THE UNIVERSITY OF HULL

Theoretical Investigations into Competition, Regulation, and Integration in

Transport Networks

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by

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This thesis is dedicated to Mum, Dad, Tanya, James, Rachael, Grace, and to those past, but remaining in my thoughts; never forgotten, Nanny and Gaggae – thank you.

Abstract

This thesis consists of three parts. In the first part, we review the literature and some of the key issues in UK transport. We identify a need to discourage car use and the role that public transport plays in this. We discuss the various options available to policymakers to reduce problems of congestion and pollution. We note how the emphasis on deregulation and competition to promote public transport, and discourage car use, have had perverse side effects. In some cases, public transport services have become disintegrated; resulting in reductions in flexibility and increasing the generalised cost of travelling – making public transport less attractive. This raises an important question: how do we encourage a greater degree of service integration without undoing the gains from competition? The second part of the thesis, explores this issue using a theoretical transport network model. We find that various regimes involving private firms are likely to lead to the provision of an integrated ticketing system, but that not all such regimes are socially desirable. We consider how the configuration of regulatory policy may steer the private firms to produce more socially desirable outcomes.

The deregulation of elements of the UK public transport network has often led to situations approaching local monopoly. The third part of this thesis investigates the private (monopoly) incentive to offer joined-up services relative to the social incentive. The more complete the service provision, the closer the match with consumer's preferences, and the lower the generalised cost of travel. We find the monopolist does not always choose the socially desirable level of service, even when economically viable, but it may be possible to induce this provision through entry or threats of entry on a sub-set of the network.

The thesis ends with a summary of the main results and suggestions for further work.

KEY DEFINITIONS

Single Journey: A journey consisting of one trip.

Return Ticket: A journey consisting of two parts: an outward and an inward trip.

Single-Service Demand: This is a demand for a return journey that is made up of travellers with known transport arrangements so they travel outwards and inwards on the same route or at a set time, which they are aware of prior to buying their ticket allowing them to make use of one predetermined service.

Cross-Service Demand: This is a demand for a return journey that is made up of travellers who are either unsure of their travel arrangements; preferring flexibility in their inward and outward route or time so that they can use whichever service is most convenient; or they know prior to buying their ticket that they wish to take their inward trip on one service and their outward trip on a separate service.

Integrated Ticket: This is a ticket that allows travel on the transport services of more than one operator on the **same mode.**

Inter-available Ticket: This is a ticket that allows travel on the transport services of more than one operator on **several modes.**

Integration: In general economic terms this refers to such measures as horizontal and vertical integration amongst firms that combine previous separate decision-making units into a single unit. However, in transport the term, integration, has been muddled by the publication of the DETR (1998), which was followed by DETR (2000). These studies talk about "integrated transport" and an "integrated approach" without really ever defining the term precisely so that "integrated transport" is used as a blanket term for a number of measures that the UK Government proposed and does not simply refer to encouraging horizontal/vertical integration between firms. "Integrated transport" actually refers to

policies that aim for coordination in transport; be it by encouraging cooperation or integration in economic terms. Examples of "integrated transport" include investment in transport interchanges so both bus and train stations share the same building, and encouraging the availability of integrated and inter-available tickets. When possible in this thesis we will attempt to refer to integration, coordination and cooperation as separate concepts. One obvious exception is that of the "integrated ticket" that our above definition states can only be used on one mode of transport. However, this use of the term integrated ticket is widespread and despite the inconsistency we will continue with the accepted use.

Cooperation: Whilst integration refers to separate decision-making units merging, cooperation is about these units communicating and agreeing to actions. These actions can then lead to a coordinated set of outcomes.

Coordination: We shall define what is meant by coordination using a simple example. We would say that a bus that arrived at a train station with just enough time for travellers to catch a train is an example of coordination. A bus that was too late or left too much time would not be coordinated. However, the term is not always simply defined as in terms of timetable coordination for a bus journey this concept can be ambiguous. We would not call a timetable that saw two companies' buses that run the same route arrive at a bus stop at the same time coordinated; although in one sense of the word it would be. Instead we call a timetable coordinated if the buses of the two companies were evenly spaced through the day so the buses arrived at the bus stop with equal wait times. Another example of coordination would be a firm accepting other firm's tickets such as is the case with integrated and inter-available ticketing.[1] This is undoubtedly one of the aims of "transport integration", whether this comes about through co-ordination or co-operation.

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[1] Recall that our definitions allow separate firms to accept these tickets but the integrated ticket is used on one mode whilst the inter-available ticket can be used across several modes.

Generalised Cost: The generalised cost of using a mode of transport is the total sum of the costs of using that mode; including both monetary and non-monetary costs. The generalised cost is made up of more than the price of a ticket on a bus or train or the cost of the petrol used in a car journey. The generalised cost of a journey by bus includes the ticket price, journey time costs, interchange costs, and wait-time costs. The generalised cost of travel varies between people (e.g. by income level) and modes – most people will tend to dislike walking more than in-car travel time and waiting time is generally disliked more than either.

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INTRODUCTION

Deregulation and the promotion of competition were popular policy responses to transport problems in the UK during the 1980s and early 1990s. The idea that such policies would stimulate improvements in the price and quality of services, and that a more consumer-focused industry would provide a range of services to better match traveller needs was extremely optimistic. Unfortunately, other aspects of transport and environmental policy at the time also relied upon successful implementation of this marketisation of public transport, since a credible and attractive alternative to the car was seen as a key tool in alleviating issues such as rising congestion and pollution. Whilst there were some notable successes of this policy, for instance, considerable cost efficiencies due to competition, there were also a number of counterproductive consequences. In particular, deregulation of public transport services often resulted in fragmentation and discord of provision, which increased information costs, reduced the flexibility of the services, and raised the generalised cost of travel. The lack of coordination was formally recognised by the government in the late 1990s (DETR, 1997), but the major part of the resulting integrated transport plan never had serious financial support and consequently little has come of it.

This thesis is concerned with how policy can be engineered to both encourage service integration, and retain the benefits of competition. We propose two theoretical frameworks within which we analyse policy options available to help stimulate greater completeness and integration of public transport services.

In Part A of this thesis (Chapters 2 and 3) we review the literature and key issues in UK transport, and investigate the relevant economic models and results. In Chapter 2 we look, in detail, at the trends and policies within the UK that have led to the fall in bus, and until recently the stagnation of railway, use and take a critical view of the various policy options available. We examine problems associated with continued under-use of public transport, and discuss the (associated) reasons for, and effects of, increased use of the car.

A reduction in the generalised cost of public transport, making it a more attractive alternative to the car, is identified as being of central importance. We investigate the reasons for the failures of past policy approaches to achieve this. To be able to use the findings from Chapter 2 and to make policy suggestions we to consider ways to represent these issues in a tractable form, so in Chapter 3 we introduce the various theoretical modelling approaches that can be employed to help analyse the issues of transport integration. These are broadly microeconomic in nature – largely deriving them from the literature of industrial organisation.

In Part B (Chapters 4 – 8), we examine the incentives and social benefits of integrated ticketing on a transport network. In Chapter 4 we apply the network model of Economides and Salop (1992) to transport, so we can investigate the effect and intuition the model can bring, as well as its short-comings when applied to transport. This understanding allows us to adapt Chapter 4's theoretical model to make it applicable for the analysis of integrated ticketing issues in public transport, which we do in Chapter 5. We investigate the public and private benefits using profits and a social welfare function. This model is essentially an extension of the Cournot (1838) model. We then generalise Chapter 5's model in Chapter 6 by introducing a (non-zero) conjectural variation term. Three regimes are examined in detail and conclusions about profit and social desirability are drawn. Chapter 7 builds on the "benchmark" model of Chapter 5 by allowing for demand asymmetries: the functional specifications for the demands for cross-service tickets and single-service tickets are allowed to vary. Essentially, we allow the willingness to pay and the price elasticities to vary between the cross-service demands and single-service demands. The private and social rankings of various ticketing regimes are then revisited. In Chapter 8 we take the results from Chapters 4, 6, and 7 and compare them with those of Chapter 5 so we can consider how the changes we make to the model affect the conclusions.

In Part C (Chapters 9 and 10) we turn our attention to the 'completeness' of the provision of network services. The first chapter of Part C establishes a model that will be used to consider whether the appropriate incentives exist for a private monopolist to provide the socially desirable level of service provision. We derive and compare the equilibrium price, quantity, and profit outcomes on the network under a social planner regime and a monopoly regime. In Chapter 10 we extend the model and investigate how entry on a sub-set of the network can affect the equilibrium variables and discuss the relative merits of such policy interventions.

The final part concludes the thesis by summarising the results and indicating areas for possible future research.

PART A

ISSUES IN UK TRANSPORT AND REVIEW OF LITERATURE

CHAPTER TWO

AN INTRODUCTION TO THE ISSUES IN UK TRANSPORT

2.1. Introduction

In this thesis we will concentrate on identifying and exploring measures that can be used to promote UK public transport use. This chapter looks at the reasons for discouraging private transport use due to the problems of congestion and pollution. The past deregulation of transport industries has not been entirely successful and has led to unforeseen problems. In some cases this has resulted in a number of services becoming uncoordinated; posing policymakers with a problem: how can they increase coordination without removing the benefits of increased competition? We highlight possible ways of achieving this with the intention of further investigation in later chapters. We also draw attention to the possibility that following deregulation some areas have been left with situations close to monopoly transport provision.

This chapter begins by looking at falling public transport patronage and how this contrasts with the rise in the popularity of private transport to form the basis of why bus and railway use needs to increase. This then leads us to consider the reasons behind the trend so we use academic literature, data, and policy documents to investigate the deregulation of the bus industry and the privatisation of railways, and this analysis highlights the possibility that coordination could be used to increase flexibility and reduce the generalised cost of transport services. We investigate ways of coordinating transport by looking at government proposals and the impact of current schemes. The results of these strategies are not entirely conclusive, so we identify coordination issues that require clarification in the later chapters of this thesis – particularly with reference to integrated ticketing. We also find that past policy decisions may have resulted in some geographical

areas having situations approaching a monopoly in transport services and this will also be investigated in subsequent chapters.

In the following section we look at modal trends in UK transport over the last 50-years. We will identify a substantial growth in the use of the private mode relative to the public transport modes' patronage. In Section 2.3 we consider the reasons for this increase in car use, with a view to understanding why individuals are choosing to use their cars more. In Section 2.4 we will examine why it is necessary to promote public transport use by looking at the problems that excessive car use brings. In Section 2.5 we investigate policies that can be used to encourage bus and rail patronage whilst possibly reducing car use. In particular, we consider the potential of an integrated transport system to promote public transport. In Section 2.6 we consider policies that impact on cars users in a direct attempt to persuade them to use their car less. In Section 2.7 we summarise the chapter and focus the thesis on the ways of reversing the problems of deregulation and privatisation – stating the reasons for this thesis focusing on integrated ticketing and the provision of transport services.

2.2 Modal Trends in UK Transport

Before we establish how to promote public transport we should first look at trends in UK mode use. The main types of travel we shall consider in our analysis are train, bus, coach, and car. Figure 2.1 shows a large rise in the number of cars that have been licensed during the last half a century in the UK. An increase in the amount of licenses does not translate into an increase in car usage in the UK, but such a rise can be seen in Figure 2.2, which shows the number of kilometres that are travelled using the car along with similar figures for buses and coaches, and trains. We can see that there is a contrast between

increased car use and falling bus use as over the period. Car use has drastically risen. but since 1962 the amount of passenger-kilometres travelled using the bus has fallen every year

Figure 2. 1: Private Cars Licensed at The End of Year (Thousands) 1952-2004
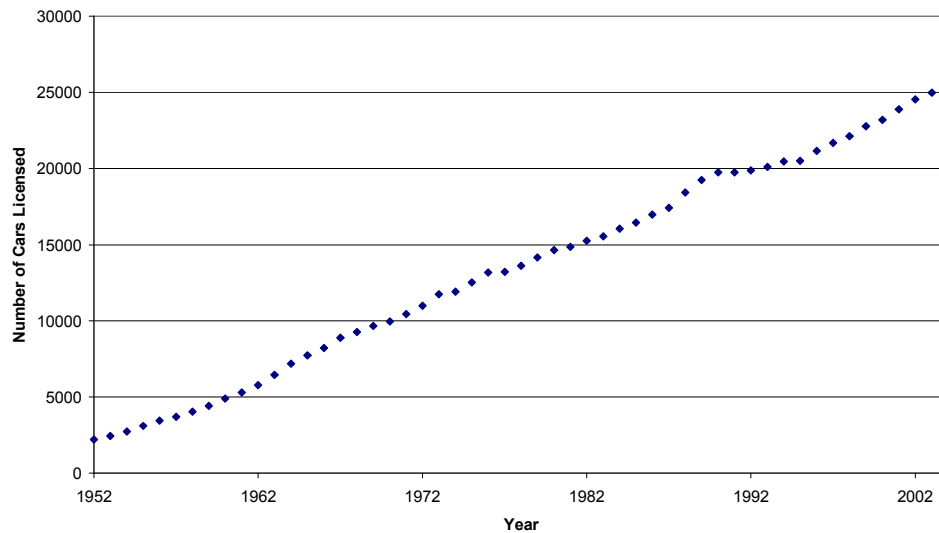
until the recent levelling off. Additionally Figure 2.2 indicates that rail use has remained

Figure 2. 2: Passenger Kilometres Various Modes
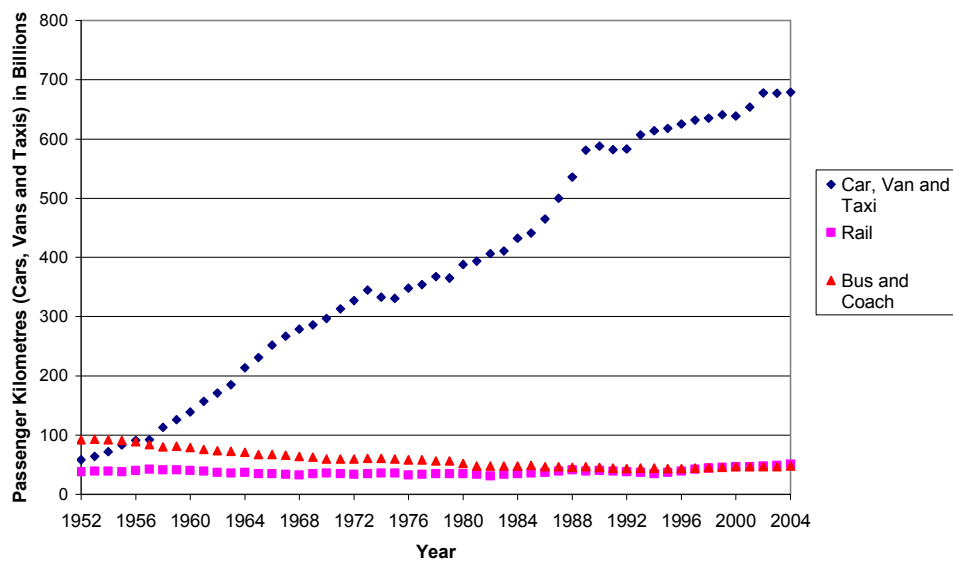
relatively constant, until recently when there has been an increase. Over the past 50-years the UK has experienced a slight shift from public transport to private transport and a considerable growth in the additional use of the car. We will investigate the recent trends in buses and railways more closely in subsequent sections.

If we are to consider ways of combating increased car use and the associated problems we need an indication as to whether the trends seen above are set to continue. Figure 2.4 shows that since 1975/76 the percentage of people with full car driving licences has increased for all age ranges and this will likely mean a larger number of drivers in the future. For example, the larger numbers of drivers at 40-49 year old drivers now will lead to

Figure 2. 3: Full Car Driving Licence Holders By Age
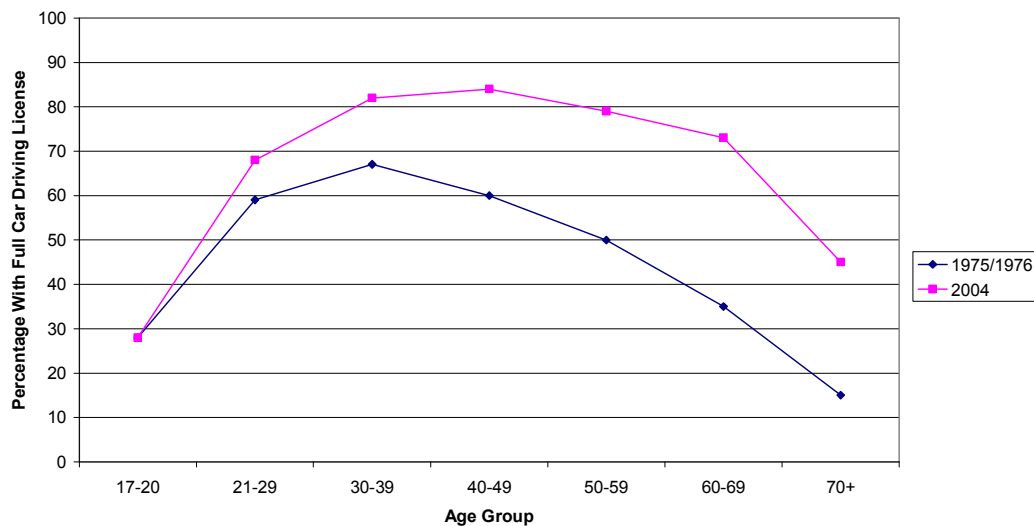
a relatively larger number of 60-69 year old license holders in twenty years, and when this is applied across age ranges then future cross-sections will, in particular, fill more of Figure 2.3 to the top right and top left of the graph – thus it is likely that there will be a greater percentage of all age ranges able to drive.

The Department of Transport (1989a) predicts car ownership will rise by 52-71%, and car use will increase 72-113% between 1990 and 2025. Romilly et al (1998), who use error correction model forecasts, support the likelihood of increased car ownership but find that the growth in car ownership would be at a peak in the year 2000 – that is car ownership will continue to rise, but at a falling rate. However, Figure 2.1 would seem to contradict Romilly's suggestion of a fall in the growth rate post-2000 as it shows the number of cars licensed has continue to rise with no reduction in the growth rate. A more recent study by Whelan (2007) uses a discrete choice model to predict that the average number of cars per household is likely to rise from 1.08 in 2001 to 1.24 in 2031, with the total number of vehicles increasing 42% to 36.35 million over the same period. Although, the predictions vary in method and magnitude the general consensus seems to be that the shift towards private transport, possibly with a small accompanied shift away from public transport, will continue for the foreseeable future.

Increased car use potentially brings many problems that we shall explore in Section 2.4. First, let us investigate the reasons behind increasing car use as this should identify areas that can be targeted by policies that encourage public transport use.

2.3 Explanation of Modal Trends

We establish an upward trend in car use in the UK. To discuss and recommend suitable policies to promote public transport we must first understand why it is that people have switched to private transport.

2.3.1 The Car as the Primary Modal Choice

In this section will investigate reasons why the car seems to have become the main mode of transport. We shall look at travellers' reasoning and why individuals favour car use. By understanding this we can evaluate the impact of past policy changes and consider how the relative attractiveness of public transport could be improved. Much of this concentrates on everyday decision-making, but we seek to establish a structured approach and then consider how policy can be used to affect modal-choice decisions.

2.3.1.1 Rising Incomes, Falling Costs and Car Use

Figure 2.4 shows that incomes in the UK have, mostly, been increasing and this could influence individuals` modal choices. De Jong and Gunn (2001) find that general traffic has an income elasticity of 1.2, so as income goes up by 10% then traffic goes up by 12%.

Figure 2.4: UK Real GDP per Capita, 1960 - 2005
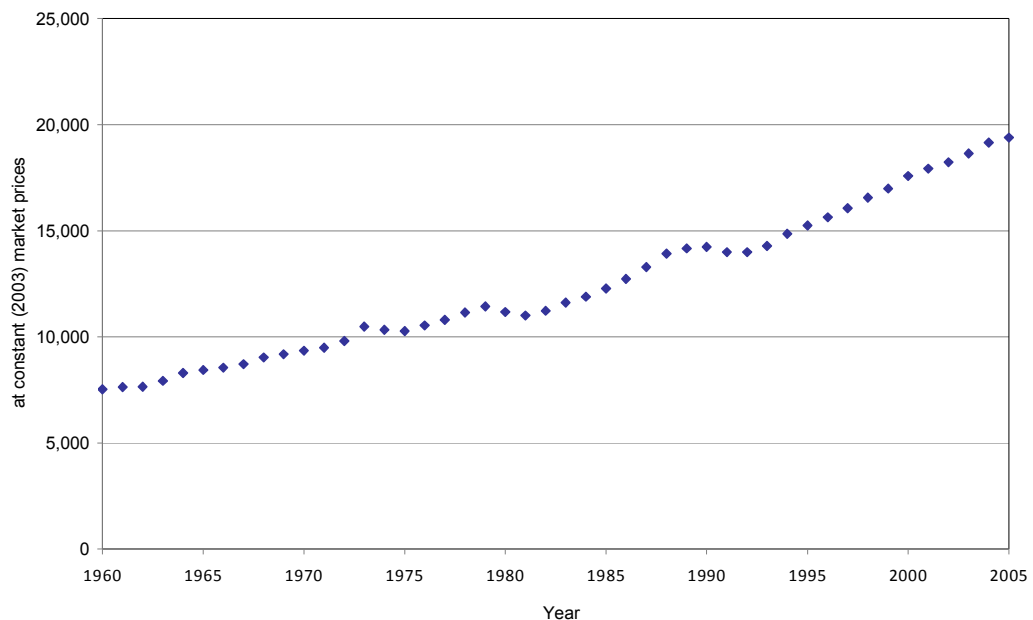
SOURCE: Economic History Services (2007)

If we apply this to the data from Figure 2.4, where there has been a 27% rise in GDP per capita between 1995 and 2005, it implies a 32% rise in traffic over that ten year period.

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Dargay (2004) uses cross-sectional data from the annual UK Family Expenditure Survey (FES) to investigate the effects of changes in various parameters on car ownership and use. The study finds that car use does increase with income. Naturally, they also find that car ownership increases with income, but the elasticity with respect to increasing income is greater than elasticity with respect to falling income. They conclude that rising incomes make it easier for households to own cars but when income decreases the households are unlikely to return to non-car ownership. This would complement and add to the effects we will discuss in the next few subsections.

Glaister (2002) shows that the composite index, which representing all costs of motoring, has been constant relative to the Retail Price Index since 1964 and contradicts the popular view that motorists continually face increasing car travel expenses. Falling costs of motoring and the increased incomes we saw above means that car use is clearly getting more affordable – resulting in more car use.

2.3.1.2 Goods and Services Whose Consumption Depends on Car Use

Simply owning a car increases the viable range of travel destinations for the individual. This means that with private transport there are an increasing number of goods and services, such as out-of-town developments, that are now more accessible. Conversely, as car ownership and use increases then accessibility to out-of-town developments increases so the number of out-of-town developments grows. However, this then reinforces the need for cars to be able to access such facilities. Big out-of-town complexes bring certain economies of scale and scope that allow a multitude of services beyond that which could be provided by smaller local services. A recent UK All-Party Parliamentary Group for Small Shops (2006) report suggests that corner shops are being forced out of business

by supermarkets. For many this erosion of local shopping facilities mean that travel by car is slowly becoming vital for individuals to be able to purchase everyday groceries.

As local services become both less, attractive and, prevalent compared to services that require travel then more individuals become attracted to car use. Often out-of-town developments will have public transport links, but using these when shopping can be extremely awkward. The perceived private generalised cost of using the car to visit such services could seem a very much cheaper alternative. Mackett (2003) presents examples of studies that suggest urban form impacts on travellers' mode choice. As facilities needed for everyday life become harder to access by walking or the use of public transport then this will lead to people increasingly using their car. Indeed, Mackett's (2001) study recommends local shops and facilities should be encouraged as a method of combating increasing car use.

Another problem that arises as facilities become harder to access without a car is that those with no private transport become excluded. This social exclusion can also bring about a range of problems such as crime and ill-health, so it could be in the best interests of society to either ensure local services are available or to provide a viable method of public transport to the non-local services.

There are also reasons other than the impact of urban form why individuals are more likely to own and use cars. In the next section we shall continue our look at the possible reasons behind increased car use.

2.3.1.3 Cognitive Car Distortion

Once an individual has purchased a car they may no longer make rational comparisons between modes for every trip. A rudimentary example is that an individual

may simply think: "Now I have paid for this car, I may as well put it to use." The decision to purchase a car may simply be the individual making their modal choice for the foreseeable future. This type of thinking sees the individual saving on future decision-making and information costs – possibly some reasons why there is increasing car use, but car owners could still asses their options concerning the choice of mode even after purchasing their car.

Even if we are to assume that the individual makes a rational modal-choice decision for every trip, the ownership of a car may still cause the individual to exhibit some cognitive distortion that favours car use. Let us consider the costs a car owner perceives when they wish to make a journey. The car owner is likely to only take the petrol and journey time costs into account as they calculate their generalised (short-run marginal) cost of using their car – a car owner once a car is purchased would no longer need to consider the cost of buying the car (the long-run marginal cost). If the same person were to calculate their generalised (short-run marginal) cost of using public transport for the same journey they would perceive costs due to ticket price, journey-time, interchange, and wait-time. We can see that the perceived private short-run marginal cost of the car would be lower than the alternative, public transport's.

The fact travellers choose the mode with the lowest short-run private marginal cost suggests that car owners are in-fact being well-behaved economists. However, there are some costs that are not taken into account such as those that do not directly affect the individual, but are borne by other car users or members of society. Such costs include pollution, congestion, accident costs, and extra wear-and-tear cost for car usage – these unperceived costs do not include the cost of purchasing, or the depreciation of, the car. We

shall see in Section 2.4 that the pollution, congestion, and accident costs associated with public transport are generally much lower than those involving the cars.

Cole (1998) provides us with an interesting mode comparison using the perceived cost of travelling from central London to central Paris. Depending slightly on the method of crossing the channel in the car (Le Shuttle, Hoverspeed or Stena Ferry), the actual cost of car use can be around double the perceived cost. Once the full costs are taken into account travelling by coach or the train between London and Paris is a more attractive proposition. This illustrates the possibility that the costs perceived by the individual may be unfairly favouring the private mode over public transport – suggesting a bias in favour of car use.

It is also important to account for the non-monetary costs and travellers' perception of these. Generally, individuals object to time spent travelling (although Lyons et al (2007) seek to disprove this) but for the main part they generally dislike walking time or in-vehicle waiting time more than in-vehicle journey time. Often people are willing to pay more to forego walking or in-vehicle waiting, whilst in-vehicle waiting cost also varies by mode. The UK Commission for Integrated Transport (2007) attempt to allow for the comfort and personal space offered by the private mode by using an in-vehicle time of "one times actual" for cars and "1.1 times actual" for public transport, reflecting the fact that travel by car is relatively less costly to the individual than public transport.

Travellers' valuation of time also varies between modes. According to Cole (1998) it is rail passengers who have the highest value of working time – with car drivers second highest. The DfT (2007) calculate the value of working time and these values can be found in Table 2.1 with the rail passenger, again, having the largest value. The value of working time is calculated using the price of the person's labour so, it is positively related to income; those who earn more tend to have a higher value of working time, although there are also

Table 2.1: Value of Working Time Per Person by Mode (£ per hour at 2002 prices)

	Working Time Per Person
Rail Passenger	£36.96
Car Driver	£26.43
Car Passenger	£18.94
PSV (Bus) Passenger	£20.22
Cyclist	£17.00

SOURCE: DfT (2007), Table 1.

other factors to consider. Generally, individuals with higher incomes are more likely to favour quicker methods of travel as the time they spend travelling is time that they are not earning money. Rail travel is fairly quick and allows the user to work, so this could be why rail passengers have a higher valuation of time. Additionally, car use is a fairly quick method of travel as there is no need for interchange, so this attracts people who tend to have a higher valuation of time and explains the positive relationship we discussed in 2.3.1.1 concerning car use and income.

The traveller's bias, with respect to the car, could be seen as having two elements. The first is some cognitive distortion that makes the individual believe a car journey is cheaper than it actually is. The second is that the full cost of the car is not borne by the traveller usually as some costs of a car journey, such as congestion and pollution, will fall upon fellow car travellers or other people. Basically, the cost to the individual of a trip in a car does not fully reflect the cost imposed on society. The possibility that car travel is not correctly priced, relative to other modes, be it privately or socially, is an argument used in support of road pricing and we shall discuss this in Section 2.6.1.

The arguments that individuals' decision making is biased towards car use suggests that policies directed at lowering car use in favour of public alternatives may well need to emphasise decreasing the perceived generalised costs of travel by public transport. This is, as we shall see, one of the main reasons behind the coordination of transport services policy.

Mackett (2003) finds public transport improvements that decrease the perceived generalised cost of travel by public transport are the mostly likely action that will result in drivers switching from car to public transport use. Let us continue the discussion of modal decisions that see individuals favour car use.

2.3.1.4 Car Use and Social Positioning

Akerlof (1997) focuses on some ideas that also help explain the trend towards car use. Initially he considers a model of a status-giving good using a linear utility function that causes an individual to lose "happiness" as they fall behind people in their consumption of the status-giving good. This model predicts the over-consumption of the status good from a social perspective. If we were to consider the car to be status-giving then this may help to explain why people use cars beyond the socially desirable level. It is difficult to justify the number of car trips or kilometres is in someway status giving, but the status aspect could be interpreted as car ownership or the value of the car owned. It is also possible that the use of other modes could give the user status; examples include airline, or first class rail, travel.

Akerlof's (1997) also proposes an alternative model that uses conformism and in this individuals lose utility when their use of a good does not match others' choices. Let us consider this in relative terms, so that an individual would lose utility if the percentage of the trips they made in their car was lower than others. For example, if an individual was to go to work by bus while their colleagues went to work in a car, then that individual may feel unhappy when they realise they are in the minority who travel to work by public transport. The next day the person rectifies this by travelling to work in their car.

Similar arguments can also be used concerning the purchase of cars. In the past people may have felt "status pain" for not having a car but now one-car families may lose

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utility when their neighbours or workmates all have two or more cars. Undoubtedly, this "status effect" will have diminished in recent years[1] but it is likely it would have had a major part to play in adding to increasing car ownership and use over time.

Akerlof finds the conformist model leads to people copying the "status quo" and results in either an underproduction (if the norm is less consumption) or overproduction (if the norm is greater consumption) of the conformist good. In the case of transport it seems to be likely that the norm is to travel more by car, so we get excessive car use.

The status model suggests positioning public transport as the cheap or cheaper mode may, in fact, reduce patronage. However, if public transport could become perceived as high quality then the status model would suggest it might increase patronage. The more realistic conformism model suggests if a policy was successful in encouraging individuals to use public transport then others could follow. Both status and conformism models suggest that individuals' modal decisions can be influenced so that transport policy can have a positive impact on patronage.

2.3.1.5 The Car as the Primary Modal Choice Conclusion

We have seen in this section that there are various possible reasons for increasing car use from changing attitudes to increasing incomes. However, it seems possible that if public transport services can be seen to be improved then the resulting fall in the perceived generalised cost will likely promote its use. Past policy decisions aimed at increasing public transport usage have not solely been based on reducing the cost of public transport to the consumer and we will see this in the next section.

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[1] This "status effect" may now be more pronounced in the particular model of car purchased.

Previous changes in UK transport structures have meant schemes were aimed at increasing the attractiveness of public transport by making bus and rail industries more sensitive to consumer demands. However, these policies, such as the deregulation of the bus industry and the privatisation of the railway industry, were not always entirely successful and we intend to investigate bus deregulation and rail privatisation in more detail. This will examine the intention of the policies, detail the various failings and successes, highlight ways to correct the problems, and suggest which actions are necessary. The analysis will requires us to look at the history of each mode in the UK to help us understand why the decisions to make structural changes were deemed necessary and may indicate further reasons why the UK continues to shift towards car use.

2.3.2 Pre-Deregulation UK Transport

Before the 1980s the UK public transport was dominated by publicly owned monopolies that were heavily regulated. All bus services were provided under a licensing system with the National Bus Company and its various subsidiaries operating throughout the UK. This state-owned company was not the only transport provider with London buses operating in Greater London, and Passenger Transport Authorities in urban areas, plus a few municipal bus companies in certain towns and cities. However, the licensing system meant that the incumbent operator in any location was protected from competition and was often a local bus monopoly. The railways were solely operated by British Rail, who was initially formed in early post-war period to operate services, as well as owning all infrastructure and vehicles.

It was thought that these 'protected' monopolies were the best way to ensure the provision of a comprehensive (due to cross-subsidisation of loss-making routes with

profitable ones), coordinated, reliable, and safe public transport system. However, it was the failure of this "regulation model" to attract travellers that led to the deregulation of buses in 1985, and the privatisation of the railway in 1993. The hope was that the removal of regulations and state-owned monopolies would result in public transport that remained comprehensive, coordinated, reliable, and safe, but more effective and, hence, better utilised by travellers.

We will now take a closer look at bus deregulation in the next section before moving on to look at the railways in Section 2.3.4. Our general focus will be on the changes, and associated results, in the regulation and structure outside of London. It is not clear that the system that has evolved, and appears to work well in London, is a model that would be successful in the rest of the country. For this reason we choose not to look at London in detail as it is difficult to tell whether these systems would be appropriate for a national transport strategy or if the situation is representative of other areas in the UK.

2.3.3 The Decline of the Bus Industry and Bus Deregulation

The introduction of the 1985 Transport Act (H.M. Government, 1985) was aimed at addressing the long-term decline of bus passenger kilometres (see Figure 2.2). It was hoped that shifting the bus industry away from a licence system, which allowed the licensing authorities control over services, timetables, and fares, would result in a number of benefits. One aim of the removal of the licensing was to encourage competition by decreasing the protection that the incumbent operator often received. It was believed that the removal of the licence system would lead to a fall in costs, prices, and increase quality, hence attracting more travellers and reverse the declining trend in bus use. There was also a belief that ticketing and timetable coordination between firms could be achieved whilst encouraging

competition. In this section we shall see that many of the expectations were not always achieved.

The improvements that bus deregulation would bring were expected to come about in a variety of ways. Glaister (1985) suggests that some innovation was hoped for and, in particular, there was an emphasis on developing high-quality, high-fare minibus services that would be more flexible and more tailored to travellers needs than previous. Nash (1993) blames the existence of cross-subsidies for much of the regulated era's high prices and low service levels associated with heavily demanded routes. Nash also believes some routes had excess service provision and a profit motive in the bus industry would not only remove the inefficient cross-subsidies, but also encourage optimal provision of services as competition was expected to take place on the more heavily demanded routes.

Many of the expected results of bus deregulation depended on the "alleged" contestable nature of the bus market. It was believed this contestability would ensure that the improvements from deregulation would not be isolated to routes where competition would prevail, such as the heavily demanded routes, but would also impact on lesser demanded routes.

2.3.3.1 Buses and Contestable Market Theory

One of the main arguments against the bus deregulation policies was based on the view that the bus industry was not contestable. Evans (1990) goes so far as to suggest the bus market was deregulated on the assumption that it was a contestable market. Contestable market theory states credible potential entry can limit the extent of market power, and the potential increases in price that are associated, within an industry, but this theory remains controversial and has been the subject of much criticism.

The main problem with contestable market theory is the market has to meet a number of specific requirements for entry to be credible, and to influence the incumbents' pricing. Dodgson and Katsoulacos (1988, 1990) believe such requirements are not met in bus industry. They point out two main differences between the bus industry and the theoretically-based contestable market. The first they point out is that a large investment to purchase vehicles is required upon entry in the bus industry, although this view is opposed by Nash (1993) and Button (1988). The second is it is possible incumbent firms could quickly alter their prices in response to entry to reduce any entrant's profit. The sunk costs and possible incumbent price adjustments act as a barrier to entering the bus industry because they lessen the chances that an entrant will be able to make a profit and reduces the credibility of entry.

The possibility that the purchase cost of vehicles is a valid barrier to entry is problematic as entrants can rent buses and the majority of a bus purchase price can be retrieved upon leaving the market by selling the bus. However, both Blattner (1973) and Spence (1977, 1979) state another general criticism of contestable market theory, which would seem valid in the bus industry. Their observation is that excess capacity acts as an entry deterrent because it signals to an entrant that the incumbent can react to any entry by decreasing prices and using the excess capacity. This reduces the chance that any firm could successfully enter the market, and thus acts as a barrier to the firm's entry. A feature of many transport industries is the need to run with some excess capacity due to the temporal and peak nature of demands, so this could reduce the entry threat in the bus industry.

Nash (1993) disagrees with Dodgson and Katsoulacos's suggestion that sunk costs are a barrier to entry as he does not believe "physical capital" is an issue but, instead,

focuses on asymmetric information that favours the incumbent, increasing the entry costs and manifest as another barrier. In particular, he asserts that publicity is an issue favouring the incumbent, pointing to Button's (1988) findings and the advantages the incumbent possesses in terms of experience; again, an entry barrier that reduces the credible entry threat.

The factors discussed above tend to mean that the bus industry is far removed from the specific assumption characterising a contestable market. Any thought that the deregulation of buses may lead to an overall contestable market is arguably a major oversight. On routes where only one firm is present then it is unlikely that a credible entry threat exists, so an unchecked monopoly supplier could be in operation. Additionally, Langridge and Sealey (2000) provide a more up-to-date look at the decision to privatise the bus industry and state that the question regarding the potential contestability of the bus market is immaterial as the splitting of the National Bus Company up did not lead to a contestable market scenario.

The possible effects of bus deregulation were subject to much more discussion and modelling than simply the possibility of contestable markets. However, we believe this is a key issue especially given how the deregulated bus industry has developed. We shall now explore what actually happened when de-regulation took place.

2.3.3.2 Bus Patronage After Bus Deregulation

Figure 2.5 reports Shire and Metropolitan passenger kilometres and vehicle kilometres for local stage bus services. Vehicle kilometres have increased in both

Metropolitan and Shire counties since bus deregulation in 1984, and we can see this increase has been more notable in the Shire counties[2]. The rise in vehicle kilometres could

Figure 2. 5: Passenger Kilometres (billions) and Vehicle Kilometres (millions) for Local Stage Bus Services by Area
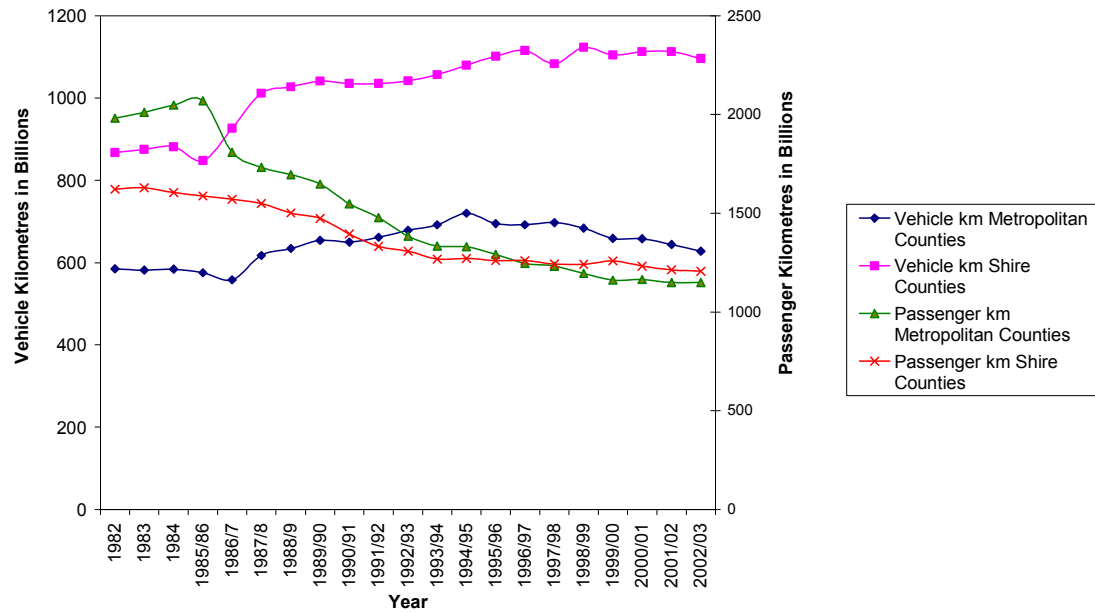
be attributed to two contrasting explanations. Initially, it may seem logical to reason that there has been an increase in the frequency of buses and thus an increase in the level of service. However, it may be that buses travel longer more complicated routes so that a poorer level of service is provided. Nash (1993) focuses on what caused the increase in vehicle kilometres and concludes these increases are mostly due to a big increase in the number of operators in medium sized towns and cities – thus an increase in frequency in such areas is the main cause of the increase in vehicle kilometres. Something that we should not forget when considering frequency is that an increase may also lead to

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[2] More recent statistics call these counties "other areas."

congestion such as that caused by the outbreak of a "bus war" on Manchester's 192 bus route (BBC, 2006a).

Contrasting the apparent increase in bus services is that both area types experience a decline in the number of passenger kilometres with the biggest decrease in patronage in the Metropolitan counties. The Metropolitan counties suffer a sharp dip in passenger kilometres following bus deregulation with patronage levels eventually falling below that of Shire counties. This drop is quite surprising as we would expect these Metropolitan counties to have denser demands and it was on such routes that the Department of Transport (1984) expects in terms of improvements in bus services and patronage.

Nash (1993) states that the falling Metropolitan bus use may be as a result of increased car ownership. However, it is possible people turned to the car as deregulation had adversely affected their perception or experience of bus travel. Mackie and Preston (1988) provide some support for this view. They suggest instability and lack of information in the timetabling due to deregulation may have led to a "loss of public confidence" in the bus as a mode of travel. The loss of faith in the bus service could be seen in terms of an increased perceived generalised cost of travel by bus so that the car became a more attractive mode of travel. In Evans' (1990) bus deregulation case studies it seems there has been limited impact of competition on bus patronage. If anything the results Evans presents indicate there has been a fall in bus patronage following bus deregulation and this is supported more broadly in Figure 2.5.

2.3.3.3 Bus Prices and Competition After Bus Deregulation

Let us look at the changes in fares since deregulation. It is reasonable to suppose that a fall in patronage might be preceded or accompanied by an increase in fare. Figure 2.6

Figure 2. 6: Real Bus Fare Indices by Local Stage Area (1984=100)
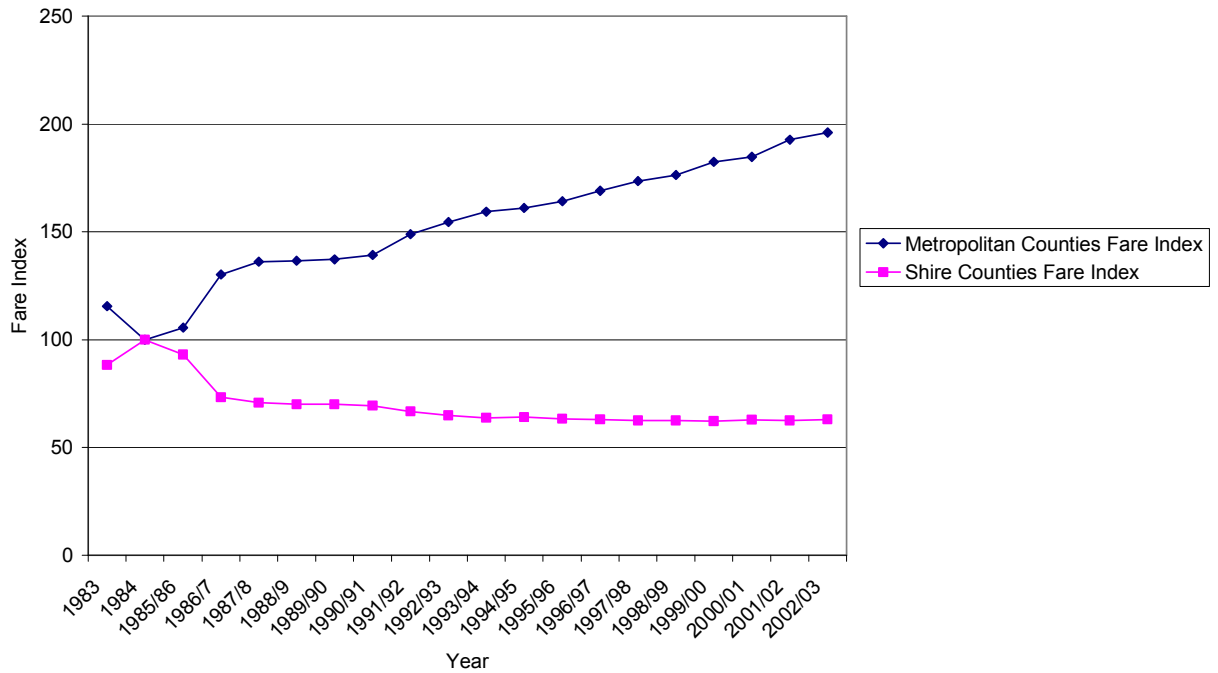
shows real fares over the same period. The price rise of the former would seem, at least

partially, to be the blame for the decrease in passenger kilometres. This major rise in fares

for Metropolitan areas may be linked with the restriction on the local authorities' ability to

give subsidies to bus services. Nash (1993) suggests Metropolitan local authorities formerly

offered such subsidies to ensure low fares and this would mean some of the loss of

patronage is due to the restriction on the ability to subsidise; not simply the absence of

competitive forces following bus deregulation. However, an increase in fare cannot be

blamed for the Shire counties reduction in the passenger kilometres so other factors such as

falling coordination may had a part to play – we discuss this in Section 2.3.3.6.

It is interesting to note that it is on the denser, metropolitan routes that the pre-

deregulation predictions suggested would most likely see the effects of price competition

and have a fall in prices; this has clearly not occurred. Evidence of price competition in the

bus industry seems to be hard to come by with little literature considering competition.

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Gomez-Ibanez and Meyer (1993) argue that "active competition" had been limited as they find only around 3% of bus-kilometres had direct "on-the-road" competition, although this is for the period immediately after deregulation. It may not be reasonable to expect immediate competition following the policy change and Evans (1990) tries to clarify the effects on competition of deregulation over time. He points to Balcombe et al (1988) who states the level of competition on individual routes (or "on-the-road" competition) had increased 300% following the year of deregulation. Despite the massive rise it only results in a fairly unimpressive figure of 9% direct "on-the-road" competition. Even less remarkable, as Evans (1990) points out, is that by 1990 the figure was already in decline. He moves on to assert the existence of competition tended to be the exception rather than the rule.

Nash (1993) suggests if any competition has taken place it has been on service levels. Our simple look at vehicle kilometres in the previous section, which saw us find bus kilometres increased post-deregulation, would tend to support this view. Evans (1990) looks at three case studies in Preston, Lancaster, and Stockton-on-Tees for a three-year post-deregulation period where competition prevailed. All three case studies show that when competition exists there is a significant increase in scheduled bus-kilometres. Despite choosing areas in which direct "on-the-road" competition took place, Evans finds the fares set pre-deregulation were maintained under competition. According to Evans the firms on allegedly "competitive" routes had actually colluded to coordinate fare increases. He emphasises this point by asserting that fares had not fallen on routes with higher demand, contrary to what the Department of Transport (1984) expects. Evans observes that non-competitive routes do not have significantly differing fares compared to competitive routes, although he does admit there were some cases of price decreases such as a brief decrease in fares on one route in the Stockton case study and another on return fares in Lancaster.

Evans (1990) in a similar vein to Nash (1993) believes that the most sustainable

competition is when competitors compete over frequency despite the possibility that such a

strategy results in lower profits.

Evans (1990) suggests the pay-off from entry is either the hope the entrant may drive

the incumbent from the route or a collusive agreement with the incumbent may be found so

frequencies can be reduced. Both these reasons indicate the possibility that there will be

effective monopoly operation on some bus routes. Evans admits entry has been a rare

occurrence and more recently Langridge and Sealey (2000) talk about the re-oligopolisation

of the bus industry and hint at a drop in competition following de-regulation. It seems that

competition and falling fares have been seen sparingly in the bus industry, so that

monopoly operation of some bus services is a real possibility.

2.3.3.4 Bus Operating Costs After Bus Deregulation

Let us now investigate the trend in bus operating costs over the chosen period. Many

of the studies that focus on the effects of bus deregulation state if any gain in welfare is to

be made then a significant decrease in costs would have to be achieved. Figure 2.7 shows

cost in pence per vehicle kilometres and highlights a major success of deregulation as there

are reductions in costs for both Metropolitan and Shire areas. Evans (1990) observes that

three years after deregulation costs had dropped by around 20%.

We, of course, should be careful as not all cost savings are necessarily good for the

industry because it could mean that service quality has fallen and this could result in lower

patronage levels. Other caveats concerning the apparent fall in bus industry costs include

Figure 2. 7: Bus Industry Real Costs (2004 Prices)

Legend:
- Costs Metropolitan Counties Excluding Depreciation
- Costs Shire Counties Excluding Depreciation
- Costs Metropolitan Counties Excluding Depreciation"
- Costs Shire Counties Including Depreciation"

Y-axis: Pence Per Vehicle Mile
X-axis: Year (1985/86 to 2002/03)
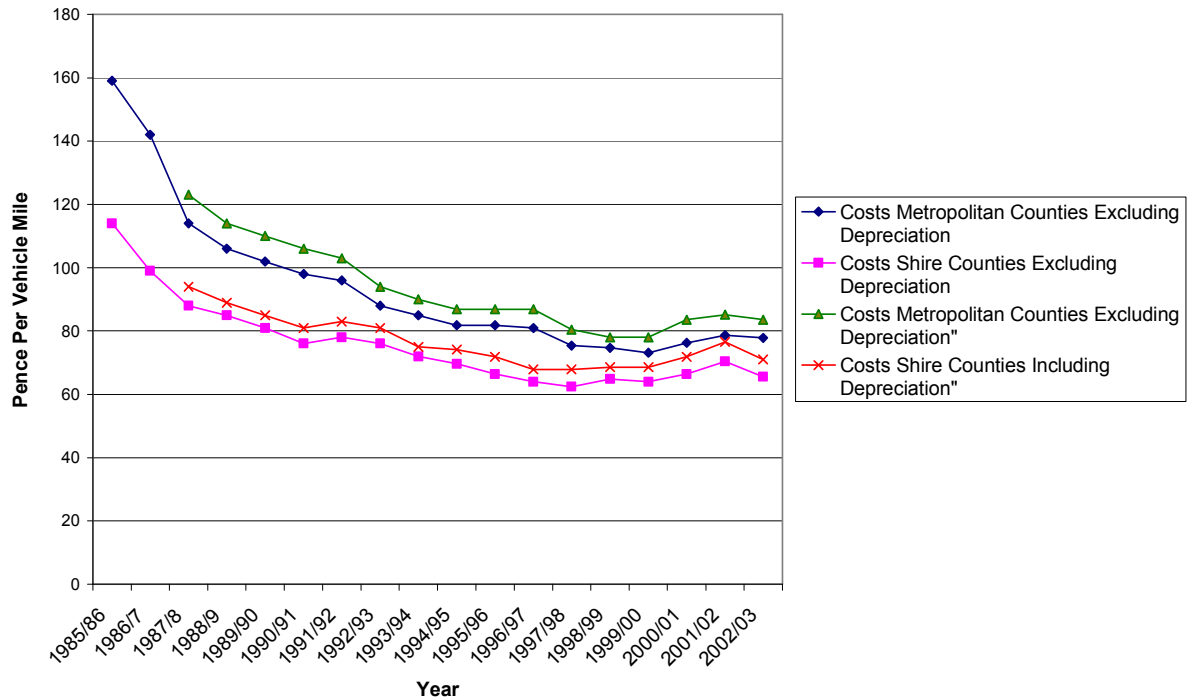
SOURCE: DoT (1995) Table 5.7 a and b, and DoT (2004c), Table 6.26 a and b.

the possibility that some cost reductions can also be attributed to shifting certain cost burdens away from bus companies. Such a suggestion is made by Tyson (1989), who emphasise that the cost of some bus stations, which remain the responsibility of Passenger Transport Executives and local authorities, are not included in the above calculations so the reduction in costs may have been over-estimated.

Nash (1993) asserts there has been a real cost saving and attributes this to the possibility of contestability. This is not as unrealistic as previous contestable market arguments as he believes it is the result of competitive tendering in the subsidised sector where the threat of entry is more realistic. Heseltine and Silcock (1990) find around a 30% fall in costs following deregulation in Metropolitan areas. These figures should be regarded somewhat tentatively as they ignore the costs incurred by the Passenger Transport Executives, who as previously stated retained some costs that were not transferred during deregulation.

The major savings within the Metropolitan PTCs seem to come from improvements in the productivity of staff with many bus managers identifying the falling subsidies as the dominant downward pressure on costs. This accounts for 19% of the total 30% savings. Heseltine and Silcock (1990) attribute other major cost reductions to falling wages, extending vehicle life, and falling fuel costs. Recent increasing fuel prices may partially explain an increase costs in the last four years of our analysis. Heseltine and Silcock's (1990) results show the changes to the bus industry in the 1980s did lead to a fall in costs and cannot be simply explained by costs being shifted away from bus companies.

The cost savings in Shire areas may explain the drop in price experienced there, but

Figure 2.8: Public Transport Support for Local Bus Services in
Great Britain Outside London
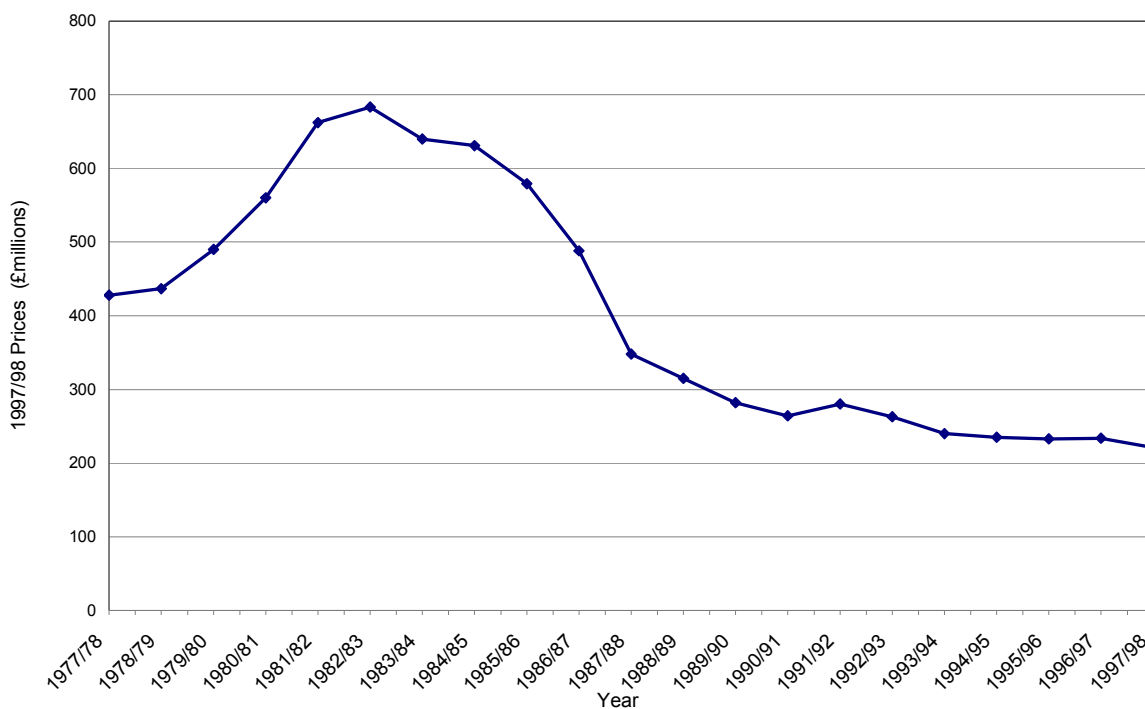
SOURCE: Focus on Public Transport - Great Britain: 1999
Edition (DETR, 1999), Table 20.

Metropolitan area fares increased despite costs falling. However, this could be explained by the fall in subsidies that accompanied bus deregulation as seen in Figure 2.8. The graph shows that the support for local bus services approximately halved from the beginning of

the period, 1977/78, to the end of the period, 1997/98, and the bus companies would have needed to make up for this fall in income. However, a drop in subsidies may only partly explain the divergence between costs and prices as the relaxation of regulations could have led to market power, in turn, resulting in higher prices.

Of the trends we investigate following bus deregulation only bus costs and Shire county fares show any significant improvement. The cost saving is an effective welfare gain in the bus industry and means that despite falling passenger kilometres in Metropolitan and Shire counties, there could have been an overall welfare improvement. Evans (1990) sums up his findings for his case studies by considering a welfare analysis of the results of deregulation. His results are sensitive to his assumptions, but he does find it possible that deregulation resulted in a net welfare gain. He points out competition decreased previously substantial profits, so that even if overall welfare did not increase then there could have been a net transfer from operators to users.

Evans suggests potential entrants have had very little effect on monopoly operators – again, contradicting what you would find in a contestable market – and means monopoly powers in some areas are not limited so operators are able raise prices; decreasing welfare. However, he also observes that industry profits, after the initial three years, had been low.

Evans' results are based on the figures from just three years after deregulation took place. Ireland (1991) suggests the bus market may take a little longer than this to adjust to the new long run equilibrium. He not only considers a longer period but introduces a different model of deregulation and his results are more suggestive of a fall in welfare due to deregulation and he highlights two reasons for this. First, entrants fail to take account of the fact their entry reduces the economies of scale of other operators. His model predicts that prices will only fall rarely and this seems to match the results of bus deregulation.

Secondly, he finds that operators do not reduce prices as they had no incentive to attract more consumers from the car.

Before continuing we need to consider another feature of the bus industry. We have already introduced some points regarding the effect bus deregulation has had on bus coordination, but now we will consider it more explicitly. A well-coordinated bus industry is vital to attract and maintain travellers so reduced coordination due to deregulation could explain much of the loss of patronage, which could have contributed to the increased use of the car.

2.3.3.5 Coordination After Bus Deregulation

A possible impact of bus deregulation that requires further exploration is the effect on coordination and levels of service. The disruption in the timetabling due to handing over of control and the introduction of competition may have led to passenger uncertainty, which would have resulted in decreased patronage.

Part of the fall in bus patronage can undoubtedly be attributed to the bus fare increases. However, Tyson (1989) stresses the importance of information and how it has a major influence upon patronage. He states pre-deregulation timetables were formally distributed with transparent, well-publicised changes that many people would be aware of and the distribution of such new information by word of mouth was effective. Following deregulation he finds that various bus operators adopted different ways of providing timetable information varying from printed timetables to nothing at all. The confusion in the provision of timetables meant that, in some cases, the Passenger Transport Executives (PTEs) had to step in and provide an information service. Tyson finds telephone enquiry facilities with computer database assistance were required to keep travellers informed and that £3 million per year was needed for PTEs to provide this service, such expenses are not

included in the cost calculations we saw in 2.3.3.5. The fact a traveller may have needed to actively investigate timetables adds to the generalised cost of bus travel.

The authorities are aware of the need for information to be available and transparent, as well as a relatively stable timetable. Following deregulation bus companies were allowed to enter a market if they agree to provide a service for 42 days. Additionally, 42 days was the period that an incumbent operator was required to give notice on when cancelling a service or when altering any of its timetabled services by more than five minutes (DfT, 2002). This "five minute" rule allows current operators to make changes to the timetable up to five minutes either side of the scheduled time without approval from the traffic commissioner. It was hoped that these rules would bring some stability to the market while also allowing freedom of entry for competition. Recently, there has been an increase in the 42-day notice period for new services, alterations, and cancellations extended to 56 days (OfT, 2006), as stability in the market had become an issue.

Some commentators believe that the 56-day notice periods is too short; adding unnecessary volatility in timetabling and would like to have seen the 56-day notice increased further to encourage coordination but others, like Nash (1993), see it as a barrier to entry. He believes more should be done to make the bus market more competitive, so prices can fall. However, attempts to encourage price decreases with competition or contestability could further impact upon the level of coordination in the bus industry. Nash suggests the notice of registration should be abolished and restrictive practices should be more firmly opposed.

Other authors believe that some practices amongst bus companies that could be regarded as restrictive and potentially leading to higher prices, such as timetable coordination and integrated ticketing, could actually be beneficial to bus travellers. The instability effects of bus deregulation could have had a major negative influence upon bus

patronage. The resulting confusion could have led many to substitute away bus usage to car usage. Relearning a formerly accepted and stable timetable adds to the generalised cost of bus travel, as do any doubts concerning reliability and stability of new bus schedules.

Coordination between firms is not simply concerned with the timetable as the loss of coordination can also adversely affect other practices that may be beneficial to travellers, such as integrated ticketing. Tyson (1989) points out one of the immediate effects of bus deregulation in the Tyne and Wear region was that operators refused to accept tickets of other operators, thus inter-available and integrated ticketing were no longer available. Some operators did seek agreements to accept other operators' tickets but were discouraged by the possibility they would be contradicting competition law. Inter-available and integrated ticketing is an issue we shall investigate further in Section 2.5.2 and it will later form a major part of this thesis.

Tyson (1989) explores the impact of deregulation upon service coordination in the Metropolitan areas outside of London. He points out coordination is an ill-defined concept and explains it as: "a means of attempting to maximise consumer benefit from a given public transport network by administrative rather than market process."[3] The effects of deregulation have not always seemed to be in the interest of coordination despite it being hoped competition could provide a series of benefits including that of a coordinated network.

Another form of coordination is addressed when Evans' (1987) considers "scheduling efficiency," which is the theoretical wait-time of the average passenger if the bus schedule has regular headways as a percentage of the actual average waiting times. Evans (1990) points out the increases in bus frequencies do not necessarily lead to a fall in passenger waiting times and competition may lead to a "bunching" of buses – where buses do not

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[3] Tyson (1989), page 284.

arrive at regular intervals but are concentrated such as when each firms' bus schedules exactly match. When there are irregular bus wait time intervals an increase in bus frequencies would not necessarily lead to a fall in the average wait-time of passenger at all, and we would consider this to be an uncoordinated timetable; as coordination amongst firms would ensure schedules had regular wait times. Evans (1990) shows that following deregulation the "scheduling efficiency" falls by 4–8%; confirming that bus deregulation led to an actual decrease in timetable coordination.

Tyson points out that following deregulation and the resulting increase in the number of operators then it was unlikely the coordination of timetables and inter-availability of tickets could be continued. However, whilst finding coordination and service level problems on less remunerative routes, he observes very little decline in the level of timetable coordination and the level of inter-mode coordination on more profitable routes. It is likely that the increased frequency of buses can mean that buses operate so often that the coordination of the times becomes inevitable, even without actual operator cooperation. Although, Tyson does note that where two firms are competing on a single route it is very rare to find the timetable is coordinated.

Another area of concern for Tyson is that some bus stations require operators to pay a charge for their use, so this is an incentive for operators not to use them and led to some bus operators terminating services away from bus stations. This practice meant the service level fell as facilities at bus stops are fewer than those at bus stations, and another rise in the generalised cost of travel.

The problems Tyson's observes concerning bus coordination have serious implications and could have led to the fall in passenger numbers that we observe. Tyson's study took place when operators were still acclimatising to bus market and improvements to coordination have since almost definitely been made. However, it remains possible some

of the upheaval, uncertainty, and coordination problems brought about by deregulation may have permanently altered travellers' behaviour leading to a switch to car use, which could have persisted. In hindsight, and as we have seen, any process that increased the use of the car should be considered to be problematic.

Let us now return to Nash's (1993) point that more should be done to encourage competition. Beesley (1990) presents a paper concerning anti-competitive behaviour within the bus industry following deregulation. This study concentrates on the effect of bus deregulation on fares and contradicts many of Tyson's (1989) points on how coordination may be improved in the bus industry. It seems there are two differing goals for bus deregulation. Beesley's view is that competition is important in decreasing costs and prices, so that bus patronage would increase. However, we have seen this focus during bus deregulation and it had mixed success. The other view is that coordination is of the up-most importance and should be the main focus because bus travellers' welfare depends greatly on costs other than the fare. Tyson would support the view that coordination is vital to ease travellers' non-fare elements of generalised costs.

These two opposing views of how best to focus the bus industry (coordination of competition) are prevalent throughout the literature. For instance, some commentators make a case for the removal of requirements to register services as an anti-competitive, barrier to entry and the removal of this would make market access easier; leading to downward pressure on prices, and, hence, a drop in the perceived generalised cost. However, removal of the registration requirements may lead to further confusion within the timetabling of buses and so further affecting the problem of coordination among bus companies to result in increases in the perceived generalised costs.

We suggest that one factor in the growth of car usage and the decline of public transport could have been because of a widening of the gap between the perceived costs of

car use (decreasing) and the perceived cost of bus use (increasing), with Bus Deregulation causing some of the latter. Policies encouraging competition and contestability amongst bus firms may have lead to decreases in ticket prices, but it has at the expense of coordination amongst firms, which may have increased the generalised cost of travel and decreased perceived quality. Even worse is the possibility that there have been no effective fare decreases despite major negative impacts on coordination.

Also relevant to the debate about competition and coordination is the focus on the advantages and disadvantages of collusion. Beesley (1990) believes collusion to be harmful to welfare, and it has been proven under certain circumstances (we shall see this in more depth in Chapter 3), but points made by Tyson (1989), as discussed above, may shed some doubt upon whether collusion is harmful in the bus industry. Collusion may bring with it cooperation in the bus industry and lead to some improvements particularly due to coordination between firms.

The benefits of coordination need clarifying as the issue with collusion concerning integrated ticketing is one example that could differ from Beesley's (1990) view, and is of particular interest, especially in light of recent policy that we shall explore in Section 2.5.3. Beesley suggests that legislation exists to deal with cases of anti-competitive behaviour and it should be used. He states that within the UK each case is considered on its own merits, but he believes a rule of thumb should be established. We suggest a rule of thumb is not so easily established within an industry that may benefit from some coordination amongst the operators and Part B will conclude otherwise. Beesley admits there is room for merger within the bus industry as there are several small bus companies. Mergers (i.e. actual economic integration) could bring substantial gains to the bus industry as this could lead to increased coordination between firms and their services. However, the net gains will depend greatly on the interdependencies between firms and their prices. The benefits of

cooperation and competition, and their interrelationship, is an interesting area that requires further exploration, which we intend to partially address when we consider integration ticketing in Part B.

2.3.3.6 Bus Deregulation Conclusion

It is difficult to come to a general conclusion concerning the effects of the changes to the UK bus industry during the 1980s on overall welfare. Ireland (1991) states that there needed to be a large cost saving to make up for the loss of coordination and, although, cost savings did undoubtedly occur it is debatable whether these were of a magnitude large enough to generate an overall increase in welfare. What is worrying is the general fall in passenger kilometres and, outside the general assessment of bus deregulation, we also need to account for the impact that shifting travellers to car use has had on pollution and congestion.

Despite the cost savings from deregulation it would seem the experience of the bus passenger probably worsened. There has been the loss of integrated ticketing and other coordination problems, which have reduced flexibility and mean, unless fares have fallen by a large amount, that the generalised cost of travel by bus increased. As, in most cases, fares did not decrease – in fact, many increased – so it is likely that the perceived generalised cost of bus travel has risen leading some to substitute public modes of transport for the private modes.

There is also the possibility that some areas have experienced a move from a regulated monopoly situation to one that is approaching a private unregulated monopoly, and the seeming lack of contestability means that they are free to set prices relatively unconstrained. It remains to be seen whether these monopolists will have the incentives to provide socially desirable service levels, or whether competition, or the threat of it, could

provide a 'better' outcome. Again, a major concern of this thesis is how to address this question.

The change in policy in the 1980s also had a major impact on the provision of information to the traveller. Bus travellers who knew the bus times and routes pre-regulation would face significant information costs post-deregulation. Not only would their previous knowledge possibly be incorrect, but due to the lack of timetable provision then this information would be costly to re-acquire – again, resulting in an increase in the perceived generalised cost of travelling by bus. The generalised cost of bus travel undoubtedly rose due to the deregulation and the other bus industry changes, whilst the actual and perceived generalised cost of car travel fell, so that bus travel's attractiveness decreased relative to the car's. It is likely this partly contributed to the increased car use we observe in Section 2.2. Any policy that is to encourage individuals back on to the bus must concentrate on decreasing the perceived generalised cost of using the buses and this may not simply be achieved by decreasing bus prices. Recently, Lyons and Harman (2002) further highlight the important role that the provision of information has on encouraging users to switch from car use.

Despite the possibility that welfare may have improved *within* the bus industry, the increased generalised cost from increased fares, reduced frequencies and coordination, in particular the absence of integrated ticketing, due to changes that took place during the 1980s contributed to rising car use. This, in turn, would have contributed to the problems of congestion, pollution, and accidents that we shall see in Section 2.4.

2.3.4 The UK Railways

For a long time the UK railways have been seen by many as the most likely source of answers to the problem of excessive car use. It was hoped that the privatisation of the UK

railways in 1993 could ensure the railways would finally provide a solution to the UK transport problem. However, more than ten years later there are still questions being asked concerning the structure of the railways and whether the train is a viable alternative to the car.

One stance is that the railway network should be as extensive as possible with costs covered by government subsidies. Another popular view is profit-orientated, where only routes that at least breakeven are provided unless the service is in someway of general economic and social benefit. The effects of these perspectives can be traced through the last 50-years with the latter standpoint coming to the fore with privatisation, where it was hoped the market could provide the railways with leadership to reconcile the problems and provide a viable solution. This section will assess the impact of privatisation and whether the railways are capable of solving the UK transport problem, as well as highlighting areas of interest.

2.3.4.1 Historical Perspective of UK Railways

Prior to the decision to privatise Britain's railway it had been run for 40-years as almost completely a monopoly by British Rail. The British Railways were originally nationalised in 1948, as Welsby and Nichols (1999) state, because investment and management were needed to rebuild a system that had been neglected during the war years. Welsby and Nichols admit that during the end of the 1950s and the beginnings of the 1960s, this funding was forthcoming but substantial money was being lost, which threw the viability of the modernisation programme into doubt. These losses called for a new strategy and led to the decision to transfer control over to the British Railway's Board. The pressure to produce a commercially viable railway led to the appointment of Dr Beeching as chairman of the Railway Board and resulted in the "Beeching report" (British Railways

Board, 1963a and 1963b). One of the major arguments/recommendations of this report was that less efficient parts of the rail network were to be closed down in an attempt to cut costs and the general finances could then be concentrated on more passenger intensive parts of the network.

A change in government in 1964, to one that saw the social advantages of certain routes meant some routes that were seen as socially beneficial would survive thanks to a subsidy. However, many routes and services were closed as they were not deemed profitable or desirable enough. Despite these closures, the railway would over the next 30-years still struggle to stay within its budget. The 1980s saw further problems, much of which could be blamed on the under-funding of the infrastructure. Welsby and Nichols (1999) cite the then railway management's reference to a "crumbling edge of quality" and the introduction of another attempt at reform. Joy (1998) refers to this 1980s reform as "sectorisation," where the railway was divided into Intercity, Provincial, London and South East, and Freight sectors. Welsby and Nichols (1999) suggest that during the early 1980s British Rail (BR) were far from being responsive to the market, with Government intervention and setting of clear performance and financial targets.[4] However, at the end of the decade Welsby and Nichols point out vast improvements were being made and the subsidy requirement was dropping. Hence, despite various regimes and the continued existence of a dichotomy concerning how best to operate the railways, the train industry still looked to be making improvements towards Government set goals.

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[4] During which the Government made it clear that British Rail's focus should be upon the railways. Leading to the first "mini-wave" of privatisations where British Transport Hotels, Sealink and British Rail Engineering Ltd. During were sold during the 1980s. The financial restrictions also saw BR enter a number of joint ventures with the private sector such as the Heathrow Express service, although these did not become common until the early 1990s.

2.3.4.2 Railway Privatisation

The economic recession of the late 1980s/early 1990s would soon take its toll on BR's more profitable routes and by 1991/2 the cash requirement of the industry was £2bn. To see how dependent BR were on the Government we need to look further at the subsidy

Table 2.2: BR Government Financing

Study	Year	Type	Subsidy Amount
Dodgson (1996)	1990/91	Subsidy Requirement	£700 million
Nash (2000)	1989/90	Total Grant	£705 million
	1991/92	Total Grant	£1035 million
	1992/93	Total Grant	£1243 million
Davies (2000)	1993/94	Revenue and capital grants	Just below £1 billion

requirement. This cash requirement figure is a good indicator of the money BR was losing, but what of the dependence on Government financing? Table 2.2 reports the various estimates of the financing that BR needed from the Government. The further increases in the cash and subsidy requirements had a part to play in the decision to privatise. It should be noted privatisation had already been decided on by the time the figures for 1992/93 and 1993/94 were calculated, but they indicate the growing cash/subsidy requirements of BR.

Welsby and Nichols (1999) suggest the prevailing thought was, despite some services making money, that profitable services would degenerate to the level of unprofitable routes if they were both run by the same operator. This is hardly the most cohesive of ideas and Joy (1998) suggests the New Opportunity for Railways (Department of Transport, 1992) seems unsure about the effect privatisation would have on the Government's role, but there was a determination to break-up BR. Joy also draws attention to the Government's insistence that the nationalised BR was "insulated from the demands of the market" despite the fact it was the Government who had been doing so by setting a Public Service Obligation since 1984 – something it would continue to do in the post-privatisation era.

The Government hoped the change in structure would lead to a reduced subsidy requirement, whilst also hoping to remove a potential source of problematic public and political discourse from their area of perceived influence. Welsby and Nichols (1999) clarify the Government's reasons for privatisation in economic terms. They cite three primary reasons with the first of these being the possibility that private sector entrepreneurs would lead to a more innovative development of the railways. This could then be supplemented by the introduction of competition in services with decreases on price and increases in quality. The final reason was that privatisation would lead to the railway becoming independent of government financing as the new era would attract private sector funding.

The commitment to privatisation is confirmed by Department of Transport (1992) but, despite the idea of railway privatisation being firmly fixed, the "how" remained open to debate. Even after the 1993 Railways Act (H.M. Government, 1993) paved the way for privatisation the new structure was still open to question, and there is still some debate concerning the structure of the railway industry. We shall now focus on how the UK railway was privatised before looking at the various problems with the chosen structure, focusing on the effects of instigating off-the-rails, and on-rails, competition.

The main decision that was made by the Government of the time was to separate the owner of the infrastructure from those companies that were to operate the services. The "new" infrastructure owner "Railtrack" was not allowed to run railway services unless in the extremely unlikely situation the regulator allowed it. The creation of a single infrastructure owner is normally motivated by a "natural monopoly" argument with the main crux of this argument being it is extremely inefficient to have two or more duplicate networks.

Starkie (1996) states "natural monopolies" as those markets that can meet the demands of a market cheaper with one firm than it can with two firms. It is clear this isn't true for the entire industry, but as Starkie claims it is likely to be true for the railway infrastructure. He dismisses the possibility of train infrastructure competition as any entrant investment would be large, and come at substantial risk as if the incumbent track-operator began to behave efficiently following the entrant's investment then the entrant would have little chance of any return. In addition to this there are safety and coordination problems that could occur should infrastructure be subject to some competition or splitting. For these reasons it was decided that the ownership of the railway track infrastructure would be a monopoly with some regulation.

It was originally planned for the infrastructure to remain in Government hands, but by May 1996 Railtrack was privatised. As Railtrack was the sole operator of the rail infrastructure a regulator had to be appointed to prevent Railtrack abusing its monopoly power. The regulator set the prices that Railtrack could charge to the Train Operating Companies (TOCs) for access to the railway network and decided upon a RPI-X regime, where Railtrack's charges were to be reduced by an average of 8% in real terms during 1995-96, before reducing at a level of 2% for each of the next five years.

An issue concerning the structure of these access charges is the degree the charge can vary depending on the level of use. Thomas and McMahon (2004) point out that around 92% of Railtrack's access charge were fixed, hence only 8% of Railtrack's revenue would increase if the level of use increases. This has the advantage of making Railtrack's revenues stable in times of low demand, but does bring into question Railtrack's incentives to invest. A report by James and McHardy (1999) show the actual level of underinvestment in the railway infrastructure. Although, Thomas and McMahon highlight that if the TOCs did wish to run additional services then Railtrack could individually negotiate increases to their

fixed charge. Despite this they state the variable income element was too low and an increase in traffic would see Railtrack's costs rising more than the income it would receive. Hence it is unlikely that Railtrack actually had the incentive to invest in track. The possibility that Railtrack were allowed to negotiate extra charges was further criticised by the rail regulator for not only reducing transparency of charging, but leading to transaction costs and information asymmetries – giving rise to the fear that a negotiated framework could lead to discrimination.

Nash (2002) suggests the regulator did think it might be desirable to have a higher variable element when it set the charge in 1994 (Office of the Rail Regulator, 1994). However, the regulator concluded it was not in the short term interest, yet they continued to consider the matter with a change, eventually, made during the regulator's periodic review in 2000 (Office of the Rail Regulator, 2000). The regulator raised the variable element of the charges to around 14% and also attempted to remove any need for individual negotiations; perhaps going a little way to correct for the previous problem of investment incentives.

The complications of allowing charges to vary with usage continue to raise issues about how to best encourage infrastructure investment and whether such an incentive can exist in the current railway structure. The setting of infrastructure prices could also have a major effect on the subsidy requirement of the industry, but first let us look at the other changes privatisation brought about.

25 Train Operating Companies (TOCs) franchises were set up that would be responsible for operation and marketing of the passenger services. These aforementioned TOCs would not only purchase access to the track from Railtrack, but would also lease rolling stock from the Rolling Stock Operating Companies (ROSCOs), who would be the firms responsible for major train maintenance and investment. These franchises were then

subject to a bidding process from firms wishing to operate the various routes. Depending upon the profitability of the routes within each of the 25 franchises, the routes could be bid for at a premium or for a subsidy requirement[5].

The impression was given that the highest bidder (or lowest subsidy) was the chosen franchise winner in each case. It was hoped this competition for the market could help reduce the subsidy requirement of the railway. However, the success in the competition for the market relied greatly on a number of the decisions the Government made when setting up the new railway structure. The level of the access charges set by the rail regulator would have a major impact on the size of these bids. If prices were perceived to be high then bids for the twenty-five franchises would have been expected to be much lower. Welsby and Nichols (1999) look at this issue, but fail to come to anything conclusive due to the possibility that the regulator are able to call for further price reviews if new information becomes available.

The level of the access charge was not the only other variable that could influence the success of the competition for the market. Another major influence was the length of the franchise contract as the longer the franchise lasted the more likely the firm would able to subvert on-rail-competitive pressure and increasing its chances of making a profit, but reducing the possible consumer gains. Shorter franchise lengths may encourage on-rail-competition; however, they could have major effects on management decisions and throw into doubt some of the Government's hopes of entrepreneurial innovation on the railways.

There was also a need to ensure franchise winners did not make profits by simply reducing frequency, reliability, or quality particularly on routes where little on-rail-competition existed. This led to various contracts that stipulated service levels but, again,

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[5] The government was committed to ensuring that it would not only be profitable routes that were served, hence the continuation of subsidies.

led the railway industry being insulated from demands of the market. Welsby and Nichols (1999) point out this practice results in a minimum service specification, which requires TOCs to keep to a level of service that was very similar to the BR timetable. The eventual result of the franchise length discussion led to various initial franchise lengths being offered with most between five and 15 years, while the Rail Regulator had the right to cancel any agreement if levels of service quality fell below agreed levels.

We shall now briefly highlight some of the other changes due to rail privatisation before returning to an in-depth look at the development of the conflicting on-the-rails and off-the-rails (or competition for the market) ideals.

According to Davies (2000) the other major reforms within the railway industry due to rail privatisation were the creation of six freight companies, thirteen infrastructure maintenance units and track renewal companies, three ROSCOs and a number of support companies. The separation of rolling stock ownership from the TOCs could also create another investment perversion as Nicholls and Welsby (1999) point out. Many franchise agreements assumed passenger growth, therefore likely increases in the amount of vehicles will be needed by the TOCs. Which of the two parties, the TOCs or the ROSCOs should invest in this expansion is unclear.

2.3.4.3 On-Rail-Competition and Competition for the Market

Two of the seemingly opposing elements in the privatisation of the UK Railways are the need for on-rail-competition and off-rail-competition (competition for the market). If the Government implemented a system that promoted too much on-rail-competition then this would reduce the firms' potential profits and result in lower, and less, bids for the franchises – i.e. poor off-rail-competition which would mean the Government's aim of reducing the subsidy requirement would not be met.

Let us take a closer look at what happened to the railway's subsidy requirement. The period of change had an immediate impact on the need for subsidies. The initial change over of ownership and pricing regimes – Railtrack came into being in 1994 but franchises did not take over completely from the Passenger Transport Executives (PTEs) until 1997 – led to an increase in the government grant to the Passenger Transport Executives. Davies (2000) estimates the PTE revenue grant to BR rose by £230m between 1993/94 and 1994/95. Preston (2000) reports the total subsidy requirement of the TOCs was £2.1bn in 1996/7 and compares it to the £545mn revenue subsidy[6] the railways received in 1993/94. He then looks at the franchises and how much each is worth in the final year of its franchise period (that varies from five to 15 years) and showed the subsidy requirement of the TOCs eventually reduces to £530mn – a figure that is just below the to 1993/94 figure. Welsby and Nichols (1999) compare the total subsidy requirements before and after the privatisation re-structuring and find the subsidy requirement almost doubles. The figure for 1993/94 was £1092mn and a year later it became £2132mn. Each of these studies show that the Government's burden in the support of the TOCs initially increased after privatisation, but Preston (2000) shows this eventually fell below pre-privatisation levels by the end of the first set of franchises.

Welsby and Nichols (1999) ask a more complicated question: has the "privatisation project" resulted in an improvement for the public sector? This is exceptionally hard to prove and recent developments suggest the Government had to provide further subsidies to certain firms. Of course, any conclusion on the success or failure on the finance side is premature and is, perhaps, one to be answered another few decades down the line. Joy

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[6] This is the revenue subsidy only and hence a lot lower than the cash and subsidy requirement of the whole industry that we discussed earlier. The revenue subsidy is the equivalent of the TOC subsidies when the industry was in public hands. Preston (2000) points out that the total subsidy requirement was around £1086mn in 1993/94.

(1998); however, suggests that no regime will be a complete success until tough decisions are made and certain services cease to run, and even then he believes this may not reduce Railtrack's (now Network Rail) costs to a manageable level. Preston (2000) narrows his focus to concentrate on whether this off-the-rails competition has been successful and finds it has reduced the subsidy for most of the franchises; leading to a welfare gain. Preston's work concentrates on the first round of franchise agreements and it would be interesting to see how the latest franchising agreements compare.

As we have already stated off-the-rails competition (competition for the market) may come at the expense of on-the-rails competition (competition in the market). By separating the TOCs into franchises there was some hope that the parts of these franchises that did overlap would see some improvements in price and service levels due to on-the-rails competition. However, these improvements would be limited by the extent the franchises introduced monopolies on other routes and the possibility of a complementary monopoly problem. The welfare gains depend on whether the inevitable complementarities within a competitive system are more than matched by the substitutibilities as a whole – this is a discussion which we shall see in more detail in the next chapter.

Another problem is how to define the boundaries of the industry. 15-out-of-the-25 original successful franchise bidders were bus companies and Preston (2000) points out that this raised some concern on routes where the railway franchise holder was also a major bus-firm owner. It seems highly likely that some areas not only have a bus or train monopoly, but an actual overall "private" public transport monopoly. However, this also raises an issue concerning coordination of services. Whilst from a static monopoly viewpoint it may not be ideal that bus and train routes are operated by the same owner, it may give some incentive for the firm to encourage coordination of the modes. Increased interconnection between the buses and trains could lead to improved public transport.

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Preston highlights a case in Oxford where the Go-Ahead Group operates Thames Trains and one of the major bus firms.

The fragmentation of passenger services within the railway industry may have been expected to bring about problems fragmentation of services. However, the fact that integrated ticketing in rail travel is still the norm, rather than the exception, shows there has been some continued coordination in the railway industry – in contrast with the experience in the bus sector.

It is often the market for advance tickets where the possibility of competition between firms is strongest. Buying a ticket on the day of travel often means the traveller has no choice, but to purchase the integrated ticket that allows them to travel on any trains or routes, which carry them towards their final destination, ignoring extreme diversions. However, advance purchase tickets can require the traveller to pick the preferred route and time of departure. This has more room for competition as individuals can actually choose which firm they travel with. Without knowledge of costs it is difficult to tell the level of effective competition, but advance tickets are significantly cheaper than normal tickets – we shall see this in more detail in Section 2.5.3. Often a firm's route that requires some interchange will be priced cheaper than a firm that provides a direct service, other examples of cheaper tickets are for services that are slower or less frequent. Some complaints have been levelled at this form of ticketing as it requires travellers to plan ahead.

There has been some call for the introduction of competition across all tickets potentially leading to the removal of the integrated ticket. This would cause major disruption to the network and a significant increase in information costs that fall on the traveller. Preston looks at a model of on-the-rails competition and finds on-rail-competition could result in falling welfares unless it is carefully controlled to avoid "cream-skimming" of certain routes. The model also predicts on-rails-competition is likely to lead to a higher

subsidy requirement. Preston's evidence supports the possibility that competition could

lead to allocative inefficiencies, especially a lack of integrated ticketing[7].

2.3.4.4 Railway Privatisation and Rail Use

Up to now we have concentrated on many of the negative aspects of railway

privatisation but what we have not focused on and what was not clear from Figure 2.2 is

that since privatisation there would appear to have been some increase in train travel.

Figure 2.9 shows the passenger kilometres for the British railways between 1980 and

2004/05, this excludes London Underground (unlike Figure 2.2) as this was not subject to

Figure 2.9: National Rail Passenger Kilometres
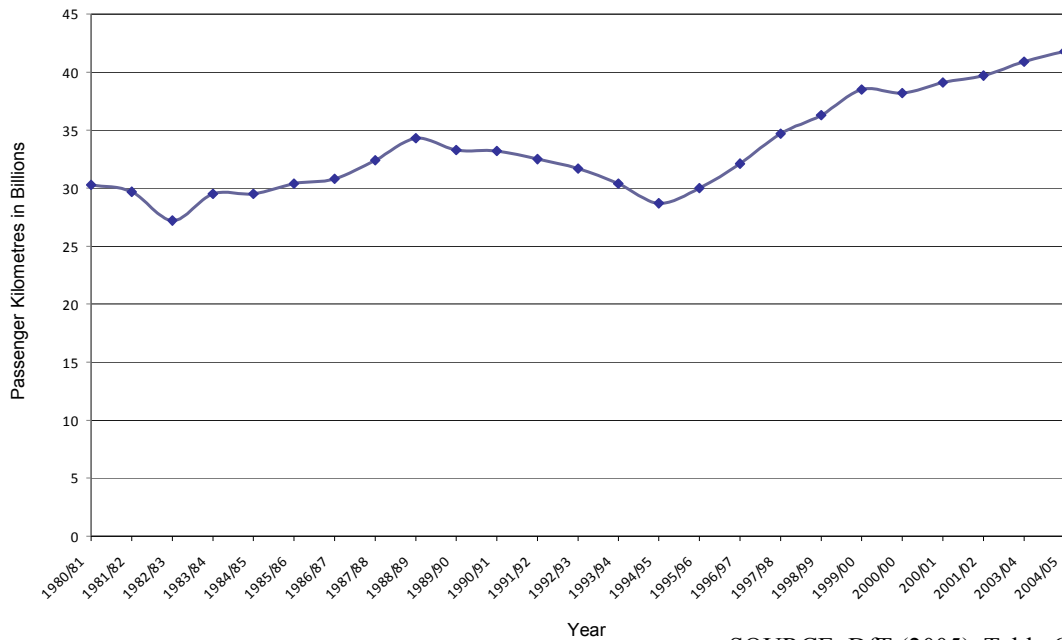
SOURCE: DfT (2005), Table 6.1

the privatisation the National Railways were. We can see that almost immediately

following privatisation rail use in passenger kilometres begins an upturn. In 2004/05 rail

use was higher than it had been since before the Second World War. While it would be

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[7] We mentioned the impact on integrated ticketing, which are tickets that are accepted across one mode. Inter-available ticketing is a system where tickets are accepted across modes. In this one ticket could be used on both buses and trains.

short sighted to assign all this growth to rail privatisation, it would be fair to conclude that it has not harmed the patronage of the railway.

2.3.4.5 Railway Privatisation Conclusion

The privatisation of the railway system has been far from a complete disaster, although there are still many issues surrounding long-term strategy and the set-up of the privatised system; it could be argued with the growth in patronage that the railways are now in as good a state as ever. However, whether the railways offer a realistic solution to the UK's problem of increased car use remains in serious doubt. As current operations stand the network is at full capacity and investments are needed to increase this capacity but the railways are extremely unlikely to receive such levels of finance.

Despite the lack of money it would be unfair to say privatisation has led to increased perceived generalised costs of travel on the UK's railways, but it also seems unlikely this perceived generalised cost has decreased. Due to a structure that concentrates on securing a fall in the subsidy requirement and the continued coordination, which could be seen as desirable, we see very little scope for actual competition on much of the railway.

The structure of privatisation has meant the coordination of services has not been disrupted and this element of the generalised cost of railway travel has seen little change – in contrast to the bus industry where it is likely the generalised cost increased following deregulation. The railways have managed to maintain a level of integrated ticketing and coordination, but possibly at the expense of potential competition.

One particular point of interest is the provision of integrated ticketing, in the bus industry this was outlawed during deregulation, but in the railways integrated ticketing was seen to be an important feature. Some of the reason for this may result from differences in the demand structures of the two industries; however, the existence of rail integrated

ticketing further underlines the need to investigate integrated ticketing. The gains from integrated ticketing and coordination at the possible expense of competition is clearly an area needing further research and we shall do this in Part B of this thesis.

A similarity that the privatisation of railways shares with the deregulation of buses is the possibility of a private monopoly running some services, although railways do continue to be regulated. Whether a sole operator will provide socially desirable service provision and prices, and whether the regulator can successfully intervene to ensure the socially desirable result is provided, is an area that requires further exploration. Additionally, the combined privatisation of railways and deregulation of buses has lead to the possibility of some areas experiencing a monopoly operator of public transport services.

Recently, there is growing concern at the high price of tickets in the UK railways industry. Increases in the ticket prices in January 2006 probably have little to do with failure of competition, but more to do with capacity constraints. Despite an increase of expenditure on investment on national rail infrastructure from £762 million in 1993/94 to £4722 million in 2003/04 (DfT, 2005) there is still perceived to be a lack infrastructure investment. Current investment seems to be maintaining the current system – actual track length has barely increased since privatisation – and this means the Government's aim of increased rail travel to ease the burden of increased car use is very unlikely. Not only can the railways not bear the strain of car uses switching modes, but the rise in prices may see individuals continue to switch to car use.

2.3.5 Modal Choice Conclusion

Earlier we established a likely bias in perception, and value, of the generalised cost of using the car relative to other modes. Bus Deregulation did little to reduce the perceived generalised cost of bus travel and probably led to an actual increase – as it seems ticket

prices have risen whilst a lack of coordination has also led to further costs falling on the bus traveller. Although, the actual impact of certain coordination policies require further exploration – we investigate the impact of one such policy: integrated ticketing using an economic model in Part B of this thesis – it would seem likely a fall in coordination is partly responsible for a continued decrease in bus patronage and could even result in increased car use.

The experience of rail privatisation, in which coordination and integrated ticketing were maintained, contrasts sharply with that of Bus deregulation. However, the constraint on rail finances has and continues to impact on the railway's ability to encourage people onto public transport. The railways are capacity constrained and this will, almost inevitably, result in higher ticket prices, so the generalised cost of travel by rail may increase despite railway privatisation maintaining a coordinated system. It is likely that these traditional policies, both railway privatisation and bus deregulation, coupled with the situation these modes find themselves in, have had little success in reversing the reduction in public transport use. Often these policies have resulted in private monopolies operating some areas public transports services. Before we look at policies to solve these issues let us explore the reasons why increased car use is problematic.

2.4 The Problems of Car Use

Up to this point we have simply indicated the social desirability of a shift to public transport use. In this section we shall consider the problems associated with the private transport modes that motivate the need to promote bus and train use. The total cost to society (the social cost) of a trip is not always fully represented by the private cost of the trip to the individual. The use of the car has two main externalities: congestion and pollution. We will now look at the problems the car brings in relation to these as well as

looking at car user behaviour concerning accidents, social exclusion and, beginning with, pollution.

2.4.1 Pollution

In recent years concern for the environment has been brought to the forefront of the political agenda in the face of growing evidence of global warming. The relationship between economic growth and the environment is still uncertain. However, what is emerging is a new focus on sustainability, so that the environment and future economic development is not sacrificed for expansion today. This has lead to an effort to optimise and reduce the damage on the environment, leading to pressure on national Governments.

In the 1997 Kyoto agreement (UNFCCC, 1997) the UK agreed to lower their emission of greenhouse gases (carbon dioxide, methane, nitrous oxide, hydroflourocarbons, perfluorocarbons and sulphur hexafluoride); reducing emissions by five percent of 1990 levels in the period 2008-2012. Current UK Government projections show it is likely such

Table 2.3: Actual and Forecast Levels of UK CO2 Emissions (Millions of Tonnes of Carbon)

	1990	1995	2000	2005	*2010*	*2020*
Road Transport	29.7	30.1	31.7	32.4	*34.5*	*38.2*
Other Transport	2.3	2.2	2.1	1.8	*1.9*	*2.1*
Rest of Economy	133	121	119	118	*106*	*104*
Total	165	154	153	152	*142*	*144*

SOURCE: DfT (2005), Table 3.7.

levels will be met, see Table 2.3 – where emissions in 2010 are predicted to be over 13% lower that the 1990 level.

In 1990 road transport was responsible for 18% of all carbon dioxide emissions, but by 2020 this figure is estimated to be 27%. Table 2.3 shows how road transport has been responsible for a large amount of the UK's carbon dioxide and – whilst the amount of carbon dioxide pollution the rest of the economy emits is expected to fall – the emissions

from road transport are predicted to increase. The lowering of road transport emissions could help to reduce emissions to below Kyoto agreement levels[8] and this could be achieved by shifting car users to comparatively (per person) less polluting forms of travel like bus or rail.

In addition to increased CO2 emissions, car use also makes up around, 48% of nitrous oxide emissions, 60% of carbon monoxide emissions; and 28% of volatile organic compound emissions (DfT, 2004c). The amount and variety of the pollution gives an indication of the damage road transport is inflicting on our environment and Delucchi (2000) highlights this by pointing out the vast vehicle pollutants and their effects, such as air pollution (leading to health effects, reduced visibility, crop losses, material damage forest damage amongst others), climate change, noise, and water pollution (such as fuel spilling into water). He states that the environmental cost of vehicles in the US is in the region of 1-10% of US 1991 GDP. This is between $60 billion and $600 billion at 2003 prices[9]. Eyre et al (1997) concludes damages to health and environment from vehicle emissions are significant when compared to the cost of fuel in the UK. This is despite particularly high fuel prices in the UK, partly due to taxation; in 1997 the Government increased the fuel tax escalator to 6% from 5%. This seems to confirm that transport, and in particular the car, as a major contributor to environmental damage. Pollution is not the only problem brought about by the car and we shall now investigate the congestion phenomenon.

2.4.2 Congestion

DfT (2004c) states there has been a 20% increase in the traffic on major roads between 1993 and 2002. The increase in road use and the resulting reduction in traffic

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[8] There is growing international pressure for the UK to become a leading example in the reduction of emissions.
[9] Using GDP at Current Prices from OECD (2003).

speeds is a major issue in the UK with congestion becoming such a problem in Durham and London that congestion charging systems were introduced. The UK Commission for Integrated Transport (2005) state that in the year 2000 35 local authorities admitted an interest in adopting road pricing schemes. We shall return to such schemes and their impacts in Section 2.6.2, when we investigate road pricing schemes in the UK.

Congestion is often thought of nothing more than a mere inconvenience to the individual, but when aggregated you find major losses to firms and individuals in the UK. The CBI (2003) make a crude estimate that congestion is costing the UK around £20 billion a year. Additionally, as congestion leads to cars running at low speeds it means the engine works less efficiently, so more pollution is produced. Congestion results in a number of problems from wasted time to increased pollution and is another reason to seek more efficient use of the car. The next problem we consider is accidents involving cars.

2.4.3 Accidents

Something car users seem to underestimate relative to public transport is the probability of accidents. Table 2.4 shows the number of passenger deaths (including the driver) per billion passenger kilometres for cars, buses or coaches, and rail. We can see the rate of death as a direct result of car travel is higher than buses and coach, or rail. Of course, a fully rational car user would take account of the probability of an accident, but many drivers have an unwavering faith that their skill as a driver will mean they avoid collisions, when it may not be the case. This belief may mean that car travellers do not take full account of the fact that they are more likely to have an accident than those using public transport.

Even if a car traveller does take the probability of an accident into full consideration

Table 2.4: Passengers Deaths per Billion Passenger Kilometres By Mode

Year	Car	Bus and Coach	Rail
1994	3	0.5	0.4
1995	2.9	0.8	0.2
1996	3	0.2	0.4
1997	2.9	0.3	0.5
1998	2.8	0.4	0.4
1999	2.7	0.2	0.9
2000	2.7	0.3	0.4
2001	2.8	0.2	0.2
2002	2.7	0.4	0.4
2003	2.7	0.2	0.1

Source: DfT (2005), Table 1.7.

they may not take account of the cost that they impose on others if they do have an accident; either in terms of injuries to other parties, damage to vehicles, or the resulting congestion. So again we see the car has an actual cost of use above that of bus and coach, or rail use – another cost that is underestimated by car users. Next we shall consider a reason why it is important to maintain public transport as a viable mode of transport.

2.4.4 Social Inclusion

Despite the positive publicity in favour of the car, what is often neglected is that there continues to be individuals who do not own cars. In 2000, there were still 27% of households that did not regularly use a car (DfT, 2005, Table 9.15) and 29% of the adult population of Great Britain did not hold valid driving licences in the period 1998-2000 (DfT, 2005, Table 9.16). Non-drivers may be fewer than drivers, but this does not mean there aren't substantial numbers of individuals who do not have the "freedom" brought about by car ownership. These individuals could be seen as being socially excluded from certain activities and this can affect both physical and mental health. The issue of social inclusion draws attention to this issue and is defined as:

"Social inclusion is the process by which efforts are made to ensure that everyone, regardless of their experiences and circumstances, can achieve their potential in life. To achieve inclusion income and employment are necessary but not sufficient. An inclusive society is also characterised by a striving for reduced inequality, a balance between individuals' rights and duties and increased social cohesion."

(Centre for Economic and Social Inclusion, 2002, www.cesi.org.uk)

The benefits of following such a policy, other than paying lip service to the idea of fairness, is difficult to ascertain. Theoretically, it could be argued socially inclusive policies lead to less crime and better health. More specifically if all people have the ability to take active part in society then certain people may not succumb to the temptation of crime, illness due to economic, physical inactivity, or lack of access to vital goods and services.

In this section we are referring to individuals who are dissuaded from an active role in society due to a lack of access to the car and should not be confused with active members of society who are encouraged to use their car less; meaning they get more exercise as walking becomes the alternative. It is difficult to actually clarify the advantages of social inclusion, but it seems sensible that an included individual is more likely to be a happy one relative to those who are involuntary excluded. This means, even ignoring the possible links of social exclusion to crime and illness, a policy of social inclusion has certain political merit and hence the importance of policies that seek to improve public transport to avoid social exclusion.

In Section 2.3.1.2 we highlighted the possibility that the growth of out-of-town supermarkets may have increased social inclusion, as well as increasing car ownership and car use. The emphasis the UK Government place upon discouraging certain out-of-town retail developments is mainly down to the concern that they adversely affect poorer

people's health. It is thought out-of-town retail developments may restrict some people's access to key foods as larger supermarkets often lead to smaller, local sellers of produce closing so that people with no private transport may find it difficult to buy certain key foods. Wrigley et al (2002) points to the publication of two reports, Acheson (1998), and the Social Exclusion Unit (1998), as key reasons for the UK Government wishing to discourage out-of-town developments. Of course, if these out-of-town developments can not be discouraged then a good public transport system would be vital in allowing all access to goods and services that are vital to maintain a healthy life-style.

The problems of social exclusion mean any policy aimed at forcing individuals away from car use, by increasing the perceive generalised cost, without improvements in public transport modes may have major effects on health and crime, and other political goals. It is important to have a good public transport alternative and if policies to reduce car use are to be pursued it is even more important to have a good viable alternative in a public transport system to allow travellers to reach their destinations. Policies aimed at reducing car use should seek to reduce unnecessary journeys, but it is inevitable they will prevent some individuals making important journeys. It is vital that individuals dissuaded from car use can still make these important journeys for social inclusion and economic growth issues.

2.4.5 The Problem of Car Use Conclusion

To sum up, currently in the UK there is a need to encourage public transport use due to the problems of excessive car use, such as congestion and pollution, as well as helping the socially excluded. To increase public transport use it is important that these transport systems are improved. In Section 2.3 we found problems in the coordination of services and increased generalised cost of public transport use, so the ways of achieving coordination of public transport without removing the benefits of competition needs investigation. The

coordination issue is closely linked to the idea of "transport integration" and this has been a key word in policy during the last ten years.

2.5 The Focus on Public Transport and Transport Integration

In 1998, the UK Government decided to shift the focus of public transport with a hope of improving it. DETR (1998) introduces the "integrated transport" term and announced a new transport emphasis on to an "integrated approach," as well as the formation of an independent body: the Commission for Integrated Transport. The commission's main responsibility was to provide advice and review progress of the White Paper's goals. The "integrated transport" vision is extended by the DETR (2000), which announces a £180 billion funding for transport from 2001/02 to 2010/11. Both white papers tend to explain "integrated transport" by example rather than clearly setting out a definition that would be worked towards. Indeed, Glaister (2002) criticises the forerunner, "Developing an Integrated Transport Policy," (DETR, 1997) along with the DETR (1998) for failing to clearly define "integrated transport." This lack of definition means we must clarify what integrated transport is.

2.5.1 What is Integrated Transport?

The simplest way of explaining what may be meant by "integrated transport" is using an example of a journey that requires a change of mode. Imagine an individual who wants to make a train journey and uses the bus to reach the train station. An example of an "integrated transport" policy could be described as one that attempts to leave the traveller with the minimum amount of time (without rushing) for the change of mode to take place so that there is no time wasted waiting around.

This can lead to many ideas of how to realise "transport integration" and reduce the cost of a trip by public transport to the traveller such as: reduce distances between bus and train stations to reduce the interchange time required, allowing train tickets to be bought alongside bus tickets to, again, reduce interchange time required, increasing the reliability and punctuality of both modes to reduce traveller uncertainty. Some of these ideas are actually being put into place with transport interchanges (combining bus and train stations) being set-up in the cities of Doncaster and Hull, amongst others. Some train tickets include bus travel to the city centre if the train station is outside the city centre; the city of Norwich has introduced such an inter-available ticketing scheme. The basic idea of these schemes is to reduce the actual and perceived generalised cost of travel by public transport. By reducing the generalised cost the attractiveness of public transport is increased and this should result in increased public transport use.

Another way to look at "integration of transport" is the combination of decision-making across varying bodies that are related to transport. The DfT (2004b) look at the effects this kind transport integration brings and summarises the results. The report looks at 40 case-studies, where partnerships had been formed between local authorities and groups with other interests, with a view to improving "transport integration." The reference to "transport integration" in such cases can vary from cross-modal to simple group planning.

The partnerships explored may have been diverse but the report places them into three main categories: inter-authority partnerships (e.g. between transport authorities and education authorities), general transport partnerships (e.g. between transport authorities and transport providers), and partnerships between local authorities and external non-transport interests (e.g. NHS Trusts). The conclusions of the report seem to be fairly mixed, but with a suggestion that partnerships in all three categories could have a future. In particular, they highlight inter-authority partnerships that improved services whilst also bringing about cost

savings. They draw attention to the possibility of inter-available and integrated ticketing, which increased service levels whilst bringing about "little or no long term financial cost" (DfT, 2004b, Executive Summary, page ii) – this is a ticketing system we highlight in 2.3.3 and 2.3.4..

Many of the policies detailed above have been labelled "integrated transport," but in reality they are a combination of attempts to foster coordination, be it through actual integration or simply cooperation among transport firms. These policies do not simply refer to the economic definition of integration in terms of vertical of horizontal integration. We will attempt to be precise when referring to these during this thesis.

2.5.2 Inter-Available and Integrated Ticketing

In Section 2.5.1 we introduce the idea of "integrated transport" and the possible policies this might imply. Most of the focus on "integration of transport services" is to improve coordination and cause the perceived generalised cost of travel by public transport to fall. One suggestion from UK policymakers to achieve integration is to encourage inter-available and integrated ticketing. An "integrated ticket" refers to one ticket that allows travel on the transport services of more than one operator on the **same mode**. The best example of this is when we consider the deregulated bus industry and two bus companies operating services on the same route. In this case an integrated ticket could be a return ticket that allows the traveller to use one firm for their outward journey and another for their inward journey. An "inter-available ticket" refers to one ticket that allows travel on the transport services of more than one operator on **several modes**. If we return to our example from the start of the section regarding the traveller, who uses both the bus and the train on their journey, then an inter-available ticket in this context would be the traveller purchasing one ticket that can be used on both modes.

The possibility of coordinated ticketing and its impact upon lowering the perceived generalised cost of travel by public transport is one that shows great promise. However, it is possible it could reduce the level of competition and lead to increased prices, so it requires further exploration and we shall undertake this using an economic model with a focus on integrated ticketing with transport in Part B.

2.5.3 UK Ticketing

Before we can undergo any economic modelling of integrated ticketing, it will be useful to investigate the ticketing trends within the UK. These empirical observations will be important when building an economic model in Part B. For definitions of the various tickets please refer to page 4.

We previously stated that UK bus deregulation led to a loss of coordination within the industry with inter-available and integrated ticketing becoming less common. However, with the idea of "integrated transport" the possible advantages inter-available and integrated ticketing may bring are, again, beginning to being explored. Transport 2000 (2005) calls for greater encouragement of integrated ticketing by modifying bus competition law to allow it. DfT (2001) introduces the Public Transport Ticketing Scheme's Block Exemption and addresses the aforementioned lack of inter-available and integrated ticketing by allowing multi-operator travel cards, multi-operator individual tickets, inter-available, and short distance, and long distance add-ons – the details can be found in Office of Fair Trading (2005).

The block exemption allows firms to offer integrated and inter-available tickets as long as firms continue to provide and set individual ticket prices separately and at different price levels. However, in the Office of Fair Trading (2006) there is some ambiguity concerning the guidelines for the level of collusion firms are allowed if they wish to

provide integrated/inter-available tickets. A clarification on this is awaiting an Office of Fair Trading consultation, but there is some suggestion that the condition forcing the existence of separately priced individual tickets could be removed.

In 2003 a Task and Finish Group began investigating the issues of inter-available and integrated ticketing (DfT, 2004a). They highlight Manchester, Oxford, and Nottingham as places where interesting schemes are developing. The study suggests integrated or inter-available tickets are good for public transport users and also consider how these schemes can be developed to the advantage of society – this is again is something we wish to clarify during our modelling in Part B. Obviously, the flexibility and time savings offered by inter-available and integrated ticketing could lead to reductions in the public transport's perceived generalised cost of travel, but this form of ticketing could also lead to collusion and price increases – we will seek to clarify the theoretical link between collusion and prices in Chapter 3. We now intend to look not only at the information concerning inter-available and integrated ticketing, but also consider other ticket types such as single and return tickets.

To begin it is necessary that we relax the focus on non-London transport systems as the capital has several examples of integrated ticketing of more general interest. This area should be considered a special case as buses have remained under local authority control unlike the rest of the UK. Central London also has its own underground train service and some parts are served by a tram. Similar to the buses the London Underground had until recently been entirely publicly controlled.

Let us look at the London Underground ticketing schemes. The prices for the single tickets and the Oyster pre-pay system are reported in Table 2.5, with the Oyster system having different prices for peak times (7am to 7pm) and non-peak times (7pm to 7am).

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Table 2.5: London Underground Single Ticket Prices (as of 5th April 2006)

	Ordinary Single (Adult)	Ordinary Single (Child)	Oyster Pre-Pay (7am-7pm)	Oyster Pre-Pay (Other Times)
Zone 1	£3.00	£1.50	£1.50	£1.50
Zone 2,3,4,5,6.	£3.00	£1.50	£1.00	£1.00
Zone 1-2	£3.00	£1.50	£2.00	£1.50
Zone 2-3, 3-4, 4-5, 5-6	£3.00	£1.50	£1.00	£1.00
Zone 1-3, 1-4	£3.00	£1.50	£2.50	£2.00
Zone 2-4, 3-5, 4-6, 2-5, 3-6, 2-6	£3.00	£1.50	£1.80	£1.00
Zone 1-5, 1-6	£4.00	£2.00	£3.50	£2.00

SOURCE: Transport for London (2006)

Oyster pre-paying is a permanent card which costs £3, which is returnable if the card is given back. Travellers can use the internet or underground machines to add credit to this card and it is then swiped upon entering and leaving underground stations. The card calculates the appropriate fare and this is deducted from the balance on the card without the need to queue for tickets. There is also a need for price capping in this system to ensure day

Table 2.6: London Underground Travel Card Prices (as of 5th April 2006)

	Day Travel Card				Oyster Day Price Cap	
	Adult		**Child**			
	Peak	**Off Peak**	**Peak**	**Off Peak**	**Peak**	**Off Peak**
Zones 1-2	£6.20	£4.90	£3.10	-	£5.70	£4.40
Zones 1-3	£7.20	-	£3.60	-	£6.70	£4.90
Zones 1-4	£8.40	£5.40	£4.20	-	£7.90	£4.40
Zones 1-5	£10.40	-	£5.20	-	£9.90	£5.80
Zones 1-6	£12.40	£6.30	£6.30	£2.00	£11.90	£5.80
Zones 2-6	£7.40	£4.30	£4.30	-	£6.90	£3.80

SOURCE: Transport for London (2006)

travel card prices are not exceeded. Travel card and Oyster price cap prices are reported in Table 2.6.

Oyster cards and day travel cards represent a form of integrated and inter-available ticketing as they are not only accepted on a number of underground routes, but also on

Docklands Light Railway, London buses and trams. The Oyster card takes integration and inter-availability further than the travel card as with an Oyster card the traveller no longer needs to calculate whether a day travel card may be cheaper than a series of single tickets. As soon as an Oyster card traveller spends more than £5.70 travelling in Zone 1 during the peak period the system effectively gives them a day travel card. There are also a variety of other travel cards such as family and 3-day travel cards we have not listed here.

We should, again, note the London underground and London buses were not part of train privatisation or bus deregulation policies and we are not suggesting the regime London transport operates under is the reason for the existence of these forms of integrated and inter-available tickets. The city of London is a special case and has a denser public transport demand than other UK cities as the population size is greater and travellers often have little reasonable alternative to public transport. This could then explain the London transport's ability to provide such ticketing systems.

Briefly turning our attention to London buses, we find the price of a single ticket is uniform as any journey costs £1.20, whilst Oyster card peak singles (0630 to 0930) cost £1 with an off-peak cost of £0.80. A one-day bus travel card cost £3.50, although this is not available for use on the underground and the price cap for Oyster card travel on buses during one day is £3.00.

Despite the availability of integrated and inter-available tickets there are some price differentials. London buses offer a day travel card for use only on buses during peak hours and costs £2.70 cheaper than a day travel card that allows travel on buses, the underground, light rail, and tram. Bus travel is also cheaper for those taking single journeys. Something that stands out when comparing ticket prices is the level of discount users of the Oyster card receive.

The level of discount Oyster cardholders receive has increased under the 2006 pricing regime and seems to be around 10–20% lower compared to the price of singles. This saving could be due to the fact Oyster cards relieve some queuing and have cost advantages to the underground operator. Alternatively, this could be looked upon as price discrimination. By paying £3 to purchase an Oyster card it is as if the traveller is opting for a different "tariff." It would seem likely only regular London travellers would choose the Oyster card. Therefore, London transport may be seeking to discriminate pricing between regular travellers and non-regular travellers.

Oyster card users don't just receive monetary savings, but they also save on transaction and decision-making costs as they do not have to buy several tickets or calculate whether a day travel card might be cheaper. This can be seen as a decrease in the perceived generalised cost of travel and rather than simply causing travellers to change the type of ticket they use, it may also encourage people to use London public transport over other modes.

This London public transport system is not one that may be viable in other UK cities, but serves to highlight the possibilities for ticketing systems in the UK. The introduction of the Oyster card that is effectively an integrated and inter-available ticket, could be seen as decreasing the generalised cost of travel on the London public transport.

Let us now move on to look at ticketing in the rest of the UK beginning with the railways. Standard train tickets have an integrated ticketing element within a given origin-destination setting. For example, a trip between Doncaster and London can be made using a GNER service or a Hull Trains service. Standard tickets tend to allow the traveller to use any train between the origin and destination assuming it is not too indirect. For example a journey between Hull and Peterborough allows the traveller to go via Sheffield but would not allow for the traveller to go through Manchester.

Table 2.7: Railway Journey Ticket Prices (as of 2nd May 2006)

Journey	Single Price	Return Price	Pre-Booked Price	Changes
Manchester-London	£56.10 each leg	£57.10	£12.50 1 day advance Single. Each leg. No Returns.	0
Manchester-Hull	£26.30 each leg Standard	£33.40	£20.00 TPE Standard Advance B1 Day Advance	0
Hull – London	£70.00(all)	£73.30 (all) £64.50 (Hull Trains)	£48.50 Super Advance £13.50 GNER STD Advance	0 (Hull Trains) 1 (GNER)
Hull-Lowestoft	£55.50 Standard	£64.10	£23.00 GNER STD Advance4	2 or 3
Birmingham-Leeds	£33.00 Standard	£41.30	£9.00 Value Advance Single C	0
London-Edinburgh	£93.10	£94.10	£13.50 GNER STD Advance 1	0
London-Glasgow	£93.10	£94.10	£13.50 GNER STD Advance 1	0 or 1
London-Bristol	£48.00 Saver	£49.00	£20.50 APEX Return £20.00 APEX Single	0
Sheffield-Leeds	£7.15 Standard Day	£9.70		0

Return Prices based on purchase of a Saver Return. Single Prices based on price of a Saver Single if stated no saver return exists..
All trains and times allowed unless stated.

Travellers who are willing to forego some of the "integrated ticketing" advantages can make significant savings. Large discounts are available for advance bookings – although the lack of advance bookings between Sheffield and Leeds shows that pre-booking is not available on all routes – where the traveller guarantees travel by a certain firm on a certain train. If this train is missed the traveller is required to purchase another ticket (unless the missing of the train was the fault of a train company such as a delayed connecting train). Table 2.7 shows single and return ticket prices as well as prices of pre-booked tickets and supports our previous comments on the possible savings that can be made by purchasing advanced tickets. Additionally, we see the price of a standard return

ticket and a standard single ticket can be very similar. Often it will not cost a traveller much more to make a return journey using the train rather than a single journey. However, recently there has been a trend in offering single tickets at prices far lower than a return and this can be seen on some of the routes in Table 2.7. This change in pricing policy could be as a result of the current capacity constraints that the railway industry is operating under.

It is possible to purchase season tickets for train travel that entitles unlimited travel between two stations of choice. According to GNER (2006), a season ticket that allows travel on **any** train between Leeds and London for 12 months is £9044. However, a season ticket must be bought direct from train operating companies. Even when a train operating company runs an entire route it does not necessarily mean they offer a season ticket for that route, for example no season ticket exists between Scottish stations and London.

In the airline industry most operators only offer single tickets to travellers. This effectively means for a traveller to purchase a return ticket they must purchase an outwards single and inwards single, so a return journey price is the sum of two singles. Certain airlines, such as BA, do offer an air miles scheme that give regular customers special offers. Such deals could be seen as encouraging travel by the same operator, therefore buying two tickets from the same operator may offer the traveller some gain.

Other ticketing systems which give no discount for return travellers are not uncommon and the Eurostar railway service, which connects London and Paris via the channel tunnel, uses such a pricing system. Eurostar (2006) reports a return between London and Paris is £298.00 (flexi) whilst a single is £149.00 (flexi). Additionally, most short distance bus services work on such a basis with return tickets being twice the price of a single ticket. However, with this industry there does tend to be an array of tickets that encourage the use of one operator, for example a seven-day pass allows the use of any of the operator's buses. Tickets such as these actually discourage cross-firm and cross-mode

use and force the traveller to choose between convenience, flexibility, and a financial saving.

The long distance coach and bus industry has a number of ticket pricing types on offer. Megabus and Easybus, two recent entrants into this market, sell single tickets in a similar fashion to airlines (Easybus, 2006 and Megabus, 2006). For example, Megabus (2006) provides an outward single ticket from Manchester to London for £5, although this can be £3.00 at certain times. While an inward single from London to Manchester is £5.00, again, this can be £3.00 at certain times (Megabus, 2006). This means a return between London and Manchester is £10.00, or with restricted services it could be £6.00 (Megabus, 2006). However, the National Express long distance bus operator uses a different pricing policy; they offer a return that is cheaper than purchasing two single tickets. A standard return to London from Leeds with National Express is £30 whilst a standard single is £19.20 (National Express, 2006). Terravision, who serve the market for travel between public transport stations and airports, offer a similar pricing strategy. A single ticket using Terravison (Terravision, 2006) between London Victoria and London Stansted Airport is £8.50 (£8.10 for an online booking) while a return ticket is £14 (£13.50 online booking). A single ticket between Liverpool Street and Stansted Airport is £7.00 (£6.70 online booking) while a return is £12.50 (£11.80 online booking).

Table 2.8 summarises the information we have provided here in the form of the return ticket price to single ticket price ratios; showing the highest and lowest ratios of our examples. This is by no means meant to be a complete summary of the entire transport industry as the variety of routes and tickets available make it a difficult task. However, it gives an indication of how the ratio between the return ticket price and single ticket price varies both within, and between, industries. These empirically determined values will be

Table 2.8: Ratio of Return Ticket Price to Single Ticket Price by Mode

Industry	Return Ticket Price to Single Ticket Price Ratio (lowest)	Return Ticket Price to Single Ticket Price Ratio (highest)
Railway	1.01	1.25
Airlines	2.00	2.00
Short Distance Buses	2.00	2.00
Long Distance Buses	1.56	2.00
Eurostar	2.00	2.00

important in Part B, when we seek an appropriate representation of a non-integrated

ticketing system. Table 2.8 highlights that while airlines and Eurostar don't offer discounts

on returns we can see that other industries do.

We should also note that despite the existence of cheap returns when using the same

firm there is little evidence of inter-available or integrated ticketing in transport outside of

London or the railway industry – in fact, we saw that such ticketing is mostly outlawed in

the bus industry. The lack of such ticketing arrangements could be a reason why there is

much consultation and debate concerning the encouragement of the coordination of

transport services as there is great room for the reductions in generalised costs by providing

services such as integrated or inter-available ticketing. However, the effects of integrated

and inter-available tickets need clarifying due to the possible anti-competitive effects that

may result from firm cooperation resulting in collusion. .

2.5.4 The Focus on Public Transport and Transport Integration Conclusion

So far in this chapter we have seen that with increased car use there also comes

problems of congestion and pollution, and this means there is a need to promote public

transport. However, the major policy changes of bus deregulation and train privatisation did

have some impact on increasing train use, but bus use reduced. It is clear improvements

need to be made to public transport, in particular the bus industry. The bus industry needs

to decrease its generalised cost of travel to make it a real alternative to the car. In this

section we highlighted a possible way to reduce the generalised cost of travel was the introduction of integrated and inter-available ticketing. Transport services outside of London and the railway industry show little signs of offering "integrated ticketing" and there is particular room for such improvements to be made in bus services. The possible effects of "integrated ticketing" on the actual price of the ticket remain relatively unexplored, and are a concern we shall further consider in Part B.

The coordination of public transport is not the only suggestion when it comes to improving public transport. There have been several other suggested ways of improving public transport from "bus lanes" to "guided buses," and the somewhat bizarre "Skyrail" idea (Skyrail, 2003).

The bus lane policy, a lane on a road that only buses can enter, has so far been unpopular. The idea is that car drivers sitting in traffic would see the buses (and all the people onboard) pass them in bus lane and this should motivate a change of mode as it not only decreases the travel time by bus, and reduces the generalised cost of bus travel, but forces the car traveller to observe the bus passing them so that they are encouraged to switch modes. Unfortunately, this has so far backfired, with the effect being one of dislike for the bus lane policymakers rather than causing a change of mode. Another result of the bus lane policy is it leads to more congestion as cars have fewer lanes for them to move in. It seems that encouraging transport coordination through integrated ticketing or other means may be a preferable policy.

Improvements in public transport can not be the only policy undertaken. There is a need for car user costs to reflect the price that a car trip imposes on society; such efforts will dissuade car use rather than simply make other modes more attractive. We now consider other policies that are available to decision makers.

2.6. Policies Aimed at Car Owners

Whilst improving public transport can, in part, provide a way to reverse the trends in mode use, motorists may need further policies that encourage more efficient use of the car. In this section we intend to focus on these policies.

2.6.1 Fuel Pricing

One way in which UK policymakers have sought to influence the behaviour of motorists is by increasing fuel prices. Earlier we reported that Glaister (2002) finds that the composite index representing all costs of motoring has been constant relative to RPI since 1964 – despite the fuel price index (relative to RPI) in the UK having increased. Attempts to decrease the affordability of car use through increasing fuel costs have been offset by reductions in costs, such as falling car prices.

There does seem to be some good news in the attempt to control traffic and car use. Glaister, in his review of the literature suggests the long-term fuel price traffic elasticity is of the order -0.3 and the long-term fuel price fuel consumption elasticity of around -0.7. He explains that the long-term fuel price fuel consumption elasticity is greater because as time goes on people are able to change the way they drive and the fuel consumption of their cars. Another survey of car elasticities by De Jong and Gunn (2001) supports this view and suggests that a long-term fuel price car kilometre elasticity of between -0.20 and -0.41 depending on the purpose of the trip. The same survey suggests the number of trips taken is less responsive to fuel prices with a long term elasticity of between -0.07 and -0.17 again depending on the purpose of the trip. These figures exclude trips for educational purposes, which have a -0.40 long term elasticity. Both Glaister (2002) and De Jong and Gunn (2001) studies suggest increased fuel taxes can reduce both traffic and fuel consumption, although more so in the latter case. A problem that arises is, as we saw earlier, traffic was found to

have an income elasticity of around 1.2 (De Jong and Gunn, 2001), so that with rising

incomes, such as we saw in Figure 2.4, we can expect traffic to rise – counteracting the

effect of any increase in fuel taxes or other policies aimed at reducing car use and

congestion.

Increasing fuel taxes will also have the advantage that it can increase the

Government's revenue. This revenue could then be spent on other transport schemes to

improve public transport such as those mentioned in the previous section. Unfortunately,

the fact that fuel taxes increase government revenue attracts some scepticism concerning

the Government's true motives for increasing fuel taxes. There seems to be little political

viability in the continued fuel taxation rises and the UK public's fuel strikes in response to

increased fuel prices in September 2000 demonstrate the unpopularity of such a policy.

Such tax measure may therefore not be used as prevalently in the future, in fact, such a

strong negative response could limit any plans to increase fuel charges to motorists for

sometime.

2.6.2 Road Pricing

One of the most interesting UK transport policy developments in recent years has

been the introduction of road pricing to London, albeit not in an economist's ideal format,

with the introduction of a cordon toll. In Section 2.4 we saw that car use led to some

negative externalities, such as congestion and pollution, but how should these costs be

internalised?

Clearly, the marginal social cost (MSC) of using the car is greater than the marginal

private cost (MPC) of using the car due to congestion and pollution. The simple economic

representation of this is shown in Figure 2.10, assuming that marginal private benefit (MPB)

Figure 2. 10: Private and Social Optimum Prior to Road Pricing
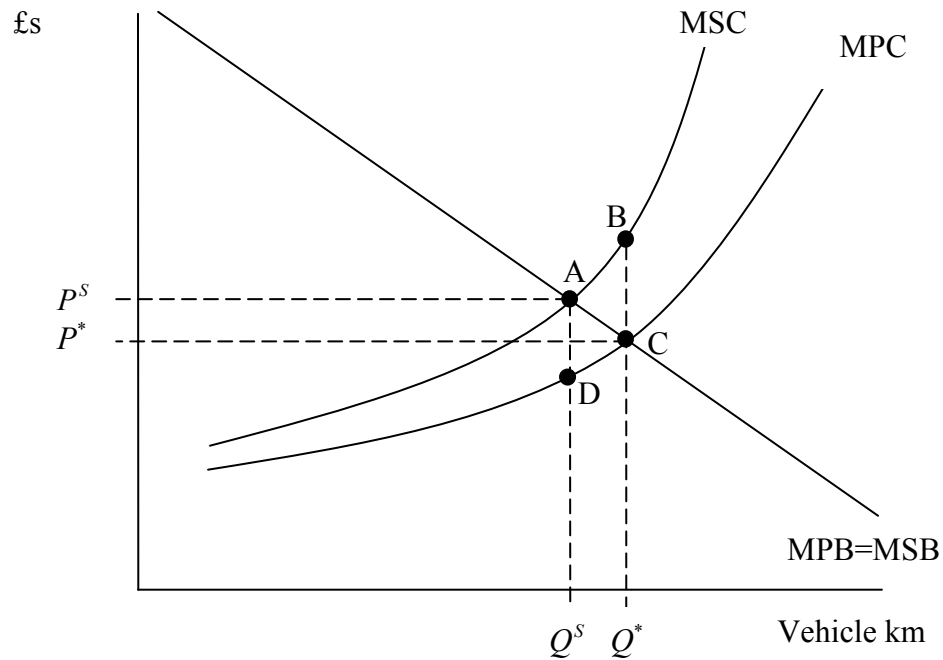
equals marginal social benefit (MSB). The market equilibrium is represented by Q^* and P^* whilst the social optimum is represented by Q^S and P^S.

We see that the social optimum has a lower level of vehicle kilometres associated with it than the market level. The area ABC represents the efficiency loss, as at Q^* the cost for a society would be at B whilst the private cost is at C. To remove this inefficiency we need to increase the marginal private cost until it crosses the MPB at A and this can be done by introducing AD as a charge per vehicle kilometre, so the MPC shifts upwards as in Figure 2.11; where we can see that the new AD charge per vehicle kilometre results in a market equilibrium that is also a social optimum.

Road pricing theory may be relatively simple but it is hard to implement as such schemes would take a large investment in technology on a national scale. The London cordon toll has taken the simpler and cheaper direction. Those who enter central London on

a week day between hours of 7am and 6.30pm have to pay a £8.00 (£10 if payment is late in the day[10]) charge. Transport for London (2006) reports, one year on, that the scheme is a

Figure 2. 11: Private and Social Optimum After The Introduction of Road Pricing
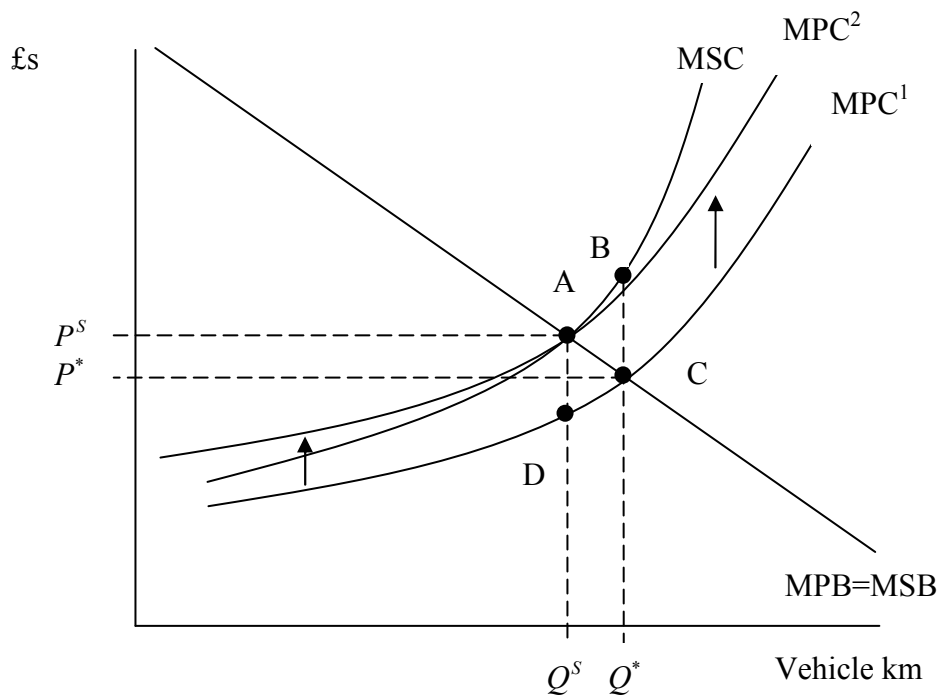
success in reducing congestion with delays falling by an average of 25% within the charging zone. They also state the revenue for the first year of the scheme is £68 million, which is less than half the expected £180 million. However, despite the revenue being less than expected the support for the scheme has increased with 60% said to be in favour after the scheme was implemented, while just 20% were against. The charging area has recently increased to cover Chelsea, Kensington, and Westminster.

The city of Durham has also introduced a cordon toll of £2 charge for vehicles entering a certain, albeit small, area between 10 am and 4 pm, Monday to Saturday. Since implementation the Durham County Council (2003) reports an 85% reduction in vehicular

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[10] Although, this is due to a change so individuals can pay up to a day later without paying the higher tariff.

traffic with a 21% increase in number of people believing the road user charge is a good idea meaning that 70% of individuals are in favour of the charge.

In December 2003, the UK's first toll motorway (Toll M6) opened. The level of the toll varies on the class of the vehicle (class 1 being a motorbike, class 2 being a car etc.), the time (day or night), and the toll area entered. For example a car entering the main toll plazas would cost £3 in the day and £2 at night. This system, as it varies with vehicle type and time of day, would seem to represent the closest the UK has so far got to the economist's ideal road pricing system, but unfortunately toll booths are used, and these can cause congestion as cars have to queue to enter the toll zone.

The UK is not the only nation to introduce a form of road pricing. The BBC (2006b) reports schemes exist all over the world, such as those in the cities of Singapore, Oslo, and Trondheim. The Singapore scheme uses a more complicated system that relies on electronic systems and many believe this is the future for UK congestion charging. Germany and Switzerland both have distance-based systems using advanced technology and approximate the "economist's system"; certainly more so than cordon tolls. So far there is no country that uses a national road charging system, so travellers are able to avoid tolls if they use roads that are not covered by the charge. Toll systems similar to the UK M6 system exist in France (Autoroutes), Australia (Melbourne City Link) and Canada (Toronto).

Despite road pricing seemingly being effective in the reduction of traffic it is clear this is not a policy that should be used by in isolation. There are issues associated with road pricing that can lead to problems of fairness and social exclusion. Improved public transport must also be provided to ensure people continue to make necessary journeys and those that cannot afford to pay a congestion charge are not isolated. If a road pricing scheme is to be politically acceptable then there will need to be perceived improvements in public transport or a similar situation to the UK's fuel protests may arise.

An interesting effect that the Durham scheme (Durham County Council, 2003) reports is a 10% increase in pedestrian activity. This figure may be higher than you would expect from larger cordon toll areas as the walk in the Durham scheme to avoid the toll is only short. The increase in walking, and increased physical activity, raises the possibility that forcibly reducing car use can lead to health benefits, although we also have to be wary of those that may become socially excluded as we mentioned in Section 2.4.4. Reduced car use could therefore have important consequences for the nation's health, particularly in a time when there are increasing concerns about obesity, its effects on health, and the burden it may place on the National Heath Services. The UK Government is looking to improve people's health and an increase in the number of people walking would have some health benefits.

2.6.3 Other Policies Aimed at Increasing Road Use Costs

Road pricing is not the only scheme being proposed to reduce traffic on the roads in the UK. Recently, Nottingham has been seeking to introduce a work-place levy by charging a price (estimated to be £185 per year in 2010) on most parking spaces at work within the city boundaries (Nottingham County Council, 2007). It is hoped this will help pay for a new tram system that should improve public transport. Another set of schemes that have been used are Vehicle Quota Systems, where certain cars aren't allowed to be driven on certain days, and these are in place throughout the world.

Singapore, in addition to road pricing, operates a Vehicle Quota System, which was introduced in 1990. The Singapore scheme means that before a car owner is allowed to use the car on the road they have to bid for a Certificate of Entitlement, that allows the holder to use the car for 10 years, in an auction. The number of licences that are available for new cars is controlled so that car ownership can be limited.

In the 1980s, Athens introduced a measure to quell its problems with congestion by introducing a system that allowed city centre to cars with odd numbered licence plates on odd dates and to those cars with even numbered license plates on even dates. Unfortunately, this resulted in people purchasing two cars; one with an even numbered licence plate and another with an odd numbered licence plate so that an individual could use a car in the centre on all days. Mexico City when faced with pollution problems in the 1980s also introduced a similar scheme; cars were regulated so each car in the city could not enter the city on one day per week with the number plate used to determine this. Eskeland and Tarhan (1995) show that such quota systems can actually lead to increased car use.

Methods of encouraging car sharing has also been used to ease the problems of car use with some schemes attempting to encourage it by charging fees to vehicles with low occupancy rates, but this has lead to people charging to simply be a passenger on a trip for a fee less than the charge. The UK Government has not introduced any mandatory scheme, but seems to be happy to encourage car sharing. The www.liftshare.org website (Liftshare, 2006) was set up in 1997 to help facilitate and encourage lift sharing (for a small fee). Such websites or firms bring to mind a modern equivalent of hitchhiking, but hopefully the fears and dangers of such a system can be alleviated by some screening process.

Many of the schemes, which attempt to simply force people to stop driving, often fail as there are ways around the schemes. These avoidance tactics often result in making the congestion and pollution problem worse as we saw with the Athens scheme. Such behaviour limits the desirability of Vehicle Quota Systems and suggests that market systems, such as road pricing coupled with improvements to public transport, are a preferable alternative.

2.6.4 Political Viability of Increasing Road Use Costs

There is a fine balance that needs to be struck between discouraging unnecessary journeys and encouraging more socially acceptable modes of transport, and not preventing people from access to goods and services, work-places or, indeed, harming the countries economic development. Some systems discussed reduce car use and have the added advantage of being able to increase government revenues. This revenue could be spent on other methods that complement the reduction in car use. Improving public transport give people, who have been isolated by schemes such as road pricing, a viable travel alternative. In all, it is advisable that a mixed strategy of both improved public transport and road pricing should be pursued and this would have the added advantage that would minimise the potential harm to economic growth. While travellers should be discouraged from using the car they should be encouraged into public transport use, so that they can continue to make journey's that are necessary. Therefore policies to encourage coordinated transport including integrated ticketing and optimal network provision are as important as road pricing initiatives.

2.7. Summary of Chapter

We explore the modal trends in UK transport to find that there has been a substantial and sustained increase in the use of the car, and we look at the reasons for the increased use of the car, along with the problems it brings. We find that the car is used because the perceived generalised cost of private transport is lower than public transport and we consider how the policies of deregulation and privatisation of public transport modes may have the resulted in the reduction of the coordination of transport services.

The fragmentation leads to an increase in the perceived generalised cost of public transport, so that the gap between the generalised cost of public transport and that of private

transport has widened. Policymakers have two choices with which they can improve the situation: decrease the generalised cost of public transport modes or increase the generalised cost of car travel. The possibility of a road pricing system shows great promise in being able to decreased car use, but there are a number of issues that mean that road pricing may not be politically viable on its own. If road pricing is to be introduced then it is clear public transport will need to be improved. UK policymakers have suggested the coordination of transport services as a way to improve bus and train services, and one such scheme could be the introduction of integrated and inter-available ticketing. However, this poses the question: how to encourage the integrated and inter-available ticketing without removing the benefits of competition? This is a question we seek to answer in Part B.

We have also seen that the deregulation and privatisation of public transport have left some areas with a monopoly or near monopoly provider of public transport services. This sole operation of routes may not always result in the desirable interconnection of services in a transport network. We shall consider this in more detail in Part C and, in particular, look at ways the regulator could ensure monopolists provide the socially desirable level of network provision.

CHAPTER THREE

REVIEW OF THEORETICAL LITERATURE

3.1 Introduction

In the previous chapter we looked at the various issues in UK transport and found that there was a need to expand the use of public transport. In order, to achieve this expansion we identified the need to improve the coordination of public transport, so the generalised cost of using the mode would be reduced. In particular, one method to attain coordination is to encourage integrated and inter-available ticketing, but the introduction of such ticketing types should not come at the expense of competitive forces within the markets for the public transport modes. We also identified that the deregulation and privatisation of transport may have resulted in near monopoly operators of public transport in some areas.

To be able to focus on the questions, concerning integrated or inter-available ticketing and near monopoly provision of public transport, we intend to introduce two theoretical models in this thesis. To allow us to produce appropriate economic representations of public transport we need to be able to use appropriate techniques – in this Chapter we explore simple economic modelling that we use to build our own models. An understanding of the intuition and working behind these economic models will also be needed, so that we can interpret our models' results to enable us to make sensible policy recommendations. We shall also investigate more complicated economic models of networks and transport as this will provide further ways of modelling the problems of integrated/ticketing and near monopoly provision of transport

In the following section we look at some basic micro-economic theory that will be important when we model the research questions that we have highlighted. This insight will also be useful when we look for the intuition behind our models. We explore Cournot's (1838) techniques and observations regarding competition, as well as Stackelberg's (1934)

extension. In Section 3.3 we consider network models by first looking at more findings from Cournot (1838), whose observations regarding competition and complementary monopoly will be important in this thesis. We shall also look at Spengler's (1950) extension of Cournot's (1838) model before moving on to Economides and Salop's (1992) model. This study will provide us with a particularly useful framework when considering transport networks, such as when we consider integrated ticketing in Part B. In Section 3.4 we consider Economides and Woroch's (1992) model of network interconnection. This study will be useful when we consider the effect of a monopoly operator on service provision in a transport network in Part C. In Section 3.5, we introduce network literature with specific application in transport. These works will show us techniques that are particularly relevant to transport and the interpretation of results will. In Section 3.6 we will consider conjectural variations; how and why they are used in economic models as well as looking at the advantages and disadvantages of this approach. In Section 3.7 we consider studies involving airlines hub-and-spoke networks, as they will later help us to approach the formulation of a service provision model, as we do in Part C of this thesis. In Section 3.8 we look at suitable ways of accurately calculating welfare to allow us to consider the preferences of society, while Section 3.9 deals with the possible problems when calculating welfare. Finally, in Section 3.10 we summarise the chapter and specify the importance of this chapter's model in terms of Parts B and C.

3.2 Microeconomic Theory

We begin our exploration of the theoretical literature by reviewing Cournot's (1838) model of duopoly (and oligopoly). Along with Cournot's (1838) model of complementary monopoly –introduced later on in the chapter – we have two models of non-cooperative behaviour that provide the foundations for most of the relevant literature, and, indeed, both feature in the new theoretical work proposed in later chapters of this thesis.

Cournot's most well-known contribution to industrial theory is undoubtedly the "Cournot Duopoly" model. However, we present here the more general case of n-firm Cournot Oligopoly, of which Cournot Duopoly is a special case with $n = 2$, and also appears in Cournot's (1838) work.

Consider an industry in which there are n identical firms supplying a homogenous good. The inverse demand for the good is $P(Q)$, where P is the industry price, Q is the total quantity of the good traded, and $P'(Q) < 0$. In the Cournot model, firms make simultaneous decisions over their choice variable, quantity. Each firm i ($i=1,..., n$) sets its output, q_i, yielding an industry output:

$$Q = q_i + \sum q_j . \qquad\qquad (i \neq j = 1,...,n). \qquad\qquad (3.1)$$

Equilibrium price, which is common to all the firms, is then determined by the market via the inverse demand function. Although, Cournot's original model assumes away production costs, we include them here for greater generality. Given symmetry, each firm faces the same marginal cost, c, and fixed cost, F. Hence, the profit for the ith firm is:

$$\pi_i = Pq_i - cq_i - F , \qquad\qquad (i = 1,...,n). \qquad\qquad (3.2)$$

Using (3.1) in (3.2) we have:

$$\pi_i = q_i\{P(q_i + \sum q_j) - c\} - F , \qquad\qquad (i \neq j = 1,...,n). \qquad\qquad (3.3)$$

An essential assumption of the Cournot model is that firms make their quantity choices "independently" according to the rule that they do not expect any of the other firms to respond to a change in their output (i.e. $\partial q_j / \partial q_i = 0$). Given this, maximising (3.3) with respect to own output, q_i, we get:

$$P(q_i + \sum q_j) + q_i P'(q_i + \sum q_j) - c = 0 , \qquad\qquad \text{where } i \neq j = 1,2 . \qquad\qquad (3.4)$$

This equation represents the implicit reaction function for firm i. Given symmetry, summing (3.4) over all i firms yields the following equilibrium condition:

$$nP(Q) + QP'(Q) - nc = 0 . \qquad\qquad (3.5)$$

This equilibrium represents the implicit relationship between industry quantity and the number of firms under symmetric n-firm Cournot oligopoly. One interesting question to ask is how the equilibrium under Cournot oligopoly compares with that under a situation of monopoly. To see this, we can set $n = 1$ in (3.5), which yields the relevant implicit expression for monopoly output:

$$P(Q) + QP'(Q) - c = 0. \tag{3.6}$$

Cournot concludes that the equilibrium industry quantity (price) in (3.6) is lower (higher) than that in (3.5) for $n > 1$ and that, more generally, in (3.5) equilibrium quantity (price) is increasing (decreasing) in n; lending support to the view that increasing the number of competitors has beneficial effects for consumers.

The reason for this is because there is a horizontal externality between the firms providing substitutes in Cournot duopoly that causes the firms to concentrate on their own output and not the industry's. This horizontal externality results in the higher outputs and lower prices compared to the joint ownership regime, which internalises the horizontal externality that causes the "competition effect." Joint ownership forces the firm to concentrate on industry output, allowing maximum profit across the industry and leading to higher prices compared to Cournot duopoly. Cournot's analysis also shows that as n, number of firms, increases then quantity (price) further increase (decrease). This is the basis for the assumption that introducing competition into a market leads to falling prices.

Cournot competition is a way of modelling oligopolies, particularly when the specific conditions of Bertrand competition, where firms price at marginal cost and make no profits, are not met. Cournot competition has been used to model a number of industries and scenarios, and Tirole (2002) explores various applications of the Cournot model to industrial economic problems.

The Cournot (1838) model assumes that both firms make simultaneously decision about their quantities. This assumption might not always be appropriate, as in some

industries it might be reasonable to assume that a firm will be able to make its output decision prices before another. The Stackelberg (1934) model incorporates what is known as the first mover advantage.

We can consider a Stackelberg model by using a two-firm industry where the incumbent firm chooses its quantity before an entrant, after which the incumbent's quantity is fixed. An entrant observes the incumbent's output decision and then chooses its own quantity. The incumbent firm when making its initial decision will be able to logically pre-empt the entrants' decision-making, so the incumbent takes the s reaction function of the entrant into account when maximising its own profit. This is known as the first mover advantage, because the firm that moves first can set a level of quantity that limits the quantity of the firm that moves second. It can be shown that the first mover, or incumbent in our example, will make a greater profit as a result of this advantage.

We should also point out that Cournot (1838) pricing takes place when the firms compete over quantity. However, there is the possibility that firms are competing using prices and this is called Bertrand (1883) competition – when firms are identical and produce homogenous goods using constant-returns to scale technology – which results in the firms pricing at marginal cost. Although, this behaviour does mean that no firm makes a profit.

In Section 2.3.3.1 we mentioned the results of Spence's (1977, 1979) entry deterrence models with reference to the bus industry and they use a version of the Stackelberg model but with capacity as the decision variable instead of quantity. They show that an incumbent firm can use idle capacity as a way of deterring entry.

3.3 Network Theory

Returning again to Cournot's model, although he sets out the advantages for splitting up monopolies into separate competitive firms, he also proposes another result. Cournot considers two goods, (1) and (2), where each good is supplied by a separate firm, who are

monopolists in the production of its good, but the goods are only of use when jointly consumed as good (12).

Cournot finds that the price of a composite commodity will be higher if firms are separated compared to when a monopoly controls the industry. Cournot generalises this result over *n* firms, so that with complementary goods it is possible that the separation of ownership will lead to higher prices. In fact, as the joint ownership regime is split into smaller firms each providing a complement, the magnitude of the vertical externality grows and causes firms to charge higher prices compared to the joint ownership regime. This is because there is a vertical externality between the complements in a split ownership regime that causes firms to concentrate on their own output and not the industries. This has an opposite effect to the horizontal externality as it causes firms in split ownership regimes to charge higher prices compared to joint ownership regimes because the joint ownership regime internalises the vertical externality, which causes this "complementary monopoly effect". The joint ownership firm concentrates on industry output, maximising profit across the industry, and leads to lower prices compared to a duopoly.

The Cournot "complementary monopoly" result shows that price decreases are not inevitable following the separation of a monopolist. The result of any split depends, as Cournot's model shows, on whether the goods produced are substitutes or complements for each other. If the goods are substitutes (complements) then the splitting of a monopolist will result in lower (higher) prices.

To ensure we explain each situation carefully we discuss both Cournot's competition and complementary monopoly models, but Sonnenschein (1968) has shown by reinterpreting the variables that these two theories are in fact one. This means that the criticisms of Cournot's duopoly model are also criticisms of his complementary monopoly model.

Spengler (1950) considers Cournot's (1838) complementary monopoly observation and models it by introducing a sequential strategic game and shows, once again, that consumers and firms may prefer a monopoly regime to a separate ownership regime.

We have introduced some basic modelling techniques and shown how these have been used to model realistic, yet basic, economic scenarios. We use the Cournot model in both Parts B and C of the thesis, whilst we shall use a sequential technique, much like Stackelberg and Spengler, in an extension of our integrated ticketing model and this is in Section 5.5. The results and intuition from these models are also useful as they are present in more complicated models that we intend to investigate, before moving on to model situations ourselves. We shall now look at extension of Cournot (1838) into a simple network scenario.

Economides and Salop[1] (1992) extend another Cournot (1838) result by considering a complicated network consisting of multiple producers and differentiated brands, assuming that components are fully compatible and the number of brands of each component is exogenous. This model has a network made of goods that are both complements and substitutes – thus it combines elements of both Cournot's (1838) competition and complementary monopoly observations. The model interests us not just for the results and intuition, but for the modelling framework used so we shall consider it in some detail.

In the simplest example of the *ES* network there are two components: A and B, as shown in Figure 3.1. Each component has two brands: 1 and 2; between which there is no cost to the consumer or the producer of interconnecting the two brands. The price of component A_i is p_i and the price of component B_i is q_i, where $i = 1, 2$. Each component is sold as part of a composite good, so the demand for A_i would be the demand for composite good A_iB_1 and the demand for composite good A_iB_2. This demand framework means that

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[1] From here to be referred to as *ES*.

Figure 3. 1: ES Demand Framework
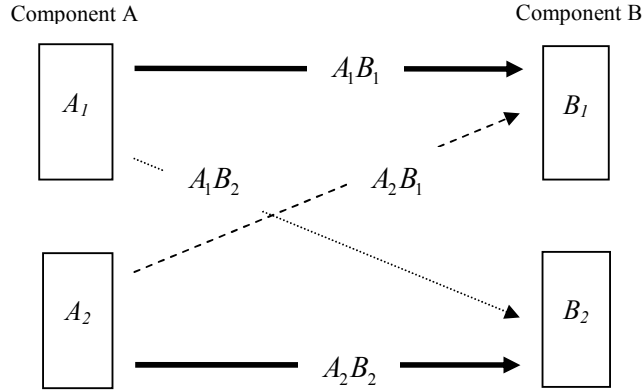
there are a total of four possible combinations of goods, although *ES* do generalise the

model to look at an infinite number of brands.

They assume that the demand system is symmetric and illustrate their results with a

linear demand system:

$$D_{11} = a - bs_{11} + cs_{12} + ds_{21} + es_{22}, \tag{3.7a}$$

$$D_{12} = a - bs_{12} + cs_{11} + ds_{22} + es_{21}, \tag{3.7b}$$

$$D_{21} = a - bs_{21} + cs_{22} + ds_{11} + es_{12}, \tag{3.7c}$$

$$D_{22} = a - bs_{22} + cs_{21} + ds_{12} + es_{11}, \tag{3.7d}$$

where D_{ij} is the demand for component $A_i B_j$ ($i, j = 1, 2$). The price of each composite is

made up of the price of each component so that $s_{ij} = p_i + q_j$. $s_{22} = p_2 + q_2$. *ES* assume that

$a, b, c, e > 0$, so the demand for composite goods is negatively related to its own price and

positive related to other composite good prices, so that the composite goods are substitutes,

although we should be aware that there are also complementarities between the goods.

To be consistent with consumer theory they assume the goods are gross substitutes

so $b > c + d + e$. Gross substitutes describe a situation where as the price of one good

increases the "Marshallian" demand for the other good increases – the "Marshallian"

demand is the demand as a function prices and incomes as opposed to "Hicksian" demand

that are a function of prices and utility. The condition $b > c + d + e$ means that that an equal

increase in the prices of all composite goods, an effective increase in total price, will lead to a fall in demand for each composite good so total demand will also fall. This condition also means that as the price of one composite good increases then the demands for the three other goods also increases. In the *ES* model there is no possibility of discounted return tickets thus there is no price discrimination.

They begin by looking at a separate ownership regime, where each component brand is owned by a different firm. They assume that marginal costs are zero so that the four separate profit functions become:

$$\pi_{A_1} = p_1 D^{A_1},$$ (3.8a)

$$\pi_{A_2} = p_2 D^{A_2},$$ (3.8b)

$$\pi_{B_1} = q_1 D^{B_1},$$ (3.8c)

$$\pi_{B_2} = q_2 D^{B_2}.$$ (3.8d)

Maximising (3.8) and solving the resulting first-order conditions for prices gives the independent ownership equilibrium component price:

$$s^I = \frac{a(2b-c-d)}{(b-c)(b-d)+(2b-c-d)(b-c-d-e)}.$$ (3.9)

Next *ES* consider the possibility of joint ownership, or a network monopolist. The profit function becomes:

$$\pi_{A_1} = p_1 D^{A_1} + p_2 D^{A_2} + q_1 D^{B_1} + q_2 D^{B_2}.$$ (3.10)

Once again, profit maximising and solving the system of equations for prices gives the joint ownership equilibrium component price:

$$s^J = \frac{a}{2(b-c-d-e)}.$$ (3.11)

They then compare (3.9) and (3.11) and this leads them to the proposition:

ES Proposition 1. *Prices are higher in joint ownership than in independent ownership if and only if the composite goods are close substitutes.*

We can see that Cournot's complementary monopoly observation may or may not hold in a more complicated network depending on the level of substitutability of the products. This makes intuitive sense as when the composite goods become closer substitutes then the effect of the substitutability will be stronger compared to that of the complementarity. As we explained earlier it is the substitutability (complementarity) that causes firms to charge lower (higher) prices when a monopolist is separated.

ES then move on to consider composite good competition, where each of the composite goods is owned by a different firm so that profit functions become:

$$\pi_{11} = s_{11}D_{11}, \tag{3.12a}$$

$$\pi_{12} = s_{12}D_{12}, \tag{3.12b}$$

$$\pi_{21} = s_{21}D_{21}, \tag{3.12c}$$

$$\pi_{22} = s_{22}D_{22}. \tag{3.12d}$$

Once again maximising profit and solving the system of reaction functions for prices gives the composite good competition equilibrium component price:

$$s^C = \frac{a}{(2b-c-d-e)}. \tag{3.13}$$

They then compare (3.9) and (3.11) with (3.13), showing that composite good competition prices are lower than in independent ownership and in joint ownership.

Next *ES* consider the possibility of parallel vertical integration with goods A_i and B_i integrated into one firm that is denoted as i, where $i = 1, 2$. Importantly, *ES* assume that there is no price discrimination, so that consumers, who buy both components from the same firm, pay the same price as one who purchases components from different firms[2]. The profit functions become:

$$\pi_1 = p_1 D^{A_1} + q_1 D^{B_1}, \tag{3.14a}$$

$$\pi_2 = p_2 D^{A_2} + q_2 D^{B_2}. \tag{3.14b}$$

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[2] Note that in Section 2.5.3 that we found that this may not always be the case in transport industries.

Once again, profit maximisation and solving the first-order conditions for prices gives parallel vertical integration's equilibrium component price:

$$s^V = \frac{4a(b+e)}{4(2b-2c-d-e)(2b-2d-c-e)-9(b-c-d-e)^2}.$$ (3.15)

They then compare (3.15) with (3.9) to find that $s^V < s^I$ and this leads them to their second proposition:

ES Proposition 2. *Prices are always lower in parallel vertical integration than in independent ownership.*

It seems that parallel vertical integration internalises some of the vertical externalities that lead to the complementarities having an upwards effect on the prices in independent ownership.

ES also compare (3.15) with (3.11) and this leads them to another proposition:

ES Proposition 3. *Prices are higher in joint ownership than in parallel vertical integration if and only if the composite goods are close substitutes.*

Whilst parallel vertical integration removes some of the complementarities produced from the splitting of joint ownership it does not remove all of them, leaving vertical externalities between components A_1 and B_2, and between components A_2 and B_1. These remaining externalities mean that prices in parallel vertical integration are higher than in joint ownership unless the substitutibilities are sufficiently high. Comparisons of (3.15) and (3.13), lead them to their Proposition 4:

ES Proposition 4. *Prices are higher in parallel vertical integration than in composite goods competition.*

Composite good competition internalises all the vertical externalities that the complementarities act on to result in higher prices and none of the horizontal externalities that the substitutabilities act on to produce lower prices.

We can see that the *ES* model produces some interesting results that extend Cournot's (1838) simple network. The framework for this seems to have some promising characteristics that could be applicable to transport modelling. Indeed, we use it as the basis to explore the impacts of integrated ticketing and do so in Part B; beginning in Chapter 4 by applying the *ES* model to transport so that we can focus on its suitability. Chapter 5 then augments the basic *ES* demand framework to consider integrated ticketing in more detail.

3.4 Conjectural Variations

Another technique that is linked to Cournot's modelling is that of conjectural variations. In the following sections we will be looking at transport modelling by James (1998), and McHardy and Trotter (2006), who both use conjectural variations. A conjectural variation term is a parameter that measures how a firm expects another firm to react to a change its strategic choice variable. The value of this conjecture impacts on how the firms set own prices to maximise profit.

The following is an example of a price conjectural variation term:

$$\gamma_{ij} = \frac{dP_i}{dP_j}.$$ (3.16)

This conjectural variation term measures firm *i*'s expectation of the response in P_i to a change in P_j and can be interpreted as the implicit collusiveness of the industry. Joint profit maximisation is characterised by $\gamma = 1$, whilst lower values of γ represent increasingly competitive pricing behaviour. Independent pricing is characterised by $\gamma = 0$, and values of $\gamma < 0$ imply accommodating pricing behaviour.

The origin of the conjectural variations approach is credited to Bowley (1924), although the "conjectural variations" term is due to Frisch (1933). The conjectural variations approach is not completely without its opposition: Makowsky (1987), Shapiro (1989), and Lindh (1992) all criticise its use on the grounds of logic and of consistency. Not

only is the conjectural variation parameter empirically difficult to estimate but values of the conjectural variation parameter other than 0 and 1 are subject to the criticism that behaviour is irrational. Conjectural variations also has its supporters: e.g. Bresnahan (1981), Martin (1993), Fraser (1994), and Dixon and Somma (2003).

Due to the criticisms the conjectural variations approach should be used with caution. In Chapter 6 we will use a conjectural variations approach to check the robustness of our integrated ticketing model to possible collusion amongst firms based on if firms act in the way prescribed by conjectural variations.

3.5 Network Interconnection

Part C of the thesis will concentrate on the private monopolies and their impact on network interconnection so we now focus on the modelling of network interconnection – these models could also prove useful when we look at integrated ticketing. We shall begin by examining a study of network interconnection before we move on to look at commentators who consider network provision in transport. This work is slightly different to the modelling of public transport forms but the techniques and results will still be useful. Let us first consider Economides and Woroch's (1992) model that deals with general interconnection issues.

Figure 3. 2: Network Interconnection Model
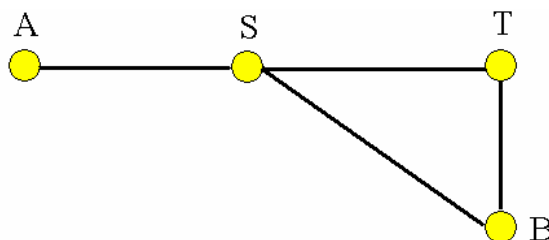
Economides and Woroch (1992) look at the private and social incentives of network interconnection under the structure seen in Figure 3.2. The only demand in this network is between the two end nodes A and B, and these two can be linked using components AS, ST,

TB, and SB. Economides and Woroch[3] call S and T *switches* so that the indirect service between A and B includes the *gateway,* ST, as well as routes, AS and BT, whilst the direct service simply comprises of AS and SB.

Let us look at the modelling process involved. They begin with a consumer's quadratic utility function:

$$U(D_{ASB}, D_{ASTB}) = \alpha D_{ASB} + \alpha' D_{ASTB} - \left[\beta(D_{ASB})^2 + \beta'(D_{ASTB})^2 + 2\gamma D_{ASB} D_{ASTB}\right]/2, \qquad (3.17)$$

where D_{ASB} is the demand for the direct route ASB and D_{ASTB} is the demand for indirect route ASTB. The consumer maximises:

$$U(D_{ASB}, D_{ASTB}) - \left[p_{ASB} D_{ASB} + (q_{AS} + q_{STB}) D_{ASTB}\right], \qquad (3.18)$$

where p_{ASB} is the price of route ASB, q_{AS} is the price of route AS and q_{STB} is the price of route STB. They use utility maximisation to get inverse demands before inverting to give rise to the following two demand curves that are linear in own prices and cross prices;

$$D_{ASB} = a - bp_{ASB} + c(q_{AS} + q_{STB}), \qquad (3.19a)$$

$$D_{ASTB} = a' - b'(q_{AS} + q_{STB}) + cp_{ASB}, \qquad (3.19b)$$

where $a = (\alpha\beta' - \alpha'\gamma)/(\beta\beta' - \gamma^2)$, $a' = (\alpha'\beta - \alpha\gamma)/(\beta\beta' - \gamma^2)$, $b = \beta'/(\beta\beta' - \gamma^2)$, $b = \beta/(\beta\beta' - \gamma^2)$, and $c = \gamma/(\beta\beta' - \gamma^2)$ with α, β, and γ as coefficients on the various terms in the utility function. This assumes $a, a', b, b' > 0$ and $c > 0$ so $\beta\beta' > \gamma^2$ which in turn implies $bb' > c$ where c measures the cross-price effect. These provide EW with some useful constraints which they can use to limit to set of plausible results.

We can see in the demands that the route STB and route ASTB are substitutes. The profit functions of the firms contain a fixed cost component, but there is the assumption that marginal cost is zero. The rest of the study's techniques do not go beyond what we already seen so we will concentrate on the results of the various regimes.

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[3] From here to be referred to as *EW.*

The first regime that they consider is inter-modal competition where network 1

provides ASB and network 2 provides STB. In this regime network 2 has to purchase access

to AS from network 1. *EW* compare profits with and without interconnection, and find that

both firms desire network interconnection assuming the fixed cost of running a route is not

large. They then calculate profits, consumer surplus, and welfare for the interconnected

scenario, but there is no specific attempt to compare the welfare with a non-interconnection

scenario. Instead, the main focus is similar to the other studies we consider as it

concentrates on comparing the prices of the various regimes rather than actual welfare.

EW find if the demands for ASB and ASTB are approximately equal then p_{ASB} is

higher than q_{AS}. However, if D_{ASTB} is smaller than D_{ASB} and inelastic then p_{ASB} can be

below q_{AS}, so network 1 charges a lower price to its individual customers for the service

ASB than it does to network 2 for link AS – thus the existence of discriminatory

interconnection. The owner of the direct route, ASB, will charge a lower price for AS or

ASTB depending on which has the lower elasticity of demand. In addition, they find that if

there are two networks, with one owning AS and the other owning BT, then the ownership

of ST is of no value to either firm. All of these results are so far intuitively appealing if not

particularly surprising.

The next regime that *EW* consider is when a network monopolist owns the whole

network. They find that society is better off when a network monopolist controls the

network rather than when there are separate firms. This result further extends Cournot's

complementary monopoly observations and is, again, due to a network monopolist

internalising the vertical externalities arising from the complementarities in the model.

EW then look at an alternative version of inter-model competition where instead of

simple Cournot competition they introduce the possibility of a sequential game by using the

Stackelberg model. They assume the owner of ASB is the leader and moves first, and find

that it is now possible that network 2 would be foreclosed – the owner of ASB would not

allow the owner of network 2 access to AS. However, this is ultimately an undesirable strategy for the owner of ASB as the loss from removing a vertical partner is greater than the gains of removing a horizontal competitor.

Next they consider the possibility of competitive interconnection where network 1 controls ASB, network 2 controls BT, and network 3 owns ST. They compare these results with those of inter-modal competition, and find the sale of ST by network 1, resulting in this competitive interconnection regime, would be profitable. However, it reduces profits for the pre-existing network and total industry profits, while also increasing prices and, hence a fall in welfare.

EW look at further disintegration of the ownership structure in an independent ownership scenario. Here network 1 owns AS, network 2 owns STB and network 3 owns SB. They, again, compare the equilibrium results with those of the inter-modal competition scenario and find that prices of end-to-end services are higher in the new scenario and total welfare is lower. This would seem to generalise Cournot's complementary monopoly observations regarding increased prices as a result of the splitting of a monopolist who produces complementary goods. They also find that industry profits are higher (lower) when the direct (ASB) and indirect (ASTB) goods are close (poor) substitutes.

Next *EW* consider the possibility of independent ownership with competitive interconnection. This scenario begins with the independent ownership scenario and considers the effects of the sale of ST by network STB. They find that the effect of this change leads to the prices of ASB and ASTB increasing; resulting in reduced industry profits, consumer surplus, and total welfare. In addition, they also find that network STB would be better off selling the link (ST) to a third party if the demand on ASTB is relatively elastic.

The final scenario they consider is a comparison of competitive interconnection and independent ownership with competitive interconnection. This comparison is basically

modelled as the sale of SB by network ASB. They find that the sale of SB, turning the competitive interconnection into an independent ownership with competitive interconnection regime, by network ASB would result in higher prices for routes ASB, BT, and ST, but a lower price for AS. The overall price of service ASTB would increase (decrease) depending on how relatively elastic (inelastic) the demand was on the direct ASB route.

The results all tend to suggest that welfare and profits fall as the network disintegrates, hence we see Cournot's complementary monopoly observations in a more complicated network. *EW* also find that the networks are always willing to provide interconnection. Although, the results do not seem too interesting the model does provide a useful framework for comparing networks, and dealing with interconnection and service provision issues. This highlights the need for any model to have a legitimate set of regimes that is fully considered to give interesting and relevant results.

An area that remains unexplored is the possibility that more than one firm could operate along a single route. *EW* only introduces competition through the possibility of an indirect service alongside a direct service. It would seem likely that more than one firm would want to offer a single component. If we take an internet connection network as an example, then a computer operating system would be required and is offered by Microsoft and UNIX, whilst an internet browser is also needed and these are provided by Microsoft, Netscape, Opera, and Mozilla. This would be a particularly relevant question to ask in a transport context. Bus or train operators have, following bus deregulation and train privatisation, been able to run competing services on the same route.

Despite firms always preferring to provide interconnection *EW* never consider whether society will prefer the provision of an interconnecting network. Welfares or prices between network connection and non-network connection are never presented. Additionally, the set-up of the *EW* model, although a useful look at the possible provision of network

interconnection, is not always appropriate when considering transport. The demand is only for the end nodes, whilst in transport we would likely find individual demands for each component of the system. Additionally, the consumer in the *EW* model is not influenced by distance and this, in particular, is an important parameter when modelling transport. To provide us with a more specific view of transport modelling we shall look at studies considering transport in more detail and this will demonstrate methods that are appropriate to this thesis.

3.6 Transport Network Modelling

We now intend to look at several studies that focus on transport and this will provide us with further modelling techniques as well as applications of previously discussed models. Some studies may also incorporate ways of dealing with the specifics of transport that could highlight ways we could adapt models to fit the industries we shall model in Parts B and C.

We begin our look with an extremely simple network with a novel twist by James (1998). He sets up a model of frequency, entry, and predation in the bus industry. The primary choice variable in this model is frequency. This is not the choice variable that we intend to focus on in this thesis – in Parts B and C we shall use price as the main choice variable. However, this model is still appropriate as it focuses on ways of adapting models to a transport mode along with an exploration of a variety of regimes, where firms still seek to maximise profit subject to a single variable, albeit a frequency variable. This model also introduces the use of conjectural variations – which we saw in Section 3.4 – into a transport model.

James looks at the features of the UK bus industry following deregulation in October 1986. The model set-up is simple with passengers travelling between two points in a single direction over a time-cycle of length, *T*. This basically means that he considers a single route, but with passengers arriving at uniform point on a "clock". All bus operators are

identical except from costs, whilst there is no fare competition. Passengers are assumed to board the first bus that arrives at their starting point. This assumption may seem overly simple, but when considering bus passenger's behaviour it is not unreasonable especially as much of the evidence from bus deregulation seemed to have suggested a lack of price competition in the industry.

Q is the total number of passengers for any given level of total service frequency (V) and is uniformly distributed over T. It is assumed that a unit increase in service frequency would lead to a one unit increase in total passenger numbers. The model's concern is with one incumbent operator with a first mover advantage facing possible entry from one entrant. The overall service frequency after entry is:

$$V = v_m + v_e,$$ (3.20)

where v_m is the service frequency of the incumbent and v_e is the service frequency of the entrant. James assumes the inverse demand function is linear, taking the form:

$$p = 1 - Q + gV,$$ (3.21)

where p is price and g is the sensitivity of service frequency to price.

The two firms differ in cost structure as the incumbent is assumed only to have a marginal cost (c), whilst the entrant has a sunk cost of entry (F) and a marginal cost (c) at the same level as the incumbent. Costs for the incumbent and entrant, respectively, become:

$$C_m = cv_m,$$ (3.22)

$$C_e = cv_e + F.$$ (3.23)

James uses (3.21), (3.22) and (3.23) to form the profit functions of the incumbent and entrant, respectively:

$$\pi_m = (1 - V + gV - c)v_m,$$ (3.24)

$$\pi_e = (1 - V + gV - c)v_e - F.$$ (3.25)

He then derives the first-order conditions for profit maximisation with frequency being the choice variable – remembering there is no price competition. The incumbent's first-order condition becomes:

$$1 - (\lambda_e + 2)(1-g)v_m - (1-g)v_c - c = 0, \tag{3.26}$$

where $\lambda_e = \frac{dv_e}{dv_m}$ is the conjectural variation term that measures how much the incumbent expects the entrant's frequency will change when the incumbent themselves changes their own frequency. The entrant's first-order condition becomes:

$$1 - (\lambda_m + 2)(1-g)v_e - (1-g)v_m - c = 0, \tag{3.27}$$

where $\lambda_m = \frac{dv_m}{dv_e}$ is the conjectural variation term that measures how much the entrant expects the incumbent's frequency to change when the entrant changes their own frequency. The assumed values of the conjectural variations term can then be substituted into this so that the model represents the structural forms of an industry and it is then possible to derive the reaction functions.

James begins with the analysis of the Stackelberg equilibrium, where the incumbent is

Figure 3.3: James' Indifferent Fixed Cost ($l = 0$) and g
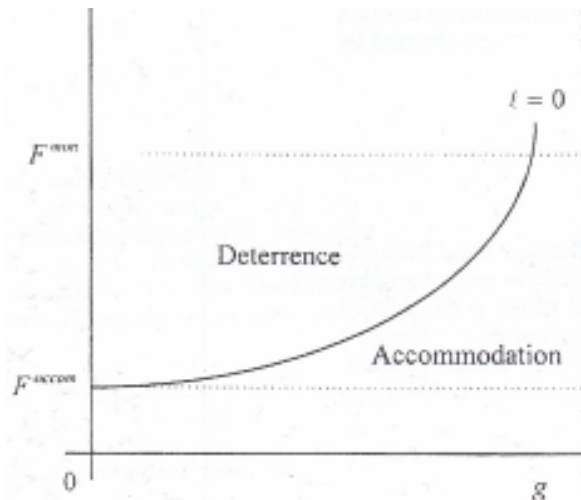
(James, 1998, Figure 1, Page 46.)

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the leader and the entrant is the Cournot follower, and finds that the incumbent will provide a greater level of service frequency.[4] Using this analysis James looks at values of F that would lead to the formation of a contour, where the incumbent is indifferent between entry accommodation and blockading. This contour shows that as g increases then entry accommodation becomes more likely due to a positive relationship between the indifferent fixed cost and g (shown in Figure 3.3). James looks further into the reasons for this and concludes that entry deterrence is extremely difficult for a large incumbent. He also suggests it is likely that large incumbents will co-exist with a series of smaller incumbents operating on a small fraction of the total network and finds that his results generalises to other demand and cost conditions.

This model, despite its primary concern being with firms choosing a value of frequency that maximises profits, provides us with a valuable look at a simple transport network. It incorporates ways of introducing a conjectural variations parameter, as well as examples of various regimes appropriate to transport, and a method of comparing them by calculating an indifferent fixed cost contour. We now move on to a more complicated model set-up that also introduces further transport regime types.

Another theoretical model that looks at transport networks, although this time focusing on a rail network, is Else and James (1995). They look at the privatisation of rail services, but with a network more complicated than James (1998) as they have a linear rail network between A and C, and an intermediate station B. They assume the demand function for round trip journeys is;

$$Q = 1 - P_f, \tag{3.28}$$

where Q is the quantity and P_f is the generalised cost of travel, not just the fare and is defined:

$$P_f = P + S, \tag{3.29}$$

---

[4] There is also a discontinuity where the incumbent provides such a large frequency that entry is blockaded and the entrant provides no service.

where P is the fare and S is all other parts of generalised cost. The generalised cost includes factors such as wait time, with Else and James (1995) considering it to be inversely related to the service frequency (V) – as service frequency increases then the wait time of passengers should decrease and so too does the generalised cost of travel. They do not believe that the relationship between service frequency and demand would be linear, so they introduce a parameter g that they use to model the diminishing returns between service frequency and demand. Therefore, demand becomes:

$$Q = 1 - P + gV^{\mu}. \tag{3.30}$$

The next important step in the set up of the model is the assumption of the cost structure. They suggest that the marginal cost per passenger is likely to be close to zero and so emphasis that the main marginal cost is linked to the frequency variable. The cost function becomes:

$$C = (a + b)V + F, \tag{3.31}$$

where a and b are coefficients on the frequency variable and F is the fixed cost. To begin they use this and the demand equation to calculate the welfare maximising values of Q and V assuming that welfare maximising point is where price equals marginal cost per passenger, which is when price is zero, to give:

$$Q_W = \frac{2(a + b)}{2(a + b) - g^2}, \tag{3.32}$$

$$V_W = \left[\frac{g}{2(a + b) - g^2} \right]^2. \tag{3.33}$$

These welfare maximising equilibrium values of Q and V are when all costs are covered by subsidies. They then assume that subsidies are restricted and calculate a second-best welfare solution that has a lower quantity and service frequency. Further analyse shows that a profit-maximising monopolist has even lower values of quantity and frequency.

Therefore, in this model we can see that welfare decreases as we move from the social planner to the monopolist.

Else and James then look at a situation where the ownership of the track and train operations are split into two monopolies. They find that this vertical disintegration of the railway leads to a lower price, but with a reduced level of service and less passengers – so this regime leads to a lower value of welfare than under the monopoly. This is a result similar to Cournot's complementary monopoly findings.

The next situation they consider is where one firm owns services from A to B and another owns services from B to C. Here they make the assumption that all passengers travel from A to C, therefore there is no demand between A and B, and B and C, so that services A and B, and B and C are complements. Under this split ownership they find that the fares are higher and the number of passengers is lower than with an integrated monopolist. A situation that, again, can be related back to Cournot's complementary monopoly observations.

Else and James examine one further scenario, when they look at the latter situation, but with another firm controlling the infrastructure. This could represent the reality of the post-privatisation era that we saw in Section 2.3.4 with Railtrack (now Network Rail) operating the track and Train Operating Companies running services. In this scenario they find that there are reductions in frequency levels and passenger numbers, but the effect on fare is ambiguous. They conclude that, generally, disintegration of network leads to poorer welfare outcomes, although when comparing the latter two regimes the results are ambiguous.

The study provides another application of Cournot's complementary monopoly observation, again showing how important the observation is to network models. The regimes Else and James consider provide us with further examples of how to model and

compare various ownership types within transport. The fact the model has specifics relative to the railway industry further demonstrates methods to stylise a model to the industry.

McHardy and Trotter (2006) consider a model of airport regulation and the relationship between aeronautical and non-aeronautical services. They consider Starkie's (2001) suggestion that aeronautical and non-aeronautical services should be regulated in different ways – an effective move from single-till to dual-till regulation – but McHardy and Trotter point out that there is an implicit relationship between the two services that mean there may be complementarities and that separating the regulation of the two services may result in airports having an incentive that is not in the consumer interest.

McHardy and Trotter use a simple linear demand for round trip air travel between two countries (foreign and domestic) of the form:

$$Q = \alpha - \beta p ,$$ (3.34)

where α and β are both positive. They assume that the total price, p, is:

$$p = p_a + p_d + p_f ,$$ (3.35)

where p_a is price charged by airlines, p_d is the price charged per passenger by the domestic airport, and p_f is the price charged per passenger by the foreign airport. Profit for the ith air port is:

$$\pi_i = (p_i - c_i)Q ,$$ (3.36)

where c is the long-run constant marginal cost. They solve this for the three sectors; introducing a conjectural variation parameter between the three sectors, and a conjectural variation parameter between competing airlines. McHardy and Trotter find the gains associated with moving from independent to perfectly collusive behaviours between the three sectors, in terms of reduced total price, are greater than the gains from moving from perfectly collusive behaviour to independent behaviour in the airline market. The model also shows that some of the gains (i.e. lower prices) from introducing competition amongst airlines are lost as airports do not pass on the lower prices they receive to consumers;

basically the monopoly airport increases its charges and "absorbs" some of the gains themselves.

McHardy and Trotter use this model to consider the implications of a change in regulation of airports may have. A key finding is that the complementarities between aeronautical services and non-aeronautical services arising from a change from single-till to dual-till regulation could damage welfare should the aeronautical and non-aeronautical services be close complements. This model, again, incorporates many of the factors that drive Cournot's (1838) competitive and complementary monopoly observations.

Yang and Kin (2000) consider a solitary bus route with a capacity constraint and produce a model that they then use to calculate demand, service quality, profit, and social welfare for a variety of monopoly and competitive regimes. They use these comparisons to highlight when and under what circumstances a bus route is preferred by firms, passengers, and society. The approach concentrates on the complicated visualisation of the problem rather than the recommendations and intuition that result from the model. This limits the usefulness of the paper in the terms of this thesis' focus, but it does underline the need for some simplicity in modelling to produce useful results.

3.7 Hub-and-Spoke Networks

In Section 3.4 we presented a study by Economides and Woroch (1992) that examines network interconnection under a number of ownership regimes. However, we find that despite an interesting set-up that the demand system was not appropriate for transport modelling, so we need to investigate models that consider a similar issue but with a more appropriate demand system.

A set of transport studies that look specifically at how to model, and ascertain, the optimal network set-up is present in airline literature. This literature attempts to explain why hub-and-spoke airlines have become the dominant form of operation. In the airline industry

many firms find it efficient to operate a hub-and-spoke operation, where initially travellers are flown to a main airport or hub before flying from this hub to one of the several smaller airports that it serves.

Hub-and-spoke systems require travellers to make an interchange to get to destinations other than main hubs. The operation of this type of network has seen the loss of many direct routes, but airlines argue that this allows them to provide an overall greater number of origin-destination journeys. So whilst direct routes become less prevalent with hub-and-spoke operations it could allow the airlines to have increased their service provision, and this reasoning is something that various studies attempt to clarify. These studies have some relation to our consideration of service interconnection and the incentives for a private monopolist in Part C despite our focus being public transport.

Shy (2001) sets up a simple hub-and-spoke model where passenger's utility functions do not have a distance parameter, but there are two types of passenger: high-value-of-time passengers and low value of time passengers. In his model there are three nodes (A, B, and C); a fully connected (FC) network has all three nodes provided for or connected by a route, whilst the hub-and-spoke (HS) network connects the three nodes with two routes. If an individual wishes to travel from A to C they would have to travel via B.

Shy finds that as long as the cost-per-flight is low enough then the fully connecting set-up is more profitable than hub-and-spoke configuration for a network monopoly. However, Shy states it is likely airline/airport networks have a large cost element, such as the renting of departure and arrival gates at local airports, as well as landing fees. There is also the assumption that individuals who purchase tickets to fly from A to C cannot get off at the hub B and this allows the price of flying from A to C to be less than the price of flying from A to B, or B to C. Shy does not consider whether society would prefer for FC or HS networks to exist.

Shy does investigate the effects of the introduction of competition and finds the incumbent would allow the entrant to run all direct services for the high-value-of-time customers, whilst the incumbent runs a hub-and-spoke network on the remaining two routes.

Brueckner and Spiller (1991) set up a network that is more complicated than Shy as it has four nodes. This hub-and-spoke system can be seen in Figure 3.4 with cities A, B, and C

Figure 3.4: Bruecker and Spiller's (1991) Hub-and-Spoke Network
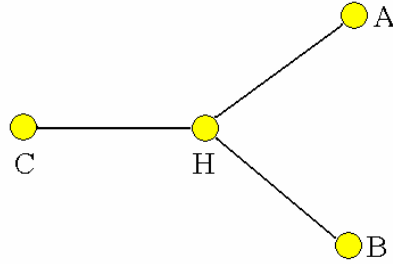
equidistant from the hub, H. They assumed the inverse demand function for round trip travel in any city-pair market i to j is given by $D(Q_{ij})$, where Q_{AH} represents the number of passengers travelling along AH in both directions, from A to H and from H to A. The monopolist's profit function becomes:

$$\pi^M = R(Q_{AH}) + R(Q_{BH}) + R(Q_{CH}) + R(Q_{AB}) + R(Q_{AC}) + R(Q_{BC})$$
$$- c(Q_{AH} + Q_{AB} + Q_{AC}) - c(Q_{BH} + Q_{AB} + Q_{BC}) - c(Q_{CH} + Q_{AC} + Q_{BC})$$
(3.37)

where $R(Q_{ij}) = Q_{ij}D(Q_{ij})$. Assuming that marginal revenue and marginal cost are:

$$R'(Q) \equiv \alpha - Q,$$
(3.38)

$$c'(Q) \equiv 1 - \theta Q,$$
(3.39)

where Q is the total number of passengers travelling and θ is an arbitrary parameter that determines the relationship between quantity and marginal cost. The first-order conditions from (3.37) are then solved for equilibrium quantities, which when rearranged become:

$$Q_{AB} = Q_{AC} = Q_{BC} = \frac{2 - \alpha(1 + \theta)}{5\theta - 1},$$
(3.40a)

$$Q_{AH} = \frac{\alpha - 1 + \theta(Q_{AB} + Q_{AC})}{1 - \theta},$$
(3.40b)

$$Q_{BH} = \frac{\alpha - 1 + \theta(Q_{AB} + Q_{BC})}{1 - \theta},$$
(3.40c)

$$Q_{CH} = \frac{\alpha - 1 + \theta(Q_{AC} + Q_{BC})}{1 - \theta}.$$ (3.40d)

Brueckner and Spiller then look at the implications of other types of competition in this model and the effects on welfare. The first structure they consider is inter-hub competition, where another airline serves city D connected to hub K that connects to A and B; in essence – this is the addition of a mirror image to the right-hand side of Figure 3.4. This airline has the same cost function and same city-pair demand functions as the original. Due to the structure, competition between the two airlines now exists in the AB market with one airline providing service AHB and another AKB. They find that the main determinant of whether price is higher (lower) than in the monopoly regime is whether θ is greater (lower) than 0.152. They also find inter-hub competition can harm passengers in other markets served by the hub-and-spoke system as prices rise in markets AC, BC, AH, BH, and CH.

The next situation they explore is "direct competition" where a small airline provides a non-stop service between A and B. They also look at "leg competition", where an airline operates on A to H and carries no connecting traffic. From their simulations they find "direct competition" leads to a lower price and increased traffic, when compared with monopoly. Markets AC, BC, AH, BH, and CH also benefit if increasing returns are weak and demand is high, otherwise welfare suffers. In "leg competition" they find welfare is higher in AH, but AB, AC, BC, BH, and CH suffer from higher fares and lower traffic. The study suggests that competition can result in some harmful network effects that lead to lowering welfares – although there are interesting effects on the various components within the model and such results being compatible with Cournot's (1838) complementary monopoly observations.

The demand system that both Shy (2001), and Brueckner and Spiller (1991), use do not include distance as a parameter and thus will not be useful in our modelling, but in their approaches they do highlight many avenues for our research. Shy does not make reference

to the possible welfare effects from the various network types. This slightly undermines the usefulness of his conclusions. Welfare results are definitely an area that should be explored within Parts B and C of this thesis. However, Brueckner and Spiller's structure is of use and the various regimes they consider show the possible structures that can be considered in a network model. These, again, should be applicable to the public transport industry that we are to model despite needing to alter the cost side.

Zhang (1996) looks at further hub-and-spoke network issues by considering the "fortress hubs" phenomenon. "Fortress hubs" are where airlines operate different hub-and-spoke networks and compete for traffic between non-hub cities via trans-hub connecting services, but retain a local monopoly in the spoke segments from their hubs.

Zhang constructs a situation where there are four nodes, with each airline operating on three of these, whilst one is exclusively their own service and the two others are shared. Airlines have the option of entering the other firms' markets. This competition brings about a negative network effect as entering into the other market may reduce the entrant's profit in its own hub-and-spoke market. This shows an extension of the hub-and-spoke view and the result, again, stems from Cournot's (1838) complementary monopoly.

Another model that looks at the hub-and-spoke set up is Encauo et al (1995), who examine a network that links three nodes compromising of two direct connections and one indirect connection with two airlines operating. These two airlines have asymmetric traffic rights and compete sequentially. The airlines begin by, first, setting their departure times, and, secondly, setting price.

Encauo et al find that deregulating a route that is part of a complicated network can affect the equilibrium on other routes of the network. The possibility that deregulating one route can have a major effect on network profit, and welfare, results suggests that policy decisions on one route of a network can have large impacts on the outcomes of the network. In this particular model, deregulation on one route has a negative welfare effect, as Encauo

et al suggest, an opposite result to what is usually associated with competition. Again, we see negative network effects from attempts to introduce competition and it is a result linked to Cournot's complementary monopoly observations.

Even in more complicated hub-and-spoke literature the complementary monopoly effects that Cournot observes still have a major impact. The hub-and-spoke literature indicate many ways to consider networks and the ways that competition can be introduced, either between competing networks, or competition upon certain routes. Encauo et al's suggestion that policy decisions on one route of a complicated network could have a major influence on profit and welfare results opens up an interesting angle for network policy consideration and is a regime type worth considering in Part C.

3.8 Welfare Calculation

The models we introduce in this chapter have made conclusions based on comparisons of the outcomes of the situations they have proposed using a number of methods. Whilst the well-being of firms is often represented by calculating the profit there are a number of ways of considering the how well off society is. Economides and Salop use a simple comparison of prices and quantities to make inferences concerning social welfare. For the most part an increase in price and a resulting fall in quantity can be considered to lead to a fall in welfare, but in our models this may not always be the case. Therefore, we need to investigate ways of calculating welfare.

The simplest way to envisage the concept of economic welfare is to use Marshall's (1920) method. Let us assume that the equilibrium price in a one good industry is p^*, and the equilibrium quantity is q^*. Figure 3.5 shows such a situation. The area $0\,p^*\mathrm{M}q^*$ is total industry profit and the area $p^*\mathrm{aM}$ is the industry's consumer surplus. If we assume costs are zero then profit can be calculated using:

$$\pi_I = p^* q^*. \tag{3.41}$$

This can then be used to represent the firms' preferences as has been the case in many of the studies we have looked at in this chapter.

Marshall (1920) considers consumer surplus to be the difference between what the consumer has to pay to purchase the good and what they are willing to pay. If we consider

Figure 3.5: Welfare, Consumer Surplus and Profit
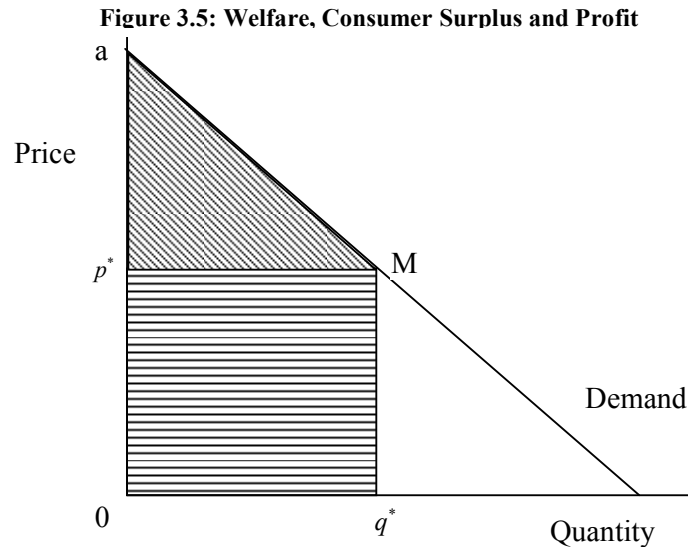
the demand curve to represent the consumer's "willingness to pay then using Figure 3.5 we can define consumer surplus:

$$. \; CS = \frac{1}{2} q^* \left(a - p^* \right), \tag{3.42}$$

where a is the demand curve's intercept with the y-axis. Adding the shaded triangle underneath the demand curve and above the price (p^*) together with the shaded square below the price (p^*) gives the total welfare arising from this industry. The equation for total industry welfare is:

$$W_I = p^* q^* + \frac{1}{2} q^* \left(a - p^* \right). \tag{3.43}$$

We can work out the welfare arising from an industry if we know the equilibrium price, quantity, and the demand curve's intercept with the vertical axis. With multiple good industries we can simply work out the welfare arising from the production of each good and sum them to give us the total welfare.

McHardy (2006) uses a similar method to look at the welfare effects of a complementary monopoly. He introduces a measurement of complementary monopoly deadweight welfare loss that that allows him to consider how the number of complementary monopolists, amongst other parameters, affects the level of dead-weight welfare loss. McHardy (2006) eventually concludes an industry that has complementary interdependencies, such as those we saw in give rise to vertical externalities in Cournot's complementary monopoly model, might lead to large dead-weight welfare losses when moving from a monopoly regime to a complementary monopoly regime, and this loss increases with the number of firms introduced.

3.9 Welfare Calculation Problems and Alternatives

In the previous section we proposed a simple method of calculating the absolute welfare in an economic model. However, there are a number of problems of calculating absolute welfare the way we describe in Section 3.8.

We should be aware that the consumer surplus, and therefore welfare calculation, above is just an estimate and may not give an accurate overall value of the welfare of society. Indeed, Hicks (1943) seeks to explore the concept of welfare further by introducing four other measures of consumer surplus following a change in prices: compensating variation, compensating surplus, equivalent variation and equivalent surplus. The calculation of these consumer surplus definitions is based on the compensated demand curve and the results differ from Marshall's consumers surplus method – unless, As Ng (1979) points out, the income effect is small when the compensated demand curve is equivalent to the "normal" demand curve.

Despite the problems with the Marshall's consumer surplus definition Ng (1979) does suggest that it is preferable when looking to measure the consumer's gain from a change, rather than an absolute level of welfare. In this thesis will we not be seeking to estimate the

actual welfare of society, but we will concentrate on whether the travellers gain or lose following a regime or price change; thus we adopt the Marshall method of calculating consumer surplus. However, there are some further problems that we should be aware of.

The first problem is that Marshall's method treats all individuals equally. Ng (1979) points to Pigou's (1912) suggestion that the transfer of income from a richer man to a poorer man increases the overall sum of satisfaction. This is the case if we place a higher value upon the poorer man satisfying his lesser fulfilled wants; that is a greater utility may be gained by the poorer person purchasing an extra £1 worth of their wants compared to a richer person purchasing an extra £1 of their wants. Thus a £1 gain in income does not equate to the same welfare gain for all individuals. One way to correct for this issue is to use a social welfare function that specifies welfare across a range of individuals. However, for the purposes of this thesis it is impractical and the assumptions over the properties of this could have significant effect on the results, so that our conclusions would be subject to the form we assume welfare to take rather than the variables we wish to investigate in our models.

Another problem is that in transport economics we often find that the traveller's cost is not simply the fare incurred, but also includes the non-monetary costs that make up the generalised cost. We will for the most part in our models of Part B and C assume that other aspects of the generalised cost remain the same between the various regimes; often their arbitrary nature and tractability issues give us good reason to do this, thus justifying our focus on comparisons using the measure of welfare method described.

The third additional issue is where the structure of the model does not enable the calculation of welfare in the way that Section 3.8 suggests. One possibility is that the demands are dependent on the prices of other goods and this path dependency problem can result in the miscalculation of total welfare. Ideally in such cases the cumulative demands should be disentangled so that each demand is dependent on its own price, but if this is not

possible it may be necessary to either integrate demands in an attempt to calculate welfare, which could produce complicated equations that lack intuition. The alternative is to find another way in which to rank welfare. Additionally, the model may be in such a form that the calculation of welfare will not be possible. In this case an alternative way of representing the preferences of society has to be found. Whilst we would prefer to calculate welfare for the models in this thesis we realise this may not always be the case. Indeed, in Chapter 5, we find that the calculation of welfare to be problematic, so during that chapter we suggest an alternative using a social planner's preference function, which uses the total patronage and average price to allow us to make inferences concerning the ranking of welfares.

In transport the profit maximising firm(s) may not always be the operator(s) and, in Section 2.3, we saw that public transport has been run by public bodies such as the pre-deregulation bus industry when in some areas the Passenger Transport Executives were the main operator. To represent such organisations we will use a first-best welfare maximising social planner – who we define as an organisation that seeks to maximise total welfare with no constraints. The first-best social planner optimal pricing rule is:

$$P = MC. \tag{3.44}$$

where P is price and MC is marginal cost. Although, this produces the maximum welfare we should be aware that it can lead to negative profits and thus may not be viable, such as when a social planner faces a large fixed cost.

3.10 Summary of Chapter

In this theoretical literature review we introduce a number of studies and techniques that will be useful when we consider the research questions we raise in Chapter 2; concerning integrated ticketing and monopoly network provision, with Part B considering the former and Part C the latter. Initially we introduced Cournot's (1838) models of

competition and complementary monopoly, and the effects observed in these two models constantly appear throughout the economic modelling literature we investigate. These observations will inevitably appear in our models in Parts B and C of this thesis.

We also presented the models of Stackelberg and Spengler, both model firm interactions when a firm has a first mover advantage, and we will introduce such an extension of our integrated ticketing model in Section 5.5. Another model that we found particularly useful in our review of is that of Economides and Salop, who extend Cournot's analyse of competition and complementary monopoly in a simple network model. In Chapter 4 we will apply this model to the transport industry before using the Economides and Salop demand system further when we model integrated ticketing in Chapter 5. After we look at Economides and Salop model we consider the conjectural variations approach, which is another modelling concept related to Cournot. This is an approach we will use to extend Chapter 5's integrated ticketing model and we will consider this in Chapter 6.

We look at the model of Economides and Woroch and this type of modelling, along with the hub-and-spoke modelling we introduce, is useful for when we look at monopoly network interconnection in Part C. As this thesis is primarily concerned with public transport we look at the models of James (1998), James and Else (1995), and McHardy and Trotter (2006). These studies suggest ways of specifically modelling transport demands and regimes types that are related to transport industries, and will be useful in the models of both Parts B and C. Parts B and C will require consideration of the best course of action for society, in addition to considering the firms' preferences using profit, so we propose ways of calculating overall welfare.

PART B

INTEGRATED TICKETING

CHAPTER FOUR

AN INTERPRETATION OF INTEGRATED TICKETING

4.1. Introduction

In Chapter 2, we saw that one policy being discussed to improve public transport was that of the increased use of inter-available and integrated ticketing. It is hoped that such a ticketing reform could improve flexibility of public transport and reduce the generalised cost of the mode. This improvement in public transport could then offset some of the results of deregulation, which in some cases have led to the disintegration of services and reduced service flexibility. However, there is a danger that the various integrated ticketing schemes could encourage collusion, so in Part B of the thesis, beginning in this chapter, we seek clarify the effects that the forms of integrated ticketing could have on fares, patronage, and profits.

As some of the integrated ticketing schemes can become complicated, particularly when more than one firm is involved, we begin in this chapter by adapting the ES model to a transport network. This allows us (and the reader) to explore the simplest set-up of the model and gain an elementary understanding before we build a more complex model to explore the issues we highlight; thus this chapter focuses, specifically, on the suitability of the model for, and how the model can be adapted to best represent, the transport industry.

In the following section we establish the *ES* model, which we introduced during Chapter 3, in a transport context. Subsequent sections consider the equilibrium price, quantity, and profit outcomes on the network under the network regimes: joint ownership, independent ownership, composite good competition, parallel vertical integration, and

optimal regulation. Finally, in Section 4.8, we conclude the results of the various regime

comparisons, and evaluate the model.

4.2. The Model

Let us consider the *ES* model from Section 3.3 in terms of a single-route transport

system. We assume the services are differentiated not by branding, but by the time of

travel. Travellers make return journeys, so that they have a composite demand for

travel, Q_{ij}. There are no travellers who wish to travel only in one direction and there are no

single tickets. Therefore, component A becomes the outward leg of a journey, for

simplicity, let there be two distinct outward services, O_i ($i = 1, 2$). Component B is now the

inward leg and, again, there are two inward services, I_j ($j = 1, 2$), with fares p_i and q_j,

respectively.

For the purpose of characterising specific demands, Q_{ij}, we refer to the round-trip

price, P_{ij}.

$$P_{ij} = p_i + q_j. \tag{4.1}$$

If we now simplify the *ES* system of demands assuming that β is own-price

coefficient and δ is the cross price coefficient across all alternative services to *ij*. All the

alternative services are equally good substitutes[1] for all services so that Q_{ij}:

$$Q_{ij} = \alpha - \beta P_{ij} + \sum_{mn \neq ij} \delta P_{mn}. \tag{4.2}$$

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[1] It is important to recognise that δ indicates the degree to which services are differentiated and might realistically be expected to feature as a strategic choice variable of a firm rather then be parametric as it is here.

A corollary of this symmetry is that any benefits of the integrated ticket, in terms of improved flexibility of travel, are ignored. Similar to the *ES* model the effect of the round trip's own price is negative, as $\beta > 0$, so the round trips are substitutes.

The symmetry our assumptions introduce means that the total demand and welfare can be calculated. We also assume the cross-price co-efficient to be positive and unitary across all alternative services to *ij* and this aids us when calculating welfare. We maintain the *ES* set-up, so that we have a system of gross substitutes and this means that we require that:

$$\beta > 3. \tag{4.3}$$

To aid tractability, and without loss of generality, we now normalise the framework with the parameterisation:

$$\delta = 1. \tag{4.4}$$

4.3. Joint Ownership

We saw in Chapter 2 that following bus deregulation and train privatisation that it was possible that there could be a monopoly operator along bus and train routes. It is also possible that the introduction of integrated ticketing will allow public transport operators to collude over the setting of ticket prices. If we make the strong assumption that integrated ticketing leads to perfect price collusion then we can use the joint ownership regime as a loose approximation of an integrated ticketing regime.

We are aware integrated ticketing may not lead to perfect collusion and, even then, regulators may seek to place some bounds on the collusive arrangements of integrated ticketing – the forms and levels of collusion that result from integrated ticketing is considered in Chapter 5 and the extension in Chapter 6. However, for now we shall use

joint ownership as an approximation of an integrated ticketing regime and compare its outcomes with the other regimes to see whether an integrated ticketing regime would be of benefit to firms, society, or both.

The transport operator is a simple network monopolist, who maximises profits of the general form:

$$\Pi^J = \sum p_i Q_{ij} + \sum q_i Q_{ij} .$$
(4.5)

ES's assumption that the marginal cost (per passenger) is zero is not an entirely unreasonable assumption in the context of transport, when the are no capacity constraints – as this means an extra passenger does not result in the provision of another carriage or service and the resulting increase in cost.

Substituting (4.2) into (4.5) and maximising with respect to p_i and q_i yields the equilibrium prices for round-trip journeys as the first-order conditions are not independent:

$$P^J = p^J + q^J = \frac{\alpha}{2(\beta - 3)} .$$
(4.6)

Substituting (4.6) into the relevant demand equation (4.2) yields the equilibrium quantities:

$$Q_{ij}^J = \frac{\alpha}{2} .$$
(4.7)

Substituting (4.6) and (4.7) into (4.5) gives us total industry profit:

$$\Pi^J = \frac{\alpha^2}{\beta - 3} .$$
(4.8)

In Section 3.8, we presented a method of calculating consumer surplus and we use that method here so total consumer surplus is:

$$CS = \frac{1}{2}\left(\frac{4\alpha}{\beta - 3} - P\right)Q^T .$$
(4.9)

Summing across quantities and substituting this along with (4.6) into (4.9) gives:

$$CS^J = \frac{7\alpha^2}{2(\beta-3)}.$$

(4.10)

In Section 3.8, we defined total welfare as profit and consumer surplus so welfare can be calculated using:

$$W^J = \text{Profit} + CS.$$

(4.11)

Using (4.8) and (4.10) in (4.11) we can now calculate total welfare arising from the Joint Ownership regime:

$$W^J = \frac{9\alpha^2}{2(\beta-3)}.$$

(4.12)

4.4. Independent Ownership

In this regime each of the components or services is operated by a separate transport company. One firm operates O_1, a second firm operates O_2, a third firm operates I_1, and a fourth firm operates I_2. This is a little unlikely in transport as for a vehicle to be able to make the same outward journey the next day it will have to make an inward journey to get back to its origin, whether travellers are using it or not – so as this inward journey has to be made it would make financial sense for travellers to be accepted if they pay more than the marginal cost they impose. However, this does make a useful benchmark case to compare to Cournot's (1838) complementary monopoly model, although the regimes in future chapters should be more applicable to the public transport industry.

Profit for firms i and j are given in general terms by:

$$\pi_{O_i} = \sum p_i Q^{ij},$$

(4.13a)

$$\pi_{I_j} = \sum q_j Q^{ij}.$$

(4.13b)

Substituting (4.2) into (4.13a) and (4.13b) and maximising with respect to p_i and q_i yields the equilibrium prices:

$$p^I = \frac{\alpha}{3\beta - 7},$$ (4.14a)

$$q^I = \frac{\alpha}{3\beta - 7},$$ (4.14b)

$$P^I = \frac{2\alpha}{3\beta - 7}.$$ (4.14c)

To ensure (4.14) is positive the denominator needs to be greater than zero and this is clearly the case given (4.3). Substituting (4.14) into (4.13) and summing across firms gives total industry profit:

$$\Pi^I = \frac{8\alpha^2 (\beta - 1)}{(3\beta - 7)^2}.$$ (4.15)

Summing across quantities and substituting this, and (4.14c) into (4.10) gives:

$$CS^I = \frac{4\alpha^2 (5\beta - 11)(\beta - 2)}{(\beta - 3)(3\beta - 7)^2}.$$ (4.16)

Finally, using (4.15) and (4.16) in (4.11) we have equilibrium welfare for the independent ownership regime:

$$W^I = \frac{(7\beta - 17)4\alpha^2 (\beta - 2)}{(\beta - 3)(3\beta - 7)^2}.$$ (4.17)

A comparison between the independent ownership regime and the joint ownership regime using equilibrium profits and welfare leads us to Proposition 4.1:

Proposition 4.1. *(i) The firms prefer a joint ownership regime to an independent ownership regime:* $\Pi^J > \Pi^I$. *(ii) Society prefers a joint ownership regime to an independent ownership regime:* $W^J > W^I$.

The joint ownership regime internalises the vertical externalities that result in the complementarities in the independent ownership regime causing higher prices, so that profits and welfares are lower in independent ownership. This follows almost directly from Cournot's observations on competition and on complementary monopoly that we presented in Chapter 3.

Proposition 4.1 follows *ES* Proposition 1 that is seen in Chapter 3. *ES* Proposition 1 finds that prices in joint ownership are higher only when the composite goods are close substitutes. Our normalisation of the demand system, so that $\delta = 1$ and imposing $\beta > 3$ ensure the services are never good enough substitutes to result in the existence of a horizontal externality, so prices in a joint ownership system are never higher than a independent ownership system.

4.5 Composite Good Competition

This is where each firm operates one of the return journeys, so there will be four firms each operating one of route pairs; $O_1 I_1$, $O_1 I_2$, $O_2 I_1$, and $O_2 I_2$. This seems to be a reasonable structure for a transport system, and can probably be witnessed on both bus and train routes. Each transport operator would run an inward and outward journey with departure times differing between the operator's services. This structure implies a restriction upon the traveller's freedom; the traveller must get an inward vehicle from the same company that provided them with the outward journey.[2] This is clearly not a form of integrated ticket and is the type of ticketing that the Government may be attempting to discourage.

The profit for firm *m* is given by:

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[2] This structure may provide further product differentiation but that is not a concern of this chapter.

$$\Pi_m = r_{ij} Q^{ij}. \tag{4.18}$$

Substituting (4.2) into (4.18) and maximising with respect to r_{ij} yields composite price:

$$P^C = \frac{\alpha}{2\beta - 3}. \tag{4.19}$$

Substituting (4.19) into (4.2), yields the following equilibrium expression for quantity:

$$Q^C = \frac{\alpha\beta}{2\beta - 3}. \tag{4.20}$$

Substituting (4.19) and (4.20) into (4.18) gives us total industry profits:

$$\Pi^C = \frac{4\alpha^2 \beta}{(2\beta - 3)^2}. \tag{4.21}$$

Substituting total quantity and (4.19) into (4.8) gives:

$$CS^C = \frac{2\alpha^2 (7\beta - 9)\beta}{(\beta - 3)(2\beta - 3)^2}. \tag{4.22}$$

Using (4.21), (4.22) in (4.11) we have equilibrium welfare under the composite good competition regime:

$$W^C = \frac{6\alpha^2 (3\beta - 5)\beta}{(\beta - 3)(2\beta - 3)^2}. \tag{4.23}$$

A comparison between the composite good competition regime and the joint ownership regime using equilibrium profits and welfare leads us to Proposition 4.2:

Proposition 4.2. *(i) The firms prefer a joint ownership regime to a composite good competition regime: $\Pi^J > \Pi^C$. (ii) Society prefers a composite good competition regime to a joint ownership regime: $W^C > W^J$.*

The splitting of the regime into a composite good competition results in the existence of non-internalised substitutibilities, leading to horizontal externalities, that result in the regime producing prices that are below those found in the joint ownership regime. These

lower fares mean that profits are lower and welfare is higher in the composite good

competition compared to joint ownership regimes.

4.6 Parallel Vertical Integration

In parallel vertical integration the services O_1 and I_1 are run by one transport operator

whilst O_2 and I_2 are both produced by another operator. We should make it clear that both

firm's services continue to be compatible, so the passenger can use a combination of all

services at no extra cost. In Chapter 2 we see that bus deregulation and rail privatisation

means that it is possible to find two operators on a single network in UK bus and rail

industries.

The general form of the profit function for firm m is given by:

$$\Pi_m^V = p_i \sum_{i,j=1,2} Q^{ij} + q_i \sum_{i,j=1,2} Q^{ij} . \tag{4.24}$$

Substituting in (4.2) and maximising (4.24) with respect to p_i and q_i yields the equilibrium

prices:

$$p^V = \frac{2\alpha}{(7\beta - 17)}, \tag{4.25a}$$

$$q^V = \frac{2\alpha}{(7\beta - 17)}, \tag{4.25b}$$

$$P^V = \frac{4\alpha}{(7\beta - 17)}. \tag{4.25c}$$

When $\beta > 3$ the condition for non-negative pricing is met (i.e. the denominator is greater

than zero).

Substituting (4.25c) into (4.2), yields the following expression for equilibrium

quantity:

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$$Q^V = \frac{\alpha(3\beta - 5)}{(7\beta^2 - 17)}.$$ (4.26)

Using (4.25c) and (4.26) in (4.24) before summing across firms gives total industry profit:

$$\Pi^V = \frac{16\alpha^2(3\beta - 5)}{(7\beta - 17)^2}.$$ (4.27)

Substituting (4.25c) and total quantity into (4.9) gives:

$$CS^V = \frac{16\alpha^2(3\beta - 7)(3\beta - 5)}{(\beta - 3)(7\beta - 17)^2}$$ (4.28)

Using (4.27) and (4.28) in (4.11) gives the total equilibrium welfare under the vertically integrated regime:

$$W^V = \frac{32\alpha^2(3\beta - 5)(2\beta - 5)}{(\beta - 3)(7\beta - 17)^2}.$$ (4.29)

A comparison between the parallel vertical integration regime and the network joint ownership regime using equilibrium profits and welfare leads us to Proposition 4.3:

Proposition 4.3. *(i) The firms prefer a joint ownership regime to a parallel vertical integration regime:* $\Pi^J > \Pi^V$. *(ii) Society prefers a joint ownership regime to a parallel vertical integration regime when $\beta > 7$: $W^J > W^V$ when $\beta > 7$.*

The splitting of the joint ownership regime into a parallel vertical integration regime gives rise to both substitutibilities and complementarities to result in prices being either higher or lower than the joint ownership regime depending on which dominates. Profit are below those from joint ownership, but the effect on welfare is ambiguous – if β is low then substitutabilities dominate to give lower prices and higher welfares in the parallel vertical integration regime. However, if β is high, then the complementarities dominate, and give higher prices and lower welfares in the parallel vertical integration regime.

Proposition 4.3 follows from ES' Proposition 3 that was seen in Chapter 3 and states that prices in joint ownership are higher than in parallel vertical integration regime only when the composite goods are close substitutes, although we compare welfare rather than prices.

4.7. Optimal Regulation

Let us introduce the possibility of a transport operator maximising the total welfare on the network. The case of the optimal regulation within this model is somewhat trivial. The first best welfare solution is for price to be set equal to marginal cost, this marginal cost is zero and thus prices are zero:

$$p^{OR} = q^{OR} = P^{OR} = 0 . \tag{4.30}$$

Substituting (4.30) into (4.2) yields the demand:

$$Q^{OR} = \alpha . \tag{4.31}$$

As the optimal regulation is charging a price of zero the total profit becomes:

$$\Pi^{OR} = 0 . \tag{4.32}$$

Summing across quantities and substituting this and (4.32) into (4.9) gives:

$$CS^{OR} = \frac{8\alpha^2}{(\beta - 3)} . \tag{4.33}$$

Using (4.32) and (4.33) in (4.11) gives the total welfare under the optimal regulation regime:

$$W^{OR} = \frac{8\alpha^2}{(\beta - 3)} . \tag{4.34}$$

A comparison between the optimal regulation regime and the network joint ownership regime using equilibrium profits and welfare leads us to Proposition 4.4:

Proposition 4.4. *(i) The firms prefer a joint ownership regime to an optimal regulation regime:* $\Pi^J > \Pi^{OR}$. *(ii) Society prefers an optimal regulation regime to a joint ownership regime:* $W^{OR} > W^J$.

This is a sensible result as the optimal regulation regime by definition maximises welfare over the whole demands of the regime whereas the joint ownership maximises profit.

4.8. Conclusion

The main conclusion we draw from using the *ES* model in a transport application along with some simplifying assumptions is one of caution regarding the results. However, the focus of this chapter was with the understanding that the intuition behind the model and to highlight content for the following chapters in Part B – this is where we will begin our conclusion.

Firstly, we use the joint ownership regime as a simple proxy for integrated ticketing. On a basic level this approximation may seem reasonable as the introduction of integrated ticketing is likely to encourage some collusion, but it is unlikely to result in the firms colluding perfectly. The model used in this chapter clearly does not allow for, the variety of, revenue splitting arrangements, or pricing agreements, which may arise as a result of allowing integrated ticketing. Indeed, the possible collusive effects concerning the level of collusion and type provide a promising avenue for development of the model. Whatever regime type involving the various agreements should be appropriate in the context of public transport.

Secondly, consumers in an integrated ticket scheme may behave differently compared to those in a non-integrated ticket situation, so we will need to ensure we account for this in

the demand system we introduce. It is likely that we will need to introduce one demand system type for integrated ticketing regimes and another for non-integrated ticketing regimes. Additionally, the regime types that we use in the model should be appropriate in the context of public transport.

We also introduce some simplifying assumptions to the *ES* model in this chapter and they make the calculation of welfare possible, unlike the original *ES* model where comparisons of total quantity and average price had to be made. These assumptions should be useful in future chapters to ensure that the models we propose are as tractable as possible.

A quick look at the results of this model show that encouraging integrated ticketing, as represented by a joint ownership regime, may not necessarily bring about beneficial results for society. The joint ownership regime, despite always being preferred by the firms, only provides a welfare that is superior to the parallel vertical integration regime and this is dependent on β being large. In terms of this model it suggests that integrated ticketing may not be a policy that is best for society and that it may not lead to a fall in the generalised cost of public transport use. Of course, we again sound a note of caution regarding these results as the model is not entirely ideal. However, the basic demand framework shows promise, and by ensuring the problems are rectified the model we will consider in the following chapter will be representative of transport and integrated ticketing.

CHAPTER FIVE

THE ECONOMICS OF INTEGRATED TICKETING

5.1. Introduction

In this part of the thesis, we undertake a theoretical exploration into the issue of

integrated ticketing on a transport network with a view to reducing the generalised cost of

public transport and so promoting its use. In Chapter 4 we used the *ES* model that we

introduced and applied to a transport network scenario in Chapter 3. We found that the

model was not entirely capable of representing an integrated ticketing public transport

structure. However, the demand system, along with the simplifications we made, could be

used to produce a sensible and tractable model.

We now develop the basic *ES* model to generate a framework, which has direct

applicability to the. We are interested in understanding how different integrated ticketing

policies may help make public transport more attractive, but without removing the benefits

of competition. More specifically, we shall introduce two main demand structures; one

demand structure, when an integrated ticket is provided, and another, when no integrated

ticket is provided. These demand structures are consistent to allow meaningful comparisons

between the regime types in terms of profits, prices, and quantities.

We begin by considering the effects that the introduction of integrated ticketing has

on a monopoly regime before moving on to look at the case of network duopolists. When

integrated ticketing is introduced into a duopoly regime there become a number of ways in

which the integrated ticket price can be decided on: from the regulator insisting on

independent pricing to allowing the firms to agree on prices using some kind of price rule.

It is also possible that the decision of the two firms regarding the integrated ticket price

may be made before they set their stand-alone prices. We model such situations and compare between integrated ticket regimes and with non-integrated ticketing regimes.

In the following section we set out the modified version of *ES*. We also suggest a method of comparing different regime outcomes from a social perspective. In Section 5.3 we consider the case of the network monopolist; deriving the profit maximising values of price and quantity to determine the circumstances under which a monopolist may or may not choose to offer an integrated ticket, and also whether society agrees. In Section 5.4 we derive a model of a simultaneous symmetric network duopoly; calculate profit maximising values of price and quantity for several duopoly regime types, and examine the scenarios that the firms or society prefer. In Section 5.5, we develop the previous section's symmetric network duopoly to include sequential decision-making and compare this with the previously derived regime outcomes. In Section 5.6, we take a more detailed look at the comparisons between the regimes to ascertain the level of improvement that the various regimes may give in terms of profit or welfare. Finally, in Section 5.7, we conclude our findings, make recommendations concerning integrated ticketing, evaluate the model, and consider possible extensions for the next two chapters.

5.2. Integrated Ticketing

Let us consider a single-route transport system, which faces demands for travel that are differentiated. For the purpose of characterising specific demands, Q_{ij}, we refer to the round-trip price, P_{ij}. For completion purposes let us once again show that the demand Q_{ij} is linear in its own price and also in the round-trip prices of all other possible service combinations:

$$Q_{ij} = \alpha - \beta P_{ij} + \sum_{mn \neq ij} \delta P_{mn} \,. \tag{5.1}$$

All the appropriate caveats and explanations from Chapter 4 still apply.

In Chapter 4 we consider the monopoly regime to be a proxy for an integrated ticketing system, but we find it to be unrealistic and over-simplified. This meant that the usefulness of any of the previous chapter's results is limited as the demand system remains the same, whether an integrated ticket is provided or not. As we expect the travellers to react differently to integrated tickets than to a non-integrated ticketing system, we need to represent the demand system when an integrated ticket is, and when it is not available, so that we can compare the two situations.

Unlike the previous chapter, it is necessary to specify the options available to those preferring cross-service travel in the absence of an integrated ticket option. One possibility would be to introduce single tickets into the model, but this would result in intractability. Therefore the benchmark we adopt, when no integrated ticket option is available, is that passengers wishing to travel across the services must purchase a round-trip ticket for each stage of the journey; one round-trip ticket for the outbound service and one round-trip ticket for another inbound service, hence the price of travelling across the service (P_{mn}^1) is:

$$P_{mn}^1 = P_{mm} + P_{nn}, \tag{5.2}$$

where P_{mm} is the price of ticket using a round-trip service mm and P_{nn} is the price of the ticket using service nn. This assumption allows us to assume away single tickets. Although assuming that travel cross-service involves the purchase of two round-trip tickets, one for each service that is travelled on, may at first seem an extreme one as normally a traveller could in reality buy two single tickets. However, as we saw in Chapter 2, there is some evidence to suggest that a one-way (or single) ticket is indeed approximately equal in price to a round-trip (or return) ticket so that (5.2) could be empirically supported.

Using (5.2) the demands become:

$$Q^1_{mm} = \alpha - \beta P_{mm} + \delta P_{nn} + 2\delta(P_{mm} + P_{nn}), \tag{5.3a}$$

$$Q^1_{mn} = \alpha - (\beta - \delta)(P_{mm} + P_{nn}) + \delta(P_{mm} + P_{nn}). \tag{5.3b}$$

Given (5.3b), the following restriction is required to ensure a system of gross substitutes:[1]

$$\beta > 5\delta. \tag{5.4}$$

In Chapter 2, we saw that not all industries' return tickets are twice the price of a single ticket and it is possible that single tickets are available for less than a return ticket. Although, no single tickets are offered in the model we can alter our non-integrating ticketing demand system to give a structure that approximates this. Therefore, we also include another non-integrating ticket scenario where the purchase of a round-trip ticket for service *mm* and a round-trip ticket for service *nn* scenario comes with a 25% discount. Under this assumption the price of travelling across the services (P^2_{mn}) is:

$$P^2_{mn} = \frac{3}{4}(P_{mm} + P_{nn}). \tag{5.5}$$

We may find that this represents a situation that we are more likely to find if we look at ticketing behaviour in the transport industry. The demand structure becomes:

$$Q^2_{mm} = \alpha - \beta P_{mm} + \delta P_{nn} + \frac{3}{2}\delta(P_{mm} + P_{nn}), \tag{5.6a}$$

$$Q^2_{mn} = \alpha - \frac{3}{4}(\beta - \delta)(P_{mm} + P_{nn}) + \delta(P_{mm} + P_{nn}). \tag{5.6b}$$

Given (5.6b), this means a new restriction is required to ensure a system of gross substitutes:

$$\beta > 4\delta. \tag{5.7}$$

As in Chapter 3 to aid tractability, and without loss of generality, we now normalise the framework with the parameterisation:

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[1] The reasons for this were explained in Chapter 3.

$$\delta = 1 .$$
(5.8)

This means that we are assuming that the cross-price elasticities are constant and fixed, but without this the model would become intractable. Theoretically, δ could be a function of β, as a large value of β means a high own price elasticity and this could be linked to high cross-price elasticities. This means that our model is somewhat restricted as even making δ a function of β in the model would make finding a solution problematic.

Let us now briefly turn our attention to costs. As the structure of the model, and thus the number of physical services, is constant over all regimes, and if we assume that the provision of integrated ticketing can be undertaken at no extra cost, as they state in DfT (2004b, Executive Summary, page ii), then we can assume that fixed costs will not impact on the results of the model. We therefore set fixed costs equal to zero. Further, given the marginal passenger costs for most public transport systems are very low, then for simplicity, we take them to be zero. Finally, since journey distance is not a consideration in the present context, we take marginal distance costs to be zero, too.[2]

The case of the welfare-maximising social planner is a trivial one. With zero marginal costs, a welfare-maximising social planner will set the price for each round trip equal to zero. However, throughout this analysis the reference to the first-best outcome is not always possible as given the inter-relationship between demands it is difficult to calculate surplus while maintaining tractability. To make recommendations for what society would prefer we need some indicator of welfare and this means we have to introduce a social planner's welfare function.

If, following a change in regime, all prices across the network moved in one direction whilst the quantities moved in the opposite direction, it would be straightforward to draw

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[2] Note that the marginal cost assumptions are especially plausible in the short run, when operators are committed to a given timetable irrespective of demand.

conclusions about the welfare superiority of one regime over another. Unfortunately, this will not always be the case. Nevertheless, given that one of the central motivations for this analysis is to identify regimes which help to increase the patronage of public transport, a regime that results in a higher total patronage across the network should be considered superior to one with a lower patronage. However, this is not the only variable that may result in an improvement from the perspective of a traveller, indeed a decrease in the average ticket price might also be a favourable indicator for a regime in itself. Using just price or quantity would be overly simplistic, but a mixture of the two would give additional, clarity and, depth to our insight into the preference of regimes. We propose using a combination of both total patronage levels and average prices to give us a social planner's preference function.

These assumptions are not too unrealistic as a patronage focus was taken when the maximisation of passenger-miles was adopted as a target by London Transport (see Glaister and Collings (1978) and the references therein). This is also put forward by Sir Peter Parker (1978), when Chairman of British Rail, in his 1978 Haldane Lecture. An "output-related profits levy," which would reward faster growth of output was one regulatory mechanism considered when British Telecom was privatised in 1984, and a (weighted) average price is the focus of the 'RPI–X' regulation. To some extent these are complementary objectives, but including them both in the social planner's objective function allows for instances where, for example, increased output is due to general economic growth and not to any action of transport operators. We can summarise this function by:

$$S(\tilde{Q}, \tilde{P}), \qquad\qquad S_{\tilde{Q}} > 0,\ S_{\tilde{Q}\tilde{Q}} < 0,\ S_{\tilde{P}} < 0,\ S_{\tilde{P}\tilde{P}} < 0, \qquad\qquad (5.9)$$

where $\tilde{Q}$ is the total patronage on the network and $\tilde{P}$ is the average (per passenger) fare. Subscripts denote partial derivatives.

Given the above discussion, we suggest that the weight on the former term would be strictly greater than that on the latter. We refer to a regime that improves both terms, $S(+, -)$, as *strictly* superior, whilst one regime is *weakly* superior to another regime if $S(+, +)$, i.e. total patronage increases, but (despite this) there is a rise in the average passenger cost. Conversely, a decrease in the average passenger cost should not dominate a decrease in total patronage, and we therefore describe such a regime, $S(-, -)$, as *weakly* inferior. This seems reasonable in a climate when we are seeking to decrease car use and increase public transport use[3]. Finally, a regime that has a lower patronage and higher average passenger cost, $S(-, +)$ is *strictly* inferior. Whilst we stress importance on total patronage we are keen to for average price to be part of the social planner's preference function to give clarity, depth and realism.

It would also be possible to look at the gains and losses from regime change by using the formula calculated from 0.5 (old demand + new demand), but we believe that the social planner's function gives us a more consistent and deeper consideration of the impact of the regime changes we intend to present. Having established this social planner framework let us move on to the specific regime discussions.

5.3. Network Monopoly

5.3.1 Network Monopoly Without Integrated Ticketing (1)

In this section we consider the equilibrium prices and outputs in a situation of network monopoly where all services are provided by a single profit-maximising firm. We examine three regimes: the network monopolist does not provide integrated ticketing (M1) and we assume (5.4), a network monopolist provides an integrated ticket (M2), and the

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[3] We acknowledge that those extra public transport users will not just be switching from car use.

network monopolist does not provide integrated ticketing (M3) assuming (5.7). Beginning with regime M1, the network monopolist's profit in general terms is given by:

$$\Pi^{M1} = \sum_{m=1,2} P_{mm} Q_{mm} + \sum_{m \neq n=1,2} P_{mn} Q_{mn} \ . \tag{5.10}$$

Substituting (5.3) in (5.10) and maximising with respect to P_{11} and P_{22} yields the following equilibrium prices for the single and cross-services, respectively:

$$P_{mm}^{M1} = \frac{3\alpha}{2(5\beta - 13)} , \tag{5.11a}$$

$$P_{mn}^{M1} = P_{mm}^{M1} + P_{nn}^{M1} = \frac{3\alpha}{(5\beta - 13)} \ . \tag{5.11b}$$

Substituting (5.11) into the relevant demand functions (5.3), yields the equilibrium quantities of single-service and cross-service journeys, respectively:

$$Q_{mm}^{M1} = \frac{\alpha(7\beta - 11)}{2(5\beta - 13)} , \tag{5.12a}$$

$$Q_{mn}^{M1} = \frac{\alpha(2\beta - 7)}{5\beta - 13} \ . \tag{5.12b}$$

Inspection of (5.12a) and (5.12b) shows that $Q_{mm}^{M1} > Q_{mn}^{M1}$, as we expect given the cross-service pricing rule (5.2).

Finally, using (5.11) and (5.12) in (5.10) gives:

$$\tilde{\Pi}^{M1} = \frac{9\alpha^2}{2(5\beta - 13)} \ . \tag{5.13}$$

To be able to apply the social planner's objective function that we introduce at the end of Section 5.2, we need to calculate the total patronage on the network ($\tilde{Q}$) and the average (per passenger) fare ($\tilde{P}$). They are calculated using:

$$\tilde{Q} = 2\left(Q_{mm} + Q_{mn}\right), \tag{5.14}$$

$$\tilde{P} = \frac{2}{\tilde{Q}} \left(P_{mm} Q_{mm} + P_{mn} Q_{mn} \right). \tag{5.15}$$

Substituting (5.12) in (5.14) and (5.11) in (5.15) yields:

$$\tilde{Q}^{M1} = \frac{\alpha \left(11\beta - 25 \right)}{\left(5\beta - 13 \right)}, \tag{5.16}$$

$$\tilde{P}^{M1} = \frac{9\alpha}{2 \left(11\beta - 25 \right)}. \tag{5.17}$$

5.3.2 Network Monopoly With Integrated Ticketing

We now consider how the monopoly equilibrium changes when integrated tickets are introduced and to do this we need to consider the demand structure when an integrated ticket is available. Integrated ticketing will allow cross-service travel without the need to purchase two separate round-trip tickets. Let P_x be the price for the integrated ticket so the relevant demand functions are now:

$$Q_{mm} = \alpha - \beta P_{mm} + P_{nn} + 2P_x, \tag{5.18a}$$

$$Q_{mn} = \alpha - \beta P_x + P_x + P_{mm} + P_{nn}. \tag{5.18b}$$

The network monopolist's profit, in general terms, is now given by:

$$\Pi^{M2} = \sum_{m=1,2} P_{mm} Q_{mm} + P_x \sum_{m \neq n = 1,2} Q_{mn}. \tag{5.19}$$

Substituting (5.18) in (5.19) and maximising with respect to P_{11}, P_{22}, and P_x yields the following equilibrium prices for the single-service and integrated ticket, respectively:

$$P_{mm}^{M2} = P_x^{M2} = \frac{\alpha}{2(\beta - 3)}. \tag{5.20}$$

The network monopolist does not discriminate on price across the different ticket types due to the symmetry of the model. Substituting (5.20) into (5.18), yields the following equilibrium expression for quantity demanded of each ticket type:

$$Q_{mm}^{M2} = Q_{mn}^{M2} = \frac{\alpha}{2}. \tag{5.21}$$

Using (5.20) and (5.21) in (5.19), we have the equilibrium profit under regime M2:

$$\tilde{\Pi}^{M2} = \frac{\alpha^2}{\beta - 3}. \tag{5.22}$$

We now use (5.21) and (5.22) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{M2} = 2\alpha, \tag{5.23}$$

$$\tilde{P}^{M2} = \frac{\alpha}{2(\beta - 3)}. \tag{5.24}$$

A comparison between M1 and M2 using equilibrium profits, average prices, and total quantities leads us to Proposition 5.1:

Proposition 5.1. *(i) The network monopolist always prefers the integrated ticketing regime M2 over regime M1: $\Pi^{M2} > \Pi^{M1}$. (ii) The social planner strictly prefers regime M1 over regime M2: $S(\tilde{Q}^{M1}, \tilde{P}^{M1}) \succ S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.*

5.3.3 Network Monopoly Without Integrated Ticketing (2)

Now let us consider the network monopolist who does not provide integrated tickets, but this time assuming that there is a 25% discount available for the purchase of two round-trip tickets, so the pricing rule is (5.5).

Profit is, again, given by (5.10) and then we substitute in (5.6) before maximising with respect to P_{11} and P_{22} to yield the following equilibrium prices for the single and cross-services, respectively:

$$P_{mm}^{M3} = \frac{5\alpha}{2(13\beta - 37)},$$ (5.25a)

$$P_{mn}^{M3} = \frac{3}{2} P_{mm}^{M3} = \frac{15\alpha}{2(13\beta - 37)}.$$ (5.25b)

Substituting (5.25) into the relevant demand functions (5.6) yields the equilibrium quantities of single-service and cross-service journeys, respectively:

$$Q_{mm}^{M3} = \frac{\alpha(8\beta - 17)}{(13\beta - 37)},$$ (5.26a)

$$Q_{mn}^{M3} = \frac{\alpha(11\beta - 39)}{2(13\beta - 37)}.$$ (5.26b)

Inspection of (5.26a) and (5.26b) shows that $Q_{mm}^{M3} > Q_{mn}^{M3}$, as we would expect given the cross-service pricing rule (5.5).

Using (5.25) and (5.26) in (5.5):

$$\tilde{\Pi}^{M3} = \frac{25\alpha^2}{2(13\beta - 37)}.$$ (5.27)

We now use (5.26) and (5.27) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{M3} = \frac{\alpha(27\beta - 73)}{(13\beta - 37)},$$ (5.28)

$$\tilde{P}^{M3} = \frac{25\alpha}{2(27\beta - 73)}.$$ (5.29)

A comparison between M2 and M3 leads us to Proposition 5.2:

Proposition 5.2. *(i) The network monopolist always prefers the integrated ticketing regime M2 over regime M3:* $\Pi^{M2} > \Pi^{M3}$. *(ii) The social planner strictly prefers regime M3 over regime M2:* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succ S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

The rationale for Propositions 5.1 and 5.2 is straightforward: in the absence of an integrated ticket and given the "double price" or "one-and-a-half" cross-service penalty the network monopolist is forced to charge a very low fare on the single-service round trip in order to make profit. The "double price" or "one-and-a-half" effect penalises the network monopolist harshly against increasing the single-service price. Hence the non-integrated ticket regimes M1 and M3 have lower prices and profits, but higher quantities and welfare than the integrated ticketing regime M2.

5.4. Network Duopoly

In this section, we examine the effects of introducing strategic interaction in the model with a duopoly structure, where two separate firms run two services each. Firm *m* provides outward leg *m* and inward leg *m,* so that it provides a substitute single-service operation: firm *m* provides Q_{mm} ($m \neq n = 1,2$). As travellers are able to use whichever services they wish they can combine the services of the two firms. The firm *m* would thus provide the inward leg of cross-service operation Q_{mn} and the outward leg of cross-service operation Q_{nm} ($m \neq n = 1,2$).

We begin, as in Section 5.3, by considering a regime, D1, in which the duopolists do not provide cross-service tickets and the "price rule" is assumed to be (5.2). In regime D2, the duopolists are allowed to collude on a "price rule" for the integrated ticket price (not the actual ticket price), but no other collusion is allowed when setting single-service prices. In regime D3, the duopolists provide an integrated ticket and are required to set the price for

their component of the integrated ticket independently with no collusion allowed. In regime D4 no integrated ticket is available and the "price rule" is assumed to be (5.5). Finally, in regime I1 there is integrated ticketing and firms can collude in setting the integrated ticket price, but the firms are required to set it in advance of setting their own single-service tickets.

5.4.1 Network Duopoly Without Integrated Ticketing (1)

The relevant demands for regime D1 follow from (5.3), with firm i setting P_{mm} ($m \neq n = 1, 2$). Profit for firm m is given in general terms by:

$$\Pi_m^{D1} = P_{mm}Q_{mm} + (P_{mm} + P_{nn})Q_{mn}, \qquad (m \neq n = 1, 2). \qquad (5.30)$$

Maximising (5.30) with respect to P_{mm}, yields the following expression for the equilibrium duopoly price:

$$P_{mm}^{D1} = \frac{3\alpha}{8\beta - 19}, \qquad (m \neq n = 1, 2), \qquad (5.31a)$$

$$P_{mn}^{D1} = \frac{6\alpha}{8\beta - 19}. \qquad (5.31b)$$

Substituting (5.31) into (5.3), yields, respectively, the equilibrium quantities demanded of the single and cross-services:

$$Q_{mm}^{D1} = \frac{\alpha(5\beta - 4)}{8\beta - 19}, \qquad (5.32a)$$

$$Q_{mn}^{D1} = \frac{\alpha(2\beta - 7)}{8\beta - 19} \qquad (m \neq n = 1, 2). \qquad (5.32b)$$

Again, as would be expected, $Q_{mm}^{D1} > Q_{mn}^{D1}$.

Substituting (5.31) and (5.32) into (5.30) and summing over both firms, aggregate profit in regime D1 is:

$$\tilde{\Pi}^{D1} = \frac{54\alpha^2(\beta-2)}{(8\beta-19)^2}. \tag{5.33}$$

We now use (5.32) and (5.33) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{D1} = \frac{2\alpha(7\beta-11)}{(8\beta-19)}, \tag{5.34}$$

$$\tilde{P}^{D1} = \frac{27\alpha(\beta-2)}{(8\beta-19)(7\beta-11)}. \tag{5.35}$$

A comparison M1 and leads us to Proposition 5.3:

Proposition 5.3. *(i) The firms prefer regime M1 (joint profit maximisation) over regime D1: $\Pi^{M1} > \Pi^{D1}$. (ii) The social planner strictly prefers regime M1 over regime D1: $S(\tilde{Q}^{M1}, \tilde{P}^{M1}) \succ S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.*

Regime M1 internalises the complementarities between demands *mm* and *mn* that result in the vertical externalities in regime D1, which causes the non- integrated ticketing duopoly regime to price at the value above the monopoly integrated ticketing regime does. The monopoly regime is therefore able to maximise profit across the industry resulting in higher profits than the duopoly regime. This increased price in regime D1 also leads to lower quantities and, therefore, the social planner strictly prefers regime M1

If we compare M2 and D1 we find the total profit arising from monopoly regime M2 is greater than the total profit arising from duopoly regime D1. We can also see the total patronage arising from monopoly regime M2 is greater than the total patronage arising from duopoly regime D1 when $\beta > 8$. Additionally, the average price arising from monopoly regime M2 is greater than the average price arising from duopoly regime D1.

Hence when $\beta \leq 8$ the social planner strictly prefers D1 to M2, but when $\beta > 8$ the social planner weakly prefers M2 to D1.

5.4.2 Network Duopoly With Simultaneous Integrated Ticketing

We now introduce integrated ticketing into the duopoly model. In regime D2 an integrated ticket is made available, but firms are only able, or allowed (as a regulator may enforce such an arrangement), to collude on the price of this ticket by using a price rule, whilst they independently set their respective single-service prices. The general expression for profit on the cross-service operation is given by:[4]

$$\Pi_x^{D2} = P_x(Q_{mn} + Q_{nm}), \qquad\qquad (m \neq n). \qquad\qquad (5.36)$$

Substituting (5.18b) into (5.36) and maximising with respect to

$P_x (= P_{nm} = P_{mn}; m \neq n = 1,\, 2)$ yields the following expression for the integrated ticket price in terms of the single-service prices, P_{mm} ($m = 1,\, 2$):

$$P_x = \frac{\alpha + P_{mm} + P_{nn}}{2(\beta - 1)}. \qquad\qquad (5.37)$$

As the firms have agreed a rule for maximising joint profit on the cross-service travel using P_x (given the single-service ticket prices); each firm now chooses its single-service price by maximising its own profit independently taking (5.37) as given. Assuming each firm takes an equal share of the profits from the integrated ticket, the general expression for the profit of firm m is:

$$\Pi_m^{D2} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}), \qquad\qquad (m \neq n = 1,2). \qquad\qquad (5.38)$$

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[4] Note, given the equilibrium prices for the integrated ticket always exceed those for the single-service ticket, only passengers wishing to travel cross-service will purchase the integrated ticket: the two are synonymous in this model.

Maximising (5.38) with respect to P_{mm} and solving using (5.37) gives the following

equilibrium prices:

$$P_x^{D2} = \frac{\alpha(2\beta^2 - \beta - 2)}{2(2\beta^2 - 3\beta - 3)(\beta - 1)},$$ (5.39a)

$$P_{mm}^{D2} = \frac{\alpha(2\beta + 1)}{2(2\beta^2 - 3\beta - 3)}.$$ (5.39b)

Substituting (5.39) in (5.18), yields the equilibrium demands for single-service and cross-

service, respectively:

$$Q_{mm}^{D2} = \frac{\alpha(2\beta^3 - 3\beta^2 - 2\beta + 1)}{2(2\beta^3 - 5\beta^2 + 3)},$$ (5.40a)

$$Q_{mn}^{D2} = \frac{\alpha(2\beta^2 - \beta - 2)}{2(2\beta^2 - 3\beta - 3)}.$$ (5.40b)

Using (5.39) and (5.40) in (5.36) and summing over both firms, aggregate profit

across the network is:

$$\tilde{\Pi}^{D2} = \frac{\alpha^2(8\beta^4 - 8\beta^3 - 14\beta^2 + 4\beta + 5)}{2(2\beta^3 - 5\beta^2 + 3)(2\beta^2 - 3\beta - 3)}.$$ (5.41)

We now use (5.40) and (5.41) to form total patronage on the network ($\tilde{Q}$) and average (per

passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{D2} = \frac{\alpha(4\beta^3 - 6\beta^2 - 3\beta + 3)}{(2\beta^3 - 5\beta^2 + 3)},$$ (5.42)

$$\tilde{P}^{D2} = \frac{\alpha(8\beta^4 - 8\beta^3 - 14\beta^2 + 4\beta + 5)}{2(2\beta^2 - 3\beta - 3)(4\beta^3 - 6\beta^2 - 3\beta + 3)}.$$ (5.43)

If we compare M1 and D2 we find the total profit arising from duopoly regime D2 is

greater than the total profit arising from monopoly regime M1. We can also see the total

patronage arising from monopoly regime M1 is greater than the total patronage arising

from duopoly regime D2 if $\beta > 7.68$. Additionally, the average price arising from duopoly regime D2 is greater than the average price arising from monopoly regime M1. When $\beta < 7.68$ the social planner weakly prefers D2 to M1 but when $\beta > 7.68$ the social planner strictly prefers M1 to D2.

A comparison between D1 and D2 leads us to Proposition 5.4:

Proposition 5.4. *(i) The firms prefer regime D2 over regime D1:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D1}$. *(ii) The social planner weakly prefers regime D2 over regime D1:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succeq S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.

Regime D1 in the absence of an integrated ticket, and given the "double price" cross-service penalty, the non-integrated ticketing duopolists are forced to charge a very low fare on the single-service round trip in order to make profit. The "double price" effect penalises the non-integrated ticketing duopolists harshly against increasing the single-service price. Hence the non-integrated ticket regimes D1 has lower average prices and profits than the integrated ticketing regime D2. However, as D2 does not use a "double price" rule, but offers an integrated ticket for cross-service travel; resulting in a greater total patronage, which leads to the social planner preferring regime D2 over D1.

A comparison between M2 and D2 leads us to Proposition 5.5:

Proposition 5.5. *(i) The firms prefer regime M2 over regime D2:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D2}$. *(ii) The social planner strictly prefers regime D2 over regime M2:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succeq S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

The network monopoly or joint ownership regime internalises the horizontal externality that arises from the existence of substitutabilities in regime D2, which cause the integrated ticketing duopolists to charge lower prices than regime M2. The lower prices in regime D2 result in lower profits, but patronages are higher.

If we compare M3 and D2 we see that the total profit arising from duopoly regime D2 is greater than the total profit arising from monopoly regime M3 when $\beta > 4.89$. We can

also see the total patronage arising from duopoly regime D2 is greater than the total patronage arising from monopoly regime M3. Additionally, the average price arising from monopoly regime M3 is greater than the average price arising from duopoly regime D2. The social planner always strictly prefers duopoly regime D2 to monopoly regime M3.

5.4.3 Network Duopoly With Independently Priced Integrated Ticketing

We now introduce regime D3, where an integrated ticket is provided, but the duopolists are not allowed, perhaps by a regulatory ruling, to collude on any aspect of pricing in the network. What this amounts to is a situation of independent pricing on the two firms components of the cross-service ticket even though the integrated ticket is sold to the traveller as one ticket: each firm m sets the price of its component, P_{xm}, of the integrated ticket price. The integrated ticket price is the sum of these two component prices:

$$P_m = \sum P_{xm} \qquad\qquad (m = 1,\ 2). \qquad\qquad (5.44)$$

Given (5.44) the general expression for the profit of firm m is given by, $\Pi_m{}^{D3}$:

$$\Pi_m{}^{D3} = P_{mm}Q_{mm} + P_{xm}(Q_{mn} + Q_{nm}) \qquad\qquad (m \neq n = 1,2). \qquad\qquad (5.45)$$

Using (5.18) in (5.45) and maximising with respect to P_{xm} and P_{mm} for $m = 1,\ 2$ yields the following equilibrium expressions for the cross-service and single-service ticket prices, respectively:

$$P_x^{D3} = P_{x1}^{D3} + P_{x2}^{D3} = \frac{4\alpha}{3(2\beta - 5)}, \qquad\qquad (5.46a)$$

$$P_{mm}^{D3} = \frac{\alpha}{2\beta - 5}. \qquad\qquad (5.46b)$$

Using (5.46) in (5.18) yields the following equilibrium expressions for cross-service and single-service ticket prices, respectively:

$$Q_{mn}^{D3} = \frac{\alpha}{3}, \tag{5.47a}$$

$$Q_{mm}^{D3} = \frac{\alpha(3\beta-4)}{3(2\beta-5)}. \tag{5.47b}$$

Profit across the network then follows from substituting (5.46) and (5.47) into (5.45) and summing across the two firms:

$$\tilde{\Pi}^{D3} = \frac{2\alpha^2(17\beta-32)}{9(2\beta-5)^2}. \tag{5.48}$$

We now use (5.47) and (5.48) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{D3} = \frac{2\alpha(5\beta-9)}{3(2\beta-5)}, \tag{5.49}$$

$$\tilde{P}^{D3} = \frac{\alpha(17\beta-32)}{3(2\beta-5)(5\beta-9)}. \tag{5.50}$$

If we compare M1 and D3 we find the total profit arising from duopoly regime D3 is greater than the total profit arising from monopoly regime M1. We can also see the total patronage arising from monopoly regime M1 is greater than the total patronage arising from duopoly regime D3. The average price arising from duopoly regime D3 is greater than the average price arising from monopoly regime M1. The social planner always strictly prefers monopoly M1 to duopoly D3.

A comparison between D1 and D3 using leads us to Proposition 5.6:

Proposition 5.6. *(i) The firms prefer regime D3 over regime D1:* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{D1}$. *(ii) The social planner strictly prefers regime D1 over regime D3:* $S(\tilde{Q}^{D3}, \tilde{P}^{D3}) \succ S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.

The "double price" effect, in regime D1, penalises the non-integrated ticketing duopolists harshly if they increase the single-service price. Therefore, regime D1 has lower

prices and profits than the integrated ticketing regime D3, but due to the lower prices it has a higher, total patronage and welfare. A comparison between D3 and M2 leads us to Proposition 5.7:

Proposition 5.7. *(i) The firms prefer regime M2 over regime D3:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D3}$. *(ii) The social planner strictly (weakly) prefers regime M2 over regime D3 if* $\beta > 7.29$

$(6 < \beta \leq 7.29)$: $S(\tilde{Q}^{M2}, \tilde{P}^{M2}) \underset{(-)}{\succ} S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

When β is high then the complementarities in D3 dominate and give rise to vertical externalities, which are internalised in regime M2 and cause average prices in D3 to be above those in M2. These high prices result in lower profits and patronages than in regime M2. However, when β is low then the substitutibilities in D3 dominate and give rise to horizontal externalities – internalised in regime M2 – that cause average prices in D3 to be below those in M2. The low prices result in lower profits and higher patronages than in regime M2.

The total profit arising from duopoly regime D3 is greater than the total profit arising from monopoly regime M3. We can see the total patronage arising from monopoly regime M3 is greater than the total patronage arising from duopoly regime D3. The average price arising from monopoly regime D3 is greater than the average price arising from monopoly regime M3. When $\beta > 4.47$ then the social planner strictly prefers monopoly M3 to D3 otherwise the social planner weakly prefers D3 to M3.

A comparison D2 and the D3 leads us to Proposition 5.8:

Proposition 5.8. *(i) The firms prefer regime D2 over regime D3 if* $\beta > 6.51$, *otherwise regime D3 is preferred to D2:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D3}$ ($\tilde{\Pi}^{D2} \leq \tilde{\Pi}^{D3}$) *if* $\beta > 6.51$ ($\beta \leq 6.51$). *(ii) The social planner strictly prefers regime D2 over regime D3:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succ S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

When β is low, substitutabilities give rise to a horizontal externality that causes firms in D2 to decrease prices below that in D3 to result in lower profits and higher patronages for regime D2 when β is low. The complementarities that exist due to the separation of integrated ticketing prices in regime D3 are increasing with β, but the substitutibilities in regime D2 are decreasing with β. As β increases the vertical externality in D3 – resulting from the increased complementarities – increases, leading to higher prices, whilst the substitutibilities, and the reduced prices in D2, fall away. These relative price changes, when β increases, lead to profits in D2 being greater than D3 despite retaining lower overall prices and higher patronages.

5.4.4 Network Duopoly Without Integrated Ticketing (2)

Now let us consider another network duopoly that does not provide integrated tickets, but this time assuming that there is a 25% discount available for the purchase of two round-trip tickets so the pricing rule is (5.5). The relevant demands for regime D4 follow from (5.6), with firm i setting P_{mm} ($m \neq n = 1, 2$). The profit for firm m is given by:

$$\Pi_m^{D4} = P_{mm}Q_{mm} + \frac{3}{4}(P_{mm} + P_{nn})Q_{mn}, \qquad (m \neq n = 1, 2). \qquad (5.51)$$

Substituting (5.6) into (5.51) and maximising with respect to P_{mm} and P_{mm}, yields the following expression for the equilibrium duopoly price:

$$P_{mm}^{D4} = \frac{7\alpha}{(17\beta - 43)}, \qquad (m \neq n = 1, 2), \qquad (5.52a)$$

$$P_{mn}^{D4} = \frac{21\alpha}{2(17\beta - 43)}. \qquad (5.52b)$$

Substituting (5.52) into (5.3), yields, respectively, the equilibrium quantities demanded of the single and cross-services:

$$Q_{mm}^{D4} = \frac{5\alpha(2\beta - 3)}{(17\beta - 43)}, \tag{5.53a}$$

$$Q_{mn}^{D4} = \frac{\alpha(13\beta - 37)}{2(17\beta - 43)}, \qquad (m \neq n = 1, 2). \tag{5.53b}$$

Again, as would be expected, $Q_{mm}^{D4} > Q_{mn}^{D4}$.

Finally, substituting (5.52) and (5.53) into (5.51) and summing over both firms, aggregate profit in regime D1 is:

$$\tilde{\Pi}^{D4} = \frac{7\alpha^2(79\beta - 171)}{2(17\beta - 43)^2}. \tag{5.54}$$

We now use (5.53) and (5.54) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{D4} = \frac{\alpha(33\beta - 67)}{(17\beta - 43)}, \tag{5.55}$$

$$\tilde{P}^{D4} = \frac{7\alpha(79\beta - 171)}{2(17\beta - 43)(33\beta - 67)}. \tag{5.56}$$

If we compare M2 and D4 we find the total profit arising from monopoly regime M2 is greater than the total profit arising from duopoly regime D4. We can see the total patronage arising from monopoly regime M2 is greater than the total profit arising from duopoly regime D4 when $\beta > 19$. The average price arising from monopoly regime M2 is greater than the average price arising from duopoly regime D4. When $\beta < 19$ the social planner strictly prefers D4 to M2 and when $\beta > 19$ then the social planner weakly prefers M2 to D4.

A comparison between M3 and D4 leads us to Proposition 5.12:

Proposition 5.9. *(i) Both firms prefer regime M3 over regime D4:* $\Pi^{M3} > \Pi^{D4}$. *(ii) The social planner strictly prefers regime M3 over regime D4 if* $\beta > 7.34$:

$$S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succ S(\tilde{Q}^{D4}, \tilde{P}^{D4})$$

In the duopoly regimes where no integrated ticket is provided there exists a complementarity between the demands for *mm* and *mn,* whilst a monopoly regime internalises the resulting vertical externality, and it leads to higher prices in the duopoly regime (compared to the monopoly regime). These higher prices in D4 lead to lower profits and when $\beta > 7.34$ lower patronages.

A comparison between D2 and D4 leads us to Proposition 5.10:

Proposition 5.10. *(i) The firms prefer regime D2 over regime D4 if* $\beta \geq 4.76$:

$\tilde{\Pi}^{D2} > \tilde{\Pi}^{D4}$ *if* $\beta \geq 4.76$. *(ii) The social planner strictly prefers regime D2 over regime D4:*

$$S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succ S(\tilde{Q}^{D4}, \tilde{P}^{D4}).$$

Regime D4 contains complementarities leading to vertical externalities that are internalised in the integrated ticket regime D2. These vertical externalities cause the non-integrated ticketing duopoly firms to raise their prices above those in D2 – a regime which also has a substitutibility that means prices are smaller – and results in lower patronages in D4 (compared to D2). Additionally, when $\beta \geq 4.76$ then the complementarities have such an effect that profits in D4 fall below those in D2, which are below those in D4 due to substitutibilities in D2 when β is low.

A comparison between D3 and D4 leads us to Proposition 5.11:

Proposition 5.11. *(i) The firms prefer regime D3 over regime D4:* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{D4}$. *(ii) The social planner strictly prefers regime D4 over regime D3:* $S(\tilde{Q}^{D4}, \tilde{P}^{D4}) \succ S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

The explanation for this follows from the explanation for Proposition 5.6.

5.5 Network Duopoly With Pre-Emptive Integrated Ticketing

Now, we introduce regime I1 where we again allow the duopolists to collude on the integrated ticket price, but, in contrast with regime D2, the integrated ticket price itself can now be set in advance of the firms making their choices about their own single-service ticket prices. This means that there will now be some sequential firm interaction, such as those that we saw in the models of Stackelberg and Spengler during Chapter 3. Regime I1 is in regulatory terms less restrictive that regime D2, but allows the firms to impose greater constraints on their own second-period behaviour.

If, in the first period, the firms set prices for the integrated ticket and aim to maximise total profit, given their expectation of their own (independent) behaviour, when setting period-two single-service ticket prices. This means that in stage-two the firms will each attempt to maximise their own profit by setting their own single-service price taking the integrated price as given. The relevant general expression for the profit of firm m is given by (5.38). Substituting (5.18) and maximising with respect to P_{mm} gives the following expression of firm m's optimal choice of P_{mm}, in terms of P_x and P_{nn}:

$$P_{mm} = \frac{\alpha + P_{nn} + 3P_x}{2\beta}.$$ (5.57)

Solving (5.57) simultaneously across the two firms, we have:

$$P_{mm} = P_{nn} = \frac{\alpha + 3P_x}{2\beta - 1}.$$ (5.58)

The equilibrium expression (5.58) is the reaction function of the firms indicating their profit maximising choice of P_{mm} in terms of P_x. Differentiating (5.58) with respect to P_x, we arrive at the following expression for the slope of the reaction function, γ:

$$\gamma = \frac{3}{2\beta - 1}.$$ (5.59)

The firms can now exploit their knowledge of their second-stage reaction to the first-stage price agreement, P_x, in order to commit themselves to a more 'collusive' second-stage price game via strategic pre-commitment through P_x. The first-stage problem is to identify the level of P_x that maximises joint profit across the network given (5.58). Profit across the network in general terms is given by:

$$\tilde{\Pi}^{I1} = P_{mm}Q_{mm} + P_{nn}Q_{nn} + P_x(Q_{mn} + Q_{nm}), \qquad (m \neq n = 1, 2). \qquad (5.60)$$

Substituting (5.18) in (5.60) and maximising with respect to P_x, recognising that P_{mm} is a function of P_x, through (5.58), we have:

$$P_x = \frac{\alpha(2\beta + 3)}{2(2\beta^2 - 3\beta - 5)}. \qquad (5.61a)$$

Substituting (5.61a) into (5.58) gives the equilibrium second-stage single-service price:

$$P_{mm} = \frac{\alpha(2\beta + 1)}{2(2\beta^2 - 3\beta - 5)}. \qquad (5.61b)$$

Using (5.61) in (5.18) yields the equilibrium levels of demand for the single-service and cross-service tickets in regime I1, respectively:

$$Q_{mn} = \frac{\alpha}{2}, \qquad (5.62a)$$

$$Q_{mm} = \frac{\alpha(2\beta - 3)}{2(2\beta - 5)}. \qquad (5.62b)$$

Aggregate profit across the network under this regime then follows from substitution of (5.61) and (5.62) into (5.60):

$$\tilde{\Pi}^{I1} = \frac{\alpha^2(4\beta^2 - 4\beta - 9)}{(2\beta^2 - 3\beta - 5)(2\beta - 5)}. \qquad (5.63)$$

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We now use (5.62) and (5.63) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{I1} = \frac{4\alpha(\beta - 2)}{(2\beta - 5)},$$

(5.64)

$$\tilde{P}^{I1} = \frac{\alpha(4\beta^2 - 4\beta - 9)}{4(2\beta^2 - 3\beta - 5)(\beta - 2)}.$$

(5.65)

A comparison between I1 and all the other regimes we have introduced (M1, M2, M3, D1, D2, D3, and D4) leads us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when* $\beta < 4.78$ *where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ ($n = 1, 3$) *and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ ($m = 1, 2, 3, 4$) *unless* $\beta < 4.78$ *when* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. *(ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, ($k = M1, D2$) *and* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

Regime D1 contains complementarities that lead to the existence of a vertical externality, which are internalised on I1 to result in regime D1 having higher prices than regime I1. These prices lead to D1 having lower profits and lower patronages than I1.

Integrated ticketing regime I1 sees the introduction of some strategic collusion between the two firms beyond that occurring which occurs in integrated ticketing regime D2. This increased collusion in regime I1 means more of the horizontal externality that results from the substitutibilities is internalised in regime I1 to lead to regime D2 having lower prices. These lower prices result in regime D2 having lower profits and higher patronages than regime I1.

Regime I1 contains substitutabilities that lead to the existence of a horizontal externality, but this horizontal externality is internalised in integrated ticketing regime M2.

This horizontal externality results in the integrated ticketing regimes I1 having lower prices than M2 and thus lowers profits but higher patronage.

There are complementarities in regime D3; not existing in I1 that lead to vertical externalities and result in higher prices, compared to I1, and mean that profits and patronages are higher in I1.

5.6 Comparisons

In previous sections we have simply been interested in finding the regimes that are preferred by the firms and the social planner, but in this section we will attempt to look at relative differences between the regimes. Expanding the profit and welfare rankings we establish with our propositions will give us further understanding of the differences between the regimes and whether any regulatory enforced change would be worthwhile for society, or subject to opposition from firms. The relative profits of the firms (in terms of M2), total patronages (in terms of M1), and average prices (in terms of M2) can be found in Figures 5.1, 5.2, and 5.3, respectively, for values above $\beta > 5$ as this is where all regimes are valid.

For simplicity we include all the regimes on the diagrams, but it should noted that we cannot fairly compare the results of M1 with M3 or D4, or M3 with M1 and D1 as they have different underlying assumptions.

In Propositions 5.1, 5.2, 5.4, 5.6, 5.12 we find that regime D1 has inferior profits compared to regimes M1, D2, D3, and I1, respectively and Figure 5.1 confirms that D1 offers by far the lowest profit. D1, when compared with M2 – a 17% lower profit for low values of β and a 26% lower profit for high levels of β. M1 offers a relatively low profit – see Propositions 5.1 and 5.12 – and this is due to similar reasons as D1's inferior profit as

no integrated ticket is offered; meaning that the firm loses surplus as it does not maximise profit over the full range of demands. The result of not offering an integrated ticket does lead to significantly lower average prices and higher total patronages making it an attractive proposition for any regulator compared to most other regimes. However, due to such low relative profits it would be unlikely that the monopolist would accept non-integrated ticketing regimes and they could take part in activities to ensure such a regime does not come to pass.

Figure 5.1 illustrates Propositions 5.1, 5.2, 5.5, 5.7, and 5.12 to show that M2 is the regime that offers the highest profit as none of the other regimes overtake it. However, regimes I1, D2, and D3 do offer reasonably high profit alternatives with the former two regimes giving profits very close to M2 at larger values of β. This could mean that depending on the welfares offered by I1 and D2 that they may give the regulator a viable alternative to the integrated ticketing monopolist (M2).

If we now consider total patronage, shown in Figure 5.2, we see that for low values of β then D2 provides the larger patronages with M1 second, whilst for higher values of β M1 offers the highest total patronage with D2 second. Other high patronages come from regimes M3 and I1, particularly for high values of β, where each regime comes with a less than a 10% drop in patronages compared to the highest, M1. One regime to be avoid if a regulator seeks high traveller numbers is that of D3, which has extremely low patronages whatever the values of β – also, neither D1 or M2 look particularly promising in terms of the number of passengers.

When considering welfare, patronage is only part of the story and, in line with our social planner welfare function, we also look at the relative rankings of average prices between the regimes. It is the non-integrated ticketing regime M1 that offers by far the

Figure 5.1 Total Firm Profit By Regime Relative to Regime M2 (M2=1)
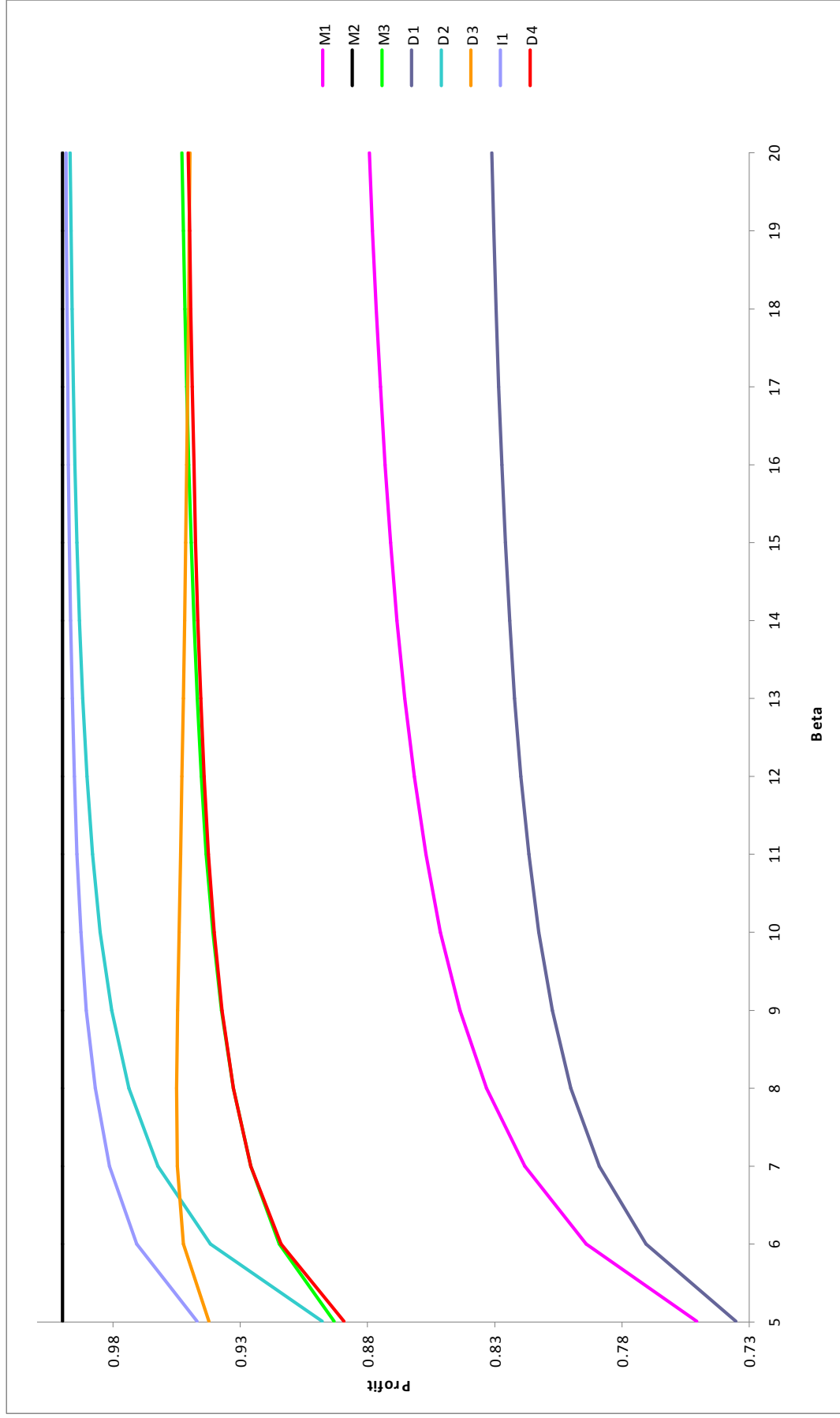
Figure 5.2 Total Patronage By Regime Relative to Regime M1 (M1=1)
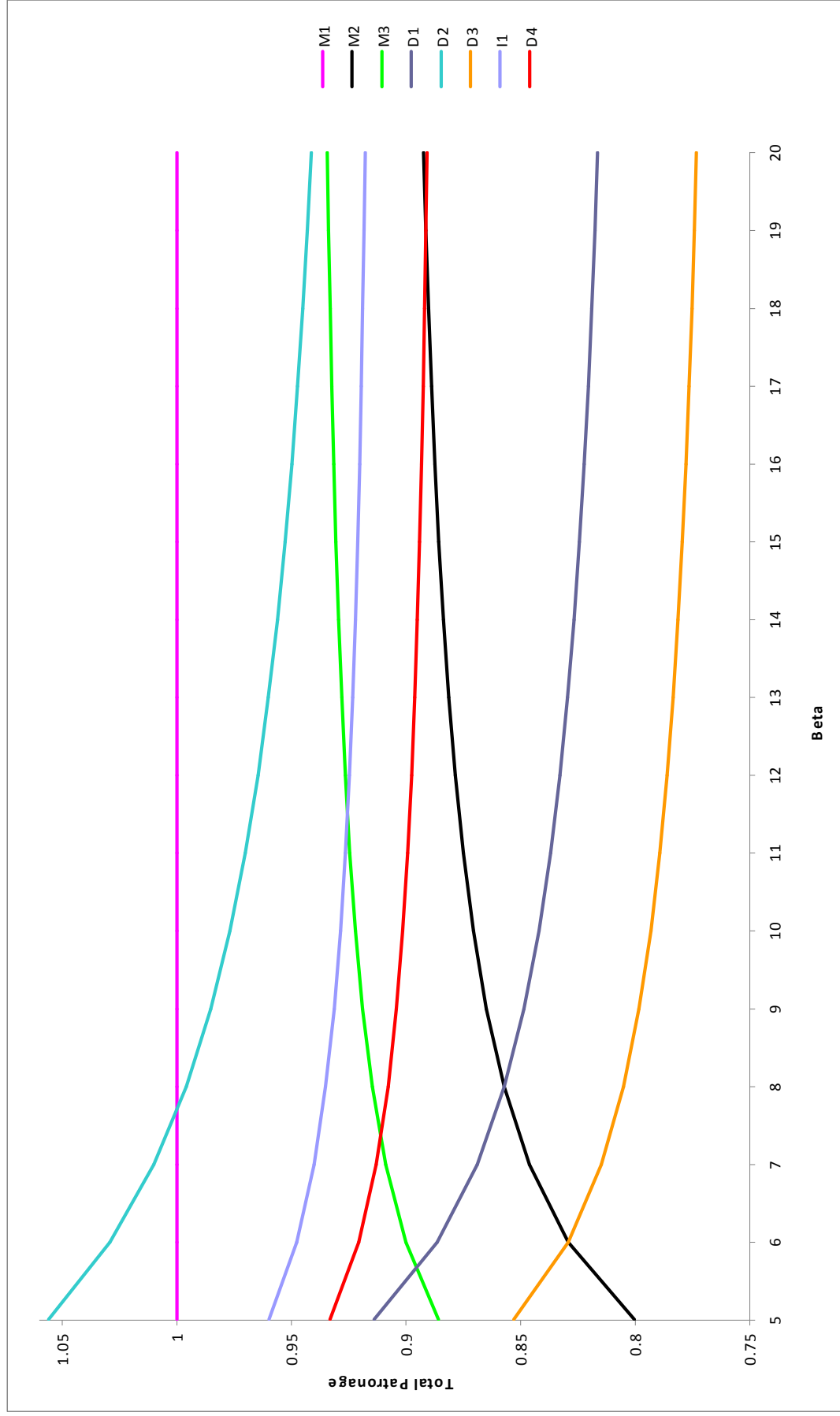
Figure 5.3 Average Price By Regime Relative to Regime M2 (M2=1)

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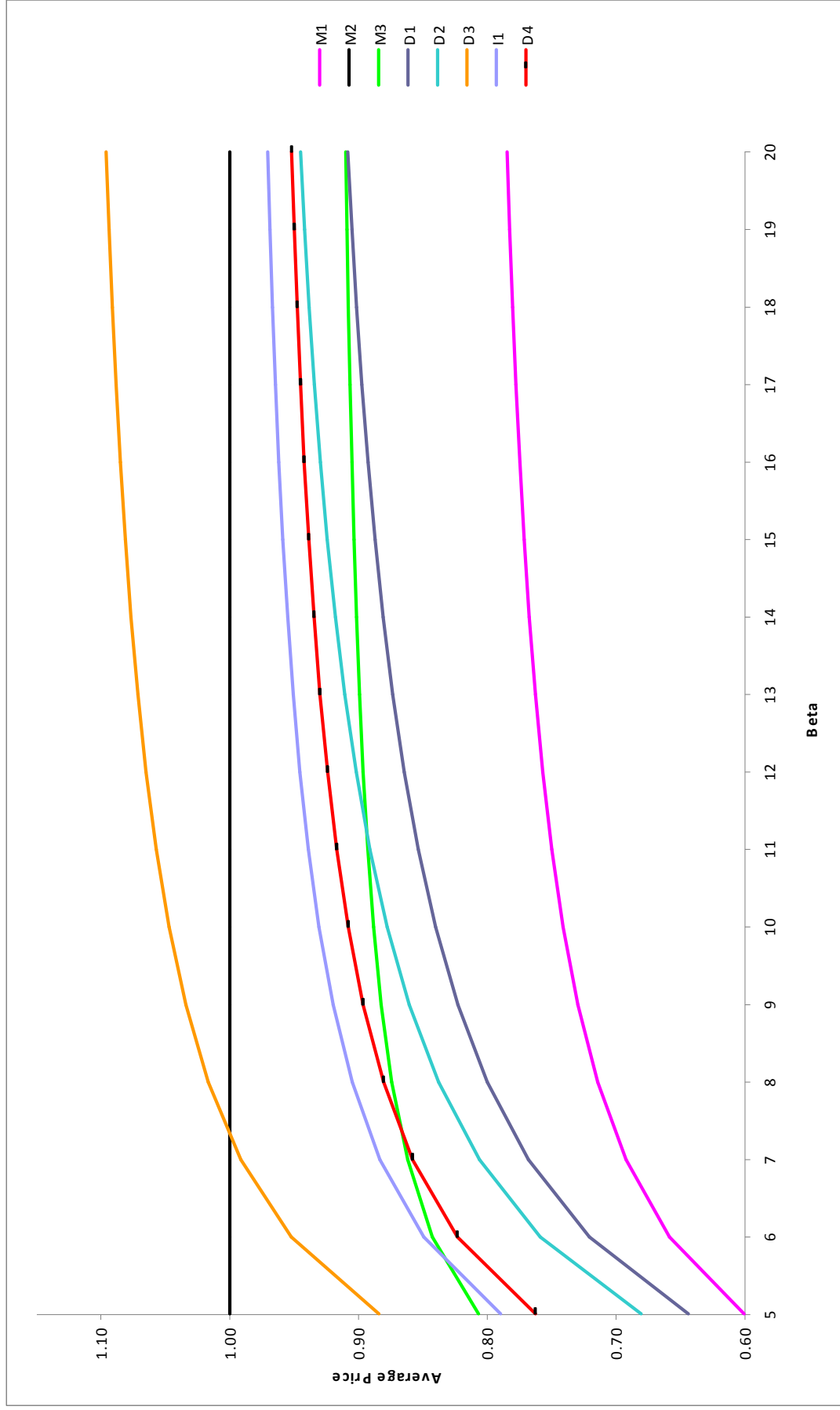
lowest average price – although, we did note early that this regime does offer significantly lower profits. D1 is the next best alternative, but for higher values of β it only offers a less than 15% lower average price than the highest, M2. Another non-integrated ticket regime offers the third lowest price, only slightly higher than D1 for high values of β. Surprisingly given the relatively high patronages it offers, I1 has quite a high average price as only M2 and D3 have average prices above it.

Whilst our propositions indicate that M1 is by far the best for society our profit comparisons show that this offers a relatively low profit. Similarly the firms preferred regime, M2, suffers from low patronages and high prices, and could mean that regulator looks to compromise by finding a regime that is acceptable to firms and society. In terms of profit, we highlight that both regimes D2 and I1 seem to offer attractive alternatives depending on the relative average prices and total patronages. It is regime D2 that would seem to offer both a higher patronage and a lower average price, although if the regime is not deemed acceptable then I1 may provide a reasonable alternative.

5.7 Conclusion

In this chapter we explore a simple model of integrated ticketing for both monopoly and duopoly markets, from the point of view of both the firm(s) involved and a welfare-maximising social planner. It is possible that the firms will not always act in the best interests of society and this may restrict the promotion of public transport so intervention in the market will be justified. We summarise the position here.

First, firms always prefer to offer an integrated ticket except if β is low, when D4 is preferred to regime D2. Society prefers the monopolist not to introduce integrated ticketing, but otherwise at least weakly prefers integrated ticketing when the market is a duopoly

unless the duopoly has independent pricing (D3). We see that in certain situations integrated ticketing could be successful at increasing the patronage of public transport. Additionally, integrated ticketing in a duopoly scenario may, depending on the circumstances and regime type, be preferred to a non-integrated ticketing monopoly.

Secondly, and not surprisingly, the firms generally prefer to be able to collude to some extent. The exception is if β is quite low – that is, when travel is relatively inelastic to (own) price – when the independent pricing regime D3 is preferred to the more collusive regime D2. Society prefers some or complete collusion over independent pricing, and the firms agree. However, society prefers limited collusion to perfect collusion: D2 and I1 are both better than M2, but, in both cases, the firms disagree. Society also prefers some collusion to independent pricing: I1 and D2 are strictly preferred to D3 with the firms agreeing in both cases. Of the partial collusion alternatives D2 is strictly preferred to I1, but the firms disagree.

Overall, society's best choice is M1 if $\beta > 7.68$, i.e. the own-price elasticity is high, and D2 if $\beta \leq 7.68$, the own-price elasticity is low. However, the profit arising from M1 is so low relative to other regimes we can expect any imposition of such a regime to be met with opposition from the firm. The firms' best choice is M2, but this offers fairly poor results in terms of low total patronage and high average price. Any regulator deciding to impose this regime will do so against the wishes of the social planner.

It becomes clear that any preferable regime from the point of view of the regulator must represent a compromise between the firm(s) and the social planner. Our look at relative levels of profit, total patronage, and average pricing brings us to the firm's other best choices: I1 and D2; the latter would represent one of the social planner's preferred choices. These two regimes – both allow firms to set prices according to an agreed rule –

offer relatively high profits, so that the firms will be less likely to seek to influence the regime type, whilst giving comparably high total patronage with low prices.

The fact M1 is, for higher own-price elasticities, the preferred choice of the social planner calls into question whether an integrated ticketing regime is what is best for society and the promotion of public transport. However, it would be unlikely that any such regime would be accepted by the firm(s), but policymakers will find integrated ticketing regimes will offer a useful compromise between the profits and the goals of the social planner. We must also recall that regime M1 forces the cross-service traveller to purchase two tickets as no integrated ticket is available and will burden the traveller with an extra transaction cost – one that was not included in the model for tractability reasons – that would mean welfare in the regime may be smaller than our estimate.

If we consider the situation in the UK following bus deregulation and the block exemption – not that this model is exclusively concerned with buses – the regime that best represents this situation is D3 and we identify this as often giving the lowest welfare. If firms were allowed to collude (moving from D3 to D2 or I1) on integrated ticket prices, as the recent Office of Fair Trading consultation suggests, the model shows that this could result in improvements in average prices and total patronage (as seen in Figure 5.2 and 5.3). It seems integrated ticketing could be successful in the promotion of public transport use, but we do acknowledge that any switch to public transport use will not just consist of car users, as people who use other modes (e.g. walking) may also switch.

This model presents us with a number of interesting policy results. However, it is important to extend the model, so that we can enhance the conclusions, increase our understanding of the issues the model raises, and to take into account the effects that other parameters may have. These extensions will be undertaken in the next two chapters, beginning by the introduction of a conjectural variations term in Chapter 6.

CHAPTER SIX

THE ECONOMICS OF INTEGRATED TICKETING WITH NON-ZERO CONJECTURAL VARIATIONS

6.1. Introduction

In Chapter 5, we establish a model of integrated ticketing in a transport network, which we use to examine the private and social outcomes that we derived from various regimes. One of the main aims of the previous chapter was to investigate whether integrated ticketing could encourage public transport use and we found, depending on the form of integrated ticketing, that it was possible that integrated ticketing could lead to lower prices and higher patronages in public transport.

This chapter will examine the outcomes of a more generalised framework to find whether the results are robust in the face of the introduction of a new behavioural parameter. In Chapter 3, we discuss conjectural variations and we will in the current chapter use this approach to extend Chapter 5's model to include a conjectural variation term. By introducing such a term we can further investigate how the different behavioural assumptions between firms may impact on integrated ticketing profits and welfare. We will concentrate on whether the conjectural variations term, under the assumption that firms behave in such a manner, has affected the results of Chapter 5. The problems with conjectural variations means the set of regimes we consider will have to be carefully adapted and our refinement of regimes concentrates on those that provide integrated ticketing. We can then use these to investigate how the price collusion assumed to take place in conjectural variations may change firms' and society's preferred regimes.

In the following section we add a conjectural variations parameter to the model we introduce in Chapter 5 before deriving and comparing the prices, quantities, profits, and welfares of integrated ticketing regimes. In Section 6.3 we bring together the results of the various regime comparisons and refer to the recommendations concerning integrated ticketing that we introduce in Chapter 2 and consider further in Chapter 5.

6.2 Price Strategies and Regulation

In this section we take three regimes from Chapter 5, add a conjectural variation term and consider the equilibrium prices and outputs under them. Table 6.1 summarises the relationships between Chapter 6's regimes and Chapter 5's regimes. Originally an R3 was included in this investigation but upon close inspection this regime was not valid.

Table 6. 1: Chapter 5 and Chapter 6 Regimes		
Name of Regime	**Chapter 5**	**Chapter 6 Cournot Conjecture**
Network Monopoly With Integrated Ticketing	M2	R1
Network Duopoly With Independently Priced Integrated Ticketing	D3	R4
Network Duopoly With Pre-Emptive Integrated Ticketing	I1	R2

6.2.1 Network Monopoly With Integrated Ticketing

Before we move on to situation that are altered by the introduction of non-zero conjectures let us now, for ease of reference, restate the model using the monopoly situation as an example. The network monopolist's profit in general terms, is given by:

$$\Pi^{R1} = \sum_{i=1,2} \sum_{j=1,2} P_{ij} Q_{ij} .$$
(6.1)

Let P_x be the price for the integrated ticket:

$$P_x = P_{ij},\qquad\qquad\qquad (i \neq j = 1,\, 2).\qquad\qquad\qquad (6.2)$$

The relevant demand functions are:

$$Q_{mm} = \alpha - \beta P_{mm} + P_{nn} + 2P_x,\qquad\qquad\qquad (6.3a)$$

$$Q_{mn} = \alpha - \beta P_x + P_x + P_{mm} + P_{nn}.\qquad\qquad\qquad (6.3b)$$

To ensure a system of gross substitutes we impose the condition:

$$\beta > 3.\qquad\qquad\qquad (6.4)$$

Our model set-up remains the same and that non-zero conjectures do not alter the situation faced by the monopolist as there is only one firm, so we arrive at the same prices, quantities and profits that we found in 5.3.2 – to prevent the constant need to refer to the previous chapter let us repeat the M2 results here as regime R1:

$$P_{mm}^{R1} = P_x^{R1} = \frac{\alpha}{2(\beta - 3)}.\qquad\qquad\qquad (6.5)$$

$$Q_{ij}^{R1} = Q_{ij}^{R1} = \frac{\alpha}{2}\qquad\qquad (\forall i, j = 1,\, 2).\qquad\qquad\qquad (6.6)$$

$$\Pi^{R1} = \frac{\alpha^2}{\beta - 3}.\qquad\qquad\qquad (6.7)$$

$$\tilde{Q}^{R1} = 2\alpha,\qquad\qquad\qquad (6.8)$$

$$\tilde{P}^{R1} = \frac{\alpha}{2(\beta - 3)}.\qquad\qquad\qquad (6.9)$$

6.2.2 Network Duopoly With Pre-Emptive Integrated Ticketing

We now examine regime R2, where the duopolists are allowed to collude in setting the price of the integrated ticket. Recall that this regime is a Stackelberg game, where the two firms are Cournot followers in the second period.

The relevant demands for regime R2 follow from (6.3) and firm 1 sets P_{11} and firm 2 sets P_{22}. Assuming each firm takes an equal share of the profits from the integrated ticket sales, the general expression for the profit of firm i is given by:

$$\Pi_i^{R2} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}), \qquad (m=i,\ n=j). \qquad (6.10)$$

Substituting (6.3) into (6.10) and maximising with respect to P_{mm} to give the following expression of firm i's optimal choice of P_{mm}, in terms of P_x and P_{nn}:

$$P_{mm} = \frac{\alpha + P_{nn} + (3 + \gamma_{R2})P_x}{2\beta - \gamma_{R2}}, \qquad (6.11)$$

where $\gamma_{R2} = \gamma_{ij} = dP_{mm}/dP_{nn}$ ($i=n,\ j=m$) is a common conjectural variation term, which measures firm i's expectation of the response in P_{mm} to a change in P_{nn} for regime R2, and can be interpreted as the implicit collusiveness of the industry. We allow conjectural variation term to vary between the regime types.

Solving (6.11) simultaneously across the two firms, we have:

$$P_{mm} = P_{nn} = \frac{\alpha + (3 + \gamma_{R2})P_x}{2\beta - \gamma_{R2} - 1}. \qquad (6.12)$$

The equilibrium expression (6.12) is the reaction function of the firms indicating their profit maximising choice of P_{mm} in terms of P_x. If we differentiate (6.12) with respect to P_x we arrive at the following expression for the slope of the reaction function, ξ:

$$\xi = \frac{3 + \gamma_{R2}}{2\beta - \gamma_{R2} - 1}. \qquad (6.13)$$

Profit across the network in general terms is given by:

$$\tilde{\Pi}^{R2} = P_{mm}Q_{mm} + P_{nn}Q_{nn} + P_x(Q_{mn} + Q_{nm}) \quad (m \neq n = 1,\ 2). \qquad (6.14)$$

Substituting (6.12) in (6.14) and maximising with respect to P_x, recognising that P_{mm} is a function of P_x, through (6.14), we have:

$$P_x = \frac{\alpha(2\beta - \gamma_{R2} + 3)}{2(2\beta^2 - 3\beta - \beta\gamma_{R2} - \gamma_{R2} - 5)}.$$ (6.15a)

Substituting (6.15a) into (6.12) gives the equilibrium second-stage single-service price:

$$P_{mm} = \frac{\alpha(2\beta + \gamma_{R2} + 1)}{2(2\beta^2 - 3\beta - \beta\gamma_{R2} - \gamma_{R2} - 5)}.$$ (6.15b)

Using (6.15) in (6.3) yields the equilibrium levels of demand for the single-service and cross-service tickets in regime R2, respectively:

$$Q_{mn} = \frac{\alpha}{2},$$ (6.16a)

$$Q_{mm} = \frac{\alpha(2\beta - 3 - 3\gamma_{R2})}{2(2\beta - 5 - \gamma_{R2})}.$$ (6.16b)

Aggregate profit across the network under this regime then follows from substitution of (6.15) and (6.16) into (6.14):

$$\tilde{\Pi}^{R2} = \frac{\alpha^2\left(4\beta^2 - 4\beta - 9 - 4\beta\gamma_{R2} - \gamma_{R2}^2 - 2\gamma_{R2}\right)}{\left(2\beta^2 - 3\beta - 5 - \beta\gamma_{R2} - \gamma_{R2}\right)\left(2\beta - 5 - \gamma_{R2}\right)}.$$ (6.17)

We now use (6.16) and (6.17) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{R2} = \frac{4\alpha(\beta - 2 - \gamma_{R2})}{(2\beta - 5 - \gamma_{R2})},$$ (6.18)

$$\tilde{P}^{R2} = \frac{\alpha\left(4\beta^2 - 4\beta - 9 - 4\beta\gamma_{R2} - \gamma_{R2}^2 - 2\gamma_{R2}\right)}{4\left(2\beta^2 - 3\beta - 5 - \beta\gamma_{R2} - \gamma_{R2}\right)(\beta - 2 - \gamma_{R2})}.$$ (6.19)

A comparison between R1 (or M2) and R2 leads us to Proposition 6.1:

Proposition 6.1. *(i) The firms prefer regime R1 (joint profit maximisation) over regime R2 when $\gamma_{R2} < 1$: $\Pi^{R1} > \Pi^{R2}$ when $\gamma_{R2} < 1$. (ii) The social planner strictly prefers regime R2 over regime R1 when $\gamma_{R2} < 1$: $S(\tilde{Q}^{R2}, \tilde{P}^{R2}) \succ S(\tilde{Q}^{R1}, \tilde{P}^{R1})$ when $\gamma_{R2} < 1$.*

The only change here is when $\gamma_{R2} = 1$, the duopolists are perfectly colluding and this means the duopoly regimes become equivalent to the monopoly or joint ownership regime, so the duopoly regime produces the same prices, patronages and profits as the monopoly or joint ownership regime; meaning the relevant part of Proposition 5.12 stands. The explanation on page 174 concerning I1 and M2 also applies here.

6.2.3 Network Duopoly With Independently Priced Integrated Ticketing

Let us return to a regime where the duopolists are now allowed to collude on any aspect of pricing. In this regime, R4, we should note we have assumed no relation between P_{xm} and P_{mm}, so there is no conjectural variation term between P_{mm} and P_{xn}. However, there is a relation between P_{xm} and P_{xn}, this is assumed to be γ_{R4} - the same as the relation between P_{mm} and P_{nn}. What this amounts to is a situation of independent pricing on components of the cross-service ticket: each firm *m* sets the price of its component, P_{xm}, of the integrated ticket price. The integrated ticket price is the sum of these two component prices:

$$P_m = \sum P_{xm}, \qquad\qquad (m = 1,\ 2). \qquad\qquad (6.20)$$

Given (6.20) the general expression for the profit of firm *m* is given by:

$$\Pi_m^{R4} = P_{mm}Q_{mm} + P_{xm}(Q_{mn} + Q_{nm}), \qquad (m \neq n = 1, 2). \qquad (6.21)$$

Using (6.3) and (6.20) in (6.21) and maximising with respect to P_{xm} and P_{xn} for $m = 1, 2$ yields the following equilibrium expressions for the cross-service and single-service ticket prices, respectively:

$$P_x^{R4} = P_{x1}^{R4} + P_{x2}^{R4} = \frac{4\alpha}{(2\beta - 5 - \gamma_{R4})(3 + \gamma_{R4})},\tag{6.22a}$$

$$P_{mm}^{R4} = \frac{\alpha}{(2\beta - 5 - \gamma_{R4})},\tag{6.22b}$$

where $\gamma_{R4} = \gamma_{ij} = dP_{mm} / dP_{nn}$ ($i = n,\ j = m$) is a common conjectural variation term. As the R4 regime is set-up differently to the R2 regime it is likely that the level of price collusion will vary between the two regimes, so we have separate conjecture terms (γ_{R4} and γ_{R4}). As regime R4 forces independent pricing we may expect the collusion in this regime to be lower than in regime R2, which allows collusion on integrated ticket prices. Using (6.22) in (6.3) yields the following equilibrium expressions for cross-service and single-service ticket prices, respectively:

$$Q_{mm}^{R4} = \frac{\alpha(3\beta - 4 + \beta\gamma_{R4} - 7\gamma_{R4} + \beta\gamma_{R4})}{(2\beta - 5 - \gamma_{R4})(3 + \gamma_{R4})},\tag{6.23a}$$

$$Q_{mn}^{R4} = \frac{\alpha(1 + \gamma_{R4})}{(3 + \gamma_{R4})}.\tag{6.23b}$$

Profit across the network then follows from substituting (6.22) and (6.23) into (6.21), and summing across the two firms:

$$\tilde{\Pi}^{R4} = \frac{2\alpha^2[14\gamma_{R4}\beta - 49\gamma_{R4} - 14\gamma_{R4}^2 + 17\beta - 32 - \gamma_{R4}^3 + \beta\gamma_{R4}^2]}{(2\beta^2 - 5 - \gamma_{R4})^2(3 + \gamma_{R4})^2}.\tag{6.24}$$

We now use (6.23) and (6.24) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1::

$$\tilde{Q}^{R4} = \frac{2\alpha\left(3\gamma_{R4}\beta - 13\gamma_{R4} - 2\gamma_{R4}{}^2 - 5\beta + 9\right)}{\left(2\beta^2 - 5 - \gamma_{R4}\right)\left(3 + \gamma_{R4}\right)}, \tag{6.25}$$

$$\tilde{P}^{R4} = \frac{\alpha[14\gamma_{R4}\beta - 49\gamma_{R4} - 14\gamma_{R4}{}^2 + 17\beta - 32 - \gamma_{R4}{}^3 + \beta\gamma_{R4}{}^2]}{\left(2\beta - 5 - \gamma_{R4}\right)\left(3 + \gamma_{R4}\right)\left(3\gamma_{R4}\beta - 13\gamma_{R4} - 2\gamma_{R4}{}^2 - 5\beta + 9\right)}. \tag{6.26}$$

A comparison between R1 and leads us to Proposition 6.2:

Proposition 6.2. *(i) The firms prefer regime R1 over regime R4 when $\gamma_{R4} < 1$:*

$\Pi^{R1} > \Pi^{R4}$ *when $\gamma_{R4} < 1$. (ii) The social planner strictly prefers regime R4 over regime R1 when β is low and $\gamma_{R4} < 1$: $S(\tilde{Q}^{R4}, \tilde{P}^{R4}) \succ S(\tilde{Q}^{R1}, \tilde{P}^{R1})$ when β is low and $\gamma_{R4} < 1$.*

The only change here compared to the result in the previous chapter is when $\gamma_{R2} = 1$, and the explanation for this follows from Proposition 6.1 – Proposition 6.2 is equivalent to Proposition 5.7 and the explanation follows from that (see page 168). The introduction of γ_{R4} sees the prices and quantities of the two regimes get closer together as γ_{R4} increases, but it does not change the eventual result.

A comparison between R2 and R4 leads us to Proposition 6.3:

Proposition 6.3. *(i) If β is large or if R2 is significantly more collusive than R4 then the firms prefer regime R2 over regime R4: $\Pi^{R2} > \Pi^{R4}$ if β is large or γ_{R2} is significantly larger than γ_{R4}. (ii) The social planner strictly prefers regime R4 over regime R2 when β is small and γ_{R2} is significantly larger than γ_{R4} : $S(\tilde{Q}^{R4}, \tilde{P}^{R4}) \succ S(\tilde{Q}^{R2}, \tilde{P}^{R2})$ when β is small and γ_{R2} is significantly larger than γ_{R4}.*

When γ_{R2} is significantly larger than γ_{R4} then this proposition contradicts part of

Proposition 5.12, which compares I1 and D3. As γ_{R2} increases above γ_{R4} the collusiveness

of regime R2 increases, so the firms internalise the horizontal externality that causes them

to decrease prices and this results in patronages falling and profits increasing. When β is

small and the collusiveness of regime R2 is high (relative to R4) this results in the

horizontal externalities in I1, that result in part of Proposition 5.12, being internalised in

R2, so that prices and profits are higher in R2 (compared to R4) while patronage is lower.

We should also note that when $\gamma_{R2} = 1$ and $\gamma_{R4} = 1$ then both duopolies yield results

equivalent to the monopolist.

6.3 Conclusion

In this chapter we extend Chapter 5's model by adding a conjectural variations model

and examining the effects. This chapter is particularly relevant to the possibility that

integrated ticketing erodes the effects of competition as we introduce a firm behavioural

parameter that allows us to consider the possible impacts of increased collusion due to

integrated ticketing.

Other than the effect on welfare comparisons when $\gamma_{R2} = 1$, and $\gamma_{R4} = 1$ the

conjectural variation term has its main impact on the comparison between R2 and R4, and

means Proposition 5.12 of the previous chapter is contradicted. The structure of regime R2

allows firms to communicate and set a price for the integrated ticket, but regime R4 forces

the firms to set all prices independently. The increased cooperation between firms in R2

could lead to effective price collusion compared to regime R4 and we allow for this

collusiveness, by considering the results when γ_{R2} is bigger than γ_{R4}. When γ_{R2} is

significantly larger than γ_{R4} then firms will prefer regime R2 to regime R4, whatever the

value of β. However, if β is small and the collusiveness of regime R2 is greater than the collusiveness of R4, then the social planner will prefer regime R4 to R2. Therefore the regulator could, if they believe β to be small and that too much cooperation leads to price collusion, prefer to induce regime R4 as it would be the preferred choice of society in such a situation.

The overall profit and welfare rankings are therefore dependent on the level of collusion in the regimes and β. Society's best choice could be R2 or R4 depending on the value of the parameters except when $\gamma = 1$, where all regimes yield the same profits and welfares. However, the firms' best choice is unaffected when $\gamma < 1$ and remains R1 (M2).

CHAPTER SEVEN

THE ECONOMICS OF INTEGRATED TICKETING WITH DEMAND

ASYMMETRIES

7.1 Introduction

In Chapter 5 we establish a model of integrated ticketing in a transport network, which we then extend in Chapter 6 by introducing a conjectural variations term. The extension in the previous chapter finds that, for the most part, the results were unaffected by the introduction of a firm behavioural parameter.

We now wish to check the robustness of another aspect of Chapter 5's integrated ticketing model by making the single-service and cross-service demands asymmetrical to examine the impacts on private and social preferences, and whether this affects Chapter 5's results. In this chapter we relax the symmetry in the demand types as it would seem likely that the demand for cross-service (i.e. the integrated tickets) will be, due to the flexibility it offers the traveller, larger or more robust to price changes than a normal single-service ticket. We allow the willingness to pay to be larger and the price elasticities to be smaller, for a cross-service ticket. Given that we are investigating the asymmetries in demand it would also be relevant to consider asymmetries in the cross-price elasticities, but this would result in the model becoming intractable.

In the following section we add an asymmetry in α to the model that we introduce in Chapter 5, so that the cross-service demand level is higher that the single-service demand level. We then calculate and compare the prices, quantities, profits, and welfares of integrated ticketing monopoly and duopoly regimes with this demand level asymmetry. In Section 7.3, we consider the main results of the change in the demand structure. In Section

7.4, we look at another alteration to Chapter 5's model by adding an asymmetry in β, so that the cross-service demand is less responsive to a change in own-price than the single-service demand is. We then compare the prices, quantities, profits, and welfares of integrated ticketing monopoly and duopoly regimes with this asymmetry in the own-price effect. Finally, in Section 7.5, we consider the main results of the change in the demand structure.

7.2 The Model With Asymmetric Demand Levels

In this section we take four regimes from Chapter 5, add an asymmetry in the value of α, and consider the equilibrium prices and outputs. Table 7.1 summarises the relationships between Chapter 5's regimes and this section of Chapter 7's regimes.

Table 7. 1: Chapter 5 and Chapter 7 Regimes (1)

Name of Regime	Chapter 5	Chapter 7 Asymmetry in α
Network Monopoly With Integrated Ticketing	M2	S1
Network Duopoly With Simultaneous Integrated Ticketing	D2	S3
Network Duopoly With Independently Priced Integrated Ticketing	D3	S4
Network Duopoly With Pre-Emptive Integrated Ticketing	I1	S2

7.2.1 Network Monopoly With Integrated Ticketing

Let us first investigate the results of varying the level of α between cross-service and single-service demands. As before:

$$\Pi^{S1} = \sum_{i=1,2} \sum_{j=1,2} P_{ij} Q_{ij} . \tag{7.1}$$

Let P_x be the price for the integrated ticket:

$$P_x = P_{ij}, \qquad\qquad\qquad (i \neq j = 1, 2). \qquad\qquad (7.2)$$

The relevant demand functions now become:

$$Q_{mm} = \alpha - \beta P_{mm} + P_{nn} + 2P_x, \qquad\qquad (7.3a)$$

$$Q_{mn} = \alpha_x - \beta P_x + P_x + P_{mm} + P_{nn}, \qquad\qquad (7.3b)$$

where we set $\alpha_x > \alpha$, as cross-service ticket allows the traveller to combine any outward service with any inward service and this flexibility means it could be in more demand. We are no longer interested in the non-integrated ticketing regimes, so the above two equations both imply the following condition to ensure a system of gross substitutes:

$$\beta > 3. \qquad\qquad (7.4)$$

Substituting (7.3) into (7.1) and maximising with respect to P_{11}, P_{22} and P_x yields the following equilibrium prices for the single-service and integrated ticket, respectively:

$$P_{mm}^{S1} = \frac{\alpha(\beta-1)+2\alpha_x}{2(\beta+1)(\beta-3)}, \qquad\qquad (7.5a)$$

$$P_x^{S1} = \frac{\alpha_x(\beta-1)+2\alpha}{2(\beta+1)(\beta-3)}. \qquad\qquad (7.5b)$$

We can now see the network monopolist discriminates on price across the two ticket types as $\alpha_x > \alpha$, the price of the integrated ticket is higher. Substituting (7.5) into (7.3), yields the following equilibrium expression for quantity demanded of each ticket type:

$$Q_{mm}^{S1} = \frac{\alpha}{2}, \qquad\qquad (\forall i, j = 1, 2), \qquad\qquad (7.6a)$$

$$Q_{mm}^{S1} = \frac{\alpha_x}{2}, \qquad\qquad (\forall i, j = 1, 2). \qquad\qquad (7.6b)$$

Finally, using (7.5) and (7.6) in (7.1), we have the equilibrium profit under regime S1:

$$\tilde{\Pi}^{S1} = \frac{\left(\beta\alpha^2 - \alpha^2 + 4\alpha\alpha_x + \beta\alpha_x^2 - \alpha_x^2\right)}{2(\beta+1)(\beta-3)}. \tag{7.7}$$

We now use (7.6) and (7.7) to form total patronage on the network ($\tilde{Q}$) and average

(per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S1} = \alpha + \alpha_x, \tag{7.8}$$

$$\tilde{P}^{S1} = \frac{\left(\beta\alpha^2 - \alpha^2 + 4\alpha\alpha_x + \alpha_x^2\beta - \alpha_x^2\right)}{2(\beta+1)(\beta-3)(\alpha+\alpha_x)}. \tag{7.9}$$

7.2.2 Network Duopoly With Pre-Emptive Integrated Ticketing

This regime is one where there are two distinct periods. In the first, the two

duopolists collude to set the price of the integrated ticket. In the second period, the firms

attempt to maximise their profits on their single-service operations taking the integrated

ticket price as given.

The relevant demands for regime S2 follow from (7.3) with firm 1 setting P_{11} and

firm 2 setting P_{22}. Assuming each firm takes an equal share of the profits from the

integrated ticket, the general expression for the profit of firm i is given by:

$$\Pi_i^{S2} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}), \qquad (m = i, \ n = j). \tag{7.10}$$

Substituting (7.3) into (7.10) and maximising with respect to P_{mm} gives the following

expression relating firm i's optimal choice of P_{mm} in terms of P_x and P_{nn}:

$$P_{mm} = \frac{\alpha + P_{nn} + 3P_x}{2\beta}. \tag{7.11}$$

Solving (7.11) simultaneously across the two firms, we have:

$$P_{mm} = P_{nn} = \frac{\alpha + 3P_x}{2\beta - 1}. \tag{7.12}$$

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The equilibrium expression (7.12) is the reaction function of the firms indicating their profit maximising choice of P_{mm} in terms of P_x. Differentiating (7.12) with respect to P_x we arrive at the following expression for the slope of the reaction function, ξ:

$$\xi = \frac{3}{2\beta - 1}. \tag{7.13}$$

Profit across the network in general terms is given by:

$$\tilde{\Pi}^{S2} = P_{mm}Q_{mm} + P_{nn}Q_{nn} + P_x(Q_{mn} + Q_{nm}) \quad (m \neq n = 1,\, 2). \tag{7.14}$$

Substituting (7.12) in (7.14) and maximising with respect to P_x, recognising that P_{mm} is a function of P_x, through (7.13), we have:

$$P_x = \frac{\alpha(4\alpha + 2\alpha_x\beta - \alpha_x)}{2\left(2\beta^2 - 3\beta - 5\right)}. \tag{7.15a}$$

Substituting (7.15) into (7.12) gives the equilibrium second-stage single-service price:

$$P_{mm} = \frac{\alpha(2\alpha\beta - 2\alpha + 3\alpha_x)}{2(2\beta^2 - 3\beta - 5)}. \tag{7.15b}$$

Using (7.15) in (7.3) yields the equilibrium levels of demand for the single-service and cross-service tickets in regime R2, respectively:

$$Q_{mn} = \frac{\alpha_x}{2}, \tag{7.16a}$$

$$Q_{mm} = \frac{(2\alpha\beta - 4\alpha + \alpha_x)}{2(2\beta - 5)}. \tag{7.16b}$$

Aggregate profit across the network under this regime then follows from substitution of (7.15) and (7.16) into (7.10):

$$\tilde{\Pi}^{S2} = \frac{\left(8\alpha_x\beta\alpha - 17\alpha_x\alpha + 2\alpha_x^2\beta^2 - 6\alpha_x^2\beta + 4\alpha_x^2 + 2\alpha^2\beta^2 - 6\alpha^2\beta + 4\alpha^2\right)}{\left(2\beta^2 - 3\beta - 5\right)(2\beta - 5)}. \tag{7.17}$$

We now use (7.16) and (7.17) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1::

$$\tilde{Q}^{S2} = \frac{2\left(\beta\alpha - 2\alpha - 2\alpha_x + \beta\alpha_x\right)}{\left(2\beta - 5\right)}, \tag{7.18}$$

$$\tilde{P}^{S2} = \frac{\left(8\beta\alpha_x\alpha - 17\alpha_x\alpha + 2\alpha_x^2\beta^2 - 6\alpha_x^2\beta + 4\alpha_x^2 + 2\alpha^2\beta^2 - 6\alpha^2\beta + 4\alpha^2\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(\beta\alpha - 2\alpha - 2\alpha_x + \beta\alpha_x\right)}. \tag{7.19}$$

A comparison between S1 and S2 leads us to Proposition 7.2.1:

Proposition 7.2.1. *The firms prefer the network monopoly regime S1 over regime S2:* $\Pi^{S1} > \Pi^{S2}$. *(ii) The social planner strictly prefers regime S2 over regime S1:* $S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ S(\tilde{Q}^{S1}, \tilde{P}^{S1})$.

Proposition 7.2.1 is equivalent to part of Chapter 5's Proposition 5.12 so the introduction of higher demands in integrated ticketing does not change the results between S1 and S2, the explanation therefore follows from Proposition 5.12 (on page 174 concerning regimes I1 and M2).

7.2.3 Network Duopoly With Simultaneous Integrated Ticketing

In regime S3, where the firms collude to jointly maximise profit on the cross-service demands, the general expression for profit on the cross-service operation is given by:[1]

$$\Pi_x^{S3} = P_x(Q_{mn} + Q_{nm}), \qquad (m \neq n). \tag{7.20}$$

Substituting (7.3) into (7.20) and maximising with respect to $P_x (= P_{mm} = P_{mn}; m \neq n = 1, 2)$ yields the following expression for the integrated ticket price in terms of the single-service

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[1] Note, given the equilibrium prices for the integrated ticket always exceed those for the single-service ticket, only passengers wishing to travel cross-service will purchase the integrated ticket: the two are synonymous in this model.

prices, P_{mm} ($m = 1, 2$):

$$P_x = \frac{a_x + P_{mm} + P_{nn}}{2(\beta - 1)}.$$ (7.21)

Assuming each firm takes an equal share of the profits from the integrated ticket, the general expression for the profit of firm i is given by:

$$\Pi_i^{S3} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}) \qquad (m = i, \ n = j).$$ (7.22)

Maximising (7.22) with respect to P_{mm} and solving using (7.21) gives the following equilibrium prices:

$$P_x^{S3} = \frac{(2\alpha_x\beta^2 - 3\alpha_x\beta + 2\alpha\beta - 2\alpha)}{2(2\beta^2 - 3\beta - 3)(\beta - 1)},$$ (7.23a)

$$P_{mm}^{S3} = \frac{(2\alpha\beta - 2\alpha + 3\alpha_x)}{2(2\beta^2 - 3\beta - 3)}.$$ (7.23b)

Using (7.23) in (7.3) yields the following equilibrium demand for cross-service and single-service ticket prices, respectively:

$$Q_{mm}^{S3} = \frac{(2\alpha\beta^3 - 4\alpha\beta^2 + 4\alpha - 2\alpha\beta + \alpha_x\beta^2 - 3\alpha_x)}{2(2\beta^3 - 5\beta^2 + 3)},$$ (7.24a)

$$Q_{mn}^{S3} = \frac{(2\alpha_x\beta^2 - 3\alpha_x\beta + 2\alpha\beta - 2\alpha)}{2(2\beta^2 - 3\beta - 3)}.$$ (7.24b)

Using (7.23) and (7.24) in (7.22) and summing over both firms, aggregate profit across the network is:

$$\tilde{\Pi}^{S3} = \frac{\begin{array}{l} \alpha^2(4\beta^4 - 12\beta^3 + 8\beta^2 + 4\beta\alpha - 4) \\ + \alpha_x(4\alpha_x\beta^4 - 12\alpha_x\beta^3 + 12\alpha_x\beta^2 - 34\beta^2 - 9\alpha_x) \\ + \alpha_x\alpha(16\beta^3 + 18) \end{array}}{2(2\beta^3 - 5\beta^2 + 3)(2\beta^2 - 3\beta - 3)}.$$ (7.25)

We now use (7.24) and (7.25) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S3} = \frac{\left(2\alpha_x\beta^3 + 2\alpha\beta^3 - 2\alpha\beta^2 - 4\alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 6\alpha - 3\alpha_x\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)}, \tag{7.26}$$

$$\tilde{P}^{S3} = \frac{\begin{array}{c} \alpha^2(4\beta^4 - 12\beta^3 + 8\beta^2 + 4\beta - 4) \\ +\alpha_x(4\beta^4 - 12\alpha_x\,\beta^3 + 12\alpha_x\,\beta^2 - 34\beta^2 - 9\alpha_x \\ +\alpha_x\alpha(16\beta^3 + 18) \end{array}}{2\left(2\beta^2 - 3\beta - 3\right)}. \tag{7.27}$$

$$\left(2\alpha_x\beta^3 + 2\alpha\beta^3 - 2\alpha\beta^2 - 4\alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 6\alpha - 3\alpha_x\right)$$

A comparison between S1 and S3 leads us to Proposition 7.2.2:

Proposition 7.2.2. *The firms prefer the network monopoly regime S1 over regime S3:* $\Pi^{S1} > \Pi^{S3}$ *. (ii) The social planner strictly prefers regime S3 over regime S1:* $S(\tilde{Q}^{S3}, \tilde{P}^{S3}) \succ S(\tilde{Q}^{S1}, \tilde{P}^{S1})$.

Proposition 7.2.2 is equivalent to Chapter 5's Proposition 5.5, so the results do not change with the asymmetrical demand and the explanation also follows from Proposition 5.5 (on page 165).

A comparison between S2 and S3 using equilibrium profits, average prices, and total quantities leads us to Proposition 7.2.3:

Proposition 7.2.3. *The firms prefer the regime S2 over regime S3:* $\Pi^{S2} > \Pi^{S3}$. *(ii) The social planner strictly prefers regime S3 over regime S2:* $S(\tilde{Q}^{S3}, \tilde{P}^{S3}) \succ S(\tilde{Q}^{S2}, \tilde{P}^{S2})$.

Proposition 7.2.3 is equivalent to part of Chapter 5's Proposition 5.12, so once the introduction of an asymmetry in the demand level does not impact on our results and the explanation follows from Proposition 5.12 (on page 174 concerning I1 and D2).

7.2.4 Network Duopoly With Independently Priced Integrated Ticketing

In regime S4 duopolists are not allowed to collude on any aspect of pricing in the

network. The integrated ticket price is the sum of these two component prices:

$$P_m = \sum P_{xm} \qquad (m = 1, 2). \qquad (7.28)$$

Given (7.28) the general expression for the profit of firm m is given by, Π_m^{S4}:

$$\Pi_m^{S4} = P_{mm}Q_{mm} + P_{xm}(Q_{mn} + Q_{nm}) \qquad (m \neq n = 1, 2). \qquad (7.29)$$

Using (7.18) in (7.29) and maximising with respect to P_{xm} and P_{mm} for $m = 1, 2$ yields the

following equilibrium expressions for the cross-service and single-service ticket prices,

respectively:

$$P_x^{S4} = P_{x1}^{S4} + P_{x2}^{S4} = \frac{4\alpha_x \beta + 6\alpha - 2\alpha_x}{3(2\beta - 5)(\beta + 1)}, \qquad (7.30a)$$

$$P_{mm}^{S4} = \frac{\alpha\beta - \alpha + 2\alpha_x}{(2\beta - 5)(\beta + 1)}. \qquad (7.30b)$$

Substituting (7.30) in (7.3), yields the equilibrium demands for single-service and cross-

service, respectively:

$$Q_{mn}^{S4} = \frac{\alpha_x}{3}, \qquad (7.31a)$$

$$Q_{mm}^{S4} = \frac{(3\alpha\beta - 6\alpha + 2\alpha_x)}{3(2\beta - 5)}. \qquad (7.31b)$$

Profit across the network then follows from substituting (7.30) and (7.31) into (7.29)

and summing across the two firms:

$$\tilde{\Pi}^{S4} = \frac{2\left(8\alpha_x^2\beta^2 - 24\alpha_x^2\beta + 36\alpha_x\alpha\beta - 72\alpha_x\alpha + 22\alpha_x^2 + 9\alpha^2\beta^2 - 27\alpha^2\beta + 18\alpha^2\right)}{9(2\beta - 5)^2(\beta + 1)}. \qquad (7.32)$$

We now use (7.31) and (7.32) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S4} = \frac{2\left(2\alpha_x\beta - 3\alpha_x + 3\alpha\beta - 6\alpha\right)}{3\left(2\beta - 5\right)},\tag{7.33}$$

$$\tilde{P}^{S4} = \frac{\left(8\alpha_x^2\beta^2 - 24\alpha_x^2\beta + 36\alpha_x\alpha\beta - 72\alpha_x\alpha + 22\alpha_x^2 + 9\alpha^2\beta^2 - 27\alpha^2\beta + 18\alpha^2\right)}{3\left(2\beta - 5\right)\left(\beta + 1\right)\left(2\alpha_x\beta - 3\alpha_x + 3\alpha\beta - 6\alpha\right)}.\tag{7.34}$$

A comparison between S1 and S4 leads us to Proposition 7.2.4:

Proposition 7.2.4. *The firms prefer the network monopoly regime S1 over regime S4:* $\Pi^{S1} > \Pi^{S4}$. *(ii) The social planner strictly prefers, strictly prefers regime S1 (S4) over regime S4 (S1) when β is large (small):* $S(\tilde{Q}^{S1}, \tilde{P}^{S1}) \succ (\prec) S(\tilde{Q}^{S4}, \tilde{P}^{S4})$ *when β is large (small).*

Proposition 7.2.4 is equivalent to Chapter 5's Proposition 5.7, so the explanation follows from Proposition 5.7 (on page 168).

A comparison S2 and S4 leads us to Proposition 7.2.5:

Proposition 7.2.5. *The firms prefer the duopoly regime S2 (S4) over regime S4 (S2) when β is large (small):* $\Pi^{S2} > (<) \Pi^{S4}$ *when β is large (small). (ii) The social planner strictly prefers regime S2 over regime S4:* $S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ S(\tilde{Q}^{S4}, \tilde{P}^{S4})$.

Proposition 7.2.5 is equivalent to Chapter 5's Proposition 5.12 and the explanation follows from there (on page 175 concerning I1 and D3).

A comparison between S3 and S4 leads us to Proposition 7.2.6:

Proposition 7.2.6. *The firms prefer the network monopoly regime S3 over regime S4 when β is large:* $\Pi^{S3} > (<) \Pi^{S4}$ *when β is large (small). (ii) The social planner strictly*

(weakly) prefers regime S3 (S4) over regime S4 (S3) if β is large (small):

$S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ (\preceq) S(\tilde{Q}^{S4}, \tilde{P}^{S4})$ *if β is large (small).*

Proposition 7.2.6 is equivalent to Chapter 5's Proposition 5.8 so the explanation follows (on page 168).

7.3. Demand Level Asymmetries: Conclusion

In this investigation, we further explore Chapter 5's model of integrated ticketing for both monopoly and duopoly markets from the point of view of both the firm(s) involved and a welfare-maximising social planner by introducing the term α_x and allowing $\alpha_x > \alpha$. However, allowing $\alpha_x > \alpha$ has no impact on the comparisons that we highlighted in Chapter 5. Each of the propositions is consistent with those in Chapter 5. It seems that α and α_x have no real influence over the complementarities and substitutabilities that drive the model. In Chapter 5, we find that α had no impact on the results and introducing an asymmetry does not alter this.

7.4 Model With Asymmetries in Own-Price Elasticities of Demand

In this section we take four regimes from Chapter 5, add an asymmetry in the value

Table 7. 2: Chapter 5 and Chapter 7 Regimes (2)

Name of Regime	Chapter 5	Chapter 7 Asymmetry in β
Network Monopoly With Integrated Ticketing	M2	S5
Network Duopoly With Simultaneous Integrated Ticketing	D2	S7
Network Duopoly With Independently Priced Integrated Ticketing	D3	S8
Network Duopoly With Pre-Emptive Integrated Ticketing	I1	S6

of β, and consider the equilibrium prices and outputs.[2] Table 7.2 summarises the

relationships between Chapter 5's regimes and this section of Chapter 7's regimes.

7.4.1 Network Monopoly With Integrated Ticketing

Let us now investigate the results of a lower β for the demand of integrated tickets.

As before:

$$\Pi^{S5} = \sum_{i=1,2} \sum_{j=1,2} P_{ij} Q_{ij} . \tag{7.35}$$

Let P_x be the price for the integrated ticket:

$$P_x = P_{ij}, \qquad\qquad (i \neq j = 1, 2). \tag{7.36}$$

The relevant demand functions now become:

$$Q_{mm} = \alpha - \beta P_{mm} + P_{nn} + 2P_x, \tag{7.37a}$$

$$Q_{mn} = \alpha - \beta_x P_x + P_x + P_{mm} + P_{nn} . \tag{7.37b}$$

where $\beta > \beta_x$. A cross-service ticket allows the purchaser to combine any outward service

with any inward service and this flexibility means travellers would be less likely to reduce

cross-service demand when its price rises, thus a lower price elasticity for the cross-service

ticket and $\beta > \beta_x$ assumption embodies it. This along with a condition to ensure a system

of gross substitutes gives:

$$\beta > \beta_x > 3 . \tag{7.38}$$

Substituting (7.38) into (7.35) and maximising with respect to P_{11}, P_{22}, and P_x yields the

following equilibrium prices for the single-service and integrated ticket, respectively:

$$P_{mm}^{S5} = \frac{\alpha(\beta_x + 1)}{2(\beta\beta_x - \beta - \beta_x - 3)}, \tag{7.39a}$$

---

[2] Note: α reverts back to the same form as in Chapter 5

$$P_x^{S5} = \frac{\alpha(\beta+1)}{2(\beta\beta_x - \beta - \beta_x - 3)}.$$
(7.39b)

We can now see the network monopolist discriminates on price across the different ticket types as $\beta > \beta_x > 3$ it results in price of the integrated ticket being higher. Substituting (7.39) into (7.37), yields the following equilibrium expression for quantity demanded of each ticket type:

$$Q_{mm}^{S5} = Q_{mm}^{S5} = \frac{\alpha}{2}, \qquad (\forall i, j = 1, 2).$$
(7.40)

Finally, using (7.39) and (7.40) in (7.35), we have the equilibrium profit under regime R1:

$$\tilde{\Pi}^{S5} = \frac{\alpha^2(\beta_x + 2 + \beta)}{2(\beta\beta_x - \beta - \beta_x - 3)}.$$
(7.41)

We now use (7.40) and (7.41) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S5} = 2\alpha,$$
(7.42)

$$\tilde{P}^{S5} = \frac{\alpha(\beta_x + \beta + 2)}{4(\beta\beta_x - \beta - \beta_x - 3)}.$$
(7.43)

7.4.2 Network Duopoly With Pre-Emptive Integrated Ticketing

The relevant demands for regime S6 follow from (7.37) and firm 1 sets P_{11} and firm 2 sets P_{22}. Assuming each firm takes an equal share of the profits from the integrated ticket, the general expression for the profit of firm i is given by:

$$\Pi_i^{S6} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}), \qquad (m = i, n = j).$$
(7.44)

Substituting in (7.37) to (7.44) and maximising with respect to P_{mm} gives the following

expression relating firm i's optimal choice of P_{mm} in terms of P_x and P_{nn}:

$$P_{mm} = \frac{\alpha + P_{nn} + 3P_x}{2\beta}.$$

(7.45)

Solving (7.45) simultaneously across the two firms, we have:

$$P_{mm} = P_{nn} = \frac{\alpha + 3P_x}{2\beta - 1}.$$

(7.46)

The equilibrium expression (7.46) is the reaction function of the firms indicating their profit

maximising choice of P_{mm} in terms of P_x. Differentiating (7.46) with respect to P_x we

arrive at the following expression for the slope of the reaction function, ξ:

$$\xi = \frac{3}{2\beta - 1}.$$

(7.47)

Profit across the network in general terms is given by:

$$\tilde{\Pi}^{S6} = P_{mm}Q_{mm} + P_{nn}Q_{nn} + P_x(Q_{mn} + Q_{nm}) \quad (m \neq n = 1,\, 2).$$

(7.48)

Substituting (7.46) in (7.48) and maximising with respect to P_x, recognising that P_{mm} is a

function of P_x, through (7.47), we have:

$$P_x = \frac{\alpha(2\beta + 3)}{2(2\beta\beta_x - 2\beta - \beta_x - 5)}.$$

(7.49a)

Substituting (7.49) into (7.46) gives the equilibrium second-stage single-service price:

$$P_{mm} = \frac{\alpha(2\beta_x + 1)}{2(2\beta\beta_x - 2\beta - \beta_x - 5)}.$$

(7.49b)

Using (7.49) in (7.37) yields the equilibrium levels of demand for the single-service and

cross-service tickets in regime R2, respectively:

$$Q_{mn} = \frac{\alpha}{2}, \qquad (7.50a)$$

$$Q_{mm} = \frac{(2\beta\beta_x - \beta - 3)}{2(2\beta\beta_x - \beta_x - 2\beta - 5)}. \qquad (7.50b)$$

Aggregate profit across the network under this regime then follows from substitution of (7.50) and (7.49) into (7.48):

$$\tilde{\Pi}^{S6} = \frac{\alpha^2 \left(4\beta^2\beta_x + 4\beta\beta_x^2 + 4\beta\beta_x - 4\beta^2 - 17\beta - 9\beta_x - 18\right)}{2\left(2\beta\beta_x - 2\beta - \beta_x - 5\right)^2}. \qquad (7.51)$$

We now use (7.50) and (7.51) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S6} = \frac{\alpha\left(4\beta\beta_x - 3\beta - \beta_x - 8\right)}{\left(2\beta\beta_x - 2\beta - \beta_x - 5\right)}, \qquad (7.52)$$

$$\tilde{P}^{S6} = \frac{\alpha\left(4\beta^2\beta_x + 4\beta\beta_x^2 + 4\beta\beta_x - 4\beta^2 - 17\beta - 9\beta_x - 18\right)}{2\left(2\beta\beta_x - 2\beta - \beta_x - 5\right)\left(4\beta\beta_x - 3\beta - \beta_x - 8\right)}. \qquad (7.53)$$

A comparison between S5 and S6 leads us to Proposition 7.2.1:

Proposition 7.4.1. *(i) The firms prefer the network monopoly regime S5 over regime S6:*
$\Pi^{S5} > \Pi^{S6}$. *(ii) The social planner strictly prefers regime S6 over regime S5:*
$S(\tilde{Q}^{S6}, \tilde{P}^{S6}) \succ S(\tilde{Q}^{S5}, \tilde{P}^{S5})$.

The Proposition 7.4.1 is equivalent to part Chapter 5's Proposition 5.12, so the explanation follows from Proposition 5.12 (concerning I1 and M2 on page 174).

7.4.3 Network Duopoly With Simultaneous Integrated Ticketing

In regime S7, the firms collude to jointly maximise profit on the cross-service

demands and the general expression for cross-service operation profit is:[3]

$$\Pi_x^{S7} = P_x(Q_{mn} + Q_{nm}) \qquad\qquad (m \neq n). \qquad\qquad (7.54)$$

Substituting (7.37) into (7.54) and maximising with respect to

$P_x(= P_{nm} = P_{mn}; m \neq n = 1, 2)$ yields the following expression for the integrated ticket price

in terms of the single-service prices, P_{mm} ($m = 1, 2$):

$$P_x = \frac{\alpha + P_{mm} + P_{nn}}{2(\beta_x - 1)}. \qquad\qquad (7.55)$$

Assuming each firm takes an equal share of the profits from the integrated ticket, the

general expression for the profit of firm i is given by:

$$\Pi_i^{S7} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}), \qquad\qquad (m = i, n = j). \qquad\qquad (7.56)$$

Substituting (7.37) in (7.56) and maximising with respect to P_{mm}, and solving using (7.55),

gives the following equilibrium prices:

$$P_x^{S7} = \frac{(2\beta\beta_x + \beta_x - 2\beta - 2)}{2\left(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3\right)}, \qquad\qquad (7.57a)$$

$$P_{mm}^{S7} = \frac{\alpha(2\beta_x^2 - \beta - 1)}{2\left(2\beta\beta_x^2 - 4\beta\beta_x - \beta_x^2 + 3\beta - 3\beta_x + 3\right)}. \qquad\qquad (7.57b)$$

Using (7.57) in (7.37) yields the following equilibrium demand for cross-service and

single-service ticket prices, respectively:

$$Q_{mm}^{S7} = \frac{\alpha(2\beta\beta_x^2 - 4\beta\beta_x + \beta^2 - 4\beta_x + 1 + 2\beta)}{2(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta_x - \beta_x^2 + 3 - 3\beta_x)}, \qquad\qquad (7.58a)$$

$$Q_{mn}^{S7} = \frac{\alpha(2\beta\beta_x^2 + \beta_x^2 - 4\beta\beta_x - 3\beta_x + 2 + 2\beta)}{2(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta_x - \beta_x^2 + 3 - 3\beta_x)}. \qquad\qquad (7.58b)$$

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[3] Note, given the equilibrium prices for the integrated ticket always exceed those for the single-service ticket, only passengers wishing to travel cross-service will purchase an integrated ticket: the two are synonymous in this model.

Using (7.57) and (7.58) in (7.56) and summing over both firms, aggregate profit across the network is:

$$\tilde{\Pi}^{S7} = \frac{\alpha^2[\beta(-\beta^2-7\beta-11)}{\beta_x(-7\beta_x^2-3\beta_x+12)-5}$$

$$\tilde{\Pi}^{S7} = \frac{\beta\beta_x(+4\beta\ \beta_x^2-12\beta\ \beta_x+16\beta+4\beta_x^3-4\beta_x^2-14\beta_x+28)]}{2\left(2\beta\beta_x^2-4\beta\beta_x+3\beta-\beta_x^2-3\beta_x+3\right)^2}. \tag{7.59}$$

We now use (7.58) and (7.59) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S7} = \frac{\alpha\left(4\beta\beta_x^2-8\beta\beta_x+\beta^2+4\beta+\beta_x^2-7\beta_x+3\right)}{\left(2\beta\beta_x^2-4\beta\beta_x+3\beta-\beta_x^2-3\beta_x+3\right)}, \tag{7.60}$$

$$\tilde{P}^{S7} = \frac{\begin{array}{c}\alpha[\beta(-\beta^2-7\beta-11)\\ \beta_x(-7\beta_x^2-3\beta_x+12)-5\\ +\beta\beta_x(4\beta\ \beta_x^2-12\beta\ \beta_x+16\beta+4\beta_x^3-4\beta_x^2-14\beta_x+28)]\end{array}}{\begin{array}{c}\left(2\beta\beta_x-4\beta\beta_x+3\beta-\beta_x^2-3\beta_x+3\right)\\ \left(4\beta\beta_x^2-8\beta\beta_x+\beta^2+4\beta+\beta_x^2-7\beta_x+3\right)\end{array}} \tag{7.61}$$

A comparison between S5 and S7 leads us to Proposition 7.2.2:

Proposition 7.4.2. *(i) The firms prefer the network monopoly regime S5 over duopoly regime S7: $\Pi^{S5} > \Pi^{S7}$. (ii) The social planner strictly prefers regime S7 over regime S5:*

$S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S5}, \tilde{P}^{S5})$.

The Propositions 7.4.2 is equivalent to Chapter 5's Proposition 5.5 and the explanation follows from Proposition 5.5 (on page 165).

A comparison between S6 and leads us to Proposition 7.2.3:

Proposition 7.4.3. *(i) The firms prefer the network monopoly regime S6 over duopoly regime S7: $\Pi^{S6} > \Pi^{S7}$. (ii) The social planner strictly prefers regime S7 over regime S6:*

$S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S6}, \tilde{P}^{S6})$.

The Proposition 7.4.3 is equivalent to part of Chapter 5's Propositions 5.12 and the explanation follows from Proposition 5.12 (concerning I1 and D2 on page 174).

7.4.4 Network Duopoly With Independently Priced Integrated Ticketing

We now introduce regime S8, where the duopolists are not allowed to collude on any aspect of pricing in the network. The integrated ticket price is the sum of these two component prices:

$$P_m = \sum P_{xm}, \qquad\qquad (m = 1, 2). \qquad\qquad (7.62)$$

Given (7.62) the general expression for the profit of firm m is given by, $\Pi_m{}^{S4}$:

$$\Pi_m{}^{S8} = P_{mm}Q_{mm} + P_{xm}(Q_{mn} + Q_{nm}), \qquad (m \neq n = 1, 2). \qquad (7.63)$$

Using (7.37) and (7.62) in (7.63) and maximising with respect to P_{xm} and P_{mm} for $m = 1, 2$ yields the following equilibrium expressions for the cross-service and single-service ticket prices, respectively:

$$P_x^{S8} = P_{x1}^{S8} + P_{x2}^{S8} = \frac{4\alpha(\beta+1)}{3(2\beta\beta_x - \beta_x - 2\beta - 5)}, \qquad (7.64a)$$

$$P_{mm}^{S8} = \frac{\alpha(\beta_x + 1)}{(2\beta\beta_x - \beta_x - 2\beta - 5)}. \qquad (7.64b)$$

Substituting (7.64) in (7.37), yields the equilibrium demands for single-service and cross-service, respectively:

$$Q_{mn}^{S8} = \frac{\alpha}{3}, \qquad (7.65a)$$

$$Q_{mm}^{S8} = \frac{(3\beta\beta_x - \beta - 4)}{3(2\beta\beta_x - \beta_x - 2\beta - 5)}. \qquad (7.65b)$$

Profit across the network follows from substituting (7.64) and (7.65) into (7.63) and summing across the two firms:

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$$\tilde{\Pi}^{S8} = \frac{2\alpha^2 \left(9\beta\beta_x^2 + 10\beta\beta_x + 8\beta^2\beta_x - 8\beta^2 - 16\beta_x - 31\beta - 32\right)}{9\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)^2}. \tag{7.66}$$

We now use (7.65) and (7.67) to form total patronage on the network ($\tilde{Q}$) and average (per passenger) fare ($\tilde{P}$) as we did at the end of 5.3.1:

$$\tilde{Q}^{S8} = \frac{2\alpha\left(5\beta\beta_x - \beta_x - 3\beta - 9\right)}{3\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)}, \tag{7.67}$$

$$\tilde{P}^{S8} = \frac{\alpha\left(9\beta\beta_x^2 + 10\beta\beta_x + 8\beta^2\beta_x - 8\beta^2 - 16\beta_x - 31\beta - 32\right)}{3\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)\left(5\beta\beta_x - \beta_x - 3\beta - 9\right)}. \tag{7.68}$$

A comparison between S5 and S8 leads us to Proposition 7.2.4:

Proposition 7.4.4. *(i) The firms prefer regime S5 over regime S8: $\tilde{\Pi}^{S5} > \tilde{\Pi}^{S8}$ ii) The social planner at least weakly prefers regime S8 to regime to S5 when $\beta_x < \beta < 6$ or when β is high and β_x is low. When $\beta > \beta_x > 6$ the social planner at least weakly prefers regime S5 to S8: $S(\tilde{Q}^{S8}, \tilde{P}^{S8}) \succeq S(\tilde{Q}^{S5}, \tilde{P}^{S5})$ when $\beta_x < \beta < 6$ or when β is high and is β_x low. $S(\tilde{Q}^{S5}, \tilde{P}^{S5}) \succeq S(\tilde{Q}^{S8}, \tilde{P}^{S8})$ when $\beta > \beta_x > 6$.*

The first part of Proposition 7.2.4 is equivalent to the first part of Chapter 5's Proposition 5.7, so the explanation follows from Proposition 5.7 (on page 168). However, when β is high and β_x is low there is an additional element in the proposition as S8 (D3) is preferred by the social planner to S5 (D4), where as previous this was only the case when β was low. When both β and β_x are high then the complementarities in S8 dominate and give rise to vertical externalities, which are internalised in regime S5, that cause average prices in S8 to be above those in S5. These high prices result in lower profits and patronages than in regime S5. However, when β and β_x are low or when just β_x is low the substitutibilities in S8's dominate and give rise to horizontal externalities, which are

internalised in regime S5, that cause average prices in S8 to be below those in S5. These low prices result in lower profits and higher patronages than in regime S5.

A comparison between S6 and S8 leads us to Proposition 7.2.5:

Proposition 7.4.5 *(i) The firms prefer regime S6 over regime S8 when $\beta > \beta_x > 4.78$: $\tilde{\Pi}^{S8} > \tilde{\Pi}^{S6}$ when $\beta > \beta_x > 4.78$ ii) The social planner at least strictly prefers regime S6 to regime to S8: $S(\tilde{Q}^{S6}, \tilde{P}^{S6}) \succ S(\tilde{Q}^{S8}, \tilde{P}^{S8})$.*

The first part of Proposition 7.2.5 is equivalent to part of Chapter 5's Proposition 5.12 and the explanation follows from Proposition 5.12 (concerning I1 and D3 on page 175).

A comparison between S7 and S8 leads us to Proposition 7.2.6:

Proposition 7.4.6 *(i) The firms prefer regime S7(S8) over regime S8(S7) when $\beta > \beta_x > 6.51$ ($\beta_x < \beta < 6.51$ or β is high and β_x is low) : $\tilde{\Pi}^{S7} > \tilde{\Pi}^{S8}$ ($\tilde{\Pi}^{S8} > \tilde{\Pi}^{S7}$) when $\beta > \beta_x > 6.51$ ($\beta_x < \beta < 6.51$ or β is high and β_x is low). ii) The social planner strictly prefers regime S7 to regime to S8: $S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S8}, \tilde{P}^{S8})$.*

The second part of Proposition 7.2.6 is equivalent to Chapter 5's Proposition 5.8, so the explanation also follows (on page 168). However, when β is high and β_x is low there is an additional element as S8 (D3) is preferred by the social planner to S7 (D2), previously this was only the case when β was low. When β and β_x are low or just β_x is low then substitutabilities give rise to a horizontal externality that causes firms in S7 to decrease prices below that in S8 and result in lower profits and higher patronages for regime D2. When β and β_x both grow then the explanation follows from Proposition 5.8 (on page 168).

7.5 Own-Price Asymmetry: Conclusion

In this investigation we further explore Chapter 5's simple model of integrated ticketing for both monopoly and duopoly markets by introducing the term β_x and allowing $\beta > \beta_x$. During this section we clarify the effect of introducing β and β_x on the results of Chapter 5.

Allowing $\beta > \beta_x$ has no real impact on comparisons between S5 (M2) and S6 (I1), S5 (M2) and S7 (D2), S6 (I1) and S7 (D2), and S6 and S8. However, the relative size of β and β_x does give additional elements to the profit comparison between duopoly regimes S7 (I1) and S8 (D3), and the welfare comparison between S5 (M2) and S8 (D3). The firms now prefer S8 (D3) to S7 (I1) not only when β is small, but also when β is high and β_x is low. An asymmetry in β sees the social planner will weakly prefer S8 (D3) to S5 (M2) when β and β_x are small and also when β is high and is β_x low.

Overall, again, the firm's best choice is S5 (M2) and society's best choice is S7 (D2). When β is small or β is high and β_x is low, then any duopoly regime that results from the splitting up of the monopoly regime will be preferred by society. This adds to Chapter 5's result that an integrated ticketing duopoly is "generally" (from the view of society) better at promoting public transport use than an integrated ticketing monopoly. This chapter, again, highlight that the regulator must be fully aware of the market's characteristics before making their decision concerning what is best for the society.

CHAPTER EIGHT

INTEGRATED TICKETING –

INTER-CHAPTER COMPARISONS

8.1. Introduction

In this part of the thesis, we have been looking at a theoretical model of integrated ticketing on a transport network, so that we can examine whether integrated ticketing leads to the promotion of public transport. In Chapter 4, we apply *ES*'s modelling framework to a transport network. Chapter 5 built on this framework and develops a model of integrated ticketing in transport. In Chapter 6 and Chapter 7, we extend Chapter 5's model to include a firm behavioural parameter, and allow both price elasticities and demand levels to vary between cross-service and single-service demands, respectively. The four chapters in Part B provide many observations and recommendations concerning the circumstances where forms of integrated ticketing are preferable to firms and society, creating options that the regulator can explore by encouraging the regime they find best for a given situation.

The various outcomes of the chapters also require some inter-chapter comparisons and we will do this here. To begin we summarise the results from the Chapters 4, 5, 6, and 7 in Tables 8.1, 8.2, 8.3, 8.4, and 8.5, respectively. We then go on to explain and highlight the main differences between these chapters as well as explaining their importance.

In the following section, Section 8.3 and Section 8.4 we compare Chapters 4 and 5, Chapters 5 and 6, and Chapters 6 and 7, respectively. Finally, Section 8.5 will conclude and sum up the results of the comparisons.

Table 8. 1: Chapter 4 Summary

	Independent Ownership	Composite Good Competition	Parallel Vertical Integration	Optimal Regulation
Joint Ownership	$\Pi^M > \Pi^{SO}$ $W^M > W^{SO}$	$\Pi^M > \Pi^{RO}$ $W^{RO} > W^M$	$\Pi^M > \Pi^V$ $W^M > W^V$ if $\beta > 7$	$\Pi^M > \Pi^{SP}$ $W^{SP} > W^M$

Table 8. 2: Chapter 5 Summary

Regime Ranking with Double Price Rule: Non-Integrated Demands (5.3)

Profit Comparison (1ˢᵗ place represents the highest profit)

$4 \le \beta \le 4.78$	$4.78 \le \beta \le 6.51$	$\beta > 6.51$
M2	M2	M2
D3	I1	I1
I1	D3	D2
D2	D2	D3
	M1 (not valid if $\beta \le 5$)	M1
	D1 (not valid if $\beta \le 5$)	D1

Welfare Comparison (1ˢᵗ placed represents the highest welfare)

$5 < \beta \le 6$	$6 < \beta \le 7.68$	$7.68 < \beta \le 8$	$8 < \beta$
D2	D2	M1	M1
M1	M1	D2	D2
I1	I1	I1	I1
D1	D1	D1	M2
D3	M2	M2	D1
M2	D3	D3	D3

Regime Ranking with One-and-a-Half Price Rule: Non-Integrated Demands (5.6)

Profit Comparison (1ˢᵗ place represents the highest profit)

$4 < \beta \le 4.76$	$4.76 \le \beta \le 4.78$	$4.78 \le \beta < 4.89$	$4.89 \le \beta \le 6.51$	$6.51 < \beta$
M2	M2	M2	M2	M2
D3	D3	I1	I1	I1
I1	I1	D3	D3	D2
M3	M3	M3	D2	D3
D4	D2	D2	M3	M3
D2	D4	D4	D4	D4

Welfare Comparison (1ˢᵗ placed represents the highest welfare)

$4 < \beta < 4.47$	$4.47 < \beta \le 6$	$6 < \beta < 7.34$	$7.34 < \beta$
D2	M2	M2	M2
I1	D3	I1	I1
D4	I1	D3	D3
D3	M3	M3	D2
M3	D2	D2	M3
D2	D4	D4	D4

Table 8. 3: Chapter 6 Summary

	R2 (I1)	R4 (D3)
R1 (M2)	$\Pi^{R1} > \Pi^{R2}$ if $\gamma_{R2} < 1$ $W^{R2} \succ W^{R1}$ if $\gamma_{R2} < 1$	$\Pi^{R1} > \Pi^{R4}$ if $\gamma_{R4} < 1$ $W^{R4} \succ W^{R1}$ if β is high and γ is low.
R4 (D3)	-	$\Pi^{R2} > \Pi^{R4}$ if β is large or γ_{R2} is significantly larger than γ_{R4} $W^{R4} \succ W^{R2}$ when β is small and γ_{R2} is significantly larger than γ_{R4}

Table 8. 4: Chapter 7 Summary (1)

	S2 (D2)	S3 (I1)	S4 (D3)
S1 (M2)	$\Pi^{S1} > \Pi^{S2}$ $W^{S2} \succ W^{S1}$	$\Pi^{S1} > \Pi^{S3}$ $W^{S3} \underset{\sim}{\succ}(\succ)W^{S1}$ if $\alpha < (\geq)0.55$ and α_x is low.	$\Pi^{S1} > \Pi^{S4}$ $W^{S1} \succ W^{S4}$ if β is large
S2 (I1)	-	$\Pi^{S2} > \Pi^{S3}$ $W^{S3} \succ W^{S2}$	$\Pi^{S2} > \Pi^{S4}$ if β is large $W^{S1} \succ W^{S4}$
S3 (D2)	-	-	$\Pi^{S3} > \Pi^{S4}$ if β is large $W^{S3} \succ W^{S4}$

Table 8. 5: Chapter 7 Summary (2)

	S6 (D2)	S7 (I1)	S8 (D3)
S5 (M2)	$\Pi^{S5} > \Pi^{S6}$ $W^{S6} \succ W^{S5}$	$\Pi^{S5} > \Pi^{S7}$ $W^{S7} \succ W^{S5}$	$\Pi^{S5} > \Pi^{S8}$ $W^{S5} \succ W^{S8}$ if $\beta_x < \beta < 6$ or β_x is small and β is large $W^{S5} \succeq W^{S8}$ if $\beta > \beta_x \geq 6$
S6 (I1)	-	$\Pi^{S6} > \Pi^{S7}$ $W^{S7} \succ W^{S6}$	$\Pi^{S6} > \Pi^{S8}$ if $\beta > \beta_x > 4.78$ $W^{S6} \succ W^{S8}$
S7 (D2)	-	-	$\Pi^{S7} > \Pi^{S8}$ if $\beta > \beta_x > 6.51$ $W^{S7} \succ W^{S8}$

8.2 Chapter 4 and Chapter 5 Comparison

The results from Chapter 4 can be found in Table 8.1. In Chapter 4's model we establish an appropriate framework, which we could then extend to transport and integrated ticketing. The use of the joint ownership regime as an approximation of an integrated ticket regime highlights the possible effects that the introduction of an integrated ticketing regime may have. The welfare in Chapter 4's model arising from a monopoly or joint ownership

regime is greater than the welfare arising from a separate ownership and we also find the welfare arising from composite good competition to be larger than the welfare arising from the monopoly or joint ownership regime. These are two results that embody Cournot's (1838) observations regarding competition and complementary monopoly that we explained in Chapter 3.

Chapter 4 underlines the effects that would be important when we consider integrated ticketing. However, the assumption that the joint ownership or monopoly regime was an approximation of integrated ticketing is overly simplistic and unrealistic. It ignores the impact an integrated ticket has on demands, the possibility that a non-monopoly regime could provide integrated tickets, and the chance that a monopoly regime may not provide integrated tickets – these are issues we seek to address in Chapter 5, which uses Chapter 4's demand framework, including several of the simplifications we made to the model, but with a more reasonable representation of a transport network with integrated ticketing plus a set of more realistic regimes. Chapter 5 leads to a more complicated set of results that we summarise using profit and welfare rankings in Table 8.2.

The difference in regimes and demand structure make direct comparisons difficult, but we can still see the importance of Counot's observations. Unsurprisingly, we find that firms, again, prefer a monopoly regime to duopoly regime. The complicated results concerning whether integrated ticketing or non-integrated ticketing is preferred serve to further highlight the over-simplification in Chapter 4. During Chapter 5 we see the emergence of the possibility that completely independent pricing in regime D3 can lead to inferior welfare results and this can be considered a similar result, and process, to when we find that the is monopoly ownership is preferred to independent ownership in Chapter 4 – see appropriate cells in Table 8.1 and 8.2. In Chapter 5 we also find it is possible that a well-structured integrated ticketing duopoly regime (i.e. D2) can lead to superior welfare

results and thus the encouragement of public transport use. This result, and the intuition, is comparable to the welfare gains a route operation regime has over the monopoly regime in Chapter 4 – again, see appropriate cells in Table 8.1 and 8.2. The possibility that integrated ticketing could lead to lower prices and increased patronages, as found in Chapter 5, stands in contrast to the result we found in Chapter 4.

8.3 Chapter 5 and Chapter 6 Comparison

Chapter 5 establishes a model of integrating ticketing in a transport network in Chapter 6 we alter the model to include a conjectural variations term. We should note that the chapter's conclusions are based on the assumption that price collusion takes place where firms behave in the way prescribed by conjectural variations. We choose to introduce a conjectural variations term in the network duopoly with independently priced integrated ticketing D3, and network duopoly with pre-emptive integrated ticketing I1 regimes – a network monopoly with integrated ticketing M2 regime is included, but by definition has no conjecture as there is only one firm.

The conjectural variation term has a more significant impact is on Chapter 5's result in the comparison between R2 (Chapter 6's regime I1) and R4 (D3). The first part of Proposition 5.12 states that firms prefer regime D3 (R4) to I1 (R2) when $\beta < 4.78$ – we summarise in the appropriate cell of Table 8.3. However, once we introduce a conjecture variation term it would be reasonable to assume that the price collusion in regime R2 (I1) is significantly greater than that in R4 (D3), because regime R2 (I1) allows firms to communicate in the setting of the integrated ticket price, while in regime R4 (D3) all price decisions are separate. Once we allow γ_{R2} to be significantly larger than γ_{R4} we find that, even when β is small, that firms prefer regime R2 (I1) to R4 (D3). By introducing the

conjecture and allowing γ_{R2} to be greater than γ_{R4}, we are further exploring what may happen when integrated ticketing is allowed and in this case the communication in regime R2 (I1) could lead to price collusion, which results in higher profits for the duopolists.

The second part of Chapter 5's Proposition 5.12 implies the social planner prefers regime I1 (R2) to regime D3 (R4). However, in Chapter 6 we find that, if β is small while the collusiveness of regime R2 (I1) is significantly greater than the collusiveness of R4, (D3) then the social planner will prefer regime R4 (D3) to R2 (I1). In Chapter 5, the introduction of a conjecture results in the possibility that the social planner prefers R4 (D3) to R2 (I1). This means that the regulator, if they believe that allowing firms to cooperate when setting the integrated ticket price leads to price collusion, will prefer an independently pricing duopoly (R4/D3).

8.4 Chapter 5 and Chapter 7 Comparison

Chapter 7 extends the model we establish in Chapter 5, so that the cross-service demands has either a higher demand level or a lower own-price effect. We focus on integrated ticket regimes to see if the private or social ranking of the regime's change.

When we introduce the term α_x and allow $\alpha_x > \alpha$ we find the results are the same as Chapter 4's, but when we allow $\beta > \beta_x$ then the relative sizes of β and β_x do have an impact on results of the profit comparison concerning duopoly regimes S7 (I1) and S8 (D3), and the welfare comparison between S5(M2) and S8 (D3). In Chapter 5, we find that D3 (S8) is preferred by firms to I1 (S7) only when β was small – see Table 8.2. In Chapter 7, the firms prefer S8 (D3) to S7 (I1) not only when β is small, but also when β is high and β_x is low – see Table 8.5. We also find that with an asymmetry in β that the social planner will weakly prefer S8 (D3) to S5 (M2) when β is small, and when β is high and

is β_x low. In Chapter 5 the social planner prefers D3 (S8) to M2 (S5) only when β was small. Again, it seems likely that integrated ticketing can encourage public transport use, but as some of the results are dependant on parameter values it highlights the need for the regulator to be careful before choosing the structure of the integrated ticketing regime.

8.5 Conclusion

In this chapter we compare the results from Chapters 4, 6, and 7 with those in the integrated ticketing model that we introduced in Chapter 5. The results from Chapter 4 embody many of Cournot's competition and complementary monopoly observations concerning vertical and horizontal externalities, which highlight the mechanisms that would drive the results of Chapter 5's model. Chapters 6 and 7 look to extend Chapter 5's modelling by adding various parameters to the model and the results of these later chapters confirm that for the most part the results from Chapter 5 are robust. However, some results we find vary and this means that a regulator needs to examine the circumstances – including the current regime, the size of each demand, the own-price elasticity of each demand, and the current and possible collusion – before deciding which particular course of action is best.

PART C

TRANSPORT INTERCONNECTIVITY

CHAPTER NINE

TRANSPORT INTERCONNECTIVITY

9.1. Introduction

In this part of the thesis, we undertake a theoretical exploration into the issue of transport interconnectivity. During Part A we state there is a need to encourage public transport use in the UK and that the recent deregulation of some transport services has resulted in some areas having a local monopoly.

In this chapter we establish a model that can be used to consider whether the incentives exist for a private monopolist to give the socially desirable level of service provision. We introduce a social planner regime as a benchmark that we use to represent the preference of society and to investigate whether the private sector, or more specifically a private monopolist, would produce the network set-up that is desirable for society. We use many of the approaches and techniques that we present in Chapter 3.

In the following section we introduce a circular city model of a simple transport network. Section 9.3 considers the benchmark scenario of the welfare-maximising social planner. We then in Section 9.4 look at a profit-maximising network monopolist comparing the results with those of a social planner. Finally, in Section 9.5, we highlight the main result of the chapter and propose how we can investigate the issue that arises from it by introducing the possibility of entry – we do so in the next chapter.

9.2. The Network

Consider a circular city, as illustrated in Figure 9.1, with three public transport services along routes 1, 2 and 3 between the three origins and destinations r, s and t. This

Figure 9.1: Transport Interconnectivity Framework
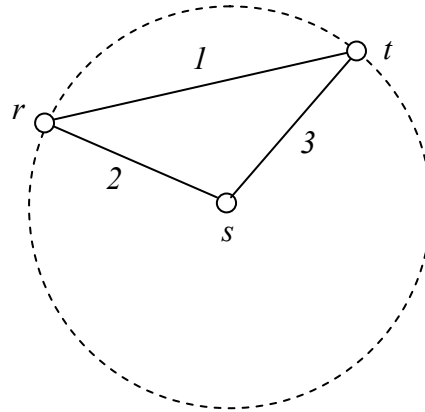
set-up serves demands for radial travel between the city centre (s) and the perimeter of the city (points r and t) as well as cross-city travel (between r and t). The simple network, therefore, allows for direct travel between points on the perimeter along route 1 as well as indirect travel between these points using routes 2 and 3. For passengers wishing to travel between r and t, the combined services along routes 2 and 3 now provide a substitute for route 1: for these passengers, services on routes 2 and 3 are complements.

The regulatory interest in this chapter concerns how to ensure the public transport provider operates the direct cross-city service (route 1) when it is socially optimal to do so, given the existence of the combined (substitute) services along routes 2 and 3.

Assumption 9.1: Services on radial routes 2 and 3 are always provided.

It is later shown that if services on route 1 are provided then the transport operator will optimally charge such a fare for these services that there is no demand for cross-city travel via routes 2 and 3. An important consequence of this is that demand for indirect travel only occurs when services on the corresponding direct route are not provided, hence from Assumption 9.1 no combination of services (other than 2 and 3) will be jointly consumed in this framework and we disregard the possibility that either radial route 2 or 3 is not provided – in Section 3 we justify this assumption.

By definition, routes 2 and 3 are of equal length (the radius of the circular city), which for simplicity is normalised to unity. Therefore, route 1 is the chord joining r and t with length x. In this framework, x lies in the range $0 < x < 2$.

Suppose the relevant public transport provider charges a uniform fare f_i for travel on route I ($i = 1,2,3$). Let the demand for direct travel along route i be given by:

$$Q_1 = \alpha - x - f_1,$$ (9.1a)

$$Q_j = \beta - 1 - f_j,$$ ($j = 2,3$). (9.1b)

where α and β are positive constants. To ensure that (9.1a) and (9.1b) are positive, at least at zero fares, and given $0 < x < 2$, let:

$$\alpha \geq 2, \quad \beta > 1.$$ (9.2)

For simplicity, we assume that the psychological passenger cost per unit distance is unity; thus for passengers on routes 2 and 3 (which have unit length) the relevant cost is also unity, whilst for passengers of route 1 it is x. Wes also assume that all services travel at an equal (constant) speed; hence there is no need to introduce a separate time-cost parameter in the generalised travel cost. It follows that the generalised cost of direct travel along mn ($m \neq n = r,s,t$), G_{mn}, is given by:

$$G_{rt} = f_1 + x, \quad G_{rs} = f_2 + 1, \quad G_{st} = f_3 + 1.$$ (9.3)

Clearly each journey can be undertaken directly using one route or indirectly using the remaining two routes. This chapter allows the provision of services along route 1 to be an option for the relevant service provider. In order to compare the gains to the relevant operator with and without services on route 1, and to incorporate the fact that pricing decisions on route 1 must be undertaken in the knowledge that too high a fare may divert passengers onto routes 2 and 3, it is necessary to consider the demands for the alternative

journey rt via s. Assuming that there is no interchange penalty, the generalised cost for a

passenger making indirect travel along mn ($m \neq n \neq l = r,s,t$), $G_{m\ln}$, is given by:

$$G_{rst} = 2 + f_2 + f_3, \quad G_{str} = 1 + x + f_1 + f_3, \quad G_{trs} = 1 + x + f_1 + f_2.$$ (9.4)

If f_1 is prohibitively high or services on route 1 are not provided, then all rt travel is

diverted through routes 2 and 3. Hence, total demand for travel on route j (direct and

indirect), $\hat{Q}_j$, becomes:

$$\hat{Q}_j = \alpha + \beta - 3 - 2f_j - f_k, \qquad (j \neq k = 2,3).$$ (9.5)

In terms of the cost structure of the model, it is assumed for simplicity that public

transport provision on a route has a zero marginal cost per passenger, but a non-zero

operating cost that is proportional to the length of that route. This means that there are no

capital costs in the model. Setting F as the operating cost per unit distance, it follows that

the operating cost for routes 2 and 3 (which both have unit length) is F whilst for route 1

the operating cost is Fx.

9.3. First-Best Social Planner

This section considers the conditions under which a first-best social planner would

choose to provide services along route 1. In Part B of this thesis we consider how prices

and quantities changed with regime changes, but now we are concerned with the desirable

network set-up for society, so we use a social planner regime to represent the preferences of

society. With the social planner engaging in marginal-cost pricing, our assumption of zero

marginal cost implies a fare of zero on each route:

$$f_i^S = 0, \qquad (\forall i = 1,2,3).$$ (9.6)

It follows immediately that network revenue under the social planner is zero and hence welfare is derived solely from consumer surplus. In the case where the social planner provides services on route 1, consumer surplus on each route, C_i^S, follows from (1):[1]

$$C_1^S = \frac{1}{2}(\alpha - x)^2 ,$$
(9.7a)

$$C_j^S = \frac{1}{2}(\beta - 1)^2 , \qquad (j = 2,3).$$
(9.7b)

Total consumer surplus across the system, C^S, is then

$$C^S = \sum_{i=1}^{3} C_i^S = \frac{1}{2}(\alpha - x)^2 + (\beta - 1)^2 .$$
(9.8)

Welfare under the social planner with route 1 provided, W^s, is just consumer surplus minus the operating costs of operating three routes:

$$W^S = C^S - (2 + x)F = \frac{1}{2}(\alpha - x)^2 + (\beta - 1)^2 - (2 + x)F.$$
(9.9)

If the social planner does not provide route 1, consumer surplus is measured in relation to (9.5). However, to calculate welfare correctly we need to separate demands into travellers, who are diverted from route 1, (Q_1'), direct route 2 travellers and direct route 3 travellers (Q_2' and Q_2' respectively):

$$Q_1' = \alpha - 2 - (f_2 + f_3),$$
(9.10a)

$$Q_2' = \beta - 1 - f_2,$$
(9.10b)

$$Q_3' = \beta - 1 - f_3.$$
(9.10c)

Aggregate consumer surplus, $\hat{C}^S$, is then

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[1] Note, with zero fares on all routes and given $x < 2$, it follows that the generalised cost of travel for *rt* passengers would always be lower for direct travel on route 1 than indirect travel via routes 2 and 3. Hence (1) provides the relevant demands for this case.

$$\hat{C}^S = \sum_{j=2}^{3} \hat{C}_j = \frac{1}{2}(\alpha - 2)^2 + (\beta - 1)^2. \tag{9.11}$$

Welfare for the case of the social planner with routes 2 and 3, $\hat{W}^S$, becomes:

$$\hat{W}^S = \frac{1}{2}(\alpha - 2)^2 + (\beta - 1)^2 - 2F. \tag{9.12}$$

To ensure that $\hat{W}^S > 0$ we require:

$$F < \frac{1}{4}[(\alpha - 2)^2 + 2(\beta - 1)^2].$$

In order for the framework to be consistent with route 1 being the marginal route from the social planner's perspective it must be the case that this combination of routes should be at least weakly preferred to any other pair-combination. By comparing the social planner's welfare-maximising decisions when providing any other pair-combination, we can derive Lemma 9.1:

Lemma 9.1. *The welfare-maximising social planner will weakly prefer to supply routes 2 and 3 over any other pair-combination if*

$$F > \frac{2(\alpha - 1) + x(x + 1 - \alpha - \beta)}{(x - 1)}. \tag{9.13}$$

The main purpose of Lemma 9.1 is to ensure the social planner's view is that route 1 is the marginal route, as we set out in Assumption 9.1. Given (9.13), it is therefore not necessary for us to impose a further constraint on the system to ensure that parameter values satisfy the voluntary provision of these services.

Comparing the welfare from the social planner's complete network (with route 1) with the welfare from the social planner's incomplete network (without route 1) leads us to Proposition 9.1:

Proposition 9.1. *The welfare-maximising social planner would prefer to provide a complete network if the operating cost per unit distance satisfies the inequality:*

$$F < \frac{1}{2x}\left[(\alpha - x)^2 - (\alpha - 2)^2\right]. \tag{9.14}$$

Proposition 9.1 gives a value of the operating cost that ensures the social planner will provide a complete network. If the level of operating costs were the same as the RHS of (9.14) then the social planner would be indifferent between supplying route 1 and not supplying route 1.

Considering the parameters in (9.14) yields Corollaries 9.1 and 9.2.

Corollary 9.1. *If the operating cost per unit distance is zero, the welfare maximising social planner will always provide a complete network.*

Corollary 9.2. *The welfare-maximising social planner would prefer to provide a complete network for an increasing range of operating cost of operation per unit distance as α (x) rises (falls).*

As x falls the benefit that society gets from a direct route increases, which means that the social planner would increasingly want to supply it. As α increases the number of travellers who wish to travel along route 1 increases and means that the surplus the social planner gains from the provision of a complete network increases.

Since the social planner is the benchmark case, we set (9.14) as an equality, giving the level of operating cost (per unit distance), which makes the social planner indifferent between providing a service on route 1 and not providing a service, $\tilde{F}^S$:

$$\tilde{F}^S = \frac{1}{2x}\left[(\alpha - x)^2 - (\alpha - 2)^2\right]. \tag{9.15}$$

This is defined as the social planner's threshold operating cost. For any $F < \tilde{F}^S$ the social planner would choose to supply a complete network.

9.4. Network Monopoly

The case of network monopoly is more complicated than that of the social planner, since the former has to select the network structure (whether or not to provide route 1) and the optimum combination of fares, whilst the latter's price policy is independent of choice of routes. Matters are, however, made more straightforward by the symmetrical nature of routes 2 and 3. It follows that whatever the monopolist's choice of network configuration and fare structure, the optimal fare for route 2 will be the same as that for route 3. Dealing with the scenario where the network monopolist supplies a complete network, the general expression for network profit, Π^M, is then:

$$\Pi^M = f_1(\alpha - x - f_1) + 2f_j(\beta - 1 - f_j) - (2 + x)F .$$ (9.16)

where f_j is the common fare on routes 2 and 3. Maximising profit yields the following fares:

$$f_1 = \frac{\alpha - x}{2},$$ (9.17a)

$$f_j = \frac{\beta - 1}{2} .$$ (9.17b)

Substituting (9.17) into (9.16) gives the reduced-form network monopoly profit:

$$\Pi^M = \frac{1}{4}(\alpha - x)^2 + \frac{1}{2}(\beta - 1)^2 - (2 + x)F .$$ (9.18)

If, however, the network monopolist supplies an incomplete network (omitting route 1), the general expression for network profit is, $\hat{\Pi}^M$:

$$\hat{\Pi}^M = 2f_j(\alpha + \beta - 3 - 3f_j) - 2F .$$ (9.19)

Profit maximisation yields the following fares:

$$f_j = \frac{1}{6}(\alpha + \beta - 3) .$$ (9.20)

Substituting (9.20) into (9.19) gives the reduced-form expression for maximum monopoly profit with an incomplete network:

$$\hat{\Pi}^M = \frac{1}{6}(\alpha + \beta - 3)^2 - 2F. \tag{9.21}$$

Comparing the profits of the incomplete network and the complete network leads us to Proposition 9.2.

Proposition 9.2. *The network monopolist would prefer to provide a complete network if the operating cost per unit distance satisfies the inequality:*

$$F < \frac{\alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x}. \tag{9.22}$$

This basis of the proposition is similar to Proposition 9.1, but now instead of the social planner we have a network monopolist. If (9.22) were an equality (instead of an inequality) then network monopolist would be indifferent between supplying route 1 and not supplying route 1, and we call this the network monopolist's threshold operating cost.

Corollary 9.3. *If the operating cost per unit distance is zero, the network monopolist will always prefer to provide a complete network.*

Like the social planner, the network monopolist would always prefer to supply a complete network if there is no operating cost, as they will always be able to extra surplus from route 1 travellers. In the absence of operating cost the private monopolist will provide the socially-preferred network.

If a fixed cost is present it can be seen that equations (9.14) and (9.22) do not coincide; hence, the monopolist will not always provide a complete network when a social planner would. To illustrate this point and to understand the different impacts of the variables, we can consider the monopolist's decisions when it faces the social planner's threshold operating cost ($\tilde{F}^S$) – in other words, investigate the effects of the variables on

the monopolist's network provision when they face the social planner's threshold operating cost. This leads us to Proposition 9.3 and 9.4:

Proposition 9.3. *If faced with the social planner's threshold operating cost then the range of values of the operating cost per unit distance for which the monopolist will provide a complete network increases (decreases) as x increases (decreases).*

When the monopolist is faced with an operating cost, which makes the social planner indifferent between providing route 1 and not providing route 1, they do not always agree with the social planner – this is highlighted by the impact the variables have on the monopolist's provision of route 1. One such variable is x, as x increases the value of the social planner's threshold operating cost decreases so the cost that the monopolist faces as a result of providing a complete network falls and this leads to increased profits from the complete provision despite the revenue falling as a result of the increases in x. As x increases the monopolist finds a greater range of values of the operating cost that it would provide a complete network for (whilst the social planner remains indifferent).

Proposition 9.4 considers the effects of the other variables.

Proposition 9.4. *(i) The monopolist, when faced with the social planner's threshold operating cost per unit distance, will, if β is high compared to α, offer a complete network for an increasing (decreasing) range of operating cost per unit distance as α decreases (increases) and as β increases (decreases). (ii) The monopolist, when faced with the social planner's threshold operating cost per unit distance, will, if β is low compared to α, offer a complete network for an increasing (decreasing) range of operating cost per unit distance for as α increases (decreases) and as β decreases (increases).*

When β is low compared to α the relative importance of rt direct travellers to the monopolist is higher and as α increases further then the producer surplus that the

monopolist gains from providing route 1 increases; meaning complete network provision is preferable for the monopolist. However, if β is high compared to α then the extra social planner's threshold operating cost that is incurred by the monopolist in the provision of route 1 has a larger impact. In this situation, an increase in α increases the social planner's threshold operating cost and reduces the monopolist's profit from supplying a complete network. As α gets higher then the provision of an incomplete network becomes attractive because the extra gain in revenue from a complete network due to increased α is not as big as the increase in the cost.

9.5 Conclusions

The main result of this chapter, in which we establish a circular city model and use it to investigate network service provision, is that when there is cost associated with operating a route then a network monopolist would not always offer a complete network when a social planner would – as highlighted by the varying effects of the variables in Propositions 9.3 and 9.4. Following the deregulation of transport in the UK many public transport networks are no longer under the control of a social planner and some areas find situations similar to monopolist operation.

The possibility that monopolist provision of the network may differ from that of a social planner, whilst not surprising, does mean that we need to investigate ways of improving the coincidence of what the market may provide and what society prefers. In the next chapter we shall explore another ownership regime to investigate whether the introduction of competition on a sub-section of the network can cause the monopolist to alter their behaviour to ultimately result in network interconnection more akin to that which a social planner would provide.

CHAPTER TEN

TRANSPORT INTERCONNECTIVITY WITH REGULATION AND COMPETITION

10.1. Introduction

In Chapter 9, we establish a network model of transport interconnectivity and consider a benchmark social planner regime and a network monopoly regime. We find that when an operating cost is present that a monopolist and social planner may not agree on the interconnectivity of the network.

In this chapter, we extend the model by introducing the possibility that a regulator allows entry on a sub-section of the network. This creates a new set of potential scenarios, which we can investigate and enables us to make further policy comment concerning how a monopolist behaves if entry is allowed. We can now consider the effect this has on the profit of the former network monopolist (now the incumbent). If profit is severely reduced by entry then it is possible that the incumbent may attempt to block entry and it could be achieved by using the network configuration; we look at how and when this could happen.

In the following section we use the demand system that we introduce in Chapter 9 to calculate prices, quantities, profits, and welfares for a route 3 entry regime. Finally, in Section 10.3, we conclude the results of the regime comparisons. We make reference to the recommendations concerning integrated transport we introduce in Chapter 2.

10.2 Entry on a Sub-Set of the Network: Price Competition

In this section we consider the impact upon the network monopolist's fare structure and network configuration when rival firms enter the network. Initially we shall look at the

effect on network provision and the monopolist's behaviour when entry is allowed onto route 1. Then we move on to look at entry on the other routes and pose the question "will the introduction of a rival on route 3 cause the network monopolist (henceforth, incumbent) to pursue a complete network (when previously incomplete)?" by considering the case where entry on route 3 leads to a Cournot quantity game on that route. The analysis is restricted to only those cases when the operating costs of operation are low enough to accommodate n firms on route 3 under the Cournot regime. This means that the incumbent can in the provision of route 1 use it as a strategic tool to reduce profitability on route 3 and hence exclude the entrant(s).

As a simple starting point let us assume that one entrant is allowed[1] to provide route 1, but the incumbent (previously the monopolist) is confined to only providing routes 2 and 3. The profit function for the incumbent and entrant become, respectively:

$$\Pi^M = 2f_j(\beta - 1 - f_j) - 2F, \tag{10.1a}$$

$$\Pi^{E_{R1}} = f_1(\alpha - x - f_1) - xF. \tag{10.1b}$$

Maximising profits yields the same fares as in the Chapter 9 in (9.17a) and substituting these into (10.1) gives us the reduced form profits for the incumbent and entrant, respectively:

$$\Pi^I = \frac{1}{2}(\beta - 1)^2 - 2F, \tag{10.2a}$$

$$\Pi^{E_{R1}} = \frac{1}{4}(\alpha - x)^2 - xF. \tag{10.2b}$$

---

[1] Let us assume a regulator can allow or forbid access to the network, although this is not currently the case in the bus industry in the UK.

The provision of route 1 would then be based upon whether the entrant made a positive profit, so their indifferent operating cost is:

$$\tilde{F}^{E_{R1}} = \frac{1}{4x}(\alpha - x)^2 .$$ (10.3)

Clearly (10.3) is not the same as the monopolist's indifferent fixed cost (9.22) and a comparison between (10.3) and (9.22) yields Proposition 10.1:

Proposition 10.1. *When entry is allowed on route 1 and restricted to a single entrant, with competition over price, and an operating cost, then the entrant (monopolist) will provide a complete network for a greater range of operating cost per unit distance than the monopolist (entrant), if $\alpha > \beta$ ($\alpha < \beta$).*

If an entrant is allowed on to the network then they will provide a complete network when a monopolist would not if route 1 was to have a higher level of demand than routes 2 and 3 – meaning that the introduction of an entrant could improve the provision of a complete network. However, if the demand for routes 2 and 3 is higher then it is possible that the entrant would choose not to provide route 1 when the monopolist would; hence, possibly, reducing the provision of a complete network. This means that we should investigate *n* firm entry on route 1 to ascertain the effects of allowing more entrants.

If the incumbent is allowed to enter on 1, while also providing routes 2 and 3, alongside the entrant with competition over prices prevailing then we will have Bertrand (1883) style competition that will result in prices equalling marginal cost, *MC*:

$$f_1 = MC = 0 .$$ (10.4)

When a complete network is provided assuming symmetry amongst firms the quantity demanded on route one for the incumbent and the entrants would be:

$$q_1^I = q_1^E = \frac{\alpha - x}{n_1} .$$ (10.5)

where is n_1 the total number of firms on route 1. Incumbent's fares on routes 2 and 3 are given by (9.17b), so profits for the incumbent and the entrant therefore become, respectively:

$$\Pi^I = \frac{(\beta - 1)^2}{2} - F(2 + x),$$ (10.6a)

$$. \Pi^E = -Fx .$$ (10.6b)

We can see that (10.6) is always negative if there is a positive fixed cost, so the entrant will never provide route 1. If the incumbent chooses not to provide route 1 when there is entry then the entrant's profit remains (10.6) and the incumbent's profit becomes:

$$\hat{\Pi}^I = \frac{(\beta - 1)^2}{2} - 2F$$ (10.7)

This leads us to Proposition 10.2:

Proposition 10.2. *When entry is allowed on route 1, with competition over price, and there is an operating cost, then neither the entrant nor the incumbent will provide route 1, so there is always incomplete network provision.*

With entry on route 1 resulting in Bertrand competition, then the fare on route 1 becomes zero, so an incumbent or an entrant that provides route 1 will make no revenue to cover the extra operating costs of a complete network. The lack of entry on route 1 means that an incumbent providing an incomplete network does not lose travellers to a firm on route 1, so the incumbent will not provide route 1 as it can make greater profits with an incomplete network.

Additionally, this also means that if more than one entrant were allowed on route 1, but the incumbent continued to be confined simply to routes 2 and 3, then route 1 would not be provided as Bertrand competition ensures zero prices and negative profits for the entrant.

This provides the regulator with something of a problem. If they can ensure a single entrant on route 1, then the move towards entry and away from the monopoly may increase the range of variables where a complete network will occur. However, if two or more firms (including the incumbent) enter route 1 then this will result in incomplete network provision.

Now let us focus our attention on route 3 and allow n-firm entry (including the incumbent), so that we have Bertrand (1883) style competition, which will result in prices equalling marginal cost, MC,:

$$f_3 = MC = 0. \tag{10.8}$$

When a complete network is provided assuming symmetry amongst firms the quantity demanded on route three for the incumbent, $q_3{}^I$, and the entrants, $q_3{}^E$, would be:

$$q_3{}^I = q_3{}^E = \frac{\beta - 1}{n}. \tag{10.9}$$

Fares along routes 1 and 2 would be given by (9.17a) and (9.17b) and result in profits becoming:

$$\Pi^I = \frac{(\alpha - x)^2}{4} + \frac{(\beta - 1)^2}{2} - F(2 + x), \tag{10.10a}$$

$$\hat{\Pi}^E = -F. \tag{10.10b}$$

We see that when $F > 0$ that the entrant would never actually enter as (10.10b) would be negative.

When an incomplete network is provided, marginal cost pricing on route 3 results in the profits for the incumbent and entrant becoming, respectively:

$$\hat{\Pi}^I = f_2(\alpha + \beta - 3 - 2f_2) - 2F, \tag{10.11a}$$

$$\hat{\Pi}^E = -F. \tag{10.11b}$$

Again, when $F > 0$ the entrant will never actually enter as (10.11b) would be negative. Maximising (10.11a) with respect to f_2 and re-arranging:

$$f_2 = \frac{\alpha + \beta - 3}{4}.$$

(10.12)

Substituting this back into (10.21a) gives:

$$\hat{\Pi}^I = \frac{1}{8}(\alpha + \beta - 3)^2 - 2F.$$

(10.13)

Subtracting (10.13) from (10.10a) gives us the following condition for the incumbent providing a complete network:

$$F < \frac{1}{8x}(\alpha^2 + 2x^2 - 4\alpha x + 3\beta^2 - 2\beta - 2\alpha\beta + 6\alpha - 5).$$

(10.14)

This leads to Proposition 10.3:

Proposition 10.3. *When entry is permitted on route 2 or 3 with competition over price, then it can ensure that the incumbent provides a complete network, where a monopolist would not, in accordance with the social planner's preference.*

Despite the entrant never actually entering route 3 as it would make a negative profit, the possibility of competition can force the monopolist to provide a complete network, when it would otherwise optimally prefer to provide an incomplete network in opposition to the social planner desiring a complete network. Sustainable entry threat on route 3 (or route 2) would inevitably mean that the former monopolist's (now an incumbent) profits would drop. It is possible these profits fall to a level below that of a monopolist providing an incomplete network where entry is not viable. The incumbent would prefer to once again become a monopolist, but one that provides a complete network rather than face entry on an incomplete network.

10.3 Entry on a Sub-Set of the Network: Quantity Competition

So far in this Chapter and the previous we have concentrated on price as the firms'
decision variable with the previous section modelling what happens if entry onto a part of
the network results in price competition. We saw that this resulted in Bertrand competition,
where price was set equal to marginal cost (zero in our case), but this may not always be
intuitively appealing especially when a fixed cost results in losses.

We now wish to investigate the results of quantity (Cournot) competition on the route
which experiences entry. If we interpret the quantity variable as a choice the firms makes
with regards to capacity, and that this capacity is constrained, then the Cournot competition
(reduced-form) profits are the same as the profits from a price game.[2]

Allowing the incumbent to provide a service along route 1 in addition to n_1^E number
of entrants gives the general expression for the incumbent providing a complete network,
using (9.1) is:

$$\Pi^I = q_1^I (\alpha - x - Q_1) + f_2 (\beta - 1 - f_2) + f_3 (\beta - 1 - f_3) - F(2 + x). \tag{10.15}$$

If n_1^E is the total number of entrants on route 1 then total demand for route 1, Q_1, can be
defined by:

$$Q_1 = q_1^I + n_1^E q_1^{E_i}, \tag{10.16}$$

where q_1^E is the demand met by the entrant(s). Profit maximisation implies the first-order
conditions (9.17b) and given the Cournot assumption, on route 3:

$$\alpha - x - 2q_1^I - n_1^E q_1^{E_i} = 0. \tag{10.17}$$

Given i entrants on route 1:

$$\Pi^{E_i} = q_1^{E_i} (\alpha - x - Q_1) - Fx. \tag{10.18}$$

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[2] Subject to caveats made by Davidson and Deneckere (1986).

This implies that the entrant faces the same operating cost as the incumbent. Relevant first-order condition for profit maximisation:

$$\alpha - x - q_1^I - (1 + n_1^E)q_1^{E_i} .$$ (10.19)

Using (10.16), (10.17) and (10.19), the equilibrium outputs of the two firms, total demand for travel on route 3 and the equilibrium fare are, respectively:

$$q_1^{E_i} = (a - x)/(1 + n),$$ (10.20a)

$$q_1^I = (a - x)/(1 + n),$$ (10.20b)

$$Q_1 = n(a - x)/(1 + n),$$ (10.20c)

$$f_1 = (a - x)/(1 + n),$$ (10.20d)

If the regulator could restrict entry to one entrant then it is simple to see that these fares (and profits) would be the same as the case when we have price competition with a single entrant on route 1. Substituting (10.20) into (10.15) and (10.18) yields profits:

$$\Pi^I = (\alpha - x)^2 /(1 + n)^2 + (\beta - 1)^2 / 2 - F(2 + x),$$ (10.21a)

$$\Pi^{E_i} = (a - x)^2 /(1 + n)^2 - Fx .$$ (10.21b)

If, on the other hand, the incumbent chooses not to provide services on route 1, but the entrant(s) does then we do not get the switch in demands as the rival firms offer route 1; thus the incumbents profit becomes:

$$\hat{\Pi}^I = (\beta - 1)^2 / 2 - 2F .$$ (10.22)

So now with no incumbent on route 1 and given i entrants on route 3:

$$\hat{\Pi}^{E_i} = q_1^{E_i}(\alpha - x - Q_1) - Fx .$$ (10.23)

As $q_1^I = 0$, Q_1 can now be defined as

$$Q_1 = n_1^E q_1^{E_i} .$$ (10.24)

This implies that the entrant faces the same operating cost as the incumbent. The relevant first-order condition for profit maximisation is:

$$\alpha - x - (1 + n_1^E)q_1^{E_i}.$$ (10.25)

Using (10.23), (10.24) and (10.25), the equilibrium outputs of the two firms, total demand for travel on route 3 and the equilibrium fare are, respectively:

$$q_1^{E_i} = \frac{(\alpha - x)}{\left(1 + n^{E_i}\right)},$$ (10.26a)

$$Q_1 = \frac{n^{E_i}(\alpha - x)}{\left(1 + n^{E_i}\right)},$$ (10.26b)

$$f_1 = \frac{(\alpha - x)}{\left(1 + n^{E_i}\right)}.$$ (10.26c)

Substituting (10.26) into (10.23) yields profits:

$$\hat{\Pi}^{E_i} = \frac{(\alpha - x)^2}{\left(1 + n^{E_i}\right)^2} - Fx$$ (10.27)

Comparing (10.21) and (10.27) leads us to Proposition 10.4.

Proposition 10.4. *The entrant will provide route 1 for a greater range of operating cost per unit distance than the incumbent.*

Whilst an the incumbent has to take account of the other routes it provides an entrant is at times able to make enough revenue on route 1 to more than cover the cost of provision. This leads to the entrant providing route 1 for some values of the operating cost of operation per unit distance that the incumbent does not. However, we cannot naturally assume that this results in a level of network interconnection that is seen as favourable by the social planner.

Proposition 10.5. *Entry on route 1 can ensure that a complete network is provided, where a monopolist would not, in accordance with the social planner's preference.*

We saw in the previous chapter that when facing a operating cost that the monopolist does not always provide the socially desirable level of interconnection using 1. Entry on route 1 increases the range of operating cost that route 1 is provided for and this can coincide with the social planner's preference.

Now let us allow entry on route 3, but this time where quantity competition takes place so let the general expression for the incumbent providing a complete network, using (9.1) is:

$$\Pi^I = f_1(\alpha - x - f_1) + f_2(\beta - 1 - f_2) + q_3^I(\beta - 1 - Q_3) - F(2 + x), \quad (10.28)$$

where q_3^I is the demand fulfilled by the incumbent. If n^E is the total number of entrants then total demand for route $3, Q_3$, can be defined by:

$$Q_3 = q_3^I + n^E q_3^E, \quad (10.29)$$

where q_3^E is the demand fulfilled by the entrant(s). Profit maximisation implies the first-order conditions (9.17a) and (9.17b) for $j = 2$ and given the Cournot assumption, on route 3:

$$\beta - 1 - 2q_3^I - n^E q_3^E = 0. \quad (10.30)$$

Given i entrants on route 3:

$$\Pi^{E_i} = q_3^{E_i}(\beta - 1 - Q_3) - F. \quad (10.31)$$

This implies that the entrant faces the same operating cost as the incumbent. The relevant first-order condition for profit maximisation is:

$$\beta - 1 - q_3^I - (1 + n^E)q_3^{E_i}. \quad (10.32)$$

Using (10.29), (10.30) and (10.32), the equilibrium outputs of the two firms, total demand

for travel on route 3 and the equilibrium fare are, respectively:

$$q_3^{E_i} = \frac{(\beta - 1)}{(1 + n)},$$ (10.33a)

$$q_3^I = \frac{(\beta - 1)}{(1 + n)},$$ (10.33b)

$$Q_3 = \frac{n(\beta - 1)}{(1 + n)},$$ (10.33c)

$$f_3 = \frac{(\beta - 1)}{(1 + n)},$$ (10.33d)

where $n = 1 + n^E$. The equilibrium quantity on route 1 and 2:

$$Q_1 = \frac{\alpha - x}{2},$$ (10.34a)

$$Q_2 = \frac{\beta - 1}{2}.$$ (10.34b)

Substituting (10.33) and (10.34) into (10.29) and (10.31) yields profits:

$$\Pi^I = \frac{(\alpha - x)^2}{4} + \frac{(\beta - 1)^2}{4} + \frac{(\beta - 1)^2}{(1 + n)^2} - F(2 + x),$$ (10.35a)

$$\Pi^{Ei} = \frac{(\beta - 1)^2}{(1 + n)^2} - F.$$ (10.35b)

If, on the other hand, the incumbent chooses not to provide services on route 1,

demands on routes 2 and 3 become interlinked by the diverted route 1 passengers. Recall

that when route 1 is not provided that demand on route 3, Q_3:

$$Q_3 = \alpha + \beta - 3 - f_2 - 2f_3.$$ (10.36)

Rearranging (10.36) yields:

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$$f_3 = \frac{1}{2}\left(\alpha + \beta - 3 - f_2 - Q_3\right). \tag{10.37}$$

The profit for the incumbent and entrant, respectively:

$$\hat{\Pi}^I = f_2\left(\alpha + \beta - 3 - 2f_2 - f_3\right) + \left(\frac{q_3^I}{2}\right)\left(\alpha + \beta - 3 - f_2 - Q_3\right) - 2F, \tag{10.38a}$$

$$\hat{\Pi}^{E_i} = \left(\frac{q_3^{E_i}}{2}\right)\left(\alpha + \beta - 3 - f_2 - Q_3\right) - F. \tag{10.38b}$$

Maximising incumbent and entrant profit with respect to output on route 3 gives the following first-order conditions:

$$\alpha + \beta - 3 - f_2 - 2q_3^I - n^E q_3^{E_i} = 0, \tag{10.39a}$$

$$\alpha + \beta - 3 - f_2 - q_3^I - (1 + n^E)q_3^{E_i} = 0. \tag{10.39b}$$

Rearranging (10.39) and solving them for as a system of simultaneous equations, then using (10.30) and (10.37) along with $n^E = n - 1$ gives the following equilibrium quantities and fares:

$$q_3^I = \left(\frac{1}{n+1}\right)\left(\alpha + \beta - 3 - f_2\right), \tag{10.40a}$$

$$q_3^{E_i} = \left(\frac{1}{n+1}\right)\left(\alpha + \beta - 3 - f_2\right), \tag{10.40b}$$

$$Q_3 = \left(\frac{n}{n+1}\right)\left(\alpha + \beta - 3 - f_2\right), \tag{10.40c}$$

$$f_3 = \left(\frac{1}{2(n+1)}\right)\left(\alpha + \beta - 3 - f_2\right). \tag{10.40d}$$

Using (10.40a) and (10.40c) in (10.38a):

$$\hat{\Pi}^I = \left[\frac{f_2}{2(n+1)}\right]\left[(2n+1)(\alpha + \beta - 3) - (4n+3)f_2\right] + \frac{n(\alpha + \beta - 3 - f_2)^2}{2(n+1)^2} - 2F. \tag{10.41}$$

Maximising (10.41) with respect to f_2 and re-arranging:

$$f_2 = \frac{\left[(2n^2 + n + 1)(\alpha + \beta - 3)\right]}{2(4n^2 + 6n + 3)}.$$

(10.42)

Substituting (10.42) in (10.40):

$$q_3^I = \frac{(6n + 5)(\alpha + \beta - 3)}{2(4n^2 + 6n + 3)},$$

(10.43a)

$$q_3^E = \frac{(6n + 5)(\alpha + \beta - 3)}{2(4n^2 + 6n + 3)},$$

(10.43b)

$$Q_3 = \frac{n(6n + 5)(\alpha + \beta - 3)}{2(4n^2 + 6n + 3)},$$

(10.43c)

$$f_3 = \frac{n(6n + 5)(\alpha + \beta - 3)}{4(4n^2 + 6n + 3)}.$$

(10.43d)

Using (10.43), the equilibrium quantity on route 2 becomes:

$$Q_2 = \frac{(8n^2 + 26n + 13)(\alpha + \beta - 3)}{4(4n^2 - 6n + 3)}.$$

(10.44)

Substituting (10.40) and (10.39) into (10.32) gives profits for the incumbent and entrant respectively:

$$\hat{\Pi}^I = \frac{(16n^4 + 96n^3 + 120n^2 + 64n + 13)(\alpha + \beta - 3)^2}{8(4n^2 + 6n + 3)^2} - 2F,$$

(10.45a)

$$\hat{\Pi}^E = \frac{n(6n + 5)^2(\alpha + \beta - 3)^2}{8(4n^2 + 6n + 3)^2} - F.$$

(10.45b)

Now, compare the incumbent's results by subtracting (10.45a) from (9.21) and solve for F:

$$F = \left[\frac{1}{4x}(\alpha - x)^2 + \frac{1}{2x}(\beta - 1)^2 \right]$$
$$- \frac{\left(16n^4 + 96n^3 + 120n^2 + 64n + 13\right)(\alpha + \beta - 3)^2}{8\left(4n^2 + 6n + 3\right)^2} \cdot \qquad (10.46)$$

This gives us the operating cost that makes the incumbent indifferent between wanting a complete network with no entry and an incomplete network with entry. The incumbent will provide a complete network if the operating cost per unit distance satisfies the inequality:

$$F < \left[\frac{1}{4x}(\alpha - x)^2 + \frac{1}{2x}(\beta - 1)^2 \right]$$
$$- \frac{\left(16n^4 + 96n^3 + 120n^2 + 64n + 13\right)(\alpha + \beta - 3)^2}{8x\left(4n^2 + 6n + 3\right)^2} \cdot \qquad (10.47)$$

Entry is sustainable on route 3 when the incumbent provides an incomplete network if:

$$F \le \frac{n(6n+5)^2(\alpha + \beta - 3)^2}{8\left(4n^2 + 6n + 3\right)^2}. \qquad (10.48)$$

Entry is not profitable on route 3 when the incumbent provides a complete network if:

$$F > (\beta - 1)^2 / (1 + n)^2. \qquad (10.49)$$

Now by comparing (9.22), (10.47), (10.48), and (10.49), it is possible to see that viable areas exist that give rise to Proposition 10.6.

Proposition 10.6. *Entry on route 2 or 3 where competition takes place over quantities can ensure the monopolist acquiesces with the social planner and provides a complete network without viable entry.*

We can see that it is possible competition can force the monopolist to provide a complete network, when it would otherwise optimally prefer to provide an incomplete network in opposition to the social planner desiring a complete network. Sustainable entry on route 3 (or route 2) would inevitably mean that the former monopolist's (now an

incumbent) profits would drop. It is possible these profits fall to a level below that of a monopolist providing an incomplete network where entry is not viable. The incumbent would prefer to once again become a monopolist, but one that provides a complete network rather than face entry on an incomplete network.

10.4. Conclusion

In this chapter we explore two further ownership regimes extending Chapter 9's look at a social planner and network monopoly regimes in a simple circular city. In Chapter 9, we find that once we introduce an operating cost that a profit maximising monopoly regime may not always provide the level of service provision that a welfare maximising social planner would prefer. During this chapter we investigate whether it may be possible for a regulator to influence the monopolist's behaviour.

We find two ways in which a regulator can ensure that a complete network is offered when it would be socially beneficial to do so. Firstly, if the regulator believes that competition upon constrained capacities will occur it can take direct action by allowing entry on route 1; this can cause the incumbent to provide a complete network, in agreement with the social planner, when the monopolist would not have previously done so. However, if entry on takes the form of Bertrand competition, and it cannot be ensured that only one (non-incumbent) firm will enter, then it will result in route 1 not being provided. In the case of Bertrand competition it is possible for the regulator to induce the social planner's preference of a complete network by allowing entry on route 2 or 3. Credible entry threats on these routes can cause the monopolist to find it preferable to agree with the social planner and provide a complete network.

Whilst this chapter has focus on "allowing" entry on to the network, the reality of the situation in the UK is that competition is openly encouraged, particularly in the bus industry where entry is allowed assuming they comply with certain rules, such as the entrant being able to provide a service for a 56 day minimum. The results of this chapter would tend to support Nash's (1993) view that the barriers to entry, such as the aforementioned "56 day rule", should be reduced so that competition can be actively encouraged. It may also be necessary to not simply alter the rules with regards to who can enter and when, but also to consider other ways to encourage entry. Our model finds that private outcomes do not always match with the preferences of society; perhaps, subsidies could offer a solution. A subsidy could be offered to firms for providing particular routes, if they are considered in the social interest. The situation with regards to the railways is more complicated as the addition of train services is problematic due to the capacity constraint, whilst the Government already sets a minimum service requirement.

PART D

CONCLUSION AND SUMMARY

CHAPTER ELEVEN

CONCLUSIONS AND SUMMARY

11.1 Introduction

We begin this thesis by investigating the modal trends in transport and use this to partly establish the need to seek a switch from private to public transport. A closer look at deregulation and privatisation of public transport shows they had, in some cases, led to the disintegration of transport services and that the resulting increase in generalised cost needs to be reversed. However, the improvements in public transport need to occur without reducing the benefits of the competition. We consider one such measure, integrated ticketing, using theoretical modelling. Additionally, we consider another of the results of deregulation and privatisation: that it left some areas with a local monopoly and whether there are appropriate incentives for such a private firm to provide a joined-up network.

In this chapter, we sum up the findings from the previous chapters by presenting the main results from each chapter and highlighting their significance. We will also examine the models, so we can make plausible suggestions for future extensions. Once we have looked at the main results we shall bring all the conclusions together in a final summary.

In the following section we present the main results of Part B and relate them back to the policy that we investigated in Chapter 2. In Section 11.3 we repeat this process for Part C's results. In Section 11.4 we provide a final summary on the work in this thesis.

11.2 Part B Conclusion and Summary

In Chapter 4 we applied Economides and Salop's model to a transport network with the intention of clarifying the issues that later chapters would focus on. We found that it

was the substitutibilities and complementarities within regimes that drove the results. When substitutibilities dominate a split ownership regime, as in the case of composite good competition, then a joint ownership (Chapter 4's approximation of an integrated ticketing regime) will not be preferred by society. This means that integrated ticketing would not be successful at promoting public transport and a composite good competition regime should be encouraged, as the firms in this split ownership regime charge lower prices than a joint ownership regime. However, the results of this model are problematic due to its clear unsuitability in modelling the problem we wish to explore.

The demand framework in Chapter 4 is promising, but the regimes, and their lack of application to transport, limit the usefulness of the conclusions. In Chapter 5, we make the model more suitable for use with integrated ticketing in transport to allow us to investigate strategic interaction between duopoly firms and compare the results using profits and welfare – with the welfare comparison requiring the introduction of a social planner's objective function, so we could rank price and quantity combinations.

We find that if the own-price elasticity of demand was low then the society's best choice is integrated ticketing regime D2. Conversely, if the own-price elasticity is high then non-integrated ticketing regime M1 represents the social planner's preferable choice. However, the importance of reaching a compromise between firm and society's preferences was highlighted, and led to the recommending of two regime types combining the aim of comparatively high profits and welfares: D2 or I1.

If we consider the situation in the UK following bus deregulation, where integrated ticketing is allowed, but firms still have to produce separately priced tickets (DfT, 2001a) then in our model this is best represented by regime D3. In Chapter 5 we found that D3 gives the lowest welfare yet it was possible to make welfare improvements by allowing

firms to communicate on integrated ticket prices (i.e. moving from regime D3 to D2 or I1), as proposed with the removal of the block exemption (Office of Fair Trading, 2006).

The chapter recommendations are based on a mix of profit and welfare focuses, but the regulator must ultimately decide whether a profit, welfare, or a trade-off between the two is a focus as the private and social outcomes differ. Some of the preferred welfare solutions may be immensely opposed by firms as profits can be significantly below that of other plausible regimes and this may mean a regulator has to find a compromise.

We find that the results depend greatly on the own-price elasticity of demand; if it is low then it might be that an integrated ticketing monopoly regime will be societies preferred outcome. We could relate this to the UK and, in particular, the provision of integrated ticketing in central London. With a road congestion charge the own-price elasticity of public transport in central London is likely to be lower than that of other UK cities, as most travellers have fewer viable alternative modes of travel. Where the own-price elasticity is low then the prevalence of integrated ticketing will likely be the society's preferred outcome.

There are also areas or routes where the car presents a viable alternative to public transport option; thus a high public transport own-price elasticity, meaning there could only be a small welfare gain from the introduction of integrated ticketing. However, outlawing integrated ticketing in such places could adversely affect profits. This may mean the firm(s) attempts to influence the regulator's decision and a compromise may be to provide integrated ticketing in a duopoly, so local planners and regulators should have the knowledge and power to impose regime changes on public transport when they decide it is for the best.

Rising congestion in many UK cities and proposed cordon tolls, or even the possibility of a nationwide road pricing scheme – problems of car travel, and policy options

that we saw in Chapter 3 – could result in a fall in the own-price elasticity of public transport. This would increase the possibility that integrated ticketing could lead to public transport use increases and adds further support to the view that an integrated transport should be part of a portfolio of policies that include road pricing.

Our model does have some limitations mostly due to tractability reasons. Firstly, we do not consider the generalised cost that the traveller faces and our welfare analysis is simply based upon the changes in prices and quantities (or fares and patronages). Certain non-integrated ticketing regimes could imply a higher non-monetary cost falling on the traveller as they will have to spend time purchasing two-tickets. We acknowledge this in our conclusions to Chapter 5 as decreasing the desirability of regime M1 (and M3, D1, and D4) from the social planner's perspective. The second issue is the cross-price elasticities of the various routes are assumed to be constant and unitary. It would seem sensible for these to be allowed to vary separately and with changes in the own-price elasticity, but for tractability purposes we were unable to account for these changes within this model.

In Chapter 6, where we extend Chapter 5's model by adding a conjectural variations term, we propose that due to the nature of the integrated ticketing agreement in regime R2 that firms will be able to communicate to a greater extent, so that price collusion in regime R2 is likely to be greater than R4. If price collusion is greater in R2 then we find regime R4 could, if the own-price elasticity of demand is small, be preferable to R2 by the social planner. This contradicts the result in Chapter 5, where D3 (R4) is inferior to I1 (R2) by the social planner, and poses the question: how much firms should be allowed to coordinate in the setting of the integrated ticket price?

If a regulator finds the own-price elasticity to be low and believes allowing integrated ticketing arrangements could lead to effective price collusion then they should force the firms to set all prices separately. This is particularly relevant to the Office of Fair

Trading's (2006) current consultation that proposes to reduce the restrictions on integrated ticketing in the bus industry as it suggests that removing the current block exemption could, in areas with a low own-price elasticity of demand of public transport, result in lower welfares. This contrasts with a finding in Chapter 5 that supports the removal of the block exemption; underlining the need for local regulators to observe levels price-elasticities, in addition to having powers to act accordingly, and also suggests the need to monitor and take action, should harmful collusion between firms be found.

One of the main issues with introducing conjectural variations into the model is the general criticism conjectural variations have received in terms of rationality and consistency. However, we have found it useful to consider the potential impact collusion – if firms acted in the ways assumed by conjectural variations – may have upon our results.

In Chapter 7, we look at another extension of Chapter 5's model. In Chapter 5 the demand for cross-service tickets and single-service tickets were symmetric, but in Chapter 7 we allow for some asymmetry. Introducing an asymmetry in the size of demands maintains the ranking we find in Chapter 5's model. However, the asymmetry in the own-price elasticities produce more significant results. This, again, highlights the complicated nature of the regulator's decision as not only will they have to consider the own-price elasticity of public transport before making a decision on policy, but they will have to examine the own-price elasticities of cross-service and single-service demands.

Allowing asymmetry in demand levels and own-price elasticities of cross and single-service demands, again, raises the issue that the cross-price elasticities of the various routes are fixed. It would seem sensible that these cross-price elasticities were allowed to vary separately between integrated and non-integrated ticket demands, especially given the assumed changes in the own-price elasticity. The relationships between the routes embodied by the cross-price elasticities could be important factors concerning the pricing

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structure of the integrated ticket, and by not accounting for this we could miss a factor that could alter the policy response. However, due to tractability problems we were unable to account for these relationships n this model.

The chapters in Part B build, explore, and extend a model of integrated ticketing in transport. The importance of the value of β is clear as it is very important in factor when determining when integrated ticketing should be encouraged and what structure the regulator should allow. In each chapter the variety of possible outcome rankings by the different parties means that it is not sensible to propose a general policy rule. Any regulator should fully explore the current regime, the impact of potential regimes, the collusion between firms, and the own-price elasticity of demand as these are factors that vary between UK cities, and areas.

The model suggests that simply amending current regulation to lower the restrictions on integrated ticketing will not be adequate. In some situations, integrated ticketing should be actively encouraged as a way of promoting public transport. Of course, any arrangements between firms will need to be assessed and monitored by local regulators to ensure it does not lead to anti-competitive effects. If integrated ticketing is found detrimental to the promotion of public transport use, then it should be possible for the regulator to affect a change by forcing a different regime, although they should be wary of the potential effect this may have upon firms' profits.

Part B focuses on integrated ticketing and looks at situations where a change in ticketing structure can improve public transport, but integrated ticketing is not the only way method that can be used. In some cases we found that non-integrated ticketing was preferred by society as the resulting "ad hoc" rule limits the price that the firm(s) could charge. Another approach could be to introduce price controls not just on "same service" tickets, but also on "cross-service" tickets. This has been, and continues to be, a method of

controlling anti-competitive behaviour in other industries, so it could easily be adapted to the transport industry. Of course, one of our findings was that profits were severely reduced by non-integrated ticketing, so we assume that a price control would not be met favourably by firms – suggesting that perhaps integrated ticketing regimes that give a compromise between private and social interests could be extremely useful policy tools.

We should also point out that there are no single trips made in our model, but these remain a form of ticketing used in transport industries. The exclusion made sense in terms of the tractability of the model, but it should be taken account of in our policy recommendations. Even if an integrated ticket is offered it is plausible that travellers will still prefer the flexibility of only using a single ticket. One example could be if they have a chance of making one leg of the journey in a car (such as the occasional offer of a lift to or from work by a colleague). In this case the traveller would not only prefer a single ticket to a return or integrated ticket, particularly the latter if it comes at an even higher price. We do attempt to account for this in the model in our non-integrated ticket regimes, but this enforces an "ad hoc" rule, which could result in the overly low prices for non-integrated ticket regimes – thus the lack of single tickets could severely alter the applicability of our conclusions.

There are many ways in which the model in Part A can be extended. The possibility of integrated ticketing brings with it several interesting impacts, and not only on the transport industry. Generalisations to mainstream industrial economics may be plausible, although our social planner's objective function has a focus on total patronage, which may not be acceptable in all industries. Extending the model to account for possible frequency disincentives that may arise from integrated ticketing could be difficult to implement, but is definitely a concept that could be explored. It may be possible to incorporate, James' (1998) model, that we introduced in Section 3.6, where passengers arrived at a uniform

point on a clock, or Else and James's (1995) that included a service frequency term. Chapter 5's model can also be altered to include other regimes and types of firm behaviour that could arise from firms providing both single-service and integrated tickets.

A look at the possible impacts of introducing a cost-side to the model should also not be ruled out. We have acknowledged that there are also possible cost savings to the individual of integrated ticketing, such as the reduction of transactions costs, but it may also have some effect on the cost of the firm. Chapter 2 highlights the impact that deregulation had on the cost of bus companies and the introduction of integrated ticketing may also have some effects on the cost of firms. It is feasible that integrated ticket may result in some savings, such as ticket sales staff and ticket printing, although it could take a large one-off investment to introduce machines and procedures capable of taking integrated ticketing. If the savings to the firm are great then integrated ticketing regimes could result in bigger profits that we see in Part B. Alternatively, if the initial investment is large then integrated ticket regimes may lead to lower profits and result in firms disliking integrated ticketing regimes more than we account for. Integrated ticketing could also impact on the firms' incentives regarding cost; whether this would lead to upward or downward pressure on cost is something the study could ascertain.

Additionally, it may also be possible to look at an increased set of regimes. Whilst we concentrate on how a private regime can be affected by the introduction of integrated ticketing, it may be appropriate to consider how an overall public transport planner subject to a fixed cost would view the introduction of integrated ticketing, as it could be argued that such an example exists in London. Other possible regimes could also be appropriate including the further possibility of entry into the market.

11.3 Part C Conclusion and Summary

In Chapter 9 we establish a model that considers whether incentives exist for a monopolist to provide the desirable level of service in a transport network with a focus on the profit and welfare outcomes for a social planner and a network monopolist. The monopolist, like the social planner, will always provide a complete network if there is no operating cost. However, if there is an operating cost the profit maximising monopoly regime does not always provide a complete service where a welfare maximising social planner finds it desirable to. During Chapter 10 we extend the model from Chapter 9 by allowing entry on sub-sections of the network to find that entry can increase the provision of the complete network in line with the preferences of the social planner. However, it varies with where the entry is, the scale of the entry, and the type of competition that prevails.

One issue with this model is, again, that non-monetary costs of travel are assumed away and this affects our welfare measure, although it is only used in regime comparisons. The introduction of an interchange penalty, which increases the likelihood of travellers preferring route 1, would make the model more realistic and undoubtedly impact on our results. However, there is a danger of results being too sensitive to another arbitrary value (we have already accounted for one in the value of operating costs), as well as causing issues of tractability.

The network is also fairly simple and closed, so that we only consider the three routes, and this could impact on the usefulness of the conclusions as in reality other routes will exist; thus route 1 may not be "marginal" route as there could be other services. This focus on route 1 may also be unrealistic as travellers are likely to be able to use a number of route combinations to reach their destination. This means the regulators decision may not

be as simple as just attempting to ensure route 1 is provided, but that every route is provided.

Again, extensions of Part C's model could involve some use of James' (1998) model or including a service quality variable such as James and Else (1995). We should also not rule out ways of extending the model in terms of entry to provide further interesting results and room for further exploration. The model refers specifically to a transport network, so a generalisation to industrial economics would be difficult. However, the model with some changes may yield interesting results if applied to other industries.

11.4 Final Summary

In this thesis we explore two issues that arise due to problems in past transport policy change. The models we propose suggest a number of possible improvements that could be made. We find that a regulator has a number of potential policies and actions that, although dependent on the situation, could improve public transport flexibility and provision to result in improved public transport patronage. These public transport improvements could play a part in attracting travellers away from using private transport and help alleviate congestion and pollution.

Appendices

Appendix A: Chapter 4 Proofs

A.1 Separate Ownership and Joint Ownership

Let us compare separate ownership and joint ownership profits and welfares. Beginning with profits if we subtract (4.15) from (4.8):

$$\Pi^J - \Pi^I = \frac{\alpha^2\left(\beta^2 - 10\beta + 25\right)}{(\beta-3)(3\beta-7)^2}. \tag{A.1}$$

Due to the restrictions we place on the parameters we know that the quantities and prices in this network are positive. This means the denominators and numerators of individual prices and quantities, total quantities, average prices, and profits are always positive. When we compare total quantities, average prices, and profits by subtracting one from the other we know that the denominator will always be positive. The result of this is that when trying to determine the sign of the comparison we only need to concentrate upon the numerator. To simplify this even further, we are also aware that α is always positive, so it is the term in the brackets that determines whether the function is positive or negative. To work out the sign of the equation we only need to find the value of the bracket. We can see that (A.1) is always positive except when $\beta = 5$ when (A.1) is zero, so the firms always prefer the network monopoly regime to the independent ownership regime.

Now let us compare welfares. If we subtract (4.17) from (4.11):

$$W^J - W^I = \frac{\alpha^2\left(25\beta^2 - 130\beta + 169\right)}{2(\beta-3)(3\beta-7)^2}. \tag{A.2}$$

Given (4.3) we can see that (A.2) is always positive. Society will prefer an independent ownership regime over joint ownership (or our proxy for integrated ticketing) regime. The results from (A.1) and (A.2) results lead us to Proposition 4.1:

Proposition 4.1. *(i) The firms prefer a joint ownership regime to an independent ownership regime:* $\Pi^J > \Pi^I$. *(ii) Society prefers a joint ownership regime to an independent ownership regime:* $W^J > W^I$.

A.2 Composite Good Competition and Joint Ownership

We can now composite good competition and joint ownership profits and welfares. Beginning with profits if we subtract (4.21) from (4.8):

$$\Pi^J - \Pi^C = \frac{9\alpha^2}{(\beta-3)(2\beta-3)^2}.$$ (A.3)

We can see that (A.3) is always positive, so the firms always prefer the network monopoly regime to the route operation regime.

Now let us compare welfares. If we subtract (4.12) from (4.23):

$$W^C - W^J = \frac{3\alpha^2(16\beta-27)}{2(\beta-3)(2\beta-3)^2}.$$ (A.4)

We can see that given (A.3) then (A.4) is always positive. Society will prefer a composite good competition regime over a joint ownership (or our proxy for integrated ticketing) regime. The results from (A.3) and (A.4) lead us to Proposition 4.2:

Proposition 4.2. *(i) The firms prefer a joint ownership regime to a composite good competition regime:* $\Pi^J > \Pi^C$. *(ii) Society prefers a composite good competition regime to a joint ownership regime:* $W^C > W^J$.

A.3 Parallel Vertical Integration and Joint Ownership

We can now compare parallel vertical integration and joint ownership profits and welfares. Beginning with profits if we subtract (4.27) from (4.8):

$$\Pi^J - \Pi^V = \frac{\alpha^2 \left(\beta^2 - 14\beta + 49 \right)}{(\beta - 3)(7\beta - 17)^2}. \tag{A.5}$$

We can see that (A.5) is always positive except if $\beta = 7$ when it zero, so the firms always prefer the joint ownership regime to the parallel vertical integration regime.

Now let us compare welfares. If we subtract (4.29) from (4.12):

$$W^J - W^V = \frac{3\alpha^2 \left(57\beta^2 - 542\beta + 1001 \right)}{2(\beta - 3)(7\beta - 17)^2}. \tag{A.6}$$

We can see that (A.6) is positive when $\beta > 7$. When $\beta > 7$ society will prefer a network joint ownership regime over a parallel vertical integration regime. The results from (A.5) and (A.6) lead us to Proposition 4.3:

Proposition 4.3. *(i) The firms prefer a joint ownership regime to a parallel vertical integration regime: $\Pi^J > \Pi^V$. (ii) Society prefers a joint ownership regime to a parallel vertical integration regime when $\beta > 7$: $W^J > W^V$ when $\beta > 7$.*

A.4. Optimal Regulation and Network Monopoly Ownership

We can now compare optimal regulation and joint ownership profits and welfares. Beginning with profits if we subtract (4.32) from (4.8):

$$\Pi^J - \Pi^{OR} = \frac{\alpha^2}{(\beta - 3)}. \tag{A.7}$$

We can see that (A.7.) is always positive so the firms always prefer the joint ownership regime to the optimal regulation regime.

Now let us compare welfares. If we subtract (4.34) from (4.12):

$$W^{OR} - W^J = \frac{7\alpha^2}{2(\beta - 3)}. \tag{A.8}$$

We can see that (A.8) is always positive so society will prefer a network monopoly ownership regime over a social planner regime. The results from (A.7) and (A.8) lead us to Proposition 4.4:

Proposition 4.4. *(i) The firms prefer a joint ownership regime to an optimal regulation regime:* $\Pi^J > \Pi^{OR}$. *(ii) Society prefers an optimal regulation regime to a joint ownership regime:* $W^{OR} > W^J$.

Appendix B: Chapter 5 Proofs

B.1. Network Monopolies Without Integrated Ticketing (M1 and M3) and Network Monopolist With Integrated Ticketing (M2)

Let us compare M1 and M2 using profits, total quantities and average prices. Beginning with profits; if we subtract (5.13) from (5.22) we yield:

$$\tilde{\Pi}^{M2} - \tilde{\Pi}^{M1} = \frac{\alpha^2 (\beta+1)}{2(\beta-3)(5\beta-13)}. \tag{B.1}$$

Assuming (5.4) then (B.1) is clearly positive. Hence, we can see that the total profits arising from network monopoly regime M2 are greater than the total profits arising from network monopoly regime M1.

Moving on to look at welfare, this requires total patronage across the network and average (per passenger) fare comparisons. Subtracting (5.23) from (5.16) and (5.17) from (5.24) gives:

$$\tilde{Q}^{M1} - \tilde{Q}^{M2} = \frac{\alpha(\beta+1)}{(5\beta-13)}, \tag{B.2}$$

$$\tilde{P}^{M2} - \tilde{P}^{M1} = \frac{\alpha(\beta+1)}{(\beta-3)(11\beta-25)}. \tag{B.3}$$

Assuming (5.4) we can see that both (B.2) and (B.3) are both positive. Hence, we can see that the total patronage arising from network monopoly regime M1 is greater than the total patronage arising from network monopoly regime M2. We can also see that the average price arising from network monopoly regime M2 is greater than the average price arising from network monopoly regime M1. The social planner always strictly prefers regime network monopoly regime M1 to network monopoly regime M2. The results from (B.1), (B.2), and (B.3) lead us to Proposition 5.1:

Proposition 5.1. *(i) The Network Monopolist always prefers the integrated ticketing regime M2 over regime M1:* $\Pi^{M2} > \Pi^{M1}$. *(ii) The social planner strictly prefers regime M1 over regime M2:* $S(\tilde{Q}^{M1}, \tilde{P}^{M1}) \succ S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

We can now M3 and M2. Let us begin with profits; if we subtract (5.27) from (5.22) we yield:

$$\Pi^{M2} - \Pi^{M3} = \frac{\alpha^2(\beta+1)}{2(13\beta-37)(\beta-3)}. \tag{B.4}$$

Assuming (5.7) then (B.4) is clearly positive. Hence, we can see that the total profits arising from network monopoly regime M2 are greater than the total patronage arising from network monopoly regime M3.

Moving on to look at welfare; subtracting (5.23) from (5.28), and (5.29) from (5.24) gives:

$$\tilde{Q}^{M3} - \tilde{Q}^{M2} = \frac{\alpha(\beta+1)}{(13\beta-37)}, \tag{B.5}$$

$$\tilde{P}^{M2} - \tilde{P}^{M3} = \frac{\alpha(\beta+1)}{4(\beta-3)(27\beta-73)}. \tag{B.6}$$

Once again, assuming (5.7), it is simple to see that both (B.5) and (B.6) are positive. Hence, we can see that the total patronage arising from network monopoly regime M3 is greater than the total patronage arising from network monopoly regime M2. We can also see that the average price arising from network monopoly regime M2 is greater than the average price arising from network monopoly regime M3. The social planner always strictly prefers network monopoly regime M3 to network monopoly regime M2. The social planner always strictly prefers regime network monopoly regime M1 to network monopoly regime M2. The results from (B.4), (B.5), and (B.6) lead us to Proposition 5.2:

Proposition 5.2. *(i) The network monopolist always prefers the integrated ticketing regime M2 over regime M3:* $\Pi^{M2} > \Pi^{M3}$. *(ii) The social planner strictly prefers regime M3 over regime M2:* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succ S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

B.2. Network Duopoly Without Integrated Ticketing (D1 and D4) and Network Monopolist Without Integrated Ticketing (M1 and M3)

We can now compare the M1 and D1. Beginning with profits; if we subtract (5.33) from (5.13) we yield:

$$\Pi^{M1} - \Pi^{D1} = \frac{9\alpha^2 \left(4\beta^2 - 28\beta + 49\right)}{2\left(5\beta - 13\right)\left(8\beta - 19\right)^2}. \tag{B.7}$$

Assuming (5.4) then (B.7) is clearly positive. Hence, we can see that the total profit arising from network monopoly regime M1 is greater than the total profit arising from network duopoly regime D1.

Moving on to look at welfare; subtracting (5.34) from (5.16), and (5.17) from (5.35) gives:

$$\tilde{Q}^{M1} - \tilde{Q}^{D1} = \frac{9\alpha\left(2\beta^2 - 13\beta + 21\right)}{\left(5\beta - 13\right)\left(8\beta - 19\right)}, \tag{B.8}$$

$$\tilde{P}^{D1} - \tilde{P}^{M1} = \frac{9\alpha\left(10\beta^2 - 61\beta + 91\right)}{2\left(8\beta - 19\right)\left(7\beta - 11\right)\left(11\beta - 25\right)}. \tag{B.9}$$

Once again, assuming (5.4), it is simple to see that both (B.8) and (B.9) are positive. Hence, we can see that the total patronage arising from network monopoly regime M1 is greater than the total patronage arising from network duopoly regime D1. We can also see that the average price arising from network duopoly regime D1 is greater than the average price arising from network monopoly regime M1. The social planner always strictly prefers

network monopoly regime M1 to network duopoly regime D1. The results from (B.7), (B.8), and (B.9) lead us to Proposition 5.3:

Proposition 5.3. *(i) The firms prefer regime M1 (joint profit maximisation) over regime D1:* $\Pi^{M1} > \Pi^{D1}$. *(ii) The social planner strictly prefers regime M1 over regime D1:* $S(\tilde{Q}^{M1}, \tilde{P}^{M1}) \succ S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.

We can now compare D4 and M3. Beginning with profits; if we subtract (5.54) from (5.27) we yield:

$$\Pi^{M3} - \Pi^{D4} = \frac{2\alpha^2 \left(9\beta^2 - 132\beta + 484\right)}{\left(13\beta - 37\right)\left(17\beta - 43\right)^2}. \qquad (B.10)$$

Assuming (5.7) then (B.10) is clearly positive. Hence, we can see the total profit arising from network monopoly regime M3 is greater than the total profit arising from network duopoly regime D4.

Moving on to look at welfare; thus subtracting (5.55) from (5.28), and (5.29) from (5.56) gives:

$$\tilde{Q}^{M3} - \tilde{Q}^{D4} = \frac{10\alpha \left(3\beta^2 - 31\beta + 66\right)}{\left(13\beta - 37\right)\left(17\beta - 43\right)}, \qquad (B.11)$$

$$\tilde{P}^{D4} - \tilde{P}^{M3} = \frac{5\alpha \left(5005\beta^2 - 26992\beta + 36103\right)}{4\left(27\beta - 73\right)\left(13\beta - 37\right)\left(79\beta - 198\right)}. \qquad (B.12)$$

For (B.11) to be positive it requires that $\beta > 7.34$ and assuming (5.7) it is simple to see that (B.12) is positive. Hence, we can see the total patronage arising from network monopoly regime M3 is greater than the total patronage arising from network duopoly regime D4 when $\beta > 7.34$. We can also see the average price arising from network duopoly regime D4 is greater than the average price arising from network monopoly regime M3. When $\beta > 7.34$ then the social planner strictly prefers network monopoly regime M3 to network

duopoly regime D4. When $\beta \leq 7.34$ the network duopoly regime D4 is weakly preferred to network monopoly regime M3. The results from (B.10), (B.11), and (B.12) lead us to Proposition 5.9:

Proposition 5.9. *(i) Both firms prefer regime M3 over regime D4:* $\Pi^{M3} > \Pi^{D4}$. *(ii) The social planner strictly prefers regime M3 over regime D4 if $\beta > 7.34$:*

$$S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succ S(\tilde{Q}^{D4}, \tilde{P}^{D4})$$

B.3. Network Duopoly Without Integrated Ticketing (D1 and D4) and Network Monopolist With Integrated Ticketing (M2)

We can now compare D1 and M2. Beginning with profits; if we subtract (5.33) from (5.22) we yield:

$$\Pi^{M2} - \Pi^{D1} = \frac{\alpha^2 \left(10\beta^2 - 34\beta + 37\right)}{\left(8\beta - 19\right)^2 \left(\beta - 3\right)}. \tag{B.13}$$

Assuming (5.4) then (B.13) is clearly positive. Hence, we can see that the total profit arising from network monopoly regime M2 is greater than the total profit arising from network duopoly regime D1.

Moving on to look at welfare; subtracting (5.34) from (5.23), and (5.35) from (5.24) gives:

$$\tilde{Q}^{M2} - \tilde{Q}^{D1} = \frac{2\alpha\left(\beta - 8\right)}{\left(8\beta - 19\right)}, \tag{B.14}$$

$$\tilde{P}^{M2} - \tilde{P}^{D1} = \frac{\alpha\left(2\beta^2 + 49\beta - 115\right)}{2\left(8\beta - 19\right)\left(7\beta - 11\right)\left(\beta - 3\right)}. \tag{B.15}$$

For (B.14) to be positive we require that $\beta > 8$ whilst (B.15) is always positive. Hence, we can see that the total patronage arising from network monopoly regime M2 is greater than

the total patronage arising from network duopoly regime D1 when $\beta > 8$. We can also see that the average price arising from network monopoly regime M2 is greater than the average price arising from network duopoly regime D1. Hence, when $\beta < 8$ the social planner strictly prefers D1 to M2, but when $\beta > 8$ the social planner weakly prefers M2 to D1.

We can now compare D4 and M2. Beginning with profits; if we subtract (5.54) from (5.22) we yield:

$$\Pi^{M2} - \Pi^{D4} = \frac{\alpha^2 \left(25\beta^2 - 68\beta + 107 \right)}{2 \left(17\beta - 43 \right)^2 \left(\beta - 3 \right)}.$$
(B.16)

Assuming (5.4) then (B.16) is clearly positive. Hence, we can see the total profit arising from network monopoly regime M2 is greater than the total profit arising from network duopoly regime D4.

Moving on to look at welfare; subtracting (5.55) from (5.23), and (5.56) from (5.24) gives:

$$\tilde{Q}^{M2} - \tilde{Q}^{D4} = \frac{\alpha \left(\beta - 19 \right)}{\left(17\beta - 43 \right)},$$
(B.17)

$$\tilde{P}^{M2} - \tilde{P}^{D4} = \frac{\alpha \left(4\beta^2 + 149\beta - 355 \right)}{\left(17\beta - 43 \right) \left(33\beta - 67 \right) \left(\beta - 3 \right)}.$$
(B.18)

We require that $\beta > 19$ for (B.17) to be positive while assuming (5.7) means that (B.18) is positive. Hence, we can see the total patronage arising from network monopoly regime M2 is greater than the total patronage arising from network duopoly regime D4 when $\beta > 19$. We can also see the average price arising from network monopoly regime M2 is greater than the average price arising from network duopoly regime D4. When $\beta < 19$ the social

planner strictly prefers D4 to M2 and when $\beta > 19$ then the social planner weakly prefers M2 to D4.

B.4. Network Duopoly With Simultaneous Integrated Ticketing (D2) and Network Monopolist Without Integrated Ticketing (M1 and M3)

We can now compare D2 and M1. Beginning with profits; if we subtract (5.13) from (5.41) we yield:

$$\Pi^{D2} - \Pi^{M1} = \frac{\alpha^2 \left(4\beta^5 - 47\beta^3 + 13\beta^3 + 54\beta + 16\right)}{2\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)\left(5\beta - 13\right)}. \tag{B.19}$$

Assuming (5.4) then (B.19) is clearly positive. Hence, we can see that the total profit arising from network duopoly regime D2 is greater than the total profit arising from network monopoly regime M1.

Moving on to look at welfare; subtracting (5.42) from (5.16), and (5.17) from (5.43) gives:

$$\tilde{Q}^{M1} - \tilde{Q}^{D2} = \frac{\alpha \left(2\beta^4 - 23\beta^3 + 62\beta^2 - 21\beta - 36\right)}{\left(5\beta - 13\right)\left(2\beta^3 - 5\beta^2 + 3\right)}, \tag{B.20}$$

$$\tilde{P}^{D2} - \tilde{P}^{M1} = \frac{\alpha \left(16\beta^5 - 72\beta^4 + 46\beta^3 + 97\beta^2 - 45\beta - 44\right)}{2\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)\left(11\beta - 25\right)}. \tag{B.21}$$

For (B.20) to be positive we require that $\beta > 7.68$ whilst (B.21) is always positive. Hence, we can see that the total patronage arising from network monopoly regime M1 is greater than the total patronage arising from network duopoly regime D2 if $\beta > 7.68$. We can also see that the average price arising from network duopoly regime D2 is greater than the average price arising from network monopoly regime M1. When $\beta < 7.68$ the social

planner weakly prefers D2 to M1 but when $\beta > 7.68$ the social planner strictly prefers M1 to D2.

We can now compare D2 and M3. Beginning with profits; if we subtract (5.27) from (5.41) we yield:

$$\Pi^{D2} - \Pi^{M3} = \frac{\alpha^2 \left(4\beta^5 - 111\beta^3 + 45\beta^2 + 142\beta + 40\right)}{2\left(13\beta - 37\right)\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)} \ . \tag{B.22}$$

If $\beta > 4.89$ then (B.22) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D2 is greater than the total profit arising from network monopoly regime M3 when $\beta > 4.89$.

Moving on to look at welfare; subtracting (5.28) from (5.42), and (5.43) from (5.29) gives:

$$\tilde{Q}^{D2} - \tilde{Q}^{M3} = \frac{\alpha \left(216\beta^4 - 958\beta^3 + 839\beta^2 + 600\beta - 513\right)}{2\left(2\beta^3 - 5\beta^2 + 3\right)\left(27\beta - 73\right)}, \tag{B.23}$$

$$\tilde{P}^{M3} - \tilde{P}^{D2} = \frac{-\alpha \left(16\beta^5 - 200\beta^4 + 206\beta^3 + 305\beta^2 - 157\beta - 140\right)}{2\left(27\beta - 73\right)\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)} \ . \tag{B.24}$$

Once again, assuming (5.4) it is simple to see that both (B.23) and (B.24) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the total patronage arising from network monopoly regime M3. We can also see the average price arising from network monopoly regime M3 is greater than the average price arising from network duopoly regime D2. The social planner always strictly prefers network duopoly regime D2 to network monopoly regime M3.

B.5. Network Duopoly With Simultaneous Integrated Ticketing (D2) and Network Monopolist With Integrated Ticketing (M2)

We can now compare the D2 and M2. Beginning with profits; if we subtract (5.41) from (5.22) we yield:

$$\Pi^{M2} - \Pi^{D2} = \frac{\alpha^2 \left(8\beta^3 - 4\beta^2 - 11\beta - 3\right)}{2(\beta - 3)\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)}. \tag{B.25}$$

Assuming (5.7) then (B.25) is clearly positive. Hence, we can see the total profit arising from network monopoly regime M2 is greater than the total profit arising from network duopoly regime D2.

Moving on to look at welfare; subtracting (5.23) from (5.42), and (5.43) from (5.24) gives:

$$\tilde{Q}^{D2} - \tilde{Q}^{M2} = \frac{\alpha\left(4\beta^2 - 3\beta - 3\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)}, \tag{B.26}$$

$$\tilde{P}^{M2} - \tilde{P}^{D2} = \frac{\alpha\left(8\beta^4 - 10\beta^3 - 13\beta^2 + 7\beta + 6\right)}{2(\beta - 3)\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)}. \tag{B.27}$$

Once again, assuming (5.7), it is simple to see that both (B.26) and (B.27) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the average price arising from network monopoly regime M2. We can also see that the average price arising from network monopoly regime M2 is greater than the average price arising from network duopoly regime D2. The social planner always strictly prefers network duopoly regime M2 to network duopoly regime D2. The results from (B.25), (B.26), and (B.27) lead us to Proposition 5.5:

Proposition 5.5. *(i) The firms prefer regime M2 over regime D2:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D2}$. *(ii) The social planner strictly prefers regime D2 over regime M2:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succeq S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

B.6. Network Duopoly Without Integrated Ticketing (D1 and D4) and Network Duopoly With Simultaneous Integrated Ticketing (D2)

We can now compare D2 and D1. Beginning with profits; if we subtract (5.33) from (5.41) we yield:

$$\Pi^{D2} - \Pi^{D1} = \frac{\alpha^2 \left(80\beta^6 - 352\beta^5 - 4\beta^4 + 1300\beta^3 - 442\beta^2 - 1048\beta - 139\right)}{2(8\beta-19)^2 \left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)} . \tag{B.28}$$

Assuming (5.4) then (B.28) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D2 is greater than the total profit arising from network duopoly regime D1.

Moving on to look at welfare; subtracting (5.34) from (5.42) and (5.35) from (5.43) gives:

$$\tilde{Q}^{D2} - \tilde{Q}^{D1} = \frac{\alpha \left(4\beta^4 - 10\beta^3 - 20\beta^2 + 39\beta + 9\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)\left(8\beta - 19\right)}, \tag{B.29}$$

$$\tilde{P}^{D2} - \tilde{P}^{D1} = \frac{\alpha \left(16\beta^6 - 56\beta^5 + 64\beta^4 - 136\beta^3 + 34\beta^2 + 217\beta + 73\right)}{2(8\beta - 19)(7\beta - 11)\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)} . \tag{B.30}$$

Once again, assuming (5.4), it is simple to see that both (B.29) and (B.30) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the total patronage arising from network duopoly regime D1. We can also see the average price arising from network duopoly regime D2 is greater than the average price arising from network monopoly regime D1. The social planner always weakly prefers network duopoly regime D2 to network duopoly regime D1. The results from (B.28), (B.29), and (B.30) lead us to Proposition 5.4:

Proposition 5.4. *(i) The firms prefer regime D2 over regime D1:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D1}$. *(ii) The social planner weakly prefers regime D2 over regime D1:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succeq S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.

We can now compare D2 and D4. Beginning with profits; if we subtract (5.54) from (5.41) we yield:

$$\Pi^{D2} - \Pi^{D4} = \frac{\alpha^2 \left(100\beta^6 - 372\beta^5 - 1687\beta^4 + 5992\beta^3 - 175\beta^2 - 5710\beta - 1528\right)}{2\left(2\beta^3 - 5\beta^2 + 3\right)\left(17\beta - 43\right)^2\left(2\beta^2 - 3\beta - 3\right)} \quad \text{.(B.31)}$$

If $\beta \geq 4.76$ then (B.31) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D2 is greater than the total profit arising from network duopoly regime D4.

Moving on to look at welfare; subtracting (5.55) from (5.42), and (5.43) from (5.56) gives:

$$\tilde{Q}^{D2} - \tilde{Q}^{D4} = \frac{\alpha\left(2\beta^4 + 25\beta^3 - 128\beta^2 + 81\beta + 72\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)\left(17\beta - 43\right)}, \quad \text{(B.32)}$$

$$\tilde{P}^{D4} - \tilde{P}^{D2} = \frac{\alpha\left(-64\beta^6 + 2104\beta^5 - 6930\beta^4 + 3241\beta^3 + 8260\beta^2 - 3711\beta - 3632\right)}{2\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)\left(17\beta - 43\right)\left(33\beta - 67\right)}. \quad \text{(B.33)}$$

Once again, assuming (5.7), it is simple to see that both (B.32) and (B.33) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the total patronage arising from network duopoly regime D4. We can also see the average price arising from network duopoly regime D4 is greater than the average price arising from network duopoly regime D2. The social planner always strictly prefers network duopoly regime D2 to network duopoly regime D4. The results from (B.31), (B.32) and (B.33) lead us to Proposition 5.10:

Proposition 5.10. *(i) The firms prefer regime D2 over regime D4 if $\beta \geq 4.76$:*

$\tilde{\Pi}^{D2} > \tilde{\Pi}^{D4}$ *if $\beta \geq 4.76$. (ii) The social planner strictly prefers regime D2 over regime D4:*

$S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succ S(\tilde{Q}^{D4}, \tilde{P}^{D4})$.

B.7. Network Duopoly With Independently Priced Integrated Ticketing (D3) and Monopolist Without Integrated Ticketing (M1 and M3)

We can now compare D3 and M1. Beginning with profits; if we subtract (5.13) from (5.48) we yield:

$$\Pi^{D3} - \Pi^{M1} = \frac{\alpha^2 \left(16\beta^2 + 96\beta - 361\right)}{18\left(5\beta - 13\right)\left(2\beta - 5\right)^2}.$$

(B.34)

Assuming (5.4) then (B.34) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D3 is greater than the total profit arising from network monopoly regime M1.

Moving on to look at welfare; subtracting (5.49) from (5.16), and (5.17) from (5.50) gives:

$$\tilde{Q}^{M1} - \tilde{Q}^{D3} = \frac{\alpha \left(16\beta^2 - 95\beta + 141\right)}{3\left(5\beta - 13\right)\left(2\beta - 5\right)},$$

(B.35)

$$\tilde{P}^{D3} - \tilde{P}^{M1} = \frac{\alpha \left(104\beta^2 - 393\beta + 385\right)}{6\left(11\beta - 25\right)\left(2\beta - 5\right)\left(5\beta - 9\right)}.$$

(B.36)

Once again, assuming (5.4) it is simple to see that both (B.35) and (B.36) are positive. Hence, we can see the total patronage arising from network monopoly regime M1 is greater than the total patronage arising from network duopoly regime D3. We can also see the average price arising from network duopoly regime D3 is greater than the total profit arising from network monopoly regime M1. The social planner always strictly prefers network monopoly M1 to network duopoly D3.

We can now D3 and M3. Beginning with profits; if we subtract (5.27) from (5.48) we yield:

$$\Pi^{D3} - \Pi^{M3} = \frac{\alpha^2 \left(320\beta - 16\beta^2 - 889\right)}{18\left(13\beta - 37\right)\left(2\beta - 5\right)^2}.$$

(B.37)

Assuming (5.7) then (B.37) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D3 is greater than the total profit arising from network monopoly regime M3.

Moving on to look at welfare; subtracting (5.49) from (5.28) and (5.29) from (5.50) gives:

$$\tilde{Q}^{M3} - \tilde{Q}^{D3} = \frac{\alpha\left(32\beta^2 - 239\beta + 429\right)}{3\left(13\beta - 37\right)\left(2\beta - 5\right)}, \tag{B.38}$$

$$\tilde{P}^{D3} - \tilde{P}^{M3} = \frac{\alpha\left(168\beta^2 - 985\beta + 1297\right)}{6\left(2\beta - 5\right)\left(5\beta - 9\right)\left(27\beta - 73\right)}. \tag{B.39}$$

If $\beta > 4.47$ then (B.38) is positive and, assuming (5.7), it is simple to see that (B.39) is positive. Hence, we can see the total patronage arising from network monopoly regime M3 is greater than the total patronage arising from network duopoly regime D3. We can also see the average price arising from network monopoly regime D3 is greater than the average price arising from network monopoly regime M3. When $\beta > 4.47$ then the social planner strictly prefers network monopoly M3 to D3 otherwise the social planner weakly prefers D3 to M3.

B.8. Network Duopoly With Independently Priced Integrated Ticketing (D3) and Monopolist With Integrated Ticketing (M2)

We can now compare D3 and network M2. Beginning with profits; if we subtract (5.48) from (5.22) we yield:

$$\Pi^{M2} - \Pi^{D3} = \frac{\alpha^2\left(2\beta^2 - 14\beta + 33\right)}{9\left(\beta - 3\right)\left(2\beta - 5\right)^2}. \tag{B.40}$$

Assuming (5.7) then (B.40) is clearly positive. Hence, we can see the total profit arising from network monopoly regime M2 is greater than the total profit arising from network duopoly regime D3.

Moving on to look at welfare; subtracting (5.49) from (5.23), and (5.24) from (5.50) gives:

$$\tilde{Q}^{M2} - \tilde{Q}^{D3} = \frac{2\alpha(\beta-6)}{3(2\beta-5)}, \tag{B.41}$$

$$\tilde{P}^{D3} - \tilde{P}^{M2} = \frac{\alpha(4\beta^2 - 37\beta + 57)}{6(\beta-3)(2\beta-5)(5\beta-9)}. \tag{B.42}$$

For (B.41) to be positive we require that $\beta > 6$, whilst (B.42) requires $\beta > 7.29$ to be positive. Hence, we can see the total patronage arising from network monopoly regime M2 is greater than the total patronage arising from network duopoly regime D3, when $\beta > 6$. We can also see the average price arising from network duopoly regime D3 is greater than the average price arising from network duopoly regime M2 when $\beta > 7.29$. When $\beta < 6$ the social planner strictly prefers D3 to M2. However, when $6 < \beta < 7.29$ then the social planner weakly prefers M2 to D3 and when $\beta > 7.29$ the social planner strictly prefers M2 to D3. The results from (B.40), (B.41), and (B.42) lead us to Proposition 5.7:

Proposition 5.7. *(i) The firms prefer regime M2 over regime D3:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D3}$. *(ii) The social planner strictly (weakly) prefers regime M2 over regime D3 if* $\beta > 7.29$

$$(6 < \beta \leq 7.29): S(\tilde{Q}^{M2}, \tilde{P}^{M2}) \underset{(-)}{\succ} S(\tilde{Q}^{D3}, \tilde{P}^{D3}).$$

B.9. Network Duopoly With Independently Priced Integrated Ticketing (D3) and Network Duopoly Without Integrated Ticketing (D1 and D4)

We can now compare D3 and D1. Beginning with profits; if we subtract (5.33) from (5.48) we yield:

$$\Pi^{D3} - \Pi^{D1} = \frac{4\alpha^2 \left(58\beta^3 - 206\beta^2 + 35\beta + 299\right)}{9(2\beta - 5)^2 \left(8\beta^2 - 19\right)^2} \ . \tag{B.43}$$

Assuming (5.4) then (B.43) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D3 is greater than the total profit arising from network duopoly regime D1.

Moving on to look at welfare; subtracting (5.49) from (5.34), and (5.35) from (5.50) gives:

$$\tilde{Q}^{D1} - \tilde{Q}^{D3} = \frac{4\alpha \left(\beta^2 - 2\beta - 3\right)}{3(2\beta - 5)(8\beta - 19)} , \tag{B.44}$$

$$\tilde{P}^{D3} - \tilde{P}^{D1} = \frac{2\alpha \left(71\beta^3 - 223\beta^2 + 7\beta + 301\right)}{3(2\beta - 5)(5\beta - 9)(8\beta - 19)(7\beta - 11)} . \tag{B.45}$$

Once again, assuming (5.4), it is simple to see that both (B.44) and (B.45) are positive. Hence, we can see the total patronage arising from network duopoly regime D3 is greater than the total patronage arising from network duopoly regime D1. We can also see the average price arising from network duopoly regime D3 is greater than the average price arising from network duopoly regime D1. The results from (B.43), (B.44), and (B.45) lead us to Proposition 5.6:

Proposition 5.6. *(i) The firms prefer regime D3 over regime D1: $\tilde{\Pi}^{D3} > \tilde{\Pi}^{D1}$. (ii) The social planner strictly prefers regime D1 over regime D3: $S(\tilde{Q}^{D3}, \tilde{P}^{D3}) \succ S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.*

We can now compare D3 and D4. Beginning with profits; if we subtract (5.54) from (5.48) we get:

$$\Pi^{D3} - \Pi^{D4} = \frac{-\alpha^2 \left(256\beta^3 - 6224\beta^2 + 27017\beta - 32653\right)}{18\left(17\beta - 43\right)^2 \left(2\beta - 5\right)^2} .$$ (B.46)

Assuming (5.7) then (B.46) is clearly positive. Hence, we can see the total profit arising from network duopoly regime D3 is greater than the total profit arising from network duopoly regime D4.

Moving on to look at welfare; subtracting (5.49) from (5.55), and (5.50) from (5.56) gives:

$$\tilde{Q}^{D4} - \tilde{Q}^{D3} = \frac{7\alpha\left(4\beta^2 - 23\beta + 33\right)}{3\left(2\beta - 5\right)\left(17\beta - 43\right)},$$ (B.47)

$$\tilde{P}^{D3} - \tilde{P}^{D4} = \frac{\alpha\left(2484\beta^3 - 15629\beta^2 + 32598\beta - 22789\right)}{6\left(2\beta - 5\right)\left(5\beta - 9\right)\left(17\beta - 43\right)\left(33\beta - 67\right)} .$$ (B.48)

Once again, assuming (5.7), it is simple to see that both (B.47) and (B.48) are positive. Hence, we can see the total patronage arising from network duopoly regime D4 is greater than the total patronage arising from network duopoly regime D3. We can also see the average price arising from network monopoly regime D3 is greater than the average price arising from network duopoly regime D4. The social planner always strictly prefers network duopoly regime D4 to network duopoly regime D3. The results from (B.46), (B.47), and (B.48) lead us to Proposition 5.11:

Proposition 5.11. *(i) The firms prefer regime D3 over regime D4:* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{D4}$. *(ii) The social planner strictly prefers regime D4 over regime D3:* $S(\tilde{Q}^{D4}, \tilde{P}^{D4}) \succ S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

B.10. Network Duopoly With Simultaneous Integrated Ticketing (D2) and Network Duopoly With Independently Priced Integrated Ticketing (D3)

We can now compare D2 and D3. Beginning with profits; if we subtract (5.48) from (5.41) we yield:

$$\Pi^{D2} - \Pi^{D3} = \frac{\alpha^2 \left(16\beta^6 - 128\beta^5 + 76\beta^4 + 588\beta^3 - 390\beta^2 - 540\beta - 27\right)}{18\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)\left(2\beta - 5\right)^2} . \tag{B.49}$$

(B.49) is clearly positive when $\beta > 6.51$. Hence, we can see the total profit arising from network duopoly regime D2 is greater than the total profit arising from network duopoly regime D3 when $\beta > 6.51$.

Moving on to look at welfare; subtracting (5.49) from (5.42), and (5.43) from (5.50) gives:

$$\tilde{Q}^{D2} - \tilde{Q}^{D3} = \frac{\alpha\left(4\beta^4 - 10\beta^3 - 18\beta^2 + 33\beta + 9\right)}{3\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta - 5\right)}, \tag{B.50}$$

$$\tilde{P}^{D3} - \tilde{P}^{D2} = \frac{\alpha\left(32\beta^6 - 56\beta^5 - 156\beta^4 + 276\beta^3 + 144\beta^2 - 201\beta - 99\right)}{6\left(2\beta - 5\right)\left(5\beta - 9\right)\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)} . \tag{B.51}$$

Once again, assuming (5.4) it is simple to see that both (B.50) and (B.51) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the total patronage arising from network duopoly regime D3. We can also see the average price arising from network duopoly regime D3 is greater than the average price arising from network duopoly regime D2. The social planner always prefers network duopoly regime D2 to network duopoly regime D3. The results from (B.51), (B.52), and (B.51) lead us to Proposition 5.8:

Proposition 5.8. *(i) The firms prefer regime D2 over regime D3 if* $\beta > 6.51$*, otherwise regime D3 is preferred to D2:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D3}$ *(* $\tilde{\Pi}^{D2} \leq \tilde{\Pi}^{D3}$ *) if* $\beta > 6.51$ *(* $\beta \leq 6.51$ *).* *(ii) The social planner strictly prefers regime D2 over regime D3:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succ S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

B.11. Network Duopoly With Pre-Emptive Integrated Ticketing (I1) and Network Monopoly Without Integrated Ticketing (M1 and M3)

We can now compare the I1 and M1. Beginning with profits; if we subtract (5.13) from (5.63) we yield:

$$\Pi^{I1} - \Pi^{M1} = \frac{\alpha^2 \left(4\beta^3 - 31\beta + 9\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(2\beta - 5\right)\left(5\beta - 13\right)} . \tag{B.52}$$

Assuming (5.4) then (B.52) is clearly positive. Hence, we can see the total profit arising from network duopoly regime I1 is greater than the total profit arising from network monopoly regime M1

Moving on to look at welfare; subtracting (5.64) from (5.16), and (5.17) from (5.65) gives:

$$\tilde{Q}^{M1} - \tilde{Q}^{I1} = \frac{\alpha \left(2\beta^2 - 13\beta + 21\right)}{\left(5\beta - 13\right)\left(2\beta - 5\right)} , \tag{B.53}$$

$$\tilde{P}^{I1} - \tilde{P}^{M1} = \frac{\alpha \left(8\beta^3 - 18\beta^2 - 17\beta + 45\right)}{4\left(11\beta - 25\right)\left(\beta - 2\right)\left(2\beta^2 - 3\beta - 5\right)} . \tag{B.54}$$

Once again, assuming (5.4) it is simple to see that both (B.53) and (B.54) are positive. Hence, we can see the total patronage arising from network monopoly regime M1 is greater than the total patronage arising from network duopoly regime I1. We can also see the average price arising from network duopoly regime I1 is greater than the average arising from network monopoly regime M1. The social planner always prefers network monopoly

regime M1 to network duopoly regime I1. The results from (B.52), (B.53), and (B.54) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when* $\beta < 4.78$ *where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ *(* $n = 1, 3$ *) and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ *(* $m = 1, 2, 3, 4$ *) unless* $\beta < 4.78$ *when* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. *(ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^{k}, \tilde{P}^{k}) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, *(* $k = M1, D2$ *) and* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

We can now compare I1 and M3. Beginning with profits; if we subtract (5.27) from (5.63) we yield:

$$\Pi^{I1} - \Pi^{M3} = \frac{\alpha^2 \left(4\beta^3 - 63\beta + 41\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(2\beta - 5\right)\left(13\beta - 37\right)} \ . \tag{B.55}$$

Assuming (5.7) then (B.55) is clearly positive. Hence, we can see the total profit arising from network duopoly regime I1 is greater than the total profit arising from network monopoly regime M3.

Moving on to look at welfare; subtracting (5.64) from (5.28), and (5.29) from (5.65) gives:

$$\tilde{Q}^{M3} - \tilde{Q}^{I1} = \frac{-\alpha \left(2\beta^2 - 29\beta + 69\right)}{\left(13\beta - 37\right)\left(2\beta - 5\right)}, \tag{B.56}$$

$$\tilde{P}^{I1} - \tilde{P}^{M3} = \frac{\alpha \left(8\beta^3 - 50\beta^2 - \beta + 157\right)}{4\left(2\beta^2 - 3\beta - 5\right)\left(\beta - 2\right)\left(27\beta - 73\right)} \ . \tag{B.57}$$

Once again, assuming (5.7), it is simple to see that both (B.56) and (B.57) are positive. Hence, we can see the total patronage arising from network duopoly regime I1 is greater than the total patronage arising from network duopoly regime M3. We can also see the average price arising from network duopoly regime I1 is greater than the average price

arising from network monopoly regime M3. Therefore, the social planner weakly prefers

network duopoly regime I1 to network duopoly regime M3. The results from (B.55),

(B.56), and (B.57) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless*

when $\beta < 4.78$ where D3 is preferred to I1: $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ ($n = 1, 3$) and $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$

($m = 1, 2, 3, 4$) unless $\beta < 4.78$ when $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. (ii) The social planner strictly prefers M1

and D2 to I1, and weakly prefer regime M3 to I1: $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, ($k = M1, D2$)

and $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

B.12. Network Duopoly With Pre-Emptive Integrated Ticketing (I1) and Network Monopoly With Integrated Ticketing (M2)

We can now compare I1 and M2. Beginning with profits; if we subtract (5.63) from

(5.22) we yield:

$$\Pi^{M2} - \Pi^{I1} = \frac{2\alpha^2(\beta-1)}{(2\beta^2 - 3\beta - 5)\ (\beta-3)(2\beta-5)}. \tag{B.58}$$

Assuming (5.4) then (B.58) is clearly positive. Hence, we can see the total profit arising

from network monopoly regime M2 is greater than the total profit arising from network

duopoly regime I1.

Moving on to look at welfare; subtracting (5.23) from (5.64), and (5.65) from (5.24)

gives:

$$\tilde{Q}^{I1} - \tilde{Q}^{M2} = \frac{2\alpha}{(2\beta-5)}, \tag{B.59}$$

$$\tilde{P}^{M2} - \tilde{P}^{I1} = \frac{\alpha(2\beta^2 - \beta - 7)}{4(\beta-3)(\beta-2)(2\beta^2 - 3\beta - 5)}. \tag{B.60}$$

Once again, assuming (5.4), it is simple to see that both (B.59) and (B.60) are positive. Hence, we can see the total patronage arising from network duopoly regime I1 is greater than the total patronage arising from network monopoly regime M2. We can also see the average price arising from network monopoly regime M2 is greater than the average price arising from network duopoly regime I1. The social planner always strictly prefers network duopoly I1 to network monopoly M2. The results from (B.58), (B.59), and (B.60) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when* $\beta < 4.78$ *where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ ($n = 1, 3$) *and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ ($m = 1, 2, 3, 4$) *unless* $\beta < 4.78$ *when* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. *(ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^{k}, \tilde{P}^{k}) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, ($k = M1, D2$) *and* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

B.13. Network Duopoly With Pre-Emptive Integrated Ticketing (I1) and Network Duopoly Without Integrated Ticketing (D1 and D4)

We can now compare I1 and D1. Beginning with profits; if we subtract (5.33) from (5.63) we yield:

$$\Pi^{I1} - \Pi^{D1} = \frac{\alpha^2 \left(40\beta^4 - 176\beta^3 + 86\beta^2 + 482\beta - 549 \right)}{\left(2\beta^2 - 3\beta - 5 \right)\left(2\beta - 5 \right)\left(8\beta - 19 \right)^2}. \tag{B.61}$$

Assuming (5.4) then (B.61) is clearly positive. Hence, we can see the total profit arising from network duopoly regime I1 is greater than the total profit arising from network duopoly regime D1

Moving on to look at welfare; subtracting (5.34) from (5.64), and (5.35) from (5.65) gives:

$$\tilde{Q}^{I1} - \tilde{Q}^{D1} = \frac{2\alpha\left(2\beta^2 - 13\beta + 21\right)}{(2\beta - 5)(8\beta - 19)}, \tag{B.62}$$

$$\tilde{P}^{I1} - \tilde{P}^{D1} = \frac{\alpha\left(8\beta^4 + 80\beta^3 - 404\beta^2 + 289\beta + 279\right)}{4(\beta - 2)(7\beta - 11)(8\beta - 19)\left(2\beta^2 - 3\beta - 5\right)}. \tag{B.63}$$

Once again, assuming (5.4), it is simple to see that both (B.62) and (B.63) are positive.

Hence, we can see the total patronage arising from network duopoly regime I1 is greater

than the total profit arising from network duopoly regime D1. We can also see the average

price arising from network duopoly regime I1 is greater than the average price arising from

network duopoly regime D1 This means that the social planner weakly prefers I1 to D1.

The results from (B.61), (B.62), and (B.63) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless*

when $\beta < 4.78$ *where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ $(n = 1, 3)$ *and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$

$(m = 1, 2, 3, 4)$ *unless* $\beta < 4.78$ *when* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. *(ii) The social planner strictly prefers M1*

and D2 to I1, and weakly prefer regime M3 to I1: $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, $(k = M1, D2)$

and $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

We can now compare I1 and D4. Beginning with profits; if we subtract (5.54) from

(5.63) we yield:

$$\Pi^{I1} - \Pi^{D4} = \frac{\alpha^2\left(100\beta^4 - 372\beta^3 - 631\beta^2 + 3684\beta - 3357\right)}{2(17\beta - 43)^2\left(2\beta^2 - 3\beta - 5\right)(2\beta - 5)}. \tag{B.64}$$

Assuming (5.7) then (B.64) is clearly positive. Hence, we can see the total profit arising

from network duopoly regime I1 is greater than the total profit arising from network

duopoly regime D4.

Moving on to look at welfare; subtracting (5.55) from (5.64) and (5.56) from (5.65):

$$\tilde{Q}^{I1} - \tilde{Q}^{D4} = \frac{\alpha\left(2\beta^2 - 9\beta + 9\right)}{\left(17\beta - 43\right)\left(2\beta - 5\right)}, \tag{B.65}$$

$$\tilde{P}^{I1} - \tilde{P}^{D4} = \frac{\alpha\left(32\beta^4 + 54\beta^3 - 1157\beta^2 + 2832\beta - 1989\right)}{4\left(17\beta - 43\right)\left(33\beta - 67\right)\left(\beta - 2\right)\left(2\beta^2 - 3\beta - 5\right)}. \tag{B.66}$$

Once again, assuming (5.7), it is simple to see that both (B.65) and (B.66) are positive. Hence, we can see the total patronage arising from network duopoly regime I1 is greater than the total patronage arising from network duopoly regime D4. We can also see the average price arising from network duopoly regime I1 is greater than the average price arising from network duopoly regime D4. The social planner always weakly prefers network duopoly regime I1 to network duopoly regime D4. The results from (B.64), (B.65), and (B.66) lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when $\beta < 4.78$ where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ *($n = 1, 3$) and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ *($m = 1, 2, 3, 4$) unless $\beta < 4.78$ when $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. (ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, *($k = M1, D2$) and $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.*

B.14. Network Duopoly With Pre-Emptive Integrated Ticketing (I1) and Network Duopoly With Simultaneous Integrated Ticketing (D2)

We can now compare I1 and D2. Beginning with profits; if we subtract (5.41) from (5.63) we yield:

$$\Pi^{I1} - \Pi^{D2} = \frac{\alpha^2\left(16\beta^5 - 16\beta^4 - 88\beta^3 + 32\beta^2 + 109\beta + 37\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)\left(2\beta - 5\right)}. \tag{B.67}$$

Assuming (5.7) then (B.67) is clearly positive. Hence, we can see the total profit arising from network duopoly regime I1 is greater than the total profit arising from network duopoly regime D2

Moving on to look at welfare; subtracting (5.64) from (5.42), and (5.43) from (5.65) gives:

$$\tilde{Q}^{I1} - \tilde{Q}^{D4} = \frac{\alpha\left(4\beta^3 - 16\beta^2 + 9\beta + 9\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta - 5\right)}, \tag{B.68}$$

$$\tilde{P}^{I1} - \tilde{P}^{D2} = \frac{\alpha\left(16\beta^6 - 48\beta^5 - 8\beta^4 + 92\beta^3 + 9\beta^2 - 54\beta - 19\right)}{4\left(2\beta^2 - 3\beta - 5\right)\left(\beta - 2\right)\left(2\beta^2 - 3\beta - 3\right)\left(4\beta^3 - 6\beta^2 - 3\beta + 3\right)}. \tag{B.69}$$

Once again, assuming (5.7), it is simple to see that both (B.68) and (B.69) are positive. Hence, we can see the total patronage arising from network duopoly regime D2 is greater than the total patronage arising from network duopoly regime I1. We can also see the average price arising from network duopoly regime I1 is greater than the average price arising from network duopoly regime D2 This means that the social planner strictly prefers network duopoly regime D2 to network duopoly regime I1. The results from (B.67), (B.68), and (B.69) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when* $\beta < 4.78$ *where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ *($n = 1, 3$) and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ *($m = 1, 2, 3, 4$) unless* $\beta < 4.78$ *when* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. *(ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$, *($k = M1, D2$) and* $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.

B.15. Network Duopoly With Pre-Emptive Integrated Ticketing (I1) and Network Duopoly With Independently Priced Integrated Ticketing (D3)

We can now compare I1) and D3. Beginning with profits; if we subtract (5.48) from (5.63) we yield:

$$\Pi^{I1} - \Pi^{D3} = \frac{\alpha^2 \left(2\beta^2 - 6\beta - 17\right)}{9\left(2\beta - 5\right)^2 \left(\beta + 1\right)} \; . \tag{B.70}$$

If $\beta \geq 4.78$ then (B.70) is clearly positive. Hence, we can see the total profit arising from network duopoly regime I1 is greater than the total profit arising from network duopoly regime D3 when $\beta \geq 4.78$.

Moving on to look at welfare, this requires total patronage across the network and average (per passenger) fare comparisons, thus subtracting (5.49) from (5.64), and (5.65) from (5.50) gives:

$$\tilde{Q}^{I1} - \tilde{Q}^{D3} = \frac{2\alpha\left(\beta - 3\right)}{3\left(2\beta - 5\right)} , \tag{B.71}$$

$$\tilde{P}^{D3} - \tilde{P}^{I1} = \frac{\alpha\left(8\beta^3 - 28\beta^2 + 19\beta + 13\right)}{12\left(2\beta - 5\right)\left(5\beta - 9\right)\left(\beta - 2\right)\left(\beta + 1\right)} . \tag{B.72}$$

Once again, assuming (5.7), it is simple to see that both (B.71) and (B.72) are positive. Hence, we can see the total patronage arising from network duopoly regime I1 is greater than the total patronage arising from network duopoly regime D3. We can also see the average price arising from network duopoly regime D3 is greater than the average price arising from network duopoly regime I1 The social planner always prefers network duopoly regime I1 to network duopoly regime D3. The results from (B.70), (B.71), and (B.72) partly lead us to Proposition 5.12:

Proposition 5.12. *(i) The firms prefer regime I1 over all other regimes except M2 unless when $\beta < 4.78$ where D3 is preferred to I1:* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Mn}$ *($n = 1, 3$) and* $\tilde{\Pi}^{I1} > \tilde{\Pi}^{Dm}$ *($m = 1, 2, 3, 4$) unless $\beta < 4.78$ when $\tilde{\Pi}^{D3} > \tilde{\Pi}^{I1}$. (ii) The social planner strictly prefers M1 and D2 to I1, and weakly prefer regime M3 to I1:* $S(\tilde{Q}^k, \tilde{P}^k) \succ S(\tilde{Q}^{I1}, \tilde{P}^{I1})$*, ($k = M1, D2$) and $S(\tilde{Q}^{M3}, \tilde{P}^{M3}) \succeq S(\tilde{Q}^{I1}, \tilde{P}^{I1})$.*

Appendix C: Chapter 6 Proofs

C.1. Network Duopoly With Pre-Emptive Integrated Ticketing (R2) and Network Monopoly With Integrated Ticketing (R1)

We can now compare R2 and R1. Beginning with profits; if we subtract (6.17) from (6.7) we yield:

$$\Pi^{R1} - \Pi^{R2} = \frac{2\alpha^2 \left(\beta - 1 - \gamma_{R2}^2 + \gamma_{R2}^2 \beta + 2\gamma_{R2} - 2\beta\gamma_{R2}\right)}{\left(\beta - 3\right)\left(2\beta^2 - 3\beta - 5 - \beta\gamma_{R2} - \gamma_{R2}\right)\left(2\beta - 5 - \gamma_{R2}\right)}. \tag{C.1}$$

Assuming (6.4), we can see this is always positive, except where $\gamma_{R2} = 1$ when (C.1) is zero. Hence, we can see that, when $\gamma_{R2} < 1$, the total profit arising from network monopoly regime R1 is always greater than R2. When there is perfect collusion ($\gamma_{R2} = 1$) between the firms they are acting as a network monopoly and thus (6.20) becomes zero. This is also intuitively correct as the network monopoly maximises the profit for all the demands and should have superior total profits to all regimes using the same demands.

Moving on to look at welfare; subtracting (6.8) from (6.18), and (6.19) from (6.9) gives:

$$\tilde{Q}^{R2} - \tilde{Q}^{R1} = \frac{2\alpha\left(1 - \gamma_{R2}\right)}{\left(2\beta - 5 - \gamma_{R2}\right)}, \tag{C.2}$$

$$\tilde{P}^{R1} - \tilde{P}^{R2} = \frac{\alpha\left(2\beta^2 - \beta - 7 - \gamma_{R2}^2 + 8\gamma_{R2} - 2\beta^2\gamma_{R2} + 3\beta\gamma_{R2}^2 - 2\beta\gamma_{R2}\right)}{4\left(\beta - 3\right)\left(2\beta^2 - 3\beta - 5 - \gamma_{R2} - \beta\gamma_{R2}\right)\left(2\beta - 5 - \gamma_{R2}\right)}. \tag{C.3}$$

Once again, assuming (6.4), it is simple to see that both (C.2) and (C.3) are positive, except when $\gamma_{R2} = 1$, which results in C.2 and C.3 both becoming zero. Hence, we can see that the total patronage arising from network duopoly regime R2 is greater than the total patronage arising from network monopoly regime R1. We can also see that the average price arising

from network monopoly regime R1 is greater than the average price arising from network duopoly regime R2. The social planner strictly prefers network duopoly regime R2 to network monopoly regime R1. The results from (C.1), (C.2), and (C.3) lead us to Proposition 6.1:

Proposition 6.1. *(i) The firms prefer regime R1 (joint profit maximisation) over regime R2 when* $\gamma_{R2} < 1$*:* $\Pi^{R1} > \Pi^{R2}$ *when* $\gamma_{R2} < 1$*. (ii) The social planner's decision is unaffected by the introduction of* γ_{R2}*:* $S(\tilde{Q}^{R2}, \tilde{P}^{R2}) \succ S(\tilde{Q}^{R1}, \tilde{P}^{R1})$ *when* $\gamma_{R2} < 1$*.*

C.2. Network Duopoly With Independently Priced Integrated Ticketing (R4) and Network Monopoly With Integrated Ticketing (R1)

We can now compare R4 and R1. Let us compare regimes R1 and R4. Beginning with profits, if we subtract (6.24) from (6.7) we yield:

$$\Pi^{R1} - \Pi^{R4} = \frac{\alpha^2[\beta(2\beta - 14 + 26\gamma_{R4} - 4\gamma_{R4}\beta - 10\gamma_{R4}^2 + 2\gamma_{R4}^2\beta - 2\gamma_{R4}^3)}{+\gamma(10\gamma_{R4} - 54 + 10\gamma_{R4}^2 + \gamma_{R4}^3) + 33]}{(\beta - 3)(2\beta - 5 - \gamma_{R4})^2(3 + \gamma_{R4})^2}. \tag{C.4}$$

Assuming (6.4), we can see that (C.4) is always positive, except where $\gamma_{R4} = 1$ when (C.4) is zero. Hence, we can see that when $\gamma_{R4} < 1$ the total profit arising from network monopoly regime R1 is always greater than R4. When there is perfect collusion ($\gamma_{R4} = 1$) between the firms they are acting as a network monopoly and thus (6.20) becomes zero. This is also intuitively correct as the network monopoly maximises the profit for all the demands and should have superior total profits to all regimes using the same demands.

Moving on to look at welfare; subtracting (6.25) from (6.8), and (6.9) from (6.26) gives:

$$\tilde{Q}^{R1} - \tilde{Q}^{R4} = \frac{2\alpha\left(\beta - 6 + 5\gamma_{R4} - \beta\gamma_{R4} + \gamma_{R4}^{2}\right)}{\left(2\beta - 5 - \gamma_{R4}\right)\left(3 + \gamma_{R4}\right)}, \tag{C.5}$$

$$\tilde{P}^{R4} - \tilde{P}^{R1} = \frac{\begin{aligned}&\alpha[\beta(-\gamma_{R4} + 33\gamma_{R4}^{2} + 4\beta - 37 + 5\gamma_{R4}^{3} - 4\gamma_{R4}^{2}\beta)\\ &+\gamma_{R4}(-2\gamma_{R4} - 23\gamma_{R4}^{2} - 59\gamma_{R4} + 27) + 57]\end{aligned}}{2\left(2\beta - 5 - \gamma_{R4}\right)\left(3 + \gamma_{R4}\right)\left(\beta - 3\right)} \cdot \left(3\gamma_{R4}\beta + 5\beta - 9 - 13\gamma_{R4} - 2\gamma_{R4}^{2}\right) \tag{C.6}$$

The signs of (C.5) and (C.6) are ambiguous, except when $\gamma_{R4} = 1$ where they become zero.

To ascertain whether R1 or R4 is preferred by the social we therefore look at the bracketed

parts of the numerators of (C.5) and (C.6) in order to find the sign of the equation. We take

the bracketed parts of the numerators of (C.5) and (C.6), and plot them in 3D using the

feasible values for β and γ_{R4}. Figures C.1 and C.2 have little intuition other than clarifying

the sign of the equation; a plot of the equation would also complicate matters as we would

have four variables represent. Figure C.1 shows that for low values of β and γ_{R4} then (C.5)

is negative. As β increases so does the likelihood of (C.5) being positive unless the value

of γ_{R4} is extremely high. Hence, we can see that for high values of β then the total

Figure C.1: Total Quantity Comparison Between R1 and R4 (Bracketed Part of Numerators Only)
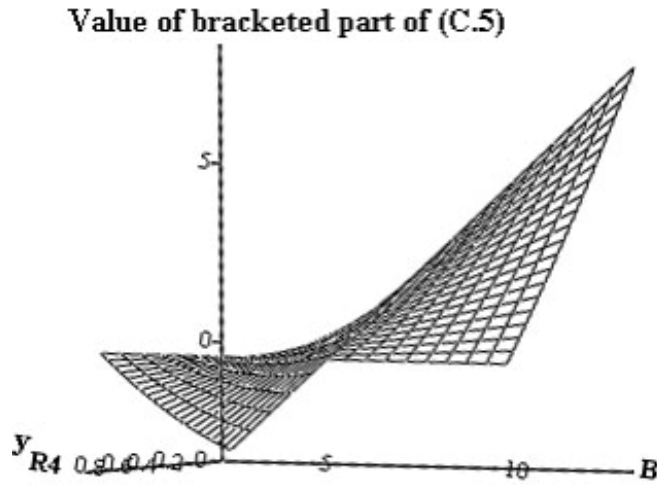
patronage arising from regime network monopoly regime R1 is greater than the total

patronage arising from regime R4. Figure C.2 shows that (C.6) tends to be negative unless $\beta > 7$ and γ is low. Hence, we can see that, when $\beta < 7$ and $\gamma_{R4} < 1$, the average price

Figure C. 2: Average Price Comparison Between R1 and R4 (Bracketed Part of Numerators Only)
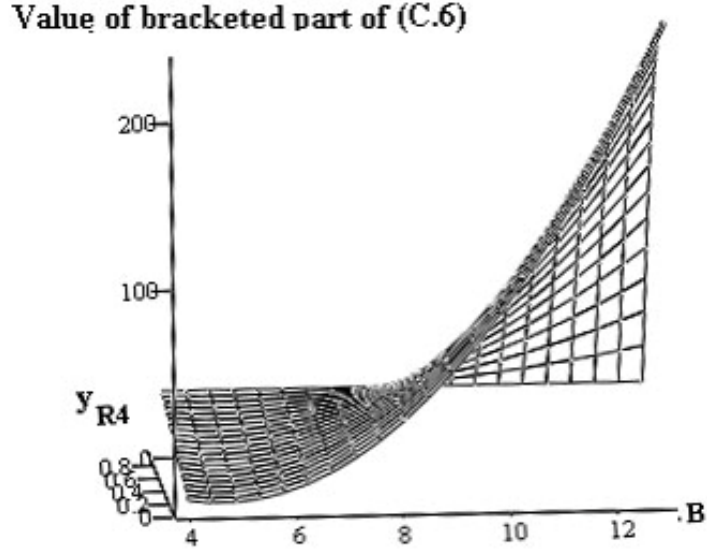
Value of bracketed part of (C.6)

arising from network monopoly regime R1 is greater than the average price arising from

network duopoly regime R4. When $\beta > 7$ the average price arising from network

monopoly regime R1 is lower than the average price arising from network duopoly regime

R4. The social planner strictly prefers regime R4 over regime R1, when β is low

and $\gamma_{R4} < 1$. However, the social planner strictly prefers regime R1 over regime R4, when

β is high or $\gamma_{R4} < 1$. The results from (C.4), (C.5), and (C.6) lead us to Proposition 6.2:

Proposition 6.2. *(i) The firms prefer regime R1 over regime R4 when $\gamma_{R4} < 1$:*

$\Pi^{R1} > \Pi^{R4}$ *when* $\gamma_{R4} < 1$. *(ii) The social planner strictly prefers regime R4 over regime R1*

when β is low and $\gamma_{R4} < 1$: $S(\tilde{Q}^{R4}, \tilde{P}^{R4}) \succ S(\tilde{Q}^{R1}, \tilde{P}^{R1})$ when β is low and $\gamma_{R4} < 1$.

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C.3. Network Duopoly With Pre-Emptive Integrated Ticketing (R2) and Network Duopoly With Independently Priced Integrated Ticketing (R4)

We can now compare R2 and R4. Beginning with profits, if we subtract (6.24) from (6.17) we yield:

$$\Pi^{R2} - \Pi^{R4} = \frac{\begin{aligned}&\alpha^2[(4\beta^2 - 4\beta - 4\gamma_{R2}\beta - 9 - 2\gamma_{R2} - \gamma_{R2}{}^2)(3 + \gamma_{R4})^2 \\ &(2\beta - 5 - \gamma_{R4})^2 \\ &-2(14\gamma_{R4}\beta - 49\gamma_{R4} - 14\gamma_{R4}{}^2 + 17\beta - 32 - \gamma_{R4}{}^3 + \gamma_{R4}{}^2\beta) \\ &(2\beta^2 - 3\beta - 5 - \gamma_{R2}\beta - \gamma_{R2})(2\beta - \gamma_{R2} - 5)]\end{aligned}}{\begin{aligned}&\left(2\beta^2 - 3\beta - 5 - \gamma_{R2}\beta - \gamma_{R2}\right)\left(2\beta - 5 - \gamma_{R2}\right) \\ &\left(3 + \gamma_{R4}\right)^2 \left(2\beta - 5 - \gamma_{R4}\right)^2\end{aligned}} . \qquad \text{(C.7)}$$

The sign of C.7 is ambiguous except when both $\gamma_{R2} = 1$ and $\gamma_{R4} = 1$, then (C.7) equals zero. Table C1 shows a simulation of the bracketed part of the numerator of (C.7) as it is this part of the numerator that defines whether this function is positive or negative; remembering that our main interest is to ascertain which of R2 or R4's profits are larger. In this table, we produce simulations for the bracketed part of (C.7) when $0 < \gamma_{R2} < 1$ (on the horizontal heading) and $0 < \gamma_{R4} < 1$ (on the vertical heading) for various sensible values of β ($\beta = 3.1$, $\beta = 4$, $\beta = 8$ and $\beta = 12$). There is no intuition to be found in these simulations other than where the calculated value is positive or negative that therefore indicates whether the profit for R2 or R4 is larger.

Table C 1: Simulation of Bracketed Part of Numerator of (C.7)

For Values of B=3.1

γ_{R4}	γ_{R2} 0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-15.56	8.69	25.03	33.45	33.96	26.55	17.22
0.25	-34.02	-8.69	9.35	20.09	23.54	19.70	13.09
0.4	-47.97	-22.78	-4.07	8.16	13.91	13.19	9.11
0.55	-55.69	-32.13	-14.05	-1.42	5.74	7.43	5.53
0.7	-55.25	-35.12	-19.25	-7.62	-0.24	2.89	2.61
0.85	-44.54	-29.96	-18.21	-9.30	-3.21	0.05	0.65
0.95	-30.59	-21.05	-13.28	-7.29	-3.07	-0.62	0.02

For Values of B=4

γ_{R4}	γ_{R2} 0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-65.80	20.58	86.85	133.01	159.07	165.03	157.82
0.25	-143.26	-49.98	23.01	75.70	108.11	120.21	117.01
0.4	-209.63	-111.35	-33.29	24.54	62.14	79.51	79.85
0.55	-262.25	-161.11	-79.87	-18.53	22.90	44.44	47.74
0.7	-297.99	-196.42	-114.16	-51.20	-7.54	16.81	22.32
0.85	-313.30	-214.04	-133.20	-70.80	-26.83	-1.30	5.49
0.95	-310.20	-214.15	-135.80	-75.15	-32.19	-6.93	0.08

For Values of B=8

γ_{R4}	γ_{R2} 0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	5872.73	6424.43	6835.46	7105.82	7235.50	7224.51	7139.03
0.25	3063.87	3741.63	4267.77	4642.31	4865.23	4936.55	4899.87
0.4	774.11	1563.33	2190.28	2654.96	2957.37	3097.50	3100.77
0.55	-1016.21	-129.75	584.08	1125.27	1493.84	1689.77	1724.48
0.7	-2324.18	-1354.43	-567.33	37.10	458.88	698.00	755.93
0.85	-3164.33	-2125.00	-1277.99	-623.31	-160.94	109.09	182.27
0.95	-3470.53	-2392.35	-1512.72	-831.65	-349.12	-65.15	13.85

For Values of B=12

γ_{R4}	γ_{R2} 0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	56732.98	57782.08	58461.57	58771.44	58711.70	58282.34	57790.76
0.25	36317.02	37848.18	38976.34	39701.53	40023.72	39942.94	39665.20
0.4	19784.98	21739.77	23258.11	24339.99	24985.43	25194.42	25091.27
0.55	7005.50	9327.23	11179.01	12560.82	13472.68	13914.57	13948.08
0.7	-2146.53	487.09	2617.25	4243.95	5367.19	5986.97	6120.45
0.85	-7789.98	-4897.99	-2542.89	-724.71	556.57	1300.95	1498.92
0.95	-9658.95	-6623.72	-4147.63	-2230.68	-872.88	-74.22	147.59

We are mostly concerned with values where $\gamma_{R2} > \gamma_{R4}$ as the structure of R2 makes it likely to be more collusive than R4. We can see that, when $\gamma_{R2} > \gamma_{R4}$ and the gap between the two grows, then the function tends to become positive; the likelihood (C.7) is positive increases as the gap between γ_{R2} and γ_{R4} gets bigger. We can also see that, when β is large, that the function also tends to be positive. This means if R2 is significantly more collusive than R4 then the profit from regime R2 will be bigger than that of R4.

Moving on to look at welfare; subtracting (6.25) from (6.18), and (6.19) from (6.26) gives:

$$\tilde{Q}^{R4} - \tilde{Q}^{R2} = \frac{\begin{array}{l} 2\alpha[\gamma_{R4}(6\gamma_{R4} + 33 - 17\beta + 2\beta^2 - 2\gamma_{R4}\beta) \\ + \gamma_{R2}(7\beta - 21) - 15 - 2\beta^2 + \gamma_{R4}\beta\gamma_{R2} + 11\beta - 3\gamma_{R4}\gamma_{R2}] \end{array}}{(3+\gamma_{R4})(2\beta - 5 - \gamma_{R4})(2\beta - \gamma_{R2} - 5)}, \tag{C.8}$$

$$\tilde{P}^{R4} - \tilde{P}^{R2} = \frac{\begin{array}{l} \alpha[4\left(2\beta^2 - 3\beta - 5 - \beta\gamma_{R2} - \gamma_{R2}\right)\left(\beta - 2 - \gamma_{R2}\right) \\ (14\gamma_{R4}\beta - 49\gamma_{R4} - 14\gamma_{R4}^2 + 17\beta - 32 - \gamma_{R4}^3 + \gamma_{R4}^2\beta) \\ -(3+\gamma_{R4})\left(3\gamma_{R4}\beta - 13\gamma_{R4} - 2\gamma_{R4}^2 + 5\beta - 9\right) \\ (2\beta - 5 - \gamma_{R4})(4\beta^2 - 4\beta - 4\gamma_{R2}\beta - 9 - 2\gamma_{R2} - \gamma_{R2}^2)] \end{array}}{\begin{array}{l} 4(3+\gamma_{R4})\left(3\gamma_{R4}\beta - 13\gamma_{R4} - 2\gamma_{R4}^2 + 5\beta - 9\right)(2\beta - 5 - \gamma_{R4}) \\ \left(2\beta^2 - 3\beta - 5 + \beta\gamma_{R2} - \gamma_{R2}\right)\left(\beta - 2 - \gamma_{R2}\right) \end{array}}. \tag{C.9}$$

The sign of C.8 is ambiguous except when both $\gamma_{R2} = 1$ and $\gamma_{R4} = 1$, then (C.8) equals zero.

Table C2 shows a table of the bracketed part of the numerator of (C.8) as it is this part of the numerator that defines whether this function is positive or negative; remembering that our main interest is to ascertain which of R2 or R4's total patronages is larger. In this table, we produce simulations for the bracketed part of (C.8), when $0 < \gamma_{R2} < 1$ (on the horizontal heading) and $0 < \gamma_{R4} < 1$ (on the vertical heading) for various sensible values of β ($\beta = 3.1$, $\beta = 4$, $\beta = 8$ and $\beta = 12$).

We can see that when β is large then (C.8) is negative. When β is small and the gap

Table C.2: Simulation of Bracketed Part of Numerator of (C.8)
For Values of B=3.1

γ_{R4}	γ_{R2} 0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-0.09	0.01	0.12	0.23	0.33	0.44	0.51
0.25	-0.17	-0.06	0.04	0.15	0.26	0.37	0.44
0.4	-0.26	-0.15	-0.04	0.07	0.18	0.29	0.37
0.55	-0.36	-0.25	-0.13	-0.02	0.09	0.20	0.28
0.7	-0.47	-0.35	-0.24	-0.12	-0.01	0.11	0.19
0.85	-0.59	-0.47	-0.35	-0.23	-0.12	0.00	0.08
0.95	-0.67	-0.55	-0.43	-0.31	-0.19	-0.07	0.01

For Values of B=4

γ_{R2}

γ_{R4}	0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-2.54	-1.47	-0.41	0.66	1.72	2.79	3.50
0.25	-3.08	-1.99	-0.90	0.19	1.27	2.36	3.09
0.4	-3.71	-2.60	-1.49	-0.38	0.73	1.84	2.58
0.55	-4.42	-3.29	-2.16	-1.03	0.11	1.24	1.99
0.7	-5.23	-4.08	-2.92	-1.77	-0.61	0.54	1.31
0.85	-6.13	-4.95	-3.78	-2.60	-1.42	-0.24	0.54
0.95	-6.78	-5.59	-4.40	-3.20	-2.01	-0.82	-0.02

For Values of B=8

γ_{R2}

γ_{R4}	0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-48.70	-43.37	-38.05	-32.72	-27.40	-22.07	-18.52
0.25	-45.39	-39.95	-34.51	-29.08	-23.64	-18.20	-14.58
0.4	-42.53	-36.98	-31.43	-25.88	-20.33	-14.78	-11.08
0.55	-40.12	-34.46	-28.80	-23.14	-17.47	-11.81	-8.04
0.7	-38.17	-32.39	-26.62	-20.84	-15.07	-9.29	-5.44
0.85	-36.66	-30.77	-24.88	-19.00	-13.11	-7.22	-3.30
0.95	-35.90	-29.94	-23.98	-18.02	-12.05	-6.09	-2.12

For Values of B=12

γ_{R2}

γ_{R4}	0.11	0.26	0.41	0.56	0.71	0.86	0.96
0.1	-152.45	-142.87	-133.28	-123.70	-114.11	-104.53	-98.14
0.25	-135.70	-125.91	-116.12	-106.34	-96.55	-86.76	-80.24
0.4	-119.75	-109.76	-99.77	-89.78	-79.79	-69.80	-63.14
0.55	-104.62	-94.43	-84.24	-74.04	-63.85	-53.66	-46.86
0.7	-90.30	-79.90	-69.51	-59.11	-48.72	-38.32	-31.39
0.85	-76.78	-66.19	-55.59	-44.99	-34.39	-23.80	-16.73
0.95	-68.22	-57.49	-46.76	-36.03	-25.29	-14.56	-7.41

between γ_{R2} and γ_{R4} is small, then (C.8) is negative. However, when β is small and γ_{R2}

is significantly larger than γ_{R4}, then (C.8) can be positive.

Table C.3 shows the values of the bracketed part of the numerator of (C.9) as it is this

part of the numerator that defines whether this function is positive or negative;

remembering that our main interest is to ascertain which of R2 or R4's average prices is

larger. In this table, we produce simulations for the bracketed part of (C.9) when

$0 < \gamma_{R2} < 1$ (on the horizontal heading) and $0 < \gamma_{R4} < 1$ (on the vertical heading) for various

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Table C. 3: Simulation of Bracketed Part of Numerator of (C.9)

For Values of B=3.1

γ_{R4}	γ_{R2}						
	0.11	**0.26**	**0.41**	**0.56**	**0.71**	**0.86**	**0.96**
0.1	32.70	-16.24	-49.46	-66.96	-68.73	-54.79	-36.77
0.25	69.35	18.31	-18.25	-40.32	-47.91	-41.01	-28.36
0.4	97.26	46.52	8.65	-16.36	-28.50	-27.78	-20.16
0.55	112.97	65.49	28.86	3.08	-11.85	-15.94	-12.64
0.7	112.65	71.98	39.74	15.94	0.56	-6.38	-6.32
0.85	92.11	62.44	38.38	19.93	7.11	-0.11	-1.80
0.95	64.92	45.26	29.11	16.46	7.33	1.71	-0.09

For Values of B=4

γ_{R4}	γ_{R2}						
	0.11	**0.26**	**0.41**	**0.56**	**0.71**	**0.86**	**0.96**
0.1	378.36	139.52	-61.98	-226.14	-352.96	-442.44	-481.35
0.25	539.60	285.81	69.70	-108.74	-249.51	-352.61	-400.41
0.4	693.71	428.30	200.40	10.01	-142.87	-258.23	-314.30
0.55	835.79	562.55	326.15	126.60	-36.11	-161.97	-225.40
0.7	960.03	683.30	442.23	236.82	67.08	-67.01	-136.59
0.85	1059.71	784.46	543.15	335.79	162.36	22.88	-51.25
0.95	1108.81	837.64	598.96	392.77	219.08	77.89	1.81

For Values of B=8

γ_{R4}	γ_{R2}						
	0.11	**0.26**	**0.41**	**0.56**	**0.71**	**0.86**	**0.96**
0.1	25565.42	21254.21	17170.37	13313.90	9684.79	6283.05	4141.53
0.25	27446.17	22784.01	18368.07	14198.36	10274.87	6597.61	4282.90
0.4	28896.03	23903.58	19175.67	14712.30	10513.46	6579.16	4103.26
0.55	29972.61	24669.82	19649.28	14910.99	10454.93	6281.12	3655.39
0.7	30728.39	25134.62	19840.13	14844.94	10149.03	5752.42	2987.62
0.85	31210.74	25344.83	19794.55	14559.90	9640.88	5037.49	2143.90
0.95	31401.45	25364.42	19653.50	14268.67	9209.94	4477.31	1503.38

For Values of B=12

γ_{R4}	γ_{R2}						
	0.11	**0.26**	**0.41**	**0.56**	**0.71**	**0.86**	**0.96**
0.1	183395.93	165918.99	149012.80	132677.37	116912.69	101718.76	91906.56
0.25	186750.39	167799.09	149474.84	131777.63	114707.47	98264.34	87650.61
0.4	185146.73	164803.98	145144.60	126168.60	107875.96	90266.70	78906.84
0.55	178933.09	157278.29	136363.13	116187.63	96751.76	78055.55	66002.32
0.7	168445.13	145554.37	123459.38	102160.17	81656.72	61949.04	49252.69
0.85	154006.03	129952.28	106750.18	84399.74	62900.94	42253.80	28962.19
0.95	142338.83	117548.83	93647.56	70635.01	48511.19	27276.09	13613.10

sensible values of β ($\beta = 3.1$, $\beta = 4$, $\beta = 8$ and $\beta = 12$).

Table C.3 shows for reasonable values of β that (C.9) is always positive. We can see that, when $\gamma_{R2} > \gamma_{R4}$ and the gap between the two grow, then the function tends to be negative; the likelihood (C.9) is negative increases as the gap between γ_{R2} and γ_{R4} gets bigger. We can also see that when β is large that the function also tends to be positive. This means, if R2 is significantly more collusive R4 and β is small, then the average price from regime R2 will be bigger than that of R4.

The results from (C.7), (C.8) and (C.9) lead us to Proposition 6.3:

Proposition 6.3. *(i) If β is large or if R2 is significantly more collusive than R4 then the firms prefer regime R2 over regime R4: $\Pi^{R2} > \Pi^{R4}$ if β is large or γ_{R2} is significantly larger than γ_{R4}. (ii) The social planner strictly prefers regime R4 over regime R2 when β is small and γ_{R2} is significantly larger than γ_{R4} : $S(\tilde{Q}^{R4}, \tilde{P}^{R4}) \succ S(\tilde{Q}^{R2}, \tilde{P}^{R2})$ when β is small and γ_{R2} is significantly larger than γ_{R4}.*

Appendix D: Chapter 7 Proofs

D.1.1 Network Duopoly With Pre-Emptive Integrated Ticketing (S2) and Network Monopoly With Integrated Ticketing (S1)

We can now compare S2 and S1. Beginning with profits; if we subtract (7.17) from (7.7) we yield:

$$\tilde{\Pi}^{S1} - \tilde{\Pi}^{S2} = \frac{\left(\alpha_x^2\beta + \beta\alpha^2 + 2\alpha\alpha_x\beta - 2\alpha\alpha_x - \alpha^2 - \alpha_x^2\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(2\beta - 5\right)\left(\beta - 3\right)}. \tag{D.1}$$

Assuming (7.4) we can see (D.1) is always positive. The network monopoly regime S1 makes greater total profits than network duopoly regime S2.

Moving on to look at welfare; subtracting (7.8) from (7.18), and (7.19) from (7.9) gives:

$$\tilde{Q}^{S2} - \tilde{Q}^{S1} = \frac{\alpha_x + \alpha}{\left(2\beta - 5\right)}, \tag{D.2}$$

$$\tilde{P}^{S1} - \tilde{P}^{S2} = \frac{\left(\alpha^2\beta^2 - 3\beta\alpha^2 + 2\alpha^2 + 5\alpha\alpha_x\beta - 11\alpha\alpha_x + \alpha_x^2\beta^2 - 3\alpha_x^2\beta + 2\alpha_x^2\right)}{2\left(2\beta^2 - 3\beta - 5\right)\left(\alpha_x\beta - 2\alpha - 2\alpha_x + \alpha_x\beta\right)\left(\beta - 3\right)}. \tag{D.3}$$

Once again, assuming (7.4), it is simple to find both (D.2) and (D.3) are positive. Hence, we can see that the total patronage for network duopoly regime S2 is larger than the total patronage for network duopoly regime S1. We can also see that the average price from network monopoly regime S1 is greater than the average price from network duopoly regime S2. The social planner strictly prefers network duopoly regime S2 over network monopoly regime S1. The results from (D.1), (D.2), and (D.3) lead us to Proposition 7.2.1:

Proposition 7.2.1. *The firms prefer the network monopoly regime S1 over regime S2:* $\Pi^{S1} > \Pi^{S2}$. *(ii) The social planner strictly prefers regime S2 over regime S1:* $S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ S(\tilde{Q}^{S1}, \tilde{P}^{S1})$.

D.1.2 Network Duopoly With Simultaneous Integrated Ticketing (S3) and Network Monopoly With Integrated Ticketing (S1)

We can now compare S3 and network S1. Beginning with profits; if we subtract (7.25) from (7.7) we yield:

$$\tilde{\Pi}^{S1} - \tilde{\Pi}^{S3} = \frac{\begin{array}{c}\alpha^2(6\beta^2 + 4\beta - 12\beta^3 - 3 + 5\beta^4) \\ +\alpha_x^2(\beta^4 - 18 + 15\beta^2 - 18\beta) \\ +\alpha\alpha_x(18 - 36\beta^2 + 16\beta^3 + 2\beta^4)\end{array}}{2(2\beta^2 - 3\beta - 3)(2\beta^3 - 5\beta^2 + 3)(\beta + 1)(\beta - 3)}. \tag{D.4}$$

Assuming (7.4) we can see (D.4) is always positive. The network monopoly regime S1 makes greater total profits than network duopoly regime S3.

Moving on to look at welfare; subtracting (7.8) from (7.26), and (7.27) from (7.9) gives:

$$\tilde{Q}^{S3} - \tilde{Q}^{S1} = \frac{\left(3\alpha\beta^2 + \alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 3\alpha - 6\alpha_x\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)}, \tag{D.5}$$

$$\tilde{P}^{S1} - \tilde{P}^{S3} = \frac{\begin{array}{c}\alpha^3(6\beta^5 - 22\beta^4 + 24\beta^3 + 6 - 14\beta) \\ +\alpha_x^3(48\beta^2 - 9\beta - 36 + 2\beta^4 - 27\beta^3 + 2\beta^5) \\ +\alpha^2\alpha_x(63\beta^2 + 32\beta^4 - 107\beta^3 - 39 + 2\beta^5 + 49\beta) \\ +\alpha\alpha_x(64\alpha_x\beta^3 - 123\alpha_x\beta^2 - 16\alpha_x\beta^4 + 6\alpha_x\beta^5 + 81\alpha_x)\end{array}}{2(2\beta^2 - 3\beta - 3)(\beta + 1)(\alpha + \alpha_x)(\beta - 3)}. \tag{D.6}$$
$$\left(2\alpha_x\beta^3 + 2\alpha\beta^3 - 2\alpha\beta^2 - 4\alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 6\alpha - 3\alpha_x\right)$$

Once again, assuming (7.4) and $\alpha_x > \alpha$, it can be shown that (D.5) is positive. However, (D.6) is more complicated, so Table D1 shows values of the numerator of (D.6)

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Table D. 1: Simulation of Numerator of (D.6)

For Values of $\alpha = 0.1$

β	α_x 0.11	1.11	2.11
3.01	2.72	520.46	3171.15
4.01	132.99	240604.06	2776028.17
5.01	11811.52	214949054.50	4714160164.86
6.01	1805210.76	331483516383.86	13819431689906.60
7.01	437778951.20	811183193027423.00	64284558664449200.00
8.01	158389024950.72	2961552561758600000.00	446135358035742000000.00

For Values of $\alpha = 1$

β	α_x 1.01	2.01	3.01
3.01	2387.40	8013.49	18944.96
4.01	9959331.02	66495474.44	235387727.88
5.01	80955223734.87	1075675656300.13	5702213619375.80
6.01	1135977268987650.00	30038676071377800.00	238458807972830000.00
7.01	25294438483314300000.00	1331100580717280000000.00	15823913204410400000000.00

For Values of $\alpha = 6$

β	α_x 6.01	7.01	8.01
3.01	509295.6373	647308.9338	808252.1845
4.01	75791267407.19	112357595977.72	160306412293.44
5.01	21995712964004100.00	38033374382429900.00	62005157174345300.00
6.01	11019633637200200000000.00	22224782360604100000000.00	41401394471987200000000.00

as it is this numerator that defines whether this function is positive or negative; remembering that our main interest is to ascertain which of S1 or S3's average price is larger. In the table, we produce simulations for the numerator of (D.6) for values of α_x (on the horizontal heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$. There is no other intuition to be found in these simulations other than where the calculated value is positive or negative; that therefore indicates whether the average price for S1 or S3 is larger – this is true of all the tables in this appendix.

Table D.1 shows that (D.6) is always positive. Hence, we see that the total patronage from network duopoly regime S3 is greater than the total patronage from network monopoly regime S1, and the average price from network monopoly regime S1 is greater

than the average price from network duopoly regime S3. The social planner always strictly

prefers network duopoly regime S3 to network monopoly regime S1. The results from

(D.4), (D.5), and (D.6) lead us to Proposition 7.2.2:

Proposition 7.2.2. *The firms prefer the network monopoly regime S1 over regime S3.:*

$\Pi^{S1} > \Pi^{S3}$. *(ii) The social planner strictly prefers regime S3 over regime S1:*

$S(\tilde{Q}^{S3}, \tilde{P}^{S3}) \succ S(\tilde{Q}^{S1}, \tilde{P}^{S1})$.

D.1.3 Network Duopoly With Integrated Ticketing (S2) and Network Duopoly With Integrated Ticketing (S3)

We can now compare S2 and S3. Beginning with profits; if we subtract (7.25) from

(7.17) we yield:

$$\tilde{\Pi}^{S2} - \tilde{\Pi}^{S3} = \frac{\begin{array}{l} \alpha^2(16\beta^5 - 44\beta - 44\beta^2 + 124\beta^3 + 28 - 80\beta^4) \\ +\alpha_x^2(81\beta - 204\beta^2 + 60\beta^3 + 153) \\ +\alpha\alpha_x(72\beta - 272\beta^3 + 280\beta^2 + 64\beta^4 - 144) \end{array}}{2(2\beta^2 - 3\beta - 5)(2\beta^3 - 5\beta^2 + 3)(2\beta^2 - 3\beta - 3)(2\beta - 5)}. \tag{D.7}$$

As the numerator of (D.7) defines whether the function is positive or negative, we use

simulations to ascertain the sign – shown in Table D.2, for values of α_x (on the horizontal

heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

Table D. 2: Simulation of Numerator of (D.7)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	18.94781	322.2059	995.0712	2037.544	3449.624	5231.311
4.01	146.2276	2004.778	5996.011	12119.93	20376.53	30765.81
5.01	530.5366	6043.538	17523.5	34970.42	58384.3	87765.14
6.01	1410.695	13710.84	38603.43	76088.45	126165.9	188835.8
7.01	3133.235	26540.36	72676.6	141542	233136.4	347460
8.01	6171.601	46346.28	123617.9	237986.6	389452.2	578014.8
9.01	11145.35	75242.47	195755.7	372684.9	606030.2	895791.6
10.1	19696.83	119956.7	304174.7	572351.1	924485.6	1360578
11.01	30222.97	170375.1	423315	789042.7	1267558	1858862

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For Values of $\alpha = 1$

β	α_x 1.1	2.1	3.1	4.1	5.1	6.1
3.01	978.3825	2015.083	3421.392	5197.307	7342.83	9857.961
4.01	8356.624	15425.63	24627.33	35961.7	49428.76	65028.51
5.01	32704.59	55613.02	84488.42	119330.8	160140.1	206916.4
6.01	92134.06	147136.4	214731.2	294918.4	387698	493070.1
7.01	214145.6	325479.7	459542.9	616335.2	795856.6	998107.1
8.01	437548.7	638971.7	877491.7	1153109	1465823	1815634
9.01	814381.5	1150706	1543447	1992603	2498176	3060165
10.1	1479160	2028384	2661566	3378706	4179805	5064862
11.01	2314153	3106619	4011874	5029916	6160746	7404364

For Values of $\alpha = 6$

β	α_x 6.1	7.1	8.1	9.1	10.1	11.1
3.01	32712.28	37823.66	43304.65	49155.25	55375.46	61965.27
4.01	283097	319113	357261.7	397543.1	439957.2	484503.9
5.01	1118336	1237886	1363403	1494886	1632337	1775754
6.01	3172282	3464519	3769348	4086769	4416783	4759390
7.01	7412176	8011993	8634539	9279814	9947819	10638552
8.01	15207767	16305014	17439359	18610800	19819338	21064973
9.01	28400436	30249135	32154250	34115781	36133728	38208091
10.1	51738515	54781984	57909411	61120796	64416139	67795440
11.01	81115383	85531819	90061043	94703055	99457854	1.04E+08

Assuming (7.4) and $\alpha_x > \alpha$, we can see from the table that (D.7) is always positive.

We see that the network duopoly regime S2 always makes greater total profits than network duopoly regime S3.

Moving on to look at welfare; subtracting (7.18) from (7.26), and (7.27) from (7.19) gives:

$$\tilde{Q}^{S3} - \tilde{Q}^{S2} = \frac{\left(4\alpha\beta^3 - 22\alpha\beta^2 + 36\alpha\beta - 18\alpha + 27\alpha_x + 6\alpha_x\beta^2 - 27\alpha_x\beta\right)}{\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta - 5\right)}, \tag{D.8}$$

$$\tilde{P}^{S2} - \tilde{P}^{S3} = \frac{\begin{aligned}&\alpha^3(72\beta - 116\beta^3 - 52\beta^5 + 8\beta^6 + 124\beta^4 - 4\beta^2 - 32)\\ &+\alpha_x^3(12\beta^5 - 45\beta - 177\beta^2 - 78\beta^4 + 192\beta^3 + 126)\\ &+\alpha^2\alpha_x(202 - 252\beta - 294\beta^4 + 52\beta^5 + 544\beta^3)\\ &+\alpha\alpha_x^2(442\beta^2 + 8\beta^6 + 171\beta - 60\beta^5 - 528\beta^3 - 315 + 240\beta^4)\end{aligned}}{2(2\beta^2 - 3\beta - 3)(2\beta^2 - 3\beta - 5)(\alpha_x\beta + \alpha\beta - 2\alpha - 2\alpha_x)} \cdot \tag{D.9}$$

$$(2\alpha_x\beta^3 + 2\alpha\beta^3 - 2\alpha\beta^2 - 4\alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 6\alpha + 3\alpha_x)$$

As the numerators of (D.8) and (D.9) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.3 and D.4, for values of α_x (on the horizontal heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

Using Tables D.3 and D.4, as well as assuming (7.4) and $\alpha_x > \alpha$, it is simple to see that both (D.8) and (D.9) are positive. Hence, we can see that the total patronage from network duopoly regime S3 is greater than the total patronage arising from network

Table D. 3: Simulation of Numerator of (D.8)

For Values of $\alpha = 0.1$

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	0.03026	0.12086	0.21146	0.30206	0.39266	0.48326
4.01	6.09438	21.30498	36.51558	51.72618	66.93678	82.14738
5.01	19.7825	62.1131	104.4437	146.7743	189.1049	231.4355
6.01	43.49462	124.9452	206.3958	287.8464	369.297	450.7476
7.01	79.63074	212.2013	344.7719	477.3425	609.9131	742.4837
8.01	130.5909	326.2815	521.9721	717.6627	913.3533	1109.044
9.01	198.775	469.5856	740.3962	1011.207	1282.017	1552.828
10.1	295.5304	661.8904	1028.25	1394.61	1760.97	2127.33
11.01	396.4152	853.4658	1310.516	1767.567	2224.618	2681.668

For Values of $\alpha = 1$

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	0.221064	0.311664	0.402264	0.492864	0.583464	0.674064
4.01	47.25426	62.46486	77.67546	92.88606	108.0967	123.3073
5.01	159.7275	202.0581	244.3887	286.7193	329.0499	371.3805
6.01	361.6407	443.0913	524.5419	605.9925	687.4431	768.8937
7.01	676.9939	809.5645	942.1351	1074.706	1207.276	1339.847
8.01	1129.787	1325.478	1521.168	1716.859	1912.549	2108.24
9.01	1744.02	2014.831	2285.641	2556.452	2827.263	3098.073
10.1	2625.58	2991.94	3358.3	3724.66	4091.02	4457.38
11.01	3552.807	4009.857	4466.908	4923.958	5381.009	5838.06

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For Values of α =6

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	1.281084	1.371684	1.462284	1.552884	1.643484	1.734084
4.01	275.9203	291.1309	306.3415	321.5521	336.7627	351.9733
5.01	937.1995	979.5301	1021.861	1064.191	1106.522	1148.852
6.01	2129.119	2210.569	2292.02	2373.47	2454.921	2536.372
7.01	3995.678	4128.248	4260.819	4393.39	4525.96	4658.531
8.01	6680.877	6876.568	7072.258	7267.949	7463.639	7659.33
9.01	10328.72	10599.53	10870.34	11141.15	11411.96	11682.77
10.1	15570.3	15936.66	16303.02	16669.38	17035.74	17402.1
11.01	21088.31	21545.37	22002.42	22459.47	22916.52	23373.57

monopoly regime S2. We can also see that the average price from network monopoly

regime S2 is greater than the average price from network duopoly regime S3. The social

planner always strictly prefers network duopoly regime S3 to network monopoly regime S2.

The results from (D.7), (D.8), and (D.9) lead us to Proposition 7.2.3:

Table D. 4: Simulation of Numerator of (D.9)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	10.2571682	447.020372	2343.28454	6810.17909	14958.83344	27900.377
4.01	68.6419481	4122.20801	22110.3341	64555.2368	141979.1327	264904.238
5.01	319.620504	19822.0798	106067.747	309081.335	678887.5534	1265511.12
6.01	1094.61525	66793.1962	354622.505	1029406.07	2255967.409	4199130.06
7.01	3032.90763	180032.049	946426.128	2734393.12	5976110.987	11103757.7
8.01	7229.28615	416407.174	2165746.78	6225289.85	13565078.17	25155153.5
9.01	15410.4943	861616.461	4432634.94	12674168.9	27531921.48	50951595.6
10.1	31902.0683	1729425.6	8792775.62	24995798.9	54112342.06	99916251.9
11.01	55054.437	2913066.13	14668776.8	41500268.9	89585624.51	165102926

For Values of α =1

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	4568.04304	11048.6639	21124.1132	35905.5202	56504.01438	84030.7252
4.01	27751.2475	74751.2348	156926.878	284800.394	468893.9989	719729.91
5.01	128734.1	348522.356	740498.05	1354685.89	2241110.596	3449796.87
6.01	444988.003	1193252.01	2531070.37	4623266.62	7634664.292	11730086.9
7.01	1244636.93	3304450.49	6979651.48	12702417.9	20904927.66	32019358.8
8.01	2990426.16	7872396.68	16552570.9	30000990.5	49187697.36	75082733.2
9.01	6416420.61	16773447.9	35116140.5	63390201.5	103541333.8	157515240
10.1	13360736.9	34707888.2	72352221.8	130067584	211627822.9	320806784
11.01	23151180.3	59876677.3	124416149	222947679	361649347.7	546699239

For Values of $\alpha = 6$

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	897572.426	1079547.72	1276980.77	1490982.71	1722664.663	1973137.76
4.01	5419988.16	6605530.92	7944255.42	9446683.89	11123338.53	12984741.6
5.01	25190495.5	30605879.8	36779252.7	43760639.1	51600063.66	60347551
6.01	87280819	105591095	126532167	150268860	176965996	206788400
7.01	244588755	294869212	352465016	417808342	491331371	573466279
8.01	588521076	707585268	844103452	999045671	1173381966	1368082378
9.01	1264206568	1516747081	1806500221	2135411688	2505427187	2918492421
10.1	2635015898	3155607464	3753225967	4431645254	5194639171	6045981566
11.01	4568951497	5464812243	6493577387	7661425011	8974533197	1.0439E+10

Proposition 7.2.3. *The firms prefer the regime S2 over regime S3:* $\Pi^{S2} > \Pi^{S3}$. *(ii) The social planner strictly prefers regime S3 over regime S2:* $S(\tilde{Q}^{S3}, \tilde{P}^{S3}) \succ S(\tilde{Q}^{S2}, \tilde{P}^{S2})$.

D.1.4 Network Duopoly with Independent Integrated Ticketing (S4) and Network Monopoly with Integrated Ticketing (S1)

We can now compare S4 and S1. Beginning with profits; if we subtract (7.32) from (7.7) we yield:

$$\tilde{\Pi}^{S1} - \tilde{\Pi}^{S4} = \frac{\left(9\alpha^2\beta - 9\alpha^2 + 36\alpha\alpha_x + 4\alpha_x^2\beta^3 - 24\alpha_x^2\beta^2 + 29\alpha_x^2\beta + 39\alpha_x^2\right)}{18(\beta+1)(\beta-3)(2\beta-5)^2}. \qquad (D.10)$$

Assuming (7.4) we can see (D.10) is always positive. Hence the network monopoly regime S1 makes greater total profits than network duopoly regime S4.

Moving on to look at welfare; subtracting (7.33) from (7.8), and (7.9) from (7.34) gives:

$$\tilde{Q}^{S1} - \tilde{Q}^{S4} = \frac{\left(2\alpha_x\beta - 3\alpha - 9\alpha_x\right)}{3(2\beta-5)}, \qquad (D.11)$$

$$\tilde{P}^{S4} - \tilde{P}^{S1} = \frac{\begin{aligned}&\alpha^3(27\beta - 9\beta^2 - 18) + \alpha_x^3(95\beta - 87 + 4\beta^3 - 36\beta^2)\\&+\alpha^2\alpha_x(6\beta^3 - 48\beta^2 + 69\beta + 9) + \alpha\alpha_x^2(27\beta^2 - 151\beta - 2\beta^3 + 210)\end{aligned}}{6(2\beta-5)(\beta+1)(\alpha+\alpha_x)(\beta-3)} \\ \left(2\alpha_x\beta - 3\alpha_x + 3\alpha\beta - 6\alpha\right). \qquad (D.12)$$

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As the numerators of (D.11) and (D.12) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.5 and D.6, for values of α_x (on the horizontal heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

Table D. 5: Simulation of Numerator of (D.11)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	-0.896	-3.876	-6.856	-9.836	-12.816	-15.796
4.01	-0.496	-1.476	-2.456	-3.436	-4.416	-5.396
5.01	-0.096	0.924	1.944	2.964	3.984	5.004
6.01	0.304	3.324	6.344	9.364	12.384	15.404
7.01	0.704	5.724	10.744	15.764	20.784	25.804
8.01	1.104	8.124	15.144	22.164	29.184	36.204
9.01	1.504	10.524	19.544	28.564	37.584	46.604
10.1	1.94	13.14	24.34	35.54	46.74	57.94
11.01	2.304	15.324	28.344	41.364	54.384	67.404

For Values of α =1

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	-6.278	-9.258	-12.238	-15.218	-18.198	-21.178
4.01	-4.078	-5.058	-6.038	-7.018	-7.998	-8.978
5.01	-1.878	-0.858	0.162	1.182	2.202	3.222
6.01	0.322	3.342	6.362	9.382	12.402	15.422
7.01	2.522	7.542	12.562	17.582	22.602	27.622
8.01	4.722	11.742	18.762	25.782	32.802	39.822
9.01	6.922	15.942	24.962	33.982	43.002	52.022
10.1	9.32	20.52	31.72	42.92	54.12	65.32
11.01	11.322	24.342	37.362	50.382	63.402	76.422

For Values of α =6

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	-36.178	-39.158	-42.138	-45.118	-48.098	-51.078
4.01	-23.978	-24.958	-25.938	-26.918	-27.898	-28.878
5.01	-11.778	-10.758	-9.738	-8.718	-7.698	-6.678
6.01	0.422	3.442	6.462	9.482	12.502	15.522
7.01	12.622	17.642	22.662	27.682	32.702	37.722
8.01	24.822	31.842	38.862	45.882	52.902	59.922
9.01	37.022	46.042	55.062	64.082	73.102	82.122
10.1	50.32	61.52	72.72	83.92	95.12	106.32
11.01	61.422	74.442	87.462	100.482	113.502	126.522

It can be shown that (D.11) is positive if β is large – see table D.5. However, if β is small then (D.11) is negative. Hence, we can see, for large values of β and a small difference between α_x and α, the total patronage in network monopoly regime S1 is greater than the total patronage in network duopoly regime S4. However, if β is small

Table D. 6: Simulation of Numerator of (D.12)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	-0.49	-39.84	-220.61	-651.58	-1441.54	-2699.25
4.01	-0.83	-60.92	-333.54	-980.73	-2164.56	-4047.07
5.01	-0.87	-38.71	-184.47	-508.03	-1079.30	-1968.14
6.01	-0.41	66.98	477.14	1541.81	3572.70	6881.53
7.01	0.79	296.31	1901.82	5944.09	13549.88	25845.97
8.01	2.93	689.47	4340.11	13474.10	30610.71	58269.19
9.01	6.23	1286.63	8042.54	24907.15	56513.64	107495.21
10.1	11.40	2217.03	13812.04	42724.51	96882.50	184214.07
11.01	17.18	3253.64	20241.97	62583.52	141879.64	269731.70

For Values of α =1

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	-168.29	-540.80	-1250.61	-2406.50	-4117.24	-6491.62
4.01	-308.87	-911.31	-2034.68	-3841.03	-6492.41	-10150.89
5.01	-374.88	-947.84	-1908.18	-3325.79	-5270.56	-7812.36
6.01	-309.28	-405.46	-159.87	739.19	2603.44	5744.61
7.01	-55.07	960.79	3921.52	9953.90	20184.68	35740.64
8.01	444.79	3395.84	11047.27	25918.33	50528.27	87396.35
9.01	1247.32	7144.64	21928.63	50232.45	96689.31	165932.37
10.1	2533.73	13014.64	38901.61	88122.71	168606.02	288279.59
11.01	3988.49	19563.27	57803.24	130309.75	248684.15	424527.81

For Values of α =6

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	-32196.96	-40846.00	-50920.53	-62529.33	-75781.18	-90784.85
4.01	-59853.59	-74116.01	-90612.72	-109505.78	-130957.24	-155129.14
5.01	-73963.57	-88458.97	-104894.29	-123339.40	-143864.19	-166538.56
6.01	-63852.88	-69712.95	-75236.96	-80113.20	-84029.94	-86675.47
7.01	-18847.50	-3715.99	16887.52	44089.80	79017.62	122797.75
8.01	71726.59	123693.84	190007.42	273186.59	375750.60	500218.72
9.01	218543.43	326678.49	462651.00	631094.15	836641.11	1083925.07
10.1	455160.06	650630.36	894932.72	1195995.21	1561745.91	2000112.87
11.01	723601.41	1016020.03	1380622.27	1829009.51	2372783.09	3023544.38

relative to the gap between α_x and α, the total patronage from network duopoly regime S4 is greater than the total patronage from network monopoly regime S1.

It can be shown that (D.12) is positive, if β is large relative to the gap between α_x and α – see Table D.6. If β is small relative to the gap between α_x and α, then (D.12) will be negative. Hence, we can see that if β is large relative to the gap between α_x and α then the average price arising from network duopoly regime S4 is greater than the average price arising from network monopoly regime S1. If β is small relative to the gap between α_x and α, then the average price from network monopoly regime S1 is greater than the average price from network duopoly regime S4. The social planner strictly prefers network monopoly regime S1 to network duopoly regime S4, when β is large compared to the gap between α_x and α. However, the social planner strictly prefers network duopoly regime S4 to network monopoly regime S1, when β is small compared to the gap between α_x and α. The results from (D.10), (D.11), and (D.12) lead us to Proposition 7.2.4:

Proposition 7.2.4. *The firms prefer the network monopoly regime S1 over regime S4:* $\Pi^{S1} > \Pi^{S4}$. *(ii) The social planner strictly prefers, strictly prefers regime S1 (S4) over regime S4 (S1) when β is large:* $S(\tilde{Q}^{S1}, \tilde{P}^{S1}) \succ (\prec) S(\tilde{Q}^{S4}, \tilde{P}^{S4})$ *when β is large (small).*

D.1.5 Network Duopoly with Pre-Emptive Integrated Ticketing (S2) and Network Duopoly with Independent Integrated Ticketing (S4)

We can now compare S2 and S4. Beginning with profits, if we subtract (7.32) from (7.17) we yield:

$$\tilde{\Pi}^{S2} - \tilde{\Pi}^{S4} = \frac{\alpha_x \left(2\alpha_x \beta^2 - 9\alpha - 8\alpha_x - 6\alpha_x \beta \right)}{9(\beta+1)(2\beta-5)^2}. \tag{D.13}$$

As the numerator of (D.13) defines whether the function is positive or negative, we use simulations to ascertain the sign – shown in Table D7, for values of α_x (on the horizontal heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

From Table D.7 we can see (D.13) is positive, unless β is small relative to the gap between α_x and α. The network duopoly regime S2 makes greater total profits than

Table D. 7: Simulation of Numerator of (D.13)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	-2.49	-10.43	-18.37	-26.31	-34.25	-42.19
4.01	-0.88	-0.78	-0.68	-0.58	-0.48	-0.38
5.01	1.53	13.67	25.81	37.95	50.09	62.23
6.01	4.74	32.92	61.10	89.28	117.46	145.64
7.01	8.74	56.96	105.18	153.40	201.62	249.85
8.01	13.55	85.81	158.07	230.33	302.59	374.85
9.01	19.16	119.46	219.76	320.06	420.36	520.66
10.1	26.18	161.60	297.02	432.44	567.86	703.28
11.01	32.78	201.16	369.54	537.92	706.30	874.68

For Values of α =1

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	-17.73	-25.67	-33.61	-41.55	-49.49	-57.43
4.01	-8.89	-8.79	-8.69	-8.59	-8.49	-8.39
5.01	4.35	16.49	28.63	40.77	52.92	65.06
6.01	22.00	50.18	78.36	106.54	134.72	162.90
7.01	44.04	92.26	140.48	188.70	236.92	285.14
8.01	70.49	142.75	215.01	287.27	359.53	431.79
9.01	101.33	201.63	301.93	402.23	502.53	602.83
10.1	139.96	275.38	410.80	546.22	681.64	817.06
11.01	176.22	344.60	512.98	681.36	849.74	1018.12

For Values of α =6

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	-102.43	-110.37	-118.31	-126.25	-134.19	-142.13
4.01	-53.39	-53.29	-53.19	-53.09	-52.99	-52.89
5.01	20.06	32.20	44.34	56.48	68.62	80.76
6.01	117.90	146.08	174.26	202.44	230.62	258.80
7.01	240.14	288.36	336.58	384.80	433.02	481.24
8.01	386.79	459.05	531.31	603.57	675.83	748.09
9.01	557.83	658.13	758.43	858.73	959.03	1059.33
10.1	772.06	907.48	1042.90	1178.32	1313.74	1449.16
11.01	973.12	1141.50	1309.88	1478.26	1646.64	1815.02

network duopoly regime S4 when β is large relative to the gap between α_x and α.

However, we can see that the network duopoly regime S4 makes greater total profits than

network duopoly regime S2 when β is small.

Moving on to look at welfare; subtracting (7.33) from (7.18), and (7.19) from (7.34)

gives:

$$\tilde{Q}^{S2} - \tilde{Q}^{S4} = \frac{2\alpha_x(\beta-3)}{3(2\beta-5)},$$
(D.14)

$$\tilde{P}^{S4} - \tilde{P}^{S2} = \frac{\begin{aligned}&\alpha_x[\alpha^2(75\beta-54+6\beta^3-36\beta^2)\\&+\alpha_x^2(4\beta^3+62\beta-52-26\beta^2)\\&+\alpha\alpha_x(34\beta^2-118\beta-2\beta^3+119)]\end{aligned}}{6(\alpha\beta-2\alpha-2\alpha_x+\alpha_x\beta)(2\beta^2-3\beta-5)} \cdot$$
$$(2\alpha_x\beta-3\alpha_x+3\alpha\beta-6\alpha)$$
(D.15)

Whilst it is simple to see that (D.14) is positive given (7.4), but (D.15) is complicated. As

the numerator of (D.15) defines whether the function is positive or negative, we use

simulations to ascertain the sign – shown in Table D8, for values of α_x (on the horizontal

heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

Table D.8 shows that (D.15) is always positive, so not only do we see that the total

patronage under network duopoly regime S2 is always greater than that of network duopoly

Table D. 8: Simulation of Numerator of (D.15)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	0.32	17.67	97.10	287.45	637.57	1196.32
4.01	1.64	78.73	431.07	1277.42	2836.57	5327.27
5.01	4.84	227.79	1261.65	3760.56	8378.67	15770.11
6.01	10.79	508.91	2846.50	8522.54	19035.95	35885.69
7.01	20.35	966.14	5443.30	16349.01	36580.47	69034.88
8.01	34.38	1643.56	9309.69	28025.64	62784.29	118578.54
9.01	53.74	2585.23	14703.34	44338.09	99419.49	187877.53
10.1	81.94	3964.34	22624.36	68320.86	153312.72	289858.80
11.01	111.94	5437.57	31103.10	94013.13	211072.23	399185.00

For Values of $\alpha = 1$

β	α_x 1.1	2.1	3.1	4.1	5.1	6.1
3.01	41.93	171.13	437.55	890.03	1577.43	2548.58
4.01	185.69	733.05	1867.00	3806.30	6769.73	10976.07
5.01	491.99	1944.31	5029.76	10402.49	18716.63	30626.34
6.01	1017.85	4049.84	10637.11	22278.59	40473.23	66719.99
7.01	1820.29	7294.60	19400.30	41034.58	75094.64	124477.67
8.01	2956.34	11923.52	32030.60	68270.45	125635.95	209119.98
9.01	4483.01	18181.55	49239.27	105586.17	195152.27	325867.56
10.1	6658.94	27145.82	74046.19	159618.91	296122.86	495816.88
11.01	8936.35	36564.72	100236.79	216857.14	403330.36	676561.03

For Values of $\alpha = 6$

β	α_x 6.1	7.1	8.1	9.1	10.1	11.1
3.01	6052.23	8545.27	11592.98	15244.20	19547.78	24552.56
4.01	24475.30	34613.44	47067.83	62057.26	79800.50	100516.32
5.01	60026.81	85611.03	117396.29	156036.74	202186.51	256499.74
6.01	116792.76	168031.99	232358.62	311271.61	406269.90	518852.44
7.01	198859.18	288370.26	401735.09	541850.87	711614.79	913924.05
8.01	310312.10	453119.78	635305.95	861863.49	1137785.28	1468064.20
9.01	455237.54	668774.51	942851.48	1285398.46	1704345.47	2207622.50
10.1	656126.95	969440.56	1373811.36	1881498.21	2504759.99	3255855.54
11.01	861850.07	1278775.33	1818987.58	2499391.39	3336891.36	4348392.06

regime S4, but the average price under regime network duopoly regime S4 is always greater than that of network duopoly regime S2. The social planner always strictly prefers the network duopoly regime S2 to network duopoly regime S4. The results from (D.13), (D.14), and (D.15) lead us to Proposition 7.2.5:

Proposition 7.2.5. *The firms prefer the duopoly regime S2 (S4) over regime S4 (S2) when* β *is large (small):* $\Pi^{S1} > (<) \Pi^{S4}$ *when* β *is large (small). (ii) The social planner strictly prefers regime S2 over regime S4:* $S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ S(\tilde{Q}^{S4}, \tilde{P}^{S4})$.

D.1.6 Network Duopoly with Simultaneous Integrated Ticketing (S3) and Network Duopoly with Independent Integrated Ticketing (S4)

We can now compare S3 and S4. Beginning with profits, if we subtract (7.32) from (7.25) we yield:

$$\tilde{\Pi}^{S3} - \tilde{\Pi}^{S4} = \frac{\begin{aligned}&\alpha^2(396\beta^2 - 1116\beta^3 - 252 - 144\beta^5 + 396\beta + 720\beta^4) \\ &\alpha_x^2(232\beta^4 - 1233 + 164\beta^5 + 16\beta^7 + 1572\beta^2 - 972\beta^3 - 112\beta^6) \\ &+ \alpha\alpha_x(2286\beta^3 + 1458 - 486\beta - 72\beta^5 - 2898\beta^2 - 288\beta^4) \\ &-\alpha_x^2 477\beta)\end{aligned}}{18\left(2\beta^3 - 5\beta^2 + 3\right)\left(2\beta^2 - 3\beta - 3\right)(\beta+1)(2\beta - 5)^2} . \qquad \text{(D.16)}$$

As the numerator of (D.16) defines whether the function is positive or negative, we use

simulations to ascertain the sign – shown in Table D9, for values of α_x (on the horizontal

heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$. From Table D.9

we can see (D.16) is positive if β is large. Firms prefer network duopoly regime S3 to

network duopoly regime S4, if β is large compared to the gap between α_x and α. Firms

prefer network duopoly regime S4 to network duopoly regime S3, if β is small compared

Table D. 9: Simulation of Numerator of (D.16)

For Values of $\alpha = 0.1$

β	α_x 0.2	1.2	2.2	3.2	4.2
3.01	-244.72	-4765.64	-14980.73	-30890.00	-52493.44
4.01	-1626.94	-19695.96	-56605.12	-112354.41	-186943.84
5.01	-2238.26	81745.02	313554.74	693190.92	1220653.54
6.01	12193.81	914542.12	3184557.41	6822239.66	11827588.88
7.01	87689.79	4290981.06	14680561.82	31256432.09	54018591.85
8.01	334323.55	14394871.67	48909514.24	103878251.27	179301082.75
9.01	974687.89	39537540.45	133867226.09	283963744.80	489827096.58
10.1	2600561.81	101786790.73	343883063.74	728889380.83	1256805742.00
11.01	5319294.07	204359414.06	689626084.71	1461119306.00	2518839077.94

For Values of $\alpha = 1$

β	α_x 1.1	2.1	3.1	4.1	5.1
3.01	-11714.04	-26174.62	-46329.38	-72178.32	-103721.43
4.01	-92484.10	-171437.55	-269231.14	-385864.86	-521338.73
5.01	-254586.89	-213016.94	-23620.54	313602.32	798651.62
6.01	-193347.44	1444734.07	4450482.55	8823898.00	14564980.42
7.01	1283107.45	9910056.84	24723295.72	45722824.10	72908641.98
8.01	7214755.31	37368126.40	87975591.95	159037151.95	250552806.41
9.01	23939431.86	108427058.91	248681519.03	444702812.23	696490938.50
10.1	68353887.12	288501938.26	651560033.47	1157528172.78	1806406356.17
11.01	144509557.56	589287384.05	1320291761.19	2337522688.97	3640980167.40

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	α_x				
β	6.1	7.1	8.1	9.1	10.1
3.01	-387576.61	-457257.57	-532632.71	-613702.03	-700465.52
4.01	-3123182.31	-3540382.61	-3976423.06	-4431303.63	-4905024.35
5.01	-9049620.09	-9243679.70	-9289912.87	-9188319.58	-8938899.85
6.01	-9652293.65	-3926805.48	3166349.65	11627171.76	21455660.83
7.01	30363740.56	63566568.37	102955685.68	148531092.49	200292788.79
8.01	202508981.50	322066924.68	462078962.31	622545094.39	803465320.93
9.01	698977769.71	1038603032.78	1433995128.92	1885154058.14	2392079820.43
10.1	2032524604.64	2924682779.18	3959750997.81	5137729260.52	6458617567.32
11.01	4333128737.61	6143116044.61	8239329902.25	10621770310.54	13290437269.48

to the gap between α_x and α.

Moving on to look at welfare; subtracting (7.33) from (7.26), and (7.27) from (7.34) gives:

$$\tilde{Q}^{S3} - \tilde{Q}^{S4} = \frac{\begin{array}{l}\alpha(108\beta - 54 + 12\beta^3 - 66\beta^2) \\ +\alpha_x(4\beta^4 - 75\beta + 63 + 48\beta^2 - 22\beta^3)\end{array}}{3(2\beta^3 - 5\beta^2 + 3)(2\beta - 5)}, \tag{D.17}$$

$$\tilde{P}^{S4} - \tilde{P}^{S3} = \frac{\begin{array}{l}\alpha^3(648\beta - 36\beta^2 + 72\beta^6 - 1044\beta^3 - 288 + 1116\beta^4 - 468\beta^5) \\ +\alpha_x^{~3}(801 + 1356\beta^3 - 128\beta^6 - 852\beta^2 + 16\beta^7 + 476\beta^5 - 1024\beta^4) \\ +\alpha^2\alpha_x\beta(24\beta^6 - 1428\beta + 4212\beta^2 + 924\beta^4 - 2892\beta^3 - 180\beta^5) \\ +\alpha\alpha_x^{~2}\beta(2920\beta^3 + 212\beta^5 + 2259\beta - 8\beta^6 - 1144\beta^4 + 1647) \\ +1476\alpha^2\alpha_x - 2088\alpha\alpha_x^{~2} - 459\beta\alpha_x^{~3} - 2136\alpha^2\alpha_x\beta - 4104\alpha\alpha_x^{~2}\beta^3\end{array}}{6(2\alpha_x\beta - 3\alpha_x + 3\alpha\beta - 6\alpha)(\beta + 1)(2\beta - 5)(2\beta^2 - 3\beta - 3)} \cdot \tag{D.18}$$
$$\left(2\alpha_x\beta^3 + 2\alpha\beta^3 - 2\alpha\beta^2 - 4\alpha_x\beta^2 - 6\alpha\beta + 3\alpha_x\beta + 6\alpha - 3\alpha_x\right)$$

As the numerators of (D.17) and (D.18) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.10 and D.11, for values of α_x (on the horizontal heading) and β (on the vertical heading) when $\alpha = 0.1$, $\alpha = 1$, and $\alpha = 6$.

From Table D.10 we see that (D.17) is positive, if β is large and (D.18) is always positive. Hence, we can see that the total patronage from network duopoly regime S3 is

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Table D. 10: Simulation of Numerator of (D.17)

For Values of α =0.1

β	α_x					
	0.2	1.2	2.2	3.2	4.2	5.2
3.01	-29.36	-28.84	-28.33	-27.81	-27.29	-26.78
4.01	-0.18	149.60	299.39	449.18	598.96	748.75
5.01	113.97	759.55	1405.13	2050.71	2696.29	3341.88
6.01	380.49	2169.35	3958.21	5747.07	7535.93	9324.80
7.01	885.97	4862.56	8839.14	12815.73	16792.31	20768.90
8.01	1736.20	9437.92	17139.63	24841.35	32543.07	40244.78
9.01	3056.18	16609.39	30162.60	43715.81	57269.02	70822.23
10.1	5199.70	28359.22	51518.74	74678.25	97837.77	120997.29
11.01	7701.34	42172.48	76643.61	111114.74	145585.87	180057.01

For Values of α =1

β	α_x					
	1.1	2.1	3.1	4.1	5.1	6.1
3.01	-294.05	-293.53	-293.01	-292.50	-291.98	-291.46
4.01	-136.65	13.14	162.93	312.71	462.50	612.29
5.01	558.65	1204.23	1849.82	2495.40	3140.98	3786.56
6.01	2194.90	3983.76	5772.63	7561.49	9350.35	11139.21
7.01	5280.76	9257.35	13233.94	17210.52	21187.11	25163.69
8.01	10430.49	18132.21	25833.92	33535.64	41237.35	48939.07
9.01	18363.94	31917.15	45470.36	59023.57	72576.78	86129.99
10.1	31153.42	54312.94	77472.46	100631.98	123791.50	146951.01
11.01	45989.42	80460.55	114931.68	149402.82	183873.95	218345.08

For Values of α =6

β	α_x					
	6.1	7.1	8.1	9.1	10.1	11.1
3.01	-1764.54	-1764.03	-1763.51	-1762.99	-1762.48	-1761.96
4.01	-894.77	-744.99	-595.20	-445.41	-295.63	-145.84
5.01	3029.12	3674.70	4320.28	4965.86	5611.45	6257.03
6.01	12274.99	14063.85	15852.71	17641.57	19430.43	21219.29
7.01	29696.29	33672.87	37649.46	41626.05	45602.63	49579.22
8.01	58732.08	66433.79	74135.51	81837.23	89538.94	97240.66
9.01	103407.02	116960.23	130513.44	144066.65	157619.86	171173.07
10.1	175340.77	198500.29	221659.81	244819.33	267978.85	291138.37
11.01	258700.95	293172.08	327643.21	362114.35	396585.48	431056.61

greater than the total patronage from network duopoly regime S4, when β is large. We can

also see that the average price from network duopoly regime S4 is always larger than the

average price from network duopoly regime S3. The social planner strictly (weakly) prefers

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Table D. 11: Simulation of Numerator of (D.18)

For Values of α =0.1

β	α_x 0.2	1.2	2.2	3.2
3.01	3832.38	28764.52	75898.27	162367.28
4.01	82574.46	566973.46	1329438.66	2586169.02
5.01	638064.60	4303857.31	9857105.42	18788325.53
6.01	3186674.77	21271452.34	48043024.06	90446611.61
7.01	12151702.61	80417742.18	179384034.55	333818372.19
8.01	38359999.16	251922872.49	555331865.37	1021556453.70
9.01	105127684.93	685698252.17	1494719957.52	2718674432.97
10.1	278439513.58	1804161544.40	3888660327.88	6990010130.76
11.01	579515210.90	3736633068.98	7984460947.11	14219695425.93

For Values of α =1

β	α_x 1.1	2.1	3.1	4.1
3.01	38483.70	107721.25	229351.78	420508.95
4.01	649672.43	1644298.35	3279216.44	5770625.66
5.01	4651718.74	11373840.83	22214402.96	38663921.70
6.01	22061877.11	52730100.88	101508571.57	175342510.85
7.01	80906354.80	189785718.89	360466465.51	617716387.14
8.01	247795984.58	571809777.67	1071886954.46	1820996990.88
9.01	663192025.10	1508272192.33	2791695583.90	4699943831.79
10.1	1721230869.01	3860102006.21	7051805429.71	11754415406.24
11.01	3534012094.94	7843726461.89	14183244881.28	23449263933.74

For Values of α =6

β	α_x 6.1	7.1	8.1	9.1
3.01	3435085.58	4530890.97	5846819.19	7400003.87
4.01	39804910.16	52361699.88	67571148.30	85649454.39
5.01	237187760.24	312074630.73	403474300.24	512877285.37
6.01	959546584.59	1266604205.61	1644124588.88	2099052956.10
7.01	3038181007.87	4026834474.75	5250073378.23	6732665510.80
8.01	8125699828.96	10813083940.60	14155516236.88	18225966193.74
9.01	19209136034.50	25652949609.22	33701173553.97	43540289500.75
10.1	44101925817.87	59084671478.66	77865002390.37	100900992819.71
11.01	82520168220.20	110788381008.92	146303586918.87	189962482530.65

network duopoly regime S3 (S4) to network duopoly regime S4 (S3) if β is large (small).

The results from (D.16), (D.17), and (D.18) lead us to Proposition 7.2.6:

Proposition 7.2.6. *The firms prefer the network monopoly regime S3 over regime S4 when* β *is large:* $\Pi^{S3} > (<) \Pi^{S4}$ *when* β *is large (small). (ii) The social planner strictly*

(weakly) prefers regime S3 (S4) over regime S4 (S3) if β is large (small):

$S(\tilde{Q}^{S2}, \tilde{P}^{S2}) \succ (\prec) S(\tilde{Q}^{S4}, \tilde{P}^{S4})$ *if β is large (small).*

D.2.1 Network Duopoly with Pre-Emptive Integrated Ticketing (S6) and Network Monopoly with Integrated Ticketing (S5)

We can now compare the network duopoly S6 and S5. Beginning with profits; if we subtract (7.51) from (7.41) we yield:

$$\tilde{\Pi}^{S5} - \tilde{\Pi}^{S6} = \frac{\alpha^2 \left(2\beta\beta_x^2 + 3\beta_x^2 + \beta^2\beta_x + \beta_x^3 + 2\beta\beta_x - \beta^2 - 4\beta - 4\right)}{2\left(\beta\beta_x - \beta - \beta_x - 3\right)\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)^2}. \tag{D.19}$$

Assuming (7.38), we find (D.19) is always positive. Hence, we can see that the total profit in the network monopoly regime S5 is greater than total profit in the network duopoly regime S6.

Moving on to look at welfare; subtracting (7.42) from (7.52), and (7.53) from (7.43) gives:

$$\tilde{Q}^{S6} - \tilde{Q}^{S5} = \frac{\alpha(\beta + \beta_x + 2)}{\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)}, \tag{D.20}$$

$$\tilde{P}^{S5} - \tilde{P}^{S6} = \frac{\alpha[\beta(-15\beta - 36 - 2\beta^2) - 28 + \beta_x(\beta_x^2 - 24 - 3\beta_x) + \beta\beta_x(4\beta\beta_x + 2\beta^2 + 2\beta_x^2 - 10 + 8\beta_x + 5\beta)]}{4\left(\beta_x\beta - \beta_x - \beta - 3\right)\left(2\beta_x\beta - 2\beta - \beta_x - 5\right)\left(4\beta_x\beta - \beta_x - 3\beta - 8\right)}. \tag{D.21}$$

Whilst it is simple to see that (D.20) is positive given (7.38), but (D.21) is complicated. As the numerator of (D.21) defines whether the function is positive or negative, we use simulations to ascertain the sign – shown in Table D.12, for values of β_x (on the horizontal heading) and β (on the vertical heading).

We can see that the total patronage on network duopoly regime S6 is greater than that on network monopoly regime S5 and from Table D.12. We see that the average price on

Table D. 12: Simulation of the Bracketed Part of Numerator of (D.21)

β	β_x					
	3.11	4.11	5.11	6.11	7.11	8.11
3.01	581.03	1316.83	2340.39	3693.83	5419.26	7558.80
4.01	1083.95	2258.46	3842.21	5889.32	8453.90	11590.07
5.01	1766.87	3495.98	5771.80	8660.46	12228.07	16540.75
6.01	2655.13	5066.71	8178.48	12068.57	16815.09	22496.17
7.01	3774.04	7007.97	11111.57	16174.97	22288.28	29541.63
8.01	5148.92	9357.08	14620.39	21040.98	28720.97	37762.46

network monopoly regime S5 is greater than that on network duopoly regime S6. The

social planner strictly prefers network duopoly regime S6 to network monopoly regime S5.

The results from (D.19), (D.20), and (D.21) lead us to Proposition 7.4.1:

Proposition 7.4.1. *(i) The firms prefer the network monopoly regime S5 over regime*

S6: $\Pi^{S5} > \Pi^{S6}$. *(ii) The social planner strictly prefers regime S6 over regime S5:*

$S(\tilde{Q}^{S6}, \tilde{P}^{S6}) \succ S(\tilde{Q}^{S5}, \tilde{P}^{S5})$.

D.2.2 Network Duopoly With Simultaneous Integrated Ticketing (S7) and Network Monopoly With Integrated Ticketing (S5)

We can now compare S7 and network S5. Beginning with profits, if we subtract (7.41) from (7.59) we yield:

$$\tilde{\Pi}^{S5} - \tilde{\Pi}^{S7} = \frac{\alpha^2[\beta(7-\beta^2+4\beta-\beta^3)+3 +\beta_x(\beta_x^4+\beta_x^3+4-9\beta_x^2-9\beta_x) +\beta\beta_x(4\beta_x^3+\beta^3+6+2\beta\beta_x^2-2\beta^2-\beta-8\beta_x-2\beta_x^2)]}{2(\beta\beta_x-\beta-\beta_x-3)(2\beta\beta_x^2-4\beta\beta_x+3\beta-\beta_x^2-3\beta_x+3)^2}. \qquad \textbf{(D.22)}$$

As the numerator of (D.22) defines whether the function is positive or negative, we use simulations to ascertain the sign – shown in Table D.13, for values of β_x (on the horizontal heading) and β (on the vertical heading).

From Table D.13 we can see that (D.22) is positive. Hence, we can see that profit in network monopoly regime S5 is greater than profit in network duopoly regime S7.

Table D. 13: Simulation of the Bracketed Part of Numerator of (D.22)

β	β_x 3.11	4.11	5.11	6.11	7.11	8.11
3.01	1361.17	4667.67	12083.44	26359.96	51174.87	91251.95
4.01	2155.79	6746.47	16546.16	34821.12	65859.80	115092.78
5.01	3310.16	9487.26	22071.73	44868.64	82801.23	142030.89
6.01	5009.36	13171.34	29037.69	56976.30	102569.19	172732.56
7.01	7489.08	18154.65	37920.24	71740.52	125880.34	208034.69
8.01	11035.69	24867.78	49294.19	89880.38	153598.00	248944.82

Moving on to look at welfare; subtracting (7.42) from (7.60), and (7.61) from (7.43) gives:

$$\tilde{Q}^{S7} - \tilde{Q}^{S5} = \frac{\alpha\left(\beta^2 + 3\beta_x^2 - 2\beta - \beta_x - 3\right)}{\left(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3\right)},$$
(D.23)

$$\tilde{P}^{S5} - \tilde{P}^{S7} = \frac{\begin{aligned}&\alpha[\beta(\beta^3 - 13\beta + \beta^2 - 25) - 12\\&+\beta_x(11 - 12\beta_x^3 - 19\beta_x^2 + 18\beta_x - \beta_x^4)\\&+\beta\beta_x(2\beta_x^2\beta^2 + 33 + 4\beta\,\beta_x)\\&+\beta^2\beta_x(20 - 4\beta - 5\beta\,\beta_x - 2\beta^2 + 2\beta^2\beta_x)\\&+3\beta\beta_x^4 + 6\beta^2\beta_x^4 + 6\beta\beta_x^5 + 32\beta\beta_x^2 - 15\beta^2\beta_x^3 - 31\beta\beta_x^3]\end{aligned}}{4\left(\beta_x\beta - \beta_x - \beta - 3\right)\left(2\beta_x^2\beta - 4\beta\beta_x + 3\beta - 3\beta_x - \beta_x^2 + 3\right)} \cdot \\ \left(4\beta_x^2\beta - 8\beta\beta_x + 4\beta + \beta^2 + \beta_x^2 + 3\right)$$
(D.24)

As the numerators of (D.23) and (D.24) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.14 and D.15, for values of β_x (on the horizontal heading) and β (on the vertical heading).

We can see from Tables D.14 and D.15, and assuming (7.38), that it is simple to show (D.23) and (D.24) are positive. As (D.23) is always positive, the total

Table D. 14: Simulation of the Bracketed Part of Numerator of (D.23)

β	β_x					
	3.11	**4.11**	**5.11**	**6.11**	**7.11**	**8.11**
3.01	25.95	46.61	73.27	105.93	144.59	189.25
4.01	30.97	51.63	78.29	110.95	149.61	194.27
5.01	37.99	58.65	85.31	117.97	156.63	201.29
6.01	47.01	67.67	94.33	126.99	165.65	210.31
7.01	58.03	78.69	105.35	138.01	176.67	221.33
8.01	71.05	91.71	118.37	151.03	189.69	234.35

Table D. 15: Simulation of the Bracketed Part of Numerator of (D.24)

β	β_x					
	3.11	**4.11**	**5.11**	**6.11**	**7.11**	**8.11**
3.01	5631.80	24218.02	73294.52	179394.69	380746.48	729319.61
4.01	11008.66	42415.31	121054.38	285820.79	592066.15	1114365.89
5.01	19543.82	68343.65	184576.57	420935.47	851619.99	1575823.68
6.01	32768.23	105112.05	269217.06	593082.52	1171552.49	2130523.04
7.01	52551.85	156467.09	381363.93	812128.39	1566117.35	2798085.82
8.01	81103.59	226792.87	528437.34	1089462.18	2051677.51	3600925.65

patronage from network duopoly regime S7 is greater than the total patronage from network monopoly regime S5. The social planner always strictly prefers network duopoly regime S7 over network monopoly regime S5. The results from (D.22), (D.23), and (D.24) lead us to Proposition 7.4.2:

Proposition 7.4.2. *(i) The firms prefer the network monopoly regime S5 over duopoly regime S7: $\Pi^{S5} > \Pi^{S7}$. (ii) The social planner strictly prefers regime S7 over regime S5:* $S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S5}, \tilde{P}^{S5})$.

D.2.3 Network Duopoly With Integrated Ticketing (S7) and Network Duopoly With Pre-Emptive Integrated Ticketing (S6)

We can now compare the S7 and S6. Beginning with profits, if we subtract (7.59) from (7.51) we yield:

$$\tilde{\Pi}^{S6} - \tilde{\Pi}^{S7} = \frac{\begin{aligned} \alpha^2 [\beta(4\beta^4 + 12\beta^3 - 89\beta - 16\beta^2 - 102) - 37 \\ + \beta_x(\beta_x^3 + 58\beta_x^2 - 7 + 68\beta_x - 2\beta_x^4) \\ + \beta\beta_x(26 - 19\beta\ \beta_x) \\ + \beta^2\beta_x(91 - 31\beta\ \beta_x + 62\beta - 8\beta^3 + 4\beta^3\beta_x - 4\beta^2 - 8\beta^2\beta_x) \\ + \beta\beta_x^3(-8 - 43\beta_x - 28\beta + 4\beta^2\beta_x + 8\beta\ \beta_x^2 + 8\beta^2\beta_x^2 - 8\beta\ \beta_x) \\ + 64\beta\beta_x^2] \end{aligned}}{2(2\beta\beta_x - 2\beta - \beta_x - 5)^2 (2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3)^2}. \qquad \text{(D.25)}$$

As the numerator of (D.25) defines whether the function is positive or negative, we use

simulations to ascertain the sign – shown in Table D.16, for values of β_x (on the horizontal

heading) and β (on the vertical heading).

Table D. 16: Simulation of the Bracketed Part of Numerator of (D.25)

β	β_x					
	3.11	**4.11**	**5.11**	**6.11**	**7.11**	**8.11**
3.01	19348.58	48733.74	96989.26	167225.18	261349.15	379826.43
4.01	47236.32	118584.68	236327.49	409943.97	647710.93	956462.78
5.01	101312.97	252541.36	499543.78	861921.26	1358072.40	2004953.44
6.01	199446.09	492796.51	964630.47	1649287.44	2579904.48	3788176.25
7.01	368342.05	901452.47	1744985.93	2953500.97	4580353.73	6677457.97
8.01	645683.06	1565163.84	2997522.78	5023882.15	7724161.81	11176839.23

From Table D.16 we can see that (D.25) is always positive. Hence, we can see that

the total profit from network duopoly regime S6 is greater than the total profit from

network duopoly regime S7.

Moving on to look at welfare; subtracting (7.52) from (7.60), and (7.61) from (7.53)

gives:

$$\tilde{Q}^{S7} - \tilde{Q}^{S6} = \frac{\begin{aligned} \alpha[\beta(7 - 4\beta - 2\beta^2) + 9 \\ + \beta_x(11\beta_x - 2\beta_x^2 - 9\beta_x) \\ + \beta\beta_x(6 - \beta - \beta\ \beta_x + 4\beta_x^2 - 7\beta_x + 2\beta^2)] \end{aligned}}{(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta x + 3)(2\beta\beta_x - \beta_x - 2\beta - 5)}, \qquad \text{(D.26)}$$

$$\tilde{P}^{S6} - \tilde{P}^{S7} = \frac{\begin{aligned}\alpha[\beta(64 - 12\beta - 70\beta^2 - 38\beta^3 - 6\beta^4) + 38 \\ + \beta_x(76\beta_x^3 + 44 + 46\beta_x^2 - 139\beta_x + 16\beta_x^4) \\ + \beta\beta_x(95 - 359\beta\ \beta_x + 126\beta^2\beta_x^2) \\ + \beta\beta_x(181\beta_x^2 + 152\beta + 95 - 359\beta\beta_x + 301\beta\beta_x^2 + 22\beta_x^3) \\ + \beta\beta_x(-420\beta_x - 56\beta^2\beta_x^3 - 68\beta^3\beta_x + 4\beta^3\beta_x^2 + 8\beta^2\beta_x^4 - 60\beta\beta_x^3) \\ + \beta\beta_x(174\beta - 140\beta^2\beta_x + 126\beta^2\beta_x^2 - 8\beta_x^5 + 16\beta\beta_x^5 + 8\beta^4\beta_x^2) \\ + \beta\beta_x(14\beta^4\beta_x - 16\beta^4\beta_x - 20\beta\beta_x^4 - 60\beta_x^4 + 87\beta^3)]\end{aligned}}{2(2\beta_x\beta - \beta_x - 2\beta - 5)(4\beta\beta_x - 3\beta - \beta_x - 8)} \quad . \qquad \text{(D.27)}$$

$$\left(2\beta_x^2\beta - 4\beta\beta_x - \beta_x^2 + 3\beta - 3\beta_x + 3\right)$$

$$\left(4\beta_x^2\beta - 8\beta\beta_x + 4\beta - 7\beta_x + \beta^2 + \beta_x^2 + 3\right)$$

As the numerators of (D.26) and (D.27) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.17 and D.18, for values of β_x (on the horizontal heading) and β (on the vertical heading).

Table D. 17: Simulation of the Bracketed Part of Numerator of (D.26)

| β | β_x | | | | | |
	3.11	4.11	5.11	6.11	7.11	8.11
3.01	85.65	272.71	629.09	1215.04	2090.80	3316.61
4.01	203.14	519.76	1076.31	1957.02	3246.14	5027.91
5.01	388.60	866.47	1650.85	2850.01	4572.17	6925.58
6.01	667.35	1350.14	2402.05	3955.32	6142.21	9094.93
7.01	1064.72	2008.10	3379.21	5334.29	8029.57	11621.30
8.01	1606.00	2877.67	4631.66	7048.22	10307.59	14590.00

Table D. 18: Simulation of the Bracketed Part of Numerator of (D.27)

| β | β_x | | | | | |
	3.11	4.11	5.11	6.11	7.11	8.11
3.01	49630.34	302287.30	1268265.32	4095195.76	10956994.83	25526812.40
4.01	192287.54	874907.34	3149677.81	9368798.15	23903978.05	54052429.76
5.01	567944.30	2124243.33	6690502.34	18337655.22	44512307.48	97523323.59
6.01	1406316.29	4625917.46	13063807.32	33115005.54	76290669.69	161438589.74
7.01	3059240.68	9252314.24	24029367.60	56835475.18	124377990.28	253740018.69
8.01	6026511.80	17234455.83	42054773.06	93864373.13	195875629.72	385307665.51

From Tables D.17 and D.18, we can see that (D.26) and (D.27) are positive. Hence, we can see that the total patronage from network duopoly regime S7 is greater than the total patronage from network duopoly regime S6. The average price of network duopoly regime

S6 is greater than the average price of network duopoly regime S7. The results from (D.25), (D.26), and (D.27) lead us to Proposition 7.4.3:

Proposition 7.4.3. *(i) The firms prefer the network monopoly regime S6 over duopoly regime S7:* $\Pi^{S6} > \Pi^{S7}$. *(ii) The social planner strictly prefers regime S7 over regime S6:* $S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S6}, \tilde{P}^{S6})$.

D.2.4 Network Duopoly With Independent Integrated Ticketing (S8) and Network Monopoly With Integrated Ticketing (S5)

We can now compare S8 and S5. Beginning with profits, if we subtract (7.66) from (7.41) we yield:

$$\tilde{\Pi}^{S5} - \tilde{\Pi}^{S8} = \frac{\alpha^2[\beta(4\beta^2 + 85 + 32\beta) + 66}{18(\beta\beta_x - \beta - \beta_x - 3)(2\beta\beta_x^2 - 2\beta - \beta_x - 5)^2}.$$

$$+ \beta_x(85 + 44\beta_x - 9\beta_x^2)$$
$$+ \beta\beta_x(42 + 5\beta_x - 24\beta - 8\beta^2 + 4\beta^2\beta_x - 8\beta\beta_x)]$$

(D.28)

As the numerator of (D.28) defines whether the function is positive or negative, we use simulations to ascertain the sign – shown in Table D.19, for values of β_x (on the horizontal heading) and β (on the vertical heading).

From Table D.19 we can see that (D.28) is positive. Hence we can see that the total profit from network monopoly regime S5 is always greater than the total profit from

Table D. 19: Simulation of the Bracketed Part of Numerator of (D.28)

β	β_x					
	3.11	4.11	5.11	6.11	7.11	8.11
3.01	678.07	790.38	872.05	869.09	727.49	393.26
4.01	762.19	1155.59	1713.72	2382.58	3108.17	3836.48
5.01	1034.74	2048.09	3582.02	5582.51	7995.57	10767.20
6.01	1602.58	3700.02	6882.34	11095.56	16285.67	22398.68
7.01	2572.56	6343.49	12020.11	19548.43	28874.44	39944.15
8.01	4051.53	10210.64	19400.74	31567.81	46657.85	64616.88

network duopoly regime S8.

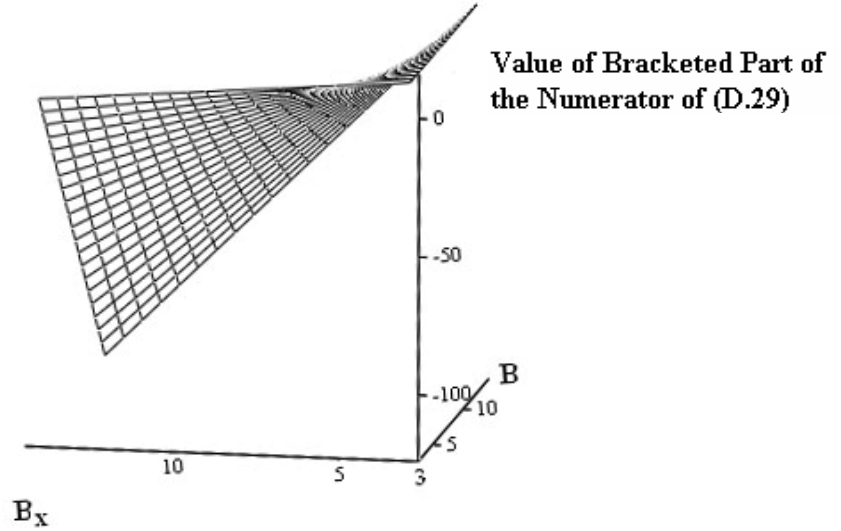Moving on to look at welfare; subtracting (7.42) from (7.67), and (7.68) from (7.43) gives:

$$\tilde{Q}^{S8} - \tilde{Q}^{S5} = \frac{2\alpha(2\beta_x + 3\beta - \beta\beta_x + 6)}{3(2\beta\beta_x - \beta_x - 2\beta - 5)},$$ (D.29)

$$\tilde{P}^{S5} - \tilde{P}^{S8} = \frac{\begin{array}{l}\alpha[\beta(85\beta + 14\beta^2 + 167) + 114 \\ + \beta_x(101 - 3\beta_x^2 + 16\beta_x) \\ + \beta\beta_x(27 - 36\beta - 19\beta\beta_x - 15\beta_x^2 + 6\beta\beta_x^2 + 2\beta^2\beta_x - 16\beta^2 - 59\beta_x]\end{array}}{12(\beta_x\beta - \beta_x - \beta - 3)(2\beta\beta_x - 2\beta - \beta_x - 5)(5\beta_x\beta - \beta_x - 3\beta - 9)}.$$ (D.30)

Now, to define the signs of (D.29) and (D.30) we plot 3D graphs of the bracketed part of

Figure D.1: Plot of Bracketed Part of the Numerator of (D.29)


the numerators – see Figure D.1 and D.2, with β and β_x making up the x- and z-axis, and the value of the bracket on the y-axis.

(D.29) is clearly ambiguous as Figure D.1 shows that, when both β and β_x are high, (D.29) becomes negative. When β is significantly higher than β_x, it may or may not cause (D.29) to be negative. Hence, when both β and β_x are low, we can conclude that the total patronage in regime S8 is greater than the network monopoly regime S5. However, when

both β and β_x are high, then the total patronage from network monopoly regime S5 is greater than the network duopoly regime S8. Although, we can also see that, if β and β_x are low, it is possible that the total patronage from network duopoly regime S8 remains higher than the total patronage from the network monopoly regime S5.

In Figure D.2, we see that if the values of both β and β_x are low, then the (D.30) is negative, but once β becomes large it becomes more likely that (D.30) is positive and this likelihood increases as β_x grows. However, it should be noted that if β is large and β_x is

Figure D. 2: Plot of Bracketed Part of the Numerator of (D.30)
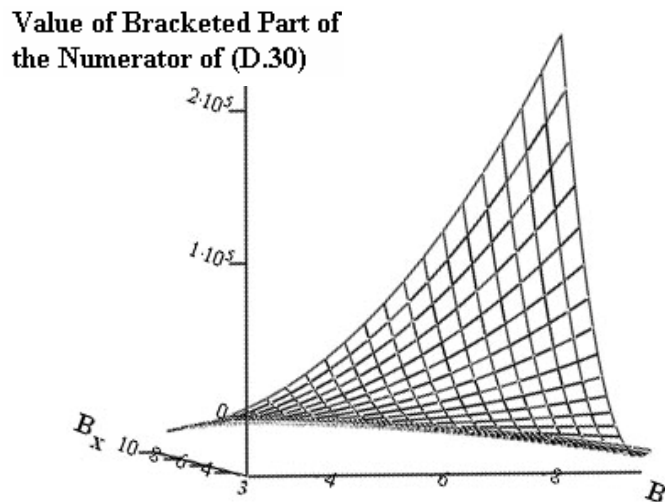
small, it is possible that (D.30) could be negative. Hence, when both β and β_x are high we can see that the average price in network duopoly regime S8 is greater than the network monopoly regime S5. However, when the both β and β_x are low then the average price from network monopoly regime S5 is greater than the average price from network duopoly regime S8. Although, if β is high and β_x is low, it is possible that the average price from network monopoly regime S5 remains higher than the average price from the network duopoly regime S8. The results from (D.28), (D.28), and (D.30) lead us to Proposition 7.4.4:

Proposition 7.4.4 *(i) The firms prefer regime S5 over regime S8:* $\tilde{\Pi}^{S5} > \tilde{\Pi}^{S8}$. *ii) The social planner at least weakly prefers regime S8 to regime to S5 when* $\beta_x < \beta < 6$ *or when* β *is high and is* β_x *low. When* $\beta > \beta_x > 6$, *when the social planner at least weakly prefers regime S5 to S8:* $S(\tilde{Q}^{S8}, \tilde{P}^{S8}) \succeq S(\tilde{Q}^{S5}, \tilde{P}^{S5})$ *when* $\beta_x < \beta < 6$ *or when* β *is high and is* β_x *low.* $S(\tilde{Q}^{S5}, \tilde{P}^{S5}) \succeq S(\tilde{Q}^{S8}, \tilde{P}^{S8})$ *when* $\beta > \beta_x > 6$.

D.2.5 Network Duopoly With Independent Integrated Ticketing (S8) and Network Duopoly With Pre-Emptive Integrated Ticketing (S6)

We can now compare S8 and S6. Beginning with profits, if we subtract (7.66) from (7.51) we yield:

$$\tilde{\Pi}^{S6} - \tilde{\Pi}^{S8} = \frac{\alpha^2 \left(4\beta^2 \beta_x - 4\beta\beta_x - 17\beta_x - 29\beta - 4\beta^2 - 34 \right)}{18 \left(2\beta\beta_x^2 - 2\beta - \beta_x - 5 \right)^2}. \tag{D.31}$$

Now, to define the sign of (D.31) we plot a 3D graph of the bracketed part of the numerator – see Figure D.3, with β and β_x making up the x- and z-axis, and the value of the bracket on the y-axis.

Figure D.3 shows that for low values ($\beta_x < \beta < 4.78$) of both β and β_x then (D.31) is negative, but once β becomes large ($\beta_x > \beta > 4.78$) it becomes more likely that (D.31) is positive and this likelihood increases as β_x grows. Hence, we can see that the total profit from network duopoly regime S6 is greater than total profit from network duopoly regime S8, when $\beta_x > \beta > 4.78$. However, we can see that the total profit arising from network duopoly regime S6 is greater than the total profit arising from network duopoly regime S8, if $\beta < \beta_x < 4.78$.

Figure D. 3: Plot of Bracketed Part of the Numerator of (D.31)
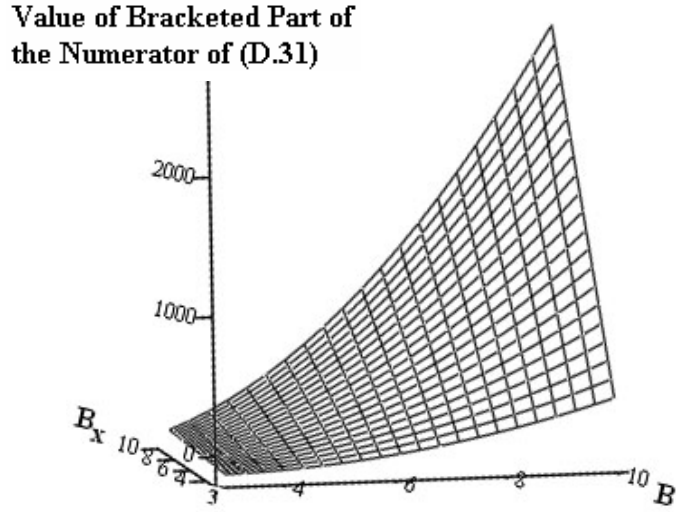
Moving on to look at welfare; subtracting (7.67) from (7.52), and (7.53) from (7.68) gives:

$$\tilde{Q}^{S6} - \tilde{Q}^{S8} = \frac{\alpha\left(2\beta\beta_x - 3\beta - \beta_x - 6\right)}{3\left(2\beta\beta_x - \beta_x - 2\beta - 5\right)},$$
(D.32)

$$\tilde{P}^{S8} - \tilde{P}^{S6} = \frac{\begin{aligned}&\alpha[\beta(12\beta^2 + 53\beta + 67) + 26\\ &+ \beta_x(5\beta_x + 23)\\ &+ \beta\beta_x(4\beta^2\beta_x - 12 - 2\beta\beta_x - 37\beta_x - 33\beta - 16\beta^2 - 6\beta_x^2 + 12\beta\beta_x^2)]\end{aligned}}{6\left(5\beta_x\beta - \beta_x - 3\beta - 9\right)\left(2\beta\beta_x - 2\beta - \beta_x - 5\right)\left(4\beta_x\beta - \beta_x - 3\beta - 8\right)}.$$
(D.33)

As the numerators of (D.32) and (D.33) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.20 and D.21, for values of β_x (on the horizontal heading) and β (on the vertical heading).

From Table D.20 we can see that the total patronage from network duopoly regime

Table D. 20: Simulation of the Bracketed Part of Numerator of (D.32)

β	β_x					
	3.11	4.11	5.11	6.11	7.11	8.11
3.01	0.58	5.60	10.62	15.64	20.66	25.68
4.01	3.80	10.82	17.84	24.86	31.88	38.90
5.01	7.02	16.04	25.06	34.08	43.10	52.12
6.01	10.24	21.26	32.28	43.30	54.32	65.34
7.01	13.46	26.48	39.50	52.52	65.54	78.56
8.01	16.68	31.70	46.72	61.74	76.76	91.78

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Table D. 21: Simulation of the Bracketed Part of Numerator of (D.33)

β	β_x 3.11	4.11	5.11	6.11	7.11	8.11
3.01	1285.56	3993.07	8905.47	16566.73	27520.82	42311.70
4.01	2861.20	8514.26	18497.22	33823.47	55506.44	84559.52
5.01	5043.15	14801.09	31790.25	57637.46	93969.59	142413.47
6.01	7836.98	22936.41	48992.69	88390.11	143512.96	216745.52
7.01	11248.28	33003.07	70312.67	126462.82	204739.24	308427.65
8.01	15282.59	45083.91	95958.33	172237.01	278251.12	418331.83

S6 is always greater than total patronage from network duopoly regime S8. We can also see from Table D.21 that average price in network duopoly regime S8 is always greater than average price in network duopoly regime S6. The social planner always strictly prefers network duopoly regime S6 over network duopoly regime S8. The results from (D.31), (D.32), and (D.33) lead us to Proposition 7.4.5:

Proposition 7.4.5 *(i) The firms prefer regime S6 over regime 8 when $\beta > \beta_x > 4.78$:*

$\tilde{\Pi}^{S8} > \tilde{\Pi}^{S6}$ *when $\beta > \beta_x > 4.78$. ii) The social planner at least strictly prefers regime S6 to regime to S8: $S(\tilde{Q}^{S6}, \tilde{P}^{S6}) \succ S(\tilde{Q}^{S8}, \tilde{P}^{S8})$.*

D.2.6 Network Duopoly With Integrated Ticketing (S8) and Network Duopoly With Simultaneous Integrated Ticketing (S7)

We can now compare S8 and S7. Beginning with profits, if we subtract (7.66) from (7.59) we yield:

$$
\tilde{\Pi}^{S7} - \tilde{\Pi}^{S8} = \frac{\begin{aligned}
&-\alpha^2[\beta(189\beta^2 + 36\beta^4 + 63\beta + 144\beta^3 - 45) - 27 \\
&+\beta_x(408\beta_x + 145\beta_x^3 + 777\beta_x^2 - \beta_x^4 - 522) \\
&+\beta\beta_x(80\beta^3\beta_x^3 - 416\beta^2\beta_x^2 + 1159\beta\beta_x - 1374) \\
&+\beta^4\beta_x(+136\beta_x - 168 - 175\beta_x^2 - 72\beta + 36\beta) \\
&+\beta^2\beta_x(725\beta\ \beta_x - 1098 - 342\beta) \\
&+\beta_x^5(32\beta^3 - 16\beta^4 + 120\beta^2 - 64\beta) \\
&+\beta\beta_x^2(897 + 88\beta^2\beta_x^2 - 304\beta\beta_x + 488\beta_x - 538\beta_x^2) \\
&-360\beta^2\beta_x^4)]
\end{aligned}}{18\left(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3\right)^2 \left(2\beta\beta_x^2 - 2\beta - \beta_x - 5\right)^2}. \qquad (D.34)
$$

Now, to define the sign of (D.34) we plot a 3D graph of the bracketed part of the numerator – see Figure D.4, with β and β_x making up the x- and z-axis, and the value of the bracket on the y-axis.

From Figure D.4 we can see that (D.34) starts out at zero before becoming negative

Figure D. 4: Plot of Bracketed Part of the Numerator of (D.34)
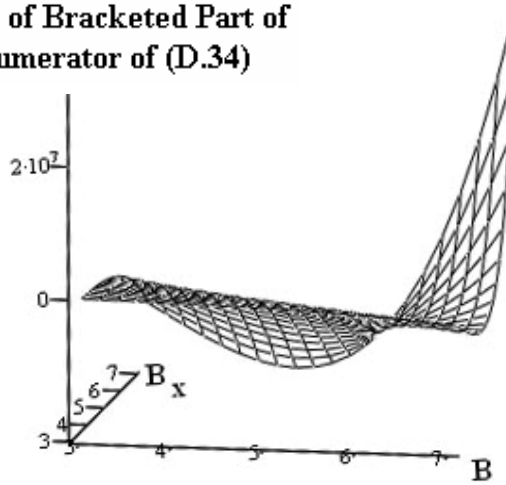
and it isn't until both β and β_x reach over 6.51 that the function becomes positive. We can also see that, if β is high and β_x is low, it is possible that the function remains negative. We need to be a little wary here as a large part of the plot that is negative and looks to continue to be negative, when $\beta_x > \beta$, but this is a case that is not of interest to us. Hence,

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we can see that, when β and β_x are below 6.51, the profit from network duopoly regime S8 is greater than profit from network duopoly regime S7. When both β and β_x increase above 6.51, then the total profit from network duopoly regime S7 is greater than profit from network duopoly regime S8. It should also be noted that, if β is large but β_x remains low, then it is possible that the total profit from network duopoly regime S7 is smaller than total profit from network duopoly regime S8.

Moving on to look at welfare; subtracting (7.67) from (7.60), and (7.61) from (7.68) gives:

$$\tilde{Q}^{S7} - \tilde{Q}^{S8} = \frac{\alpha \begin{pmatrix} \beta(-21\beta - 6\beta^2 - 6) + 9 \\ +\beta_x(48 - 18\beta_x - 5\beta_x^2) \\ +\beta\beta_x(54 + 6\beta^2 - 32\beta_x + 15\beta - 20\beta\beta_x + 8\beta_x^2 + 4\beta\ \beta_x^2) \end{pmatrix}}{3(2\beta\beta_x - \beta_x - 2\beta - 5)(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3)}, \tag{D.35}$$

$$\tilde{P}^{S8} - \tilde{P}^{S7} = \frac{\begin{aligned}&\alpha[\beta(78 - 432\beta^2 - 282\beta - 201\beta^3 - 30\beta^4) + 99 \\ &+\beta_x(53\beta_x^4 + 239\beta_x^3 - 107\beta_x^2 - 468\beta_x + 222) \\ &+\beta\beta_x(754\beta^2\beta_x^2 - 1888\beta\ \beta_x - 52\beta^3\beta_x^3 + 708) \\ &+\beta^5\beta_x(64 - 66\beta_x + 32\beta_x^2) \\ &+\beta^2\beta_x(1194\beta + 498\beta^2 - 1166\beta\beta_x - 481\beta^2\beta_x + 168\beta^2\beta_x^2 + 1246) \\ &+\beta\beta_x^5(24\beta^2\beta_x + 8\beta^3 - 138\beta - 187 - 52\beta^2 - 30\beta_x + 48\beta_x\beta) \\ &+\beta\beta_x^2(1116\beta\beta_x - 1641 - 6\beta\beta_x^2 + 479\beta_x - 178\beta^2\beta_x^2 + 161\beta_x^2)]\end{aligned}}{\begin{aligned}&6(5\beta_x\beta - \beta_x - 3\beta - 9)(2\beta\beta_x - 2\beta - \beta_x - 5) \\ &(2\beta\beta_x^2 - 4\beta\beta_x + 3\beta - \beta_x^2 - 3\beta_x + 3) \\ &(4\beta_x^2\beta - 8\beta\beta_x + 4\beta - 7\beta_x + \beta^2 + \beta_x^2 + 3)\end{aligned}}. \tag{D.36}$$

As the numerators of (D.35) and (D.36) define whether the functions are positive or negative, we use simulations to ascertain the signs – shown in Table D.22 and D.23, for values of β_x (on the horizontal heading) and β (on the vertical heading).

Using Tables D.22 and D.23 and assuming (7.38) it is simple to show that (D.35) and (D.36) are positive. Hence, we can see that the total patronage arising from network

duopoly regime S8 is larger than the total patronage arising from network duopoly regime

S7. We can also see that the average price arising from network duopoly regime S8 is

Table D. 22: Simulation of the Bracketed Part of Numerator of (D.35)

β	β_x					
	3.11	**4.11**	**5.11**	**6.11**	**7.11**	**8.11**
3.01	3534.30	6703.91	11371.48	17868.94	26528.21	37681.22
4.01	6481.14	12235.57	20594.50	32106.33	47319.46	66782.30
5.01	10411.40	19528.58	32634.08	50540.77	74061.54	104009.26
6.01	15401.04	28694.92	47638.19	73356.23	106974.40	149618.06
7.01	21526.04	39846.52	65754.79	100736.67	146278.01	203864.66
8.01	28862.33	53095.37	87131.83	132866.04	192192.33	267005.02

Table D. 23: Simulation of the Bracketed Part of Numerator of (D.36)

β	β_x					
	3.11	**4.11**	**5.11**	**6.11**	**7.11**	**8.11**
3.01	212197.95	1700020.26	8017406.10	27458602.01	76080405.96	181435089.16
4.01	720672.54	4793010.97	20828293.39	68373563.84	184757098.36	433466937.21
5.01	1885768.58	10877386.29	44097594.42	139562307.51	369172328.96	854461560.92
6.01	4259261.61	21846824.62	83041609.25	253644354.34	657079200.27	1500883758.90
7.01	8670554.34	40398784.23	144696989.20	426815814.86	1082640338.73	2439965337.85
8.01	16285866.46	70195280.52	238261955.91	679472943.98	1687458720.44	3751291183.31

larger than that which arises from network duopoly regime S7. The results from (D.34),

(D.35), and (D.36) lead us to Proposition 7.4.6:

Proposition 7.4.6 *(i) The firms prefer regime S7(S8) over regime S8(S7)*

when $\beta > \beta_x > 6.51$ ($\beta_x < \beta < 6.51$ or β is high and β_x is low) : $\tilde{\Pi}^{S7} > \tilde{\Pi}^{S8}$ ($\tilde{\Pi}^{S8} > \tilde{\Pi}^{S7}$)

when $\beta > \beta_x > 6.51$ ($\beta_x < \beta < 6.51$ or β is high and β_x is low). ii) The social planner

strictly prefers regime S7 to regime to S8: $S(\tilde{Q}^{S7}, \tilde{P}^{S7}) \succ S(\tilde{Q}^{S8}, \tilde{P}^{S8})$.

Appendix E: Chapter 9 Proofs

E.1. Proof of Lemma 9.1

Let us suppose that the social planner provides routes 1 and 3 or routes 1 and 2 instead of routes 2 and 3; thus the separate demands would become:

$$Q_1^{"} = \alpha - x - f_1, \tag{E.1a}$$

$$Q_l^{"} = \beta - 1 - f_l, \tag{E.1b}$$

$$Q_m^{"} = \beta - 1 - x - f_m, \qquad\qquad l \neq m = 2,3. \tag{E.1c}$$

Given that a welfare-maximising social planner charges a zero fare, the consumer surplus for each market can be calculated; total consumer surplus, $\hat{C}^{S'}$, is

$$\hat{C}^{S'} = \sum_{j=2}^{3} \hat{C}_j = \frac{1}{2}[(\alpha - x)^2 + (\beta - 1)^2 + (\beta - 1 - x)^2]. \tag{E.2}$$

Welfare for the case of the social planner with the alternative two-route system, 1 and 2 or 1 and 3, $\hat{W}^{S'}$, is then:

$$\hat{W}_{12}^{S'} = \hat{W}_{13}^{S'} = \frac{1}{2}[(\alpha - x)^2 + (\beta - 1)^2 + (\beta - 1 - x)^2] - F(1 + x). \tag{E.3}$$

The social planner will prefer to supply the combination of routes 2 and 3 rather than any other dual combination if $\hat{W}_{12}^{S'} = \hat{W}_{13}^{S'} < \hat{W}_{23}^{S}$. Subtracting (E.3) from (9.12):

$$\hat{W}_{23}^{S} - \hat{W}_{12}^{S'} = 2(1 - \alpha) + F(x - 1) + x(\alpha + \beta - x - 1) > 0. \tag{E.4}$$

Solving the inequality for F gives us the value of F that ensures the social planner will weakly prefer to supply routes 2 and 3 over any other pair combination:

$$F > \frac{2(\alpha - 1) + x(x + 1 - \alpha - \beta)}{(x - 1)}. \tag{9.13}$$

This leads to Lemma 9.1.

Lemma 9.1. *The welfare maximising social planner will weakly prefer to supply routes 2 and 3 over any other pair combination if:*

$$F > \frac{2(\alpha - 1) + x(x + 1 - \alpha - \beta)}{(x - 1)}.$$ (9.13)

E.2. Proof of Proposition 9.1

Compare the social planner's complete network and incomplete network by subtracting (9.12) from (9.9):

$$W^S - \hat{W}^S = (\alpha - x)^2 - (\alpha - 2)^2 - 2Fx.$$ (E.5)

Rearranging this in terms of F gives:

$$F = \frac{1}{2x}\left[(\alpha - x)^2 - (\alpha - 2)^2\right].$$ (E.6)

This leads to Proposition 9.1

Proposition 9.1. *The welfare maximising social planner would optimally provide a complete network if the fixed operating cost per unit distance satisfies the inequality:*

$$F < \frac{1}{2x}\left[(\alpha - x)^2 - (\alpha - 2)^2\right].$$ (9.14)

E.3. Proof of Corollary 9.1

Given (9.2) and that $0 < x < 2$, we can see that all the elements of (9.14) are positive; that is $(\alpha - x)^2 > 0$, $(\alpha - 2)^2 > 0$, and $2x > 0$. We can also see that (9.2) and $0 < x < 2$ mean that $(\alpha - x)^2 > (\alpha - 2)^2$ and this leads us to Corollary 9.1:

Corollary 9.1. *If the fixed cost of operation per unit distance is zero, the welfare maximising social planner will always provide a complete network.*

E.4. Proof of Corollary 9.2

If we now partially differentiate the bracketed part of the R.H.S[1] of (9.14) with respect to x, and then α we have:

$$\partial / \partial x = \frac{(x^2 - 4\alpha + 4)}{2x^2},$$

(E.7a)

$$\partial / \partial \alpha = \frac{(2 - x)}{x}.$$

(E.7b)

Using (9.2) we can see that (E.7a) is negative whilst (E.7b) is always positive. Taking into account the fact that the range of values, which satisfy the inequality (9.14), increases as the RHS of (9.14) increases. This leads to Corollary 9.2.

Corollary 9.2. *The welfare-maximising social planner would prefer to provide a complete network for an increasing (decreasing) range of F of as α (x) rises.*

E.5 Proof of Proposition 9.2

Subtracting (9.21) from (9.18) gives:

$$\Pi^M - \hat{\Pi}^M = \frac{\alpha^2 - 6\alpha x + 3x^2 + 4\beta^2 - 4\alpha\beta - 12 - 12Fx}{12}.$$

(E.8)

If we set F to zero we can see that (E.8) is always positive (see Corollary 9.3). However, when F is positive, the monopoly will provide a complete network if the fixed operating cost per unit distance satisfies the inequality:

$$F < \frac{\alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x}.$$

(9.22)

This leads us to Proposition 9.3.

Proposition 9.2: *The network monopolist will provide a complete network if the fixed operating cost per unit distance satisfies the inequality:*

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[1] This part is what the sign of the equation is dependent on.

$$F < \frac{\alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x}. \tag{9.22}$$

E.6. Proof of Corollary 9.3

A simulation could show the range of values that equation (9.22) takes and would

show that $\dfrac{\alpha^2 - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x} > 0$. However, a more definitive proof that

$\dfrac{\alpha^2 - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x} > 0$ is available. From (9.22):

$$f = \alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta. \tag{E.9}$$

Note that f is monotonic in x in the relevant range (0, 2) as per the following:

$$\frac{\delta f}{\delta x} = -6\alpha + 6x < 0.$$

Therefore, we are only interested in properties of f in $f(\alpha, \beta, \bar{x})$. If we form a bordered

Hessian, $|B|$, then:

$$|B_1| = \begin{vmatrix} 0 & A \\ A & 2 \end{vmatrix}$$
$$|B_1| = -(2\alpha + 12 - 6x - 4\beta)^2$$
$$|B_1| < 0$$

$$|B_2| = \begin{vmatrix} 0 & A & B \\ A & 2 & -4 \\ B & -4 & 8 \end{vmatrix}$$
$$|B_2| = -1152 + 1152x - 288x^2$$

Using $0 < x < 2$ gives:

$$|B_3| < 0$$

where $A = 2\alpha + 12 - 6x - 4\beta$ and $B = 4(2\beta - \alpha)$. Hence it is quasiconvex in the relevant

range $x\varepsilon(0,2)$.

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If we now look at the partial derivatives of (E.8) and set them equal to zero:

$$\frac{\partial f}{\partial x} = -6\alpha + 6x = 0,$$

$$\frac{\delta f}{\delta \alpha} = 2\alpha + 12 - 6x - 4\beta = 0,$$

$$\frac{\delta f}{\delta \beta} = 4(2\beta - \alpha) = 0.$$

Then by substitution we reveal a stationary point, that we know is a minimum due to the function being quasiconvex, when $x = 2$, $\alpha = 2$ and $\beta = 1$, where (E.9) is zero; meaning that when $x\varepsilon(0,2)$, $\alpha > 2$, and $\beta > 1$ then the function is greater than zero. Therefore, the RHS of (9.22) is positive in the relevant range; meaning when fixed cost is zero that the monopolist's profit from providing route 1 is greater than when the monopolist does not provide route 1 and results in Corollary 9.3:

Corollary 9.3. *If the fixed cost of operation per unit distance is zero, the welfare maximising social planner will always provide a complete network.*

E.7 Proof of Proposition 9.3 and 9.4

The monopolist is indifferent between supplying a complete network and an incomplete network with fixed cost $\tilde{F}^S$ if $\xi^M(\tilde{F}^{SP}) = \Pi^M(\tilde{F}^{SP}) - \hat{\Pi}^M(\tilde{F}^{SP}) = 0$. Using (9.18), (9.21), and (9.12) solved for F, in this yields:

$$\xi^M(\tilde{F}) = (\alpha^2 + 6\alpha x - 3x^2 + 4\beta^2 + 12 - 12\alpha - 4\alpha\beta) = 0. \tag{E.10}$$

If we now partially differentiate the R.H.S of (E.10) with respect to x, α, and then β we have:

$$\partial \xi^M(\tilde{F})/\partial x = 6(\alpha - x), \tag{E.11a}$$

$$\partial \xi^M (\tilde{F}) / \partial \alpha = 2(\alpha + 3x - 6 - 2\beta), \tag{E.11b}$$

$$\partial \xi^M (\tilde{F}) / \partial \beta = 4(2\beta - \alpha). \tag{E.11c}$$

Using (9.2) we can see that (E.11a) is always positive; however, the others are more ambiguous. (E.11b) is negative if all parameters are low or if $\beta > \frac{1}{2}\alpha$ and (E.11c) is positive if $\beta > \frac{1}{2}\alpha$. This leads to Proposition 9.3 and 9.4.

Proposition 9.3. *If faced with the social planner's threshold operating cost then the range of values of the operating cost per unit distance for which the monopolist will provide a complete network increases (decreases) as x increases (decreases).*

Proposition 9.4. *(i) The monopolist, when faced with the social planner's threshold operating cost per unit distance, will, if β is high compared to α, offer a complete network for an increasing (decreasing) range of operating cost per unit distance as α decreases (increases) and as β increases (decreases). (ii) The monopolist, when faced with the social planner's threshold operating cost per unit distance, will, if β is low compared to α, offer a complete network for an increasing (decreasing) range of operating cost per unit distance for as α increases (decreases) and as β decreases (increases).*

Appendix F: Chapter 10 Proofs

F.1 Proof of Proposition 10.1

Subtracting equation (10.3) from (9.22) gives:

$$\tilde{F}^M - \tilde{F}^E = \frac{6\alpha + 2\beta^2 - \alpha^2 - 2\alpha\beta - 6}{6x} \tag{F.1}$$

If (F.1) is positive then (9.22) is greater than (10.3) and the monopolist will provide a complete network for a greater range of the operating cost per unit distance. Alternatively, if (F.1) is negative then (9.22) is less than (10.3), so the entrant will provide a complete

Table F.1: Simulation of Values (F.1)

For Values of x=0.5

	α											
β	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	5.1	5.5	6.1	6.6
1.1	-0.20	-0.02	0.00	-0.15	-0.47	-0.95	-1.60	-2.42	-3.40	-4.55	-5.87	-7.35
1.6	0.33	0.35	0.20	-0.12	-0.60	-1.25	-2.07	-3.05	-4.20	-5.52	-7.00	-8.65
2.1	1.20	1.05	0.73	0.25	-0.40	-1.22	-2.20	-3.35	-4.67	-6.15	-7.80	-9.62
2.6	2.40	2.08	1.60	0.95	0.13	-0.85	-2.00	-3.32	-4.80	-6.45	-8.27	-10.25
3.1	3.93	3.45	2.80	1.98	1.00	-0.15	-1.47	-2.95	-4.60	-6.42	-8.40	-10.55
3.6	5.80	5.15	4.33	3.35	2.20	0.88	-0.60	-2.25	-4.07	-6.05	-8.20	-10.52
4.1	8.00	7.18	6.20	5.05	3.73	2.25	0.60	-1.22	-3.20	-5.35	-7.67	-10.15
4.6	10.53	9.55	8.40	7.08	5.60	3.95	2.13	0.15	-2.00	-4.32	-6.80	-9.45
5.1	13.40	12.25	10.93	9.45	7.80	5.98	4.00	1.85	-0.47	-2.95	-5.60	-8.42
5.6	16.60	15.28	13.80	12.15	10.33	8.35	6.20	3.88	1.40	-1.25	-4.07	-7.05
6.1	20.13	18.65	17.00	15.18	13.20	11.05	8.73	6.25	3.60	0.78	-2.20	-5.35
6.6	24.00	22.35	20.53	18.55	16.40	14.08	11.60	8.95	6.13	3.15	0.00	-3.32

For Values of x=1

	α											
β	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	5.1	5.5	6.1	6.6
1.1	-0.10	-0.01	0.00	-0.08	-0.24	-0.48	-0.80	-1.21	-1.70	-2.28	-2.94	-3.68
1.6	0.17	0.17	0.10	-0.06	-0.30	-0.63	-1.04	-1.53	-2.10	-2.76	-3.50	-4.33
2.1	0.60	0.52	0.37	0.12	-0.20	-0.61	-1.10	-1.68	-2.34	-3.08	-3.90	-4.81
2.6	1.20	1.04	0.80	0.47	0.07	-0.43	-1.00	-1.66	-2.40	-3.23	-4.14	-5.13
3.1	1.97	1.72	1.40	0.99	0.50	-0.08	-0.73	-1.48	-2.30	-3.21	-4.20	-5.28
3.6	2.90	2.57	2.17	1.67	1.10	0.44	-0.30	-1.13	-2.04	-3.03	-4.10	-5.26
4.1	4.00	3.59	3.10	2.52	1.87	1.12	0.30	-0.61	-1.60	-2.68	-3.84	-5.08
4.6	5.27	4.77	4.20	3.54	2.80	1.97	1.07	0.07	-1.00	-2.16	-3.40	-4.73
5.1	6.70	6.12	5.47	4.72	3.90	2.99	2.00	0.92	-0.24	-1.48	-2.80	-4.21
5.6	8.30	7.64	6.90	6.07	5.17	4.17	3.10	1.94	0.70	-0.63	-2.04	-3.53
6.1	10.07	9.32	8.50	7.59	6.60	5.52	4.37	3.12	1.80	0.39	-1.10	-2.68
6.6	12.00	11.17	10.27	9.27	8.20	7.04	5.80	4.47	3.07	1.57	0.00	-1.66

For Values of x=1.5

β	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	5.1	5.5	6.1	6.6
1.1	-0.07	-0.01	0.00	-0.05	-0.16	-0.32	-0.53	-0.81	-1.13	-1.52	-1.96	-2.45
1.6	0.11	0.12	0.07	-0.04	-0.20	-0.42	-0.69	-1.02	-1.40	-1.84	-2.33	-2.88
2.1	0.40	0.35	0.24	0.08	-0.13	-0.41	-0.73	-1.12	-1.56	-2.05	-2.60	-3.21
2.6	0.80	0.69	0.53	0.32	0.04	-0.28	-0.67	-1.11	-1.60	-2.15	-2.76	-3.42
3.1	1.31	1.15	0.93	0.66	0.33	-0.05	-0.49	-0.98	-1.53	-2.14	-2.80	-3.52
3.6	1.93	1.72	1.44	1.12	0.73	0.29	-0.20	-0.75	-1.36	-2.02	-2.73	-3.51
4.1	2.67	2.39	2.07	1.68	1.24	0.75	0.20	-0.41	-1.07	-1.78	-2.56	-3.38
4.6	3.51	3.18	2.80	2.36	1.87	1.32	0.71	0.05	-0.67	-1.44	-2.27	-3.15
5.1	4.47	4.08	3.64	3.15	2.60	1.99	1.33	0.62	-0.16	-0.98	-1.87	-2.81
5.6	5.53	5.09	4.60	4.05	3.44	2.78	2.07	1.29	0.47	-0.42	-1.36	-2.35
6.1	6.71	6.22	5.67	5.06	4.40	3.68	2.91	2.08	1.20	0.26	-0.73	-1.78
6.6	8.00	7.45	6.84	6.18	5.47	4.69	3.87	2.98	2.04	1.05	0.00	-1.11

network for a greater range of the operating cost per unit distance. Table F.1 shows a simulation of (F.1) for a range of values of α, β, and x. We can see that there tends to be an even split, so that if $\alpha > \beta$ ($\alpha < \beta$) then (F.1) is negative (positive) and the entrant (monopolist) will provide a complete network for a greater range of values than the monopolist (entrant). This leads directly to Proposition 10.1:

Proposition 10.1. *When entry is allowed on route 1 and restricted to a single entrant, with competition over price, and an operating cost, then the entrant (monopolist) will provide a complete network for a greater range of operating cost per unit distance than the monopolist (entrant), if $\alpha > \beta$ ($\alpha < \beta$).*

F.2 Proof of Proposition 10.2

$$\Pi^E = -Fx,$$ (10.6a)

$$\Pi^E = \frac{(\beta-1)^2}{2} - F(2+x).$$ (10.6b)

If the incumbent chooses not to provide route 1 the entrant's profit remains (10.3), then the incumbent's profit becomes:

$$\hat{\Pi}^{E} = \frac{(\beta-1)^{2}}{2} - 2F \tag{10.7}$$

When $F > 0$ we can see that (10.6a) is negative so the entrant would not enter and (10.7) is greater than (10.6b) so the incumbent would always prefer to provide an incomplete network. This leads us to proposition 10.2.

Proposition 10.2. *When entry is allowed on route 1, with competition over price, and there is an operating cost, then neither the entrant nor the incumbent will provide route 1, so there is always incomplete network provision.*

F.3 Proof of Proposition 10.3

Figure F.1: RHS of (9.15), (9.22), and (10.24) When $x = 0.2$ and $\alpha = 5$
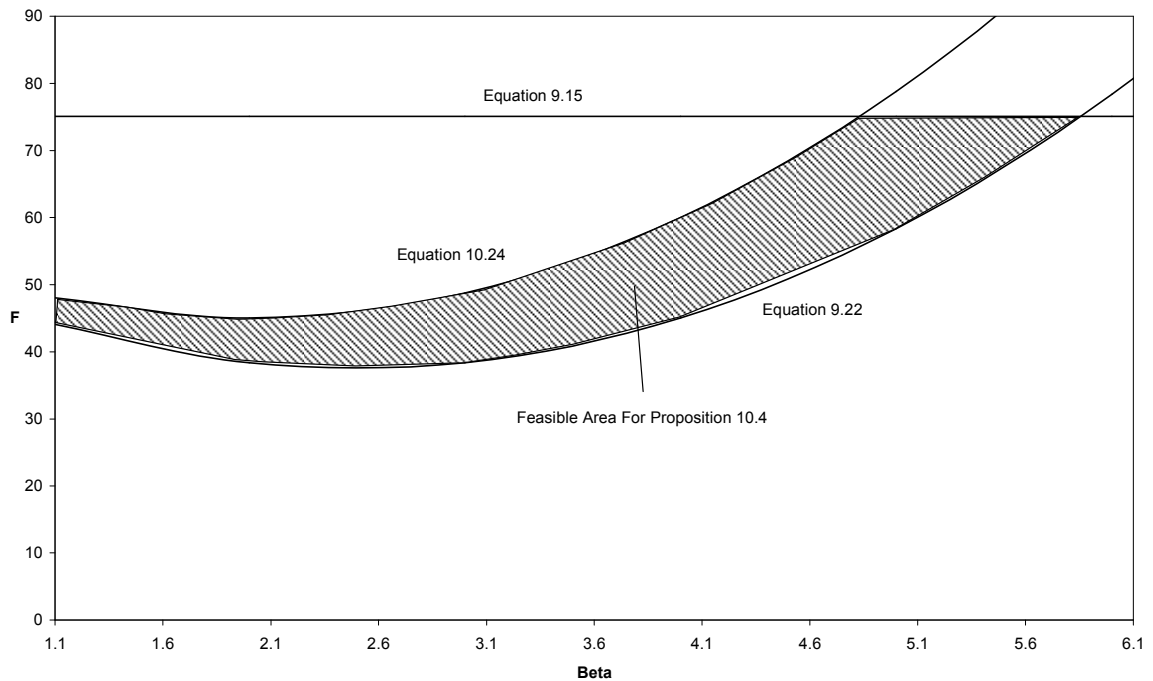
Producing simulations of the RHS of (9.15), the RHS of (9.22) and (10.14)[1], and then creating graphs shows that when x and α is large then viable areas exist where

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[1] More specifically we are looking for values above the RHS of equation (9.22), below the RHS of (9.19) and below the RHS of (F.4).

allowing entry on route 1 results in the monopolist and the social planner agreeing in the provision of a complete network, but without entry. Such an area is shown below the lines traced by the RHS of (9.15) and (10.24), and above the line traced by the RHS of (9.22).

Figure F.2: RHS of (9.15), (9.22), and (10.24) When $x = 0.2$ and $\alpha = 7$
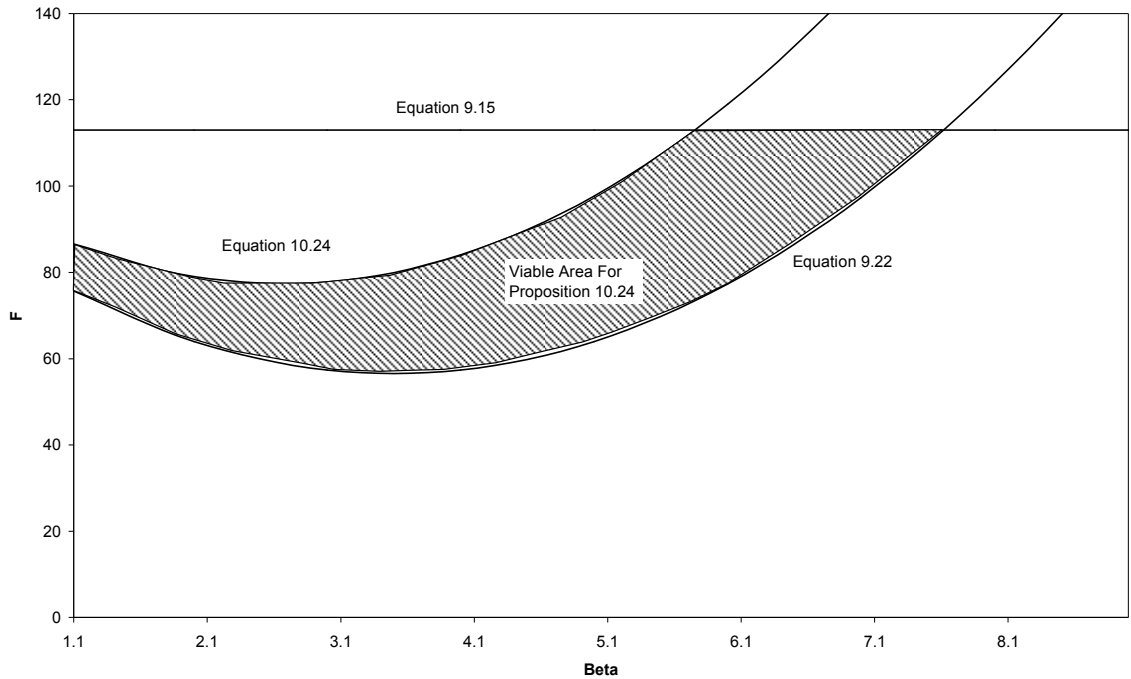
Examples of which can be seen in the Figures F.1 and F.2, and lead to Proposition 10.4:

Proposition 10.3. *When there is entry is allowed on route 1 with competition over price then it can ensure that a complete network is provided, where a monopolist would not, in accordance with the social planner's preference.*

F.4 Proof of Proposition 10.4

The incumbent's threshold fixed cost in the provision of route 1 is calculated by subtracting (10.22) from (10.21a), is:

$$\tilde{F}^{I_1} = \frac{(\alpha - x)^2}{(1+n)^2 x}.$$

(F.2)

The incumbent would provide route 1 if:

$$F^{I_1} < \frac{(\alpha - x)^2}{(1+n)^2 x}. \tag{F.3}$$

The entrant's threshold fixed cost if the incumbent did not provide route 1 would, from (10.27), be:

$$\tilde{F}^{E_1} = \frac{(\alpha - x)^2}{(1+n^{E_1})^2 x}. \tag{F.4}$$

The entrant would thus provide route 1 when the incumbent does not if:

$$F^{E_1} < \frac{(\alpha - x)^2}{(1+n^{E_1})^2 x}. \tag{F.5}$$

As $n^{E_1} < n$ we can see that (F.2) would always be smaller than (F.4) – thus the entrant provides route 1 for a greater range of F than the incumbent does. This leads to Proposition 10.4:

Proposition 10.4. *The entrant will provide route 1 for a greater range of fixed cost of operation per unit distance than the incumbent.*

F.5 Proof of Proposition 10.5

Producing simulations of the RHS of (9.15), RHS of (9.22) and (F.5), and then creating graphs shows that when x and n are small, and α is large then viable areas exist where allowing entry on route 1 results in the monopolist and the social planner agreeing in the provision of a complete network, but without entry. Such an area is shown below equation the lines traced by the RHS of (9.15) and the equation (F.5), and above the line traced by the RHS of (9.22). Examples of which can be seen in the Figures F.3 and F.4, and lead to Proposition 10.5:

Proposition 10.5. *Entry on route 1 can ensure that a complete network is provided, where a monopolist would not, in accordance with the social planner's preference.*

Figure F.3: RHS of (9.15), (9.22), and (F.5) When $n = 2$, $x = 0.2$ and $\alpha = 5$
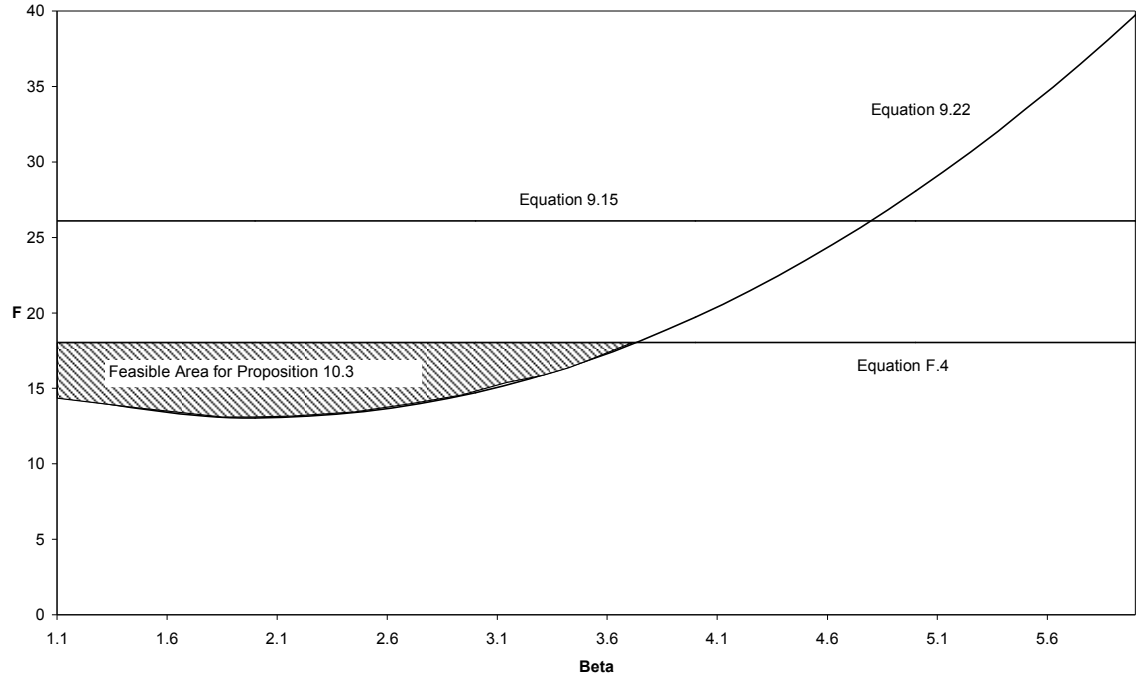
Figure F.4: RHS of (9.15), (9.22), and (F.5) When $n = 2$, $x = 0.2$ and $\alpha = 7$
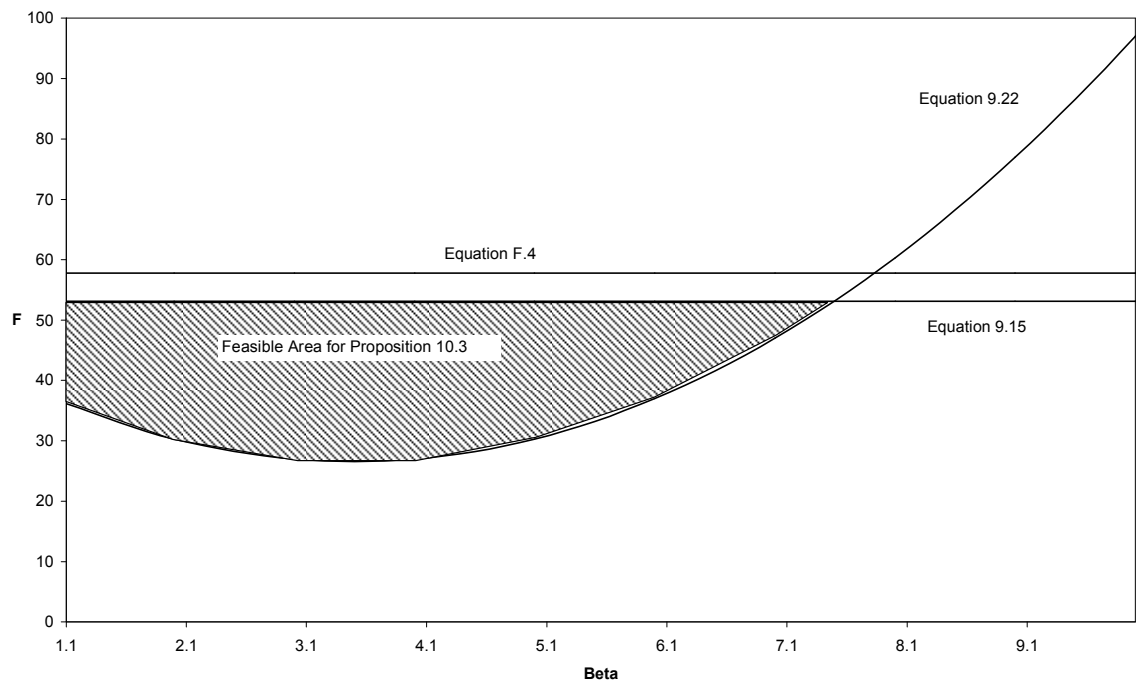
F.6 Proof of Proposition 10.6

Compare the incumbent's results by subtracting (10.45a) from (9.21) and solve for F:

$$F = \left[\frac{1}{4x}(\alpha - x)^2 + \frac{1}{2x}(\beta - 1)^2 \right]$$
$$- \frac{\left(16n^4 + 96n^3 + 120n^2 + 64n + 13\right)(\alpha + \beta - 3)^2}{8\left(4n^2 + 6n + 3\right)^2}. \tag{10.46}$$

This gives us the fixed cost that makes the incumbent indifferent between wanting a complete network with no entry and an incomplete network with entry. The incumbent will provide a complete network if the fixed operating cost per unit distance satisfies the inequality:

$$F < \left[\frac{1}{4x}(\alpha - x)^2 + \frac{1}{2x}(\beta - 1)^2 \right]$$
$$- \frac{\left(16n^4 + 96n^3 + 120n^2 + 64n + 13\right)(\alpha + \beta - 3)^2}{8x\left(4n^2 + 6n + 3\right)^2}. \tag{10.47}$$

Entry is sustainable on route 3 when the incumbent supplies an incomplete network if:

$$F \leq \frac{n(6n+5)^2(\alpha + \beta - 3)^2}{8\left(4n^2 + 6n + 3\right)^2}. \tag{10.49}$$

Entry is not profitable on route 3 when the incumbent supplies a complete network if:

$$F > \frac{(\beta - 1)^2}{(1 + n)^2}. \tag{10.49}$$

Producing simulations of the RHS of (9.22), (10.47), (10.49), and (10.49)^2, and then creating graphs shows that, when x is small, viable areas exist where the monopolist and the

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[2] More specifically we are looking for values above the RHS of equation (9.22), above the RHS of above (F.8), below the RHS of (F.6) and below the RHS of (F.7).

Figure F.5: RHS of (9.22), (10.47), (10.48), and (10.49) When $n = 2$, $x = 0.2$, **and** $\alpha = 5$
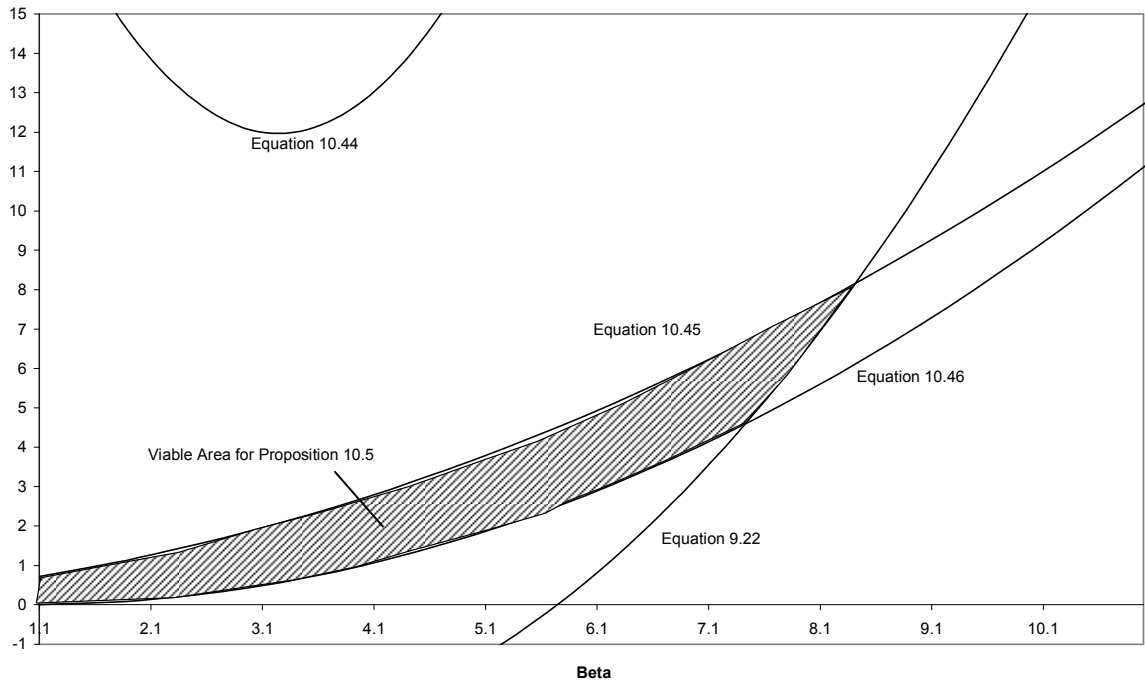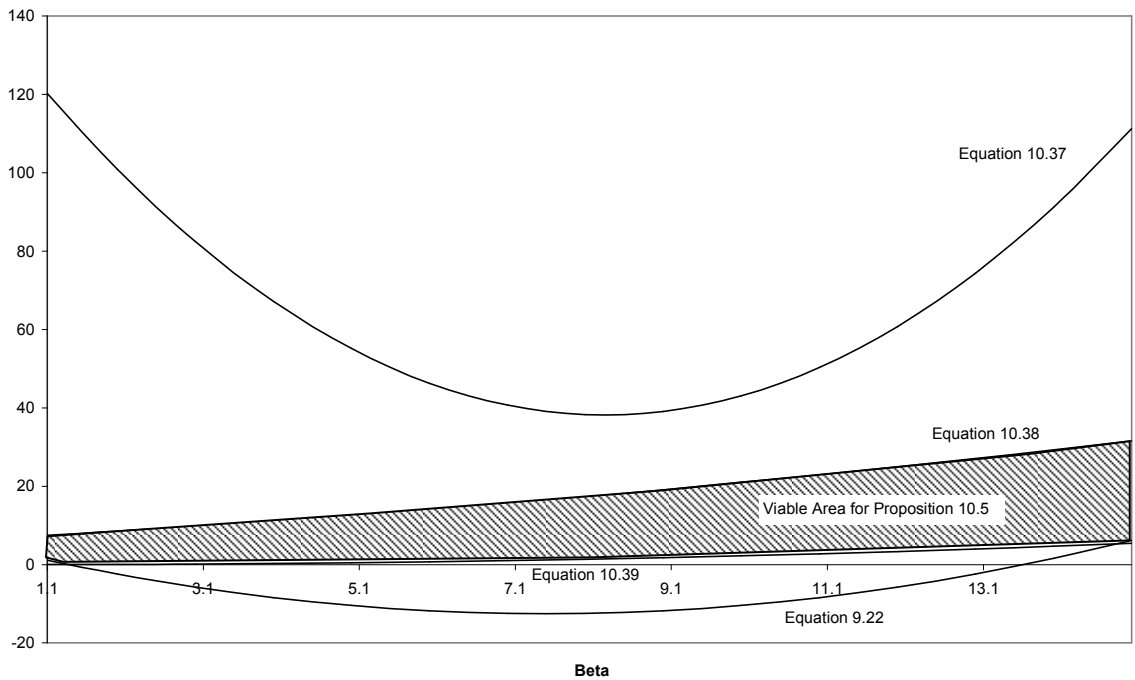

Figure F.6: RHS of (9.22), (10.47), (10.48), and (10.49) When $n = 5$, $x = 0.2$, **and** $\alpha = 10$

social planner agree in the provision of a complete network, but without entry. Such an area

exists below the lines traced by (10.47) and (10.48), and above the lines traced by (10.49)

and the RHS of (9.22) – examples of which can be seen in the Figure F.5 and Figure F.6.

These lead us to Proposition 10.6:

Proposition 10.6. *Entry on route 2 or 3 can ensure the monopolist acquiesces with the social planner and provides a complete network without viable entry.*

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Summary of Thesis submitted for PhD degree

By Michael Matthew Reynolds

on

Theoretical Investigations into Competition, Regulation, and Integration in

Transport Networks

This thesis consists of three parts. In the first part, we review the literature and some of the key issues in UK transport. We identify a need to discourage car use and the role that public transport plays in this. We discuss the various options available to policymakers to reduce problems of congestion and pollution. We note how the emphasis on deregulation and competition to promote public transport, and discourage car use, have had perverse side effects. In some cases, public transport services have become disintegrated; resulting in reductions in flexibility and increasing the generalised cost of travelling – making public transport less attractive. This raises an important question: how do we encourage a greater degree of service integration without undoing the gains from competition? The second part of the thesis, explores this issue using a theoretical transport network model. We find that various regimes involving private firms are likely to lead to the provision of an integrated ticketing system, but that not all such regimes are socially desirable. We consider how the configuration of regulatory policy may steer the private firms to produce more socially desirable outcomes.

The deregulation of elements of the UK public transport network has often led to situations approaching local monopoly. The third part of this thesis investigates the private (monopoly) incentive to offer joined-up services relative to the social incentive. The more complete the service provision, the closer the match with consumer's preferences, and the lower the generalised cost of travel. We find the monopolist does not always choose the socially desirable level of service, even when economically viable, but it may be possible to induce this provision through entry or threats of entry on a sub-set of the network.

The thesis ends with a summary of the main results and suggestions for further work.