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Three Essays in Fiscal Policy

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Abstract

This thesis presents three papers on fiscal policy.

The first paper "Macroeconomic impacts of fiscal policy shocks in UK; a DSGE analysis" (joint with Keshab Bhattarai) uses an estimated new-Keynesian dynamic stochastic general equilibrium (DSGE) model to analyse the effects of fiscal policy in the UK. We show that positive shocks in government consumption and investment result in the highest stimulus in the short term, whereas the capital tax cut and the positive public investment shock in the longer horizon. On the government's expenditure side public investment remains the most stimulating instrument even if we allow for a constant elasticity of substitution index of private and public consumption in utility. We also find that, the nominal and real frictions present in the model tend to influence stronger the level of labour and capital tax multipliers and less public expenditure multipliers.

The second paper "Credit constraints, the housing market, and fiscal policy" investigates the effects of fiscal policy in an estimated new-Keynesian open-economy DSGE model with a housing market and indebted households. We show that house prices drop following a negative shock to government transfers, and a positive shock to public spending, public investment and taxes. The results reveal that the financial deregulation increases the sensitivity of fundamentals to fiscal policy. In particular, in the case of a stimulus, the financial deregulation contributes to a weakening of multipliers in the case of government consumption and investment and tends to improve multipliers for public transfers and tax cuts.

The third paper "Who is afraid of austerity? The redistributive impact of fiscal policy in a DSGE framework" (joint with Richard McManus and Gulcin Ozkan) explores the distributional consequences of fiscal austerity using a medium scale new-Keynesian DSGE model with a richly specified fiscal sector. We show that agents who are credit constrained are most exposed to austerity in contrast to agents with full access to financial markets. This is particularly true in the case of rises in taxes on labour income and cuts in transfers. In general, tax based consolidations exhibit more conflict than spending based ones. Our results also reveal that the distributive impacts of fiscal consolidations are amplified the longer the austerity persists; the slower the policy reversal and when monetary policy reaches its zero lower bound.

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Preface

Since the seminal work of Kydland and Prescott (1982) there has been remarkable progress in the development and estimation of dynamic stochastic general equilibrium (DSGE) models which are now used by key policymaking institutions, including the Bank of England, the European Central Bank, the European Commission, the International Monetary Fund, the Organisation for Economic Co-operation and Development, the U.S. Federal Reserve and many others. After the 2008-2009 global financial crisis and the subsequent revival of interest in discretionary fiscal policy, DSGE models have become widely used in analysing the fiscal policy dynamics. In this thesis, we contribute to this literature.

The first paper "Macroeconomic impacts of fiscal policy shocks in UK; a DSGE analysis.", (joint with Keshab Bhattarai) is the first attempt to evaluate the effects of fiscal policy in the UK with the use of a new-Keynesian DSGE model. The paper investigates the qualitative and quantitative effects of fiscal policy on the key macroeconomic variables. As in Smets and Wouters (2003) and Christiano *et al.* (2005) the model incorporates a number of real and nominal frictions which enable a better fit with the data and more realistic dynamics. Real frictions stem from external habit formation in private consumption, investment adjustment costs, capital utilisation cost, and fixed costs in intermediate goods production. Nominal frictions include staggered price and wage mechanism *a la* Calvo (1983), and a partial indexation of price and wage contracts. Following seminal papers of Campbell and Mankiw (1989) and Mankiw (2000), we incorporate a heterogeneity of agents regarding their access to financial markets: Ricardian consumers who own the

entire capital stock of the economy and possess access to the financial markets, and non-Ricardian households, who do not possess access to financial markets and simply consume their total disposable income arising from labour and transfers. The monetary authority sets the nominal interest rate following a Taylor type rule and the introduction of six fiscal policy instruments into this framework enables an empirical investigation of the effects of changes in these instruments and of their contribution to business cycle fluctuations in the UK. The fiscal policy instruments include public consumption, investment and transfers and distortionary taxation on consumption and on labour and capital income.

Our results indicate that the persistent government consumption and investment shocks are the most stimulating in the short term (the impact multiplier is equal respectively to 0.99 and 1.07). In the medium and longer horizon the capital tax cut and the public investment shock result in the highest multipliers (the present value cumulative 5 year multiplier totals -1.05 and 1.02 respectively). The government transfers result in a relatively lower multiplier when compared to the remaining fiscal policy instruments, mainly as a result of the low share of non-Ricardian households.

In contrast to Coenen *et al.* (2013) we find private and public consumption in UK to be Edgeworth substitutes, therefore for a public consumption shock, presence of a constant elasticity of substitution index of private and public consumption in utility strengthen the negative wealth effect in the economy. Government consumption yields multipliers larger than one, once temporary fiscal stimulus is at place.

Regarding the role of nominal and real frictions, we show that higher levels of nominal and real frictions tend to influence more labour and capital taxes and less the government expenditure multipliers.

In the second paper entitled "Credit constraints, the housing market, and fiscal policy" we develop and estimate a new-Keynesian open-economy DSGE model with a housing market. The model goes one step beyond Mankiw (2000), and Gali *et al.* (2007), and assumes two types of households *a la* Iacoviello (2005). The heterogeneity of households reflects the fact that impatient households discount the future more heavily than patient households, which implies that in the equilibrium the former become borrowers and the later lenders. Housing is the only collaterable good in the model and the maximum level of borrowings is limited by the expected real value of housing and the exogenous in the model downpayment share. Surprisingly, in the context of the 2008-2009 global financial crisis where debt and indebted households were the key feature, the impatient households have received relatively less attention in the literature.

On the production side, there are two sectors: one sector producing a tradable, non-residential good, and the second, producing a non-tradable, residential good. We assume perfect competition in the residential market following Barsky *et al.* (2007) and Iacoviello and Neri (2010), but allow for monopolistic competition in the non-residential sector. The small open-economy setting is similar to that of Adolfson *et al.* (2007). The interaction with the rest of the world comprises trade of goods and riskless bonds. The four final goods, private and public consumption and investment, are produced by combining the domestic and imported homogenous good. Monetary policy is implemented by means of a

Taylor type rule whereas the fiscal policy with the help of six fiscal policy rules which respond to the cyclical changes in debt and GDP. The fiscal policy instruments include: public consumption, investment, transfers and taxes on consumption, labour and capital. The model features a number of nominal and real frictions: external habit formation in private consumption, investment adjustment costs, capital utilisation cost, fixed costs in intermediate goods production, staggered price and wage mechanism *a la* Calvo (1983), partial indexation of price and wage contracts, and imperfect international financial markets. Nominal rigidities in the exporting and importing sectors imply an incomplete exchange rate pass-through to both export and import prices.

We show that house prices drop on impact following a positive shock to public spending, public investment and taxes and a negative shock to public transfers. Moreover, our results indicate that the financial deregulation increased the sensitivity of the house price to fiscal policy, which is in line with Bernanke and Gertler (1995), and Muellbauer and Murphy (2008). Therefore, when fiscal policy leads to the house price increase, the collateral role of property allows impatient households for further spending. On the other hand, when fiscal policy results in a drop of the house price, a reduction of available credit leads to a decline in credit-constrained households' expenditure. Consequently, the financial deregulation leads to a weakening of multipliers in the case of government consumption and government investment and tends to improve multipliers for public transfers and tax cuts.

Third paper "Who is afraid of austerity? The redistributive impact of fiscal policy in a DSGE framework" (joint with Richard McManus and Gulcin Ozkan) explores the distributional impact of fiscal austerity in a standard new-Keynesian DSGE model extended by

a rich fiscal policy setup and inclusion of non-Ricardian households. This paper makes two distinct contributions. The first is to provide a comprehensive examination of the distributive consequences of fiscal austerity, which has received very little attention in the existing literature. Our second contribution lies in the scope of our fiscal policy analysis; we examine a much richer set of fiscal instruments than has been provided in the existing literature on fiscal consolidation.[1] In addition to public consumption, income and lump-sum taxes that are widely explored in previous work, we incorporate capital taxes, consumption taxes, social security contributions as well as public employment and public investment as sources of fiscal adjustment packages. A clear motivation for adopting this extended set of fiscal instruments is provided by the structure of fiscal policy packages enacted in the wake of 2008 financial crisis that made use of a large number of fiscal tools including all the items in the set of fiscal policy instruments used in this paper.

Our findings can be summarized as follows. First, we find that fiscal austerity has a wide range of distributional outcomes that are determined by the composition of initial fiscal action. Also, the welfare consequences of fiscal consolidations are unevenly distributed among agents with more detrimental impact on credit constrained households than those with full access to credit markets. For instance, in four out of eight sets of fiscal experiments, fiscal consolidation reduces the welfare of the credit constrained households more than the Ricardian ones (transfer payments, labour income tax, consumption tax and employers social security contributions based consolidations). Similarly, when fiscal austerity

[1] One exception is Coenen *et al*. (2013) who extend the ECB's New Area-Wide Model to include a wide variety of fiscal instruments. The number of fiscal instruments is the same in our paper but while they include taxes on dividends, we have government employment.

is beneficial to both types of agents, Ricardian households always gain more in relative terms (government consumption, public investment and public employment based consolidations). In contrast, a rise in capital taxes is the only fiscal action that reduces Ricardian household's welfare more than that of the credit constrained households.

Second, our results reveal that the form of policy reversal - to neutralize the impact of austerity on debt is plays a key role in determining the welfare implications of initial austerity. For instance, fiscal consolidation based on a fall in transfers is good for Ricardian agents if it is reversed by a fall in employers social security contributions but bad if the policy reversal is through a rise in public employment. Third, we also show that the distributive impact of fiscal policy is amplified the longer the austerity persists; the slower the policy reversal and when monetary policy reaches its zero lower bound. This is of empirical relevance to much of the current debate as many long term shocks are used to pay off existing debt in a period of a liquidity trap.

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Chapter 1
Macroeconomic Impacts of Fiscal Policy Shocks in UK; a DSGE Analysis

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Abstract: *In this paper we use an estimated new-Keynesian dynamic stochastic general equilibrium (DSGE) model to analyse the implications of fiscal policy in the UK economy. In the context of GDP multipliers, we find that shocks in government consumption and investment are the most stimulating in the short term. In the medium and longer horizon the capital tax cut and the public investment shock result in the highest multipliers. We show that on the government's expenditure side public investment remains the most stimulating instrument even if we allow for a constant elasticity of substitution index between private and public consumption in utility. The parameter estimates indicate that public investment, consumption and capital taxes play the most important role in controlling for the government debt over the sample period, whereas capital tax rates and government investment characterise significant procyclical response to GDP.*

Keywords: *fiscal policy shocks, DSGE model, UK economy*

JEL Classification: *E32, E63*

1.1 Introduction

Fiscal policy has been used extensively for a long time to stabilise the economy and to foster more efficient, fairer and equitable societies. This is among the reasons why there exists a long tradition in the analysis of fiscal policy in the UK. It was at the heart of the revenue neutral tax exercise of Ramsey (1927) and macroeconomic analysis of Keynes (1936). Following these works was a path-breaking analysis on the optimal tax rule by Mirrlees (1971). In practical terms, the Institute of Fiscal Studies published Meade (1978) on the burden of direct taxes and Mirrlees *et al.* (2010) on both direct and indirect taxes. Whereas the macro impacts of fiscal policy have been studied using various analytical frameworks (Holly and Weale, 2000), the burden of direct and indirect taxes on households in terms of the Hicksian equivalent and compensating variations in the welfare taxes to households have been measured using a general equilibrium analysis (Bhattarai and Whalley, 1999). Tax-benefit models have been used to measure the impacts of taxes and benefits on the labour supply (Brewer *et al.*, 2009). The Green Budgets of the IFS have regularly reported on the impacts of taxes on economic growth, inequality, and welfare. The conclusions of the most of above studies are mainly derived from comparative static analysis.

The UK economy has been growing secularly in the past several centuries but is frequently disturbed by transitional shocks arising either from the demand or supply side, or from both sides of the economy. The dynamic stochastic general equilibrium (DSGE) models developed recently aim to assess the impacts of such shocks on the transitional dynamics of the economy. Despite a fairly large body of the literature on DSGE modelling, few studies analyse the impacts of such shocks in the UK. Doing so are Batini *et al.* (2003),

the Bank of England Quarterly Model (BEQM) by Harrison *et al.* (2005), DiCecio and Nelson (2007), Faccini *et al.* (2011), Gorts and Tsoukalas (2011), Harrison and Oomen (2010), and Millard (2011).[2] This paper fits into the recent DSGE developments for the UK economy and contributes through its extensive analysis of fiscal policy.

The fiscal stimulus, a growing debt-to-GDP ratio and the budget deficit have brought much attention in the recent years. Some of the developments in the analysis of fiscal policy by means of the DSGE models include: Coenen and Straub (2005), Lopez-Salido and Rabanal (2006), Gali *et al.* (2007), Forni *et al.* (2009), Ratto *et al.* (2009), Cogan *et al.* (2010), Leeper (2010), Christiano *et al.* (2011), Eggertson (2011), Drautzburg and Uhlig (2011), Coenen *et al.* (2013). In the context of fiscal policy, models by Coenen and Straub (2005) and Gali *et al.* (2007) focus primarily on the implications of government spending, and deficit is adjusted using lump-sum taxes. Our paper can be easily distinguished from Forni *et al.* (2009), Ratto *et al.* (2009), Lopez-Salido and Rabanal (2006), in the context of fiscal policy specification or its granularity. Coenen *et al.* (2013) focus on the implications of European Economic Recovery Plan (EERP), whereas Cogan *et al.* (2010) and Uhlig and Drautzburg (2011) on the the American Recovery and Reinvestment Act (ARRA). Christiano *et al.* (2011) and Eggertson (2011) study the effects of fiscal stimulus at the zero lower bound. The important caveat of Leeper *et al.* (2010), which we avoid in this paper, is that they do not include the interaction between the fiscal and monetary policies. We

[2] In the context of theoretical model specifications, DiCecio and Nelson (2007) estimate the closed-economy model of Christiano *et al.* (2005) on the UK data. Harrison and Oomen (2010) estimate the open-economy model similar to Christiano *et al.* (2005), Smets and Wouters (2003), and Adolfson *et al.* (2007). Faccini *et al.* (2011) implement a search and matching framework into a labour market of a DSGE model. Millard (2011) extends the Harrison and Oomen (2010) model and incorporates three consumption goods: non-energy output, petrol and utilities. Gorts and Tsoukalas (2011) fit the extended investment sector into a medium-size DSGE model.

also make the distinction between productive and non-productive government spending and incorporate into the model nominal frictions in prices and wages. The model also clearly differs from the long-run growth analysis contained in multi-household and multi-sector dynamic general equilibrium models with perfect foresight as presented in Bhattarai (2007) and staggered-price dynamic general equilibrium model with overlapping generations in Ascari and Rankin (2013).

By building fiscal policy explicitly into a new-Keynesian DSGE model, this study aims to address several questions: (1) What are the qualitative and quantitative effects of distortionary taxation (on consumption and on labour and capital income) and government expenditure (consumption, investment and transfers) on the key macroeconomic variables in the UK? (2) How in the historical context has fiscal policy been used in controlling for debt? (3) What is the difference in implications of productive and non-productive public spending? (4) What parameters are crucial for determining the fiscal policy effectiveness?

To answer the above questions, we use a Dynamic Stochastic General Equilibrium Model for the UK economy. The introduction of six fiscal policy instruments enables an empirical investigation of the effects of changes in these instruments and of their contribution to business cycle fluctuations in the UK. We estimate the model with Bayesian methods on a linearly detrended quarterly macro time series ranging from 1987:Q1 to 2011:Q1.

The parameter estimates indicate that public investment, consumption and capital taxes play decisive role in controlling for the government debt over the sample period. Additionally capital tax rates and government investment characterise significant procyclical

responses to GDP. In contrast, the response of labour taxes to aggregate output and debt is relatively modest.

In the context of GDP multipliers, the persistent government consumption and investment shocks are the most stimulating in the short term (the impact multiplier totals 0.99 and 1.07 respectively). In the medium and longer horizon the capital tax cut and the public investment shock result in the highest multipliers (the present value cumulative 5 year multiplier totals -1.05 and 1.02 respectively). The estimate of government consumption impact multiplier is higher than the average impact multiplier of 0.89 obtained in the empirical study of fiscal policy in the UK conducted by Canova and Pappa (2011). We show that a temporary increase (in contrast to a persistent increase) in public consumption yields the GDP multiplier higher than 1. The government transfers shock results in a relatively lower multiplier when compared to the remaining fiscal policy instruments, mainly as a result of the low share of non-Ricardian households. The consumption and labour tax multipliers result in moderate multipliers in the short and medium horizon ranging from (-0.33) to (-0.67). Additionally we show, that on the government expenditure side, government investment results in the highest multipliers even in a model with non-separable non-wasteful government consumption in the utility function. The reason is that private and public consumption are Edgeworth substitutes.

Regarding the role of nominal and real frictions, we show that higher levels of nominal and real frictions tend to influence more labour and capital taxes and less the government expenditure multipliers.

This paper is organised as follows. The next section presents a theoretical model of the UK economy. Section three outlines the necessary information on the solution and estimation methods along with the discussion of calibrated parameters, prior and posterior estimates. In section four, we discuss the impulse responses and present value multipliers implied by the fiscal policy shocks. Section five presents the multipliers for permanent shocks, whereas section six presents sensitivity analysis. In section seven we focus on the variance decomposition. In section eight we discuss the alternative setup with a non-separable non-wasteful government consumption in the utility function, and in section nine we draw conclusions.

1.2 Theoretical Model

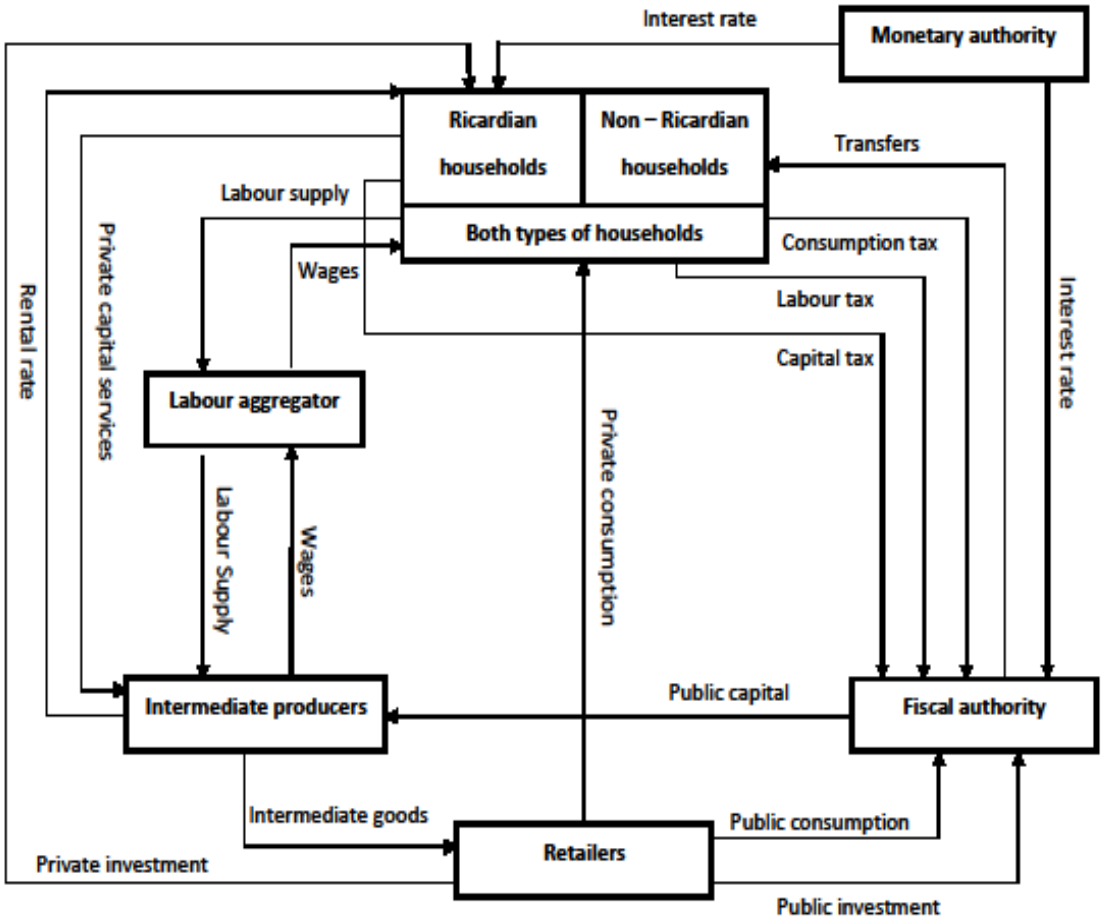
The model economy is populated by a continuum of households indexed by ι, where a share ϑ of them comprise non-Ricardian or rule-of-thumb consumers. The remaining proportion $(1-\vartheta)$ comprise Ricardian consumers, who anticipate and internalise the government's tax and borrowing behaviour and maximise their life-time utility subject to their intertemporal budget constraint. They own the entire capital stock of the economy and possess access to the financial and capital asset markets, which are assumed to be complete. Non-Ricardian households do not have access to the financial and capital markets and simply consume their total disposable income stemming from labour and transfers[3]. Firms produce differentiated

[3] Following the developments of Campbell and Mankiw (1989) and Mankiw (2000), non-Ricardian households became a common feature of fiscal policy papers.

goods, choose labour and capital inputs and set prices similarly to the method proposed by Calvo (1983).

The monetary authority sets the nominal interest rate according to a Taylor rule. Fiscal authority determines a set of policy instruments' rules in which they respond to the cyclical changes in output and debt. This model features a number of real and nominal frictions as found in Smets and Wouters (2003) and Christiano *et al.* (2005). The flow chart of the model is presented in Figure 1.1.

Fig. 1.1. Flow chart



1.2.1 Households

Ricardian households

The utility functional of each Ricardian household is represented by:

$$E_0 \sum_{t=0}^{\infty} \varepsilon_t^B \beta^t U \left(\frac{\left(C_t^\iota - H_t\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(L_t^\iota\right)^{1+\sigma_l} \right) \tag{1.1}$$

where ε_t^B represents the preference shock; the subjective discount factor β satisfies $0 < \beta < 1$; $\sigma_l \geqslant 0$ denotes the inverse Frisch elasticity of labour (L_t^ι); $\sigma_c > 0$ denotes the inverse of the intertemporal elasticity of substitution in consumption (C_t^ι); and H_t denotes the external habit variable such that $H_t = hC_{t-1}$, where $0 < h < 1$.

Each Ricardian household maximises its lifetime utility subject to the flow budget constraint, which simply states that the household's total expenditure on consumption (C_t^ι), investment in physical capital (I_t^ι) and accumulation of a portfolio of riskless one-period contingent claims $\left(b_{r,t}^\iota\right)$ must equal the household's total disposable income (inc_t^ι).

$$b_{r,t}^\iota - \frac{b_{r,t-1}^\iota}{\pi_t} + I_t^\iota + \left(1+\tau_t^c\right) C_t^\iota = inc_t^\iota \tag{1.2}$$

where π_t denotes the gross inflation rate. The presence of consumption tax τ_t^c implies that the wedge arises between the price consumer pay and the price at which producer sells. We follow the developments of Woodford (1996), and subsequently Erceg *et al.* (2000) and Christiano *et al.* (2005) and assume complete markets for the state contingent claims in consumption and in capital but not in labour. This assumption implies that consumption and capital holdings are the same across households. Consequently, $C_t^\iota = C_t$, $K_t^\iota = K_t$, $u_t^\iota = u_t$.

The total real disposable income of each Ricardian household consists of the following:

- the after tax labour income $\left(1-\tau_t^l\right) w_t^\iota L_t^\iota$, where w_t^ι represents the real wage rate, L_t^ι denotes the hours worked, and τ_t^l is the effective labour tax rate;
- the after tax return on capital $\left(1-\tau_t^k\right) r_{k,t} u_t K_{t-1}$, where $r_{k,t}$ denotes the real rate of return on capital, K_{t-1} denotes the physical stock of capital, and u_t is the capital utilisation rate. Setting the level of capital utilisation rate requires each household to incur a cost equal to $a(u_t)K_{t-1}$.We assume that $\frac{a''(u_t)}{a'(u_t)} = \kappa$. Consequently, only the dynamics of the model depend on the parameter κ. In the steady state $u = 1$.
- the income from dividends div_t;
- the interest income from the bond holdings $\left(\frac{i_{t-1} b_{r,t-1}^\iota}{\pi_t}\right)$, where i_{t-1} denotes the nominal interest rate on a one-period bond[4].

$$inc_t^\iota = \left(1-\tau_t^l\right) w_t^\iota L_t^\iota + \left(1-\tau_t^k\right) r_{k,t} u_t^\iota K_{t-1}^\iota - a(u_t^\iota) K_{t-1}^\iota + \mathrm{div}_t + \frac{i_{t-1} b_{r,t-1}^\iota}{\pi_t} \tag{1.3}$$

Physical capital accumulates in accordance with the following:

$$K_t = (1-\delta_k) K_{t-1} + F_t\left(I_t, I_{t-1}\right) \tag{1.4}$$

where: $F_t\left(I_t, I_{t-1}\right) = \left[1 - S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\right] I_t$. As in Schmitt-Grohe and Uribe (2006): $S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right) = \frac{\phi_k}{2}\left(\frac{\varepsilon_t^I I_t}{I_{t-1}} - 1\right)^2$ is the cost of the adjustment function which possesses the following properties: $S(1) = S'(1) = 0$, and $S''(1) = \phi_k > 0$, where ε_t^I denotes an investment-specific

[4] The gross nominal interest rate is represented by $R_t = 1 + i_t$.

efficiency shock. Ricardian households maximise their utility subject to the flow budget constraint, the capital accumulation function, and the demand for labour they face from the labour unions. The Lagrangian takes the following form:

$$\begin{aligned}\mathcal{L} &= E_t \sum_{t=0} \varepsilon_t^B \beta^t \left(\frac{\left(C_t^\iota - hC_{t-1}^\iota\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(L_t^\iota\right)^{1+\sigma_l} \right) \\ &+ E_t \sum_{t=0} \lambda_t \beta^t \left(\begin{array}{c} \frac{R_{t-1} b_{r,t-1}^\iota}{\pi_t} + \left(1-\tau_t^l\right) w_t^\iota L_t^\iota + \left(1-\tau_t^k\right) r_{k,t} u_t^\iota K_{t-1}^\iota \\ -a(u_t^\iota) K_{t-1}^\iota + div_t^\iota - b_{r,t}^\iota - I_t^\iota - \left(1+\tau_t^c\right) C_t^\iota \end{array} \right) \\ &+ E_t \sum_{t=0} \lambda_t Q_t \beta^t \left((1-\delta) K_{t-1}^\iota + F_t^\iota \left(I_t^\iota, I_{t-1}^\iota\right) - K_t^\iota \right)\end{aligned} \tag{1.5}$$

where: λ_t denotes the marginal utility of income; Q_t denotes the shadow price of capital.

(Subsubsubsection head:)First-order conditions of Ricardian households

The combination of first-order conditions with respect to consumption and bonds results in a standard Euler equation:

$$U_{c,t} = E_t \left[\frac{R_t}{\pi_{t+1}} \frac{\left(1+\tau_t^c\right)}{\left(1+\tau_{t+1}^c\right)} \beta U_{c,t+1} \right] \tag{1.6}$$

where: $U_{c,t}$ denotes the marginal utility of consumption $U_{c,t} = \lambda_t \left(1+\tau_t^c\right) = \varepsilon_t^B \left(C_t - H_t\right)^{-\sigma_c}$. The left-hand side of equation (1.6) represents the marginal utility cost of investing in bonds (to invest household sacrifices current consumption). The right-hand side implies that investing in bonds provides an *ex ante* real rate of return represented by $\frac{R_t}{\pi_{t+1}}$.

The first-order condition with respect to the capital utilisation rate, presented in equation (1.7), indicates that the real rental rate net of capital taxes is equal to the marginal cost of capital utilisation.

$$\left(1-\tau_t^k\right) r_{k,t} = a'(u_t) \tag{1.7}$$

A higher rate of return on capital or a lower capital tax rate implies a higher utilisation rate up to the point where extra benefits are equal to extra costs.

The first-order condition with respect to capital links the shadow price of capital between two periods:

$$Q_t = \frac{E_t \pi_{t+1}}{R_t} E_t \left[Q_{t+1}(1-\delta_k) + \left(1-\tau_{t+1}^k\right)\left(r_{k,t+1} u_{t+1}\right) - a(u_{t+1}) \right] \tag{1.8}$$

Equation (1.8) implies that price of capital is simply a present value of future net income from capital holdings. The price of capital depends positively on the expected real rental rate and the expected utilisation rate. It depends negatively on the real *ex ante* interest rate, capital taxes and the capital utilisation cost.

The first-order condition with respect to investment is presented in equation (1.9). The left-hand side of the equation represents the marginal utility cost of investment in physical capital, which is equal to the marginal utility cost of investment in bonds. An increase in investment by one unit at time t leads to an increase in the value of capital by $Q_t F_t'\left(I_t, I_{t-1}\right)$ in period t, and by $Q_{t+1}\beta F_{t+1}'\left(I_{t+1}, I_t\right)$ in period $t+1$[5].

[5] Substituting for $F_t\left(I_t, I_{t-1}\right)$ results in:

$$1 = Q_t \left[1 - \frac{\phi_k}{2} - \frac{3\phi_k}{2}\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)^2 + 2\phi_k \frac{\varepsilon_t^I I_t}{I_{t-1}} \right] + Q_{t+1}\frac{U_{c,t+1}}{U_{c,t}}\beta \left[\phi_k \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\right)^3 - \phi_k \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\right)^2 \right].$$

Now it is easy to see that $F_t'(1) = 1$ and $F_{t+1}'(1) = 0$, thus the steady state does not depend on the parameter ϕ_k and $Q = 1$.

$$\lambda_t = Q_t \lambda_t F_t' \left(I_t, I_{t-1}\right) + Q_{t+1} \beta \lambda_{t+1} E_t F_{t+1}' \left(I_{t+1}, I_t\right) \tag{1.9}$$

Non-Ricardian households

Non-Ricardian households do not save; they simply consume current, after tax income, which comprises transfers from the government, and after tax labour income.

$$\left(1 + \tau_t^c\right) C_{nr,t}^{\iota} = \left(1 - \tau_t^l\right) w_{nr,t}^{\iota} L_{nr,t}^{\iota} + TR_t^{\iota} \tag{1.10}$$

For simplicity, we follow Erceg *et al.* (2006) and assume that each non-Ricardian household sets its wage equal to the average wage of optimising households. Because all households face the same labour demand, the labour supply, and total labour income of each rule-of-thumb household are equal to the average labour supply and average labour income of forward-looking households.

1.2.2 Labour Union

In creating the wage setup, we follow Erceg *et al.* (2000) and Woodford and Benigno (2006). We assume the existence of a competitive labour union, whose only task is to transform households' differentiated labour into composite labour good in proportions that intermediate firms would demand. The composite labour is subsequently supplied to these firms. The union takes every household's wage, W_t^{ι}, as given and maximises profit according to the following equation:

$$Prof_t = W_t N_t - \int_0^1 W_t^{\iota} L_t^{\iota} d\iota \tag{1.11}$$

where L_t^ι denotes the amount of labour supplied by a household ι to the union, and W_t^ι is the corresponding wage rate for this labour; W_t is the aggregate wage index, and N_t denotes the labour index: $N_t = \left[\int_0^1 (L_t^\iota)^{\frac{\nu-1}{\nu}} d\iota\right]^{\frac{\nu}{\nu-1}}$, where $\nu > 0$ denotes the elasticity of substitution among the differentiated labour inputs. Profit maximisation results in the demand for the labour of a household ι:

$$L_t^\iota = \left(\frac{W_t^\iota}{W_t}\right)^{-\nu} N_t \tag{1.12}$$

Setting the profits of labour unions to 0 results in the aggregate wage index: $W_t = \left[\int_0^1 (W_t^\iota)^{1-\nu} di\right]^{\frac{1}{1-\nu}}$.[6] Ricardian households set nominal wages similarly to a staggered-price mechanism in Calvo (1983). In particular, within each period a fraction of forward-looking households (ϖ_w) are unable to adjust their wage rate. These households simply follow the partial indexation rule, i.e. they choose their wage in the following way: $W_t = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}$. The remaining fraction of households $(1-\varpi_w)$ that are able to set their nominal wages, maximise their utility subject to the budget constraint and the demand for labour from the labour unions. The objective takes the following form:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi_w)^l \left\{ \begin{array}{c} -\frac{1}{1+\sigma_L}\left(\left(\frac{\tilde{W}_t X_{tl}}{W_{t+l}}\right)^{-\nu} N_{t+l}\right)^{1+\sigma_L} \\ +\lambda_{t+l}^\iota \left(1-\tau_{t+l}^l\right) \frac{\tilde{W}_t}{P_{t+l}} X_{t,l} \left(\frac{\tilde{W}_t X_{tl}}{W_{t+l}}\right)^{-\nu} N_{t+l} \end{array} \right\} \tag{1.13}$$

where $X_{tl} = \pi_t \times \pi_{t+1} \times ... \times \pi_{t+l-1}$ for $l \geqslant 1$ and $X_{tl} = 1$ for $l = 0$ as in Altig *et al.* (2005). The maximisation results in equation (1.14) for newly optimised wages.

[6] This is a condition stemming from the assumption of perfect competition.

$$E_t \sum_{l=0}^{\infty} (\beta \varpi_w)^l L_{t+l}^{\iota} \lambda_{t+l} \left\{ \frac{\tilde{W}_t X_{tl}}{P_{t+l}} \frac{U_{c,t+l}}{\left(1+\tau_{t+l}^c\right)} - \frac{\nu}{(1-\nu)} \frac{U_{l,t+l}}{\left(1-\tau_{t+l}^l\right)} \right\} = 0 \tag{1.14}$$

The first-order condition implies that Ricardian households set their wages so that the present value of the marginal utility of income from an additional unit of labour is equal to the markup over the present value of the marginal disutility of working. When all households are able to negotiate their wage contracts each period, wage becomes as follows: $\frac{\tilde{W}_t}{P_t} = \frac{\nu}{(1-\nu)} \frac{U_{l,t}(1+\tau_t^c)}{U_{c,t}\left(1-\tau_t^l\right)}$. Finally, the wage index can be transformed into the following:

$$W_t = \left[(1-\varpi_w) \tilde{W}_t^{1-\nu} + \varpi_w \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{1.15}$$

1.2.3 Firms

Composite consumption good producer

The competitive producer of the final good purchases differentiated goods from intermediate producers and combines them into one single consumption good (Y_t). $Y_{i,t}$ denotes the intermediate firm output, and $P_{j,t}$ is the corresponding price. The final good is produced with the following technology: $Y_t = \left[\int_0^1 Y_{j,t}^{\frac{s-1}{s}} dj \right]^{\frac{s}{s-1}}$, where $s > 0$ denotes the elasticity of substitution among the differentiated outputs of intermediate firms. Producer maximises profit as follows:

$$\Pi_t = P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj = P_t \left[\int_0^1 Y_{j,t}^{\frac{s-1}{s}} dj \right]^{\frac{s}{s-1}} - \int_0^1 P_{j,t} Y_{j,t} dj \tag{1.16}$$

The first-order condition results in a demand function for intermediate goods:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^s Y_t \tag{1.17}$$

The zero profit condition implies that the price index is represented by the following equation: $P_t = \left[\int_0^1 P_{j,t}^{1-s} dj\right]^{\frac{1}{1-s}}$.

Intermediate good production sector.

Each monopolistic intermediate producer indexed by j uses the following production function:

$$Y_{j,t} = \varepsilon_t^A (u_t K_{j,t-1})^\alpha N_{j,t}^{1-\alpha} \left(K_{g,t-1}\right)^{\alpha_g} - fc \tag{1.18}$$

where fc denotes a fixed cost of production[7], K_g denotes public capital, and ε_t^A is a total factor productivity shock. Firms rent capital $(K_{j,t-1})$ and labour $(N_{j,t})$, for which they pay respectively a nominal rental rate $(R_{k,t})$ and a wage rate (W_t). Monopolistic companies face the following cost-minimization problem:

$$\begin{aligned} &\underset{K_{j,t-1}, N_{j,t}}{min} \; W_t N_{j,t} + R_{k,t} u_t K_{j,t-1} \\ &-\lambda_t^P P_{j,t} \left(Y_{j,t} - \varepsilon_t^A (u_t K_{j,t-1})^\alpha N_{j,t}^{1-\alpha} \left(K_{g,t-1}\right)^{\alpha_g} + fc\right) \end{aligned} \tag{1.19}$$

[7] Fixed cost is a standard characteristic of DSGE models; see for example Smets and Wouters (2003, 2007), Christiano *et al.* (2005), Adolfson *et al.* (2008). The fixed cost share of the output is waisted. The presence of fixed costs ensures that in the steady state, firms' profits are equal to zero, which is consistent with Basu and Fernald (1994), Hall (1988) and Rotemberg and Woodford (1995).

From the combination of first-order conditions, we obtain the wage rental ratio (equation 1.20), which implies that the capital to labour ratio across all of the monopolistic producers remains the same.

$$\frac{K_t}{N_t} = \frac{K_{j,t}}{N_{j,t}} = \frac{\alpha}{(1-\alpha)} \frac{W_t}{u_t R_{k,t}} \tag{1.20}$$

The nominal marginal cost is represented by the following:

$$P_t mc_t = (\frac{1}{1-\alpha})^{1-\alpha} (\frac{1}{\alpha})^{\alpha} \left(\varepsilon_t^A\right)^{-1} K_{g,t-1}^{-\alpha_g} (W_t)^{1-\alpha} (R_{k,t})^{\alpha} \tag{1.21}$$

The marginal cost increases as the wage rate and the rate of return on capital increase. A positive total factor productivity shock along with an increase in public capital leads to a decrease in the marginal costs.

Price setting

The intermediate firm profit maximisation problem can be transformed to the following:

$$Prof_t = P_{j,t} Y_{j,t} - mc_t P_t \left(Y_{j,t} + fc\right) = \left[\frac{P_{j,t}}{P_t} - mc_t\right] P_t Y_{j,t} - P_t mc_t fc$$

where: $Y_{j,t} = (\frac{P_{j,t}}{P_t})^s Y_t$.[8] Intermediate good producers set prices similarly to the mechanism presented in Calvo (1983). In particular, during every period, a share of these firms (ϖ) are not able to reoptimise their price.[9] These firms simply follow the partial indexation

[8] From this equation, it follows that for profits to be equal to 0 in the steady state, $fc = \frac{(1-mc)}{mc} Y$.

[9] When ϖ is equal to 1, all companies are able to reoptimize their prices, and when ϖ is equal to 0, none of the companies is able to reoptimise its price.

rule, i.e.: $P_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} P_{j,t-1}$. The remaining fraction of companies $(1-\omega)$ choose P_t to maximise the objective:

$$Prof_t = E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\widetilde{P}_t X_{tl}}{P_{t+l}} - mc_{t+l} \right] P_{t+l} Y_{j,t+l} - P_{t+l} mc_{t+l} fc \tag{1.22}$$

subject to the demand (equation 1.17). Maximisation results in equation (1.23) for newly optimised prices:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\widetilde{P}_t X_{tl}}{P_{t+l}} - \frac{s}{1-s} mc_{t+l} \right] P_{t+l} Y_{j,t+l} = 0 \tag{1.23}$$

In the case that all firms are allowed to reoptimise their prices, the above condition reduces to: $\widetilde{P}_t = \frac{s}{s-1} P_t mc_t$, which indicates that the optimised price is equal to a markup over the marginal costs. In addition, $(\beta\varpi)^l \lambda_{t+l}$ denotes a discount factor of future profits for firms. Here, λ_t denotes the Lagrange multiplier on the Ricardian household's budget constraint and is treated by firms as exogenous. The price index $P_t = \left[\int_0^1 P_{j,t}^{1-s} dj\right]^{\frac{1}{1-s}}$ can be rewritten as:

$$P_t = \left[(1-\varpi)\, \widetilde{P}_t^{1-s} + \varpi \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} P_{t-1} \right)^{1-s} \right]^{\frac{1}{1-s}} \tag{1.24}$$

1.2.4 Fiscal and Monetary Policy

Equation (1.25) provides the government budget constraint, which requires the total expenditure of government on consumption (G), investment (IG), transfers (TR), and the

repayment of last-period debt with interests $\left(\frac{R_{t-1}}{\pi_t}\right) b_{t-1}$, to be equal to the revenue from taxes $\left(\tau_t^c C_{T,t} + \tau_t^l w_t L_t + \tau_t^k r_{k,t} u_t K_{t-1}\right)$ and new bond issuance (b_t).

$$\tau_t^c C_{T,t} + \tau_t^l w_t L_t + \tau_t^k r_{k,t} u_t K_{t-1} + b_t = \left(\frac{R_{t-1}}{\pi_t}\right) b_{t-1} + G_t + IG_t + TR_t \tag{1.25}$$

The public capital accumulation equation is represented by:

$$K_{g,t} = (1 - \delta_{k,g}) K_{g,t-1} + IG_t \tag{1.26}$$

We assume that the government instruments respond countercyclically to movements in debt and GDP, whereas taxes respond to them procyclically. Fiscal policy instruments rules are set similarly to those used by Leeper *et al.* (2010). Firstly, expenditure instruments respond countercyclically to GDP deviations from steady-state, whereas taxes respond to them procyclically; therefore fiscal instruments play a role of automatic stabilizers. Secondly, fiscal instruments keep real debt dynamics under control in order not to allow for high debt to GDP ratios[10]. This form of fiscal policy implies that it is countercyclical. Six fiscal instruments that are linked to the debts $\left(\hat{b}_{t-1}\right)$ and GDP $\left(\hat{Y}_t\right)$ as follows[11]:
Public spending $\left(\hat{G}_t\right)$ process:

$$\hat{G}_t = -\phi_{b,g} \hat{b}_{t-1} - \phi_{g,y} \hat{Y}_t + \varepsilon_{g,t} \tag{1.27}$$

Public investment $\left(\widehat{IG}_t\right)$:

[10] Romer and Romer (2010) find for example that most of the tax changes in the USA are motivated by: (1) a change in government spending, (2) other factors likely to affect output in in the close future, (3) budget deficit, (4) higher growth.

[11] Hats over variables denote deviations from the steady state.

$$\widehat{IG}_t = -\phi_{b,ig}\hat{b}_{t-1} - \phi_{ig,y}\hat{Y}_t + \varepsilon_{ig,t} \tag{1.28}$$

Transfers $\left(\widehat{TR}_t\right)$:

$$\widehat{TR}_t = -\phi_{b,tr}\hat{b}_{t-1} - \phi_{y,tr}\hat{Y}_t + \varepsilon_{tr,t} \tag{1.29}$$

Consumption tax rate $(\hat{\tau}_t^c)$:

$$\hat{\tau}_t^c = \phi_{b,\tau^c}\hat{b}_{t-1} + \phi_{y,\tau^c}\hat{Y}_t + \varepsilon_{\tau^c,t} \tag{1.30}$$

Labour income tax rate $\left(\hat{\tau}_t^l\right)$:

$$\hat{\tau}_t^l = \phi_{b,\tau^l}\hat{b}_{t-1} + \phi_{y,\tau^l}\hat{Y}_t + \varepsilon_{\tau^l,t} \tag{1.31}$$

Capital income tax rate $\left(\hat{\tau}_t^k\right)$:

$$\hat{\tau}_t^k = \phi_{b,\tau^k}\hat{b}_{t-1} + \phi_{y,\tau^k}\hat{Y}_t + \varepsilon_{\tau^k,t} \tag{1.32}$$

where $\varepsilon_{x,t}$ for $x = \{G, IG, TR, \tau^c, \tau^k\tau^l\}$ denote fiscal shocks which affect the spending $\{G, IG, TR\}$ and the revenue $\{\tau^c, \tau^k\tau^l\}$ sides of the government[12]. Finally, we follow the approach set in the monetary policy literature, and assume that the i.i.d. error terms in the above fiscal policy rules constitute an unexpected changes in policy which is an analogous approach to errors in the the case of Taylor rule.[13]

[12] We have attempted to use setup with fiscal policy responding to the one-period lagged output, but it yielded a lower marginal likelihood than the benchmark scenario.

[13] see Forni *et al.* (2009).

Nominal interest rate $\left(\hat{R}_t\right)$ follows a Taylor type rule that links it to its own lag term $\left(\hat{R}_{t-1}\right)$, inflation $(\hat{\pi}_t)$ and oputput gap $\left(\hat{Y}_t - \hat{Y}_{t-1}\right)$ as measured by coefficients ρ, ρ_π, and ρ_y:

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\,\rho_\pi\hat{\pi}_t + (1-\rho)\,\rho_y\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \eta_t^m \tag{1.33}$$

where η_t^m denotes an i.i.d.normal error term on the interest rate rule.

1.2.5 Aggregation and Market clearing

The aggregate quantity, expressed in per-capita terms, of any household quantity variable Z_t^ι, is represented by $Z_t = \int_0^1 Z_t^\iota d\iota = (1-\theta)Z_t^\iota + \theta Z_t^\iota$, as all members of each household choose identical allocations in equilibrium. The final goods market is in equilibrium when the aggregate supply equals the aggregate public and private demand for consumption and investment goods. The labour market is in equilibrium when the total labour demanded by the intermediate firms equals total labour supplied by households at a wage rate (W_t). The capital rental market is in equilibrium when capital supplied by Ricardian households is equal to the capital demanded by intermediate producers at a market rental rate $(R_{k,t})$. The goods market clearing condition is represented by:

$$Y_t - a(u_t)K_{t-1} = C_{T,t} + G_t + I_t + IG_t \tag{1.34}$$

where: $C_{T,t} = \vartheta C_{nr,t} + (1-\vartheta)\,C_t$

Capital market clearing condition is represented by:

$$\int_0^1 K_{j,t} dj = (1-\vartheta) K_t^{\iota} \tag{1.35}$$

The relation between labour demand and labour supply can be derived from equation (1.12). Integrating the equation over all households we obtain:

$$L_{nr,t} = L_t = L_t^S = \int_0^1 \left(\frac{W_t^{\iota}}{W_t}\right)^{-\nu} d\iota N_t \tag{1.36}$$

where L_t^S denotes labour supply and labour demand is given by: $N_t = \int_0^1 N_{j,t} dj$. Denoting $o_t = \int_0^1 \left(\frac{W_t^{\iota}}{W_t}\right)^{-\nu} d\iota$, the relation between labour demand and supply can be summarised by:

$$L_t^S = o_t N_t \tag{1.37}$$

The bond market can be summarized by:

$$b_t = (1-\vartheta) b_{r,t} \tag{1.38}$$

Log-linearized equations describing the equilibrium of the model are represented in Appendix $(1.A)$.

1.3 Bayesian Estimation

We use perturbation techniques to solve the model and Bayesian methods to estimate it[14]. Sims's $csminwel$ function is used as the optimiser for the mode' computation. The ac-

[14] For solution, estimation and necessary calculations, we use Dynare 4.2.4 by Adjemian *et al.* (2011) and MATLAB.

ceptance ratios obtained in the Metropolis-Hasting algorithm simulation are approximately 0.25, which is in line with the range of ratios proposed in the literature. Model simulations starts with the initial values of variable and priors on parameters. Time series generated by these DSGE model simulations are then used to estimate posteriors of these parameters using the Bayesian likelihood functions. Model is simulated with these updated values of parameters to minimise the errors between the actual and model generated series. This process continues till the parameters are close enough to replicate the actual series meeting the convergence criteria set for the model. Results presented in this paper are based on 250000 iterations with an optimal acceptance rate of 0.234 which seems to be a standard in the DSGE literature as explained in Gelman *et al.* (1997).

In estimation, we use twelve data series for the period from 1987:Q1 to 2011:Q1. The length of the sample period is determined by the availability of the tax data. The time series used in the estimation comprise per capita: private consumption, GDP, private investment, hours, wages, inflation, government consumption, government investment, transfers, and effective tax revenue from consumption, labour and capital (see Appendix $1.B$ for more details on the dataset). In the baseline estimation, data are detrended with their linear trends.

1.3.1 Calibration

Most of the parameters related to the steady state are calibrated and their values are presented in Table (1.1). The discount factor (β) is set to 0.99, which implies a steady state annual real interest rate of 4 per cent as Harrison and Oomen (2010). We fix the depreci-

ation rate of public capital (δ_g) at 0.015, which results in an annual depreciation of public capital of 6 per cent. This value together with the ratio of public investment to GDP pins down the ratio of public capital stock to annual GDP at 0.32 to match the ONS data on the public capital stock and public investment. The calibration of δ_g, together with the share of public investment expenditure in GDP (see below), pins down the steady state ratio of public capital to annualised GDP at around 0.33. For the depreciation rate of private capital we choose $\delta = 0.025$, which implies an annual depreciation of 10 per cent as Harrison and Oomen (2010). The steady state wage markup parameter $\left(\frac{\nu}{\nu-1}\right)$ is set to 1.05 per cent as in Christiano *et al.* (2005). The share of capital in the production function (α) is calibrated to 0.30, which results in a steady state share of labour income in total output of 70 per cent as Harrison and Oomen (2010). The calibration of α, together with the capital tax rate $\left(\tau^k\right)$, and the value of private capital depreciation (δ) , fixes the share of private investment expenditure in GDP at roughly 15 per cent as in the data. Also the ratio of private consumption to GDP fits the data for the sample period. The elasticity of output to public capital (σ_g) is set to 0.01. We select the effective tax rates so that they match the corresponding rates implied by the data for the sample period. This implies 20 per cent for the VAT (consumption tax rate: τ^c), and 29 per cent for the labour and capital tax rates $\left(\tau^l, \tau^k\right)$. A share of public consumption and public investment in GDP is calibrated respectively at 20 per cent and 2 per cent to match their empirical counterparts over the sample period. The endogenous public transfers to GDP ratio is pinned down at approximately 17 per cent.

Table 1.1. Calibrated parameters and steady state ratios

Share/Paramenter	Definition	Value	Data
A. Expenditure shares			
C/GDP	Private consumption to GDP ratio	0.63	0.64
I/GDP	Private investment to GDP ratio	0.15	0.15
G/GDP	Gov. consumption to GDP ratio	0.20	0.20
IG/GDP	Gov. investment to GDP ratio	0.02	0.02
B. Production sector			
α	Share of capital in production function	0.30	
δ	Private capital depreciation rate	0.025	
δ_g	Public capital depreciation rate	0.015	
σ_g	Elasticity of output to government investment	0.01	
C. Taxes and fiscal policy			
τ^c	Consumption tax rate	0.20	0.20
τ^l	Labour tax rate	0.29	0.29
τ^k	Capital tax rate	0.29	0.29
TR/GDP	Transfers to GDP ratio	0.17	0.16
b/GDP	Annualised gov. debt to GDP ratio	0.60	0.51
D. Other calibrated parameters			
β	Discount factor	0.99	
$\frac{\nu}{\nu-1}$	Steady state wage markup	1.05	

The GDP expenditure shares in Section A do not add to 1 due to the data rounding and the fact that model is in a closed economy setup.

1.3.2 Prior Distributions

Assumptions about priors are presented in Tables (1.8) and (1.9) in Appendix $(1.C)$. Priors are set in line with the previous studies. We use inverse gamma priors for standard deviations of shocks, with a prior mean set to 0.01, except for investment shocks where we set the prior means to 0.1. We select a beta distribution for shock persistence parameters with prior means set to 0.8 and standard deviations of 0.1. We select normal distribution priors for all fiscal policy response parameters.[15] The parameters controlling the response of fiscal policy instruments to GDP have a relatively loose prior means set to 0.5 with stan-

[15] Leeper *et al.* (2009) select a gamma distribution and therefore allow only for positive estimates.

dard deviations of 0.5, whereas debt aversion parameters have prior means set to 0.2 and standard deviations of 0.1. We use a beta distribution for the share of non-Ricardian households with a prior mean of 0.3, close to the estimates of Ratto *et al.* (2009) and Coenen *et al.* (2013). We choose the prior mean of the habit formation parameter to 0.7, similarly to the estimate of Harrison and Oomen (2010) for the UK. The prior mean of constant relative risk aversion is set to 0.66, as in Harrison and Oomen (2010). For the prior mean of the inverse Frisch elasticity of labour, we choose a value of 1, as in Christiano *et al.* (2005). Turning to the monetary policy rule, for the degree of interest rate smoothing parameter we select a beta distribution with a prior mean equal to 0.7, whereas for the Taylor rule coefficients on inflation and output we select a normal distribution and set prior means respectively at 1.5 and 0.125. The prior means for the price and wage indexation parameters are set to 0.3, whereas the prior mean of the Calvo price and wage stickiness parameters is fixed at 0.5 with a standard deviation of 0.1. Finally, we select the normal distribution prior for the capital adjustment cost with its mean set to 4, as in Smets and Wouters (2003), and choose a normal distribution prior for the utilisation parameter with a mean of 0.8 and a standard deviation of 0.2.

1.3.3 Posterior Estimates

The details of posterior estimates are presented in Tables (1.8) and (1.9) in Appendix $(1.C)$. According to the results, agents exhibit a moderate degree of habit formation in consumption $(b = 0.66)$, which is similar to the value of 0.59 found in Millard (2011) and 0.69 in Harison and Oomen (2010). The inverse of the intertemporal elasticity of substitution co-

efficient is estimated to 0.93. The above estimates imply that the elasticity of Ricardian households' consumption with respect to the short-term ex ante real interest rate is equal to 0.37 and is close to the value of 0.47 obtained by Harrison and Oomen (2010) and somewhat lower than 0.66 found in Millard (2011).[16]

The investment adjustment cost parameter is estimated to 6.41, similar to 6.71, a value obtained for the euro area by Adolfson *et al* (2008) in a closed economy model. The parameter governs the transmission mechanism from the price of installed capital to investment.[17] The parameter can be interpreted as the inverse elasticity of investment with respect to an increase in the installed capital.[18] Its estimate implies that a 1 per cent increase in the price of capital is followed by a $\frac{1}{\phi(1-\beta)} = 15.60$ per cent increase in investment. Smets and Wouters (2003) estimate this elasticity at 16 per cent for the euro area, whereas Christiano *et al.* (2005) estimate it at 38 per cent for the USA.

The capital utilisation adjustment parameter $(\kappa = 0.81)$ can be defined as the inverse elasticity of utilisation with respect to the rental rate of capital net of capital taxes. Our result is higher than the value of 0.46 obtained by Millard (2011) and the value of 0.56 by Harrison and Oomen (2010) and is closer to 0.85 in Smets and Wouters (2007) and 0.77 in Edge *et al.* (2003) for the USA. The fixed cost parameter estimate ($\varphi_y = 1.65$) is slightly higher than 1.46 in Christiano *et al.* (2005) and 1.50 in Smets and Wouters (2003).[19]

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[16] This result stems from the transformation of the consumption equation to $\hat{C}_t = b\hat{C}_{t-1} - \frac{(1-b)}{\sigma_c}\sum_{i=0}^{+\infty}\left(R_{t+i} - \pi_{t+i+1}\right)$.

[17] The presence of investment adjustment costs in this form improves the performance of the model by inducing hump-shape responses of the investment (for more discussion, please see Burnside *et al.*, 2004 and Christiano *et al.*, 2005).

[18] Disregarding shocks to the investment technology, the investment equation can be transformed to $\hat{I}_t = \hat{I}_{t-1} - \frac{1}{\phi}\sum_{i=0}^{+\infty}\beta^i\hat{Q}_{t+i}$.

[19] It can be shown that for $fc = \frac{1-mc}{mc}Y$, and $mc = \frac{1}{x}$, where x is a markup;

The estimate of a price stickiness parameter (0.59) implies that prices change roughly every 2.5 quarters. The Frish elasticity of labour supply is equal to 0.83, which is consistent with macroeconomic estimates, and implies that the labour supply is relatively elastic with respect to the changes in real wages. The estimate of the wage stickiness parameter (0.56) implies that wages adjust on average approximately nine months, which is similar to the results found by Millard (2011).

Estimates of monetary policy parameters take the following values: persistence parameter, (0.72); response to inflation, (1.64); and the response to the output parameter, (0.21) and are in line with previous UK data estimates. The share of non-Ricardian households is estimated to be 0.33 which is consistent with estimates for EU and US (see for example Ratto *et al.*, 2009).

Turning to the fiscal policy parameters, the most persistent fiscal policy shocks are the government consumption shock with a half life of 33 months, and the transfers shock with a half life of 22 months. Tax shocks feature lower persistence with a half life oscillating around $(10) - (12)$ months. The least persistent is the public investment shock with a half life of only 2 months.

The estimates of fiscal policy parameters are presented in Table (1.9). The 90 per cent confidence intervals of some of them (response of government transfers, consumption taxes, and labour taxes to debt and GDP) include 0 which implies that they were not used systematically in the controlling for debt and GDP.

$\varphi_y = \left(1 + \frac{fc}{Y}\right) = \left(1 + \frac{(1-mc)Y}{mcY}\right) = x.$

It also implies an elasticity of substitution between intermediate goods equal to 2.5 as $\varkappa = \frac{s}{s-1}$ where (s) denotes the elasticity of substitution.

The parameter estimates imply that the government investment, consumption and capital taxes played the most important role in controlling the government debt over the sample period. The results indicate that capital tax rates and government investment have significant procyclical response to GDP. In contrast, labour taxes do not respond strongly to the aggregate output.

1.3.4 Model Comparison

Table 1.2. Model comparison

	Marginal Likelihood
BVAR(1)	2128.64
BVAR(2)	2224.80
BVAR(3)	2215.66
BVAR(4)	2223.00
BVAR(5)	2268.31
BVAR(6)	2265.95
DSGE model	2375.71

To assess the fit of the model we compare it to the BVAR models of lag order from 1 to 6.[20] In the BVAR setup, we follow Juillard *et al.* (2006) and Ratto *et al.* (2009) and set the prior decay parameter to 0.5, the tightness of the prior parameter to 3, the parameter determining the weight on own-persistence is set to 2, and the parameter determining the degree of co-persistence to 5. Table (1.2) presents the marginal likelihood of each of the estimates and indicates that the model has better fit than the presented BVAR. Similarly to Juillard *et al.* (2006) and Ratto *et al.* (2009) our model yields better fir then the BVARs.

[20] For the discussion of BVAR see Doan *et al.* (1987) and Villemot (2011).

1.4 Impulse Responses and Fiscal Multipliers

This section presents impulse response functions and present value multipliers of the fiscal policy shocks. To enable comparability across the graphs, for the impulse of public spending, investment, and transfers, we use shock equal to a 1 per cent of the steady state value of GDP.[21] In the case of tax rates we calibrate a standard deviation of shock to the initial change in the particular tax revenue is equal to a 1 percent of the steady state value of GDP.

On each graph presenting impulse responses, the horizontal axis denotes time in quarters, and the vertical axis denotes the percentage deviation from the steady state. For each shock, we also provide cumulative present value multipliers of GDP, private consumption and private investment for the first quarter (impact multiplier), four quarters, twelve quarters, twenty quarters and long horizon multipliers (one thousand quarters). Multipliers are calculated with the following formula[22]:

$$PV = \frac{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta Y_{t+j}}{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta X_{t+j}} \tag{1.39}$$

where $X_t = \left\{G, IG, TR, \tau_{inc}^{c}, \tau_{inc}^{k}\tau_{inc}^{l}\right\}$.[23]

[21] GDP is normalised to 1.

[22] The present value multipliers were firstly used by Mountford and Uhlig (2009) and since then have become widely used (see for example: Leeper et al., 2010, Coenen et al. ,2013). This type of multipliers is preferred over the "ahead multipliers" for two reasons. Firstly, they incorporate the full dynamics associated with fiscal shocks and secondly, they appropriately discount future macroeconomic effects.

[23] For example for $k = 2, Y_{t-1} = 0, G_{t-1} = 0$ the present value multiplier can be presented in the following form; for $\hat{Y}_t = \frac{Y_t - Y}{Y} \Rightarrow \hat{Y}_t Y + Y = Y_t \Rightarrow Y_t = Y\left(\hat{Y}_t + 1\right)$ we obtain:

$$PV = \frac{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta Y_{t+j}}{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta G_{t+j}} = \frac{\hat{Y}_t + \frac{\hat{Y}_{t+1} - \hat{Y}_t}{R(\hat{R}_t+1)} + \frac{\hat{Y}_{t+2} - \hat{Y}_{t+1}}{R(\hat{R}_t+1) * R(\hat{R}_{t+1}+1)}}{\hat{G}_t + \frac{\hat{G}_{t+1} - \hat{G}_t}{R(\hat{R}_t+1)} + \frac{\hat{G}_{t+2} - \hat{G}_{t+1}}{R(\hat{R}_t+1) * R(\hat{R}_{t+1}+1)}} \frac{Y}{G}$$

where we have assumed $\hat{Y}_{t-1} = 0$ and $\hat{G}_{t-1} = 0$

1.4.1 Public consumption

The dynamics implied by the fiscal policy instruments' shocks are present in Figures $(1.2 - 1.4)$. The model predicts that the government spending shock results in a persistent increase in the government's demand for goods. The increased demand for goods leads subsequently to a higher capital utilisation and an increase in a demand for labour, which puts upward pressure on the capital rental rate and the wage rate.[24] An increase of marginal cost results in a higher inflation. Both, higher inflation and an increase in output oblige the central bank to increase the nominal interest rate.

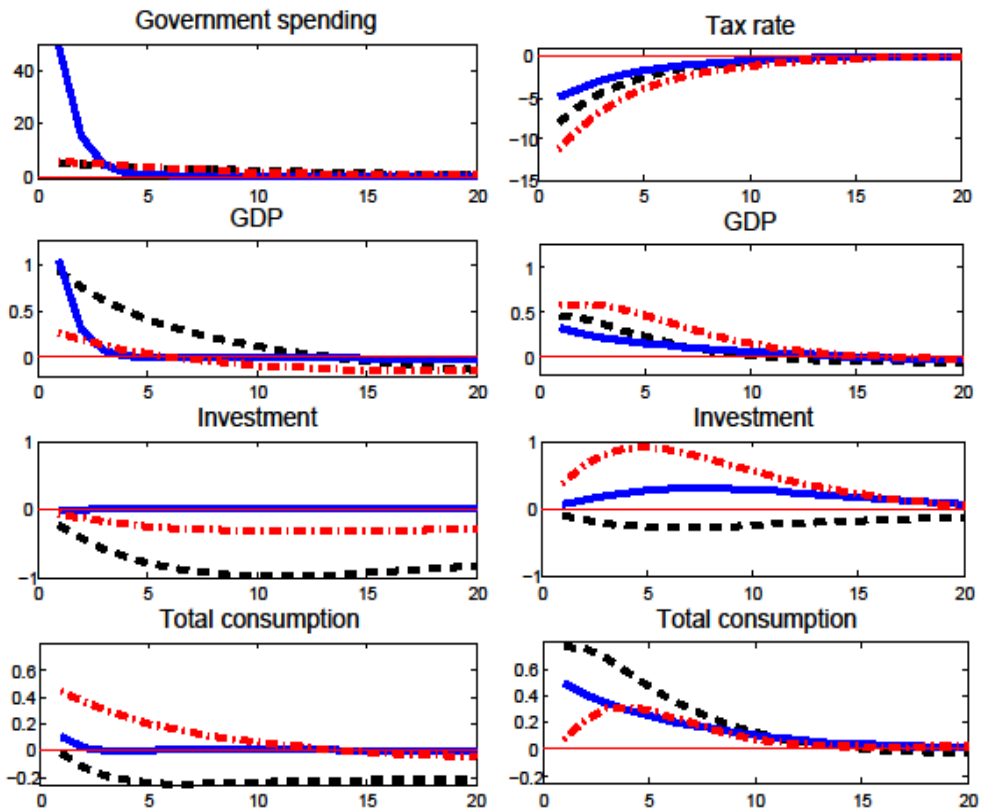
Consumption of non-Ricardian households increases due to an increase in the labour income. Forward-looking households cut on the interest rate sensitive consumption and investment. Total consumption also decreases due to the fall in consumption of Ricardian households, which account for the major proportion of the GDP. An increase in government spending leads to an increase in the government's deficit, debt, and contraction in the remaining fiscal policy instruments.

Our results can be compared with the empirical study of fiscal policy in the UK conducted by Perotti (2005),[25] who estimates the impulse responses of private investment to be significantly negative, which is in line with the results implied by our model. Perotti (2005) also obtains a positive and significant response of the *ex ante* real interest rate,

[24] Consumption of Ricardian households decreases strongly on impact what makes them more willing to supply labour. This effect prevails over the higher labour demand, therefore the wage rate decreases.

[25] Perotti (2005) uses the Structural Vector Autoregression (SVAR) approach to estimate, among others, the effect of a government spending shock on key macroeconomic variables in the US, the UK and the euro area. He divides his sample into two parts, one from 1963:1 to 1979:4 and the second from 1980:1 to 2001:2, and reports cumulative responses in the fourth and twelfth quarters. We compare our results with the results based on the sample from 1980:1 to 2001:2, as this period is closer to our sample.

Fig. 1.2. Impulse responses

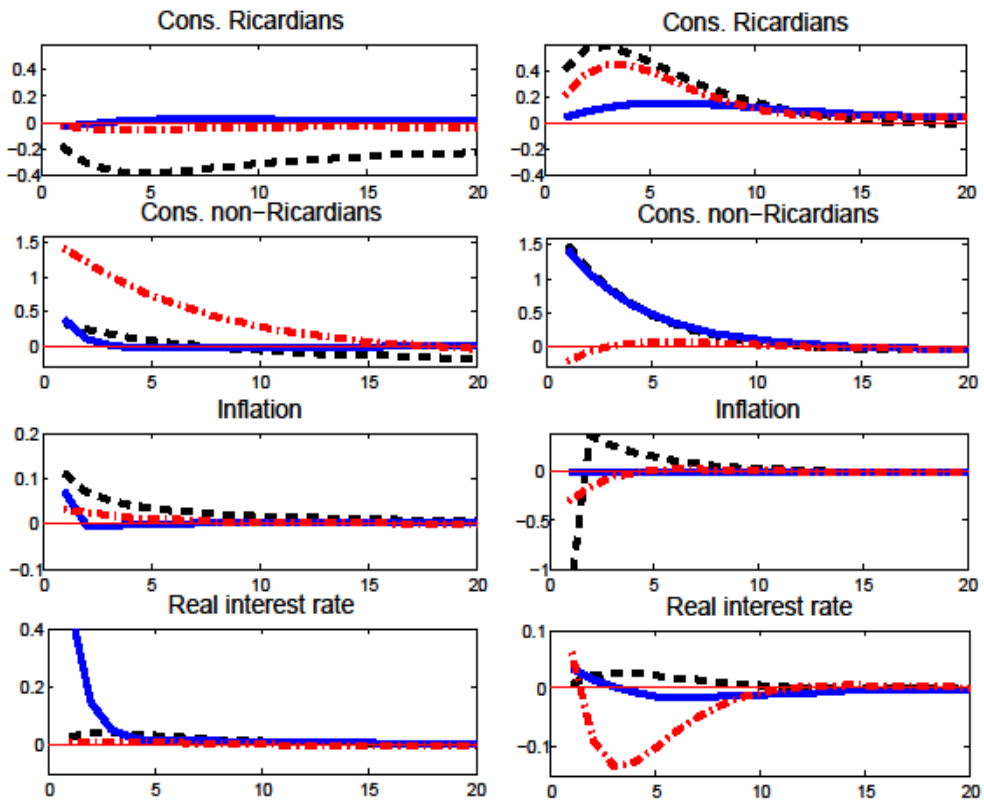


The left column of the graph illustrates the impulse responses of spending shocks, whereas the right column of the graph presents the impulse responses of tax cuts. On the left column the blue line (-) relates to public investment, the black line (–) ralates to public consumption, and the red line (-.) to public transfers. On the right column the blue line (-) relates to labour income tax cut, the black line (–) ralates to consumption tax cut, and the red line (-.) to capital tax cut.

which is in line with this study and a negative response of inflation, which is not the case here. Perotti (2005) estimates the cumulative response of consumption to be negative in the fourth quarter but positive in the twelfth quarter. In both cases, the responses are not statistically significant.[26]

[26] The response of consumption to the government spending shock is discussed widely in the literature. Our result is similar to those of Harrison *et al.* (2005), Harrison and Oomen (2010) and models estimated on euro area data (see for example Coenen and Straub, 2007, and Ratto *et al.*, 2009). It must be noted that it differs from Gali *et al.* (2007) who built a small DSGE model in which they obtain a positive response of consumption to a government spending shock. However, these authors assume flexible wages and calibrate the weight on the non-Ricardian households to 0.5. Our sensitivity analysis indicates that it is possible to obtain a positive response of consumption by imposing flexible wages and increasing the weight of non-

Fig. 1.3. Impulse responses for tax shocks



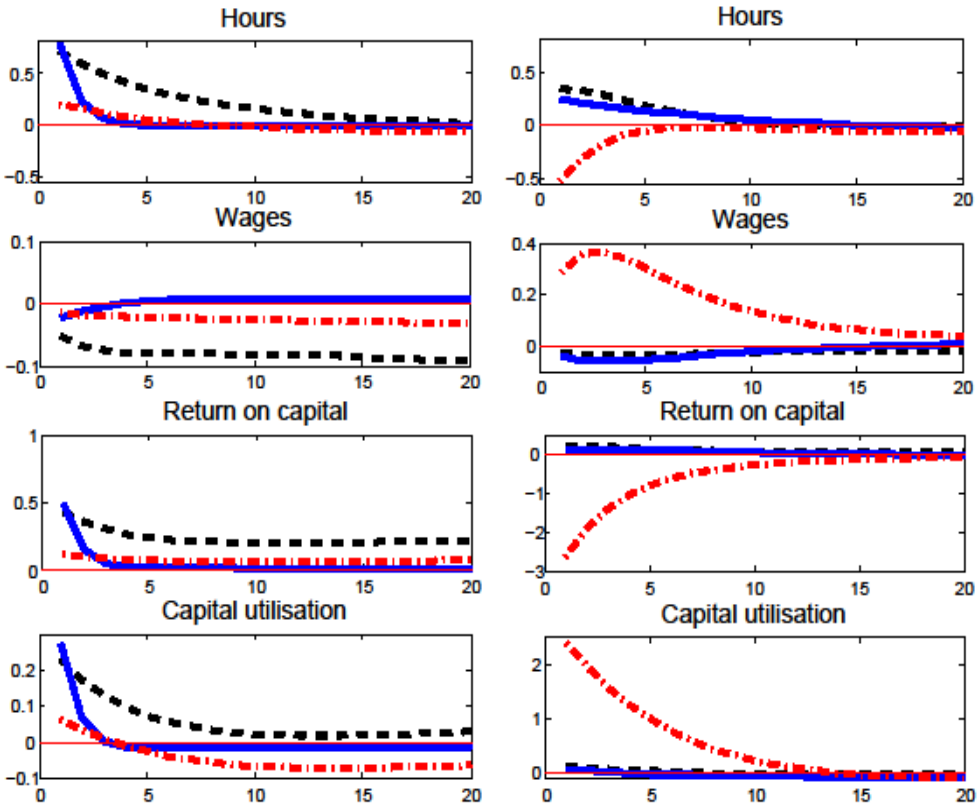
The left column of the graph illustrates the impulse responses of spending shocks, whereas the right column of the graph presents the impulse responses of tax cuts. On the left column the blue line (-) relates to public investment, the black line (–) ralates to public consumption, and the red line (-.) to public transfers. On the right column the blue line (-) relates to labour income tax cut, the black line (–) ralates to consumption tax cut, and the red line (-.) to capital tax cut.

Table (1.3) provides present value multipliers of output, consumption and investment implied by shocks to the government spending, investment and transfers. The magnitudes of the government spending multipliers differ significantly in the literature. Ramey (2011) conducts a literature review on the impact of government spending on GDP in the USA and concludes that the estimates of deficit-financed government spending multiplier lie somewhere between 0.8 and 1.5. In the context of the UK, we estimate the impact government

Ricardian households at the same time.

spending multiplier on GDP to 0.99. This result is slightly higher than the average impact multiplier obtained in an empirical study on fiscal policy in the UK conducted by Canova and Pappa (2011), (0.89). When longer horizon is considered, the size of multipliers become smaller. Present value multiplier of consumption and investment remains negative over the long horizon in analysis. This finding is consistent with Ramey (2012), who reports that government consumption crowds out total private expenditure on consumption and investment.

Fig. 1.4. Impulse responses for tax shocks



The left column of the graph illustrates the impulse responses of spending shocks, whereas the right column of the graph presents the impulse responses of tax cuts. On the left column the blue line (-) relates to public investment, the black line (–) ralates to public consumption, and the red line (-.) to public transfers. On the right column the blue line (-) relates to labour income tax cut, the black line (–) ralates to consumption tax cut, and the red line (-.) to capital tax cut.

1.4.2 Public investment

The main difference between the effects of the public investment and public consumption shock is that the former, apart from an increase in the aggregate demand leads also to a rise in the public capital, which subsequently results in an increase in the supply of monopolistic producers.[27] The initial increase in the capital utilisation rate and the demand for labour is stronger than in the case of the government consumption shock. The rental rate of capital increases on impact, and the wage rate is above the steady state level just after four quarters (the increase in the wage rate is related to an increase in Ricardian's consumption, which causes that households are less willing to supply labour, driving; therefore, the wage rate up).

The marginal cost initially increases stronger than in the case of a public consumption shock. The increase in the *ex ante* real interest rate is also larger only at the outset. Afterwards, as public capital accumulates, marginal cost, inflation and the nominal interest rate are below the levels implied by the government consumption shock.

The interest rate sensitive private investment is initially crowded out but is above the steady state level after less than 2 quarters, (the four quarters cumulative multiplier is already positive). Traum and Yang (2010) and Baxter and King (1993) obtain a positive response of investment in the USA. Sensitivity analysis reveals that for higher values of the elasticity of output to the public capital, an increase in public investment results in corresponding increase in its private investment.

[27] The presence of public capital in the production function implies that an increase in public investment is analogous to an increase in total factor productivity.

Table 1.3. Public spending cumulative multipliers

	Quarters				
	1	4	12	20	1000
Fiscal multipliers of public consumption					
GDP	0.99	0.84	0.62	0.46	-1.08
Private consumption	-0.01	-0.10	-0.20	-0.28	-1.34
Private investment	-0.04	-0.08	-0.19	-0.26	-0.77
Fiscal multipliers of public investment					
GDP	1.07	1.05	1.04	1.02	1.08
Private consumption	0.07	0.06	0.10	0.13	0.30
Private investment	-0.00	0.00	0.03	0.04	0.07
Fiscal multipliers of public transfers					
GDP	0.27	0.21	0.06	-0.09	-1.15
Private consumption	0.28	0.26	0.23	0.19	-0.21
Private investment	-0.01	-0.03	-0.07	-0.11	-0.30
Fiscal multipliers of consumption tax					
GDP	-0.51	-0.66	-0.67	-0.55	0.23
Private consumption	-0.54	-0.74	-0.90	-0.90	-0.57
Private investment	0.02	0.05	0.12	0.17	0.31
Fiscal multipliers of labour tax					
GDP	-0.33	-0.36	-0.42	-0.40	0.09
Private consumption	-0.33	-0.36	-0.44	-0.47	-0.39
Private investment	-0.01	-0.04	-0.11	-0.15	-0.06
Fiscal multipliers of capital tax					
GDP	-0.58	-0.84	-1.07	-1.05	-0.73
Private consumption	-0.05	-0.21	-0.33	-0.35	-0.39
Private investment	-0.06	-0.16	-0.31	-0.35	-0.26

Simiraly to Ratto *et al.* (2009) we obtain a positive response of total consumption to a public investmetnt shock.[28] The main reason behind this is that the response of Ricardian households' consumption is positive in just after four quarters and that the initial crowding out effect is small. The consumption of rule-of-thumb households increases and in the medium horizon remains above the level of consumption implied by the public con-

[28] Ratto *et al.* (2009) estimate their model on the euro area data.

sumption shock. An increase of the elasticity of output to public capital, or the share of non-Ricardian households results in an even stronger response of total consumption.

The values of government investment multipliers are higher than that implied by government consumption, and remain above 1 over the analysed horizon. The effects on private expenditure are positive and the highest out of all public spending instruments.

1.4.3 Public transfers

The direct implication of a transfer shock is an increase in consumption of non-Ricardian households as these households are the only beneficiaries of government transfers in the model. Subsequent increase in demand leads to a short-lived increase in the capital utilisation and the labour demand. The capital rental rate increases, whereas the wage rate decreases. In the context of Ricardian households' expenditure, the increase in the real interest rate implies that they cut on consumption and investment.

The GDP multiplier is less favourable in the short and long term when compared to multipliers implied by the remaining government spending shocks. The reason is that transfers have positive influence just on consumption of non-Ricardian households who comprise only a fraction of total consumption. The sensitivity analysis presented in Section 5 indicates that an increase of the share of non-Ricardian households leads to higher multipliers implied by the shock to transfers.

1.4.4 Consumption tax

A decrease in consumption tax rate results in a fall in consumer prices lasting approximately 7 months. Consequently, the consumption of both optimising and non-optimising households increases. Higher demand for goods, implied by the consumption tax cut, results in an increase of the demand for labour and a higher capital utilisation.

A decrease in the consumption tax leads to an increase in output. Government debt increases; therefore, the total government's expenditure on transfers, public consumption and investment decreases. A decrease in the consumption tax leads to a fall in private investment and the multiplier remains significant in the medium and the long term. This has subsequently negative effect on the economy.

Table (1.3) includes present value multipliers for consumption tax shocks. In the short term, a consumption tax cut induces relatively high multipliers for consumption and GDP.[29] Whereas the consumption multiplier remains larger over time, the GDP multiplier drops significantly in the longest horizon considered.

1.4.5 Capital tax

The instant effect of a decrease in the capital tax rate is the reallocation of production inputs from labour to capital, which results in a higher capital utilisation and a lower labour demand. The lower demand for labour puts downward pressure on the wage, which is more than offset by the increase of consumption of Ricardian households. The marginal cost de-

[29] Note that in contrast to public spending multipliers where plus is a desired sign of a multiplier (increase in spending leads to an increase in GDP), in the case of taxes minus is the desired sign (an increase in a tax leads to fall in GDP).

creases as a result of the fall in the rental rate of capital. This is followed by a decrease in inflation and the nominal interest rate. Consumption of non-optimising households decreases slightly initially due to a drop in labour income, whereas the interest-rate-sensitive consumption of Ricardian households increases. Investment increases significantly as a result of high increase in the discounted rental rates.

Table (1.3) presents multipliers of the capital tax, which can be compared to the results of Leeper *et al.* (2010) and Leeper *et al.* (2009). On impact, multipliers of consumption and investment remain rather modest, to increase substantially in the longer term. GDP multiplies remain high irrespective of the period of consideration.

1.4.6 Labour tax

The instant effect of a decrease in the labour tax is the reallocation of production inputs from capital to labour, leading to an increase of a labour demand and an decrease of capital utilisation. The labour tax cut has a positive impact on GDP and households' disposable income. The consumption of both types of households increases. The increase among the non-optimising households is stronger, as labour income is the main determinant of their consumption. Inflation falls as a result of a lower marginal cost. Because the monetary authority places more weight on inflation than on GDP, the nominal interest rate falls. The model predicts that as a result of a decrease in the labour tax, private investment increases. A decrease in the labour tax results in a lower expenditure of government on consumption, transfers and investment.

In the short horizon, the present value multiplier is relatively small for investment, and yields higher values for GDP and consumption. In the longest horizon considered GDP multiplier becomes positive.

1.5 Permanent Fiscal Shocks

Table (1.4) presents GDP multipliers for a permanent change in the fiscal policy[30]. The table implies that for a higher shock persistence, the GDP multiplier increases for the labour and capital tax cuts and decreases for public expenditure shocks and the consumption tax cut. The reason for a decrease in the public spending shocks is that for permanent shocks the negative wealth effect is larger, which results in a stronger crowding out effect of Ricardian households expenditure. On the other hand, higher persistence of production taxes causes that Ricardian households invest and consume more, therefore, the GDP multiplier increases.

Table 1.4. Present value GDP multipliers for permanent shocks

GDP multipliers	Quarters				
	1	4	12	20	1000
Public consumption	0.92	0.76	0.60	0.52	-0.77
Public investment	0.91	0.75	0.57	0.49	0.04
Public transfers	0.25	0.18	0.04	-0.06	-1.28
Consumption tax	-0.40	-0.44	-0.45	-0.40	0.40
Labour tax	-0.46	-0.55	-0.63	-0.62	-0.07
Capital tax	-0.84	-1.31	-1.78	-1.93	-2.19

[30] For further discussion of fiscal shocks' persistence and their effect on GDP multipliers see Coenen *et al.* (2012) and Roeger and in 't Veld (2009).

1.6 Sensitivity Analysis

Each parameter in the model governs a particular channel through which fiscal policy shocks translate into the economy. Table (1.5) presents multipliers of the frictionless model where all frictions are turned off i.e. $\vartheta = h = \omega = \omega_w = \phi = 1/\kappa = 0$. The values differ significantly from those in Section (1.4). As can be noted, apart from the labour tax multiplier, all the multipliers deteriorate.[31] Below we analyse the reasons behind it and discuss the sensitivity of GDP multipliers to the key six parameters of the model. We turn on each parameter at a time in a frictionless model and examine the effect on fiscal policy multiplier. Each figure in Appendix $(1.C)$ illustrates the sensitivity of an 'impact' multiplier and a 5 year present value cumulative multiplier.

Table 1.5. Present value GDP multipliers for a frictionless model

GDP multipliers	Quarters				
	1	4	12	20	1000
Public consumption	0.31	0.26	0.12	-0.05	-2.56
Public investment	0.07	-0.04	0.32	0.54	-1.41
Public transfers	-0.04	-0.08	-0.19	-0.31	-1.32
Consumption tax	-0.18	-0.10	0.16	0.40	1.73
Capital tax	-0.24	-0.25	-0.25	-0.22	0.16
Labour tax	-1.41	-1.50	-1.77	-2.02	-2.87

The share of non-Ricardian households (ϑ)

Fig. (1.5) presents a sensitivity analysis for ϑ. In the model economy the share of non-Ricardian households determines the behaviour of total consumption, which takes the form

[31] The public spending multiplier yields similar values to the model in Bai et al. (2011); see Dydra and Rios-Rull (2012)

of Ricadian households' consumption for $\vartheta = 0$, and that of non-Ricardian households for $\vartheta = 1$. As we increase the value of ϑ, and when the consumption response of rule-of-thumb households is higher than that of forward-looking households, the total consumption multiplier and subsequently the GDP multiplier increase. Subsequently as we increase the parameter, the impact GDP multiplier increases for all the fiscal instruments.

The habit formation (h)

An increase in the level of habit formation (see Fig. 1.6) has two implications for the model. First, the weight on the past consumption increases, and second, the elasticity of consumption with respect to the real interest rate decreases. As a result, impulse responses take a hump-shaped form (for further discussion see Fuhrer, 2000, Woodford, 2003, and Christiano *et al.*, 2005). Subsequently, higher levels of habit formation imply lower short-term and higher medium and long-term multipliers of consumption in absolute terms. Therefore, looking at the impact multipliers, in the case of a shock resulting in a positive response of Ricardian households' consumption (a decrease in consumption and labour tax rates and transfer shock), as the value of the parameter increases, the short term responses of consumption become smaller, which results in a lower short term GDP multiplier. However, in the case of shocks resulting in a negative response of consumption (government consumption, investment, and capital tax), as h increases, in the short term the response of consumption decreases in absolute terms, which implies a higher GDP multiplier.

Price stickiness (ω)

The price stickiness parameter governs the size of the elasticity of inflation with respect to marginal costs. As Dixon and Rankin (1994) indicate price stickiness makes any policy that influences aggregate demand effective. When the price stickiness parameter is increased (see Fig. 1.7), the transmission mechanism from marginal costs to inflation is abated (the elasticity of inflation with respect to the marginal costs decreases). Consequently, for shocks resulting in the marginal cost increase, as the parameter's value increases, an increase in inflation becomes smaller, which implies a lower response of the nominal interest rate and, subsequently a less contractionary effect on the economy. Hence, for shocks which induce a general increase in marginal costs within the 5 years period (consumption and capital tax, and public consumption, investment), higher levels of price stickiness result in a higher GDP multiplier (as the transmission mechanism to inflation is abated). However, in the case of the shocks which result in a decrease in marginal costs (labour tax), higher levels of price stickiness result in lower GDP multipliers. In the case of transfers, the 5 years present value multiplier remains unchanged.

Wage stickiness (ω_w)

The wage stickiness parameter governs the size of the elasticity of the wage rate with respect to the wage markup (the difference between the real wage and the wage that would prevail under the flexible wage setup). A higher level of parameter indicates lower elasticity; therefore, wage becomes less dependent on the markup. Changes in the wage are passed on primarily through the labour income (mainly implies changes in the consumption

of the rule-of-thumb customers) and through the inflation channel (mainly implies changes in the interest rate sensitive consumption and the investment of optimising households).

In the longer term, for the capital tax cut and the positive transfer shock, higher level of wage stickiness magnifies the multiplier effect, whereas for the remaining fiscal instruments higher levels of wage stickiness tends to lower multipliers as observed in Fig. 1.8.

Investment adjustment cost (ϕ)

The investment adjustment cost parameter governs the size of the elasticity of investment with respect to the Tobin's Q. The higher the parameter, the lower the elasticity, therefore investment becomes less dependent on the price of capital. Consequently, for shocks resulting in a positive response of price of capital, (capital and labour taxes), higher values of the parameter decrease the the response of investment, leading subsequently to lower multipliers. On the other hand, for shocks resulting in a negative response of Tobin's Q (consumption taxes, government consumption, investment and transfers), higher values of the parameter decrease in absolute terms the response of investment, leading subsequently to higher multipliers as can be seen in Fig. (1.9).

Capital utilisation rate $(1/\kappa)$

The capital utilisation adjustment parameter (κ) determines the elasticity of utilisation with respect to the rental rate of capital net of capital taxes. As $\kappa \to \infty$ the model is characterised by the full capital utilisation, whereas when $\kappa \to 0$ then capital utilisation becomes

more responsive to changes in rental rate. Presence of capital utilisation in the model dampens the fluctuations in marginal costs as fluctuations in rental rate of capital are smaller i.e. the lower the parameter κ the lower the deviation of marginal cost in absolute terms and subsequently inflation and real rate, also in absolute terms. Subsequently in the longer horizon, as the parameter κ decreases ($1/\kappa$ increases along the horizontal axis in Fig. 1.10) the multiplier of government spending, investment and capital and consumption taxes increases, whereas the multiplier of transfers and labour taxes decreases.

1.7 Variance Decomposition

Table 1.6. Variance decomposition in percents

shock	t=4			t=20			t=100		
	Y	C	I	Y	C	I	Y	C	I
ε^G	0.98	0.04	0.04	0.42	0.17	0.19	0.61	0.40	0.29
ε^{IG}	10.01	0.09	0.00	3.42	0.05	0.00	2.46	0.04	0.00
ε^{TR}	0.24	0.80	0.01	0.20	0.44	0.07	0.62	0.49	0.15
ε^{τ^c}	0.88	2.59	0.02	0.36	1.48	0.03	0.31	1.03	0.04
ε^{τ^k}	1.89	0.34	0.24	0.96	0.29	0.27	0.71	0.20	0.26
ε^{τ^l}	0.36	0.85	0.01	0.16	0.51	0.04	0.15	0.35	0.05

The contribution of fiscal policy shocks to a forecast error variance of GDP, consumption, and investment is presented by model horizons in Table (1.6). In the short term, all fiscal shocks account for approximately 14 per cent of total variance. The most significant contributions are from the government investment shock (10 per cent), and the capital tax shock (1.89 per cent). The government consumption shock accounts for about 1 per cent of total variance. When the horizon is extended, the contribution of fiscal policy shocks decreases: in the medium term it explains 6 per cent of total variance, whereas in the long-term hori-

zon 5 per cent of the variance. Demand and supply sice shocks, mainly to preferences of households and technologies of firms account for the above 70 percent of the variance in output.

1.8 Extension

Public goods and investment generate positive externality to both households and firms. It is not a surprising result that public consumption yields lower multipliers than public investment due to the supply side effects public investment. Assuming that the households do not receive utility from the public goods is not realistic. Therefore we relax this assumptions here by extending the benchmark setup by assuming that government consumption is no longer treated as a wasteful spending by households but enters in their non-separable utility function which now becomes:

$$E_0 \sum_{t=0}^{\infty} \varepsilon_t^B \beta^t \left(\frac{1}{1-\sigma_c} X_t^{(1-\sigma_c)} - \frac{1}{1+\sigma_L} \left(L_t^{\iota}\right)^{1+\sigma_L} \right) \tag{1.40}$$

$$X_t = \left[a \left(C_t - H_{c,t}\right)^{\frac{s-1}{s}} + (1-a) \left(G_t - H_{g,t}\right)^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \tag{1.41}$$

where X_t denotes effective consumption, $H_c = h_c C_{t-1}$ and $H_g = h_g G_{t-1}$. This specification similar to that of Bouakez and Rebei (2007) and Leeper *et al*. (2009). The implementation of public consumption into the utility as in equation (1.40) results in changes in two equations; first is the consumption equation of Ricardian households (1.42), and the second is the equation determining the wage markup (1.43)

$$\begin{aligned}
\hat{C}_t &= n * \left\{ \begin{array}{c} s\left[\hat{R}_t - E_t\hat{\pi}_{c,t+1} + E_t\hat{\varepsilon}^B_{t+1} - \hat{\varepsilon}^B_t\right] \\ +\frac{(1-f)(1-s\sigma_c)}{1-h_g}\left[E_t\hat{G}_{t+1} - (1+h_g)\hat{G}_t + h_g\hat{G}_{t-1}\right] \end{array} \right\} \\
&\quad + \frac{1}{1+h_c}E_t\hat{C}r_{t+1} + \frac{h_c}{1+h_c}\hat{C}r_{t-1} \qquad (1.42) \\
X^w_t &= \hat{w}_t - \sigma_l\hat{L}_t - \frac{1-(1-s\sigma_c)f}{s(1-h)}\left[\hat{C}_t + h\hat{C}_{t-1}\right] \\
&\quad + \frac{(1-f)(1-s\sigma_c)}{s(1-h)}\left[\hat{G}_t - h\hat{G}_{t-1}\right] - \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t - \frac{\tau^l}{1-\tau^l}\hat{\tau}^l_t \qquad (1.43)
\end{aligned}$$

where:

$$\begin{aligned}
n &= \left[\frac{1}{(1-s\sigma_c)f-1}\right]\frac{(1-h_c)}{(1+h_c)} < 0 \\
f &= \frac{a\left(C(1-h_c)\right)^{\frac{s-1}{s}}}{a\left(C(1-h_c)\right)^{\frac{s-1}{s}} + (1-a)\left(G(1-h_g)\right)^{\frac{s-1}{s}}} \in \ <0,1> \text{ and,} \\
\hat{\pi}_{c,t+1} &= \hat{\pi}_{t+1} - \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{t+1}
\end{aligned}$$

The elasticity of substitution between public and private consumption (s) takes values from 0 (perfect complements) to ∞ (perfect substitutes). Equation (1.42) implies that as the elasticity of substitution increases $(s \to \infty)$ the weight on the *ex ante* real interest rate in the consumption units increases, whereas the weight on the government consumption $\left(\frac{(1-f)(1-s\sigma_c)}{(1-h_g)}\right)$ decreases. Therefore, for higher levels of the elasticity the *ex ante* real exchange rate remains a dominant force in determination of Ricardian households' consumption. When $s > \frac{1}{\sigma_c}$ private and public spending are Edgeworth substitutes and increase in public consumption leads to an even stronger decrease in private expenditure of Ricardian households. Public and private consumption are Edgeworth complements when $s < \frac{1}{\sigma_c}$, and in that case increase in public consumption have positive impact on private

consumption through the analysed channel. Similar effects can be noted in the case of the wage markup. When $s > \frac{1}{\sigma_c}$ any increase in public spending has a negative effect on the wage. On the other hand, small values of s imply a positive effect of public consumption on the wage, therefore, an increase in the wage is associated with an increase in the consumption expenditure. Interestingly, strong complementarity of public and private consumption can imply an increase in the wage rate and consumption.

The second parameter determining the effects of public consumption expenditure on private is the share of public consumption in total consumption $0 < (1-a) < 1$. When $a = 0 \Rightarrow f = 0$ total consumption of Ricardian households is public and in contrast when $a = 1 \Rightarrow f = 1$ total consumption is private. Clearly the lower the level of a the higher the effects of public on private consumption, as $\frac{(1-f)(1-s\sigma_c)}{(1-h_g)}$ increases.

Finally, positive effect of public on private consumption can be achieved by assuming high value of habit formation in government consumption (h_g) which results in a significant weight on public consumption variables (see equation 1.42 and 1.43).

Turning to the estimation of the parameters related to the new utility functional, we set the prior mean for the share of public good in consumption (a) to 0.75, a level calibrated by Coenen *et al.* (2013) for the euro area. We set the beta prior mean for habit formation (h_g) to 0.7 and a standard deviation of 0.1. For the elasticity of substitution (s) we select gamma distribution prior with a mean of 1 and a standard deviation of 0.5. The results of estimation are presented in Appendix $(1.C)$. The habit formation parameter is estimated to 0.65, the elasticity of substitution to 1.15, and the share parameter to 0.76. Ta-

ble (1.7) shows present value GDP multipliers for the estimated model with government consumption in utility.

Table 1.7. Present value GDP multipliers

GDP multipliers	Quarters				
	1	4	12	20	1000
Public consumption	0.96	0.81	0.60	0.44	-1.12
Public investment	1.07	1.05	1.04	1.03	1.10
Public transfers	0.27	0.21	0.06	-0.08	-1.12
Consumption tax	-0.53	-0.68	-0.70	-0.57	0.18
Labour tax	-0.33	-0.36	-0.43	-0.41	0.07
Capital tax	-0.58	-0.85	-1.08	-1.06	-0.76

Multipliers do not differ much on the benchmark calibration implying that model with non-separable non-wasteful public consumption in utility does not introduce significant changes in contrast to results of Coenen *et al.* (2013) for the euro area. In fact, the estimates imply that the GDP multiplier for the public consumption shock is lower than in the case when public consumption does not enter utility because $(1 - s\sigma_c) < 0$. For the remaining instruments the effect is negligible and naturally improving multipliers.

1.9 Conclusions

This paper analyses the implications of fiscal policy in the UK economy in an estimated DSGE model.[32] The parameter estimates indicate that public investment, consumption and capital taxes play the most important role in controlling for the government debt over

[32] The model in the paper can be extended in two possible ways. First, an important extension is to include an open economy, and a second to have search and matching frictions and transition probabilities between various states in the labour market to see how these extensions influence the fiscal policy.

the sample period. Additionally capital tax rates and government investment characterise significant procyclical response to GDP. In contrast, we find that the response of labour taxes to aggregate output and debt is relatively modest.

GDP multipliers, vary among six fiscal policy shocks from short to long horizons. Whie the government consumption and investment are the most stimulating fiscal instruments in the short term (the impact multiplier totals 0.99 and 1.07 respectively), the capital tax cut and the public investment shock result in the highest multipliers in the medium and longer horizons (the present value cumulative 5 year multiplier totals -1.05 and 1.02 respectively). The government transfers yield relatively lower multipliers when compared to the remaining fiscal policy instruments, mainly as a result of the low share of non-Ricardian households.

Additionally, we estimate a model with non-separable non-wasteful government consumption in the utility function. In contrast to findings of Bourakez and Rebei (2007) for the US, and Coenen *et al.* (2013) for the euro area, our results indicate that the alternative setup does not have significant implications for results. Private and public consumption are Edgeworth substitutes, therefore, the estimates imply that the GDP multiplier for the public consumption shock is slightly lower than in the case when public consumption does not enter utility.

Regarding the role of nominal and real frictions, we show that those tend to influence more labour and capital tax and less government expenditure multipliers.

Finally, forecast error variance decomposition reveals that fiscal policy shocks, public spending, investment and transfers in the spending side and taxes on capital and labour

income and consumption on the revenue side, represent approximately 14 per cent of variance in the short term and for approximately 5 percent of variance in the long run.

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1.A Log-Linearised System of Equations

This section presents a log-linearised system of equations. The capital letters without subscript t denote the steady state values, whereas a hat over a variable denotes its log-deviations from the steady state.

1.A.1 Households:

$$\hat{C}_t = \frac{E_t\hat{C}_{t+1}}{1+h} + \frac{h\hat{C}_{t-1}}{1+h} - \frac{1}{\sigma_c}\frac{1-h}{1+h}E_t\left[R_t - \hat{\pi}_{t+1} + \frac{\tau^c}{1+\tau^c}\left(\hat{\tau}^c_t - E_t\hat{\tau}^c_{t+1}\right) + \hat{\varepsilon}^B_{t+1} - \hat{\varepsilon}^B_t\right] \tag{44}$$

$$\hat{Q}_t = -\hat{R}_t + E_t\hat{\pi}_{t+1} + \frac{1}{1-\delta+(1-\tau^k)r_k}E_t\left[(1-\delta)\hat{Q}_{t+1} + r_k\left(1-\tau^k\right)\left(\hat{r}_{k,t+1} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_{t+1}\right)\right] \tag{45}$$

$$\hat{I}_t = \frac{\hat{Q}_t}{\phi(1+\beta)} + \frac{\hat{I}_{t-1}}{1+\beta} + \frac{\beta E_t\hat{I}_{t+1}}{1+\beta} + \frac{1}{1+\beta}E_t\left(\beta\hat{\varepsilon}^I_{t+1} - \hat{\varepsilon}^I_t\right) \tag{46}$$

$$\hat{u}_t = \frac{1}{\kappa}\left[\hat{r}_{k,t} - \frac{\tau^k}{\left(1-\tau^k\right)}\hat{\tau}^k_t\right] \tag{47}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t \tag{48}$$

$$\begin{aligned}\hat{C}_{nr,t} = {} & \left(1-\tau^l\right)\frac{wL}{(1+\tau^c)C_{nr}} * \left(\hat{w}^\tau_t + \hat{L}_t - \frac{\tau^l}{1-\tau^l}\hat{\tau}^l_t\right) \\ & + \frac{TR}{(1+\tau^c)C_{nr}}\widehat{TR}_t - \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t\end{aligned} \tag{49}$$

$$\begin{aligned}\hat{w}_t = {} & \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \\ & + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\varpi^w)(1-\varpi^w)}{\left(1+\frac{(1+\varsigma_w)\sigma_l}{\varsigma_w}\right)\varpi^w}\left(X^w_t + \varepsilon^w_t\right)\end{aligned} \tag{50}$$

$$X^w_t = \hat{w}_t - \sigma_l\hat{L}_t - \frac{1}{1-b}\left(\hat{C}_t - b\hat{C}_{t-1}\right) - \frac{\tau^l}{1+\tau^l}\hat{\tau}^l_t + \hat{\varepsilon}^L_t - \frac{\tau^c}{1+\tau^c}\tau^c_t \tag{51}$$

$$\hat{\pi}_{c,t} = \hat{\pi}_t + \frac{\tau^c}{1+\tau^c}\left(\hat{\tau}^c_t - \hat{\tau}^c_{t-1}\right) \tag{52}$$

1.A.2 Firms:

$$\hat{Y}_t = \varphi_y \left[\hat{\varepsilon}_t^A + \alpha \hat{K}_{t-1} + \alpha \hat{u}_t + (1-\alpha) \hat{L}_t + \sigma_g \hat{K}_{g,t-1} \right] \tag{53}$$

$$\hat{L}_t = \hat{u}_t + \hat{r}_t + \hat{K}_{t-1} - \hat{w}_t \tag{54}$$

$$\widehat{mc}_t = (1-\alpha) \hat{w}_t + \alpha \hat{r}_t - \hat{\varepsilon}_t^A - \sigma_g \hat{K}_{g,t-1} \tag{55}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{t-1} + \frac{(1-\beta\varpi)(1-\varpi)}{\varpi(1+\beta\gamma_p)} (\widehat{mc}_t + \hat{\varepsilon}_t^p) \tag{56}$$

1.A.3 Government:

$$\hat{G}_{rev,t} = \tau^c \frac{C_T}{Y} \left(\hat{\tau}_t^c + \hat{C}_{T,t} \right) + \tau^l \frac{wL}{Y} \left(\hat{\tau}_t^l + \hat{w}_t + \hat{L}_t \right) + \tau^k \frac{r_k K}{Y} \left(\hat{\tau}_t^k + \hat{r}_{k,t} + \hat{u}_t + \hat{K}_{t-1} \right) \tag{57}$$

$$\hat{G}_{rev,t} = R \frac{b}{Y} \left(\hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) - \frac{b}{Y} \hat{b}_t + \frac{G}{Y} \hat{G}_t + \frac{IG}{Y} \widehat{IG}_t + \frac{TR}{Y} \widehat{TR}_t \tag{58}$$

$$\hat{K}_{g,t} = (1-\delta) \hat{K}_{g,t-1} + \delta \widehat{IG}_t \tag{59}$$

1.A.4 General equilibrium conditions:

$$\hat{Y}_t = \frac{C_T}{Y} \hat{C}_{T,t} + \frac{I}{Y} \hat{I}_t + \frac{G}{Y} \hat{G}_t + \frac{IG}{Y} \widehat{IG}_t + (1-\tau^k) \frac{r_k K}{Y} \times \hat{u}_t \tag{60}$$

$$C_T \hat{C}_{T,t} = (1-\lambda) C \hat{C}_t + \lambda C_{nr} \hat{C}_{nr,t} \tag{61}$$

The above equations plus the equations specifying fiscal and monetary policy in the text (equations $1.27 - 1.33$, which are already in the log-linear form) comprise the system of equations which is subsequently solved and estimated.

1.B Data Description and Prior and Posterior Distribution

In order to estimate the model, twelve data series are used: GDP, consumption, investment, wages, inflation, hours, government consumption, government investment, effective consumption, labour and capital tax rates, and transfers. The data are from Office for National Statistics webpage (ONS), and cover period from 1987:Q1 to 2011:Q1. While some of the data series can be obtained directly from the ONS, other including effective tax rates, and transfers were calculated closely following methods such these of Mendoza *et al.* (1994), Jones (2002), and Leeper *et al.* (2010). To derive the effective tax rates on labour, $\left(\tau^{l}\right)$, and capital, $\left(\tau^{k}\right)$, firstly the average tax rate on income, (τ^{i}) is calculated . The reason for it being that the ONS does not distinguish between labour and capital income taxes.

The average income tax rate:

$$\tau^{i} = \frac{IT + OCT}{W + PI + GOS + MI} \tag{1.62}$$

where IT denotes income taxes paid by households (HHLDs) and non-profit institutions serving households (NPISH) [QWMQ]; OCT stands for other current taxes paid by HHLDs & NPISH [NVCO]; W denotes wages and salaries of HHLDs & NPISH [QWLW]; PI denotes property income of HHLDs & NPISH [QWME]; GOS denotes gross operat-

ing surplus of HHLDs & NPISH [QWLS];MI stands for gross mixed income of HHLDs & NPISH [QWLT];

The effective labour tax rate:

$$\tau^l = \frac{(W + 0.5 * MI) * \tau^i + ESC}{W + ESC} \tag{1.63}$$

where ESC stands for employers social contributions of HHLDs & NPISH [QWLX]; (denominator comprises compensations of employees)

The effective capital tax rate:

$$\tau^k = \frac{(PI + GOS + 0.5 * MI) * \tau^i + (ITG + OCTG - IT - OCT) + CT + OTP}{OS} \tag{1.64}$$

where ITG stands for current taxes on income received by general government (GG) [NMZJ]; $OCTG$ stands for other current taxes received by the GG [NVCM]; CT denotes capital taxes of HHLDs & NPISH [NSSO]; OTP stands for other taxes on production [NMYD];OCT stands for other current taxes OS stands for gross operating surplus of the whole economy;

The effective consumption tax rate:

$$\tau^c = \frac{TTP}{C - TTP} \tag{1.65}$$

where TTP stands for total taxes on products (GG) [NVCC]; C stands for final consumption expenditure of HHLDs & NPISH [NSSG].

Transfers:

$$TR^M = TR_t + \left[TC_t^M + TL_t^M + TK_t^M - T\,\mathrm{Re}\,s_t\right] \tag{1.66}$$

where $\left[TC_t^M + TL_t^M + TK_{t-1}^M - T\,\mathrm{Re}\,s_t\right]$ is a tax residual. TR_t represents the sum of: social benefits other than social transfers in kind (GG) [NNAD]; other current transfers (GG) [NNAN]; subsidies (GG) [NMRL]; total capital transfers (GG) [NNBC]; $TC_t^M + TL_t^M + TK_t^M$ denotes total tax revenue. $T\,\mathrm{Re}\,s_t$ represents total resources and totals sum of gross operating surplus (GG) [NMXV]; total taxes on production and import received (GG) [NMYE]; other taxes on production (GG) [NMYD]; property income received (GG) [NMYU]; current taxes on income and wealth (GG) [NMZL]; total social contributions (GG) [NMZR]; other current transfers (GG) [NNAA]; total capital transfers receivable (GG) [NNAY]

Government investment: government gross fixed capital formation [NNBF];

Government consumption: total final consumption expenditure by general government [NMRK];

Gross domestic product government investment+goverment consumption+private consumption+private investment;

Private investment: total gross fixed capital formation [NPQS]- government investment;

Consumption: final consumption by households [ABJQ]+ final consumption by non-profit institutions [HAYE]

Wages: compensation of employees [DTWM];

Hours: Actual hours worked [Labour Force Survey];

Inflation: The gross inflation is defined using the consumption deflator of at market prices deflator of consumption and NPISH [UKEA]

Definition of variables:

$$X = ln\left(\frac{x}{pop}\right) * 100 \tag{1.67}$$

where x = government investment, government consumption, transfers, GDP, private consumption, private investment, wages, hours; and pop is defined as all persons aged 16 and over table A02 [Labour Force Survey Summary]. All data are demeaned with their linear trend.

1.C Sensitivity Figures, Priors and Posteriors

Fig. 1.5. Sensitivity of GDP multipliers to the share of non-Ricardian households

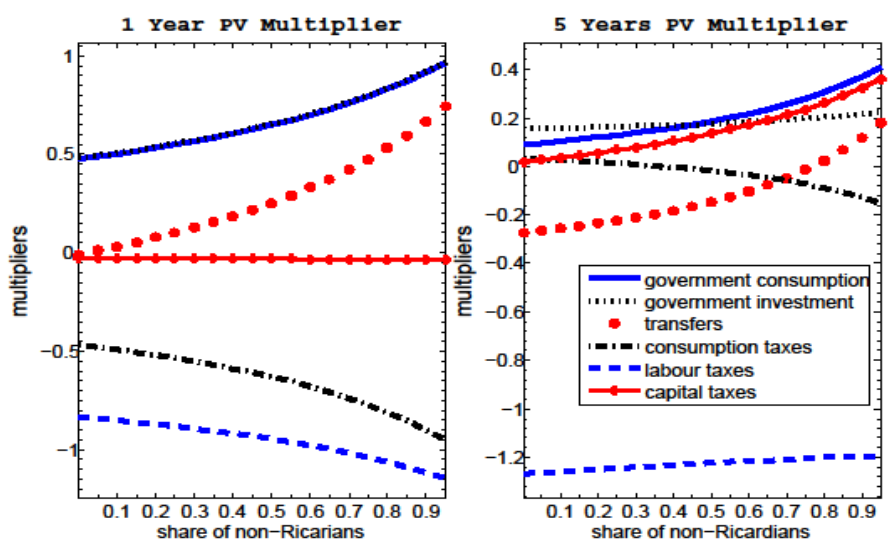

Fig. 1.6. Sensitivity of GDP multipliers to habit formation

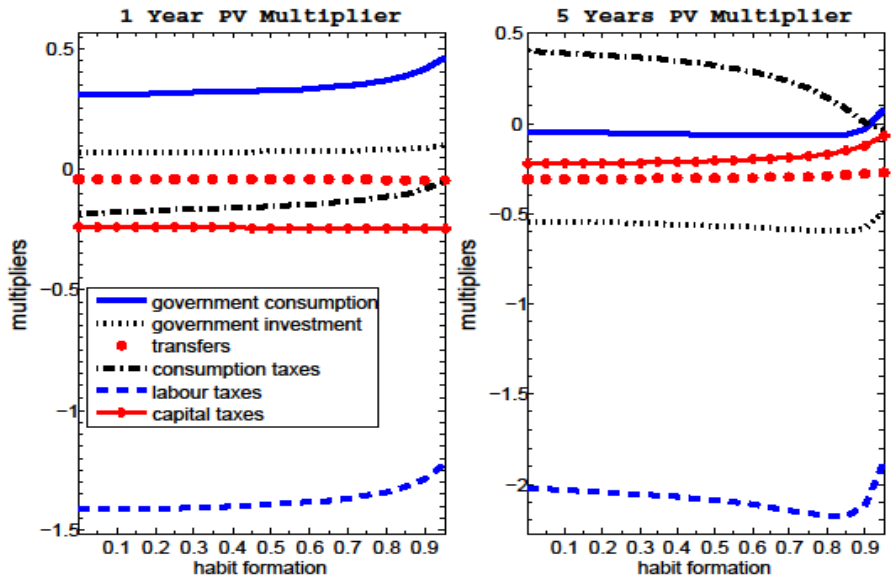

Fig. 1.7. Sensitivity of GDP multipliers to price stickiness

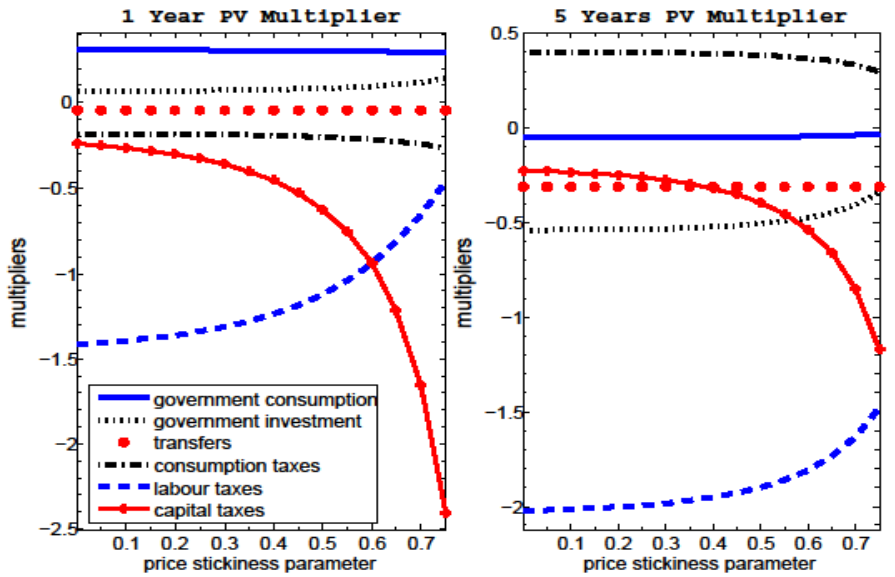

Fig. 1.8. Sensitivity of GDP multipliers to wage stickiness

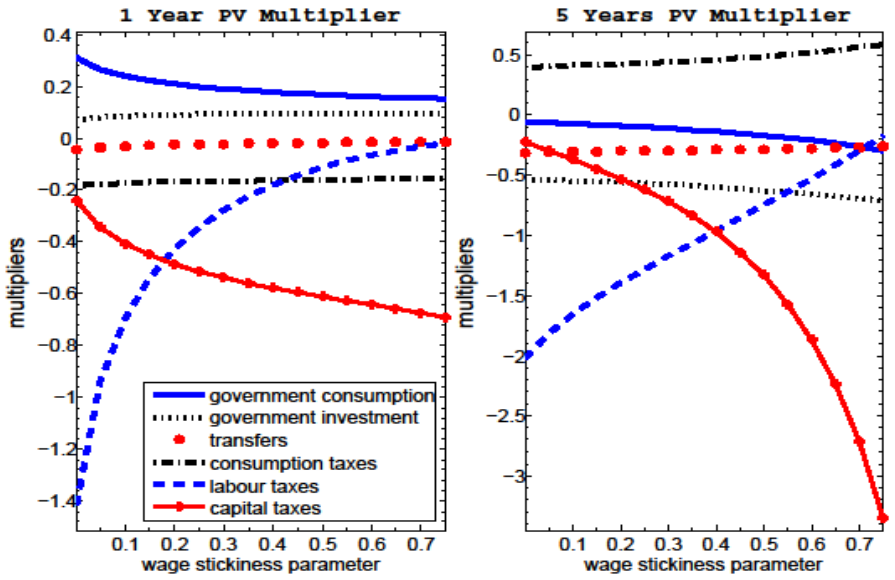

Fig. 1.9. Sensitivity of GDP multipliers to investment adjustment cost

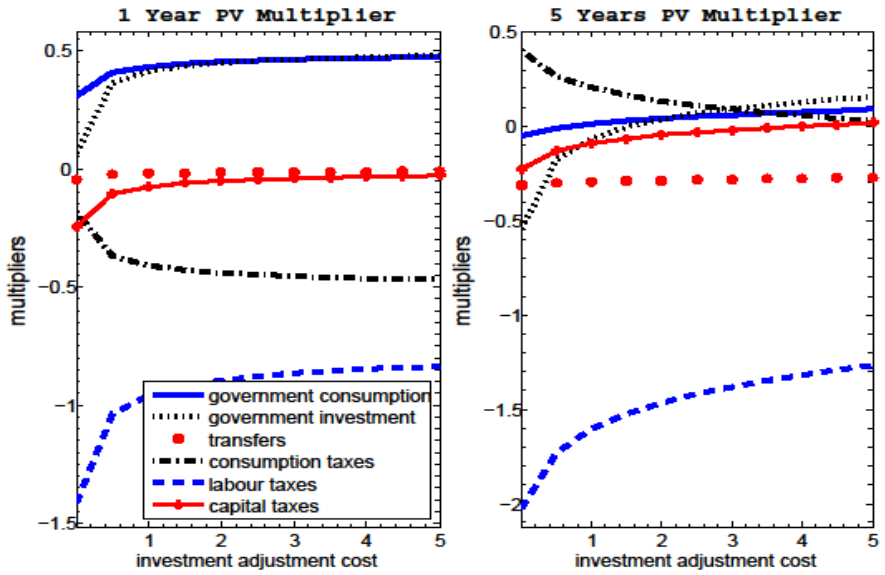

Fig. 1.10. Sensitivity of GDP multipliers to capital utilisation

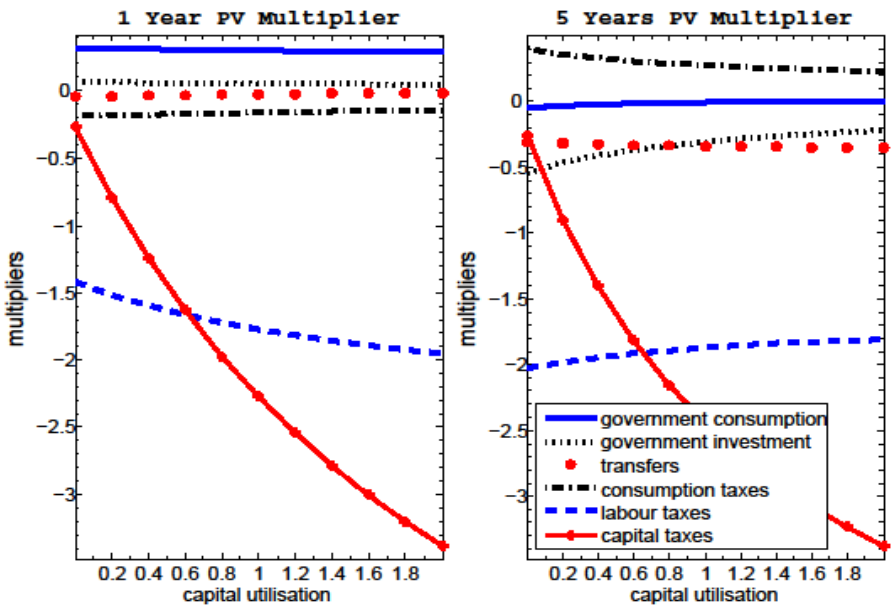

Table 1.8. Priors and posteriors

	Prior distribution			Est. max. post.		Post. distribution MH			Post. distribution MH		
Parameter's name	type	mean	st. err.	mode	st. err.	mean	conf. interval		mean	conf. interval	
				without gov. cons. in utility					gov. cons. in utility		
invest. adj. cost	normal	4.0	1.5	6.205	1.125	6.405	4.589	8.209	6.434	4.576	8.269
capital util. cost	normal	0.8	0.2	0.805	0.187	0.813	0.501	1.113	0.811	0.513	1.119
inv. elast. of labour	normal	1.0	0.35	1.129	0.316	0.202	0.699	1.704	1.216	0.722	1.710
CRRA coefficient	normal	0.66	0.2	0.947	0.156	0.930	0.676	1.199	0.918	0.645	1.190
price index	beta	0.3	0.15	0.034	0.045	0.084	0.003	0.170	0.090	0.003	0.185
wage index	beta	0.3	0.15	0.243	0.114	0.264	0.083	0.438	0.266	0.086	0.442
calvo prices	beta	0.5	0.1	0.590	0.048	0.590	0.500	0.682	0.589	0.500	0.683
calvo wages	beta	0.5	0.1	0.548	0.058	0.557	0.450	0.658	0.557	0.456	0.657
habit formation	beta	0.7	0.1	0.618	0.070	0.662	0.518	0.815	0.653	0.526	0.792
fixed cost	normal	1.15	0.1	1.666	0.078	1.655	1.557	1.782	1.652	1.548	1.778
sh. of non-Ricardians	beta	0.3	0.1	0.291	0.291	0.327	0.207	0.451	0.329	0.211	0.452
elas. of substitution	gamma	1.00	0.5						1.145	0.587	1.753
gov. cons. share parameter	beta	0.75	0.1						0.761	0.608	0.917
gov. cons. habit	beta	0.70	0.1						0.683	0.523	0.849

Table 1.9. Priors and posteriors

	Prior distribution			Est. max. post.		Post. distribution MH			Post. distribution MH		
Parameter's name	type	mean	st. err.	mode	st. err.	mean	conf. interval		mean	conf. interval	
				without gov. cons. in utility					gov. cons. in utility		
σ productivity	inv. gamma	0.01	inf	0.007	0.001	0.007	0.006	0.008	0.008	0.007	0.008
σ preferences	inv. gamma	0.01	inf	0.041	0.007	0.053	0.033	0.075	0.051	0.033	0.070
σ wage markup	inv. gamma	0.01	inf	0.369	0.131	0.456	0.187	0.713	0.455	0.186	0.714
σ price markup	inv. gamma	0.01	inf	0.014	0.002	0.016	0.011	0.021	0.016	0.011	0.021
σ cons. tax	inv. gamma	0.01	inf	0.035	0.003	0.036	0.032	0.041	0.036	0.032	0.041
σ capital tax	inv. gamma	0.01	inf	0.051	0.004	0.053	0.046	0.059	0.053	0.046	0.059
σ labour tax	inv. gamma	0.01	inf	0.022	0.002	0.023	0.020	0.025	0.023	0.020	0.025
σ investment	inv. gamma	0.10	inf	0.066	0.006	0.069	0.059	0.078	0.069	0.059	0.078
σ transfers	inv. gamma	0.01	inf	0.029	0.002	0.029	0.026	0.033	0.029	0.026	0.033
σ gov. inv.	inv. gamma	0.10	inf	0.535	0.039	0.544	0.480	0.609	0.546	0.480	0.612
σ mon. policy	inv. gamma	0.01	inf	0.007	0.001	0.007	0.006	0.008	0.007	0.006	0.008
σ gov. cons.	inv. gamma	0.01	inf	0.013	0.001	0.013	0.011	0.015	0.013	0.011	0.015
AR(1) tfp.	beta	0.80	0.1	0.971	0.021	0.943	0.889	0.993	0.944	0.890	0.993
AR(1) pref.	beta	0.80	0.1	0.760	0.057	0.705	0.557	0.852	0.717	0.591	0.849
AR(1) inv.	beta	0.80	0.1	0.245	0.064	0.263	0.157	0.368	0.265	0.159	0.367
AR(1) wage	beta	0.80	0.1	0.374	0.073	0.380	0.263	0.499	0.382	0.258	0.496
AR(1) price	beta	0.80	0.1	0.963	0.027	0.906	0.801	0.995	0.901	0.792	0.992
AR(1) gov. cons.	beta	0.80	0.1	0.933	0.028	0.925	0.880	0.972	0.926	0.882	0.973
AR(1) gov. inv.	beta	0.80	0.1	0.302	0.068	0.311	0.200	0.420	0.313	0.203	0.426
AR(1) transfers	beta	0.80	0.1	0.880	0.049	0.876	0.804	0.954	0.875	0.802	0.953
AR(1) capital tax	beta	0.80	0.1	0.790	0.054	0.786	0.701	0.874	0.785	0.700	0.872
AR(1) labour tax	beta	0.80	0.1	0.742	0.080	0.775	0.655	0.895	0.775	0.656	0.896
AR(1) cons. tax	beta	0.80	0.1	0.742	0.070	0.752	0.640	0.867	0.748	0.634	0.861
gov. cons. resp. to. gdp	normal	0.5	0.5	0.227	0.107	0.246	0.068	0.422	0.257	0.060	0.446
gov. inv. resp. to gdp	normal	0.5	0.5	0.922	0.482	0.917	0.121	1.688	0.905	0.122	1.703
trans. resp. to gdp	normal	0.5	0.5	0.258	0.210	0.294	-0.055	0.647	0.284	-0.062	0.638
cons. tax resp. to gdp	normal	0.5	0.5	0.146	0.241	0.155	-0.250	0.561	0.160	-0.248	0.561
cap. tax resp. to gdp	normal	0.5	0.5	0.611	0.292	0.629	0.150	1.123	0.621	0.130	1.114
lab. tax resp. to gdp	normal	0.5	0.5	0.028	0.153	0.085	-0.196	0.355	0.092	-0.176	0.374
gov. con. resp. to debt	normal	0.2	0.1	0.126	0.098	0.156	0.059	0.251	0.157	0.060	0.257
gov. inv. resp. to debt	normal	0.2	0.1	0.195	0.085	0.201	0.020	0.363	0.197	0.034	0.354
trans. resp. to debt	normal	0.2	0.1	0.065	0.063	0.078	-0.056	0.211	0.077	-0.057	0.213
con. tax resp. to debt	normal	0.2	0.1	0.074	0.070	0.100	-0.016	0.217	0.102	-0.019	0.217
cap. tax resp. to debt	normal	0.2	0.1	0.171	0.090	0.177	0.032	0.321	0.173	0.027	0.318
lab. tax resp. to debt	normal	0.2	0.1	0.013	0.063	0.055	-0.071	0.185	0.053	-0.070	0.182
AR (1) nom. int. rate	beta	0.70	0.1	0.707	0.032	0.719	0.667	0.774	0.721	0.669	0.771
inflation response	normal	1.50	0.1	1.642	0.092	1.635	1.484	1.787	1.636	1.482	1.785
output response	normal	0.125	0.1	0.198	0.090	0.214	0.067	0.359	0.218	0.067	0.367

1.D Dynare code

1.D.1 Dynare Code for the impulse responses of the model

```
var R mc tao_c tao_k tao_l c cr cnr q omega ro I w L y b kg g Ig trans pi pip k e_v e_tr e_ig
e_tc e_tk e_tl e_a e_l e_i e_pi e_n chi_w tao_c_inc tao_k_inc tao_l_inc;

    // R - nominal interest rate
    // mc - real marginal cost
    // tao_c - consumption tax rate
    // tao_k - capital tax rate
    // tao_l - labour tax rate
    // c - total consumption
    // cr - consumption of Ricardian households
    // cnr - ronsumption of non-Ricardian households
    // q - Tobin's Q
    // omega - capital utilisation rate
    // ro - return on capital
    // I - investment
    // w - wage
    // L - labour
    // y - output
    // b - bonds
    // kg - public capital
```

```
// g - government spending
// Ig - government investment
// trans - transfers
// pi - consumer price inflation
// pip - producer price inflation
// k - private capital
// e_v - public spending shock
// e_tr - transfers shock
// e_ig - government investment shock
// e_tc - consumption tax shock
// e_tk - capital tax shock
// e_tl - labour tax shock
// e_a - tfp shock
// e_l - wage push-up shock
// e_i - investment shock
// e_pi - producer price push-up shock
// e_n - preferences shock
// chi_w - monetary policy shock

varexo tc at nt lt pit wt it vt ig tr tk tl;

parameters fc std_vt omega_w gamma_w e_wi gamma_p sigma_c r_bar kappa beta
alfa delta omega_p fi bb rho b_pi b_y sigma_l rho_a sigma_g delta_g phi_g phi_ig rho_i
```

```
std_at std_lt std_it std_pit std_nt rho_n std_wt R_bar cy_bar Iy_bar gy_bar Igy_bar try_bar
wLy_bar rky_bar by_bar tao_cbar tao_kbar tao_lbar std_tc std_tl std_tk std_tr std_ig psi_g
psi_ig phi_tr psi_l phi_tc psi_tc phi_tk psi_tk phi_tl psi_tl sh ela_ll ela_lc ela_lk ela_g
ela_ig epa_tr ela_tr rho_l rho_p;

    //calibrated parameters

    delta=0.025; // depreciation rate of private capital
    delta_g=0.015; // depreciation rate of public capital
    e_wi=0.050; // wage markup
    beta=0.990; // discount rate
    alfa=0.300; // share of capital in production
    sigma_g=0.010; // the elasticity of output to public capital
    fi = 6.405 ; // capital investment adjustment cost
    kappa = 0.8133 ; // capital utilisation parameter
    sigma_l = 1.2023 ; // inverse elasticity of labour
    sigma_c = 0.9298 ; // CRRA
    gamma_p = 0.0838 ; // price indexation
    gamma_w = 0.2639 ; // wage indexation
    omega_p = 0.5897 ; // share of firms changing the price each period
    omega_w = 0.5566 ; // share of households changing the wage each period
    bb = 0.6623 ; // habit formation
    fc = 1.6549 ; // fixed cost
```

```
sh = 0.3266 ; // share of Impatient households

// monetary policy
rho = 0.7185 ;
b_pi = 1.6350 ;
b_y = 0.2140 ;

// shocks AR (1) coefficients
rho_a = 0.9432 ;
rho_n = 0.7047 ;
rho_i = 0.2633 ;
rho_l = 0.3804 ;
rho_p = 0.9055 ;
phi_g = 0.9246 ;
phi_ig = 0.3110 ;
phi_tr = 0.8760 ;
phi_tk = 0.7863 ;
phi_tl = 0.7746 ;
phi_tc = 0.7521 ;

// response to GDP
psi_g = 0.2457 ;
psi_ig = 0.9173 ;
psi_l = 0.2941 ;
```

```
psi_tc = 0.1552 ;

psi_tk = 0.6287 ;

psi_tl = 0.0847 ;

// response to debt

ela_g = 0.1562 ;

ela_ig = 0.2007 ;

ela_tr = 0.0776 ;

ela_ll = 0.0552 ;

ela_lc = 0.0998 ;

ela_lk = 0.1765 ;

epa_tr = 0.0000 ;

// standard deviations of shocks

std_at = 0.0074 ;

std_nt = 0.0527 ;

std_lt = 0.4556 ;

std_pit = 0.0159 ;

std_tc = 0.0796 ;

std_tk = 0.1149 ;

std_tl = 0.0493 ;

std_it = 0.0685 ;

std_tr = 0.0583 ;
```

```
std_ig = 0.5 ;

std_wt = 0.0069 ;

std_vt = 0.05 ;

//calibrated steady state values

tao_cbar=0.20; // consumption tax

tao_kbar=0.29; // capital tax

tao_lbar=0.29; // labour tax

gy_bar =0.20; // government spending/ GDP

Igy_bar =0.02; // government investment/GDP

by_bar =2.40; // debt to GDP ratio

R_bar=1/beta; // nominal interest rate

rky_bar=alfa; // capital income/GDP

wLy_bar=1-alfa; // labour income/GDP

r_bar=(1/(1-tao_kbar))*(1/beta-(1-delta)); // rental rate

Iy_bar=delta*rky_bar/r_bar; // Investment/GDP ratio

cy_bar=1-gy_bar-Iy_bar-Igy_bar; // consumption/GDP ratio

try_bar=tao_cbar*cy_bar+tao_lbar*wLy_bar+tao_kbar*rky_bar-gy_bar-Igy_bar

-R_bar*by_bar+by_bar; // transfers/GDP ratio

model(linear);

// 1. consumption of non-Ricardian households

(1+tao_cbar)*cy_bar*(cnr+(tao_cbar/(1+tao_cbar))*tao_c)=

wLy_bar*((1-tao_lbar)*(w+L)-tao_lbar*tao_l)+try_bar*trans;
```

```
// 2. consumption of Ricardian households
cr=((bb/(1+bb))*cr(-1))+((1/(1+bb))*cr(+1))-((1-bb)/((1+bb)*sigma_c))*(R)
+((1-bb)/((1+bb)*sigma_c))*pip(+1)+((1-bb)/((1+bb)*sigma_c))*(e_n-e_n(+1))
-((1-bb)/((1+bb)*sigma_c))*(tao_cbar/(1+tao_cbar))*(tao_c-tao_c(+1));

// First order conditions of Ricardian households
// 3. w.r.t capital
q=-R+pip(+1)+((1-delta)/(1-delta+(1-tao_kbar)*r_bar))*q(+1)+((1-tao_kbar)*
r_bar/(1-delta+(1-tao_kbar)*r_bar))*(ro(+1)-(tao_kbar/(1-tao_kbar))*tao_k(+1));

// 4. w.r.t investment
I=(1/(fi*(1+beta)))*q+(1/(1+beta))*I(-1)+(beta/(1+beta))*I(+1)-(1/((1+beta)))
*(beta*e_i(+1)-e_i);

// 5. w.r.t capital utilisation
omega=(1/kappa)*(ro-(tao_kbar/(1-tao_kbar))*tao_k);

// 6. wage equation
w=(beta/(1+beta))*w(+1)+(1/(1+beta))*w(-1)+(beta/(1+beta))*pip(+1)
-((1+beta*gamma_w)/(1+beta))*pip+(gamma_w/(1+beta))*pip(-1)
-((((1-omega_w)*(1-beta*omega_w))/((1+((1+e_wi)/(e_wi))*sigma_l)*omega_w))
*(1/(1+beta)))*(w-sigma_l*L-(sigma_c/(1-bb))*(cr-bb*cr(-1))-(tao_lbar/(1-tao_lbar))
*tao_l-(tao_cbar/(1+tao_cbar))*tao_c+e_l);

// 7. private capital accumulation equation
```

```
k=(1-delta)*k(-1)+delta*(I);

// 8. public capita accumulation equation

kg=(1-delta_g)*kg(-1)+delta_g*Ig;

// 9. output of firms

y=fc*(e_a+alfa*k(-1)+alfa*omega+(1-alfa)*L+sigma_g*kg(-1));

// 10. combination of first order conditions of firms

ro+omega+k(-1)=w+L;

// 11. real marginal costs

mc=-e_a+(1-alfa)*w+alfa*ro-sigma_g*kg(-1);

// 12. hybrid new-Keynesian Philips curve for producers price inflation

pip=(beta/(1+beta*gamma_p))*pip(+1)+(gamma_p/(1+beta*gamma_p))*pip(-1)

+((((1-omega_p)*(1-beta*omega_p))/(omega_p))*(1/(1+beta*gamma_p)))

*(mc+e_pi);

// 13. consumer price inflation

pi=pip+(tao_cbar/(1+tao_cbar))*(tao_c-tao_c(-1));

// 14. resource constraint

y=(cy_bar)*c+(Iy_bar)*I+(1-tao_kbar)*(rky_bar)*omega+gy_bar*g+Igy_bar*Ig;

// 15. total consumption

c=(1-sh)*cr+sh*cnr;
```

```
// 16. monetary policy rule:
R=rho*R(-1)+(1-rho)*b_y*(y-y(-1))+(1-rho)*b_pi*(pip)+chi_w;

// 17-22 fiscal policy rules:
g=-(psi_g*y+ela_g*b(-1))+e_v;
Ig=-(psi_ig*y+ela_ig*b(-1))+e_ig;
trans=-(psi_l*y+ela_tr*b(-1)+epa_tr*L)+e_tr;
tao_c=(ela_lc*(b(-1))+psi_tc*(y))-e_tc;
tao_k=(ela_lk*(b(-1))+psi_tk*(y))-e_tk;
tao_l=(ela_ll*(b(-1))+psi_tl*(y))-e_tl;

// 23. government budget constraint
g*gy_bar+Igy_bar*Ig+try_bar*trans+by_bar*R_bar*(b(-1)+R(-1)-pip)=
by_bar*b+tao_cbar*cy_bar*(c+tao_c)+tao_lbar*wLy_bar*(w+L+tao_l)
+tao_kbar*rky_bar*(ro+omega+k(-1)+tao_k);

// Tax incomes
tao_c_inc=(c+tao_c);
tao_k_inc=(ro+omega+k(-1)+tao_k);
tao_l_inc=(w+L+tao_l);

// shocks
e_tc=phi_tc*e_tc(-1)+tc;
e_tk=phi_tk*e_tk(-1)+tk;
e_tl=phi_tl*e_tl(-1)+tl;
```

```
e_tr=phi_tr*e_tr(-1)+tr;
e_ig=phi_ig*e_ig(-1)+ig;
e_a=rho_a*e_a(-1)+at;
e_i=rho_i*e_i(-1)+it;
e_l=rho_l*e_l(-1)+lt;
e_pi=rho_p*e_pi(-1)+pit;
e_n=rho_n*e_n(-1)+nt;
chi_w=wt;
e_v=phi_g*e_v(-1)+vt;
end;

steady;

check;

shocks;
var tc ; stderr std_tc;
var tk ; stderr std_tk;
var tl ; stderr std_tl;
var ig ; stderr std_ig;
var vt ; stderr std_vt;
var tr ; stderr std_tr;
var at ; stderr std_at;
var it ; stderr std_it;
```

```
var lt ; stderr std_lt;

var pit; stderr std_pit;

var nt ; stderr std_nt;

var wt ; stderr std_wt;

end;

stoch_simul(order=1,irf=20);

//estimation(datafile=datases3008,mh_nblocks=4,mh_replic=250000

,mh_jscale=0.31,lik_init=2,mode_compute=4) ;

+
```

Chapter 2
Credit Constraints, the Housing Market, and Fiscal Policy

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Abstract: *This paper investigates the effects of fiscal policy on house prices. We develop and estimate a new-Keynesian open-economy dynamic stochastic general equilibrium (DSGE) model with a housing market and heterogeneous households. We show that house prices drop in response to positive shocks to government spending, government investment and taxes and negative shocks to public transfers. The results reveal that the financial deregulation results in a higher sensitivity of fundamentals to fiscal policy. The influence of the financial deregulation on the GDP multipliers depends on whether the fiscal policy results in an increase or decrease in the price of the durable good.*

2.1 Introduction

The housing market plays an important economic role due to its twofold function: the role of a shelter and of an investment allowing for more consumption in the future. Residential property comprises a significant part of households' balance sheets. For example, in the UK in 2011 housing wealth constituted 47 per cent of total assets owned by households and Non-Profit Institutions Serving Households (NPISH), whereas the total financial wealth constituted 50 per cent.

The influence of housing wealth on consumption has been analysed among others by Skinner (1989), Case (1992), Benjamin *et al.* (2004), Case *et al.* (2005), and Campbell & Cocco (2007). Case *et al.* (2005) use panel data from fourteen countries and find that a 10 per cent increase in housing wealth is followed by a 1 to 1.1 per cent increase in consumption. Benjamin *et al.* (2004) use quarterly data from the USA for the period from 1951Q1 to 2001Q4. They find that a 1 dollar increase in housing wealth leads to an 8 cents increase in consumption. Campbell and Cocco (2005) in a study based on micro data from the UK find that the elasticity of consumption with respect to wealth totals 1.7 for older households.[33]

The globalisation of financial markets along with the development of credit channels over the past decades has increased the sensitivity of house prices to monetary policy and other macroeconomic fundamentals at national and global levels (Bernanke and Gertler, 1995, and Muellbauer and Murphy, 2008). The financial accelerator (see for example Bernanke *et al.*,1996), causes upswings and downswings of assets prices and the

[33] For further discussion of housing and consumption see Muellbauer (2007).

value of the real estate. When assets prices are increasing, the collateral role of property allows households for further spending. On the other hand, when assets prices are decreasing, a credit crunch leads to an even stronger reduction of available credit and further declines in assets prices (Muellbauer, 2005). Nine out of eleven recessions in the USA, including the last one, have been triggered by a contraction in housing investment. After the 2008-09 global financial crisis, the weakness in the housing market remained a concern for the economic recovery in both the US and the UK for a long time.

The literature clearly indicates that house prices have important implications for the consumption expenditure and therefore for the aggregate demand. Considering the extent of recent fiscal stimulus and austerity plans, it is surprising that this channel has received such little attention. In this paper we extend the work of Iacoviello (2005) and Iacoviello and Neri (2010) in two dimensions. First, we incorporate an open-economy and second an extensive public sector. DSGE model with a housing market and analyse the effects of fiscal policy on the housing market and the expenditure decisions of indebted (impatient) households.[34,35]

Following Campbell and Mankiw (1989) and Mankiw (2000) rule-of-thumb consumers became a common feature of fiscal policy papers. The key characteristics of these

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[34] Papers which consider impatient households in the context of fiscal policy analysis include: Callegari (2007), Roeger and in't Veld (2009), Andres *et al.* (2012), and Kollmann *et al.* (2013).

Callegari (2007) and Andrés *et al.* (2012) focus solely on the implications of government spending shock. Roeger and in't Veld (2009) concentrate on the differences in GDP multipliers implied by a temporary and permanent fiscal policy shocks under various specifications of the model (with and without impatient households). Finally, Kollmann *et al.* (2013) analyse the impact of government support for banks and the bank asset losses.

[35] The model is estimated on the quarterly data from the UK for the period from 1987Q2 to 2011Q1. We assess the empirical strength of the model by comparison of its marginal likelihood with that of 8 BVAR models of lag order from 1 to 8. The results indicate that the DSGE model has better fit than related BVARs.

agents include a lack of access to financial markets, a zero net worth, and spending on consumption their total disposable income each period. Surprisingly, especially in the context of the 2008-2009 global financial crisis, where debt and indebted households were the key feature, the impatient households have received relatively less attention. The consumption decision of indebted households is different than that of rule-of-thumb households as it depends not only on the current income, but also on their holdings of durables (houses), and its prices.

In the context of the housing market we show that house prices drop on impact following positive shocks to public spending, investment and taxes and a negative shock to public transfers. The response of the house price to a government consumption shock is similar to that for the UK in the empirical study of Afonso and Sousa (2009).[36]

Moreover, the results reveal that the financial deregulation tends to increase the sensitivity of fundamentals to fiscal policy. If fiscal policy results in an increase of the house price, then financial deregulation results in the strengthening of the effect, because the collateral role of property allows households for further spending. On the other hand, when fiscal policy results in a drop of the house price, then the 'credit squeeze' weakens the fiscal stimulus, because of the lower borrowing possibilities of the credit constrained households. Consequently, financial deregulation weakens GDP multipliers implied by public consumption and investment and tends to improve GDP multipliers for public transfers and tax cuts.

[36] Afonso and Sousa (2009) use a VAR to analyse the effects of government spending shock and government revenue shock on house and stock prices in the U.S, U.K., Germany, and Italy.

The rest of the paper is organised as follows. The next section comprises the theoretical model. Section 3 focuses on the model calibration and estimation. Section 4 brings attention to the model implications of fiscal policy. Section 5 analyses the financial deregulation effects for the fiscal policy. Finally Section 6 concludes.

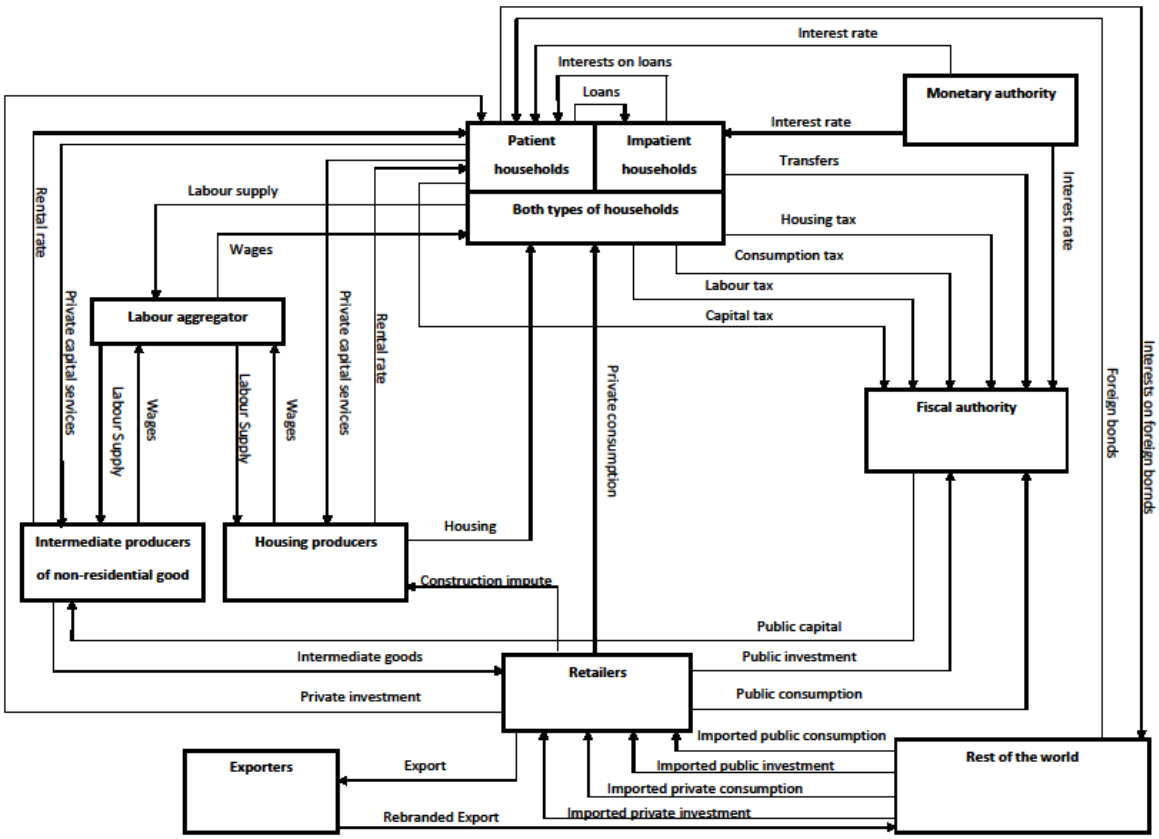
2.2 Theoretical Model

In this paper we develop a small open-economy DSGE model that shares features with Iacoviello (2005), Adolfson *et al.* (2007), Smets and Wouters (2007), and Iacoviello and Neri (2010). The model economy is inhabited by two types of households: lenders and borrowers. Lenders, or patient households, can freely optimise, and possess access to a broad array of financial and real assets: domestic and international bonds, loans to impatient households, housing, and physical capital investment in residential and non-residential production sector. Lenders, along with borrowers, supply differentiated labour to production firms via a labour union which transforms differentiated labour into a homogenous good. Labour income along with transfers from the government are the main sources of income for credit constrained, or impatient households who comprise the poorer part of society. These households discount the future more heavily than patient households, which implies that in the equilibrium they are net borrowers and the borrowing constraint linked to the value of their housing is binding.

On the production side, there are two sectors: one sector producing a tradable, non-residential good, and the second, producing a non-tradable, residential good. Whereas we allow for monopolistic competition in the non-residential sector, we assume perfect com-

petition in the residential market following Barsky *et al.* (2007) and Iacoviello and Neri (2010). Monetary policy is implemented by means of a Taylor type rule and the fiscal policy by means of six rules for fiscal policy instruments: public consumption, investment, transfers and taxes on consumption, labour and capital. The model features a number of nominal and real frictions, which improve the fit with the data. These include among others: sticky prices and wages, habit formations, capital utilisation, or investment adjustment costs. Prime $(')$ distinguish impatient households from patient, and subscript c distinguishes variables related to the non-residential sector from variables related to the residential sector (subscript h). Flow chart of the model is presented in Figure (2.11).

Fig. 2.11. Flow Chart



2.2.1 Production Sector

The non-residential good production sector features a hierarchical, nested structure. The monopolistic producers are situated at the bottom; they use public and private capital, labour from patient and impatient households, and produce differentiated goods, which are sold to a composite good producer. The composite good producer combines the differentiated goods into: a homogenous private and public consumption good and investment (purchased by the final firm); housing construction intermediate input (sold to the producer of housing); and export goods (purchased by monopolistic exporters). A final firm is located on the top of the non-residential good production sector. It combines domestically produced homogenous investment and consumption goods with imported counterparts, and sells it as a final public and private consumption and investment good to households and the government.

The residential good production sector consists of a competitive firm, which uses private labour and capital and the intermediate input to produce residential goods, sold directly to households.

Producer of final, private and public, consumption and investment

Final private consumption, $C_{T,t}$, and final private investment, $I_{T,t}$, are purchased by the heterogeneous agents respectively at prices $P_{c,t}$ and $P_{i,t}$, whereas final public consumption, $G_{T,t}$, and public investment, $IG_{T,t}$, are purchased by the government at prices $P_{g,t}$ and $P_{ig,t}$ respectively. Final private investment is sold in the form of one-for-one transformed residential capital investment, $I_{h,t}$, and non-residential capital investment, $I_{c,t}$, so

that $I_{T,t} = I_{h,t} + I_{c,t}$, and the price of $I_{c,t}$ and $I_{h,t}$ is $P_{i,t}$[37]. All four, final private and public goods, $C_{T,t}$, $I_{T,t}$,$G_{T,t}$, and $IG_{T,t}$ are produced by four competitive branches of the final good firm, each of which specialises in the production of only one type of the final good.

To produce the final consumption or investment good $(N_{T,t})$, each branch combines the domestic homogenous good, $N_{d,t}$, with its imported counterpart, $N_{m,t}$, where $N \in \{C, I, G, IG\}$, and subscripts d and m represent respectively domestically produced and imported goods. The technology used in production of final goods is represented by a constant elasticity of substitution function (CES):

$$N_{T,t} = \left(a_n^{\frac{1}{s_n}} N_{d,t}^{\frac{s_n-1}{s_n}} + (1-a_n)^{\frac{1}{s_n}} N_{m,t}^{\frac{s_n-1}{s_n}} \right)^{\frac{s_n}{s_n-1}} \tag{2.1}$$

where $n \in \{c, i, g, ig\}$, $0 < (1-a_n) < 1$ denotes a share of imported consumption or investment, and s_n represents an elasticity of substitution between domestically produced and imported good. Each branch chooses $N_{d,t}$, and $N_{m,t}$, to maximise profits of the following form:

$$Prof_{n,t} = P_{n,t}N_{T,t} - P_t N_{d,t} - P_{n,m,t}N_{m,t} \tag{2.2}$$

where $P_{n,m,t}$ represents the price of imported consumption or investment, and P_t denotes a price of the domestically produced homogenous good[38]. Profit maximisation results in the demand functions for $N_{d,t}$ and $N_{m,t}$ presented below:

---

[37] Residential capital investment, I_h, should not be confused with housing investment denoted as HI. I_h represents investment in machinery and tools used for the production of HI.

[38] $C_{d,t}$ and $I_{d,t}$ are sold at P_t, as both are one-for-one transformations of the homogenous output of the composite firm which production process is described in section describing composite producer.

$$N_{d,t} = a_n \left(\frac{P_t}{P_{n,t}} \right)^{-s_n} N_{T,t} \tag{2.3}$$

$$N_{m,t} = (1 - a_n) \left(\frac{P_{n,m,t}}{P_{n,t}} \right)^{-s_n} N_{T,t} \tag{2.4}$$

The zero profit condition implies that the consumption and investment price indexes are presented respectively by:

$$P_{n,t} = \left[a_n \left(P_t \right)^{1-s_n} + (1 - a_n) \left(P_{n,m,t} \right)^{1-s_n} \right]^{\frac{1}{1-s_n}} \tag{2.5}$$

Producers of non-residential output

Composite producer

Y_t denotes the homogenous output of the composite producer of the domestic non-residential good and is converteded one-for-one into the homogenous: private and public consumption and investment goods, housing construction input, CO_t, and export, EX_t. As a result the price of $I_{d,t}, C_{d,t}, IG_{d,t}, G_{d,t}, CO_t,$ and EX_t is equal to the price of Y_t and is represented by P_t[39]. The producer of final, public and private, consumption and investment goods purchases $I_{d,t}, C_{d,t}, IG_{d,t},$ and $G_{d,t}$, the producer of the housing good purchases construction input, $CO_t,$ and finally monopolistic exporters buy the export good EX_t.

$$Y_t - a(u_{c,t})K_{c,t-1} - a(u_{h,t})K_{h,t-1} = I_{d,t} + C_{d,t} + IG_{d,t} + G_{d,t} + CO_t + EX_t \tag{2.6}$$

[39] $a(u_{c,t})K_{c,t-1}$ and $a(u_{h,t})K_{h,t-1}$ denote a sector specific capital utilisation costs and are discussed in more details in the section describing patient households.

The homogenous output Y_t, is produced from the differentiated outputs of intermediate producers by means of the following CES technology: $Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\upsilon-1}{\upsilon}} dj\right]^{\frac{\upsilon}{\upsilon-1}}$, where $\upsilon > 0$ denotes the elasticity of substitution among the differentiated outputs of intermediate firms and $Y_{j,t}$ represents an output of the jth producer. The composite firm chooses $Y_{j,t}$ which maximises profit of the form:

$$Prof_t = P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{2.7}$$

where $P_{j,t}$ denotes a price of output of the intermediate firm j. The first-order condition results in the demand equation for the output of intermediate producer j:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\upsilon} Y_t \tag{2.8}$$

The zero profit condition implies that the price index is given by $P_t = \left[\int_0^1 P_{j,t}^{1-\upsilon} dj\right]^{\frac{1}{1-\upsilon}}$.

Monopolistic producers

The intermediate producers use public and private capital, and labour supplied by patient and impatient households to produce differentiated non-residential goods, which are subsequently sold to the composite good producer. Production function of jth monopolistic producer is represented by a Cobb-Douglas type function:

$$Y_{j,t} = \varepsilon_t^{Ac} (u_{c,j,t} K_{c,j,t-1})^{\alpha} \left(N_{c,j,t}^{b_1} N_{c,j,t}'^{1-b_1}\right)^{1-\alpha} K_{g,j,t-1}^{\sigma_g} - \Phi \tag{2.9}$$

where $K_{g,j,t-1}$ and $K_{c,j,t-1}$ denote respectively public and private capital, $u_{c,j,t}$ represents a specific non-residential sector utilisation rate of private capital, $N_{c,j,t}$ and $N'_{c,j,t}$

denote respectively labour supplied by lenders and borrowers, and ε_t^{Ac} is the sector specific total factor productivity shock. Φ stands for a fixed cost of production, $0 < \alpha < 1$ denotes a share of capital in production, $0 < b_1 < 1$ is a share of hours in production supplied by patient households, and σ_g represents the elasticity of output with respect to the public capital. Monopolistic producers choose $K_{c,j,t-1}$, $N_{c,j,t}$, and $N'_{c,j,t}$ to minimise the total costs subject to the available technology:

$$\begin{aligned} \underset{K_{j,t-1},\, N_{j,t},\, N'_{j,t}}{min} \; & W_{c,t}N_{c,j,t} + W'_{c,t}N'_{c,j,t} + R_{k,c,t}\omega_{c,t}K_{c,j,t-1} - \\ & \lambda_{p,t}P_{j,t}\left(Y_{j,t} - \varepsilon_t^{Ac}(u_{c,j,t}K_{c,j,t-1})^{\alpha}\left(N_{c,j,t}^{b_1}N_{c,j,t}'^{1-b_1}\right)^{1-\alpha}K_{g,t-1}^{\sigma_g} + \Phi\right) \end{aligned} \tag{2.10}$$

where $\lambda_{p,t}$ is a Lagrange multiplier, and $W_{c,t}$, $W'_{c,t}$, $R_{k,c,t}$ denote respectively wage rate of lenders, borrowers and a rate of return on capital. All prices of inputs of production are taken by firms as given. First order conditions of firms result in:

$$\begin{aligned} W_{c,t}N_{c,j,t} &= b_1(1-\alpha)\lambda_{p,t}P_{j,t}(Y_{j,t}+\Phi) \\ W'_{c,t}N'_{c,j,t} &= (1-b_1)(1-\alpha)\lambda_{p,t}P_{j,t}(Y_{j,t}+\Phi) \\ u_{c,t}R_{k,c,t}K_{c,j,t-1} &= \alpha\lambda_{p,t}P_{j,t}(Y_{j,t}+\Phi) \end{aligned}$$

where $\lambda_{p,t}P_{j,t} = MC_t$ denotes a nominal marginal cost represented by:

$$MC_t = cons\left(\varepsilon_t^{Ac}\right)^{-1}R_{k,c,t}^{\alpha}W_{c,t}^{b_1(1-\alpha)}W_{c,t}'^{(1-b_1)(1-\alpha)}K_{g,t-1}^{-\sigma_g} \tag{2.11}$$

where $cons = \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{b_1(1-\alpha)}\right)^{1-\alpha}\left(\frac{b_1}{1-b_1}\right)^{(1-b_1)(1-\alpha)}$. Monopolistic producers are price setters, and prices are subject to Calvo (1983) frictions. In particular, each period a share of

companies $(0 < \varpi < 1)$ is not able to reoptimise its price. These companies simply set their prices by applying an indexation rule: $P_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma} P_{j,t-1}$. The remaining share of companies, $(1 - \varpi)$, choose price $\bar{P}_t$ to maximises the objective of the form:

$$Prof_t = E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\tilde{P}_t \Pi_{tl}}{P_{t+l}} - mc_{t+l} \right] P_{t+l} Y_{j,t+l} - MC_{t+l} \left(Y_{j,t} + \Phi \right) \tag{2.12}$$

subject to the demand presented in equation (2.8). λ_t denotes a Lagrange multiplier on the patient household budget constraint, $(\beta\varpi)^l$ denotes discount factor of future profits for firms and $\Pi_{tl} = \pi_t \times \pi_{t+1} \times ... \times \pi_{t+l-1}$ for $l \geqslant 1$, and $\Pi_{tl} = 1$ for $l = 0$ as in Altig *et al.* (2005). Profits maximisation results in the equation for newly optimised prices:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\tilde{P}_t X_{tl} - \frac{\upsilon}{\upsilon - 1} P_{t+l} mc_{t+l} \right] Y_{j,t+l} = 0 \tag{2.13}$$

The first-order condition implies that the price set by monopolistic producers is a function of expected future nominal marginal costs. In the environment where all firms are allowed to set their prices, the newly optimised price is a markup over the marginal costs: $\tilde{P}_t = \frac{\upsilon}{\upsilon-1} MC_t$.

Exporters

Composite producer

The competitive producer of the homogenous export good faces a foreign demand for its output in the form of: $X_t = \left(\frac{P_{x,t}}{P_t^*}\right) Y_t^*$, where $P_{x,t}$ denotes the foreign currency price of the exported good, Y_t^* represents a foreign output, and P_t^* is a foreign price index. The ho-

mogenous export good X_t, is produced from the differentiated export good, $X_{j,t}$, by means of the CES technology: $X_t = \left[\int_0^1 X_{j,t}^{\frac{\upsilon_x - 1}{\upsilon_x}} dj\right]^{\frac{\upsilon_x}{\upsilon_x - 1}}$, where $\upsilon_x > 0$ denotes the elasticity of substitution among the differentiated export goods. The composite firm chooses $X_{j,t}$ and maximises profit:

$$Prof_{x,t} = P_{x,t}X_t - \int_0^1 P_{x,j,t}X_{j,t}dj \tag{2.14}$$

The first-order condition results in the demand equation for the output of monopolistic producer j:

$$X_{j,t} = \left(\frac{P_{x,t}}{P_{x,j,t}}\right)^{\upsilon_x} X_t \tag{2.15}$$

The price index is represented by $P_{x,t} = \left[\int_0^1 P_{x,j,t}^{1-\upsilon_x} dj\right]^{\frac{1}{1-\upsilon_x}}$.

Monopolistic producers

Monopolistic exporters simply buy output (EX_t) from the composite producer of the domestic homogenous good at price P_t, rebrand it, and sell it to the competitive producer of the homogenous export good at a price $P_{x,t}$. The real marginal cost is therefore represented by:

$$mc_{x,t} = \frac{MC_{x,t}}{P_{x,t}S_t} = \frac{P_t}{P_{x,t}S_t} \tag{2.16}$$

To introduce incomplete exchange rate pass-through we let the monopolistic exporters set prices subject to Calvo (1983) frictions. In particular each period a share of companies $0 < \varpi_x < 1$, is not able to reoptimise its price. They simply set prices by means of the

indexation rule: $P_{x,j,t} = \left(\frac{P_{x,t}}{P_{x,t}}\right)^{\gamma_x} P_{x,j,t-1}$. The remaining share of companies $(1-\varpi_x)$, choose price $P_{x,t}$ and maximise an objective of the form:

$$Prof_{x,j,t} = E_t \sum_{l=0}^{\infty} (\beta\varpi_x)^l \lambda_{t+l} \left[\frac{P_{x,t} S_{t+l} \Pi_{x,tl}}{P_{x,t} S_{t+l}} - mc_{x,t+l} \right] P_{x,t} S_{t+l} X_{j,t+l} \tag{2.17}$$

where $X_{j,t}$ is defined in equation (2.15), and $\Pi_{x,tl} = \pi_t^{\gamma_x} \times \pi_{t+1}^{\gamma_x} \times ... \times \pi_{t+l-1}^{\gamma_x}$ for $l \geqslant 1$ and $\Pi_{x,tl} = 1$ for $l = 0$. Time t profits are simply represented by $\left[\frac{P_{x,j,t}}{P_{x,t}} - mc_{x,t}\right] P_{x,t} S_t X_{j,t}$ and are equal to 0 in the steady state. The maximisation results in the equation for a newly optimised export good price:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi_x)^l \lambda_{t+l} \left[P_{x,t} S_{t+l} \Pi_{x,tl} - \left(\frac{\upsilon_x}{\upsilon_x - 1} \right) P_{x,t} S_{t+l} mc_{x,t+l} \right] X_{t+l} = 0 \tag{2.18}$$

The first-order condition implies that the price set by companies is a function of expected future nominal marginal costs. In the environment where all firms are allowed to reset their prices, the export price expressed in terms of domestic prices is simply a markup over nominal marginal costs: $P_{x,t} S_t = \left(\frac{\upsilon_x}{\upsilon_x - 1}\right) MC_{x,t}$.

Importers

Composite producer

There are four homogenous imported goods in the model economy: private consumption $C_{m,t}$, private investment $I_{m,t}$, public consumption $G_{m,t}$, and public investment $IG_{m,t}$. All of them are purchased by the producer of final consumption and investment.

The producer of the homogenous imported goods consists of four competitive branches, each of which specialises in production of one of four homogenous imported goods. The four branches purchase from monopolistic importers differentiated imported consumption and investment goods $N_{m,j,t}$ where as above $N \in \{C, I, G, IG\}$, $n \in \{c, i, g, ig\}$, and j denotes a particular monopolistic intermediate importer. The homogenous imported good $N_{m,t}$, is produced from differentiated goods, $N_{m,j,t}$, by means of the following CES technology: $N_{m,t} = \left[\int_0^1 N_{m,j,t}^{\frac{\upsilon_{n,m}-1}{\upsilon_{n,m}}} dj\right]^{\frac{\upsilon_{n,m}}{\upsilon_{n,m}-1}}$, where $\upsilon_{n,m} > 0$ denotes the corresponding elasticity of substitution among the differentiated imported goods. The composite firm chooses $N_{m,j,t}$ and maximises profit of the following form:

$$Prof_{n,t} = P_{n,m,t}N_{m,t} - \int_0^1 P_{n,m,j,t}N_{m,j,t}dj \tag{2.19}$$

The first-order condition results in the demand equation for the output of a monopolistic producer of imported differentiated good j:

$$N_{m,j,t} = \left(\frac{P_{n,m,t}}{P_{n,m,j,t}}\right)^{\upsilon_{n,m}} N_{m,t} \tag{2.20}$$

Monopolistic producers

The monopolistic importers operate in four different sectors. Each importer specialises in importing only one good N. They buy goods abroad, rebrand and sell it to the composite imported goods producer. The companies pay for goods in the foreign currency, therefore, the real marginal cost is represented by: $mc_{n,m,t} = \frac{P_t^* * S_t}{P_{n,m,t}}$. To introduce the incomplete exchange rate pass-through only fraction $0 < (1 - \varpi_{n,m}) < 1$ of importers can

adjust prices every period. Those who cannot adjust prices, simply follow an indexation rule $P_{n,m,t} = \left(\frac{P_{n,m,t-1}}{P_{n,m,t-2}}\right)^{\upsilon_{n,m}} P_{n,m,t-1}$. Remaining companies choose $P_{n,m,t}$ to maximise profits of the following form:

$$Prof_{n,m,t} = E_t \sum_{l=0}^{\infty} (\beta \varpi_{n,m})^l \lambda_{t+l} \left[\frac{\tilde{P}_{n,m,t} \Pi_{n,m,tl}}{P_{n,m,t+l}} - mc_{n,m,t+l} \right] P_{n,m,t+l} N_{m,j,t+l} \quad (2.21)$$

subject to the demand presented in equation (2.20), and where $\Pi_{n,m,tl} = \pi_t^{\gamma_{n,m}} \times \pi_{t+1}^{\gamma_{n,m}} \times ... \times \pi_{t+l-1}^{\gamma_{n,m}}$ for $l \geqslant 1$, and $\Pi_{n,m,tl} = 1$ for $l = 0$. Result of this maximisation is the following:

$$E_t \sum_{l=0}^{\infty} (\beta \varpi_{n,m})^l \lambda_{t+l} \left[\frac{\tilde{P}_{n,m,j,t}}{P_{n,m,t+l}} - \frac{\upsilon_{n,m}}{\upsilon_{n,m} - 1} mc_{n,m,t+l} \right] P_{n,m,t+l} N_{m,j,t+l} = 0 \quad (2.22)$$

The first-order conditions imply that the price set by importing companies is a function of expected future marginal costs. In the environment that all firms can set their prices, the import price is a markup over the marginal costs: $\tilde{P}_{n,m,j,t} = \frac{\upsilon_{n,m}}{\upsilon_{n,m}-1} MC_{n,m,t}$. Time t profits for price setters are represented by $\left[\frac{\tilde{P}_{n,m}}{P_{n,m,t}} - mc_{n,m,t}\right] P_{n,m,t} N_{m,j,t}$ and are positive in the long run.

Producer of residential output

Homogenous output of the composite residential good producer HI_t is directly purchased by patient and impatient households:

$$HI_t = IH_t + IH'_t = H_t + H'_t - (1 - \delta_h) \left(H_{t-1} + H'_{t-1} \right) \quad (2.23)$$

where IH_t denotes residential investment of patient households, IH'_t represents residential investment of impatient households, H_t and H'_t denote respectively stock of housing owned by lenders and borrowers, and δ_h represents the depreciation rate of housing. The residential good producer operates in a perfectly competitive environment and uses capital, $K_{h,t}$, labour from both type of households denoted respectively by $N_{h,t}$ and $N'_{h,t}$, and homogenous output of domestic non-residential producers, CO_t, to produce the residential output. The company uses a Cobb-Douglas production function:

$$HI_t = \varepsilon_t^{A_h}(u_{h,t}K_{h,t-1})^{\alpha_h}\left(N_{h,t}^{b_1}N_{h,t}^{\prime 1-b_1}\right)^{1-\alpha_h-\alpha_{co}} CO_{t-1}^{\alpha_{co}} \tag{2.24}$$

where α_h denotes a share of capital in production, and α_{co} represents a share of non-residential output in the production, $u_{h,t}$ denotes the utilisation rate of capital specific to the residential sector, and $\varepsilon_t^{A_h}$ is a sector specific total factor productivity shock. The company maximizes profit:

$$\begin{aligned} Prof_{h,t} &= \varepsilon_t^{A_h}(u_{h,t}K_{h,t-1})^{\alpha_h}\left(N_{h,t}^{b_1}N_{h,t}^{\prime 1-b_1}\right)^{(1-\alpha_h-\alpha_{co})} CO_{t-1}^{\alpha_{co}} \\ &\quad -W_tN_{h,t} - W'_{h,t}N'_{h,t} - R_{k,h,t}u_{h,t}K_{h,t-1} - P_tCO_t \end{aligned} \tag{2.25}$$

where $W_{c,t}$, $W'_{c,t}$, $R_{k,h,t}$ denote respectively prices of lenders' labour, borrowers' labour, and a rate of return on capital. All prices are taken by firms as given. First order conditions of the housing producing firm are represented by:

$$
\begin{aligned}
W_t N_{h,t} &= b_1 \left(1 - \alpha_h - \alpha_{co}\right) P_{h,t} HI_t \\
W'_{h,t} N'_{h,t} &= \left(1 - b_1\right) \left(1 - \alpha_h - \alpha_{co}\right) P_{h,t} HI_t \\
R_{k,h,t} u_{h,t} K_{h,t-1} &= \alpha_h P_{h,t} HI_t \\
P_t CO_t &= \alpha_{co} P_{h,t} HI_t
\end{aligned}
$$

2.2.2 Households

The economy is inhabited by a continuum of households indexed by τ. Patient households can freely optimise and have access to all available financial and capital markets. Impatient households discount the future more heavily which implies that they become borrowers in the equilibrium, are allowed to purchase only non-residential consumption good and housing bundle, and their maximum amount of borrowing is constrained by a borrowing constraint linked to the expected value of their assets.

Patient Households

The utility functional of each patient household takes the form:

$$
E_0 \sum_{t=0}^{\infty} \varepsilon_t^B \beta^t \left(\ln\left(C_t^\tau - HC_t^\tau\right) + \varepsilon_t^H j \ln\left(H_t^\tau - HH_t^\tau\right) - \frac{\varepsilon_t^L}{1+\sigma_L} \left[L_{h,t}^{\tau(1+\varsigma)} + L_{c,t}^{\tau(1+\varsigma)} \right]^{\frac{1+\sigma_L}{1+\varsigma}} \right) \tag{2.26}
$$

where $0 < \beta < 1$ denotes the subjective discount factor; j represents a weight on housing in utility; $\sigma_L \geqslant 0$ is the inverse Frisch elasticity of labour; $\varsigma \geqslant 0$ denotes the inverse elasticity of substitution across hours in the two sectors; HC_t^τ and HH_t^τ represent external

habit variables so that $HC_t^\tau = h_c C_{t-1}^\tau$, and $HH_t^\tau = h_h H_t^\tau$, where $\{h_c, h_h\} \in < 0, 1 >$; ε_t^B, stands for the shock to intertemporal preferences, ε_t^L stands for the labour supply shock, and ε_t^Hdenotes the housing preference shock.

Disposable income and expenditure

Total real disposable income of each lender consists of:

- the after tax labour income $\left(1 - \tau_t^l\right)\left[w_{c,t} L_{c,t}^\tau + w_{h,t} L_{h,t}^\tau\right]$, where τ_t^l denotes the effective labour tax rate;
- the after tax capital income $\left(1 - \tau_t^k\right)\left[r_{c,k,t} u_{c,t}^\tau K_{c,t-1}^\tau + r_{h,k,t} u_h^\tau K_{h,t-1}^\tau\right]$, where τ_t^k represents the effective capital tax rate;
- the interests income from holdings of government bonds, and deposits $\frac{i_{t-1}}{\pi_t}\left[b_{g,t-1}^\tau + D_{t-1}^\tau\right]$, where i_{t-1} denotes the nominal interest rate on the one period bond, and π_t denotes a gross inflation[40];
- the interests income from holdings of foreign bonds $\frac{i_{t-1}^* S_t P_{t-1}^* b_{f,t-1}^\tau risk_{t-1}}{P_t}$ where i_{t-1}^* denotes the nominal interest rate on the one period bond, P_{t-1}^* denotes the foreign price, and S_t the nominal exchange rate;
- the dividends income div_t^τ;

[40] The gross nominal interest rate is represented by $R_t = 1 + i_t$.

The real disposable income is reduced by a cost of capital utilisation represented by: $a(u_{i,t})K_{i,t-1}$ for $i\epsilon\{c,h\}$ [41] .

Every period, each Patient household τ, decides on the allocation of its resources between consumption and the accumulation of financial and non-financial assets. The non-residential consumption bundle is purchased at a consumer price $P_{c,t}\left(1+\tau_t^c\right)$, where τ_t^c denotes the effective consumption tax rate. Housing is purchased at $\left(1+\tau_t^h\right)P_{h,t}$ where τ_t^h denotes the effective residential tax rate. The net acquisition of residential property at time t is represented by:

$$P_{h,t}\left[\left(1+\tau_t^h\right)H_t^\tau-(1-\delta_h)H_{t-1}^\tau\right] \tag{2.27}$$

The investment in both types of capital investment increases the stock of physical capital which accumulates in accordance to:

$$K_{i,t}=(1-\delta_{i,k})K_{i,t-1}+\varepsilon_{i,t}^I\left[1-S\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right]I_{i,t} \tag{2.28}$$

where $i\epsilon\{c,h\}$, and the quadratic cost of adjustment function is represented by $S\left(\frac{I_{i,t}}{I_{i,t-1}}\right)=\frac{\phi_{i,k}}{2}(\frac{I_{i,t}}{I_{i,t-1}}-1)^2$. $\phi_{i,k}>0$ denotes the inverse elasticity of investment with respect to installed capital. Finally, net accumulation of financial assets comprises:

- deposits in banks $D_t^\tau-\frac{D_{t-1}^\tau}{\pi_{t-1}}$;
- acquisition of government bonds, $b_{g,t}^\tau-\frac{b_{g,t-1}}{\pi_{t-1}}$;
- acquisition of foreign bonds $\frac{S_tB_{f,t}^\tau}{P_t}-\frac{S_tB_{f,t-1}^\tau risk_{t-1}}{P_t}$, where $risk$ is a premium on the foreign bond holdings following Adolfson *et al.* (2007) and Benigno (2009);

[41] $a(u_{i,t})$ represents the cost function originating from changes in the capital utilization rate. We set $a(u_{i,t})=\left(1-\tau^k\right)r_i\left[\frac{1}{2}\kappa u_{i,t}^2+(1-\kappa)u_{i,t}+(\frac{\kappa}{2}-1)\right]$, similarly to Iacoviello and Neri (2010). Consequently only dynamics of the model depend on the parameter κ. In the steady state $u_i=1$.

$risk_t = \exp\left(-\varkappa\left(aa_t - aa\right) + \eta_t^r\right)$, and $aa_t = \frac{S_t B_{f,t}^{\tau}}{P_t}$ denotes the real aggregate net foreign asset position.

For period t, the flow budget constraint can be summarized to:

$$\begin{aligned}
& p_{i,t} I_{c,t}^{\tau} + p_{i,t} I_{h,t}^{\tau} + p_{c,t}\left(1+\tau_t^c\right) C_t^{\tau} + q_t\left[\left(1+\tau_t^h\right) H_t^{\tau} - (1-\delta_h) H_{t-1}^{\tau}\right] \\
& - \left(D_t^{\tau} - \frac{D_{t-1}^{\tau}}{\pi_{t-1}}\right) + \left(b_{g,t}^{\tau} - \frac{b_{g,t-1}}{\pi_{t-1}}\right) + \left(\frac{S_t B_{f,t}^{\tau}}{P_t} - \frac{S_t B_{f,t-1}^{\tau} risk_{t-1}}{P_t}\right) \\
= \; & \left(1-\tau_t^l\right)\left(w_{c,t} L_{c,t}^{\tau} + w_{h,t} L_{h,t}^{\tau}\right) + \left(1-\tau_t^k\right)\left[r_{c,k,t} u_{c,t}^{\tau} K_{c,t-1}^{\tau} + r_{h,k,t} u_h^{\tau} K_{h,t-1}^{\tau}\right] \\
& + div_t^{\tau} + \frac{i_{t-1}}{\pi_t}\left(b_{g,t-1}^{\tau} - D_{t-1}^{\tau}\right) + \left(\frac{i_{t-1}^{*} S_t B_{f,t-1}^{\tau} risk_{t-1}}{P_t}\right) \\
& - a(u_{h,t}^{\tau}) K_{h,t-1}^{\tau} - a(u_{c,t}^{\tau}) K_{c,t-1}^{\tau}
\end{aligned} \tag{2.29}$$

where the relative prices are denoted by a lower case p; the relative price of housing is represented by $q_t = \frac{P_{h,t}}{P_t}$. Each Ricardian household maximizes the utility (2.26) subject to the capital accumulation equation (2.28) , the budget constraint (2.29) and the demand for labour (2.46).[42]

First Order Conditions of Patient Households

First order conditions with respect to endogenous choice variables of Patient households are represented by:

[42] λ_t is the Lagrange multiplier on the budget constraint, $\lambda_t Q_{i,t}$ is the Lagrange multiplier on the capital accumulation equation, and $L_{i,t}^{\tau} = \left(\frac{W_{i,t}^{\tau}}{W_{i,t}}\right)^{-\nu} N_{i,t}$ represents the labour demand for $i \epsilon \{c, h\}$. The wage setting is further section below.

$$C_t \quad : \quad \lambda_t = \frac{U_{c,t}}{p_{c,t}\left(1+\tau_t^c\right)} \tag{2.30}$$

$$b_{g,t} \quad : \quad \lambda_t = E_t\left[\beta\lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{2.31}$$

$$B_t^* \quad : \quad \lambda_t = E_t\left[\beta\lambda_{t+1}\frac{R_t^*}{E_t\pi_{t+1}}\frac{S_{t+1}risk_t}{S_t}\right] \tag{2.32}$$

$$I_{i,t} \quad : \quad \lambda_t = E_t\left[\frac{\lambda_t Q_{i,t}F_t'\left(I_{i,t},I_{i,t-1}\right)+\beta E_t\lambda_{t+1}Q_{i,t+1}F_{t+1}'\left(I_{i,t},I_{i,t-1}\right)}{p_{i,t}}\right] \tag{2.33}$$

$$H_t \quad : \quad \lambda_t = E_t\left[\frac{U_{h,t}+\beta\lambda_{t+1}q_{t+1}\left(1-\delta_h\right)}{q_t\left(1+\tau_t^h\right)}\right] \tag{2.34}$$

$$K_{i,t} \quad : \quad Q_{i,t} = \frac{E_t\pi_{t+1}}{R_t}E_t\left[\begin{array}{c} Q_{i,t+1}\left(1-\delta_{i,k}\right) \\ +\left(1-\tau_{t+1}^k\right)r_{i,k,t+1}u_{i,t+1}-a\left(u_{i,t+1}\right)\end{array}\right] \tag{2.35}$$

$$u_{i,t} \quad : \quad a'\left(u_{i,t}\right) = \left(1-\tau_t^k\right)r_{i,k,t} \tag{2.36}$$

where $F_t\left(I_{i,t},I_{i,t-1}\right) = \left[1-S\left(\frac{\varepsilon_{i,t}^I I_{i,t}}{I_{i,t-1}}\right)\right]I_{i,t}$

The first-order condition with respect to consumption, presented in equation (2.30), indicates that with an extra unit of income spent on consumption, the level of utility increases by $\frac{U_{c,t}}{p_{c,t}(1+\tau_t^c)}$, where U_c denotes the marginal utility of consumption.

As an alternative to consumption, each patient household can choose to invest in domestic or foreign bonds, physical capital, or residential property. The outcome of such decisions is presented in equations $(2.31-2.34)$. The left hand side of the mentioned conditions represents the marginal utility cost of investment in a given asset, (to invest household resigns from the current consumption) and the right hand side comprises the return on investment which is naturally equal across all the types of investments. Consequently:

- from the investment in government bonds and deposits household receives *ex ante* real interest rate $E_t\left(\frac{R_t}{\Pi_{t+1}}\right)$;

- the return from the investment in foreign bonds is represented by $E_t\left(\frac{R_t^*}{\pi_{t+1}}\frac{S_{t+1}risk_t}{S_t}\right)$. The combination of the first-order condition with respect to the domestic and foreign bonds results in the modified uncovered interest rate parity condition:

$$S_t R_t = E_t S_{t+1} R_t^* risk_t \tag{2.37}$$

- from the investment in capital, each household receives a rise in the value of capital by $Q_{i,t}F_t'\left(I_{i,t}, I_{i,t-1}\right)$ in period t, and by $E_t Q_{i,t+1}\beta F_{t+1}'\left(I_{i,t+1}, I_{i,t}\right)$ in period $t+1$;
- from the investment in housing, each household receives a direct increase in the utility $U_{h,t}$, and the value of non-depreciated housing next period represented by $\beta q_{h,t+1}(1-\delta_h)$.

The first-order condition with respect to capital is presented in equation (2.35). It implies that the present value of capital depends positively on its future value adjusted for depreciation, expected rental rate of capital and utilisation rate. Higher expected capital utilisation costs and the *ex ante* real interest rate decrease the value of capital. The first-order condition with respect to the capital utilisation rate is presented in equation (2.36). This equation indicates that the rate of return on a particular capital good has to be equal to the marginal cost of capital utilisation.

Impatient Households

Following Campbell and Mankiw (1989) and Mankiw (2000) rule-of -thumb consumers became a common feature of fiscal policy papers. The key characteristics of these households include a lack of access to financial markets, a lack of optimisation, and consumption of total disposable income every period. Surprisingly, especially in the context of recent financial crisis where debt and indebted households were the key feature, the credit constraint households received relatively less attention. The consumption decision of impatient households is different than that of rule-of-thumb consumers as it depends not only on the current income, but also on their holdings of the durable good and its value. The utility functional of a borrower is represented by:

$$E_0 \sum_{t=0}^{\infty} \gamma^t \left(\ln\left(C_t'' - HC_t''\right) + \varepsilon_t^H j \ln\left(H_t'' - HH_t''\right) - \frac{\varepsilon_t^L}{1+\sigma_L'} \left[L_{h,t}''^{(1+\varsigma')} + L_{c,t}''^{(1+\varsigma')} \right]^{\frac{1+\sigma_L'}{1+\varsigma'}} \right) \tag{2.38}$$

where $0 < \gamma < 1$ denotes the subjective discount factor, $\sigma_L' \geqslant 0$ represents inverse Frisch elasticity of labour; $\varsigma \geqslant 0$ denotes the inverse elasticity of substitution across hours in the two sectors; and HC_t'' and HH_t'' represent external habit variables so that $HC_t'' = h_c' C_{t-1}''$, and $HH_t'' = h_h' H_t''$, where $\{h_c', h_h'\} \in < 0, 1 >$;

Disposable income and expenditure possibilities

Each impatient household faces a flow budget constraint which states that the after taxes labour income $\left(1 - \tau_t^l\right)\left(w_{c,t}'' L_{c,t}'' + w_{h,t}'' L_{h,t}''\right)$ plus government transfers TR_t, diminished by the interest payments on loans $\frac{r_t}{\pi_t} LO_{t-1}''$ has to be equal to a consumption expendi-

ture, $p_{c,t}\left(1+\tau_t^c\right)C_t'^{\iota}$, a net acquisition of housing, $q_t\left[\left(1+\tau_t^h\right)H_t'^{\iota}-(1-\delta_h)H_{t-1}'^{\iota}\right]$, and a net increment of loans $\left(LO_t'^{\iota}-\frac{LO_{t-1}'^{\iota}}{\pi_t}\right)$.

$$\begin{aligned} & p_{c,t}\left(1+\tau_t^c\right)C_t'^{\iota}+q_t\left[\left(1+\tau_t^h\right)H_t'^{\iota}-(1-\delta_h)H_{t-1}'^{\iota}\right]-LO_t'^{\iota} \\ = \; & \left(1-\tau_t^l\right)\left(w_{c,t}'^{\iota}L_{c_b,t}'^{\iota}+w_{h,t}'^{\iota}L_{h,t}'^{\iota}\right)+TR_t-\frac{R_{t-1}}{\pi_t}LO_{t-1}'^{\iota} \end{aligned} \tag{2.39}$$

The maximum amount of borrowing is limited by the borrowing constraint linked to the expected value of housing:

$$LO_t' \leq (1-\vartheta)(1-\delta_h)E_t\left[q_{t+1}H_t'\frac{\pi_{t+1}}{R_t}\right] \tag{2.40}$$

where ϑ stands for the share of a downpayment. The borrowing constraint simply states that the level of the loan with interest due, has to be equal or lower than the expected, discounted value of a home, adjusted for the downpayment $(1-\vartheta)(1-\delta_h)E_t\left[q_{t+1}H_t'\right]\frac{\pi_{t+1}}{R_t}$. To keep the borrowing constraint binding in and around the steady state, we follow Iacoviello (2005) and set γ so that $\gamma<\beta$. As a result in the steady state a Lagrange multiplier on the borrowing constraint is positive, $\lambda_b=1-\frac{\gamma}{\beta}>0$, which is a condition required to keep the borrowing constraint binding as can be seen in equation (2.42).

First Order Conditions of Impatient Households

First order conditions of impatient households are represented by:

$$C'_t \quad : \quad \lambda'_t = \frac{U'_{c,t}}{p_{c,t}\left(1+\tau^c_t\right)} \tag{2.41}$$

$$LO'_t \quad : \quad \lambda'_t = \lambda'_t \lambda_{b,t} + \lambda'_{t+1}\gamma \frac{R_t}{\pi_{t+1}} \tag{2.42}$$

$$H'_t \quad : \quad \lambda'_t = \frac{U'_{h,t} + \lambda'_{t+1}\gamma q_{t+1}(1-\delta_h) + \lambda'_t \lambda_{b,t}(1-\vartheta)(1-\delta_h)E_t\left[\frac{q_{t+1}\pi_{t+1}}{R_t}\right]}{q_t\left(1+\tau^h_t\right)} \tag{2.43}$$

The first-order condition with respect to consumption presented in equation (2.41), implies that one extra unit of income allocated for consumption increases the level of utility by $\frac{U'_{c,t}}{p_{c,t}(1+\tau^c_t)}$, where $U'_{c,t}$ denotes the marginal utility of consumption of impatient households. The first-order condition with respect to the borrowings, presented in equation (2.42), reduces to the standard Euler equation when $\lambda_{b,t} = 0$, i.e. when borrowing constraint is not binding. When the borrowing constraint is binding, $\lambda_{b,t} > 0$, the condition implies that repaying the loan by one unit (left hand side of equation) increases the potential level of borrowings by $\lambda'_t \lambda_{b,t}$, and implies that households will not have to repay in the future: $\lambda'_{t+1}\gamma\frac{R_{t+1}}{\pi_t}$ (right hand side of equation). The first-order condition with respect to housing implies that with one extra unit of income spent on housing, a household receives a direct increase in the utility $U'_{h,t}$, the expected increase of utility in the case of a resale of housing next period $\lambda'_{t+1}\gamma q_{h,t+1}(1-\delta_h)$, and the increase in the utility stemming from the fact that the house can be used as a collateral thus current consumption can be increased $\left(\lambda'_t \lambda_{b,t}(1-\vartheta)(1-\delta_h)E_t\left[\frac{q_{t+1}\pi_{t+1}}{R_t}\right]\right)$.

The Wage Setting

Labour Union

There are four types of labour in the model (two types of households supply two types of labour), which is supplied by households to one of four competitive branches of the labour union. Each of the branches specialises in the transformation of a certain type of differentiated labour, into a composite labour good, which is subsequently supplied to monopolistic firms. Each branch, depending on the specialisation, chooses $L^{\iota}_{i,t}$ or $L'^{\iota}_{i,t}$ for $i\epsilon\{c,h\}$ to maximise profits defined as:

$$Prof_{L,i,t} = W_{i,t}N_{i,t} - \int_0^1 W^{\iota}_{i,t}L^{\iota}_{i,t}di \tag{2.44}$$

$$Prof_{L',i,t} = W'_{i,t}N'_{i,t} - \int_0^1 W'^{\iota}_{i,t}L'^{\iota}_{i,t}di \tag{2.45}$$

where $W_{i,t}$ and $W'_{i,t}$ denote type i aggregate wage indexes; and $N_{i,t}$ and $N'_{i,t}$ are type i aggregate labour index such that: $N_{i,t} = \left[\int_0^1 \left(L^{\iota}_{i,t}\right)^{\frac{\upsilon_{i,w}-1}{\upsilon_{i,w}}} di\right]^{\frac{\upsilon_{i,w}}{\upsilon_{i,w}-1}}$ and $N'_{i,t} = \left[\int_0^1 \left(L'^{\iota}_{i,t}\right)^{\frac{\upsilon_{i,w}-1}{\upsilon_{i,w}}} di\right]^{\frac{\upsilon_{i,w}}{\upsilon_{i,w}-1}}$, and $\upsilon_{i,w}$ denotes the elasticity of substitution among differentiated labour inputs. Transformation of first-order conditions results in the demand functions for the differentiated labour:

$$L^{\iota}_{i,t} = \left(\frac{W^{\iota}_{i,t}}{W_{i,t}}\right)^{-\upsilon_{i,w}} N_{i,t} \tag{2.46}$$

$$L'^{\iota}_{i,t} = \left(\frac{W'^{\iota}_{i,t}}{W'_{i,t}}\right)^{-\upsilon_{i,w}} N'_{i,t} \tag{2.47}$$

The zero profit conditions imply that the wage indexes are as follows: $W_{i,t} = \left[\int_0^1 \left(W_{i,t}^{\iota}\right)^{1-\upsilon_{i,w}} di\right]^{\frac{1}{1-\upsilon_{i,w}}}$, and $W'_{i,t} = \left[\int_0^1 \left(W'^{\iota}_{i,t}\right)^{1-\upsilon_{i,w}} di\right]^{\frac{1}{1-\upsilon_{i,w}}}$.

Households Wage Decisions

Wages, similarly to prices are subject to Calvo frictions. In particular every period a share $0 < (1-\varpi_w) < 1$ of households are able to reoptimise their wages and the remaining share ϖ_w are not. We assume that the fraction of households that cannot reoptimise their wages is identical across all types of households. Households which are not able to set their prices simply follow the partial indexation rules i.e: $W_{i,t} = \left(\frac{P_{c,t-1}}{P_{c,t-2}}\right)^{\gamma_w} W_{i,t-1}$, and $W'_{i,t} = \left(\frac{P_{c,t-1}}{P_{c,t-2}}\right)^{\gamma_w} W'_{i,t-1}$. Households that are able to set their wages choose $\tilde{W}_t$ and $\tilde{W}'_t$ and maximise the objectives of the following form:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi_w)^l \left\{ \begin{array}{c} -\frac{\varepsilon_t^L}{1+\sigma_L} \left[\begin{array}{c} \left(\left(\frac{\tilde{W}_{c,t}X_{tl}}{W_{t+l}}\right)^{-\upsilon_{cw}} N_{c,t+l}\right)^{(1+\varsigma)} \\ + \left(\left(\frac{\tilde{W}_{h,t}X_{tl}}{W_{t+l}}\right)^{-\upsilon_{h,w}} N_{h,t+l}\right)^{(1+\varsigma)} \end{array} \right]^{\frac{1+\sigma_L}{1+\varsigma'}} \\ +\lambda_{i,t+l}^{\iota} \left(1-\tau_t^l\right) \frac{\tilde{W}_{i,t}}{P_{t+l}} X_{i,tl} \left(\frac{\tilde{W}_{i,t}X_{tl}}{W_{i,t+l}}\right)^{-\upsilon_{i,w}} N_{i,t+l} \end{array} \right\} \tag{2.48}$$

$$E_t \sum_{l=0}^{\infty} (\gamma\varpi_w)^l \left\{ \begin{array}{c} -\frac{\varepsilon_t^L}{1+\sigma'_L} \left[\begin{array}{c} \left(\left(\frac{\tilde{W}'_{c,t}X_{tl}}{W_{t+l}}\right)^{-\upsilon_{cw}} N'_{c,t+l}\right)^{(1+\varsigma')} \\ + \left(\left(\frac{\tilde{W}'_{h,t}X_{tl}}{W_{t+l}}\right)^{-\upsilon_{h,w}} N'_{h,t+l}\right)^{(1+\varsigma')} \end{array} \right]^{\frac{1+\sigma'_L}{1+\varsigma'}} \\ +\lambda'^{\iota}_{i,t+l} \left(1-\tau_t^l\right) \frac{\tilde{W}'_{i,t}}{P_{t+l}} X_{i,tl} \left(\frac{\tilde{W}'_{i,t}X_{tl}}{W'_{i,t+l}}\right)^{-\upsilon_{i,w}} N'_{i,t+l} \end{array} \right\} \tag{2.49}$$

where $X_{tl} = \pi_t \times \pi_{t+1} \times ... \times \pi_{t+l-1}$ for $l \geqslant 1$ and $X_{tl} = 1$ for $l = 0$ as in Christiano *et.al.* (2005). The first order conditions result in the equations for newly optimised wages:

$$\sum_{l=0}^{\infty}(\beta\varpi_w)^l L^{\iota}_{i,t+l}\lambda_{t+l}\left\{\frac{\tilde{W}_{i,t}X_{tl}}{P_{t+l}}\lambda_{t+l}-\frac{\upsilon_{i,w}}{(1-\upsilon_{i,w})}U_{i,l,t+l}\right\} = 0 \tag{2.50}$$

$$\sum_{l=0}^{\infty}(\gamma\varpi_w)^l L'^{\iota}_{i,t+l}\lambda_{t+l}\left\{\frac{\tilde{W}'_{i,t}X_{tl}}{P_{t+l}}\lambda'_{t+l}-\frac{\upsilon_{i,w}}{(1-\upsilon_{i,w})}U'_{i,l,t+l}\right\} = 0 \tag{2.51}$$

The first-order conditions imply that households set wages so that the present value of the marginal utility of income from an additional unit of labour is equal to the markup over the present value of the marginal disutility of work. When all households are able to renegotiate their wage contracts each period, wage becomes as follows: $\frac{\tilde{W}_{i,t}}{P_t} = \frac{\upsilon_{i,w}}{(\upsilon_{i,w}-1)}\frac{U_{l,i,t}}{U_{c,t}}\frac{p_{c,t}(1+\tau^c_t)}{\left(1-\tau^L_t\right)}$ and $\frac{\tilde{W}'_{i,t}}{P_t} = \frac{\upsilon_{i,w}}{(\upsilon_{i,w}-1)}\frac{U'_{l,i,t}}{U'_{c,t}}\frac{p_{c,t}(1+\tau^c_t)}{\left(1-\tau^L_t\right)}$. Taking partial wage indexation into consideration, the wage indexes can be transformed into the following:

$$W_{i,t} = \left[(1-\varpi_w)\tilde{W}_{i,t}^{1-\upsilon_{i,w}} + \varpi_w\left(\pi^{\gamma_w}_{t-1}W_{i,t-1}\right)^{1-\upsilon_{i,w}}\right]^{\frac{1}{1-\upsilon_{i,w}}} \tag{2.52}$$

$$W'_{i,t} = \left[(1-\varpi_w)\tilde{W}'^{1-\upsilon_{i,w}}_{i,t} + \varpi_w\left(\pi^{\gamma_w}_{t-1}W'_{i,t-1}\right)^{1-\upsilon_{i,w}}\right]^{\frac{1}{1-\upsilon_{i,w}}} \tag{2.53}$$

2.2.3 Fiscal and Monetary Policy

The nominal interest rate follows a Taylor-type rule:

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\left[\rho_\pi\hat{\pi}_{c,t} + \rho_y\left(\widehat{GDPP}_t - \widehat{GDPP}_{t-1}\right)\right] + \eta^m_t \tag{2.54}$$

where $\hat{\eta}^m_t \sim N(0,\sigma^2_z)$ are $i.i.d.$- normally distributed errors, and as in Iacoviello and Neri (2010) monetary policy does not respond to movements in house prices. Therefore: $\widehat{GDPP}_t = \frac{Y}{GDPP}\hat{Y}_t + \frac{qHI}{GDPP}\widehat{HI}_t - \frac{M}{GDPP}\hat{M}_t$.

Equation (2.55) presents a government budget constraint, which requires the total expenditure of government on consumption G_T, investment IG_T, transfers TR and the repayment of last-period debt with interests $\left(\frac{R_{t-1}}{\pi_t}\right) b_{t-1}$, must be equal to the revenue from taxes and new bond issuance.

$$\begin{aligned} & \tau_t^c p_c \left(C_t + C_t'\right) + \tau_t^h q \left(H_t + H_t'\right) + \tau_t^k r_{k,c,t} u_{c,t} K_{c,t-1} + \tau_t^k r_{k,h,t} u_{h,t} K_{h,t-1} \\ & + \tau_t^l \left(w_{c,t} L_{c,t} + w_{c,t}' L_{c,t}' + w_{h,t} L_{h,t} + w_{h,t}' L_{h,t}'\right) + b_t \\ = & \left(\frac{R_{t-1}}{\pi_t}\right) b_{t-1} + p_{g,t} G_{T,t} + p_{ig,t} IG_{T,t} + TR_t \end{aligned} \tag{2.55}$$

The public capital accumulation equation is represented by:

$$K_{g,t} = \left(1 - \delta_{k,g}\right) K_{g,t-1} + IG_{T,t} \tag{2.56}$$

The fiscal policy instruments rules are set similarly to Leeper *et al.* (2010). We assume that the government expenditure instruments respond counter cyclically to the movements in debt and GDP, whereas tax rates respond to them procyclically.

$$\hat{Z}_t = \phi_{z,g} \hat{b}_{t-1} + \phi_{z,GDP} \widehat{GDP}_t + e_{z,t} \tag{2.57}$$

where $z = \{g, ig, tr, \tau^c, \tau^k \tau^l\}$ and $Z = \{-G, -IG, -TR, \tau^c, \tau^k \tau^l\}$. Fiscal policy shocks affect the revenue and spending sides of the government. All the shocks follow first-order autoregressive processes: $\hat{e}_{z,t} = \rho^z \hat{e}_{z,t-1} + \hat{\eta}_{x,t}$ where $\hat{\eta}_{z,t} \sim N\left(0, \sigma_z^2\right)$ are $i.i.d.$-normally distributed errors.

2.2.4 Aggregation and Market clearing

The output of non-residential goods market consists of public and private investment, exports and construction imputes. The final goods market is in equilibrium when the aggregate supply equals the aggregate public and private demand for goods. The housing market produces new homes. It is in equilibrium when the supply of new houses equals the expenditure of households on these. The foreign bond market is in equilibrium when the net export equals the households' holdings of foreign bonds. The labour market is in equilibrium when for each type of labour, the total labour demanded by the intermediate firms equals total labour supplied by households at a wage rate set by unions. The capital rental market is in equilibrium when for each type of capital, capital supplied by patient households is equal to the capital demanded by intermediate producers at a market rental rate. The borrowing market is in equilibrium when total loans of patient households equal total borrowings of impatient households. The non-residential and residential goods market clearing conditions are represented respectively by:

$$Y_t - a(u_{h,t})K_{c,t-1} - a(u_{c,t})K_{h,t-1} = I_{d,t} + C_{d,t} + IG_{d,t} + G_{d,t} + CO_t + EX_t \quad (2.59)$$

$$HI_t = IH_t + IH'_t = H_t + H'_t - (1-\delta_h)\left(H_{t-1} + H'_{t-1}\right)$$

Market clearing for both types of capital are represented respectively by:

$$\int_0^1 K_{c,j,t} dj = \int_0^1 K^{\iota}_{c,t} d\iota$$

$$K_{h,t} = \int_0^1 K_{h,t}^{\iota} d\iota$$

The relation between labour demand and labour supply can be derived from equations (2.46) and (2.47). Integrating the equation over all households we obtain:

$$\begin{aligned}
\int_0^1 L_{i,t}^{\iota} &= L_{i,t}^{S} = \int_0^1 \left(\frac{W_{i,t}^{\iota}}{W_{i,t}}\right)^{-\upsilon_{i,w}} d\iota N_{i,t} \\
\int_0^1 L_{i,t}^{\prime\iota} &= L_{i,t}^{\prime S} = \int_0^1 \left(\frac{W_{i,t}^{\prime\iota}}{W_{i,t}^{\prime}}\right)^{-\upsilon_{i,w}} d\iota N_{i,t}^{\prime}
\end{aligned}$$

where $i\epsilon\{c,h\}$; $L_{i,t}^{S}$ and $L_{i,t}^{\prime S}$ denote labour supply, and labour demand is represented by: $N_{i,t} = \int_0^1 N_{i,j,t} dj$ and $N_{i,t}^{\prime} = \int_0^1 N_{i,j,t}^{\prime} dj$. Denoting $o_{i,t} = \int_0^1 \left(\frac{W_{i,t}^{\iota}}{W_{i,t}}\right)^{-\upsilon_{i,w}} d\iota$, and $o_{i,t}^{\prime} = \int_0^1 \left(\frac{W_{i,t}^{\iota}}{W_{i,t}}\right)^{-\upsilon_{i,w}} d\iota$, the relation between labour demand and supply can be summarised by:

$$\begin{aligned}
L_{i,t}^{S} &= o_{i,t} N_{i,t} \\
L_{i,t}^{\prime S} &= o_{i,t}^{\prime} N_{i,t}^{\prime}
\end{aligned}$$

Market clearing in the loan merket is represented by:

$$\int_0^1 LO_t^{\prime\iota} d\iota = \int_0^1 D_t^{\iota} d\iota$$

The evolution of net foreign assets at the aggregate level satisfies

$$S_t B_{f,t}^{\iota} + S_t P_t^{*} \left(C_{m,t} + I_{m,t} + G_{m,t} + IG_{m,t}\right) = S_t P_{x,t} EX_t + S_t B_{f,t-1}^{\iota} R_{t-1}^{*} risk_{t-1} \quad (2.60)$$

The current account is represented by:

$$CA_t = \frac{S_t B^{\iota}_{f,t}}{P_t} - \frac{S_t B^{\iota}_{f,t-1}}{P_t} = \frac{TB_t}{P_t} + \frac{\left(R^*_{t-1} - 1\right) S_t B^{\iota}_{f,t-1} risk_{t-1}}{P_t}$$

where TB denotes the trade balane given by:

$$TB_t = P_{x,t} S_t EX_t - P^*_t S_t \left(C_{m,t} + I_{m,t} + G_{m,t} + IG_{m,t}\right)$$

The steady-state and the log-linearized equations describing the equilibrium of the mode are represented respectively in Appendix $(2.A)$ and $(2.B)$

2.3 Bayesian Estimation

Perturbation techniques are used to solve the model and Bayesian methods to estimate it[43]. The model is estimated on quarterly data from the UK for the period from 1987Q2 to 2011Q1. Sims's $csminwel$ function is used as the optimiser for the mode computation. The acceptance ratios obtained in the Metropolis-Hasting algorithm simulation are approximately 0.24, which is in line with Gelman *et al.* (1997) who indicate an optimal acceptance rate of 0.234. Appendix $(2.C)$ describes data, data sources and measurement equations.

2.3.1 Calibration

Table (2.10) presents key steady state ratios of the model. Some of the parameters determining the steady state are estimated, which slightly affects the ratios.

[43] For solution, estimation and necessary calculations, we use Dynare 4.2.4 by Adjemian *et al.* (2011) and MATLAB.

Table 2.10. Steady state ratios

Share	Description	Value	Data
A. GDP expenditure shares			
$p_c C_T / GDP$	Private consumption to GDP	0.66	0.64
$p_i I_T / GDP$	Private business investment to GDP	0.13	0.12
qHI / GDP	Residential investment to GDP	0.04	0.03
$(p_g G_T + p_{ig} IG_T) / GDP$	Public expenditure to GDP	0.22	0.22
EX / GDP	Export to GDP	0.19	0.26
$\left(\begin{array}{c} p_{c,m} C_m + p_{i,m} I_m + \\ + p_{g,m} G_m + p_{ig,m} IG_m \end{array} \right) / GDP$	Import to GDP	0.24	0.28
B. Capital ratios			
$q(H + H') / (4 \times GDP)$	Housing wealth to annual GDP	1.05	0.89
$(K_c + K_h) / (4 \times GDP)$	Business capital to annual GDP	1.13	1.03
$K_g / (4 \times GDP)$	Public capital to annual GDP	0.32	0.31
$\left(\begin{array}{c} q(H + H') + \\ + K_c + K_h + K_g \end{array} \right) / (4 \times GDP)$	Total capital to annual GDP	2.50	2.22
$K_h / (qH + qH' + K_c + K_h + K_g)$	Capital in the housing sector to total capital	0.01	0.01
C. Government expenditure in detail			
$p_g G_T / GDP$	Public consumption to GDP	0.20	0.20
$p_{ig} IG_T / GDP$	Public investment to GDP	0.02	0.02
TR / GDP	Public transfers to GDP	0.24	0.13
$B / (4 \times GDP)$	Government debt to annual GDP	0.72	0.51

The GDP expenditure shares in Section A do not add to 1 due to the data rounding. Data used for calculation of first four capital ratios are from: Vaze et al. (2003), Griffin et al. (2006), and ONS (2010), whereas data used for calculation of the last capital ratio from O'Mahony and Timmer (2009). Data used for other shares are from The Office for National Statistics webpage.

We choose the discount factor of patient households $\beta = 0.99$, which implies a steady state nominal interest rate of 4 per cent. The discount factor of impatient households, γ, is set respectively at 0.96 as in Darracq Pariès and Notarpietro (2008). We calibrate a weight on housing in the utility function (j) to 0.14, which together with the value for the depreciation rate of housing $(\delta_h = 0.01)$ pins down the share of residential investment to GDP at roughly 4 per cent as in the data. The share of capital in the non-residential production function (α) is set to 0.3 as in Smets and Wouters (2003) and Harrison and Oomen (2010). This implies that the share of labour income from the non-residential sector is equal to

70 per cent. To maintain the same labour intensity as in the housing production sector we choose the share of capital $\alpha_h = 0.2$, similar to Darracq Pariès and Notarpietro (2008) for the EU, and the share of non-residential inputs $(\alpha_{co} = 0.1)$, as in Iacoviello and Neri (2010). Following the existing literature, the depreciation rate in the non-residential sector $(\delta_{k,c})$ is set to 0.025. The depreciation rate of the capital used in the residential construction sector $(\delta_{k,h})$ is calibrated at a slightly higher level (0.04) to reflect the information in the EU KLEMENS data (see O'Mahony and Timmer, 2009). The above calibration together with the values for the capital tax rate and a markup in the investment sector (see below) implies that business capital stock comprises roughly 113 per cent of annual GDP.

Turning to the fiscal policy parameters, we set the consumption and labour effective tax rates to match the rates implied by the data for the sample period. This implies values of 20 per cent for the consumption tax rate, and 29 per cent for the labour tax rate. The effective tax on housing $\left(\tau^h = 0.5\%\right)$ is calibrated as in Harrison *et al.* (2005). We calibrate the level of the effective capital tax rate $\left(\tau^k = 0.4\right)$ slightly higher than the level implied by the data, to match more accurately the ratio of business investment to GDP. The ratios of public consumption and public investment to GDP are calibrated to match their empirical counterparts over the sample period. This implies a share of public consumption in GDP at 20 per cent and the share of public investment in GDP at 2 per cent. We set a share of transfers at a higher level $(tr = 24\%)$ so that the endogenous annualized public debt to GDP ratio is pinned down at a reasonable level (0.71). Finally, the depreciation rate of public capital $(\delta_g = 0.015)$, together with the ratio of public investment to GDP pins down the ratio of public capital stock to annual GDP at 0.32 to match the ONS data on the public

capital stock and public investment. The elasticity of public capital to output (σ_g) is set to 0.01 which is similar to the value calibrated by Straub and Tcharkov (2005) for the US and the euro area.

Turning to the open-economy parameters, we calibrate the share parameters in the CES functions in line with the input-output analytical tables' data[44]. The share of imports in consumption equals approximately 22 per cent, the share of imports in private investment equals approximately 32 per cent, the share of imports in public consumption equals around 11 per cent, and the share of imports in public investment we assume to be roughly 16 per cent. Table (2.10) indicates that the value of imports is larger than the value of exports. This is due to the fact that values in the Table (2.10) are at consumer prices; therefore they include already markups imposed by domestic importers. At the border the value of export is equal to the value of import $(19\%$ of GDP$)$ so that the balanced trade is ensured in the steady state. The share of exports and imports in GDP is smaller than that implied in the data mainly due to the fact that imported exports are not modelled.

Finally, we calibrate the wage markup in the non-residential sector to 1.05 as in Christiano *et al.* (2005), and set the average LTV for the constrained households to 0.85 as in Iacoviello and Neri (2010) which is a reasonable choice for the UK. The average loan to value ratio in the UK for the period between 1987 and 2010 was oscillating at around 68 per cent, whereas those for first time buyers at 81 per cent[45].

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[44] We set the share of imports' parameters in the CES functions to match roughly the ratios implied by the input-output analytical tables from 1990, 1995 2005.

[45] Data from: The Department for Communities and Local Government; live tables on housing market and house prices, Table 513.

2.3.2 Prior Selection

The assumptions about priors are presented in Appendix $(2.D)$. We set them in line with the existing literature. We choose gamma priors with means of 0.15 and standard deviations of 0.1 for all the markups. The prior means for the indexation parameters are set to 0.3, whereas the prior means of the Calvo price and wage stickiness are centred at 0.5, with a standard deviation of 0.1. For the elasticities of substitution between the imported and domestically produced goods we select inverse gamma priors with a mean of 1.5 as in Adolfson *et al.* (2007).

We use a beta distribution for the share of patient households with a mean that is equal to 0.65 and a standard deviation of 0.1, as in Iacoviello and Neri (2010). We set the means of habit formations in consumption to 0.7, close to the estimates in Harrison *et al.* (2010). For the habit formation in housing we set a prior mean to 0.5 and a standard deviation of 0.1. For parameters related to the disutility of working $(\sigma_L$ and $\sigma'_L)$ and the inverse elasticity of substitution across hours in the two sectors $(\varsigma$ and $\varsigma')$, we follow closely Iacoviello and Neri (2010) and set values respectively at: $\sigma_L = \sigma'_L = 0.5$, and $\varsigma = \varsigma' = 1$, with standard deviations equal to 0.1.

We select priors with normal distribution for the fiscal policy response parameters. For the debt aversion parameters, we set a prior mean to 0.2 and a standard deviation to 0.1, whereas for GDP controlling parameters we choose a prior mean equal to 0.5 and a standard deviation that is equal to 0.5. For the monetary policy rule, we centre the degree of interest rate smoothing prior at 0.7, and for the Taylor rule coefficient on inflation and output we choose a normal distribution and set means respectively at 1.5 and 0.125. We

select a normal distribution prior for the capital adjustment cost parameter with a mean of 4, as in Smets and Wouters (2003), and choose a normal prior for the capital utilisation parameter with a mean of 0.8 and a standard deviation of 0.2.

For the autoregressive parameters of shocks, we choose a beta distribution prior with a mean of 0.8 and a standard deviation of 0.1 with the exception of investment shocks where we set a prior equal to 0.5. For all the shocks we choose inverse gamma priors with a mean of 0.01 with the exception of investment, housing demand, and cost and wage push up shocks for which we select the mean of 0.1. Following Adolfson *et al.* (2007) we select inverse gamma prior for the risk premium parameter with a mean of 0.01.

2.3.3 Posterior Estimates

The details of posterior estimates are presented in Appendix $(2.D)$. The parameter determining the domestic price stickiness is estimated at a level of 0.72. This implies that domestic prices adjusts roughly once every four quarters to changes in the marginal cost[46]. Similarly to Millard (2011), we obtain lower price stickiness in the importing sector. The estimates imply that prices of imports and exports react to the changes in markups roughly every $1-2$ quarters. Low estimates of all the price indexation parameters, ranging from 0.08 to 0.13, indicate that the estimated Philips curves are mostly forward-looking.

The estimate of the wage stickiness parameter in the non-residential sector ($\omega_w = 0.86$, similar to Harrison and Oomen, 2010) indicates that the wage in this sector reacts to the changes in the markup once every 7 quarters. The wage in the residential sector adjusts

[46] The positive estimates of price indexation parameters imply that prices change every period.

to the changes in markup once every 5 months, which points to a higher efficiency in this sector. This seems to be consistent with our assumption of flexible prices in the housing production sector. The wage indexation parameters are estimated also at relatively low levels: at 0.27 and 0.26 for non-residential and residential wage respectively.

The estimate of the domestic markup (43.7%) is close to Smets and Wouters (2003). It implies that the elasticity of substitution of domestically produced goods is equal to 3.3. Regarding the markups in the importing industries, those are higher in the private than in the public sector, suggesting that the willingness to substitute among the imported goods in the private sector is relatively lower than in the case of public sector.

Similarly to Iacoviello and Neri (2010) we obtain larger consumption habit for lenders. According to our estimates patient agents exhibit a degree of habit formation in consumption equal to $0.78,$ which is a bit higher than 0.69 in Harison and Oomen (2010) and close to Di Cecio and Nelson (2007), who similarly to us assume log utility. Impatient households exhibit relatively high habit formation of 0.95. The habit formation in the housing consumption is estimated at 0.63 for patient households and 0.48 for impatient households respectively.

The share of impatient households' labour in production is estimated to be 22%. This is close to result obtained by Iacoviello and Neri (2010) for the USA.

The estimates of the investment adjustment cost parameters imply that a one per cent increase in the price of capital in the non-residential and in the residential sector is followed respectively by approximately 20%, and 30% increase in the relevant investment. Smets and Wouters (2003) estimate this elasticity at 16% for the euro area, whereas Christiano *et*

al. (2005) estimate it at 38% for the USA. The estimate of the capital utilisation adjustment parameter $(\kappa = 1.47)$ implies that a one percent increase in the net rental rate of capital results in a 0.68% increase in the capital utilisation.

Turning to the fiscal policy parameters, we find that public investment expenditure along with taxes on consumption and labour control relatively stronger for the debt than the remaining instruments. In controlling for GDP fluctuations we find labour taxes and public investment respond the strongest and public consumption the weakest. Regarding the fiscal policy shocks, government consumption yields the highest persistence, whereas the public investment the lowest.

The estimates of monetary policy parameters take the following values: persistence parameter $\rho = 0.54$; the response to inflation $\rho_\pi = 1.63$; and the response to the output $\rho_y = 0.20$ and are in line with previous studies.

2.3.4 Model Comparison

In order to assess the empirical strength of the model we compare its fit with BVARs as in Smets and Wouters (2003) and Juliard *et al.* (2006). The marginal likelihoods of 8 BVAR models of lag order from 1 to 8 are presented in Table (2.11)[47]. The last row presents the likelihood of our DSGE model estimated over the sample from 1989:Q3 to 2011:Q1. The results indicate that the DSGE model has better fit than the related BVARs.

[47] We follow Juillard et al. (2006) and Ratto et al. (2009) in setting the relevant parameters. We choose the prior decay parameter to 0.5, the tightness of the prior parameter to 3, the parameter determining the weight on own-persistence to 2, and the parameter determining the degree of co-persistence to 5.

Table 2.11. Comparison of marginal likelihoods with BVAR models

	Marginal Likelihood
BVAR(1)	3005.23
BVAR(2)	3486.05
BVAR(3)	3570.07
BVAR(4)	3771.58
BVAR(5)	3820.56
BVAR(6)	3893.52
BVAR(7)	3877.50
BVAR(8)	3867.81
DSGE model	3897.50

2.4 Model Implications of Fiscal Policy

In this section we look at the model implications of fiscal policy. First, we provide theoretical considerations and then we focus on dynamics and multipliers. The motivation for including the theoretical part is to help understand the dynamics of the model.

2.4.1 Note on the Decision on the Consumption of Durables and Non-Durables

Below we discuss the decision process of both types of households regarding the expenditure on a non-durable consumption bundle and a durable housing good.

Patient households

-Non-residential consumption decision

Neglecting the preference shocks, the equation determining the lender's consumption of non-durable goods is standard in its form and is represented by:

$$\hat{C}_t = \frac{E_t\hat{C}_{t+1}}{1+h} + \frac{h\hat{C}_{t-1}}{1+h} - \frac{1-h}{1+h}\left(R_t - E_t\hat{\pi}_{ct,t+1}\right)$$

where a hat above the variable denotes its log deviation from the steady state, and $\hat{\pi}_{ct,t+1}$ denotes the consumer price inflation[48]. For patient households the determinants of consumption comprise: the *ex ante* real interest rate (expressed in the units of consumer prices), and the weighted future and past consumption. Higher levels of habit formation imply that consumption depends less on the real interest rate and the consumption expectations, but more on the lagged consumption. Subsequently, the response of non-durable consumption becomes hump shaped (for more details see Chriastiano *et al.* ,2005).

Housing demand and the user cost

The equation linking the consumption of a durable good with the consumption of a non-durable good stems from the first-order condition with respect to housing and is represented by:

$$H_t = \frac{jC_t}{Z_{ct,t}}$$

where:

$$Z_{ct,t} = \frac{U_h}{U_c} = \left(1+\tau^h\right) q_{ct,t} - E_t\left[\frac{\pi_{c,t+1}}{R_t} q_{ct,t+1}\left(1-\delta_h\right)\right]$$

[48] Note that: $\pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}}$, whereas $\pi_{ct,t} = \frac{(1+\tau_t^c)P_{c,t}}{\left(1+\tau_{t-1}^c\right)P_{c,t-1}}$. The reason that we distinguish between $(\pi_{ct,t})$, and $(\pi_{c,t})$ is that the first is the price paid by households, and the second is the price targeted by the central bank (gross of consumption tax). Also:
$q_t = \frac{P_{h,t}}{P_t}$; and $q_{ct,t} = \frac{P_{h,t}}{(1+\tau_t^c)P_{c,t}}$

$Z_{ct,t}$ denotes a lender's user cost of a one unit of a durable good (expressed in the units of consumer prices), $\left(1+\tau^h\right) q_{ct,t}$ denotes the total time t expenditure on a one unit of housing, and $\frac{\pi_{c,t+1}}{R_t} q_{ct,t+1}\left(1-\delta_h\right)$ represents discounted expected resale value of a one unit of housing. The user cost of the durable good log-linearises to:

$$\begin{aligned}\hat{Z}_{ct,t} &= \frac{1+\tau^h}{\left[1+\tau^h-\left(1-\delta_h\right)\beta\right]}\left(\hat{q}_{ct,t}+\frac{\tau^h}{1+\tau^h}\hat{\tau}_t^h\right) \\ &\quad -\frac{\left(1-\delta_h\right)\beta}{\left[1+\tau^h-\left(1-\delta_h\right)\beta\right]} E_t\left(\hat{q}_{ct,t+1}-\left(\hat{R}_t-\hat{\pi}_{ct,t+1}\right)\right) \qquad (2.61)\end{aligned}$$

Equation (2.61) indicates that the borrower's user cost of housing depends positively on the current price of housing, the housing tax, the *ex ante* real interest rate, and negatively on the expected price of housing.

If we assume housing services to be a non-durable good $(\delta_h = 1)$ the housing condition simplifies to: $\left(1+\tau_t^h\right) q_t H_t = j p_{c,t}\left(1+\tau_t^c\right) C_t$, where the level of j determines the ratio of the non-residential to the residential consumption expenditure. Naturally, housing is a durable good, $\delta_h \in (0,1)$, and once purchased, its undepreciated part can be used next period. Hence, the decision on consumption of housing services requires equating the marginal rate of substitution of housing for consumption to the user cost of durables.

The shadow value of housing

In order to show how the housing price is determined we follow Barsky *et al.* (2007) and Monacelli (2009). For this reason we denote a shadow value of one unit of durables as $V_t = \left(1+\tau_t^h\right) q_{h,t}\lambda_t$, which log-linearises to:

$$\begin{aligned}\hat{V}_t &= \frac{\tau^h}{1+\tau^h}\hat{\tau}_t^h + \hat{q}_t + \hat{\lambda}_t \\ &= \frac{1+\tau^h-\beta\left(1-\delta_h\right)}{1+\tau^h}E_t\sum\nolimits_{j=0}^{+\infty}\left(\frac{\beta\left(1-\delta_h\right)}{1+\tau^h}\right)^j\hat{U}_{h,t+j} \\ &\quad -\frac{\tau^h}{1+\tau^h}E_t\sum\nolimits_{j=0}^{+\infty}\left(\frac{\beta\left(1-\delta_h\right)}{1+\tau^h}\right)^{j+1}\hat{\tau}_{t+1+j}^h \qquad (2.62)\end{aligned}$$

In the model, tax on housing is simply an AR(1) process, it does not respond neither to debt or to GDP, therefore $\frac{\tau^h}{1+\tau^h}E_t\sum_{j=0}^{+\infty}\left(\frac{\beta(1-\delta_h)}{1+\tau^h}\right)^{j+1}\hat{\tau}_{t+1+j}^h = 0$. Similarly, the other term on the right hand side of equation (2.62) is equal approximately to zero. The reason is that $\hat{U}_h$ is determined by the stock of durables $\hat{H}$, which on the other hand depends on housing investment $\widehat{IH}_t$. Because the ratio of housing stock to housing investment is high (in the steady state $\frac{1}{\delta_h} = \frac{H}{IH}$), then even a large housing investment induces a small effect on the housing stock and thus utility stemming from housing services. Therefore $E_t\sum_{j=0}^{+\infty}\left(\frac{\beta(1-\delta_h)}{1+\tau^h}\right)^j\hat{U}_{h,t+j}$ is a small number, approximately equal to 0. Given that the right hand side of (2.62) is equal approximately 0, it must be also true that $\hat{V}_t \approx 0$, therefore $\hat{q}_t$ and $\hat{\lambda}_t$ have opposite signs, i.e. when $\hat{\lambda}_t$ is positive, then $\hat{q}_t$ is negative, and when $\hat{\lambda}_t$ is negative, then $\hat{q}_t$ is positive.

Impatient households

Consumption of residential and non-residential good

The decision of impatient households regarding the expenditure on a non-durable consumption bundle, and a durable housing good can be simplified to two equations. We

start the analysis by treating housing as a non-durable good i.e. we set $\delta_h = 1$. The two equations then become:

$$\begin{aligned} p_{c,t}\left(1+\tau_t^c\right)C_t' + q_t\left(1+\tau_t^h\right)H_t' &= L_{inc,t}' + TR_t \\ q_t H_t'\left(1+\tau_t^h\right) &= j p_{c,t}\left(1+\tau_t^c\right)C_t' \end{aligned}$$

where we use $L_{inc,t}' = \left(1-\tau_t^l\right)\left(w_{c_b,t}' L_{c_b,t}' + w_{h_b,t}' L_{h_b,t}'\right)$. In that limited situation, the credit constraint households become rule-of-thumb households (see Gali *et al.*, 2007) with an access to two types of consumption bundles, one with flexible prices and the other with sticky prices. A lack of the durable good in the balance sheet implies that impatient households can no longer borrow, which implies that they do not participate in the financial markets. They simply spend their whole disposable income stemming from labour and transfers on the consumption of both types of goods. The expenditure ratio is pinned down by the parameter j. The share of income spent on non-residential consumption totals $\frac{1}{(1+j)}$, whereas that on housing $\frac{j}{(1+j)}$.

For the benchmark calibration two equations determining the expenditure of impatient households are represented by:

$$\begin{aligned} p_{c,t}\left(1+\tau_t^c\right)C_t' + q_t\left(1+\tau_t^h\right)H_t' &= L_{inc,t}' + TR_t + \tau(1-\delta_h)q_t H_{t-1}' \\ &\quad + (1-\vartheta)(1-\delta_h)E_t\left[q_{t+1}H_t'\frac{\pi_{t+1}}{R_t}\right] \\ Z_{c,t}' H_t' &= jC_t' \end{aligned}$$

Similarly to patient households, the expenditure of borrowers on housing depends on non-residential consumption and the user cost of housing. Impatient households choose the bundle of durables so that the marginal rate of substitution of housing for consumption equals the user cost of housing (Z'_{ct}) which is represented by:

$$\begin{aligned} Z'_{ct,t} &= \frac{U'_h}{U'_c} = \left(1+\tau^h\right) q_{ct,t} - \tau\left(1-\delta_h\right) E_t\left[q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}\left(1-\lambda_{b,t}\right)\right] \\ &\quad - \left(1-\vartheta\right)\left(1-\delta_h\right) E_t\left[q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}\right] \end{aligned}$$

where $\left(1-\vartheta\right)\left(1-\delta_h\right) E_t\left[q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}\right]$ denotes the present value of borrowing for lenders, and $\vartheta\left(1-\delta_h\right)\left(1-\lambda_{b,t}\right) E_t\left(q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}\right)$ denotes the present value of the downpayment for borrowers. $\frac{R_t}{E_t\pi_{t+1}}\frac{1}{\left(1-\lambda_{b,t}\right)}$ can be treated as a discount rate of credit constrained households. From equation (2.42) we can see that: $\frac{\lambda'_t}{E_t\lambda'_{t+1}\gamma} = \frac{R_t}{E_t\pi_{t+1}}\frac{1}{\left(1-\lambda_{b,t}\right)}$, therefore when $\lambda_{b,t} > 0$, which is the case in the model, the discount rate of impatient households is higher than that prevailing interest rate in the market, which implies that impatient households are always keen to borrow. The user cost log-linearises to:

$$\begin{aligned} \hat{Z}'_t &= \frac{1+\tau^h}{\{1+\tau^h-\psi\}}\left(\hat{q}_{ct,t} + \frac{\tau^h}{1+\tau^h}\hat{\tau}^h_t\right) + \frac{\vartheta\gamma\left(1-\delta_h\right)\left(\beta-\gamma\right)}{\{1+\tau^h-\psi\}}\hat{\lambda}_{b,t} \\ &\quad - \frac{\psi}{\{1+\tau^h-\psi\}} E_t\left(\hat{q}_{ct,t+1} - \left(\hat{R}_t - \hat{\pi}_{ct,t+1}\right)\right) \end{aligned} \tag{2.63}$$

where $\psi = \left[\beta + \vartheta\left(\gamma-\beta\right)\right]\left(1-\delta_h\right)$. Equation (2.63) implies that the impatient households' user cost of housing depends positively on the shadow value of borrowing, the current price of housing, the housing tax, the *ex ante* real interest rate and negatively on the expected price of housing. The user cost of housing for impatient households simplifies

to the user cost of housing of patient households for $\gamma = \beta$. The difference stems mainly from the presence of the shadow value of borrowing which is a variable signalling tightening or loosening of the borrowing constraint. The aggregate expenditure on non-residential consumption and on housing services depends on the after tax income from labour and transfers, $L'_{inc,t} + TR_t$; present value of last period downpayment, $\vartheta(1-\delta_h)q_t H'_{t-1}$; and the value of the constraint, $(1-\vartheta)(1-\delta_h)E_t\left[q_{t+1}H'_t\frac{\pi_{t+1}}{R_t}\right]$.

The shadow value of housing

The shadow value of a durable good for impatient households is represented by: $V'_t = \left(1+\tau^h_t\right)q_t\lambda'_t$. Using the first-order condition with respect to housing it log-linearises to:

$$\begin{aligned}\hat{V}'_t &= \frac{\tau^h}{1+\tau^h}\hat{\tau}^h_t + \hat{q}_t + \hat{\lambda}'_t \qquad (2.64)\\ &= \frac{1+\tau^h - \left[(\beta-\gamma)(1-\vartheta)+\gamma\right](1-\delta_h)}{1+\tau^h-(\beta-\gamma)(1-\vartheta)(1-\delta_h)}E_t\left\{\sum\nolimits_{j=0}^{+\infty}\left(\frac{\beta(1-\delta_h)}{1+\tau^h}\right)^j\hat{U}'_{h,t+j}\right\}\\ &\quad -E_t\left\{\sum\nolimits_{j=0}^{+\infty}\left(\frac{\gamma(1-\delta_h)}{K(1+\tau^h)}\right)^j\hat{K}_{t+j} + \frac{\tau^h}{1+\tau^h}\sum\nolimits_{j=0}^{+\infty}\left(\frac{\gamma(1-\delta_h)}{K(1+\tau^h)}\right)^{j+1}\hat{\tau}^h_{t+1+j}\right\}\end{aligned}$$

where:

$$\hat{K}_t = -\frac{(\beta-\gamma)(1-\vartheta)(1-\delta_h)}{\left(1+\tau^h\right)-(\beta-\gamma)(1-\vartheta)(1-\delta_h)}E_t\left(\hat{\lambda}_{b,t} - \frac{\tau^h}{\left(1+\tau^h\right)}\hat{\tau}^h_t - \left(\hat{R}_t - \hat{\pi}_{h,t+1}\right)\right)$$

Equation (2.64) simplifies to the shadow value of housing for patient households when $\beta = \gamma$[49]. The key difference with respect to the shadow value of housing for patient households is the presence of $E_t\left\{\sum_{j=0}^{+\infty}\left(\frac{\gamma(1-\delta_h)}{K\left(1+\tau^h\right)}\right)^j\hat{K}_{t+j}\right\}$. K depends on the shadow

[49] When $\beta = \gamma$, then $K = 1$, $\hat{K}_t = 0$, and $V' = V$.

value of borrowing, nominal interest rate and housing inflation, which do fluctuate in a response to fiscal policy shocks. Subsequently, the shadow value of housing for impatient households, $\hat{V}_t'$, may be significantly different from 0 which implies that $\hat{q}_t$, and $\hat{\lambda}_t'$ do not have to have to have opposite signs.

2.4.2 Impulse Response Functions and Fiscal Multipliers

This section presents impulse response functions and present value multipliers of fiscal policy shocks. Impulse responses are presented for the case of a stimulus, i.e. for public consumption, investment and transfers an impulse is represented by a positive one standard deviation shock, whereas for tax rates the impulse is represented by a negative one standard deviation shock. Appendix $(2.D)$ presents the benchmark dynamics along with the sensitivity analysis.

Government expenditure

Government Consumption

The dynamics implied by a government consumption shock along with the sensitivity analysis are presented in Fig. (2.12) and (2.18). The increase in a public demand for the non-residential goods is the most persistent out of all fiscal policy shocks. It stimulates the economy, results in a higher debt as other spending instrument, but also in a higher demand for production inputs in the non-residential sector, which puts upward pressure on

Table 2.12. Government expenditure cumulative multipliers

	Quarters					
	1	4	12	20	40	1000
Fiscal multipliers of Public Consumption						
GDP	0.72	0.67	0.58	0.51	0.34	-0.10
Total consumption	-0.08	-0.15	-0.25	-0.31	-0.45	-0.82
Consumption - Lenders	-0.04	-0.09	-0.16	-0.19	-0.27	-0.44
Consumption - Borrowers	-0.03	-0.06	-0.10	-0.13	-0.18	-0.34
Business Investment	-0.03	-0.06	-0.12	-0.15	-0.18	-0.25
Residential Investment	-0.07	-0.05	-0.04	-0.04	-0.04	-0.05
Fiscal Multipliers of Public investment						
GDP	0.78	0.89	1.10	1.22	1.41	1.85
Total consumption	-0.01	-0.01	0.03	0.08	0.22	0.53
Consumption - Lenders	-0.01	-0.1	0.06	0.14	0.27	0.47
Consumption - Borrowers	-0.01	-0.02	-0.09	-0.10	-0.13	-0.05
Business Investment	0.00	0.02	0.12	0.18	0.23	0.27
Residential Investment	-0.03	-0.02	-0.00	0.01	0.01	0.03
Fiscal Multipliers of Public Transfers						
GDP	0.05	0.05	0.08	0.06	-0.09	-0.48
Total consumption	0.07	0.17	0.35	0.38	0.28	0.04
Consumption - Lenders	-0.02	-0.06	-0.13	-0.18	-0.25	-0.38
Consumption - Borrowers	0.10	0.23	0.48	0.56	0.53	0.41
Business Investment	-0.03	-0.07	-0.16	-0.20	-0.20	-0.23
Residential Investment	0.02	-0.01	-0.04	-0.04	-0.02	-0.04

the wage and rental rate of capital in this sector.[50] The real interest rate of the economy naturally increases as a result of a positive public spending shock.

The expenditure decision of patient households is mostly determined by the response of the real interest rate. As the real interest rate increases, lenders cut on the interest rate sensitive non-residential consumption and investment in physical capital. To understand why housing investment increases, it is essential to recollect the definition of the shadow value of housing $\left(\hat{V}_t = \frac{\tau^h}{1+\tau^h}\hat{\tau}_t^h + \hat{q}_{ct,t} + \hat{U}_{c,t} \approx 0\right)$. In a response to a positive government consumption shock the housing tax remains constant $\left(\frac{\tau^h}{1+\tau^h}\hat{\tau}_t^h = 0\right)$, therefore, as

[50] A drop in consumption causes that households are more willing to supply labour. In fact, this effect prevails over the labour demand effect thus wage decreases.

consumption falls $\left(\hat{C}_t < 0\right)$, the marginal utility of it increases $\left(\hat{U}_{c,t} > 0\right)$, which leads to a drop in a house price on impact $(\hat{q}_{ct,t} < 0)$. The lender's user cost of housing drops, as the house price decrease prevails over the real interest rate increase, leading to an increase in lender's demand for housing. Over the longer horizon, as the house price returns to the steady state, lender's demand for housing fades away.[51]

Turning to impatient households, their after tax labour income is the only source of income that increases in response to a public consumption shock. Transfers decrease in response to an increase in government debt and GDP. The drop in a house price implies lower net wealth and the borrowing limit and thus further stifles the spending. As a result constrained households are forced to cut on their expenditure on both goods. The response of borrowers' housing bundle returns to the steady state quicker than that of non-residential consumption, as the public spending shock leads to a persistent increase of the price of private consumption, but a drop in house prices. As house prices slowly return to the equilibrium level the borrower's housing wealth slowly regains its value. In the longer horizon, as fiscal contraction takes over, the decrease in the labour income dominates and the expenditure of borrowers remains below the steady state levels.

[51] The empirical evidence on the response of house price to the government consumption shock is limited and boils down to two papers: Afonso and Sousa (2009), and Khan and Reza (2013). The response of house prices to the public expenditure shock is similar to the one in the empirical study of Afonso and Sousa (2009) for UK. On the other hand, Khan and Reza (2013) obtain a positive response of house price to government consumption shock for the USA in the VAR analysis. Subsequently, authors incorporate public consumption into the utility (as in the first chaprter of the thesis) of Iacoviello and Neri (2010) model. They calibrate the model so that it results in a positive response of Patient households' consumption and subsequently house price to public spending shock. The estimation of the model with a public consumption in the first chapter of the thesis implies that a positive response of Patient households' consumption and therefore, a positive response of the house price is not possible for the case of UK.

Turning to the open-economy variables, the trade balance deteriorates on impact and the real exchange rate depreciates after three quarters. Table (2.12) indicates that the impact GDP multiplier is below 1; it remains positive for a longer period, but becomes negative in the longest horizon considered. Clearly in the short and long horizon a persistent increase in government consumption crowds out consumption expenditure of patient households stronger than that of the credit-constrained. Fig. (2.12) and (2.18) also presents the sensitivity of results to the real and nominal frictions in the model.[52] The assumption of flexible prices implies that a public consumption stimulus is less effective, i.e. it results in a lower GDP multiplier than in the case when prices are sticky. The reason is that: firstly prices can adjust stronger on impact, and secondly a higher increase of the real interest rate implies a stronger crowding out effect on the lenders expenditure. The assumption of flexible wages also results in a lower GDP multiplier; it implies that a government consumption shock results in a higher on impact increase in the marginal cost. This subsequently leads to a stronger response of the nominal and real interest rate, which translates into a larger crowding out of lenders' investment and consumption, and a lower price of the durable good. The labour income of impatient households increases stronger on impact, however, its effect on the aggregate expenditure are relatively small. Finally, an increase in the persistence of government consumption to a permanent shock $\left(\rho_g = 1\right)$ increases the negative wealth effect and in general leads to lower GDP multipliers in short and longer horizon,

[52] In Appendix $(2.E)$ we present also the sensitivity analysis, analogous to the one described in detail in Section (1.6). We switch off all the frictions i.e.$\frac{1}{\kappa} = h'_c = h'_h = h_c = h_h = (1 - b_1) = \phi_{k,c} = \phi_{k,h} = \varpi_{c,w} = \varpi_{h,w} = \gamma_{h,w} = \gamma_{c,w} = \varpi_p = \gamma_p = \varpi_n = \gamma_n = 0$, for $n \in \{c, m; i, m; ig, m; g, m\}$ and then turn on each friction at a time. Table (2.19) presents the present value multipliers. In general the changes of multipliers are in line with the those presented in Chapter 1.

as can be seen in Table (2.18) in Appendix $(2.E)$.[53] The exclusion of investment adjustment cost leads to a significant drop in investment. The increase in the real interest rate in the longer term, and a stronger crowding out effect on consumption of Patient households result in stronger decrease in the house price. Figure (2.18) indicates that the presence of habit formation leads to a slightly smaller decrease in house price, whereas the implications of capital utilisation for the government consumption shock are rather negligible. Finally, Figure (2.24) in Appendix $(2.E)$ indicates that as Monetary Authority responds stronger to inflation and GDP growth, which leads to a higher real interest rate, the drop in the house price in responce to government consumption shock becomes stronger.

Government Investment

In the case of a public investment shock two things need to be noted. First, the public investment shock features the lowest persistence out of all fiscal policy shocks, and second, apart from an increase in the aggregate demand it also results in an increase in the supply side of the economy. The reason is that government investment shock triggers the public capital accumulation which improves the productivity of the model economy. Subsequently, the real interest rate increases less than in the case of the public consumption shock, and the negative wealth effect of public expenditure is much lower in this case. This can be observed when comparing the multipliers of public consumption and investment shocks. Indeed the crowding out of private activity is much smaller. In fact consumption of

[53] Table (2.18) presents GDP multipliers for a permanent change in the fiscal policy. The implications of an increased persisternce of shocks are the same as in Chapter 1. For permanent shocks, the GDP multiplier increases for the labour and capital tax cuts and decreases for public expenditure shocks and the consumption tax cut.

lenders is above the steady state level after less than four quarters. The impact on business investment expenditure is positive. As in the case of a public consumption shock house prices drop on impact, but this drop is smaller. Moreover house prices are above the steady state level after three quarters. The drop in the lender's user cost of housing implies an increase in the housing demand of these households.

The labour income of impatient households increases strongly on impact. The increase is, however, outweighed by a strong decrease in transfers that react to an increase in debt and GDP. Also the drop in a house price on impact lowers the net wealth and the borrowing limit, therefore impatient households are forced to cut on the expenditure on both: a non-residential consumption good and a housing bundle.

Turning to the open-economy variables Fig. (2.13) and (2.19) indicate that the real exchange rate depreciates on impact and is above the steady state level over a long time. The trade balance deteriorates strongly on impact to be above the steady state after just $(2-3)$ quarters.

The impact GDP multiplier is less than one, it increases above one in the medium horizon, and remains on a high level over the longer horizon. Regarding households consumption expenditure, in contrast to a public consumption shock, the effects of a government investment shock are more favourable for patient households. In fact, in the longer horizon, a government investment shock has a positive effect on their private expenditure. Residential investment is crowded out in the short term, but in the long term multiplier takes the positive sign.

The implications of sensitivity analysis are in line to those in the case of public consumption shock. Because the government investment shock yields low persistence, the effects of permanent shocks are strongly visible in Figure (2.13). Indeed, as Table (2.18) indicates the drop in present value multipliers is rather significant as the infinite multiplier decreases from 1.85 to 0.10.

Government Transfers

A government transfer shock (Fig. 2.14 an 2.20) differs from the other spending shocks in the way it spills over to economy. Whereas public consumption and investment shocks result in a direct increase of purchases of non-residential output, the government transfers shock also triggers the demand for a durable good.

In the model economy the only beneficiaries of transfers are the credit-constrained households, therefore an increase in transfers translates into the broader economy via their expenditure on the consumption of a non-residential bundle and a housing good. High demand for the housing bundle implies that a flexible house price increases on impact to be below the steady state level after less than five quarters. A higher persistence of transfers shocks leads to stronger increase in house price (see Fig. 2.14). An increase in the net housing wealth of impatient households implies that the shadow price of borrowings signals the loosening of the collateral constraint and impatient households increase even stronger their consumption expenditure.

'On impact' increase in the house price along with a higher real interest rate implies that the lender's user cost of housing increases strongly on impact, which subsequently re-

sults in a drop of lender's housing demand. Interest rate sensitive business investment and consumption of Patient households are also crowded out. Nonetheless, total consumption increases. Regarding the open-economy variables, the real exchange rate depreciates and the trade balance deteriorates. The aggregate effect on GDP is the smallest out of all government spending instruments. A shock to transfers is the only public spending shock that results in a positive multiplier for impatient household's consumption (in the short and long horizon), and a positive impact multiplier for residential investment (in the short horizon).

As can be noted from Figure (2.14) and (2.20) an important frictions for the result is the wage stickiness and habit formation. For flexible wages (Fig. 2.14) as a result of transfers' shock, marginal cost jumps strongly, leading therefore to a higher real interest rate, crowding out of investment and consumption, and a drop in house price. Also, following the reasoning of Section (1.6), a decrease in habit formation leads to a stronger decrease in consumption of Lenders and an increase in consumption of Borrowes (see Fig. 2.20). Both effects can be seen as resulting in a decrease of house price on impact. First, through the mechanism related to the shadow value of housing and explained above, and second via the interest rate mechanism as higher consumption results in higher GDP multiplier and therefore higher real interest rate.

Government receipts

Consumption Tax

Table 2.13. Tax multipliers

	Quarters					
	1	4	12	20	40	1000
Fiscal Multipliers of Consumption Tax						
GDP	-0.08	-0.17	-0.32	-0.35	-0.25	0.08
Total consumption	-0.13	-0.30	-0.56	-0.61	-0.57	-0.45
Consumption - Lenders	-0.10	-0.21	-0.37	-0.42	-0.42	-0.39
Consumption - Borrowers	-0.03	-0.09	-0.19	-0.19	-0.15	-0.07
Business Investment	0.02	0.05	014	0.17	0.17	0.21
Residential Investment	0.02	0.03	0.04	0.05	-0.04	-0.03
Fiscal Multipliers of Labour Tax						
GDP	-0.08	-0.12	-0.18	-0.19	-0.10	0.28
Total consumption	-0.03	-0.06	-0.14	-0.16	-0.16	-0.03
Consumption - Lenders	-0.01	-0.02	-0.06	-0.10	-0.15	-0.11
Consumption - Borrowers	-0.02	-0.04	-0.07	-0.06	-0.00	0.08
Business Investment	-0.00	-0.01	-0.04	-0.07	-0.09	-0.03
Residential Investment	-0.06	-0.06	-0.06	-0.07	-0.03	-0.03
Fiscal Multipliers of Capital Tax						
GDP	-0.63	-0.86	-1.11	-1.23	-1.36	-1.39
Total consumption	-0.03	-0.07	-0.12	-0.17	-0.32	-0.45
Consumption - Lenders	-0.04	-0.11	-0.24	-0.33	-0.50	-0.58
Consumption - Borrowers	0.01	0.04	0.12	0.16	0.18	0.13
Business Investment	-0.09	-0.23	-0.50	-0.61	-0.60	-0.50
Residential Investment	-0.09	-0.08	-0.09	-0.10	-0.07	-0.08

A consumption tax cut (Fig. 2.15 and 2.21) results in an increase in debt and a decrease of consumer prices which implies an increase in the non-residential consumption of both types of households. A decrease in the consumer prices along with higher consumption of lenders imply an increase of a real house price, which puts upward pressure on the user cost of housing which indeed increases on impact for both types of households. The user cost of housing for impatient households is below the steady state level after less than two quarters. The decrease in the shadow value of borrowing which signals loosening of the borrowing constraint, along with higher consumption expenditure cause that impatient households increase their expenditure on housing.

Patient households cut on both business and residential investment. The real exchange rate depreciates on impact, whereas the trade balance deteriorates. Table 2.13 indicates that the cumulative GDP multiplier remains relatively small with a peak roughly at a 5th year. In the longer horizon it becomes positive. The consumption tax cut also yields a stronger multiplier for the consumption of patient households compared to that of impatient ones. Residential investment is crowded out in the short term, but crowded in in the longer term.

The sensitivity analysis implies that setting nominal rigidities in the wage market to 0 has relatively strong implications for the short term GDP multiplier. Also, higher shock persistence influences stronger the consumption of impatient households than that of lenders, as borrowers use the financial accelerator mechanism to increase their current spending. Finally, lack of capital utilisation leads to a significant drop in private investment, which subsequently translate into an on impact drop in the real GDP.

Labour Tax

A decrease in the labour tax (Fig. 2.16 and 2.22) results in an increase in debt and the reallocation of production inputs from capital to labour and a subsequent significant increase in the labour income of impatient households. Higher income of borrowers implies an increase in the demand of these households for both types of goods. Business investment and consumption of patient households increase as a result of a drop in the real interest rate. The house price increase pushes up the lender's user cost of housing leading to a drop of lender's housing demand. Following the labour tax cut, the real exchange rate depreciates.

In aggregate the implications of a labour tax cut are similar to the effect of a consumption tax cut. The GDP multiplier remains similar in the short and long run. In the case of a labour tax cut, consumption of households increases less in the short and term than in the case of consumption tax cut.

Capital Tax

A capital tax rate cut (Fig. 2.17 and 2.23) results in an increase in debt and reallocation of production inputs from labour to capital, which causes a higher capital utilisation and a lower labour demand.[54] The marginal cost decreases as a result of the decrease of the rental rate of capital. This is followed by a drop in inflation and the nominal and real interest rate. The instant effect of the reallocation in the production sector is a decrease in households' labour income. Therefore even though the house price increases, impatient households are forced to cut on the expenditure on both goods. Business investment increases significantly as a result of the high increase in the discounted rental rates. The aggregate consumption also increases.The increase in consumption and business investment is consistent with Gomes *et al.* (2013). The aggregate residential investment increases following the capital tax cut. Turning to the open-economy, the real exchange rate depreciates on impact. The effect of a capital tax cut on GDP remains significant in the short and long run. Figure (2.17) indicates that if capital tax is decreased permanently, the house price would decrease.

[54] In the case of two most stimulating fiscal instruments, i.e. capital tax and government investment, the increase in government debt is the shortest (it is repaid after approximatelly 4 years) and the increase in both the residential investment and the accumulation of housing assets is long lasting.

2.5 Financial Market Deregulation

In this section we pay closer attention to the financial deregulation. Whereas there exist a significant literature on the role of financial markets in the great moderation, there is little focus on the effects of financial market developments on the effectiveness of fiscal policy. [55]

Table 2.14. Government expenditure multipliers for the rate of a loan downpayment equal to 0.025

	Quarters					
	1	4	12	20	40	1000
Fiscal multipliers of Public Consumption						
GDP	0.68	0.63	0.55	0.47	0.31	-0.18
Total consumption	-0.15	-0.20	-0.28	-0.33	-0.47	-0.88
Consumption - Lenders	-0.04	-0.09	-0.16	-0.20	-0.28	-0.47
Consumption - Borrowers	-0.10	-0.11	-0.12	-0.14	-0.19	-0.37
Business Investment	-0.03	-0.06	-0.12	-0.16	-0.19	-0.27
Residential Investment	-0.05	-0.04	-0.04	-0.04	-0.03	-0.05
Fiscal Multipliers of Public investment						
GDP	0.77	0.86	1.05	1.16	1.34	1.76
Total consumption	-0.03	-0.04	-0.01	0.05	0.17	0.47
Consumption - Lenders	-0.01	-0.01	0.05	0.12	0.25	0.44
Consumption - Borrowers	-0.02	-0.06	-0.12	-0.15	-0.16	-0.08
Business Investment	0.00	0.02	0.10	0.16	0.21	0.25
Residential Investment	-0.03	-0.01	0.00	0.01	0.01	0.03
Fiscal Multipliers of Public Transfers						
GDP	0.10	0.15	0.22	0.21	0.10	-0.24
Total consumption	0.21	0.35	0.48	0.49	0.42	0.21
Consumption - Lenders	-0.02	-0.05	-0.10	-0.13	-0.18	-0.29
Consumption - Borrowers	0.23	0.41	0.60	0.63	0.61	0.51
Business Investment	-0.02	-0.06	-0.12	-0.15	-0.15	-0.17
Residential Investment	-0.04	-0.07	-0.06	-0.05	-0.02	-0.04

[55] Whereas most of the literature point on the significant role of the financial markets in the great moderation (see for example Campbell and Hercowitz, 2005; Iacoviello and Pavan, 2013;), Den Haan and Sterk (2011) indicate that the role of financial markets was relatively limited.

In this section we bring attention to the parameter determining the value of downpayment (ϑ). We assume this parameter to be constant in our model; however, in the real economy, developments in the financial markets may influence the level of this parameter. Over the longer horizon, for instance, as financial markets become more developed and liberalized, the value of ϑ decreases (see for example Campbell and Hercowitz, 2005). In the short horizon the level of ϑ may also vary: i.e. in the times of boom, when financial markets are characterised by optimism, positive expectations, and high liquidity the level of ϑ for impatient households may be lower, and thus contributing to further stimulus (for some evidence see Mian and Sufi, 2011). On the other hand, in the case of the credit crunch the level of required downpayment may increase drastically. As ϑ decreases, impatient households become more constrained, their net wealth $\left(\vartheta(1-\delta_h)q_{h,t}H'_{t-1}\right)$ decreases and the value of possible borrowing increases $(1-\vartheta)(1-\delta_h)E_t\left[q_{h,t+1}H'_t\frac{\pi_{t+1}}{R_t}\right]$. Also, the user cost of housing depends less on $\vartheta(1-\delta_h)q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}(1-\lambda_{b,t})$ and more on $(1-\vartheta)(1-\delta_h)q_{ct,t+1}\frac{\pi_{ct,t+1}}{R_t}$.

Tables (2.14) and (2.15) provide present value multipliers for the case when the required value of downpayment is set to 0.025 of the expected value of the collateral. The multipliers indicate that the implications depend significantly on the nature of the fiscal policy, in particular whether it results in a house price increase or decrease. Let us consider, for instance, the case of a positive government consumption shock.

As indicated in the previous section the house price decreases following the government consumption shock which results in a lower value of housing wealth for impatient households. Subsequently, the more constrained are the impatient households, i.e. the

higher the level of borrowings in their spending, the more severe are the implications of public consumption shock for them, implying, therefore, larger on impact drop in the expenditure of impatient households on a non-residential consumption good and a housing bundle.

Once we decrease the level of required downpament from 0.15 to 0.025, then depending on the horizon over which the GDP multiplier is considered, it decreases by $(0.04 - 0.08)$ and this decrease is mainly caused by the drop in borrowers' expenditure (the impact consumption multiplier decreases from -0.03 to -0.10).

When the fiscal policy shock results in a house price increase, the financial accelerator implies that the expenditure of impatient households increases stronger on impact translating into higher GDP multipliers. For instance, in the case of a public transfer shock, once we decrease the level of downpayment from 0.15 to 0.025, the impact GDP multiplier increases from 0.05 to 0.10, the cumulative GDP multiplier for three years increases from 0.08 to 0.22, whereas the negative infinite multiplier reduces by approximately a half. To conclude, the effects of fiscal deregulation lead to a weakening of GDP multipliers in the case of government consumption and government investment and tend to improve multipliers for public transfers and the tax cuts.[56]

[56] The analysis of the excess returns on housing and other assets in response to financial deregulation is an interesting extension of the analysis.

Table 2.15. Tax multipliers for the rate of a loan downpayment equal to 0.025

	Quarters					
	1	4	12	20	40	1000
Fiscal Multipliers of Consumption Tax						
GDP	-0.08	-0.20	-0.37	-0.40	-0.32	-0.01
Total consumption	-0.15	-0.35	-0.59	-0.64	-0.61	-0.51
Consumption - Lenders	-0.10	-0.21	-0.38	-0.44	-0.45	-0.42
Consumption - Borrowers	-0.06	-0.14	-0.21	-0.20	-0.16	-0.08
Business Investment	0.02	0.05	0.12	0.15	0.15	0.19
Residential Investment	0.03	0.04	0.04	0.05	-0.04	-0.03
Fiscal Multipliers of Labour Tax						
GDP	-0.10	-0.15	-0.24	-0.25	-0.19	0.17
Total consumption	-0.06	-0.11	-0.17	-0.19	-0.20	-0.09
Consumption - Lenders	-0.01	-0.02	-0.07	-0.11	-0.17	-0.14
Consumption - Borrowers	-0.06	-0.09	-0.10	-0.07	-0.02	0.06
Business Investment	-0.00	-0.01	-0.05	-0.09	-0.11	-0.05
Residential Investment	-0.04	-0.05	-0.06	-0.07	-0.03	-0.03
Fiscal Multipliers of Capital Tax						
GDP	-0.65	-0.86	-1.09	-1.21	-1.33	-1.37
Total consumption	-0.05	-0.06	-0.10	-0.15	-0.31	-0.43
Consumption - Lenders	-0.04	-0.11	-0.24	-0.32	-0.48	-0.56
Consumption - Borrowers	-0.01	0.05	0.14	0.17	0.19	0.15
Business Investment	-0.09	-0.23	-0.49	-0.59	-0.58	-0.49
Residential Investment	-0.09	-0.09	-0.09	-0.11	-0.07	-0.07

2.6 Conclusions

This paper presents an open-economy DSGE model with a housing market and credit constrained households. The model is estimated on the UK time series from for the period from 1987Q2 to 2011Q1 and fits the data well. We show that house prices fall after an increase in government spending, government investment and taxes and the decrease in transfers. Residential investment increases the strongest following a capital tax cut.

We show that the implications of the deregulation in financial market depend significantly on the nature of the fiscal policy, in particular whether it results in a house price increase or decrease. Subsequently, the effects of financial deregulation lead to a weakening

of GDP multipliers in the case of an increase in government consumption and government investment and improvement of multipliers for public transfers increase and tax cuts.

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2.A Steady State

2.A.1 Assumptions regarding the steady state:

- Habit formation in consumption and housing disappears i.e. $h_c = h_h = h'_c = h'_h = 0$
- Holdings of foreign bonds are equal to 0 in the steady state i.e. $b_f = 0$
- Import is equal to the export i.e. $EX = C_m + I_m + G_m + IG_m$
- The nominal exchange rate is normalised to one i.e. $S = 1$
- Domestic price is equal to the world price i.e. $P = P^*$
- All the stochastic processes are zero.

2.A.2 Prices

Price of export

Once we have imposed that in the steady state export equals import, from the evolution of the net foreign assets we get:

$$P_x = P^* = P \tag{2.65}$$

Price of private consumption

Relative price of private consumption is represented by:

$$p_c = \left[a_c + (1 - a_c)\, p_{c,m}^{1-s_c}\right]^{\frac{1}{1-s_c}}$$

where $p_c = \frac{P_c}{P}$ and $p_{c,m} = \frac{P_{c,m}}{P}$. The first-order condition of firms importing consumption becomes in the steady state: $p_{c,m} = \frac{\upsilon_{c,m}}{\upsilon_{c,m}-1}\frac{P^* \times S}{P}$. Using equation the assumption that $P^* = P$ and $S = 1$, we obtain:

$$p_{c,m} = \frac{\upsilon_{c,m}}{\upsilon_{c,m} - 1} \tag{2.66}$$

Let $x_{c,m} = \frac{\upsilon_{c,m}}{\upsilon_{c,m}-1}$ denotes a markup, where $\upsilon_{c,m}$ is the elasticity of substitution between imported consumption goods and $x_{c,m} = p_{c,m}$.

Substituting for $p_{c,m} = \frac{\upsilon_{c,m}}{\upsilon_{c,m}-1}$ into $p_c = \left[a_c + (1 - a_c)\, p_{c,m}^{1-s_c}\right]^{\frac{1}{1-s_c}}$ we obtain the following:

$$p_c = \left[a_c + (1 - a_c)\left(\frac{\upsilon_{c,m}}{\upsilon_{c,m} - 1}\right)^{1-s_c}\right]^{\frac{1}{1-s_c}} \tag{2.67}$$

Price of private investment, public consumption and investment

Analogous results to the above are obtained for the private investment (p_i), public investment (p_{ig}), and public consumption (p_g). Therefore for $n \in \{i, ig, g\}$ we obtain:

$$p_{n,m} = \frac{\upsilon_{n,m}}{\upsilon_{n,m} - 1} \tag{2.68}$$

$$p_n = \left[a_n + (1 - a_n)\left(\frac{\upsilon_{n,m}}{\upsilon_{n,m} - 1}\right)^{1-s_n}\right]^{\frac{1}{1-s_n}} \tag{2.69}$$

where $p_n = \frac{P_n}{P}$, $p_{n,m} = \frac{P_{n,m}}{P}$, the markup is given by $x_{n,m} = \frac{\upsilon_{n,m}}{\upsilon_{n,m}-1}$.

2.A.3 Firms

Home country manufacturers

In the steady state the real profit of domestic manufactures is equal to 0. The production function and first-order conditions become in the steady state:

$$Y = mcK_c^{\alpha}\left(N_c^{b_1}N_c'^{1-b_1}\right)^{1-\alpha}K_g^{\sigma_g} \tag{2.70}$$

$$N_c w_c = b_1(1-\alpha)Y \tag{2.71}$$

$$w_c' N_c' = (1-b_1)(1-\alpha)Y \tag{2.72}$$

$$K_c r_{k,c} = \alpha Y \tag{2.73}$$

$$mc = \frac{1}{x} \tag{2.74}$$

Where equations $(2.71)-(2.73)$ are the first-order conditions stemming from the cost minimisation and equation (2.74) represents the condition stemming from the price setting: where x is a markup.

Producers of residential good

In the steady state, the production function and first-order conditions of the residential good firm sector are represented by:

$$HI = K_h^{\alpha_h} \left(N_h^{b_1} N_h'^{1-b_1}\right)^{1-\alpha_h-\alpha_l-\alpha_m} M^{\alpha_{co}} \tag{2.75}$$

$$w_h N_h = b_1(1-\alpha_h-\alpha_m)qHI \tag{2.76}$$

$$w_h' N_h' = (1-b_1)(1-\alpha_h-\alpha_m)qHI \tag{2.77}$$

$$K_h r_{k,h} = \alpha_h qHI \tag{2.78}$$

$$M = \alpha_M qHI \tag{2.79}$$

Importers

The nominal profits of importers are positive in the steady state and are given by $Prof_{n,m} = [1 - mc_{n,m}] P_{n,m} Y_{n,m}$ where $n \in \{c, i, ig, g\}$ and $N \in \{C, I, IG, G\}$. Using the first-order condition of importers i.e. $mc_{n,m} = \frac{1}{x_{n,m}} = \frac{1}{p_{n,m}}$ we obtain profits of importers equal to:

$$prof_{n,m} = [p_{n,m} - 1] N_m \tag{2.81}$$

$$N_m = (1-a_n)\left(\frac{P_{n,m}}{P_n}\right)^{-s_n} N_T \tag{2.82}$$

where $prof_{n,m} = \frac{Prof_{n,m}}{P}$.

Exporters

Nominal profits of exporters are equal to: $Prof_{x,t} = [1 - mc_x] P_x X$. Using the fact that $mc_x = \frac{MC_x}{P_x S} = \frac{P}{P_x S} = 1$ we get:

$$prof_x = 0 \tag{2.82}$$

2.A.4 Patient households

In the steady state $\pi = \frac{P}{P} = 1$, therefore the first-order condition with respect to the domestic bond holdings becomes:

$$R = \frac{1}{\beta} \tag{2.83}$$

From the first-order condition with respect to the foreign bonds holdings we get $R = R^{*}risk$, which after taking into consideration that premium on the foreign bond holdings is equal to one $risk = \exp\left(-\phi_a\left(aa - aa\right) + \phi\right) = \exp\left(0\right) = 1$ becomes

$$R^{*} = R \tag{2.84}$$

From the definition of the new purchases of foreign assets $aa = \frac{SP^{*}b^{F,\iota}}{P}$:

$$aa = b^{F,\iota} = 0 \tag{2.85}$$

From the first-order conditions with respect to the capital utilisation we obtain the following:

$$u_c = u_k = 1 \tag{2.86}$$

From the first-order conditions with respect to the both types of capital we get:

$$Q_c = Q_h = p_i \tag{2.87}$$

The combination of equation (2.73) with the first-order condition with respect to investment: $r_{k,h} = \frac{p_i}{1-\tau^k}\left[\frac{1}{\beta} - \left(1 - \delta_{k,c}\right)\right]$ results in:

$$\frac{K_c}{Y} = \frac{\alpha}{\frac{p_i}{1-\tau^k}\left[\frac{1}{\beta} - (1-\delta_{k,c})\right]} = \zeta_1 \tag{2.88}$$

The combination of equation (2.78) with the first-order condition with respect to investment $r_{k,h} = \frac{p_i}{1-\tau^k}\left[\frac{1}{\beta} - (1-\delta_{k,h})\right]$ yields:

$$\frac{K^h}{qHI} = \frac{\alpha_h}{\frac{p_i}{1-\tau^k}\left[\frac{1}{\beta} - (1-\delta_{k,h})\right]} = \zeta_2 \tag{2.89}$$

The capital accumulation equations in the steady state reduce to:

$$I_c = \delta_{k,c} K_c \tag{2.90}$$

$$I_h = \delta_{k,h} K_h \tag{2.91}$$

The first-order condition with respect to housing results in:

$$\frac{qH}{C} = \frac{jp_c\left(1+\tau^c\right)\left(1-b\right)}{\left[(1+\tau^h) - \beta(1-\delta_h)\right]} = \zeta_3 \tag{2.92}$$

2.A.5 Impatient households

The combination of first-order conditions of impatient households results in the following:

$$\frac{qH'}{C'} = \frac{jp_c\left(1+\tau^c\right)\left(1-b_{\prime}\right)}{\left\{\left[(1+\tau^h) - \gamma(1-\delta_h)\right] - \left(\frac{1}{R_m} - \gamma\right)(1-\vartheta)(1-\delta_h)\right\}} = \zeta_4 \tag{2.93}$$

Finally $LO = D$ and:

$$LO\,(R-1) = D\,(R-1) = (R-1)\,(1-\vartheta)(1-\delta_h)\left[\frac{qH'}{R_t}\right] = \zeta_5 qH' \tag{2.94}$$

Using $qH = \zeta_3 C$, $qH' = \zeta_4 C'$ and the housing accumulation equation $HI = H + H' - (1-\delta_h)(H+H')$ we get:

$$qHI = \delta_h\,(\zeta_3 C + \zeta_4 C') \tag{2.95}$$

2.A.6 Government

The government budget constraint is represented by:

$$\begin{aligned} & \tau^c p_c\,(C+C') + \tau^l\,(w_c L_c + w_h L_h + w'_c L'_c + w'_h L'_h) + \\ & \tau^k\,(r_{k,c} K_c + r_{k,h} K_h) + \tau^h q\,(H+H') - (p_g G + p_{ig} IG + TR) \\ = \; & (R-1)\,b_G \end{aligned} \tag{2.96}$$

2.A.7 Hours

The combination of first-order conditions of labour unions and the relevant firm's conditions we obtain:

$$N_{c,t} = \left[\frac{\frac{b_1(1-\alpha)}{\varkappa^w p_c mc(1-b)}\frac{Y}{C}\frac{\left(1+\tau^l\right)}{(1-\tau^c)}}{CX}\right]^{\frac{1}{1+\sigma_L}} \tag{2.97}$$

$$N'_{c,t} = \left[\frac{\frac{(1-b_1)(1-\alpha)Y}{\varkappa^w p_c C'(1-b)}\frac{\left(1+\tau^l\right)}{(1-\tau^c)}}{CX}\right]^{\frac{1}{1+\sigma'_L}} \tag{2.98}$$

$$N_{h,t} = \left(\frac{\frac{b_1(1-\alpha_h-\alpha_{CO}-\alpha_L)qHI}{\varkappa^w p_c C(1-b)}\frac{\left(1+\tau^l\right)}{(1-\tau^c)}}{HX}\right)^{\frac{1}{1+\sigma_L}} \tag{2.99}$$

$$N'_{h,t} = \left(\frac{\frac{(1-b_1)(1-\alpha_h-\alpha_{CO}-\alpha_L)qHI}{\varkappa^w p_c C'(1-b)}\frac{\left(1+\tau^l\right)}{(1-\tau^c)}}{HX}\right)^{\frac{1}{1+\sigma'_L}} \tag{2.100}$$

$$CX = \left[1+\frac{(1-\alpha_h-\alpha_{CO})}{(1-\alpha)}\frac{qHI}{Y}\right]^{\frac{\sigma'_L-\xi'_L}{1+\xi'_L}} \tag{2.101}$$

$$HX = \left[1+\frac{(1-\alpha)}{(1-\alpha_h-\alpha_{CO})}\frac{Y}{qHI}\right]^{\frac{\sigma'_L-\xi'_L}{1+\xi'_L}} \tag{2.102}$$

To find the value of N_c, N'_c, N_h, and $N\prime_h$ we need to find $\frac{Y}{C}$, $\frac{Y}{C'}$, $\frac{qHI}{C}$, $\frac{qHI}{C'}$, which we do in the next section.

2.A.8 Budget constraints

The budget constraints of patient and impatient households in the steady state are respectively given by:

$$
\begin{aligned}
& p_i I_c + p_i I_h + p_c \left(1+\tau^c\right) C + q\left[\left(1+\tau_t^h\right) H - (1-\delta_h) H\right] + b_G \qquad (2.103)\\
= \; & D - RD + Rb_G + \left(1-\tau^l\right)\left(w_c L_c + w_h L_h\right)\\
& + \left(1-\tau^k\right)\left[r_{c,k} K_c + r_{h,k} K_h\right] + \text{div}\\
& p_c \left(1+\tau^c\right) C' + q\left(\left(1+\tau^h\right) H' - (1-\delta_h) H'\right) \qquad (2.104)\\
= \; & LO' - RLO' + \left(1-\tau^l\right)\left(w_c' L_c' + w_h' L_h'\right) + TR
\end{aligned}
$$

Further, plugging the relevant equations into the above budget constraints results in:

$$
\begin{aligned}
\varsigma_1 Y &= -\varsigma_2 C + \varsigma_3 C' \qquad (2.105)\\
\varsigma_4 C' &= \varsigma_5 Y + \varsigma_6 C \qquad (2.106)
\end{aligned}
$$

where:

$$
\begin{aligned}
\varsigma_1 &= \left\{ \begin{array}{c} \left(p_i \delta_{k,c} - r_{k,c} - \digamma_1 \delta_{k,c}\right)\zeta_1 + p_g g + p_{ig} ig - (sh-1)\, tr \\ - \left[\tau^l (1-b_1) + b_1\right](1-\alpha) - \left(\digamma_3 ig + \digamma_4 g\right) \end{array} \right\} \qquad (2.107)\\
\varsigma_2 &= \left[p_c - F_2 + \left[\begin{array}{c} \left(p_I \delta_{k,h} - r_{k,h}\right)\zeta_2 + 1 - \digamma_1 \delta_{k,h} \zeta_2 - \\ \left(\tau^l (1-b_1) + b_1\right)(1-\alpha_h - \alpha_m) \end{array} \right] \delta_h \zeta_3 \right] \qquad (2.108)\\
\varsigma_3 &= \left\{ \begin{array}{c} \left(\zeta_5 + \tau^h\right)\zeta_4 + \tau^c p_c + \digamma_2 + \\ \left[\begin{array}{c} -\left(p_i \delta_{k,h} - r_{k,h}\right)\zeta_2 + \digamma_1 \delta_{k,h} \zeta_2 \\ + \left(\tau^l (1-b_1) + b_1\right)(1-\alpha_h - \alpha_m) \end{array} \right] \delta_h \zeta_4 \end{array} \right\} \qquad (2.109)\\
\varsigma_4 &= \left\{ \begin{array}{c} p_c \left(1+\tau^c\right) + \zeta_4 \left(\delta_h + \tau^h\right) + \zeta_5 \zeta_4 - \\ \left(1-\tau^l\right)(1-b_1)(1-\alpha_h - \alpha_m)\delta_h \zeta_4 \end{array} \right\} \qquad (2.110)\\
\varsigma_5 &= \left\{ \left(1-\tau^l\right)(1-b_1)(1-\alpha) + (1-sh)\, tr \right\} \qquad (2.111)\\
\varsigma_6 &= \left(1-\tau^l\right)(1-b_1)(1-\alpha_h - \alpha_m)\delta_h \zeta_3 \qquad (2.112)
\end{aligned}
$$

where:

$$\mathcal{F}_1 = [p_{i,m} - 1](1 - a_i)\left(\frac{P_{i,m}}{P_i}\right)^{-s_i}$$
$$\mathcal{F}_2 = [p_{c,m} - 1](1 - a_c)\left(\frac{P_{c,m}}{P_c}\right)^{-s_c}$$
$$\mathcal{F}_3 = [p_{ig,m} - 1](1 - a_{ig})\left(\frac{P_{ig,m}}{P_{ig}}\right)^{-s_{ig}}$$
$$\mathcal{F}_4 = [p_{g,m} - 1](1 - a_g)\left(\frac{P_{g,m}}{P_g}\right)^{-s_g}$$

Solving the two equations we obtain two ratios:

$$\frac{C'}{Y} = \frac{\varsigma_6\varsigma_1 - \varsigma_5\varsigma_2}{\varsigma_3\varsigma_6 - \varsigma_2\varsigma_4} \tag{2.113}$$
$$\frac{C}{Y} = \frac{\varsigma_1\varsigma_4 - \varsigma_3\varsigma_5}{\varsigma_3\varsigma_6 - \varsigma_2\varsigma_4} \tag{2.114}$$

Subsequently we can solve for all the remaining variables. For instance output in the non-residential sector and wages are represented by:

$$Y = (mc)^{\frac{1}{1-\alpha-\sigma_g}}(\zeta_1)^{\frac{\alpha}{1-\alpha-\sigma_g}}\left(N_c^{b_1}N_c'^{1-b_1}\right)^{\frac{1-\alpha}{1-\alpha-\sigma_g}}\left(\frac{i_g}{\delta_g}\right)^{\frac{\sigma_g}{1-\alpha-\sigma_g}} \tag{2.115}$$
$$w_c = \frac{b_1(1-\alpha)Y}{N_c} \tag{2.116}$$
$$w_c' = \frac{(1-b_1)(1-\alpha)Y}{N_c'} \tag{2.117}$$
$$w_h = q\frac{b_1(1-\alpha_h-\alpha_m)HI}{N_h} \tag{2.118}$$
$$w_h' = q\frac{(1-b_1)(1-\alpha_h-\alpha_m)HI}{N_h'} \tag{2.119}$$

2.B Log-Linearised System of Equations:

2.B.1 Patient households

$$\hat{U}_{c,t} = \hat{R}_t - E_t\hat{\pi}_{t+1} + p_{c,t} - p_{c,t+1} + \frac{\tau^c}{1+\tau^c}\left(\tau^c_t - \tau^c_{t+1}\right) + \hat{U}_{c,t+1} \tag{2.120}$$

$$\hat{I}_{c,t} = \frac{\hat{Q}_{c,t} - \hat{p}_{i,t}}{\phi_{k,c}(1+\beta)} + \frac{\beta E_t \hat{I}_{c,t+1}}{(1+\beta)} + \frac{\hat{I}_{c,t-1}}{(1+\beta)} - \frac{1}{(1+\beta)}\left(\varepsilon^{I_c}_t - \beta E_t \hat{\varepsilon}^{I_c}_{t+1}\right) \tag{2.121}$$

$$\hat{I}_{h,t} = \frac{\hat{Q}_{h,t} - \hat{p}_{i,t}}{\phi_{k,h}(1+\beta)} + \frac{\beta E_t \hat{I}_{h,t+1}}{(1+\beta)} + \frac{\hat{I}_{h,t-1}}{(1+\beta)} \tag{2.122}$$

$$\begin{aligned}\hat{Q}_{c,t} = & -\hat{R}_t + E_t\hat{\pi}_{t+1} + \frac{(1-\delta_{k,c})}{1-\delta_{k,c}+(1-\tau^k)r_{k,c}}\left(E_t\hat{Q}_{c,t+1}\right) + \\ & + \frac{r_{k,c}\left(1-\tau^k\right)}{1-\delta_{k,c}+(1-\tau^k)r_{k,c}} E_t\left(\hat{r}_{k,c,t+1} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_{t+1}\right)\end{aligned} \tag{2.123}$$

$$\begin{aligned}\hat{Q}_{h,t} = & -\hat{R}_t + E_t\hat{\pi}_{t+1} + \frac{(1-\delta_{k,h})}{1-\delta_{k,h}+(1-\tau^k)r_{k,h}}\left(E_t\hat{Q}_{h,t+1}\right) + \\ & + \frac{r_{k,h}\left(1-\tau^k\right)}{1-\delta_{k,h}+(1-\tau^k)r_{k,h}} E_t\left(\hat{r}_{k,h,t+1} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_{t+1}\right)\end{aligned} \tag{2.124}$$

$$\hat{U}_{h,t} = \frac{1+\tau^h}{1+\tau^h-\beta(1-\delta_h)}\left[\begin{array}{c}\hat{\lambda}_t + \hat{q}_t + \frac{\tau^h}{1+\tau^h}\hat{\tau}^h_t \\ -\frac{\beta(1-\delta_h)}{1+\tau^h}E_t\left(\hat{\lambda}_{t+1} + \hat{q}_{t+1}\right)\end{array}\right] \tag{2.125}$$

$$\hat{u}_{c,t} = \frac{1}{\kappa}\left[\hat{r}_{k,c,t} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_t\right] \tag{2.126}$$

$$\hat{u}_{h,t} = \frac{1}{\kappa}\left[\hat{r}_{k,h,t} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_t\right] \tag{2.127}$$

$$\hat{R}_t = \left(\hat{S}_{t+1} - \hat{S}_t\right) + \hat{R}^*_t - \varkappa\widehat{aa}_t + \hat{\varepsilon}^r_t \tag{2.128}$$

$$\begin{aligned}\hat{w}_{c,t} = & \frac{\beta}{1+\beta}E\hat{w}_{c,t+1} + \frac{1}{1+\beta}\hat{w}_{c,t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_{c,w}}{1+\beta}\hat{\pi}_t \\ & + \frac{\gamma_{c,w}}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\varpi_{c,w})(1-\varpi_{c,w})}{\left(1+\frac{(1+\varsigma_{c,w})\sigma_L}{\varsigma_{c,w}}\right)\varpi_{c,w}}\left(\hat{X}_{c,w,t}\right)\end{aligned} \tag{2.129}$$

$$\hat{X}_{c,w,t} = \hat{w}_{c,t} - \sigma_L\hat{L}_{c,t} - \frac{\hat{C}_t - b\hat{C}_{t-1}}{1-b} + \hat{\varepsilon}^L_t - \frac{\tau^l}{1+\tau^l}\hat{\tau}^l_t - \frac{\tau^c}{1+\tau^c}\tau^c_t - \hat{p}_{c,t} \tag{2.130}$$

$$\begin{aligned}\hat{w}_{h,t} = & \frac{\beta}{1+\beta}E\hat{w}_{h,t+1} + \frac{1}{1+\beta}\hat{w}_{h,t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_{h,w}}{1+\beta}\hat{\pi}_t \\ & + \frac{\gamma_{h,w}}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\varpi_{h,w})(1-\varpi_{h,w})}{\left(1+\frac{(1+\varsigma_{h,w})\sigma_L}{\varsigma_{h,w}}\right)\varpi_{h,w}}\left(\hat{X}_{h,w,t}\right)\end{aligned} \tag{2.131}$$

$$\hat{X}_{h,w,t} = \hat{w}^h_t - \sigma_L\hat{L}_{h,t} - \frac{\hat{C}_t - b\hat{C}_{t-1}}{1-b} + \hat{\varepsilon}^L_t - \frac{\tau^l}{1+\tau^l}\hat{\tau}^l_t - \frac{\tau^c}{1+\tau^c}\tau^c_t - \hat{p}_{c,t} \tag{2.132}$$

2.B.2 Impatient households

$$
\begin{aligned}
C'C'_t &= LO'\widehat{LO}'_t - qH'\left(\hat{q}_t + \hat{H}'_t\right) + (1-\delta_h)qH'\left(\hat{q}_t + \hat{H}'_{t-1}\right) && (2.133)\\
&\quad -RLO'\left(\hat{R}_{t-1} - \hat{\pi}_t + \widehat{LO}'_{t-1}\right) + w'_c L'_c\left(\hat{w}'_{c,t} + \hat{L}'_{c,t}\right) + w'_h L'_h\left(\hat{w}'_{h,t} + \hat{L}'_{h,t}\right) \\
\widehat{LO}'_t &= E_t\hat{q}_{t+1} + \hat{H}'_t + E_t\hat{\pi}_{t+1} - \hat{R}_t && (2.134)\\
\hat{U}'_{c,t} &= \hat{\lambda}'_t + p_{c,t} + \frac{\tau^c}{1+\tau^c}\tau^c_t && (2.135)\\
\hat{\lambda}_{b,t} &= \frac{\gamma R}{1-\gamma R}E_t\left[\hat{\lambda}'_t - E\hat{\lambda}'_{t+1} - \hat{R}_t + \hat{\pi}_{t+1}\right] && (2.136)\\
\hat{U}'_{h,t} &= \frac{1}{\left\{1 - \frac{1-\delta_h}{1+\tau^h}\left[\gamma + \left(\frac{1}{R} - \gamma\right)(1-\vartheta)\right]\right\}} \times && (2.137)\\
&\quad \left\{\begin{array}{c} \left(\hat{\lambda}'_t + \hat{q}_t + \frac{\tau^h}{1+\tau^h}\hat{\tau}^h_t\right) - \gamma\frac{(1-\delta_h)}{\left(1+\tau^h\right)}E_t\left(\hat{\lambda}'_{t+1} + \hat{q}_{t+1}\right) \\ -\left(\frac{1}{R} - \gamma\right)\frac{(1-\vartheta)(1-\delta_h)}{1+\tau^h}E_t\left(\hat{\lambda}'_t + \hat{\lambda}_{b,t} + \hat{q}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t\right) \end{array}\right\} \\
\hat{w}'_{c,t} &= \frac{\gamma}{1+\gamma}E\hat{w}'_{c,t+1} + \frac{1}{1+\gamma}\hat{w}'_{c,t-1} + \frac{\gamma}{1+\gamma}E_t\hat{\pi}_{t+1} - \frac{1+\gamma\gamma_{c,w}}{1+\gamma}\hat{\pi}_t && (2.138)\\
&\quad + \frac{\gamma_{c,w}}{1+\gamma}\hat{\pi}_{t-1} - \frac{1}{1+\gamma}\frac{(1-\gamma\varpi_{c,w})(1-\varpi_{c,w})}{\left(1 + \frac{(1+\varsigma_{c,w})\sigma_L}{\varsigma_{c,w}}\right)\varpi_{c,w}}\hat{X}'_{c,w,t} \\
\hat{X}'_{c,w,t} &= \hat{w}'_{c,t} - \sigma_L\hat{L}'_{c,t} - \frac{\hat{C}'_t - b\hat{C}'_{t-1}}{1-b} + \hat{\varepsilon}^L_t - \frac{\tau^l}{1+\tau^l}\hat{\tau}^l_t - \frac{\tau^c}{1+\tau^c}\tau^c_t - \hat{p}_{c,t} - \hat{\varepsilon}^w_t && (2.139)\\
\hat{w}'_{h,t} &= \frac{\gamma}{1+\gamma}E\hat{w}'_{h,t+1} + \frac{1}{1+\gamma}\hat{w}'_{h,t-1} + \frac{\gamma}{1+\gamma}E_t\hat{\pi}_{t+1} - \frac{1+\gamma\gamma_{h,w}}{1+\gamma}\hat{\pi}_t && (2.140)\\
&\quad + \frac{\gamma_{h,w}}{1+\gamma}\hat{\pi}_{t-1} - \frac{1}{1+\gamma}\frac{(1-\gamma\varpi_{h,w})(1-\varpi_{h,w})}{\left(1 + \frac{(1+\varsigma_{h,w})\sigma_L}{\varsigma_{h,w}}\right)\varpi_{h,w}}\hat{X}'_{h,w,t} \\
\hat{X}'_{h,w,t} &= \hat{w}'_{h,t} - \sigma_L\hat{L}'_{h,t} - \frac{\hat{C}'_t - b\hat{C}'_{t-1}}{1-b} + \hat{\varepsilon}^L_t - \frac{\tau^l}{1+\tau^l}\hat{\tau}^l_t - \frac{\tau^c}{1+\tau^c}\tau^c_t - \hat{p}_{c,t} - \hat{\varepsilon}^w_t && (2.141)
\end{aligned}
$$

2.B.3 Production Sector

Non-residential good producers

$$\hat{Y}_t = \phi_c \left[\begin{array}{l} \hat{\varepsilon}_t^{A_c} + \alpha(\hat{u}_{c,t} + \hat{K}_{c,t-1}) + \sigma_g \hat{K}_{g,t-1} \\ + (1-\alpha)\left(b_1 \hat{N}_{c,t} + (1-b_1)\hat{N}'_{c,t}\right) \end{array} \right] \tag{2.142}$$

$$\hat{r}_{k,c,t} = \widehat{mc}_t + \frac{\hat{Y}_t}{\phi_c} - \hat{K}_{c,t-1} - \hat{u}_{c,t} \tag{2.143}$$

$$\hat{w}'_{c,t} = \widehat{mc}_t + \frac{\hat{Y}_t}{\phi_c} - \hat{N}'_{c,t} \tag{2.144}$$

$$\hat{w}_{c,t} = \widehat{mc}_t + \frac{\hat{Y}_t}{\phi_c} - \hat{N}_{c,t} \tag{2.145}$$

$$\hat{\pi}_t = \frac{1}{1+\beta\gamma_p}\left(\beta E_t \hat{\pi}_{t+1} + \gamma_p \hat{\pi}_{t-1} + \frac{(1-\beta\varpi_p)(1-\varpi_p)}{\varpi_p}\left(\widehat{mc}_{p,t} + \hat{\varepsilon}_t^p\right)\right) \tag{2.146}$$

Residential good producers

$$\widehat{HI}_t = \hat{\varepsilon}_t^{A_h} + \alpha_h\left(\hat{K}_{h,t-1} + \hat{u}_{h,t}\right) + \alpha_{co}\hat{M}_t + (1-\alpha_h-\alpha_{co})\left(b_1\hat{N}_{h,t} + (1-b_1)\hat{N}'_{h,t}\right) \tag{2.147}$$

$$\hat{w}_{h,t} = \hat{q}_t + \widehat{HI}_t - \hat{N}_{h,t} \tag{2.148}$$

$$\hat{w}'_{h,t} = \hat{q}_t + \widehat{HI}_t - \hat{N}'_{h,t} \tag{2.149}$$

$$\hat{M}_t = \hat{q}_t + \widehat{HI}_t \tag{2.150}$$

$$\hat{r}_{k,h,t} = \hat{q}_t + \widehat{HI}_t - \hat{K}_{h,t-1} - \hat{u}_{h,t} \tag{2.151}$$

Exporters

$$\hat{\pi}_{x,t} = \frac{\beta E_t \hat{\pi}_{x,t+1}}{1+\beta\gamma_x} + \frac{\gamma_x \hat{\pi}_{x,t-1}}{1+\beta\gamma_x} + \psi_x\left(\widehat{mc}_{x,t} + \hat{\varepsilon}_t^{p,x}\right) \tag{2.152}$$

$$\widehat{mc}_{x,t} = \widehat{mc}_{x,t-1} + \hat{\pi}_t - \hat{\pi}_{x,t} - \hat{S}_t + \hat{S}_{t-1} \tag{2.153}$$

Importers

$$\hat{\pi}_{c,m,t} = \frac{\beta E_t \hat{\pi}_{c,m,t+1}}{1+\beta\gamma_{c,m}} + \frac{\gamma_{c,m}\hat{\pi}_{c,m,t-1}}{1+\beta\gamma_{c,m}} + \psi_{c,m}\left(\widehat{mc}_{c,m,t} + \hat{\varepsilon}_t^{c,m}\right) \tag{2.154}$$

$$\widehat{mc}_{c,m,t} = \widehat{mc}_{c,m,t-1} + \hat{\pi}_t^* - \hat{\pi}_{c,m,t} + \hat{S}_t - \hat{S}_{t-1} \tag{2.155}$$

$$\hat{\pi}_{i,m,t} = \frac{\beta E_t \hat{\pi}_{i,m,t+1}}{1+\beta\gamma_{i,m}} + \frac{\gamma_{i,m}\hat{\pi}_{i,m,t-1}}{1+\beta\gamma_{i,m}} + \psi_{i,m}\left(\widehat{mc}_{i,m,t} + \hat{\varepsilon}_t^{i,m}\right) \tag{2.156}$$

$$\widehat{mc}_{i,m,t} = \widehat{mc}_{i,m,t-1} + \hat{\pi}_t^* - \hat{\pi}_{i,m,t} + \hat{S}_t - \hat{S}_{t-1} \tag{2.157}$$

$$\hat{\pi}_{ig,m,t} = \frac{\beta E_t \hat{\pi}_{ig,m,t+1}}{1+\beta\gamma_{ig,m}} + \frac{\gamma_{ig,m}\hat{\pi}_{ig,m,t-1}}{1+\beta\gamma_{ig,m}} + \psi_{ig,m}\left(\widehat{mc}_{ig,m,t} + \hat{\varepsilon}_t^{ig,m}\right) \tag{2.158}$$

$$\widehat{mc}_{ig,m,t} = \widehat{mc}_{ig,m,t-1} + \hat{\pi}_t^* - \hat{\pi}_{ig,m,t} + \hat{S}_t - \hat{S}_{t-1} \tag{2.159}$$

$$\hat{\pi}_{g,m,t} = \frac{\beta E_t \hat{\pi}_{g,m,t+1}}{1+\beta\gamma_{g,m}} + \frac{\gamma_{g,m}\hat{\pi}_{g,m,t-1}}{1+\beta\gamma_{g,m}} + \psi_{g,m}\left(\widehat{mc}_{g,m,t} + \hat{\varepsilon}_t^{g,m}\right) \tag{2.160}$$

$$\widehat{mc}_{g,m,t} = \widehat{mc}_{g,m,t-1} + \hat{\pi}_t^* - \hat{\pi}_{g,m,t} + \hat{S}_t - \hat{S}_{t-1} \tag{2.161}$$

where $\psi_n = \frac{(1-\beta\varpi_n)(1-\varpi_n)}{\varpi_n(1+\beta\gamma_n)}$ for $n \in \{c,m;i,m;ig,m;g,m\}$

2.B.4 Fiscal and monetary policy

$$\begin{aligned}
&\tau^c p_c C_T\left(\hat{\tau}_t^c + \hat{p}_{c,t} + \hat{C}_{T,t}\right) + \\
&+\tau^l w_c L_c\left(\hat{\tau}_t^l + \hat{w}_{c,t} + \hat{L}_{c,t}\right) + \tau^l w_h L_h\left(\hat{\tau}_t^l + \hat{w}_{h,t} + \hat{L}_{h,t}\right) + \\
&+\tau^l w_c' L_c'\left(\hat{\tau}_t^l + \hat{w}_{c,t}' + \hat{L}_{c,t}'\right) + \tau^l w_h' L_h'\left(\hat{\tau}_t^l + \hat{w}_{h,t}' + \hat{L}_{h,t}'\right) + \\
&+\tau^k r_{k,c} K_c\left(\hat{\tau}_t^k + \hat{r}_{k,c,t} + \hat{u}_{c,t} + \hat{K}_{c,t-1}\right) + \tau^h q H'\left(\hat{\tau}_t^h + \hat{q}_t + \hat{H}_t'\right) + \\
&+\tau^k r_{k,h} K_h\left(\hat{\tau}_t^k + \hat{r}_{k,h,t} + \hat{u}_{h,t} + \hat{K}_{h,t-1}\right) + \tau^h q H\left(\hat{\tau}_t^h + \hat{q}_t + \hat{H}_t\right) + \\
=\ & Rb\left(\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t\right) + p_g G\left(\hat{p}_{g,t} + \hat{G}_t\right) + p_{ig} IG\left(\hat{p}_{ig,t} + \widehat{IG}_t\right) + TR\widehat{tr}_t - b\hat{b}_t
\end{aligned} \tag{2.162}$$

$$\hat{g}_t = -\phi_{b,g}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{g,t} \tag{2.163}$$

$$\widehat{Ig}_t = -\phi_{b,ig}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{ig,t} \tag{2.164}$$

$$\widehat{tr}_t = -\phi_{b,tr}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{tr,t} \tag{2.165}$$

$$\hat{\tau}_t^c = \phi_{b,\tau^c}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{\tau^c,t} \tag{2.166}$$

$$\hat{\tau}_t^l = \phi_{b,\tau^l}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{\tau^l,t} \tag{2.167}$$

$$\hat{\tau}_t^k = \phi_{b,\tau^k}\hat{b}_{t-1} - \phi_{GDP,g}\widehat{GDP}_t + e_{\tau^k,t} \tag{2.168}$$

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\left[\rho_\pi\hat{\pi}_{c,t} + \rho_y\left(\widehat{GDPP}_t - \widehat{GDPP}_{t-1}\right)\right] + \hat{\eta}_t^m \tag{2.169}$$

2.B.5 General equilibrium and aggregation conditions

$$\begin{aligned}\hat{Y}_t = {} & \frac{CO}{Y}\widehat{CO}_t + \frac{I_d}{Y}\hat{I}_{d,t} + \frac{C_d}{Y}\hat{C}_{d,t} + \frac{IG_d}{Y}\widehat{IG}_{d,t} + \frac{G_d}{Y}\hat{G}_{d,t} + \\ & + \frac{Y^*}{Y}\left(-s_f * p_{x,t} + y_t^*\right) + \left(1-\tau^k\right)\frac{r_{k,c}K_c}{Y}\hat{u}_{c,t} + \left(1-\tau^k\right)\frac{r_{k,h}K_h}{Y}\hat{u}_{h,t}\end{aligned} \tag{2.170}$$

$$HI\widehat{HI}_t = H\hat{H}_t + H'\hat{H}'_t - (1-\delta_h)\left(H\hat{H}_{t-1} + H'\hat{H}'_{t-1}\right) \tag{2.171}$$

$$GDP\widehat{GDP}_t = Y\hat{Y}_t + qHI\left(\hat{q}_t + \widehat{HI}_t\right) - CO\widehat{CO}_t \tag{2.172}$$

$$\hat{C}_{T,t} = \frac{C}{C_T}\hat{C}_t + \frac{C'}{C_T}\hat{C}'_t \tag{2.173}$$

$$\hat{I}_{T,t} = \frac{I_c}{I_T}\hat{I}_{c,t} + \frac{I_h}{I_T}\hat{I}_{h,t} \tag{2.174}$$

$$\hat{D}_t = \widehat{LO}_t \tag{2.175}$$

$$\hat{R}_t = \hat{R}_{m,t} \tag{2.176}$$

2.B.6 Other

Foreign assets accumulation equation

$$\begin{aligned}\hat{a}_t &= -C_m\left(-\widehat{mc}_{x,t} - \hat{p}_{x,t} - s_c\left(\hat{p}_{c,m,t} - p_{c,t}\right) + \hat{C}_{T,t}\right) \\ &\quad -IG_m\left(-\widehat{mc}_{x,t} - \hat{p}_{x,t} - s_{ig}\left(\hat{p}_{ig,m,t} - \hat{p}_{ig,t}\right) + \widehat{IG}_{T,t}\right) \\ &\quad -G_m\left(-\widehat{mc}_{x,t} - \hat{p}_{x,t} - s_g\left(\hat{p}_{g,m,t} - \hat{p}_{G,t}\right) + \hat{G}_{T,t}\right) + R\hat{a}_{t-1} \\ &\quad -I_m\left(-\widehat{mc}_{x,t} - \hat{p}_{x,t} - s_i\left(\hat{p}_{i,m,t} - \hat{p}_{i,t}\right) + \hat{I}_{T,t}\right) + Y^*\left(-\widehat{mc}_{x,t} - s_f p_{x,t} + Y_t^*\right)\end{aligned} \tag{2.177}$$

Capital accumulation equations

$$\hat{K}_{c,t} = \left(1-\delta_{k,c}\right)\hat{K}_{c,t-1} + \delta_{k,c}\hat{I}_{c,t} \tag{2.178}$$

$$\hat{K}_{h,t} = \left(1-\delta_{k,h}\right)\hat{K}_{h,t-1} + \delta_{k,h}\hat{I}_{h,t} \tag{2.179}$$

$$\hat{K}_{g,t} = (1-\delta_{k,g})\hat{K}_{g,t-1} + \delta_{k,g}\hat{I}_{g,t} \tag{2.180}$$

Marginal utilities

$$\hat{U}_{h,t} = \hat{\varepsilon}_t^B + \hat{\jmath}_t - \frac{\hat{H}_t - h_h\hat{H}_{t-1}}{1-h_h} \tag{2.181}$$

$$\hat{U}_{c,t} = \hat{\varepsilon}_t^B - \frac{\hat{C}_t - h_c\hat{C}_{t-1}}{1-h_c} \tag{2.182}$$

$$\hat{U}'_{h,t} = \hat{\varepsilon}_t^B + \hat{\jmath}_t - \frac{\hat{H}'_t - h_h\hat{H}'_{t-1}}{1-h_h} \tag{2.183}$$

$$\hat{U}'_{c,t} = \hat{\varepsilon}_t^B - \frac{\hat{C}'_t - h_c\hat{C}'_{t-1}}{1-h_c} \tag{2.184}$$

Private Consumption price inflation

$$\hat{\pi}_{c,t} = \frac{a_c}{p_c^{1-s_c}}\left(\hat{\pi}_t - p_{c,t-1}\right) + \left(1 - \frac{a_c}{p_c^{1-s_c}}\right)\left(\hat{\pi}_{c,m,t} + \hat{p}_{c,m,t-1} - p_{c,t-1}\right) \tag{2.185}$$

$$\hat{\pi}_{i,t} = \frac{a_i}{p_i^{1-s_i}}\left(\hat{\pi}_t - \hat{p}_{i,t-1}\right) + \left(1 - \frac{a_i}{p_i^{1-s_i}}\right)\left(\hat{\pi}_{i,m,t} + \hat{p}_{i,m,t-1} - \hat{p}_{i,t-1}\right) \tag{2.186}$$

$$\hat{\pi}_{ig,t} = \frac{a_{ig}}{p_{ig}^{1-s_{ig}}}\left(\hat{\pi}_t - \hat{p}_{ig,t-1}\right) + \left(1 - \frac{a_{ig}}{p_{ig}^{1-s_{ig}}}\right)\left(\hat{\pi}_{ig,m,t} + \hat{p}_{ig,m,t-1} - \hat{p}_{ig,t-1}\right) \tag{2.187}$$

$$\hat{\pi}_{g,t} = \frac{a_g}{p_g^{1-s_g}}\left(\hat{\pi}_t - \hat{p}_{g,t-1}\right) + \left(1 - \frac{a_g}{p_g^{1-s_g}}\right)\left(\hat{\pi}_{g,m,t} + \hat{p}_{g,m,t-1} - \hat{p}_{g,t-1}\right) \tag{2.188}$$

Relative prices:

$$\hat{p}_{i,t} = \hat{p}_{i,t} + \hat{\pi}_{i,t} - \hat{\pi}_t \tag{2.189}$$

$$p_{c,t} = p_{c,t} + \hat{\pi}_{c,t} - \hat{\pi}_t \tag{2.190}$$

$$\hat{p}_{ig,t} = \hat{p}_{ig,t-1} + \hat{\pi}_{ig,t} - \hat{\pi}_t \tag{2.191}$$

$$\hat{p}_{g,t} = \hat{p}_{g,t-1} + \hat{\pi}_{g,t} - \hat{\pi}_t \tag{2.192}$$

$$\hat{p}_{x,t} = \hat{p}_{x,t-1} + \hat{\pi}_{x,t} - \hat{\pi}_t^* \tag{2.193}$$

$$\hat{p}_{c,m,t} = \hat{p}_{c,m,t-1} + \hat{\pi}_{c,m,t} - \hat{\pi}_t \tag{2.194}$$

$$\hat{p}_{i,m,t} = \hat{p}_{i,m,t-1} + \hat{\pi}_{i,m,t} - \hat{\pi}_t \tag{2.195}$$

$$\hat{p}_{g,m,t} = \hat{p}_{g,m,t-1} + \hat{\pi}_{g,m,t} - \hat{\pi}_t \tag{2.196}$$

$$\hat{p}_{ig,m,t} = \hat{p}_{ig,m,t-1} + \hat{\pi}_{ig,m,t} - \hat{\pi}_t \tag{2.197}$$

Shocks

$$\hat{\varepsilon}_t^z = \rho^z \hat{\varepsilon}_{t-1}^z + \hat{\eta}_{z,t} \tag{2.198}$$

where $z\epsilon\{A_c; A_h; B; H; I_c; c, m; g, m; i, m; ig, m; p; p, x; r; w\}$ and $\hat{\eta}_{z,t} \sim N(0, \sigma_z^2)$ are $i.i.d.$- normally distributed errors. Following Harrison and Oomen (2010) and Millard (2011) we hard-coded the foreign shock processes into the model that was estimated. The shock processes are taken from Millard (2011) and are represented by:

$$\hat{y}_t^* = 0.9061\hat{y}_t^* + \hat{\eta}_t^{y^*}, \sigma_{y^*} = 0.0142 \tag{2.199}$$

$$\hat{\pi}_t^* = 0.8991\hat{\pi}_t^* + \hat{\eta}_t^{\pi^*}, \sigma_{\pi^*} = 0.0075 \tag{2.200}$$

$$\hat{R}_t^* = 0.8738\hat{R}_t^* + \hat{\eta}_t^{R^*}, \sigma_{y^*} = 0.0012 \tag{2.201}$$

2.C Data and Matching Equations

In order to estimate the model twenty time series are used: GDP, private consumption, private business investment, dwelling investment, wages, hours, public consumption, public investment, GDP deflator, private consumption deflator, public consumption deflator, public investment deflator, business investment deflator, effective consumption, labour and capital tax rates, transfers, and effective exchange rate. The data are from the Bank of England (BoE), Office for National Statistics webpage (ONS), and Bhattarai and Trzeciakiewicz (2013) and cover period from 1987:Q2 to 2011:Q1.

Definition of variables is represented by:

$$X = ln\left(\frac{x}{pop}\right) * 100 \tag{2.202}$$

where x = government investment, government consumption, transfers, GDP, private consumption, private investment, wages, hours; and pop is defined as all persons aged 16 and over table A02 [Labour Force Survey Summary]. All data are demeaned with their linear trend.

GDP at market prices, real ABMI.Q

$$\mathring{GDP}_t = \frac{y}{GDP}\hat{y}_t + \frac{qHI}{GDP}\left(\hat{q}_t + \widehat{HI}_t\right) - \frac{M}{GDP}\hat{M}_t \tag{2.203}$$

Real private consumption (including NPISH) ABJR.Q+HAYO.Q

$$\mathring{C}_t = \hat{C}_{T,t} \tag{2.204}$$

Gross Fixed Capital Formation: real business investment, NPEL.Q

$$\mathring{I}_t = \hat{I}_{T,t} \tag{2.205}$$

Gross Fixed Capital Formation: real dwelling investment DFEA.Q+DKQH.Q

$$\mathring{HI}_t = HI_t \tag{2.206}$$

Gross Fixed Capital Formation: real government investment DLWF.Q

$$\mathring{I}_{g,t} = \hat{I}_{g,t} \tag{2.207}$$

Real government consumption, NMRY.Q

$$\mathring{G}_{g,t} = \hat{G}_t \tag{2.208}$$

Actual weekly hours of work YBUS.Q The Labour Force Survey

$$\begin{aligned}\mathring{N}_t &= \frac{N_{h,t}^{b_1} N\prime_{h,t}^{1-b_1}}{N_h^{b_1} N\prime_h^{1-b_1} + N_c^{b_1} N\prime_c^{1-b_1}} \left(b_1 \hat{N}_{h,t} + (1-b_1)\, \hat{N}'_{h,t}\right) \\ &\quad + \frac{N_{c,t}^{b_1} N\prime_{c,t}^{1-b_1}}{N_h^{b_1} N\prime_h^{1-b_1} + N_c^{b_1} N\prime_c^{1-b_1}} \left(b_1 N_{c,j,t} + (1-b_1)\, N'_{c,j,t}\right) \qquad (2.209)\end{aligned}$$

Wages: Compensation of employees DTWM.Q + 0.5 of mixed income ROYH.Q divided by the GDP deflator.

$$\begin{aligned}\mathring{W}_t &= \frac{W_c}{W_c + W_h + W'_c + W'_h} \hat{W}_{c,t} + \frac{W_h}{W_c + W_h + W'_c + W'_h} \hat{W}_{h,t} \\ &\quad + \frac{W'_c}{W_c + W_h + W'_c + W'_h} \hat{W}'_{c,t} + \frac{W_h}{W_c + W_h + W'_c + W'_h} \hat{W}_{h,t} \qquad (2.210)\end{aligned}$$

Deflator domestic producers YBHA.Q/ABMI.Q

$$\mathring{P}_t^{def_i} = d\hat{p}_t \qquad (2.211)$$

$\pi_t = d\hat{p}_t - d\hat{p}_{t-1}$

Deflator private consumption (ABJQ.Q+HAYE.Q)/(ABJR.Q+HAYO.Q)

$$\mathring{P}_{c,t}^{def_i} = d\hat{p}_{c,t} + \frac{\tau^c}{1+\tau^c} \tau_t^c \qquad (2.212)$$

$\pi_{c,t} = d\hat{p}_{c,t} + \frac{\tau^c}{1+\tau^c} \tau_t^c - d\hat{p}_{c,t-1} - \frac{\tau^c}{1+\tau^c} \tau_{t-1}^c$

Deflator private investment NPEK.Q/NPEL.Q

$$\mathring{P}_{i,t}^{def_i} = d\hat{p}_{i,t} \qquad (2.213)$$

$\pi_{i,t} = d\hat{p}_{i,t} - d\hat{p}_{i,t-1}$

Deflator public consumption NMRP.Q/NMRY.Q

$$\mathring{P}_{g,t}^{def_i} = d\hat{p}_{g,t} \tag{2.214}$$

$$\pi_{g,t} = d\hat{p}_{g,t} - d\hat{p}_{g,t-1}$$

Deflator public investment RPZG.Q/DLWF.Q

$$\mathring{P}_{ig,t}^{def_i} = d\hat{p}_{ig,t} \tag{2.215}$$

$$\pi_{ig,t} = d\hat{p}_{ig,t} - d\hat{p}_{ig,t-1}$$

Dwelling investment deflator: (GGAG.Q+DKQG.Q)/(DFEA.Q+DKQH.Q)

$$\mathring{P}_{h,t} = \hat{q}_t + \hat{P}_t \tag{2.216}$$

Nominal interest rate: 1+Quarterly average rate of discount, 3 month Treasury bills, Sterling (IUQAAJNB)

$$\mathring{R}_t = \hat{R}_t \tag{2.217}$$

Nominal exchange rate growth rate = difference of Quarterly average Effective exchange rate index, Sterling (XUQABK67)

Transfers and taxes are taken from Bhattarai and Trzeciakiewicz (2013)

2.D Impulse Responses, Priors and Posteriors

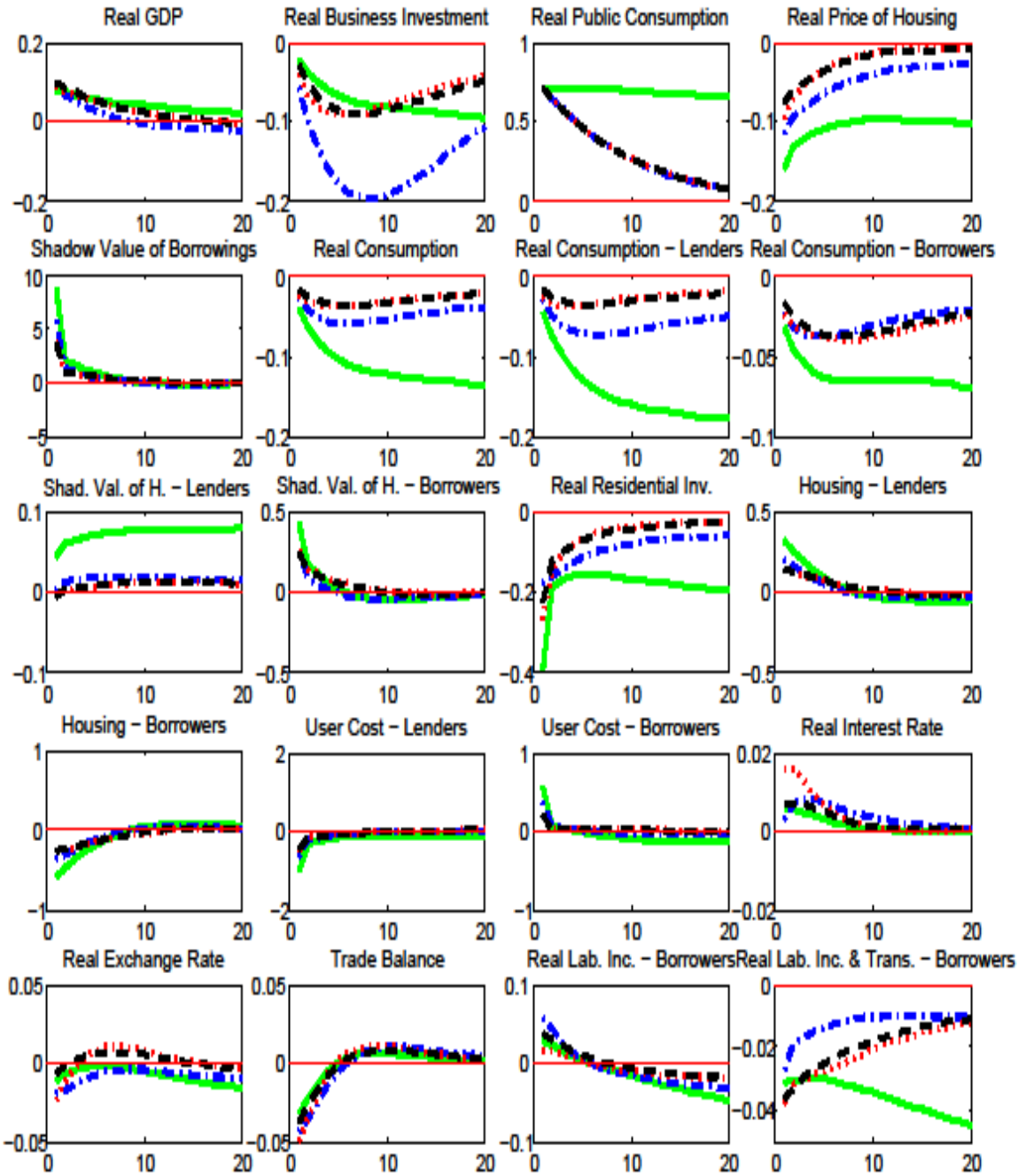
Table 2.16. Priors and Posteriors

Parameter	Prior distribution			Est. max. post.		Post. distribution MH		
	type	mean	st. err.	mode	st. err.	mean	conf. interval	
σ tfp non-resid	inv. gamma	0.01	inf	0.005	0.000	0.005	0.005	0.006
σ tfp resid	inv. gamma	0.01	inf	0.030	0.002	0.031	0.027	0.035
σ investment	inv. gamma	0.10	inf	0.081	0.007	0.085	0.072	0.097
σ preferences	inv. gamma	0.01	inf	0.057	0.007	0.060	0.046	0.073
σ housing demand	inv. gamma	0.10	inf	0.066	0.019	0.079	0.046	0.108
σ risk premium shock	inv. gamma	0.01	inf	0.004	0.001	0.005	0.004	0.006
σ wage markup	inv. gamma	0.10	inf	0.088	0.009	0.095	0.079	0.112
σ domestic prod price	inv. gamma	0.10	inf	0.041	0.015	0.047	0.023	0.070
σ export price	inv. gamma	0.10	inf	0.052	0.006	0.055	0.043	0.066
σ import price investment	inv. gamma	0.10	inf	0.116	0.025	0.139	0.078	0.209
σ import price consumption	inv. gamma	0.10	inf	0.049	0.004	0.051	0.043	0.059
σ import price gov. cons.	inv. gamma	0.10	inf	0.109	0.016	0.124	0.089	0.160
σ import price gov. inv.	inv. gamma	0.10	inf	0.328	0.057	0.356	0.260	0.456
σ mon. policy	inv. gamma	0.01	inf	0.010	0.001	0.010	0.008	0.011
σ gov. cons.	inv. gamma	0.01	inf	0.007	0.001	0.007	0.006	0.008
σ gov. inv.	inv. gamma	0.10	inf	0.520	0.038	0.528	0.465	0.590
σ transfers	inv. gamma	0.01	inf	0.028	0.002	0.029	0.025	0.032
σ cons. tax	inv. gamma	0.01	inf	0.034	0.003	0.035	0.031	0.039
σ capital tax	inv. gamma	0.01	inf	0.022	0.002	0.023	0.020	0.026
σ labour tax	inv. gamma	0.01	inf	0.050	0.004	0.051	0.045	0.056
AR(1) tfp non-resid	beta	0.80	0.10	0.975	0.011	0.966	0.944	0.991
AR(1) tfp resid	beta	0.80	0.10	0.998	0.005	0.995	0.991	0.999
AR(1) investment	beta	0.50	0.10	0.240	0.066	0.258	0.149	0.367
AR(1) preferences	beta	0.80	0.10	0.781	0.044	0.784	0.714	0.855
AR(1) housing demand	beta	0.80	0.10	0.942	0.027	0.938	0.902	0.976
AR(1) risk premium	beta	0.80	0.10	0.887	0.037	0.900	0.849	0.955
AR(1) wage markup	beta	0.80	0.10	0.959	0.020	0.935	0.886	0.984
AR(1) domestic prod price	beta	0.80	0.10	0.572	0.101	0.542	0.397	0.689
AR(1) export price	beta	0.80	0.10	0.966	0.021	0.947	0.908	0.988
AR(1) import price investment	beta	0.80	0.10	0.873	0.041	0.851	0.778	0.928
AR(1) import price consumption	beta	0.80	0.10	0.991	0.006	0.986	0.974	0.998
AR(1) import price gov. cons.	beta	0.80	0.10	0.885	0.054	0.859	0.764	0.958
AR(1) import price gov. inv.	beta	0.80	0.10	0.654	0.094	0.632	0.477	0.782
AR(1) gov. cons.	beta	0.80	0.10	0.909	0.039	0.902	0.844	0.966
AR(1) gov. inv.	beta	0.50	0.10	0.265	0.062	0.273	0.172	0.384
AR(1) transfers	beta	0.80	0.10	0.869	0.049	0.873	0.800	0.947
AR(1) cons. tax	beta	0.80	0.10	0.804	0.080	0.813	0.701	0.934
AR(1) capital tax	beta	0.80	0.10	0.856	0.070	0.850	0.754	0.949
AR(1) labour tax	beta	0.80	0.10	0.793	0.056	0.796	0.709	0.884
gov. con. resp. debt	normal	0.20	0.10	0.082	0.038	0.093	0.032	0.157
gov. inv. resp. to debt	normal	0.20	0.10	0.175	0.098	0.179	0.016	0.329
trans. resp. to debt	normal	0.20	0.10	0.037	0.074	0.044	-0.080	0.165
con. tax resp. to debt	normal	0.20	0.10	0.218	0.079	0.225	0.099	0.359
cap. tax resp. to debt	normal	0.20	0.10	0.079	0.093	0.086	-0.049	0.216
lab. tax resp. to debt	normal	0.20	0.10	0.161	0.090	0.182	0.047	0.321

Table 2.17. Priors and Posteriors

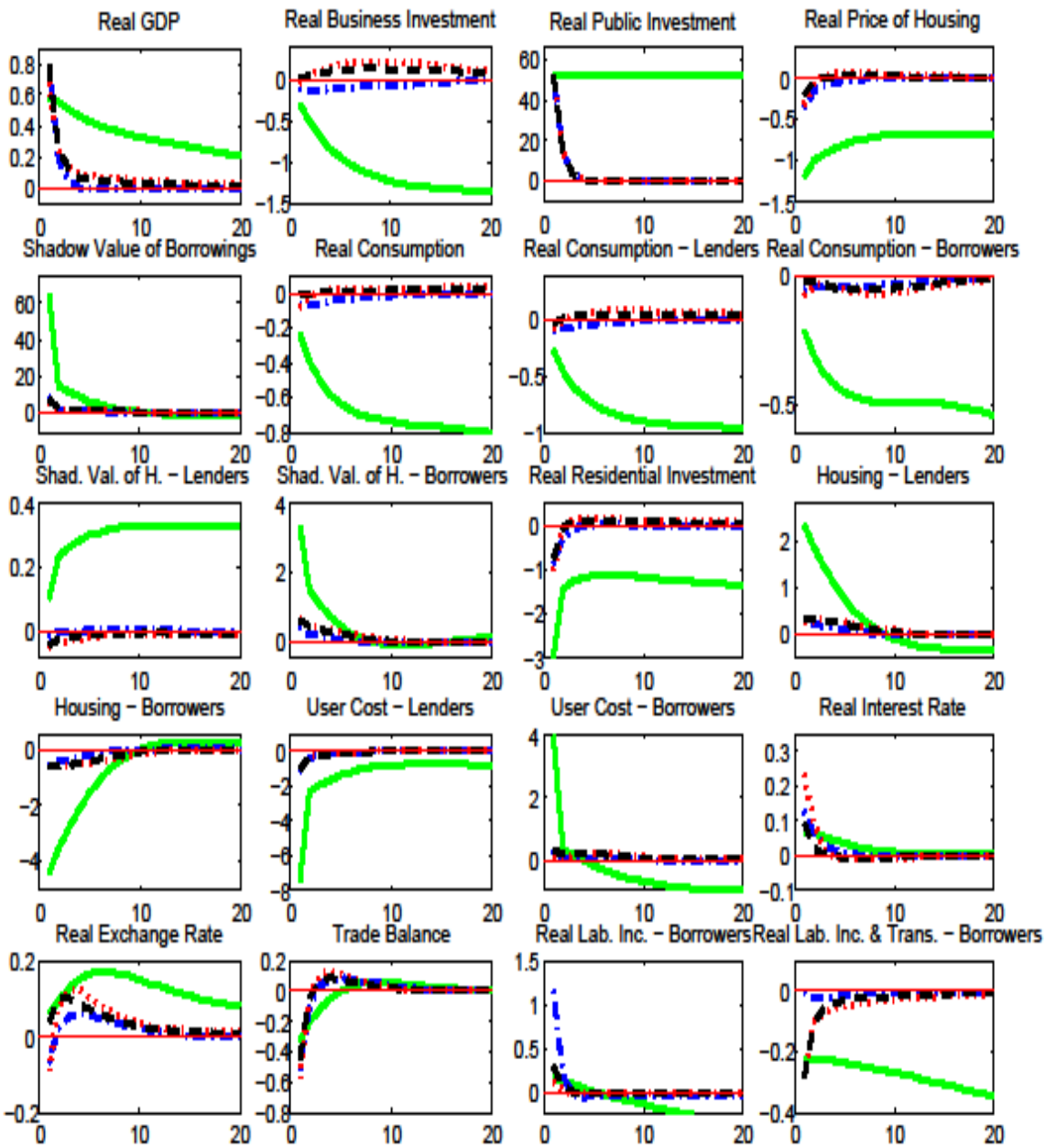
Parameter	Prior distribution			Est. max. post.		Post. distribution MH		
	type	mean	st. err.	mode	st. err.	mean	conf. interval	
AR (1) nom. int. rate	beta	0.70	0.10	0.524	0.041	0.544	0.479	0.613
inflation response	normal	1.50	0.10	1.623	0.080	1.630	1.491	1.759
output response	normal	0.125	0.10	0.178	0.076	0.203	0.071	0.329
gov. con. resp. GDP	normal	0.50	0.50	0.195	0.111	0.197	0.022	0.366
gov. inv. resp. to GDP	normal	0.50	0.50	1.088	0.485	1.075	0.286	1.830
trans. resp. to GDP	normal	0.50	0.50	0.700	0.300	0.722	0.215	1.188
con. tax resp. to GDP	normal	0.50	0.50	0.714	0.315	0.727	0.203	1.235
cap. tax resp. to GDP	normal	0.50	0.50	0.578	0.317	0.637	0.142	1.110
lab. tax resp. to GDP	normal	0.50	0.50	1.132	0.353	1.137	0.572	1.704
calvo domestic prices	beta	0.50	0.10	0.723	0.048	0.719	0.645	0.794
price index.	beta	0.30	0.15	0.030	0.034	0.081	0.006	0.162
calvo import priv. cons. prices	beta	0.50	0.10	0.229	0.042	0.233	0.162	0.307
price import priv cons. index.	beta	0.30	0.15	0.075	0.060	0.125	0.013	0.240
calvo import priv. inv. prices	beta	0.50	0.10	0.321	0.075	0.343	0.207	0.479
price import priv. inv. index.	beta	0.30	0.15	0.080	0.068	0.128	0.013	0.244
calvo import pub. cons. prices	beta	0.50	0.10	0.250	0.069	0.267	0.147	0.387
price import pub cons. index.	beta	0.30	0.15	0.074	0.061	0.123	0.010	0.231
calvo import pub. inv. prices	beta	0.50	0.10	0.231	0.072	0.246	0.138	0.356
price import pub. inv. index.	beta	0.30	0.15	0.073	0.061	0.119	0.011	0.227
calvo export prices	beta	0.50	0.10	0.248	0.056	0.254	0.163	0.345
price export index.	beta	0.30	0.15	0.073	0.059	0.118	0.011	0.223
calvo wages non-residential	beta	0.50	0.10	0.863	0.020	0.851	0.817	0.888
wage non-residential index.	beta	0.30	0.15	0.267	0.134	0.290	0.081	0.480
calvo wages residential	beta	0.50	0.10	0.243	0.046	0.291	0.054	0.511
wage residential index.	beta	0.30	0.15	0.259	0.171	0.256	0.177	0.334
invest. adj. cost non-resid	normal	4.00	1.50	4.536	1.194	4.904	3.021	6.825
invest. adj. cost resid	normal	4.00	1.50	3.275	1.729	3.470	0.868	5.849
capital util. adj. cost	normal	0.8	0.20	1.471	0.156	1.476	1.216	1.732
inv. elast. of labour Patients	normal	0.50	0.10	0.319	0.044	0.364	0.269	0.457
inv. elast. of labour Impatients	normal	0.50	0.10	0.674	0.069	0.663	0.535	0.791
inv. elast. of hours in 2 sec. Pat.	normal	1.00	0.10	1.065	0.095	1.083	0.929	1.235
inv. elast. of hours in 2 sec. Imp.	normal	1.00	0.10	0.968	0.098	0.965	0.799	1.128
habit Patient consumption	beta	0.70	0.10	0.776	0.032	0.779	0.727	0.833
habit Impatient consumption	beta	0.70	0.10	0.958	0.009	0.949	0.926	0.973
habit Patient housing	beta	0.50	0.10	0.760	0.095	0.628	0.434	0.840
habit Impatient housing	beta	0.50	0.10	0.494	0.103	0.477	0.320	0.629
risk premium	inv gamma	0.01	2.00	0.022	0.011	0.039	0.011	0.067
sh.of Impatient labour	beta	0.65	0.1	0.748	0.047	0.779	0.688	0.878
elast. of subs. priv. cons.	inv gamma	1.50	2.00	0.553	0.116	0.644	0.412	0.877
elast. of subs. priv. inv.	inv gamma	1.50	2.00	2.963	0.524	2.904	1.881	4.016
elast. of subs. pub. cons.	inv gamma	1.50	2.00	1.403	0.545	1.559	0.715	2.348
elast. of subs. pub. inv.	inv gamma	1.50	2.00	0.714	0.236	0.896	0.424	1.419
elast. of subs. foreign	inv gamma	1.50	2.00	1.681	0.179	1.811	1.432	2.196
markup domestic	gamma	0.15	0.10	0.399	0.124	0.436	0.221	0.649
markup import priv. cons.	gamma	0.15	0.10	0.155	0.115	0.211	0.034	0.379
markup import priv. inv.	gamma	0.15	0.10	0.432	0.158	0.467	0.165	0.768
markup import pub. cons.	gamma	0.15	0.10	0.087	0.077	0.147	0.015	0.281
markup import pub. inv.	gamma	0.15	0.10	0.052	0.047	0.091	0.005	0.175
markup housing wage	gamma	0.15	0.05	0.170	0.048	0.186	0.105	0.266

Fig. 2.12. Impulse - responses to a positive one standard deviation public consumption shock



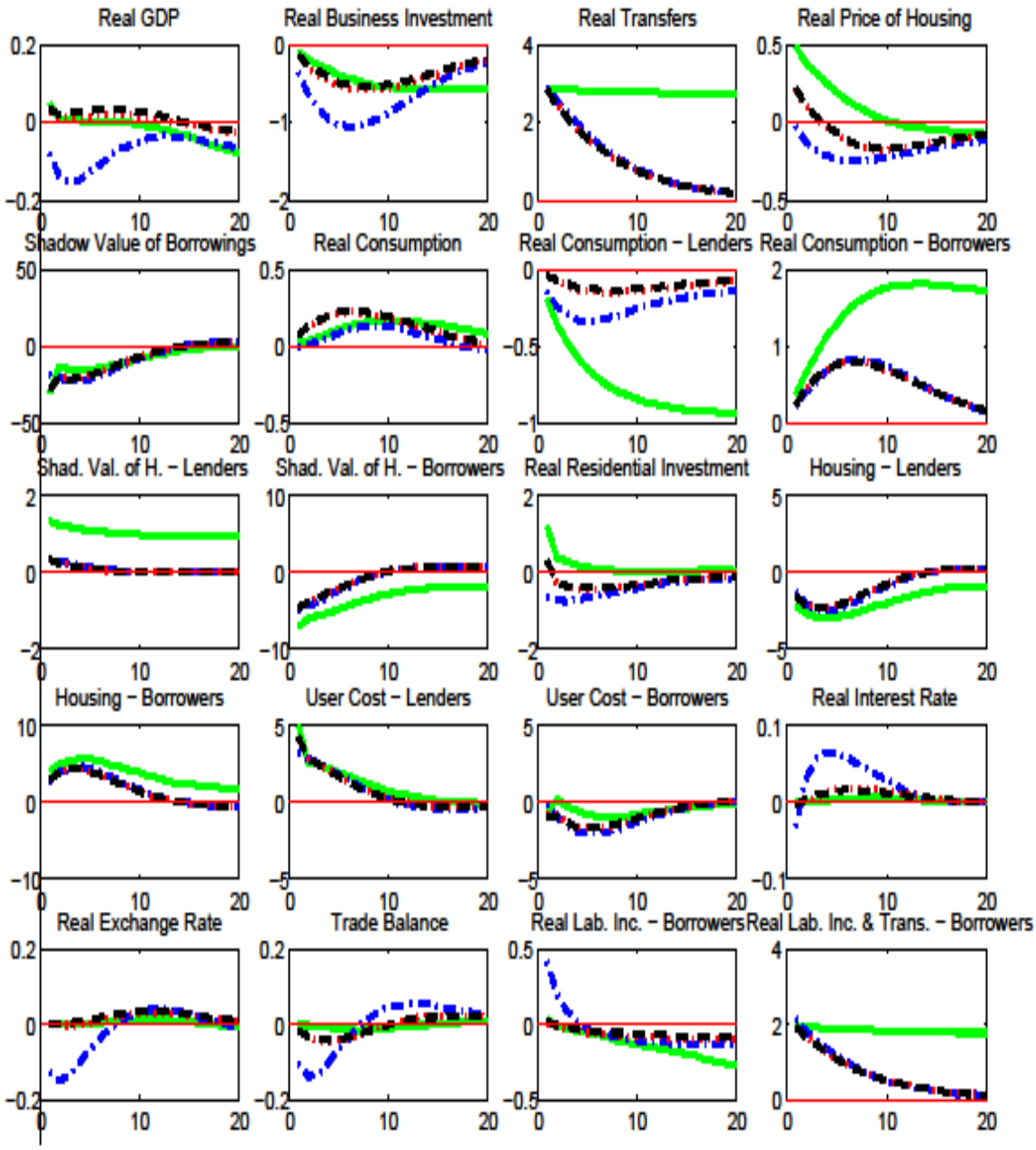
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.13. Impulse - responses to a positive one standard deviation public investment shock



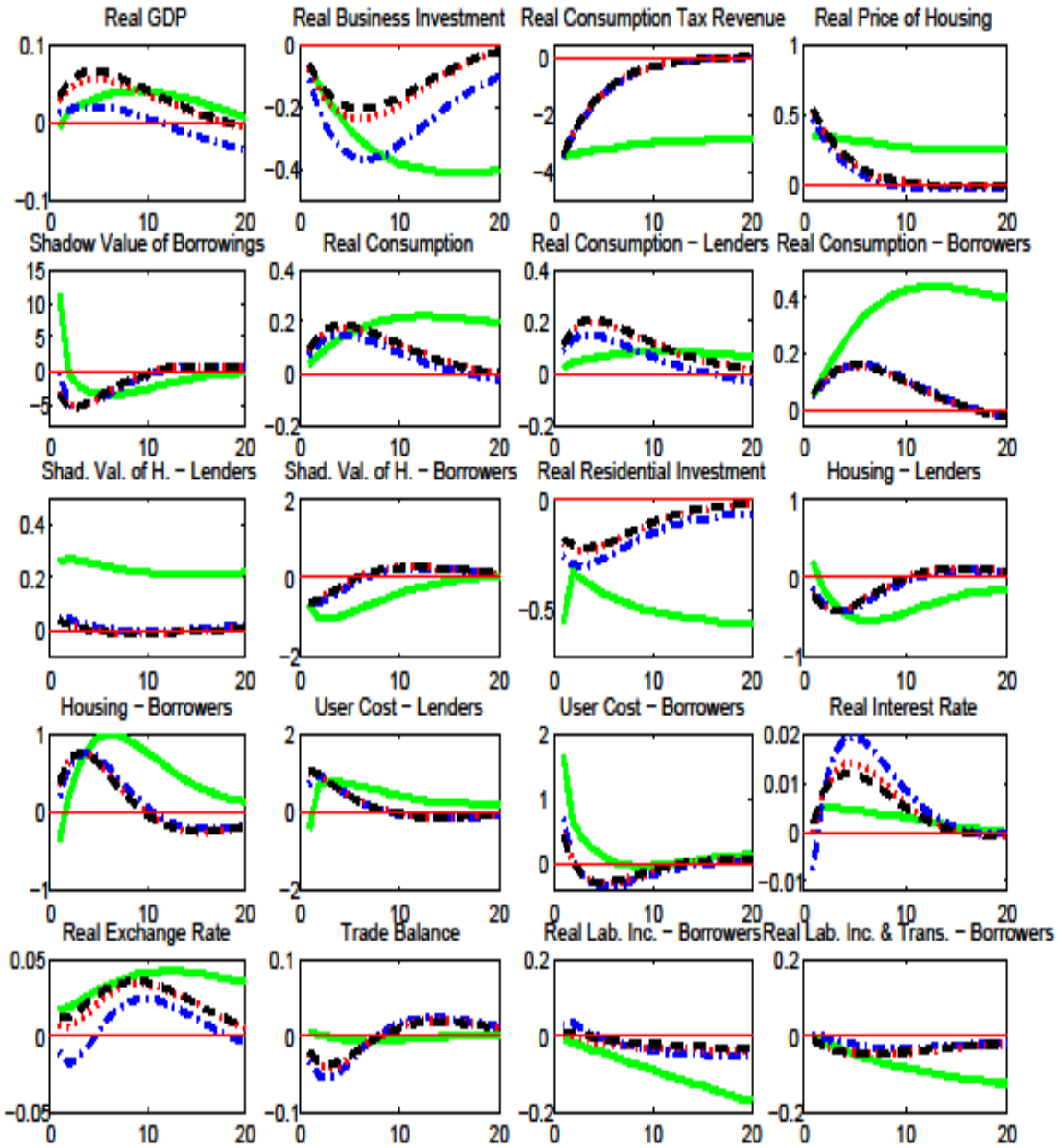
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.14. Impulse - responses to a positive one standard deviation public transfers shock



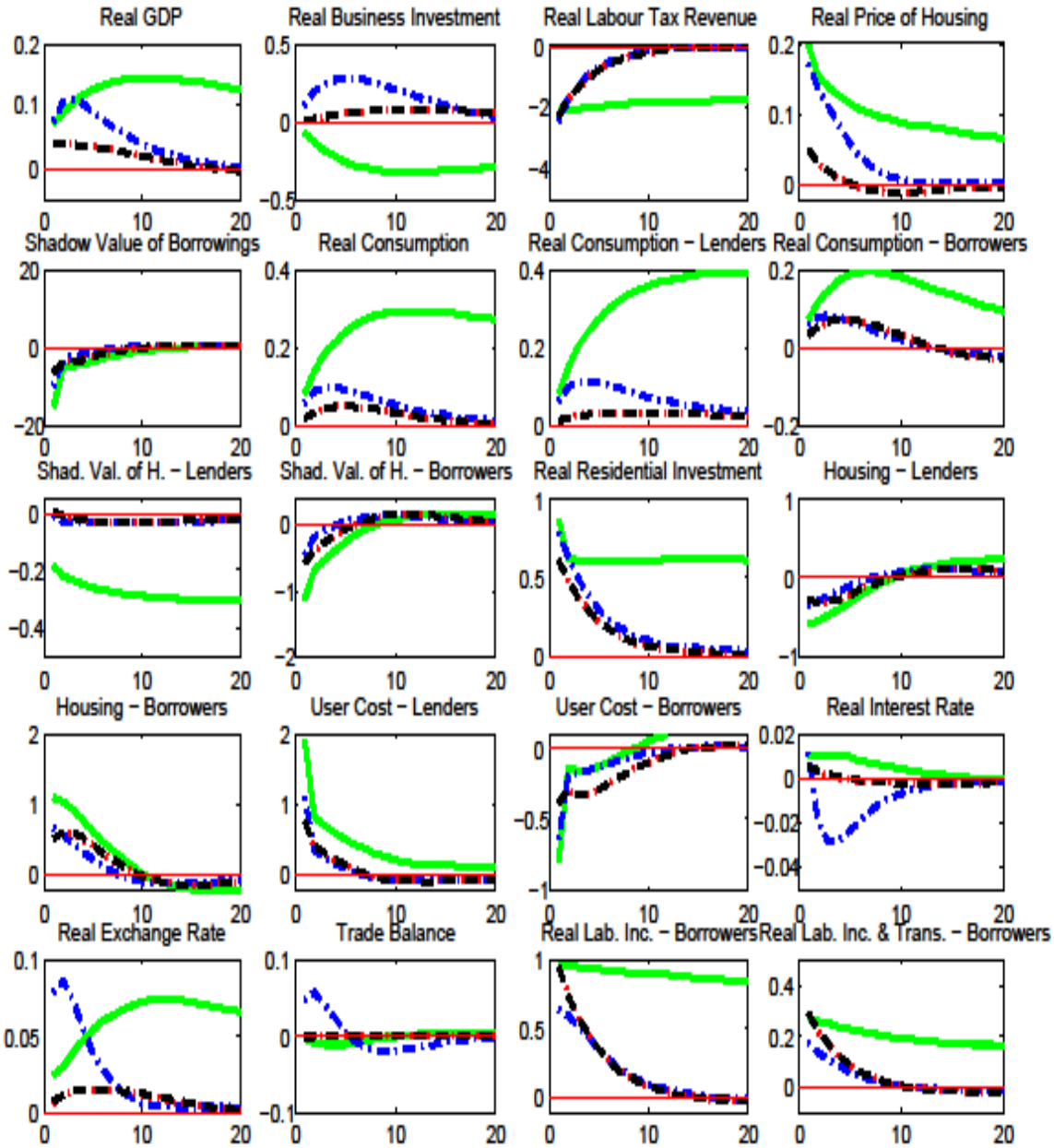
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.15. Impulse - responses to a negative one standard deviation consumption tax shock



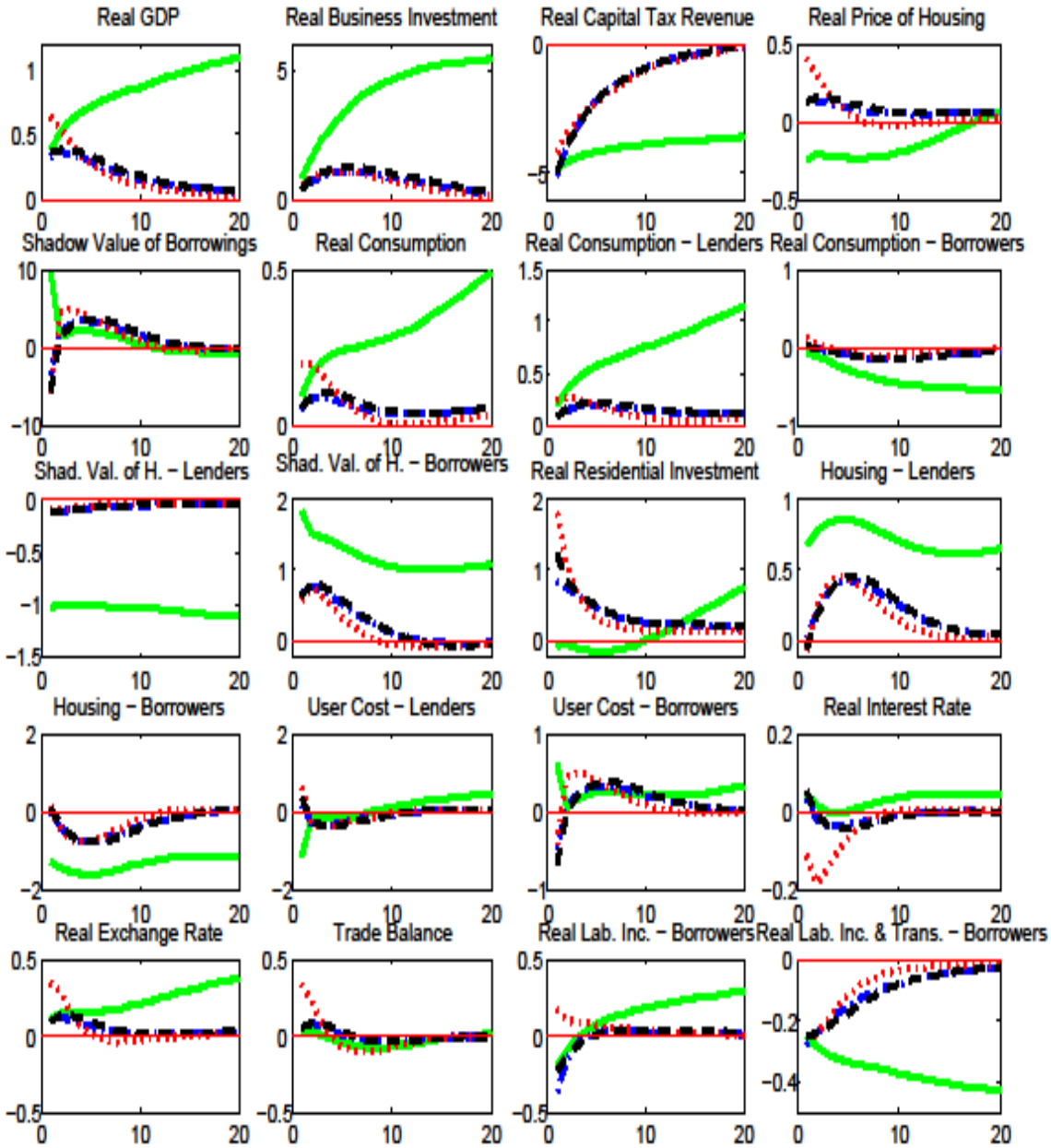
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.16. Impulse - responses to a negative one standard deviation labour tax shock



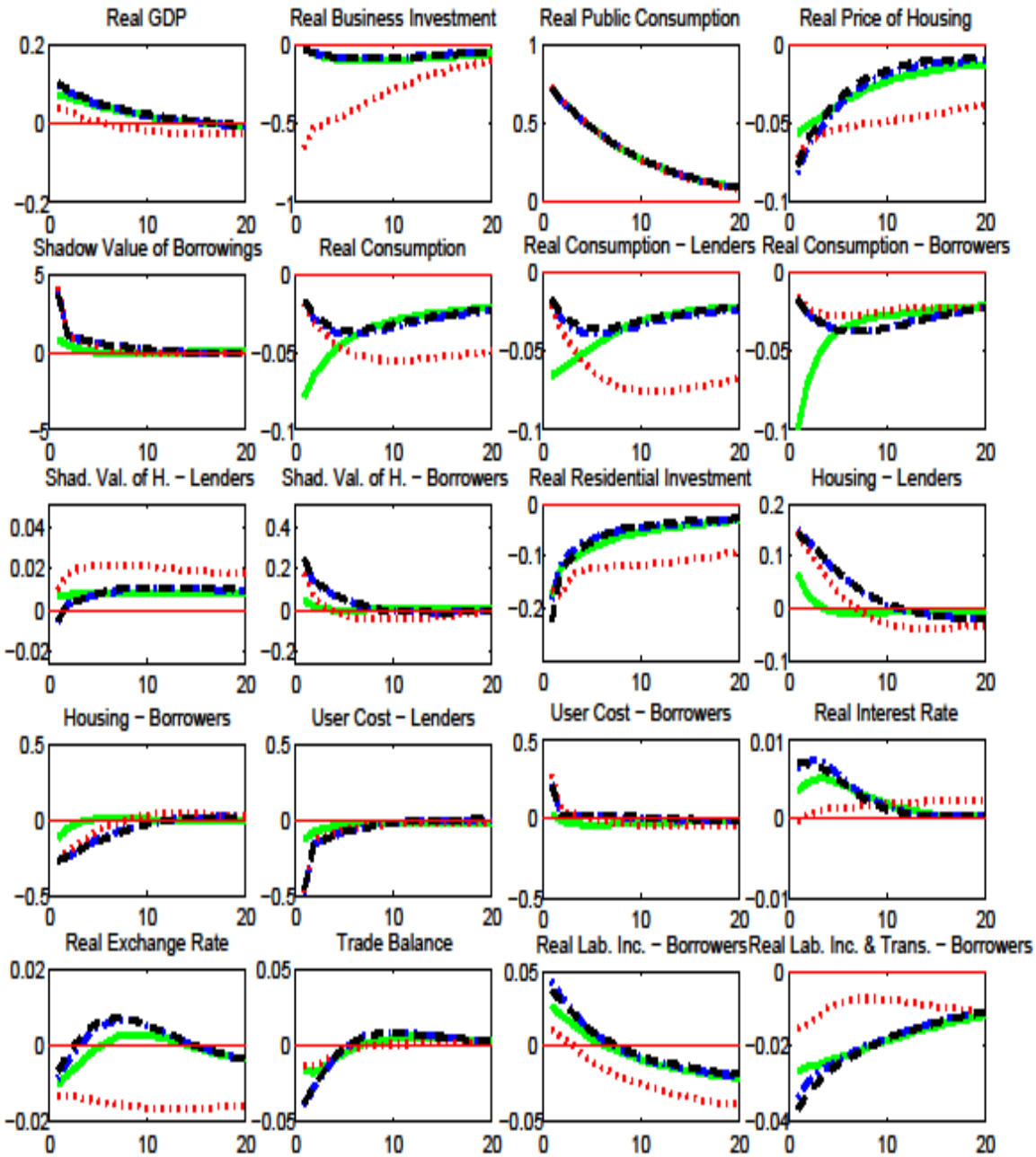
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.17. Impulse - responses to a negative one standard deviation capital tax shock



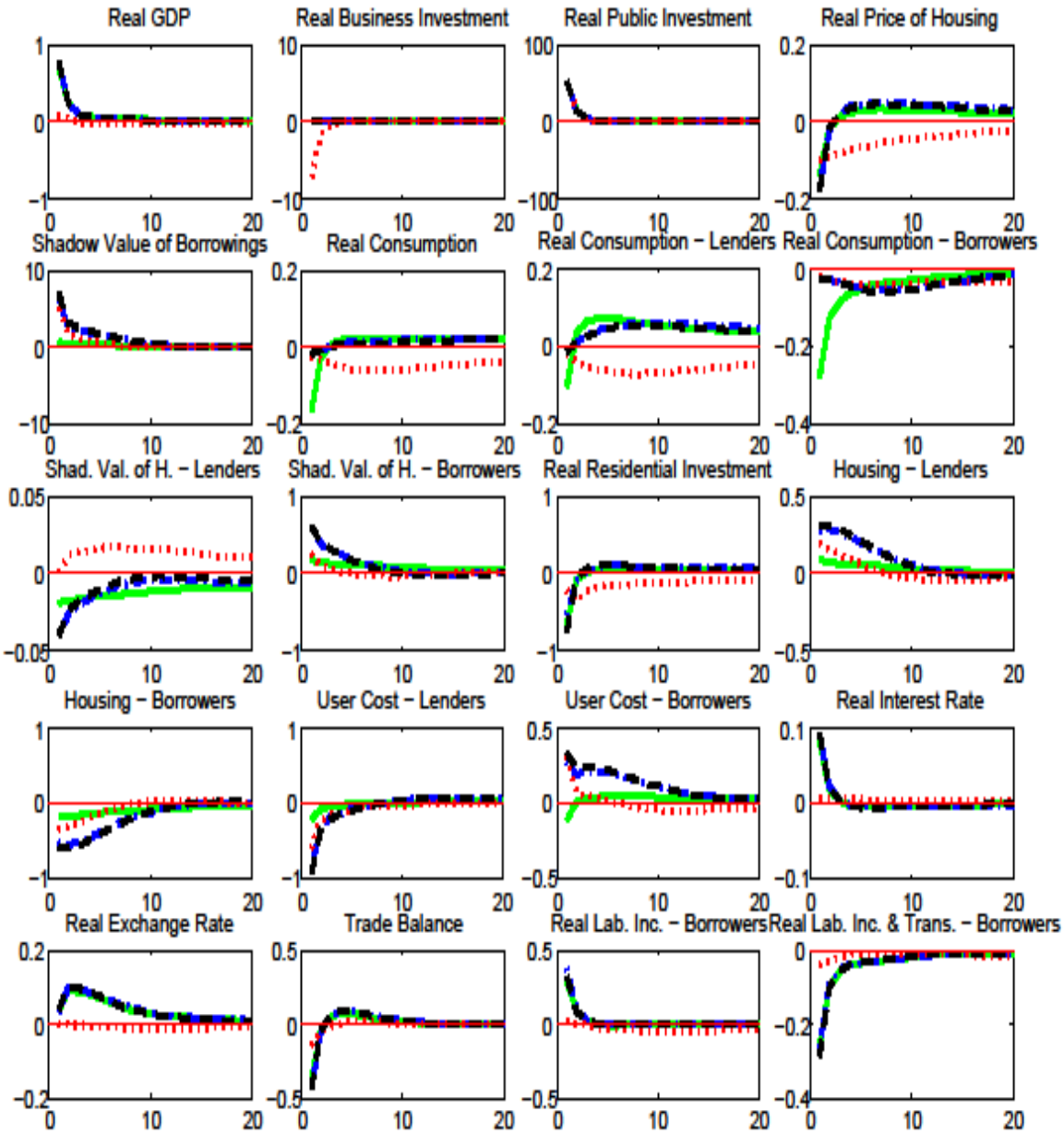
benchmark, (−−); flexible prices, (.); flexble wages, (−.); higher persistance , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.18. Impulse - responses to a positive one standard deviation public consumption shock



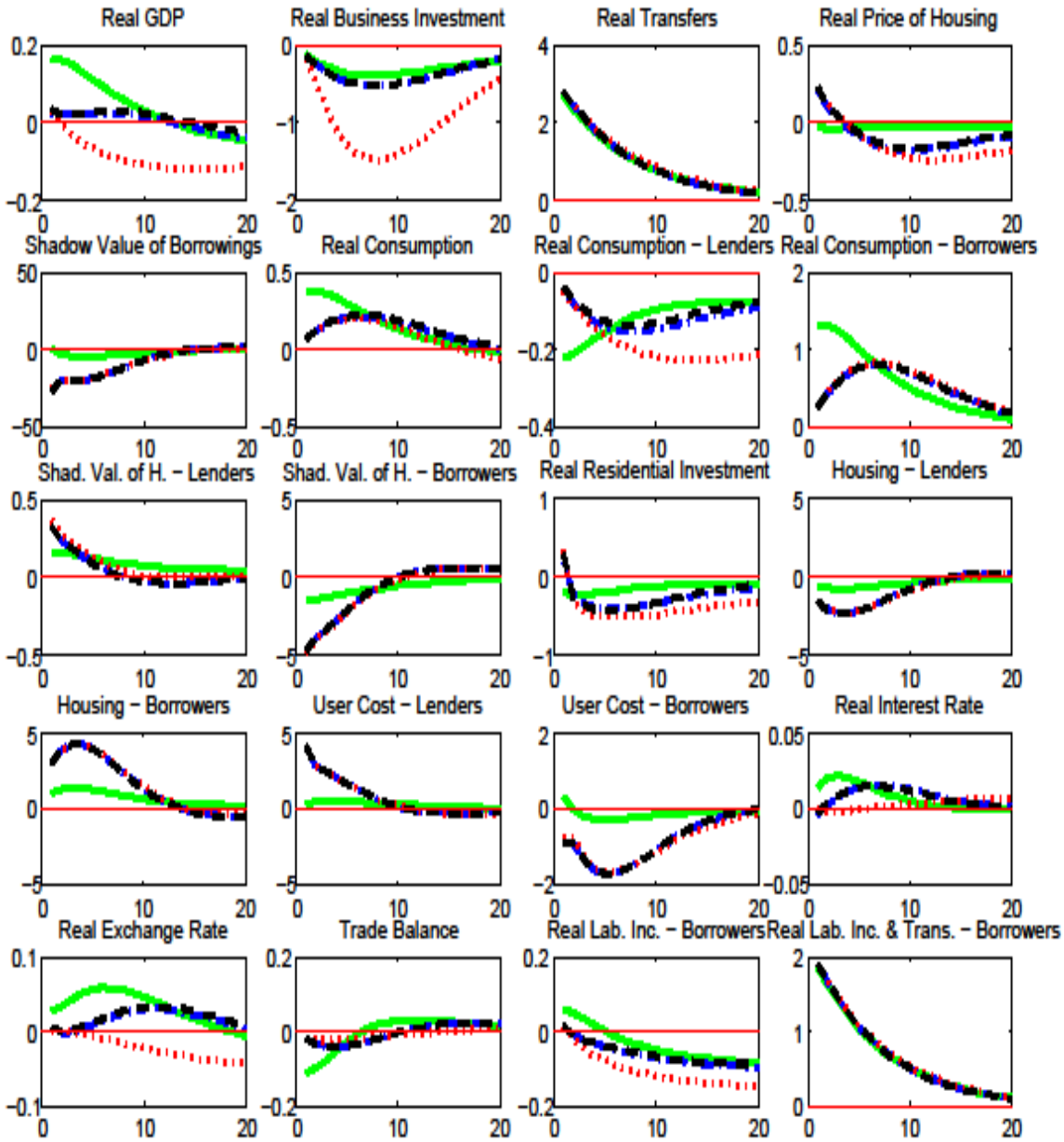
benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.19. Impulse - responses to a positive one standard deviation public investment shock



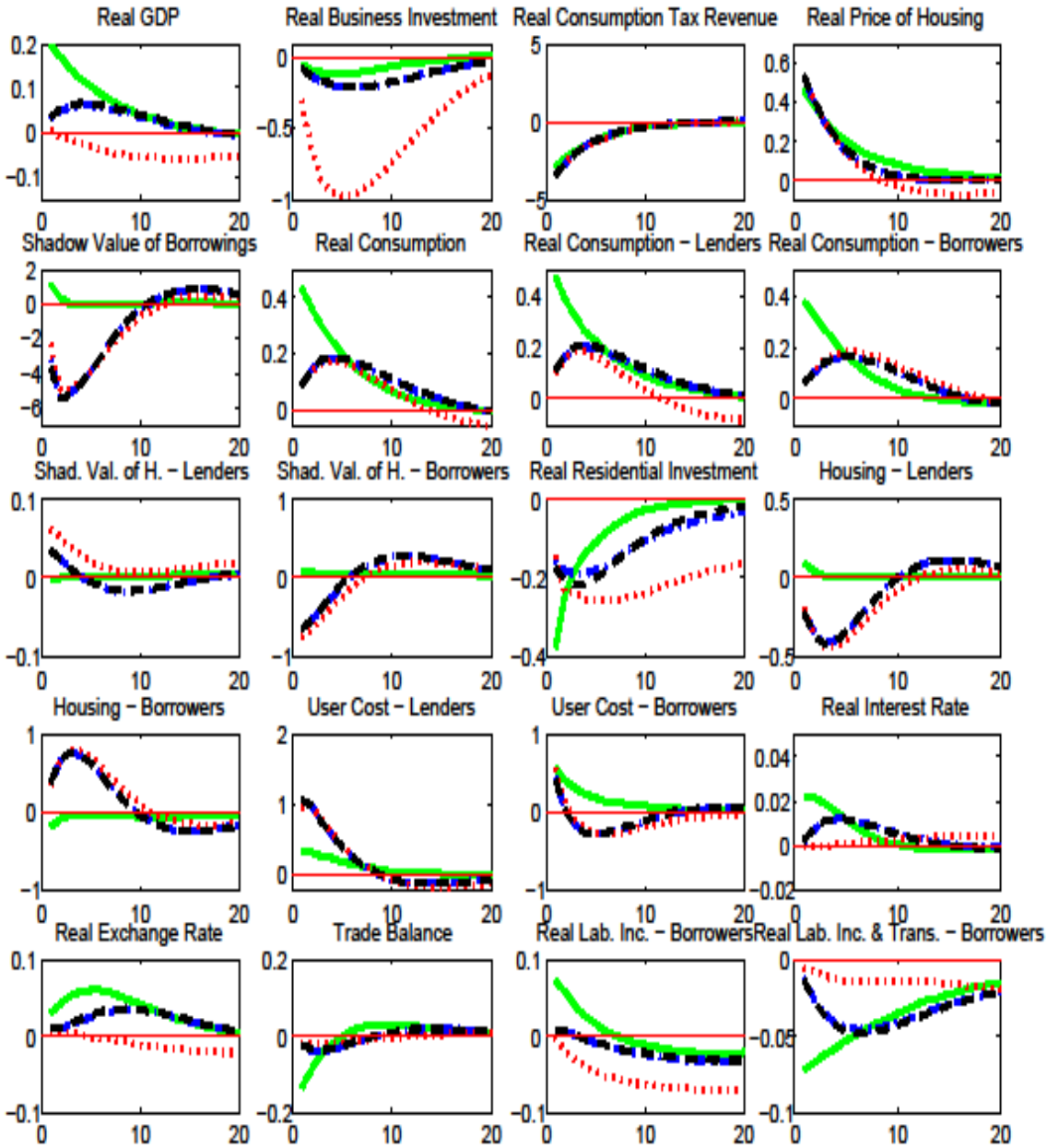
benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.20. Impulse - responses to a positive one standard deviation public transfers shock



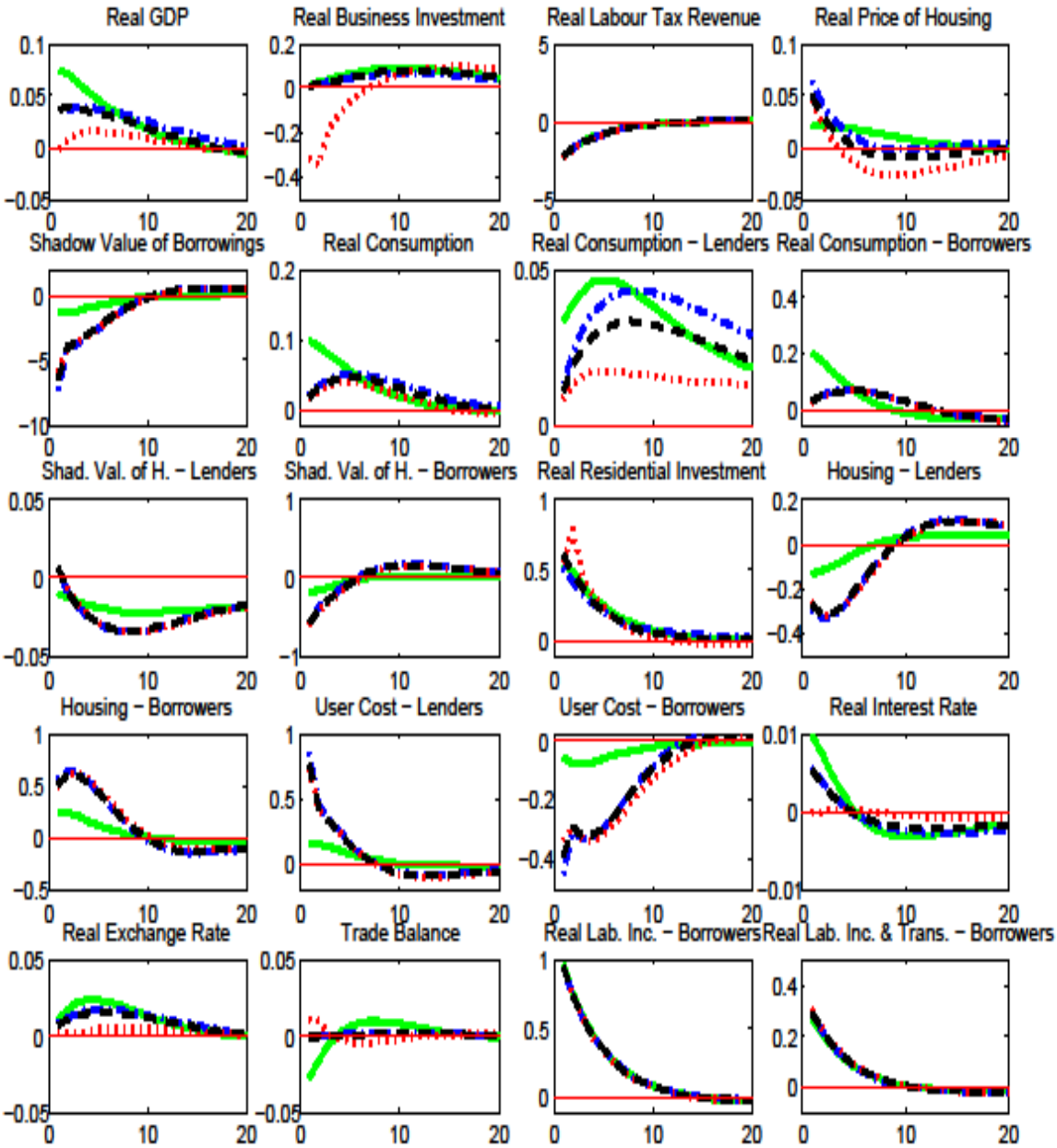
benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.21. Impulse - responses to a negative one standard deviation consumption tax shock



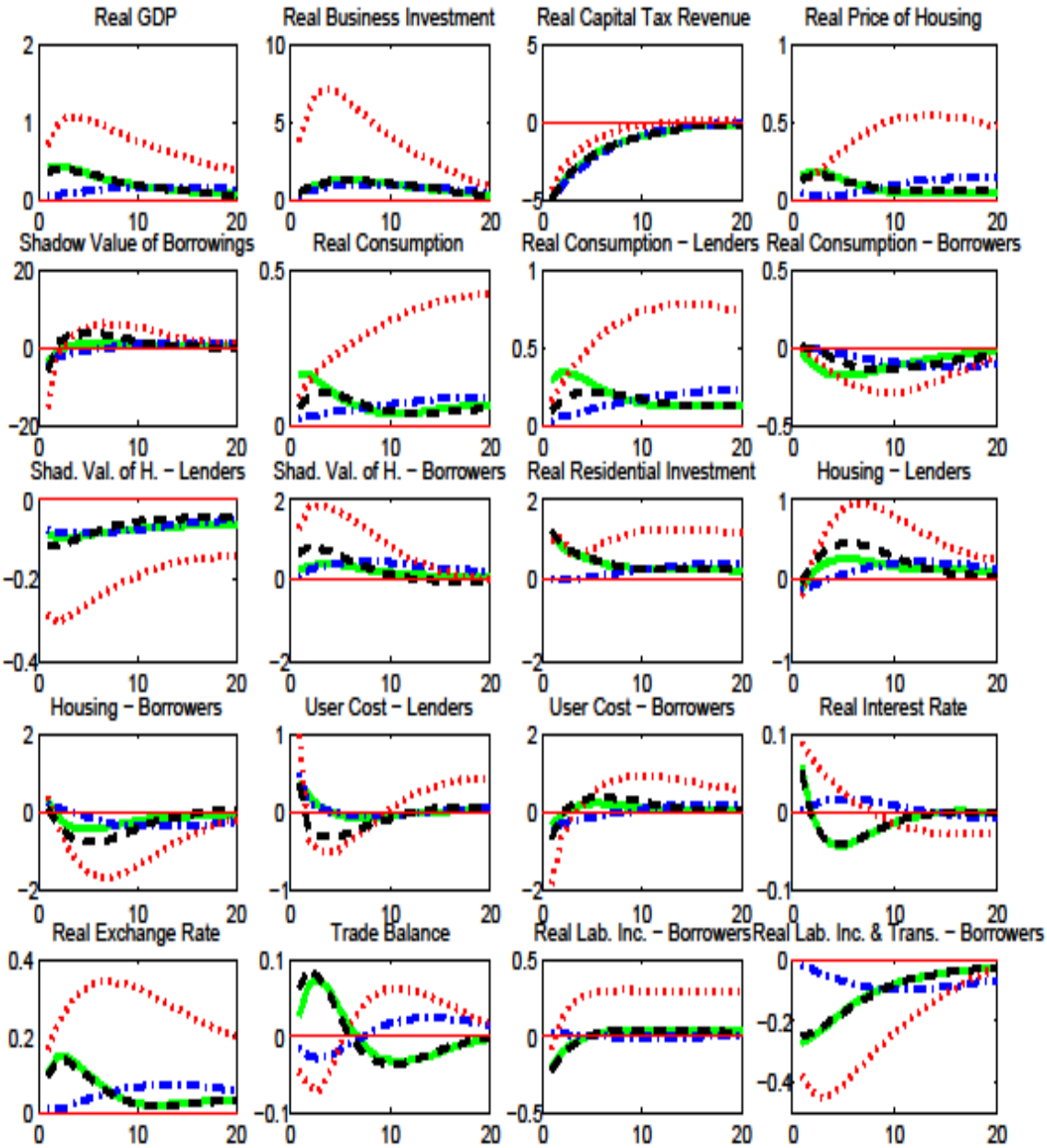
benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.22. Impulse - responses to a negative one standard deviation labour tax shock



benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

Fig. 2.23. Impulse - responses to a negative one standard deviation capital tax shock



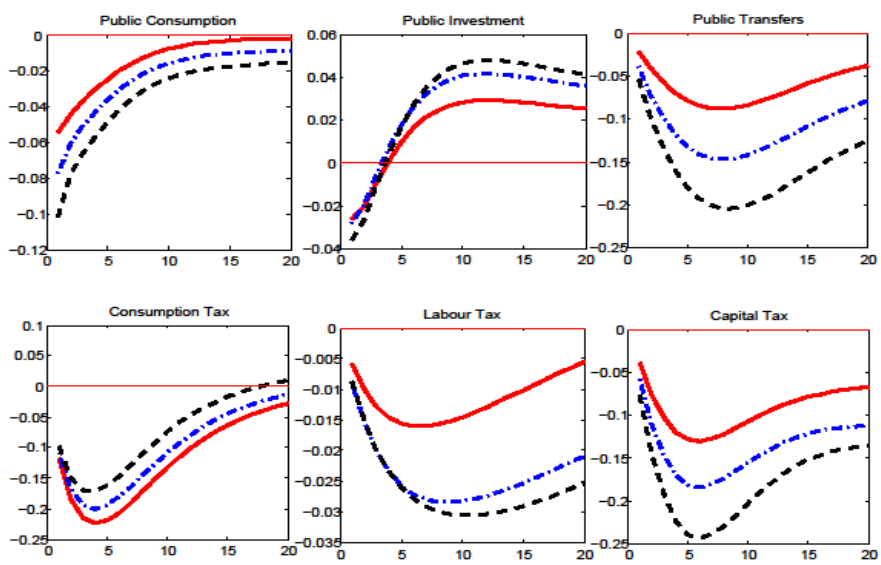
benchmark, (−−); no investment adjustment cost, (.); no capital utilisation, (−.); no habit , (−); impulse responses are presented as percentage deviations from the steady state

2.E Sensitivity

Table 2.18. Present value GDP multipliers for permanent shocks

GDP multipliers	Quarters					
	1	4	12	20	40	1000
Public consumption	0.59	0.52	0.39	0.32	0.19	-0.54
Public investment	0.56	0.52	0.41	0.35	0.26	0.10
Public transfers	0.07	0.03	0.01	-0.02	-0.14	-0.77
Consumption tax	0.01	-0.03	-0.07	-0.07	0.01	0.67
Labour tax	-0.17	-0.23	-0.32	-0.34	-0.32	0.05
Capital tax	-0.67	-1.01	-1.48	-1.78	-2.28	-3.31

Fig. 2.24. Impulse - responses of house price to changes in the monetary policy



benchmark, $(-.)$; $0.75 * \rho_\pi; 0.75 * \rho_y$, $(-)$; $1.5 * \rho_\pi; 1.5 * \rho_y$,$(.)$;

Table 2.19. Sensitivity Analysis

	Frictionless Economy	$\varpi_{c,w} = 0.851$ $\varpi_{h,w} = 0.256$	$h_c = 0.779$ $h'_c = 0.949$ $h_h = 0.628$ $h'_h = 0.477$	$\varpi_{g,m} = 0.267$ $\varpi_{i,m} = 0.343$ $\varpi_p = 0.719$ $\varpi_{g,m} = 0.246$ $\varpi_{c,m} = 0.233$ $\varpi_x = 0.254$	$\phi_{k,c} = 4.904$ $\phi_{k,h} = 3.470$	$\kappa = 1.476$
Fiscal Multipliers of Public Consumption						
1	0.24	0.16	0.40	0.26	0.29	0.22
4	0.17	0.11	0.25	0.19	0.26	0.16
12	0.01	-0.08	0.02	0.02	0.14	0.02
20	-0.17	-0.26	-0.18	-0.15	-0.01	-0.13
40	-0.64	-0.63	-0.69	-0.60	-0.42	-0.54
1000	-2.46	-1.74	-2.69	-2.31	-1.89	-2.13
Fiscal Multipliers of Public Investment						
1	0.07	0.11	0.10	0.09	0.24	0.06
4	-0.05	-0.06	-0.01	0.01	0.25	-0.02
12	-0.35	-0.39	-0.35	-0.26	0.15	-0.22
20	-0.61	-0.56	-0.63	-0.48	0.03	-0.40
40	-1.04	-0.68	-1.12	-0.87	-0.19	-0.72
1000	-1.83	-0.77	-1.99	-1.52	0.49	-1.30
Fiscal Multipliers of Public transfers						
1	-0.05	0.04	-0.54	0.01	0.01	-0.07
4	-0.13	-0.02	-0.60	-0.10	-0.03	-0.15
12	-0.33	-0.23	-0.67	-0.31	-0.17	-0.32
20	-0.53	-0.42	-0.79	-0.51	-0.35	-0.50
40	-0.96	-0.76	-1.19	-0.93	-0.74	-0.88
1000	-1.95	-1.37	-2.22	-1.88	-1.63	-1.81
Fiscal Multipliers of Consumption Tax						
1	-0.15	-0.06	0.06	-0.11	-0.23	-0.14
4	-0.08	0.03	0.10	-0.06	-0.21	-0.08
12	0.13	0.31	0.18	0.14	-0.08	0.10
20	0.36	0.57	0.35	0.36	0.10	0.29
40	0.90	1.02	0.85	0.89	0.53	0.75
1000	2.82	2.29	2.61	2.74	1.91	2.35
Fiscal Multipliers of Labour Tax						
1	-0.88	-0.06	-0.74	-0.35	-0.71	-0.98
4	-0.95	-0.07	-0.87	-0.66	-0.72	-1.02
12	-1.15	-0.08	-1.20	-0.89	-0.78	-1.13
20	-1.33	-0.07	-1.47	-1.02	-0.86	-1.22
40	-1.63	0.01	-1.85	-1.21	-1.00	-1.39
1000	-1.90	0.46	-2.15	-1.38	-1.13	-1.59
Fiscal Multipliers of Capital Tax						
1	-0.22	-0.74	-0.33	-0.90	-0.01	-1.41
4	-0.27	-1.12	-0.34	-0.85	-0.01	-1.48
12	-0.35	-2.47	-0.40	-0.83	-0.03	-1.65
20	-0.40	-4.04	-0.46	-0.97	-0.03	-1.83
40	-0.48	-6.39	-0.55	-1.24	0.01	-2.19
1000	-0.55	-7.07	-0.64	-1.68	0.18	-2.92

2.F Dynare code

```
var b trans tao_h G IG Kg tao_k tao_c tao_l C_prim C I_h I_c e_j lambda R
    pi K_c K_h Q_c Q_h e_I_c q r_k_c r_k_h lambda_prim e_ltv b_prim w_c L_c e_l
    w_h L_h w_c_prim w_h_prim L_c_prim L_h_prim y e_ac e_pi e_pi_e e_pi_m_c
    e_pi_m_i HI e_ah e_g e_m H H_prim e_n mc p_I p_c S p_e pi_e mc_e aa p_m_c
    pi_m_c pi_m_i p_m_I e_ig e_tc e_tk e_tl pi_g_m e_pi_g p_m_g p_g
    pi_IG_m e_pi_IG p_m_IG p_IG e_tr UU C_T I_T res_inv H_w H_wprim H_wt
    GDP tao_kinc tao_linc tao_cinc growth_rate cons_hp lambda_b
    U_c U_c_prim U_h U_h_prim user_cost user_cost_prim trade_bb rr Limpinctr
    Limpextr KK VV_prim VV rex H_t;
    // variables
    // b government debt
    // G government consumption
    // IG government investment
    // trans government transfers
    // tao_k tax on capital income
    // tao_c tax on consumption
    // tao_l tax on labour income
    // tao_h tax on housing
    // Kg public capital
    // K_c private non-residential capital
    // K_h private residential capital
```

```
// Q_c price of non-residential capital

// Q_h price of residential capital

// I_h housing capital investment

// I_c non-housing capital investment

// r_k_c rate of return on housing capital

// r_k_h rate of return on non-housing capital

// lambda shadow value of income for Patient households

// lambda_prim shadow value of income for Impatient households

// C consumption - Patient households

// C_prim consumption - Impatient households

// H housing - Patient households

// H_prim consumption - Impatient households

// L_c labour for non-residential sector - Patient households

// L_c_prim labour for non-residential sector - Impatient households

// L_h labour for residential sector - Patient households

// L_h_prim labour for residential sector - Impatient households

// w_c wage for non-residential sector - Patient households

// w_c_prim wage for non-residential sector - Impatient households

// w_h wage for residential sector - Patient households

// w_h_prim wage for residential sector - Impatient households

// R nominal rate of return

// y non-residential output
```

```
// HI residential investment

// q house price

// mc marginal cost non-residential sector

// mc_e marginal cost - exporters

// aa net foreign assets

// S nominal exchange rate difference

// b_prim borrowings of Impatient households

// UU GDP for interesst rule

// p_c relative price of private consumption

// p_I relative price of private investment

// p_g relative price of public consumption

// p_IG relative price of public investment

// p_e relative price of exports

// pi inflation of domestically produced goods

// pi_e inflation of exported goods

// pi_m_c inflation of imported, private consumption goods

// pi_m_i inflation of imported, private investment goods

// pi_g_m inflation of imported, public consumption goods

// pi_IG_m inflation of imported, public investment goods

// p_m_c price of imported, private consumption goods

// p_m_I price of imported, private investment goods

// p_m_g price of imported, public consumption goods
```

```
// p_m_IG price of imported, public consumption goods

// e_g shock to public spending

// e_ig shock to public investment

// e_tr shock to public transfers

// e_tc shock to consumption tax

// e_tk shock to capital tax

// e_tl shock to labour tax

// e_pi cost push-up shock domestic prices

// e_pi_m_c cost push-up shock imported private consumption prices

// e_pi_m_i cost push-up shock imported private investment prices

// e_pi_g cost push-up shock imported public consumption prices

// e_pi_IG cost push-up shock imported public investment prices

// e_pi_e cost push-up shock exports

// e_l wage push-up shock

// e_ac productivity - non-residential sector

// e_ah productivity - residential sector

// e_m Taylor rule

// e_n preference shock

// e_j housing preference shock

// e_I_c private non-residential investment shock

// e_ltv risk premium shock

// all variables starting with ln_ denote observable variavles
```

```
varexo lgt ligt ltr tc tl tk lnt lpimc laht lact lit lpimi lpit ljt lpict llt lpig lpiIG ltvt lpiet;
parameters sigma_lprim X_mc X_mg X_mi X_mig XX_c_bar rho_j rho_i
rho_ah rho_pimi rho_pi rho_g ela_gcy ela_gcb rho_l rho_pimc jej sigma_g
tao_hbar tao_cbar tao_kbar tao_lbar gamma_r omega_r rho_n alfa_m
hab_c sigma_c hab_c_prim sigma_c_prim ig gg sh tr delta_g rho_piet beta
delta_h kappa gamma tao rho_e hab_h sigma_h hab_h_prim sigma_h_prim
delta_k_c delta_k_h phi_c phi_h omega_wc omega_wh gamma_wc gamma_wh sigma_l
zeta psi_c be_1 alfa omega_p gamma_p omega_e gamma_e gamma_m_i omega_m_i
gamma_m_c omega_m_c psi_h alfa_h gamma_h omega_h a_c s_c a_i s_i s_f rho
b_y b_pi b_x std_act rho_ac std_yst std_rst std_pist phi_aa std_gt rho_ig
ela_igb phi_tc ela_tc rho_ltv rho_ih ela_tk ela_tl ela_tr phi_tk phi_tl
gamma_g omega_g rho_pi_g a_g s_g X_mbar X_b gamma_IG omega_IG rho_pi_IG
a_IG s_IG phi_tr hab eta_gcb eta_igb eta_tc eta_tl eta_tk eta_tr zeta_h
elel elel_prim no_cost;
// estimated parameters:
// markups
X_mig = 0.091 ;
X_mg = 0.147 ;
X_mc = 0.211 ;
X_mi = 0.467 ;
XX_c_bar = 0.436 ;
// staggered prices
```

```
omega_IG = 0.246 ;
omega_g = 0.267 ;
omega_p = 0.719 ;
omega_m_c = 0.233 ;
omega_m_i = 0.343 ;
omega_e = 0.254 ;
// price indexation
gamma_IG = 0.119 ;
gamma_g = 0.123 ;
gamma_p = 0.081 ;
gamma_m_c = 0.125 ;
gamma_m_i = 0.128 ;
gamma_e = 0.118 ;
// staggered wages
omega_wc = 0.851 ;
omega_wh = 0.256 ;
//wage indexation
gamma_wc = 0.290 ;
gamma_wh = 0.291 ;
// habit
hab_c = 0.779 ;
hab_h = 0.628 ;
```

```
hab_c_prim = 0.949 ;

hab_h_prim = 0.477 ;

// steady state habit

hab = 0.000 ;

// fiscal policy responce to debt

ela_gcb = 0.093 ;

ela_igb = 0.179 ;

ela_tc = 0.225 ;

ela_tl = 0.182 ;

ela_tk = 0.086 ;

ela_tr = 0.044 ;

// fiscal policy responce to GDP

eta_gcb = 0.197 ;

eta_igb = 1.075 ;

eta_tc = 0.727 ;

eta_tl = 1.137 ;

eta_tk = 0.637 ;

eta_tr = 0.722 ;

// monetary policy

rho = 0.544 ;

b_pi = 1.631 ;

b_y = 0.203 ;
```

```
// persistence shocks
//fiscal policy
rho_g = 0.902 ;
rho_ig = 0.273 ;
phi_tr = 0.873 ;
phi_tc = 0.813 ;
phi_tk = 0.850 ;
phi_tl = 0.796 ;
//other
rho_ac = 0.966 ;
rho_ah = 0.995 ;
rho_i = 0.258 ;
rho_l = 0.935 ;
rho_pi = 0.542 ;
rho_n = 0.784 ;
rho_pi_g = 0.859 ;
rho_pi_IG = 0.632 ;
rho_j = 0.938 ;
rho_ltv = 0.900 ;
rho_pimi = 0.851 ;
rho_pimc = 0.986 ;
rho_piet = 0.947 ;
```

```
// wage markup

zeta_h = 0.186 ;

// capital utilisation

kappa = 1.476 ;

// share of Patient households

be_1 = 0.779 ;

// risk premium parameter

phi_aa = 0.039 ;

// investment adjustment cost

phi_c = 4.904 ;

phi_h = 3.470 ;

// disutility of working

sigma_l = 0.364 ;

sigma_lprim = 0.663 ;

// inverse elasticity of substitution across hours in the two sectors

elel = 1.083 ;

elel_prim = 0.965 ;

// elasticity of substitution be tween imported and domestic goods

s_IG = 0.896 ;

s_g = 1.559 ;

s_c = 0.644 ;

s_i = 2.904 ;
```

```
s_f = 1.811 ;

// calibrated parameters

// wage markup

zeta = 0.050 ;

// price of exports

p_e_bar = 1.000;

// calibrated shocks

std_yst = 0.000142;

std_rst = 0.00012;

std_pist = 0.00075;

// utility weight on housing

jej = 0.140;

// share of transfers that goes to Patient households

sh = 0.000;

// tax rates

tao_kbar = 0.400;

tao_lbar = 0.290;

tao_cbar = 0.200;

tao_hbar = 0.005;

// depreciation

delta_g = 0.015;

delta_k_c = 0.025;
```

```
delta_k_h = 0.040;

delta_h = 0.010;

// elasticity of output to public capital

sigma_g = 0.010;

// shares of public consumption, investment and transfers

gg = 0.200;

ig = 0.020;

tr = 0.240;

// shares of capital in production function

alfa = 0.300;

alfa_h = 0.200;

alfa_m = 0.100;

// discount factors

beta = 0.990;

gamma = 0.960;

// downpayment

tao = 0.150;

// shares of imports in the CES functions

a_c = 0.740;

a_i = 0.630;

a_g = 0.860;

a_IG = 0.800;
```

```
no_cost=1;
// load hab_cparam hab_c;
// set_param_value('hab_c',hab_c);
// load hab_hparam hab_h;
// set_param_value('hab_h',hab_h);
// load hab_c_primparam hab_c_prim;
// set_param_value('hab_c_prim',hab_c_prim);
// load hab_h_primparam hab_h_prim;
// set_param_value('hab_h_prim',hab_h_prim);
// load kappaparam kappa;
// set_param_value('kappa',kappa);
// load no_costparam no_cost;
// set_param_value('no_cost',no_cost);
model (linear);
// steady state
//wage markups
# X_w=zeta+1;
# X_wh=zeta_h+1;
// import prices
# p_IG_barm=X_mig+1;
# p_g_barm=X_mg+1;
# p_I_barm=X_mi+1;
```

```
# p_c_barm=X_mc+1;

// domestic markup

# X_c_bar=XX_c_bar+1;

// prices

# p_c_bar=(a_c+(1-a_c)*(p_c_barm^(1-s_c)))^(1/(1-s_c));

# p_g_bar=(a_g+(1-a_g)*(p_g_barm^(1-s_g)))^(1/(1-s_g));

# p_I_bar=(a_i+(1-a_i)*(p_I_barm^(1-s_i)))^(1/(1-s_i));

# p_IG_bar=(a_IG+(1-a_IG)*(p_IG_barm^(1-s_IG)))^(1/(1-s_IG));

// definitions

# F_1=(p_I_barm-1)*(1-a_i)*((p_I_barm/p_I_bar)^(-s_i));

# F_2=(p_c_barm-1)*(1-a_c)*((p_c_barm/p_c_bar)^(-s_c));

# F_11=(p_g_barm-1)*(1-a_g)*((p_g_barm/p_g_bar)^(-s_g));

# F_22=(p_IG_barm-1)*(1-a_IG)*((p_IG_barm/p_IG_bar)^(-s_IG));

// nominal interest rate

# R_bar=1/beta;

//return on capitals

# r_k_h_bar=(p_I_bar/(1-tao_kbar))*(1/beta-(1-delta_k_h));

# r_k_c_bar=(p_I_bar/(1-tao_kbar))*(1/beta-(1-delta_k_c));

//definitions

# zeta_11=alfa/((R_bar-(1-delta_k_c))*(p_I_bar/(1-tao_kbar)));

# zeta_22=alfa_h/((R_bar-(1-delta_k_h))*(p_I_bar/(1-tao_kbar)));

# zeta_33=(jej*p_c_bar*(1+tao_cbar)*(1-hab))/((1+tao_hbar-beta*(1-delta_h)));
```

```
# zeta_44=(jej*p_c_bar*(1+tao_cbar)*(1-hab))/((1+tao_hbar-gamma*(1-delta_h))-
(1/R_bar-gamma)*(1-tao)*(1-delta_h));
# zeta_55=(R_bar-1)*(1-tao)*(1-delta_h)*(1/R_bar);
# stigma_11=-((F_11*gg+F_22*ig+F_1*delta_k_c*zeta_11)+
(r_k_c_bar-p_I_bar*delta_k_c)*zeta_11-gg*p_g_bar-ig*p_IG_bar+(sh-1)*tr
+((tao_lbar*(1-be_1)+be_1)*(1-alfa)));
# stigma_22=(-(F_2+F_1*delta_k_h*delta_h*zeta_22*zeta_33)+p_c_bar
+(-((tao_lbar*(1-be_1)+be_1)*(1-alfa_h-alfa_m))+
(-r_k_h_bar+p_I_bar*delta_k_h)*zeta_22+1)*delta_h*zeta_33);
#stigma_33=((F_1*delta_k_h*delta_h*zeta_22*zeta_44+F_2)+zeta_44*
(zeta_55+tao_hbar)+tao_cbar*p_c_bar+(((tao_lbar*(1-be_1)+be_1)*
(1-alfa_h-alfa_m))+((r_k_h_bar-p_I_bar*delta_k_h)*zeta_22))*delta_h*zeta_44);
# stigma_44=(1+tao_cbar)*p_c_bar+(delta_h+tao_hbar)*zeta_44+zeta_44
*zeta_55-(1-be_1)*(1-tao_lbar)*(1-alfa_h-alfa_m)*delta_h*zeta_44;
# stigma_55=(1-tao_lbar)*(1-be_1)*(1-alfa)+(1-sh)*tr;
# stigma_66=(1-tao_lbar)*(1-be_1)*(1-alfa_h-alfa_m)*delta_h*zeta_33;
//ratios
# YC=1/((stigma_11*stigma_44-stigma_33*stigma_55)
/(stigma_33*stigma_66-stigma_44*stigma_22));
# YCprim=1/((-stigma_55*stigma_22+stigma_66*stigma_11)
/(stigma_33*stigma_66-stigma_44*stigma_22));
# CCprim=YCprim/YC;
```

```
# CprimC=1/CCprim;
# qHIc=delta_h*(zeta_33+zeta_44*CprimC);
# qHIcprim=delta_h*(zeta_33*CCprim+zeta_44);
# qHIy=delta_h*(zeta_33*(1/YC)+zeta_44*(1/YCprim));
# HX=(1+((1-alfa)/(1-alfa_h-alfa_m))*qHIy)^((sigma_l-elel)/(1+elel));
# HXprim=(1+((1-alfa)/(1-alfa_h-alfa_m))*qHIy)^((sigma_lprim-elel_prim)
/(1+elel_prim));
# CX=(1+((1-alfa_h-alfa_m)/(1-alfa))*(1/qHIy))^((sigma_l-elel)/(1+elel));
# CXprim=(1+((1-alfa_h-alfa_m)/(1-alfa))*(1/qHIy))^((sigma_lprim-elel_prim)
/(1+elel_prim));
# N_c_bar=((((be_1*(1-alfa))/(X_w*p_c_bar*(1-hab)))*YC*
((1+tao_lbar)/(1+tao_cbar)))/CX)^(1/(1+sigma_l));
# N_cprim_bar=(((((1-be_1)*(1-alfa))/(X_w*p_c_bar*(1-hab)))*YCprim
*((1+tao_lbar)/(1+tao_cbar)))/CXprim)^(1/(1+sigma_lprim));
# N_h_bar=((((be_1*(1-alfa_h-alfa_m))/(X_wh*p_c_bar*(1-hab)))*qHIc
*((1+tao_lbar)/(1+tao_cbar)))/HX)^(1/(1+sigma_l));
# N_hprim_bar=(((((1-be_1)*(1-alfa_h-alfa_m))/(X_wh*p_c_bar*(1-hab)))
*qHIcprim*((1+tao_lbar)/(1+tao_cbar)))/HXprim)^(1/(1+sigma_lprim));
# Y_bar=((1/X_c_bar)^(1/(1-alfa-sigma_g)))*(zeta_11^(alfa/(1-alfa-sigma_g)))
*(((N_c_bar^be_1)*(N_cprim_bar^(1-be_1)))^((1-alfa)/(1-alfa-sigma_g)))
*((ig/delta_g)^((sigma_g)/(1-alfa-sigma_g)));
# K_c_bar=zeta_11*Y_bar;
```

```
# C_bar=((stigma_11*stigma_44-stigma_33*stigma_55)/
(stigma_33*stigma_66-stigma_44*stigma_22))*Y_bar;
# Cprim_bar=((-stigma_55*stigma_22+stigma_66*stigma_11)/
(stigma_33*stigma_66-stigma_44*stigma_22))*Y_bar;
# qHI=delta_h*(zeta_33*C_bar+zeta_44*Cprim_bar);
# HI_bar=((zeta_22*qHI)^alfa_h)*(((N_h_bar^be_1)*
(N_hprim_bar^(1-be_1)))^(1-alfa_h-alfa_m))*((alfa_m*qHI)^alfa_m);
# q_bar=qHI/HI_bar;
# K_h_bar=zeta_22*qHI;
# I_h_bar=delta_k_h*K_h_bar;
# I_c_bar=delta_k_c*K_c_bar;
# H_bar=(zeta_33*C_bar)/q_bar;
# Hprim_bar=(zeta_44*Cprim_bar)/q_bar;
# bondprim_bar=(1-tao)*(1-delta_h)*(((q_bar*Hprim_bar)/R_bar));
# bond_bar=-bondprim_bar;
# w_cprim_bar=((1-be_1)*(1-alfa)*Y_bar)/(N_cprim_bar);
# w_hprim_bar=q_bar*(((1-be_1)*(1-alfa_h-alfa_m)*HI_bar)/(N_hprim_bar));
# w_c_bar=(be_1*(1-alfa)*Y_bar)/(N_c_bar);
# w_h_bar=q_bar*((be_1*(1-alfa_h-alfa_m)*HI_bar)/(N_h_bar));
#C_Tbar=((a_c^(1/s_c))*(((a_c)*((1/p_c_bar)^(-s_c))*(C_bar+Cprim_bar))^
((s_c-1)/s_c))+((1-a_c)^(1/s_c))*(((1-a_c)*((p_c_barm/p_c_bar)^(-s_c))
*(C_bar+Cprim_bar))^((s_c-1)/s_c)))^(s_c/(s_c-1));
```

```
# I_Tbar=((a_i^(1/s_i))*(((a_i)*((1/p_I_bar)^(-s_i))*(I_c_bar+I_h_bar))^
((s_i-1)/s_i))+((1-a_i)^(1/s_i))*(((1-a_i)*((p_I_barm/p_I_bar)^(-s_i))*
(I_c_bar+I_h_bar))^((s_i-1)/s_i)))^(s_i/(s_i-1));
#Y_star_bar=((1-a_c)*((p_c_barm/p_c_bar)^(-s_c))*(C_bar+Cprim_bar))+
((1-a_i)*((p_I_barm/p_I_bar)^(-s_i))*(I_c_bar+I_h_bar))+(1-a_g)*
((p_g_barm/p_g_bar)^(-s_g))*(gg*Y_bar)+(1-a_IG)*((p_IG_barm/p_IG_bar)
^(-s_IG))*(ig*Y_bar);
#B_bar=(tao_hbar*q_bar*(H_bar+Hprim_bar)+tao_cbar*p_c_bar*C_Tbar+
tao_kbar*(r_k_c_bar*K_c_bar+r_k_h_bar*K_h_bar)+tao_lbar*(w_c_bar
*N_c_bar+w_cprim_bar*N_cprim_bar+w_h_bar*N_h_bar+w_hprim_bar
*N_hprim_bar)-tr*Y_bar-p_IG_bar*ig*Y_bar-p_g_bar*gg*Y_bar)/(R_bar-1);
# GDP_bar=Y_bar+qHI-alfa_m*qHI;
# FF=tao_lbar*w_c_bar*N_c_bar+tao_lbar*w_h_bar*N_h_bar+tao_lbar
*w_cprim_bar*N_cprim_bar+tao_lbar*w_hprim_bar*N_hprim_bar;
# K_bar=1-(beta-gamma)*(1-tao)*(1-delta_h)*(1/(1+tao_hbar));
# Vprim_bar=jej/(Hprim_bar*(K_bar-gamma*(1-delta_h)/(1+tao_hbar)));
# IG_bar=ig*Y_bar;
# G_bar=gg*Y_bar;
# G_m_bar=(1-a_g)*((p_g_barm/p_g_bar)^(-s_g))*(G_bar);
# IG_m_bar=(1-a_IG)*((p_IG_barm/p_IG_bar)^(-s_IG))*(IG_bar);
# G_d_bar=(a_g)*((1/p_g_bar)^(-s_g))*(G_bar);
# IG_d_bar=(a_IG)*((1/p_IG_bar)^(-s_IG))*(IG_bar);
```

```
# C_m_bar=(1-a_c)*((p_c_barm/p_c_bar)^(-s_c))*(C_bar+Cprim_bar);

# I_m_bar=(1-a_i)*((p_I_barm/p_I_bar)^(-s_i))*(I_c_bar+I_h_bar);

# TR_bar=tr*Y_bar;

//Patient households

// First order conditions of Patient households

// 1. w.r.t consumption

e_n-((C-hab_c*C(-1))/(1-hab_c))=lambda+p_c+(tao_cbar/(1+tao_cbar))*tao_c;

// 2. w.r.t bonds

lambda=R-pi(+1)+lambda(+1);

// 3,4. w.r.t investment

((phi_c*(1+beta))*I_c-(I_c(-1)/(1+beta)+beta*I_c(+1)/(1+beta)

-(e_I_c-beta*e_I_c(+1))/(1+beta))*(phi_c*(1+beta)))*no_cost=(Q_c-p_I);

((phi_h*(1+beta))*I_h-(I_h(-1)/(1+beta)+beta*I_h(+1)

/(1+beta))*(phi_h*(1+beta)))*no_cost=(Q_h-p_I);

// 5. w.r.t housing

(1+tao_hbar)*(lambda+q+(tao_hbar/(1+tao_hbar))*tao_h)=(1+tao_hbar-beta*(1-delta_h))

*(e_n+e_j-((H-hab_h*H(-1))/(1-hab_h)))+beta*(1-delta_h)*(lambda(+1)+q(+1));

// 6,7 w.r.t capital

Q_c=pi(+1)-R+((1-delta_k_c)/(1-delta_k_c+(1-tao_kbar)*r_k_c_bar))*Q_c(+1)

+(((1-tao_kbar)*r_k_c_bar)/(1-delta_k_c+(1-tao_kbar)*r_k_c_bar))

*(r_k_c(+1)-(tao_kbar/(1-tao_kbar))*tao_k(+1));

Q_h=pi(+1)-R+((1-delta_k_h)/(1-delta_k_h+(1-tao_kbar)*r_k_h_bar))*Q_h(+1)+
```

```
(((1-tao_kbar)*r_k_h_bar)/(1-delta_k_h+(1-tao_kbar)*r_k_h_bar))*
(r_k_h(+1)-(tao_kbar/(1-tao_kbar))*tao_k(+1));
// 8. U.I.P condition
R=S(+1)-phi_aa*aa+e_ltv;
// 9,10. capital accumulation equations
K_c=(1-delta_k_c)*K_c(-1)+delta_k_c*I_c;
K_h=(1-delta_k_h)*K_h(-1)+delta_k_h*I_h;
// Impatient households
// 11. budget constraint
-bondprim_bar*b_prim+p_c_bar*(1+tao_cbar)*Cprim_bar*(p_c+C_prim
+(tao_cbar/(1+tao_cbar)*tao_c))+q_bar*Hprim_bar*(1+tao_hbar)*(H_prim+q
+(tao_hbar/(1+tao_hbar))*tao_h)-(1-delta_h)*q_bar*Hprim_bar*(q+H_prim(-1))=
-R_bar*bondprim_bar*(R(-1)+b_prim(-1)-pi)+(1-tao_lbar)*w_cprim_bar*N_cprim_bar*
(w_c_prim+L_c_prim-(tao_lbar/(1-tao_lbar)*tao_l))+(1-tao_lbar)*w_hprim_bar*
N_hprim_bar*(w_h_prim+L_h_prim-(tao_lbar/(1-tao_lbar)*tao_l))+(tr*Y_bar)*trans;
//First order conditions of Impatient households
// 12. w.r.t consumption
e_n-((C_prim-hab_c_prim*C_prim(-1))/(1-hab_c_prim))=
lambda_prim+p_c+(tao_cbar/(1+tao_cbar))*tao_c;
// 13. w.r.t housing
q+lambda_prim+(tao_hbar/(1+tao_hbar))*tao_h=(1-(1/R_bar-gamma)*(1-tao)
*(1-delta_h)/(1+tao_h)-gamma*(1-delta_h)/(1+tao_h))*(e_n+e_j
```

```
-((H_prim-hab_h_prim*H_prim(-1))/(1-hab_h_prim)))+((1/R_bar-gamma)*(1-tao)
*(1-delta_h)/(1+tao_h))*(((gamma*R_bar/(1-gamma*R_bar))*(lambda_prim-
lambda_prim(+1)-R+pi(+1)))+lambda_prim+q(+1)+pi(+1)-R)
+(gamma*(1-delta_h)/(1+tao_h))*(lambda_prim(+1)+q(+1));
// 14. borrowing constraint
b_prim=q(+1)+H_prim+pi(+1)-R;
// 15,16,17,18. wage equations
w_c=(beta/(1+beta))*w_c(+1)+(1/(1+beta))*w_c(-1)+(beta/(1+beta))*pi(+1)-
((1+beta*gamma_wc)/(1+beta))*pi+(gamma_wc/(1+beta))*pi(-1)-((((1-omega_wc)
*(1-beta*omega_wc))/((1+((1+zeta)/(zeta))*sigma_l)*omega_wc))*(1/(1+beta)))*
(w_c-elel*L_c+(sigma_l-elel)*(((L_c*N_c_bar^(1+elel))/(N_c_bar^(1+elel)
+N_h_bar^(1+elel)))+((L_h*N_h_bar^(1+elel))/(N_c_bar^(1+elel)
+N_h_bar^(1+elel))))-((C-hab_c*C(-1))/(1-hab_c))
-(tao_lbar/(1-tao_lbar))*tao_l-(tao_cbar/(1+tao_cbar))*tao_c-p_c+e_l);
w_h=(beta/(1+beta))*w_h(+1)+(1/(1+beta))*w_h(-1)+(beta/(1+beta))*pi(+1)-
((1+beta*gamma_wh)/(1+beta))*pi+(gamma_wh/(1+beta))*pi(-1)-((((1-omega_wh)*
(1-beta*omega_wh))/((1+((1+zeta_h)/(zeta_h))*sigma_l)*omega_wh))*(1/(1+beta)))*
(w_h-elel*L_h+(sigma_l-elel)*(((L_c*N_c_bar^(1+elel))/(N_c_bar^(1+elel)
+N_h_bar^(1+elel)))+((L_h*N_h_bar^(1+elel))/(N_c_bar^(1+elel)+N_h_bar^
(1+elel))))-((C-hab_c*C(-1))/(1-hab_c))
-(tao_lbar/(1-tao_lbar))*tao_l-(tao_cbar/(1+tao_cbar))*tao_c-p_c+e_l);
w_c_prim=(gamma/(1+gamma))*w_c_prim(+1)+(1/(1+gamma))*w_c_prim(-1)+
```

```
(gamma/(1+gamma))*pi(+1)-((1+gamma*gamma_wc)/(1+gamma))*pi+(gamma_wc
/(1+gamma))*pi(-1)-((((1-omega_wc)*(1-gamma*omega_wc))/((1+((1+zeta)/(zeta))
*sigma_lprim)*omega_wc))*(1/(1+gamma)))*(w_c_prim-elel_prim*L_c_prim
+(sigma_lprim-elel_prim)*(((L_c_prim*N_cprim_bar^(1+elel_prim))/(N_cprim_bar
^(1+elel_prim)+N_hprim_bar^(1+elel_prim)))+((L_h_prim*N_hprim_bar^(1+
elel_prim))/(N_cprim_bar^(1+elel_prim)+N_hprim_bar^(1+elel_prim))))-((C_prim-

hab_c_prim*C_prim(-1))/(1-hab_c_prim))-(tao_lbar/(1-tao_lbar))*tao_l-(tao_cbar/
(1+tao_cbar))*tao_c-p_c+e_l);
w_h_prim=(gamma/(1+gamma))*w_h_prim(+1)+(1/(1+gamma))*w_h_prim(-1)+
(gamma/(1+gamma))*pi(+1)-((1+gamma*gamma_wh)/(1+gamma))*pi+(gamma_wh
/(1+gamma))*pi(-1)-((((1-omega_wh)*(1-gamma*omega_wh))/((1+((1+zeta_h)/
(zeta_h))*sigma_lprim)*omega_wh))*(1/(1+gamma)))*(w_h_prim-elel_prim*
L_h_prim+(sigma_lprim-elel_prim)*(((L_c_prim*N_cprim_bar^(1+elel_prim))
/(N_cprim_bar^(1+elel_prim)+N_hprim_bar^(1+elel_prim)))+((L_h_prim
*N_hprim_bar^(1+elel_prim))/(N_cprim_bar^(1+elel_prim)+N_hprim_bar^
(1+elel_prim))))-((C_prim-hab_c_prim*C_prim(-1))/(1-hab_c_prim))-(tao_lbar/
(1-tao_lbar))*tao_l-(tao_cbar/(1+tao_cbar))*tao_c-p_c+e_l);
// PRODUCTION
//domestic producers of non-residential goods
// 19. production function
y=X_c_bar*(e_ac+alfa*(K_c(-1)+(((1/kappa)*(r_k_c-(tao_kbar/(1-tao_kbar))*tao_k))))
```

```
+(1-alfa)*(be_1*L_c+(1-be_1)*L_c_prim)+sigma_g*Kg);
// 20,21,22. first order conditions
r_k_c+K_c(-1)+(((1/kappa)*(r_k_c-(tao_kbar/(1-tao_kbar))*tao_k)))=mc+y/X_c_bar;
w_c+L_c =mc+y/X_c_bar;
w_c_prim+L_c_prim=mc+y/X_c_bar;
// 23. NKPC
pi=(beta/(1+beta*gamma_p))*pi(+1)+(gamma_p/(1+beta*gamma_p))*pi(-1)
+((((1-omega_p)*(1-beta*omega_p))/(omega_p))*(1/(1+beta*gamma_p)))*(mc+e_pi);
// exporters
// 24. NKPC
pi_e=(beta/(1+beta*gamma_e))*pi_e(+1)+(gamma_e/(1+beta*gamma_e))*pi_e(-1)
+((((1-omega_e)*(1-beta*omega_e))/(omega_e))*(1/(1+beta*gamma_e)))*(mc_e+e_pi_e);
// 25. marginal costs
mc_e=mc_e(-1)+pi-pi_e-S;
// 26. importers of the private consumption good
pi_m_c=(beta/(1+beta*gamma_m_c))*pi_m_c(+1)+(gamma_m_c/(1+beta*gamma_m_c))
*pi_m_c(-1)+((((1-omega_m_c)*(1-beta*omega_m_c))/(omega_m_c))
*(1/(1+beta*gamma_m_c)))*(-mc_e-p_e-p_m_c+e_pi_m_c);
// 27. importers of the private investment good
pi_m_i=(beta/(1+beta*gamma_m_i))*pi_m_i(+1)+(gamma_m_i/(1+beta*gamma_m_i))
*pi_m_i(-1)+((((1-omega_m_i)*(1-beta*omega_m_i))/(omega_m_i))
*(1/(1+beta*gamma_m_i)))*(-mc_e-p_e-p_m_I+e_pi_m_i);
```

```
// 28. importers of public consumption good
pi_g_m=(beta/(1+beta*gamma_g))*pi_g_m(+1)+(gamma_g/(1+beta*gamma_g))
*pi_g_m(-1)+((((1-omega_g)*(1-beta*omega_g))/(omega_g))
*(1/(1+beta*gamma_g)))*(-mc_e-p_e-p_m_g+e_pi_g);
// 29. importers of public investment good
pi_IG_m=(beta/(1+beta*gamma_IG))*pi_IG_m(+1)+(gamma_IG/(1+beta*gamma_IG))
*pi_IG_m(-1)+((((1-omega_IG)*(1-beta*omega_IG))/(omega_IG))
*(1/(1+beta*gamma_IG)))*(-mc_e-p_e-p_m_IG+e_pi_IG);
// producers of housing
// 30. production function
HI=e_ah+alfa_h*(K_h(-1)+(((1/kappa)*(r_k_h-(tao_kbar/(1-tao_kbar))
*tao_k))))+(1-alfa_h-alfa_m)*(be_1*L_h+(1-be_1)*L_h_prim)+alfa_m*(q+HI);
// 31,32,33. first order conditions
r_k_h+K_h(-1)+(((1/kappa)*(r_k_h-(tao_kbar/(1-tao_kbar))*tao_k)))=q+HI;
w_h+L_h=q+HI;
w_h_prim+L_h_prim=q+HI;
// relative prices
// 34. price of private investment divided by the price of home produced goods
p_I=p_I(-1)+(a_i*((1/p_I_bar)^(1-s_i))*(pi-p_I(-1))
+(1-a_i*((1/p_I_bar)^(1-s_i)))*(pi_m_i+p_m_I(-1)-p_I(-1)))-pi;
// 35. price of private consumption divided by the price of home produced goods
p_c=p_c(-1)+(a_c*((1/p_c_bar)^(1-s_c))*(pi-p_c(-1))
```

```
+(1-a_c*((1/p_c_bar)^(1-s_c)))*(pi_m_c+p_m_c(-1)-p_c(-1)))-pi;
// 36. price of public consumption divided by the price of home produced goods
p_g=p_g(-1)+(a_g*((1/p_g_bar)^(1-s_g))*(pi-p_g(-1))
+(1-a_g*((1/p_g_bar)^(1-s_g)))*(pi_g_m+p_m_g(-1)-p_g(-1)))-pi;
// 37. price of public investment divided by the price of home produced goods
p_IG=p_IG(-1)+(a_IG*((1/p_IG_bar)^(1-s_IG))*(pi-p_IG(-1))
+(1-a_IG*((1/p_IG_bar)^(1-s_IG)))*(pi_IG_m+p_m_IG(-1)-p_IG(-1)))-pi;
// 38. price of export divided by the price of foreign produced goods
p_e=p_e(-1)+pi_e;
// 39. price of imported private consumption divided by the price of domestic goods
p_m_c=p_m_c(-1)+pi_m_c-pi;
// 40. price of imported private investment divided by the price of domestic goods
p_m_I=p_m_I(-1)+pi_m_i-pi;
// 41. price of imported public consumption divided by the price of domestic goods
p_m_g=p_m_g(-1)+pi_g_m-pi;
// 42. price of imported public investment divided by the price of domestic goods
p_m_IG=p_m_IG(-1)+pi_IG_m-pi;
// 43,44. market clearing
H_bar*H+Hprim_bar*H_prim=(1-delta_h)
*(H_bar*H(-1)+Hprim_bar*H_prim(-1))+HI_bar*HI;
y=(alfa_m*qHI/Y_bar)*(q+HI)+(((a_i)*((1/p_I_bar)^(-s_i))*(I_c_bar+I_h_bar))/Y_bar)
*(s_i*p_I+((I_h_bar/I_Tbar)*I_h+(I_c_bar/I_Tbar)*I_c))+(((a_c)*((1/p_c_bar)^(-s_c))
```

```
*(C_bar+Cprim_bar))/Y_bar)*(s_c*p_c+((Cprim_bar/C_Tbar)*C_prim
+(C_bar/C_Tbar)*C))+(((a_IG)*((1/p_IG_bar)^(-s_IG))*(ig*Y_bar))/Y_bar)
*(s_IG*p_IG+IG)+(((a_g)*((1/p_g_bar)^(-s_g))*(gg*Y_bar))/Y_bar)*(s_g*p_g+G)
+(Y_star_bar/Y_bar)*(-s_f*p_e)+(1-tao_kbar)*r_k_c_bar*K_c_bar*(((1/kappa)
*(r_k_c-(tao_kbar/(1-tao_kbar))*tao_k)))/Y_bar+(1-tao_kbar)*r_k_h_bar*K_h_bar
*(((1/kappa)*(r_k_h-(tao_kbar/(1-tao_kbar))*tao_k)))/Y_bar;
// 45. foreign assets accumulation equation
aa=-((1-a_IG)*((p_IG_barm/p_IG_bar)^(-s_IG))*(ig*Y_bar))*(-mc_e-p_e-s_IG
*(p_m_IG-p_IG)+IG)-((1-a_g)*((p_g_barm/p_g_bar)^(-s_g))*(gg*Y_bar))*
(-mc_e-p_e-s_g*(p_m_g-p_g)+G)+Y_star_bar*(-mc_e-s_f*p_e)-((1-a_c)*
((p_c_barm/p_c_bar)^(-s_c))*(C_bar+Cprim_bar))*(-mc_e-p_e-s_c*(p_m_c-p_c)
+((Cprim_bar/C_Tbar)*C_prim+(C_bar/C_Tbar)*C))-((1-a_i)*((p_I_barm/p_I_bar)
^(-s_i))*(I_c_bar+I_h_bar))*(-mc_e-p_e-s_i*(p_m_I-p_I)+((I_h_bar/I_Tbar)*I_h
+(I_c_bar/I_Tbar)*I_c))+R_bar*(aa(-1));
// 46,47. monetary policy
R=rho*R(-1)+(1-rho)*b_y*(UU-UU(-1))+(1-rho)*b_pi*(a_c*((1/p_c_bar)^(1-s_c))
*(pi-p_c(-1))+(1-a_c*((1/p_c_bar)^(1-s_c)))*(pi_m_c+p_m_c(-1)-p_c(-1)))+e_m;
UU=(Y_bar*y+qHI*(HI)-alfa_m*qHI*(q+HI))/GDP_bar;
// 48-56. fiscal policy
p_IG_bar*(ig*Y_bar)*(IG+p_IG)+p_g_bar*(gg*Y_bar)*(G+p_g)+(tr*Y_bar)*trans
+B_bar*R_bar*(b(-1)+R(-1)-pi)=B_bar*b+tao_cbar*p_c_bar*C_Tbar
*(((Cprim_bar/C_Tbar)*C_prim+(C_bar/C_Tbar)*C)+p_c+tao_c)+tao_lbar*w_c_bar
```

```
*N_c_bar*(w_c+L_c+tao_l)+tao_lbar*w_h_bar*N_h_bar*(w_h+L_h+tao_l)+tao_lbar
*w_cprim_bar*N_cprim_bar*(w_c_prim+L_c_prim+tao_l)+tao_lbar*w_hprim_bar
*N_hprim_bar*(w_h_prim+L_h_prim+tao_l)+tao_kbar*r_k_c_bar*K_c_bar*(r_k_c
+(((1/kappa)*(r_k_c-(tao_kbar/(1-tao_kbar))*tao_k)))+K_c(-1)+tao_k)+tao_kbar
*r_k_h_bar*K_h_bar*(r_k_h+(((1/kappa)*(r_k_h-(tao_kbar/(1-tao_kbar))*tao_k)))
+K_h(-1)+tao_k)+tao_hbar*q_bar*H_bar*(tao_h+H+q)
+tao_hbar*q_bar*Hprim_bar*(tao_h+H_prim+q);
Kg=(1-delta_g)*Kg(-1)+delta_g*IG;
G=-ela_gcb*b(-1)-eta_gcb*((Y_bar*y+qHI*(q+HI)-alfa_m*qHI*(q+HI))/GDP_bar)+e_g;
IG=-ela_igb*b(-1)-eta_igb*((Y_bar*y+qHI*(q+HI)-alfa_m*qHI*(q+HI))/GDP_bar)+e_ig;
trans=-ela_tr*b(-1)-eta_tr*((Y_bar*y+qHI*(q+HI)-alfa_m*qHI*(q+HI))/GDP_bar)+e_tr;
tao_c=ela_tc*(b(-1))+eta_tc*((Y_bar*y+qHI*(q+HI)
-alfa_m*qHI*(q+HI))/GDP_bar)-e_tc;
tao_k=ela_tk*(b(-1))+eta_tk*((Y_bar*y+qHI*(q+HI)
-alfa_m*qHI*(q+HI))/GDP_bar)-e_tk;
tao_l=ela_tl*b(-1)+eta_tl*((Y_bar*y+qHI*(q+HI)-alfa_m*qHI*(q+HI))/GDP_bar)-e_tl;
tao_h=0;
// 57-76. shocks
e_pi_g=rho_pi_g*e_pi_g(-1)+lpig;
e_pi_IG=rho_pi_IG*e_pi_IG(-1)+lpiIG;
e_tr=phi_tr*e_tr(-1)+ltr;
e_tc=phi_tc*e_tc(-1)+tc;
```

```
e_tk=phi_tk*e_tk(-1)+tk;

e_tl=phi_tl*e_tl(-1)+tl;

e_j=rho_j*e_j(-1)+ljt;

e_n=rho_n*e_n(-1)+lnt;

e_I_c=rho_i*e_I_c(-1)+lit;

e_ltv=rho_ltv*e_ltv(-1)+ltvt;

e_l=rho_l*e_l(-1)+llt;

e_ac=rho_ac*e_ac(-1)+lact;

e_ah=rho_ah*e_ah(-1)+laht;

e_pi=rho_pi*e_pi(-1)+lpit;

e_pi_e=rho_piet*e_pi_e(-1)+lpiet;

e_pi_m_c=rho_pimc*e_pi_m_c(-1)+lpimc;

e_pi_m_i=rho_pimi*e_pi_m_i(-1)+lpimi;

e_g=rho_g*e_g(-1)+lgt;

e_ig=rho_ig*e_ig(-1)+ligt;

e_m=lpict;

// 77. total consumption

C_T=(Cprim_bar/C_Tbar)*C_prim+(C_bar/C_Tbar)*C;

// 78. total investment

I_T=(I_h_bar/I_Tbar)*I_h+(I_c_bar/I_Tbar)*I_c;

// 79. GDP

GDP_bar*GDP=Y_bar*y+qHI*(q+HI)-alfa_m*qHI*(q+HI);
```

```
// 80. Utility of consumption of Patient households

U_c= e_n-((C-hab_c*C(-1))/(1-hab_c));

// 81. Utility of consumption of Impatient households

U_c_prim= e_n-((C_prim-hab_c_prim*C_prim(-1))/(1-hab_c_prim));

// 82. Utility of consumption of Patient households

U_h= e_n+e_j-((H-hab_h*H(-1))/(1-hab_h));

// 83. Utility of housing of Impatient households

U_h_prim= e_n+e_j-((H_prim-hab_h_prim*H_prim(-1))/(1-hab_h_prim));

// 84. residential investment

res_inv=q+HI;

// 85-86. value of residential assets

H_w=q+H;

H_wprim=q+H_prim;

// 87. growth rate of housing stock

growth_rate=H_t-H_t(-1);

H_t=(H_bar*(H)+Hprim_bar*(H_prim))/(H_bar+Hprim_bar);

// 88. relative house price

cons_hp=q+(tao_hbar/(1+tao_hbar))*tao_h-p_c-((tao_cbar/(1+tao_cbar))*tao_c);

// 89. shadow value of borrowings

lambda_b=(gamma*R_bar/(1-gamma*R_bar))*(lambda_prim-lambda_prim(+1)-R+pi(+1));

// 90-92. shadow Value of Housing - Borrowers and Lenders

KK=-(((beta-gamma)*(1-tao)*(1-delta_h))/((1-tao_hbar)-(beta-gamma)*(1-tao)
```

```
*(1-delta_h)))*(lambda_b+q(+1)-(tao_hbar/(1+tao_hbar))*tao_h-q-R+pi(+1));
VV=(((1+tao_h)-beta*(1-delta_h))/(1+tao_h))*(U_h)+(beta*(1-delta_h)
/(1+tao_h))*(VV(+1)-(tao_hbar/(1+tao_hbar))*tao_h(+1));
VV_prim=(jej/(Hprim_bar*K_bar*Vprim_bar))*U_h_prim+(gamma*(1-delta_h)
/(K_bar*(1+tao_h)))*(VV_prim(+1)-(tao_hbar/(1+tao_hbar))*tao_h(+1))-KK;
// 93. real interest rate
rr=R-pi(+1);
// 94. trade balance
trade_bb=-IG_m_bar*(-mc_e-p_e-s_IG*(p_m_IG-p_IG)+IG)-G_m_bar*
(-mc_e-p_e-s_g*(p_m_g-p_g)+G)+Y_star_bar*(-mc_e-s_f*p_e)-C_m_bar
*(-mc_e-p_e-s_c*(p_m_c-p_c)+C_T)-I_m_bar*(-mc_e-p_e-s_i*(p_m_I-p_I)+I_T);
// 95. income of Impatient households
((1-tao_lbar)*w_cprim_bar*N_cprim_bar+(1-tao_lbar)*w_hprim_bar*N_hprim_bar
+TR_bar)*Limpinctr=(1-tao_lbar)*w_cprim_bar*N_cprim_bar*(w_c_prim+L_c_prim
-(tao_lbar/(1-tao_lbar)*tao_l))+(1-tao_lbar)*w_hprim_bar*N_hprim_bar*(w_h_prim
+L_h_prim-(tao_lbar/(1-tao_lbar)*tao_l))+TR_bar*trans;
((1-tao_lbar)*w_cprim_bar*N_cprim_bar+(1-tao_lbar)*w_hprim_bar*N_hprim_bar)
*Limpextr=(1-tao_lbar)*w_cprim_bar*N_cprim_bar*(w_c_prim+L_c_prim-(tao_lbar
/(1-tao_lbar)*tao_l))+(1-tao_lbar)*w_hprim_bar*N_hprim_bar*(w_h_prim+L_h_prim
-(tao_lbar/(1-tao_lbar)*tao_l));
// 96-97. user costs
user_cost=U_h-U_c;
```

```
user_cost_prim=U_h_prim-U_c_prim;
// 98. real exchange rate
rex=-(1-a_c)*(p_c_barm^(-(1-s_c)))*p_m_c-mc_e-p_e;
// 99-101. tax incomes
tao_linc=tao_lbar*w_c_bar*N_c_bar*(w_c+L_c+tao_l)/FF+tao_lbar*w_h_bar
*N_h_bar*(w_h+L_h+tao_l)/FF+tao_lbar*w_cprim_bar*N_cprim_bar*
(w_c_prim+L_c_prim+tao_l)/FF+tao_lbar*w_hprim_bar*N_hprim_bar
*(w_h_prim+L_h_prim+tao_l)/FF;
tao_kinc=(tao_kbar*r_k_c_bar*K_c_bar/(tao_kbar*r_k_c_bar*K_c_bar+tao_kbar
*r_k_h_bar*K_h_bar))*(r_k_c+((1/kappa)*(r_k_c-(tao_kbar/(1-tao_kbar))*tao_k))
+K_c(-1)+tao_k)+(tao_kbar*r_k_h_bar*K_h_bar/(tao_kbar*r_k_c_bar*K_c_bar
+tao_kbar*r_k_h_bar*K_h_bar))*(r_k_h+((1/kappa)*(r_k_h-(tao_kbar/(1-tao_kbar))
*tao_k))+K_h(-1)+tao_k);
tao_cinc=C_T+p_c+tao_c;
// 102. total value of housing assets
H_wt=(H_bar*(q+H)+Hprim_bar*(q+H_prim))/(H_bar+Hprim_bar);
end;
steady;
check;
shocks;
var lact ; stderr 0.0053 ;
var ltvt ; stderr 0.0049 ;
```

```
var tc ; stderr 0.0352 ;
var ltr ; stderr 0.0285 ;
var tl ; stderr 0.0228 ;
var tk ; stderr 0.0505 ;
var laht ; stderr 0.0309 ;
var lit ; stderr 0.0851 ;
var lpict ; stderr 0.0097 ;
var lpit ; stderr 0.0493 ;
var llt ; stderr 0.0951 ;
var lpiet ; stderr 0.0541 ;
var lpimi ; stderr 0.1308 ;
var lpimc ; stderr 0.0506 ;
var lnt ; stderr 0.0598 ;
var ljt ; stderr 0.0777 ;
var lgt ; stderr 0.0073 ;
var ligt ; stderr 0.5308 ;
var lpig ; stderr 0.1201 ;
var lpiIG ; stderr 0.3760 ;
end;
stoch_simul(order=1, irf=20,nograph);
// estimation(datafile=data_1807, mh_nblocks=4,mh_replic=200000,
mh_jscale=0.21,lik_init=2,mode_compute=4) ;
```

Chapter 3
Who is Afraid of Austerity? The Redistributive Impact of Fiscal Policy in a DSGE Framework

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Abstract*: This paper explores the distributional consequences of fiscal austerity, which are, surprisingly, ignored in the existing theoretical literature on fiscal adjustments. We construct a medium scale New Keynesian dynamic stochastic general equilibrium (DSGE) model with sufficient detail to enable us consider a rich set of fiscal instruments. We find that those agents who are credit constrained are most exposed to austerity as opposed to those with full access to capital markets. This is particularly true in the case of cuts in transfer payments and rises in labour income taxes. In general, tax based consolidations exhibit more conflict than spending based ones, with clear implications for the successful completion of each type of programmes. Our results also reveal that the distributive impacts of fiscal consolidations are amplified the longer the austerity persists; the slower the policy reversal and when monetary policy reaches its zero lower bound.*

Keywords: fiscal austerity; welfare; redistribution.

JEL Classification: E65; H2; H3.

3.1 Introduction

There has been a great revival of interest in fiscal policy issues in both policy and academic circles following the 2008-09 global financial crisis. Substantial fiscal stimulus packages that were put in place in response to the crisis were followed by much reduced fiscal revenues during the subsequent Great Recession, leading to a clear loss of fiscal discipline, particularly in advanced economies.[57] For example, debt to GDP ratios reached an average of 92.3 per cent in OECD countries with 89 per cent in the UK, 99 per cent in the US, 126 per cent in Italy and 210 per cent in Japan in 2009 (see, OECD Economic Outlook, 2012).

Efforts towards a better understanding of fiscal policy dynamics particularly in the aftermath of a financial crisis have already led to a substantial and growing literature. Given the seriousness of the downturn in global economic activity since 2008, recent work has primarily focussed on the output implications of fiscal policy and thus on the size of fiscal multipliers. This line of work has identified a wide range of fiscal multipliers, varying from 1.6 (Romer and Bernstein, 2009) to much smaller figures that are close to zero (Cogan *et al.* 2010). It was also shown that fiscal multipliers are larger when monetary policy is accommodative (Coenen *et al.* 2013); when the zero lower bound on interest rates binds (Christiano *et al.* 2011, Eggertson, 2011, and Erceg *et al.* 2012); under fixed exchange rate regimes (Ilzetski *et al.* 2013 and Born *et al.* 2013); and when the share of credit constrained consumers is high (Cogan *et al.* 2010 and Drautzburg and Uhlig, 2011).[58]

[57] The fiscal stimulus programmes were as large as 5.9 per cent of GDP in the US, 3.3 per cent on average in OECD and 4.8 per cent in China in 2008 among many others (see, for example, OECD Economic Outlook, 2009).

[58] This range of multipliers is in line with the empirical literature which has also established that the economic circumstances fiscal policy is conducted in plays a key role on its impact; see, for example, Auerbach and Gorodnichenko (2013) and Corsetti *et al.* (2012).

The fiscal sustainability issues that surfaced soon after the adoption of the large fiscal stimulus packages, as mentioned above, have forced policy reversals in most advanced economies. The resulting fiscal consolidation has proved difficult both politically and economically in many countries. For example, challenges fiscal austerity posed for different sections of the society have been at the top of the political agenda in both the US and the UK since 2010. A key question related to fiscal adjustment is, therefore, how the cost of consolidation is distributed. Although the effectiveness of fiscal programs, both stimulus and consolidation, is widely explored in existing work, the distributional impact of fiscal policy is largely ignored.[59] This is somewhat surprising given the clear distributional implications of fiscal austerity, as is evident from the recent policy discussions.[60] It is also widely acknowledged that austerity programs that are viewed as 'unfair' are unlikely to succeed (see, for example, IMF, 2012). Agnello and Sousa (2012) and IMF (2012) present empirical evidence suggesting that periods of fiscal consolidation are associated with increases in income inequality by examining adjustments in OECD countries between 1970 and 2010; Mulas-Granados (2005) obtain similar results studying a sample of EU countries between 1960 and 2000, and find that the composition of consolidations play a key role on their consequences for income inequality.

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[59] To the best of our knowledge, there are two exceptions to this: Drautzburg and Uhlig (2011) and McManus (2013). The former finds that the specific policy of the American Recovery and Reinvestment Act was detrimental to all agents over the lifetime of the policy unless the future is discounted at empirically inappropriate high levels. Positive impacts of the policy are the main focus of the paper however, and the normative impacts are not fully considered. The latter finds that counter-cyclical policy, especially those targeting government spending, is to the benefit of credit constrained agents and the detriment to the unconstrained. However, in contrast to our paper, the latter excludes many empirically relevant instruments within the framework of a smaller model.

[60] For example, one of the most repeated statements on fiscal austerity in the UK has been that of the Chancellor of Exchequer who maintained that 'we're all in this together'.

This paper attempts to explore the distributional impact of fiscal austerity by utilizing a dynamic stochastic general equilibrium model (DSGE) with real and nominal frictions. To that end, our framework incorporates heterogeneity of agents regarding access to credit into a medium scale New Keynesian DSGE model. Ricardian consumers own the entire capital stock of the economy and possess access to the financial markets, which are assumed to be complete. Non-Ricardian households, on the other hand, simply consume their total disposable income arising from labour and transfers. Firms produce differentiated goods, choose labour and capital inputs and set prices similar to the method proposed by Calvo (1983). The monetary authority sets the nominal interest rate according to a Taylor rule. The fiscal authority has a set of policy instruments at its disposal with which to respond to the cyclical changes in debt.

We begin by examining the sizes and the signs of fiscal multipliers, which establishes the basis of our welfare analysis while allowing us to compare the performance of our benchmark model with those in the existing literature. We then present a comprehensive welfare analysis to explore the distributional implications of fiscal policy in contrast to the great majority of existing studies. This is done by simulating our benchmark model in a large number of fiscal experiments, each portraying fiscal consolidation and is designed as a policy package initiated by a positive shock in each tax category and a negative shock in each expenditure one.

This paper makes two distinct contributions. The first is to provide a comprehensive examination of the distributive consequences of fiscal austerity, which has received very little attention in the existing literature, as stated above. Our second contribution lies in the

scope of our fiscal policy analysis; we examine a much richer set of fiscal instruments than has been provided in the existing literature on fiscal consolidation.[61] In addition to public consumption, public investment, income and lump-sum taxes that are widely explored in previous work, we incorporate capital taxes, consumption taxes, social security contributions as well as public employment as sources of fiscal adjustment packages. A clear motivation for adopting this extended set of fiscal instruments is provided by the structure of fiscal policy packages enacted in the wake of 2008 financial crisis that made use of a large number of fiscal tools including all the items in the set of fiscal policy instruments used in our paper.

Our findings can be summarized as follows. First, we find that fiscal austerity has a wide range of distributional outcomes that are determined by the composition of initial fiscal action. Also, the welfare consequences of fiscal consolidations are unevenly distributed among agents with more detrimental impact on credit constrained households than those with full access to credit markets. For example, in four out of eight sets of fiscal experiments, fiscal consolidation reduces the welfare of the credit constrained households more than the Ricardian ones (transfer payments, labour income tax, consumption tax and employers social security contributions based consolidations). Similarly, when fiscal austerity is beneficial to both types of agents, Ricardian households always gain more in relative terms (government consumption, public investment and public employment based consol-

[61] One exception is Coenen *et al*. (2013) who extend the ECB's New Area-Wide Model to include a wide variety of fiscal instruments. The number of fiscal instruments is the same in our paper but whereas they include taxes on dividends we have government employment.

idations). In contrast, a rise in capital taxes is the only fiscal action that reduces Ricardian household's welfare more than that of the credit constrained households.

Second, our results reveal that the form of policy reversal - to neutralize the impact of austerity on debt is also of key importance in determining the welfare implications of initial austerity. For instance, fiscal consolidation based on a fall in transfers is good for Ricardian agents if it is reversed by a fall in employers' social security contributions but bad if the policy reversal is through a rise in public employment. Third, we also show that the distributive impact of fiscal policy is amplified the longer the austerity persists; the slower the policy reversal and when monetary policy reaches its zero lower bound. This is of empirical relevance to much of the current debate as many long term shocks are used to pay off existing debt in a period of a liquidity trap.

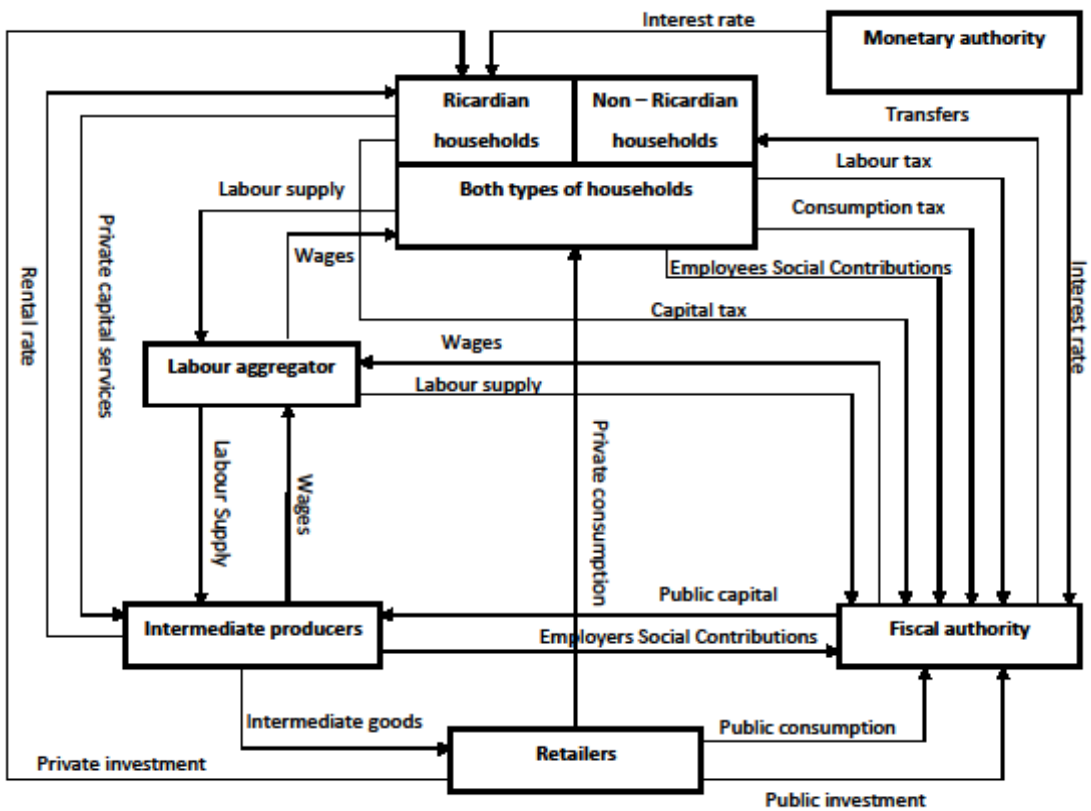
The rest of the paper is organised as follows. Section 2 sets out the benchmark model. Both the sizes of multipliers and the distributive consequences of fiscal austerity are explored in Section 3. A number of extensions and robustness checks are also presented in Section 3. Finally Section 4 provides conclusions and policy implications.

3.2 Theoretical Model

Our benchmark model shares many features with Smets and Wouters (2003), Christiano *et al.* (2005) and Bhattarai and Trzeciakiewicz (2012) featuring nominal rigidities in price and wage setting, real frictions in adjustment costs and monopolistic competition, and distortionary taxation on labour, capital and consumption. The economy is populated by a continuum of households indexed by h, a share of which, $(1-\theta)$, have access to capital

markets (Ricardian households) and the remainder, θ, do not (non-Ricardian households); two types of firms producing final and intermediate goods respectively, and a fiscal and a monetary authority. Figure (3.25) presents the flow chart of the model.

Fig. 3.25. Flow chart



3.2.1 Households

Utility for both types of household is assumed to be the same and evolves according to:

$$E_0 \sum_{t=0}^{\infty} \varepsilon_t^b \left(\beta^i\right)^t U \left(\ln \left(C_t^i(h)\right) - \frac{1}{1+\sigma_l} \left(L_t^i(h)\right)^{1+\sigma_l} \right) \tag{3.1}$$

where E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, σ_l denotes the inverse of the Frisch labour supply elasticity, C_t and L_t denote consumption and Labour respectively, and ε_t^b represents a first-order autoregressive exogenous shock process to preferences. Superscript i differentiates variables between Ricardian ($i = R$) and non-Ricardian ($i = NR$) households.

Ricardian Households

Each period Ricardian household, h, faces a budget constraint which states that the household's total expenditure on consumption, C_t^R, investment in physical capital, I_t, and accumulation of a portfolio of riskless one-period contingent claims, B_t^R, must equal the household's total disposable income:

$$\begin{aligned}(1+\tau_t^c)\, C_t^R(h) + I_t(h) + B_t^R(h) \;=\; & \left(1-\tau_t^l-\tau_t^{ee}\right) w_t\,(h)\, L_t^R\,(h) + div_t\,(h) \\ & + \left[\left(1-\tau_t^k\right) r_{k,t} u_t\,(h) - a(u_t\,(h))\right] \bar{K}_{t-1}\,(h) \\ & + \frac{(1+i_{t-1})B_{t-1}^R(h)}{\pi_t} + T_t\,(h) \qquad (3.2)\end{aligned}$$

where τ_t^c, τ_t^l and τ_t^{ee} represent taxes on consumption, labour income and employee social security contributions, and w_t the real wage; div_t represents dividends paid out of firms' profits; τ_t^k is a tax on capital, $r_{k,t}$ the real return on capital services, u_t the capital utilisation rate where the cost of capital utilization is given by $a(u_t)\bar{K}_{t-1}$, and $\bar{K}_{t-1}$ the stock of physical capital; i_{t-1} represents the net nominal interest rate on one-period bonds, π_t the gross inflation rate, and the gross nominal interest rate is given by $R_t = 1 + i_t$; and T_t represents a lump sum transfer. Following Christiano *et al.* (2005), we assume complete

markets for the state contingent claims in consumption and capital but not in labour, which implies that consumption and capital holdings are the same across Ricardian households: consequently, $C_t^R(h) = C_t^R$, $K_t^R(h) = K_t$.

In line with most of the existing literature, we maintain that physical capital accumulates in accordance with:

$$\bar{K}_t = (1-\delta_k)\bar{K}_{t-1} + \left[1 - S\left(\varepsilon_t^i \frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3.3}$$

where we follow Schmitt-Grohe and Uribe (2006) and define the cost of investment adjustment function as $S\left(I_t/I_{t-1}\right) = [(\phi_k/2)(I_t/I_{t-1}-1)^2]$ and ε_t^i is an investment specific first-order autoregressive shock process.[62] Each Ricardian household maximises utility (3.1) subject to the flow budget constraint (3.2), the capital accumulation function (3.3), and the labour demand from the labour unions discussed below. The Lagrangian takes the following form:

$$\begin{aligned}
\mathcal{L} \;=\; & E_t \sum_{t=0} \varepsilon_t^b \left(\beta^i\right)^t \left(\ln\left(C_t^i(h)\right) - \frac{1}{1+\sigma_l}\left(L_t^i(h)\right)^{1+\sigma_l}\right) \\
& + E_t \sum_{t=0} \lambda_t \left(\beta^i\right)^t \left(\begin{array}{c} \left(1-\tau_t^l-\tau_t^{ee}\right) w_t\left(h\right) L_t^R\left(h\right) + \left[\left(1-\tau_t^k\right) r_{k,t} u_t\left(h\right) - a(u_t\left(h\right))\right] \bar{K}_{t-1}\left(h\right) \\ + div_t\left(h\right) + \frac{(1+i_{t-1})B_{t-1}^R(h)}{\pi_t} + T_t\left(h\right) - \left(1+\tau_t^c\right) C_t^R(h) - I_t(h) - B_t^R(h) \end{array}\right) \\
& + E_t \sum_{t=0} \lambda_t Q_t \left(\beta^i\right)^t \left((1-\delta_k)\bar{K}_{t-1} + \left[1 - S\left(\varepsilon_t^i \frac{I_t}{I_{t-1}}\right)\right] I_t - \bar{K}_t\right)
\end{aligned}$$

where: λ_t denotes the marginal utility of income; Q_t denotes the shadow price of capital.

First-order conditions of Ricardian households

[62] Where $S\left(1\right) = S'\left(1\right) = 0$, and $S''\left(1\right) = \phi_k > 0$ are assumed for the adjustment cost function process: as a result the steady state does not depend on parameter ϕ_k.

The first order condition with respect to consumption results in:

$$C_t^{-1} = \lambda_t \left(1 + \tau_t^c\right) \tag{3.4}$$

where λ_t represents the Lagrange multiplier on the budget constraint. The first-order conditions with respect to the bond holdings results in the standard Euler equation:

$$\lambda_t = E_t \left[\frac{R_t}{\Pi_{t+1}} \beta \lambda_{t+1} \right] \tag{3.5}$$

The left-hand side of equation (3.5) represents the marginal utility cost of investing in bonds. The right-hand side implies that investing in bonds provides an *ex ante* real rate of return represented by R_t/π_{t+1}. The first-order condition with respect to the capital utilisation rate, presented in equation (3.6), indicates that the real rental rate net of capital taxes is equal to the marginal cost of capital utilisation:

$$\left(1 - \tau_t^k\right) r_{k,t} = a'(u_t) \tag{3.6}$$

A higher rate of return on capital or a lower capital tax implies a higher utilisation rate up to the point where extra benefits are equal to extra costs. The first-order condition with respect to capital links the shadow price of capital between two periods:

$$Q_t = \frac{E_t \pi_{t+1}}{R_t} E_t \left[Q_{t+1}(1 - \delta_k) + \left(1 - \tau_{t+1}^k\right) \left(r_{k,t+1} u_{t+1}\right) - a(u_{t+1}) \right] \tag{3.7}$$

Equation (3.7) implies that the price of capital is simply the present value of future net income from capital holdings. The price of capital depends positively on the expected

real rental rate and the expected utilisation rate and depends negatively on the real *ex ante* interest rate, capital taxes and the capital utilisation cost.

The first-order condition with respect to investment is given by:

$$\lambda_t = Q_t \lambda_t F_t' \left(I_t, I_{t-1}\right) + Q_{t+1} \beta \lambda_{t+1} \beta E_t F_{t+1}' \left(I_{t+1}, I_t\right) \tag{3.8}$$

where the left-hand side represents the marginal utility cost of investment in physical capital, which is equal to the marginal utility cost of investment in bonds. An increase in investment by one unit at time t leads to an increase in the value of capital by $Q_t F_t' \left(I_t, I_{t-1}\right)$ in period t, and by $Q_{t+1} \beta F_{t+1}' \left(I_{t+1}, I_t\right)$ in period $t+1$.[63]

Non-Ricardian households

As discussed above, non-Ricardian households are credit constrained agents who simply consume current after-tax income which comprises of after-tax labour income and transfers.[64] This behaviour can be rationalised in a setting where non-Ricardian households are more impatient than Ricardian households, $\beta^R > \beta^{NR}$, and can default on their debt up to the value of their collateral (see for example Iacloviello, 2005). With no durable goods in the non-Ricardian utility function, impatience prohibits the accumulation of collateral and

[63] Substituting for $F_t \left(I_t, I_{t-1}\right)$ results in:
$1 = Q_t \left[1 - \frac{\phi_k}{2} - \frac{3\phi_k}{2} \left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)^2 + 2\phi_k \frac{\varepsilon_t^I I_t}{I_{t-1}}\right] + Q_{t+1} \frac{U_{c,t+1}}{U_{c,t}} \beta \left[\phi_k \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\right)^3 - \phi_k \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\right)^2\right]$.
Therefore $F_t' \left(1\right) = 1$ and $F_{t+1}' \left(1\right) = 0$, thus the steady state does not depend on the parameter ϕ_k and $Q = 1$.

[64] In what follows the terms ‘non-Ricardian’, ‘rule-of-thumb’ and ‘credit constrained’ are used interchangeably.

as such non-Ricardians are prohibited from engaging in bond and capital markets.[65] The budget constraint of non-Ricardian households is therefore:

$$(1+\tau_t^c)\, C_t^{NR}(h) = \left(1-\tau_t^l-\tau_t^{ee}\right) w_t(h)\, L_t^{NR}(h) + T_t(h) \tag{3.9}$$

Following Erceg *et al.* (2006) we assume that each non-Ricardian household sets its wage equal to the average wage of optimising households (discussed below). Given that all households face the same labour demand, the labour supply and total labour income of each Ricardian and non-Ricardian households will be the same: by extension, consumption for all rule-of-thumb agents will also be the same ($C_t^{NR}(h) = C_t^{NR}$).

3.2.2 Wage-setting behaviour

As in Erceg *et al.* (2000) we consider a competitive labour union that transforms households' differentiated labour into composite labour which is subsequently supplied to private intermediate firms and the public sector. The technology used in this transformation is defined by:

$$L_t = \left[\int_0^1 \left(L_t(h)\right)^{\frac{\nu-1}{\nu}} dh\right]^{\frac{\nu}{\nu-1}} \tag{3.10}$$

where $\nu > 0$ is the elasticity of substitution among the differentiated labour inputs and L_t the aggregate labour index. The union takes every household's wage, $W_t(h)$, as given and maximises profit Π_t^U:

[65] Existing literature provides two sources of motivation for introducing rule-of-thumb consumers; first is the lack of evidence for consumption smoothing in the face of income fluctuations (see for example Campbell,1989); and second the observation that an important fraction of households have near-zero net worth (see for example Wolff, 1998, and Mankiw, 2000).

$$\Pi_t^U = W_t L_t - \int_0^1 W_t(h) L_t(h) di \tag{3.11}$$

where $L_t(h)$ denotes the amount of labour supplied by household h to the union, and $W_t(h)$ is the corresponding wage rate for the labour: W_t is the aggregate wage index. Profit maximisation results in the following demand for household h's labour:

$$L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\nu} L_t \tag{3.12}$$

Setting the profits of labour unions to zero, due to the prevailing perfect competition in the composite labour market, results in the aggregate wage index:

$$W_t = \left[\int_0^1 \left(W_t(h)\right)^{1-\nu} di\right]^{1/(1-\nu)} \tag{3.13}$$

Nominal wages are set in a staggered-price mechanism as in Calvo (1983), where every period, each Ricardian household faces a fixed probability $(1 - \varpi_W)$ of being able to adjust the nominal wage. The household then sets nominal wages to maximize expected future utility subject to labour demand from firms. Those who cannot reoptimize set wages in accordance with the indexation rule, $W_t = \pi_t^{\gamma_w} W_{t-1}$, where $\gamma_w \in \langle 0, 1\rangle$ is a parameter that measures the degree of wage indexation. The objective is to maximise the following with respect to $\tilde{W}_t$:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi_w)^l \left\{ \begin{array}{c} -\frac{1}{1+\sigma_L}\left(\left(\frac{\tilde{W}_t X_{tl}}{W_{t+l}}\right)^{-\nu} L_{t+l}\right)^{1+\sigma_L} \\ +\lambda_{t+l}^{\tau}\left(1-\tau_{t+l}^{l}\right)\frac{\tilde{W}_t}{P_{t+l}} X_{t,l}\left(\frac{\tilde{W}_t X_{tl}}{W_{t+l}}\right)^{-\nu} L_{t+l} \end{array} \right\} \tag{3.14}$$

where $X_{tl} = \pi_t \times \pi_{t+1} \times ... \times \pi_{t+l-1}$ for $l \geqslant 1$ and $X_{tl} = 1$ for $l = 0$ as in Altig *et al.* (2005). The maximisation results in:

$$E_t \sum_{l=0}^{\infty} (\beta \varpi_w)^l L_{t+l}^{\tau} \lambda_{t+l} \left\{ \frac{\tilde{W}_t X_{tl}}{P_{t+l}} \frac{U_{c,t+l}}{\left(1+\tau_{t+l}^c\right)} - \frac{\nu}{(1-\nu)} \frac{U_{l,t+l}}{\left(1-\tau_{t+l}^l\right)} \right\} = 0 \tag{3.15}$$

The first-order condition implies that Ricardian households set their wages so that the present value of the marginal utility of income from an additional unit of labour is equal to the markup over the present value of the marginal disutility of working. When all households are able to negotiate their wage contracts each period, the prevailing wage is $\tilde{W}_t/P_t = (\nu/(1-\nu))(U_{l,t}(1+\tau_t^c)/U_{c,t}\left(1-\tau_t^l\right))$. Finally, the wage index can be transformed into the following:

$$W_t = \left[(1-\varpi_w) \tilde{W}_t^{1-\nu} + \varpi_w \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{3.16}$$

3.2.3 Production

A competitive final good producer purchases differentiated goods from intermediate producers and combines them into one single consumption good. The final good, Y_t^P, is produced by aggregating the intermediate goods, $Y_{j,t}^P$, with technology:

$$Y_t^P = \left[\int_0^1 \left(Y_{j,t}^P\right)^{\frac{s-1}{s}} dj \right]^{\frac{s}{s-1}} \tag{3.17}$$

Profit in the final good sector, Π_t^F, can be stated as:

$$\Pi_t^F = P_t Y_t^P - \int_0^1 P_{j,t} Y_{j,t}^P dj \tag{3.18}$$

where $P_{j,t}$ is the price of the intermediate good j. Standard demand functions for intermediate goods and a zero profit condition for prices can be derived, as was performed for labour unions.

The intermediate good production sector is populated by monopolistic firms indexed by j that use the following production function:

$$Y_{j,t}^{P} = \varepsilon_t^a \left(K_{j,t-1}\right)^{\alpha} \left(L_{j,t}^{P}\right)^{1-\alpha} \left(K_{j,t-1}^{G}\right)^{\alpha_G} - \Phi \tag{3.19}$$

where K_G denotes public capital, Φ represents a fixed cost of production, and ε_t^a represents total factor productivity shock that follows a first-order autoregressive process. Firms rent capital services $K_{j,t-1}$, and incur a cost of labour equal to $(1+\tau_t^{er})\,W_t$ where τ_t^{er} denotes employers social security contributions. As is standard in the new-Keynesian framework, intermediate-good sector firms face three constraints: the production function, a demand constraint, and price rigidity determined by a Calvo (1983) mechanism. Each firm acts to minimise its total costs, $(1+\tau_t^{er})W_t L_{j,t}^{P} + R_{k,t}K_{j,t-1}$, subject to the production function (3.19). Monopolistic companies face the following cost-minimization problem:

$$\begin{aligned} &\underset{K_{j,t-1},\, L_{j,t}^{P}}{min} \quad (1+\tau_t^{er})W_t L_{j,t}^{P} + R_{k,t}K_{j,t-1} \\ &-\lambda_t^{P} P_{j,t} \left(Y_{j,t}^{P} - \varepsilon_t^a \left(K_{j,t-1}\right)^{\alpha} \left(L_{j,t}^{P}\right)^{1-\alpha} \left(K_{j,t-1}^{G}\right)^{\alpha_G} + \Phi\right) \end{aligned}$$

The nominal marginal cost is represented by the following:

$$P_t mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\varepsilon_t^A\right)^{-1} K_{g,t-1}^{-\alpha_g} \left(\left(1+\tau_t^{er}\right) W_t\right)^{1-\alpha} \left(R_{k,t}\right)^{\alpha} \tag{3.20}$$

Intermediate goods producers act as price setters where each period a given firm faces a constant probability, $(1-\varpi)$, of being able to reoptimise its nominal price. Those who can, maximize expected future profits at these prices:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\widetilde{P}_t X_{tl}}{P_{t+l}} - mc_{t+l} \right] P_{t+l} Y_{j,t+l} - P_{t+l} mc_{t+l} fc \tag{3.21}$$

subject to the standard demand ($Y_{j,t} = (P_{j,t}/P_t)^s Y_t$) and maximisation results in:

$$E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\widetilde{P}_t X_{tl}}{P_{t+l}} - \frac{s}{1-s} mc_{t+l} \right] P_{t+l} Y_{j,t+l} = 0 \tag{3.22}$$

In the case that all firms are allowed to reoptimise their prices, the above condition reduces to, $\widetilde{P}_t = (s/(s-1)) P_t mc_t$, which indicates that the optimised price is equal to a markup over the marginal costs. In addition, $(\beta\varpi)^l \lambda_{t+l}$ denotes a discount factor of future profits for firms. Here λ_t denotes the Lagrange multiplier on the Ricardian household's budget constraint and is treated by firms as exogenous. The price index can be rewritten as:

$$P_t = \left[(1-\varpi) \widetilde{P}_t^{1-s} + \varpi \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} P_{t-1} \right)^{1-s} \right]^{\frac{1}{1-s}} \tag{3.23}$$

3.2.4 Macroeconomic policy

The government budget constraint requires that total expenditure on government consumption of final goods, G_t^C, public investment, I_t^G, and public employment, L_t^G be paid through either taxes or transactions in the bond market:

$$G_t^C + I_t^G + (1+\tau_t^{er})\, w_t L_t^G = \left(B_t(1+\eta_{B,t}) - \frac{(1+i_{t-1})B_{t-1}}{\pi_t}\right) + \tau_t^c C_t + T_t + \left(\tau_t^l + \tau_t^{ee} + \tau_t^{er}\right) w_t L_t + \tau_t^k r_{k,t} u_t K_{t-1} \tag{3.24}$$

where $G_t = G_t^C + (1+\tau_t^{er})\, w_t L_t^G$. Following Cavallo (2007) and Forni *et al.* (2009) we assume that government consumption of final goods and services does not enter the utility of households and that the production of final goods by private sector firms is not affected in any way by government expenditure on labour and final goods and services. These assumptions imply that the both types of government spending are pure waste. Furthermore, we assume that the wage rate prevailing in the public sector is exactly the same as the one in the private sector. Moreover, hours in both sectors are perfect substitutes in the utility function of households and can be moved costlessly across the two sectors. Furthermore, $\eta_{b,t}$ represents an i.i.d. exogenous shock to government borrowing, which can either represent a change in spending, tax revenue, or borrowing conditions, exogenous to the model. This, for example, could take the form of an exogenous rise in spending (e.g. a bank bail out), or a revenue windfall. Public capital accumulates according to:

$$K_t^G = (1-\delta_k^G)K_{t-1}^G + I_t^G \tag{3.25}$$

which is equivalent to the accumulation of private capital in (3.3) but without cost to adjustment (as is common in the literature) and where δ_k^G represents depreciation specific to public capital.

We maintain that the nine fiscal instruments in steady state ensure a non-increasing level of debt and out of steady state these instruments respond to maintain the solvency condition of the government:[66]

$$\hat{x}_t = \phi_{b,x}\hat{b}_{t-1} + e_{x,t} \tag{3.26}$$

where $X = \left\{\tau^c, \tau^k, \tau^l, \tau^{er}, \tau^{ee}, G, I^G, L^G, T\right\}$ and where hatted variables represent log deviations of variables from steady state values. Fiscal instruments only respond to changes in debt and therefore a positive shock to debt initiates fiscal consolidation: a fall in spending instruments and a rise in tax instruments.

As standard in the literature, the monetary authority sets nominal interest rates (R_t) by following a Taylor rule which responds to both output and inflation with some persistence:

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\left(\rho_\pi\hat{\pi}_t + \rho_y\hat{Y}_t\right) \tag{3.27}$$

where ρ is the interest rate smoothing parameter and $\eta_{R,t}$ represents an i.i.d. shock to the nominal interest rate: all other variables are as defined earlier.

3.2.5 Aggregation and Market clearing

The aggregate quantity, expressed in per-capita terms, of any household quantity variable $Z_t(h)$, is represented by $Z_t = \int_0^1 Z_t(h)\,dh = (1-\theta)Z_t^R(h) + \theta Z_t^{NR}(h)$, as all members of each household choose identical allocations in equilibrium. The final goods market is

[66] Note that labour income taxes and employees social security contributions enter the model in the same way hence we drop the latter in our simulations and focus on the former.

in equilibrium when the aggregate supply equals the aggregate public and private demand for consumption and investment goods. The labour market is in equilibrium when the total labour demanded by the intermediate firms equals total labour supplied by households at a wage rate (W_t). The capital rental market is in equilibrium when capital supplied by Ricardian households is equal to the capital demanded by intermediate producers at a market rental rate $(R_{k,t})$. The equilibrium conditions are represented by:

Total output is the sum of private and public sector output where thegoods market clearing condition is given by:

$$Y_t = C_t + G_t + I_t + I_t^G + a(u_t)\bar{K}_{t-1} \tag{3.28}$$

where: $C_t = \theta C_t^{NR} + (1-\theta)\, C_t^R$. Market clearing in capital market is represented by:

$$\int_0^1 K_{j,t} dj = (1-\theta)\, u_t \bar{K}_t\,(h) \tag{3.29}$$

The relation between labour demand and labour supply can be derived from equation (3.12). Integrating the equation over all households we obtain:

$$L_t^{NR} = L_t^R = L_t^S = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\nu} dh L_t \tag{3.30}$$

where L_t^S denotes labour supply and labour demand is given by: $L_t = L_t^P + L_t^G$. Denoting $o_t = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\nu} dh$, the relation between labour demand and supply can be summarised by:

$$L_t^S = o_t L_t \tag{3.31}$$

The bond market can be summarized by:

$$B_t = (1 - \theta)\, B_t^R \tag{3.32}$$

Log-linearized system of equations and the steady state are presented respectively in Appendix $(3.A)$ and $(3.B)$

3.3 Distributive Impact of Fiscal Policy

In order to explore the distributional implications of fiscal policy we perform simulations on the model shocking each of our eight fiscal instruments in turn.[67] We chose to examine the case of fiscal austerity in our simulations due to the policy relevance of fiscal consolidation for the current policy debate not just in the US and the UK but also in a number of eurozone countries. Parallel to the much of the existing literature we start off by examining the sizes and the signs of fiscal multipliers. Although the effectiveness of fiscal policy is not our main focus, this exercise establishes the basis of our welfare analysis while allowing us to compare the performance of our benchmark model with those in the existing literature. We then present a comprehensive welfare analysis regarding the non-Ricardian and Ricardian households.

3.3.1 Calibration and Welfare Calculation

We follow a calibration procedure in line with the existing literature with common parameters fixed in a standard way, as is listed in Table (3.20) and (3.21). Steady state tax rates

[67] Note that labour income taxes and employees social security contributions enter the model in the same way.

Table 3.20. Calibration I

Share/parameter	Description	Value
Expenditure shares		
C/Y	Private consumption to GDP	0.65
G/Y	Public consumption to GDP	0.2
I/Y	Private investment to GDP	0.13
I^G/Y	Public investment to GDP	0.02
Preferences		
β	Discount factor	0.99
σ_l	Inverse Frisch elasticity	2
θ	Share of non-Ricardian households	0.3
Technology		
δ_k	Depreciation rate: private capital	0.025
δ_k^G	Depreciation rate: public capital	0.02
α	Share of capital in production	0.35
ϕ_k	Investment adjustment cost parameter	5
κ	Capital utilisation adjustment parameter	0.6
ϖ	Stickiness in prices	0.75
ϖ_W	Stickiness in wages	0.5
γ_p	Price indexation	0.15
γ_w	Wage indexation	0.15
s	Elasticity of substitution in consumption	7.67
ν	Elasticity of substitution in labour	7.67
Φ	Fixed costs in production	0.15

on consumption, capital, labour income and employee and employer social security contributions (τ^c, τ^k, τ^l, τ^{ee} and τ^{er}) are set at 0.2, 0.4, 0.18, 0.05 and 0.07 respectively and the level of government debt in steady state is set at 60 per cent of output. We select a slightly lower value of the depreciation of public capital compared to private capital with $\delta_k^G = 0.02$, and we fix the share of public employment in total employment at 0.15. The elasticity of public capital in the production function, σ_G, is set at 0.02 which is slightly higher than the value calibrated by Straub and Tcharkov (2007) for the US and the euro area. We fix the share of public investment in GDP at 0.02, whereas the share of public consumption at 0.2. This calibration implies the ratio of private investment to GDP is 0.13 whereas private consumption to GDP is 0.65.

Table 3.21. Calibration II

Share/parameter	Description	Value
Monetary policy		
ρ	Monetary policy persistence	0.8
ρ_π	Inflation Taylor rule weight	1.5
ρ_y	Output Taylor rule weight	0.125
Fiscal policy		
τ^c	Steady state consumption tax	0.2
τ^k	Steady state capital tax	0.4
τ^l	Steady state labour income tax	0.18
τ^{ee}	Steady state employee social security tax	0.05
τ^{er}	Steady state employer social security tax	0.07
ϕ_T	Responsiveness of transfers to hours	0.7
b/Y	Government debt to annual GDP	60%
α_G	Elasticity of public capital in production	0.02
wL^G/Y	Share of public to total employment	0.15

The persistence of fiscal shock parameters are all set equal to 0.85 which represents a half-life of the shock of one year. The debt aversion parameters for each individual fiscal experiment are set such that the half-life of existing government debt is equal to three and a half years, a prudent parameter within the context of the existing literature (see, for example, Leeper *et al.* 2010).[68] Finally, the share of non-Ricardian consumers is set equal to 0.3 in line with those in the existing literature.

Our welfare calculations follow Woodford (2003) and are performed by deriving a welfare criterion based on a second order Taylor series expansion of the utility function around the non-stochastic steady state values. This procedure provides a criterion expressed as the equivalent one period consumption loss, as a proportion of steady state consumption,

[68] In practice this means normalising the debt aversion parameters to correct for the different importance of each fiscal instrument in steady state. This level of debt aversion suggests that the debt impact of the initial fiscal policy change is below 1 per cent of its steady state level in approximately 8 years.

that leaves the agent indifferent between living through the shock or the one period consumption loss. The resulting expression for welfare is of the following form[69]:

$$\begin{aligned} W^i &= E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^i + \frac{1-\sigma_c}{2} \left(c_t^i\right)^2 \right) \\ &- \frac{1-\alpha}{1+\tau^{er}} \frac{1}{\mu^w} \frac{1-\tau^l-\tau^{ee}}{1+\tau^c} \frac{1}{\gamma_c} E_0 \sum_{t=0}^{\infty} \beta^t \left(l_t^i + \frac{1+\sigma_l}{2} \left(l_t^i\right)^2 \right) \end{aligned} \tag{3.33}$$

where i denotes the type of the consumer, $\gamma_c = C/Y$, and $\mu^w = \nu(\nu-1)$, and c and l denote respectively consumption and hours in terms of log deviations. This criterion provides disaggregate calculations for each set of households in the economy. The coefficient attached to the movements in employment implies that the higher the steady state share of consumption in output, the share of capital in production, and steady state distortions the less averse agents are to working more. The welfare function as presented above considers the whole lifetime of agents however the conduct of fiscal policy is often performed with shorter time horizons. For this purpose, as an illustration, the results presented below will restrict the welfare calculation to 20 quarters: this is reflective of a 5 year political cycle. We check the sensitivity of our results with respect to this restriction as part of our robustness analysis.[70]

[69] Please see Appendix $(3.C)$ for more details on the welfare measure.

[70] The basis for the 5 years welfare calculation is that, in accordance to the Fixed-term Parliaments Act 2011, the period between one general election and the next is fixed at 5 years, unless Parliament votes to hold an election sooner. As a sensitivity analysis we provide the life-time welfare measure for 250 years.

3.3.2 The Positive Impact of Fiscal Policy

The fiscal shocks are normalised such that each represents an austerity shock of one percent of steady state output. Determinacy requires that debt aversion parameters, $\phi_{b,i}$, be large enough to ensure that government debt is repaid over the long term[71]. In what follows the debt aversion parameters are set such that each instrument takes an equal share in rebalancing government debt.

Tax shocks

Fig. (3.26) presents the responses of the economy to four separate tax shocks. In each case a rise in taxes leads to a fall in output, the biggest initial impact resulting from a rise in the consumption tax[72]. Over the medium run capital tax rises have the biggest cumulative impact because they directly reduce the productive capacity of the economy as well as reducing investment demand. The medium term impact with respect to output from movements in labour income taxes and employer social security contributions are similar but the latter has a strong initial inertia due to the stickiness of prices.[73]

Aggregate consumption is initially most exposed to rises in consumption taxes leading to declines in both Ricardian and non-Ricardian consumption. The fall in the latter is larger than the fall in the former as credit constrained agents are unable to smooth their consumption through bond market transactions. Non-Ricardian households are relatively

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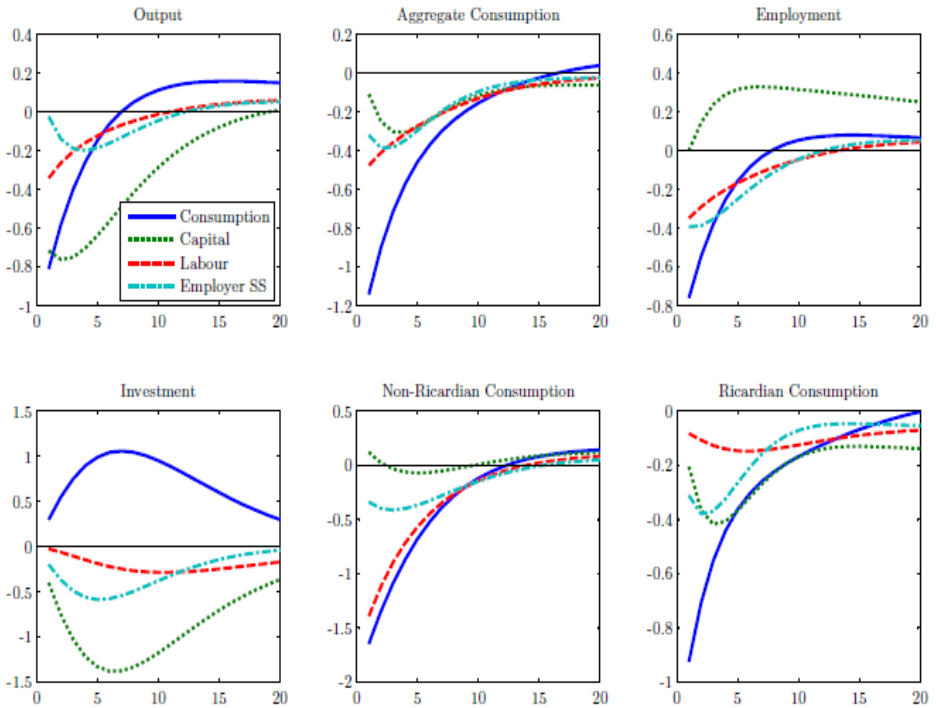
[71] An impulse response of the stock of debt in response to each policy shock is present in Appendix $(3.D)$.

[72] Presence of habit formation in the model leads to a smaller initial impact.

[73] The time profile of our tax shocks is consistent with the empirical literature: see for example Cloyne (2013).

Fig. 3.26. Dynamics from tax shocks



Dynamics achieved through making a shock to individual taxes equivalent to one percent point of steady state output and through an effective debt aversion of 0.14 leading to a half life of existing debt of three and a half years. This effective debt aversion is shared equally across all fiscal instruments. The normalisation of shocks means that the experiment for both labour income taxes and employees social security contributions are equivalent and therefore the latter have been excluded from the analysis for brevity.

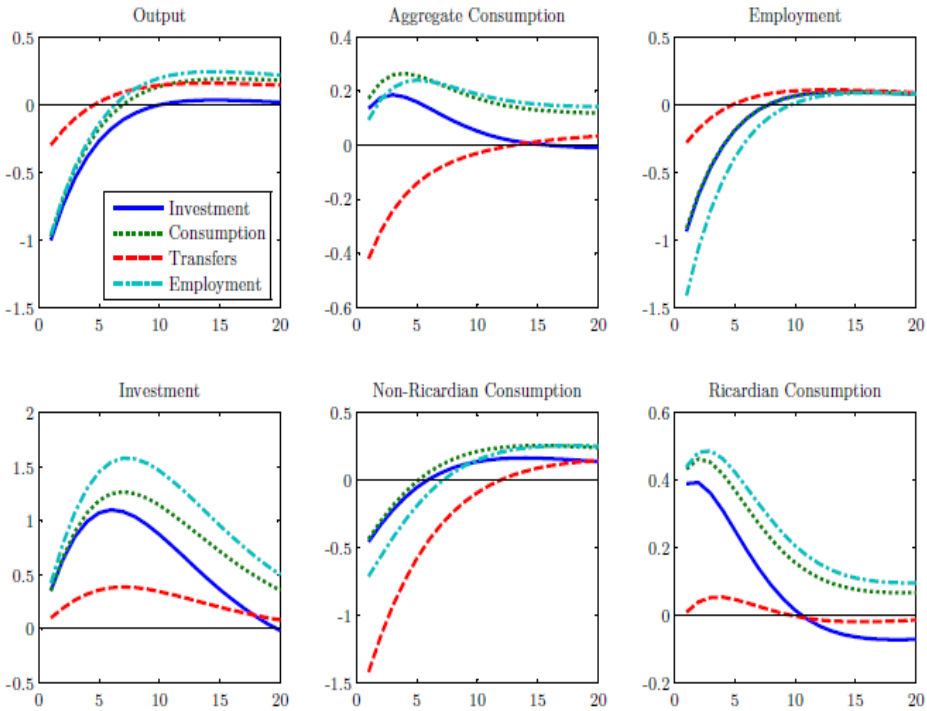
unexposed to movements in capital taxes. This is because the labour income remains relatively unchanged as the increase in the labour demand is mostly offset by the decrease in the wages.

Spending shocks

Fig. (3.27) repeats the same exercise for innovations in public consumption, investment, employment and transfers. As is the case with tax shocks, the initial fall in each spending category is set as equivalent to one percentage point of steady state output. Reductions in all four government spending categories lead to a decline in output, however these effects

are short lived, diminishing as the policy is slowly reversed. In response to the shocks both aggregate private consumption and investment increase 'crowding in' the withdrawal of government demand, which increases the productive capacity of the economy. As the shock diminishes the additional productive capacity is used to meet increases in demand at lower prices and therefore output rises above steady state levels.

Fig. 3.27. Dynamics from spending shocks



Dynamics achieved through making a shock to individual spending instruments equivalent to one percent point of steady state output and through an effective debt aversion of 0.14 leading to a half life of existing debt of three and a half years. This effective debt aversion is shared equally across all fiscal instruments.

Nonetheless, this aggregate analysis hides a big disparity in the disaggregate paths of consumption. A clear asymmetry in the response of consumption by non-Ricardian versus Ricardian households emerges in Fig. (3.27). A decline in all four categories of govern-

ment spending induces credit constrained consumers to reduce their consumption while that of Ricardian households increases. This is because non-Ricardian households are exposed to current movements in output because they drive movements in labour demand and subsequently wages. Hence fiscal austerity leading to a fall in output and a subsequent fall in wages reduces non-Ricardian consumption. In contrast, there is an increase in Ricardian consumption following the fall in real interest rate that the cuts in spending bring about.[74]

Fiscal multipliers

The short-run and the long-run multipliers in each of the eight experiments are presented in Table (3.22).

Table 3.22. Present value multipliers

Instrument	Impact multiplier	One year	Five year
Consumption tax	-0.95	-0.75	-0.14
Capital tax	-0.68	-0.94	-1.17
Labour income tax	-0.35	-0.31	-0.17
Government investment	1.00	0.85	0.53
Government consumption	0.97	0.78	0.15
Transfers	0.31	0.20	-0.21
Employers' social security	-0.02	-0.18	-0.17
Public employment	0.89	0.70	0.01

Dynamics achieved through an effective debt aversion of 0.14 leading to a half life of existing debt of three and a half years. This effective debt aversion is shared equally across all fiscal instruments. Impact multipliers represent where n=1, one year multipliers where n=4 and five year multipliers where n=20. Tax multipliers have been modified such that positive values represent a rise in taxes leading to a fall in output.

As is clear from Table (3.22), the immediate impact of all austerity measures is greater than their long-term impact, with the exception of capital taxes and employer social secu-

[74] Note that consumption of Ricardian households is given by:
$\hat{C^R}_t = -\sum_{m=0}^{\infty} E_t \left(R_{t+m} - \hat{\Pi}_{t+1+m}\right) - \tau^c/(1+\tau^c) \sum_{m=0}^{\infty} E_t \left(\hat{\tau}^c_{t+m} - \hat{\tau}^c_{t+1+m}\right)$

rity contributions. That is, impact multipliers are bigger than one-year multipliers which are bigger than five-year multipliers. This is because the shock slowly diminishes and is reversed as the borrowing converges to its steady state level. Austerity has the largest immediate output cost if it is carried out by a reduction in government consumption, public investment and public employment or a rise in consumption taxes. This is because all these instruments have a direct impact either on output or output demand. Interestingly, the reverse is true after five years; with the exception of public investment these instruments have the smallest five year multipliers. There is a timing trade off present whereby those policies which have the most immediate effect also have the smallest effect over the medium term. This follows from the fact that fiscal policy which directly impacts the production side of the economy can have long lasting effects - by changing the productive capacity of the economy- while policies directly impacting consumption are short-lived though immediate.

3.3.3 Welfare Effects

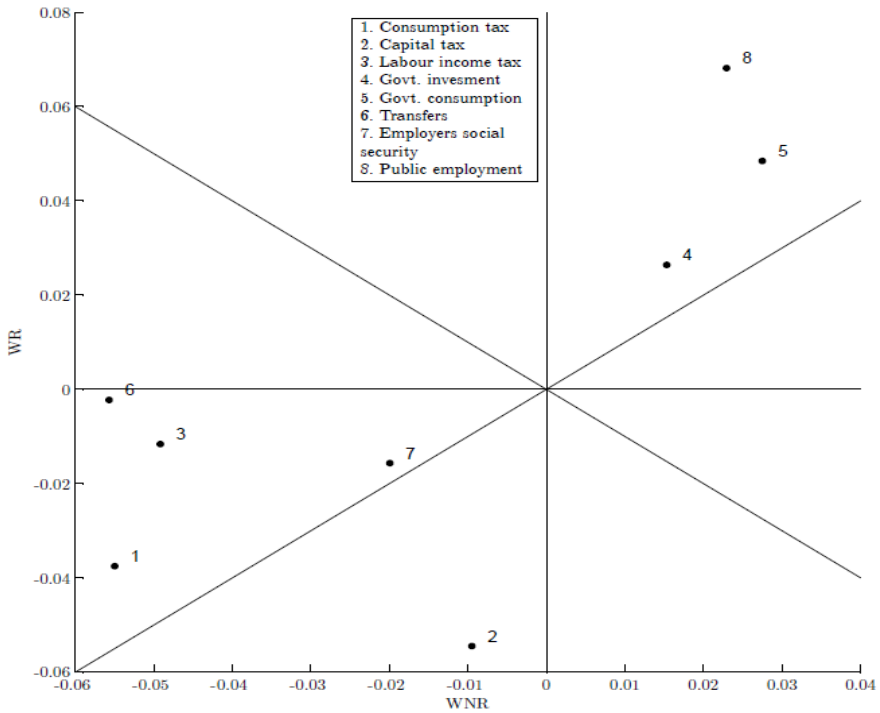
We now turn to the implications of fiscal austerity on the welfare of the non-Ricardian and Ricardian households in each of the eight experiments, as described in equation (3.33).

Welfare implications of fiscal austerity

Fig. (3.28) plots coordinates of welfare outcomes in eight austerity experiments. For illustrative purposes a positive and a negative 45 degree line are drawn where the former

represents equal welfare impacts across the two agents (fully fair and equitable) and the latter the reverse (maximum conflict). Other points indicate varying degrees of conflict.

Fig. 3.28. Benchmark welfare average values



Both axis represent movements of welfare as a proportion of one period steady state consumption: 'WNR' and 'WR' represents non-Ricardian and Ricardian welfare respectively. Lines representing positive and negative 45 degree lines have also been included to aid analysis. Dynamics achieved through the benchmark calibration and through shocks to fiscal instruments which are the equivalent of one percent of steady state output.

Fig. (3.28) shows that austerity produces a wide range of distributional outcomes although, in general, the welfare consequences are unevenly shared by the two types of agents. More specifically, austerity tends to harm non-Ricardian households more than the Ricardian households. This follows from the fact that all points bar one lie to the left of the upward sloping 45 degree line. For example, only 8, 4 and 5 (reductions in government consumption, investment and employment) are welfare improving for both

types of households, nonetheless, Ricardians benefit from 8, 4 and 5 much more than non-Ricardians in relative terms. Similarly, consolidation by cutting transfers, or raising the labour income tax reduces Ricardian welfare mildly (points 6 and 3) while reducing that of non-Ricardians substantially more. Also a rise in consumption taxes (point 1) although welfare reducing for both agents, decreases the welfare of non-Ricardians nearly twice as much as that of Ricardians.

There are, however, two exceptions to this pattern; points 2 and 7. Point 2, fiscal austerity through a rise in capital taxes, is one policy that reduces Ricardian welfare substantially while being only mildly disliked by non-Ricardians. This is not only because non-Ricardian agents do not own capital but also the rise in capital taxes leads to a rise in employment as production resources are switched from capital to labour. The resulting increase in employment brings about an increase in the disposable income and therefore consumption to non-Ricardian agents. The other exception to the above pattern is Point 7, which denotes fiscal consolidation through a reduction in employers' social security contributions. As is seen from the position of 7 this policy is (almost) equally disliked by both types of households, where the magnitude of welfare movements is small as the rigidities in the labour market soften the impact.

It is clear from Fig. (3.28) and impulse responses implied in Fig. (3.26) and Fig. (3.27) that the policies which cause the most hardship for non-Ricardian agents are those which cut transfers (6) or raise taxes on consumption (1) and labour income (3). In contrast, among these policies only a rise in consumption taxes really impacts Ricardian agents, although even in this case the hardship is skewed significantly towards non-Ricardian house-

holds. Fiscal consolidation through direct spending policies, such as cut backs in government consumption, investment and employment are associated with welfare improvements for both agents. This is the substitution effect usually found in fully-Ricardian models in reverse. A fall in government demand leads to a fall in output and in general equilibrium lower employment needs. This leads, at least in aggregate, to a fall in employment met with a rise in consumption as agents substitute some of this rise in leisure. Fig. (3.27) highlights this mechanism for the Ricardian households in our model. For the non-Ricardian households the drop in employment and wages results in lower consumption. They see a slight increase in welfare as a result of these experiments as the fall in employment outweighs the fall in consumption, however the benefits accrue more to Ricardian households. A drop in public investment is associated with the smallest gains for both agents within this category because although consumption increases in aggregate, it increases by less compared to other policies because of the impact of the fall in public investment on the productive side of the economy. A fall in public employment is associated with the biggest welfare gains, in aggregate, from these policies as it has a greater impact on cutting aggregate employment in the economy, as is seen in Fig. (3.27).

The above discussion suggests that the composition of fiscal consolidation package plays a key role in the nature of the distributional conflict that is faced by the two types of households. Also important to note is the link between the aggregate impact of each policy, as measured by multipliers in Table (3.22), and its distributional implications. Tracing the distributional outcomes in Fig. (3.28) back to the size of multipliers in Table (3.22), reveals that those policies that appear as fairest involve greatest total cost in the form of

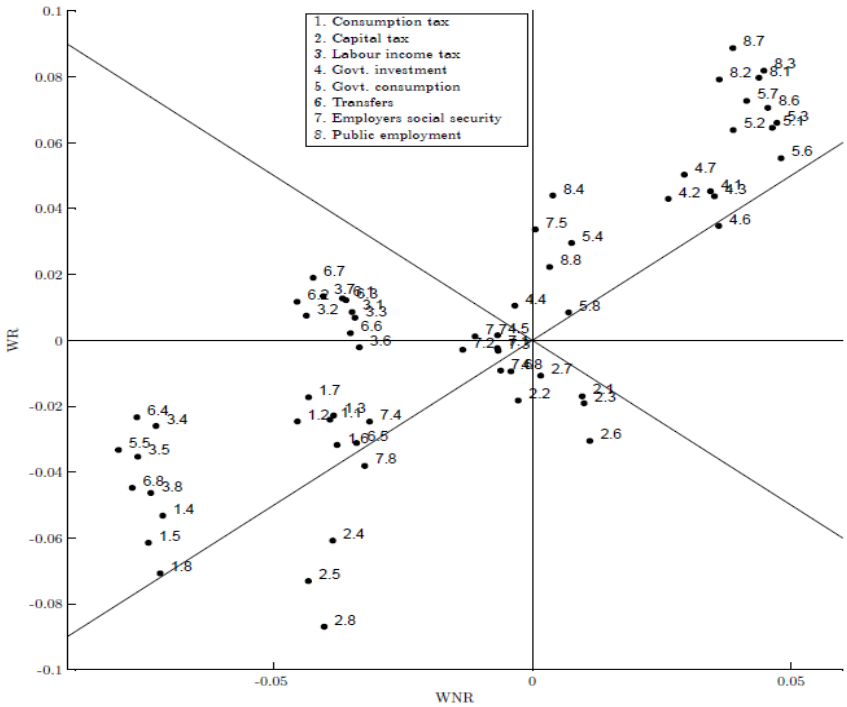
output losses immediately following the fiscal shock. For instance, the instruments with highest impact multipliers; consumption tax (1), public investment (4) and government consumption (5); are all positioned relatively closer to the upward sloping 45 degree line. In other words, there appears to be a trade off between polarization and aggregate output cost arising from fiscal austerity; appeasing conflict between the two agents comes at the cost of greater aggregate output loss. Our distributional outcomes as presented by Fig. (3.28) also provide a possible explanation for the existing empirical findings regarding the composition of fiscal adjustments. It has been shown that fiscal adjustments based on tax rises are less likely to be long-lasting than those based on spending ones (see, for example, Alesina and Perotti, 1996). Our results point to an important feature of tax based consolidations; they all reduce welfare of both types of consumers but disproportionately more of non-Ricardian ones thus exhibit greater conflict among different types of households than those based on spending cuts.

Lets now turn to the reversing of the initial austerity measure in each experiment. The initial austerity measure is reversed with each fiscal instrument separately, so for example the fiscal austerity in public consumption is reversed separately by means of each out of eight fiscal instruments. We run further experiments on each austerity package to explore the role of the exact pattern of reversing the fiscal stance on its welfare implications. We consider the same eight instruments as means of decumulating the fiscal surplus created by the austerity, leading to sixty four separate experiments.[75] The two numbers on each point

[75] It should be noted that most fiscal austerity packages that were adopted in the aftermath of the fiscal stimuli of 2009 aimed at reducing the deficits rather than accumulating surpluses in contrast to the case in our analysis where the initial steady state fiscal balance is zero. Nonetheless, the principle and the dynamics of reversing the initial policy remain the same.

in Fig. (3.29) refer to, respectively, the source of fiscal austerity and the instrument that is used in reversing the initial policy. For instance, point 3.6 in Fig. (3.29) represents a fiscal policy package that is based on a rise in labour income tax (3) as its source, reversed by a rise in transfers (6).

Fig. 3.29. Benchmark welfare



Both axis represent movements of welfare as a proportion of one period steady state consumption: 'WNR' and 'WR' represents non-Ricardian and Ricardian welfare respectively. Lines representing positive and negative 45 degree lines have also been included to aid analysis. Dynamics achieved through the benchmark calibration and through shocks to fiscal instruments which are the equivalent of one percent of steady state output.

Although the results presented in Fig. (3.29) are in general in line with those of Fig. (3.28), the position of the policy pairs in Fig. (3.29) reveals additional insights into the welfare implications of each fiscal package; there is considerable variation in the welfare of agents in individual fiscal austerity programs depending upon how the policy is reversed.

For example, a fiscal consolidation based on cuts in transfers is welfare improving for Ricardian agents if this is reversed through rises in government consumption or cuts in capital taxes and employers social security contributions. Yet the same fiscal action - a reduction in transfer payments - reduces the Ricardian household's welfare if the policy is reversed through a rise in employers' social security contributions or a rise in public employment. Likewise, an initial rise in capital taxes leads to welfare improvements for non-Ricardians but only if it is not repaid over the medium run through rises in government consumption, investment or employment.

Notwithstanding these observations, in general, most other policy pairs reveal small movements in welfare from the average position and these small movements are in line with the general results presented above. This is because the future is discounted by both agents and thus the medium term reversal of policy is of secondary consideration for agents under these circumstances.

3.3.4 Extensions

This section extends our welfare analysis to five cases. The choice of the first four cases is, in most part, motivated by the fiscal experiences of countries in the wake of the recent global financial crisis. The fourth case generalizes the above results by considering a lifetime welfare measure.

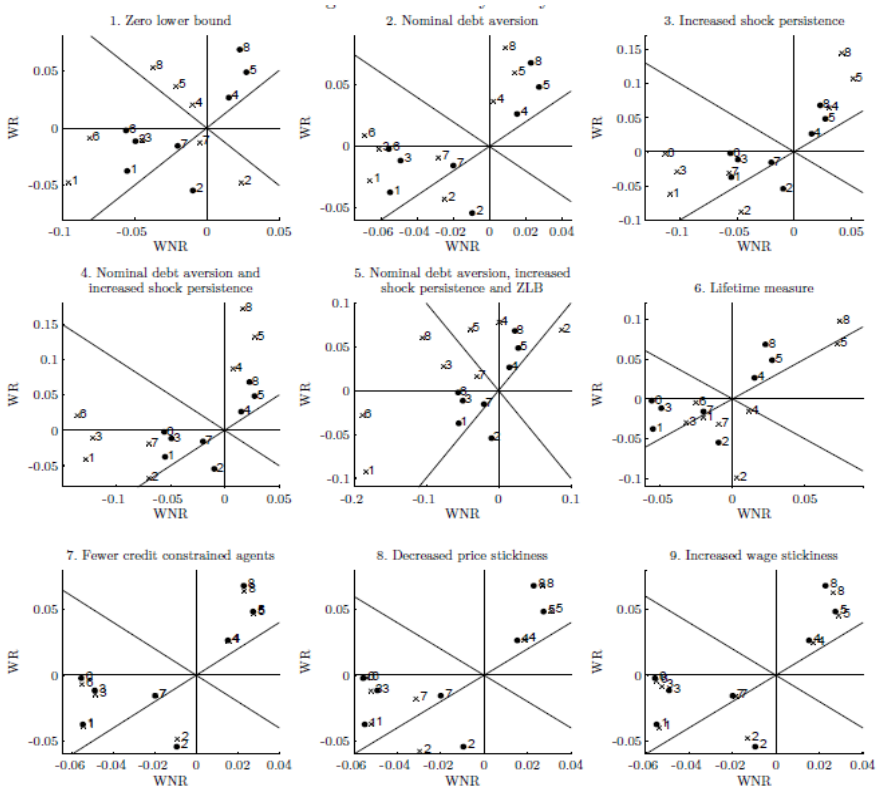
Fiscal policy at the zero lower bound

A characteristic that has been prevalent in the recent recession for which has received much academic attention is that monetary policy has been operating at its lower bound where nominal interest rates reach or are close to zero. Under such a scenario fiscal multipliers are shown to increase as the contractionary impact of higher interest rates associated with higher levels of output are removed (see for example Eggertson, 2011, Christiano *et al*, 2011 and Hall, 2011). When this is imposed on our model the impact of austerity shocks are deeper in five of our eight experiments compared to when the zero lower bound is not binding. However, rises in capital taxes, income taxes and employer social security contributions have a lower impact now than under normal times as these shocks increase inflation in the economy and therefore drive down real interest rates as the nominal interest rate is fixed. This increases Ricardian consumption which has an expansionary impact on output: a common finding as noted in Eggertson (2011).

As is seen from the first pane in Fig. (3.30), when monetary policy is at its zero lower bound the welfare results from above move in a south-westerly direction representing losses for both agents from the benchmark case.[76] However, these losses are felt more strongly by the non-Ricardian agents who experience significant welfare movements whereas the impact on the Ricardian households is small. All policies with the exception of a rise in capital taxes lead to welfare losses for credit constrained households whereas in only two

[76] The zero lower bound on the nominal interest rate is implemented by means of the algorithm described in Holden and Paetz (2012). The method is based on the introduction of shadow price shocks which hit the nominal interest rate whenever it violates the constraint. Therefore, if the zero lower bound condition on nominal interest rate is violated, a simple quadratic optimisation program is solved. The optimisation yields the linear combination of shadow price shocks which increases the nominal interest rate back to the bound. If the zero lower bound constraint is already binding, the shadow-price shocks are zero.

Fig. 3.30. Sensitivity analysis



Experiments equivalent to those presented in Figure 3 are represented with solid circles whereas those represented with a cross represent new results as a consequence of the change(s) in parameters. The different panels in the figure represent different extensions or sensitivity tests on the model as described in Section 3. The numbers used to notate different fiscal experiments are the same as those used above in Figures 3 and 4.

polices are Ricardian households significantly hurt. Non-Ricardian agents are now seen to loose from cuts in government spending on consumption, investment and employment as these cuts cause deflationary pressure and therefore raise real interest rates leading to a fall in the expenditure of Ricardian households, and subsequently a deeper recession. The exceptions to these general conclusions are when production taxes on labour and capital are raised for the reasons discussed above.

Overall, clearer distributional consequences are observed when austerity is set in the empirically appropriate zero lower bound with a significant skew in the distribution

of losses. Further, stimulus in this situation would lead to a disproportionate beneficial impact for non-Ricardian households if centred on government spending, transfers and consumption taxes.

Degree of debt aversion

Due to the modelling assumptions it was necessary for all positive debt arising from the austerity to be repaid. This means that the advantages of medium term reversals of fiscal shocks seen above are unlikely to hold in the current policy environment where austerity is motivated to drive down existing debt. One way to account for this is to shorten the time horizon over which the welfare calculations are taken as is done above: this discounts medium term reversals. Another way would be to reduce the debt aversion parameters, $\phi_{b,i}$, in the model, meaning that positive levels of debt resulting from the austerity shocks are repaid at a negligible rate. The second pane in Fig. (3.30) presents new welfare outcomes where debt aversion parameters are set to their lowest possible levels that still allow a determinable model, meaning that positive levels of debt from the austerity shocks are repaid at a negligible rate (in practice, a quarter of what was adopted for our benchmark results).

Through removing the benefits of medium term reversals, the welfare results move in a north-westerly direction leading to further welfare losses for non-Ricardian agents and smaller welfare losses for Ricardian agents. In fact, Ricardian agents now benefit in four of the eight austerity experiments. This highlights the contrast between the two agents experiences as a result of a fiscal shock. Ricardian agents can insulate themselves from

the shock and can gain as a result of the austere fiscal actions reducing the role of the government. The only grace of the austerity from the non-Ricardian perspective is when these actions are reversed: if this does not occur then the damaging impact of austerity on these agents is amplified.

Persistence of fiscal shocks

In the rhetoric of the recent austerity programmes there has been discussion on the adjustments being more permanent than the calibrations of ρ_x employed to obtain our results above would suggest. The third pane of Fig. (3.30) clearly indicates that a rise in the persistence (to $\rho_x = 0.95$) of the shocks in the fiscal experiments significantly magnify the quantitative results. However, this magnification occurs in respect only to the loses of the non-Ricardian agents from the tax based policies and to Ricardian agents in respect to the gains they receive from spending based policies. That is, the unequal distributions of austere policies are amplified the longer the austerity persists.

As opposed to the movements observed when nominal debt aversion is employed (unambiguously north-west), the points move in a direction further away from the origin (south-west or north-east depending on the starting point) representing an amplification of results. This reflects and reinforces the message that non-Ricardian households can gain when fiscal austerity is reversed, but if this does not occur then the impact of austerity can be large for these constrained households.

The fourth pane in Fig. (3.30) represents the effect of both an increase in shock persistence and the adoption of nominal debt aversion: these results are a combination of the

two previous extensions argued to be empirically appropriate in the current climate. All points now move significantly to the west representing amplifications of welfare losses for non-Ricardian households, and with the exception of austerity through rises in capital taxes all points move north representing gains for Ricardian households, at least from the benchmark results; the unequal burden of austerity is heightened. This is further amplified when these calibrations are taken to a model which also has the monetary zero lower bound imposed: the fifth pane in Fig. (3.30). These three extensions provide a clear commentary on the welfare consequences of austerity and also to some extent predict the political outbreak that such policies have inspired in the current climate.

The effectiveness of public infrastructure investment

There appears to be widespread support for increasing public capital either through stimulus packages at the start of the recession or as an antidote in a broader package of net austerity. For example, the IMF has recently called on the UK to bring forward public investment projects whilst praising the overall commitment to fiscal austerity.[77] The results presented above however suggest that increases in public expenditure decrease rather than increase welfare of agents as it increases employment for these individuals whilst reducing private consumption.

There therefore appears to be a disconnect between the political rhetoric in support of public infrastructure projects and the desirability of them on the grounds of welfare in our analysis. One possible way to reconcile this disconnect is to explore whether the elasticity

[77] See, for example, the Financial Times, 22 May 2013.

of government capital in the production function, α_G, is calibrated too low. However, this calibration needs to be extended up to 0.1 to lead to any significant losses to be observed from decreases in government investment: an infeasible level given the existing literature. Another way to reconcile this is through a review of the multipliers in Table (3.22) which demonstrates that these policies are associated with high aggregate impacts over both the short and medium run. Governing politicians are likely to value positive improvements in the aggregate economy as it will reflect well on their policies and management.

Lifetime welfare measure

As was stated above, our welfare assessments above are based on policies over a short political horizon, of only 20 quarters. We now turn to the lifetime welfare measure - welfare derived by agents from the separate austerity experiments over the whole lifetime of the policy- incorporating the full impact of reversing the fiscal position. The sixth pane in Fig. (3.30) reveals that for those policies which harm non-Ricardian households the most (rises in consumption and labour income taxes and cuts to transfers) there is a north-easterly movement in welfare as a result of extending the horizon over which welfare assessments are made. This reflects smaller welfare movements in total and is a consequence of the stimulating policies performed in order to unwind the fiscal position. There is also an easterly movement in those policies which harm non-Ricardian households the least: rises in capital taxes and employers social security contributions. Credit constrained households are now seen to gain over the lifetime of these rises in production taxes. However, there is minimal movement along the y-axis reflecting that Ricardian households, the holders

of capital, still lose from these policies. Finally, those austerity policies which benefit both households continue to benefit both households when the lifetime of the policy is considered: the gains are amplified as a result of the stimulating impact of the reversal of the fiscal austerity. The only exception to this is where cuts to government investment over the long run have a dampening effect on Ricardian households, as they reduce the productive capacity of the economy.

This demonstrates that there is an intuitive timing of welfare movements between the two households. Whereas Ricardian households have the ability to smooth the impact of shocks in the short run, non-Ricardian households do not and therefore experience significant initial welfare losses as a result of austerity. As fiscal austerity is reversed non-Ricardian agents tend to benefit from the stimulating measures that this incurs. As a result, the conflict between the two agents diminishes, although, with the exception of austerity measures which target the productive side of the economy, the welfare results of Ricardian households dominate those of non-Ricardian households. However, this discussion is dependent upon the modelling assumption that the positive debt levels resulting from the austerity will be repaid and not used to reduce the debt, as discussed above. This is something which is unlikely in current environment.

3.3.5 Sensitivity Analysis

In this section we present a series of robustness checks with respect to a number of our baseline calibration values.

Proportion of credit constrained consumers

Given our focus on distributional issues a key parameter of interest is the proportion of credit constrained consumers. Whether a change in this ratio has any significant impact on welfare outcomes can be examined by inspecting the seventh pane in Fig. (3.30) where θ is halved from 0.3 to 0.15. The aggregate impact of fiscal shocks are sensitive to the share of non-Ricardian consumer as at higher levels the impacts are increased. However, the individual impacts on the two types of household remain quantitatively unchanged at reasonable values of this parameter.

Price and wage stickiness

It is commonly agreed that price and wage stickiness are significant determinants of the size of fiscal multipliers (see, for example, Coenen *et al*. 2013). It is therefore important to check the sensitivity of our welfare results with respect to the changes in price and wage stickiness. This is presented in the eighth ($\varpi = \varpi_W = 0.5$) and ninth ($\varpi = \varpi_W = 0.75$) pane in Fig. (3.30) where the results are seen not be sensitive to the stickiness in prices and wages. The directions of qualitative movements are not unique to each policy because the impact of stickiness is different for different policies however quantitative impacts are small. The only time this is not true is when very low stickiness magnifies the impact to non-Ricardian households of policies involving rises in employer social security contributions and capital taxes. For these policies, the reduction of stickiness in wages leads to a quicker downward response of these from the austere actions leading to

a hastening of consumption of non-Ricardian households. However, Ricardian households are relatively impervious to this. Moreover, the results from other experiments remain almost unchanged.

Distortions in welfare calculation

Finally, it could be argued that through including the distortions in the model in the welfare calculation (3.33) the case against austerity is amplified as the reductions in welfare as a result of lower levels of employment are discounted. Ignoring these distortions within the welfare calculations has a negligible qualitative and quantitative impact on the results.

We also carry out a number of other sensitivity analyses (not reported) including further experiments on parameters governing price and wage stickiness, debt aversion, public capital, the Frisch labour supply elasticity and the adjustment cost to capital. Further, Drautzburg and Uhlig (2011) argue that non-Ricardian households have higher discount factors citing empirical evidence that poorer households discount the future more. Again, increases within a reasonable range for higher non-Ricardian discount rates have a negligible qualitative and quantitative impact on the results.

3.4 Conclusions

This paper explored the distributional impact of fiscal austerity by utilizing a medium scale DSGE model with nominal rigidities in price and wage setting, real frictions such as investment adjustment costs and monopolistic competition and distortionary taxation on labour, capital and consumption. The model economy analysed here consists of two types

of agents, those with access to capital markets (Ricardian) and those who are credit constrained (non-Ricardian), two types of firms (final good and intermediate good producing firms) and both fiscal and monetary authorities[78]. We explore the distributional implications of fiscal austerity by examining the welfare consequences of eight different fiscal adjustment packages incorporating a much wider set of fiscal instruments than studies previously in this literature.

Our main results are as follows. First, we find that fiscal austerity gives rise to a variety of distributional outcomes, determined by the composition of fiscal adjustment. In general, austerity tends to harm credit constrained households more than those with full access to capital markets. This is particularly the case with fiscal contractions based on cuts in transfer payments and increases in consumption and income taxes. Tax based fiscal consolidations reduce welfare of both types of households but disproportionately more of non-Ricardian households thus exhibit greater conflict between the two types of agents. This aspect of tax based consolidations versus spending based ones (which improve welfare for both type of agents) may have important implications for the continuity of these programs. Indeed, existing empirical literature on fiscal adjustments present evidence for tax based consolidations to be shorter lived than spending based ones. Finally, we also show that the distributive impact of fiscal policy is amplified the longer the austerity persists; the faster the policy reversal and when monetary policy reaches its zero lower bound.

Our findings also point to a clear trade off between austerity policies which cause the most harm to the short run growth of the economy (cuts in government spending) and

[78] An interesting extension of the paper is to conduct the similar analysis in a multi-households environment.

those which cause the most harm to agents (rises in taxes). Given the preoccupation with GDP figures and the severity of the current downturn in those advanced economies who have adopted austerity (notably the UK and the Eurozone), this trade-off between growth and distributional consequences of fiscal consolidation is likely to pose serious challenges to policymakers in many countries.

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3.A Log-Linearised System of Equations

In this section we present log linearized system of equations.

3.A.1 Households:

$$\hat{C^R}_t = E_t\hat{C^R}_{t+1} - R_t + E_t\hat{\pi}_{t+1} - \frac{\tau^c}{1+\tau^c}\left(\hat{\tau}^c_t - E_t\hat{\tau}^c_{t+1}\right) \tag{3.34}$$

$$\hat{Q}_t = -\hat{R}_t + E_t\hat{\pi}_{t+1} + \frac{(1-\delta)}{1-\delta+(1-\tau^k)\,r_k}\left(E_t\hat{Q}_{t+1}\right) + \tag{3.35}$$
$$+\frac{r_k\left(1-\tau^k\right)}{1-\delta+\left(1-\tau^k\right)r_k}E_t\left(\hat{r}_{k,t+1} - \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_{t+1}\right)$$

$$\hat{I}_t = \frac{\hat{Q}_t}{\phi\,(1+\beta)} + \frac{\hat{I}_{t-1}}{(1+\beta)} + \frac{\beta E_t\hat{I}_{t+1}}{(1+\beta)} \tag{3.36}$$

$$\hat{u}_t = \frac{1}{\kappa}\left[\hat{r}_{k,t} - \frac{\tau^k}{\left(1-\tau^k\right)}\hat{\tau}^k_t\right] \tag{3.37}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t \tag{3.38}$$

$$\hat{C}^{NR}_t = \left(1-\tau^l-\tau^{ee}\right)\frac{wL}{(1+\tau^c)\,C^{NR}} * \left(\hat{w}^\tau_t + \hat{L}_t - \frac{\tau^l}{1-\tau^l-\tau^{ee}}\hat{\tau}^l_t - \frac{\tau^{ee}}{1-\tau^l-\tau^{ee}}\hat{\tau}^{ee}_t\right) +$$
$$+\frac{T}{(1+\tau^c)\,C^{NR}}\widehat{T}_t - \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t \tag{3.39}$$

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} -$$
$$-\frac{1}{1+\beta}\frac{\left(1-\beta\varpi^w\right)\left(1-\varpi^w\right)}{\left(1+\frac{(1+\varsigma_w)\sigma_L}{\varsigma_w}\right)\varpi^w}X^w_t \tag{3.40}$$

$$X^w_t = \hat{w}_t - \sigma_L\hat{L}_t - \hat{C}_t - \frac{\tau^l}{1-\tau^l-\tau^{ee}}\hat{\tau}^l_t - \frac{\tau^{ee}}{1-\tau^{ee}-\tau^l}\hat{\tau}^{ee}_t - \frac{\tau^c}{1+\tau^c}\tau^c_t \tag{3.41}$$

3.A.2 Firms:

$$\hat{Y}_t^P = \varphi_y \left[\alpha_p \hat{K}_{t-1} + \alpha_p \hat{u}_t + (1-\alpha)\,\hat{L}_t^P + \sigma_g \hat{K}_{t-1}^g\right] \tag{3.42}$$

$$\hat{L}_t^P = \hat{u}_t + \hat{r}_t + \hat{K}_{t-1} - \hat{w}_t \tag{3.43}$$

$$\widehat{mc}_t = (1-\alpha_p)\left(\hat{w}_t + \frac{\tau^{er}}{1+\tau^{er}}\hat{\tau}_t^{er}\right) + \alpha_p \hat{r}_t - \sigma_g \hat{K}_{t-1}^g \tag{3.44}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \frac{(1-\beta\varpi)(1-\varpi)}{\varpi\left(1+\beta\gamma_p\right)}\widehat{mc}_t \tag{3.45}$$

$$\hat{\pi}_{c,t} = \hat{\pi}_t + \frac{\tau^c}{1+\tau^c}\left(\hat{\tau}_t^c - \hat{\tau}_{t-1}^c\right) \tag{3.46}$$

3.A.3 Government:

$$G_{rev,t} = \tau^c \frac{C}{Y}\left(\hat{\tau}_t^c + \hat{C}_t\right) + \tau^l \frac{wL}{Y}\left(\hat{\tau}_t^l + \hat{w}_t + \hat{L}_t\right) + \tau^k \frac{r_k K}{Y}\left(\hat{\tau}_t^k + \hat{r}_{k,t} + \hat{u}_t + \hat{K}_{t-1}\right) +$$

$$+ \tau^{ee}\frac{wL}{Y}\left(\hat{\tau}_t^{ee} + \hat{w}_t + \hat{L}_t\right) + \tau^{er}\frac{wL}{Y}\left(\hat{\tau}_t^{er} + \hat{w}_t + \hat{L}_t\right) \tag{3.47}$$

$$G_{rev,t} = R\frac{b}{Y}\left(\hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1}\right) - \frac{b}{Y}\hat{b}_t + \frac{G}{Y}\hat{G}_t + \frac{I^G}{Y}\hat{I}_t^G + \frac{T}{Y}T_t +$$

$$+ (1+\tau^{er})\frac{wL^G}{Y}\left(\frac{\tau^{er}}{1+\tau^{er}}\hat{\tau}_t^{er} + \hat{w}_t + \hat{L}_t^G\right) \tag{3.48}$$

$$\hat{K}_t^g = (1-\delta)\hat{K}_{t-1}^g + \delta \hat{I}_t^g \tag{3.49}$$

3.A.4 General equilibrium conditions:

$$\hat{Y}=\frac{Y^P}{Y}\hat{Y}^P+(1+\tau^{er})\frac{wL^G}{Y}\left(\frac{\tau^{er}}{1+\tau^{er}}\hat{\tau}_t^{er}+\hat{w}_t+\hat{L}_t^G\right) \tag{3.50}$$

$$\hat{Y}_t^P=\frac{C^T}{Y^P}\hat{C}_t^T+\frac{I}{Y^P}\hat{I}_t+\frac{G}{Y^P}\hat{G}_t+\frac{I^g}{Y^P}\hat{I}_t^g+(1-\tau^k)\frac{r_kK}{Y^P}*\hat{u}_t \tag{3.51}$$

$$C\hat{C}_t=(1-\lambda)\,C^R\widehat{C^R}_t+\lambda C^{NR}\hat{C}_t^{NR} \tag{3.52}$$

$$\hat{L}_t=\left(1-\frac{L^G}{L}\right)\hat{L}_t^P+\frac{L^G}{L}\hat{L}_t^G \tag{3.53}$$

The above equations plus the equations specifying fiscal and monetary policy in the text (equations 3.13-3.14, which are already in the log-linear form) comprise the system of equations.

3.B Steady State

In this section we focus on the steady state of the model. We denote the share of public labour in total labour as $\vartheta = N^P/N$; the share of government consumption in GDP as $g = \frac{G}{Y}$; the share of government investment to GDP as $ig = IG/Y$; the share of debt to GDP ratio as $by = B/Y$. Finally we denote the share of the capital income in the economy to $\alpha_T = r_kK/Y$. From equation the $Y_t = Y_t^P + (1+\tau_t^{er})\,w_tN_t^G$ we find the proportion of private and public output in GDP:

$$\frac{Y^P}{Y} = 1-\vartheta\,(1-\alpha_T) \tag{3.54}$$

$$\frac{(1+\tau^{er})\,wL^G}{Y} = \vartheta\,(1-\alpha_T) \tag{3.55}$$

From the Euler equation we get the steady state value of the interest rate:

$$R = \frac{1}{\beta} \tag{3.56}$$

From equation (3.17) we obtain the rental rate of capital:

$$r_k = \frac{1}{1-\tau_t^k}\left[\frac{1}{\beta} - (1-\delta_k)\right] \tag{3.57}$$

From the capital accumulation equation we get ratio of private investment to GDP:

$$\frac{I}{Y} = \delta_k \frac{\alpha_T}{r_k} \tag{3.58}$$

Using the above and the aggregate resource constraint we obtain the ratio of private consumption to output:

$$\frac{C}{Y} = 1 - \delta\frac{\alpha_T}{r_k} - g - ig \tag{3.59}$$

The share of government consumption in output net of government employment outlays is given by:

$$\frac{G_c}{Y} = g - \vartheta\left(1-\alpha_T\right) \tag{3.60}$$

From the government budget constraint we get the ratio of lump sum taxes or transfers to GDP:

$$\begin{aligned}\frac{T}{Y} &= \tau^c\left(1-\delta\frac{\alpha_T}{r_k}-g-ig\right)+\left(\tau^l+\tau^{ee}+\tau^{er}\right)\left(\frac{1-\alpha_T}{1+\tau^{er}}\right)\\ &\quad +\tau^k\alpha_T+by\left(\frac{1}{\beta}-1\right)-g-ig \end{aligned} \tag{3.61}$$

The share of capital in private output is given by:

$$\alpha^P=\frac{\alpha_T}{1-\vartheta\left(1-\alpha_T\right)} \tag{3.62}$$

Finally we stipulate that in the steady state $C/Y=C^R/Y=C^{NR}/Y$.

3.C Welfare Calculation

To derive the welfare functions for each household, a second order Taylor series expansion around steady state values of the period utility function (assumed to be identical across households) is performed, such as that performed in Woodford (2003).

$$\begin{aligned}U_t^i-U^i &\approx U_c\left(C_t^i-C^i\right)+U_l\left(L_t^i-L^i\right)+\frac{1}{2}U_{cc}\left(C_t^i-C^i\right)^2\\ &\quad +\frac{1}{2}U_{ll}\left(L_t^i-L^i\right)^2 \end{aligned} \tag{3.63}$$

where $U_x=\partial U(.)/\partial x$, and the separability of consumption and employment means $U_{cl}=0$. Let upper case letters with no time subscripts represent steady state values. Manipulating the above provides:

$$\begin{aligned}
U_t^i - U^i &\approx U_c C^i \left(\frac{C_t^i - C^i}{C^i}\right) + U_l L^i \left(\frac{L_t^i - L^i}{L^i}\right) \\
&\quad + \frac{1}{2} U_{cc} \left(C^i\right)^2 \left(\frac{C_t^i - C^i}{\left(C^i\right)^2}\right)^2 + \frac{1}{2} U_{nn} \left(N^i\right)^2 \left(\frac{N_t^i - N^i}{\left(N^i\right)^2}\right)^2 \\
U_t^i - U^i &\approx U_c C^i \left[\left(\frac{C_t^i - C^i}{C^i}\right) + \frac{1}{2}\frac{U_{cc}}{U_c} C^i \left(\frac{C_t^i - C^i}{\left(C^i\right)^2}\right)^2\right] \\
&\quad + U_l L^i \left[\left(\frac{L_t^i - L^i}{L^i}\right) + \frac{1}{2}\frac{U_{ll}}{U_l} L^i \left(\frac{L_t^i - L^i}{\left(L^i\right)^2}\right)^2\right] \qquad (3.64)
\end{aligned}$$

And dividing through by $U_c C^i$ provides:

$$\begin{aligned}
\frac{U_t^i - U^i}{U_c C^i} &\approx \left[\left(\frac{C_t^i - C^i}{C^i}\right) + \frac{1}{2}\frac{U_{cc}}{U_c} C^i \left(\frac{C_t^i - C^i}{\left(C^i\right)^2}\right)^2\right] \\
&\quad + \frac{U_l L^i}{U_c C^i} \left[\left(\frac{L_t^i - L^i}{L^i}\right) + \frac{1}{2}\frac{U_{ll}}{U_l} L^i \left(\frac{L_t^i - L^i}{\left(L^i\right)^2}\right)^2\right] \qquad (3.65)
\end{aligned}$$

Distortions in the model coming from monopolistically competitive labour and goods markets, distortionary taxes on consumption and labour and employee and employer social security contributions create a wedge in general equilibrium between the marginal product of labour (MPL) and the marginal rate of substitution for households (MRS):

$$\mu^w MRS \frac{1 + \tau_t^c}{1 - \tau_t^l - \tau_t^{ee}} = \frac{1 - \alpha}{1 + \tau^{er}} \frac{Y}{\mu^p L} \qquad (3.66)$$

Where the marginal rate of substitution is given by $MRS = -U_l(.)/U_c(.)$, and where $Y = (K)^\alpha \left(L^P\right)^{1-\alpha} \left(K^G\right)^{\alpha_G} - \Phi = \frac{(K)^\alpha \left(L^P\right)^{1-\alpha} \left(K^G\right)^{\alpha_G}}{\mu^p}$. Substituting these conditions into the Taylor series expansion above gives:

$$\begin{aligned}\frac{U_t^i - U}{U_c C^i} &\approx \left[\left(\frac{C_t^i - C^i}{C^i}\right) + \frac{1}{2}\frac{U_{cc}}{U_c}C^i\left(\frac{C_t^i - C^i}{(C^i)^2}\right)^2\right] \\ &\quad - \frac{1-\alpha}{1+\tau^{er}}\frac{1}{\mu^w}\frac{1-\tau^l-\tau^{ee}}{1+\tau^c}\frac{Y}{C^i}\left[\left(\frac{L_t^i - L^i}{L^i}\right) + \frac{1}{2}\frac{U_{ll}}{U_l}L^i\left(\frac{L_t^i - L^i}{(L^i)^2}\right)^2\right]\end{aligned}$$

Which from the specified utility function gives:

$$\begin{aligned}\frac{U_t^i - U}{U_c C} &\approx \left[\left(\frac{C_t^i - C}{C}\right) - \frac{1}{2}\left(\frac{C_t^i - C}{(C)^2}\right)^2\right] \\ &\quad - \frac{1-\alpha}{1+\tau^{er}}\frac{1}{\mu^w}\frac{1-\tau^l-\tau^{ee}}{1+\tau^c}\frac{Y}{C^i}\left[\left(\frac{L_t^i - L^i}{L^i}\right) + \frac{\sigma_l}{2}\left(\frac{L_t^i - L}{(L)^2}\right)^2\right]\end{aligned}$$

The above can be written in terms of log deviations as:

$$\begin{aligned}\frac{U_t^i - U}{U_c C} &\approx \left[c_t^i + \frac{1}{2}\left(c_t^i\right)^2 - \frac{1}{2}\left(c_t^i\right)^2\right] \\ &\quad - \frac{1-\alpha}{1+\tau^{er}}\frac{1}{\mu^w}\frac{1-\tau^l-\tau^{ee}}{1+\tau^c}\frac{Y}{C^i}\left[l_t^i + \frac{1}{2}\left(l_t^i\right)^2 + \frac{\sigma_l}{2}\left(l_t^i\right)^2\right]\end{aligned}$$

And simplified to:

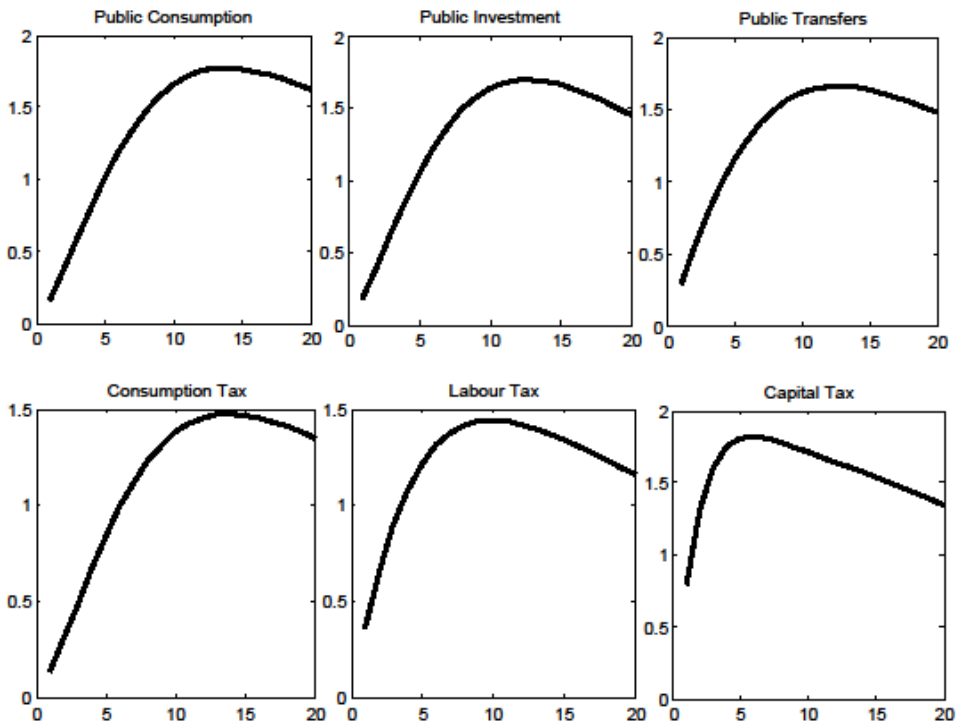
$$\frac{U_t^i - U}{U_c C} \approx \left[c_t^i\right] - \frac{1-\alpha}{1+\tau^{er}}\frac{1}{\mu^w}\frac{1-\tau^l-\tau^{ee}}{1+\tau^c}\frac{Y}{C^i}\left[l_t^i + \frac{1+\sigma_l}{2}\left(l_t^i\right)^2\right]$$

Finally, welfare losses for each type of household can be defined as the sum of lifetime utility lost expressed as a fraction of steady state consumption as:

$$\begin{aligned}W^i &= E_0\sum_{t=0}^{\infty}\left(\beta^i\right)^t\left(\frac{U_t^i - U}{U_c C}\right) \\ W^i &= E_0\sum_{t=0}^{\infty}\left(\beta^i\right)^t\left(\left[c_t^i\right] - \frac{1-\alpha}{1+\tau^{er}}\frac{1}{\mu^w}\frac{1-\tau^l-\tau^{ee}}{1+\tau^c}\frac{Y}{C^i}E_0\sum_{t=0}^{\infty}\left(\beta^i\right)^t\left(l_t^i + \frac{1+\sigma_l}{2}\left(l_t^i\right)^2\right)\right)\end{aligned}$$

3.D Debt dynamics

Fig. 3.31. Debt dynamics



3.E Dynare code

```
var y_p L_g L_p e_fsc e_hsc tao_fsc tao_hsc R mc tao_c tao_k tao_l c cr cnr q omega ro
    I w L y b kg g Ig trans pi pip k e_v e_tr e_ig e_tc e_tk e_tl e_a e_l e_i e_pi e_n chi_w
    g_rev tao_c_inc tao_k_inc tao_l_inc e_tlg tao_fsc_inc pub_sp;
    // R - nominal interest rate
    // mc - real marginal cost
```

```
// tao_c - consumption tax rate

// tao_k - capital tax rate

// tao_l - labour tax rate

// c - total consumption

// cr - consumption of Ricardian households

// cnr - ronsumption of non-Ricardian households

// q - Tobin's Q

// omega - capital utilisation rate

// ro - return on capital

// I - investment

// w - wage

// L - labour

// y - output

// b - bonds

// kg - public capital

// g - government spending

// Ig - government investment

// trans - transfers

// pi - consumer price inflation

// pip - producer price inflation

// k - private capital

// e_v - public spending shock
```

```
// e_tr - transfers shock
// e_ig - government investment shock
// e_tc - consumption tax shock
// e_tk - capital tax shock
// e_tl - labour tax shock
// e_a - tfp shock
// e_l - wage push-up shock
// e_i - investment shock
// e_pi - producer price push-up shock
// e_n - preferences shock
// chi_w - monetary policy shock
varexo tlg tfsc thsc tk ig tl tc tr vt at it lt pit nt wt;
parameters gcy_bar std_tlg phi_tlg theta wLgy_bar y_py_bar alfa_p tao_hscbar ela_hsc
phi_hsc std_hsc tao_fscbar ela_fsc phi_fsc std_fsc taoy_bar ul std_tt phi_t fc std_vt
omega_w gamma_w e_wi gamma_p sigma_c r_bar kappa beta
alfa delta omega_p fi bb rho b_pi b_y sigma_l rho_a sigma_g delta_g phi_g phi_ig
rho_i std_at std_lt std_it std_pit std_nt rho_n std_wt R_bar cy_bar Iy_bar
gy_bar Igy_bar try_bar wLy_bar rky_bar by_bar tao_cbar tao_kbar tao_lbar
std_tc std_tl std_tk std_tr std_ig phi_tr phi_tc
phi_tk phi_tl sh ela_ll ela_lc ela_lk ela_g ela_ig epa_tr ela_tr ela_tlg
x;
x = 1;
```

```
// %%%%%%%% calibrated parameters %%%%%%%%%%%%%%%

delta = 0.025 ; // depreciation rate of private capital

delta_g = 0.02 ; // depreciation rate of public capital

e_wi = 0.15 ; // wage markup

beta = 0.99 ; // discount rate

alfa = 0.31 ; // share of capital in production

sigma_g = 0.02 ; // the elasticity of output to public capital

//calculated steady state values

R_bar = 1.0101 ; // bonds rate of return

gy_bar = 0.2 ; // government consumption to gdp ratio

Igy_bar = 0.02 ; // government investment to gdp ratio

tao_cbar = 0.20 ; // consumption tax rate

tao_kbar = 0.40 ; // capital tax rate

tao_lbar = 0.18 ; // labour tax rate

by_bar = 2.4 ; // debt to gdp ratio

tao_hscbar=0.05;

tao_fscbar=0.07;

theta=0.15 ; // share of public employment in total employment

wLy_bar = (1-alfa)/(1+tao_fscbar) ; // share of labour income

wLgy_bar = theta*wLy_bar ; // share of government expenditure on hours in GDP

y_py_bar=1-theta*(1+tao_fscbar)*wLy_bar ; // share of private output in GDP

r_bar = (1/(1-tao_kbar))*(R_bar-1+delta) ; // rate of return on capital
```

```
alfa_p=1-wLy_bar*(1/y_py_bar)*(1-theta)*(1+tao_fscbar) ; //share of private capital
in private production function
gcy_bar=gy_bar-(1+tao_fscbar)*wLgy_bar ;// government consumption to
GDP ratio (adjusted by government expenditure on hours)
rky_bar = alfa_p*y_py_bar ; // share of capital income in GDP
Iy_bar = delta*rky_bar/r_bar ; // investment to GDP ratio
cy_bar = 1-Iy_bar-gy_bar-Igy_bar ; // consumption to GDP ratio
try_bar = tao_cbar*cy_bar+(tao_lbar+tao_fscbar+tao_hscbar)*wLy_bar+tao_kbar
*rky_bar-by_bar*(R_bar-1)-gy_bar-Igy_bar ; // government transfers to gdp ratio
fi = 5 ; // the inverse elasticity of investment with respect to an increase in the
installed capital
kappa = 0.6 ; // the inverse elasticity of utilisation with respect to the rental rate
of capital
sigma_l = 2 ; // the inverse elasticity of labour supply
sigma_c = 1 ; // the inverse of the intertemporal elasticity of sub-stitution
gamma_p = 0.15 ; // indexation prices
gamma_w = 0.15 ; // indexation wages
omega_p = 0.75 ; // price stickiness parameter
omega_w = 0.5 ; // wage stickiness parameter
bb = 0.0 ; // habit
fc = 1.15 ; // share of fixed costs in production
sh = 0.3 ; // share of non_Ricardian households in consumption
```

```
// monetary policy - Taylor rule
rho = 0.8 ; // smoothing parameter
b_pi = 1.5 ; // responce to inflation
b_y = 0.125 ; // responce to output
// fiscal policy (responce to debt)
ela_g = 0.181/x ; // gov. cons.
ela_ig = 0.875/x ; // gov. inv.
ela_tr = 0.086/x ; // transfers
ela_lc = 0.135/x ; // consumption tax
ela_lk = 0.141/x ; // capital tax
ela_ll = 0.151/x ; // labour tax
ela_fsc = 0.456/x ; //Employers' social security
ela_hsc = 0.0 ; //Employees' social security
ela_tlg = 0.235/x ; //Public employment
epa_tr = 0.7001 ; // responce of transfers to hours worked
//fiscal policy shocks
phi_g = 0.85 ; // AR(1) government consumption shock
phi_ig = 0.85 ; // AR(1) government investment shock
phi_tr = 0.85 ; // AR(1) government transfers shock
phi_tk = 0.85 ; // AR(1) capital taxes shock
phi_tl = 0.85 ; // AR(1) labour taxes shock
phi_tc = 0.85 ; // AR(1) consumption taxes shock
```

```
phi_hsc = 0.85 ;

phi_fsc = 0.85 ;

phi_tlg = 0.85 ;

std_tc = 0.0341 ; // std consumption taxes shock

std_tk = 0.0547 ; // std capital taxes shock

std_tl = 0.0172 ; // std labour taxes shock

std_tr = 0.0581 ; // std transfers shock

std_ig = 0.0029 ; // std government investment shock

std_vt = 0.0108 ; // std government spending shock

std_fsc = 0.03 ;

std_hsc = 0.03 ;

std_tlg = 0.03 ;

//remaining shocks

rho_a = 0.9367 ; // AR(1) tfp shock

rho_n = 0.7265 ; // AR(1) preference shock

rho_i = 0.1159 ; // AR(1) investment shock

std_at = 0.0084 ; // std tfp shock

std_nt = 0.0051 ; // std preference shock

std_lt = 0.0034 ; // std labour supply shock

std_pit = 0.0082 ; // std cost push-up shock

std_it = 0.0577 ; // std investment shock

std_wt = 0.0029 ; // std monetary policy shock
```

```
model(linear);
// 1. consumption of non-Ricardian households
(1+tao_cbar)*cy_bar*(cnr+(tao_cbar/(1+tao_cbar))*tao_c)=wLy_bar*((1-tao_lbar
-tao_hscbar)*(w+L)-tao_lbar*tao_l-tao_hscbar*tao_hsc)+try_bar*trans;
// 2. consumption of Ricardian households
cr=((bb/(1+bb))*cr(-1))+((1/(1+bb))*cr(+1))-((1-bb)/((1+bb)*sigma_c))*(R)+((1-bb)
/((1+bb)*sigma_c))*pi(+1)+((1-bb)/((1+bb)*sigma_c))*(e_n-e_n(+1))-((1-bb)/
((1+bb)*sigma_c))*(tao_cbar/(1+tao_cbar))*(tao_c-tao_c(+1));
// First order conditions of Ricardian households
// 3. w.r.t capital
q=-R+pi(+1)+((1-delta)/(1-delta+(1-tao_kbar)*r_bar))*q(+1)+((1-tao_kbar)*r_bar
/(1-delta+(1-tao_kbar)*r_bar))*(ro(+1)-(tao_kbar/(1-tao_kbar))*tao_k(+1));
// 4. w.r.t investment
I=((1/(fi*(1+beta)))*q+(1/(1+beta))*I(-1)+(beta/(1+beta))*I(+1)
-(1/((1+beta)))*(beta*e_i(+1)-e_i));
// 5. w.r.t capital utilisation
omega=(1/kappa)*(ro-(tao_kbar/(1-tao_kbar))*tao_k);
// 6. wage equation
w=(beta/(1+beta))*w(+1)+(1/(1+beta))*w(-1)+(beta/(1+beta))*pi(+1)-((1+beta
*gamma_w)/(1+beta))*pi+(gamma_w/(1+beta))*pi(-1)-((((1-omega_w)*(1-beta
*omega_w))/((1+((1+e_wi)/(e_wi))*sigma_l)*omega_w))*(1/(1+beta)))*(w-
sigma_l*L-(sigma_c/(1-bb))*(cr-bb*cr(-1))-(tao_lbar/(1-tao_lbar-tao_hscbar))
```

```
*tao_l-(tao_hscbar/(1-tao_lbar-tao_hscbar))*tao_hsc-(tao_cbar/(1+tao_cbar))
*tao_c)+e_l;
// 7. private capital accumulation equation
k=(1-delta)*k(-1)+delta*(I);
// 8. public capita accumulation equation
kg=(1-delta_g)*kg(-1)+delta_g*Ig;
// 9. output of firms
y_p=fc*(e_a+alfa_p*k(-1)+alfa_p*omega+(1-alfa_p)*L_p+sigma_g*kg(-1));
// 10. combination of first order conditions of firms
//ro+omega+k(-1)=w+L;
// 11. real marginal costs
//mc=-e_a+(1-alfa_p)*w+alfa_p*ro-sigma_g*kg(-1);
ro=mc+e_a+(alfa_p-1)*(k(-1)+omega)+(1-alfa_p)*L_p+sigma_g*kg(-1);
w+(tao_fscbar/(1+tao_fscbar))*tao_fsc=mc+e_a+(alfa_p)*(k(-1)+omega)
-alfa_p*L_p+sigma_g*kg(-1);
// 12. hybrid new-Keynesian Philips curve for producers price inflation
pip=(beta/(1+beta*gamma_p))*pip(+1)+(gamma_p/(1+beta*gamma_p))*pip(-1)
+((((1-omega_p)*(1-beta*omega_p))/(omega_p))*(1/(1+beta*gamma_p)))
*(mc)+e_pi;
// 13. consumer price inflation
pi=pip;
// 14. resource constraint
```

```
y-(1+tao_fscbar)*wLgy_bar*((tao_fscbar/(1+tao_fscbar))*tao_fsc+w+L_g)
=(cy_bar)*c+(Iy_bar)*I+(1-tao_kbar)*(rky_bar)*omega+gcy_bar*g+Igy_bar*Ig;
// 15. total consumption
c=(1-sh)*cr+sh*cnr;
// 16. monetary policy rule:
R=rho*R(-1)+(1-rho)*b_y*y+(1-rho)*b_pi*pi+chi_w;
// 17-22 fiscal policy rules:
g=-(ela_g*b(-1))+e_v;
Ig=-(ela_ig*b(-1))+e_ig;
trans=-(ela_tr*b(-1)+epa_tr*L)+e_tr;
tao_c=(ela_lc*(b(-1)))-e_tc;
tao_k=(ela_lk*(b(-1)))-e_tk;
tao_l=(ela_ll*(b(-1)))-e_tl;
tao_fsc=(ela_fsc*(b(-1)))-e_fsc;
tao_hsc=(ela_hsc*(b(-1)))-e_hsc;
// 23. government budget constraint
(1+tao_fscbar)*wLgy_bar*((tao_fscbar/(1+tao_fscbar))*tao_fsc+w+L_g)
+g*gcy_bar+Igy_bar*Ig+try_bar*trans+by_bar*R_bar*(b(-1)+R(-1)-pi)=
by_bar*b+tao_cbar*cy_bar*(c+tao_c)+tao_lbar*wLy_bar*(w+L+tao_l)+
tao_fscbar*wLy_bar*(w+L+tao_fsc)+tao_hscbar*wLy_bar*(w+L+tao_hsc)
+tao_kbar*rky_bar*(ro+omega+k(-1)+tao_k);
g_rev=w+L-(tao_lbar/(1-tao_lbar))*tao_l;
```

```
tao_c_inc=(c+tao_c);

tao_k_inc=(ro+omega+k(-1)+tao_k);

tao_l_inc=(w+L+tao_l);

tao_fsc_inc=(w+L+tao_fsc);

pub_sp=(tao_fscbar/(1+tao_fscbar))*tao_fsc+w+L_g;

//Public employment

L=(1-theta)*L_p+theta*L_g;

L_g=-(ela_tlg*b(-1))+e_tlg;

y=y_py_bar*y_p+(1+tao_fscbar)*wLgy_bar*((tao_fscbar/(1+tao_fscbar))*tao_fsc+w+L_g);

// shocks

e_tc=phi_tc*e_tc(-1)+tc;

e_tk=phi_tk*e_tk(-1)+tk;

e_tl=phi_tl*e_tl(-1)+tl;

e_tr=phi_tr*e_tr(-1)+tr;

e_ig=phi_ig*e_ig(-1)+ig;

e_v=phi_g*e_v(-1)+vt;

e_hsc=phi_hsc*e_hsc(-1)+thsc;

e_fsc=phi_fsc*e_fsc(-1)+tfsc;

e_tlg=phi_tlg*e_tlg(-1)+tlg;

e_a= rho_a*e_a(-1)+at;

e_i= rho_i*e_i(-1)+it;

e_l= lt;
```

```
e_pi= pit;

e_n= rho_n*e_n(-1)+nt;

chi_w= wt;

end;

steady;

check;

shocks;

var tc; stderr 0.07722;

var tk; stderr 0.08065;

var tl; stderr 0.08615;

var ig; stderr 0.5;

var vt; stderr 0.10363;

var tr; stderr 0.04933;

var thsc; stderr 0.31013;

var tfsc; stderr 0.22152;

var tlg; stderr 0.103413;

end;

stoch_simul(order=1,irf=1000,nograph, periods=1000);
```

Chapter 4
Notes

4.1 Note on Bayesian Estimation (Chapter 1 and 2)

Bayesian estimation[79] can be perceived as a combination of maximum likelihood estimation and calibration. Calibration because of the presence of priors, which comprise weights on likelihood function so more importance is given to particular areas of parameters in subspace. Firstly, let's denote prior probability distribution as $p(\theta)$, where θ denotes parameters of the model and $p(\bullet)$ stands for probability distribution function, likelihood function as $\mathcal{L}\left(\theta|Y^T\right)$, where Y^T denotes the complete sample of data, and finally $p\left(\theta|Y^T\right)$, as a posterior distribution. Secondly, note that likelihood can be formulated as:

$$\mathcal{L}\left(\theta|Y^T\right) = p\left(Y^T|\theta\right) = p\left(Y_0|\theta\right)\prod_{t=1}^{T} p\left(Y_t|Y_{t-1},\theta\right)$$

Thirdly, in order to get posterior, $p\left(\theta|Y^T\right)$, Bayes' theorem is used which can be derived from the definition of conditional probability:

$$p\left(Y^T|\theta\right) = \frac{p\left(\theta, Y^T\right)}{p\left(\theta\right)}$$

$$p\left(\theta|Y^T\right) = \frac{p\left(\theta, Y^T\right)}{p\left(Y^T\right)}$$

[79] Notes are based on An and Schorfheide (2007), Den Haan (2011), and Mancini Griffoli (2011).

$$\Rightarrow p\left(\theta, Y^T\right) = p\left(Y^T|\theta\right) p\left(\theta\right) = p\left(\theta|Y^T\right) p\left(Y^T\right)$$

Therefore:

$$p\left(\theta|Y^T\right) = \frac{p\left(Y^T|\theta\right) p\left(\theta\right)}{p\left(Y^T\right)}$$

Since $p\left(Y^T\right)$ is constant, Bayes' theorem can be written as:

$$p\left(\theta|Y^T\right) \propto p\left(Y^T|\theta\right) p\left(\theta\right) \equiv \Bbbk\left(\theta|Y^T\right)$$

where $p\left(Y^T|\theta\right)$ stands for maximum function and $p\left(\theta\right)$ stands for prior probability distributions, and $\Bbbk\left(\theta|Y^T\right)$ stands for posterior kernel. Likelihood function is estimated with help of Kalman filter.

4.1.1 Estimation of Likelihood Function of the Model

The state space representation of the solution to the model can be rewritten in the following way :

$$\begin{aligned} \hat{x}_{t+1} &= A\hat{x}_t + Bv_{t+1} \\ \hat{y}_t &= C\hat{x}_t + w_t \end{aligned}$$

where first equation is the equation comprising the solution of the model and the second equation is the observation equation i.e. $\hat{y}$ is an observable variable, and w_t is an measurement error. Hats over variables denote that the solution is in the deviation from steady state form in case of model solution, and in case of observable variable it means that date are

detrended by means of linear trend. Moreover, $E\begin{bmatrix} v_t \\ w_t \end{bmatrix}\begin{bmatrix} v_t \\ w_t \end{bmatrix}' = \begin{matrix} Q & V \\ V & R \end{matrix}$, v_t and w_t are uncorrelated and orthogonal to y_t. Kalman filter recursion is the following:

$$\begin{aligned}
\widehat{\tilde{y}}_{t+1} &= \hat{y}_{t+1} - CE_t\hat{x}_{t+1} \\
E_t\hat{x}_{t+1} &= AE_{t-1}\hat{x}_t + K_t\widehat{\tilde{y}}_t \\
K_t &= (AP_tC' + BV)(CP_tC' + R)^{-1} \\
P_{t+1} &= AP_tA' + BQB' - K_t(AP_tC' + BV)'
\end{aligned}$$

Subsequently from the Kalman filter recursion log-likelihood is derived. With the assumption of normal distribution which has the probability distribution function:

$$p(Y_t|\theta) = \frac{1}{\sqrt{2\pi(CP_tC' + R)}}\exp\left(-\frac{\widehat{\tilde{y}}_t\widehat{\tilde{y}}_t'}{(CP_tC' + R)}\right)$$

the log-likelihood is given by:

$$\mathcal{L}\left(\theta|Y^T\right) = -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T}\left[(CP_tC' + R) - \widehat{\tilde{y}}\widehat{\tilde{y}}'(CP_tC' + R)^{-1}\right]$$

The log posterior kernel becomes then: $\ln \Bbbk\left(\theta|Y^T\right) = \ln \mathcal{L}\left(\theta|Y^T\right) + \ln p(\theta)$. Subsequently, maximizing the above log posterior kernel with respect to θ the mode of the posterior distribution is found.

4.1.2 Derivation of Posterior Distribution

At this stage only the mode of posterior distribution is known. In order to simulate posterior distribution a particular version of Markov Chain Monte Carlo (MCMC) algorithm i.e. Metropolis algorithm is employed. The employed steps are as follows:

1. Choose a starting point - posterior mode.

2. Draw θ^* from the distribution $f\left(\theta^*|\theta^i\right) = N\left(\theta^i, c\sum_m\right)$, where $\sum_m$ is the inverse of the Hessian matrix computed at the mode of the posterior distribution. θ^* is a candidate for θ^{i+1} with the probability of $q\left(\theta^{i+1}|\theta^i\right)$, and θ^i is a candidate for θ^{i+1} with probability of $1 - q\left(\theta^{i+1}|\theta^i\right)$, where $q\left(\theta^{i+1}|\theta^i\right) = \min\left[1, \frac{p\left(\theta^*, Y^T\right)}{p\left(\theta^i, Y^T\right)}\right]$, where $\frac{p\left(\theta^*, Y^T\right)}{p\left(\theta^i, Y^T\right)}$ is an acceptance ratio.

3. Accept, or discard the proposed θ^*.

4. Update mean of the drawing distribution, retain value of the parameter.

5. Repeat steps 2,3, and 4 for a chosen number of times.

6. Plot histogram of the retained values.

The idea is to search through the space of θ using appropriate size of steps. This is why the variance of and in particular the scaling parameter are of special interest in here. Increase in the scaling parameter will cause acceptance rate do decrease, and decrease in the scaling parameter will cause the acceptance ratio to increase. In case of too high acceptance ratio the Metropolis algorithm would never visit the tails of the distribution and in case of too low acceptance ratio it would take long time to converge since it can easily get stuck in the local subspaces. Literature proposes acceptance ratio in a range of 0.2-0.4. Gelman *et al.* (1997) in particular get an optimal acceptance rate of 0.234.

4.2 Note on Log-Linearisation

Log-linearization procedure isin line with the one presented in Campbell (1994) and Uhlig (1995). Variables are denoted in the following way: big letters without subscript t denote steady state values. Big letters with subscript t denote variables without any transformation. Letters with subscript t and hat above denote log deviations of particular variable from steady state. Below I present how log-linearization procedure is applied. Deviation of capital from steady state is equal:

$$\hat{K}_t = \ln K_t - \ln K$$

Though:

$$\ln K_t = \ln K + \hat{K}_t$$

Taking exponents of both sides we get:

$$e^{\ln K_t} = e^{\ln K + \hat{K}_t} = e^{\ln K} e^{\hat{K}_t}$$

Thus:

$$K_t = K e^{\hat{K}_t} \Rightarrow e^{\hat{K}_t} = \frac{K_t}{K}$$

Next step is to take the first order Taylor approximation of $e^{\hat{K}_t}$ around the steady state thus $\hat{K}_t = 0$, though we get:

$$e^{\hat{K}_t} = e^0 + e^0(\hat{K}_t - 0) = 1 + \hat{K}_t$$

thus:

$$1 + \hat{K}_t = \frac{K_t}{K} \Rightarrow K_t = K(1 + \hat{K}_t)$$

or:

$$\hat{K}_t = \frac{K_t - K}{K}$$

The variable $\hat{K}_t$ multiplied by 100 informs by what percentage capital at time t diverges from the steady state. So for example if $\hat{K}_t$ is equal 0.2 we interpret that capital is 20% above the steady state.

4.2.1 Derivation of non-linear new-Keynesian Philips curve

The first order condition of firms (resulting from price setting) is represented by

$$E_t \sum_{l=0}^{\infty} (\beta\varpi)^l \lambda_{t+l} \left[\frac{\tilde{P}_t}{P_{t+l}} X_{tl} - \varkappa mc_{t+l} \right] P_{t+l} Y_{j,t+l} = 0$$

$$Y_{j,t+l} = \left(\frac{X_{tl} \tilde{P}_{t+l}}{P_{t+l}} \right)^{\frac{\varkappa}{1-\varkappa}} Y_{t+l}$$

where

$\varkappa = \frac{s}{s-1}$, and s denotes an elasticity of substitution

$\frac{\varkappa}{1-\varkappa} = \frac{\frac{s}{s-1}}{1-\frac{s}{s-1}} = \frac{\frac{s}{s-1}}{\frac{-1}{s-1}} = -s$

Equation 1 can be represented in the following way:

$$
\begin{aligned}
& \sum_{l=0}^{\infty}(\beta\varpi)^l\lambda_{t+l}\left[\frac{\tilde{P}_t}{P_{t+l}}X_{tl}-\varkappa mc_{t+l}\right]\left(\frac{\tilde{P}_{t+l}}{P_{t+l}}\right)^{\frac{\varkappa}{1-\varkappa}}Y_{t+l} \\
= \; & \lambda_t\left[\frac{\tilde{P}_t}{P_t}-\varkappa mc_t\right]\left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\varkappa}{1-\varkappa}}P_tY_t \\
& +\beta\varpi\lambda_{t+1}\left[\frac{\tilde{P}_t}{P_{t+1}}\pi_t^{\gamma}-\varkappa mc_{t+1}\right]\left(\pi_t^{\gamma}\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+1}Y_{t+1} \\
& +(\beta\varpi)^2\lambda_{t+2}\left[\frac{\tilde{P}_t}{P_{t+2}}\pi_t^{\gamma}\pi_{t+1}^{\gamma}-\varkappa mc_{t+2}\right]\left(\pi_t^{\gamma}\pi_{t+1}^{\gamma}\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+2}Y_{t+2} \\
& +... \\
= \; & \lambda_t\frac{\tilde{P}_t}{P_t}\left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\varkappa}{1-\varkappa}}P_tY_t-\lambda_t\varkappa mc_t\left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\varkappa}{1-\varkappa}}P_tY_t \\
& +\beta\varpi\lambda_{t+1}\frac{\tilde{P}_t}{P_{t+1}}\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+1}Y_{t+1}-\beta\varpi\lambda_{t+1}\varkappa mc_{t+1}\left(\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+1}Y_{t+1} \\
& +(\beta\varpi)^2\lambda_{t+2}\frac{\tilde{P}_t}{P_{t+2}}\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+2}Y_{t+2} \\
& -\varkappa mc_{t+2}(\beta\varpi)^2\lambda_{t+2}\left(\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}}P_{t+2}Y_{t+2} \\
& +...
\end{aligned}
$$

Below I call as the 'first part' $\sum_{l=0}^{\infty}(\beta\varpi)^l\lambda_{t+l}\frac{\tilde{P}_t}{P_{t+l}}X_{tl}P_{t+l}Y_{j,t+l}$
and as the 'second part' $\sum_{l=0}^{\infty}(\beta\varpi)^l\lambda_{t+l}\varkappa mc_{t+l}P_{t+l}Y_{j,t+l}$

First part

working on first part of the expression and denoting it as $P_tx_t^1$

$$
\begin{aligned}
P_t x_t^1 &= \lambda_t \frac{\tilde{P}_t}{P_t}\left(\frac{\tilde{P}_t}{P_t}\right)^{\frac{\varkappa}{1-\varkappa}} P_t Y_t \\
&+\beta\varpi\lambda_{t+1}\frac{\tilde{P}_t}{P_{t+1}}\pi_t^{\gamma}\left(\pi_t^{\gamma}\frac{\tilde{P}_t}{P_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1} \\
&+(\beta\varpi)^2\lambda_{t+2}\frac{\tilde{P}_t}{P_{t+2}}\pi_t^{\gamma}\pi_{t+1}^{\gamma}\left(\pi_t^{\gamma}\pi_{t+1}^{\gamma}\frac{\tilde{P}_t}{P_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+...
\end{aligned}
$$

$$
\begin{aligned}
P_t x_t^1 &= \lambda_t \left(\tilde{p}_t\right)^{\frac{1}{1-\varkappa}} P_t Y_t \\
&+\beta\varpi\lambda_{t+1}\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1} \\
&+(\beta\varpi)^2\lambda_{t+2}\left(\pi_t^{\gamma}\pi_{t+1}^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}\pi_{t+2}}\right)^{\frac{1}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+...
\end{aligned}
$$

where $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$

If we forward one period the equation becomes:

$$
\begin{aligned}
P_{t+1} x_{t+1}^1 &= \lambda_{t+1}\frac{\tilde{P}_{t+1}}{P_{t+1}}\left(\frac{\tilde{P}_{t+1}}{P_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1} \\
&+\beta\varpi\lambda_{t+2}\pi_{t+1}^{\gamma}\frac{\tilde{P}_{t+1}}{P_{t+2}}\left(\pi_{t+1}^{\gamma}\frac{\tilde{P}_{t+1}}{P_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+(\beta\varpi)^2\lambda_{t+3}\pi_{t+1}^{\gamma}\pi_{t+2}^{\gamma}\frac{\tilde{P}_{t+1}}{P_{t+3}}\left(\pi_{t+1}^{\gamma}\pi_{t+2}^{\gamma}\frac{\tilde{P}_{t+1}}{P_{t+3}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+3}Y_{t+3} \\
&+...
\end{aligned}
$$

$$\begin{aligned}
P_{t+1}x^1_{t+1} &= \lambda_{t+1}\left(\tilde{p}_{t+1}\right)^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1} \\
&+\beta\varpi\lambda_{t+2}\left(\pi^{\gamma}_{t+1}\frac{\tilde{p}_{t+1}}{\pi_{t+2}}\right)^{\frac{1}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+(\beta\varpi)^2\lambda_{t+3}\left(\pi^{\gamma}_{t+1}\pi^{\gamma}_{t+2}\frac{\tilde{p}_{t+1}}{\pi_{t+2}\pi_{t+3}}\right)^{\frac{1}{1-\varkappa}} P_{t+3}Y_{t+3} \\
&+...
\end{aligned}$$

Now we are looking for an expression (w_t) that meets the following condition:

$w_t * \lambda_{t+1}(\tilde{p}_{t+1})^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1} = \beta\varpi\lambda_{t+1}\left(\pi^{\gamma}_t\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1}$

$\Rightarrow w_t = \frac{\beta\varpi\lambda_{t+1}\left(\pi^{\gamma}_t\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1}}{\lambda_{t+1}(\tilde{p}_{t+1})^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1}} = \frac{\beta\varpi\left(\pi^{\gamma}_t\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{1}{1-\varkappa}}}{(\tilde{p}_{t+1})^{\frac{1}{1-\varkappa}}} = \beta\varpi\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\varkappa}}\left(\pi^{\gamma}_t\right)^{\frac{1}{1-\varkappa}}\pi^{-\frac{1}{1-\varkappa}}_{t+1}$

thus:

$$P_t x^1_t = \lambda_t\left(\tilde{p}_t\right)^{\frac{1}{1-\varkappa}} P_tY_t + \beta\varpi\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\varkappa}}\left(\pi^{\gamma}_t\right)^{\frac{1}{1-\varkappa}}\pi^{-\frac{1}{1-\varkappa}}_{t+1}P_{t+1}x^1_{t+1}$$

$P_{t+1}x^1_{t+1} = \lambda_{t+1}\left(\tilde{p}_{t+1}\right)^{\frac{1}{1-\varkappa}} P_{t+1}Y_{t+1} + \beta\varpi\left(\frac{\tilde{p}_{t+1}}{\tilde{p}_{t+2}}\right)^{\frac{1}{1-\varkappa}}\left(\pi^{\gamma}_{t+1}\right)^{\frac{1}{1-\varkappa}}\pi^{-\frac{1}{1-\varkappa}}_{t+2}P_{t+2}x^1_{t+2}$

$P_{t+2}x^1_{t+2} = \lambda_{t+2}\left(\tilde{p}_{t+2}\right)^{\frac{1}{1-\varkappa}} P_{t+2}Y_{t+2} + \beta\varpi\left(\frac{\tilde{p}_{t+2}}{\tilde{p}_{t+3}}\right)^{\frac{1}{1-\varkappa}}\left(\pi^{\gamma}_{t+2}\right)^{\frac{1}{1-\varkappa}}\pi^{-\frac{1}{1-\varkappa}}_{t+3}P_{t+3}x^1_{t+3}$

$P_t x^1_t$ divided by P_t, gives:

$$x^1_t = \lambda_t\left(\tilde{p}_t\right)^{\frac{1}{1-\varkappa}} Y_t + \beta\varpi\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\varkappa}}\left(\pi^{\gamma}_t\right)^{\frac{1}{1-\varkappa}}\pi^{\frac{\varkappa}{1-\varkappa}}_{t+1}x^1_{t+1}$$

Second part

Working on the second part of the expression and denoting it as $P_t x^2_t$

$$
\begin{aligned}
P_t x_t^2 &= \lambda_t \varkappa mc_t \left(\tilde{p}_t\right)^{\frac{\varkappa}{1-\varkappa}} P_t Y_t \\
&+\beta\varpi\lambda_{t+1}\varkappa mc_{t+1}\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1} \\
&+(\beta\varpi)^2\lambda_{t+2}\varkappa mc_{t+2}\left(\pi_t^{\gamma}\pi_{t+1}^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}\pi_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+...
\end{aligned}
$$

$$
\begin{aligned}
P_{t+1} x_{t+1}^2 &= \lambda_{t+1} \varkappa mc_{t+1} \left(\tilde{p}_{t+1}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+1} Y_{t+1} \\
&+\beta\varpi\lambda_{t+2}\varkappa mc_{t+1}\left(\pi_{t+1}^{\gamma}\frac{\tilde{p}_{t+1}}{\pi_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+2}Y_{t+2} \\
&+(\beta\varpi)^2\lambda_{t+3}\varkappa mc_{t+3}\left(\frac{\tilde{p}_{t+1}}{\pi_{t+2}\pi_{t+3}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+3}Y_{t+3} \\
&+...
\end{aligned}
$$

I need (w_t) that meets the following condition:

$$w_t * \left[\lambda_{t+1}\varkappa mc_{t+1}(\tilde{p}_{t+1})^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1}\right] = \beta\varpi\lambda_{t+1}\varkappa mc_{t+1}\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1}$$

$$\Rightarrow w_t = \frac{\beta\varpi\lambda_{t+1}\varkappa mc_{t+1}(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}})^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1}}{\lambda_{t+1}\varkappa mc_{t+1}(\tilde{p}_{t+1})^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1}} = \frac{\beta\varpi\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}}}{(\tilde{p}_{t+1})^{\frac{\varkappa}{1-\varkappa}}} = \beta\varpi\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\pi_{t+1}\tilde{p}_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} =$$

$$\beta\varpi\left(\pi_t^{\gamma}\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} (\pi_t^{\gamma})^{\frac{\varkappa}{1-\varkappa}} \pi_{t+1}^{-\frac{\varkappa}{1-\varkappa}}$$

Thus:

$$P_t x_t^2 = \lambda_t\varkappa mc_t(\tilde{p}_t)^{\frac{\varkappa}{1-\varkappa}} P_t Y_t + \beta\varpi\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} (\pi_t^{\gamma})^{\frac{\varkappa}{1-\varkappa}} \pi_{t+1}^{-\frac{\varkappa}{1-\varkappa}} P_{t+1}x_{t+1}^2$$

$$P_{t+1}x_{t+1}^2 = \lambda_{t+1}\varkappa mc_{t+1}(\tilde{p}_{t+1})^{\frac{\varkappa}{1-\varkappa}} P_{t+1}Y_{t+1} + \beta\varpi\left(\frac{\tilde{p}_{t+1}}{\tilde{p}_{t+2}}\right)^{\frac{\varkappa}{1-\varkappa}} (\pi_{t+1}^{\gamma})^{\frac{\varkappa}{1-\varkappa}} \pi_{t+2}^{-\frac{\varkappa}{1-\varkappa}} P_{t+2}x_{t+2}^2$$

Thus (divided by P_t):

$$x_t^2 = \lambda_t \varkappa mc_t(\tilde{p}_t)^{\frac{\varkappa}{1-\varkappa}} Y_t + \beta\varpi \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} (\pi_t^\gamma)^{\frac{\varkappa}{1-\varkappa}} \pi_{t+1}^{\frac{1-2\varkappa}{1-\varkappa}} x_{t+1}^2$$

To sum up:

$$\begin{aligned} x_t^1 &= \lambda_t (\tilde{p}_t)^{\frac{1}{1-\varkappa}} Y_t + \beta\varpi \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\varkappa}} (\pi_t^\gamma)^{\frac{1}{1-\varkappa}} \pi_{t+1}^{\frac{\varkappa}{1-\varkappa}} x_{t+1}^1 \\ x_t^2 &= \lambda_t \varkappa mc_t(\tilde{p}_t)^{\frac{\varkappa}{1-\varkappa}} Y_t + \beta\varpi \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{\varkappa}{1-\varkappa}} (\pi_t^\gamma)^{\frac{\varkappa}{1-\varkappa}} \pi_{t+1}^{\frac{1-2\varkappa}{1-\varkappa}} x_{t+1}^2 \end{aligned}$$

Price index becomes:

$P_t^{\frac{1}{1-\varkappa}} = (1-\varpi)\, \tilde{P}_t^{\frac{1}{1-\varkappa}} + \varpi(\pi_{t-1}^\gamma P_{t-1})^{\frac{1}{1-\varkappa}}$

$$1 = (1-\varpi)\, p_t^{\frac{1}{1-\varkappa}} + \varpi \left(\frac{\pi_{t-1}^\gamma}{\pi_t}\right)^{\frac{1}{1-\varkappa}}$$

Now we log-linearize to check whether we get NKPC

$x_t^1 = \lambda_t (\tilde{p}_t)^{\frac{1}{1-\varkappa}} Y_t + \beta\varpi \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{\frac{1}{1-\varkappa}} (\pi_t^\gamma)^{\frac{1}{1-\varkappa}} \pi_{t+1}^{-\frac{\varkappa}{1-\varkappa}} x_{t+1}^1$

becomes in the steady state:

$x^1 = \frac{\lambda Y}{1-\beta\varpi}$

Log-linearization results in:

$x^1 (1+\hat{x}_t^1) = \lambda Y \left(1 + \hat{\lambda}_t + \frac{1}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right)$

$+\beta\varpi x^1 \left(1 + \frac{1}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\gamma}{1-\varkappa}\hat{\pi}_t - \frac{\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}\right)$

$$\begin{aligned} \hat{x}_t^1 &= (1-\beta\varpi)\left(\hat{\lambda}_t + \frac{1}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) \\ &\quad +\beta\varpi\left(\frac{1}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\gamma}{1-\varkappa}\hat{\pi}_t - \frac{\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}\right) \end{aligned}$$

Now second term:

$x_t^2 = \lambda_t \varkappa mc_t(\tilde{p}_t)^{\frac{\varkappa}{1-\varkappa}} Y_t + \beta\varpi(\frac{\tilde{p}_t}{\tilde{p}_{t+1}})^{\frac{\varkappa}{1-\varkappa}} (\pi_t^{\gamma})^{\frac{\varkappa}{1-\varkappa}} \pi_{t+1}^{\frac{1-2\varkappa}{1-\varkappa}} x_{t+1}^2$

becomes in the steady state:

$x^2 = \frac{\lambda Y}{1-\beta\varpi}$

Log-linearization results in:

$$\begin{aligned} \hat{x}_t^2 &= (1-\beta\varpi)\left(\hat{\lambda}_t + \widehat{mc}_t + \frac{\varkappa}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) \\ &\quad +\beta\varpi\left(\frac{\varkappa}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\varkappa\gamma}{1-\varkappa}\hat{\pi}_t + \frac{1-2\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}^2\right) \end{aligned}$$

Now the price index:

$1 = (1-\varpi)\, p_t^{\frac{1}{1-\varkappa}} + \varpi(\frac{\pi_{t-1}^{\gamma}}{\pi_t})^{\frac{1}{1-\varkappa}}$

$0 = (1-\varpi)\left(\frac{1}{1-\varkappa}p_t\right) + \varpi\left(\frac{1}{1-\varkappa}\left(\gamma\pi_{t-1} - \pi_t\right)\right)$

$0 = (1-\varpi)\left(\hat{p}_t\right) + \varpi\left(\left(\gamma\hat{\pi}_{t-1} - \hat{\pi}_t\right)\right)$

$$\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_t - \gamma\hat{\pi}_{t-1}\right) = \hat{p}_t$$

To sum up we have:

$$
\begin{aligned}
0 &= (1-\varpi)(\hat{p}_t) + \varpi((\gamma\hat{\pi}_{t-1} - \hat{\pi}_t)) \\
\hat{x}_t^1 &= (1-\beta\varpi)\left(\hat{\lambda}_t + \frac{1}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) \\
&\quad +\beta\varpi\left(\frac{1}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\gamma}{1-\varkappa}\hat{\pi}_t - \frac{\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}\right) \\
\hat{x}_t^2 &= (1-\beta\varpi)\left(\hat{\lambda}_t + \widehat{mc}_t + \frac{\varkappa}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) \\
&\quad +\beta\varpi\left(\frac{\varkappa}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\varkappa\gamma}{1-\varkappa}\hat{\pi}_t + \frac{1-2\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}^2\right) \\
\hat{x}_t^1 &= \hat{x}_t^2
\end{aligned}
$$

Substitute for $\hat{x}_t^1$ and $\hat{x}_t^2$ into $\hat{x}_t^1 = \hat{x}_t^2$ we get:

$$
\begin{aligned}
&(1-\beta\varpi)\left(\hat{\lambda}_t + \frac{1}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) + \beta\varpi\left(\frac{1}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\gamma}{1-\varkappa}\hat{\pi}_t - \frac{\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}\right) \\
= \;& (1-\beta\varpi)\left(\hat{\lambda}_t + \widehat{mc}_t + \frac{\varkappa}{1-\varkappa}\widehat{\tilde{p}}_t + \hat{Y}_t\right) \\
&+\beta\varpi\left(\frac{\varkappa}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right) + \frac{\varkappa\gamma}{1-\varkappa}\hat{\pi}_t + \frac{1-2\varkappa}{1-\varkappa}\hat{\pi}_{t+1} + \hat{x}_{t+1}^2\right)
\end{aligned}
$$

Simplifies to:

$$
\begin{aligned}
&\left[(1-\beta\varpi)\frac{1}{1-\varkappa} - (1-\beta\varpi)\frac{\varkappa}{1-\varkappa}\right]\widehat{\tilde{p}}_t - (1-\beta\varpi)\widehat{mc}_t + \beta\varpi\left(\frac{1}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right)\right) \\
&-\beta\varpi\left(\frac{\varkappa}{1-\varkappa}\left(\widehat{\tilde{p}}_t - \widehat{\tilde{p}}_{t+1}\right)\right) + \beta\varpi\left(\frac{\gamma}{1-\varkappa}\hat{\pi}_t - \frac{\varkappa}{1-\varkappa}\hat{\pi}_{t+1}\right) - \beta\varpi\left(\frac{\varkappa\gamma}{1-\varkappa}\hat{\pi}_t + \frac{1-2\varkappa}{1-\varkappa}\hat{\pi}_{t+1}\right) \\
= \;& 0
\end{aligned}
$$

Simplifying further we get:

$$(1-\beta\varpi)\,\widehat{\tilde{p}}_t-(1-\beta\varpi)\,\widehat{mc}_t+\beta\varpi\left(\widehat{\tilde{p}}_t-\widehat{\tilde{p}}_{t+1}\right)+\beta\varpi\left[\gamma\hat{\pi}_t-\hat{\pi}_{t+1}\right]=0$$

So now I use equation $\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_t-\gamma\hat{\pi}_{t-1}\right)=\hat{p}_t$

$$\begin{aligned}
&(1-\beta\varpi)\,\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_t-\gamma\hat{\pi}_{t-1}\right)-(1-\beta\varpi)\,\widehat{mc}_t\\
&+\beta\varpi\left(\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_t-\gamma\hat{\pi}_{t-1}\right)-\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_{t+1}-\gamma\hat{\pi}_t\right)\right)+\beta\varpi\left[\gamma\hat{\pi}_t-\hat{\pi}_{t+1}\right]\\
=\;&0
\end{aligned}$$

$$\begin{aligned}
&\frac{(1-\beta\varpi)\,\varpi}{(1-\varpi)}\left(\hat{\pi}_t-\gamma\hat{\pi}_{t-1}\right)-(1-\beta\varpi)\,\widehat{mc}_t\\
&+\beta\varpi\left(\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_t-\gamma\hat{\pi}_{t-1}\right)-\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_{t+1}-\gamma\hat{\pi}_t\right)\right)+\beta\varpi\left(\gamma\hat{\pi}_t-\hat{\pi}_{t+1}\right)\\
=\;&0
\end{aligned}$$

divide both sides by $\beta\varpi$

$$\begin{aligned}
&\frac{(1-\beta\varpi)}{(1-\varpi)\,\beta}\hat{\pi}_t-\gamma\frac{(1-\beta\varpi)}{(1-\varpi)\,\beta}\hat{\pi}_{t-1}+\frac{\varpi}{(1-\varpi)}\hat{\pi}_t\\
&-\gamma\frac{\varpi}{(1-\varpi)}\hat{\pi}_{t-1}-\frac{\varpi}{(1-\varpi)}\left(\hat{\pi}_{t+1}-\gamma\hat{\pi}_t\right)+\gamma\hat{\pi}_t-\hat{\pi}_{t+1}\\
=\;&\frac{(1-\beta\varpi)}{\beta\varpi}\widehat{mc}_t
\end{aligned}$$

$$\frac{(1-\beta\varpi)}{(1-\varpi)\,\beta}\hat{\pi}_t+\frac{\varpi}{(1-\varpi)}\hat{\pi}_t+\frac{\varpi}{(1-\varpi)}\gamma\hat{\pi}_t+\gamma\hat{\pi}_t=\frac{1}{1-\varpi}\hat{\pi}_{t+1}+\frac{\gamma}{(1-\varpi)\,\beta}\hat{\pi}_{t-1}+\frac{(1-\beta\varpi)}{\beta\varpi}\widehat{mc}_t$$

$$\left[\frac{(1-\beta\varpi)}{(1-\varpi)\beta}+\frac{\varpi}{(1-\varpi)}+\frac{\varpi\gamma}{(1-\varpi)}+\gamma\right]\hat{\pi}_t=\frac{1}{1-\varpi}\hat{\pi}_{t+1}+\frac{\gamma}{(1-\varpi)\beta}\hat{\pi}_{t-1}+\frac{(1-\beta\varpi)}{\beta\varpi}\widehat{mc}_t$$

working on: $\left[\frac{(1-\beta\varpi)}{(1-\varpi)\beta}+\frac{\varpi}{(1-\varpi)}+\frac{\varpi\gamma}{(1-\varpi)}+\gamma\right]$

$$\begin{aligned}&\frac{(1-\beta\varpi)}{(1-\varpi)\beta}+\frac{\varpi}{(1-\varpi)}+\frac{\varpi\gamma}{(1-\varpi)}+\gamma\\ =\ &\frac{(1-\beta\varpi)}{(1-\varpi)\beta}+\frac{\varpi\beta}{(1-\varpi)\beta}+\frac{\varpi\gamma\beta}{(1-\varpi)\beta}+\frac{\gamma(1-\varpi)\beta}{(1-\varpi)\beta}=\frac{1+\gamma\beta}{(1-\varpi)\beta}\end{aligned}$$

Therefore:

$$\frac{1+\gamma\beta}{(1-\varpi)\beta}\hat{\pi}_t=\frac{1}{1-\varpi}\hat{\pi}_{t+1}+\frac{\gamma}{(1-\varpi)\beta}\hat{\pi}_{t-1}+\frac{(1-\beta\varpi)}{\beta\varpi}\widehat{mc}_t$$

$$\hat{\pi}_t=\frac{(1-\varpi)\beta}{1+\gamma\beta}\frac{1}{1-\varpi}\hat{\pi}_{t+1}+\frac{(1-\varpi)\beta}{1+\gamma\beta}\frac{\gamma}{(1-\varpi)\beta}\hat{\pi}_{t-1}+\frac{(1-\varpi)\beta}{1+\gamma\beta}\frac{(1-\beta\varpi)}{\beta\varpi}\widehat{mc}_t$$

$$\hat{\pi}_t=\frac{\beta}{1+\gamma\beta}\hat{\pi}_{t+1}+\frac{\gamma}{1+\gamma\beta}\hat{\pi}_{t-1}+\frac{(1-\varpi)(1-\beta\varpi)}{(1+\gamma\beta)\varpi}\widehat{mc}_t$$

4.3 References

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