

THE UNIVERSITY OF HULL

Observer based Active Fault Tolerant Control of Descriptor Systems

Being a Thesis submitted for the Degree of Philosophy

in the University of Hull

by

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August 2013

Acknowledgement

I have been in the adventure of student life 21 years since I went to school in 1992. During this period, so many people have given me so much help. I really appreciate all of that and I want to take this opportunity to say thanks to you nice people.

First, I want to express my sincere gratitude to my supervisor, Professor Ron J Patton, without whose guidance and advice at Hull University there would be no possibility of developing this thesis. Professor Ron J Patton has created a flexible, encouraging and critical academic environment which is essential for all the outcome of my research. Personally, I am very proud to have had him as supervisor. I also want to thank my second supervisor Dr. Ming Hou for the available advice he has freely given during the seminars and thesis advisory panel meetings.

I also would like to thank all of my colleagues in the Control and Intelligent Systems Engineering (Control Group) in University of Hull, Dr. Lejun Chen, Dr. Chenglei Nie, Dr. Zheng Huang, Xiaoyu Sun, Yimeng Tang, Dr. Montadher Sami Shaker, Dr. Eshag Yousef Larbah, Xiaoran Feng, and Zhihuo Wang who have created much beautiful memories in the past three years.

Appreciations also go to my supervisor in China, Professor Hangen He who always encourages and supports me. Taking the chance, I want to say thanks to all my teachers and colleagues in China.

Thanks also go to Chinese Scholarship Council and the University of Hull, who jointly gave me the financial support of my PhD study.

Finally, I would like to express special gratitude to my family, my fiancée Yujuan Li and her family. It is really appreciated for the understanding, support and love during the long distance, long term research study.

Abstract

The active fault tolerant control (AFTC) uses the information provided by fault detection and fault diagnosis (FDD) or fault estimation (FE) systems offering an opportunity to improve the safety, reliability and survivability for complex modern systems. However, in the majority of the literature the roles of FDD/FE and reconfigurable control are described as separate design issues often using a standard state space (i.e. non-descriptor) system model approach. These separate FDD/FE and reconfigurable control designs may not achieve desired stability and robustness performance when combined within a closed-loop system.

This work describes a new approach to the integration of FE and fault compensation as a form of AFTC within the context of a descriptor system rather than standard state space system. The proposed descriptor system approach has an integrated controller and observer design strategy offering better design flexibility compared with the equivalent approach using a standard state space system. An extended state observer (ESO) is developed to achieve state and fault estimation based on a joint linear matrix inequality (LMI) approach to pole-placement and H_∞ optimization to minimize the effects of bounded exogenous disturbance and modelling uncertainty. A novel proportional derivative (PD)-ESO is introduced to achieve enhanced estimation performance, making use of the additional derivative gain. The proposed approaches are evaluated using a common numerical example adapted from the recent literature and the simulation results demonstrate clearly the feasibility and power of the integrated estimation and control AFTC strategy. The proposed AFTC design strategy is extended to an LPV descriptor system framework as a way of dealing with the robustness and stability of the system with bounded parameter variations arising from the non-linear system, where a numerical example demonstrates the feasibility of the use of the PD-ESO for FE and compensation integrated within the AFTC system.

A non-linear offshore wind turbine benchmark system is studied as an application of the proposed design strategy. The proposed AFTC scheme uses the existing industry standard wind turbine generator angular speed reference control system as a “baseline” control within the AFTC scheme. The simulation results demonstrate the added value of the new AFTC system in terms of good fault tolerance properties, compared with the existing baseline system.

List of Publications

1. Shi, F and Patton, R. J. (2012, September). Low eigenvalue sensitivity eigenstructure assignment to linear parameter varying systems. UKACC International Conference on Control, Cardiff.
2. Shi, F and Patton, R. J. (2012, September). A robust LPV fault detection approach using parametric eigenstructure assignment. UKACC International Conference on Control, Cardiff.
3. Chen, L; Shi, F and Patton, R. J (2013, October 9-11). Active fault tolerant control for hydraulic pitch system based on an off-shore wind turbine, Proceedings of the IEEE *SysTol 2013* Conference (2nd International Conference on Control and Fault-Tolerant Systems), Nice, France.
4. Shi, F and Patton, R. J (2013). Fault estimation and fault tolerant control of LPV descriptor systems, submitted to International Journal of Robust and Nonlinear Control.

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Symbols

0	Zero matrix with compatible dimensions
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{R}^n	Set of n dimensional real vectors
$\mathbb{R}^{n \times m}$	Set of n by m matrices with elements in \mathcal{R}
M^T	The transpose of the matrix M
M^{-1}	The inverse of the invertible matrix M
M^*	The conjugate transpose of M
$M > 0 (M \geq 0)$	M is positive definite (positive semidefinite)
$M < 0 (M \leq 0)$	M is negative definite (negative semidefinite)
$\ker(M)$	The kernel (or null space) of M
$\det(M)$	Determinant of matrix M
λ	An eigenvalue
$\text{Re}(\lambda)$	Real part of the eigenvalue λ
λ_i	The i^{th} eigenvalue
$\ x\ $	Euclidean norm of x , and $\ x\ = \sqrt{x^T x}$
$\bar{\sigma}\{M\}$	Denotes the maximum singular value of the matrix M
$\ G(s)\ _\infty$	$\ G(s)\ _\infty = \sup_{\omega \in \mathbb{R}} \bar{\sigma}\{G(j\omega)\}$
θ	Plant parameter vector
Θ	Compact parameter set
$\ G(\theta, s)\ _\infty$	$\ G(\theta, s)\ _\infty = \sup_{\theta \in \Theta, \omega \in \mathbb{R}} \bar{\sigma}\{G(\theta, j\omega)\}$
■	End of proof
*	Symmetric part
::	Arbitrary part

Abbreviations

AFTC	Active Fault Tolerant Control
ASO	Augmented State Observer
BMI	Bi-linear Matrix Inequality
ESO	Extended State Observer
FD	Fault Detection
FDD	Fault Detection and Diagnosis
FDI	Fault Detection and Isolation
FE	Fault Estimation
FTC	Fault Tolerant Control
LMI	Linear Matrix Inequality
LPV	Linear Parameter Varying
LTI	Linear Time Invariant
LTV	Linear Time Varying
MM	Multiple Model
IMM	Interacting Multiple Model
UIO	Unknown Input Observer
UIPIO	Unknown Input Proportional Integral Observer
PD	Proportional Derivative
PD-ESO	Proportional Derivative Extended State Observer
PFTC	Passive Fault Tolerant Control
PID	Proportional Integral Derivative
PIM	Pseudo-Inverse Method
MPIM	Modified Pseudo-Inverse Method
PIO	Proportional Integral Observer
PMIDO	Proportional Multi-Integral Derivative Observer
PMIO	Proportional Multi-Integral Observer
SMO	Sliding Mode Observer

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Chapter 1: Introduction

1.1 Background

Automatic control systems are around our lives everywhere, from washing machines, automatic toasters and personal computers to chemical processes, nuclear power stations, aircraft autopilots and wind turbines, and so on. The history of control can be traced back to ca 200 BC in Greece with the “float regulator mechanism” and the first automatic controller used in an industry process is Watt’s “fly-ball governor” in 1769 (Pidhayny, 1972; Dorf, 1991).

Both frequency domain and time domain approaches were being developed before the 1940s. Tremendous efforts were made around World War II when it became necessary to design accurate and fast response military systems like automatic airplane pilots, gun-positioning system, and radar antenna control systems. Both time domain and frequency domain approaches were considered at that time to improve system performances. The optimal control was developed in the 1960s and developed dramatically since then to improve system robustness as there is often existing modelling uncertainty or/and disturbances (Francis, 1987; Zhou, Doyle and Glover, 1996). Adaptive techniques have been significantly studied a lot since 1970s (Åström and Wittenmark, 1995; Ioannou and Sun, 1996; Feng and Lozano, 1999) to handle time varying property of dynamic systems. Discrete time systems (Oppenheim, Schaffer and Buck, 1999) become attractive with the emergence of computers with digital system. The effects of time delays also have been studied a lot (Dugard and Verriest, 1998; Gu, Chen and Kharitonov, 2003; Zhong, 2006) as it is a very common problem when considering real systems.

The demand for safety, reliability and dependability in control systems has been increasing with a steady increase in system complexity. However, complexity means that systems become more vulnerable to faults which can lead to system breakdown, failure or even disaster if not handled suitably. Thus, it is very important to develop controllers, which not only can manage system stability and desired performance in normal conditions but are also able to maintain stability and required performance or with some acceptable degradation when faults occur. Fault detection and diagnosis

(FDD) and fault tolerant control (FTC), also known as reconfigurable control or self-repairing control, have been being developed to achieve the requirement since the 1970s (Eterno, Weiss, Looze and Willsky, 1985; Stengel, 1991; Rauch, 1995; Chen and Patton, 1999; Mahmoud, Jiang and Zhang, 2003; Blanke, Kinnaert, Lunze and Staroswiecki, 2006; Ding, 2008; Ducard, 2009). Historically, FDD and FTC are specifically developed for aircraft systems (Willsky, 1976), stimulated by two aviation accident in 1970s (Frank, 1990; Blanke, Kinnaert, Lunze and Staroswiecki, 2006). FDD and FTC have stimulated various research in different industries which requires high degree of safety, reliability and availability (Isermann, 1984; Patton, 1997a; Zhang and Jiang, 2008; Hwang, Kim, Kim and Seah, 2010), such as nuclear reactor systems, chemical systems with hazardous elements, and recently developed large wind turbines which are expensive and installed remotely, for instance, in mountain zones or off-shore and hence hard and costly to maintain.



Asiana Airlines Flight 214 after crash
(Wikipedia, 2013)



A wind turbine accident
(Anonymous, 2009)

Figure 1.1 An aircraft accident and a wind turbine accident

System redundancy is, in principle, necessary in order to achieve fault-tolerance and fault diagnosis (Frank, 1990; Patton, 1997a). A conventional procedure in FTC is that a non-impaired identical alternative or redundant component, for instance, sensors, actuators, or control computers, etc., is brought into service to replace an impaired component when a fault occurs. This is known invariably as hardware redundancy. The choice of the most reliable components can be achieved with a quadruple redundant scheme to keep the system running with satisfactory performance (Patton, 1997a). However, it is not necessary to use hardware redundancy, because an alternative form, functional redundancy (or analytical redundancy) is often available. Functional redundancy is achieved by careful design or by arranging different subsystems to make

the function of these subsystems overlap. An illustration of hardware and functional sensor redundancy is depicted in Figure 1.2 in fault diagnosis framework. Making the best use of both hardware redundancy and functional redundancy provided by the systems is a major task of FTC system design. Sometimes a combination of the two forms of redundancy is necessary, for example to reduce a quadruplex redundant system to a triplex redundant system.

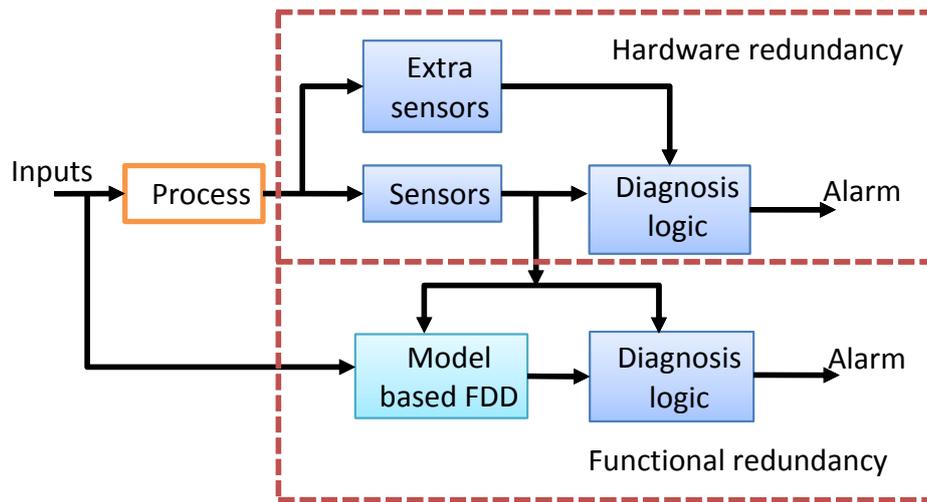


Figure 1.2 Example of hardware and functional sensor redundancy in fault diagnosis (Chen and Patton, 1999)

1.2 Modelling approaches

The models used determine the ways in which the design is conducted as well as providing a good framework for design of robust FDD and FTC schemes. Figure 1.3 illustrates the partitioning of a system (or a plant) into the main process itself and the actuator and sensors that impact upon the inputs and outputs of the system.

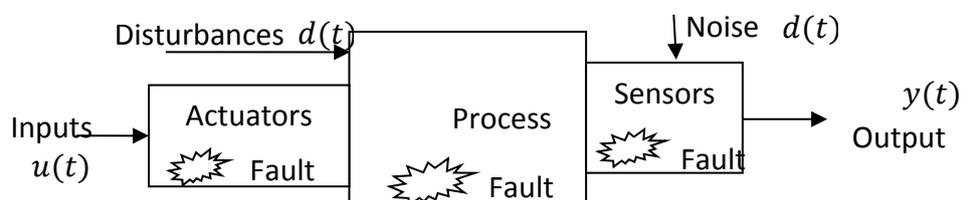


Figure 1.3 An schematic description of a dynamic system

1.2.1 Classification of faults

It is necessary to define the action of a fault in a control system context before further discussion. Actually, any abnormal change at unexpected times can be seen as a fault. As defined in (Isermann and Ballé, 1997):

A fault is an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable / usual / standard condition.

Therefore, a fault is a state that may lead to a malfunction or failure of the system. Literally, the words fault and failure have different meanings. A fault is something that will lead to performance degradation and a failure means complete loss of effectiveness. Usually, a fault may lead to a failure if not accommodated correctly and timely.

There are different standards to classify faults in the literature. According to the time dependency of fault signals, faults can be classified to abrupt faults, incipient faults, and intermittent faults (Isermann, 1997) as shown in Figure 1.4.

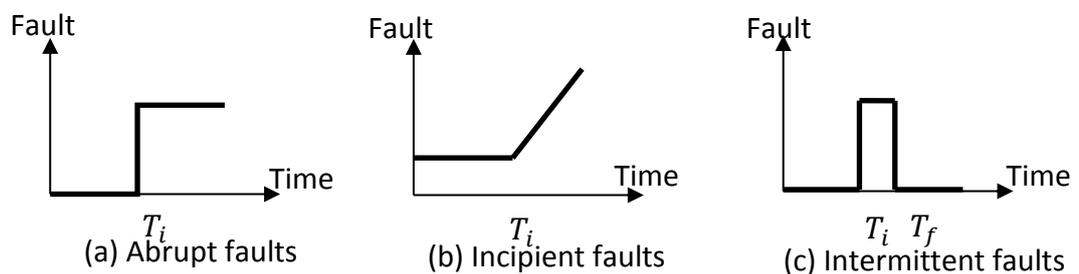


Figure 1.4 Fault classification with respect to time dependency

Abrupt faults refer to changes that occur much faster than the normal dynamics of the system and require fast detection. Incipient or soft faults are small and slowly developing. Intermittent faults appear and disappear repeatedly.

With respect to the location of faults, there are process faults, sensor faults and actuator faults as shown in Figure 1.5.

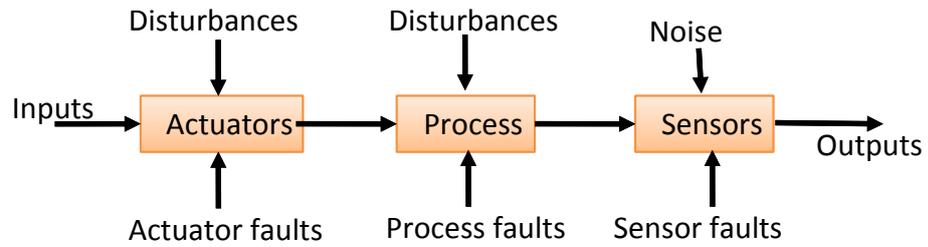


Figure 1.5 Fault classification with respect to the locations of faults

Actuator faults correspond to total or partial loss of effectiveness of input equipment, which can be motors, valves, relays, and so on. The total loss of effectiveness of an actuator fault means that the actuator produces no actuation regardless of the input applied to it. This can occur as a result of breakage, or burn out of wiring, or the actuator is stuck in one position and cannot move (Patton, 1997a; Blanke, Kinnaert, Lunze and Staroswiecki, 2006). The total loss of effectiveness is not considered in the thesis.

The partial actuator faults are cases in which the actuator becomes less effective, for instance, degradation in the actuator gain (due to a clogged or rusty valve). These faults may occur on the system gradually or abruptly. According to (Eslinger and Chandler, 1988), 20% of loss of aircraft are caused by faulty or damaged actuators.

Process faults, also referred to as component faults, arise as variations from the structure or parameters of the system itself (Patton, 1997a; Blanke, Kinnaert, Lunze and Staroswiecki, 2006; Isermann, 2006; Ding, 2008). These faults directly reflect the changes of physical parameters and in turn affect the input/output properties of the system. A wide class of possible faults can be covered in this range, e.g. change of mass, aerodynamic coefficients, damping constant, or operating condition, etc.

Sensor faults are due to incorrect readings or measurements from the real dynamic system. A sensor can be any equipment that takes a measurement or observation from the system, e.g. anemometers, accelerometers, tachometers, pressure gauges, strain gauges, etc. Sensor faults can be due to poor calibration or bias, scaling errors or a change in the sensors dynamic characteristics which cause errors on the sensors outputs, but not on the plant dynamics. With some sensor fault estimation approaches, sensor faults can be transferred to the actuator faults with a filter (Tan and Edwards, 2003;

Alwi and Edwards, 2008a) or state augmentation (Saif and Guan, 1993; Park, Rizzoni and Ribbens, 1994).

Some literature also classifies the faults as additive or multiplicative faults as shown in Figure 1.6 according to the way in which the fault is modelled (Chen and Patton, 1999; Isermann, 2006). Additive faults influence signals by an addition of an extra fault signal, whilst multiplicative faults by a product with an additional fault signal. Additive faults appear, e.g., as offsets of sensors, whereas multiplicative faults are parameter changes within a process, although some of them can also be modelled as additive faults (Ding, Zhang, Ding and Frank, 2003; Tan and Edwards, 2004).

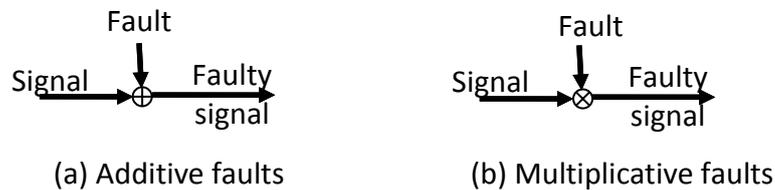


Figure 1.6 Additive faults and multiplicative faults (Isermann, 1997)

Also for this reason, the major focus of the research is on additive faults. In the study, process fault is transferred to actuator fault using the approach to transform multiplicative fault to additive fault. But, one thing should be kept in mind is that multiplicative faults are functions of the state or input variable of the system and thus will affect the system stability (Ding, Zhang, Ding and Frank, 2003; Ding, 2008).

1.2.2 Linear time invariant (LTI) systems

When using state space modelling, differential and/or algebraic equations are used to form the mathematical model of the system in terms of a state vector $x(t)$, a control input vector $u(t)$ which acts on the system actuators, a uncontrolled input $d(t)$ and an output vector $y(t)$ whose elements are properly chosen as measurable variables of the system. Taking into account the possibility that the state space system may be nonlinear a general form of the so-called state equations (1-1) combined with the output equation (1-2), are as follows:

$$h(\dot{x}(t); x(t); u(t); d(t); t) = 0 \tag{1-1}$$

$$g(x(t); u(t); d(t); y(t); t) = 0 \tag{1-2}$$

where $h(\cdot)$ and $g(\cdot)$ are general nonlinear vector functions of appropriate dimensions with respect to $\dot{x}(t)$, $x(t)$, $u(t)$, $d(t)$, and t .

A specific form of (1-1)–(1-2) is the following (Dai, 1989; Wang, Yung and Chang, 2006; Duan, 2010):

$$E(t)\dot{x}(t) = H(x(t); u(t); d(t); t) \quad (1-3)$$

$$y(t) = J(x(t); u(t); d(t); t) \quad (1-4)$$

where $t > 0$ is the time variable, H and J are vector functions with appropriate dimensions, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^h$, $d(t) \in \mathbb{R}^d$, and $u(t) \in \mathbb{R}^m$ are system states, outputs, uncontrolled inputs and control inputs, respectively. Furthermore, it is assumed that $\text{rank}(E(t)) = r \leq n$. (1-3) and (1-4) are the general nonlinear form for a descriptor system, which can also be referred to as semi-state systems, singular systems, generalised state space systems, or differential-algebraic systems (Lewis, 1986; Dai, 1989; Lam and Xu, 2006; Duan, 2010).

H and J can be linear time-invariant (LTI) functions of $x(t)$ and $u(t)$, or can be linearized around one operation point, so that system (1-3) and (1-4) will have the following time-invariant descriptor linear system structure:

$$E\dot{x}(t) = Ax(t) + Bu(t) + R_1d(t) \quad (1-5)$$

$$y(t) = Cx(t) + Du(t) + R_2d(t) \quad (1-6)$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^h$, $d(t) \in \mathbb{R}^d$, and $u(t) \in \mathbb{R}^m$ are system states, outputs, uncontrolled inputs and control inputs, respectively. The constant matrices E, A, B, C, D, R_1 and R_2 are the system coefficient matrices.

It is important to note that $R_1d(t)$ and $R_2d(t)$ can be used to represent exogenous disturbances or modelling uncertainty which have similar effects on the system (Blanke, Kinnaert, Lunze and Staroswiecki, 2006). Secondly, both disturbances and modelling uncertainty can be lumped into one disturbance vector with disturbance distribution matrices R_1 and R_2 (Chen and Patton, 1999).

When the matrix E is an identity matrix, the system of (1-5) and (1-6) is called a standard system, which has been intensively studied in linear systems theory. Obviously,

if the matrix E is square and non-singular, the system of (1-5) and (1-6) can be written in the following standard system form:

$$\dot{x}(t) = E^{-1}Ax(t) + E^{-1}Bu(t) + E^{-1}R_1d(t) \quad (1-7)$$

$$y(t) = Cx(t) + Du(t) + R_2d(t) \quad (1-8)$$

Hence, it is clear that the theory for standard linear systems analysis and synthesis can be readily applied to a linear system in the form of (1-5) and (1-6) if E^{-1} exists by first converting it into the standard form of system (1-7) and (1-8). However, a motivation for using the appropriate theory for descriptor linear systems analysis and design is to avoid the numerical problems associated with computation of the matrix inverse E^{-1} (Duan, 2010). A second motivation for studying descriptor systems is that, in practice many systems can be established efficiently by using the general descriptor system form of (1-5) and (1-6) for which E is singular, as for instance for time-delay systems (Fridman and Shaked, 2002). The focus in this work is on descriptor systems based on a square matrix E . In this thesis the terminologies of both *descriptor systems* and *square descriptor systems* are used for this case. It is suggested to refer to (Zhang, 2006) and references therein for the case of rectangular descriptor systems.

A third motivation for the use of descriptor systems methods is that when applied to the LPV modelling and system design, the so-called LPV descriptor system is an important framework for the analysis and design of FDD and FTC systems, as explained in Section 1.2.3.

Furthermore, when a system fault has occurred, the system structure changes so that the most appropriate model for this situation becomes:

$$E_f\dot{x}(t) = A_fx(t) + B_fu(t) + R_{f1}d(t) \quad (1-9)$$

$$y(t) = C_fx(t) + D_fu(t) + R_{f2}d(t) \quad (1-10)$$

Or if only additive faults are considered, the system model for this fault case is:

$$E\dot{x}(t) = Ax(t) + Bu(t) + F_1f(t) + R_1d(t) \quad (1-11)$$

$$y(t) = Cx(t) + Du(t) + F_2f(t) + R_2d(t) \quad (1-12)$$

where $f(t) \in \mathbb{R}^p$ is the vector of fault signals, and F_1 and F_2 are the corresponding fault distribution matrices. It can be seen that system (1-11) and (1-12) together represent a

generalized model of the standard system model studied by many investigators, e.g. in (Chen and Patton, 1999).

From (1-11) and (1-12), it can be seen that there may be some conflict between the uncertainty or disturbance terms $R_1d(t)$, $R_2d(t)$ and the fault terms $F_1f(t)$, $F_2f(t)$. However, the faults are distinguished from the disturbances using their different characteristics. For example, the following characteristics can be used: Noise is considered as a random signal, whilst faults are considered to be deterministic signals (e.g., a constant bias or drift) or semi deterministic (jumps oncoming at random intervals with random amplitudes) (Gertler, 1988). Furthermore, the effects of disturbances or uncertainty are usually attenuated within the robust controller designs, whilst the faults need to be compensated via more sophisticated techniques because of the severe effect of faults in the FTC design (Blanke, Kinnaert, Lunze and Staroswiecki, 2006).

1.2.3 Linear parameter varying (LPV) systems

The description of a non-linear system using a single linear model can lead to significant representation problems, especially when the non-linear system has more than one or even no unique equilibrium points in state space. In the LPV descriptor system framework, the considered model can be given as (Masubuchi, Akiyama and Saeki, 2003):

$$E\dot{x}(t) = A(\theta(t))x(t) + Bu(t) + F_1f(t) + R_1d(t) \quad (1-13)$$

$$y(t) = Cx(t) + F_2f(t) + R_2d(t) \quad (1-14)$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^h$, $f(t) \in \mathbb{R}^g$, $d(t) \in \mathbb{R}^d$, and $u(t) \in \mathbb{R}^m$ are system states, outputs, faults vector, uncontrolled inputs and control inputs, respectively. The parameter-dependent matrices $A(\cdot)$ are a function of $\theta(t)$. $E, B, C, D, R_1, R_2, F_1$ and F_2 are matrices with compatible dimensions. $\theta(t)$ is the vector of time-varying parameters belonging to a compact set: $\theta(t) \in \Theta$.

The third motivation to study descriptor systems is that some rational LPV standard systems can be easily transformed to affine LPV descriptor systems (Masubuchi, Akiyama and Saeki, 2003; Bouali, Chevrel and Yagoubi, 2006; Bouali, Yagoubi and Chevrel, 2008) which is more convenient for analysis and synthesis.

1.3 Fault detection and diagnosis (FDD)

Historically, in the control systems community the term *fault detection and diagnosis* is used for the procedures of detecting potential changes in the operation of a process system, in terms of detecting the presence of faults and diagnosing their severity. This Section outlines the terminologies most used by the control system FDD community and a general classification of model-based FDD approaches.

In the FDD design, functional redundancy is used to generate a residual signal to indicate the system states. There are different terminologies in this context to refer to the various FDD functions. The terms fault detection, fault isolation, and fault identification are defined in (Isermann and Ballé, 1997) as:

Fault detection: *Determination of the faults present in a system and the time of detection.*

Fault isolation: *Determination of the kind, location and time of detection of a fault. This follows the fault detection.*

Fault identification: *Determination of the size and time-variant behaviour of a fault. Follows fault isolation.*

Following fault detection, fault diagnosis is to determine the type, size, location and occurring time of a fault. The functions to be designed are subjective and problem related. However fault detection is absolutely necessary for any practical application. Fault identification, on the other hand, whilst undoubtedly helpful, may not be essential if no controller redesign is involved. As a result, in most literature, fault diagnosis is very often considered as fault detection and isolation (FDI) (Chen and Patton, 1999). In AFTC, the fault feature is one of the most critical items of information required for controller redesign (Patton, 1997a; Zhang and Jiang, 2008). Hence, the fault identification must be considered in an AFTC process, in which not only the fault alarm and location of the faults are required, but also the time response characteristic of each fault should be known.

A general schematic diagram of the various stages of fault diagnosis is shown in Figure 1.7. The residual generation process needs to be followed by a residual evaluation stage which is responsible for evaluating the residuals and monitoring if and where a fault has

occurred. Based on the processed residual signal, a decision can be made as to whether or not fault has occurred, or whether the residual signal changes due to some other effect, e.g. due to modelling uncertainty or disturbance.

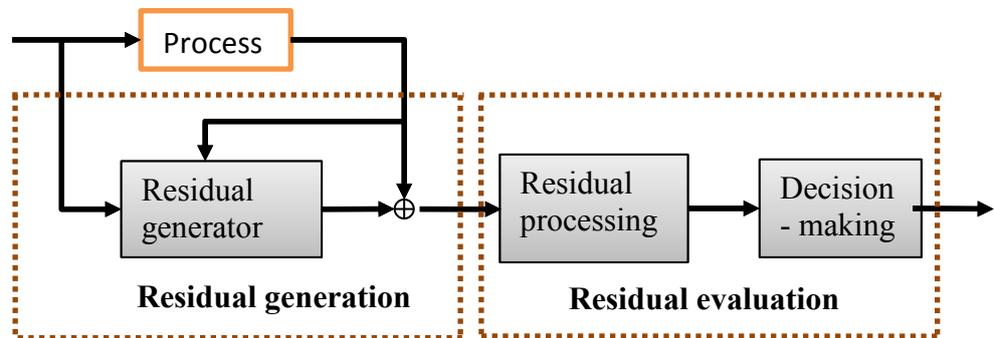


Figure 1.7 Schematic description of model-based FDD (Chen and Patton, 1999)

1.4 Fault estimation (FE)

Another important but relatively new concept is fault estimation (FE) which is introduced first to improve the detection robustness to modelling uncertainty in optimal design framework (Mangoubi, Appleby and Farrell, 1992; Stoustrup and Niemann, 2002). The topic of FE has become well accepted in the control community based on various robust estimation approaches, e.g. sliding mode estimation (Edwards, Spurgeon and Patton, 2000; Jiang, Staroswiecki and Cocquempot, 2004; Gao, Ding and Ma, 2007). If a fault can be perfectly (or accurately) estimated, all the information including type, size, location and time of occurring can be obtained. The determination of accurate fault signal values has become an attractive subject since FE gives a more direct way to achieve the fault information (detection and isolation) than the alternative use of “fault indicator” or “residual” signals. The FE approach is a more direct way to obtain this information and the fault estimates can be used directly within some AFTC strategies, for example using fault hiding and fault compensation (Blanke, Kinnaert, Lunze and Staroswiecki, 2006; Wu, Thavamani, Zhang and Blanke, 2006; Zhang, 2009; Ponsart, Theilliol and Aubrun, 2010; Nazari, Seron and De Doná, 2013).

Nevertheless, the FE problem is always accompanied by some estimation uncertainty which can be minimized according to a suitable robustness performance, for instance using H_∞ optimization. In other words, FE can be used to achieve fault detection, isolation and identification in one step instead of two or three steps, but it is essential

that a robustness problem is solved correctly. In this thesis the FE approach is used to provide fault information for fault compensation within AFTC schemes.

The advantage of model based FDD is to make full use of the information of the process. Thus, the performance would heavily depend on the accuracy of the system model (Patton, 1997b; Park and Lee, 2004), specifically for the quantitative model-based approaches. However, it is not possible to obtain a perfect model as the systems are nonlinear in nature and there is often disturbance or system uncertainty. The mismatch between the mathematic model and real system may cause some problems and enough attention should be paid. For instance, the mismatch may cause false alarms and deteriorate the performance of the system to some extent that the FDD/FE system may even become totally useless. Therefore, it could be a key issue to design a robust FDD/FE system which is insensitive to unknown inputs such as disturbances, noises on the working system and model uncertainty.

1.5 Fault tolerant control

Basically, there are two types of FTC strategies, which are active fault tolerant control (AFTC) and passive fault tolerant control (PFTC). Figure 1.8 gives a general structural comparison of AFTC and PFTC.

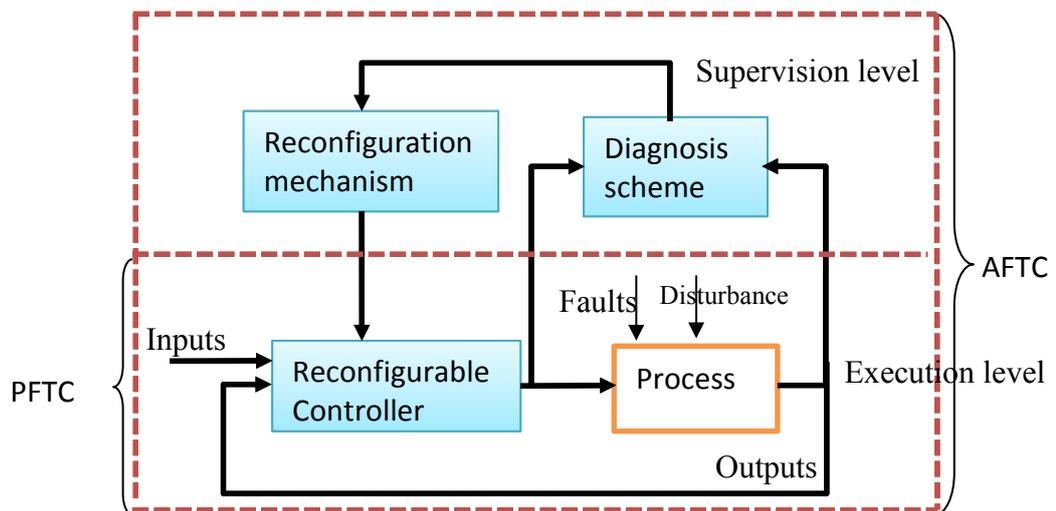


Figure 1.8 The structure of AFTC and PFTC {adapted from (Blanke, Kinnaert, Lunze and Staroswiecki, 2006)}

1.5.1 Passive fault tolerant control (PFTC)

A PFTC system is implemented with a constant feedback controller which is designed carefully using robust control techniques; and the closed-loop system remains insensitive to certain faults without use of on-line fault information. This is suitable in restricted cases, perhaps when the effects of faults are similar to those of modelling errors and disturbances. The main disadvantage of the passive approach is the less capability of handling large faults. A consideration of small faults would be realistic problems to solve within PFTC framework (Eterno, Weiss, Looze and Willsky, 1985; Patton, 1997a).

1.5.2 Active fault tolerant control (AFTC)

In contrast to PFTC systems, AFTC systems react to the system faults actively to recover or at least approximate the performance of the faulty system to the performance of the healthy system using the FDD results. As shown in Figure 1.8, on-line fault accommodation and on-line controller-reconfiguration are usually used in AFTC framework. The controllers of AFTC are generally variable in parameter or even structure. The key issues in AFTC include the design of (Zhang and Jiang, 2008):

- 1) A reconfigurable controller,
- 2) A FDD scheme which is sensitive to faults and robust to model uncertainty and disturbances, and
- 3) A reconfiguration mechanism.

These three elements of the AFTC design when considered together are expected to lead to the recovery of the system to its pre-fault performance as closely as possible in the presence of uncertainty and disturbances, given some system constraints. Significant efforts are required from advanced system analysis and controller design to achieve the overall closed-loop performance (Blanke, Izadi-Zamanabadi, Bøgh and Lunau, 1997; Patton, 1997a). In other words, the overall strategy should be considered systematically.

1.5.3 Reconfiguration mechanism

Since sensor faults will not change the system dynamics, it is not necessary to redesign the controller if the faulty output can be corrected when a sensor fault occurs. Therefore,

the most commonly used approach is *sensor fault masking* using a software sensor (Wu, Thavamani, Zhang and Blanke, 2006; Zhang, 2009), or equivalently using virtual sensors (Blanke, Kinnaert, Lunze and Staroswiecki, 2006; Ponsart, Theilliol and Aubrun, 2010; Nazari, Seron and De Doná, 2013). The idea of the software or virtual sensor is to estimate the sensor fault or the real system outputs, and then feed the fault-free output signals to the controller, thereby de-couple the effects of the faults in the feedback loop (Bennett, 1998; Blanke, Kinnaert, Lunze and Staroswiecki, 2006; Wu, Thavamani, Zhang and Blanke, 2006; Gao, Breikin and Wang, 2007; Gao and Ding, 2007c; Rothenhagen and Fuchs, 2009; Zhang, 2009; Ponsart, Theilliol and Aubrun, 2010). The key issue of this strategy is the requirement of fast and accurate fault estimation which should be robust to uncertainty and disturbances.

In contrast to sensor faults, the most used compensation law for actuator or process faults would be in the following additive form (Noura, Sauter, Hamelin and Theilliol, 2000; Zhang and Jiang, 2008):

$$u(t) = u_h(t) + u_c(t)$$

$u_h(t)$, known as *baseline controller*, is designed for the healthy condition and $u_c(t)$ is used to compensate for the impact of the faults. In the normal or healthy condition, $u_c(t)$ is identical or close to zero. Once an actuator or process fault is detected, $u_c(t)$ should be suitable to compensate for the effect of the corresponding fault.

From an AFTC point of view, the total loss of effectiveness of an actuator requires a control system reconfiguration in order to recover the loss of actuation using a redundant actuator (Patton, 1997a; Blanke, Kinnaert, Lunze and Staroswiecki, 2006). Changes in the controller parameters may not be enough to accommodate the total loss of actuator function and the controller structure or controller strategy must be redesigned on-line. The restructure or redesign is often done off-line, so that pre-computed redundant control laws and hardware systems can be selected once an actuator is known to have failed.

However, the study in this focuses on partial actuator faults which can be accommodated using the faulty actuator.

According to whether or not the reconfigurable controller is calculated on-line, the AFTC can be classified according to whether a pre-calculated controller approach is

used or whether the controller is designed on-line (Patton, 1997a; Zhang and Jiang, 2008).

1.6 Motivation and scope of the thesis

In AFTC systems design, the FDD/FE scheme is designed to operate together with control mechanisms in closed-loop. The problem, in fact, lies in the interaction between the FDD/FE and controller reconfiguration and the influence of the interaction on the entire system (Patton, 1997a; Zhang and Jiang, 2008; Ding, 2009) Generally speaking, both FDD and reconfigurable control design modules should be designed in such a way that they should work in harmony and try to reduce the adverse effects from each other.

1.6.1 Real time nature of AFTC

One critical issue of AFTC is the limited available time for FDD and controller reconfiguration. The real time nature of AFTC is an important property and should be paid enough attention on since both the FDD and controller reconfigurations are carried out on line. The time delay will impact the overall system performance. A typical time response of AFTC systems (Zhang and Jiang, 2006) is shown in Figure 1.9.

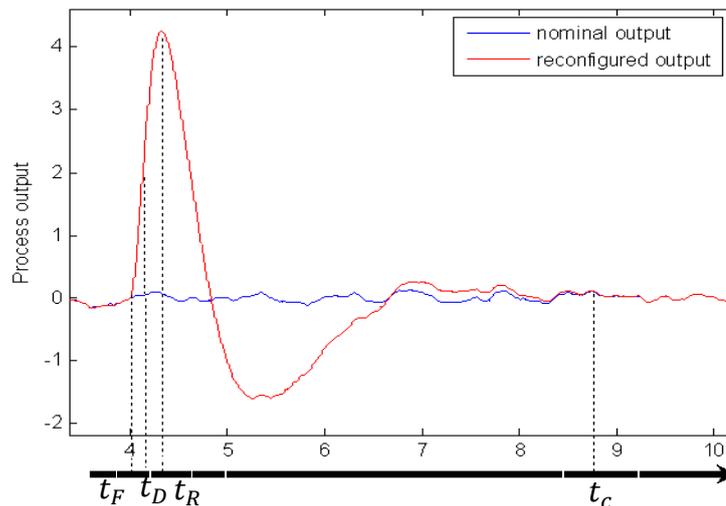


Figure 1.9 Three intervals in AFTC {adapted from (Zhang and Jiang, 2008)}

It can be seen that the response is divided into three time intervals: pre-fault, duration of fault, and post-fault. For more accuracy expression, the terms pre-fault period, transient period, and post-transition period are used to represent the three intervals. In Figure 1.7,

t_F stands for the time instant at which the fault occurs and before which is the pre-fault interval; t_D stands for fault detection time; t_R stands for control reconfiguration time; and t_C is the time instant after which all the transients due to the fault and the control system reconfiguration have settled down and a new steady-state has been reached, and the system enters the post-transition interval.

The system dynamics in the period from t_F to t_R are very complex and highly nonlinear. After the fault occurrence and before the controller has accommodated the effect of fault (i.e. the system reconfiguration), the system has operated in a faulty condition with an adverse dynamic effect (even instability).

Before the fault has been detected, there is nothing one can do to maintain fault-free (or nominal) control performance which is mainly dependent on the robustness of the baseline controller to the faulty system. Even after the detection time t_D , the isolation of the fault (i.e. to determine its location) takes some time. Furthermore, the identification of the fault in terms of its magnitude and time characteristic also takes time. There is a conflict as the ideal reconfiguration requires instantaneous and accurate fault diagnosis information. The interval between t_R and t_C represent the reconfiguration period. In this interval, the reconfigured controller has an important feedback action on the post-fault system behaviour.

Hence, besides improving the robustness of baseline controller, fast fault detection, fast diagnosis, and fast reconfiguration are essential to reduce the transient period and improve the overall performance of the post-fault system. In addition, this is an interval where the strategies of integrated FDD and reconfigurable controller can play an important role to recover required fault-free performance from the faulty system as soon as possible (Zhang and Jiang, 2006).

1.6.2 Robustness of the overall system

Besides the diagnosis speed requirements described in Section 1.5.1, another big challenge arises from the need to consider the effects of modelling uncertainty and disturbances in the overall system level (Patton, 1997a; Zhang and Jiang, 2008).

The existence of a controller changes the dynamics of a system. The observer-based monitoring approach is still well suited to monitoring faults acting in the closed-loop

system, as long as the observer is designed using a perfect system model. However, it is clear that, for the case when there is modelling uncertainty, the control system injects further uncertainty into the estimation scheme. Hence, for the closed-loop case, this uncertainty has to be accounted for carefully (Zhang and Jiang, 2008; Ding, 2009). A more complex situation arises when AFTC is considered since the observer also injects an effect of uncertainty into the control system, so that for this case both effects of uncertainty must be considered.

However, there are relatively few results on the systematic study about the role of FDD in the overall framework of AFTC and information about the way/ methodology to design FDD for AFTC systems (Patton, 1997a; Zhang and Jiang, 2008).

1.6.3 Scope of the thesis

The faults considered in the thesis are sensor faults and compensable actuator faults as explained in Section 1.5.3. Furthermore, some process faults can also be handled if they satisfy corresponding requirement using the approaches mentioned in Section 1.2.1 to transfer multiplicative faults into equivalent additive faults. More detailed information of the compensation mechanism used in the study is discussed in Chapter 5.

This thesis concerns the overall performance of AFTC systems with a simultaneous fault and state observer. More precisely, both the time domain (response speed) and frequency domain (robustness) performances are specified and a multi-objective design is used to satisfy the various real time system requirements. The basic tools to achieve the objectives are pole-placement and H_{∞} optimization.

To show a clear picture of the proposed approaches, numerical examples are studied to show the design procedure following theories proposed in each chapter. In the numerical example study, the forms of faults used are able to cover various time dependent faults given in section 1.2.1. Before the end of the thesis, a non-linear offshore wind turbine benchmark system is studied to verify the feasibility, flexibility and advantages of proposed integrated AFTC scheme and test the AFTC design strategy under varying wind speed.

1.7 Thesis structure and contributions

This Chapter introduces concepts of FTC and FDD from the beginning followed by some general classifications on the different FTC and FDD strategies. The main concepts and strategies behind some of the AFTC and FDD schemes in the literature, as well as their advantages and disadvantages are also discussed. The remainder of the thesis is arranged as follows:

Chapter 2 is devoted to a review of the main literature of FDD/FE and AFTC design, separately. Approaches to the design of controllers and state observers for both linear and nonlinear descriptor systems are outlined. Observer-based approaches to FDD and FE are outlined briefly, leading to the value of using the Extended State Observer (ESO) for combined state and FE.

This Chapter outlines some important background of well-known approaches for reconfigurable controller design, e.g. based on eigenstructure assignment, adaptive approaches, sliding model and LPV (linear parameter varying)/MM (multiple-model) approaches. The state of the art of integrated design of FDD and reconfigurable controller methods is also reviewed briefly.

In **Chapter 3**, the basic concepts and properties of descriptor systems are introduced. System analysis and synthesis are presented for descriptor systems, within an LMI framework. The study considers both the property of stabilization and robustness to disturbance to satisfy both time-domain and frequency-domain performance requirements. A novel LMI description is introduced to achieve regional pole-placement of finite eigenvalues of descriptor system with state feedback.

An important topic of observer design methods for descriptor systems, making use of simultaneous state and fault estimation via the ESO structure, is described in a multi-objective design framework via combining pole-placement design and H_∞ optimization. New ideas concerning a descriptor observer system realization are described by using a standard (i.e. not descriptor) observer design that is equivalent to the ESO of a descriptor system. A numerical example shows the design procedure and usefulness of the proposed algorithm.

Chapter 4 focuses on the design and properties of a novel state and fault estimator, the Proportional Derivative-ESO (PD-ESO) system, using a dual property that exists

between two linear square descriptor systems. Different PD-ESO augmentation strategies are proposed for dealing with sensor faults and sensor noise for estimating the fault signals, whilst minimizing the influence of disturbances or noise. A sensor noise-free system is obtained as a starting point to the PD-ESO design. The robustness to exogenous disturbance or uncertainty is also considered in an H_∞ framework, using a tutorial example given in Chapter 3.

Chapter 5 is concerned with the integrated design problem for observer-based AFTC systems to achieve robustness performance for the overall system. Whilst there is some background literature, this is an immature topic and the work comprises a strong contribution in terms of integrated design of state and FE within a reconfigurable control structure. In this work, both the ESO and the PD-ESO methods are considered in an integrated AFTC system using a two-step design procedure. The same numerical example as in Chapters 3 & 4 is considered as an illustrative comparison of the designs of two AFTC systems using the ESO and the PD-ESO, subject to identical fault scenarios.

Chapter 6 focuses on extending the descriptor systems approaches developed in Chapters 3, 4, & 5 to an LPV framework. One advantage of using an LPV systems approach is that although it handles many nonlinear and time-varying systems it nevertheless is based on a linear systems design theory.

As another new work both the ESO and PD-ESO approaches are extended to a descriptor LPV system format. Based on this, a mechanism is described for developing an integrated AFTC design scheme. The definition of strongly equivalent systems taken from the literature is applied here to a descriptor ESO problem. LPV PD-ESO with constant derivative gain is proposed considering the practical implementation. Finally, a numerical example of an AFTC system incorporating a PD-ESO within an LPV descriptor system framework is studied to illustrate the proposed integrated AFTC design procedure.

Chapter 7 focuses on an application problem of combined AFTC and FE for sustainable control of a nonlinear offshore wind turbine benchmark system. The combined LPV system integrating together the descriptor-system-based AFTC and observer-based FE is specifically adapted for the FTC requirements of the wind turbine system. With an interest of providing added-value to the wind turbine industry, the

AFTC uses the existing wind turbine generator angular speed reference control as a starting point for the baseline control design; so that the scheme can be implemented on existing systems as an additional FTC feature, rather than requiring total system re-design.

Chapter 8 summarises the work of the thesis, emphasising the original contributions in the light of the current research literature, providing a general conclusion for the research. Suggestions as to how the research can be further developed are also discussed.

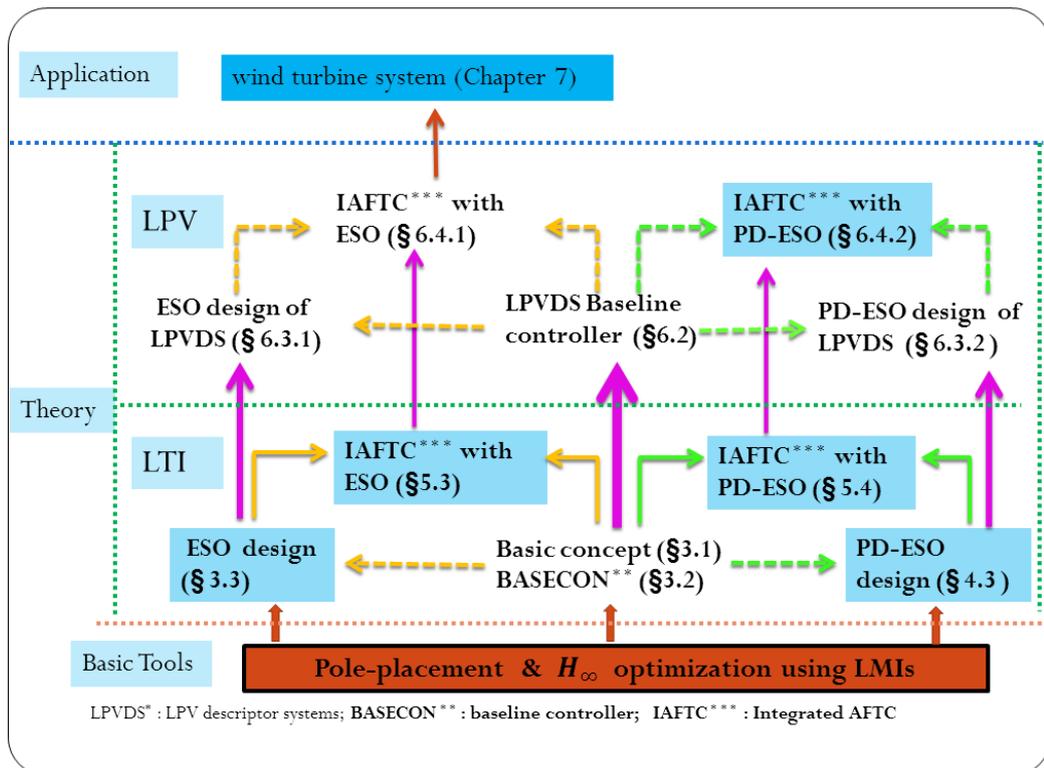


Figure 1.10 The structure of the contributions in the thesis

In summary, a diagram of the contributions is given in Figure 1.10. The purple lines denote the extensions from LTI systems to LPV systems. The yellow lines denote the thesis material involving ESO design, while the green lines denote the chapters that are concerned about PD-ESO design. The dashed lines show in the sections of the thesis that provide alternative ways to understand the design approach. The light blue background of blocks denotes the corresponding theory evaluated using numerical examples. For example, the strategy proposed in Section 6.4.2 is an extension of the approach in Section 5.4 involving PD-ESO design, or a combination of the approaches in Sections 6.2 & 6.3.2. Also, the strategy in Section 6.4.2 is evaluated using a numerical example.

Chapter 2: Literature review on FDD and reconfigurable controller design

This Chapter provides a review of the development in the field of analysis and synthesis of descriptor control and estimation, based on the use of LMI tools. Some basic approaches to feedback design for baseline controllers and baseline observers for descriptor systems is given, with a focus towards the development of FD, FE and FTC schemes. The state of the art of the use of descriptor system methods in FD and FTC is given, bearing in mind that this subject is at a very early stage of development, according to the recent literature. The literature has several rich ideas on the analysis and synthesis of standard (non-descriptor) systems, forming a basis for significant stimulus and motivation for new work in this thesis.

2.1 Analysis and synthesis approaches

The study of descriptor systems actually started from the end of 1970s (Cobb, 1983; Lewis, 1986; Dai, 1989) although the concept was first introduced in 1973 (Singh and Liu, 1973). In the literature there has been a tradition of using descriptor system realizations for economic systems (Luenberger and Arbel, 1977) and population dynamics (Campbell and Campbell, 1980). Other application fields now include electronic systems and robotics (Lewis, 1986; Duan, 2010). A number of fundamental notions and results of conventional state-space systems have been re-applied within a descriptor system context, including controllability and observability, stability and stabilization, eigenstructure assignment. Furthermore, discrete-event problems also have a descriptor system context (Zhang and Jia, 2002) as time-delay systems according to (Fridman and Shaked, 2002; Wang, Sun and Sun, 2004; Kim and Oh, 2007; Gao, Breikin and Wang, 2008).

Modern design methods using LMI tools can also be considered as appropriate for the design of discrete-event systems. Some real application examples of descriptor systems can be found in the survey of (Lewis, 1986) and the books of (Lewis, 1986; Dai, 1989; Lam and Xu, 2006; Duan, 2010).

Since the LMIs are tractable and useful (Boyd, Ghaoui, Feron and Balakrishnan, 1994; Scherer and Weiland, 2005), a lot effort has been done to provide descriptor system generalizations of LMI descriptions that usually apply to standard systems. This review focuses on approaches that make use of LMI design strategies.

2.1.1 Analysis and Controller design

It is well known that both stability and robustness are very important properties (Francis, 1987; Zhou, Doyle and Glover, 1996) of dynamic systems. To achieve robustness to disturbances or uncertainty in relation to descriptor systems, the stabilization and dynamic compensation problems of descriptor systems are solved in an LMI framework in (Masubuchi, Kamitane, Ohara and Suda, 1997). Following this, different forms of the Bounded Real Lemma for descriptor system are given in (Wang, Yung and Chang, 1998; Zhang, Xia and Shi, 2008). Analysis and synthesis with quadratic performance is proposed in (Takaba and Katayama, 1998; Rehm and Allgower, 1999). An equivalent description is given by (Zhang and Jia, 2002) for discrete time descriptor systems. Semi-definite LMI descriptions are less tractable from a numerical point of view. Hence, strict LMIs are defined in (Uezato and Ikeda, 1999) for descriptor systems to improve the computational tractability. Controller design approaches are proposed in (Chen, Zhang and Zhai, 2005) for descriptor systems with mixed H_2/H_∞ performance using non-strict LMIs. Robust and non-fragile H_∞ control is studied in (Kim and Oh, 2007) for descriptor systems with parameter uncertainty. Extended H_2 controller designs for continuous-time descriptor system are reported in (Feng, Yagoubi and Chevrel, 2012).

Eigenvalue assignment (or sometimes known as pole-placement) is an important time domain property of dynamic systems, which has been studied in (Hsiung and Lee, 1997) and strict LMIs are given in (Kuo and Fang, 2003; Marx, Koenig and Georges, 2003b; Kuo and Lee, 2004). Pole-placement in disjoint regions is discussed in (Rejichi, Bachelier, Chaabane and Mehdi, 2008) for system analysis.

An iterative approach to proportional derivative (PD) output feedback is proposed in (Lin, Wang and Lee, 2005) with norm-bounded perturbations. A robust PD state feedback controller is designed in (Ren and Zhang, 2009) with necessary and sufficient conditions for the solvability of problem given using LMIs. Robust state derivative feedback is proposed in (Faria, Assuno, Teixeira and Cardim, 2010), for descriptor systems restricted to certain bounded outputs.

An iterative approach is proposed in (Yagoubi, 2010b) for static output feedback of descriptor system. A two-step static output feedback design is reported in (Chaabane, Tadeo, Mehdi and Souissi, 2011).

2.1.2 Observer design

The analysis and design of Observers (or filter systems) can also be developed in a descriptor system format (Luenberger, 1966; 1971) and it turns out that this is an important approach to estimator design (Darouach, 2012; Ezzine, Souley Ali, Darouach and Messaoud, 2012). The general architecture of an observer is shown in the Figure 2.1. In keeping with standard classical control theory, the descriptor observer design is a dual problem of the descriptor state feedback problem (Dai, 1988; 1989). Considering the realization and implementation issues, more efforts are made to design standard full-order or reduced order observers for descriptor system (Lewis, 1986; Dai, 1988; 1989; Darouach and Boutayeb, 1995; Darouach, Zasadzinski and Hayar, 1996; Hou and Muller, 1999; Gao, 2005; Wu and Duan, 2007; Ren and Zhang, 2010; Ezzine, Souley Ali, Darouach and Messaoud, 2012).

Within the LMI framework, the reduced order observer design is investigated in (Zhou and Li, 2008) for descriptor systems determined by parameterizing the desired observer in an H_∞ or H_2 framework. Reduced order filters are designed for discrete time descriptor systems based on a generalised Sylvester matrix equation in (Xu and Lam, 2007; Darouach and Zasadzinski, 2009). A functional H_∞ filter design approach is reported in (Darouach, 2012) based on a new definition of partial impulse observability, given some sufficient conditions for the existence and stability satisfied.

With another design parameter, the PD observer design approaches have also been studied to design a standard full order observer for descriptor systems (Gao, 2005; Wu and Duan, 2007; Ren and Zhang, 2010).

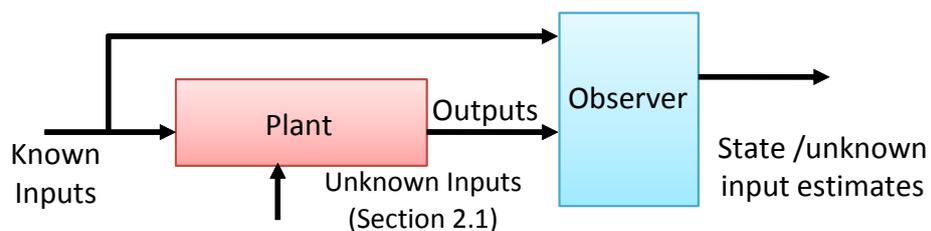


Figure 2.1 A general architecture of observer system

2.1.3 LPV approaches to non-linear systems

As one attractive solution, gain-scheduling techniques has been studied extensively , driven by a strong requirement for parameter-varying flight control systems in the aviation industry (Shamma and Athans, 1990; Rugh, 1991; Shamma and Athans, 1991; Packard and Kantner, 1996; Leith and Leithead, 1999; Rugh and Shamma, 2000). The conventional gain scheduling idea is to design different controllers for each system operating point, and then implement the family of controllers such that the gains are scheduled according to the current value of the scheduling parameter. The main advantage is to use linear techniques to address non-linear problems. A drawback of the traditional gain scheduling is the requirement of slow variation of scheduling parameter to achieve the global stability of the closed-loop system (Shamma and Athans, 1991; Wu, 1995).

Linear Parameter Varying (LPV) techniques are developed to provide desired performance globally without such restriction both for LPV standard and descriptor systems (Apkarian, Gahinet and Becker, 1995; Wu, 1995; Wu, Yang, Packard and Becker, 1995; Packard and Kantner, 1996; Apkarian and Adams, 1998; Bara, Daafouz, Ragot and Kratz, 2000; Bara, Daafouz, Kratz and Ragot, 2001; Scherer, 2001; Masubuchi, Akiyama and Saeki, 2003; Bouali, Chevrel and Yagoubi, 2006; Hamdi, Rodrigues, Mechmeche, Theilliol and Benhadj Braiek, 2009; Halalchi, Bara and Laroche, 2011). The main advantage of LPV modelling is that it facilitates the application of powerful linear design tools to complex non-linear modelling problems (Wu, 1995; Packard and Kantner, 1996; Hallouzi, Verdult, Babuska and Verhaegen, 2005) with achieve d global stability and robust performance.

LPV modelling of monitored systems has been considered for FDD/FE (Akhenak, Chadli, Maquin and Ragot, 2004; Bokor and Balas, 2004; Hallouzi, Verdult, Babuska and Verhaegen, 2005; Rodrigues, Theilliol and Sauter, 2005a; Szaszi, Marcos, Balas and Bokor, 2005; Grenaille, Henry and Zolghadri, 2008; Zolghadri, Henry and Grenaille, 2008; Hallouzi, Verhaegen and Kanev, 2009), and FTC (Rodrigues, Theilliol and Sauter, 2005b; Patton, Chen and Klinkhieo, 2012). State estimation of polytopic descriptor system are design in (Hamdi, Rodrigues, Mechmeche, Theilliol and Benhadj Braiek, 2009) with application to FDD. Multi-objective design approaches are proposed in (Yagoubi, 2010a) for parameter-dependent descriptor systems. Dilated LMI

conditions for the robust analysis of uncertain parameter-dependent descriptor systems are reported in (Bara, 2010) for the continuous-time case and the discrete-time case has been reported in (Bara, 2011). Observer-based controllers have been proposed in (Halalchi, Bara and Laroche, 2011) using dilated LMI descriptions.

2.2 Observer based FDD approaches

During the last 40 years, FDD has become a very significant research topic. Generally, FDD methods are classified as either model-based approaches or data-driven (model-free) approaches. Data-driven methods are generally based on implicit model information, whereas the so-called model-based methods use explicit model information (Calado, Korbicz, Patan, Patton and Sa da Costa, 2001). A fuzzy neural network itself can be an implicit representation of a dynamic system (Calado and Sa da Costa, 1998) or via a Neuro-Fuzzy structure (Uppal and Patton, 2005; Uppal, Patton and Witczak, 2006).

Explicit model-based FDD approaches are the most preferred and most studied (Willsky, 1976; Clark, 1978; Isermann and Ballé, 1997; Chen and Patton, 1999; Patton, Clark and Frank, 2000; Venkatasubramanian, Rengaswamy, Yin and Kavuri, 2003; Ding, 2008; Hwang, Kim, Kim and Seah, 2010; Isermann, 2011). All of the above are so-called *quantitative methods of FDD*. There are also some approaches to *qualitative FDD* design (Venkatasubramanian, Rengaswamy and Kavuri, 2003).

In the literature, there are mainly three different approaches used for FDD based on explicit quantitative modelling methods (Chow and Willsky, 1984; Gertler, 1997; Isermann, 1997; Chen and Patton, 1999; Ding, 2008; Isermann, 2011) as outlined in Figure 2.2. The following provides a brief outline of these approaches.

1. Parameter estimation approaches based FDD: This approach is developed based on system identification techniques (Isermann, 1984; Isermann and Ballé, 1997; Isermann, 2011). The faults are reflected in the physical system parameters and then the idea of the fault detection is based on the comparison between the online estimation of system parameter and the parameter of the fault-free reference model. As the system parameters are obtained, fault estimation can be achieved in some degree. A survey of most used parameter estimation approach is reported in (Isermann, 1984; 2006).

2. Observer based FDD: The pioneer work of this approach was started by (Beard, 1971; Jones, 1973; Clark, 1978) and systematic design approach can be found in (Chen and Patton, 1999; Ding, 2008). In this approach the observer is used to provide estimate of the actual system output. For a residual generator, a residual signal is generated via the weighted output estimate error between the measured output and the estimated output. The available flexibility in selecting observer structure from full order to reduced order observer and wide application has motivated the interest of this approach. Fault estimation also can be achieved with carefully designed observer gain.

3. Parity relation based FDD: In this approach, the residual signals are generated based upon consistency check on system input and output data (Chow and Willsky, 1984; Gertler, 1997; Gertler, 1998). It has been proved that the parity equation approach has some correspondence with certain types of observer-based FDD approaches (Patton and Chen, 1994; Patton, 1997a; Ding, 2008; Hwang, Kim, Kim and Seah, 2010).

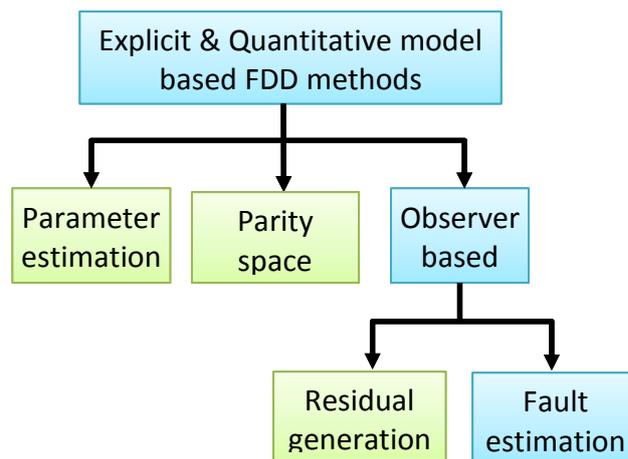


Figure 2.2 Classification of explicit & quantitative model based FDD methods

Observer based approaches for fault detection and isolation (FDI) and FDD are widely researched in a very active research field in the control systems community. This Section reviews briefly the residual design approaches (e.g. UIO and eigenstructure assignment), followed by the adaptive observer approach, sliding model observer methods, ESO/PIO and its variants, for both standard and descriptor systems.

2.2.1 Residual generator design

The robustness problem has been a main concern as there are often disturbances or uncertainty in the model-based design. One of the most successful robust fault diagnosis

strategies is the use of the disturbance decoupling principle. This can be done by using UIO or via eigenstructure assignment.

2.2.1.1 Residual generation using UIO

The idea of UIO (Watanabe and Himmelblau, 1982; Wünnenberg and Frank, 1987; Frank and Ding, 1997; Chen and Patton, 1999; Odgaard and Stoustrup, 2010) is to generate residuals reflecting the occurrence of faults and robust to disturbances or uncertainty. The residual should be at or close to zero if there is no fault and sufficiently different from zero if a fault occurs. When the system matrices satisfy some conditions, the residuals can be decoupled completely from the effects of disturbances or modelling uncertainty. The basic idea is reviewed as follows.

It is assumed that a standard system is disturbed by an additive unknown input term as follows (Chen and Patton, 1999):

$$\dot{x}(t) = Ax(t) + Bu(t) + F_1f(t) + R_1d(t) \quad (2-1)$$

$$y(t) = Cx(t) + Du(t) + F_2f(t) + R_2d(t) \quad (2-2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^h$ is the output vector, $u(t) \in \mathbb{R}^m$ is the known input vector and $d(t) \in \mathbb{R}^d$ is the unknown input or disturbance vector, $f(t) \in \mathbb{R}^p$ represents the fault vector which is considered as unknown time function. A, B, C, D, F, R_1 and R_2 are known matrices with appropriate dimensions. The matrices F_1 and F_2 are fault distribution matrices which are known.

As shown in Figure 2.3, the residual generator based on a full order observer is described as:

$$\dot{w}(t) = Fw(t) + Ky(t) + Ju(t)$$

$$r(t) = L_1w(t) + L_2y(t) + L_3u(t)$$

where $r(t) \in \mathbb{R}^p$ is a residual vector, $w(t)$ is the estimated state vector of a linear combination of original system states given by $w = T\hat{x}(t)$.

The estimation error $e(t) = w(t) - Tx(t)$ is given by:

$$\begin{aligned} \dot{e}(t) = & Fe(t) + (FT - TA + KC)x(t) + (J - TB + KD)u(t) \\ & + (KF_2 - TF_1)f(t) + (KR_2 - TR_1)d(t) \end{aligned}$$

From the above the residual vector can be re-formulated as:

$$r(t) = L_1 e(t) + (L_1 T + L_2 C)x(t) + (L_3 + L_2 D)u(t) + L_2 F_2 f(t) + L_2 R_2 d(t)$$

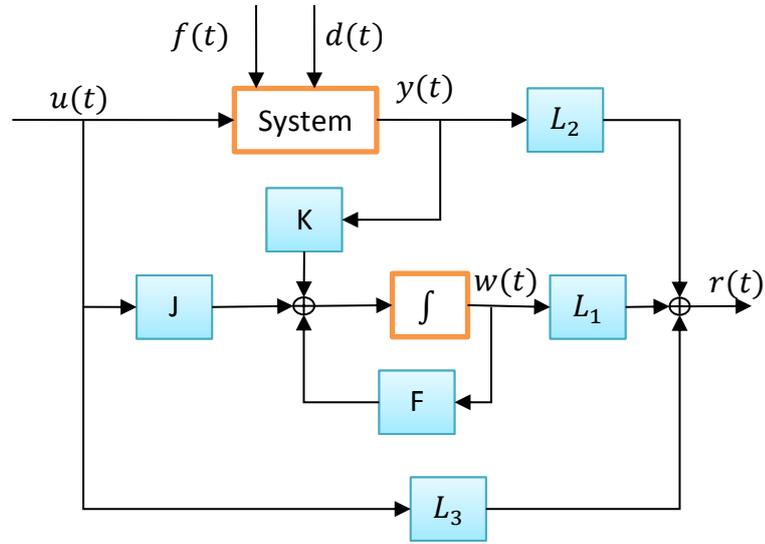


Figure 2.3 General structure of observer based residual generator
(Chen and Patton, 1999)

The goal of the UIO is to force the estimation error $e(t)$ to be independent of the disturbances or modelling uncertainty (i.e. unknown inputs) $d(t)$. The estimation error can be totally decoupled from the disturbances; from this the residual is also decoupled from the disturbances and unknown inputs. To achieve this goal, the following conditions are to be satisfied (Frank, 1990):

$$TA - FT = KC$$

$$J = TB + KD$$

$$KR_2 - TR_1 = 0$$

$$L_1 T + L_2 C = 0$$

$$L_3 + L_2 D = 0$$

$$L_2 R_2 = 0$$

where T is an unknown matrix to be designed. Once those conditions are satisfied, e and $r(t)$ can be rewritten as:

$$e(t) = Fe(t) + (KF_2 - TF_1)f(t)$$

$$r(t) = L_1 e(t) + L_2 F_2 f(t)$$

The above equations show that both the residual vector and estimation error are independent of the unknown inputs $d(t)$, and the system inputs $u(t)$. The residual can follow the fault signal if the pair (F, L_1) is observable (Patton, 1997b). A systematic design procedure can be found in (Chen and Patton, 1999).

Following this direction, the UIO for descriptor system is studied in (Koenig, 2006). Combining the UIO with PIO, a UIPIO is proposed to LPV descriptor system in (Hamdi, Rodrigues, Mechmeche, Theilliol and Braiek, 2012). The UIO approach is extended to Takagi-Sugeno descriptor systems in (Marx, Koenig and Ragot, 2007).

2.2.1.2 Residual generation using eigenstructure assignment

Instead of decoupling the effect of the disturbance in the estimation error e (as discussed above for the UIO design), the eigenstructure assignment approaches seek to design a disturbance decoupled residual due to some important roles that the eigenstructure of a linear system plays in feedback systems (observers systems in particular). Eigenstructure assignment research has been active for a long time (Andry, Shapiro and Chung, 1983; Sobel and Shapiro, 1985; Owens, 1989; Patton and Chen, 1991; Duan, 1993; Kshatriya, Annakkage, Hughes and Gole, 2010; Wang, Chang and Zhang, 2010; Cai, Hu and Duan, 2011; Zhang, 2011). It is assumed that the system is disturbed by an additive unknown input term as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ff(t) + R_1d(t) \quad (2-3)$$

$$y(t) = Cx(t) \quad (2-4)$$

The residual generator based on a full order observer is described as:

$$\hat{\dot{x}}(t) = (A - LC)\hat{x}(t) + Bu(t) + Ly(t)$$

$$\hat{y}(t) = C\hat{x}(t)$$

$$r(t) = Q[y(t) - \hat{y}(t)]$$

where $r(t) \in \mathbb{R}^p$ is the residual vector, $\hat{x}(t)$ and $\hat{y}(t)$ are state and output estimate vectors, respectively. The matrix $Q \in \mathbb{R}^{p \times h}$ is the residual weighting factor. Note that, the residual is a linear transformation of the output estimation error. Hence, the residual dimension p cannot be larger than the output dimension h . This is because the linearly-dependent extra residual components do not provide additional useful information for fault detection.

In order to make the residual $r(t)$ to be independent of the disturbance, it is necessary to null the entries in the transfer function matrix between the residual and the disturbance. That means that:

$$G_{rd}(s) = QC(sI - A + LC)^{-1}R_1d(s) = 0$$

This objective can be achieved by the following approaches (Patton and Chen, 2000):

1. Left Eigenvector Assignment: Choose Q and L such that $QCR_1 = 0$ and all rows of QC are the left eigenvectors of $A - LC$.
2. Right Eigenvector Assignment: Choose Q and L such that $QCR_1 = 0$ and all columns of R_1 are the right eigenvectors of $A - LC$.

Different design approaches can be found in (Liu and Patton, 1998; Chen and Patton, 1999; Duan, 2010) for both descriptor and standard systems. With the extended LPV eigenstructure assignment techniques (Cai, Hu and Duan, 2011; Shi and Patton, 2012a), fault detection to LPV system is studied in (Shi and Patton, 2012b).

The existence of a disturbance decoupled observer depends on certain rank conditions (Chen and Patton, 1999). In some applications, the rank conditions concerning either unstructured uncertainty or structured uncertainty are not satisfied (Jiang, Wang and Soh, 1999). Hence, it may not be possible to achieve a total decoupling between the residual and disturbances. Some methods to calculate a possible disturbance distribution matrix are proposed in Chapter 5 of (Chen and Patton, 1999).

2.2.2 Fault reconstruction with sliding mode observer

A very powerful form of sliding mode observer (SMO) is proposed in (Edwards and Spurgeon, 1998). Based on this SMO approach the faults of dynamic system are reconstructed or estimated using the concept of the *equivalent injection* signal (Edwards, Spurgeon and Patton, 2000; Tan and Edwards, 2003; Floquet, Edwards and Spurgeon, 2007; Yan and Edwards, 2007; Yan and Edwards, 2008). Furthermore, the *equivalent output injection* signal represents the average behaviour of the switching function and represents the effort necessary to maintain the motion on the sliding surface (Edwards, Spurgeon and Patton, 2000). In the SMO-based approach, the faults are estimated using the output injection concept. However, the faults must satisfy an observer matching condition which implies that the approach works well for actuator faults. However, for

the case of sensor faults the approach can be extended by using an augmented state structure involving a first order filter to transform the sensor fault problem into an equivalent actuator fault one (Tan and Edwards, 2003).

An alternative way is proposed in (Jiang, Staroswiecki and Cocquempot, 2004) for a class of non-linear systems. This approach shows that under certain geometric conditions, the original nonlinear system is transformed into two different subsystems with uncertainty. The first subsystem is in generalised observer canonical form, which is not affected by faults, whilst the second is affected by faults. An SMO is then constructed for the first subsystem to enable the estimation of the faults to be achieved from the second subsystem.

2.2.3 Fault and state estimation with extended state observer (ESO)

The problem of simultaneous state and faults estimation is very attractive as it can provide the estimates of states and faults within one design, as long as robustness and boundedness conditions are satisfied. This Subsection reviews briefly ESO and proportional integral observer (PIO) used for simultaneous state and fault estimation and shows the equivalence of these approaches.

The common idea is to design an appropriate fault estimation residual which is equal or close to the value of a fault to achieve FE. A common assumption is that the corresponding faults or unknown input signals vary slowly with time. This can be a useful restriction since there is little way to know *a priori* the dynamic information of the real faults. In the earlier literature the faults are commonly considered as constant and the FE problem for this case has a simplified form.

Consider a standard system as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ff(t) \quad (2-5)$$

$$y(t) = Cx(t) \quad (2-6)$$

A PIO (Niemann and Stoustrup, 1992) can be in the form of:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + F\hat{f}(t) + Bu(t) + L_x(y(t) - \hat{y}(t)) \quad (2-7)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (2-8)$$

$$\dot{\hat{f}}(t) = L_f(y(t) - \hat{y}(t)) \quad (2-9)$$

In the design, the PIO can be re-formulated to:

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{f}}(t) \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{f}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_x \\ L_f \end{bmatrix} (y(t) - \hat{y}(t)) \quad (2-10)$$

$$\hat{y}(t) = [C \quad 0] \begin{bmatrix} \hat{x}(t) \\ \hat{f}(t) \end{bmatrix} \quad (2-11)$$

The above observer is actually equivalent to the design of a proportional observer based on the augmented/extended state system description with assumption $\dot{f}(t) = 0$ (Yi, Xu, Han and Lam, 2001; Wang and Gao, 2003; Patton and Klinkhieo, 2009; Guo and Zhao, 2011):

$$\begin{bmatrix} \dot{x}(t) \\ \dot{f}(t) \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ f(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \quad (2-12)$$

$$\hat{y}(t) = [C \quad 0] \begin{bmatrix} x(t) \\ f(t) \end{bmatrix} \quad (2-13)$$

As it can be seen from the system description in (2-12) and (2-13), the PIO in fault estimation is actually a proportional observer with extra augmented states. In the FDD/FE and FTC contexts, the augmented states have specific meaning (the desired fault signal) as distinct from a traditional PIO. For this reason, *the name ESO is adopted in the research* to highlight the simultaneous estimation of both states and faults. Another reason is the design of the ESO and later studied PD-ESO can simply follow the established theory for descriptor system analysis and synthesis. When descriptor systems are considered, the (2-10) becomes:

$$\begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{f}}(t) \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{f}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_x \\ L_f \end{bmatrix} (y(t) - \hat{y}(t))$$

Because of the potential singularity of E , the implementation of a descriptor observer is not as easy as that of a standard observer is, and should be handled properly .

Some authors have paid a lot of attention to the design of augmented observers for joint state and fault estimation, concerning both standard systems and descriptor systems. For instance, PIO has been studied in (Busawon and Kabore, 2001; Duan, Liu and Thompson, 2001; Shafai, Pi and Nork, 2002; Marx, Koenig and Georges, 2003a; Gao

and Ho, 2004; Aguilera-González, Theilliol, Adam-Medina, Astorga-Zaragoza and Rodrigues, 2012; Hamdi, Rodrigues, Mechmeche, Theilliol and Braiek, 2012), ESO in (Yi, Xu, Han and Lam, 2001; Wang and Gao, 2003; Guo and Zhao, 2011; Liu and Li, 2012). In (Patton and Klinkhieo, 2009) this has been referred to as an augmented state observer (ASO), applicable to standard systems in their work.

The conventional ESO is conservative as it is based on the assumption that fault $f(t)$ is slowly time-varying. The main disadvantage of this method is that the fault may have unpredictable behaviour and for most of the time is not constant, which means the derivative of $f(t)$ is not zero and may also lead to high gain designs (Patton and Klinkhieo, 2010). A proportional multi-integral observer (PMIO) is developed to improve the performance in the PIO context in (Gao and Ho, 2004; Koenig, 2005; Gao and Ho, 2006; Gao and Wang, 2006; Gao, Breikin and Wang, 2007; Gao and Ding, 2007b; 2007c; Gao and Ding, 2007a; Gao, Ding and Ma, 2007; Gao, Breikin and Wang, 2008; Sami and Patton, 2012). Generally speaking, if the k^{th} derivatives of the fault signals are bounded, the original system would be augmented to an $n + pk$ order system, where n is the order of the original system and p is the number of faults.

The derivatives of the outputs are also considered to improve the performance of the FD or FE. To get fast estimation, a proportional multi-integral derivative observer (PMIDO) is proposed in (Gao and Ding, 2007b). The PMIDO introduces the derivatives of the estimation error to get faster state estimation.

The approach in (Gao, Ding and Ma, 2007) is extended to a multiple-model Takagi-Sugeno (T-S) formulation in (Sami and Patton, 2012), considering the modelling uncertainty as an important issue in the FE design and including simultaneous actuator and sensor faults for nonlinear systems. The formulation is also extended to an LPV descriptor system framework in (Aguilera-González, Theilliol, Adam-Medina, Astorga-Zaragoza and Rodrigues, 2012) dealing with both sensor and actuator faults.

2.3 Reconfigurable controller design approaches

Considering the importance of reconfigurable controller discussed in Section 1.5, various efforts have been made to develop different design schemes. However, the classification of reconfigurable controllers is still not standardised. Actually, all the methods used in normal controller design can be candidates for reconfigurable

controller design approaches. This Section is only concerned with the most studied approaches in the AFTC literature.

2.3.1 Model matching approaches

In the reconfigurable controller design step, the objective is to minimize the difference between the faulty system and the fault-free system to maintain the stability and recover the performance as much as possible.

For state feedback, the pseudo-inverse method (PIM) and the modified pseudo-inverse method (MPIM) are proposed by (Gao and Antsaklis, 1991) to get exactly the same model before and after fault occurrence. However, the PIM and MPIM require certain conditions which are restrictive and are only capable of handling state feedback cases (Jiang, 1994; Patton, 1997a). In (Jiang, 1994), an eigenstructure assignment method is proposed to design a reconfigurable control systems with the simple principle that two closed-loop system matrices share same performance if they share the same eigenstructure. A sufficient condition is given to recover the stability of the faulty system. Another improved method is proposed to design a robust reconfigurable control system with both static and dynamic output feedback in (Ashari, Sedigh and Yazdanpanah, 2005) even when the system order is changed. A particle swarm optimization algorithm is used to design the output feedback reconfigurable control law in (Zhang, Zhang, Sun and Ning, 2010).

2.3.2 Sliding mode reconfigurable control

Sliding mode reconfigurable control is also one of the most active areas. Multiple time scale reconfigurable sliding mode controller is designed in (Shtessel, Buffington and Banda, 2002) for an aircraft. A sliding mode controller is designed in the frequency domain to compensate for actuator damage without the use of FDD scheme (Hess and Wells, 2003). Using fault detection, an on-line sliding mode control allocation scheme is designed in (Corradini, Orlando and Parlangeli, 2005). With fault estimation provided by an SMO, better performance is claimed with sliding mode FTC in (Alwi and Edwards, 2008b; Edwards, Lombaerts and Smaili, 2010).

2.3.3 LPV or multiple model (MM) approaches

Another class of reconfigurable controller design is to make use of multiple-model (MM), switching or tuning scheme. Those approaches are mainly on control of systems with component faults (Tao, Joshi and Ma, 2001). In the design, faulty system is considered to a new model in multiple model approaches or the fault signal is used as scheduling parameter in LPV, switching or tuning scheme. The multiple model adaptive control has been used in an adaptive reconfigurable control scheme, such as the reconfigurable flight control schemes in (Maybeck and Pogoda, 1989; Maybeck and Stevens, 1991). The interacting multiple model (IMM) approach has been used to design an integrated fault detection and fault tolerant aircraft flight control system by (Zhan and Jiang, 1999; Zhang and Jiang, 2001a). Many authors have also designed reconfigurable flight control laws based on the multiple-model-based predictive or switching approaches, as can be seen in (Gopinathan, Boskovic, Mehra and Rago, 1998; Rago, Prasanth, Mehra and Fortenbaugh, 1998; Boskovic and Mehra, 1999; Boškovic and Mehra, 2002; Jung, Jeong, Lee and Kim, 2005).

2.4 Integrated design of FDD and Reconfigurable controller

The integration of different subsystems to obtain good FTC performance is not as straightforward a task as it first appears because of the adverse interactions between each subsystem (Eberhardt and Ward, 1999). However, the main difficulty rising from the integration of FDD and reconfigurable controller comes from the presence of a reconfiguration time-delay that may result in unstable closed-loop behaviour. Furthermore, imperfect FDD and/or fault accommodation may also lead to poor FTC performances. The question that then arises is how to balance the robustness of the closed-loop system during normal operation versus the fault sensitivity from the time of occurrence of the fault (Wu and Chen, 1996; Patton, 1997a; Wu, 1997). These requirements can actually play conflicting roles in an FTC scheme. Hence, the interaction between the FDD and the reconfigurable controller should be taken into account to ensure the subsystems work well with each other.

Basically, there are two directions in the integrated design following different design philosophies. In the fault detection field, the four-parameter-controller approach is introduced in (Nett, Jacobson and Miller, 1988; Jacobson and Nett, 1991), and

developed by (Niemann and Stoustrup, 1997; Hearn, Grimbly and Johnson, 1998; Zhang and Jiang, 2001a; Ding, 2009; Wei, 2009) or called joint designs in (Suzuki and Tomizuka, 1999) . Two different approaches are proposed in (Weng, Patton and Cui, 2008; Wang and Yang, 2009) to extend the approach to LPV systems. One LMI approach is proposed in (Davoodi, Golabi, Talebi and Momeni, 2013) to extend the approach to switched systems. Those approaches concern the effect controller has on observer or residual generator.

Following another philosophy, efforts have been made to form an integrated design approach and to evaluate the performance of the overall AFTC systems (Kobi, Nowakowski and Ragot, 1994; Tsui, 1994; Eryurek and Upadhyaya, 1995; Balle, Fischer, Fussel, Nelles and Isermann, 1998; Eberhardt and Ward, 1999; Wise, Brinker, Calise, Enns, Elgersma and Voulgaris, 1999; Noura, Theilliol and Sauter, 2000; Kim, Rizzoni and Utkin, 2001; Zhang and Jiang, 2001b; 2001a; Huang, Reklaitis and Venkatasubramanian, 2002; Zhang and Jiang, 2002; Liu, Wang and Li, 2004; Campos-Delgado, Martinez-Martinez and Zhou, 2005; Jiang and Chowdhury, 2005; Shin and Belcastro, 2006; Shin and Gregory, 2007); A seamless integration of a FDD scheme and appropriate reconfigurable control techniques still poses significant challenges in practice, and remains one important research topic (Morari and H Lee, 1999; Zhan and Jiang, 1999; Zhang and Jiang, 2001a; 2006; 2008).

There are few results on integrated design of AFTC for descriptor systems according to the literature. This is despite an interesting result using coprime factorization and Youla parameterization in (Marx, Koenig and Georges, 2004). Hence, it is clear that it is of crucial importance to study of the theory of integrated designs of AFTC using LMI tools.

Chapter 3: Extended state observer (ESO)

3.1 Introduction and preliminary mathematics

Simultaneous state and fault estimation are attracting more and more attention because both system states and fault signals are required for some AFTC schemes (Noura, Theilliol and Sauter, 2000; Gao and Ho, 2004; Gao and Ding, 2007b; Zhang, Jiang and Shi, 2009). The Extended state observer (ESO) has been a competitive candidate due to its simplicity and practicality as stated in Section 2.3.

In this Chapter, some basic concepts are defined and some baseline control design approaches are outlined within a descriptor system context prior to providing a description of the mathematical tools used for analysis and synthesis of such systems. A novel approach to pole-placement is proposed to meet requirements for time domain analysis and synthesis. The ESO design approach used in this work is presented for simultaneous estimation of system states and slowly-varying faults. It is shown that with extra state augmentation, polynomial faults can also be managed within the same framework. An alternative strategy is presented to relax the constraint conditions relating to sensor faults signals whilst also making use of additional constraints on the fault distribution matrix. Section 2.3 outlines the application of state and fault estimation to a descriptor system problem. From this it is clear that there are some problems which are not present when standard systems are considered. A discussion of these descriptor system design issues making use of equivalence forms of descriptor systems are an important aspect of Section 3.3.

The following provides some basic and essential mathematical background and concepts that relate to the analysis and design of descriptor systems, forming the basis of the mathematical approach used throughout the remainder of the thesis.

Consider an LTI descriptor system as:

$$E\dot{x}(t) = Ax(t) + Bu(t) \quad (3-1)$$

$$y(t) = Cx(t) \quad (3-2)$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^h$ and $u(t) \in \mathbb{R}^m$ are system states, outputs, and control inputs, respectively. A, B, C and E are matrices with compatible dimensions where E satisfies $\text{rank}(E) = r \leq n$. For simplicity of notation the time subscript (t) is omitted in the remainder of this thesis.

The following definitions and descriptor system concepts are presented by (Dai, 1989; Masubuchi, Kamitane, Ohara and Suda, 1997).

Definition 3.1

- (1) A pair (E, A) , referred to as a matrix pencil $(sE - A)$, is regular if $\det(sE - A)$ is not identically zero.
- (2) For a regular pair (E, A) , the finite eigenvalues of $sE - A$ are said to be the finite modes (or finite poles) of (E, A) and the corresponding right and left generalized eigenvectors satisfying $(sE - A)v_r = v_r$ or $(sE - A)^T v_l^* = v_l^*$. The infinite eigenvalues of $sE - A$ are the eigenvalues at $\lambda = 0$ of $E - \lambda A$.
- (3) Grade one infinite generalized eigenvectors of the pair (E, A) satisfy $E v_i^1 = 0$, and the corresponding infinite eigenvalues are the i^{th} non-dynamic modes. Grade k infinite generalized right eigenvectors of the matrix pencil $sE - A$ satisfy $E v_i^k = A v_i^{k-1}$, $k \geq 2$, and the corresponding infinite eigenvalues are the dynamic modes at ∞ , referred to as the *impulsive modes* of (E, A) . The non-impulsiveness of a system implies that the system is regular.
- (4) A pair (E, A) is admissible if it is regular and has neither impulsive modes nor unstable finite modes. Alternatively, a pair (E, A) is admissible if it is impulsive free and stable.

Only regular systems are considered in this study because the regularity is a necessary and sufficient condition for the existence and uniqueness of a solution of the state variable representation of the descriptor system of (3-1) and (3-2).

3.1.1 Equivalent forms

It is well known that there are no unique realizations of state space descriptor systems. The following literature provides an essential background to the material in Section 3.1.1.1 (Dai, 1989; Hinrichsen, Manthey and Helmke, 2001; Benner and Sokolov, 2006).

Here we introduce two equivalent forms from (Dai, 1989) which will be used for implementation of descriptor observer systems.

3.1.1.1 Equivalent Form 1 (EF1)

By taking a coordinate transformation $x = N_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^{n-r}$, where $N_1 \in \mathbb{R}^n$ with $M_1 \in \mathbb{R}^n$ satisfying:

$$M_1 E N_1 = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, M_1 A N_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, M_1 B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C N_1 = [C_1 \quad C_2],$$

then an equivalent linear descriptor system of (3-1) and (3-2) is obtained as:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \quad (3-3)$$

$$0 = A_{21}x_1 + A_{22}x_2 + B_2u \quad (3-4)$$

$$y = C_1x_1 + C_2x_2 \quad (3-5)$$

The M_1 and N_1 can be calculated via singular value decomposition. This equivalent form clearly reflects the physical meaning of a descriptor system. (3-3) is a differential equation composed of dynamic systems; (3-4) is an algebraic equation that represents the inter-connections between the subsystems. Thus, a descriptor system can be viewed as a composite system formed by several interconnected subsystems. Furthermore, the two state space subsystems defined by (3-3) and (3-4) reflect a layer property in some descriptor systems in which one layer has a dynamic property described by a differential equation and the second layer is represented by an algebraic interconnection equation providing constraints.

3.1.1.2 Equivalent Form 2 (EF2 or Standard Decomposition)

By taking another coordinate transformation $x = N_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^{n-r}$, where $N_2 \in \mathbb{R}^n$, with $M_2 \in \mathbb{R}^n$ and nilpotent matrix Q satisfying:

$$M_2 E N_2 = \begin{bmatrix} I_r & 0 \\ 0 & Q \end{bmatrix}, M_2 A N_2 = \begin{bmatrix} A_r & 0 \\ 0 & I_{n-r} \end{bmatrix}, M_2 B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C N_2 = [C_1 \quad C_2]$$

then the system of (3-1) and (3-2) is equivalent to:

$$\dot{x}_1 = A_r x_1 + B_1 u \quad (3-6)$$

$$Q \dot{x}_2 = x_2 + B_2 u \quad (3-7)$$

$$y = C_1x_1 + C_2x_2 \quad (3-8)$$

In this form, subsystems (3-6) and (3-7) are called slow and fast subsystem respectively. x_1 and x_2 are the slow and fast sub-states, respectively.

Although any descriptor system can be transformed to **Equivalent Form 2**, here a transformation procedure is presented which applies only to regular descriptor systems. The procedure is based on the approach proposed in (Bouali, Yagoubi and Chevrel, 2008), which is also going to be used to calculate a standard realization of an LPV descriptor system described in Chapter 6. As any regular descriptor system is equivalent to the **Equivalent Form 1**, without loss of generality, the system (3-3)-(3-5) can be considered by selecting:

$$M_2 = \begin{bmatrix} I_r & -A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}, N_2 = \begin{bmatrix} I_r & 0 \\ -A_{22}^{-1}A_{21} & I_{n-r} \end{bmatrix}.$$

It can be verified that:

$$M_2 \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N_2 = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, M_2 \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} N_2 = \begin{bmatrix} A_r & 0 \\ 0 & I_{n-r} \end{bmatrix}.$$

One important application of this equivalent form is the implementation of a descriptor observer system using standard state space system notation.

3.1.2 Preliminary of Kronecker product

The Kronecker product (Graham, 1981) is used in this study and denoted by \otimes . If A is an $m \times n$ matrix and B is a $p \times q$ matrix, then the Kronecker product $A \otimes B$ is the $mp \times nq$ block matrix given by:

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

where a_{ij} is the i^{th} row the j^{th} column in A .

The properties considered in the study include:

P1) Linearity: $A \otimes (aB) = a(A \otimes B)$

P2) Distributive with respect to addition:

$$A \otimes (B + C) = A \otimes B + A \otimes C$$

$$(A + B) \otimes C = A \otimes C + B \otimes C$$

P3) Associative: $A \otimes (B \otimes C) = (A \otimes B) \otimes C$

P4) Mixed product rule: $(AB) \otimes (CD) = (A \otimes C)(B \otimes D)$

P5) For any matrix H , there is a permutation matrix U such that (Graham, 1981; Kuo and Lee, 2004):

$$U \left(H \otimes \begin{bmatrix} A & B \\ C & D \end{bmatrix} \right) U^T = \begin{bmatrix} H \otimes A & H \otimes B \\ H \otimes C & H \otimes D \end{bmatrix}$$

From the property P5), it can be easily obtained that:

$$H \otimes \begin{bmatrix} A & B \\ C & D \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} H \otimes A & H \otimes B \\ H \otimes C & H \otimes D \end{bmatrix} < 0.$$

3.2 Baseline controller design

The analysis and synthesis of descriptor system have both attracted a lot of attention in the literature (Luenberger, 1978; Verghese, Levy and Kailath, 1981; Dai, 1989; Campbell, Nichols and Terrell, 1991; Masubuchi, Kamitane, Ohara and Suda, 1997; Uezato and Ikeda, 1999; Hou, 2004; Kuo and Lee, 2004; Marx and Ragot, 2006).

In the following the appropriate system analysis and baseline state feedback controller design is discussed. The system robustness and pole-placement constraints are considered within an LMI framework. A new LMI pole-placement description is given in Section 3.2.3. A general architecture of state feedback control systems is given in Figure 3.1 to the following system:

$$E\dot{x}(t) = Ax(t) + Bu(t) + Rd(t)$$

$$z(t) = C_{zx}x(t)$$

where C_{zx} is a predefined matrix to construct the interested performance variable $z = C_{zx}x$. C_{zx} is named H_∞ performance matrix in the study. The usual robust analysis (or synthesis) objective is to find a H_∞ performance index γ (or find a controller or observer for a given γ) satisfying $\|G\| < \gamma$ (Francis, 1987; Zhou, Doyle and Glover, 1996; Masubuchi, Kamitane, Ohara and Suda, 1997).

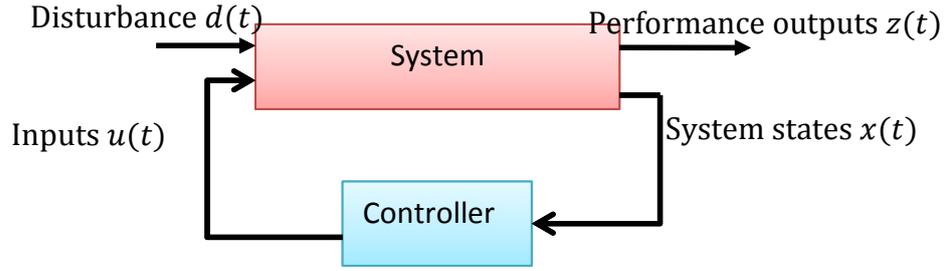


Figure 3.1 A general architecture of feedback control systems

3.2.1 Analysis using LMI tools

The LMI approach to the analysis and synthesis of standard systems has received considerable attention since the work of (Boyd, Ghaoui, Feron and Balakrishnan, 1994; Scherer and Weiland, 2005). Many efforts have also been made to develop LMI approaches for the analysis and synthesis of descriptor systems with semi-definite LMIs or strict LMIs, which are both presented below. The relationship between a semi-definite LMI and a strict LMI is also discussed. Consider Lemma 3.1 provided in the pioneering work of (Masubuchi, Kamitane, Ohara and Suda, 1997):

Lemma 3.1: A pair (E, A) is admissible if and only if there exist $P \in \mathbb{R}^{n \times n}$ such that:

$$EP = P^T E^T \geq 0 \quad (3-9)$$

$$AP + P^T A^T < 0 \quad (3-10)$$

Or equivalently,

$$E^T P^T = PE \geq 0 \quad (3-11)$$

$$A^T P^T + PA < 0 \quad (3-12)$$

Besides admissibility, robustness is another important property of dynamic systems. H_∞ optimization is widely used in the control community to measure and design the robustness of a dynamic system (Francis, 1987; Zhou, Doyle and Glover, 1996; Masubuchi, Kamitane, Ohara and Suda, 1997). When $sE - A$ is regular, the following transfer function from exogenous disturbance $d(t)$ to performance variable $z(t)$ can be obtained as:

$$G = C_{zx}(sE - A)^{-1}R \quad (3-13)$$

For standard systems, the well-known approach is to use the Bounded Real Lemma (Boyd, Ghaoui, Feron and Balakrishnan, 1994). Lemma 3.2 gives the Bounded Real Lemma for descriptor systems (Masubuchi, Kamitane, Ohara and Suda, 1997).

Lemma 3.2: The pair (E, A) is admissible and $\|G\| < \gamma$ if and only if there exists a $P \in \mathbb{R}^{n \times n}$ such that:

$$EP = P^T E^T \geq 0 \quad (3-14)$$

$$\begin{bmatrix} AP + P^T A^T & R & P^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (3-15)$$

Or equivalently:

$$E^T P^T = PE \geq 0 \quad (3-16)$$

$$\begin{bmatrix} A^T P^T + PA & PR & C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (3-17)$$

It can be seen that the LMIs (3-14) and (3-16) are not strict inequalities, whereas the LMIs (3-15) and (3-17) are strict inequalities. Considering descriptor systems, there are two approaches in the literature for handling mixed (strict and non-strict) inequalities. One approach considers the equivalent form given in Section 3.1.1 in (Masubuchi, Kamitane, Ohara and Suda, 1997). Some alternative strict LMI conditions are proposed in (Uezato and Ikeda, 1999) and these are outlined as follows using Lemma 3.3.

Lemma 3.3: (Uezato and Ikeda, 1999): The pair (E, A) is admissible and $\|G\| < \gamma$ if and only if there exists a matrix $P > 0, P \in \mathbb{R}^{n \times n}$ and a matrix $S \in \mathbb{R}^{(n-r) \times (n-r)}$, such that:

$$\begin{bmatrix} A(PE^T + USV^T) + (PE^T + USV^T)^T A^T & R & (PE^T + USV^T)^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (3-18)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively.

Inspired by the proof of Lemma 3.4 from (Uezato and Ikeda, 1999) and the Lemma 3 in (Gao and Ding, 2007b), the Lemma 3.4 shows the relationship between Lemma 3.3 and Lemma 3.2.

Lemma 3.4: All $P \in \mathbb{R}^{n \times n}$ satisfying $EP = P^T E^T \geq 0$ can be parameterized as

$$P = WE^T + USV^T \quad (3-19)$$

where $W \in \mathbb{R}^{n \times n}$ and $S \in \mathbb{R}^{(n-r) \times (n-r)}$ are parameter matrices; U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively.

Proof: Sufficiency: Using (3-19), it is derived that:

$$P^T E^T = EP = EWE^T \geq 0 \quad (3-20)$$

Necessity: As $EU = 0$ and $E^T V = 0$, based on singular value decomposition there exists a unitary matrix M and a non-singular matrix N such that:

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, U = N \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_1, V = M^T \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_2$$

where W_1 and W_2 are non-singular. Then we have:

$$E = M^{-1} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N^{-1}, \quad MEPM^T = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N^{-1} PM^T \geq 0$$

$N^{-1} PM^T$ can be partitioned as:

$$N^{-1} PM^T = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}$$

which implies that $P_{11} > 0$, P_{21} and P_{22} can be arbitrary. Furthermore:

$$P = N \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} M$$

It should be noted that:

$$\begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & \vdots \\ P_{21} & \vdots \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} P_{22} \begin{bmatrix} 0 & I_{n-r} \end{bmatrix}$$

where the \vdots can take on any form. Furthermore, it follows that:

$$\begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & \vdots \\ P_{21} & \vdots \end{bmatrix} N^T E^T M^T + N^{-1} U W_1^{-1} P_{22} W_2^{-T} V^T M^T$$

Hence:

$$P = N \begin{bmatrix} P_{11} & \vdots \\ P_{21} & \vdots \end{bmatrix} N^T E^T + U W_1^{-1} P_{22} W_2^{-T} V^T$$

Set $W = N \begin{bmatrix} P_{11} & \vdots \\ P_{21} & \vdots \end{bmatrix} N^T$, and $S = W_1^{-1} P_{22} W_2^{-T}$. Without loss of generality, set $W > 0$.

This leads to:

$$P = W E^T + U S V^T$$

This completes the proof. ■

With the above parameterization, it can be seen that the LMI description given in Lemma 3.2 can be transformed to the strict LMI conditions given in (Uezato and Ikeda, 1999) and presented in Lemma 3.3.

In a similar way to the proof in Lemma 3.4, the following parameterization Lemma can be obtained:

Lemma 3.5: All $P \in \mathbb{R}^{n \times n}$ satisfying $PE = E^T P^T \geq 0$ can be parameterized as:

$$P = E^T S + U W V^T \tag{3-21}$$

where $S \in \mathbb{R}^{n \times n}$ and $W \in \mathbb{R}^{(n-r) \times (n-r)}$ are parameter matrices; U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively.

3.2.2 State feedback control

Based on the analysis techniques presented in 3.2.1, this Subsection studies the state feedback controller design of LTI descriptor systems. The controllability of descriptor system is different from the controllability of the standard system. The following presents some useful tutorial background from (Dai, 1989) concerning the controllability conditions that apply to descriptor systems.

Definition 3.2: According to (Dai, 1989) the system of (3-1) is said to be controllable if for any $t > 0$, $x(0) \in \mathbb{R}^n$, and $w \in \mathbb{R}^n$, there exist a control u with compatible dimensions, such that $x(t) \in w$.

It should be clear that the definition of controllability of a descriptor system is a natural generalization of the controllability of a standard system.

Lemma 3.6: The descriptor system of (3-1) is said to be controllable if the following Assumptions hold simultaneously (Dai, 1989):

$$A3.1) \text{rank}([sE - A \quad B]) = n \quad s \in \mathbb{C}, s \text{ is finite}$$

$$A3.2) \text{rank}([E \quad B]) = n$$

Remark 3.1: The condition given in A3.1) is not easy to verify specifically for large descriptor systems. The following rank conditions are proposed by (Dai, 1989) as a way of testing the controllability of system (3-1), instead of using A3.1):

$$\text{rank} \left(\begin{array}{cccccccccccc} [-A & 0 & 0 & \dots & 0 & 0 & B & 0 & 0 & \dots & 0 & 0] \\ [E & A & 0 & \dots & 0 & 0 & 0 & B & 0 & \dots & 0 & 0] \\ [0 & E & A & \dots & 0 & 0 & 0 & 0 & B & \dots & 0 & 0] \\ \dots & \dots] \\ [0 & 0 & 0 & \dots & -A & 0 & 0 & 0 & 0 & \dots & B & 0] \\ [0 & 0 & 0 & \dots & E & -A & 0 & 0 & 0 & \dots & 0 & B]_{rn \times (n+m)r} \end{array} \right) = rn$$

where $r = \text{rank}(E)$.

Consider a state feedback controller $u = Kx$, then the closed-loop system is

$$E\dot{x} = (A + BK)x + Rd \quad (3-22)$$

With a simply application of Lemma 3.1, the following Lemma is obtained.

Lemma 3.7: The closed-loop pair $(E, A + BK)$ is admissible if and only if there exist P and Y with compatible dimensions such that:

$$EP = P^T E^T \geq 0 \quad (3-23)$$

$$AP + P^T A^T + BY + Y^T B^T < 0 \quad (3-24)$$

Then the state feedback control can be calculated from the solution for P and Y in (3-23) and (3-24) using $K = YP^{-1}$.

When disturbances or system uncertainty are considered, a performance variable can be defined as $z = C_z x$. When the closed-loop pair $(E, A - BK)$ is regular, the following transfer function from d to z can be obtained:

$$G_{cl} = C_z(sE - A - BK)^{-1}R \quad (3-25)$$

Based on the Bounded Real Lemma of descriptor systems, it is straightforward to obtain Lemma 3.8 as follows:

Lemma 3.8: The closed-loop pair $(E, A + BK)$ is admissible and $\|G_{cl}\| < \gamma$ if and only if there exists P with compatible dimensions such that:

$$EP = P^T E^T \geq 0 \quad (3-26)$$

$$\begin{bmatrix} AP + P^T A^T + BY + Y^T B^T & R & P^T C_{zx}^T \\ * & -\gamma & 0 \\ * & * & -\gamma \end{bmatrix} < 0 \quad (3-27)$$

Then the state feedback control can be calculated as $K = YP^{-1}$. In Lemma 3.7 and Lemma 3.8, it is possible that the solution P may singular, in which case we can modify P to $P + \varepsilon I_n$ for some arbitrary ε to obtain a non-singular P without breaking the inequalities.

With the parameterisation proposed in Lemma 3.4, we can obtain the following strict LMI conditions given in Lemma 3.9 & Lemma 3.10 for Lemma 3.7 & Lemma 3.8, respectively.

Lemma 3.9: The closed-loop pair $(E, A + BK)$ is admissible if and only if there exist matrices $P > 0, S, L$ and H with compatible dimensions such that:

$$A(PE^T + USV^T) + B(LE^T + HV^T) + * < 0 \quad (3-28)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively. Then the state feedback control can be calculated as:

$$K = (LE^T + HV^T)(PE^T + USV^T)^{-1}$$

Lemma 3.10: The pair $(E, A + BK)$ is admissible and $\|G_{cl}\| < \gamma$ if and only if there exists matrices $P > 0, S, L$ and H with compatible dimensions such that:

$$\begin{bmatrix} \Delta & R & (PE^T + USV^T)^T C_{zx}^T \\ * & -\gamma & 0 \\ * & * & -\gamma \end{bmatrix} < 0 \quad (3-29)$$

with:

$$\Delta = A(PE^T + USV^T) + B(LE^T + HV^T) + \star$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively. Then the controller gain can be calculated as:

$$K = (LE^T + HV^T)(PE^T + USV^T)^{-1}$$

In contrast to classical system analysis, the solution of the above LMIs may lead to a singular S , which in turn leads to a singular $PE^T + USV^T$. The way to avoid this singularity is to replace S by $S + \varepsilon I_{n-r}$ to obtain a non-singular $(PE^T + USV^T)$ whilst still satisfying (3-29).

3.2.3 Pole placement with state feedback

It is well known that the poles of a system play an important role in the time response of a linear system. However, it is hard to determine the exact desired pole positions of the closed-loop system. One reasonable choice is to consider regional pole-placement in controller and observer designs. The pole-placement problem can be stated in terms of suitable LMI regions (Chilali and Gahinet, 1996).

The pole-placement problem for descriptor systems is not a mature topic in contrast to the case of standard systems. Some ideas of how the pole-placement problem can be handled for descriptor systems are presented below. First, two definitions of LMI regions by (Chilali and Gahinet, 1996; Hsiung and Lee, 1997) are given, followed by two new results for unified LMI region pole-placement.

Definition 3.3 (LMI region): A subset \mathcal{D} of the complex plane is called an LMI region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{q \times q}$, and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{q \times q}$ such that:

$$\mathcal{D} := \{z \in \mathcal{C} : f_{\mathcal{D}}(z) < 0\} \quad (3-30)$$

where the characteristic function is given as:

$$f_{\mathcal{D}}(z) := \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl}z + \beta_{lk}\bar{z}]_{1 \leq k, l \leq q} \quad (3-31)$$

Definition 3.4: Let \mathcal{D} be an LMI region in the open left-half plane. Then:

- 1) \mathcal{D} is said to belong to the class \mathcal{D}_{Γ} if its characteristic function $f_{\mathcal{D}}(z)$ has $\alpha = 0$

2) \mathfrak{D} is said to belong to the class \mathfrak{D}_Λ if $\alpha \neq 0$

Lemma 3.11: (Hsiung and Lee, 1997) The pair (E, A) is admissible and \mathfrak{D}_Γ -stable if and only if there exists a matrix P with compatible dimensions such that:

$$E^T P = P^T E \geq 0 \quad (3-32)$$

$$\beta \otimes (A^T P) + \beta^T \otimes (P^T A) < 0 \quad (3-33)$$

where \otimes denotes Kronecker product.

Lemma 3.12:(Hsiung and Lee, 1997) The pair (E, A) is admissible and \mathfrak{D}_Λ -stable if and only if there exists a matrix P and a matrix Q with compatible dimensions such that:

$$E^T P = P^T E \geq 0$$

$$E^T Q = Q^T E \geq 0$$

$$A^T Q + Q^T A < 0$$

$$\alpha \otimes (E^T P) + \beta \otimes (A^T P) + \beta^T \otimes (P^T A) + I_q \otimes (E^T Q) \leq 0$$

The above LMIs are consistent with the admissible LMI design strategy of state feedback controller by H_∞ optimization. However, it should be noticed that the above inequalities are not strict LMIs because of the existence of the semi-definite inequalities (\leq). Hence, the pole-placement problem cannot easily be solved using this approach. Another drawback comes from the different LMI forms for different pole-placement regions in the complex-plane.

Unified design approaches to design LMI region pole-placement for descriptor system are discussed, for instance, sufficient descriptions are proposed to design state feedback pole-placement controller in (Marx, Koenig and Georges, 2003b; Marx and Ragot, 2006),, while sufficient and necessary conditions to analyse the pole positions are discussed in (Kuo and Lee, 2004).

Theorem 3.1 & 3.2 are presented as unified region pole-placement conditions here:

Theorem 3.1: The pair (E, A) is admissible and \mathfrak{D} stable if and only if there exists a matrix $P > 0, P \in \mathbb{R}^{n \times n}$ and a matrix $S \in \mathbb{R}^{(n-r) \times (n-r)}$ such that:

$$\alpha \otimes (EPE^T) + \beta \otimes (APE^T) + I_q \otimes (AUSV^T) + \star < 0 \quad (3-34)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively. I_q denotes the identity matrix.

Proof: (**Sufficiency**): Let v be any left generalized eigenvector associated with a finite eigenvalue λ . Pre- and post-multiplying (3-34) by $I_n \otimes v$ and $I_n \otimes v^*$ leads to:

$$\alpha \otimes (v E P E^T v^*) + \beta \otimes (v A P E^T v^*) + I_q \otimes (v A U S V^T v^*) + \star < 0 \quad (3-35)$$

From Section 3.1, since $\lambda v E = v A$, $\bar{\lambda} E^T v^* = A^T v^*$, and $EU = 0$, (3-35) then leads to:

$$\alpha \otimes (v E P E^T v^*) + \beta \otimes (\lambda v E P E^T v^*) + \beta^T \otimes (\bar{\lambda} v E P E^T v^*) < 0 \quad (3-36)$$

$P > 0$ and v is associated with a finite eigenvalue thus $v E \neq 0$, implying that $v E P E^T v^* > 0$ and λ lies within the region \mathfrak{D} .

It is of value now to prove that (3-34) implies that the system is impulsive free. From Section 3.1 based on the singular value decomposition there exist unitary matrices M and N such that:

$$\begin{aligned} M E N &= \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix}, & M A N &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ U &= N \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_1, & V &= M^T \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_2 \end{aligned}$$

where $\bar{E} = \bar{E}^T$ and W_1 and W_2 are non-singular. Assuming that (3-34) holds, it follows:

$$\begin{aligned} &\alpha \otimes \left(M^{-1} \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix} N^{-1} P N \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix} M \right) + \beta \otimes \left(M^{-1} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} N^{-1} P N \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix} M \right) \\ &+ I_q \otimes \left(M^{-1} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} N^{-1} N \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_1 S W_2^T \begin{bmatrix} 0 & I_{n-r} \end{bmatrix} M \right) + \star < 0 \quad (3-37) \end{aligned}$$

With $N^{-1} P N = (N^{-1} P N)^T = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}$, (3-37) can be re-organised as:

$$\begin{aligned} &M^T \left[\alpha \otimes \begin{bmatrix} \bar{E} P_1 \bar{E} & 0 \\ 0 & 0 \end{bmatrix} + \beta \otimes \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} P_1 \bar{E} & 0 \\ P_2^T \bar{E} & 0 \end{bmatrix} \right) + \right. \\ &\left. I_q \otimes \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & W_1 S W_2^T \end{bmatrix} \right) + \star \right] M < 0 \quad (3-38) \end{aligned}$$

Since M is non-singular, we have:

$$\alpha \otimes \begin{bmatrix} \bar{E}P_1\bar{E} & 0 \\ 0 & 0 \end{bmatrix} + \beta \otimes \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} P_1\bar{E} & 0 \\ P_2^T\bar{E} & 0 \end{bmatrix} \right) + I_q \otimes \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & W_1SW_2^T \end{bmatrix} \right) + \star < 0 \quad (3-39)$$

Suppose that the pair (E, A) is not impulsive free, which also implies that the equivalent system pair $\left(\begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)$ has impulsive modes. From Definition 3.1 there exists generalized eigenvectors v_l^2 and v_l^1 satisfying $\begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix}^T v_r^{1*} = 0$ and $\begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix}^T v_r^{2*} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T v_r^{1*}$. Pre- and post-multiplying the LMI (3-39) by $I_n \otimes v_r^1$ and $I_n \otimes v_r^{1*}$ it then follows that:

$$I_q \otimes \left(v_r^1 \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_1SW_2^T \begin{bmatrix} 0 & I_{n-r} \end{bmatrix} v_r^{1*} \right) + \star < 0$$

which can be further written as:

$$I_q \otimes \left(v_r^2 \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} W_1SW_2^T \begin{bmatrix} 0 & I_{n-r} \end{bmatrix} v_r^{1*} \right) + \star < 0$$

As $\begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then it is clear that the above inequality is contradictory. Hence, we prove that the system is impulsive free.

(Necessary): If the pair (E, A) is impulsive free, then there exist matrices M and N such that $MEN = \text{diag}(I, 0)$ and $MAN = \text{diag}(J, I)$. Moreover, if the pair (E, A) is \mathfrak{D} stable, then the eigenvalues of J lie in \mathfrak{D} . Without loss of generality and for sake of simplicity, we will consider the pair $(\text{diag}(I, 0)\text{diag}(J, I))$ in the proof of necessity. As J lie in \mathfrak{D} , there exists a matrix $P_1 > 0$ satisfying:

$$\alpha \otimes P_1 + \beta \otimes (JP_1) + \beta^T \otimes (P_1J^T) < 0$$

There is always an $S \in \mathbb{R}^{(n-r) \times (n-r)}$ such that:

$$\begin{bmatrix} \alpha \otimes P_1 + \beta \otimes (JP_1) + \beta^T \otimes (P_1J^T) & 0 \\ 0 & I_q \otimes (S + S^T) \end{bmatrix} < 0$$

which can be rewritten as:

$$\begin{bmatrix} \alpha \otimes P_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \beta \otimes (JP_1) & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \beta^T \otimes (P_1J^T) & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & I_q \otimes (S + S^T) \end{bmatrix} < 0$$

Hence, based on the property **P5**) of Kronecker product there is a U such that:

$$U \left(\alpha \otimes \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} \right) U^T = \begin{bmatrix} \alpha \otimes P_1 & 0 \\ 0 & 0 \end{bmatrix}, U \left(\beta \otimes \begin{bmatrix} JP_1 & 0 \\ 0 & 0 \end{bmatrix} \right) U^T = \begin{bmatrix} \beta \otimes (JP_1) & 0 \\ 0 & 0 \end{bmatrix}$$

$$U \left(I_q \otimes \begin{bmatrix} 0 & 0 \\ 0 & S_1 + S_1^T \end{bmatrix} \right) U^T = \begin{bmatrix} 0 & 0 \\ 0 & I_q \otimes (S + S^T) \end{bmatrix}$$

It also follows that:

$$\alpha \otimes \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \otimes \begin{bmatrix} JP_1 & 0 \\ 0 & 0 \end{bmatrix} + \beta^T \otimes \begin{bmatrix} (P_1 J^T) & 0 \\ 0 & 0 \end{bmatrix} + I_q \otimes \begin{bmatrix} 0 & 0 \\ 0 & S + S^T \end{bmatrix} < 0 \quad (3-40)$$

It can be observed that the following properties hold:

$$\begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = EPE^T$$

$$\begin{bmatrix} JP_1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} J & 0 \\ 0 & I_{n-r} \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = APE^T$$

$$\begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} = \begin{bmatrix} J & 0 \\ 0 & I_{n-r} \end{bmatrix} \begin{bmatrix} 0 \\ I_{n-r} \end{bmatrix} S \begin{bmatrix} 0 & I_{n-r} \end{bmatrix} = AUSV^T$$

From the above it can be seen that (3-40) is equivalent to (3-34), completing the proof. ■

Following the Theorem 3.1, in order to design a state feedback controller with pole-placement constraints for system (3-1) a new Theorem 3.2 is proposed:

Theorem 3.2 (pole-placement with state feedback): The pair $(E, A + BK)$ is admissible and \mathfrak{D} stable if and only if there exist matrices $P > 0$, S , L and H with compatible dimensions such that:

$$\alpha \otimes (EPE^T) + \beta \otimes (APE^T + BLE^T) + I_q \otimes (AUSV^T + BHV^T) + \star < 0 \quad (3-41)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively. Then the controller gain K is given by:

$$K = HS^{-1}(U^T U)^{-1} U^T [I_n - PE_R (E_R^T PE_R)^{-1} E_R^T] + LE_R (E_R^T PE_R)^{-1} E_R^T \quad (3-42)$$

where $E = E_L \bar{E} E_R^T$, $\bar{E} \in \mathbb{R}^{r \times r}$ is invertible and the matrices $E_L \in \mathbb{R}^{n \times r}$ and $E_R \in \mathbb{R}^{n \times r}$ are of full column rank. Alternatively, the controller gain K can be given by:

$$K = (LE^T + HV^T)(PE^T + USV^T)^{-1}$$

Proof: **(Necessity)** if the closed-loop system is \mathfrak{D} stable, based on Theorem 3.1, there exist a positive matrix P and a matrix S such that:

$$\alpha \otimes (EPE^T) + \beta \otimes ((A + BK)PE^T) + I_q \otimes ((A + BK)USV^T) + \star < 0 \quad (3-43)$$

By setting $L = KP$ and $H = KUS$, (3-41) is obtained.

(Sufficiency): It is required to show that $(PE + USV^T)$ is invertible. \bar{E} is invertible and the matrices E_L and E_R are full column rank. As a consequence, $EU = 0$ and $E^T V = 0$ implying that $E_R^T U = 0$ and $E_L^T V = 0$, respectively. Thus we have:

$$\begin{bmatrix} (E_L^T E_L)^{-1} E_L^T \\ (V^T V)^{-1} V^T \end{bmatrix} [E_L \quad V] = I_n$$

The two matrices of the left side are in $\mathbb{R}^{n \times n}$, and non-singular, with each the inverse of the other. Hence, it also follows that:

$$[E_L \quad V] \begin{bmatrix} (E_L^T E_L)^{-1} E_L^T \\ (V^T V)^{-1} V^T \end{bmatrix} = E_L (E_L^T E_L)^{-1} E_L^T + V (V^T V)^{-1} V^T = I_n$$

which implies that:

$$\begin{aligned} & \{E_L (E_L^T E_L)^{-1} \bar{E}^{-T} (E_R^T P E_R)^{-1} E_R^T \\ & + V (V^T V)^{-1} S^{-1} (U^T U)^{-1} U^T [I_n - P E_R (E_R^T P E_R)^{-1} E_R^T]\} (PE + USV^T) \\ & = I_n \end{aligned}$$

In other words:

$$\begin{aligned} (PE + USV^T)^{-1} &= E_L (E_L^T E_L)^{-1} \bar{E}^{-T} (E_R^T P E_R)^{-1} E_R^T \\ &+ V (V^T V)^{-1} S^{-1} (U^T U)^{-1} U^T [I_n - P E_R (E_R^T P E_R)^{-1} E_R^T] \end{aligned}$$

From (3-42), it follows that $KPE^T = LE^T$ and $KUS = H$, and substituting these relations into (3-41), (3-43) is obtained, completing the proof. \blacksquare

Remark 3.2: The advantage of the above LMI description is that only strict LMIs must be solved and a unique formulation embraces all LMI regions. Hence numerical tractability is improved.

3.3 ESO design of LTI systems

In the AFTC context, estimates of states are also required along with the estimates of fault signals. The fault estimation can then be used to compensate for the effects of the fault within the control system.

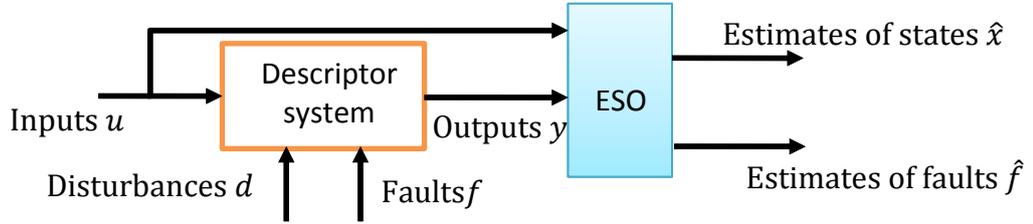


Figure 3.2 A general architecture of ESO

A general architecture of ESO is given in Figure 3.2. This Section considers design approaches for the ESO problem capable of estimating system states and fault signals simultaneously.

In comparison with Section 3.2, here a descriptor system is considered with sensor and actuator faults to achieve simultaneous fault and state estimation in the following form:

$$E\dot{x} = Ax + Bu + F_a f_a + R_1 d \quad (3-44)$$

$$y(t) = Cx + F_s f_s + R_2 d \quad (3-45)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^h$ and $u \in \mathbb{R}^m$ and $f_a \in \mathbb{R}^q$, $f_s \in \mathbb{R}^{p-q}$ are system states, outputs, known inputs and actuator and sensor fault signals, respectively. $d \in \mathbb{R}^d$ represents system uncertainty or exogenous disturbances. $A, B, C, F_a, F_s, E, R_1, R_2$ are matrices with compatible dimensions. Furthermore, $rank(E) = r \leq n$.

The system of (3-44) and (3-45) is equivalent to:

$$E\dot{x} = Ax + Bu + F_f f + R_1 d \quad (3-46)$$

$$y = Cx + D_f f + R_2 d \quad (3-47)$$

where

$$f = \begin{bmatrix} f_a \\ f_s \end{bmatrix} \in \mathbb{R}^p, D_f = [0 \quad F_s], F_f = [F_a \quad 0] \quad (3-48)$$

The above form of (3-46) and (3-47) is used in the following discussion. Following a dual approach to the state feedback design of Section 3.2, as a preliminary step to the state observer design conditions, the following definitions must be made.

Definition 3.6: System (3-1) is said to be observable if the initial condition $x(0)$ can be uniquely determined from $u(t)$ and $y(t)$, with $0 \leq t < \infty$ (Dai, 1989).

Observability here means that all the system states can be constructed using the system inputs and outputs.

Lemma 3.13: A descriptor system (E, A, C) is observable if the following Assumptions hold simultaneously (Dai, 1989):

$$\text{A3.3) } \text{rank} \left(\begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = n, s \in \mathbb{C}, s \text{ is finite}$$

$$\text{A3.4) } \text{rank} \left(\begin{bmatrix} E \\ C \end{bmatrix} \right) = n.$$

It can be observed that Definition 3.5 and the Assumptions A3.3) and A3.4) also apply to the standard system for which $E = I_n$.

3.3.1 The augmentation strategies

The basic idea of ESO is to augment the faults or disturbance signals as extra system states to obtain an augmented state vector. The simultaneous estimation can be achieved via the design of an observer corresponding to the augmented system. In the following, two augmentation methods are proposed with different assumptions.

3.3.1.1 Augmentation 1

Consider the descriptor system given in (3-46) and (3-47), with f slowly-varying (which means it is reasonable to set $\dot{f} = 0$). With the new state vector $x_a = \begin{bmatrix} x \\ f \end{bmatrix}$, the original system can be augmented as:

$$E_a \dot{x}_a = A_a x_a + B_a u + R_a d \quad (3-49)$$

$$y = C_a x_a + D d \quad (3-50)$$

where:

$$E_a = \begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix}, A_a = \begin{bmatrix} A & F_f \\ 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R_1 \\ 0 \end{bmatrix}, D = R_2, C_a = [C \quad D_f] \quad (3-51)$$

Lemma 3.14: The augmented system (E_a, A_a, C_a) is observable if A3.3), A3.4) and the following A3.5) are satisfied.

$$\text{A3.5) } \text{rank} \begin{bmatrix} A & F_f \\ C & D_f \end{bmatrix} = n + p$$

Proof:

$$\text{rank} \left(\begin{bmatrix} sE_a - A_a \\ C_a \end{bmatrix} \right) =$$

$$\text{rank} \left(\begin{bmatrix} sE - A & F_f \\ 0 & sI_p \\ C & D_f \end{bmatrix} \right) = \begin{cases} \text{rank} \begin{bmatrix} A & F_f \\ C & D_f \end{bmatrix} = n + p & s = 0 \\ \text{rank} \left(\begin{bmatrix} sE - A \\ C \end{bmatrix} \right) + p = n + p & s \neq 0 \end{cases} = n + p$$

$$\text{Also, } \text{rank} \left(\begin{bmatrix} E_a \\ C_a \end{bmatrix} \right) = \text{rank} \left(\begin{bmatrix} E & 0 \\ 0 & I_p \\ C & D_f \end{bmatrix} \right) = \text{rank} \left(\begin{bmatrix} E \\ C \end{bmatrix} \right) + p = n + p. \quad \text{Hence, the}$$

augmented system is observable. ■

Remark 3.3: A restriction of this augmentation is the assumption that the faults are varying slowly. A multi-augmentation can be carried out to extend the capability of this estimation system to account for fault signals in polynomial form (Gao and Ho, 2004; Gao, Ding and Ma, 2007; Orjuela, Marx, Ragot and Maquin, 2009). That is if the q th derivative of f , i.e. $f^{(q)}$ is bounded, the fault signals can be augmented as

$$\left. \begin{aligned} \dot{f} &= \delta_1 \\ \dot{\delta}_1 &= \delta_2 \\ &\dots \\ \dot{\delta}_{q-1} &= f^{(q)} \end{aligned} \right\} \quad (3-52)$$

The original system can be reorganized in matrix form as:

$$\begin{bmatrix} E & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{f} \\ \dot{\delta}_1 \\ \dots \\ \dot{\delta}_{q-1} \end{bmatrix} = \begin{bmatrix} A & F_f & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x \\ f \\ \delta_1 \\ \dots \\ \delta_{q-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} u + \begin{bmatrix} R_1 \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 & 0 & 0 & \dots & 1 \end{bmatrix}^T f^{(q)} \quad (3-53)$$

$$y = [C \ D_f \ 0 \ \dots \ 0][x \ f \ \delta_1 \ \dots \ \delta_{q-1}]^T + Dd \quad (3-54)$$

Then it is in a similar form as in (3-49) and (3-50).

3.3.1.2 Augmentation 2

Without loss of generality, F_s can be partitioned as $F_s = [F_{sj} \ F_{si}]$ and F_{si} is full column rank with $rank(F_{si}) = s_i$. Within this method, it is assumed that assumptions A3.3)-A3.5) are satisfied with the following assumptions:

$$A3.6): rank \left(\begin{bmatrix} E & 0 \\ C & F_{si} \end{bmatrix} \right) = n + s_i$$

$$A3.7): rank \left(\begin{bmatrix} sE - A & 0 \\ C & F_{si} \end{bmatrix} \right) = n + s_i, s \in \mathbb{C}, s \text{ is finite} .$$

Then a system can be obtained which has the same state vector and system description as given in (3-49) and (3-50) with the system matrices:

$$E_a = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I_q & 0 & 0 \\ 0 & 0 & I_{p-q-s_i} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_a = \begin{bmatrix} A & F_a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, D = R_2$$

$$C_a = [C \ 0 \ F_{sj} \ F_{si}]$$

It can be verified that $rank \left(\begin{bmatrix} E_a \\ C_a \end{bmatrix} \right) = n + p, rank \left(\begin{bmatrix} sE_a - A_a \\ C_a \end{bmatrix} \right) = n + p$, which means the alternative augmentation is observable.

Here, an academic example demonstrates that for certain system problems Assumptions A3.6) and A3.7) are not satisfied.

Example: Consider a linear system with:

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_s = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

It can be easily verified that that: $rank \left(\begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = 3, s \in \mathbb{C}, s \text{ is finite} ,$
 $rank \left(\begin{bmatrix} E \\ C \end{bmatrix} \right) = 3$, and $rank \begin{bmatrix} A & F_f \\ C & D_f \end{bmatrix} = 4$, which means the augmentation is available to

achieve simultaneous state and fault estimation. However, $\text{rank} \left(\begin{bmatrix} E & 0 \\ C & F_{si} \end{bmatrix} \right) = 3$ (instead of the required value of 4) which means that Assumption A3.6) is not satisfied, and Augmentation 2 is not an effective method to design a simultaneous state and fault observer. From this simple example, it can be concluded that the main drawback of the Augmentation 2 approach is that the design Assumptions A3.6) & A3.7) are too restrictive. However, the advantage of this approach is that the polynomial form of the sensor fault constraint is removed.

The two augmented systems have the same structure and both are observable if the corresponding Assumptions are satisfied for each method. Hence, it is appropriate here to proceed by considering the Augmentation 1 method of (3-49) and (3-50). The following approaches can be easily adapted to handle the system form of Augmentation 2 if additional Assumptions A3.6) & A3.7) are satisfied.

3.3.2 The ESO design

An augmented full order descriptor proportional observer of (3-49) and (3-50) can be designed in the following form:

$$E_a \dot{\hat{x}}_a = A_a \hat{x}_a + B_a u + L_p (\hat{y} - y) \quad (3-55)$$

$$\hat{y} = C_a \hat{x}_a \quad (3-56)$$

where \hat{y} and \hat{x}_a are estimates of the system outputs and augmented system states, respectively and the observer matrix $L_p = [L_x \quad L_f]^T$ is to be determined.

Define $e_x = x - \hat{x}$ and $e_f = f - \hat{f}$, $e_{xf} = \begin{bmatrix} e_x \\ e_f \end{bmatrix}$. With the assumption $\dot{f} = 0$, it follows that the fault estimation error is given by:

$$\dot{e}_f = -\dot{\hat{f}} = -L_f (y - \hat{y}) \quad (3-57)$$

The combined state and fault error system is as:

$$E_a \dot{e}_{xf} = A_o e_{xf} + (R_a + L_p D) d \quad (3-58)$$

where:

$$A_o = A_a + L_p C_a, E_a = \begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix}, A_a = \begin{bmatrix} A & F_f \\ 0 & 0 \end{bmatrix}, C_a = [C \quad D_f], L_p = \begin{bmatrix} L_x \\ L_f \end{bmatrix} \quad (3-59)$$

Then the observer gain L_p can be designed and the system states and estimation errors can be obtained simultaneously. Theorem 3.3 is proposed corresponding to the design of an admissible augmented observer with strict LMIs using Lemma 3.1 and the parameterization Lemma 3.5.

Theorem 3.3: The observer $(E_a, A_a + L_p C_a)$ is admissible if and only if there exist matrices $S > 0, S \in \mathbb{R}^{(n+p) \times (n+p)}$, $W \in \mathbb{R}^{(n-r) \times (n-r)}$, $J \in \mathbb{R}^{(n+p) \times h}$, and $H \in \mathbb{R}^{(n-r) \times h}$ such that:

$$E_a^T S A_a + U_a W V_a^T A_a + E_a^T J C_a + U_a H C_a + \star < 0$$

where U_a and V_a are full column rank and contain the basis vectors for $\text{Ker}(E_a)$ and $\text{Ker}(E_a^T)$ respectively. Then the observer gain can be calculated as:

$$L_p = (E_a^T S + U_a W V_a^T)^{-1} (E_a^T J + U_a H)$$

When the observer robustness is required in the design, the performance variable can be defined as $z = C_{ze} e_{xf}$, then the transfer function between the exogenous disturbance d and the estimation error e_{xf} is $G_e(s) = C_{ze} (sE_a - A_a - L_p C_a)^{-1} (R_a + L_p D)$. Theorem 3.4 can be used to improve the observer robustness to disturbances in the H_∞ framework.

Theorem 3.4: The observer $(E_a, A_a + L_p C_a)$ is admissible and $\|G_e\| < \gamma$ if and only if there exist matrices $S > 0, S \in \mathbb{R}^{(n+p) \times (n+p)}$, $W \in \mathbb{R}^{(n-r) \times (n-r)}$, $J \in \mathbb{R}^{(n+p) \times h}$, and $H \in \mathbb{R}^{(n-r) \times h}$ such that:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & C_{ze}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (3-60)$$

with:

$$\Delta_{11} = (E_a^T S + U_a W V_a^T) A_a + (E_a^T J + U_a H) C_a + \star$$

$$\Delta_{12} = (E_a^T S + U_a W V_a^T) R_a + (E_a^T J + U_a H) D$$

where U_a and V_a are full column rank and contain the basis vectors for $Ker(E_a)$ and $Ker(E_a^T)$, respectively. Then the observer gain can be calculated as:

$$L_p = (E_a^T S + U_a W V_a^T)^{-1} (E_a^T J + U_a H)$$

Theorem 3.5 can now be used to design an observer with LMI region pole-placement constraints.

Theorem 3.5: The pair $(E_a, A_a + L_p C_a)$ is admissible and \mathfrak{D} stable if and only if there exist matrices $S > 0, S \in \mathbb{R}^{(n+p) \times (n+p)}$, $W \in \mathbb{R}^{(n-r) \times (n-r)}$, $J \in \mathbb{R}^{(n+p) \times h}$, and $H \in \mathbb{R}^{(n-r) \times h}$ such that:

$$\alpha \otimes (E_a^T S E_a) + \beta \otimes (E_a^T S A_a + E_a^T J C_a) + I_q \otimes (U_a W V_a^T A_a + U_a H C_a) + \star < 0 \quad (3-61)$$

where U_a and V_a are full column rank and contain the basis vectors for $Ker(E_a)$ and $Ker(E_a^T)$, respectively. Then the observer gain L_p is given by

$$L_p = (E_a^T S + U_a W V_a^T)^{-1} (E_a^T J + U_a H).$$

It is clear that the observer design corresponding to robustness and time response requirements can be achieved in one design by combining Theorems 3.4 & Theorem 3.5 with common design variables S, H, J and W . The required robustness objective of the observer to both model uncertainty and disturbance is included in the design via LMI H_∞ optimization. The overall design is effectively an LMI procedure for achieving a multi-objective observer design.

Remark 3.4: If $\dot{f} \neq 0$ and \dot{f} is bounded, then $\dot{e}_f = \dot{f} - \hat{\dot{f}} = \dot{f} - L_f C e_x$. Hence, the observer can be rewritten as:

$$\begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} = \begin{bmatrix} A + L_x C & F_f \\ L_f C & D_f \end{bmatrix} \begin{bmatrix} e_x \\ e_f \end{bmatrix} + \begin{bmatrix} R_1 & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} d \\ \dot{f} \end{bmatrix} \quad (3-62)$$

The error system (3-62) is similar to the $\dot{f} = 0$ case of (3-49) except that here $R_a = \begin{bmatrix} R_1 & 0 \\ 0 & I_p \end{bmatrix}$. This means that Theorem 3.3-3.5 can be used a to design an ESO to estimate faults with bounded rates.

3.3.3 Implementation of a descriptor observer

It is well known in the control community that the implementation of a descriptor observer is not as easy as a standard observer. This subsection deals with the equivalence between the descriptor and standard observer problems, providing a design approach that can be realized in practice.

Following the equivalent form described in Section 3.1.1, the state observer of (3-55) and (3-56) can be re-organized in the form:

$$E_a \dot{\hat{x}}_a = (A_a + L_p C_a) \hat{x}_a + B_a u - L_p y \quad (3-63)$$

$$z = I_{n+p} \hat{x}_a \quad (3-64)$$

Noting that the descriptor ESO system is admissible, the **Equivalent Form 1** of this system is:

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + B_1 u - L_{p1} y \quad (3-65)$$

$$0 = A_{21} x_1 + A_{22} x_2 + B_2 u - L_{p2} y \quad (3-66)$$

$$z = C_{z1} x_1 + C_{z2} x_2 \quad (3-67)$$

where:

$$M_1 E_a N_1 = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, M_1 (A_a + L_p C_a) N_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, M_1 B_a = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, N_1 = [C_{z1} \quad C_{z2}]$$

Furthermore, we can obtain an **Equivalent Form 2** as:

$$\dot{\xi}_1 = A_r \xi_1 + B_r u - L_r y \quad (3-68)$$

$$0 = \xi_2 + A_{22}^{-1} B_2 u - A_{22}^{-1} L_2 y \quad (3-69)$$

$$z = C_{r1} \xi_1 + C_{r2} \xi_2 \quad (3-70)$$

where:

$$A_r = A_{11} - A_{12} A_{22}^{-1} A_{21}, B_r = B_1 - A_{12} A_{22}^{-1} B_2$$

$$L_r = L_1 - A_{12} A_{22}^{-1} L_2, C_{r1} = C_{z1} - C_{z2} A_{22}^{-1} A_{21}, C_{r2} = C_{z2}$$

which then leads to:

$$\dot{\xi}_1 = A_r \xi_1 + B_r u - L_r y \quad (3-71)$$

$$z = C_{r1} \xi_1 - C_{r2} A_{22}^{-1} B_2 u + C_{r2} A_{22}^{-1} L_2 y \quad (3-72)$$

It can be seen that the system of (3-71) and (3-72) correspond to a standard system and can be realized in practice. This approach thus facilitates a design approach that leads to a realizable system.

3.4 Illustrative example

In this Section, a numerical example is studied to illustrate the design and implementation of the ESO proposed in this Chapter to estimate system state and faults simultaneously. The numerical example is modified from (Gao and Ho, 2004; Gao and Ding, 2007b) with a sensor faults, an actuator fault, and including sensor noise and exogenous disturbance. However, in this example there are no parameter perturbations and hence the only robustness issue for the design of the observers is to have good insensitivity to the disturbance and noise. The robustness is one criterion that is achieved using H_∞ performance optimization.

3.4.1 System model

The considered descriptor system is described by:

$$E\dot{x} = Ax + Bu + F_a f_a + R_1 d$$

$$y = Cx + F_s f_s + R_2 d$$

where

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 1.5 \\ -2 & -1 \end{bmatrix}, F_a = \begin{bmatrix} 1 \\ 1.5 \\ -1 \end{bmatrix}, R_1 = \begin{bmatrix} 0.5 & 0 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}, F_s = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}, R_2 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.2 \end{bmatrix}$$

3.4.2 ESO Design

It can be easily obtained that all the Assumptions of Section 3.3 are satisfied and each of the two Augmentation Methods of Section 3.3.1 can be used to design the joint state and fault observers.

First, define $f = \begin{bmatrix} f_a \\ f_s \end{bmatrix}$, $D_f = [0 \quad F_s]$, $F_f = [F_a \quad 0]$, and the system is augmented twice to include both the fault estimate and the estimate of its time derivative:

$$E_a \dot{x}_a = A_a x_a + B_a u + R_a d \quad (3-73)$$

$$y = C_a x_a + D d \quad (3-74)$$

where:

$$x_a = \begin{bmatrix} x \\ f \\ \delta_1 \end{bmatrix}, E_a = \begin{bmatrix} E & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_a = \begin{bmatrix} A & F_f & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R_1 \\ 0 \\ 0 \end{bmatrix}, D = R_2$$

$$C_a = [C \quad D_f \quad 0]$$

The MATLAB LMI Toolbox is used to solve (3-60) and (3-61) with poles (eigenvalues) with the finite eigenvalues constrained to satisfy $-10 < Re(\lambda) < -2.5$. The H_∞ performance matrix (described in Section 3.2.1) is set to be $C_{ze} = I_7$ to reduce the disturbance influence on the state estimates and faults. The H_∞ performance index is set to be $\gamma = 1$. A feasible solution is found and the observer gain is computed as:

$$\begin{bmatrix} L_x \\ L_f \end{bmatrix}^T = L_p^T = \begin{bmatrix} 158.7534 & -30.5701 \\ 144.7162 & -27.4197 \\ 11.5831 & -1.9379 \\ 345.2376 & -66.4696 \\ -296.2988 & 52.0529 \\ 366.6322 & -70.7318 \\ -411.7776 & 67.8342 \end{bmatrix}$$

The achieved observer has one infinite eigenvalue and six finite eigenvalues according to $eig(E_a, A_a + L_p C_a)$:

$$\begin{bmatrix} -4.7879 + 3.7348i & -2.8594 + 1.9822i & -4.1563 \\ -4.7879 - 3.7348i & -2.8594 - 1.9822i & -3.6882 \end{bmatrix}$$

Another solution is obtained with Augmentation Method 2 and the observer gain obtained is:

$$\begin{bmatrix} L_x \\ L_f \end{bmatrix}^T = L_p^T = \begin{bmatrix} 1.5165 & -3.1425 \\ -0.0728 & -2.4669 \\ -2.9648 & 0.5273 \\ 8.3252 & -7.8687 \\ 16.7361 & -9.7871 \\ 20.7565 & -4.4033 \end{bmatrix}$$

This observer has two infinite eigenvalues and four stable finite eigenvalues which are:

$$\begin{bmatrix} -5.7999 + 2.4958i & -3.4734 + 0.5332i \\ -5.7999 - 2.4958i & -3.4734 - 0.5332i \end{bmatrix}$$

3.4.3 Simulation results

The ESOs are implemented with equivalent realization as discussed in Section 3.3.3 within MATLAB/SIMULINK. A sensor fault and an actuator fault are considered simultaneously. It can be seen that the fault signals shown in Figures 3.4 & 3.5 can cover the different time dependency faults as given in Section 1.2.

The simulation results of the Augmentation Method 1 are given in the Figures 3.3-3.5 and the simulation results for the Augmentation Method 2 are given in the Figures 3.6-3.8.

From the simulation results, it can be seen that both augmentation approaches work well to design the required descriptor observers for the descriptor system. Both the estimates of states and faults converge to the real values, even though the fault signal is not constant, as shown in Figures 3.3 & 3.8. It can also be seen that there is a big peak in the sensor fault estimate signal when there is a step in the actuator fault signal, which is not expected in the real system.

Comparing the results of the Augmentation Method 1 and Augmentation Method 2 the sensor fault estimate (in Figure 3.8) of the Method 2 approach is more disturbed than the result for Method 1 given in Figure 3.5, while the response in Figure 3.8 is faster since the polynomial constraint is removed. It also can be seen that the step of sensor fault signal effects actuator fault estimate less in Augmentation 2 than in Augmentation 1, when comparing Figures 3.4 and 3.7.

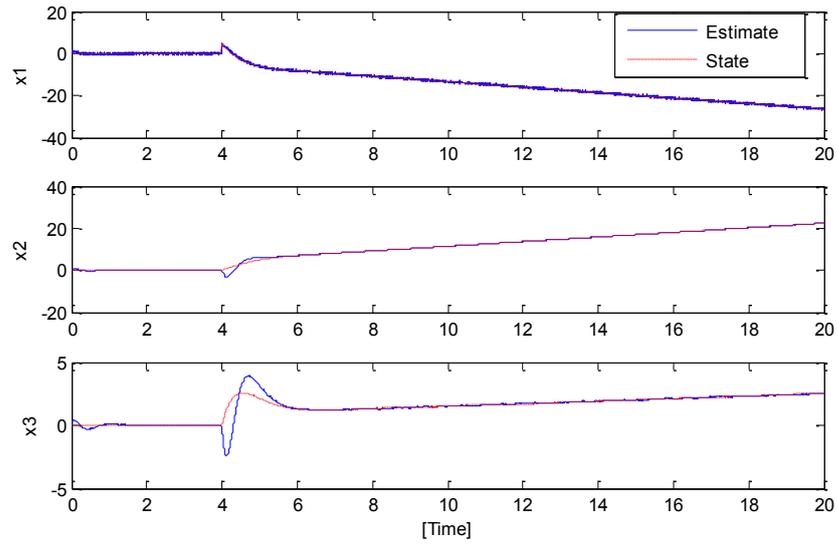


Figure 3.3 Method 1 system states and state estimates

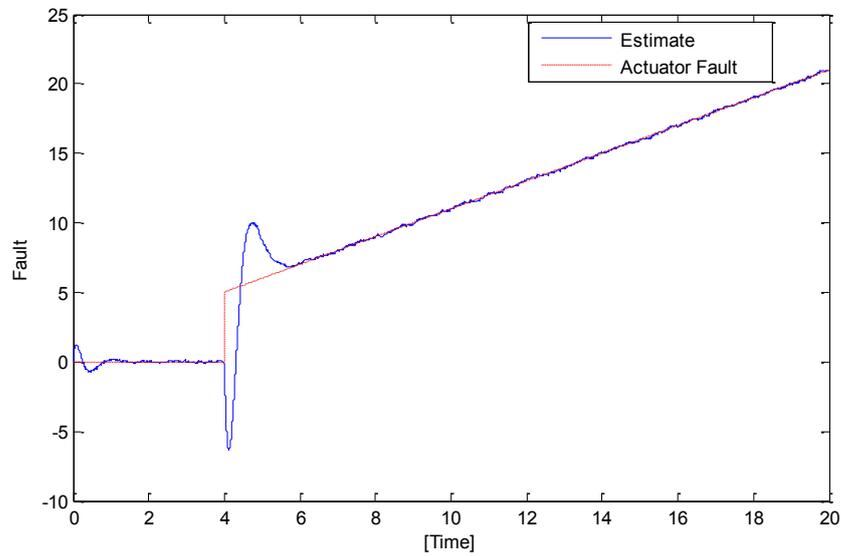


Figure 3.4 Method 1 Actuator fault and its estimate

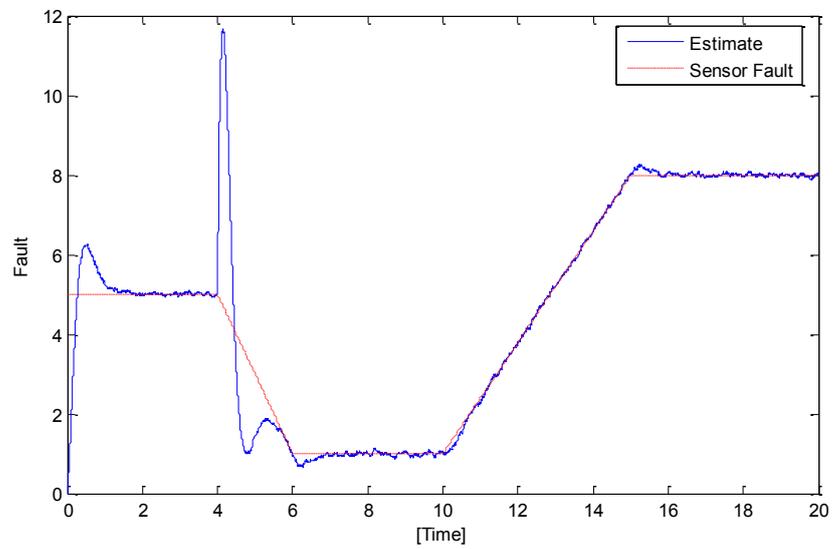


Figure 3.5 Method 1 Sensor fault and its estimate

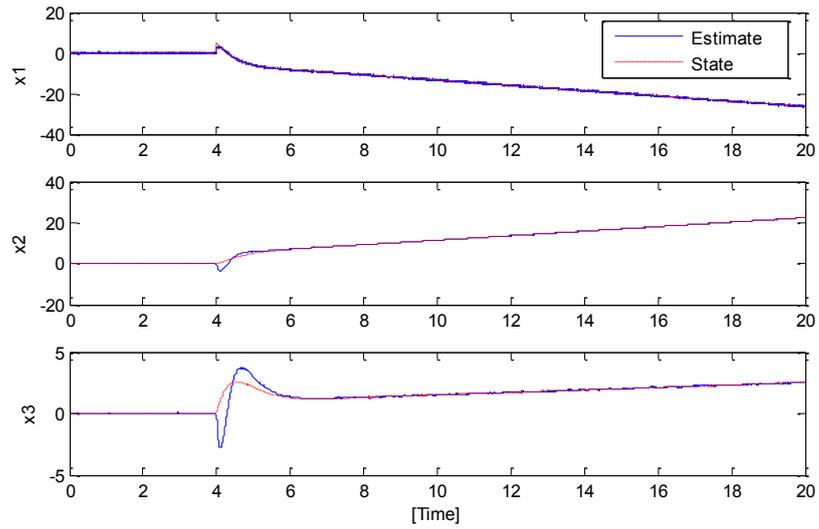


Figure 3.6 Method 2 system states and state estimates

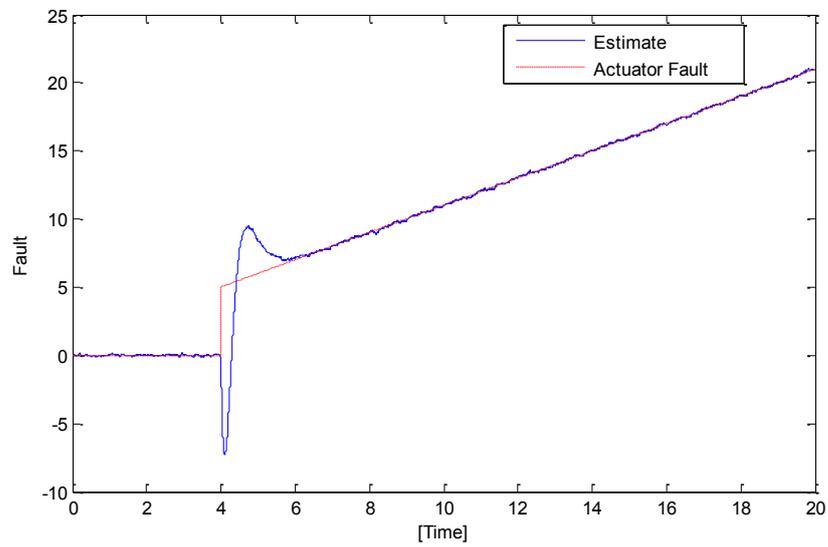


Figure 3.7 Actuator fault and its estimation with Augmentation 2

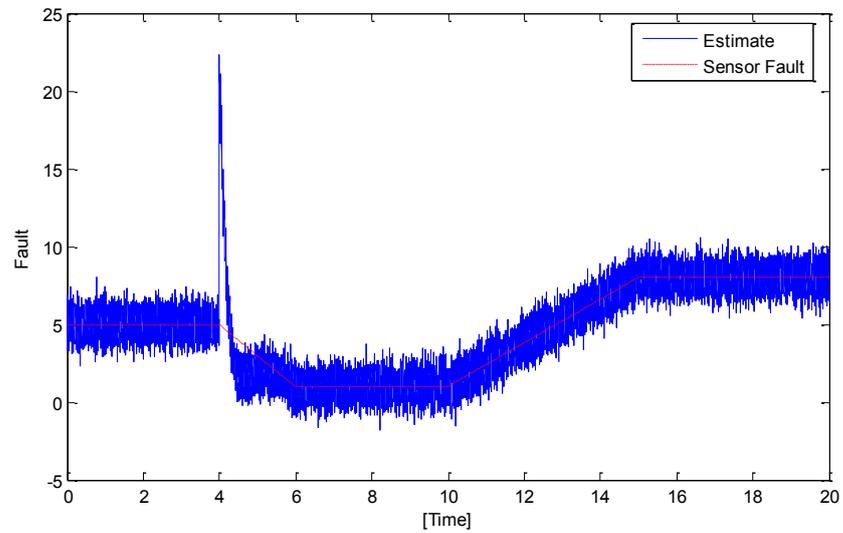


Figure 3.8 Sensor fault and its estimate with Augmentation 2

3.5 Discussion and conclusion

Sections 3.1 and 3.2 provide a description of the basic concepts and ideas of descriptor systems within the context of state feedback controller design via pole-placement in required LMI regions and LMI formulated H_∞ robust control design. New LMI design descriptions are proposed for descriptor system analysis and state feedback design. Various combinations of given LMIs can be used to analyse and synthesise the feedback system, according to different requirements

Section 3.3 shows that to estimate system states and fault signals, a descriptor ESO design problem is studied involving two augmentation strategies. In the study, both LMI region pole placement and robustness to disturbance are considered in the resulting descriptor observer design to obtain good performance. The realization of this descriptor observer is discussed in terms of the specific properties of a descriptor system. A multi-objective design approach can be achieved via suitable combinations of LMIs.

A numerical example is studied in Section 3.4 to show the design procedures and usefulness of proposed strategies.

In Chapter 4, a proportional-derivative Extended Stated Observer (PD-ESO) is discussed. It is shown that enhanced performance can be achieved using this approach with an additional design parameter arising from the use of the PD action.

Chapter 4: Proportional derivative (PD) ESO

It is well known that different combinations of Proportional-Integral-Derivative (PID) elements for controllers and observers can satisfy different requirements. Chapter 3 discusses the problem of ESO for descriptor systems using proportional gain. This Chapter considers a PD approach to the ESO problem for descriptor systems, the so-called PD-ESO, which includes the derivative of the estimation error as an additional parameter. It is shown that this approach provides an opportunity for generating state and fault estimates with enhanced properties.

In Section 4.1 a short review of recent research results is given, followed by a novel observer structure for PD-ESO design. The design approach is inspired by the duality that exists between control and observer systems for which the matrix E of the descriptor system is square, i.e. $E \in \mathbb{R}^{n \times n}$. Following the general procedure of ESO design given in Chapter 3 the design descriptions are given using an LMI framework which can be combined to meet various design objectives including regional pole-placement and H_∞ properties.

4.1 Enhanced estimation using PD observer

Recently, PD observer design has been investigated in (Gao, 2005; Wu and Duan, 2007; Ren and Zhang, 2010) for estimation of the descriptor system states. Given a descriptor system as:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + Rd(t) \\ y(t) &= Cx(t) \end{aligned}$$

The PD observer design is to obtain an observer in the following form:

$$(E - L_d C)\dot{\hat{x}} = (A + L_p C)\hat{x} - L_p y + Bu$$

A systematic method to design PD observer gains to estimate system states (where fault estimation is not considered) is given by (Gao, 2005). The method is achieved by two steps: first to find the derivative gain matrix L_d such that the matrix $E - L_d C$ is non-singular, and then to seek a stabilizing gain matrix L_p such that $(E - L_d C, A + L_p C)$ is

a stable matrix pair. In addition, based on the design of PD observers, a new parameterization of all observers for descriptor systems is developed in (Gao, 2005) based on dual co-prime factorisation. In (Wu and Duan, 2007), a type of PD observer for descriptor systems is described using an eigenstructure assignment approach. The LMI approach is discussed in (Ren and Zhang, 2010) to design a stable PD observer, for which robustness issues are not discussed and pole-placement strategies are not considered.

In the fault estimation field, the output derivatives are involved in some design approaches (Gao and Ding, 2007b; Zhang, Jiang and Shi, 2009) to achieve fast state estimation or fast fault estimation. A brief comparative review of the two approaches is given below.

Consider a descriptor system given in the form of (3-46) and (3-47) rewritten here in the form:

$$E\dot{x} = Ax + Bu + F_f f + R_1 d \quad (4-1)$$

$$y = Cx + D_f f + R_2 d \quad (4-2)$$

It is assumed that the fault signal satisfies $\dot{f} = 0$. The original system can be multi-augmented as proposed in Section 3.3 to deal with polynomial faults. In this Chapter, only the slowly varying fault case is considered.

To get fast fault estimation, a fast actuator fault observer is considered in (Zhang, Jiang and Shi, 2009) for standard (i.e. non descriptor) systems, with $D_f = 0, E = I$, as:

$$\dot{\hat{x}} = A\hat{x} + Bu + F_f \hat{f} + L_x(y - \hat{y}) \quad (4-3)$$

$$\hat{y} = C\hat{x} \quad (4-4)$$

$$\dot{\hat{f}} = L_f(y - \hat{y}) + kL_f(\dot{y} - \dot{\hat{y}}) \quad (4-5)$$

The two matrices L_x, L_f and the scalar k are to be determined. The extra term $kL_f(\dot{y} - \dot{\hat{y}})$ is introduced to achieve fast fault estimation since the original state observer parts are retained as in the standard system ESO case. LMI conditions are given to calculate the proportional observer gains.

A proportional multi-integral derivative observer (PMIDO) is proposed by (Gao and Ding, 2007b) based on the proportional multi-integral observer (PMIO). For comparison reasons, the PID-observer is given for actuator faults and sensor faults case as:

$$\dot{\hat{x}} = A\hat{x} + Bu + F_f\hat{f} + L_x(y - \hat{y}) + L_{xd}(\dot{y} - \dot{\hat{y}}) \quad (4-6)$$

$$\hat{y} = C\hat{x} + D_f\hat{f} \quad (4-7)$$

$$\dot{\hat{f}} = L_f(y - \hat{y}) \quad (4-8)$$

The three matrices L_x , L_f , and L_{xd} are to be determined. The proposed PMIDO can achieve a faster state estimation by introducing the term $L_{xd}(\dot{y} - \dot{\hat{y}})$. The so-called PMIDO is designed via orthogonal decomposition or solving a Lyapunov equation.

In the above two observer structures, either the derivative action is included in the state estimation part of the observer problem or the derivative action is included in the fault estimation part using extra parameter. These studies do not include derivative action gains in both state and fault estimation problems.

4.2 The novel PD-ESO structure

In our study, combination of a PD observer with an ESO, the PD-ESO, is proposed to achieve better performance by introducing an extra design parameter via the derivative action gain. Another advantage of PD observer design for descriptor systems is that standard observers can be achieved if the original systems are observable, as discussed in (Gao, 2005; Wu and Duan, 2007; Ren and Zhang, 2010).

Before further discussion, it is necessary to introduce an augmentation strategy which is different from the ones used in Chapter 3. The aim of the extra augmentation is to achieve a sensor noise-free formulation of the descriptor system.

Consider the following descriptor system:

$$E\dot{x} = Ax + Bu + F_f f + R_1 d_u \quad (4-9)$$

$$y = Cx + D_f f + R_2 d_s \quad (4-10)$$

To derive a descriptor system that is sensor noise-free, (4-9) and (4-10) can be augmented with a new state $\omega = R_2 d_s$ as:

$$\begin{bmatrix} E & 0 \\ 0 & I_h \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & -\rho I_h \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix} + \begin{bmatrix} F_f \\ 0 \end{bmatrix} f + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} R_1 & 0 \\ 0 & I_h \end{bmatrix} \begin{bmatrix} d_u \\ \rho R_2 d_s + R_2 \dot{d}_s \end{bmatrix}$$

$$y = [C \quad I_h] \begin{bmatrix} x \\ \omega \end{bmatrix} + D_f f$$

It can be observed that the sensor noise signals are transferred to exogenous input system disturbances, and a “noise-free” system is obtained. As a further augmentation, the sensor noise-free system can then be transformed to:

$$E_a \dot{x}_a = A_a x_a + B_a u + R_a d_D \quad (4-11)$$

$$y = C_a x_a \quad (4-12)$$

where:

$$E_a = \begin{bmatrix} E & 0 & 0 \\ 0 & I_h & 0 \\ 0 & 0 & I_p \end{bmatrix}, A_a = \begin{bmatrix} A & 0 & F_f \\ 0 & -\rho I_h & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \quad (4-13)$$

$$R_a = \begin{bmatrix} R_1 & 0 \\ 0 & I_h \\ 0 & 0 \end{bmatrix}, C_a = [C \quad I_h \quad D_f], d_D = \begin{bmatrix} d_u \\ \rho R_2 d_s + R_2 \dot{d}_s \end{bmatrix}, x_a = \begin{bmatrix} x \\ \omega \\ f \end{bmatrix}$$

Now the sensor faults and sensor noise signals are both unknown inputs affecting the system states, however their effects on the system can be handled by different augmentation schemes facilitating a way of providing an estimation of the fault in which the effect of the sensor noise is reduced. The faults and sensor noise signals often have different properties and usually different requirements apply to these different signals. For instance, a normal requirement for noise is to reduce its effect on both state and fault estimates whilst to estimate the fault signals accurately.

One should note that the augmentation techniques in PD-ESO design are different from those of the ESO design approaches as there is an additional augmentation to hide the sensor noise in the PD-ESO design. It is not necessary to provide extra augmentation of the original system as seen before in the ESO design case. Furthermore, it is a conjecture here that the “noise-hiding” will simplify the PD-ESO design dramatically.

Regarding the augmented system of (4-11) and (4-12), a PD-ESO is proposed as:

$$E_a \dot{\hat{x}}_a = A_a \hat{x}_a + B_a u + L_p (\hat{y} - y) + L_d (\dot{\hat{y}} - \dot{y}) \quad (4-14)$$

$$\hat{y} = C_a \hat{x}_a \quad (4-15)$$

where L_p, L_d in (4-16) are to be determined:

$$L_p = \begin{bmatrix} L_x \\ L_\omega \\ L_f \end{bmatrix}, L_d = \begin{bmatrix} L_{xd} \\ L_{\omega d} \\ L_{fd} \end{bmatrix} \quad (4-16)$$

To make a brief comparison with the augmentation structure given in (4-3)-(4-5) and (4-6)-(4-8), the PD-ESO of (4-14) and (4-15) can be rewritten in the following form:

$$\begin{bmatrix} E & 0 \\ 0 & I_h \end{bmatrix} \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\omega}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & -\rho I_h \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\omega} \end{bmatrix} + \begin{bmatrix} F_f \\ 0 \end{bmatrix} \hat{f} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} L_x \\ L_\omega \end{bmatrix} (\hat{y} - y) + \begin{bmatrix} L_{xd} \\ L_{\omega d} \end{bmatrix} (\dot{\hat{y}} - \dot{y}) \quad (4-17)$$

$$\hat{y} = [C \quad I_h] \begin{bmatrix} \hat{x} \\ \hat{\omega} \end{bmatrix} + D_f \hat{f} \quad (4-18)$$

$$\dot{\hat{f}} = L_f (y - \hat{y}) + L_{fd} (\dot{y} - \dot{\hat{y}}) \quad (4-19)$$

By comparing the proposed estimator with the existing observer presented in Section 4.1, some interesting results are found. For example, if $L_{xd} = 0$ and by setting $L_{fd} = kL_d$, then the proposed estimator reduces to the fast adaptive fault estimator proposed in (Zhang, Jiang and Shi, 2009); if $L_{fd} = 0$, the proposed observer reduces to observer strategy proposed in (Gao, Breikin and Wang, 2007; Gao and Ding, 2007b).

To define the estimation error system:

$$e_{x\omega} = \begin{bmatrix} x \\ \omega \end{bmatrix} - \begin{bmatrix} \hat{x} \\ \hat{\omega} \end{bmatrix}, e_f = f - \hat{f}, e_{x\omega f} = \begin{bmatrix} e_{x\omega} \\ e_f \end{bmatrix}$$

It follows that:

$$\dot{e}_f = -\dot{\hat{f}} = -L_f (y - \hat{y}) - L_d (\dot{y} - \dot{\hat{y}}) \quad (4-20)$$

The estimation error system of the proposed PD-ESO is then obtained as:

$$E_o \dot{e}_{x\omega f} = A_o e_{x\omega f} + R_a d_D \quad (4-21)$$

where:

$$A_o = A_a + L_p C_a, E_o = E_a - L_d C_a$$

Lemma 4.1 and Lemma 4.2 are introduced to prove the observability of the augmented system of (4-11) and (4-12).

Lemma 4.1: (Ren and Zhang, 2010) Given matrices E, U, V, L with appropriate dimensions, we have:

$$\max_L \{rank(E + ULV)\} = \min \left\{ rank \begin{bmatrix} E & U \\ & V \end{bmatrix}, rank \begin{bmatrix} E \\ V \end{bmatrix} \right\}$$

Lemma 4.2: If system (1) is observable, then there is an L_d such that $E_a - L_d C_a \in \mathbb{R}^{(n+h+p) \times (n+p+h)}$ is non-singular.

Proof: Suppose that $rank(E_a - L_d C_a) < n + h + p$ for all L_d . By Lemma 4.1, we have that:

$$\begin{aligned} p + h + n > \max_L \{rank(E_a - L_d C_a)\} &= \min \left\{ rank \begin{bmatrix} E_a & -I_{n+h+p} \end{bmatrix}, rank \begin{bmatrix} E_a \\ C_a \end{bmatrix} \right\} \\ &= rank \begin{bmatrix} E_a \\ C_a \end{bmatrix} = n + h + p \end{aligned}$$

which is a contradiction. So there is a L_d such that $E_a - L_d C_a$ is non-singular. ■

Therefore, by **Lemma 4.2**, there is an L_d such that E_o is invertible if the system is observable and the invertibility assumption has been used widely for instance in (Lin, Wang and Lee, 2005).

4.3 Design approaches

The duality of a square descriptor system is presented briefly, followed by a systematic design approach via designing a dual observer instead of the original one. Consider a linear square descriptor system as:

$$E\dot{x} = Ax + Rd_x \tag{4-22}$$

$$y_x = Cx \tag{4-23}$$

The dual of system (4-22) and (4-23) is given as:

$$E^T \dot{z} = A^T z + C^T d_z \tag{4-24}$$

$$y_z = R^T z \tag{4-25}$$

It is already well known that the original system of (4-22)-(4-23) and its dual system of (4-24) and (4-25) share the same stability property. The goal here is to show a duality property in terms of H_∞ performance. Given the transfer function from the disturbances to outputs as:

$$G(s) = C(sE - A)^{-1}R$$

$$G_{dual}(s) = R^T(sE^T - A^T)^{-1}C^T$$

Then it follows that:

$$\|G(s)\|_\infty = \|G^T(s)\|_\infty = \|R^T(sE^T - A^T)^{-1}C^T\|_\infty = \|G_{dual}(s)\|_\infty$$

From the above relationship, it can be seen that the dual system of (4-24) and (4-25) shares the same robustness properties with the original system of (4-22) and (4-23). Based on this observation, the following focuses on the PD-ESO observer design with the dual system $(E^T \ A^T)$ instead of the original system $(E \ A)$, whilst in the robust design the dual system $(E^T \ A^T \ R^T \ C^T)$ is considered instead of the original system $(E \ A \ C \ R)$.

4.3.1 Admissible PD-ESO design with the dual system

This Subsection considers the dual of the error system given in (4-21) (without disturbance inputs) in the following form:

$$(E_a^T - C_a^T L_d^T) \dot{z} = (A_a^T + C_a^T L_p^T) z \quad (4-26)$$

With an augmentation, the dual system (4-26) can be transformed to:

$$\begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T + C_a^T L_p^T & -E_a^T + C_a^T L_d^T \end{bmatrix} \begin{bmatrix} z \\ z_a \end{bmatrix} \quad (4-27)$$

The introduction of $z_a = \dot{z}$ may introduce impulsive modes because the continuity of z does not imply the continuity of \dot{z} . Impulsive modes in the time response of a descriptor system may be highly detrimental to the system operation. However, as pointed out in (Marx and Ragot, 2006), the following statements are equivalent:

- 1) $rank(E_o^T) = n$
- 2) The system $\left(\begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & I_{n+p+h} \\ A_o^T & -E_o^T \end{bmatrix} \right)$ is impulsive-free.

Generally, $rank(E_o^T) = n$ is a very restrictive condition. However, in the PD-ESO design, the requirement can always be satisfied as previously proved by Lemma 4.2.

With the definition:

$$E_{ea} = \begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, A_{ea} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T & -E_a^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}, L_{ea} = [L_p^T \quad L_d^T] \quad (4-28)$$

(4-27) can be re-organized as:

$$E_{ea} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = (A_{ea} + B_{ea}L_{ea}) \begin{bmatrix} z \\ z_a \end{bmatrix} \quad (4-29)$$

The problem of stabilizing the original system with PD-ESO is transferred to a state feedback problem which can be solved with the theory presented in Section 3.3.

Here, given the above proposed PD-ESO, Theorem 4.1 is given to design an admissible PD-ESO for the disturbance-free case.

Theorem 4.1: An admissible PD-ESO in the form of (4-14)–(4-15) exists if there are matrices $P > 0, P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, $S \in \mathbb{R}^{(n+p+h) \times (n+p+h)}$, $L \in \mathbb{R}^{h \times (2n+2p+2h)}$, and $H \in \mathbb{R}^{h \times (n+h+p)}$ such that:

$$A_{ea}(PE_{ea}^T + U_{ea}SV_{ea}^T) + B_{ea}(LE_{ea}^T + HV_{ea}^T) + \star < 0 \quad (4-30)$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $Ker(E_{ea})$ and $Ker(E_{ea}^T)$, respectively. Then the observer gain is calculated as:

$$\begin{bmatrix} L_p^T & L_d^T \end{bmatrix} = (LE_{ea}^T + HV_{ea}^T)(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1}$$

Based on the conditions discussed in Chapter 3, an equivalent theorem of Theorem 4.1 is proposed for the purpose of comparison.

Theorem 4.2: The closed-loop pair $(E_{ea}, A_{ea} + B_{ea}L_{ea})$ is admissible if and only if there exist $P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, and $Y \in \mathbb{R}^{c \times (2n+2p+2h)}$ such that:

$$E_{ea}P = P^T E_{ea}^T \geq 0 \quad (4-31)$$

$$A_{ea}P + P^T A_{ea}^T + B_{ea}Y + Y^T B_{ea}^T < 0 \quad (4-32)$$

The observer gain is calculated using:

$$L_{ea} = YP^{-1}$$

Considering the specific structure of E_{ea} , (4-31) can be satisfied by setting:

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

It then follows that:

$$A_{ea}P = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T & -E_a^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} = \begin{bmatrix} P_2 & P_3 \\ A_a^T P_1 - E_a^T P_2 & -E_a^T P_3 \end{bmatrix}$$

Then setting:

$$Y = [Y_1 \quad Y_2] = [L_p^T \quad L_d^T] \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} = [L_p^T P_1 + L_d^T P_2 \quad L_d^T P_3]$$

When P_3 is invertible, the variable relationship can be defined as:

$$[L_p^T \quad L_d^T] = [Y_1 \quad Y_2] \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{-1} \quad (4-33)$$

with:

$$\begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ P_3^{-1} P_2 P_1^{-1} & P_3^{-1} \end{bmatrix}$$

Then (4-33) can be rewritten as:

$$L_p^T = [Y_1 \quad Y_2] \begin{bmatrix} P_1^{-1} \\ P_3^{-1} P_2 P_1^{-1} \end{bmatrix} \quad (4-34)$$

$$L_d^T = [Y_1 \quad Y_2] \begin{bmatrix} 0 \\ P_3^{-1} \end{bmatrix} \quad (4-35)$$

So **Theorem 4.2** can be rewritten in the following form:

Theorem 4.3: There is an admissible PD-ESO if there are matrices $P_1 > 0, P_1 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$, $P_2 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$, $P_3 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$, $Y_1 \in \mathbb{R}^{h \times (n+h+p)}$ and $Y_2 \in \mathbb{R}^{h \times (n+h+p)}$ satisfying the following condition:

$$\begin{bmatrix} P_2 + P_2^T & P_1 A_a - P_2^T E_a + P_3 + Y_1^T C_a \\ \star & -P_3^T E_a - E_a^T P_3 + Y_2 C_a^T + C_a^T Y_2 \end{bmatrix} < 0 \quad (4-36)$$

The PD-ESO gain can be calculated as:

$$L_p^T = [Y_1 \quad Y_2] \begin{bmatrix} P_1^{-1} \\ -P_3^{-1}P_2P_1^{-1} \end{bmatrix} \quad (4-37)$$

$$L_d^T = [Y_1 \quad Y_2] \begin{bmatrix} 0 \\ P_3^{-1} \end{bmatrix} \quad (4-38)$$

Theorem 4.3 is exactly the one proposed in (Ren and Zhang, 2010). In the case P_3 is singular, P_3 can be modified with $P_3 + \epsilon I$, where ϵ is a small number without breaking the LMI (4-36).

4.3.2 Design with pole-placement constraints

The closed-loop poles are often required to be placed within specified regions in the complex plane to fulfil certain transient response demands as discussed in Section 3.2.3. Unfortunately, most PD observer design approaches do not consider this property with the exception of the eigenstructure assignment approach (Wu and Duan, 2007).

This Subsection presents an LMI description to design PD-ESO with regional pole placement. By applying the duality principle of linear descriptor systems presented in Subsection 4.3.1 to the pole-placement problem proposed in Chapter 3, the poles of the PD-ESO can be assigned in desired LMI regions according to Theorem 4.4.

Theorem 4.4: The pair (E_o, A_o) is admissible and \mathfrak{D} stable if and only if there exist matrices $P > 0, P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, $S \in \mathbb{R}^{(n+p+h) \times (n+p+h)}$, $Y_{10} \in \mathbb{R}^{h \times (2n+2p+2h)}$ and $Y_2 \in \mathbb{R}^{h \times (n+h+p)}$ such that:

$$\alpha \otimes (E_{ea}^T P E_{ea}) + \beta \otimes (A_{ea} P E_{ea}^T + B_{ea} Y_{10} E_{ea}^T) + I_q \otimes (A_{ea} U_{ea} S V_{ea}^T + B_{ea} Y_2 V_{ea}^T) + \star < 0 \quad (4-39)$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $\text{Ker}(E_{ea})$ and $\text{Ker}(E_{ea}^T)$, respectively. Then the PD-ESO gain can be calculated as:

$$\begin{bmatrix} L_p^T & L_d^T \end{bmatrix} = (Y_{10} E_{ea}^T + Y_2 V_{ea}^T) (P E_{ea}^T + U_{ea} S V_{ea}^T)^{-1}.$$

Furthermore, set $S = P_3$, with the partitioning of P as:

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & \ddots \end{bmatrix}$$

It follows that:

$$PE_{ea} = \begin{bmatrix} P_1 & 0 \\ P_2 & 0 \end{bmatrix}, E_{ea}^T P E_{ea} = \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix}, A_{ea} P E_{ea}^T = \begin{bmatrix} P_2 & 0 \\ A_a^T P_1 - E_a^T P_2 & 0 \end{bmatrix}$$

$$A_{ea} U_{ea} S V_{ea}^T = \begin{bmatrix} 0 & I \\ A_a^T & -E_a^T \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & P_3 \end{bmatrix} = \begin{bmatrix} 0 & P_3 \\ 0 & -E_a^T P_3 \end{bmatrix}$$

$$B_{ea} Y_{10} E_{ea}^T = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix} [Y_1 \quad Y_{12}] \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ C_a^T Y_1 & 0 \end{bmatrix}$$

$$B_{ea} Y_2 V_{ea}^T = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix} Y_2 [0 \quad I] = \begin{bmatrix} 0 & 0 \\ 0 & C_a^T Y_2 \end{bmatrix}$$

Then (4-39) can be expanded with the property P.5) of the Kronecker product to:

$$\begin{bmatrix} \alpha \otimes P_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \beta \otimes (P_2) & I_q \otimes (P_3) \\ \beta \otimes (A_a^T P_1 - E_a^T P_2 + C_a^T Y_1) & I_q \otimes (-E_a^T P_3 + C_a^T Y_2) \end{bmatrix} + \star < 0 \quad (4-40)$$

Furthermore,

$$(Y_{10} E_{ea}^T + Y_2 V_{ea}^T) = [Y_1 \quad Y_2], P E_{ea}^T + U_{ea} S V_{ea}^T = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

It then follows that:

$$\begin{bmatrix} L_p^T & L_d^T \end{bmatrix} = [Y_1 \quad Y_2] \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{-1} \quad (4-41)$$

Hence, the following Theorem 4.5 is equivalent to Theorem 4.4.

Theorem 4.5: The pair (E_o, A_o) is admissible and \mathfrak{D} stable if and only if there exist matrices $P_1 > 0, P_1 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$, $Y_1 \in \mathbb{R}^{h \times (n+h+p)}$, $P_2 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$, $P_3 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}$ and $Y_2 \in \mathbb{R}^{h \times (n+h+p)}$ such that (4-40) is satisfied, and the gains can be calculated as (4-41).

4.3.3 Robust PD-ESO design

When considering real system applications, disturbance and modelling uncertainty are always present and hence attention should be paid to the robust design problem. This Subsection considers the dual of the error system given in (4-21) with disturbance inputs in the following form:

$$E_o \dot{e}_{x\omega f} = A_o e_{x\omega f} + R_a d_D \quad (4-42)$$

The robustness problem is considered in the H_∞ framework using LMIs with the performance variable as:

$$z_e = C_{zde} e_{x\omega f} \quad (4-43)$$

where C_{zde} is the H_∞ performance matrix and $G_{zde}(s) = C_{zde}(sE_o - A_o)^{-1}R_a$ is the transfer matrix relating the exogenous disturbance d as input to the performances variable to be minimized via H_∞ optimization. With an augmentation, the dual system (4-42) and (4-43) can be transformed to:

$$E_{ea} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = (A_{ea} + B_{ea}L_{ea}) \begin{bmatrix} z \\ z_a \end{bmatrix} + R_{ea}d_D \quad (4-44)$$

$$y_z = C_{ea} \begin{bmatrix} z \\ z_a \end{bmatrix} \quad (4-45)$$

where $E_{ea}, A_{ea}, B_{ea}, L_{ea}$ are defined as in (4-28), R_{ea} and C_{ea} are defined as in (4-46).

$$R_{ea} = \begin{bmatrix} 0 \\ C_{zde}^T \end{bmatrix}, C_{ea} = [R_a^T \quad 0] \quad (4-46)$$

Theorem 4.6 is proposed to design a robust PD-ESO:

Theorem 4.6: The error system of (4-42) and (4-43) is admissible and satisfies $\|G_{zde}\| < \gamma$ if and only if there exist matrices $P > 0, P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, $S \in \mathbb{R}^{(n+p+h) \times (n+p+h)}$, $L \in \mathbb{R}^{h \times (2n+2p+2h)}$, and $H \in \mathbb{R}^{h \times (n+h+p)}$, such that:

$$\begin{bmatrix} \Delta & R_{ea} & (PE_{ea}^T + U_{ea}SV_{ea}^T)^T C_{ea}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (4-47)$$

with:

$$\Delta = A_{ea}(PE_{ea}^T + U_{ea}SV_{ea}^T) + B_{ea}(LE_{ea}^T + HV_{ea}^T) + \star$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $\text{Ker}(E_{ea})$ and $\text{Ker}(E_{ea}^T)$, respectively. Then the PD-ESO gain can be calculated as:

$$L_{ea} = (LE_{ea}^T + HV_{ea}^T)(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1}$$

In connection with either of the descriptor system design methods proposed in Section 4.3.1 or Section 4.3.2, Theorem 4.7 is obtained to design the robust PD-ESO.

Theorem 4.7: The error system of (4-29) is admissible and satisfies $\|G_{zde}\| < \gamma$ if and only if there exist matrices $P_1 > 0, P_1 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}, P_2 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}, P_3 \in \mathbb{R}^{(n+h+p) \times (n+h+p)}, Y_1 \in \mathbb{R}^{h \times (n+h+p)}$ and $Y_2 \in \mathbb{R}^{h \times (n+h+p)}$ such that the following LMI is satisfied:

$$\begin{bmatrix} P_2 + P_2^T & P_1 A - P_2^T E_a + P_3 + Y_1^T C_a & P_1 R_a & 0 \\ * & -P_3^T E_a - E_a^T P_3 + C_a^T Y_2 + Y_2^T C_a & 0 & C_o^T \\ * & * & -\gamma & 0 \\ * & * & * & -\gamma \end{bmatrix} < 0 \quad (4-48)$$

Then the PD-ESO observer gain can be calculated using (4-41).

Remark 4.1: One can see that the derivatives of the output exist in the proposed PD observer. This is clearly not desirable since more uncertainty will be introduced by calculating the derivatives on-line. This provides a motivation for removing on-line computation of the derivative signals by modifying the proposed observer. With this in mind, let $\xi = E_a \hat{x}_a - L_d(\hat{y} - y)$, then the PD-ESO can be formulated as:

$$\dot{\xi} = A_a \hat{x}_a + B_a u + L_p(\hat{y} - y) \quad (4-49)$$

$$\hat{x}_a = E_o^{-1} \xi - E_o^{-1} L_d y \quad (4-50)$$

$$\hat{y} = C_a \hat{x}_a \quad (4-51)$$

where $E_o = E_a - L_d C_a$. The output derivatives do not appear in the modified PD-ESO and hence only the original coefficient matrices are required so that the modified PD-ESO is suitable for on-line application in a real system.

4.4 Case study

Consider the example given in Section 3.4 which is modified from (Gao and Ho, 2004; Gao and Ding, 2007b) with the inclusion of an noise signal d acting through the weighting matrix R_2 .

4.4.1 System model

The considered descriptor system is the same given in Chapter 3 as:

$$E\dot{x} = Ax + Bu + F_a f_a + R_1 d$$

$$y = Cx + F_s f_s + R_2 d$$

where

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 1.5 \\ -2 & -1 \end{bmatrix}, F_a = \begin{bmatrix} 1 \\ 1.5 \\ -1 \end{bmatrix}, R_1 = \begin{bmatrix} 0.5 & 0 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}, F_s = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}, R_2 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.2 \end{bmatrix}$$

4.4.2 PD-ESO design

From the ESO design given in Section 3.4, it is known that the conditions are satisfied and there exist solutions to design the PD-ESO. With the augmentation proposed in Section 4.2, an augmented system is obtained with fault signals augmented twice. A robust PD-ESO is designed and the observer gains are computed as:

$$L_p = \begin{bmatrix} 637.3 & -449.07 & 54.851 & -142.05 & 310.59 \\ 728.06 & -849.88 & 34.926 & -562.59 & 561.3 \\ & & & -1229.8 & -1026.3 & 958.38 & -1096.8 \\ & & & -2059.4 & -14.378 & 1668.8 & -695.05 \end{bmatrix}^T$$

$$L_d = \begin{bmatrix} 4.6663 & -1.4993 & 7.9672 & -12.306 & -29.699 \\ -13.614 & -9.4569 & 5.7759 & -36.769 & -6.0393 \\ & & & -25.64 & -49.736 & 7.3071 & -8.2778 \\ & & & -32.923 & -27.895 & 1.7685 & -4.8267 \end{bmatrix}^T$$

In the design, the eigenvalues are assigned to the same LMI region as in Chapter 3, i.e. with the finite eigenvalues constrained to satisfy $-10 < \text{Re}(\lambda) < -2.5$. The achieved eigenvalues are:

$$\begin{bmatrix} -4.0608 + 3.9614i & -3.5921 + 2.5382i & -2.8587 + 0.8393i \\ -4.0608 - 3.9614i & -3.5921 - 2.5382i & -2.8587 - 0.8393i \\ -2.7330 + 0.5174i & -2.7330 - 0.5174i & -4.0349 \end{bmatrix}$$

4.4.3 Simulation results

The evaluation of the design is implemented in MATLAB/SIMULINK with the results based on different fault types, given in Figures. 4.1 to 4.3. The PD-ESO is implemented with the equivalent realization as discussed in Section 4.3.3. Different fault forms are considered.

It can be observed that the estimates of both the system states and the faults track the original variables. In Comparison with the results given in Section 3.4.3, one advantage

is that the main peak has smaller magnitude. Although the effect of noise is larger than it is when the Augmentation Method 1 is used, the effect of the noise is much smaller than it is with Augmentation Method 2, when comparing Figure 4.3 with Figure 3.5 and Figure 3.8.

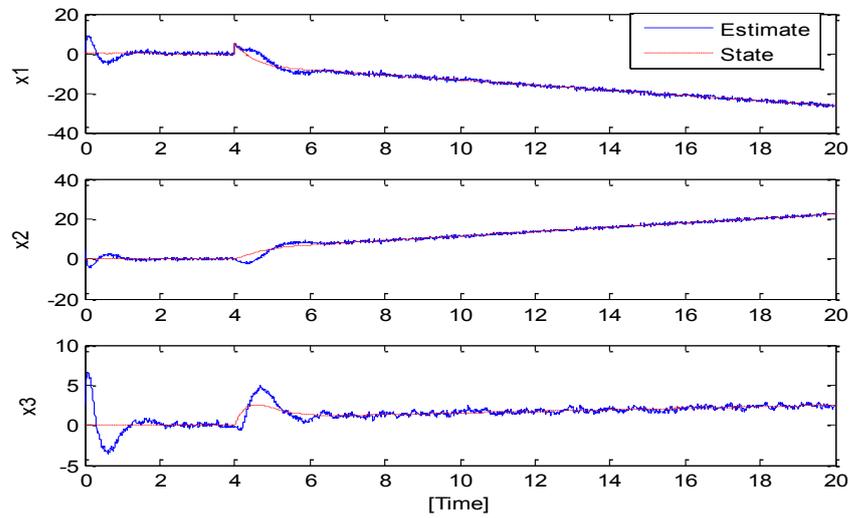


Figure 4.1 System states and their estimates

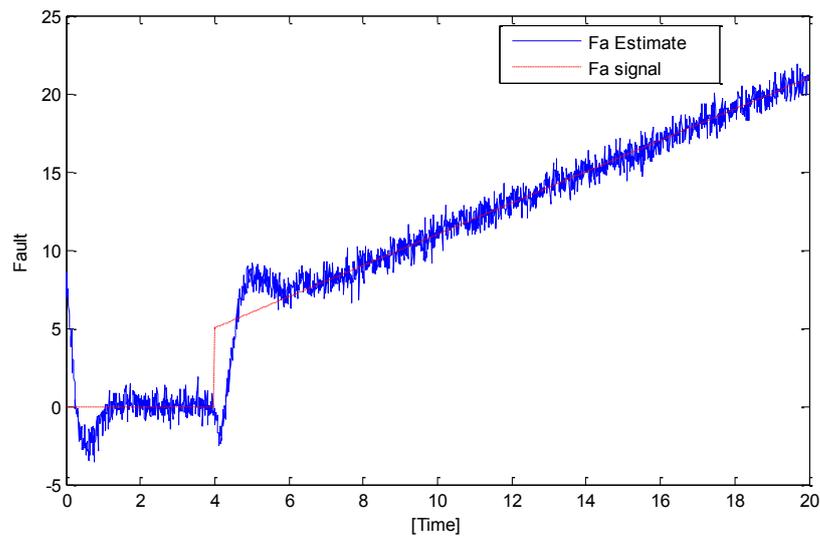


Figure 4.2 Actuator fault and its estimate

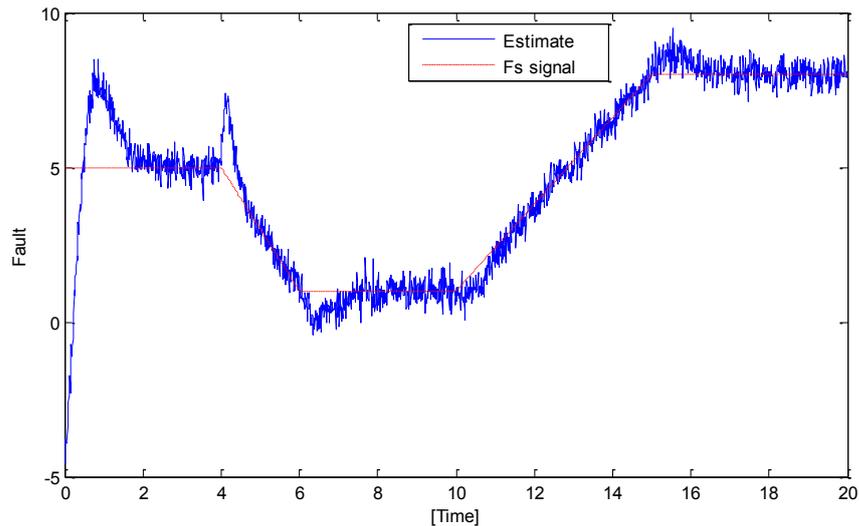


Figure 4.3 Sensor fault and its estimate

4.5 Discussion and conclusion

In this Chapter, an approach to the design of PD-ESO for a descriptor system is proposed, based on the inclusion of a design parameter used to bring the PD-ESO structure to be equivalent to a standard observer. The observer poles have been assigned using the LMI approach given in Section 4.3.2. Furthermore, the robustness to the exogenous disturbances has also been optimized using the LMI-based H_∞ procedure (see Section 3.2). As these objectives are achieved using LMIs, a combined multi-objective performance is also optimized in the LMI framework.

A numerical example is given to illustrate the design procedure. A brief comparison is made in Section 4.4.3 with the ESO results presented in Section 3.4.3.

The Chapter 5 considers an AFTC system which is designed using either the ESO or PD-ESO descriptor system representations, with a controller incorporating a fault compensation mechanism.

Chapter 5: Observer based AFTC for linear time invariant systems

Starting with an introduction to the AFTC strategy used in this study, a novel integrated design procedure is proposed. The design procedure is an adaption of well-known two-step design procedure used in observer-based controller design. The ESO and PD-ESO design approaches to this AFTC problem are considered separately in Sections 5.3 & 5.4, prior using a tutorial example to illustrate the properties of the two approaches.

5.1 The AFTC strategy

As mentioned previously in Section 2.5, there are two kinds of integrated design in the FTC community. The Chapter concerns the performance of an overall AFTC system which combines state and fault estimation with a state feedback control system in the presence of both actuator and sensor faults as well as exogenous disturbance. The design goal is to develop a control system that is capable of tolerating sensor faults and actuator faults, whilst also providing good robustness to disturbances and meeting specified performance objectives.

The basic structure of the AFTC scheme is described by (Gao and Ding, 2007b; Gao, Breikin and Wang, 2008), as shown in Figure 5.1.

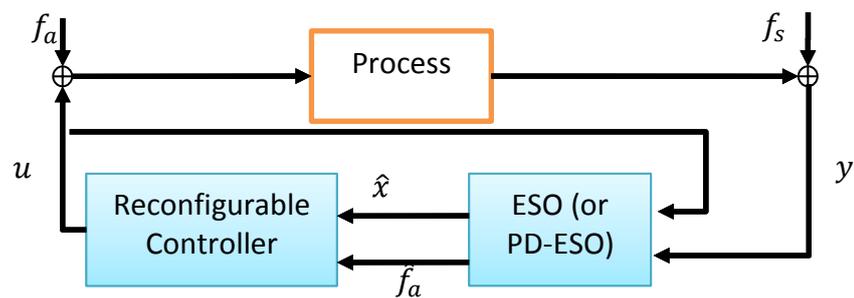


Figure 5.1 The structure of integrated AFTC {adapted from (Gao and Ding, 2007b)}

The underlying assumption for this scheme is that both sensor faults and actuator faults can be estimated using a suitable state and fault observer and their effect compensated accurately in the control system. A clear advantage of this scheme is that the state/fault observer can be based on any suitable representation as long as the states and fault

signals can be estimated simultaneously. Both the ESO and PD-ESO of Chapters 3 & 4, respectively, are capable of estimating the system states and sensor and actuator faults simultaneously.

The ESO and PD-ESO are considered separately in Sections 5.3 & 5.4 as a part of the design of integrated AFTC systems. The obvious main drawback of this scheme is that the overall system performance is affected by the observer accuracy and convergence speed, a property common to all observer-based control system methods.

Fault compensation is considered for actuator faults while fault hiding is used considered to sensor faults. As described by (Zhang and Jiang, 2008), the most commonly used actuator fault compensation strategy in an AFTC system can be generalized as:

$$u = u_h + u_c \quad (5-1)$$

where u_h is the *baseline controller* and u_c is an additive control input to compensate for the effect of fault. Based on this idea, the following observer-based AFTC scheme is proposed as a reconfigurable controller:

$$u = K\hat{x} - K_f\hat{f}_a \quad (5-2)$$

where K and K_f are to be designed, \hat{x} and \hat{f}_a are the estimates of system states and actuator faults. The additional control law $K_f\hat{f}_a$ is used to compensate for the effect of actuator faults. The controller becomes $u = K\hat{x}$ if only sensor faults are considered. When actuator faults are present, it is reasonable to assume that (Gao, Breikin and Wang, 2007; Gao and Ding, 2007a):

$$\text{rank}[B \quad F_a] = \text{rank}[B]$$

That means that there exists a matrix K_f which satisfies $F_a = BK_f$. A choice of K_f is $K_f = B^\dagger F_a$, where B^\dagger denotes the generalized inverse (pseudo-inverse) of B . Hence, the closed-loop system is obtained as:

$$E\dot{x} = Ax + BK\hat{x} - BK_f\hat{f}_a + F_af + Rd \quad (5-3)$$

Furthermore, the closed-loop system is transformed to:

$$E\dot{x} = (A + BK)x + BKe_x + F_a e_{fa} + Rd \quad (5-4)$$

where $e_{fa} = f_a - \hat{f}_a$, $e_x = x - \hat{x}$ are the estimation errors.

From a practical point of view, fault estimation and compensation can be a reasonable strategy for fault accommodation, depending on the characteristics of the expected faults. For example, an actuator offset or actuator loss of effectiveness can be considered as suitable fault scenarios for this form of compensation based AFTC, as considered in a winding machine application in (Noura, Sauter, Hamelin and Theilliol, 2000), or for a three tank system in (Noura, Theilliol and Sauter, 2000), or friction compensation in (Patton, Putra and Klinkhieo, 2010), and wind turbine actuator fault compensation in (Simani and Castaldi, 2013).

It should be noted that this kind of compensation is not always possible, e.g. for the case of a stuck fault in an actuator. That means the physical reason of the faults should be considered when choosing a compensation strategy for a real system. System decomposition is an option as presented in (Noura, Theilliol and Sauter, 2000) where only actuator faults are considered. In fact, the faulty component should be replaced as soon as possible to achieve the reliability and survivability of the entire system, but this is an aspect which is beyond the scope of this thesis.

Hereafter, it is assumed that under certain fault conditions this compensation strategy is reasonable and is hence adopted in this research. An integrated AFTC design scheme is developed in this Chapter, extended to the LPV case in Chapter 6 and applied to a wind turbine problem in Chapter 7.

5.2 Separate controller and observer designs

Before the discussion of integrated design of AFTC, this Section discusses the separate designs of the controller and observer designs within the observer-based AFTC. With the descriptor ESO proposed in Chapter 3, the estimation error system as in (3-35) is rewritten as:

$$E_a \dot{e}_{xf} = A_o e_{xf} + R_a d \quad (5-5)$$

Hence, combining the system states and the estimation error as the new system states, a closed-loop system can be obtained as follows:

$$E_{cl} \begin{bmatrix} \dot{x} \\ \dot{e}_{xf} \end{bmatrix} = A_{cl} \begin{bmatrix} x \\ e_{xf} \end{bmatrix} + R_{cl}d \quad (5-6)$$

where:

$$E_{cl} = \begin{bmatrix} E & 0 \\ 0 & E_a \end{bmatrix}, A_{cl} = \begin{bmatrix} A + BK & \Delta \\ 0 & A_o \end{bmatrix}, R_{cl} = \begin{bmatrix} R \\ R_a \end{bmatrix}, \Delta = [BK \quad F_a \quad 0_{n \times (p-q)}].$$

When exogenous disturbances or system uncertainty are not considered, one simple strategy is to design the observer and controller separately based on the separation principle (Heemels, Daafouz and Millerioux, 2009; Halalchi, Bara and Laroche, 2011). It can be seen from the structure of (E_{cl}, A_{cl}) that the eigenvalues of the matrix (E_{cl}, A_{cl}) are the direct sum of the eigenvalues of $(E, A + BK)$ and (E_o, A_o) (by analogy with the standard form of linear system). It is interesting to note that some AFTC design approaches are based on the separation principle as in (Gao and Ding, 2007b; Gao and Ding, 2007a; Zhang, Jiang and Shi, 2009; Halalchi, Bara and Laroche, 2011).

5.3 Integrated AFTC design with ESO

The main challenges of an integrated approach to AFTC design are summarised as follows. Although the robustness of the closed-loop system can be obtained to some degree, the performance of the closed-loop is not achieved and hence performance degradation should be considered (Heemels, Daafouz and Millerioux, 2009). Moreover, attention should be paid to the effect of imperfect fault estimation on the closed-loop stability and/or robustness performances due to estimation delay or disturbances (Zhang and Jiang, 2006). The challenges that accompany this integrated approach are discussed in (Chen, Tseng and Uang, 1999; Lo and Lin, 2004; Zhu and Pagilla, 2006; Ichalal, Marx, Ragot and Maquin, 2010).

However, the solution for an integrated design cannot be obtained in all cases. Generally speaking, the observer-based controller will lead to a Bilinear Matrix Inequalities (BMI) problem which is not convex and is NP-hard. Efforts to find tractable solutions to BMI problems have been on-going for many years (Goh, Turan, Safonov, Papavasilopoulos and Ly, 1994; VanAntwerp and Braatz, 2000; Wang, 2009). From an engineering point of view, a two-step procedure was first introduced by (Chen, Tseng and Uang, 1999) and studied by (Lo and Lin, 2004; Zhu and Pagilla, 2006) to achieve observer-based control designs for uncertain systems. In this approach, the controller gain is designed first via solving a pure LMI set, and then an observer gain is

designed based on the obtained controller to achieve global stability and robustness via the solution of a new LMI set. This approach is practical from an engineering point of view as each step has some connections with the real requirements. A potential problem is that the final solution may not be globally optimal. Section 5.3 and 5.4 discuss the integrated design of AFTC with the ESO and PD-ESO considered separately, and in this way the two-step design idea is adapted.

The robustness considered in this study follows the H_∞ framework as discussed in Section 3.2. Consider the closed-loop system (5-6) and define the robustness performance variable as:

$$z_{xef} = C_z \begin{bmatrix} x \\ e_{xf} \end{bmatrix} \quad (5-7)$$

where $C_z = [C_{zx} \ C_{ze}]$ is the H_∞ performance matrix. Then the transfer function can be obtained as $G_z = C_z(sE_{cl} - A_{cl})^{-1}R_{cl}$. Based on the Bound Real Lemma for a descriptor system given in Section 3.2, the closed-loop system of (5-6) and (5-7) is admissible and $\|G_z\|_\infty < \gamma$ if there exist $\mathcal{P} > 0, \mathcal{S}$ with compatible dimensions such that:

$$\begin{bmatrix} (\mathcal{P}E_{cl}^T + U_{cl}\mathcal{S}V_{cl}^T)A_{cl} + \star & (\mathcal{P}E_{cl}^T + U_{cl}\mathcal{S}V_{cl}^T)R_{cl} & C_z^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (5-8)$$

where U_{cl} and V_{cl} are full column rank and contain the basis vectors for $Ker(E_{cl})$ and $Ker(E_{cl}^T)$, respectively

Considering the structure of E_{cl} , U_{cl} and V_{cl} can be parameterized as:

$$U_{cl} = \begin{bmatrix} U & 0 \\ 0 & U_a \end{bmatrix}, V_{cl} = \begin{bmatrix} V & 0 \\ 0 & V_a \end{bmatrix}$$

where U_a and V_a are full column rank and contain the basis vectors for $Ker(E_a)$ and $Ker(E_a^T)$, U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$ respectively. Furthermore, by choosing:

$$\mathcal{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}, \mathcal{S} = \begin{bmatrix} S & 0 \\ 0 & W \end{bmatrix}$$

(5-8) can then be re-formulated as:

$$\begin{bmatrix} \Delta_{11} & \Delta & (E^T P + USV^T)R & C_{zx}^T \\ * & \Delta_{22} & (E_a^T Q + U_a W V_a^T)(R_a + L_p D) & C_{ze}^T \\ * & * & -\gamma & 0 \\ * & * & * & -\gamma \end{bmatrix} < 0 \quad (5-9)$$

with:

$$\Delta = (E^T P + USV^T)[BK \quad F_a \quad 0_{n \times (p-q)}]$$

$$\Delta_{11} = (E^T P + USV^T)(A + BK) + *, \Delta_{22} = (E_a^T Q + U_a W V_a^T)A_o + *$$

Or equivalently:

$$\begin{bmatrix} \Delta_{22} & * & \Delta_{21} & C_{ze}^T \\ \Delta & \Delta_{11} & (E^T P + USV^T)R & C_{zx}^T \\ * & * & -\gamma & 0 \\ * & * & * & -\gamma \end{bmatrix} < 0 \quad (5-10)$$

with:

$$\Delta = (E^T P + USV^T)[BK \quad F_a \quad 0_{n \times (p-q)}], \Delta_{11} = (E^T P + USV^T)(A + BK) + *$$

$$\Delta_{21} = (E_a^T Q + U_a W V_a^T)R_a + (E_a^T J + U_a H)D$$

$$\Delta_{22} = (E_a^T Q + U_a W V_a^T)A_a + (E_a^T J + U_a H)C_a + *$$

Theorem 5.1 is proposed to design an integrated AFTC with proposed structure given in Figure 5.1.

Theorem 5.1: The closed-loop system (5-6) and (5-7) is admissible and the overall system performance $\|G_z\|_\infty < \gamma$ is achieved if (5-9) or (5-10) is satisfied.

Clearly, both (5-9) and (5-10) are BMIs and cannot be solved effectively. By defining:

$$F_1 = (E_a^T Q + U_a W V_a^T)A_a + (E_a^T J + U_a H)C_a + *, F_{ci} = [\Delta^T \quad \Delta_{21} \quad C_{ze}^T]$$

$$F_c = \begin{bmatrix} \Delta_{11} & (E^T P + USV^T)R & C_{zx}^T \\ * & -\gamma & 0 \\ * & * & -\gamma \end{bmatrix}$$

$$\Delta = (E^T P + USV^T)[BK \quad F_a \quad 0_{n \times (p-q)}]$$

$$\Delta_{21} = (E_a^T Q + U_a W V_a^T)R_a + (E_a^T J + U_a H)D$$

(5-10) then leads to:

$$\begin{bmatrix} F_1 & F_{ci} \\ F_{ci}^T & F_c \end{bmatrix} < 0 \quad (5-11)$$

A two-step procedure (Zhu and Pagilla, 2006) is adopted based on the partition in (5-11):

Algorithm 5.1:

Step 1: Find a solution to:

$$\begin{aligned} & \min_{P,S,K} \gamma, \text{ subject to:} \\ & P > 0, F_c < 0 \end{aligned} \quad (5-12)$$

Step 2: Using the obtained F_c, K , find a solution to:

$$\begin{aligned} & \min_{Q,W,J,H} \Lambda, \text{ subject to:} \\ & Q > 0, \begin{bmatrix} F_1 & F_{ci} \\ F_{ci}^T & \Lambda F_c \end{bmatrix} < 0, \Lambda > 0 \end{aligned} \quad (5-13)$$

where $\Lambda = \text{diag}(\eta_1 I, \eta_2 I, \dots, \eta_n I)$.

Lemma 5.1: The optimization problems in **Step 1** and **Step 2** are feasible if (E, A, B) is controllable and (E_a, A_a, C_a) is observable.

Before the proof of Lemma 5.1, the well-known Schur Complement Lemma is recalled:

Lemma 5.2 (Schur Complement Lemma): A block matrix $\begin{bmatrix} P & M \\ M^T & Q \end{bmatrix}$ is negative definite if and only if:

$$\begin{cases} Q < 0 \\ P - MQ^{-1}M^T < 0 \end{cases}$$

The following gives the proof Lemma 5.1.

Proof: It is obvious that (5-11) is feasible if (E, A, B) is controllable. Once a solution to (5-11) is found, then it must be proved that **Step 2** is feasible. Based on the Schur Complement Lemma:

$$\begin{bmatrix} F_1 & F_{ci} \\ F_{ci}^T & \Lambda F_c \end{bmatrix} < 0, \Lambda > 0 \Leftrightarrow \Lambda F_c < 0, F_1 - F_{ci}(\Lambda F_c)^{-1}F_{ci}^T < 0, \Lambda > 0$$

From this (5-13) is feasible if $F_1 < 0, Q > 0$ is feasible. If the system (E_a, A_a, C_a) is observable, then there is an observer satisfying the robustness problem, so that $F_1 < 0, Q > 0$ is feasible. ■

This method has some merits. First, it provides a way to handle a nonlinear optimization problem. Moreover, maximization of the interconnection bounds and the introduction of the parameter Λ are of interest. However, it should be noted that (5-13) and (5-11) are not theoretically equivalent. Because of the special structure of Λ , it can be concluded that:

$$\Lambda F_c = \Lambda^{1/2} F_c \Lambda^{1/2} < 0$$

when $F_c < 0$ and $\Lambda > 0$. The inequality (5-11) is equivalent to:

$$\begin{bmatrix} I & 0 \\ 0 & \Lambda^{1/2} \end{bmatrix} \begin{bmatrix} F_1 & F_{ci} \\ F_{ci}^T & F_c \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & \Lambda^{1/2} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} F_1 & F_{oi} \Lambda^{1/2} \\ \Lambda^{1/2} F_{oi}^T & \Lambda F_c \end{bmatrix} < 0$$

However, Λ in (5-13) has been introduced in a special way (only at the (2, 2) position) to make the optimization problem feasible. Generally, there is no guarantee of the feasibility of the optimization problem after substituting the designed controller from Step 1) into Step 2) (to compute the observer gain). However by selecting a sufficiently large Λ , the feasibility of (5-13) can be guaranteed. Although, this method works well in some cases, post-design verification is necessary to assure the design performance is achieved (Zhu and Pagilla, 2006).

Based on the above discussion, the following algorithm for the integrated design is proposed.

Algorithm 5.2:

Step 1: Find a state feedback controller via solving:

$\min_{P,S,K} \gamma$, subject to:

$$P > 0, F_c < 0 \tag{5-14}$$

Step2: On obtaining the gain K , solve the following LMI optimization problem:

$$\min_{P,S} \gamma, \text{ subject to (5-11)}$$

After S, Y_1, Y_2 is obtained, L can be calculated as given in Theorem 3.3.

Remark 5.1: The conservatism of this approach may come from the fact that the gain K obtained in Step 1 may not be optimal from a global point of view. Even the pre-calculated K may lead to numerical infeasibility in Step 2. One way to overcome the drawback is to design different sub-optimal solutions in Step 1 and optimize γ in Step 2 using the different K values computed. The K , satisfying the best overall performance in some sense can then be selected.

5.4 Integrated AFTC with PD-ESO

This Section discusses the integrated AFTC design scheme with PD-ESO. The basic idea is to consider the dual system of the integrated system instead of the original system. With the same assumptions and AFTC strategy, the closed-loop system with PD-ESO is transformed to:

$$E\dot{x} = (A + BK)x + BK e_x + F_a e_{fa} + R_1 d_u \quad (5-15)$$

where $e_{fa} = f_a - \hat{f}_a$, $e_x = x - \hat{x}$ is the estimation error and the estimation error system is given by (4-21) as:

$$E_o \dot{e}_{x\omega f} = A_o e_{x\omega f} + R_a d_D \quad (5-16)$$

Then the closed-loop system is organized as:

$$\mathcal{E} \begin{bmatrix} \dot{x} \\ \dot{e}_{x\omega f} \end{bmatrix} = \mathcal{A} \begin{bmatrix} x \\ e_{x\omega f} \end{bmatrix} + \mathcal{R} d_D \quad (5-17)$$

$$z_{xe\omega f} = [C_{zx} \quad C_{zde}] \begin{bmatrix} x \\ e_{x\omega f} \end{bmatrix} \quad (5-18)$$

where $[C_{zx} \quad C_{zde}]$ is the H_∞ performance matrix and:

$$\mathcal{E} = \begin{bmatrix} E & 0 \\ 0 & E_o \end{bmatrix}, \mathcal{A} = \begin{bmatrix} A + BK & [BK \quad 0 \quad F_f] \\ 0 & A_o \end{bmatrix}, \mathcal{R} = \begin{bmatrix} R \\ R_a \end{bmatrix}, R = [R_1 \quad 0]$$

The design of the integrated system comprising the controller and the PD-ESO is based on the idea developed in Chapter 4. Consider the dual system with:

$$\mathcal{E}_d = \begin{bmatrix} E^T & 0 \\ 0 & E_o^T \end{bmatrix}, \mathcal{A}_d = \begin{bmatrix} A^T + K^T B^T & 0 \\ [BK \quad 0 \quad F_f]^T & A_o^T \end{bmatrix}, \mathcal{C}_d = \begin{bmatrix} R \\ R_a \end{bmatrix}^T, \mathcal{R}_d = [C_{zx} \quad C_{zde}]^T$$

Similar to the PD-ESO design, we propose an integrated design scheme to the augmented dual system to achieve the desired performance of the original system. The augmented system can be organized as:

$$\mathcal{E}_{da}\dot{z} = \mathcal{A}_{da}z + \mathcal{R}_{da}d_z \quad (5-19)$$

$$y_{da} = \mathcal{C}_{da}z \quad (5-20)$$

where:

$$\begin{aligned} \mathcal{E}_{da} &= \begin{bmatrix} E^T & 0 \\ 0 & E_{ea} \end{bmatrix}, E_{ea} = \begin{bmatrix} I_{n+h+p} & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{R}_{da} = \begin{bmatrix} C_{zx}^T \\ R_{ea}^T \end{bmatrix}, R_{ea}^T = \begin{bmatrix} C_{zde}^T \\ 0 \end{bmatrix} \\ \mathcal{A}_{da} &= \begin{bmatrix} A^T + K^T B^T & 0 \\ \Delta & A_{ea} \end{bmatrix} = \begin{bmatrix} A^T + K^T B^T & 0 \\ \Delta & A_{ea} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{ea} \end{bmatrix} \begin{bmatrix} 0 & L_{ea} \end{bmatrix}, L_{ea} = \begin{bmatrix} L_p^T & L_d^T \end{bmatrix} \\ A_{ea} &= \begin{bmatrix} 0 & I \\ A_o^T & -E_o^T \end{bmatrix}, \Delta = \begin{bmatrix} 0 \\ K^T B^T \\ F_f^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}, \mathcal{C}_{da} = [R^T \quad C_R^T], C_R^T = [0 \quad R_a^T] \end{aligned}$$

Based on the robustness condition presented in Section 3.2, it is known that the pair $(\mathcal{E}_{da}, \mathcal{A}_{da})$ is admissible and $\|y_{da}\|_2 < \gamma \|d_z\|_2$ for all $\|d_z\| \neq 0$ if and only if there exists P with compatible dimensions such that:

$$\mathcal{E}_{da}P = P^T \mathcal{E}_{da}^T \geq 0 \quad (5-21)$$

$$\begin{bmatrix} \mathcal{A}_{da}P + P^T \mathcal{A}_{da}^T & \mathcal{R}_{da} & P^T \mathcal{C}_{da}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (5-22)$$

Choosing a Lyapunov function in the following form:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \quad (5-23)$$

(5-22) can be explored to give:

$$\begin{bmatrix} \Delta_{11} & \mathcal{R}_{da} & \Delta_{13} \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (5-24)$$

with:

$$\Delta_{11} = \begin{bmatrix} A^T P_1 + K^T B^T P_1 & 0 \\ \Delta P_1 & (A_{ea} + B_{ea} L_{ea}) P_2 \end{bmatrix} + \star, \Delta_{13} = [R^T P_1 \quad C_R^T P_2]$$

Furthermore, it follows that:

$$\begin{bmatrix} A^T P_1 + K^T B^T P_1 + \star & \star & \star & \star \\ \Delta P_1 & (A_{ea} + B_{ea} L_{ea}) P_2 + \star & \star & \star \\ C_{zx} & R_{ea} & -\gamma & \star \\ R^T P_1 & C_R^T P_2 & 0 & -\gamma \end{bmatrix} < 0 \quad (5-25)$$

Parameterizing P_1 and P_2 with $P_1 = W_1 E^T + U S_1 V^T$, $P_2 = W_2 E_{ea}^T + U_{ea} S_2 V_{ea}^T$, then the inequality (5-25) can be re-formulated as:

$$\begin{bmatrix} \Delta_{11} & \star & \star & \star \\ \Delta_{21} & \Delta_{22} & \star & \star \\ C_{zx} & R_{ea} & -\gamma & \star \\ R^T W_1 E^T + R^T U S_1 V^T & C_R^T W_2 E_{ea}^T + C_R^T U_{ea} S_2 V_{ea}^T & 0 & -\gamma \end{bmatrix} < 0 \quad (5-26)$$

with:

$$\Delta_{11} = (A^T + K^T B^T) W_1 E^T + (A^T + K^T B^T) U S_1 V^T + \star$$

$$\Delta_{21} = \Delta (W_1 E^T + U S_1 V^T), \Delta = [0 \quad BK \quad F_f]^T$$

$$\Delta_{22} = A_{ea} W_2 E_{ea}^T + A_{ea} U_{ea} S_2 V_{ea}^T + B_{ea} Y E_{ea}^T + B_{ea} H V_{ea}^T + \star$$

Theorem 5.2: The closed-loop system of (5-17) and (5-18) is admissible and the overall system performance $\|z_{xewf}\|_2 < \gamma \|d\|_2$ is achieved if (5-26) is satisfied.

To solve the gain for the closed-loop system of (5-17) and (5-18), define:

$$F_1 = (A^T + K^T B^T) W_1 E^T + (A^T + K^T B^T) U S_1 V^T + \star$$

$$F_{ci}^T = \begin{bmatrix} \Delta_{21} \\ C_{zx} \\ R^T W_1 E^T + R^T U S_1 V^T \end{bmatrix}, F_c = \begin{bmatrix} \Delta_{22} & \star & \star \\ R_{ea} & -\gamma & \star \\ C_R^T W_2 E_{ea}^T + C_R^T U_{ea} S_2 V_{ea}^T & 0 & -\gamma \end{bmatrix}$$

$$\Delta_{21} = \Delta (W_1 E^T + U S_1 V^T), \Delta = [0 \quad BK \quad F_f]^T$$

$$\Delta_{22} = A_{ea} W_2 E_{ea}^T + A_{ea} U_{ea} S_2 V_{ea}^T + B_{ea} Y E_{ea}^T + B_{ea} H V_{ea}^T + \star$$

Inequality (5-26) can then be partitioned as follows:

$$\begin{bmatrix} F_1 & F_{ci} \\ F_{ci}^T & F_c \end{bmatrix} < 0 \quad (5-27)$$

It can be seen that (5-27) shares the same property with (5-11). Hence the same two-step design procedure proposed in Algorithm 5.2 can be applied directly to solve (5-26). The integrated designed observer-controller system can be constructed.

5.5 Case study

In this part, a numerical example modified from (Gao and Ho, 2004; Gao and Ding, 2007b) is studied to illustrate the integrated design procedure proposed in this Chapter.

5.5.1 System model

Consider the descriptor system:

$$E\dot{x} = Ax + Bu + F_a f_a + R_1 d \quad (5-28)$$

$$y = Cx + F_s f_s \quad (5-29)$$

where:

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 1.5 \\ -2 & -1 \end{bmatrix}, F_a = \begin{bmatrix} 1 \\ 1.5 \\ -1 \end{bmatrix}, R_1 = \begin{bmatrix} 0.5 \\ -1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}, F_s = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$$

5.5.2 Controller and observer design

5.5.2.1 Integrated design with ESO

Preliminary Step: Check the observability and controllability conditions of the system of (5-28) and (5-29). The conditions are satisfied and hence an AFTC system can be designed.

Step 1: Based on Step 1 given in the algorithm, a state feedback gain is calculated as:

$$K = \begin{bmatrix} -16.957 & 87.922 & 34.765 \\ 33.607 & -177.23 & -70.353 \end{bmatrix}$$

In our design, the finite eigenvalues are constrained to satisfy $-5 < Re(\lambda) < -2$ to produce a sufficiently fast time response and restrict too fast motions. The achieved H_∞ performance is $\gamma = 0.4672$.

Step2: In the observer design step, the fault signal is augmented twice to provide the capability of good estimation of faults with complex time-characteristics. And in the design, the finite eigenvalues are constrained to satisfy $-10 < Re(\lambda) < -3$. Actually, the convergence rate can be adjusted to satisfy time response requirements. With the controller gain obtained in Step 1, a set of LMIs are solved and the observer gain is calculated as :

$$L = \begin{bmatrix} 143.71 & 2.3894 & 0 & -63.405 & -13.401 & 6.3297 & 6.3354 \\ -12.587 & 40.514 & 0 & -60.121 & -53.781 & 14.647 & 8.4435 \end{bmatrix}^T$$

The achieved H_∞ performance is $\gamma = 1.5931$.

5.5.2.2 Integrated design with PD-ESO

The gains are designed following the procedure proposed in Section 5.3. Considering the integrated design based on the ESO it has been shown that the system is observable and controllable, hence an integrated AFTC system incorporating the PD-ESO exists.

Step 1: A state feedback controller is designed and the controller gain is

$$K = \begin{bmatrix} -17.168 & 90.966 & 35.802 \\ 34.029 & -183.32 & -72.426 \end{bmatrix}$$

In the design, the finite eigenvalues are constrained to satisfy $-5 < Re(\lambda) < -2$ to get fast time response and restrict too fast motions. The achieved H_∞ performance is $\gamma = 0.5104$.

Step 2: The observer gain is designed as:

$$L_p = \begin{bmatrix} 580.89 & -103.89 & 36.265 & 311.78 & 144.73 \\ 641.58 & -379.13 & 16.614 & -45.17 & 167.65 \\ -369.03 & -1288.8 & 453.41 & -1169 \\ -893.53 & -221.62 & 1027.2 & -502.93 \end{bmatrix}^T$$

$$L_d = \begin{bmatrix} 2.96 & -0.24 & 4.33 & -10.22 & -18.1 & -18.5 & -19.39 & 2.31 & 0.43 \\ -4.13 & -4.24 & 1.95 & -16.03 & -2.37 & -15.91 & -8.82 & 0.6 & -0.3 \end{bmatrix}^T$$

As pointed out in Chapter 4, the augmentation method used for the PD-ESO is different from the augmentation used for the ESO design given in subsection 5.4. For the PD-ESO problem an extra augmentation is used to obtain a sensor noise-free system. With the prescribed eigenvalue constraints, another set of LMIs is solved as discussed in Section 5.4.

5.5.3 Simulation results

The simulations both the EOS and PD-ESO-based AFTC systems are implemented in MATLAB/SIMULINK.

For the ESO case the descriptor observer is implemented with equivalent realization as discussed in Section 3.3. Both sensor and actuator faults are considered using the same signals stated in Section 3.4. The simulation results are given in Figures 5.2-5.4 below.

The integrated design with the PD-ESO is conducted with the same simulation environment and fault scenarios. The corresponding simulation results are as shown in Figures 5.5-5.7.

It can be seen that both the states and fault signals can be estimated using the proposed PD-ESO. From the state response, it can be seen that the closed-loop system is stable and the states converge to around zero whether or not there are faults acting on the system being controlled. Furthermore the required convergence rate is also satisfied, and could be adjusted further according to alternative time response requirements.

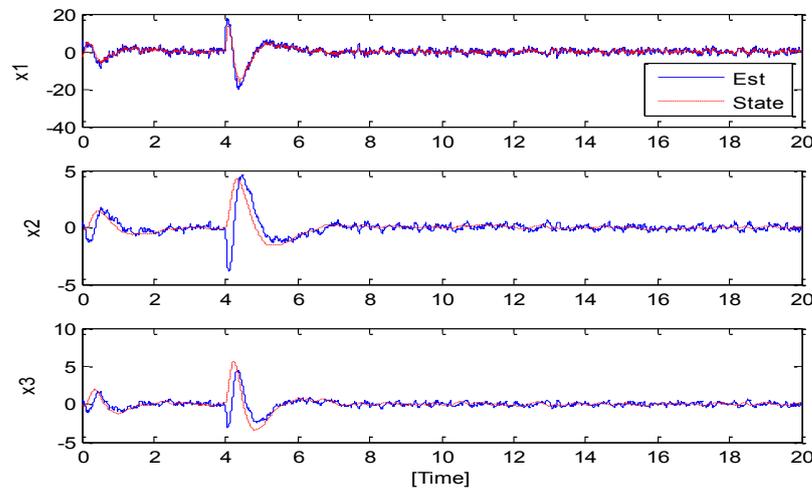


Figure 5.2 States and their estimates using integrated AFTC with ESO

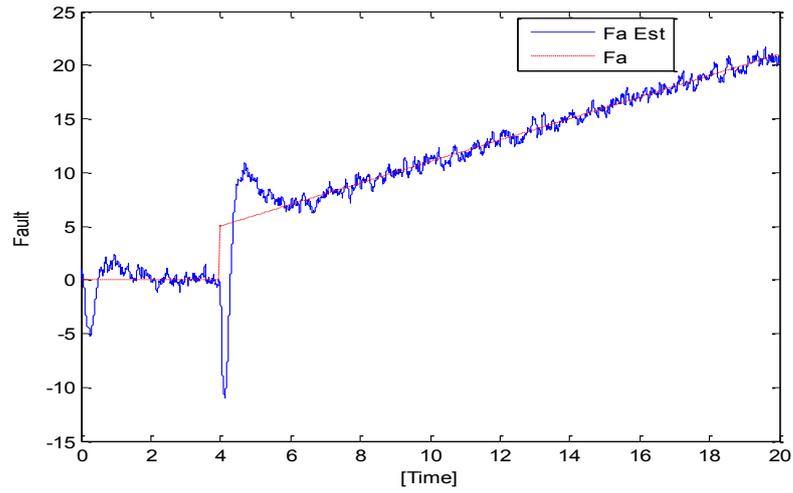


Figure 5.3 Actuator fault and its estimate using integrated AFTC with ESO

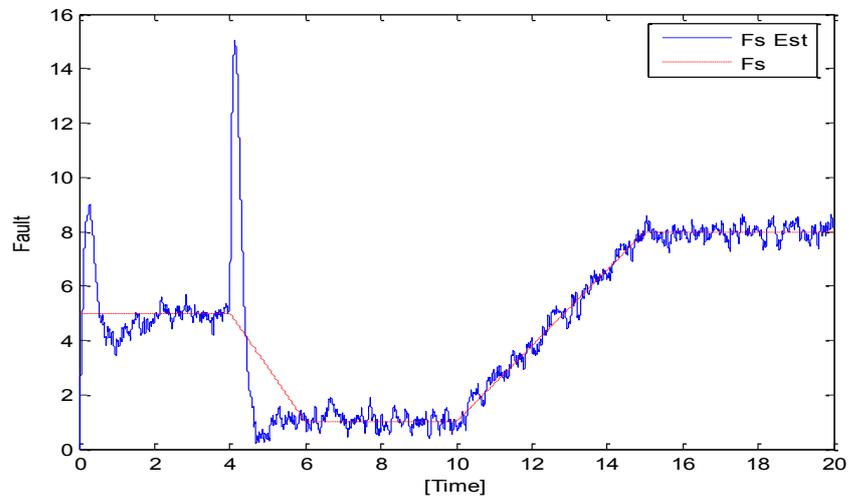


Figure 5.4 Sensor fault and its estimate using integrated AFTC with ESO

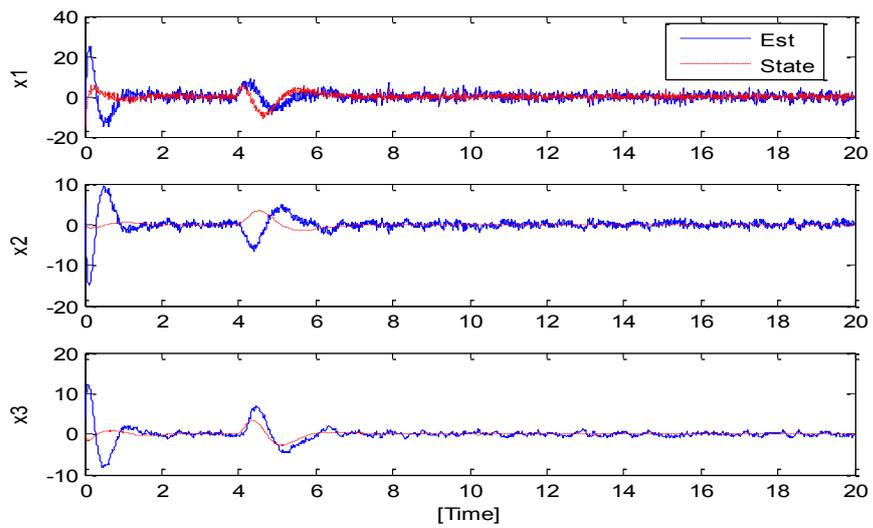


Figure 5.5 States and their estimate using integrated AFTC with PD-ESO

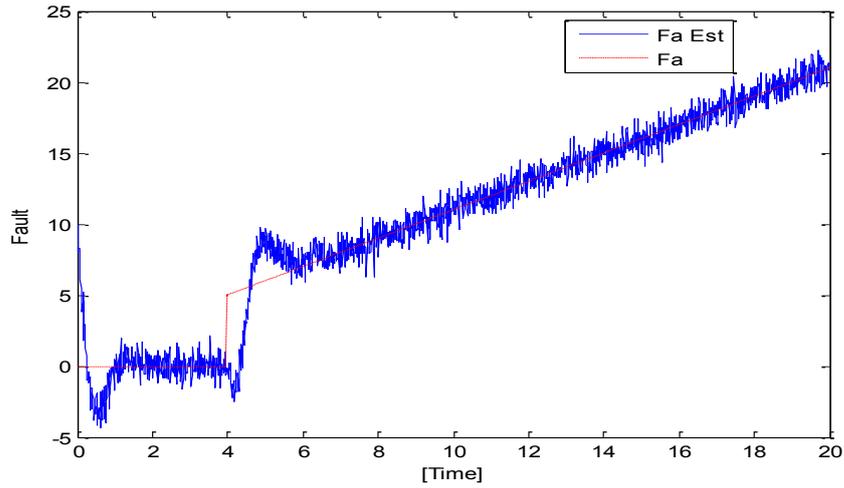


Figure 5.6 Actuator fault and its estimate using integrated AFTC with PD-ESO

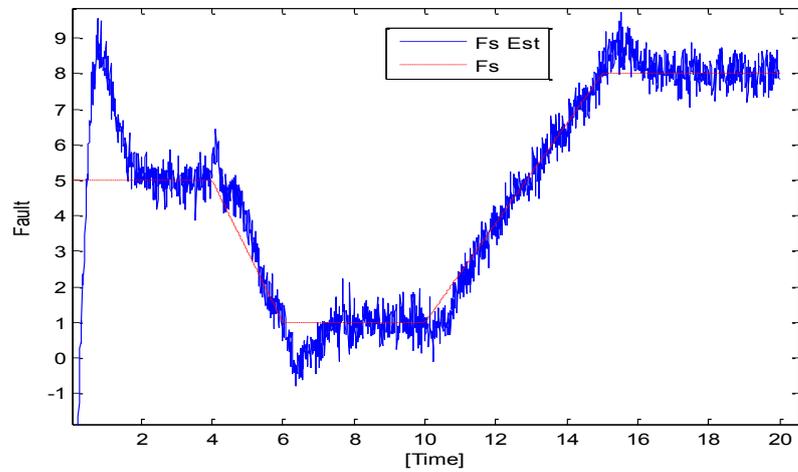


Figure 5.7 Sensor fault and its estimate using integrated AFTC with PD-ESO

5.6 Discussion and conclusion

In the Chapter, one observer based AFTC structure is presented by combining an simultaneous observer with an reconfigurable controller. Two integrated design schemes are proposed with ESO or PD-ESO, respectively for linear descriptor systems. Using the two-step design procedure, the overall robustness performance can be achieved within an H_∞ framework.

A numerical example is given to illustrate the design procedures. A brief comparison of the results from the two schemes is made in Section 5.5.3.

In Chapter 6, this design strategy is developed within an LPV system framework to account for time-varying system parametric variations, and some new problems and contributions are detailed.

Chapter 6: Observer based AFTC for linear parameter varying systems

6.1 Introduction

The design of feedback mechanisms that can be applicable to non-linear systems must be done very carefully with regard to the form of system non-linearity and the restrictions that can be imposed by using linearization. However, time-varying system approximations may be a basis for suitable representation of the original non-linear system. An early approach to achieve a representation of a time-varying system has been to use the so-called gain-scheduling design methods which are still in use today e.g. for flight control systems, principally due to their apparent simplicity. In gain-scheduling, sets of model parameters, gains etc. are pre-stored in the control systems and a selection mechanism is used to switch between the various models according to the understood non-linear behaviour.

An attractive alternative to gain-scheduling is to represent the non-linear system with a linear parameter varying (LPV) model that depends on a set of measured or estimated parameters (Leith and Leithead, 1999). The main advantage of LPV models is that they allow powerful linear design tools to be applied even to some complex non-linear systems, whilst also guaranteeing global stability over the entire working envelope (Becker, Packard, Philbrick and Balas, 1993; Becker and Packard, 1994; Wu, 1995; Packard and Kantner, 1996).

LPV modelling of monitored systems has been considered for fault diagnosis or fault estimation in (Akhenak, Chadli, Maquin and Ragot, 2004; Bokor and Balas, 2004; Hallouzi, Verdult, Babuska and Verhaegen, 2005; Rodrigues, Theilliol and Sauter, 2005a; Szaszi, Marcos, Balas and Bokor, 2005; Grenaille, Henry and Zolghadri, 2008; Zolghadri, Henry and Grenaille, 2008; Hallouzi, Verhaegen and Kanev, 2009), and FTC (Rodrigues, Theilliol and Sauter, 2005b; Patton, Chen and Klinkhieo, 2012). In particular, the LPV formulation of a descriptor system can have powerful analysis and design properties and is a suitable way of representing LPV systems (Masubuchi, Akiyama and Saeki, 2003; Masubuchi, Kato, Saeki and Ohara, 2004; Masubuchi and Suzuki, 2008). However, few studies have been concerned with LPV approaches to the

joint problems of state reconstruction and FDD/FE for descriptor systems. This is despite an interesting result on fault estimation by (Hamdi, Rodrigues, Mechmeche, Theilliol and Benhadj Braiek, 2009). A UIO design procedure is generalized to LPV descriptor systems in (Hamdi, Rodrigues, Mechmeche, Theilliol and Braiek, 2012).

This Chapter introduces the concept of LPV descriptor systems followed by an extension of the techniques developed in previous Chapter 3, 4 & 5. Some unique problems related to LPV descriptor systems are also discussed. A numerical example is used to illustrate the design of an AFTC system with integrated design of controller and observer. This uses observer-based state feedback control with a PD-ESO constructed within an LPV descriptor system framework.

6.2 Baseline controller to LPV systems

Following the LPV descriptor system formulation given in (Masubuchi, Akiyama and Saeki, 2003), consider a system with sensor and actuator faults given as:

$$E\dot{x} = A(\theta(t))x + Bu + F_a f_a + Rd \quad (6-1)$$

$$y = Cx + F_s f_s + Dd \quad (6-2)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^h$, $f_a \in \mathbb{R}^q$, $f_s \in \mathbb{R}^{p-q}$ and $d \in \mathbb{R}^d$ are the state vector, the input vector and measured output vector, actuator fault vector, sensor fault vector and disturbance vector, respectively. B, C, D, F_a, F_s, E, R are known constant matrices. $rank(E) = r \leq n$. $A(\theta(t))$ is known continuous function of a time-varying parameter vector $\theta(t)$ which satisfies:

$$\theta(t) = [\theta_1(t), \dots, \theta_{n_\theta}(t)]^T \in \Theta, \forall t \geq 0$$

where Θ is a compact set. In a similar manner to the development of an LTI descriptor system, the above system can be rewritten as:

$$E\dot{x} = A(\theta)x + Bu + F_f f + Rd \quad (6-3)$$

$$y = Cx + D_f f + Dd \quad (6-4)$$

where:

$$f = \begin{bmatrix} f_a \\ f_s \end{bmatrix}, D_f = [0 \quad F_s], F_f = [F_a \quad 0], f \in \mathbb{R}^p$$

6.2.1 State feedback Controller

In this Section the analysis and design of baseline controllers for descriptor systems given in Section 3.2 is developed within an LPV formulation. Some results are presented without proof when the result may be trivial.

Section 3.2 shows that system stability is a special case of the \mathfrak{D} stability, considering the open left-hand half of the complex plane as a stability region. Hence, the requirement of pole-placement in an LMI region of an LPV descriptor system can be considered instead of a stability condition. Lemma 6.1 provides the analysis and synthesise procedure for an LPV descriptor using LMI pole-placement regions.

Lemma 6.1: The pair $(E, A(\theta))$ is quadratically admissible and \mathfrak{D} stable if there exists a matrix $P > 0, P \in \mathbb{R}^{n \times n}$ and a matrix $S \in \mathbb{R}^{(n-r) \times (n-r)}$ such that for all θ :

$$\alpha \otimes (EPE^T) + \beta \otimes (A(\theta)PE^T) + I_q \otimes (A(\theta)USV^T) + \star < 0 \quad (6-5)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively.

Following the design procedure within H_∞ framework discussed in Section 3.2, the H_∞ performance variable is defined as $z = C_{zx}x$, which leads to transfer function to be $G(\theta, s) = C_{zx}(sE - A(\theta))^{-1}R$. The defined $G(\theta, s)$ is a measurement of the influence of disturbance on system states of (6-3)-(6-4) in H_∞ framework. As an extension of the previous study of linear descriptor systems (given in Section 3.2), the following Lemma 6.2 is introduced to handle the robustness problem of the LPV descriptor system.

Lemma 6.2: The pair $(E, A(\theta))$ is admissible and $\|G(\theta, s)\| < \gamma$ if there exists a matrix $P > 0, P \in \mathbb{R}^{n \times n}$ and a matrix $S \in \mathbb{R}^{(n-r) \times (n-r)}$ such that for all θ :

$$\begin{bmatrix} A(\theta)(PE^T + USV^T) + \star & R & (PE^T + USV^T)^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-6)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively.

Besides analysis description presented in Lemma 6.1 and 6.2, baseline control with state feedback is considered now. Consider a parameter dependent state feedback controller to system (6-3) as:

$$u = K(\theta)x \quad (6-7)$$

The fault-free closed-loop system can be organized as

$$E\dot{x} = (A(\theta) + BK(\theta))x + Rd(t) \quad (6-8)$$

For the closed-loop, $G(\theta, s) = C_{zx}(sE - A(\theta) - BK(\theta))^{-1}R$. As it is quite straightforward, there is no need to prove the following Lemmas 6.3 & 6.4, which can be used for regional pole-placement and H_∞ control of an LPV descriptor system, respectively.

Lemma 6.3 (state feedback): The pair $(E, A(\theta) + BK(\theta))$ is admissible and \mathfrak{D} stable if there exist matrices $\alpha > 0, P \in \mathbb{R}^{n \times n}$, $S \in \mathbb{R}^{(n-r) \times (n-r)}$, $L(\theta) \in \mathbb{R}^{m \times n}$ and $H(\theta) \in \mathbb{R}^{m \times (n-r)}$ such that for all θ :

$$\alpha \otimes (EPE^T) + \beta \otimes (A(\theta)PE^T + BL(\theta)E^T) + I_q \otimes (A(\theta)USV^T + BH(\theta)V^T) + \star < 0 \quad (6-9)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$ respectively. Then the controller gain K is given by:

$$K(\theta) = (L(\theta)E^T + H(\theta)V^T)(PE^T + USV^T)^{-1}$$

Lemma 6.4: The system pair $(E, A(\theta) + BK(\theta), R, C_z)$ is admissible and satisfies $\|G(\theta, s)\|_\infty < \gamma$ if there exist matrices $P > 0, P \in \mathbb{R}^{n \times n}$, $S \in \mathbb{R}^{(n-r) \times (n-r)}$, $L(\theta) \in \mathbb{R}^{m \times n}$ and $H(\theta) \in \mathbb{R}^{m \times (n-r)}$ such that for all θ :

$$\begin{bmatrix} \Delta & R & (PE^T + USV^T)^T C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-10)$$

with:

$$\Delta = A(\theta)(PE^T + USV^T) + B(L(\theta)E^T + H(\theta)V^T) + \star$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$ respectively. Then the gain K is given by:

$$K = (L(\theta)E^T + H(\theta)V^T)(PE + USV^T)^{-1}$$

In comparison with LTI descriptor systems, it can be observed that the number of LMIs is infinite for arbitrary θ in the LMI descriptions given in Lemma 6.1-6.4. Fortunately, for polytopic or affine LPV systems, the above requirement for an infinite set of LMIs can be transformed to finite dimensional LMIs, with ease of solution using the MATLAB LMITOOL box (Gahinet, Nemirovski, Laub and Chilali, 1995) or YALMIP (Lofberg, 2004). It is worth pointing out that combination of Lemma 6.3 & 6.4 with suitable parameters can be used to achieve a multi-objective design.

Remark 6.1: It can be seen that the above conditions are sufficient but not necessary. That is because of the parameter-independent Lyapunov function adopted through the above Lemmas which could lead to some conservative with reduced computational complexity. Maybe, a parameter-dependent Lyapunov function has to be adopted if no results are found satisfying the above conditions.

6.2.2 LPV observer design

Without fault considered in the system of (6-3) and (6-4), the Subsection considered the design of an LPV descriptor observer in the following form:

$$E\dot{\hat{x}} = (A(\theta) + L(\theta)C)\hat{x} + Bu - Ly \quad (6-11)$$

where \hat{x} is the estimate of x . It is well known that the observer design problem is a dual problem of the state feedback design problem. Hence, without proof, the following result is given as a background to the design of a descriptor LPV observer for LPV descriptor systems.

Lemma 6.5: The LPV descriptor system $(E, A(\theta), C)$ is observable if the following conditions hold:

$$A6.1) \text{rank} \left(\begin{bmatrix} sE - A(\theta) \\ C \end{bmatrix} \right) = ns > 0, s \text{ is finite}$$

$$A6.2) \text{rank} \left(\begin{bmatrix} E \\ C \end{bmatrix} \right) = n$$

Define the H_∞ performance variable as $z = C_e(x - \hat{x})$, then the transfer function from disturbance d to z is $G_e(\theta, s) = C_e(sE - A(\theta) - L(\theta)C)^{-1}(R + L(\theta)D)$. Lemma 6.6 is

used to propose LMI description for the design of an LPV descriptor observer, subject to \mathfrak{D} stable and robustness requirements.

Lemma 6.6: The observer pair $(E, A(\theta) + L(\theta)C)$ is \mathfrak{D} stable and $\|G_e(\theta, s)\|_\infty < \gamma$ if there exist matrices $S \in \mathbb{R}^{n \times n} > 0$, $W \in \mathbb{R}^{(n-r) \times (n-r)}$, $J(\theta) \in \mathbb{R}^{n \times h}$ and $H(\theta) \in \mathbb{R}^{(n-r) \times h}$ such that:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & C_e^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-12)$$

with:

$$\begin{aligned} \Delta_{11} &= (E^T S + U W V^T) A(\theta) + (E^T J(\theta) + U H(\theta)) C + \star \\ \Delta_{12} &= (E^T S + U W V^T) R + (E^T J(\theta) + U H(\theta)) D \\ \alpha \otimes (E^T S E) + \beta \otimes (E^T S A(\theta) + E^T J(\theta) C) + I_q \otimes (U W V^T A(\theta) + U H(\theta) C) + \star &< 0 \end{aligned} \quad (6-13)$$

where U and V are full column rank and contain the basis vectors for $\text{Ker}(E)$ and $\text{Ker}(E^T)$, respectively. Then the observer gain can be calculated as:

$$L(\theta) = (E^T S + U W V^T)^{-1} (E^T J(\theta) + U H(\theta)).$$

6.2.3 Implementation of a descriptor LPV observer

Referring to the implementation problem for descriptor observers described in Section 3.3, the implementation problem for the LPV descriptor system case follows as a natural extension. However, the equivalence property of an LTI system does not hold in the LPV system case, as discussed in (Cobb, 2006; Bouali, Yagoubi and Chevrel, 2008). Fortunately, the concept of strong equivalence is proposed in (Bouali, Yagoubi and Chevrel, 2008) for two realizations of rational descriptor systems which share the same admissibility.

Definition 6.1: Two LPV realizations $(E, A(\theta))$ and $(\bar{E}, \bar{A}(\theta))$ are strongly equivalent if there exist two continuously differentiable functions $M(\theta)$ and $N(\theta)$ such that:

- 1) $M(\theta)$ and $N(\theta)$ are non-singular matrices

- 2) The inverse of $M(\theta)$ and $N(\theta)$ are continuously differentiable and the following equations hold:

$$M(\theta)EN(\theta) = \bar{E}$$

$$M(\theta)AN(\theta) = \bar{A}$$

$$M(\theta)E \frac{d}{dt} N(\theta) = 0$$

In the following, a procedure is described to find a standard realization of a descriptor LPV observer presented as a straightforward extension of the one used in Section 3.3. For the convenience of discussion, the following variable is defined following (6-11):

$$z = I\hat{x} \quad (6-14)$$

By taking a coordinate transformation $x = N_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^{n-r}$, the observer system is strongly equivalent to:

$$\dot{x}_1 = A_{11}(\theta)x_1 + A_{12}(\theta)x_2 + B_1u - L_1y \quad (6-15)$$

$$0 = A_{21}(\theta)x_1 + A_{22}(\theta)x_2 + B_2u - L_2y \quad (6-16)$$

$$z = C_{z1}(\theta)x_1 + C_{z2}(\theta)x_2 \quad (6-17)$$

where:

$$M_1EN_1 = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, M_1A(\theta)N_1 = \begin{bmatrix} A_{11}(\theta) & A_{12}(\theta) \\ A_{21}(\theta) & A_{22}(\theta) \end{bmatrix}, M_1B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, CN_1 = [C_1 \quad C_2]$$

M_1 and N_1 can be calculated as in the LTI descriptor system case using singular value decomposition. As M_1 and N_1 are constant matrices, the system of (6-15)-(6-17) are strongly equivalent with the system of (6-11) and (6-14).

If $A_{22}^{-1}(\theta)$ exists, the following choices for invertible $M(\theta)$ and $N(\theta)$ lead to an efficient way to calculating the equivalent form:

$$M(\theta) = \begin{bmatrix} I & -A_{12}(\theta)A_{22}^{-1}(\theta) \\ 0 & A_{22}^{-1}(\theta) \end{bmatrix}, N(\theta) = \begin{bmatrix} I & 0 \\ -A_{22}^{-1}(\theta)A_{21}(\theta) & I \end{bmatrix}$$

$$M^{-1}(\theta) = \begin{bmatrix} I & A_{12}(\theta) \\ 0 & A_{22}(\theta) \end{bmatrix}, N^{-1}(\theta) = \begin{bmatrix} I & 0 \\ A_{22}^{-1}(\theta)A_{21}(\theta) & I \end{bmatrix}$$

It can be verified that:

$$M(\theta) \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N(\theta) = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},$$

$$M(\theta) \begin{bmatrix} A_{11}(\theta) & A_{12}(\theta) \\ A_{21}(\theta) & A_{22}(\theta) \end{bmatrix} N(\theta) = \begin{bmatrix} A_{11}(\theta) - A_{12}(\theta)A_{22}^{-1}(\theta)A_{21}(\theta) & 0 \\ 0 & I \end{bmatrix},$$

$$M(\theta)E \frac{d}{dt} N(\theta) = 0$$

Hence, the observer system of (6-11) and (6-14) is strongly equivalent with:

$$\dot{\xi}_1 = A_r(\theta)\xi_1 + B_r u - L_r y \quad (6-18)$$

$$0 = \xi_2 + A_{22}^{-1}(\theta)B_2 u - A_{22}^{-1}(\theta)L_2 y \quad (6-19)$$

$$z = C_{r1}\xi_1 + C_{r2}\xi_2 \quad (6-20)$$

where:

$$A_r(\theta) = A_{11}(\theta) - A_{12}(\theta)A_{22}^{-1}(\theta)A_{21}(\theta)$$

$$B_r(\theta) = B_1 - A_{12}(\theta)A_{22}^{-1}(\theta)B_2$$

$$L_r(\theta) = L_1(\theta) - A_{12}(\theta)A_{22}^{-1}(\theta)L_2(\theta)$$

$$C_{r1} = C_{z1} - C_{z2}A_{22}^{-1}(\theta)A_{21}(\theta), C_{r2} = C_{z2}$$

This leads to:

$$\dot{\xi}_1 = A_r(\theta)\xi_1 + B_r u - L_r y \quad (6-21)$$

$$z = C_{r1}\xi_1 - C_{r2}A_{22}^{-1}(\theta)B_2 u + C_{r2}A_{22}^{-1}(\theta)L_2 y \quad (6-22)$$

It can be seen that (6-21) and (6-22) is a standard system and can be implemented within a standard LPV framework. One potential problem is that $A_{22}(\theta)$ maybe is not invertible for a general descriptor system. Apart from the case of constant $A_{22}(\theta)$ or when $A_{22}(\theta)$ has a special structure with analytical inverse, an additional problem arises due to the on-line computational complexity of $A_{22}^{-1}(\theta)$ which increases dramatically with increase in dimension.

6.3 Simultaneous fault and state estimation

This Section focuses on the extension to LPV descriptor systems of two previously discussed approaches (the ESO and the PD-ESO) to extend their capability of simultaneous estimating faults and states.

6.3.1 The ESO design for LPV systems

Following the same assumption as in Chapters 3, 4 & 5 the fault signal f in (6-3)-(6-4) is assumed to be slowly-varying. An extension that is appropriate to the LPV case is stated in the Remark 6.2 in the end of this Subsection. For the case f slowly time-varying, the original system can be augmented in the following form:

$$E_a \begin{bmatrix} \dot{x} \\ \dot{f} \end{bmatrix} = A_a(\theta) \begin{bmatrix} x \\ f \end{bmatrix} + B_a u + R_a d \quad (6-23)$$

$$y = C_a \begin{bmatrix} x \\ f \end{bmatrix} + D d \quad (6-24)$$

where:

$$E_a = \begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix}, A_a(\theta) = \begin{bmatrix} A(\theta) & F_f \\ 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_a = [C \quad D_f], R_a = \begin{bmatrix} R \\ 0 \end{bmatrix} \quad (6-25)$$

The following condition is assumed for the design of the ESO and PD-ESO LPV representations of the descriptor system:

$$A6.3) \text{rank} \begin{bmatrix} A(\theta) & F_f \\ C & D_f \end{bmatrix} = n + p$$

Lemma 6.7: The augmented system (E_a, A_a, C_a) is observable with Assumptions A6.1)-A6.3).

Proof:

$$\begin{aligned} \text{rank} \left(\begin{bmatrix} E_a \\ C_a \end{bmatrix} \right) &= \text{rank} \left(\begin{bmatrix} E & 0 \\ 0 & I_p \\ C & D_f \end{bmatrix} \right) = \text{rank} \left(\begin{bmatrix} E \\ C \end{bmatrix} \right) + p = n + p \\ \text{rank} \left(\begin{bmatrix} sE_a - A_a(\theta) \\ C_a \end{bmatrix} \right) &= \text{rank} \left(\begin{bmatrix} sE - A(\theta) & F_f \\ 0 & sI_p \\ C & D_f \end{bmatrix} \right) = \\ & \begin{cases} \text{rank} \begin{bmatrix} A(\theta) & F_f \\ C & D_f \end{bmatrix} = n + p & s = 0 \\ \text{rank} \left(\begin{bmatrix} sE - A(\theta) \\ C \end{bmatrix} \right) + p = n + p & s \neq 0 \end{cases} = n + p \end{aligned}$$

Hence, the system of (E_a, A_a, C_a) is observable. ■

An LPV ESO is proposed in the following form if Assumptions A6.1), A6.2) and A6.3) are satisfied:

$$E_a \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{f}} \end{bmatrix} = A_a(\theta) \begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix} + B_a u + \begin{bmatrix} L_x(\theta) \\ L_f(\theta) \end{bmatrix} (\hat{y} - y) \quad (6-26)$$

$$\hat{y} = C_a \begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix} \quad (6-27)$$

where \hat{x} , \hat{y} and \hat{f} are estimates of the system states, outputs and unknown input signals respectively. $E_a, A_a(\theta), B_a, C_a$ are given as in (6-25). The two matrices $L_x(\theta)$ and $L_f(\theta)$ are to be determined.

Define $e_x = x - \hat{x}$ and $e_f = f - \hat{f}$. It is assumed $\dot{\hat{f}} = 0$, hence:

$$\dot{e}_f = -\dot{\hat{f}} = -L_f(\theta)C_e e_x$$

An augmented error system is obtained as:

$$E_a \begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} = A_o(\theta) \begin{bmatrix} e_x \\ e_f \end{bmatrix} + (R_a + L_p(\theta)D)d \quad (6-28)$$

where $A_o(\theta)$ and $L_p(\theta)$ are defined in (6-29) with $E_a, A_a(\theta), C_a, R_a$ given as in (6-25).

$$A_o(\theta) = A_a(\theta) + L_p(\theta)C_a, L_p(\theta) = \begin{bmatrix} L_x(\theta) \\ L_f(\theta) \end{bmatrix} \quad (6-29)$$

With the help of Lemma 6.6, Theorem 6.1 can be obtained directly.

Theorem 6.1: The observe $(E_a, A_a(\theta) + L_p(\theta)C_a)$ is \mathfrak{D} stable and $\|C_{ze}(SE_a - A_a(\theta) - L_p(\theta)C_a)^{-1}R_a\|_\infty < \gamma$ if there exist matrices $S \in \mathbb{R}^{(n+p) \times (n+p)} > 0$, $W \in \mathbb{R}^{(n-r) \times (n-r)}$, $J(\theta) \in \mathbb{R}^{(n+p) \times h}$ and $H(\theta) \in \mathbb{R}^{(n-r) \times h}$ such that:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & C_{ze}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-30)$$

with:

$$\Delta_{11} = (E_a^T S + U_a W V_a^T) A_a(\theta) + (E_a^T J(\theta) + U_a H(\theta)) C_a + \star$$

$$\Delta_{12} = (E_a^T S + U_a W V_a^T) R_a + (E_a^T J(\theta) + U_a H(\theta)) D$$

$$\alpha \otimes (E_a^T S E_a) + \beta \otimes (E_a^T S A_a(\theta) + E_a^T J(\theta) C_a) + I_q \otimes (U_a W V_a^T A_a(\theta) + U_a H(\theta) C_a) + \star < 0 \quad (6-31)$$

where U_a and V_a are full column rank and contain the basis vectors for $Ker(E_a)$ and $Ker(E_a^T)$, respectively. Then the gain $L_p(\theta)$ is given by:

$$L_p(\theta) = (E_a^T S + U_a W V_a^T)^{-1} (E_a^T J(\theta) + U_a H(\theta))$$

Remark 6.2 The original system can be augmented first if the unknown input signal f has fast variation using a multi-augmentation technique first introduced in the LTI PMIO estimator of (Gao, Ding and Ma, 2007). If the q th derivative of the f , i.e. $f^{(q)}$ is slowly time-varying, the original system of (6-3) and (6-4) can be augmented as:

$$\left. \begin{aligned} E\dot{x} &= A(\theta)x + Bu + F_f f + Rd \\ \dot{f} &= \delta_1 \\ &\dots \\ \dot{\delta}_{q-1} &= f^{(q)} \\ y &= Cx \end{aligned} \right\} \quad (6-32)$$

which can be reorganized in matrix form as:

$$\begin{bmatrix} E & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{f} \\ \dot{\delta}_1 \\ \dots \\ \dot{\delta}_{q-1} \end{bmatrix} = \begin{bmatrix} A(\theta) & F_f & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x \\ f \\ \delta_1 \\ \dots \\ \delta_{q-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} u + \begin{bmatrix} R \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 & 0 & 0 & \dots & 1 \end{bmatrix}^T f^{(q)} \quad (6-33)$$

$$y = [C \quad D_f \quad 0 \quad \dots \quad 0] [x \quad f \quad \delta_1 \quad \dots \quad \delta_{q-1}]^T + Dd \quad (6-34)$$

Then a similar augmented observer (6-26) and (6-27) can be designed with the new system matrices

Remark 6.3: It is worth noting that the Augmentation Method 2 proposed in 3.3.1.2 can be extended to LPV descriptor system as well. Similarly, F_s can be partitioned as $F_s = [F_{sj} \quad F_{si}]$ and $rank(F_{si}) = s_i$, when the Assumptions A6.1)-A6.3) are satisfied and subject to Assumptions A6.4) & A6.5) as follows:

$$\text{A6.4): } rank \left(\begin{bmatrix} E & 0 \\ C & F_{si} \end{bmatrix} \right) = n + s_i$$

$$\text{A6.5): } \text{rank} \left(\begin{bmatrix} sE - A(\theta) & 0 \\ C & F_{si} \end{bmatrix} \right) = n + s_i, s \in \mathbb{C}, s \text{ is finite.}$$

Following the Augmentation Method 2 the system matrices (6-23) and (6-24) are given by:

$$E_a = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I_q & 0 & 0 \\ 0 & 0 & I_{p-q-s_i} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_a = \begin{bmatrix} A(\theta) & F_a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D = R_2, C_a = [C \quad 0 \quad F_{sj} \quad F_{si}]$$

The observability of the augmented system is guaranteed if the Assumptions A6.1)-A6.5) are satisfied. The same restriction as discussed in Section 3.3 also applies in the LPV descriptor system context.

6.3.2 LPV PD-ESO design

This Section extends the general PD-ESO design techniques developed in Chapter 4 to LPV descriptor systems. Consider the following LPV descriptor system:

$$E\dot{x} = A(\theta)x + Bu + F_f f + R_1 d_u \quad (6-35)$$

$$y = Cx + D_f f + R_2 d_s \quad (6-36)$$

Similarly to the procedure of PD-ESO design in Chapter 4, the LPV descriptor system can be augmented as follows:

$$E_a \dot{x}_a = A_a(\theta)x_a + B_a u + R_a d_D \quad (6-37)$$

$$y = C_a x_a \quad (6-38)$$

where:

$$x_a = \begin{bmatrix} x \\ \omega \\ f \end{bmatrix}, E_a = \begin{bmatrix} E & 0 & 0 \\ 0 & I_h & 0 \\ 0 & 0 & I_p \end{bmatrix}, A_a = \begin{bmatrix} A(\theta) & 0 & F_f \\ 0 & -\rho I_h & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$$

$$R_a = \begin{bmatrix} R & 0 \\ 0 & I_h \\ 0 & 0 \end{bmatrix}, C_a = [C \quad I_h \quad D_f], d_D = \begin{bmatrix} d_u \\ \rho R_2 d_s + R_2 \dot{d}_s \end{bmatrix}$$

Then an LPV observer in the following form is proposed:

$$E_a \dot{\hat{x}}_a = A_a(\theta) \hat{x}_a + B_a u + L_p(\theta)(\hat{y} - y) + L_d(\theta)(\hat{y} - \dot{y}) \quad (6-39)$$

$$\hat{y} = C_a \hat{x}_a \quad (6-40)$$

where L_p, L_d are to be determined and:

$$L_p(\theta) = \begin{bmatrix} L_x \\ L_\omega \\ L_f \end{bmatrix}, L_d(\theta) = \begin{bmatrix} L_{xd} \\ L_{\omega d} \\ L_{fd} \end{bmatrix} \quad (6-41)$$

Define $e_{x\omega} = \begin{bmatrix} x \\ \omega \end{bmatrix} - \begin{bmatrix} \hat{x} \\ \hat{\omega} \end{bmatrix}$, $e_f = f - \hat{f}$, and $e_{x\omega f} = \begin{bmatrix} e_{x\omega} \\ e_f \end{bmatrix}$. The faults f are assumed to be slowly time varying and the augmented state estimation error system can be obtained as:

$$E_o(\theta) \dot{e}_{x\omega f} = A_o e_{x\omega f} + R_a d_D \quad (6-42)$$

with:

$$A_o(\theta) = A_a(\theta) + L_p(\theta)C_a, E_o(\theta) = E_a - L_d(\theta)C_a$$

Defining the performance variable as $z_{e\omega f} = C_{ze\omega f} e_{x\omega f}$, the dual system of the error system given in (6-42) can be obtained as:

$$(E_a^T - C_a^T L_d^T(\theta)) \dot{z} = (A_a^T(\theta) + C_a^T L_p^T(\theta))z + C_{ze\omega f}^T d_z \quad (6-43)$$

$$y_z = R_a^T z \quad (6-44)$$

With a suitable augmentation, the dual system (6-43) and (6-44) can be transformed to

$$\begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T(\theta) + C_a^T L_p^T(\theta) & -E_a^T + C_a^T L_d^T(\theta) \end{bmatrix} \begin{bmatrix} z \\ z_a \end{bmatrix} + \begin{bmatrix} 0 \\ C_{ze\omega f}^T \end{bmatrix} d_z \quad (6-45)$$

$$y_z = \begin{bmatrix} R_a^T & 0 \end{bmatrix} \begin{bmatrix} z \\ z_a \end{bmatrix} \quad (6-46)$$

Where the following are defined:

$$E_{ea} = \begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, A_{ea} = \begin{bmatrix} 0 & I_{n+p+h} \\ A_a^T(\theta) & -E_a^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}, R_{ea} = \begin{bmatrix} 0 \\ C_{ze\omega f}^T \end{bmatrix}$$

$$L_{ea}(\theta) = \begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix}, C_{ea} = \begin{bmatrix} R_a^T & 0 \end{bmatrix}$$

The system of (6-45) and (6-46) can be organized as:

$$E_{ea} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = (A_{ea}(\theta) + B_{ea}L_{ea}(\theta)) \begin{bmatrix} z \\ z_a \end{bmatrix} + R_{ea}d_z \quad (6-47)$$

$$y_z = C_{ea} \begin{bmatrix} z \\ z_a \end{bmatrix} \quad (6-48)$$

The problem of stabilizing the original system with PD-ESO is transferred to a state feedback problem which can be easily solved with the theory presented in Section 6.2. Here a theorem is proposed to design a descriptor system PD-ESO within the LPV framework.

Theorem 6.2: There is an admissible and \mathfrak{D} stable PD-ESO in the form of (6-40) and (6-41) if there exist matrices $P > 0, P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, $S \in \mathbb{R}^{(n+p+h) \times (n+p+h)}$, $L(\theta) \in \mathbb{R}^{h \times (2n+2p+2h)}$ and $H(\theta) \in \mathbb{R}^{h \times (n+h+p)}$ such that:

$$\alpha \otimes (E_{ea}^T P E_{ea}) + \beta \otimes (A_{ea}(\theta) P E_{ea}^T + B_{ea} L(\theta) E_{ea}^T) + I_q \otimes (A_{ea}(\theta) U_{ea} S V_{ea}^T + B_{ea} H(\theta) V_{ea}^T) + \star < 0 \quad (6-49)$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $\text{Ker}(E_{ea})$ and $\text{Ker}(E_{ea}^T)$, respectively. Then PD-ESO gains can be calculated as:

$$\begin{bmatrix} L_p^T(\theta) & L_d^T(\theta) \end{bmatrix} = (L(\theta) E_{ea}^T + H(\theta) V_{ea}^T) (P E_{ea}^T + U_{ea} S V_{ea}^T)^{-1}.$$

The following description is proposed to design a robust descriptor PD-ESO within the LPV system framework:

Theorem 6.3: The error system (6-29) is admissible and satisfies $\|C_{z\omega f}(S E_o - A_o(\theta))^{-1} R_a\|_\infty < \gamma$ if there exist matrices $P \in \mathbb{R}^{(2n+2p+2h) \times (2n+2p+2h)}$, $P > 0, S \in \mathbb{R}^{(n+p+h) \times (n+p+h)}$, $L(\theta) \in \mathbb{R}^{h \times (2n+2p+2h)}$ and $H(\theta) \in \mathbb{R}^{h \times (n+h+p)}$, such that:

$$\begin{bmatrix} \Delta & R & (P E_{ea}^T + U_{ea} S V_{ea}^T)^T C_{z\omega f}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-50)$$

with:

$$\Delta = A_{ea}(\theta) (P E_{ea}^T + U_{ea} S V_{ea}^T) + B_{ea} (L(\theta) E_{ea}^T + H(\theta) V_{ea}^T) + \star$$

where U_{ea} and V_{ea} are full column rank and contain the basis vectors for $\text{Ker}(E_{ea})$ and $\text{Ker}(E_{ea}^T)$, respectively. Then the PD-ESO gain can be calculated as:

$$[L_p^T(\theta) \quad L_d^T(\theta)] = (L(\theta)E_{ea}^T + H(\theta)V_{ea}^T)(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1}.$$

Remark 6.4: Extension of the PD-ESO realization technique to LPV system is quite straightforward. Let $\xi = E_a \hat{x}_a - L_d(\theta)(\hat{y} - y)$, then we can obtain an implementation of (6-40)-(6-39) as given in the following form:

$$\dot{\xi} = A_a(\theta)\hat{x}_a + B_a u + L_p(\theta)(\hat{y} - y) \quad (6-51)$$

$$\hat{x}_a = E_o^{-1}(\theta)\xi - E_o^{-1}(\theta)L_d(\theta)y \quad (6-52)$$

$$\hat{y} = C_a \hat{x}_a \quad (6-53)$$

where $E_o(\theta) = E_a - L_d(\theta)C_a$. The derivatives of outputs are not appeared in the modified PD-ESO and only original coefficient matrices are utilized; therefor the modified PD-ESO is reliable for practical application.

Remark 6.5: Calculating $(E_a - L_d(\theta)C_a)^{-1}$ on line would be a disaster with the increase of matrix dimensions. However, the inversion can be calculated off-line once L_d is parameter independent, which is practical. Inspired by the specific structure of E_{ea} , with the following partitioning:

$$L(\theta) = [L_1(\theta) \quad L_2(\theta)], P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_4 \end{bmatrix}$$

Then it follows that:

$$L(\theta)E_{ea}^T = [L_1(\theta) \quad 0], H(\theta)V_{ea}^T = [0 \quad H(\theta)]$$

$$PE_{ea}^T = \begin{bmatrix} P_1 & 0 \\ P_2^T & 0 \end{bmatrix}, U_{ea}SV_{ea}^T = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}, \begin{bmatrix} P_1 & 0 \\ P_2^T & S \end{bmatrix}^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -S^{-1}P_2^T P_1^{-1} & S^{-1} \end{bmatrix}$$

$$L(\theta)E_{ea}^T + H(\theta)V_{ea}^T = [L_1(\theta) \quad H(\theta)]$$

$$(PE_{ea}^T + U_{ea}SV_{ea}^T)^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -S^{-1}P_2^T P_1^{-1} & S^{-1} \end{bmatrix}$$

Then it follows that:

$$L_d^T(\theta) = H(\theta)S^{-1}$$

Setting H to be parameter independent, a constant gain L_d^T can be obtained. Hence, the calculation of $(E - L_d C_a)^{-1}$ can be carried out offline.

Taking above discussion into account, it is suggested to design LPV PD-ESO for observable LPV descriptor system where $rank(E) = r < n$ and design LPV ESO when $rank(E) = r = n$.

6.4 Active fault tolerant control to LPV systems

This section is devoted to extending the results in Chapter 5 to LPV descriptor system with the ESO and PD-ESO developed in last section. Additive actuator faults and sensor faults are discussed in this Section and the AFTC structure is shown in Figure 6.1. Consider a system with actuator and sensor faults as:

$$E\dot{x} = A(\theta)x + Bu + F_a f_a + Rd \quad (6-54)$$

$$y = Cx + F_s f_s + Dd \quad (6-55)$$

As discussed in Chapter 5, sensor faults are hidden by the previously developed ESO or PD-ESO. For actuator faults, the same AFTC scheme as in Chapter 5 is adopted. That is to use the additive control law with a nominal baseline controller together with FTC.

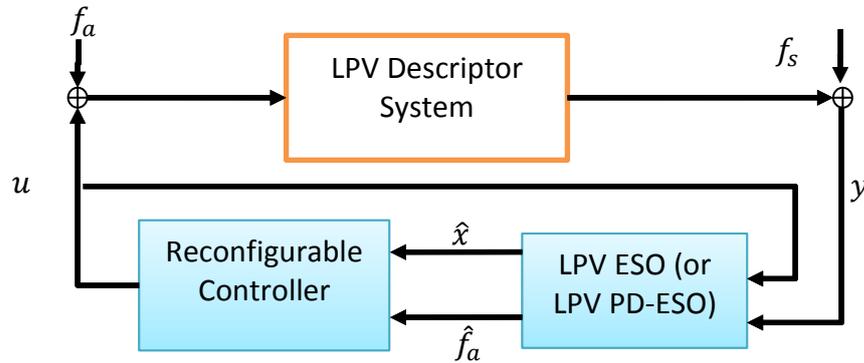


Figure 6.1 An AFTC structure of LPV descriptor systems with LPV observers

Then, based on the estimated fault signals, the controller is given in the form of:

$$u_{FTC} = K(\theta) \hat{x} - K_f \hat{f} \quad (6-56)$$

where $K(\theta) \hat{x}$ is designed to satisfy the nominal performance demand and $K_f \hat{f}$ is used to compensate for the fault influence. Then the closed-loop LPV descriptor system is obtained as:

$$E\dot{x} = A(\theta)x + BK(\theta)\hat{x} - BK_f\hat{f} + F_f f + Rd \quad (6-57)$$

With the same assumption given in Chapter 5, as:

$$\text{rank}[B \quad F_f] = \text{rank}[B]$$

This means, there exists a matrix K_f which satisfies $F_f = BK_f$. That is $K_f = B^\dagger F_f$, where B^\dagger denotes generalized inverse (pseudo-inverse) of B . The closed-loop system is transformed to:

$$E\dot{x} = (A(\theta) + BK(\theta))x + BK(\theta)e_x + F_f e_f + Rd \quad (6-58)$$

where $e_f = f - \hat{f}$, $e_x = x - \hat{x}$ is the estimation error.

With the same controller strategy, the following two subsections detail the AFTC design with ESO and PD-ESO of LPV descriptor system, respectively.

6.4.1 AFTC design with ESO

In this Subsection, the ESO of LPV descriptor system is considered to design integrated AFTC. With the ESO, the estimation error system can be obtained as:

$$E_a \dot{e}_{xf} = A_o(\theta)e_{xf} + (R_a + L_p(\theta)D)d \quad (6-59)$$

where:

$$A_o(\theta) = A_a(\theta) + L_p(\theta)C_a, C_a = [C \quad D_f] \quad (6-60)$$

$$e_{xf} = \begin{bmatrix} e_x \\ e_f \end{bmatrix}, E_a = \begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix}, R_a = \begin{bmatrix} R \\ 0 \end{bmatrix}, L_p(\theta) = \begin{bmatrix} L_x(\theta) \\ L_f(\theta) \end{bmatrix}, A_a(\theta) = \begin{bmatrix} A(\theta) & F_f \\ 0 & 0 \end{bmatrix} \quad (6-61)$$

Then the entire closed-loop system is organized as

$$E_{cl} \begin{bmatrix} \dot{x} \\ \dot{e}_{xf} \end{bmatrix} = A_{cl} \begin{bmatrix} x \\ e_{xf} \end{bmatrix} + R_{cl}d \quad (6-62)$$

where:

$$E_{cl} = \begin{bmatrix} E & 0 \\ 0 & E_a \end{bmatrix}, A_{cl} = \begin{bmatrix} A(\theta) + BK(\theta) & \Delta \\ 0 & A_o(\theta) \end{bmatrix}$$

$$\Delta = [BK(\theta) \quad F_a \quad 0_{n \times (p-q)}], R_{cl} = \begin{bmatrix} R \\ R_a + L_p(\theta)D \end{bmatrix}$$

The structure of the integrated LVP descriptor system is the same as a linear descriptor system. To enhance the robustness of the overall system within an H_∞ framework, consider the closed-loop system (6-63) and define the performance function as:

$$z_{xef} = C_z \begin{bmatrix} \dot{x} \\ \dot{e}_{xf} \end{bmatrix} \quad (6-63)$$

where $C_z = [C_{zx} \ C_{zexf}]$ is a weighting matrix. Then the transfer function from d to z_{xef} is obtained as $G_{cl}(\theta, s) = C_z(sE_{cl} - A_{cl}(\theta))^{-1}R_{cl}$.

Based on the Bound Real Lemma of LPV descriptor system, the closed-loop system of (6-62) and (6-63) is admissible and $\|G_{cl}(\theta, s)\|_\infty < \gamma$ if there exist $\mathcal{P} > 0, \mathcal{S}$ with compatible dimensions such that:

$$\begin{bmatrix} (\mathcal{P}E_{cl}^T + U_{cl}\mathcal{S}V_{cl}^T)A_{cl}(\theta) + \star & (\mathcal{P}E_{cl}^T + U_{cl}\mathcal{S}V_{cl}^T)R_{cl} & C_z^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-64)$$

where U_{cl} and V_{cl} are full column rank and contain the basis vectors for $Ker(E_{cl})$ and $Ker(E_{cl}^T)$, respectively. The de-coupled structure of E_{cl} , means that U_{cl} and V_{cl} can be parameterized as:

$$U_{cl} = \begin{bmatrix} U & 0 \\ 0 & U_a \end{bmatrix}, V_{cl} = \begin{bmatrix} V & 0 \\ 0 & V_a \end{bmatrix}$$

where U_a and V_a are full column rank and contain the basis vectors for $Ker(E_a)$ and $Ker(E_a^T)$, U and V are full column rank and contain the basis vectors for $Ker(E)$ and $Ker(E^T)$ respectively. Here, the variables \mathcal{P} and \mathcal{S} are selected as:

$$\mathcal{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}, \mathcal{S} = \begin{bmatrix} S & 0 \\ 0 & W \end{bmatrix}$$

Then (6-64) can be rewritten as:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & (E^T P + USV^T)R & C_{zx}^T \\ \star & \Delta_{22} & \Delta_{21} & C_{zexf}^T \\ \star & \star & -\gamma & 0 \\ \star & \star & \star & -\gamma \end{bmatrix} < 0 \quad (6-65)$$

with:

$$\Delta_{11}(\theta) = (E^T P + USV^T)(A(\theta) + BK(\theta)) + \star$$

$$\Delta_{12}(\theta) = (E_a^T Q + U_a W V_a^T) [BK(\theta) \quad F_f]$$

$$\Delta_{22}(\theta) = (E_a^T Q + U_a W V_a^T) (A_a(\theta) + L_p(\theta) C_a) + \star$$

$$\Delta_{21} = (E_a^T Q + U_a W V_a^T) (R_a + L_p(\theta) D)$$

In fact, (6-65) is equivalent to:

$$\begin{bmatrix} \Delta_{11}(\theta) & (E^T P + USV^T)R & C_{zx}^T & \Delta_{12}(\theta) \\ \star & -\gamma & 0 & \Delta_{21}^T \\ \star & \star & -\gamma & C_{zexf} \\ \star & \star & \star & \Delta_{22}(\theta) \end{bmatrix} < 0 \quad (6-66)$$

where $\Delta_{11}(\theta), \Delta_{12}(\theta), \Delta_{21}, \Delta_{22}(\theta)$ are given as in (6-65).

The inequality (6-66) can be re-formulated as:

$$\begin{bmatrix} F_c(\theta) & F_{ci}(\theta) \\ F_{ci}^T(\theta) & F_1(\theta) \end{bmatrix} < 0 \quad (6-67)$$

with:

$$F_1(\theta) = (QE_a^T + U_a W V_a^T) (A_a(\theta) + L_p(\theta) C_a) + \star$$

$$F_{ci}(\theta) = \begin{bmatrix} \Delta_{12}(\theta) \\ (R_a + L_p(\theta) D)^T (QE_a^T + U_a W V_a^T) \\ C_{ze} \end{bmatrix}$$

$$F_c(\theta) = \begin{bmatrix} \Delta_{11}(\theta) & (PE^T + USV^T)^T R & C_{zx}^T \\ \star & -\gamma & 0 \\ \star & \star & -\gamma \end{bmatrix}$$

where $\Delta_{12}(\theta), \Delta_{11}(\theta)$ are given as in (6-65).

The two-step design procedure developed in Chapter 5 is adopted here to solve the above descriptor system LPV problem.

Algorithm 6.1:

Step 1: Solve state feedback $K(\theta)$ according to Lemma 6.3 & 6.4 with common determine variables.

Step2: With obtained $K(\theta)$, solve the following LMI problem:

$\min_{P,S,Q,W} \gamma$, subject to (6-67).

6.4.2 AFTC design with PD-ESO

This Subsection discusses the integrated design of a PD-ESO-based AFTC problem with the same AFTC structure as illustrated in Figure 6.1. The closed-loop system with the PD-ESO is obtained as:

$$E\dot{x} = (A(\theta) + BK(\theta))x + BK(\theta)e_x + F_a e_{fa} + R_1 d_u \quad (6-68)$$

where $e_{fa} = f_a - \hat{f}_a$, $e_x = x - \hat{x}$ are the estimation errors. Combing the error system given in (6-42), then the closed-loop system is re-organized as:

$$\mathcal{E}(\theta) \begin{bmatrix} \dot{x} \\ \dot{e}_{x\omega f} \end{bmatrix} = \mathcal{A}(\theta) \begin{bmatrix} x \\ e_{x\omega f} \end{bmatrix} + \mathcal{R}d_D \quad (6-69)$$

$$z = \mathcal{C} \begin{bmatrix} x \\ e_{x\omega f} \end{bmatrix} \quad (6-70)$$

where $\mathcal{C} = [\mathcal{C}_{zx} \quad \mathcal{C}_{ze}]$ is a weighting matrix and:

$$\mathcal{E}(\theta) = \begin{bmatrix} E & 0 \\ 0 & E_o(\theta) \end{bmatrix}, \mathcal{A}(\theta) = \begin{bmatrix} A(\theta) + BK(\theta) & [BK(\theta) \quad F_f] \\ 0 & A_o(\theta) \end{bmatrix}, \mathcal{R} = \begin{bmatrix} R \\ R_a \end{bmatrix}$$

The dual system matrices of $(\mathcal{E}(\theta), \mathcal{A}(\theta), \mathcal{C}, \mathcal{R})$ would be $(\mathcal{E}_d(\theta), \mathcal{A}_d(\theta), \mathcal{C}_d, \mathcal{R}_d)$ with:

$$\mathcal{E}_d(\theta) = \begin{bmatrix} E^T & 0 \\ 0 & E_o^T(\theta) \end{bmatrix}, \mathcal{A}_d(\theta) = \begin{bmatrix} A^T(\theta) + K^T(\theta)B^T & 0 \\ [BK(\theta) \quad F_f]^T & A_o^T(\theta) \end{bmatrix}, \mathcal{C}_d = \begin{bmatrix} R \\ R_a \end{bmatrix}^T$$

$$\mathcal{R}_d = [\mathcal{C}_{zx} \quad \mathcal{C}_{ze\omega f}]^T$$

Following the design of the descriptor system LPV PD-ESO, an integrated design scheme is proposed to the augmented dual system to achieve the desired performance of the original system. The augmented system can be organized as:

$$\mathcal{E}_{da}\dot{z} = \mathcal{A}_{da}(\theta)z + \mathcal{R}_{da}d_z \quad (6-71)$$

$$y_{da} = \mathcal{C}_{da}z \quad (6-72)$$

where:

$$\mathcal{E}_{da} = \begin{bmatrix} E^T & 0 \\ 0 & E_{ea} \end{bmatrix}, E_{ea} = \begin{bmatrix} I_{n+p+h} & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{R}_{da} = \begin{bmatrix} \mathcal{C}_{zx}^T \\ \mathcal{R}_{ea}^T \end{bmatrix}, \mathcal{R}_{ea}^T = \begin{bmatrix} \mathcal{C}_{ze\omega f}^T \\ 0 \end{bmatrix}$$

$$\mathcal{A}_{da}(\theta) = \begin{bmatrix} A^T(\theta) + K^T(\theta)B^T & 0 \\ \Delta(\theta) & A_{ea}(\theta) \end{bmatrix}$$

$$A_{ea}(\theta) = \begin{bmatrix} 0 & I_{n+p+h} \\ A_o^T(\theta) & -E_o^T(\theta) \end{bmatrix}, \Delta(\theta) = \begin{bmatrix} 0 \\ K^T(\theta)B^T \\ F_f^T \end{bmatrix}, B_{ea} = \begin{bmatrix} 0 \\ C_a^T \end{bmatrix}$$

$$L_{ea}(\theta) = [L_p^T(\theta) \quad L_a^T(\theta)], C_{da} = [R^T \quad C_R^T], C_R^T = [0 \quad R_a^T]$$

Based on the robustness condition presented in Chapter 3, it is known that the pair $(\mathcal{E}_{da}, \mathcal{A}_{da}(\theta))$ is admissible for all θ and $\|y_{da}\| < \gamma\|d_z\|$ for all $\|d_z\| \neq 0$ if there exists P with compatible dimensions such that:

$$\mathcal{E}_{da}P = P^T \mathcal{E}_{da}^T \geq 0 \quad (6-73)$$

$$\begin{bmatrix} \mathcal{A}_{da}(\theta)P + P^T \mathcal{A}_{da}^T(\theta) & \mathcal{R}_{da} & P^T C_{da}^T \\ * & -\gamma & 0 \\ * & * & -\gamma \end{bmatrix} < 0 \quad (6-74)$$

Choosing a Lyapunov function with the structure as in (6-75), (6-74) can be reformulated to (6-76):

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \quad (6-75)$$

$$\begin{bmatrix} A^T(\theta)P_1 + K^T(\theta)B^T P_1 + * & * & * & * \\ \Delta(\theta)P_1 & (A_{ea}(\theta) + B_{ea}L_{ea}(\theta))P_2 + * & * & * \\ C_{zx} & R_{ea} & -\gamma & * \\ R^T P_1 & C_R^T P_2 & 0 & -\gamma \end{bmatrix} < 0 \quad (6-76)$$

Parameterizing P_1 and P_2 with $P_1 = W_1 E^T + U S_1 V^T$, $P_2 = W_2 E_{ea}^T + U_{ea} S_2 V_{ea}^T$, (6-76) can be re-organized as:

$$\begin{bmatrix} \Delta_{11}(\theta) & * & * & * \\ \Delta(\theta)(W_1 E^T + U S_1 V^T) & \Delta_{22}(\theta) & * & * \\ C_{zx} & R_{ea} & -\gamma & * \\ R^T W_1 E^T + R^T U S_1 V^T & C_R^T W_2 E_{ea}^T + C_R^T U_{ea} S_2 V_{ea}^T & 0 & -\gamma \end{bmatrix} < 0 \quad (6-77)$$

with:

$$\Delta_{11}(\theta) = (A^T(\theta) + K^T(\theta)B^T)(W_1 E^T + U S_1 V^T) + *, \Delta(\theta) = [BK(\theta) \quad F_a \quad 0_{n \times (p-q)}]$$

$$\Delta_{22}(\theta) = A_{ea}(\theta)W_2 E_{ea}^T + A_{ea}(\theta)U_{ea} S_2 V_{ea}^T + B_{ea}Y(\theta)E_{ea}^T + B_{ea}H(\theta)V_{ea}^T + *$$

It can be seen that (6-77) is an LMI if K is known. Hence the two-step design procedure can be applied to solve (6-77). Then observer gain can be calculated as:

$$L_{ea}(\theta) = (Y(\theta)E_{ea}^T + H(\theta)V_{ea}^T)(W_2E_{ea}^T + U_{ae}S_2V_{ea}^T)^{-1} \quad (6-78)$$

The algorithm proposed previously can be adapted easily to solve this problem. Hence it is not necessary to repeat the design procedure. Additionally, a parameter-independent L_d can be obtained by setting H to be parameter-independent. After the PD-ESO gain is obtained, the AFTC can be implemented with the structure depicted in Figure. 6.1.

6.5 Case study

In this Section, a numerical example shows the procedure of the integrated design for the descriptor system with an AFTC structure with PD-ESO and state feedback control, all within an LPV framework. An integrated AFTC with the ESO is considered and the design procedure is showed in Chapter 7 with application to a nonlinear offshore wind turbine benchmark model.

6.5.1 System model

Consider the following descriptor system example (Bouali, Yagoubi and Chevrel, 2008; Halalchi, Bara and Laroche, 2011):

$$E\dot{x} = A(\theta)x + Bu + F_a f_a + R_1 d \quad (6-79)$$

$$y = Cx + F_s f_s + R_2 d \quad (6-80)$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A(\theta) = \begin{bmatrix} \theta & 2 & 1 \\ 1 & -1 & 0 \\ \theta & -\theta & -2 - \theta \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = [2 \quad 1 \quad 0]^T, F_a = [2 \quad 1 \quad 0]^T, R_1 = [0.1 \quad 0.1 \quad 0]^T, F_s = 0, R_2 = 0$$

where $\theta \in [-1.5 \quad 1.5]$. In this study, only an actuator fault is considered to illustrate of the design procedure.

6.5.2 Controller and observer design

Preliminary: It can be easily obtained that the conditions are satisfied shown as:

$$A1) \text{rank} \left(\begin{bmatrix} sE - A(\theta) \\ C \end{bmatrix} \right) = 3, s > 0 \text{ for all } \theta \in [-1.5 \quad 1.5]$$

$$A2) \text{rank} \begin{pmatrix} E \\ C \end{pmatrix} = 3$$

$$A3) \text{rank} \begin{bmatrix} A & F_f \\ C & D_f \end{bmatrix} = 4$$

Step 1: As there is only one varying parameter, it is necessary and sufficient to use two vertices to cover all the possible systems in a convex set. Based on Step 1 given in the algorithm, two state feedback gains are calculated according to the system structure.

$$K_1 = [0.30089 \quad -3.2382 \quad -0.54922]$$

$$K_2 = [-2.6155 \quad -0.9643 \quad -2.8214]$$

And in this design, the finite eigenvalues are constrained to satisfy $-5 < \text{Re}(\lambda) < -0.5$ to get a suitably fast time response whilst restricting excessively fast responses. The achieved H_∞ performance is $\gamma = 0.4672$.

Step2: In the observer design step, the fault signal is augmented twice in order account for more complex possible faults than just constant faults. The finite eigenvalues are constrained to satisfy $-5 < \text{Re}(\lambda) < -1.5$. However, the convergence rate can be adjusted to satisfy the time response requirements. Considering the discussion in Section 6.3, a constant derivative gain is designed. With the obtained controller gains in last step, a set of LMIs are solved and the observer gains are calculated as:

$$L_{p1} = \begin{bmatrix} 39.056 & 1.1212 & 90.334 & 0.25934 & 0.68652 \\ -154.14 & -29.329 & -229.38 & -149.75 & -202.34 \end{bmatrix}^T$$

$$L_{p2} = \begin{bmatrix} 37.376 & 1.0861 & 90.196 & 0.0044804 & 0.35453 \\ -153.86 & -29.172 & -226.5 & -149.07 & -201.41 \end{bmatrix}^T$$

$$L_d = \begin{bmatrix} 9.8783 & 0.88448 & 28.82 & -3.437 & -2.7463 \\ -42.219 & -9.5686 & -73.954 & -26.931 & -31.457 \end{bmatrix}^T$$

The achieved H_∞ performance is $\gamma = 1.5931$.

6.5.3 Simulation results

A MATLAB/Simulink based simulation is carried out to evaluate the proposed design scheme. The original system is implemented based on an input-output equivalence (Bouali, Yagoubi and Chevrel, 2008). The observer is implemented with the equivalence form proposed in Section 6.3.2. As the system dynamics will influence the

observer performance, the fault estimation performs much better after 20s when the AFTC activated, as shown in Figure 6.4.

The disturbance is assumed to be a zero-mean Gaussian distributed band-limited white noise, parameterized with noise power=0.001 and sample time=0.01.

From the simulation results Figures 6.2-6.3, it can be seen that the proposed LPV AFTC scheme is feasible. To illustrate the advantage of the proposed scheme, with and without the AFTC activated two scenarios are considered. The simulation results are given in Figures 6.4-6.5. It is clear that the AFTC can improve the system performance dramatically.

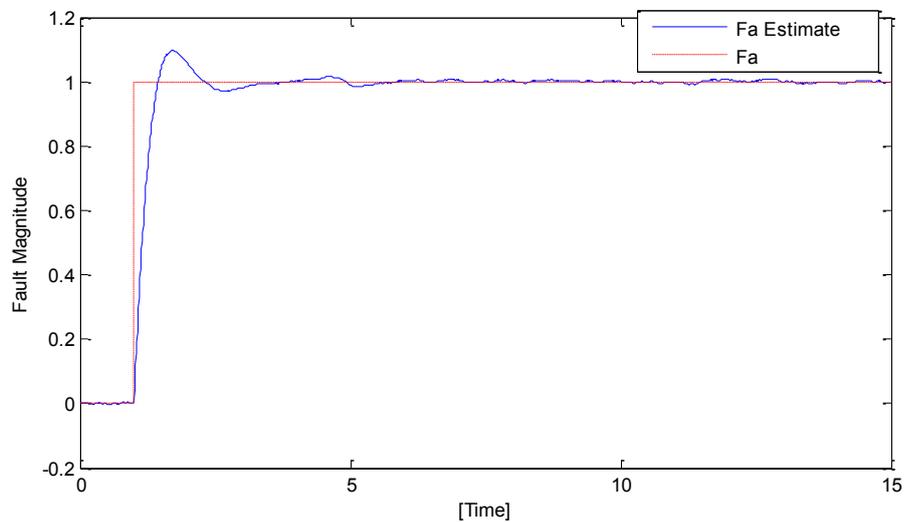


Figure 6.2 Fault signal and its estimate with AFTC activated

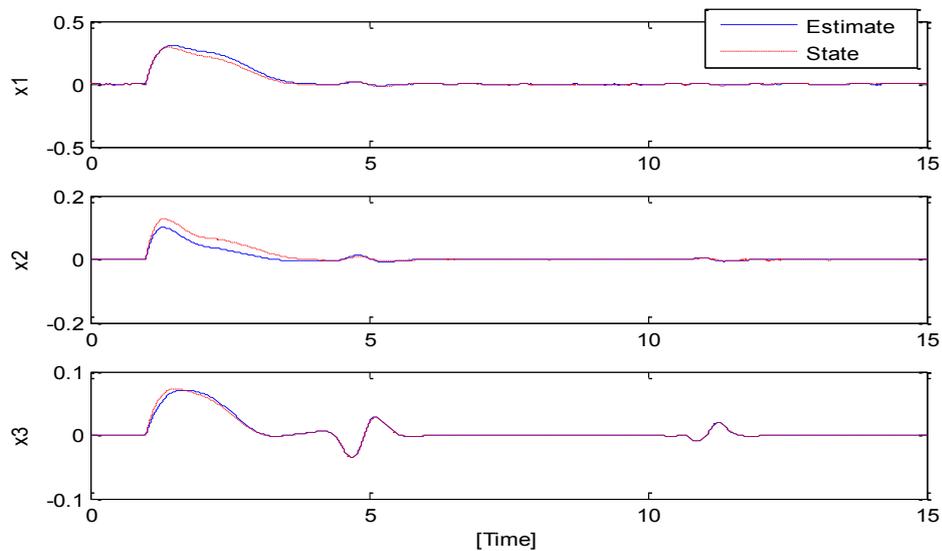


Figure 6.3 System states and their estimates with AFTC activated

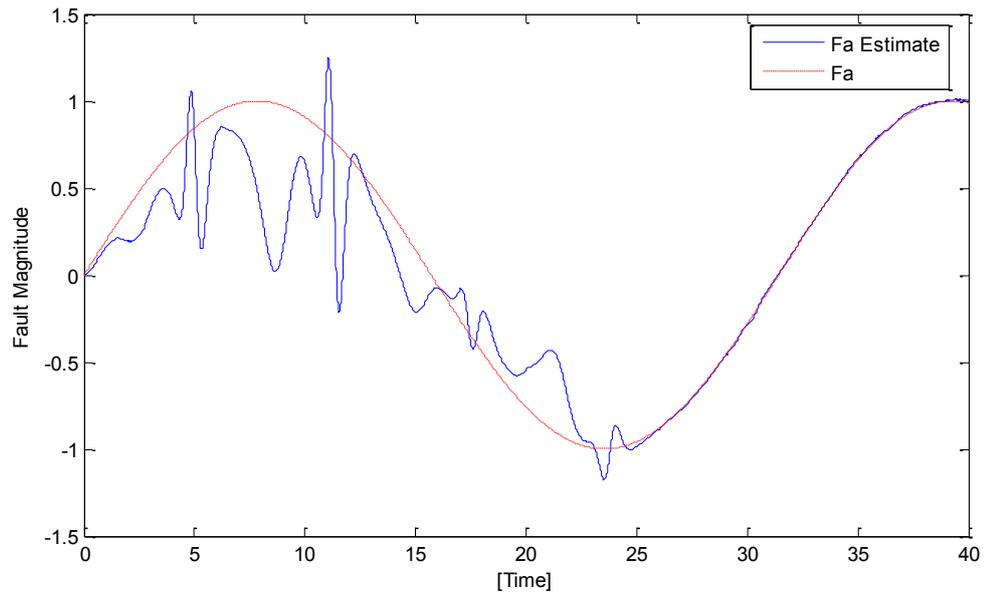


Figure 6.4 Fault signal and its estimate before and after AFTC activated at 20s

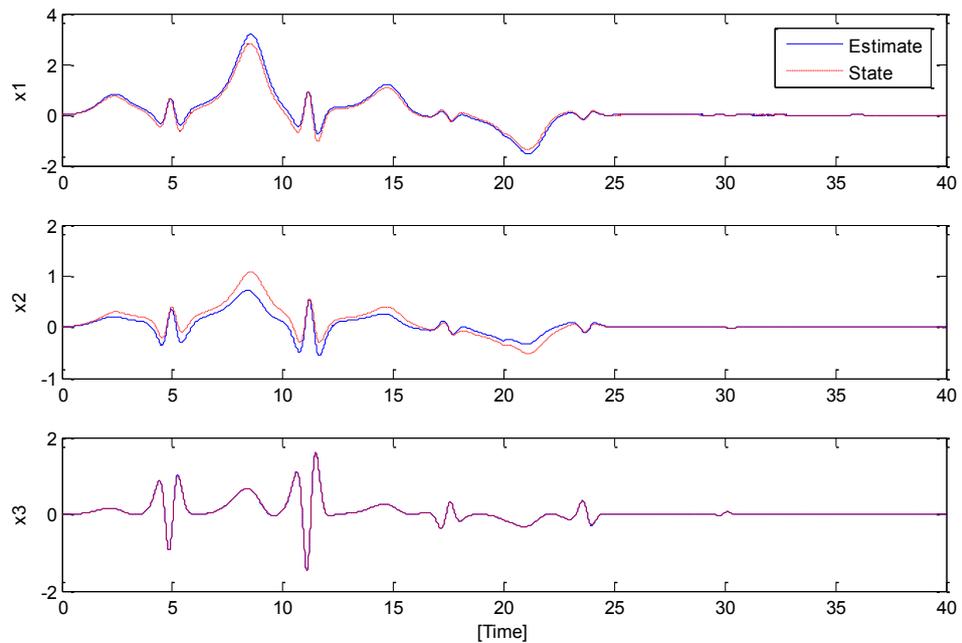


Figure 6.5 States & their estimates before and after AFTC activated at 20s

6.6 Discussion and conclusion

The design approaches for descriptor systems that have combined state estimate feedback control with a descriptor full-order observer are discussed in this Chapter based on Lyapunov theory, quadratic stability and quadratic H_∞ performance.

The ESO and PD-ESO design approaches of Chapters 3&4 are then extended to develop an LPV formulation for a linear time-varying descriptor system with parameter dependence. The pole-placement constraints and robustness to disturbance are

considered both in the controller and observer designs. The integrated AFTC design approach proposed in Chapter 5 is also extended to LPV descriptor systems. A numerical tutorial example is given to illustrate the design procedure.

Chapter 7 uses the theory developed in this Chapter considering a nonlinear offshore wind turbine benchmark system as a suitably real application example.

Chapter 7: Application to a wind turbine benchmark system

7.1 The need for AFTC of wind turbine systems

As an economically, socially as well as ecologically sustainable renewable energy, wind energy is attracting more and more attention along with the increasing awareness of protection our global environment and depletion of fossil resources. As a result, wind turbines are contributing larger and larger part of energy production in now days (Gsänger and Pitteloud, 2013) as shown in Figure 7.1, along with the increasing size of the standard wind turbine systems.

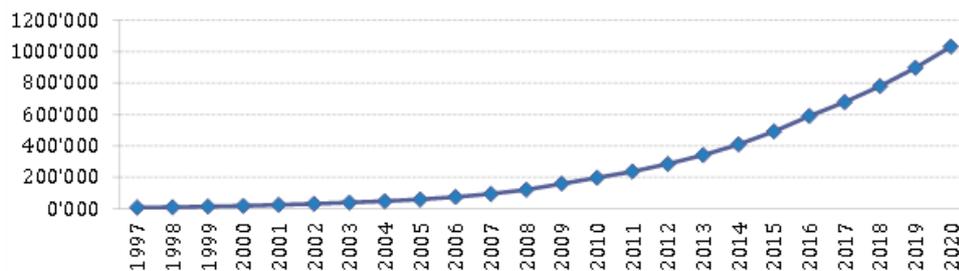


Figure 7.1 Total installed capacity of wind turbine during 1997-2020 [MW] (Gsänger and Pitteloud, 2013)

Wind turbines installed recently are expensive and far from living zones, which escalate the requirements of safety, reliability and maintainability (Bianchi, Battista and Mantz, 2007; Esbensen and Sloth, 2009; Laks, Pao and Wright, 2009; Pao and Johnson, 2011). An attractive candidate solution is to introduce fault detection and FTC techniques. As the fault detection and accommodation techniques in industry are simple and conservative, new developed and advanced fault detection and FTC techniques can improve the performance of the overall system.

This Chapter is devoted to FE and FTC of an offshore wind turbine benchmark system based on the one proposed in (Odgaard, Stoustrup and Kinnaert, 2009). This chapter adapts the descriptor system AFTC scheme within LPV framework to the wind turbine benchmark which is naturally nonlinear. The system model is depicted in Section 7.2 and base line controller is introduced in Section 7.3. Since it is common to demand to

retain the practically proved baseline controller when a more advance control scheme is employed, a bit modification is made on the previously proposed design approach in Section 7.4. Some practical problem are studied and discussed as well in this section. Section 7.5 shows the simulation results for different faults, including sensor faults, and actuator faults.

7.2 Wind turbine system description

A typical wind turbine can be depicted as in Figure 7.2. A benchmark wind turbine model described by (Odgaard, Stoustrup and Kinnaert, 2009) is considered. The purpose of the benchmark is to compare and evaluate FDI and fault accommodation designs, as well as FTC schemes with view to selecting the most promising approaches for real wind turbine system applications. The benchmark model is of a three blade horizontal wind turbine which consists of static aerodynamic, drive train, generator, converter and pitch systems.

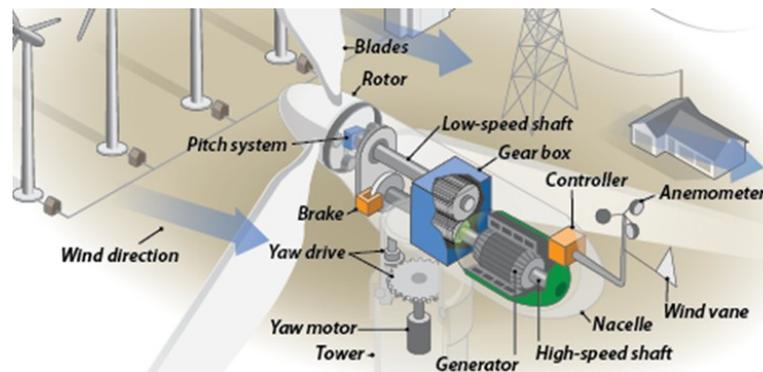


Figure 7.2 A typical wind turbine structure (Anonymous, 2013)

The goal of this study is to develop an AFTC control scheme of the wind turbine system. The wind turbine benchmark system has several faults which effectively act in different subsystems. For the purpose of design (estimation and control), an LPV model is used obtained by linearizing the non-linear wind turbine system along a suitable operating state trajectory dependent on the wind speed as scheduling parameter. Hence, the modelling uncertainty can be considered to arise mainly from uncertainty in the knowledge of the wind speed, since the effective wind speed in the rotor system is not the same as the anemometer measurement which is assumed in the benchmark.

7.2.1 Aerodynamics

The aerodynamics of the wind turbine is modelled as an aerodynamic torque $T_r(t)$ acting on the rotor blades, represented by:

$$T_r(t) = \sum_{i=1}^3 \frac{\rho \pi R^3 C_q(\lambda(t), \beta_i(t)) v_\omega^2(t)}{6} \quad (7-1)$$

where C_q is the torque coefficient table described by Figure 7.3, $\beta_i(t)$ is the pitch angle for the i^{th} rotor blade, where $i = 1, 2, 3$. ρ is the air density; R is the radius of the area swapped by the blades; $v_\omega(t)$ is the effective wind speed. This model is valid for small differences between the $\beta_i(t)$ values. When $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are equal, $T_r(t)$ is then rewritten as:

$$T_r(t) = \frac{1}{2} \rho \pi R^3 C_q(\lambda, \beta) v_\omega^2 = K C_q(\lambda, \beta) v_\omega^2 \quad (7-2)$$

Another important parameter is power coefficient table $C_p(\lambda, \beta)$, which has a relationship with $C_q(\lambda, \beta)$ (Bianchi, Battista and Mantz, 2007) as:

$$C_p(\lambda, \beta) = \lambda C_q(\lambda, \beta)$$

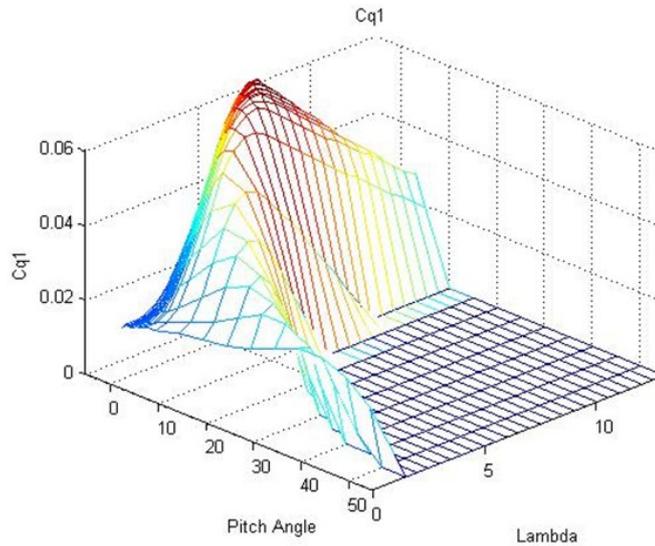


Figure 7.3 Rotor aerodynamic torque coefficient table

7.2.2 Drive train

The drive train is described as the following linear system:

$$\begin{bmatrix} \dot{\omega}_r(t) \\ \dot{\omega}_g(t) \\ \dot{\theta}_\Delta(t) \end{bmatrix} = \begin{bmatrix} -\frac{B_{dt}+B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{-K_{dt}}{J_r} \\ \frac{\eta_{dt} B_{dt}}{N_g J_g} & \frac{-\eta_{dt} B_{dt} - B_g}{N_g^2} & \frac{\eta_{dt} K_{dt}}{N_g J_g} \\ 1 & -\frac{1}{N_g} & 0 \end{bmatrix} \begin{bmatrix} \omega_r(t) \\ \omega_g(t) \\ \theta_\Delta(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{J_r} & 0 \\ 0 & -\frac{1}{J_g} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_r(t) \\ T_g(t) \end{bmatrix} \quad (7-3)$$

where is J_r the moment of inertia of the low speed shaft, K_{dt} is the torsion stiffness of the drive train, B_{dt} is the torsion damping coefficient of the drive train, B_g is the viscous friction of the high speed shaft, N_g is the gear ratio, J_g is the moment of inertia of the high speed shaft, η_{dt} is the efficiency of the drive train, and $\theta_\Delta(t)$ is the torsion angle of the drive train. The potential faults in this subsystem include faults acting in the generator and turbine rotor speeds.

7.2.3 Generator and convertor systems

The converter dynamics can be modelled by a first order transfer function.

$$\frac{T_g(s)}{T_{g,r}(s)} = \frac{\alpha_{gc}}{s + \alpha_{gc}} \quad (7-4)$$

where α_{gc} is the time parameter of the generator subsystem. The power produced by the generator is given by:

$$P_g(t) = \eta_g \omega_g(t) T_g(t) \quad (7-5)$$

The potential fault in this subsystem is an offset actuator fault.

7.2.4 Pitch system

The hydraulic pitch system is modelled as a closed-loop transfer function. In principle these are position servo systems which can be modelled quite well by a second order transfer function (Odgaard, Stoustrup and Kinnaert, 2009) as follows:

$$\frac{\beta(s)}{\beta_r(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (7-6)$$

where ω_n and ζ are the frequency and damping ration parameters, respectively. A drop of oil pressure will change the dynamics of the pitch systems. The pressure level is modelled as a convex combination of the vertices of the two parameters ω_n^2 and $\zeta\omega_n$. Hence the pitch system can be described in terms of the so-called fault effectiveness

parameter $\theta_f(t) \in [0 \ 1]$, where as $\theta_f(t) = 0$ corresponds to a fault-free actuator with $\omega_n^2 = \omega_{n0}^2, \zeta\omega_n = \zeta_0\omega_{n0}$, $\theta_f(t) = 1$ corresponds to a full fault on the actuator with $\omega_n^2 = \omega_{nf}^2, \zeta\omega_n = \zeta_f\omega_{nf}$. Hence, the parameters ω_n^2 and $\zeta\omega_n$ can be described in terms of the pitch actuator fault, as:

$$\omega_n^2 = (1 - \theta_f(t))\omega_{n0}^2 + \theta_f(t)\omega_{nf}^2 \quad (7-7)$$

$$\zeta\omega_n = (1 - \theta_f(t))\zeta_0\omega_{n0} + \theta_f(t)\zeta_f\omega_{nf} \quad (7-8)$$

From a mathematical standpoint there are no unique state space realizations for a given input-output transfer function. However, it is important here to use a state variable system based on the parameters ω_n^2 and $\zeta\omega_n$, as follows:

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \beta_r \quad (7-9)$$

$$y = [1 \ 0] \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} \quad (7-10)$$

Further, it can be rewritten as:

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_{n0}^2 & -2\zeta_0\omega_{n0} \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{n0}^2 \end{bmatrix} \beta_r + F_\beta f_\beta \quad (7-11)$$

$$f_\beta = [\omega_{nf}^2 - \omega_{n0}^2 \quad 2\zeta_f\omega_{nf} - 2\zeta_0\omega_{n0}]x_\beta\theta_f + (\omega_{nf}^2 - \omega_{n0}^2)\beta_r\theta_f, F_\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Remark 7.1: One benefit of above transformation is to simplify the design of the observer based AFTC system. However, the potential problem is that the new signal f_β may not be able to reflect well enough the severity of the original fault θ_f . One way to recover the original fault signal is:

$$\theta_f = \frac{f_\beta}{[\omega_{nf}^2 - \omega_{n0}^2 \quad 2\zeta_f\omega_{nf} - 2\zeta_0\omega_{n0}]x_\beta + (\omega_{nf}^2 - \omega_{n0}^2)\beta_r}$$

if $[\omega_{nf}^2 - \omega_{n0}^2 \quad 2\zeta_f\omega_{nf} - 2\zeta_0\omega_{n0}]x_\beta + (\omega_{nf}^2 - \omega_{n0}^2)\beta_r \neq 0$. To improve the original fault estimation accuracy, one can calculate the estimated fault over a time window of length t_Δ instead of each sampling time. In this way, the error introduced by the disturbance or noise will be reduced effectively. Moreover, the introduction of this time window can avoid singularity in the fault calculation.

7.3 Baseline controller design

The control system plays an important role in the operation of the wind turbine (Leithead, de la Salle and Reardon, 1991) and different control strategies may be considered for different wind turbines systems. As discussed in (Odgaard, Stoustrup and Kinnaert, 2009), medium and large-scale wind turbines, which are variable speed and variable pitch wind turbines, are generally designed to work in two regions –the low wind speed region (sometimes known as the partial load region) and the high wind speed region (sometimes known as the full load region). The control objective is to catch as much energy as possible in the partial load region, while the objective in the full load region is to reduce loads by producing a rated power output at a constant rotor speed.

7.3.1 Partial load operation

In the low wind speed zone, the control objective is to track the optimal point on the C_p -surface for maximizing power output. In this region, the pitch angle is set at a constant value (defined by the manufacturer) at which the maximum power coefficient is obtained. The speed of the generator is controlled by regulating the generator through the “torque reference” controller in order to obtain the optimal tip-speed ratio λ_{opt} . This is usually achieved by applying a certain generator torque as a function of the generator speed as described in (Leithead and Connor, 2000; Johnson, Pao, Balas and Fingersh, 2006):

$$T_{g,ref}(t) = \frac{\eta_{dt} \rho \pi R^5 c_{p,max}}{N_g^3 2\lambda_{opt}^3} \omega_g^2(t) - \left(B_g + \frac{\eta_{dt}}{N_g^2} B_r \right) \omega_g(t) \quad (7-12)$$

The advantage of this method is that only the measurement of the rotor or generator speed is required.

7.3.2 Full load operation

For the high wind speed zone, the desired operation of the wind turbine is to keep the rotor speed and the generator power at constant values. The main idea is to use the pitch system to control the efficiency of the aerodynamics while applying the rated generator torque. However, in order to improve tracking of the power reference and cancel steady state errors on the output power, a power controller is usually considered as well

(Leithead, de la Salle and Reardon, 1991; Leithead and Connor, 2000; Pao and Johnson, 2011). Hence, both speed control and power control are included in practice.

The speed controller is implemented as a proportional integral (PI) controller that is able to track the speed reference and cancel possible steady-state errors on the generator speed. The linear speed controller usually has the PI transfer function:

$$D_s(s) = K_{ps} \left(1 + \frac{1}{T_{is}s} \right) \quad (7-13)$$

where K_{ps} is the proportional gain and T_{is} is the integral gain.

The power controller is implemented in order to cancel possible steady state errors on the output power. The power controller is realized as a PI controller in the form:

$$D_p(s) = K_{pp} \left(1 + \frac{1}{T_{ip}s} \right) \quad (7-14)$$

where K_{pp} is the proportional gain and T_{ip} is the integral rate.

7.4 Fault estimation and observer-based AFTC design

The AFTC system makes use of the base-line control as the control system that operates in the normal condition, i.e. when it is considered that no faults are acting. Here it is proposed to use the industry standard control systems of (7-12) below rated wind speed and (7-13) and (7-14) for high wind speed operation. The baseline controller scheme (actually 3 controllers) is already known to work well for real systems and has been proved by a huge number of installed wind turbine systems in healthy conditions (Pao and Johnson, 2011). Therefore, it is reasonable to require that the baseline controller is retained in an AFTC system that has an additional control function used to compensate for the effects of possible faults. In the absence of faults the system reverts back to the baseline control action.

7.4.1 Active fault tolerant control scheme

In the study, the basic idea is to design observer-based AFTC system considering the existing baseline controller. The structure of the AFTC is shown in Figure 7.4.

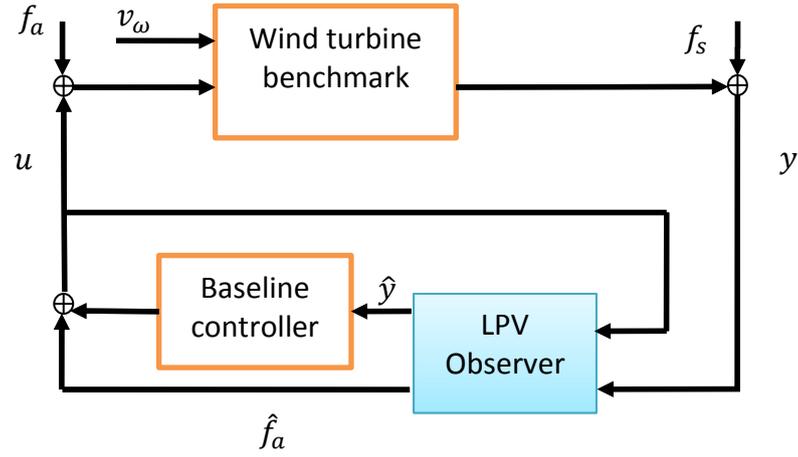


Figure 7.4 The structure of AFTC with states and faults observer

From this structure and the outline of baseline controller given presented in Section 7.3, it can be seen that both the actuator fault signals and system outputs should be estimated because of the effects of the sensor faults and sensor noise. In this study, an output feedback FTC scheme is adopted to maintain consistency with the existing practical controller based on the two step design procedure given in Section 6.4.

The controller used here has an output estimate feedback structure:

$$u_{norm} = K(\theta)\hat{y}, u_{FTC} = u_{norm} - K_f\hat{f}_a \quad (7-15)$$

where u_{norm} is the baseline controller and $K_f\hat{f}_a$ is used to compensate the effect of actuator faults. In this study, $K(\theta)$ is constant and designed using tradition gain scheduling methods which have been approved widely for real application in wind turbine systems.

7.4.2 Open-loop LPV system model

In the design, the partial derivatives of the nonlinear function for the aerodynamic torque T_r (given by (7-2)) is evaluated along the desired trajectory in terms of wind speed to obtain total derivative descriptions in terms of \tilde{T}_r , \tilde{v} , $\tilde{\beta}$, and $\tilde{\omega}_r$ indicating deviations from the design equilibrium point (EQ) values \bar{T}_r , \bar{v} , $\bar{\beta}$, and $\bar{\omega}_r$, as follows:

$$\tilde{T}_r = \left. \frac{\partial T_r}{\partial v} \right|_{EQ} \tilde{v} + \left. \frac{\partial T_r}{\partial \beta} \right|_{EQ} \tilde{\beta} + \left. \frac{\partial T_r}{\partial \omega_r} \right|_{EQ} \tilde{\omega}_r = T_{r,v}\tilde{v} + T_{r,\beta}\tilde{\beta} + T_{r,\omega_r}\tilde{\omega}_r \quad (7-16)$$

where:

$$T_{r,\omega_r} = \frac{\bar{T}_r}{\bar{\omega}_r} \frac{\partial c_q / \partial \lambda}{c_q / \lambda} \Big|_{EQ}, T_{r,v} = \frac{\bar{T}_r}{\bar{v}} \left(2 - \frac{\partial c_q / \partial \lambda}{c_q / \lambda} \Big|_{EQ} \right), T_{r,\beta} = \frac{\bar{T}_r}{\bar{\beta}} \frac{\partial c_q / \partial \beta}{c_q / \beta} \Big|_{EQ} \quad (7-17)$$

In LPV design, the set θ containing all values of θ on the operating trajectory can be selected to contain the operating locus to a strict region (Bianchi, Battista and Mantz, 2007). Furthermore, since the operating locus can be parameterized in terms of the wind speed V (Bianchi, Battista and Mantz, 2007), so that in this case the LPV model must also be parameterized in terms of V . Thus, the scheduling parameter can be defined as:

$$\theta = V \quad (7-18)$$

The partial derivatives are calculated along the normal operating trajectory (Bianchi, Battista and Mantz, 2007) and shown in Figure 7.5 together with β , ω_g and λ .

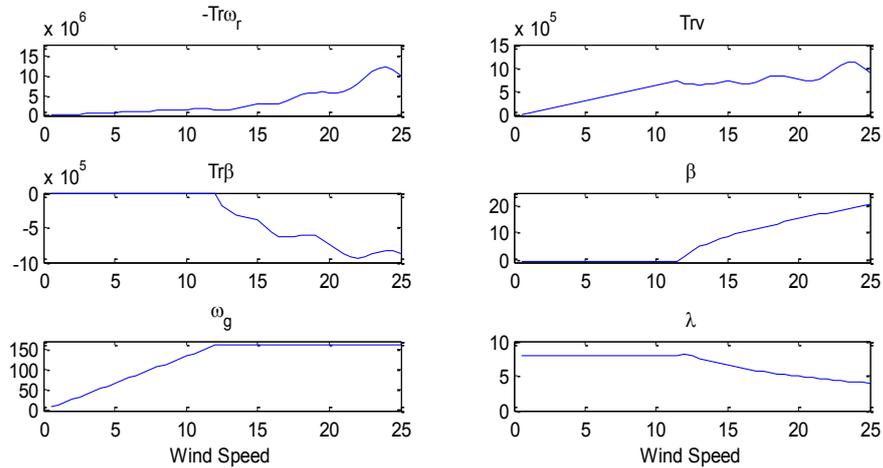


Figure 7.5 Parameters along the normal operating trajectory

It can be seen from Figure 7.5 that it is not easy to find a suitable function to fit these discontinuous relationships. Since gridding methods do not impose restrictions on the parameter dependence of the LPV model, and it is not required to derive mathematical expressions or find polynomial approximations for the gains of T_{r,ω_r} , $T_{r,\beta}$ and $T_{r,v}$. Lookup tables can be used with suitable interpolation to find the corresponding parameters during simulation or real system implementation.

From an FTC point of view, the three different blade pitch actuators may have individual faults. In addition, another variable ω_{gi} representing the integration of ω_g , is introduced to maintain consistency with the baseline controller design for this integrated AFTC design. Hence, an LPV system is proposed for the AFTC of the wind turbine system as:

$$\dot{x} = Ax + B \begin{bmatrix} \beta_r \\ T_{g,r} \end{bmatrix} + F_a f_a + Rd \quad (7-19)$$

$$y = Cx + F_s f_s + Dd \quad (7-20)$$

where:

$$x^T = [\omega_r \quad \omega_g \quad \theta_\Delta \quad \omega_{gi} \quad T_g \quad \beta_1 \quad \dot{\beta}_1 \quad \beta_2 \quad \dot{\beta}_2 \quad \beta_3 \quad \dot{\beta}_3]$$

$$A = \begin{bmatrix} A_{dt0} & \Delta_{10} & \Delta_{20} \\ 0 & A_{g0} & 0 \\ 0 & 0 & A_{ps0} \end{bmatrix}, C = \begin{bmatrix} C_{dt0} & 0 & 0 \\ 0 & C_{g0} & 0 \\ 0 & 0 & C_{ps0} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ B_{g0} & 0 \\ 0 & B_{ps0} \end{bmatrix}$$

$$A_{ps} = \begin{bmatrix} A_\beta & 0 & 0 \\ 0 & A_\beta & 0 \\ 0 & 0 & A_\beta \end{bmatrix}, A_\beta = \begin{bmatrix} 0 & 1 \\ -\omega_{n0}^2 & -2\zeta_0\omega_{n0} \end{bmatrix}$$

$$A_{dt} = \begin{bmatrix} a_{11} & \frac{B_{dt}}{N_g J_r} & \frac{-K_{dt}}{J_r} & 0 \\ \frac{\eta_{dt} B_{dt}}{N_g J_g} & a_{22} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & 0 \\ 1 & -\frac{1}{N_g} & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \Delta_{20} = \begin{bmatrix} \frac{T_{r,\beta}}{3J_r} & 0 & \frac{T_{r,\beta}}{3J_r} & 0 & \frac{T_{r,\beta}}{3J_r} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{g0} = -50, \Delta_{10} = \frac{T_{r,g}}{J_g}, a_{11} = -\frac{B_{dt} + B_r}{J_r} + \frac{T_{r,\omega_r}}{J_r}, a_{22} = -\frac{\eta_{dt} B_{dt}}{N_g^2 J_g} - \frac{B_g}{J_g}$$

$$B_{dt0} = \begin{bmatrix} \frac{T_{r,v}}{J_r} & 0 & 0 & 0 & 0 \end{bmatrix}^T, B_{g0} = -\frac{1}{J_g}, B_{ps0} = \begin{bmatrix} \omega_{n0}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_{n0}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{n0}^2 \end{bmatrix}^T$$

$$C_{g0} = 1, C_{dt0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C_{ps0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_a = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T, R = \begin{bmatrix} B_{dt0} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$D = \begin{bmatrix} 0.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .1 & .1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}^T$$

7.4.3 Integrated design of the AFTC system

In this study, the full load region is the main consideration because the pitch angles are held constant at an optimal value in the partial load region. However, in Section 7.5, it is shown that the schemes developed for the high wind speed region can also work well at low wind speeds.

In the high wind speed region, the parameters of the closed-loop system can be encapsulated by a two-vertices-polytopic. With the augmentation scheme proposed in Section 6.3, the augmented system is obtained as:

$$E_a \dot{x}_a = A_a x_a + B_a \begin{bmatrix} \beta_r \\ T_{g,r} \end{bmatrix} + R_a d \quad (7-21)$$

$$y = C_a x_a + D d \quad (7-22)$$

where:

$$x_a^T = [\omega_r \quad \omega_g \quad \theta_\Delta \quad \omega_{gi} \quad f_{rs} \quad f_{gs} \quad T_g \quad f_{tg} \quad \beta_1 \quad \dot{\beta}_1 \quad f_{\beta 1} \quad \delta_1 \quad \beta_2 \quad \dot{\beta}_2 \quad f_{\beta 2} \quad \delta_2 \quad f_{\beta s 2} \quad \beta_3 \quad \dot{\beta}_3 \quad f_{\beta s 3}]$$

$$E_a = \begin{bmatrix} E_{dt} & 0 & 0 \\ 0 & E_g & 0 \\ 0 & 0 & E_{ps} \end{bmatrix}, A_a = \begin{bmatrix} A_{dt} & \Delta_1 & \Delta_2 \\ 0 & A_g & 0 \\ 0 & 0 & A_{ps} \end{bmatrix}, C_a = \begin{bmatrix} C_{dt} & 0 & 0 \\ 0 & C_g & 0 \\ 0 & 0 & C_{ps} \end{bmatrix}, E_g = I_2$$

$$E_{\beta 1} = I_2, E_{\beta 2} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}, E_{\beta 3} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}, E_{ps} = \begin{bmatrix} E_{\beta 1} & 0 & 0 \\ 0 & E_{\beta 2} & 0 \\ 0 & 0 & E_{\beta 3} \end{bmatrix}, E_{dt} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_{\beta 1} = \begin{bmatrix} A_\beta & F_\beta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A_{\beta 2} = \begin{bmatrix} A_{\beta 1} & 0 \\ 0 & 0 \end{bmatrix}, A_{\beta 3} = \begin{bmatrix} A_\beta & 0 \\ 0 & 0 \end{bmatrix}, A_{ps} = \begin{bmatrix} A_{\beta 1} & 0 & 0 \\ 0 & A_{\beta 2} & 0 \\ 0 & 0 & A_{\beta 3} \end{bmatrix}$$

$$A_g = \begin{bmatrix} -50 & -50 \\ 0 & 0 \end{bmatrix}, A_{dt} = \begin{bmatrix} A_{dt0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} B_{dt} & 0 & 0 \\ 0 & B_g & 0 \\ 0 & 0 & B_{ps} \end{bmatrix}$$

$$\Delta_{20} = \begin{bmatrix} \frac{T_{r,\beta}}{3J_r} & 0 & 0 & 0 & \frac{T_{r,\beta}}{3J_r} & 0 & 0 & 0 & 0 & \frac{T_{r,\beta}}{3J_r} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Delta_1 = \begin{bmatrix} \frac{T_{r,g}}{J_g} & 0 \end{bmatrix},$$

$$B_{dt} = \begin{bmatrix} \frac{T_{r,v}}{J_r} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, B_g = \begin{bmatrix} -\frac{1}{J_g} & 0 \end{bmatrix}$$

$$B_{ps} = \begin{bmatrix} \omega_{n0}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{n0}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_{n0}^2 & 0 & 0 \end{bmatrix}^T, C_g = [1 \quad 0]$$

$$C_{dt} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, C_{ps} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}^T$$

Considering the application of the descriptor LPV ESO proposed in Section 6.3 to the descriptor system of (7-21) and (7-22), a descriptor LPV ESO can be obtained as:

$$E_a \dot{\hat{x}}_a = A_a(\theta) \hat{x}_a + B_a \begin{bmatrix} \beta_r \\ T_{g,r} \end{bmatrix} + L(\theta)(\hat{y} - y) \quad (7-23)$$

$$\hat{y} = C_a \hat{x}_a \quad (7-24)$$

The control strategy proposed in (7-15) is considered with baseline controller presented in section 7.2.5. With the closed-loop system states constructed with original states and estimation errors, the following closed-loop system is obtained:

$$E_{cl} \begin{bmatrix} \dot{x} \\ \dot{e}_{xf} \end{bmatrix} = A_{cl} \begin{bmatrix} x \\ e_{xf} \end{bmatrix} + R_{cl} d \quad (7-25)$$

where:

$$E_{cl} = \begin{bmatrix} I & 0 \\ 0 & E_a \end{bmatrix}, A_{cl} = \begin{bmatrix} A(\theta) + BKC & \Delta \\ 0 & A_o(\theta) \end{bmatrix}$$

$$\Delta = [BKC \quad 0 \quad F_a], R_{cl} = \begin{bmatrix} R \\ R_a + L(\theta)D \end{bmatrix}, A_o(\theta) = A_a(\theta) + L(\theta)C$$

where K corresponds to the baseline controller designed in Section 7.3, and L is the ESO gain to be determined. Furthermore, it is proposed to use a parameter independent ESO gain with a special structure as follows:

$$L = \begin{bmatrix} L_{dt} & 0 & 0 \\ 0 & L_g & 0 \\ 0 & 0 & L_{ps} \end{bmatrix}$$

where L_{dt} , L_g , L_{ps} are to be determined. One benefit from the proposed L is that this results in a parameter-independent matrix A_{22} in the **Equivalent form 1** of the closed-loop system discussed in Section 6.3, which should be clear from the special structure of E_a . Another benefit arising from using the special structure is that the problem of pole-placement of the subsystems into separate LMI regions can be simplified, as shown in the following.

Define the H_∞ performance variable as:

$$z_{xef} = C_z \begin{bmatrix} \dot{x} \\ \dot{e}_{xf} \end{bmatrix} \quad (7-26)$$

where $C_z = [C_{zx} \quad C_{zexf}]$ is a weighting matrix. Following Section 6.4, the Lyapunov function can be defined as:

$$\mathcal{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}, \mathcal{S} = \begin{bmatrix} 0 \\ W \end{bmatrix}$$

For a constant K as used in the study, substituting for \mathcal{P} and \mathcal{S} into the inequality (6-65) leads to the LMI:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & (E^T P + USV^T)R & C_{zx}^T \\ \star & \Delta_{22} & \Delta_{23} & C_{ze}^T \\ \star & \star & -\gamma & 0 \\ \star & \star & \star & -\gamma \end{bmatrix} < 0 \quad (7-27)$$

with:

$$\begin{aligned} \Delta_{11}(\theta) &= (E^T P + USV^T)(A(\theta) + BKC) + \star \\ \Delta_{12}(\theta) &= (E_a^T Q + U_a W V_a^T)[BK(\theta)C \quad 0 \quad F_{ag} \quad F_{aps}] \end{aligned}$$

$$\Delta_{22}(\theta) = (E_a^T Q + U_a W V_a^T)(A_a(\theta) + L C_a) + \star$$

$$\Delta_{23} = (E_a^T Q + U_a W V_a^T)(R_a + L D)$$

With a constant observer gain, the observer system matrix would be:

$$A_o = \begin{bmatrix} A_{dt} + L_{dt} C_{dt} & \Delta_1 & \Delta_2 \\ 0 & A_g + L_g C_g & 0 \\ 0 & 0 & A_{ps} + L_{ps} C_{ps} \end{bmatrix}$$

Furthermore:

$$E_{cl} = \begin{bmatrix} I & 0 \\ 0 & E_a \end{bmatrix}, E_a = \begin{bmatrix} E_{dt} & 0 & 0 \\ 0 & E_g & 0 \\ 0 & 0 & E_{ps} \end{bmatrix}$$

Which means that the null space of E_{cl} can be specified in terms of E_a as follows:

$$U_{cl} = \begin{bmatrix} 0 \\ U_a \end{bmatrix}, V_{cl} = \begin{bmatrix} 0 \\ V_a \end{bmatrix}, U_a = \begin{bmatrix} U_{dt} & 0 & 0 \\ 0 & U_g & 0 \\ 0 & 0 & U_{ps} \end{bmatrix}, V_a = \begin{bmatrix} V_{dt} & 0 & 0 \\ 0 & V_g & 0 \\ 0 & 0 & V_{ps} \end{bmatrix}$$

where U_{dt} and V_{dt} are full column rank and contain the basis vectors for $Ker(E_{dt})$ and $Ker(E_{dt}^T)$, respectively. U_g and V_g are full column rank and contain the basis vectors for $Ker(E_g)$ and $Ker(E_g^T)$, respectively. U_{ps} and V_{ps} are full column rank and contain the basis vectors for $Ker(E_{ps})$ and $Ker(E_{ps}^T)$, respectively. Furthermore, the structure of Q and W are specified as:

$$Q = \begin{bmatrix} Q_{dt} & 0 & 0 \\ 0 & Q_g & 0 \\ 0 & 0 & Q_{ps} \end{bmatrix}, W = \begin{bmatrix} W_1 & 0 & 0 \\ 0 & W_2 & 0 \\ 0 & 0 & W_3 \end{bmatrix}$$

Following the procedure of Section 6.4, set:

$$Y_{dt} = Q_{dt} L_{dt}, Y_g = Q_g L_g, Y_{ps} = Q_{ps} L_{ps}$$

$$H_1 = W_1 U_{a1}^T L_{dt}, H_2 = W_2 U_{a2}^T L_g, H_3 = W_3 U_{a3}^T L_{ps}$$

Then $\Delta_{22}(\theta)$ can be partitioned as:

$$\Delta_{22}(\theta) = \Delta_{22a} + \Delta_{22b}$$

with:

$$\Delta_{22a} = \begin{bmatrix} E_{dt}^T Q_{dt} A_{dt} + E_{dt}^T Y_{dt} C_{dt} & E_{dt}^T Q_{dt} \Delta_1 & E_{dt}^T Q_{dt} \Delta_2 \\ 0 & Q_g A_g + Y_g C_g & Q_g \Delta_3 \\ 0 & 0 & Q_{ps} A_{ps} + Q_{ps} C_{ps} \end{bmatrix} + \star$$

$$\Delta_{22b} = \begin{bmatrix} U_{dt} W_1 V_{dt}^T A_{dt} + U_{dt} H_1 C_{dt} & U_{dt} W_1 V_{dt}^T \Delta_1 & U_{dt} W_1 V_{dt}^T \Delta_2 \\ 0 & U_g W_2 V_g^T A_g + U_g H_2 C_g & U_g W_2 V_g^T \Delta_3 \\ 0 & 0 & F_{1b33} \end{bmatrix} + \star$$

$$\Delta_{22b33} = U_{ps} W_3 V_{ps}^T A_{ps} + U_{ps} H_3 C_{ps}$$

Furthermore, from the structure of $\Delta_{22}(\theta)$, it can be observed that the eigenvalues of one subsystem will not affect that of another subsystem. Based on the pole-placement techniques for LPV descriptor system discussed in Section 6.2, the eigenvalues of each subsystem can be assigned into corresponding desired regions. Similarly the LMIs can be obtained to constrain the subsystem poles in different regions. For example, the generator and converter subsystem must respond in a faster time-scale than the drive train subsystem, so that the relative magnitudes of the corresponding eigenvalues should be assigned to reflect this physical feature.

Then, the problem is solved using MATLAB LMITOOL box. The observer gain obtained is:

$$L_{dt1} = \begin{bmatrix} 0.079127 & -0.27673 & 3.2339 & -39.821 & 89.929 \\ 1.1142 & -4.6811 & 19.971 & -436.07 & 866.55 \\ 0.00053798 & -0.0029312 & 0.00064773 & -0.61229 & 1.5252 \\ 0.30049 & -1.5161 & -5.2508 & -53.497 & 77.101 \\ -64.93 & 64.93 & -7.38e-6 & 4.54e-5 & -9e-5 \\ -1.54e-5 & 4.46e-5 & -64.934 & 64.926 & 0.02154 \end{bmatrix}$$

$$L_g = \begin{bmatrix} -41.339 \\ -41.339 \end{bmatrix}$$

$$L_{ps} = \begin{bmatrix} -3.2697 & -15.174 & -0.62933 & 0.053112 & 2.423 & 13.213 \\ -1.3461 & -6.8059 & -10.509 & 0.21928 & -5.2258 & 54.122 \\ -2.5241 & -15.029 & -0.58687 & 0.094432 & 2.4302 & 15.616 \\ -8.7472 & 9.234 & -0.19209 & -0.067579 & -0.036529 & -0.19056 \\ 3.1015 & 4.9546 & -28.066 & -9.5366 & 5.6402 & 20.522 \\ 43.478 & 67.415 & -215.42 & -96.509 & 46.656 & 184.84 \\ 8.5534 & 14.193 & -55.379 & -21.485 & 12.928 & 41.189 \\ 35.228 & 59.995 & -175.27 & -81.372 & 46.132 & 115.29 \\ -0.0531 & -0.20429 & 9.4546 & -8.7183 & -0.081393 & -0.39747 \\ 0.19163 & 0.67649 & 0.16837 & 0.00071 & -0.98538 & -3.4362 \\ 1.3319 & -5.2952 & -9.5992 & -0.19567 & -7.4994 & 51.712 \\ -0.0517 & -0.17886 & -0.40926 & -0.066921 & -8.7343 & 9.441 \end{bmatrix}$$

7.5 Simulation results

The simulation is carried out within MATLAB/SIMULINK environment. The results are presented separately in the subsection 7.5.1. The simulations are carried out based on the wind speed time signal shown in Figure 7.6 using the principle proposed in (Burton, Sharpe, Jenkins and Bossanyi, 2001).

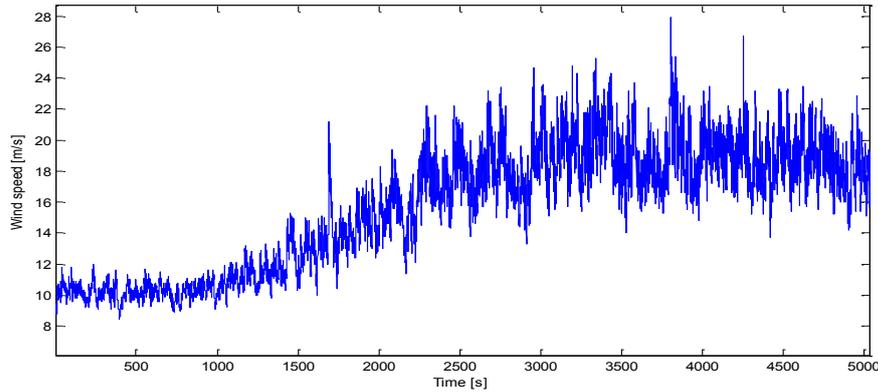


Figure 7.6 Wind speed used for evaluation

One important feature of the wind speed is that the magnitude of the disturbances becomes larger when the wind speed increases

7.5.1 Faults in pitch subsystems

The results arising from the pitch subsystem fault scenarios are presented in terms of pitch angle variations with their estimates and measured outputs (using pitch angle sensors) in Figures 7.7 -7.9. The estimation of the fault signal f_β re-defined according to (7-11), is shown in Figure 7.10. With estimated actuator fault signal, AFTC is carried out with the strategy presented in Section 7.4. The AFTC results are shown in Figures 7.11 and 7.12.

To show the improvement of the AFTC scheme in the simulation, one criteria function is defined as:

$$\gamma_\beta = \frac{\sum_{i=1}^{n_\beta} e_{FTC\beta i}}{\sum_{i=1}^{n_\beta} e_{\beta i}}$$

where n_β is the number of samples during the simulation, $e_{FTC\beta i}$ is the error of the faulty pitch angle from the fault-free angle with FTC activated, $e_{\beta i}$ is the error of the

faulty pitch angle from the fault-free pitch angle without FTC activated. Therefore, $\gamma_\beta < 1$ means there is improvement of AFTC. In the simulation, the γ_β is obtained as $\gamma_\beta = 0.8172$. Hence, the performance of the wind turbine system is improved by the AFTC when there is a pitch actuator fault.

One problem arising from re-defining the fault signal is that it is not easy to decide the severity of a fault as it is strongly coupled to the system states. The real fault signal can be constructed using the approach proposed in Section 7.2.4. The reconstructed fault is shown in the Figure 7.13 from which it is very easy to determine whether there is a fault or not and also the severity of the fault. The result given in Figure 7.13 corresponds to the case of oil with abnormally high air content – with expected fault severity of $\theta_f = 1$.

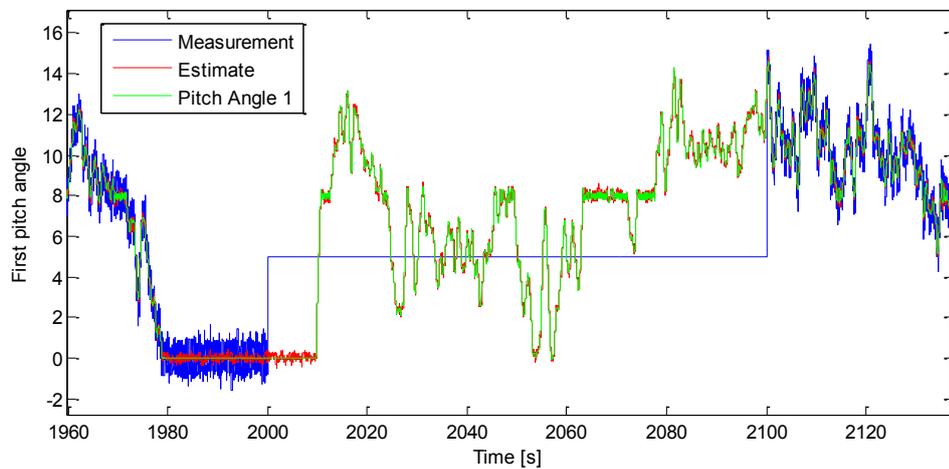


Figure 7.7 First pitch angle state, measurement and estimate with sensor fault

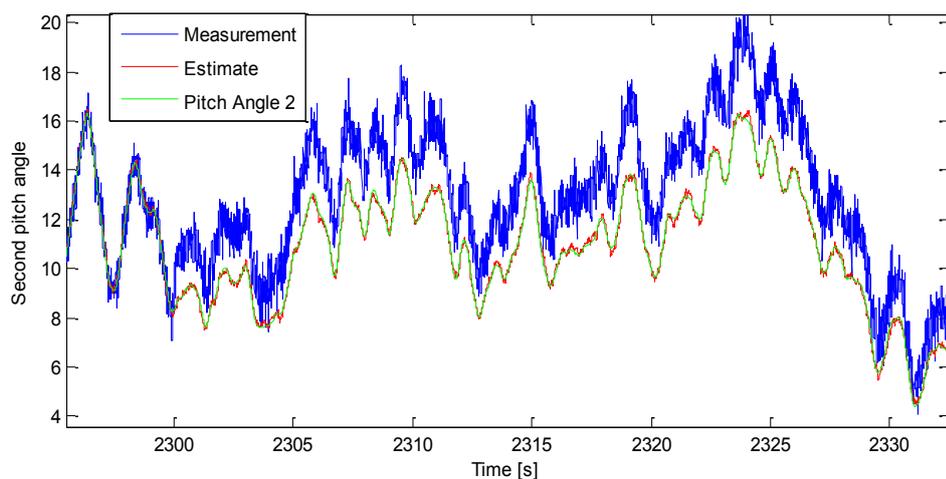


Figure 7.8 Second pitch angle state, measurement and estimate with sensor fault

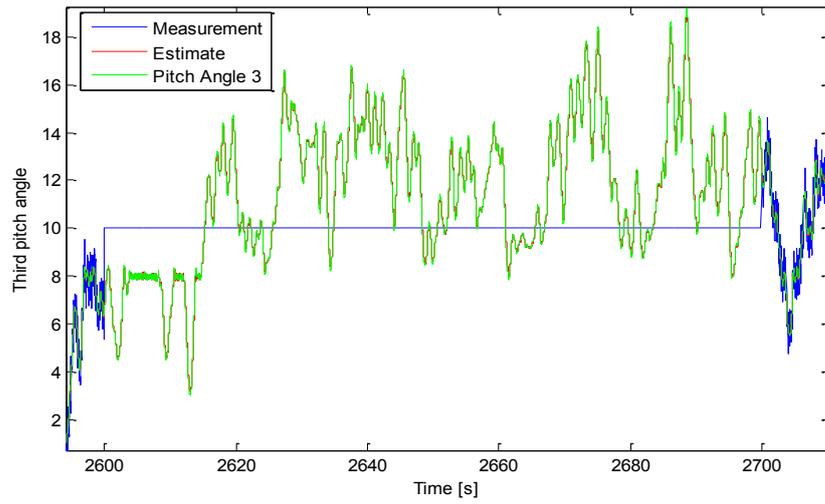


Figure 7.9 Third pitch angle state, measurement and estimate with sensor fault

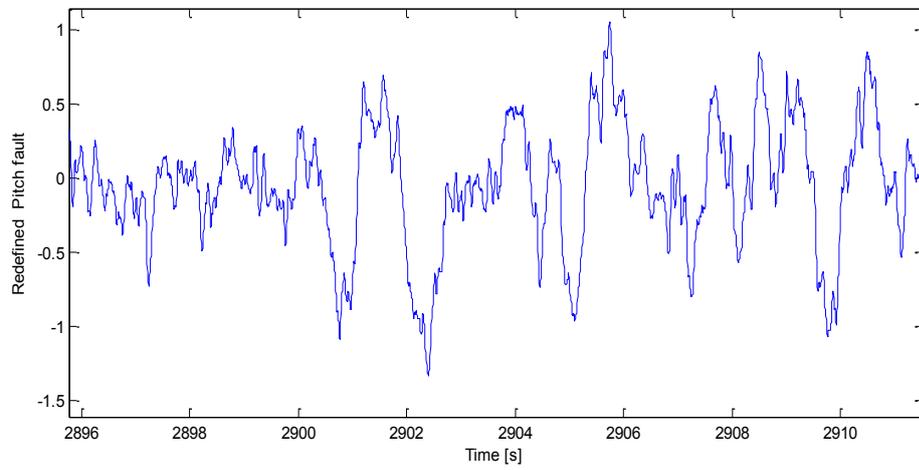


Figure 7.10 Estimate of the new defined fault f_{β}

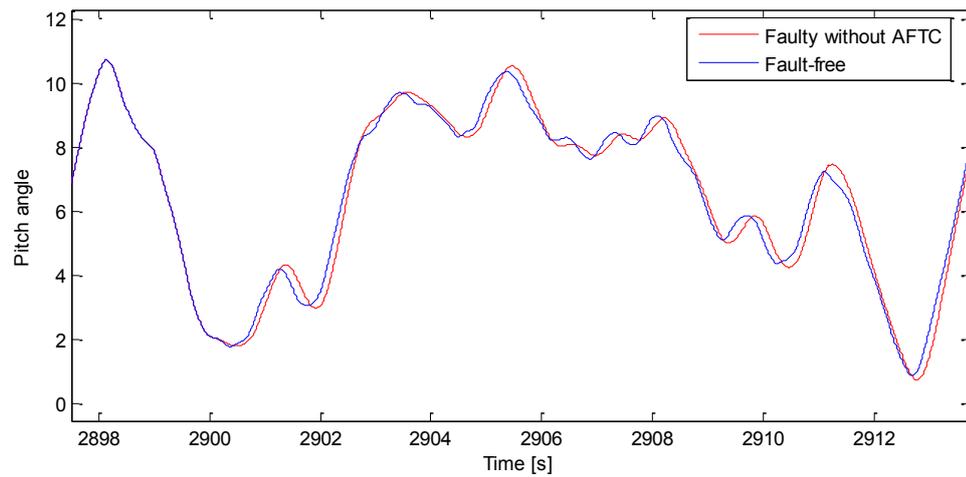


Figure 7.11 Pitch angles with fault occurring without AFTC

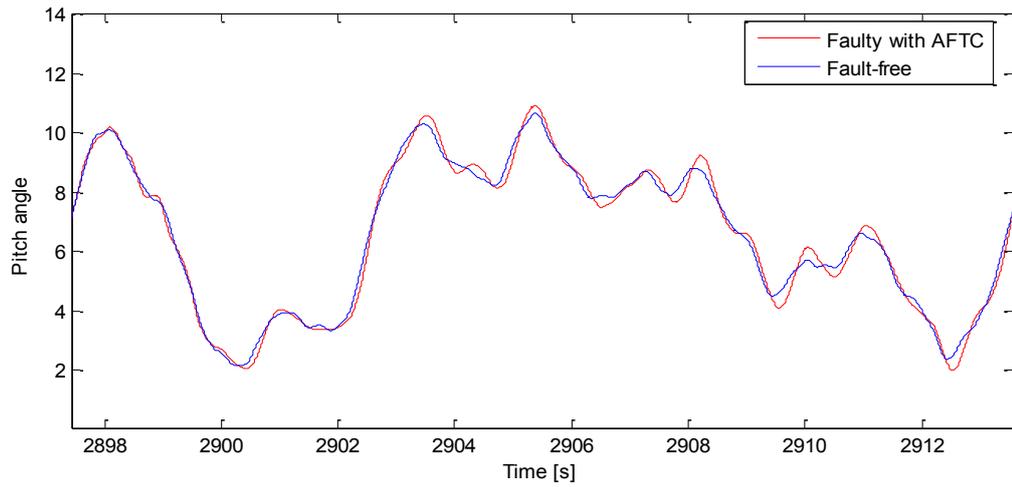


Figure 7.12 Pitch angles with AFTC activated

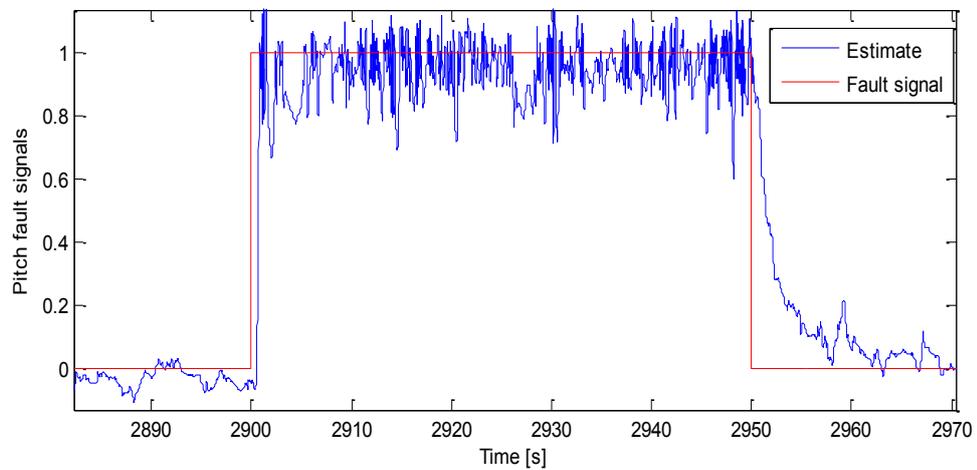


Figure 7.13 Fault θ_f and its estimate in high air content case

7.5.2 Actuator fault in generator subsystem

The estimation of the torque actuator fault is presented in Figure 7.14. it can be seen that the LPV ESO method can provide very good fault estimation, which is a significant result even though it is claimed that a 100 Nm fault is too small to be detected (Odgaard and Johnson, 2012). As the fault is small, there is no obvious improvement to be gained by using the AFTC scheme. It is important to point out that the actuator fault should be detected as early as possible to prevent the impact of faults from other subsystems, or even to prevent a gross effect on the overall system performance.

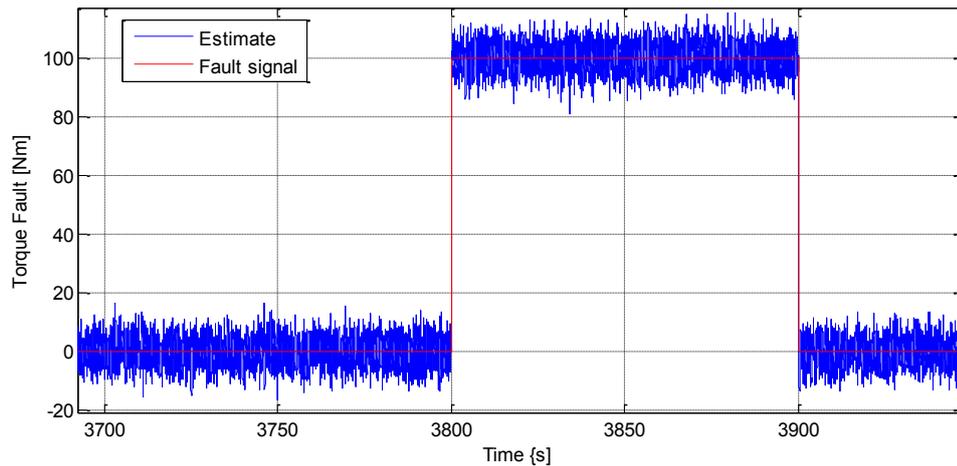


Figure 7.14 Generator torque fault and estimate

7.5.3 Generator speed sensor fault

The generator speed is simulated with a constant bias fault during 1000s-1100s. The simulation results are shown in Figure 7.15 where it can be seen that the rotor speed estimation follows the real rotor speed closely whether or not a fault has occurred.

For the result shown in Figure 7.16, the AFTC uses sensor hiding. The output power P_g response is shown (calculated from (7-5)) with and without the AFTC switched on and corresponding to different output conditions. It is clear that the quality of the output power is improved as the smoothness is an important property considering that the converter can be damaged by the large transient shown (in the red curve) by the keeping the AFTC system switched off.

The estimated rotor speed shown in Figure 7.17 shows that for the fault-free case, the estimate is closer to the real signal compared with the measurement disturbed by sensor noise. The rotor speed stuck-value sensor fault is simulated during 1500s-1600s. The estimate tracks the real signal closely even after the fault has occurred. As the rotor speed is not involved in the feedback control loop, the rotor speed sensor fault will not affect the closed-loop system performance.

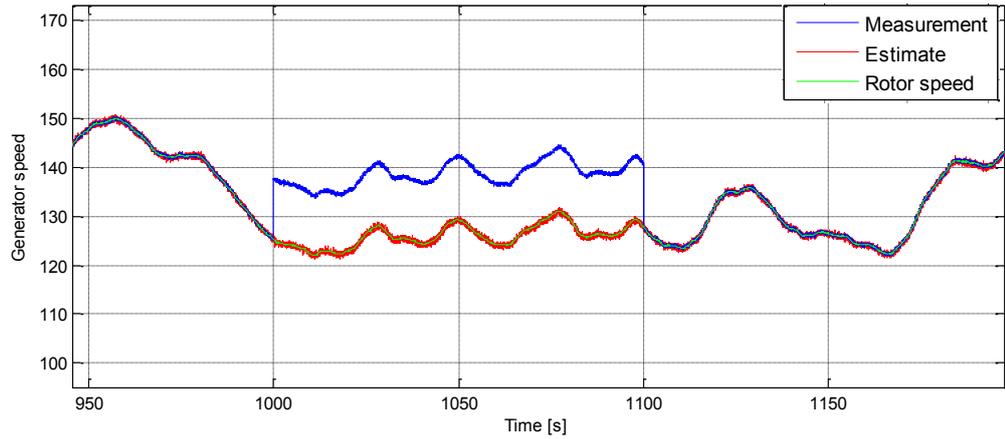


Figure 7.15 Generator speed state, measurement and estimate with sensor fault

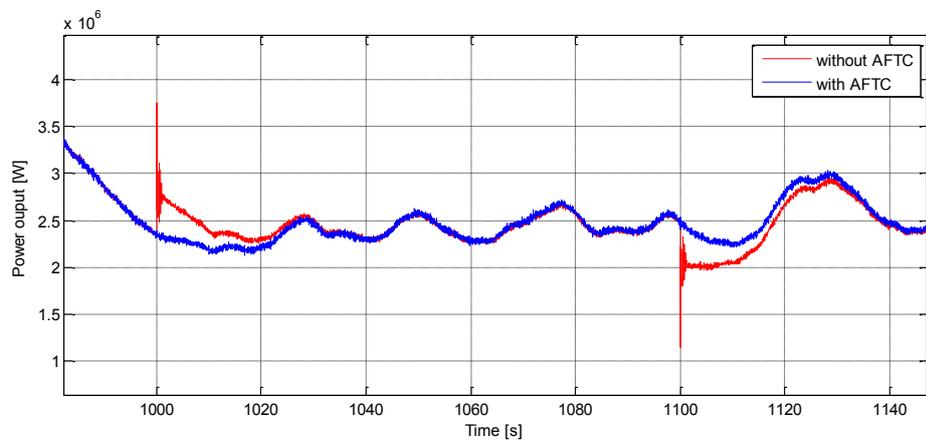


Figure 7.16 Generator speed with and without AFTC activated

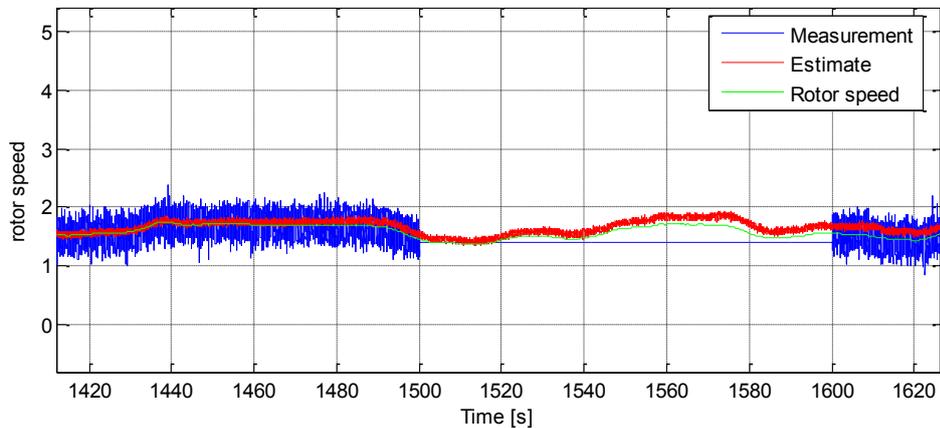


Figure 7.17 Rotor speed state, measurement and estimate

7.6 Discussion

An observer-based descriptor system AFTC scheme is designed for an offshore wind turbine system using a robust LPV framework to account for modelling uncertainty arising from (a) parameter variations in the system, (b) uncertain knowledge of the effective wind speed, and (c) sensor noise. Both the faults and the required baseline

controller system states are estimated using the proposed descriptor system LPV ESO formulated within an LPV framework. The AFTC uses an output feedback baseline controller corresponding to a typically implemented controller. It is shown that the AFTC design is capable of stabilizing both the faulty and fault-free systems. The use of a typical control system within the baseline controller structure means that the system can easily be viewed as an extension of currently used control technology, with the AFTC proving clear “added value” as a fault tolerant system, to enhance the sustainability of the wind turbine in the offshore environment.

Chapter 8: Summary and future work

8.1 Thesis summary

FTC design is attracting more attention as the designed system can function well not only in fault-free cases but also in the presences of faults to improve the safety, availability and reliability of modern control systems. This thesis concerns the overall performance of closed-loop AFTC systems with a simultaneous fault and state observer. More precisely, the response speed is considered as a time-domain performance and a defined degree of robustness, for example an H_{∞} index, is considered as a measure of frequency-domain performance. The proposed design approaches are expected to meet multi-objective design requirements that are appropriate for real systems.

The descriptor systems approach offers the potential for good design freedom, for example by combining algebraic and differential equations reflecting both the static constraints and dynamic relationships that are appropriate for these systems. It is argued that many systems can be represented in descriptor systems format in state space and this leads to the development of powerful tools for the design of both FE and control within the AFTC framework. Hence, the overall contribution of the thesis is the novel use of descriptor systems analysis and design for integrated AFTC systems.

The brief background and approach to system modelling as used in the research are given in Sections 1.1 & 1.2 followed by the definitions of the concepts of FDD and FTC accompanied with a general discussion. The challenges that arise when attempting to design an AFTC system are potential time-delays due to: FE time (i.e. time taken for the estimates to converge to realistic fault value), and controller reconfiguration time (time taken for the controller to be reconfigured to steady closed-loop operation after the fault occurrence and compensation). Furthermore, there is a need for significant robustness to bounded exogenous disturbance and bounded modelling uncertainty. Indeed, the goal is to make the reconfigurable closed-loop system to be robust against uncertainty and disturbance, whilst at the same time reconfiguring promptly when faults occur. It is the reconfigured system itself that renders the overall system to be robust against the faults, through the FE and compensation mechanism.

A review of FDD/FE and FTC is carried out in **Chapter 2**, starting from a standard state space systems framework. This sets the scene and gives a background for the development and description of the use of descriptor systems used throughout the remainder of the thesis. It is concluded that it is of both theoretical and practical importance to study the design aspects of integrated controller and observer design of AFTC schemes for descriptor systems.

Starting with the basic concepts of LTI descriptor systems, **Chapter 3** presents the design descriptions in the literature within an LMI framework. To achieve robust design of baseline controller, H_∞ optimization is considered and the Bounded Real Lemma for descriptor system is introduced. LMI descriptions for both analysis and synthesis with state feedback are given in Section 3.2.2. From the literature study, it is found that there is a lack of systematic analysis and synthesis concerning regional pole-placement methods for LTI descriptor systems. In response to this, Section 3.2.3 presents novel LMI descriptions for regional pole-placement of descriptor systems with state feedback.

It is shown that multi-objective design of time-domain and frequency-domain performance requirements can be achieved by combining the proposed LMI descriptions with the Bounded Real Lemma of descriptor systems in Section 3.2. This multi-objective strategy forms the essential basis for the design of new estimator methods described in the remainder of the thesis.

In Section 3.3, ESO design is studied to achieve simultaneous faults and system states estimation. This design approach involves simultaneous pole-placement and H_∞ robustness optimization and these objectives are also combined into a multi-objective framework using LMI tools. Section 3.3 describes two important state space augmentation strategies to descriptor system representation that are important for the development of the theory described throughout the remainder of the thesis. Augmentation 1 is capable of handling sensor and actuator faults in polynomial form. Augmentation 2 is proposed for the case where actuator fault is in polynomial form and sensor faults are not restricted to polynomial form. The benefit of removing the polynomial constraints on sensor faults in Augmentation 2 is obtained by introducing Assumptions A3.6 and A3.7.

An equivalent form of the ESO is considered to be implemented in the case E is singular in Section 3.3.3. The simulation results of a numerical example show the

usefulness and design procedure of the ESO design. One limitation of the ESO is the restriction of the polynomial form when considering actuator faults. As discussed in Section 3.4.3 for step faults, the polynomial description gives rise to errors since the fault signal has discontinuous time response behaviour. This gives rise to an impulse in the FE corresponding to the discontinuity in the fault signal.

Inspired by PD observer design approaches for fault-free systems described in the literature, a novel structure of the PD-ESO is proposed in **Chapter 4** for simultaneous state and fault estimation. The PD-ESO introduces a new design parameter, compared with the ESO, to achieve enhanced performance for simultaneous state and fault estimation. A sensor noise-free system is obtained as a starting point to the PD-ESO design. Various augmentation strategies are described that enable PD-ESO to satisfy different FE requirements, including the minimization of the effects of noise on both the state and fault estimates.

Thanks to the dual property that exists between two linear descriptor systems, a systematic approach is proposed for the PD-ESO design making use of the multi-objective LMI strategy described in Section 3.2 (outlined above). The criterion of achieving a prescribed speed of estimation response is also included in the multi-objective design of the PD-ESO.

Section 4.3 describes a way of removing the system output derivative from the estimator computation. This is achieved using the PD-ESO with a new variable introduced, giving a form of PD-ESO which is useful for practical applications. The example given in Section 3.4 is used to show that the discontinuity in the sensor fault FE signal is removed. The simulation result is compared with the result given for this example in Section 3.4.3.

Chapter 5 is concerned with the challenging new problem of integrating the observer-based FE design within the design of the AFTC system with a requirement to achieve joint robust performance in both FE and control, as well as satisfying estimation and control performances. There is very little background literature on the subject of joint or integrated design of FE and AFTC systems, hence the subject is very new. The work in this thesis on this topic comprises a strong contribution in terms of integrated design of state and FE within a reconfigurable control structure.

Inspired by the observer based controller design approaches in the literature, an observer based AFTC structure is obtained for LTI descriptor systems by combining the ESO and baseline controller. In contrast to traditional approaches to separate controller and observer designs (with all the attendant separation principle problems), an integrated AFTC scheme is proposed in this study to achieve desired robustness performance of the closed-loop system using a two-step integrated estimation/control design procedure. In a similar way, an integrated AFTC design with PD-ESO is presented in Section 5.4.

The same numerical example used in Chapters 3 & 4 is considered as an illustrative comparison of the designs of two AFTC systems using the ESO and the PD-ESO, subject to identical fault scenarios. From the simulation results, it can be seen that both of the two strategies work. Moreover, a clear advantage of the PD-ESO based integrated AFTC scheme is that no impulsive response is seen in the estimate of sensor fault.

Chapter 6 focuses on extending the descriptor systems approaches developed in Chapters 3, 4, & 5 to an LPV framework. LPV system representation is studied as an extension of the LTI cases to facilitate a way of dealing with the non-linearity or parameter variation problem. As another new work both the ESO and PD-ESO approaches are extended to a descriptor LPV system format. The definition of strongly equivalent systems taken from the literature is applied here to a descriptor ESO problem. LPV PD-ESO with constant derivative gain is proposed considering the practical implementation. Based on this, a mechanism is described for developing an integrated AFTC design scheme. Finally, a numerical example of an AFTC system incorporating a PD-ESO within an LPV descriptor system framework is studied to illustrate the proposed integrated AFTC design procedure.

The limitation of the proposed LPV approach is that a parameter-independent Lyapunov function is used (see Section 6.5). Because of this, the descriptions in the proposed theorems are sufficient but not necessary to design the corresponding integrated extended state observer and controller system.

A wind turbine benchmark is studied in **Chapter 7** as an application of AFTC approach proposed in Chapter 6. In this application study, the proposed approach is adapted to retain the industry baseline controller, which is a well-accepted generator angular speed

reference control system. Hence the proposed AFTC scheme can be applied as an added feature to the existing industry baseline controller.

A diagonal structure of the proposed state/fault observer gain is considered and separate regional pole constraints applied to the three subsystems are achieved, making use of the special dynamical structure of the wind turbine system. The AFTC system is designed in an overall system level, leading to good closed-loop robustness performance, evaluated by simulating the nonlinear wind turbine benchmark model system in the presence of changing wind operating conditions.

8.2 Future work

The possible future work following this thesis could be:

- 1) Investigate the influence of estimation delays on the overall system performances. In the design, pole-placement is considered to reduce the effects of the estimation transients on the overall system. Future work could seek to develop an effective procedure for evaluating the reconfiguration performance.
- 2) The thesis contains some ideas in Chapter 6 about ways of reducing the conservativeness of the AFTC LPV designs. However the work of Chapter 6 only considers parameter-independent Lyapunov function analysis and design. Future work can consider a parameter-dependent Lyapunov function approach as a potential option to reduce the conservativeness and improve the AFTC performance even further.
- 3) Different supervision schemes should be considered for some cases where the proposed framework is not suitable, for instance, for cases when the actuator faults should be accommodated by switching to a redundant actuator.

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