{H}} I & \mathcal{H}^T \\ \mathcal{H} & -I \end{bmatrix} < 0 \quad (6\text{-}17)$$

and

$$\begin{bmatrix} P_d & I \\ I & k_P I \end{bmatrix} > 0 \quad (6\text{-}18)$$

where $k_{\mathcal{H}}$ and k_P are a scalar variables.

Algorithm 6-4:

The same procedure as in *Algorithm 6-3* is used by replacing step 6 by:

Minimize $(\epsilon + k_{\mathcal{H}} + k_P)$ subject to $P_d > 0$, $P_1 > 0$, $P_2 > 0$, the Eqs.(6-16) , (6-17) & (6-18)

The principle of this approach when it is applied to inter-connected systems is shown in Figure 6-1.

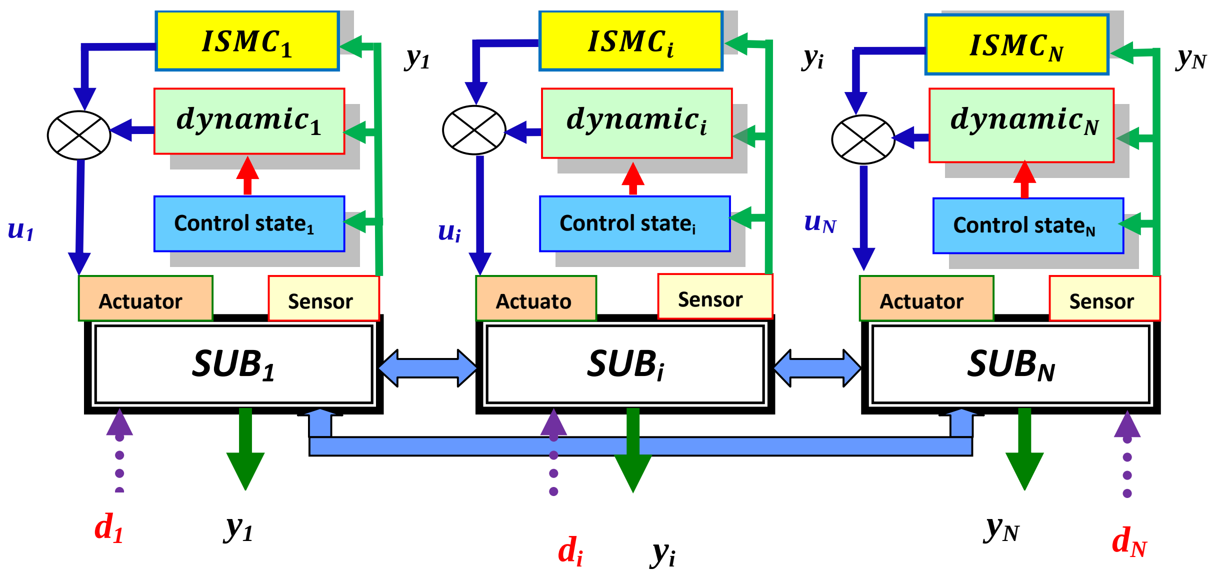

Figure 6-1: Dynamic output control of inter-connected systems via LMI+AOISMC

6.4 Application example (coupled inverted pendula)

The inverted pendulum is a very common tutorial example problem in control engineering, used to study the behaviour of unstable systems. The inverted pendulum example is analogous to a robotic arm and can thus be used to emulate an unstable robot arm or to study the motion of a standing human. In a more complex example a system comprising two coupled inverted pendula. To achieve stability forces must be applied to each of the poles of the pendula to protect them from falling down, i.e. the system is a two-dimensional control problem.

Consider two inverted pendula connected to each other by a spring, as shown in Figure 6-2. This example is considered to illustrate the proposed design technique for the case when the only available measurements are angular position, i.e. with the angular velocities remaining unmeasured.

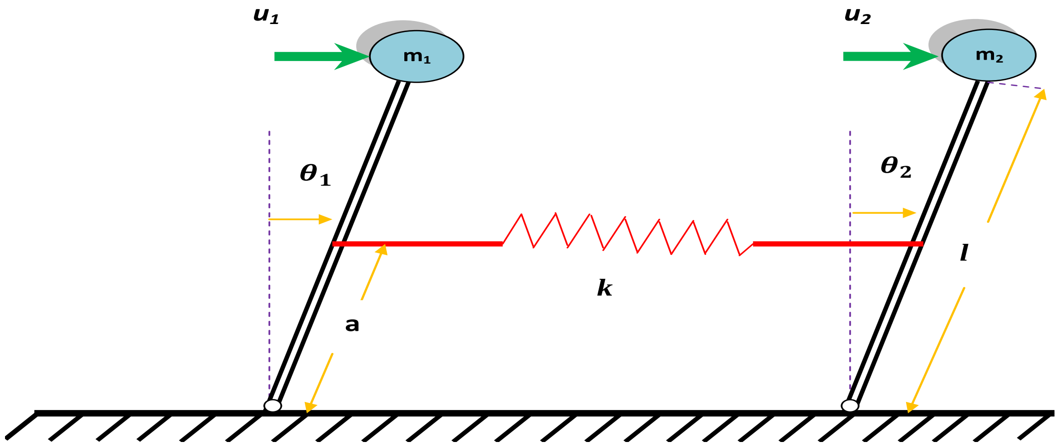

Figure 6-2 : Two coupled inverted pendula

6.4.1 Two inverted pendula system model

The motion of the two inverted pendula can be described by two inter-connected subsystems. The objective of the de-centralized is to control each pendulum with only its own information. The model is adopted from (Hua,Yuanwei,Siying and Lina, 2006).

The system contains two non-linear subsystems described as follows:

1st Subsystem:

$$\begin{bmatrix}\dot{x}_{11}(t)\\ \dot{x}_{12}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_{11}(t)\\ x_{12}(t)\end{bmatrix} + \begin{bmatrix}0 & 1\\ -\dfrac{a^2(t)}{(1+\Delta m_1)l^2} & 0\end{bmatrix}\begin{bmatrix}x_{11}(t)\\ x_{12}(t)\end{bmatrix}$$
$$+\begin{bmatrix}0\\ 1\end{bmatrix}\left[1-\frac{\Delta m_1}{(1+\Delta m_1)}\right]u_1(t) \qquad (6\text{-}19)$$
$$+\begin{bmatrix}0 & 1\\ \dfrac{a^2(t)}{(1+\Delta m_1)l^2} & 0\end{bmatrix}\begin{bmatrix}x_{21}(t)\\ x_{22}(t)\end{bmatrix} + \begin{bmatrix}0.1\\ 0.1\end{bmatrix}d_1(t)$$

2nd Subsystem:

$$\begin{bmatrix}\dot{x_{21}}(t)\\ \dot{x_{22}}(t)\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}x_{21}(t)\\x_{22}(t)\end{bmatrix}+\begin{bmatrix}0&1\\-\dfrac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{bmatrix}\begin{bmatrix}x_{21}(t)\\x_{22}(t)\end{bmatrix}$$
$$+\begin{bmatrix}0\\2\end{bmatrix}\left[1-\frac{\Delta m_2}{(1+\Delta m_2)}\right]u_2(t) \tag{6-20}$$
$$+\begin{bmatrix}0&1\\\dfrac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{bmatrix}\begin{bmatrix}x_{11}(t)\\x_{12}(t)\end{bmatrix}+\begin{bmatrix}0.1\\0.1\end{bmatrix}d_1(t)$$

where $x_{11}(t)=\theta_1$, $x_{12}(t)=\dot{\theta}_1$, $x_{21}(t)=\theta_2$, $x_{22}(t)=\dot{\theta}_2$, $|\Delta m_1|<0.1$, $|\Delta m_2|<0.05$ and $\frac{a(t)}{l}\in[0,1]$

1st Subsystem:

$$A_1=\begin{bmatrix}0&1\\1&0\end{bmatrix},B_1=\begin{bmatrix}1\\0\end{bmatrix},C_1=[1\quad 0],E_1=\begin{bmatrix}0.1\\0.1\end{bmatrix},z_1=\begin{bmatrix}0&1\\\frac{a^2(t)}{(1+\Delta m_1)l^2}&0\end{bmatrix}\begin{bmatrix}x_{21}\\x_{22}\end{bmatrix} \tag{6-21}$$
$$,\ W_1(x_1,t)=\begin{bmatrix}0&1\\\frac{-a^2(t)}{(1+\Delta m_1)l^2}&0\end{bmatrix},\ x_1(0)=\begin{bmatrix}4\\0\end{bmatrix}\text{ and }x_1(t)=\begin{bmatrix}x_{11}(t)\\x_{12}(t)\end{bmatrix}$$

2nd Subsystem:

$$A_2=\begin{bmatrix}0&1\\1&0\end{bmatrix},B_2=\begin{bmatrix}2\\0\end{bmatrix},C_2=[1\quad 0],E_2=\begin{bmatrix}0.1\\0.1\end{bmatrix},z_2=\begin{bmatrix}0&1\\\frac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{bmatrix}\begin{bmatrix}x_{11}\\x_{12}\end{bmatrix} \tag{6-22}$$
$$,W_2(x_2,t)=\begin{bmatrix}0&1\\-\frac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{bmatrix},\ x_2(0)=\begin{bmatrix}4\\0\end{bmatrix}\text{ and }x_2(t)=\begin{bmatrix}x_{21}(t)\\x_{22}(t)\end{bmatrix}$$

6.4.2 Simulation results

The subsystem dynamic controller $u_i^{Dyn}(t)$ designed using the LMI procedure of *algorithm 6-2*, leads to the gains:

1st subsystem:

$$A_{c1}=\begin{bmatrix}-2.4732&-1.6778\\-16.4386&-30.6373\end{bmatrix},\ B_{c1}=\begin{bmatrix}-0.4569\\1.0435\end{bmatrix},C_{c1}=[-1.7364\qquad 2.8465]$$
$$,\ D_{c1}=-2.2372$$

2nd subsystem:

$A_{c2} = \begin{bmatrix} -0.8173 & 0.7464 \\ -10.3240 & -21.3989 \end{bmatrix}$, $B_{c2} = \begin{bmatrix} -0.4581 \\ 0.6659 \end{bmatrix}$, $C_{c2} = [-3.8667 \quad 2.1721]$, $D_{c2} = -2.9032$

The subsystem parameter design or tuning matrices are:

$$N_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2.6 & 0 & 1 & 0 \\ 0 & 2.6 & 0 & 1 \end{bmatrix} \text{and } N_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2.8 & 0 & 1 & 0 \\ 0 & 2.8 & 0 & 1 \end{bmatrix}$$

The discontinuous control is $u_i^{ISM}(t) - \mu_i \frac{\sigma_i(y_i,t)}{\|\sigma_i(y_i,t)\| + \mathfrak{z}_i}$

where $\mu_1 = \mu_2 = 0.05$ and $\mathfrak{z}_1 = \mathfrak{z}_2 = 0.2$

The two subsystems without controls are unstable are shown in Figure 6-3 . The responses of the outputs (angles) of the two subsystems by applying dynamic control and OISMC with no faults in any subsystem are illustrated in Figure 6-4.

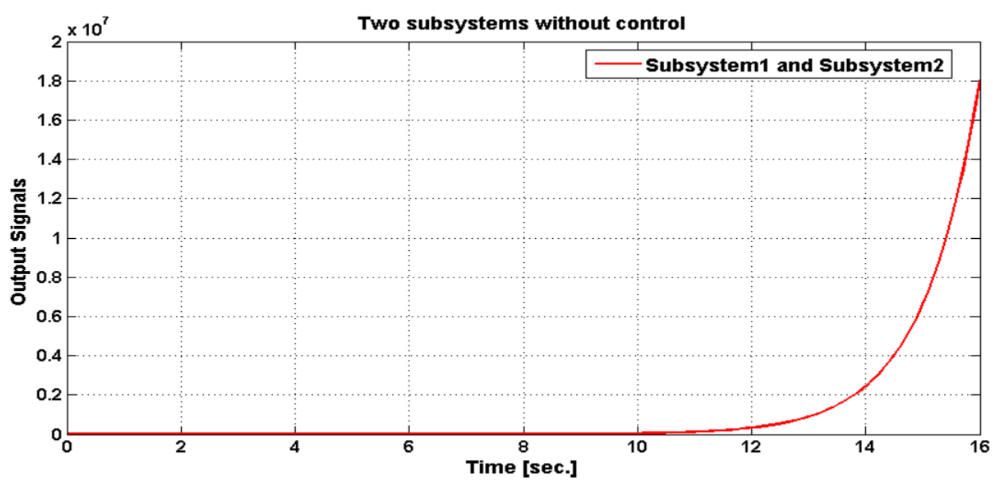

Figure 6-3: Unstable response of 1st & 2nd subsystems without control

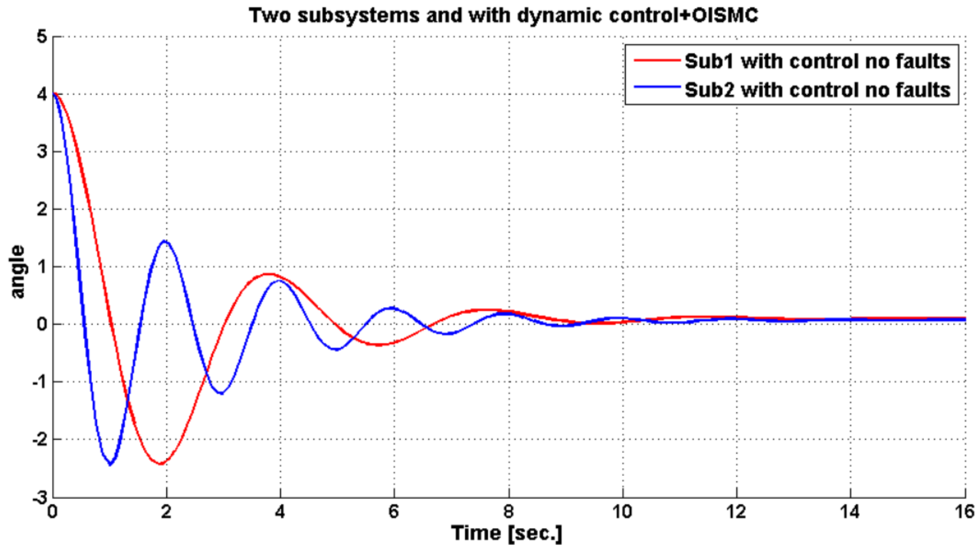

Figure 6-4: The inverted pendula angles with controller and no faults

From Figure 6-4 the two subsystems are stable and they oscillate at the beginning and reach the stable point around 8 s. Figure 6-5 shows the simulation of a 60% actuator fault in the 1st subsystem and the 2nd subsystem remains fault-free. From the simulation it is clear that both the systems are stable and the controller for the 1st subsystem decreases the affects of the faults. It is demonstrated that the OISMC can deal with a de-centralized control for some faults that are bounded.

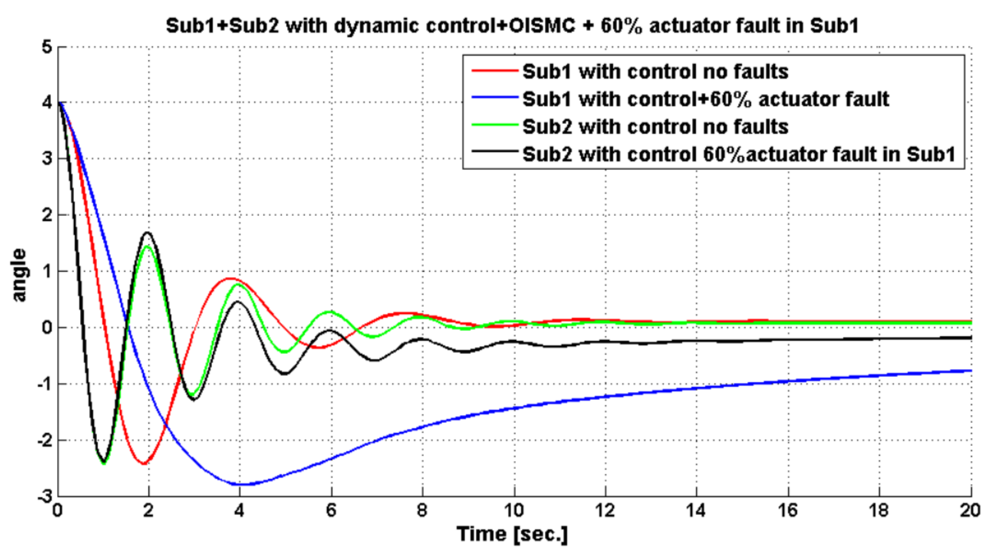

Figure 6-5: 1st subsystem with controller and 60% actuator fault

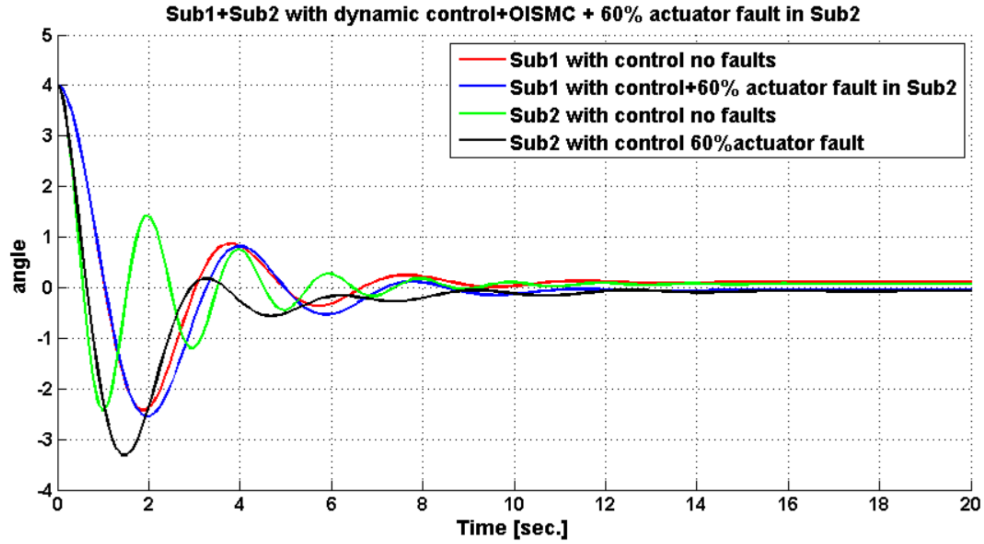

Figure 6-6: The 2nd subsystem with controller and 60% actuator fault

The same scenario is applied to the 2nd subsystem with no faults in the 1st subsystem as shown in Figure 6-6. Although the fault is a 60% actuator fault the 1st subsystem is still stable. This demonstrates the capability of the controllers to provide robust stabilization in the presence of unknown interactions.

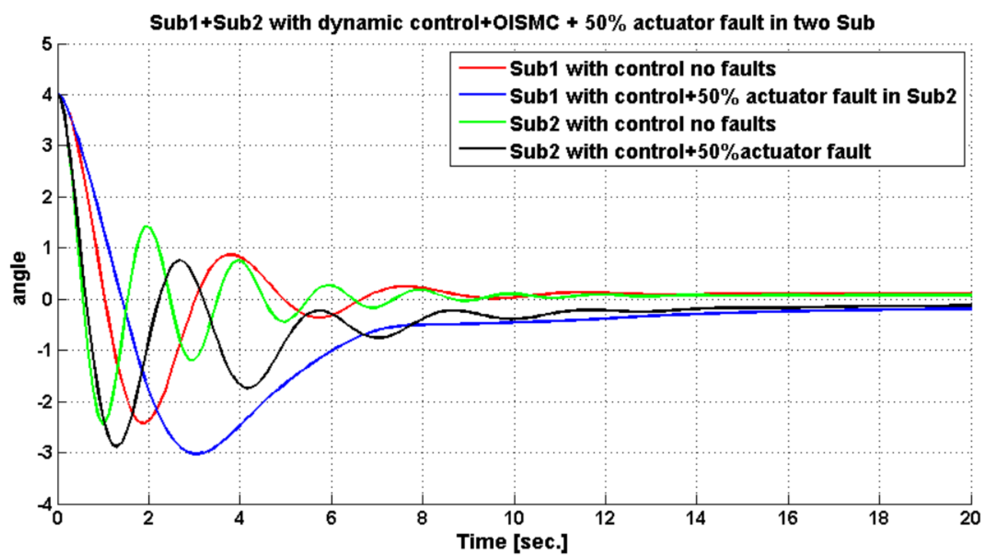

Figure 6-7: Two subsystems with controller and 50% actuator faults in each subsystem

Figure 6-7 shows the response of the output of the two subsystems for the case of 50% actuator faults in each of the subsystems. Although the faults reach 50% in the two actuators, the two subsystems remain stable. The explanation for this is the ability of the controllers to deal with these faults.

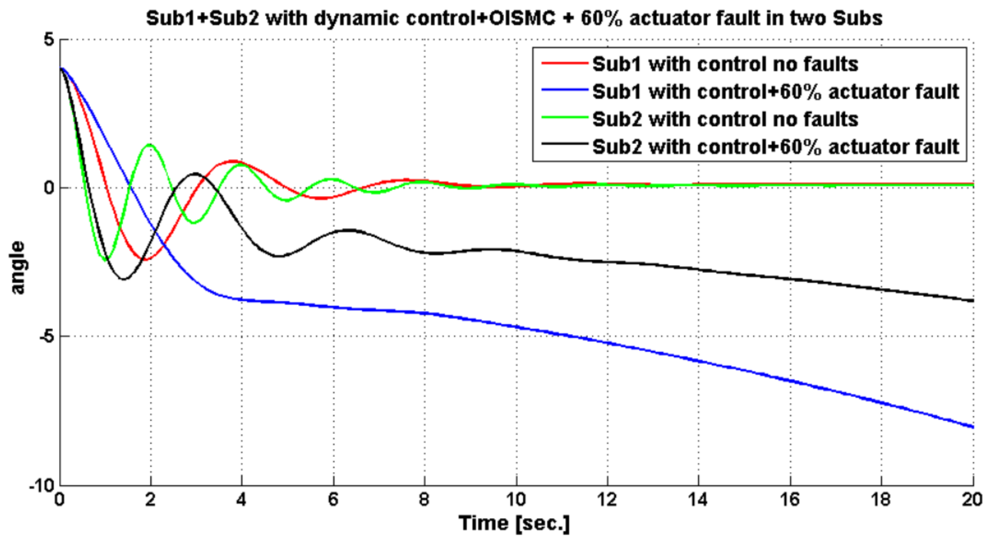

Figure 6-8 : Two subsystems with controller and 60% actuator faults in each subsystem

Figure 6-8 shows the simulation of a fault scenario for two subsystems in which each of the actuator faults is 60% and furthermore the actuator faults are 60%. It can be clearly seen that the subsystems are affected by the faults but the 1st subsystem is unstable whilst the 2nd subsystem is still stable. That means the controllers can deal with some limited or bounded actuators faults.

According to the simulation results, the following points are now highlighted as follows:

- Different actuator fault scenarios are investigated by designing a dynamic control with OISMC to decease the impacts of these faults on the closed-loop performance of the inter-connected system.
- The effects of some bounded actuator faults on the performance of the subsystem can be passively tolerated by the dynamic controller with OISMC in this example less than 60% of actuator faults.

6.5 Conclusion

The control of non-linear inter-connected systems is challenging, especially when available information about the system and measured information is limited. This Chapter shows that the use of output information to control these systems through utilization of a dynamic controller gives a greater chance and more freedom to achieve a specific performance.

A linear dynamic controller for ensuring system sliding in each subsystem of an inter-connected system can be designed via an LMI framework, in which each sliding surface is designed according to some specifications. A downside of the LMI formulation arises due to the presence of a non-linear term $\mathcal{Y}_i B_{ci} K_i C_{ci}$ in the LMI formulation as explained in Chapter 5. The effect of this can be overcome by considering these non-linear terms as described in Chapter 5.

In the de-centralized control case, the linear component of the OISMC controller is designed to deal satisfy performance and stability requirements, for example in a given subsystem. However, this linear component can also be used to compensate the subsystem unmatched uncertainties arising from modelling errors, subsystem interactions and exogenous disturbances. On the other hand, the approach proposed in this work is focused on integral sliding for output feedback systems – the OISMC.

Linear output feedback control can be achieved either using dynamical compensation control or using static output feedback. Although the dynamic compensation approach alone could give good stability and performance, when combined with OISMC some powerful robustness properties result that guarantee the compensation of matched components and bounded faults.

In this Chapter this strategy has been applied to an example of two non-linear coupled inverted pendulum systems, connected by spring. As a result of using two controllers, the dynamic output feedback and OISMC, the response of the two subsystems is perfect when there are no faults in either of the pendula subsystems. However, the controller can compensate the effects of actuator faults but up to a bounded value of these faults. In this example the control can overcome up to about 50% of actuator fault. Above that level the subsystems tend to become unstable.

Chapters 3, 4, 5 & 6 discuss new methods or new techniques to deal with fault-tolerant control in actuators. On the other hand, Chapter 7 focuses on the fault estimation in inter-connected systems, and how to construct an observer-based fault estimator to estimate an actuators fault. Hence, as long as the estimation of the fault is achieved robustly it is very easy to determine the location of the fault so that subsequent system reconfiguration after a fault has occurred can easily be achieved.

Chapter 7 : Actuator fault estimation for non-linear inter-connected systems

7.1 Introduction

The purpose of this Chapter is to investigate the potential of using robust fault estimation methods for actuator fault estimation in non-linear inter-connected systems after designing robust FTC. It is assumed that any of the methods of distributed control, based on either single level or multi-level approaches as considered in Chapters 2, 3, 4, 5 & 6 are used to achieve the stability. The challenge is to ensure that the estimation of faults in subsystem actuators is done precisely in the presence of modelling uncertainty, interactions between subsystems and disturbance. A significant number of robust fault estimation methods are available in the literature, for example using sliding mode observer estimation (Edwards,Spurgeon and Patton, 2000), proportional multiple integral observers (Ibrir, 2004, Koenig, 2005, Gao,Ding and Ma, 2007, Aiguo and Guangren, 2007). However there have been few studies that apply these methods to distributed and inter-connected system structures (Trinh and Aldeen, 1998, Klinkhieo,Patton and Kambhampati, 2008). Complex inter-connected systems have very challenging FDI design requirements due to uncertainties acting at different levels of the system (within subsystem components or at higher hierarchical levels). Hence, although fault estimation is an appealing approach to FDI the robustness problem requires careful attention. As the system complexity increases, more complex models are required, especially when uncertainties appear. It then becomes difficult to predict the fault directly online, because of the separation of the effects arising from the influences of the interactions and the uncertainties.

The problem of fault estimation for inter-connected systems is more complex than for the case of single systems since inter-connected systems are influenced not only by faults but also by subsystem inter-connections and exogenous disturbances. Whilst the literature does discuss the challenging problem of fault estimation in the presence of disturbance (Yan and Edwards, 2008) the more complex problem of fault estimation in the presence of unknown interactions has not been considered.

The distributed and inter-connected system can be viewed as a form of complex system since the number of subsystems can be significant and hence systematic procedures are required to handle the general distributed and inter-connected system problem from a fault estimation standpoint. Indeed, scientific progress in the fields of industry, energy, and several other areas of daily life has resulted in an increase in system complexity in application fields such as telecommunications networks (Boubour *et al.*, 1997), transport systems and power systems distribution networks (Tan,Crusca and Aldeen, 2008), coordinated formation flying etc (Flint,Polycarpou and Fernandez-Gaucherand, 2002), complex inter-connected process systems (Schuler,Munz and Allgower, 2012), water distribution networks (Boccelli *et al.*, 1998), high building structures (Chowdhury and Carrier, 2000) and environmental control in complex building systems.

There is a security issue associated with many complex distributed systems and furthermore the increased complexity can lead to an increase in the possibility of component faults and failures. Hence, to avoid catastrophic failure and to enhance system security and reliability as well as improve the system performance and stability some additional features beyond those of robust control are required. The detection and robust isolation of faults in system components (using robust FDI methods) with the potential of reliable FTC has become a steadily more important field of research for complex inter-connected systems (Polycarpou and Vemuri, 1995, Gertler, 1998, Wang and Yuan-Chun, 2004, Panagi and Polycarpou, 2011, Shames,Teixeira,Sandberg and Johansson, 2011, Zhang and Zhang, 2012).

It can never be guaranteed that a subsystem will operate without faults or component failures. Although many system components can have high reliability, in practice a fault could occur at any moment during the system operation. Indeed for a truly complex large-scale inter-connected system there is always a significant possibility that faults can occur in any subsystem. However, the faults may occur in the presence of interactions and other known or unknown disturbances and this means that the problem of robust FDI is very challenging, since the faults and the interactions or disturbances can have competing effects on the detection/isolation system.

Although the influence from interactions gives rise to inaccurate estimation of faults most published FDI methods dealing with complex inter-connected systems can only deal with single types of faults (without uncertainties or with very small uncertainties). These faults may lead to instability or even overall system failure.

Although FDI methods provide information about the existence of faults, they are not always capable of providing information about the size and the type of the faults and their effects on the system. On other hand by using information from the input and the output signals of the system, the faults can be estimated online (Blanke,Kinnaert,Lunze and Staroswiecki, 2006, Sun,Patton and Goupil, 2012).

For example if a fault occurs in a system actuator, fault estimation can be used not only to indicate that a fault has occurred as well as where it acts but the nature and characteristic of the fault becomes known immediately from the estimated faults signal. Since the estimation provides the time profile of the fault signal all important features of the fault become known, even if the fault estimation is subject to certain robustness requirements. It therefore can be argued that the fault estimation approach to FDI has significant advantages over the residual-based FDI method in which a residual signal is only used to determine whether or not a fault has occurred, by using a threshold. The fault estimation can be used to make some decision about the system operation, e.g. to change or adapt the controller, reconfigure the system or to use a form of hardware or analytical redundancy to recover normal system operation. The concept of an actuator fault or indeed of actuator "failure" can be appropriate for inter-connected, distributed or networked systems.

However, as for the residual-based approach to FDI the fault estimation methods also have accompanying robustness problems arising from the effects of non-linear uncertainty, modelling uncertainty, inter-connections and exogenous disturbances. All of these uncertain system effects can lead to errors in the fault estimation signals that can give rise to false alarms. The robustness problem for the residual-based approaches to FDI is also very well known, but it is very hard to achieve satisfactory robustness using this approach. Consequently, a significant number of FDI approaches now focus on robust fault estimation methods.

Clearly, early and prompt detection and isolation (and even identification) of faults can provide reliable alarm systems to prevent system damage, economic loss and even dangerous catastrophes. For example if a fault occurs in a system actuator so that it is unable to deliver the control system to actuate the system, the stability and performance of the closed-loop system can be severely affected and the fault effect may continue to develop until a system failure or catastrophe occurs. Once the fault is detected and isolated (and its severity determined) a special type of FTC system can be used to mitigate the effect of the fault either using hardware redundancy or software redundancy based on observer or estimation methods. The idea of the FTC scheme is thus to make the closed-loop system "fault tolerant", i.e. to mitigate the effects of the faults and maintain good system operation and performance (Patton, 1997a, Chen and Patton, 1999, Patton *et al.*, 2007, Halim,Edwards and Chee, 2011).

Several observer-based approaches have been proposed in fault estimation, such as unknown input observers (Aldeen,Lau and Marsh, 1998), sliding mode observers (Edwards,Spurgeon and Patton, 2000, Yan and Edwards, 2008, Orani,Pisano and Usai, 2009, Sharma and Aldeen, 2010), higher order sliding (Davila,Fridman and Levant, 2005, Orani,Pisano and Usai, 2009), the adaptive observer (Zhang,Jiang and Cocquempot, 2008, Challouf *et al.*, 2010) , fuzzy observers (Patton,Chen and Lopez-Toribio, 1998, Lopez-Toribio and Patton, 1999), H_∞ observers (Huang and Kiong, 2009) and proportional multiple integral observers (Ibrir, 2004, Witczak, 2007, Gao,Ding and Ma, 2007, Gao and Ding, 2007), and the non-linear observer (Maoa,Jianga and Shi, 2010).This Chapter focuses on the use of the proportional and multi-integral (PMI) observer approach (Ibrir, 2004, Gao,Ding and Ma, 2007) to estimate actuator faults in non-linear inter-connected systems with unknown exogenous disturbances.

It is assumed that an actuator fault signal is added to the system or the model as an unknown external signal (Chen and Patton, 1999). That can be classified according to:

1- Stuck at a specific value: means that the actuator remains at a certain value reached at a certain moment.
2- Complete failure: means that the actuator is unable to make any action at a given moment and stop completely.
3- Fault of performance: means that the actuator gives a signal less or higher than the required at a given moment (Witczak, 2007, Halim,Edwards and Chee, 2011).

The main contribution in this Chapter is the design of a Proportional Multiple Integral Observer (PMIO) to estimate or reconstruct actuator faults in non-linear inter-connected subsystems, including an approach to combining each fault estimation observer within the appropriate subsystem.

The Chapter is structured as follows. Section 7.2 describes the problem formulation. Then section 7.3 considered the proposed observer design to estimate the actuator faults. Section 7.4 illustrates a numerical example contains three inter-connected subsystems to show the design approach and simulation responses of different chosen parameters design. Finally, a conclusion is presented in Section 7.5.

7.2 Problem statement and preliminaries

The way in which actuator faults influence the dynamics of a non-linear inter-connected system can be described in state-space form via a Lipschitz non-linear system representation as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + Z_i(t) + W_i(x_i, t) + E_i d_i(t) + B_i f_i(t) \\ y_i(t) &= C_i x_i(t) \qquad i = 1, \ldots \ldots, N \end{aligned} \tag{7-1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ is are the control inputs and $y_i(t) \in \mathbb{R}^p$ is the vector of system outputs. $A_i \in \mathbb{R}^{n\times n}$ is a subsystem characteristic matrix, $B_i \in \mathbb{R}^{n\times m}$ is the subsystem control input matrix, $C_i \in \mathbb{R}^{p\times n}$ is the subsystem output matrix and $E_i \in \mathbb{R}^{n\times q}$ is the subsystem external disturbance matrix, all these matrices are known. $Z_i(t)$ denotes the interactions between subsystems. $W_i(x_i, t)$ denotes the uncertainties that satisfy the matching condition $W_i(x_i, t) = B_i Q_i(x_i, t)$ are unknown . $d_i(t)$ represents an unknown bounded disturbance.

$f_i(t) \in \mathbb{R}^k$ denotes the actuator faults where $f_i = -K(t)u_i$ and for which $K(t) = diag(K_i)$ and $0 \le K_i \le 1$. $K_i = 0$. That means the actuator is working perfectly and if $K_i = 1$ the actuator has failed completely, otherwise the fault is present.

Assumption:

The pair (A_i, B_i) is controllable and (C_i, A_i) is an observable.

7.3 Proportional and multi-integral observer design

As mentioned in Section 7.1 the estimation or reconstruction of a fault is a more powerful approach to FDI than the use of an FDI residual. The famous observer that is used to estimate faults in inter-connected systems is the sliding mode observer (the Walcott-Żak observer (Kalsi,Lian,Hui and Zak, 2009) and the Edwards and Spurgeon observer (Edwards and Spurgeon, 1998)). The main differences between these two observers are: the Walcott-Żak observer has simpler structure and it is easier to understand while the Edwards and Spurgeon observer requires a triple state transformation but on the other hand gives more system information. The main disadvantage of using the SMC observer is the sliding surface reachability problem which means that the observer does not provide sliding motion until the sliding surface is reached in state space. As a consequence if faults occur during the reaching phase the observer may be sensitive to the fault even if during the reaching phase the fault cannot be estimated. The disadvantage of the Walcott-Żak observer is that there is less design freedom when compared with the Edwards and Spurgeon observer. On the other hand the Edwards and Spurgeon observer requires a triple state transformation during the design procedure that can appear over-complicated. A consequence of the use of the triple transformation is that after designing the observer gain matrices the state system must be transferred back into the original coordinates which make the algorithm complicated and not easy to understand.

The SMC fault estimation observers clearly have some drawbacks and in this Chapter a good alternative approach is used based on the Proportional and Multi-Integral Observer (PMIO) of (Ibrir, 2004, Gao and Ding, 2007). In this work a robust approach to the PMIO has been developed to minimize the effects of subsystem interactions and uncertainties.

It is shown that this estimation approach is relatively easy to design and implement and it is also not limited by a reachability problem and does not require switching observer feedback.

Figure 7-1 illustrates the idea of the use of actuator fault estimation in non-linear inter-connected systems where it is assumed that every subsystem has a state observer for combined state and fault estimation. It is also assumed that the subsystem observers are de-centralised meaning that there are not connections between the various subsystem observers.

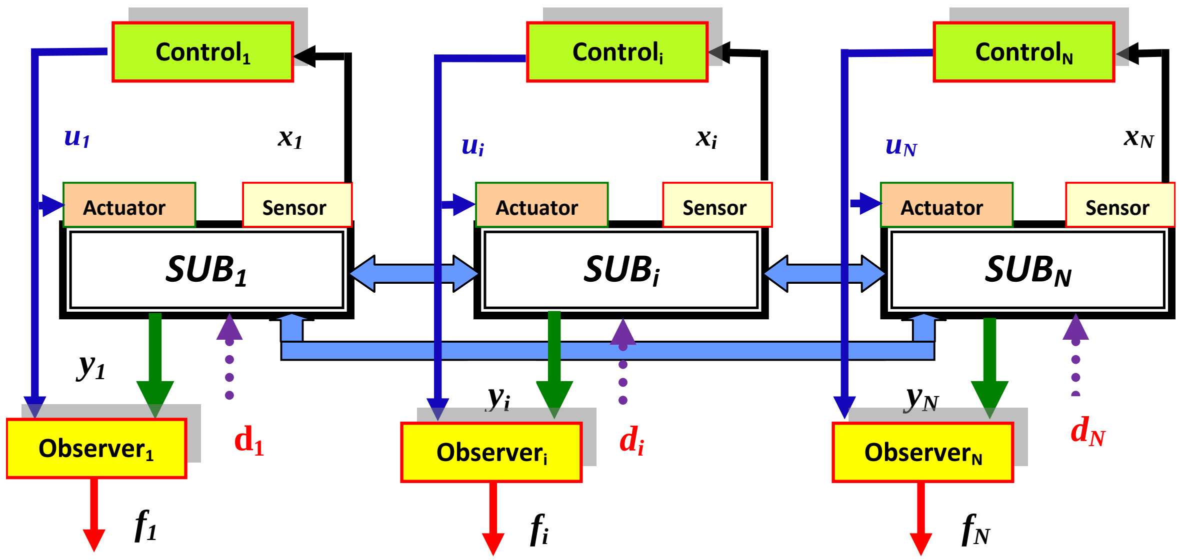

Figure 7-1: Fault estimation in inter-connected system via subsystem observers

To estimate the actuator fault assume that the r^{th} derivative of this fault is bounded according to:

$$\mathfrak{F}_{ij}(t) = f_i^{(r-1)}(t) \qquad j = 1, \dots \dots, r \tag{7-2}$$

From Eq.(7-2) the following relationships hold:

$$\begin{aligned} \dot{\mathfrak{F}}_{i1}(t) &= f_i^r(t) \\ \dot{\mathfrak{F}}_{i2}(t) &= \mathfrak{F}_{i1}(t) \\ \dot{\mathfrak{F}}_{i3}(t) &= \mathfrak{F}_{i2}(t) \\ \dot{\mathfrak{F}}_{ir}(t) &= \mathfrak{F}_{ir-1}(t) \end{aligned} \tag{7-3}$$

Combining the subsystem faults with the subsystem dynamics, i.e. by combining Eq. (7-1) and Eq. (7-3) the resulting augmented system corresponding to each subsystem can be written as:

$$\begin{aligned} \dot{\check{x}}_i(t) &= \mathcal{A}_i \check{x}_i(t) + \mathcal{B}_i u_i(t) + \mathcal{Q}_i \mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t) \\ y_i(t) &= \mathcal{C}_i \check{x}_i(t) \qquad i = 1, \dots \dots, N \end{aligned} \tag{7-4}$$

where:

$\breve{x}_i(t) = [x_i^T(t)\ ,\ \mathfrak{F}_1^T\ ,\ \mathfrak{F}_2^T\ , \dots \dots ,\ \mathfrak{F}_r^T] \in \mathbb{R}^{\breve{n}}$

$$\mathcal{A}_i = \begin{bmatrix} A_i & 0 & \cdots & 0 & B_i \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I_i & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_i & 0 \end{bmatrix} \in \mathbb{R}^{\breve{n}\times\breve{n}} \quad , \quad \mathcal{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{\breve{n}\times m} \quad , \quad \mathcal{Q}_i = \begin{bmatrix} I_i & B_i & E_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \in$$

$$\mathbb{R}^{\breve{n}\times(\breve{n}+m+h)}, \mathcal{L}_i = \begin{bmatrix} 0 \\ I_{ri} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{\breve{n}\times r}\ , \mathcal{C}_i = [\ c_i\ \ 0\ \ \ 0 \cdots 0\ \ ] \in \mathbb{R}^{p\times\breve{n}}\ \ , \mathcal{D}_i(t) = \begin{bmatrix} Z_i(t) \\ Q_i(x_i,t) \\ d_i(t) \end{bmatrix} \quad \text{and}$$

$\breve{n} = n + r$

where it is assumed that the $(\mathcal{A}_i, \mathcal{C}_i\ )$ is observerable.

Consider a classical *Luenberger* observer to estimate the combined subsystem state and actuator fault simultaneously as:

$$\dot{\hat{\breve{x}}}_i(t) = \mathcal{A}_i\hat{\breve{x}}_i(t) + \mathcal{B}_i u_i(t) + L_i(y_i(t) - \mathcal{C}_i\hat{\breve{x}}_i(t)) \tag{7-5}$$

where $\hat{\breve{x}}_i(t)$ is the estimate of the augmented state $\breve{x}_i(t)$ and L_i is observer gain. This gain is designed to decrease the effect of $\mathcal{D}_i(t)$ which contains interconnections, uncertainties and disturbances.

In the ideal case the steady error between the state and its estimation is zero but the effect of interconnections, uncertainties and disturbances may still be present, i.e. there is some residual difference between the actual and estimated state as the components in $\mathcal{D}_i(t)$ prevent the steady state estimation error from reaching zero value.

According to the design algorithm proposed by (Ibrir, 2004, Gao,Ding and Ma, 2007), suppose there is a positive-definite matrix $P_i > 0$ that is the solution of the Lyapunov equation:

$$-(\eta_i I + (\mathcal{A}_i - L_i\mathcal{C}_i))^T P_i - P_i(\eta_i I + (\mathcal{A}_i - L_i\mathcal{C}_i)) = -\mathcal{C}_i{}^T\mathcal{C}_i \tag{7-6}$$

where $-(\eta_i I + (\mathcal{A}_i - L_i\mathcal{C}_i))$ are Hurwitz. η_i are positive tuning parameters and the gain of the subsystem observer L_i can be computed as:

$$L_i = P_i{}^{-1}\mathcal{C}_i{}^T \tag{7-7}$$

Proof:

The steady-errors between the state and estimated state of each observer are $e_i(t) = \breve{x}_i(t) - \hat{\breve{x}}_i(t)$ which leads to the estimation error dynamic system (from Eqs. (7-4) and (7-5)) as:

$$\dot{e}_i = (\mathcal{A}_i - L_i\mathcal{C}_i)e_i(t) + \mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t) \tag{7-8}$$

For the case of $\mathcal{D}_i(t) \neq 0$ and $f_i^r(t) \neq 0$, the stability of the state estimation error system can be verified as follows:

Consider a Lyapunov function candidate as:

$$V_i(e_i) = e_i^T(t)P_i e_i(t) \tag{7-9}$$

The time-derivative of $V_i(e_i)$ is given by:

$$\dot{V}_i(e_i) = \dot{e}_i^T(t)P_i e_i(t) + e_i^T(t)P_i\dot{e}_i(t) \tag{7-10}$$

Substituting Eq. (7-8) into Eq. (7-10) this yields:

$$\begin{aligned}\dot{V}_i(e_i) = [(\mathcal{A}_i - L_i\mathcal{C}_i)e_i(t) + \mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t)]^T P_i e_i(t) + \\ e_i^T(t)P_i[(\mathcal{A}_i - L_i\mathcal{C}_i)e_i(t) + \mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t)]\end{aligned} \tag{7-11}$$

The Eq.(7-11) can be re-arranged as follows:

$$\begin{aligned}\dot{V}_i(e_i) = e_i^T(t)[(\mathcal{A}_i - L_i\mathcal{C}_i)^T P_i + P_i(\mathcal{A}_i - L_i\mathcal{C}_i)]e_i(t) + 2e_i^T(t)P_i(\mathcal{Q}_i\mathcal{D}_i(t) \\ + \mathcal{L}_i f_i^r(t))\end{aligned} \tag{7-12}$$

According to the Cauchy-Schwarz inequality $|x.y| \leq \|x\|.\|y\|$ the term:

$$2e_i^T(t)P_i\big(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t)\big) \leq 2\left\|e_i^T(t)P_i^{\frac{1}{2}}\right\|.\left\|P_i^{\frac{1}{2}}(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t))\right\| \tag{7-13}$$

where $P_i = P_i^{\frac{1}{2}}.P_i^{\frac{1}{2}} > 0$

The Eq.(7-13) can be re-formulated as:

$$\Re[\lambda_{j,i}(\mathcal{A}_i - L_i\mathcal{C}_i)] < -\eta_i \qquad \forall j \in (1,2,\ldots\ldots,\breve{n}) \tag{7-14}$$

Where $\lambda_{j,i}$ are the eigenvalues of $(\mathcal{A}_i - L_i\mathcal{C}_i)$ and:

$$\Re[\lambda_{max,i}(\mathcal{A}_i - L_i\mathcal{C}_i)] < -\eta_i \tag{7-15}$$

This leads to $e_i^T(t)(\mathcal{A}_i - L_i\mathcal{C}_i)^T P_i e_i(t) \leq -\eta_i V_i(e_i)$

According to Eq. (7-13) and Eq. (7-15) the time-derivative of $V_i(e_i)$, i.e. $\dot{V}_i(e_i)$ satisfies:

$$\dot{V}_i(e_i) \leq -2\eta_i V_i(e_i) + 2\left\|e_i^T(t)P_i^{\frac{1}{2}}\right\| . \left\|P_i^{\frac{1}{2}}(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t))\right\| \tag{7-16}$$

Multiplying both sides of Eq. (7-16) by $\frac{1}{2}e_i^{-T}(t)P_i^{\frac{-1}{2}}$ yields:

$$\frac{1}{2}e_i^{-T}(t)P_i^{\frac{-1}{2}}\dot{V}_i(e_i) \leq -\eta_i e_i^{-T}(t)P_i^{\frac{-1}{2}}V_i(e_i) + \left\|P_i^{\frac{1}{2}}(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t))\right\| \tag{7-17}$$

Now suppose that $\tilde{V}_i(e_i) = \sqrt{V_i(e_i)} = (e_i^T(t)P_i e_i(t))^{1/2}$ and $\dot{\tilde{V}}_i(e_i) = \frac{1}{2}[e_i^T(t)P_i e_i(t)]^{-1/2}(e_i)\dot{V}_i(e_i)$ then Eq. (7-17) can be rewritten as:

$$\dot{\tilde{V}}_i(e_i) \leq -\eta_i\tilde{V}_i(e_i) + \left\|P_i^{\frac{1}{2}}(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t))\right\| \tag{7-18}$$

The stability of the system Eq. (7-18) requires that $\dot{\tilde{V}}_i(e_i) \leq 0$ this leads to:

$$\eta_i\tilde{V}_i(e_i) = \eta_i e_i^T(t)P_i^{\frac{1}{2}} \geq \left\|P_i^{\frac{1}{2}}(\mathcal{Q}_i\mathcal{D}_i(t) + \mathcal{L}_i f_i^r(t))\right\| \tag{7-19}$$

From Eq. (7-19) the upper bound of the steady error is:

$$\|e_i\| \leq \frac{1}{\eta_i}[\|\mathcal{Q}_i\|\|\mathcal{D}_i(t)\| + \|\mathcal{L}_i\|\|f_i^r(t)\|] \tag{7-20}$$

That means the effect of interconnections, uncertainties and disturbances on the state error can be made as small as possible by increasing the value of the η_i . Hence, by choosing the values of η_i and solving Eq. (7-6) the observer will then become insensitive to any interconnections, uncertainties and disturbances.

From Eq. (7-20), the higher the value of the η_i, the less the impact of the interconnections, uncertainties and disturbances. From Eq. (7-6) as the values of η_i are increased the computed values of P_i decrease so that the value of the gains L_i increase, as a consequence of the gain equation $L_i = P_i^{-1}C_i^T$

To summarize this procedure: Increasing the η_i leads to decreasing values of the Euclidean norm $\|P_i\|$, leading to increased values of the Euclidean norm $\|L_i\|$.

Note:

From these results it can be observed that the design of a subsystem observer with high state tracking accuracy requires the use of high values of observer gain norm values, since the observer estimates the state of the system and the actuator fault simultaneously.

7.4 Numerical example

Consider the following numerical example consisting of three inter-connected non-linear subsystems, adapted from (Chou and Cheng, 2000):

1st Subsystem:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, z_1 = (\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix})$$

$$W_1(x_1, t) = \begin{bmatrix} 0 \\ 4\cos(2t)x_{11} - 2\sin(t)x_{12} \end{bmatrix}, x_1(0) = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} \text{and } x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}$$

2nd Subsystem:

$$A_2 = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, z_2 = (\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix})$$

$$W_2(x_2, t) = \begin{bmatrix} 0 \\ 2\sin(t)x_{21} + 4\cos(2t)x_{22} \end{bmatrix}, x_2(0) = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix} \text{ and } x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}$$

3rd Subsystem:

$$A_3 = \begin{bmatrix} 0 & 1 \\ -4 & 5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \quad z_3 = (\begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix})$$

$$W_3(x_3, t) = \begin{bmatrix} 0 \\ 6\sin(t)x_{31} + 2\cos(2t)x_{32} \end{bmatrix}, x_3(0) = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix} \text{ and } x_3(t) = \begin{bmatrix} x_{31}(t) \\ x_{32}(t) \end{bmatrix}$$

The subsystems without controls are unstable after using state integral sliding mode to deal with any matched components and the continuous control designed via an LMI framework as described in Section 3.3.1. Using approach the control of each subsystem within the inter-connected system is designed and the system stability and achievement of a required performance are verified.

Figure 7-2 shows the response of all subsystems with control and without any actuator fault. According to this Figure the three subsystems are stable with the controllers (LMI-based on state and ISMC).

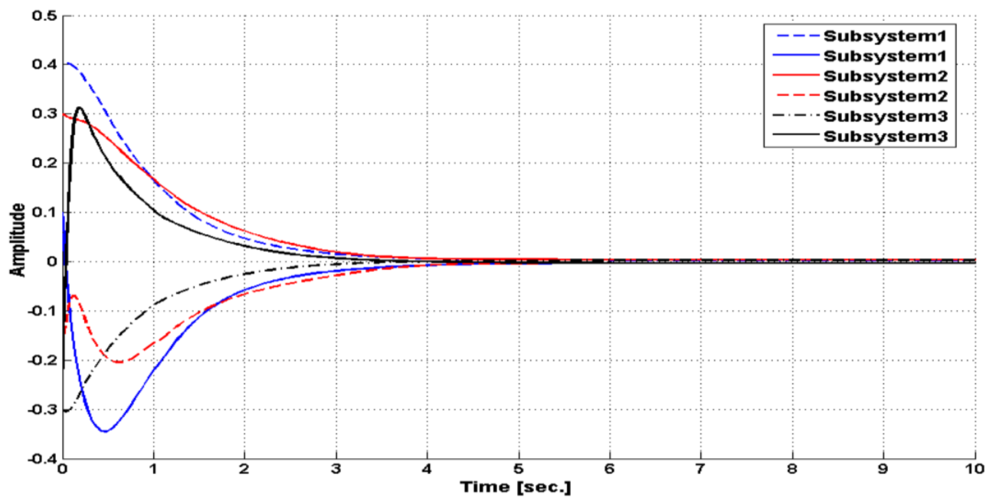

Figure 7-2: Control of inter-connected systems via LMI + ISMC each subsystem individually without any actuator faults

Suppose that a fault occurs in the actuator of the 1st subsystem and the observer is constructed with $r = 2$, the parameters of the corresponding subsystem observer are as follows:

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times4}, \mathcal{B}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4\times1} \text{ and } \mathcal{C}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{2\times4}$$

By choosing η_1=10 and solving Eq. (7-6), the gain of the observer is obtained as:

$$L_1 = \begin{bmatrix} 20 & 1 \\ 1 & 66 \\ 1.8 & 9.2 * 10^3 \\ 37.1 & 1.4 * 10^3 \end{bmatrix}$$

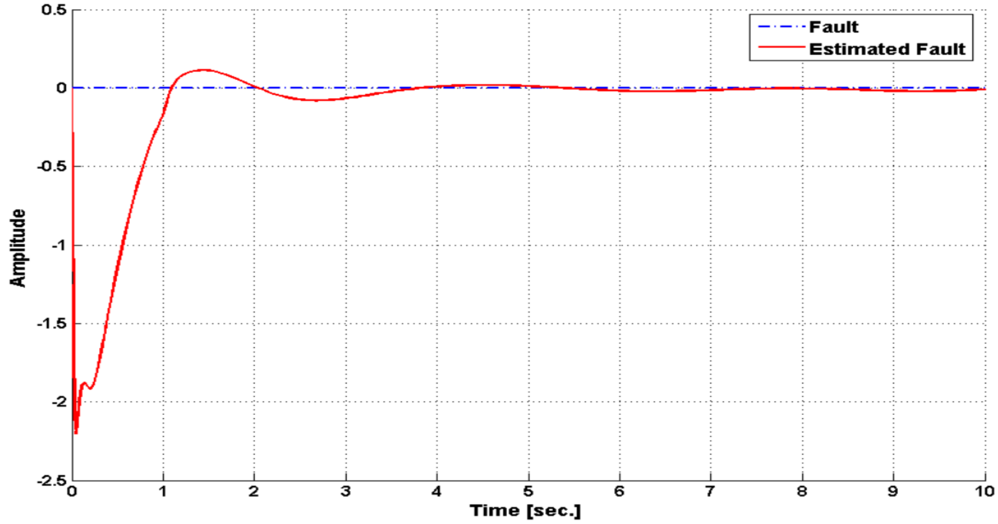

Figure 7-3: Zero actuator fault value and its estimation in 1st subsystem **(η_1=10)**

However, the norm of the observer gain $\|L_1\|$ is high but all the responses of the 1st subsystem when η_1=10 are excellent. Figures 7-3 &Figure 7-4 show the responses of the 1st estimated subsystem actuator fault and estimated subsystem states when no faults act in the other subsystems.

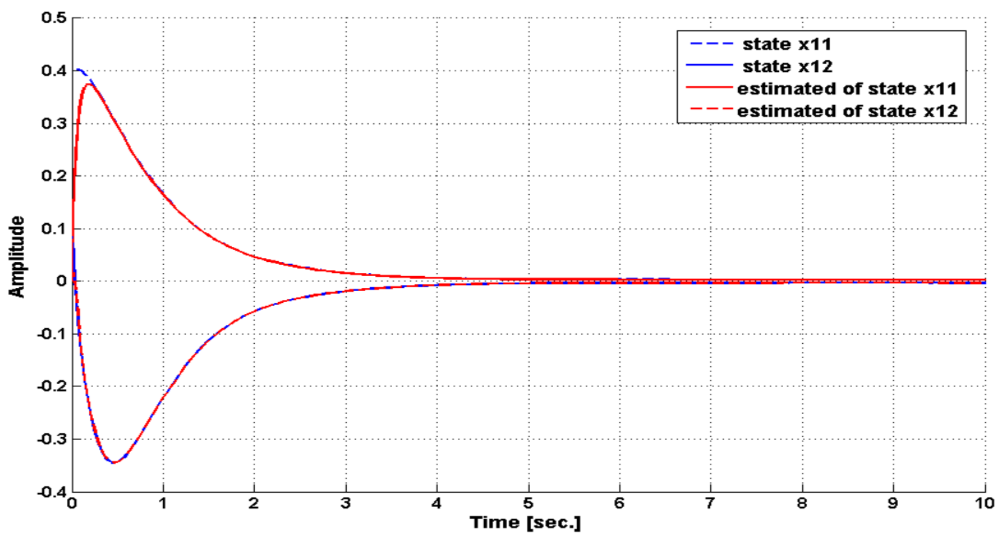

Figure 7-4: Estimated state with no faults in actuator of 1st subsystem (η_1=10)

Figure 7-5 corresponds to the situation when a 50% fault occurs in the actuator of the 1st subsystem. The time responses show that apart from a reasonable transient the observer can estimate the fault with high accuracy.

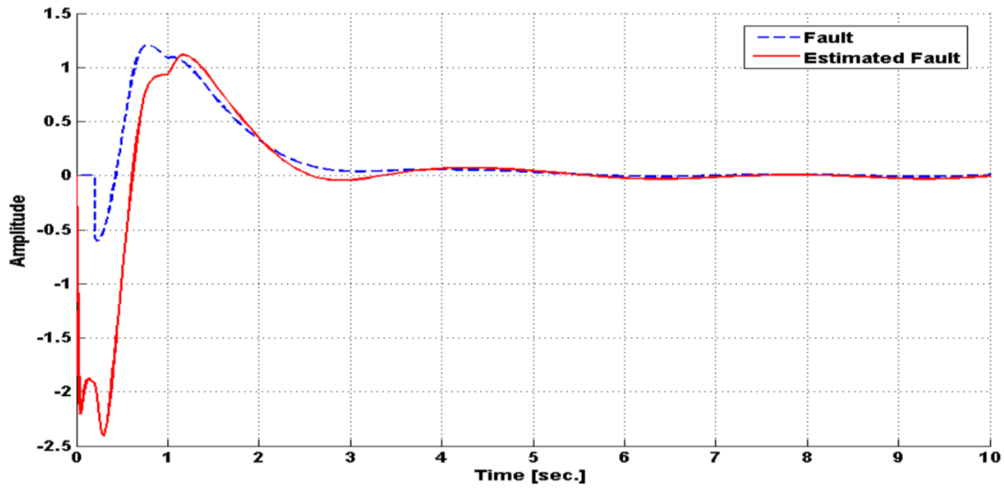

Figure 7-5 : Estimated fault with 50% of 1st subsystem actuator fault **(η_1=10)**

Figure 7-6 shows the comparison between the 1st subsystem states and their estimates with with the 50% actuator fault.

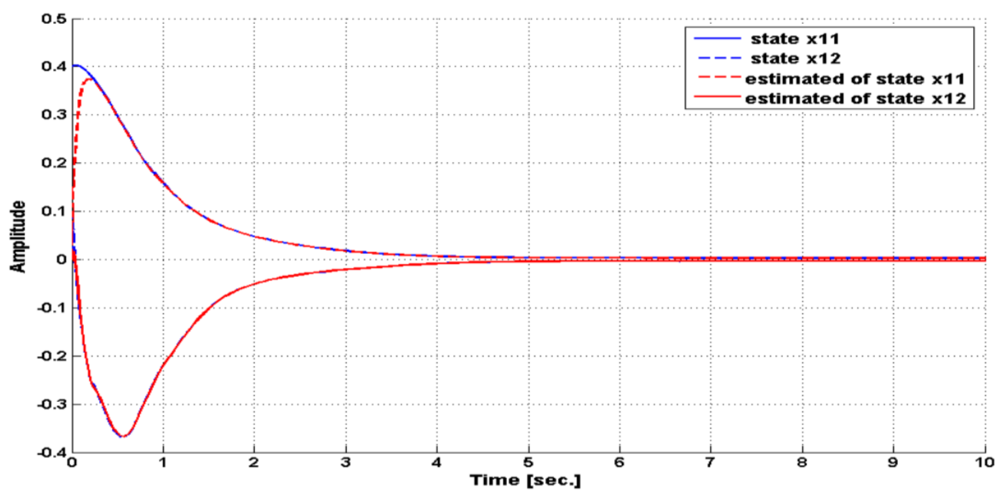

Figure 7-6: Estimated state for 50% of 1st subsystem actuator fault **(η_1**=10)

Figures 7-7, 7-8 & 7-9 show that the observer is still working well even when the actuator fault is increased to 80% with η_1=10. Although the fault increased, all the subsystems remain stable and the observers estimate the faults with good accuracy. However, the norm of the observer gain $\|L_1\|$ is unacceptably high.

The Figures 7-7, 7-8 & 7-9 show the estimated actuator fault in the 1st subsystem together with the corresponding control force and estimated states, respectively.

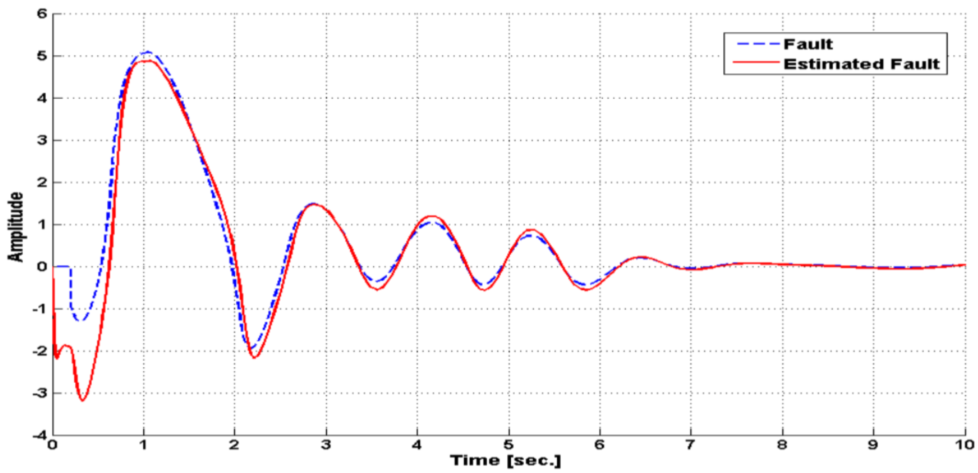

Figure 7-7: Estimated fault for 80% of 1st subsystem actuator fault ($\boldsymbol{\eta_1}$=10)

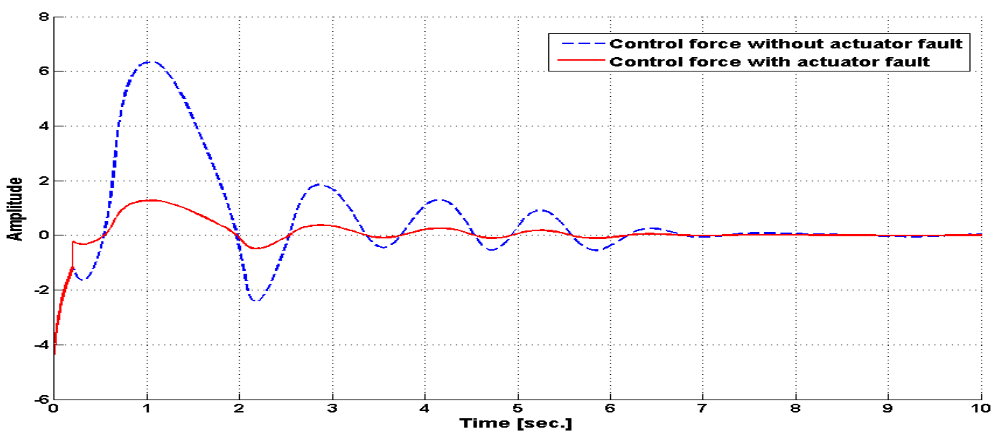

Figure 7-8: Control signal for 80% of 1st subsystem actuator fault **(** $\boldsymbol{\eta_1}$=10)

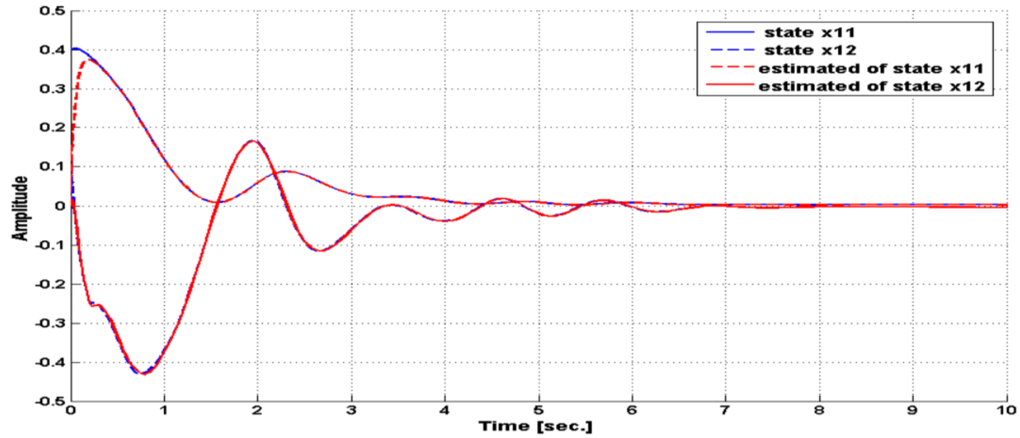

Figure 7-9 : Estimated state for 80% of 1st subsystem actuator fault (η_1=10)

However, the accuracy of the fault and state estimation remains unchanged, albeit with a small transient. However, the norm of the observer gain $\|L_1\|$ is considered too high for real application. To overcome this problem η_1 must be decreased in value. This is done by choosing a new value of η_1 , for example η_1= 4.5 and by solving Eq. (7-6) the gain of the observer then becomes:

$$L_1 = \begin{bmatrix} 9.1 & 0.99 \\ 0.99 & 32.89 \\ 1.62 & 978 \\ 15.32 & 296.7 \end{bmatrix}$$

It is clear that the gain $\|L_1\|$ has a lower norm compared with the previous gain $\|L_1\|$ computed for η_1=10. The estimated signals of the actuator fault and the states of the 1st subsystem with no faults in any subsystem are shown in Figures 7-10 & 7-11.

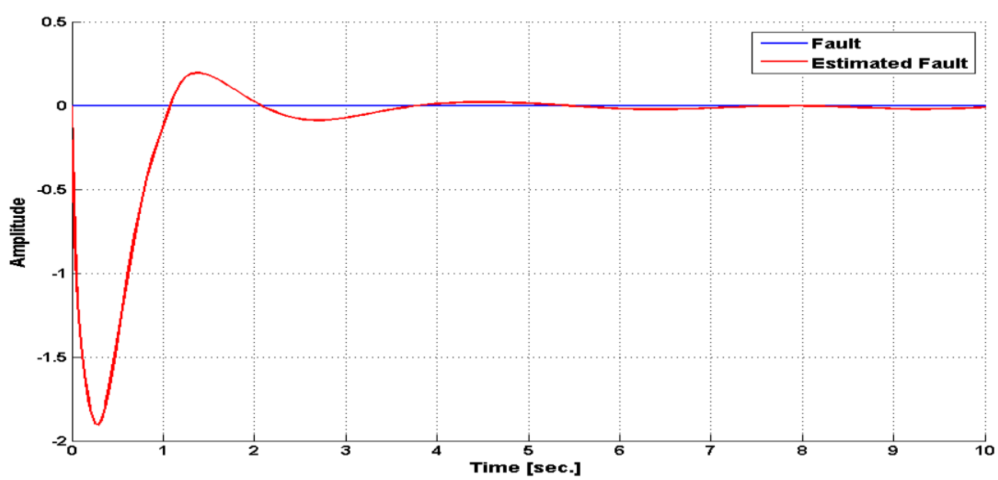

Figure 7-10: Zero actuator fault value and its estimation in 1st subsystem (η_1=4.5)

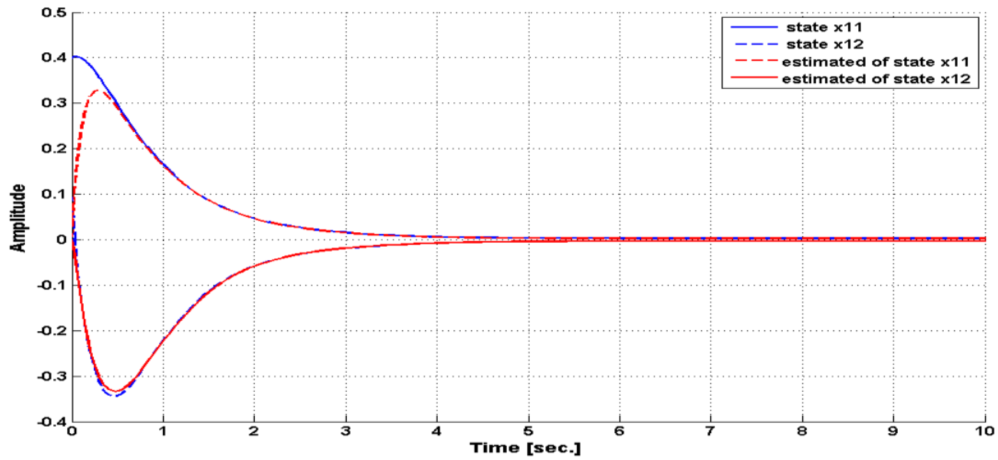

Figure 7-11: Estimated state with no faults in 1st subsystem actuator (η_1=4.5)

The same scenario of actuator faults from 50% to 80 % actuator fault with η_1=4.5 are applied to the 1st subsystem, the estimated actuator faults and estimated states compared to the original faults and original states of 1st subsystem are shown from Figure 7-12 to Figure 7-15.

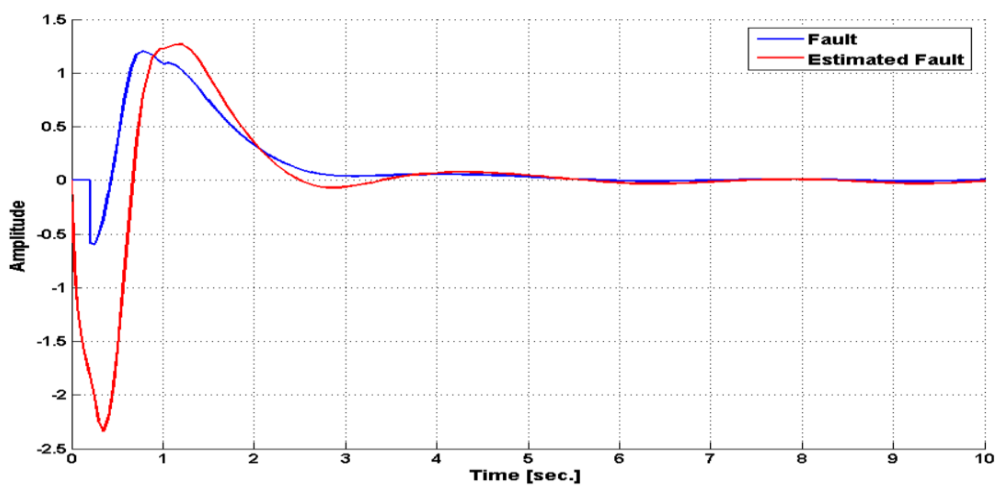

Figure 7-12: Estimated fault for 50% of 1st subsystem actuator fault (η_1= 4.5)

Figure 7-12 shows some delays at the beginning of the actuator fault estimation if this result is compared with the result with the same faults but with different η_1 as shown in Figure 7-5. It can be seen that the deviation between the fault and its estimated value is caused by the disturbance and interactions. This deviation is small and hence it can be argued that the tuning parameter η_1=4.5 gives acceptable estimation results.

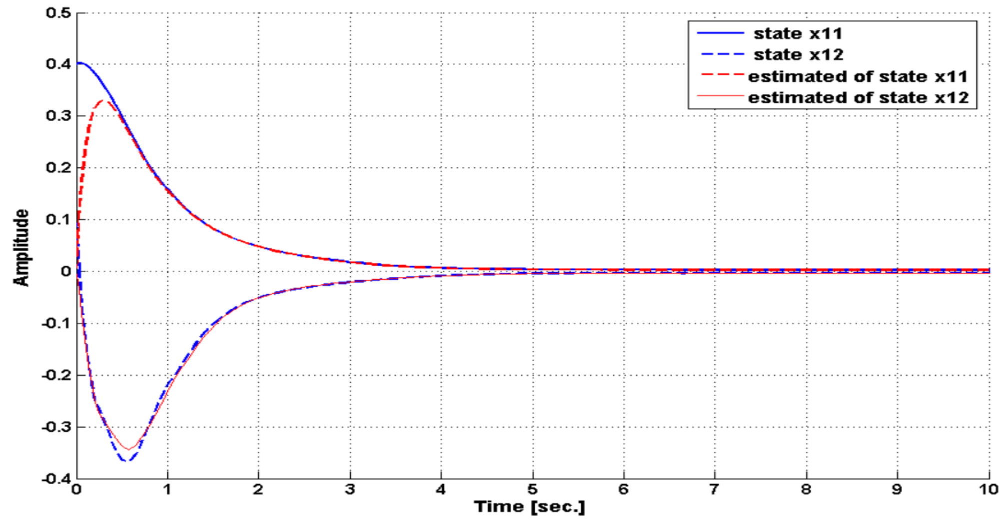

Figure 7-13: Estimated state for 50% of 1st subsystem actuator fault (η_1=4.5)

From Figure 7-14 there is some delay in fault estimated where the fault increased to 80% also the same is happened in the estimated states.

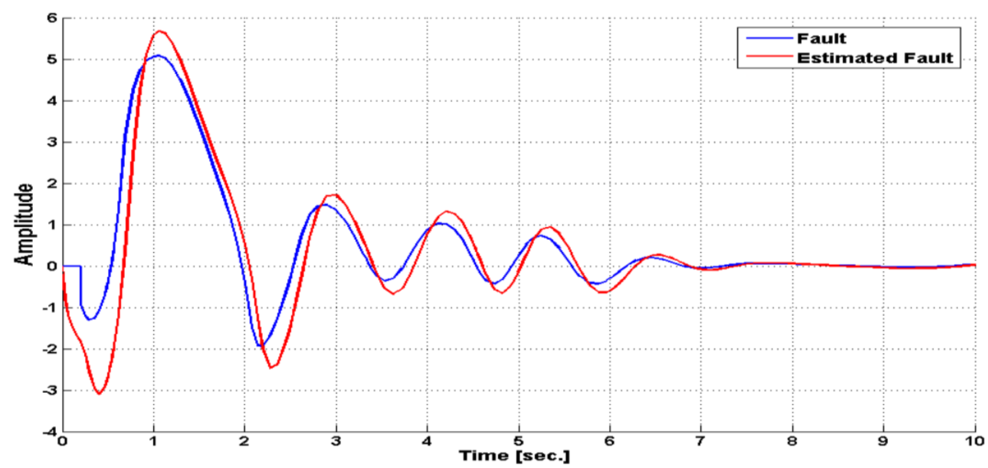

Figure 7-14: Estimated fault for 80% of 1st subsystem actuator fault (η_1=4.5)

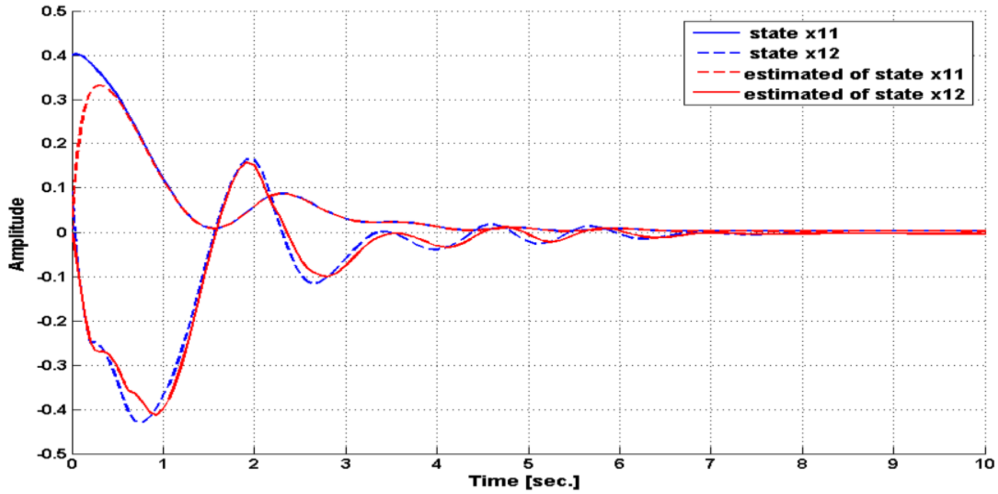

Figure 7-15: Estimated state for 80% of 1st subsystem actuator fault (η_1=4.5)

7.5 Conclusion

This Chapter has considered the problem of actuator fault estimation for non-linear inter-connected systems. Furthermore, this Chapter is concerned with the development of a suitable framework for observer design for de-centralized inter-connected systems, using an augmented observer based on the Proportional Multiple Integral Observer (PMIO) as an estimator for subsystem actuator faults. Each augmented observer is applied to a subsystem of the de-centralized system to estimate the local system states as well as the bounded actuator faults. Each subsystem observer is designed via an appropriate tuning parameter η_i used to tune the observer gain and control the accuracy of the actuator fault estimation.

According to application requirements the values of the observer gains can be designed according to the choice of parameters η_i. On-line estimation of actuator faults in inter-connected systems offers the operator an opportunity to take the most appropriate decision. Although fault estimation can be done with high accuracy the operator must define the threshold levels on the fault estimates so that when thresholds are exceeded decisive actions can be taken to reconfigure the system using available (redundant) actuators. The disadvantage of this method is that it can lead to high gain observer implementation with the possibility of unacceptable transients in the estimation errors. On the hand it is easy to design as well as easy to implement compared to SMO because it is needed only the observer's gain.

This approach is considered as the fault diagnosis and fault estimation can be used to monitor the actuator faults. This gives a freedom to choose any control design method to control the stability of subsystems and verify the desired performances.

Finally, a numerical example is used to illustrate the proposed modification to the PMIO observer where actuator fault estimation algorithms are applied in turn in this model to illustrate the validity of using this approach.

Chapter 8 : Conclusions and future work

8.1 Conclusions and summary

This thesis focuses on a study and development of control design (passive fault-tolerant control) and fault estimation based on a requirement to guarantee the stability of each subsystem as well as the whole system or compact system for linear and non-linear inter-connected systems.

The main topics of this thesis are presented as follows:

a- Hierarchical optimization control by using a two-level control strategy (Interaction Prediction Approach) in distributed systems.

b- Tackling matched components in non-linear inter-connected systems by ISMC in different cases where states, estimated states or output information are available.

c- Minimizing the effects of unmatched components and achieve specific performance as well as the stability.

d- Estimate actuator faults to take a right decision and control these faults by passive fault tolerant control.

e- Apply different control strategies to application studies and simulate the different faults scenarios.

The work presented has made some contributions within each of the topics outlined above. The thesis deals with the passive fault-tolerant control approach, based on on-line fault control depending on the ability of the controller.

A review of the literature of inter-connected or large-scale systems has been given in Chapter 1, in addition to providing the definitions and the significance of the faults, failures and the relationships between them.

Chapter 2 presents an introduction and summary of a hierarchical approach to optimization control based on liner models of inter-connected systems. Whereas the interaction prediction approach is applied to high building structure and test this method when some actuator fault accrued. Hence, this Chapter focus on the dynamic behaviour of the overall subsystems.

In Chapter 3, the new composition of ISMC and liner controller is designed by LMIs tool to control non-linear inter-connected systems based on the availability to measure all the subsystem's states. Although the subsystems are connected to each other the method can be applied to individual subsystem or whole system as one large-scale system. The topology of the subsystem can help to chose between the individual and the compact methods.

The same techniques have been applied to inter-connected systems but where the measurements of the sates are not available. That leads to use new observer based control method to estimate the states and use them to control and decrease the effects of faults, disturbances and uncertainties. The new observer based control method has been investigated in Chapter 4; also this Chapter provides a through derivation of the stability conditions that apply to the system with observer. From a practical point of view this new technique can be realized using simple gain that means easy to implement.

Although, most studies consider that all the states of the system are available but in some cases outputs are only obtainable. That leads to use it to implement static output feedback control to verify the stability in inter-connected systems. As mentioned that the non-linear system contain matched components, to use ISMC to deal with this components and decrease their effects the upper-bound of these components must be known. To overcome this problem adaptive ISMC is used in Chapter 5. In addition to use LMI directly with use output signals is not solved because the non-linear part , new technique is proposed to tackle this problem is discussed in Chapter 5. A discussion of this problem is given using the example of single power system with different values of actuator faults.

Although benefits of using dynamic control to gives freedom of design but in some case of control problems is not easy to calculate the gains of this controller especially in inter-connected systems. Chapter 6 proposed a new method to design dynamic controller depends on output signals where the same theory that is used in Chapter 5 is used but the difference is to argument the dynamic controller with the original system. The tutorial application is shown through the example of non-linear two inverted pendula, the controller gains are calculated using LMIs, whilst the faults and matched compounds that is contains disturbance and uncertainties are controlled by output ISMC. However the controller can deal with any actuators faults in specific range where the faults must be bounded.

Finally Chapter 7 presented method to estimate the actuators faults by combined an observer to each subsystem that gives the freedom to choose any suitable method to control inter-connected systems and verify the whole stability. Where the observer needs only output and control signals but during the observer design the gain observer can be tuned according to the accuracy of fault estimation. As a result high accuracy estimation needs high gain observer that gives the chose to designer according to the application.

8.2 Suggestions for future work

Although in this thesis new methods and techniques have been proposed to overcome several challenges in linear and non-linear inter-connected systems including actuator fault-tolerant control and actuator faults estimation, some developments are still required to deal with further challenges. The future work is listed below in terms of these new challenges:

1- Study active FTC in inter-connected or large-scale systems and compare it with the passive one. Also the effects of changing the controller on the whole stability.
2- Model reference de-centralized ISMC using the proposed LMI framework where in modern industrial the model reference control is more appropriate than regulating control.
3- Realize the new different methods that depend on ISMC and LMI framework in discrete time because discrete controllers are used in the real industrial application.
4- Although actuator faults have been studied in this these but sensor faults in inter-connected are needed to be considered in FTC and sensor faults estimation and study sensor faults and their influence on the subsystem itself and the other connected systems.
5- Find a new approach to estimate simultaneous actuator and sensor fault, which gives a comprehensive look at the behaviour of the subsystem and his stability.
6- Design a control which can handle the effects of actuator and sensor faults which can occur simultaneous.
7- Control reconfiguration techniques can be applied to inter-connected systems to give on-line FTC to deal with different types of faults.
8- The combination of (actuator and sensor) faults estimation and compensation together can be used to improve on-line FTC.

Finally, the thesis has demonstrated that a passive approach to FTC can be used in a powerful way to achieve good robustness and stability for both local subsystems and the overall structure of the inter-connected system. This has been achieved by choosing appropriate tools to minimize the effects of the subsystem interactions, de-coupling the effects of disturbances and isolating each subsystem from fault propagation between subsystems.

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