

**UNIVERSITY OF HULL**

**A NONPARAMETRIC APPROACH TO PRODUCTIVE  
EFFICIENCY MEASUREMENT: AN APPLICATION OF  
BOOTSTRAP DEA TO GOLD MINING**

being a dissertation submitted in partial fulfilment of the

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The responsibility for any errors in this dissertation remains mine.



## **DEDICATION**

This dissertation is dedicated to the memory of my father, Custon William Mutemererwa, who died on the 23<sup>rd</sup> April 2004.

## **ABSTRACT**

In this dissertation the technical efficiency in gold mining is investigated. To the best available knowledge, this is the first such study on gold mining, whether on a localised (one country) or for a cross-section of countries. Since the work by Farrell (1957), much work has been done using nonparametric methods such as DEA. Although extensions in DEA technique, such as bootstrapping have been available for some time, their use has been limited in comparison with the number of overall DEA studies carried out. In this dissertation both DEA and bootstrap DEA are applied to two gold mining cross sectional samples, one on Zimbabwe consisting of thirty-four mines, and the an international one which also included some Zimbabwean mines which comprise fifty-nine observations.

The main reason for carrying out the study is an interest in gold mining in general and its importance to Zimbabwe in particular. As will be noted in Chapter 2, the economic development of Zimbabwe has been linked, to a varying extent over the ages, to its growth of the gold mining sector.

The results of the dissertation provide some useful insights into the relative performances of gold mines and also some characteristics of the Zimbabwean gold mining sector. The main results indicate that gold mining is characterised mainly by technical efficiency dominating scale efficiency. This is particular relevant when the Zimbabwean mines are compared with their international counterparts. Zimbabwean mines are found to be relatively technically efficient but less so when overall efficiency

is considered. In fact they have the lowest overall efficiency scores in the international sample. The results also indicate that mines from the so-called developed mining economies, Australia, Canada, the US and South Africa are the benchmarks in terms of optimal operations. It is mines from these countries which define the overall efficiency frontier.

The results of both the samples highlight potential shortcomings in applying DEA and bootstrap extension to gold mining, both for single country and for cross-country cases. Additionally, there are possibilities, with adequate data, of relating country-specific characteristics to differences in overall efficiency among countries.

Finally there are indications that including mineralogical factors such as the recovery rate in the production technology has an effect on technical efficiency. Mines with low recovery rates tend to exhibit comparatively higher technical efficiency. The study does have some limitations, mainly because of lack of data. In particular, there were problems in coming with attributing the contribution of capital services to efficiency with the result that a different measure for the flow of capital services is used for each sample. In addition, the two samples are for different time periods. This limits comparative analysis.

## **CHAPTER 1: INTRODUCTION AND BACKGROUND TO THE STUDY**

### **1.1 Introduction**

The history of mining and that of Zimbabwe are inseparably linked. It was the lure of gold fuelled by the amazing tales of King Solomon's mines that attracted British settlers to Zimbabwe in the last part of the 19<sup>th</sup> century. Well before then, however, mining was already playing an important part in Zimbabwe cementing its interaction with the outside world. Ancient workings, some of which have been dated to as far back as the 2<sup>nd</sup> century (Jourdan, 1990), are scattered across the Zimbabwean landscape-- in fact in 21<sup>st</sup> century Zimbabwe, well over half of the major towns started off either as mining settlements or their development into urban centres largely depended on mining. The most important mining activity has been gold mining.

### **1.2 The Research Question and its Importance**

This dissertation is primarily motivated by the developments in the literature on productive efficiency particularly the use of frontier methods of efficiency estimation and the lack of application of these developments to gold mining, a subject of considerable importance to the Zimbabwean economy. As Chapter 3 makes clear, the behavioural assumptions of an eternally optimising enterprise have not been backed by any supporting evidence in empirical studies. Given this situation where there are divergences between the stated objectives and outcomes of the productive enterprises, there are some important questions to consider. How are the objectives measured and

deviations from achieving them assessed? How far from the optimum are the observed enterprises? What measures can be taken to achieve or approach this optimum and to address any identified performance shortcomings?

As Chapter 2 will reveal, gold mining is a vital cog in the wheel that is the Zimbabwean economy and has been for a long time. It is fully integrated into the global economy, with the final market of its product mainly being the London Metal Exchange (LME). It is also integrated from the inputs side in that a large proportion of its inputs, such as capital and chemicals are purchased from outside Zimbabwe. Mining is also a central plank in the transfer and adoption of new technology, either through new skills and training or acquisition of new equipment. Hence, when new plant and equipment is imported, technical skill are transferred through training at various levels in the maintenance and operation of the machinery. Gold mining is also a key export industry which has played and continues to play a large part in the generation of foreign currency, an important function in developing countries which heavily depend on external sources for capital and intermediate goods. Mining has increasingly come under scrutiny, particularly with respect to environmental impact. Given that compliance with increasingly stricter regulations will imply higher costs, improved performance and efficiency will increasingly become very important.<sup>1</sup>

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<sup>1</sup> Even in countries with weak regulatory parameters, multinational companies such as Rio Tinto, Anglo American etc. impose conditions prevailing in their home countries.

There are two primary objectives in carrying out this study, both of which are relevant to the mining of gold in Zimbabwe. The first is to estimate and analyse the productive efficiency of gold mining in Zimbabwe. The second is to analyse the same performance but this time in the context of a global sample. The first part of the study looks at a sample of Zimbabwean mines. The aim of the second approach is to compare the performance of Zimbabwean gold mines to gold mines from other countries. Both these analyses are carried out using non-parametric methods.

There are therefore two contexts to this study. The first uses a sample of anonymous gold mines from Zimbabwe from a data set extracted from the Zimbabwean Census of Production. The second is an international sample which also includes some mines from Zimbabwe. This sample is extracted from a commercially available database covering all facets of international mining, of which gold is only a part.

### **1.3 Methodology**

Given the importance of gold to Zimbabwe, there are grounds for asking several questions regarding the performance and characteristics of the gold mining sector. How do gold mines in Zimbabwe compare with those in other countries? Are there any significant differences in performances in comparison to countries with similar geology? Are there any differences with countries at same stage of economic development and which have experienced similar political and economic shocks?

There are many ways of answering these questions which range from partial measures such as labour productivity, see for example Tilton (2001) and total measures such the Malmquist productivity index or other total factor productivity measures. Recently, estimates of efficiency have increasingly been used to compare the performance of productive enterprises. Efficiency estimates have also been used to estimate a frontier of maximum attainable output for a given vector of inputs or a vector of minimum feasible inputs for a given output. In addition, efficiency estimates have also been used to identify best-practice firms which may then used as references by the inefficient ones.

As a basis for measuring efficiency and benchmarking the performance of gold mines, the non-parametric method of data envelopment analysis (DEA) will be employed. The adoption of this approach is motivated by two primary reasons. The first is the type of data available which dictates that DEA is used. DEA allows the estimation of a performance frontier and not the strictly conventional models, that is, non-frontier production models. The second is the novel application of the bootstrap in the estimation of operational efficiency. Despite its obvious appeal, there has been some reticence in its use in applied work. Some of this may possibly be a result of the computer resources required to implement the technique. However, this is less problematic given the significant falls in computing costs over the last twenty years. Another reason may be that some researchers are still to be convinced about the merits of the bootstrap (Cooper et al, 2000). Finally, there have been some controversial exchanges among researchers on how properly to implement the bootstrap. These will be discussed in Chapter 3.

Notwithstanding, the relatively slow up-take in the use of the bootstrap, there have been some innovative studies which will form the basis for this study. These are the works by Førsund et al (2006), Boame (2004) and Simar & Wilson (1998) for example.

The key methodological contribution of the bootstrap is the ability to correct for identified shortcomings in nonparametric analysis. This allows the approximation of statistical properties of what have hitherto been classified as deterministic results. In this study, there is also an attempt at looking at the applicability and problems associated with applying the bootstrap DEA to gold mining.

#### **1.4 Structure of the Study**

This study is divided into six chapters. After this introductory chapter, the study is arranged as follows: Chapter 2, consisting of two parts, gives a background to gold mining. The first part is a general description of gold mining to place the subsequent empirical work in context. The second traces the historical development of mining in Zimbabwe and highlights the major events that have taken place in making Zimbabwe a major gold producer. The problems faced and successes achieved are justify a study into the performance of the gold mining sector is analysed.

Chapter 3 examines the theory behind the study. An investigation of the both parametric and nonparametric estimation of efficiency is done. More attention is paid to the nonparametric framework which is applied here. In addition, the nonparametric enhancement though the bootstrap will be outlined and the context in which it has



hitherto been used is also investigated. Finally, a review of some previous nonparametric studies of efficiency is carried out. The focus will be on those studies which focus on DEA and mining and those that have applied the bootstrap and bias-correction to DEA. These studies will help address some data measurement and methodological problems encountered in this study.

In Chapter 4, an analysis of productive efficiency in Zimbabwean gold mining is done. The results allow the construction of a production frontier for a sample of Zimbabwean gold mines. Technical efficiency is also decomposed into its main components, these being technical efficiency and scale efficiency. The importance of each in explaining the efficiency of each mine is then analysed and general conclusions on the characteristics of the Zimbabwean gold mining industry are inferred such as the scope for scale adjustment and input savings. Finally, a test of whether the observed differences in point estimates are statistically significant is then done.

In Chapter 5, an analysis of the performance of a world-wide sample of gold mines is carried out with a view to establishing the place of Zimbabwean mines in a global context. Of interest will be which countries provide reference mines (the most efficient mines which are used in the construction of an estimated efficiency frontier). As in Chapter 4, a test of whether the observed differences in point estimates are significant is also done.

Finally in Chapter 6, some conclusions are made. Policy recommendations and directions for further research, based on perceived weaknesses in the current study, are identified.

## **1.5 Gold: A Brief History**

Gold mining is probably the most global of all mining activities, with a history stretching to ancient times and spanning across the world. Today gold metal itself is one of the most important commodities traded. Its physical (malleable and ductile) and chemical (extremely stable and resistant to most acid attacks) properties give it very diverse range of uses. These extend from the fiduciary—it is the commodity of choice for storing wealth in many societies, both advanced and less advanced—to the industrial where its physical and chemical properties are important in microelectronics and medical applications. It is the best conductor of electricity and heat and is an excellent reflector of colour. Visually, the lustrous yellow colour is rather pleasing to the eye which is an added attraction. As a result of some of these characteristics, gold has been the basis of trade since the early Egyptian and Mesopotamian civilisations and has served as a store of wealth and value for a long time. Gold has also been the source of much conflict among nations, and the basis upon which many nations, such as large parts of Africa, Latin America and the Western parts of the United States of America were forged.

In terms of historical output, the World Gold Council estimates that 125 000 metric tonnes of gold have been mined over the last six thousand years, 90% of them after

1848 after the Californian gold rush (World Gold Council, 2006). The earliest recorded mining took place in ancient in Egypt around 2000 B.C. At that time, the total world output rate was nothing more than one tonne per annum. However, from about the beginning of the 15th century, West African gold (mostly Ghana, Guinea and Mali) rose to prominence adding between five and eight tonnes annually to world output. South and Central American production was added in the 16th century. The exploitation of Russian gold and the results of the American gold rush in the 18th and 19th centuries were the quantum leaps, with world output rising to a rate of about one hundred tonnes per year. By the middle of the 19th century, this had surged to about 300 tonnes per year (World Gold Council, 2006). Australia and South African production in the late 19th century and early 20th century gave total output a further boost (World Gold Council, 2006).

The last dramatic change came in the 1980s when global output rose to an all-time peak of about 950 tonnes per annum, mainly triggered by prices of over US\$800 per ounce (World Gold Council, 2006). New technologies in exploration, mining and mineral processing added to the boom as many old and abandoned workings and other occurrences, which had hitherto been regarded as uneconomic or physically difficult to access, began to be exploited.

Today gold trading is conducted in a fiercely competitive market with a diverse range of participants from gold mines, central banks, private financial institutions, other investors and private individuals. Therefore, gold mines need to adopt clear strategies to

improve efficiency and performance in the face of these challenges. The importance of adopting these strategies can best be illustrated by looking at the effect of two notable events in Europe, the fall of the Soviet Union and the formation of the European Monetary Union (EMU). The former was followed by sudden, large-scale flows of gold onto the world market. In addition as result of the latter, European central banks, most of them seeking to comply with the provisions of European Monetary Union or simply wishing to change the composition of their portfolio of reserves, were selling significant amounts of their gold reserves. The effect was as sudden as it was dramatic; the price of gold was forced so far down, from about US\$330 to fluctuate about US\$200 in a period of six months, a level at which it stayed for about two years (Mining Journal, 1988: 1994). As a consequence, a significant number of gold mining operations worldwide found themselves to be economically unviable. By the early to mid-1990s, most were put on “care and maintenance” or simply shut down. To maintain competitiveness gold miners were forced to adopt a variety of measures which included cost cutting and more efficient use of resources and these with varying degrees of success (Mining Annual Review, 1996).

## **1.6 The Geology of Gold**

Gold is abundantly available in the earth’s crust but not always in sufficiently great concentrations, the average is 15milligrammes/tonne, for feasible extraction. However, there are occurrences of rich concentrations throughout the world which support economic exploitation. The formation of these concentrations, also called deposits,

mainly took place over long periods of time, typically millions of years. The gold which is close to the surface and, therefore, amenable to mining was formed from complex ions in molten salts which are saturated in gold, sulphur, iron, silicon etc. These molten salts migrated from huge depths through fissures and cracks in rocks, cooling and crystallising as it entered the earth's crust. Most of the cooling took place in quartz and in other complex rock formations containing sulphur, lead, zinc, platinum etc. Hence a proportion of gold mining is often associated with these other minerals. The mineralogy of gold, i.e. physical association of gold with other minerals is, therefore, quite often complex as it often occurs in a wide variety of forms and associated with many other minerals most of which are deleterious in the process of processing and purification of gold.

The commonest types of gold deposits occur in quartz-containing rocks. Gold ore can be found as disseminated and irregular particle scales, plates, veinlets or large reticulated and spongy masses. Although crystalline in nature, it rarely occurs as crystals (which are easier to process) as these require special conditions to form. There are two main types of gold deposits, vein and reef, and placer and alluvial.

Vein and reef gold occurs in granites ( a type of quartz), such as those found in the United States, Canada, Australia and Southern Africa, and in volcanic rocks, such as those found in the rest of East and West Africa, South America and South-East Asia. Reef deposits are normally deep-seated and are extracted through sophisticated mining methods. Placer and alluvial deposits resulted mainly from chemical weathering and

subsequent erosion and exposure of the gold-containing rocks. This process caused the heavy gold to become embedded in streambeds and the soils in any physical depression. Alluvial and placer deposits were the major reason for the gold rushes in the western part of the United States, South Africa and Australia. Alluvial gold, by virtue of being on the surface or just a few metres below it, was the most easily exploitable with most of it consisting of nuggets of almost pure gold. Earlier methods of extraction included panning or other gravimetric separation techniques with very little use of chemicals to separate gold from waste or gangue minerals.

### **1.7 The Gold Production Process**

There are three main stages involved in the physical production of gold. These are (a) exploration to locate economically-exploitable concentrations of gold, (b) mining where the ore is extracted from the host rock through various means such as drilling and blasting, and (c) mineral processing where the gold metal is separated from other gangue and waste materials with which it is naturally associated. The latter two stages are the relevant ones for the purposes of this study and are examined in some detail here.

Gold deposits and concentrations are found by a variety of exploration methods, from eagle-eyed prospectors (even today) and sophisticated geophysical and geochemical methods using satellite imaging etc. Once an anomalous concentration of gold is detected, core-drilling of rocks and soils and assaying of the drilled cores are the main methods of establishing the dimensions and quality of the occurrences. Once a

sufficiently rich deposit is delineated, construction of the mine infrastructure follows and access to the ore is by two main methods, open pit for shallow deposit or shafts for deep-seated ones. The mining of gold takes place by finding the most economic and easily accessible routes to the ore and these largely depend on the type of deposit. Sometimes a combination of the two is used, especially where the deposit starts from shallow levels and continues to greater depths.

Shallow-lying deposits are mainly mined by open-pit and often are heavily mechanised operations. Here the ore is exposed by removing the top “overburden” and waste layer of rock and soil. Another method, usually used in sandy deposits which are on the surface, is to use high-pressure water to free the gold from the sand or soils in which it is trapped. Deep-lying deposits, on the other hand, are usually accessed by sinking vertical or inclined shafts, normally to levels below the first layer of the ore and “driving” into the deposit. Underground mining requires more sophisticated technology to reach and extract the ore as many physical hazards such rock fall are encountered. Some of the shafts are exclusively used for ventilation, for transporting men and supplies or for transporting ore to the surface. The capacity of the mine is measured by the mass of rock material mined and moved to the surface for processing and is normally indicated by tonnes per year of ore. This capacity is determined by various factors, such as the rock strength, which affect the height and width of the horizontal accesses and also how easily ventilated the mines can be. All these variables are factored in when the mine is modelled and planned. In terms of getting the ore, the main method is by breaking up the host rock by drilling and embedding explosives into the

drilled holes. The inputs used in this process are labour (of which there are different types), trucks, earth-moving equipment and drilling machines and explosives. In terms of labour, blasting and the supervision of drilling the holes into which the explosives are inserted is only carried out by a blasting licence holder while the actual mining is only done under the supervision of someone with a formal mining engineering background.

The broken ore is raised to the surface where is sorted according to its grade. Low grade ore is normally roughly broken up and placed on lined pads. A dilute cyanide solution is poured over the surface of the broken ore. The slowly dissipating fluid dissolves the gold as it percolates down to the pads at the bottom. The pregnant (in gold) solution is then collected and taken for further processing.

High grade ore is separated into oxide, which is easier to process, and sulphide and carbonate ores (also known as refractory ore) from which it is relatively more difficult to remove the gold. The ores are then separately ground to micro-fineness. The main equipment used here are crushers and mills and these, more than anything on the mine, provide the physical constraint on how much gold ore is processed into gold<sup>2</sup>. Hence a mine cannot physically process what the mill is unable process. Oxide ore is directly sent to a “leach plant”, where, again, a weak cyanide solution percolates through the heaped ore trapping the gold. The carbonate and sulphide ores are fed into a furnace and

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<sup>2</sup> In some mines, the primary crushing is carried out underground. This however requires significant investment in ventilation infrastructure which, in some cases, may outweigh that of environmental remediation on the surface.



roasted, driving out the carbon and sulphur as gaseous oxides and then onto the leach pad.

The gold-containing complex solution is collected and sent to a cyanide leach plant. In here, the cyanide solution is passed through a chemical called activated carbon which traps the gold and lets the cyanide through. The free cyanide is then collected and recycled for re-use. The activated carbon, now laden with gold is then chemically separated from the gold, either by chemical substitution or electrolytically. As with the cyanide, the activated carbon is collected after this for re-use. Finally, the almost-pure gold is melted into bars, known as “dore bars”. These bars are finally sent to a refinery, where they are refined to 99.99 per cent purity which is what is traded on the market. There is some trade in dore gold which also takes place but it is a rather insignificant segment of the market.

The preceding discussion outlined the history, basic geology and production techniques of gold. It can be seen that the main inputs in gold mining are capital equipment, such as earthmovers, hoists, drills, crushers and mills, labour, energy and explosives. Also included is labour, both skilled (geologists and mining and metallurgical engineers) and general, such as plant operators. In mineral processing, ore, energy and chemical reagents are the main inputs. The quality of the ore, as measured by the grade of gold (normally grammes per tonne of ore) is also an important consideration in whether or not exploitation takes place. The amount of materials, that is, the reagents used depends on the grade and the mineralogical quality of the ore, the refractory ores requiring more

of the resources. The capital equipment, by extension, the physical amount of ore which can be handled by the mills and concentrators is normally fixed in the short term, constraining the amount of ore which can be raised to the surface.

## CHAPTER 2 GOLD MINING AND ZIMBABWE<sup>3</sup>

### 2.0 Introduction

This chapter has one major purpose. It will outline and emphasise the importance of gold to the economy of Zimbabwe and motivate the need to investigate the performance of this vital economic sector, first on its own and then in relation to other gold mines in the rest of the world. To put gold mining in its proper economic perspective, a historical account of the development of gold mining and its importance to the Zimbabwean economy is given. The main thread running through the whole study, therefore, is an analysis of the performance of Zimbabwean gold mining.

This chapter is divided as follows; after this introduction, Section 2.1 gives a general introduction to the gold mining in Zimbabwe. Section 2.2 deals with the pre-1923 period when the British South Africa Company (the ‘Company’) ran the colony. Section 2.3 deals with the period from 1923 when British government took over administration from the Company and gave Rhodesia<sup>4</sup> limited home rule, though in practice this was more like self-government. This period lasted until 1965. This was when, in a bid to

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<sup>3</sup> Viewing et al (1987) wrote the definitive history of mining in Zimbabwe and hence is repeatedly cited in this Chapter.

<sup>4</sup> Up until 1980, Zimbabwe was known as Rhodesia, derived from the name of the man given the royal charter by Queen Victoria to colonise it.

thwart the granting of political and economic concessions to black Rhodesians, the government made a Unilateral Declaration of Independence (UDI) from Britain. This UDI period, which lasted until Britain finally granted independence to Zimbabwe in 1980, is dealt with in Section 2.4. Section 2.5 covers the post-independence (i.e. after 1980) Zimbabwe. Finally Section 2.6 briefly discusses the post-independence mining policy.

## **2.1 Gold Mining in Zimbabwe**

This section describes the developments in the Zimbabwean gold mining industry and highlights the major milestones. The objective is to explore the development of gold mining in Zimbabwe and highlight the importance which gold has played in making Zimbabwe what it is now. The period covered is from the 1890s when fully commercial mining started until the present day. This period can be roughly divided into four stages, each corresponding roughly to a political or economic epoch.

Most of the modern mines in Zimbabwe are not the result of sophisticated and modern geological investigations. On the contrary, they are mostly located on the sites of the many ancient workings which are scattered across the landscape of Zimbabwe.

Archaeological investigations have shown that by the end of the 19<sup>th</sup> century, there were more than four thousand old mine workings which also included some workings of iron and copper (Viewing et al, 1987). Early European explorers, such as David Livingstone, reported a large number of abandoned ancient gold workings in what is now Zimbabwe. The abundance of these mines is a reflection of the both the widespread nature of the

mining activity and also the limited mining technology then possessed by the natives. The biggest technological constraints being the lack of water pumping facilities and underground roof support. There also were problems associated with processing the material, as exemplified by the abandonment of mines after the 'easy-to-process' oxide layers had been exhausted leaving behind significant quantities of refractory gold ore. Most of the mines which have become major sources of the country's wealth have developed from these ancient workings.

Evidence of mining in Zimbabwe can be traced to the beginning of the last millennium. By the 11<sup>th</sup> century Arab traders, operating between Africa and India, were purchasing gold from Zimbabwe. In the 16<sup>th</sup> century, the *Munhumutapa*, a feudal *Shona* kingdom, granted mining concessions to the Portuguese in northern Zimbabwe and parts of Mozambique, which was also part of the empire (Jourdan, 1990). This led to a trading association that lasted until the *Munhumutapa* kingdom split into two at the end of the 17<sup>th</sup> century, mainly as a result of repeated Portuguese military interventions.

The Portuguese then installed their own puppet emperor in the north of present day Zimbabwe<sup>5</sup> while in the south another kingdom known as the *Changamire* was set up. This uneasy coexistence only lasted for a short period, however. The *Changamire* invaded and routed the Portuguese and their puppets and installed their own subservient fiefdom. Their reign lasted until 1840s when other invaders, mostly the *Ndebele*, fleeing firstly the *Zulus* and, later, the *Boer* expansion (both of who were under pressure from

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<sup>5</sup> Rhodesia, the former name, and Zimbabwe are going to be interchangeably used here.

the British further to the south) overran the *Changamire* kingdom. The *Ndebele* then set up their own empire with its capital at present-day Bulawayo in the west of Zimbabwe (Jourdan, 1990).

In the 1880s, the potential for a huge gold find, particularly the perceived presence of a major reef similar to the Witwatersrand<sup>6</sup>, the 'Rand', led the British mining magnate, Cecil John Rhodes, to seek and obtain mineral concession from the *Ndebele* King, Lobengula (Parsons, 1983). In 1890, the British themselves arrived in Zimbabwe. Despite possessing nothing comparable to a second Rand, the mining industry in Zimbabwe has developed to become one of the most diversified in Africa, with over forty minerals currently being extracted. It is particularly the combination of agriculture and mining that has been the engine of Zimbabwe's economic development for the best part of the last century. At the centre of all this has been gold mining, the most common of all mining activities. In crises, Zimbabwe has mainly paid its way out of trouble using gold. As Figure 2.1 illustrates, gold mining takes palace throughout Zimbabwe.

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<sup>6</sup> Witwatersrand, literally white waters' reef, from which South Africa got the name of its currency the 'Rand', is a geological complex surrounding Johannesburg. It is the location of the largest gold mines in the world and has been the source of most of South Africa's gold riches.



- Provinces**
- 1 Masvingo
  - 2 Manicaland
  - 3 Mashonaland East
  - 4 Mashonaland Central
  - 5 Mashonaland West
  - 6 Midlands
  - 7 Matabeleland North
  - 8 Matabeleland South

**Explanation**

Industrial Mineral Deposits			Base Mineral Deposits		Principal Gold Mines
A Asbestos	F Feldspar	Li Limestone	Be Beryl	Pb Lead	G Gold
Al Bauxite	Fl Fluorspar	M Mica	Co Cobalt	Pt Platinoids	
Ba Barytes	P Phosphate	Mg Magnesite	Cr Chrome	Sn Tin	
Ca Calcite	Ka Kaolin	Si Silica	Cu Copper	Ta Tantalite	
Do Dolomite	Ky Kyanite	T Talc	Fe Iron	W Tungsten	
E Corundum	Li Lithium	V Vermiculite	Ni Nickel	Zn Zinc	
C Coal	Cl Clay	Gp Graphite			
Ma Marble	Di Diatomite	Py Pyrite			

Mineral resources of Zimbabwe  
(source, Geological Survey of Zimbabwe)

**Figure 2.1: Map of Zimbabwe**

The location of the present-day towns has also been largely determined by the presence of mining settlements. Harare, the capital city, has the Arcturus and Mazowe gold mining belts around it, see Figure 1. These mines, in turn are in the middle of one of the largest agricultural zones in Zimbabwe, to the north and east of Harare. Further, to the north of Harare lies another major town, Bindura. This town was largely founded on gold mines, the most famous one being Kimberley. Today the largest gold mine in Zimbabwe, Freda-Rebecca is located in Bindura town, again on the site of an ancient working.

In addition to gold mining, nickel mining and refining at Trojan mine, just outside Bindura, is the other pillar upon which the town rests. Just to the east of Bindura lies another gold mining centre, Shamva. Both Shamva and Bindura are also major agricultural centres, again highlighting the significant linkages between mining and other productive sectors such agriculture. The situation is similar for Bulawayo, the second city, which is surrounded by significant gold belts. In the centre of the country, in the Midlands province, the Kwekwe gold belt has been the source of most of the gold produced in the country since the 19<sup>th</sup> century. During crises, Zimbabwe has mainly paid its way out of trouble using gold. All this illustrates the lasting impact that mining, particularly of gold, has left on the economic landscape of Zimbabwe.

## **2.2 Zimbabwean Gold Mining 1890 to 1923: The Charter Company Years**

In 1890, Cecil John Rhodes' British South Africa Company (BSAC), also known as the 'Company', sent settlers into Zimbabwe. A Royal Charter, raised capital from the



London Stock Exchange (LSE) and a special concession, mentioned earlier, by Lobengula<sup>7</sup>, were the legal basis for this venture. This group of settlers was also known as the 'pioneer column'. Having heard stories of the fabulous riches of King Solomon's mines, they were in no doubt of the existence of a second Rand and they intended to secure it for themselves and for the crown (Jourdan, 1990).

Initially, the settlers carefully avoided settlements inhabited by the more war-like *Ndebele* populations, setting up settlements only where the more peaceful *Shona* lived. They took over huge swathes of land, driving the natives away in the process. When nothing remotely approaching a second Rand materialised, the settlers then started encroaching on and eventually laying claims to the *Ndebele* part of the country as well. The expropriation of land provoked bitter resentment among both *Shona* and *Ndebele*. In 1896, for the first time, these two erstwhile enemies joined hands and rebelled against the settlers, almost wiping them out, such was the unexpected nature and ferocity of the uprising. The rebellion was ruthlessly put down in 1897 with some help from South Africa and with it the colonisation of Zimbabwe was completed (Jourdan, 1990).

In anticipation of huge gold finds and mindful of the large amounts of London Stock Exchange (LSE) money it had spent in getting to Zimbabwe, the Company had

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<sup>7</sup> This was known as the Rudd Concession after the man who led the negotiations with Lobengula.

stipulated a 50 per cent free-carried equity in all mining<sup>8</sup>. This was a form of state participation since the Company was not only the administrative authority but owned all the mineral rights as well. Prospecting work was made open except in areas that were already developed such as farms and other settlements.

To encourage more prospecting activity, from 1903 small-scale mining operations (those raising less than 750 tonnes of ore per month) were allowed to be run completely privately. The Company would, instead, recoup its money from royalties on profits, using a sliding scale. For the larger mines, the free-carried equity was reduced to 30 per cent (Viewing et al, 1987). These changes were reinforced by further amendments to the regulations allowing most of the mines to pay a royalty on the gold produced (rather than the amount of ore raised). Again this was on a graduated scale with the small-scale mines (those producing up to £100 worth of gold per month) paying nothing at all while those producing up to £3 000 per month paid a royalty of 2½ per cent. Richer mines, grading about 32 grammes per tonne (g/t) paid a royalty of 7½ per cent (Viewing et al, 1987).

The second Rand remained elusive, however. As an illustration of the huge disparity between what they had found in Zimbabwe and what was happening in South Africa, in 1907 the 11 most profitable mines around Johannesburg had gross revenues of £7

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<sup>8</sup> The company thus owned 50 per cent of the mine, without having invested any money into the operations.

million<sup>9</sup>. In contrast the 11 largest mines in Zimbabwe only grossed £614 000 (Arrighi, 1973). By the early part of the 20<sup>th</sup> century, the Company had finally given up hope of discovering a second Rand. In fact, 1903 has been identified as the year when the speculative bubble in Rhodesian mining finally burst, with the primary consideration being put to the profitability of the mines rather than simply working them. In 1906, the Company started advancing loans to prospective miners. The success of this action resulted in it being made policy in 1912 (Viewing et al, 1987). This loan facility was an important feature in the development of mining during this period as it made working capital available to good prospects, especially for the purchase of consumables and the payment of wages.

By the end of 1908, there were around 250 gold mines, admittedly 175 of which were only small workings (Viewing et al, 1987.). More importantly, the emphasis was no longer on gold mining alone but other minerals as well. In 1904, Wankie Colliery was set up to exploit the vast coal reserves in the north-west tip of Zimbabwe (see Hwange on the map). In 1916, asbestos and chromite mining started in the Midlands province (Viewing, et al., 1987). The two towns of Zvishavane and Shurugwi are examples of cases where mining was the main factor in the development of centres of economic activity. Both owe their existence to these two minerals; asbestos and a large number of small and medium gold deposits for the former and chrome ore for the latter. The

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<sup>9</sup> Until 1971, the Rhodesian pound, later the dollar was on parity with sterling. Thereafter it was managed by the Reserve Bank according to a basket of currencies.

expansion of mining had a multiplier effect on the economy stimulating the agricultural sector as most of the food was obtained from there.

Gradually mining became more formalised with support services being established. In 1910, the Geological Survey (GS) was set up to provide free geological services to miners. The GS also started a systematic geological mapping exercise, resulting in the production of geological maps (such as Figure 2.1) and reports. The introduction of mining and safety regulations followed soon after.

Until the 1940s, gold mining was the most important economic activity in Zimbabwe. This was reflected in the priority given to mining discoveries over all other activities, a priority that continues to the present day. Hence, even in a prime farming area, once a mineral discovery was made, land had to be set aside to accommodate the development of the mine (with appropriate compensation being paid for the disruptions).

### **2.3 Zimbabwean Gold Mining 1923 to 1965: The Self-government Years**

In 1923, the Company's charter expired. The country became a Crown Colony again, although the mineral rights and most of the land still belonged to the Company. In reality, however, the country was a self-governing colony. This is a crucial point in that this status allowed Zimbabwe to develop in a manner different from other African colonies such as Zambia, Zaire and Kenya. Hence rather than just being a source of raw materials (and a market for processed ones) for the colonising country, the

developmental path was primarily geared towards the improvement of life for the local settlers.

The Rhodesia government pursued 'middle of the road' policies which allowed it to balance the protectionist demands of the settlers with the need to attract capital, from mostly foreign sources. The main desires of the settlers were the regulation of labour supplies and agricultural markets, and preference in government expenditure, especially in the provision of infrastructure. The government itself was more in favour of liberal laissez-faire policies (Barber, 1961). It was, however, with respect to infrastructural development that the intervention of the government was most keenly felt. In 1936, the Loan fund was used to set up an electricity link between towns in the east, the gold mining settlement of Penhalonga and the main city of Mutare, and two key towns in the Midlands, Gweru and Shurugwi (see Figure 2.1). In addition, in a bid to boost recoveries of gold from refractory ores, in 1938 a facility known as The Roasting Plant was built just outside Kwekwe. The processing of these ores was on toll basis. Finally a company, the Electrical Supply Commission (ESC) was set up to invest in the generation and distribution of electricity (Barber, 1961).

With respect to mining, the government's interventions also resulted in the establishment of major institutions to promote mining (Barber, 1961). A metallurgical testing facility, the Department of Metallurgy (MetLab) was set up in 1928. This comprised pilot plant scale testing, laboratory and assaying facilities, all these services being freely offered. Regional metallurgists were appointed to co-ordinate the regional

centres of the MetLab that were being introduced. The aim was to assist in raising recoveries of existing mines and help address the mineral processing challenges resulting from ever-deeper mines. Finally, a division of the Chief Government Mining Engineer was also set up to enforce mining safety and health regulations.

In 1933, the government bought out the mineral rights of the Company. In a bid to boost actual mining operations and diversify from gold, the exploration and production of other minerals was actively encouraged. Facilities for the hiring of plant and equipment were introduced the following year. This was in addition to the loan scheme that was still operating. To overcome shortages of skilled labour, in 1935 the government brought in Cornish miners, at the state's expense. Cash prizes were also offered to prospectors who made discoveries of new deposits (Viewing et al, 1987).

At a political level, the government took a conscious decision to divide the country into two non-competing racial groups through the legislative process. This was a huge concession to the settlers, especially those living in the rural areas who competed with natives for land and markets. This was achieved through a panoply of legislative and regulatory measures. Of these, the most effective was the Land Apportionment Act of 1933. This Act put a limit on the land available to black Rhodesians by not only enforcing a more permanent culture of cultivation but also restricting their access to more fertile land which also happened to be located in the major gold belts. This led to almost total exclusion of blacks from mining, except as sources of labour (Arrighi, 1973).

Some writers have associated with this Act the basis for declining productivity of land under cultivation by blacks and increased labour supplies to the formal sectors such as white agriculture, manufacturing and, particularly, mining, (Arrighi, 1973; Jourdan, 1991). To ensure that nothing was left to chance, a Native Registration Act and various 'pass laws' were enacted to limit the movements of blacks into the urban areas. This had the effect of enforcing a strict wage structure where the rural commercialised sectors, mainly white farming and mining, consistently paid wages that were lower than those obtaining in the urban areas. Further, to keep and maintain this pressure on wages there was a policy of active recruitment outside Zimbabwe (Arrighi, 1973).

The war in 1939 brought about an enormous transformation in the economic landscape in Rhodesia. There was a shortage of once-imported goods as the traditional sources switched to a war footing. This gave an impetus to a policy of import-substitution and with it a rapid industrialisation of the country. For the mining industry in particular, this resulted in further processing of many of the major minerals, such as asbestos and chrome. Agriculture also assumed a more commercial orientation and became more diversified, with a notable shift towards cash crops such as tobacco and cotton. Still gold retained its primacy and allowed for the payment of some war effort (Barber, 1961).

Between 1935 and 1956, structural changes in the size distribution and composition (types of minerals mined) of mines took place. For example, in 1931 there were one thousand seven hundred and fifty mines of varying sizes, while by 1956 there were only

three hundred mines of which only twelve contributed 70 per cent of output. (Viewing, 1987)

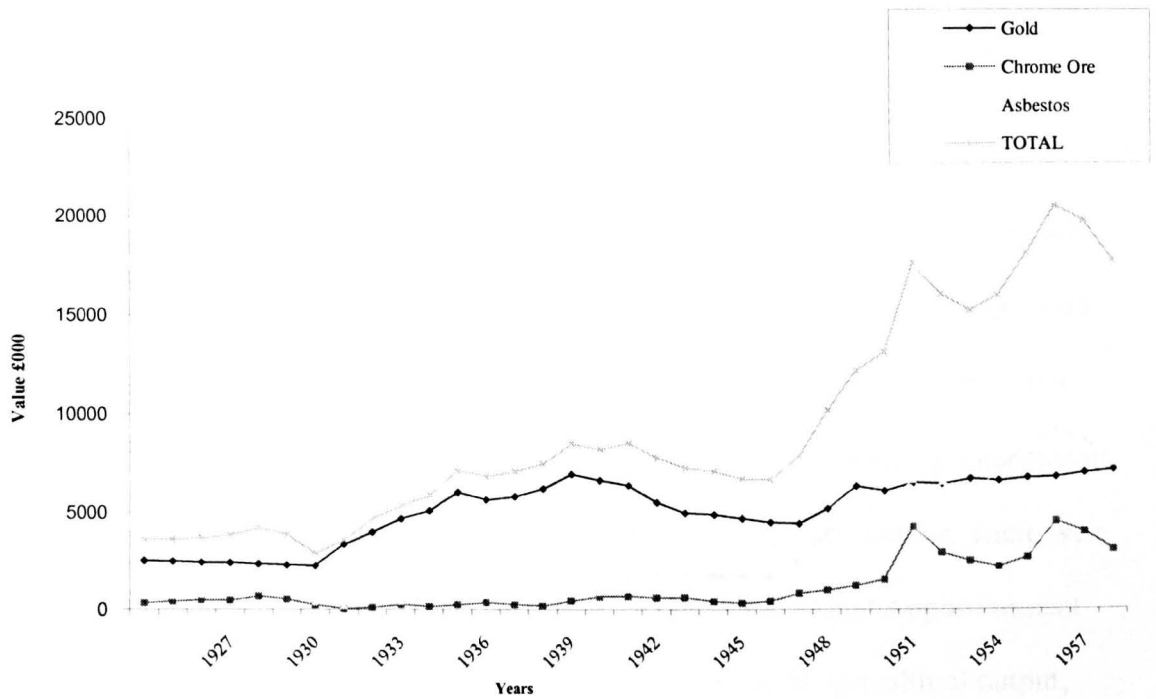
Comparatively in 1935 being over 60 per cent of the total mining output was produced by the large number of small scale mines (Barber, 1961). To a large extent this decline in the importance of the small gold worker reflected rising costs of production as depths of mines increased (the same problems which had seen off the early native miners) and deterioration in ore quality, leading to the closures/sales of most of the small and marginal gold workings as well as contiguous deposits being amalgamated into several shafts of one mine. Added to this was the increased “dollarisation” of the global economy which caused static gold prices and, consequently, increasingly lower returns from gold mining<sup>10</sup>.

The major consequence of the structural shift in the economy was that in 1952, asbestos temporarily overtook gold as the major mineral produced. Figure 2.2 illustrates this development.

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<sup>10</sup> The price of gold was fixed at US\$ 21 per ounce, a level at which it had been since the previous century.





Source: Barber (1961).

**Figure 2.2 Mineral Production in Rhodesia (1925-1959)**

Of interest in Figure 2.2 is the increased divergence, after 1948, between gold and total mineral production, this caused by the continued diversification of the mining sector. In addition, the contribution of mining to national income fell from 25 per cent in 1938 to about 5 per cent in 1960, a level around which it has ever since fluctuated. Over and above the changes that were taking place within mining itself, there were also some important economy-wide implications from the growth in size and sophistication in manufacturing and agriculture. Gold had already been overhauled by tobacco as the major earner of foreign currency. However, mining was increasingly being undertaken

by companies that could invest in deep-mining methods, in order to get at the deeper ores, and the technology to process the increasingly complex ores that were being mined as a result.

There was also increased political agitation by organised black workers and those who, despite their education, found their prospects of further advancement being hampered by the presence of a racial “glass ceiling”. In a move to forestall political activism by black politicians, the government started to rethink its policy of preventing inter-racial competition. In 1954, a bill was introduced to recognise black trade unions. There were even pledges to repeal certain of the contentious sections of the Land Apportionment Act. In addition, the state marketing boards, the major buyers of agricultural output, were to pay the same price for similar quality produce from blacks and whites (Barber, 1961).

Unfortunately, these proposed reforms were too much for the rural whites and in 1962 the reformist government was defeated in the elections. Britain, on the other hand, refused to grant independence to Rhodesia until political accommodation of the blacks had been worked out. In 1965, in an election which was basically a referendum on whether to give further concession to blacks as demanded by Britain, the right-wing Rhodesia Front (RF) won the elections. The RF victory removed any chance of reforms that would have allowed blacks to have any modicum of political influence. The RF made a Unilateral Declaration of Independence (UDI) from Britain (Arrighi, 1973: Jourdan, 1990).

## **2.4 Zimbabwean Mining 1965 to 1980: UDI and Import Substitution**

After the RF government adopted UDI, London immediately declared this move to be illegal. At Britain's behest, the United Nations imposed sanctions on Rhodesia which was especially vulnerable to sanctions, as it derived 35 to 40 per cent of its total GDP from exports (Barnekov, 1969). For a while, however, sanctions were only half-heartedly applied. In 1968, 19.6 per cent of exports and 13.9 per cent of imports were from countries not enforcing sanctions such South Africa and Portugal (and its African provinces), compared to the 12.9 per cent of exports and 19.1 per cent of imports, which came from countries partly enforcing the sanctions. The balance, 40.9 per cent of exports and 28.8 of imports, came from those countries which imposed a virtual boycott (Barnekov, 1969). In addition, it was landlocked and had no domestic sources of oil.

The British government was seemingly hoping for a significant multiplier effect and the expectations were that Rhodesia would soon be on her knees economically and therefore more amenable to reason (Arrighi, 1973; Barnekov, 1969). In 1968, Commonwealth pressure led to the UN making the sanctions against Rhodesia mandatory. However, the timing of the imposition of sanctions was tardy and as a result the effectiveness was blunted, at least initially. Therefore, by the time the sanctions were applied, the UDI government had put in place measures to lessen the impact of the sanctions. Most of the foreign exchange reserves had been moved out of London. Some research on markets had also been done in readiness for the switch from traditional ones

such as Britain. A freeze by Britain on repatriating factor income to Rhodesia actually worked in the latter's favour as there had hitherto been more of it going the other way.

Almost simultaneous with the declaration of independence, the black movements which had been agitating for political change launched a guerrilla war campaign to force, at the very least, concession from the RF government but with a much broader aim of completely supplanting the white minority political and economic power structures with majority back participation.

From the outset, therefore, the RF government faced conflicts on several fronts. For the next 15 years, until Britain finally granted independence to the modern state of Zimbabwe, the RF government defied this civil war, Britain and the international community and set the country on a course of economic development which put emphasis on import substitution industrialisation. This situation allowed the mining sector to become very important once more in the economy as will be illustrated below. Gold mining was crucial to this process as it was one of the major sources of foreign earnings through which the importation of capital and intermediate inputs was possible (Jourdan, 1990).

By the end of 1975 however, it was obvious that the economy had exhausted all the easier forms of import substitution. Shortages of capital and restricted access to technology took its toll on mining and gold production stagnated. The period 1975 to 1979 saw overall investment in the economy declining with the biggest drop recorded by the mining and metals sub-sectors, which fell by 16.4 per cent (Mlambo, 1993).

Over and beyond this, there was the strain of the civil war and the OPEC induced energy crisis that resulted in a global recession. Defence spending as a proportion of total government spending had risen from just over 28 per cent in 1969 to 34 per cent in 1975. It was to rise even further to 46 per cent by 1979. At the same time, the deficit financing which had fallen from 8 per cent in 1970, to 3 per cent in 1971 had risen to 30 per cent by 1979 (Government of Rhodesia, 1979). This was at a time when gross investment was declining, even in nominal terms, from \$468 million in 1975 to \$382 million by 1979 (Reserve Bank of Zimbabwe, 1980). Finally, the other side effects of sanctions were beginning to take their toll, in particular in the price which Rhodesia had to pay for evading sanctions. To make matters worse, Portugal had become a part of the democratic community in Europe and had given independence to Mozambique, to the east of Zimbabwe firmly closing the easiest access to the sea, through the port of Beira. Consequently the RF government and its minority white constituency gradually began to realise that negotiations with the black majority were unavoidable. After a few false starts, characterised by an internal settlement in 1978 that was not recognised by the main black movements, the RF finally gave in to majority rule at the Lancaster House talks in 1979 brokered by Britain. In 1980, Rhodesia became Zimbabwe with a democratically elected government.

## **2.5 Zimbabwean Gold Mining After 1980**

The ending of the civil war and with it the lifting of sanctions brought in a new optimism about Zimbabwe. The economy inherited by the new government, although

well-diversified, had been under immense strain for a considerable period of time. In later years, sanctions had also taken their toll on investment in new plant and equipment with most of the productive sector surviving on antiquated machinery and production techniques. The war had also left an indelible mark in that most of the mining activities had been taking place in the remote parts of the country and had at one stage or another been isolated from the main metropolitan centres (Mlambo, 1993).

In mining, low investments meant technological stagnation and poor recoveries. The new government although professedly Marxist did not embark on wide-scale nationalisation as had been feared by the private sector or, indeed, had happened in Zambia, Mozambique and Tanzania. Instead, it tried to improve the economic conditions of its majority black constituency through the economic growth which depended on very little or no disruption to the prevailing economic systems, while at the same time promoting the 'free-market' based economy (Mlambo, 19983).

Following independence, access to international markets and capital was once again allowed. In addition, resources could now be diverted from the war to production of real goods and services. An additional boost was the removal of the "sanctions-busting" premium for critical imports (Jourdan, 1990: Mlambo, 1993). In short, the government had it in its power to take advantage of the various factors that included international goodwill to embark on a bold economic strategy to move forward. Sadly, this did not happen. The economic policies did not change. The import licensing and foreign currency allocations that were more suited to the siege economy remained, although

rules were relaxed for the importation of capital and intermediate goods (Mlambo, 1993).

From the point of view of the mining sector, however, there was no specific policy announcement to try and address their concerns, which related to access to investment and technology. Hence, although the majority of them were exporters and were therefore crucial to the new dispensation by generating the scarce foreign currency that was crucial for the retooling that the economy was undertaking, mining companies continued to queue, along with others for foreign currency and import licences.

## **2.6 Post Independence Mining Policy**

All mining comes under the Mines and Minerals Act. In addition, gold mining is also regulated by the Gold Trade Act which, among other things, confers a monopoly on buying and selling of gold to the central bank, the Reserve Bank of Zimbabwe (RBZ). Exploration is carried out under a three-year licence, known as exclusive prospecting order (EPO) which gives the holder the right to search for specified minerals in a specified geographic location. This licence is renewable for one year at a time after expiry of the initial three years. There are various other government departments whose portfolios impinge on or are affected by the operations of mines. Water quality is monitored by the Ministry of Water Development under the Water Act, environmental impact is the responsibility of the Ministry of Environment and Tourism and the Ministry of Health is responsible for air quality under the Atmospheric Pollution

Prevention Act (United States Geological Survey<sup>11</sup>, 1998). All these activities are coordinated by a Mining Affairs Board which is chaired by a senior civil servant, nominally the permanent secretary, and also comprise representatives of both business and trade unions.

The new government had an immediate task at hand. Expectations among the black community had been raised by the new political dispensation. To address the racial inequalities of the past, the government launched an economic programme designed to increase equity while at the same time promoting economic growth. First was a repeal of the Land Apportionment Act.

The broad economic objectives were outlined in an economic programme, appropriately called “Growth with Equity” which, although couched in lofty socialist rhetoric, was clearly aimed at the minimising disruptions to the structure of the economy while at the same time trying to generate enough growth to address the expectations of its black constituency. This was especially so with respect to the ownership of the ‘means of production’ which remained largely in foreign hands. However, minimum wage regulations and other legislation to protect jobs reinforced the array of regulations inherited from the previous regime (Government of Zimbabwe, 1980).

Despite its obvious importance, there was very little explicit focus on mining *per se* with most government economic policy towards mining tending to be made in

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<sup>11</sup> Hereafter USGS



conjunction with other exporting sectors or as part of labour legislation. Since independence the Zimbabwean government has come up with four major economic policy statements and a number of minor ones. The first, already mentioned above, was issued in 1980 and entitled *Growth with Equity* (Government of Zimbabwe, 1980). This was a statement of intent to encourage economic growth while redressing the legacy of the previous race-based economic system. Central to this was the need not to disrupt the relatively sophisticated economic structure. Redistribution was to be achieved through price regulation and minimum wage and other labour legislation which made firing of workers difficult (Government of Zimbabwe, 1980).

The government also aimed to enhance the efficiency of the private sector, which included mining. The economy had for so long been starved of capital and new technology so it was only right that the main focus for re-igniting growth would be on encouraging investment in the productive sectors. However, the exchange and import controls remained as major constraints. The productive sectors remained starved of desperately needed investment.

There was an initial growth spurt, caused by the lifting of sanctions and an above average agricultural season. Economic growth averaged more than 14 per cent in the 1980/81 period. After that, though, the economy show signs of stagnation. The government failed to take advantage of these favourable conditions to undertake necessary reforms which would have allowed access to new investment and injection of new technology. Although the necessity for reforms seemed to have officially been

recognised, as evinced by the objectives stated in *Growth with Equity*, in reality none of this was followed up (Mlambo, 1993).

For the economy in general, the crucial point was to generate as much exports as possible. To replace the almost obsolete equipment and machinery, massive imports were required. The main problem that the new government had faced since independence was the failure to match plans with deeds on the ground. The clearest example of this can be found in the large fiscal deficit, which had been hovering above 10 per cent over the decade to 1989 (Mlambo, 1993). Throughout the 1980s, by way of Budget Statements, the government had correctly identified this huge imbalance as a primary cause of low private sector investment and high domestic inflation. Each year the government planned a gradual reduction of the deficit to more manageable levels. Yet with each new budget, the deficit stubbornly remained high.

The negative cycle of inflation and devaluation was fully played out, with prices rising over 200 per cent in the 1980s. Most of this was imported inflation (Jourdan, 1990).

For mining this period can also be described as a period of missed opportunities. Apart from Rio Tinto's \$8 million<sup>12</sup> investment in the opening up of the Renco mine, there were no other significant investments. Yet, during this time, the gold price had surged to almost US\$800 per ounce (World Gold Council, 2006). This high price had triggered an explosion in exploration activities (and investment) in countries that had similar gold

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<sup>12</sup> The total investment was £6 million sterling.

belts, such as South Africa, Canada and Australia. Canadian production, for example, rose from fifty-two tonnes in 1980 to one hundred and seventy-five tonnes in 1991, with the most significant discovery being the Hemlo fields where production started in 1985 and from which three mines now produce about 35 tonnes per annum (World Gold Council, 2006). Sadly Zimbabwe did not benefit very much from this boom. Gold production never quite matched the developments in countries with similar Archaean schistbelts (Jourdan, 1990).

The government clearly realised the value of foreign investment, yet it seemingly made no tangible effort to encourage this investment. Despite the realisation that foreign investment was crucial Zimbabwe procrastinated over the signing up to the Washington-based Multilateral Investment Guarantee Agency (MIGA)<sup>13</sup>, which guaranteed protection from forcible expropriation. The government was forced to come up with policy adjustments, this time *Foreign Investment Policy Guidelines and Procedures*. This was meant to reassure the rather sceptical foreign investors. No significant investment inflows followed however and the government was forced into making further policy changes.

The government followed *Growth with Equity with Transitional National Development Plan*, a plan which targeted overall economic growth of 8 per cent in the two years from 1982/83 to 1984/85 (Government of Zimbabwe, 1981). The main aim was the

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<sup>13</sup> Most foreign investors in large projects insisted on acceding to the MIGA protocol as a pre-requisite. The government only signed up in 1991.

encouragement and promotion of labour-intensive technologies, encourage “economically efficient” import substitution in key areas such as heavy industry, energy etc. In addition, there were also the beginnings of explicit government participation in the economy. To this end, the government set up, through Acts of Parliament, the Zimbabwe Mining Development Corporation (ZMDC) to undertake exploration and mining on behalf of the government, and the Mineral Marketing Corporation of Zimbabwe (MMCZ) for monitoring the sales of all non-gold mineral products.

However, the problem of access to foreign currency remained a problem and the state failed to move away from the rationing of it or from import controls which were enforced through import licences. Hence it was no surprise when the economy registered economic growth of -2 per cent in 1982/83 and -3 per cent for 1984/85. Non-attainment was mostly blamed on the drought which had severely affected agricultural production (Mlambo, 1993).

Realising that the initial plans had not borne fruit, a fact acknowledged in its policy statement as it launched yet another plan, this time *The First 5 Year National Development* which covered the period 1986-90. The plan called for more state participation in the economy and emphasised re-distribution of land and wealth (Government of Zimbabwe, 1985). Importantly, though, the government made no effort to increase exports and ignored calls from exporters for export incentives and the reduction of the government deficit which was inexorably rising. In 1987, however, the government allowed exporters to retain 5 to 7.5 per cent of their foreign currency

earnings for the procurement of essential spares and parts (Reserve Bank of Zimbabwe, 1988).

In 1988, the state set up a gold refinery with capacity of 90 tonnes per year, partly in the expectation of increasing gold production but mainly to rid itself of dependence on South Africa. Generally, investment in gold mining was patchy and mostly for ongoing production rather than new projects. The exceptions were the US\$5 million that Cluff spent in 1988 in the opening up of Freda-Rebecca near Bindura, and the already mentioned Rio Tinto investment at Renco (Jourdan, 1990). Freda-Rebecca was a significant event, however, because for the first time, a low-grade gold deposit had been developed into a mine in Zimbabwe. This was achieved using a new technology, called 'heap leaching' which avoided the high costs of crushing the ore.

By the end of the 1980s, however, it was obvious that there were deep-seated structural problems in the economy which had to be addressed and this would require radical policies. Lack of investment was hampering competitiveness and putting pressure on the exchange rate. This in turn had an impact on the balance of payments position, particularly as equipment, spares and intermediate goods were the major components of imports. The government was forced to go to the International Monetary Fund (IMF) and World Bank (WB) for assistance. The two Bretton Woods institutions, in turn, proposed a major structural adjustment programme, Economic Structural Adjustment Programme (ESAP) to ensure sustainability (Reserve Bank of Zimbabwe, 1991). The

main focus of this programme was put on opening the economy to external competition and the removal of controls on prices (to include interest rates), wages and imports.

The Economic Structural Adjustment Programme for 1991 to 2000 was going to run in two phases but targeted a real average growth rate of 5 per cent per annum through export-led economic expansion (Government of Zimbabwe, 1990). The market was to eliminate inefficient enterprises through competition, both external and internal. To achieve this, quantitative restrictions on imports were to be replaced with trade liberalisation. State-owned enterprises were first to be put on a commercial footing and then privatised. Finally in order to reduce inflationary pressures, the government deficit was to be reduced from 9 per cent in 1990 to 5 per cent by 1995. Part of this reduction was to be achieved by getting rid of subsidies to the parastatals a number of which were some mining companies.

Initially, the results were mixed. A drought hit Southern Africa in 1992 and caused a contraction in agriculture, and with it the plan was suffered some reversals. The government deficit rose to 10.4 per cent as a result of drought relief spending. However by 1995, the government deficit fell to 7.9 per cent with inflation falling from 42 per cent to 25 per cent during the same period (Reserve Bank of Zimbabwe, 1994). The main export sectors, manufacturing-- though hard hit by external competition--, mining and tourism in particular registered positive growth. Privatisation of loss-making state-owned enterprises however did not move according to plan and by 1995 many remained

in state hands, loss-making and a drain on fiscal resources (Zimbabwe Ministry of Mines, 1995)

As part of ESAP, in 1991, the government relaxed some of the strict controls on the economy. In particular, exporters, of which gold mines were a significant majority, were allowed to retain up to 50 per cent of their receipts (Government of Zimbabwe, 1990). This freeing of exchange rates was a welcome move, as exporting became viable once more. The ability to retain most of the foreign currency earnings also encouraged investors into the country and at last encouraged the inflow of technology.

The effect on mining investment was immediate. At the beginning of the 1990s, Zimasco, a former Union Carbide (a major US multinational) subsidiary opened Mimoso mine (platinum), initially on a trial basis. It then gradually expanded to full production. In 1994 BHP, a major Australian mining multinational, opened, Hartley, a platinum mine. BHP invested almost US\$250 million in the development of this mine and ancillary metallurgical facilities (Ministry of Mines, 1995: Reserve Bank of Zimbabwe, 1995). This was the largest single investment project in Zimbabwe's history. The climate was also such that Anglo American, a major South African mining conglomerate, also began to look at the feasibility of its own platinum properties. Exploration activities in gold, diamonds and other strategic minerals such as tantalum increased. There were a few reversals, too. In 1996, two years after it had opened Hartley, BHP announced that it had failed to achieve the targeted production rates at the mine. As a result it could not sustain the operational losses and closed down. Its junior

partner in the joint venture, Delta Gold, a joint Australian-Zimbabwean operation remained in Zimbabwe and acquired their joint assets, in particular the state d'art metallurgical complex (Reserve Bank of Zimbabwe, 2000).

Encouraged by the first positive signs, government, IMF and WB embarked on a second stage, called *Zimbabwe Programme for Economic and Social Transformation* (dubbed ZimPrest), for the period 1996-2000. Apart from harnessing the positive aspects of ESAP, this stage was also meant to put more focus on export-led growth and the creation of export processing zones. The target real annual growth rate was set at 6 per cent per (Government of Zimbabwe, 1995).

There was an element of tardiness in the implementation of this second programme, however, with the parastatals remaining in state hands and consequently still being a major drain on resources on state resources. For example, since 1992, the ZMDC, the main government investment vehicle in mining, had been set to be privatised. but was still in state hand in 1998. Of major concern, though, were squabbles with the two Bretton Woods institutions, especially with the IMF and particularly on land reform. This eventually led to a major row in 1998. The IMF withheld support for the rest of the programme when the government insisted on taking over commercially exploited land without the compensation to the previous owners. The withdrawal of the IMF support led to a return to shortages of foreign currency and with it shortages on crucial inputs such as fuel and spares parts. In 1999 a fuel crisis ensued as Zimbabwe failed to procure



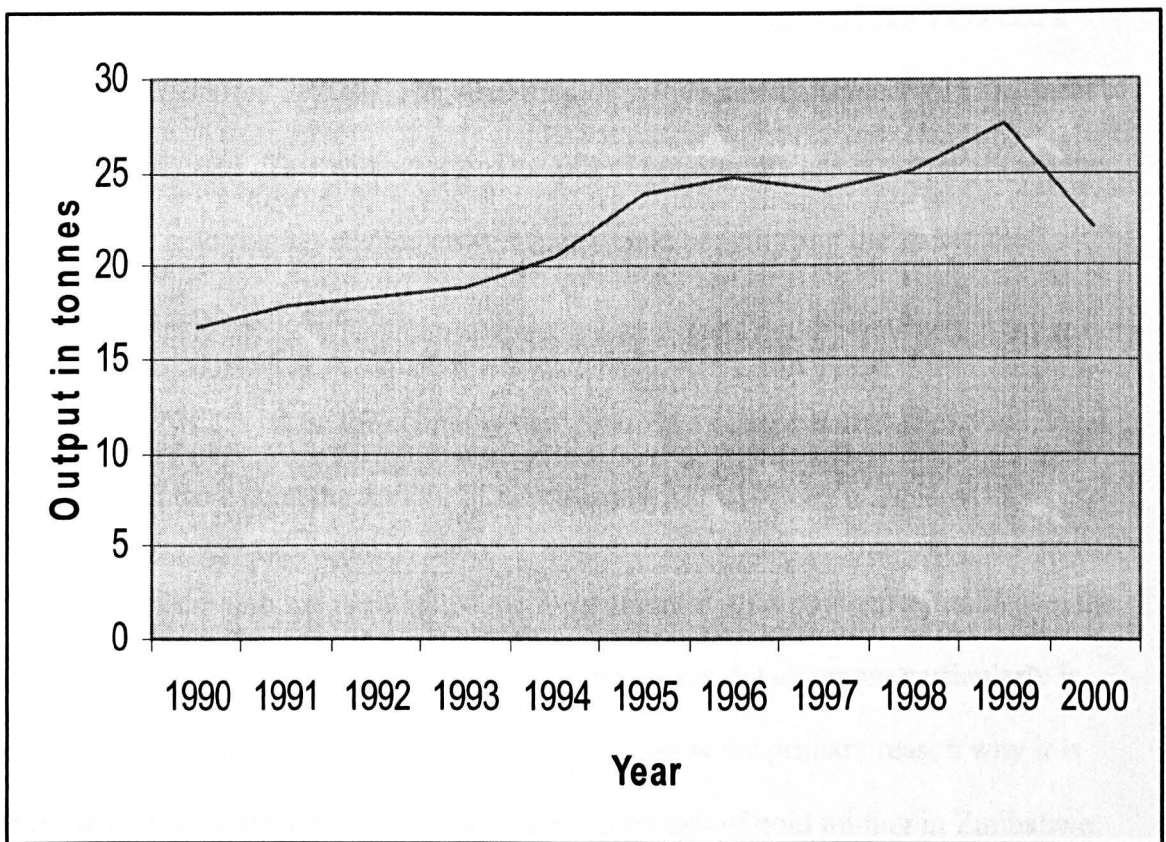
enough fuel and large parts of the productive sectors found themselves under stress (Reserve Bank of Zimbabwe, 2000).

In 1999, the government instigated land invasions by veterans of the 1970s guerrilla war. Ostensibly this was to satisfy land hunger by landless peasants. The impression was left, however, that this was the last throw of the dice for an increasingly beleaguered regime which was facing increasingly strident calls for political reforms and unprecedented unity among its political foes. This view was reinforced by the fact that commercial farmers were perceived, rightly or wrongly, as the major sources of funding for a very well-organised opposition movement which comprised trade unions and employers' organisations. The IMF and WB withdrew from Zimbabwe and they were followed then or soon after by major bilateral donors. The Zimbabwean economy went into the [now] well-documented crisis.

Before the aforementioned crisis, there was evidence that the economy had started to improve. The opening up of the economy resulted in stiff competition among suppliers to the mining industry. They (suppliers) were forced into management restructuring and adoption of new technologies and better production methods with improved use of resources (Zhou, 2000).

For gold mining in particular, there was for the first time signs of new investment. Gold mines, along with other exporters, were now being allowed to retain 60 per cent of their earnings in foreign currency. This gave them unprecedented access to foreign currency and the results were not long in coming. Significant investments were made in gold

mines, particularly by Australian and Canadian junior mining companies. Figure 2.2 shows the gradual increase in output as new mines came on stream and existing ones started benefiting from investment. For example, in 1987, Freda Rebecca mine was opened producing about 1.5 tonnes. By 1995, expansion and investment had raised this to almost 3 tonnes (USGS, 1996). Many new and existing mining companies were feeling sufficiently reassured to start bringing major investment projects on stream. An example of the new investment flowing into the sector was the Eureka mine commissioned at a cost of US\$ 25 million in 1999 (USGS, 1996).



source: USGS, Ministry of Mines (Zimbabwe).

**Figure 2.3: Zimbabwe Gold Production (1990-2000)**

In 1990, there were about five hundred registered gold workings about ninety of which produced more than 10kgs of gold per year (Zimbabwe Ministry of Mines, 1991: USGS, 1998). These relatively “large” mines produced over 90 per cent of total/ output. By 1995 the number of these large mines had risen to one hundred and twenty as old small mines were upgraded and completely new ones were brought on line. Most of the upgrading comprised the application of new technology to existing old operations (USGS, 1998). There were also new mines commissioned as a result of the more relaxed economic climate.

Figure 2.2 shows a pattern of steady growth until 1993, a surge up until 1999 and a decline which started in 2000. The beginning of this decline coincided with the onset of the economic crisis. This window (1990-2000) of opportunity has however shown the potential for gold mining in Zimbabwe which would benefit from the inevitable political and economic reforms.

While the crisis has bitten the economy hard, there is evidence that gold exports, legal or otherwise, have been the saviour of the economy.

The above discussion has highlighted the importance of gold mining to Zimbabwe, the evolution of its statehood and to its continued economic development particularly in times of severe economic and political pressure. This is the primary reason why it is important to investigate the performance and efficiency of gold mining in Zimbabwe and the world at large. The discussion has necessarily included historical and political issues which indirectly concerned gold. It is no exaggeration, however, to claim that

gold has consistently paid Zimbabwe out of trouble during period of strife and instability. Currently, Zimbabwe is in the middle of a self-made crisis which, by most reasonable calculation ought to have resulted in a major collapse. The one reason why this has not yet happened is because gold exports ( a large fraction of which are illegal) have managed to pay for enough of the essential imports required to keep the economy running albeit, at lower rates of output (Reserve Bank of Zimbabwe, 2005: Zimbabwe Ministry of Mines, 2005).

## CHAPTER 3 THEORETICAL FOUNDATIONS

### 3.0 Introduction

Since the time of classical economists, there has been much interest in measuring and comparing the performances of productive entities. David Ricardo's seminal work on comparative advantage makes use of differences in relative efficiencies, perhaps in a less specific sense than the term is now used, as a basis for trade (Hollander, 1979). Adam Smith's equally famous treatise on "The Wealth of Nations", although mainly about increasing output from a given set of resources through specialisation and division of labour, has efficiency at its centre (Heilbroner & Malone, 1986). In fact, efficiency and continual improvement in firms' performances, mainly brought about by technical progress and mechanisation, was the central theme in Karl Marx's allusions to the inevitable destruction of capitalism<sup>14</sup> (Ollman, 1975).

Most of the work on efficiency has taken place in the recent era with Farrell (1957), in particular, being credited with laying the foundations of the analysis and measurement of efficiency and productivity. A large volume of literature on the topic was stimulated, both theoretical and empirical (Simar & Wilson, 2007). This chapter outlines the tenets of the theory of production and the measurement of efficiency.

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<sup>14</sup> Rapid efficiency gains, through extensive mechanisation of the production, would inexorably increase the slice of national income accruing to capitalists, reducing both the number of people employed and their share of national income, ultimately, thus leading to discontent among the workers and peasants and, ultimately, a socialist revolution.

There will also be an analysis and discussion of relevant empirical work<sup>15</sup>. Special attention will be paid to the non-parametric mathematical programming methods which will be adopted in this study.

This chapter is subdivided into four main sections. Following this introduction, Section 3.2 deals with the theory of production and the main issues that are important in assessing and comparing performances of productive organisations. The concept of distance functions, which is useful when dealing with production frontiers and efficiency, particularly in multi-dimensional settings, is introduced. In Section 3.3, special attention is paid to the idea of technical efficiency, its treatment and measurement. This idea of efficiency follows from the distance function approach which will be also be explored.

There are two main methods of estimating efficiency in practice: the parametric methods such as stochastic frontier analysis (SFA) and non-parametric such as data envelopment analysis (DEA). These will be outlined and analysed. In measuring efficiency, what is being done is evaluating the distance of an observation from a technologically efficient frontier, whether observed or not. The relative merits of the DEA method over the SFA which largely depend on the context, particularly the availability of data, are put forward. In Section 3.4, the bootstrap, a resampling

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<sup>15</sup> The literature on DEA is truly vast as a search on “Web of Knowledge”

<http://www.webofknowledge.com/> and “Google Scholar” <http://scholar.google.com/>, among many, will indicate. Here the coverage will be restricted to published work on DEA in mining and the bootstrap DEA. In addition, a review of Banker’s work on hypothesis testing will also be done as this gives insight onto some nonparametric used the tests in this dissertation.

technique which has been proposed in order to address some of the concerns with the DEA is introduced. The bootstrap enables the approximation of the distribution of the estimated efficiency scores and for the correction of sample bias, and also allows the drawing of statistical inferences and the testing of hypotheses. Finally, in Section 3.5 a review of relevant previous studies is carried out with the principal intention of informing the estimation work in Chapters 4 and 5.

### **3.1 The Production Technology**

A production process involves the transformation of one or more inputs to produce at least one output. For example, gold mining involves the combination of labour, capital services, energy, material inputs and other services to extract and process gold ore, whose final value is expected to be greater than that of the inputs combined. The basis of production theory is the premise that, in attempting to attain certain objectives, firms always attempt to optimise the use of resources at their disposal given technology and other environmental constraints. The problem in empirical studies as noted by Färe et al (1994), is the need to make a distinction between the constraints and the failure to optimise. Hence firms will strive to produce more output from the same inputs or the same output from fewer inputs. To this end, one of the most common assumptions about producer behaviour is that the firm chooses an input vector that minimises the costs of producing a given vector of outputs. One could equally assume output maximisation at given prices for a given input vector. The approach taken in analysing producer behaviour depends on the context in which the firms are operating, on what can be observed and on what is plausible. There is a wide range of models from which to choose such as production frontier, cost frontier, profit frontier. In the measurement of

performance the choice of the model is usually dictated by the data available (Färe & Primont, 1995). The key point, though, is that firms could attain optimum either by using the least amount of resources for a given output or by maximising outputs for given inputs to achieve their stated objectives, within the constraints of the available technology and scarcity of resources.

The empirical application of the theory of production has developed rapidly in recent years as researchers have strived to accommodate situations where operation does not always mean attainment of objectives. The major point of departure, in applied studies, was the realisation that achieving objectives was by no means guaranteed. Deviations from the optimal, therefore, were no longer assumed (as hitherto had been the case) a mere result of random noise or chance. Therefore, some producers, for a variety of reasons, tend to be more successful than others. This observation was backed by, initially, anecdotal evidence-- be it casual inspection of company annual reports or commentaries in the business press that the optimistic view of firm behaviour where objectives are always met fell somewhat short of satisfactorily explaining what happened in the real world (Kumbhakar & Lovell, 2000). This has led to continuous refinements of the empirical techniques and the conclusion that, although the producers' objective may be to maximise or minimise, they do not always succeed. Specifically, failure to optimise is acknowledged to be due to more than mere random shocks (Kumbhakar & Lovell, 2000). Estimation of efficiency therefore is a means of identifying how far producers are from the frontier defined by the technology.

Once the notion of non-attainment of the optimum became widely accepted, attention turned to measuring how far the individual, observed producers were from the optimum



frontier. The production frontier in this case is then defined as the maximum feasibly achievable output given the input vector and technology. Efficiency in achieving the objectives is associated with being located on the frontier and inefficiency with being located some distance from it. The distance from the frontier represents inefficiency. Other authors (Edvardsen & Førsund, 2003) have called the study of efficiency “benchmarking”, comparing actual performance against a reference performance, known or as is mostly the case unknown. The most cited work in this area is Farrell’s 1957 paper on productive efficiency. Other key theoretical works in this regard were on distance functions by Malmquist (1953) and Shephard (1953, 1970). On the basis of these theoretical approaches, a large body of applied research has developed over the last fifty years.

Performance measures based on the frontier are directly derived from the definition of the production function. The underlying assumption is of a productive organisation or firm—the decision making unit (DMU) — capable of making optimal choices of inputs (outputs) to attain an objective, within the constraints imposed by the technology and available resources. The frontier is the limit, prescribed by prevailing technology, at which DMUs operate. The technology constraint is important, as it defines the limit of feasibility as it were, of the transformation of inputs to outputs. The only possibility of going beyond the frontier is through technical progress which is reflected in the position of the frontier shifting.

### 3.2 Nature of the Frontier

The starting point in modelling any production process is the relationship between inputs and outputs. Underlying this relationship is an assumption of a feasible technology which makes it possible to combine inputs to produce outputs. The production technology which transforms the inputs to outputs can be represented by \*a feasible set of physically attainable activities, which is denoted by  $\psi$ , or what is equivalent, a set of feasible production plans  $(x,y)$ , where  $x$  is a vector of  $p$  inputs and  $y$  is a vector of  $q$  outputs such that  $x \in \mathfrak{R}_+^p$ ,  $y \in \mathfrak{R}_+^q$ . More compactly, this can be written

$$\text{as } \psi = \{(x,y): x \in \mathfrak{R}_+^p, y \in \mathfrak{R}_+^q, x \text{ can produce } y\} \quad (3.1)$$

$\psi$  represents the technology in terms of the feasible input-output transformations and, in addition, implied input and output substitutions. (3.1) states that, to produce the output vector  $y \in \mathfrak{R}_+^q$ , a vector  $x \in \mathfrak{R}_+^p$  of inputs is required, where both  $x$  and  $y$  are vectors of dimensions  $p$  and  $q$  respectively. A further condition, grounded in the basic concept of optimising the use of scarce resources, known as “weak essentiality”, is that the production of one output requires at least one input since to produce a positive output requires positive inputs<sup>16</sup>. A rational producer will therefore be assumed to seek to minimise the level of inputs,  $x$ , used in producing a certain  $y$ .

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<sup>16</sup> This is in contrast to strong essentiality where all the inputs are essential in the production process.

There are two other alternative representations of  $\psi$ , through the input requirement set  $X(y)$  and the output correspondence set  $Y(x)$ .

$$X(y) = \{x \in \mathcal{R}_+^p \mid (x, y) \in \psi\} \quad (3.2)$$

$$Y(x) = \{y \in \mathcal{R}_+^q \mid (x, y) \in \psi\} \quad (3.3)$$

The input requirement set  $X(y)$ , consist of all the input vectors which can produce a given output vector  $y$  with the current technology. It represents the technology from an input substitution perspective. In the same vein, the output correspondence set  $Y(x)$  portrays all output vectors  $y$  that input vector  $x$  can produce and describes the [same] technology as  $X(y)$ , this time from the output substitution perspective. In fact,  $X(y)$  and  $Y(x)$  are inverse of each other, such that  $y \in Y(x)$  if and only if  $x \in X(y)$ . In short, (3.1), (3.2) and (3.3) are three ways of compactly characterising the same feasible technology which transforms  $x$  to produce  $y$ .

Given the definition of the input requirement set,  $X(y)$ , from (3.2), the estimation of efficiency is based on finding the optimal input vector  $x$  in the production of  $y$ . How efficiently an observed DMU is utilising its inputs can then be estimated, by comparing observed and potential output vectors, or, for a given output vector, comparing observed and potential input vectors. One way of applying the ideas of efficiency is through the concept of distance functions due mainly to Malmquist and Shephard (see Coelli et al, 2005). A distance function defines the [radial] distance between an observed output vector,  $y^0$  when approached from the output orientation or the observed input vector  $x^0$  when observing from the input orientation, and the production frontier. Hence there are

two orientations of the distance function in the literature, the input and output orientation. The divergence between an observed activity and a potentially achievable one is what gave rise to the notion of a distance function.

In this study, the issue of data availability and the context in which the DMUs are operating necessitates approaching the issue of efficiency from the input orientation and therefore only the input distance function will be explored here<sup>17</sup>. The phenomenon, performance in relation to the frontier defined by the technology, is the same, whether one approaches from an input or output perspective, however. According to Coelli et al (2005), “the input distance function involves the scaling of the input vector” and is derived from the input requirement set. The input distance function is defined, on the input requirement set,  $X(y)$ , as

$$D_i(x, y) = \left\{ \max \gamma : \begin{pmatrix} x \\ \gamma \end{pmatrix} \in X(y) \right\} \forall y \in \mathfrak{R}_+^q$$

$$\gamma \geq 1 \tag{3.4}.$$

The properties of distance functions are derived from the assumptions underlying the input requirement and output correspondence sets<sup>18</sup>. In particular,  $D_i(x, y)$  is non-decreasing in  $x$  and non-increasing in  $y$ . When  $x \in X(y)$ , then  $D_i(x, y) \geq 1$  and is actually

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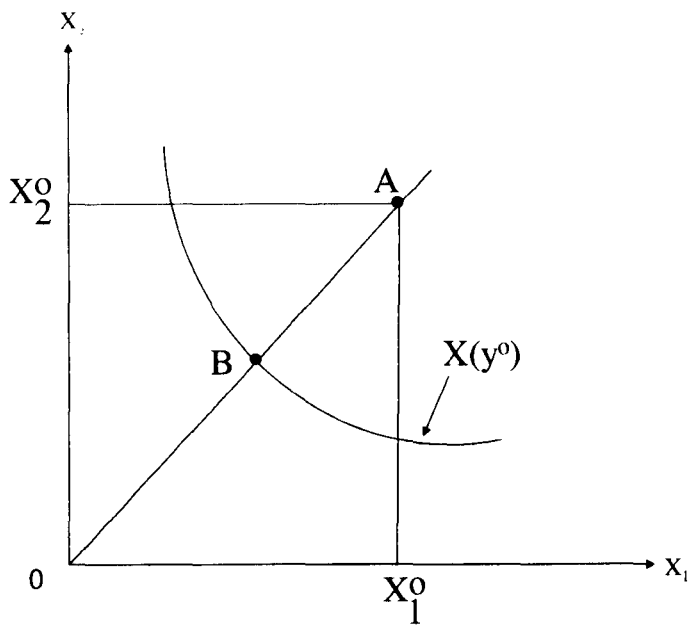
<sup>17</sup> If the technology is characterised by global CRTS, then the input and output distance functions are reciprocals. The logic used and issues being analysed are essentially the same.

<sup>18</sup> The basic assumptions include convexity of  $X(y)$  and free disposability of  $x$ . This assumption, in this case, known as weak disposability, states that if  $x$  can produce  $y$ , then  $x$  can also produce any fraction of  $y$ .

equal to 1 when DMU is exactly on the lower bound of the input requirement set; that is, when the observed DMU lies on the frontier. The capacity to model multiple outputs and inputs, without the need to qualify the model with the behavioural assumptions of profit maximisation or cost minimisation, is one of the main advantages of approaching production technology through the distance function approach.

In Figure 3.1,  $X(y)$  is represented by the area above the isoquant, with the isoquant being the lower bound and, implicitly representing the production frontier.

The distance function, in this case, is illustrated using a constant returns to scale (CRTS), two-input, one-output case but can generalise to a multi-input and multi-output case and also where returns to scale vary.



**Figure 3.1 Input Distance Function**

DMU A is observed to use input vector  $\mathbf{x}^0 = (X_1^0, X_2^0)$  to produce a single unit of output  $y^0$ , the [single] output of DMU A<sup>19</sup>. The distance function  $D_i(y^0, \mathbf{x}^0) > 1$  (also  $\gamma$  in

Equation 3.4) for observation A, is given by the ratio  $\frac{OA}{OB}$ . In this case the DMU

observed at B lies on the lower bound of  $X(y)$  and is regarded as being on the frontier,

i.e.  $D_i(y^0, \mathbf{x}^0) = 1$ . The ratio  $\frac{OA}{OB}$  reflects the extent by which A can improve its

performance by reducing its input vector while maintaining rate the output rate  $y^0$ . That

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<sup>19</sup> We follow the normal convention of using bold face to denote vectors and matrices and italics for scalars.

is, what DMU A needs to do, in order to be efficient, is in effect to move along the ray OA towards the point B, on the technologically determined frontier and that is achievable by reducing  $x^0$  while leaving  $y^0$  unchanged. Free disposability of inputs along the ray from the origin through A and B is assumed.

Distance functions have a number of uses in economics, from the construction of productivity index numbers and in the estimation and analysis of technical efficiency through the use of both the econometric and mathematical programming methods (Coelli et al, 2005). In practice the fully efficient isoquant in Figure 3.1 is unknown but can be estimated using a sample of observed DMUs using a variety of methods, including SFA and DEA.

### **3.3 Efficiency**

There are two main methods of evaluating the distance function,  $D_i(y^0, x^0)$ ; the econometric and mathematical programming methods (parametric and nonparametric methods mentioned earlier). In this section, the practical issues involved in measuring the distance function and efficiency are explored and discussed. The emphasis will be on DEA and SFA methods. In addition, a review of a selection of empirical studies using DEA and a summary of their main findings is undertaken (see footnote 2). In section 3.2, a framework through which an input-oriented measure of technical efficiency is defined was introduced. Thus described, efficiency is best viewed in terms of the distance from an observed DMU to the technologically defined frontier. For each inefficient DMU, that is, DMUs not on the frontier, there are efficient DMUs which lie

on the frontier which can be used to identify its reference point on the frontier. These efficient DMUs are known as peers. An inefficient DMU can reference a single peer or a linear combination of peers to achieve best-practice and move onto the frontier.

The distance function in (3.4) allows the formulation of an input-oriented measure of technical efficiency. This measure of technical efficiency can be represented as  $TE$

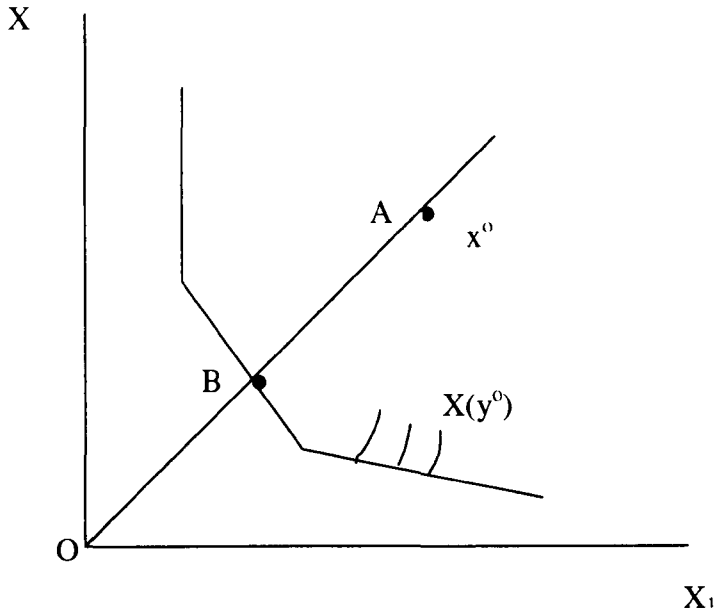
where  $TE = \frac{1}{D_i(\mathbf{x}^o, \mathbf{y}^o)}$ .

This concept of efficiency as defined is also commonly known as the Farrell input-oriented technical efficiency score, after Farrell (1957) and is illustrated in Figure 3.2 for a DMU using two inputs (denoted  $X_1$  and  $X_2$ , in this case) to produce a fixed output  $\mathbf{y}^o$ <sup>20</sup>. Since  $D_i(\mathbf{y}^o, \mathbf{x}^o) \geq 1$ , it follows that  $0 \leq TE \leq 1$ . A DMU is deemed efficient if it is on the frontier, a case where  $TE = 1$  and  $D_i(\mathbf{y}^o, \mathbf{x}^o) = 1$ . An intuitive derivation of the relationship between  $D_i(\mathbf{y}^o, \mathbf{x}^o)$  and  $TE$  is given using Figure 3.2. For a more formal treatment and proof, see Färe & Primont (1995).

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<sup>20</sup> The main difference between Figures 3.1 and 3.2 is that in the former frontier is smoothly convex and everywhere differentiable (theoretical) and the latter (more readily found in empirical studies) is a convex piece-wise linear frontier.





**Figure 3.2 Input Efficiency**

In Figure 3.2, the distance  $OA$  is also defined from the input vector,  $x^o$ , of the

observed DMU, A. For A, TE is then reformulated as  $\frac{OB}{OA}$ , such that

$OB = \frac{x^o}{D_i(y^o, x^o)}$ . Since  $TE = \frac{OB}{OA}$  and  $OA = x^o$ , this means that the input-oriented

measure of technical efficiency can also be given by

$\left( \frac{x^o}{D_i(y^o, x^o)} \right) / x^o$ . Hence,  $TE = \frac{1}{D_i(y^o, x^o)}$ , that is, technical efficiency is the

reciprocal of the distance function,  $D_i(y^o, x^o)$ . To reiterate, what has already been

stated, when  $TE = \frac{1}{D_i(y^o, x^o)} = 1$ , the observed DMU is efficient and lies

exactly on the frontier. This means that the DMU located at point B is input efficient.

If, instead,  $\frac{1}{D_i(y^o, x^o)} < 1$ , then the DMU is input-inefficient and lies above the

piecewise isoquant such as point A. The implication is that there is an opportunity to

reduce inputs while maintaining the rate of output. A case of  $\frac{1}{D_i(y^o, x^o)} > 1$

signifies a super-efficient DMU, located beyond the frontier technology. Were this to occur, then technology must have progressed beyond the currently defined technology, making  $\psi$  old technology which cannot occur if analysis is conducted accurately. As currently defined,  $\psi$ , represents the most advanced technology available, and this case is discounted by assumption.

In estimating input-oriented efficiency, interest focuses on the difference between estimated TE and full efficiency, that is  $TE=1$ , in terms of distance of the observed DMU from the frontier. There are various models and methods which have been developed for calculating the value of the distance function and hence the estimated efficiency of a DMU.

### **3.3.1 Measuring Efficiency: Econometric and Data Envelopment Analysis**

A number of issues determine the efficiency of a production unit. Lovell (1993) lists three important questions that are generally asked when measuring efficiency and analysing performance. These are, (i) how many and which inputs and outputs ought to be included in the analysis (ii), how are they to be weighted in the aggregation process (in cases where the production technology is characterised by multiple inputs

and outputs) and (iii) how is the firm's maximum potential output measured? The last question is important because estimated efficiency is always relative to a benchmark or maximal level. These questions have given rise to different means of measuring efficiency, each of which has its own weaknesses and strengths. In applied work, there are two main methods for computing technical efficiency. These are the econometric, often termed parametric analyses of which the commonest is SFA, and the mathematical programming approaches which are typically non-parametric analyses, of which the dominant method is DEA<sup>21</sup>.

The econometric and mathematical programming approaches, of which DEA is an example of mathematical programming approaches, primarily differ on two grounds: first how to take into account (or not, as the case may be) random noise and, second, the flexibility of the specified structure of technology (Lovell, 1993). In econometrics, the presence of noise is generally assumed since it cannot be ascertained that the model is always fully and correctly specified. The econometric method is generally based on a pre-specified underlying functional form representing the production technology. What Aigner, Lovell & Schmidt (1977) call "favourable as well as unfavourable external events such as luck, climate, topography and uncontrollable machine performance", are normally cited as the main sources of statistical noise. In addition, there cannot be any certainty that the data do not suffer

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<sup>21</sup> There are some parametric approaches in the mathematical programming framework, most notably due to Aigner and Chu (1968) and derivatives thereof, which use linear programming and corrected least squares to estimate the parameters. See Coelli *et al* (2005). In addition, not all nonparametric methods lead to mathematical programming.

from errors, either of sampling or of measurement. One could add to such a list of “favourable or unfavourable” external events, the political climate as characterised by both economic policies and country political risk in a particular country such as Zimbabwe and, additionally, global economic cycles which have seen the price of gold range from US\$35 to over US\$800 over the period since the gold price was freed in 1971.

The most common econometric method is SFA, which is based on the composed error model. The term composed error comes from the fact the specified residual term consists of more than one distinct component (in this case, two). The first component which captures noise, is random error, with a mean of zero and an assumed constant variance. The second term represents an estimate of efficiency and is expressed by one of several distributional forms; half-normal, truncated normal, exponential or two-parameter gamma among others (Lovell, 1993). Consequently, the assumptions made concerning the nature of the distribution of the inefficiency may result in different efficiency estimates, although the relative rankings, an important feature of efficiency measurement, of the DMUs will normally remain the same. A model of this type is generally specified by the relationship

$$y_i = f(\mathbf{x}_i; \beta) e^{(v_i - u_i)} \quad (3.5)$$

where  $i, \dots, n$  indexes the firms,  $y$  represent the single output,  $\mathbf{x}$  a vector of inputs,  $\beta$  a vector of the technology parameters and the error terms are  $u$  and  $v$ . In empirical work, a specific functional form is specified, the most common being a

transcendental logarithmic (translog) specification but here (3.5) is used for expository purposes

In this case, technical efficiency of DMU  $i$ ,  $TE_i$ , relative to the stochastic production frontier, is defined as

$$TE_i = \frac{y_i}{f(\mathbf{x}_i; \beta) e^{v_i}} = \left( e^{-u_i} \right) \quad (3.6)$$

(3.6) allows output-oriented  $TE_i$  to be inferred from an estimate of  $u_i$ . The estimation involves the compound residual approach, where the residual consists of  $u_i$  and  $v_i$ .

The computational challenge in using this approach lies in decomposing the compound residual  $e^{(v_i - u_i)}$  into its individual components,  $v_i$  and  $u_i$ . It can be noted that the expected value of  $v$  (random noise) is zero, i.e.  $E(v) = 0$  implying that on average  $E(v - u) = E(u)$ . The problem is that, although this condition holds on average (over the sample), it does not apply for each individual observation for which an efficiency score, hence decomposition, is necessary. Aigner, Lovell & Schmidt (1977), Battese & Cora (1977) and Meussen & van den Broeck (1977) provided methods for estimating (3.5) but the difficulty in partitioning the compound error term remained.

Jondrow, Lovell, Materov & Schmidt (1982) proposed a method for decomposing the compound error term, using conditional probability density theory (Bayesian analysis) to estimate  $u_i$  from the compound error term,  $(v_i - u_i)$ . In this approach,

inferences are made on the likely values of  $u_i$ , conditional on  $(v_i-u_i)$ . The mean or mode of this distribution is then used to obtain point estimates of efficiency for individual DMUs. Inserting this point estimate into (3.6) provides an estimate of DMU-specific technical efficiency,  $TE_i$ .

This brief outline of the econometric method shows its main characteristics. Its main strength is in explicitly accounting for noise. Through this one is able to distinguish inefficiency from noise. However, by having a pre-specified parametric, functional form there is a danger of specification error if the model is not correctly specified, with this specification error appearing as either noise or inefficiency. It must also be pointed out that SFA is based on a single output model.

Attention is now turned to DEA, which starts from the simple premise that the general form of the production technology is unknown. By not assuming any prior parametric form, DEA avoids combining potential specification-error and inefficiency. The data are instead “allowed to speak” (Førsund et al, 2006). This convenience comes at a cost, though, as legitimate errors of measurement and random noise are ignored. Therefore, if any noise exists, this method simply combines it with inefficiency (Schmidt, 1986).

DEA was developed by Charnes, Cooper & Rhodes (1978) and evolved directly from the work of Farrell (1957). Basic to the concept, DEA allows the estimation of efficiency relative to an estimate of a technologically determined frontier. This frontier is assumed to be the convex hull of the  $(x,y)$  observations and is constructed from the sample of observed DMUs. In its simplest form, DEA involves identifying

optimal solutions to a constrained optimisation problem for each DMU using mathematical programming.

The logic behind DEA is that in a multiple input and output setting, an estimate of efficiency is derived from the following productivity ratio,

$$\text{Efficiency} = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}$$

which, for DMU  $i$ , can also be written as

$$\frac{\sum_{r=1}^q u_r y_{ri}}{\sum_{s=1}^p v_s x_{si}} \quad (3.6)$$

where  $u_r$  is the weight of the  $r^{\text{th}}$  output ( $r = 1, 2, \dots, q$ )

$v_s$  is the weight of the  $s^{\text{th}}$  input ( $s = 1, 2, \dots, p$ )

$y_{ri}$  is the amount of the  $r^{\text{th}}$  output for DMU  $i$ , ( $i = 1, 2, \dots, n$ )

$x_{si}$  is the amount of the  $s^{\text{th}}$  input for DMU  $i$ , ( $i = 1, 2, \dots, n$ )

There is an obvious problem with the above formulation. What are the appropriate weights to each assigned input and output in aggregation? One solution could be to use arbitrary weights for each input and output. This, however, is unsatisfactory as it introduces an element of subjectivity into the choice of weights. DEA addresses this problem of weights by computing virtual inputs and outputs measures for each DMU. Each input and output is then assigned a weight which is the most favourable

for the subject DMU in maximising the efficiency ratio of that DMU. This favourable weighting system is subject to constraint that the efficiency measure cannot be greater than 1.

Using mathematical programming, the DEA efficiency score for unit  $i$ , the DMU under observation, is then estimated by a fractional programme (FP), which is obtained by maximising the ratio of weighted outputs to the ratio of weighted inputs subject to the feasibility constraint imposed by technology.

$$\text{MAX} \frac{\sum_{r=1}^q u_r y_{ir}}{\sum_{s=1}^p v_s x_{is}} \quad (3.7)$$

$$\text{subject to } 0 \leq \frac{\sum_{r=1}^q u_r y_{ir}}{\sum_{s=1}^p v_s x_{is}} \leq 1,$$

$i=1, \dots, n$ . for  $u_r, v_s \geq 0$ <sup>22</sup>. These constraints must hold for all the DMUs in the sample.

(3.7) is due to Charnes, Cooper & Rhodes (1978). The numerator of the objective function is the virtual output while the denominator is the virtual input. (3.7) is then solved for each DMU. The solution to each objective function is found by finding values  $u$  and  $v$  which maximise the value of each DMU's efficiency score, subject to

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<sup>22</sup>To impose weak essentiality,  $u_r, v_s \geq \epsilon$  is imposed where  $\epsilon$  is a small positive number.



the constraint this efficiency cannot be greater than 1. Note that there are  $n$  constraints, one for each DMU.

What DEA essentially does is to treat the inputs and outputs as fixed while varying the input and output weights to maximise the ratio, in (3.7), for each DMU. A score of less than 1 ( $<1$ ) reflects relative inefficiency where potential output is more than observed output or the potential input vector is less than that observed. The problem with (3.6) is that the solutions are non-unique, resulting in an infinite number of possible solutions for each DMU.

The non-uniqueness problem is easily addressed by converting the FP to a linear programme (LP). The set of FP in (3.7), are directly convertible, by linearisation, into tractable LP problems with unique solutions for each DMU. The simplest conversion is accomplished by setting the denominator of (3.7) equal to one. The LP for each DMU now becomes

$$\text{Maximise } \sum_{r=1}^q u_r y_{ir} \tag{3.8}$$

$$\text{subject to } \sum_{s=1}^p v_s x_{is} = 1, \sum_{r=1}^q u_r y_{ir} \leq \sum_{s=1}^p v_s x_{is},$$

where  $i = 1, \dots, n$

$$u_1, u_2, \dots, u_q \geq 0$$

$$v_1, v_2, \dots, v_p \geq 0.$$

One of the attractions of the LP is the existence of a dual to the primal problem in (3.8), known as the Dual Linear Programme (DLP)<sup>23</sup> which can be formulated as

Minimise  $\theta_i$  (3.9)

Subject to

$$\theta_i x_{is} - \sum_{i=1}^n x_{is} \lambda_i \geq 0 \text{ for } s = 1, \dots, p,$$

$$\sum_{i=1}^n y_{ri} \lambda_i \geq y_{ri} \text{ for } r = 1, \dots, q \text{ and}$$

$$\lambda_i \geq 0.$$

The dual is constructed by assigning a [dual] variable for each constraint in the primal programme. Here Min  $\theta_i$  (the dual objective function) is assigned to the first

constraint  $\sum_{s=1}^p v_s x_{is} = 1$  and  $\lambda_i$  is assigned to “ $\sum_{r=1}^q u_r y_{ir} \leq \sum_{s=1}^p v_s x_{is}$ ” (Emrouznejad, A ,

1995-2001).

This dual is known as the CCR model, after Charnes, Cooper & Rhodes (1978).

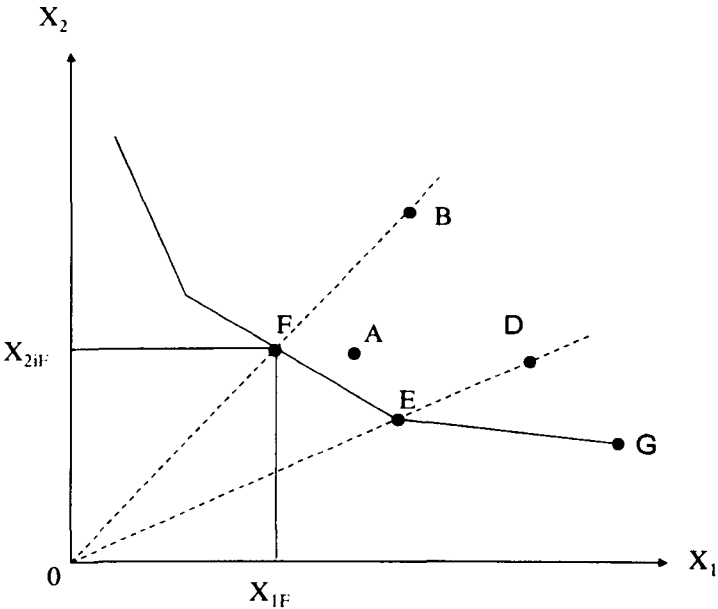
Model (3.9) has  $p + q + 1$  constraints while model (3.8) has  $n + 1$  constraints.

Since the number of DMUs in a sample,  $n$ , is generally [considerably] larger than the sum of number of inputs and outputs,  $p + q$ , the dual formulation has an added attractiveness in that there are fewer constraints than in the primal programme. The

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<sup>23</sup> This is the most commonly used DEA program and known as the “envelopment programme” while the primal-- Equation (3.7)-- is known as the “multiplier programme.”

computational efficiency of some LP methods such as the simplex has been known to fall as the size of the constraint set increases (Ganley & Cubbin, 1992). For this reason the primal (multiplier programme) has, until the recent widespread availability of cheap computing power, rarely been used in applied work. However, given that the primal offers much more information and the cost of computing is no longer a consideration, both methods are used. For expository purposes, the dual is illustrated in Figure 3.3 but the problem being analysed remains essentially the same.



**Figure 3.3 DEA Frontier: Dual Technology**

In Figure 3.3, there are six DMUs, A, B, D, E, F and G which produce a single output at rate, Y, using two inputs,  $X_1$  and  $X_2$ . Also, note that although the rate of output is the same, the rate of inputs usage is different for each DMU. DMUs G, F

and E are efficient as they are lying on the efficient convex piece-wise linear frontier. Hence, no other DMU or linear combination of the efficient DMUs G, F and E can perform better than them. DMUs A, B and D are inefficient relative to the efficient frontier, as, for the same level of output, a linear combination of the efficient DMUs, which lie on the same rays through the origin, are able to use less inputs for the same output. In particular, F produces Y with less of each of  $X_1$  and  $X_2$  than B.

The efficiency of each DMU is found, first by obtaining a reference point (recall the peers) on the frontier. Hence for DMU B, this reference point is given by F, derived by linearly combining the best practice observations i.e. those observations which lie on the frontier and E, which lies close to F. The DMU, under observation is defined as being efficient if, and only if, its efficiency score,  $\theta_i^* = 1$ . Reference points and peers will be discussed further below with reference to Figure 3.4

One drawback with the CCR programme is that it is based on the assumption of constant returns to scale everywhere on the frontier. Returns to scale are a property of the technology and may vary with firm size and, therefore, ought to be tested for rather than imposed. In fact given market imperfections, constraints on access to finance and technology, government health and safety regulations, many DMUs may never operate at the optimal scale. Assuming CRS when undertaking the DEA may confound technical inefficiency with the scale inefficiency, that is, inefficiency which is caused by operating at the wrong scale. This means that technical inefficiency may not entirely be eliminated by radially reducing inputs while maintaining output. It makes sense, therefore, to adopt a more general programme which incorporates various returns to scale and then determine the type of returns of

scale from the results. Such a general DEA, the variable returns to scale DEA (VRS DEA), was put forward by Banker, Charnes & Cooper (1984)<sup>24</sup>.

Following from (3. 9), the dual form of the VRS DEA programme is formulated as

$$\text{Minimise } \theta_i \tag{3. 10}$$

subject to

$$\sum_{i=1}^n y_{ri} \lambda_i = y_{ri} \text{ for } r = 1, \dots, q,$$

$$\theta_i x_{si} - \sum_{i=1}^n x_{si} \lambda_i = 0 \text{ for } s = 1, \dots, p,$$

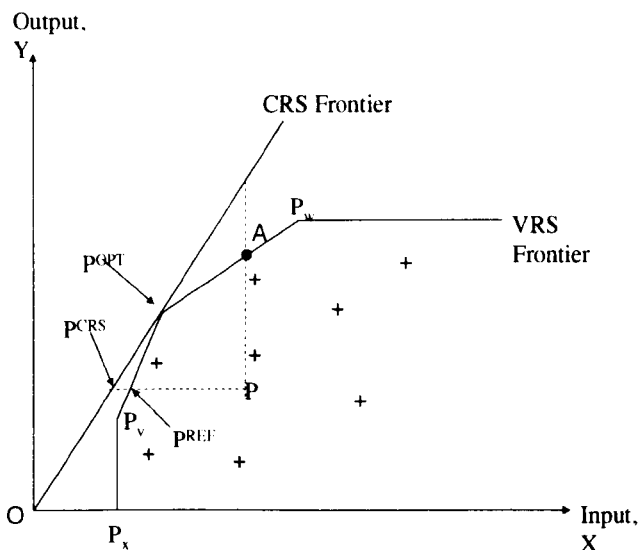
$$\sum_{i=1}^n \lambda_i = 1 \text{ and } \lambda_i \geq 0$$

As can be seen the critical difference between the BCC and the CCR is the addition of the VRS [convexity] constraint, that is  $\sum_{i=1}^n \lambda_i = 1$ , in the former. This constraint allows the frontier to be composed of increasing, constant and decreasing returns to scale segments. It allows a DMU to be benchmarked against DMUs of similar size (Coelli et al, 2005). This ensures that the frontier is composed of multiple convex combinations of best practice (Ganley & Cubbin, 1992). In Figure 3.4, the imposition of  $\sum_{i=1}^n \lambda_i = 1$  ensures that the infeasible part of the CRS ray, where the extension of

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<sup>24</sup> Also known as the BCC programme.

scale efficiency is no longer possible, is excluded. This VRS technology is best illustrated in Figure 3.4 simplified to a one-input, one-output model.



**Figure 3.4 Variable Returns to Scale DEA Technology**

The movement from Figure 3.3 to Figure 3.4 involves switching orientation from fixed output-variable inputs to variable output-one input. This change is for expository purposes and does not fundamentally alter the points being discussed. The diagram shows two technology frontiers, VRS and CRS frontier. The sign, +, denotes observed DMUs which are enveloped by the VRS frontier. For DMU, P, DEA efficiency is first defined by finding a reference point PREF, that is input minimisation without altering the output, which is obtained by using a linear programme such as (3.9). By finding solutions to the programme for all the DMU's, a [VRS] convex piece-wise frontier is obtained, which envelops all the observations as tightly as possible. The reference point

for P, as mentioned earlier, is a linear combination of estimated best practice DMUs which lie on the frontier. In using the variable output-one input orientation, a second possible optimum is identified with point A where output is maximised for the input vector at P.

The theory of linear programming, asserts that in the values of the dual programmes (3.9) and (3.10), that is, the weights,  $\lambda_i$ , are the shadow prices which correspond to the constraints limiting the efficiency of each DMU to be less than 1. These are identical to shadow prices in the primal programme (Chiang, 1986). Hence,  $\lambda_i$  corresponds to

$$\sum_{r=1}^q u_r y_{ir} \leq \sum_{s=1}^p v_s x_{is} .$$

$\lambda_i > 0$  indicates a binding constraint. Specifically, a binding constraint means a strictly positive weight corresponding to and identifying a peer for each inefficient DMU, i.e., for each observed unit  $j$  there is a composite DMU with an output of

$$\sum_{i=1}^n y_{ri} \lambda_i \quad (r=1, \dots, q) \text{ and an input of } \sum_{i=1}^n x_{is} \lambda_i \quad (s=1, \dots, p) \text{ as given in model (3.10) which}$$

is more efficient than observation  $j$  (Coelli et al, 2005). If  $j$  is efficient, the constraint is non-binding and  $\lambda_i$  will be equal to 0. This means there will be no composite DMU.  $\theta_j$ , the estimated efficiency of DMU  $j$ , represents the proportion of the input levels of  $j$  that the [efficient] composite unit would require to produce at least the output levels of  $j$  (Emrouznejad, 1995-2001).

There is another important aspect of efficiency which arises from the solutions to the dual LP in (3.9) and (3.10). This is the concept of scale efficiency. The question answered by estimating scale efficiency is whether or not each DMU is operating at its

optimal scale. To this end, CRS technical efficiency (also known as overall efficiency) is decomposable into two components, technical efficiency and scale efficiency. Using figure 3.4, this is best illustrated by using the point  $P^{OPT}$ , which is where the VRS and CRS frontiers coincide; in general this coincidence may not be a unique point but a segment. The DMU located at this point is exhibiting CRS. This is known as “technically optimal scale” (Førsund et al, 2006). In practice this means the DMU is maximising productivity,  $\frac{y}{x}$ . Scale inefficiency, therefore, measures loss of output because of not operating at the optimal scale.

The idea of the technically optimal scale can be explained using the concept of productivity, again easily explained from the one output-one input orientation of Figure 3.4. Using the basic microeconomic concept of average productivity, the scale efficiency of DMU P is represented by the ratio of the average productivity at P to the average productivity at  $P^{OPT}$ . There is, however, no guarantee that  $P^{OPT}$  is a feasible production plan. Rather,  $P^{OPT}$  merely illustrates a benchmark for comparing average productivity (or total output) which is feasible and the maximum attainable average productivity at any point on the frontier (Førsund et al, 2006). The concept of scale efficiency is similarly defined. Scale efficiency is the ratio of a DMU’s efficiency score under a CRS assumption to its score under a VRS constraint. In terms of Figure 3.4, this means projecting the DMU onto the frontier, and comparing its efficiency to that obtained at  $P^{OPT}$ . Since there are two possible optima, there are also two possible measures of scale efficiency, one in the input direction and the other in the output orientation.



Another way of looking at it is in overall efficiency terms where [input-oriented]

technical efficiency is given by the ratio  $\frac{P}{P^{REF}}$  while overall efficiency is given by

$\frac{P}{P^{CRS}}$ . The ratio of overall to technical efficiency is equal to the scale efficiency of a

given DMU. In Figure 3.4 for DMU P, this is given by the ratio  $\frac{P^{CRS}}{P^{REF}}$ . The three

efficiency measures are all bounded between 0 and 1. Finally in terms of the algebra of

(3.9) and (3.10), scale efficiency is the ratio of the sum of  $\lambda_i$  in model (3.10) to the sum

of  $\lambda_i$  in model (3.9), recalling that  $\sum_{i=1}^n \lambda_i = 1$  in model (3.10) while  $\sum_{i=1}^n \lambda_i$  is

unconstrained in model (3.9). Hence at  $P^{OPT}$ ,  $\sum_{i=1}^n \lambda_i$ , in both models is equal to 1.

However, the measurement of scale efficiency indicates only how far the DMU is from

the CRS frontier. In effect, it captures the difference (not in mathematical sense)

between overall efficiency and technical efficiency. It does not identify the nature of the

returns to scale, that is, there is no distinction between DMUs with increasing or

decreasing returns to scale.

The nature of returns to scale helps highlight another contribution by Farrell (1957).

This was to identify four production possibility sets, each reflecting the nature of returns

to scale under which a DMU is operating. From Figure 3.4, the area bounded by the

input axis, the origin O and the entire CRS frontier (or ray in 2 dimensions) is the CRS

production possibility set. The area bounded by the input axis up to the point  $P_x$ , the

folded piece-wise locus with corners at  $P_v$ ,  $P^{OPT}$ ,  $P_w$  is the VRS production possibility

frontier. The area bounded by input axis up to O, the piecewise locus with corners at O,  $P^{OPT}$  and  $P_w$  is the NIRS production possibility set. The NDRS production possibility frontier is that defined by the area bounded by the input axis up to  $P_w$ , the piecewise linear locus with points at  $P_x P_v$  and  $P^{OPT}$ . It can be seen that the four sets have zones where they coincide. More fundamentally, it can be seen that of the four sets, the frontier identified by the VRS production possibility set more tightly envelops the observations than the alternatives. This is one of the reasons why the VRS programme is the most widely used in DEA.

It can also be seen that there are five different combinations of production possibility sets. These are (a) those where all the DMUs have non decreasing returns to scale, (b) a mixture of DMUs with increasing and constant returns to scale, (c) all DMU have constant returns to scale, (d) a mixture of DMUs with constant and decreasing returns to scale and (e) all DMUs exhibiting decreasing returns to scale. These classifications will be of use when distinguishing the nature of returns to scale. Given that there are possible overlaps, the nature of returns to scale be assessed by running another DEA

programme with non-increasing returns to scale (NIRS) imposed, that is,  $\sum_{i=1}^n \lambda_i \leq 1$ . A

ratio  $\frac{\sum_{i=1}^n \lambda_i CRS}{\sum_{i=1}^n \lambda_i NIRS}$  is obtained. This ratio is, by construction, bounded by 1 below and

unconstrained above 1 (Löthgren & Tambour, 1999). To determine where the DMU is operating, the following conditions are the basis for the decision criteria. When

$$\frac{\sum_{i=1}^n \lambda_i CRS}{\sum_{i=1}^n \lambda_i NIRS} = 1$$

and when the NIRS and CRS efficiency scores are equal to 1, the DMU

is operating under constant returns to scale. When the ratio is equal to 1 but the NIRS and CRS scores are less than 1, then the DMU is operating under increasing returns to scale. Finally, when the ratio is greater than 1, the DMU is operating under decreasing returns to scale. What the criteria also illustrate are the five different combinations of the production possibility sets outlined above.

Finally, a discussion on the relative merits of the SFA and DEA is warranted. There has been much discussion about which technique to use, SFA or DEA. The comparisons have been based on the empirical results and specific contexts. A number of studies show that they substantially agree on the important issue, i.e. the estimated average efficiency (Ganley & Cubbin, 1992). These were the finding by, for example, Lovell & Wood (1992) and Ferrier & Lovell (1990). However, Ferrier & Lovell (1990) found that the rank correlations between the estimates from the two methods were poor at around 0.02 which was not significantly different from zero. Resti (1997), on the other hand, found a high rank-correlation coefficient of between 0.79 and 0.83, particularly with the CCR programmes. Murillo-Zamorano & Vega-Cervera (2001) also carried out a comparative study of parametric and non-parametric methods and concluded by calling for more work on hybrid methods which made joint use of both techniques to improve accuracy of the estimates. One cannot argue conclusively the merits of one over the other. The choice between SFA and DEA is often determined by the availability and

type of data and accuracy or lack of it is a relative point as each is accurate in its own terms.

### 3.4 The Bootstrap and DEA

In applied research, DEA has for a long time been regarded as deterministic and the efficiency scores so calculated are treated as deterministic values. In fact, Simar & Wilson (1998, 2000) have shown that this is an erroneous premise. They argue that the efficiency scores ought to be regarded as estimates of true but unobserved efficiencies, measured relative to an estimator of a true, but unknown, efficiency. Since the estimated production frontier is obtained from a finite sample the frontier is susceptible to sampling variations, as are the measures of efficiency. These will affect the characteristics of the estimated frontier such as its shape and position. The statistical properties of the frontier become important<sup>25</sup>. It follows that estimates have distributional properties which can be estimated. In addition, it is important to remember that the convex piece-wise linear frontier is based on outliers. There is the

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<sup>25</sup> An exception may be when the observations comprise a population. Hence, when we are looking at a data set of primary schools in the United Kingdom which contains *all* the primary schools, the resulting frontier is the *true* frontier (see Coelli *et al*, 2005). An alternative viewpoint on this, which primarily asserts that the Coelli *et al* (2005) statement as it stands above is inaccurate. Hammond (in private conversation) thinks this point is arguably wrong, unless the assumption is made that all one is interested in is UK primary schools. If however, the interest is on UK educational primary educational technology, then the UK primary schools as a whole are still only a sample from the population when inference is considered, i.e. each primary school has selected its position, given the technology, but these choices need not fully represent the technology.

probability, therefore, that there may be other aspects of the technology which are not being captured by these outliers. In particular sampling errors find their way into the data and must be accounted for. In order to obtain these statistical properties and to address some of the weaknesses inherent in the DEA estimates, Simar & Wilson (1998, 2000) have proposed the use of the bootstrap<sup>26</sup>.

The bootstrap is a re-sampling method, first advanced by Efron (1979) which allows the estimation of the standard deviation of the estimator obtained from [nonparametric method] methods. What the bootstrap does is mimic the data generating process (DGP) giving rise to the observed sample and allows the approximation of the unknown distribution of the phenomenon being estimated from the characteristics of the sample.

The idea behind such resampling is that, in the absence of specific information about a population from which sample data are drawn, particularly the distributional properties, a process of resampling from that sample may be used to assess, with a reasonable degree of objectivity, the sampling variations which affect the data (Young, 1994). In the bootstrap, a resampling of the sample data is used as a guide to what would happen if the population were resampled.

Another way of justifying the bootstrap is to imagine a bootstrap world and the real world (of which little or nothing is known). One can then make inferences about the real but unknown world by observing what is happening in the bootstrap world. Although there may not be exact correspondences, such as the actual magnitudes of the variables

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<sup>26</sup> Simar & Wilson (1998) have shown that the DEA estimate is this bias is non-negative in that the unknown frontier is less than or equal to the estimated DEA frontier.

being measured, it is expected that they move in parallel directions and magnitudes. In the case of DEA, the bootstrap allows inferences to be drawn about the reliability of efficiency estimates and the construction of confidence intervals for such estimates. The bootstrap method has been widely used in medical and environmental studies, saving on costly sampling and assaying. It has also recently been adopted in economics and management science, both in parametric and non-parametric analysis. See for example, Førsund et al (2006) and Gonzalez & Miles (2002) for non-parametric analysis. More will be reviewed later. For parametric applications, see Levich & Thomas(III) (1993), Horowitz (2003) Dalla & Hidalgo (2005) among others.

The bootstrap can be motivated by the following example. Suppose one has an independent and identically distributed random sample  $X = (X_1, X_2, \dots, X_n)$ , such as observations of firms or decision making units (DMU), from an unknown probability distribution,  $F$ , such that  $X_i \sim iid F$ . Consider the case of an efficiency score parameter, being a random variable,  $\theta$ , which depends on  $X$  and possibly  $F$ . Neither the true value nor the sampling properties of  $\theta$  are known. The sampling distribution of  $\theta$  can only be “guessed at” based on information provided by the observed data on  $X$ . The bootstrap is a useful tool in constructing evidence of the distribution  $\theta$ . Bootstrapping involves resampling the observed data with replacement and re-estimating the parameter of interest,  $\theta$ , each time resampling takes place. From this re-sampling procedure, an empirical estimator of  $F$ , denoted by  $\hat{F}$ , can be constructed. From the distribution of  $\hat{F}$ , inferences on the distribution of  $F$  can be made. The calculated distribution of the estimator  $\hat{\theta}$ , based on  $\hat{F}$ , therefore approximates to the true distribution of  $\theta$  if  $F = \hat{F}$ . The key point made by Efron (1979) is that, in the absence of any prior restrictions on

the form of  $F$ , any nonparametric estimator (such as  $\hat{\theta}$  obtained by DEA), which relies on  $\hat{F}$  to mimic  $F$ , must be a reasonable approximation since “ $\hat{F}$  is a central point among the class of likely  $F$ s”. The bootstrap distribution  $\hat{F}$  becomes the basis for calculating and estimating the biases, standard errors and the resulting confidence intervals of the estimates of non-parametric analysis.

There is a problem with the above procedure as it stands. Non-parametric estimation, particularly of bounded domains such as the DEA (the efficiency score is bounded between 0 and 1), has been shown to be inconsistent. It has been proved that the efficiency estimator (at the upper bound) for a naïve bootstrap (unsmoothed) is inconsistent (Efron & Tibshirani, 1993). This is because of the one-side nature of the residuals, since as the sample size increases the bias does not decrease. This case can be compared with the maximum likelihood estimator in parametric estimation, which while also biased has a bias which asymptotically falls away with an increasing sample size. This bias, as noted by Simar & Wilson (1998) and Efron & Tibshirani (1993) is not inversely related to the sample size and will therefore not disappear as the sample approaches the population but is strictly positive<sup>27</sup>. Hence their bootstrapped empirical

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<sup>27</sup> Simar & Wilson (1998) define the probability of selecting a DMU as  $(1-n^{-1})^n$ . Therefore the probability that the pseudo-sample will include the DMU-- which is the same as the probability that the bootstrap value will equal the original estimate—is  $\Pr(\hat{\theta}^* = \hat{\theta}) = 1-(1-n^{-1})^n$ .  $\lim_{n \rightarrow \infty} 1 - (1 - n^{-1})^n = 1 - e^{-1} \approx 0.632$  and not 0.

density function (EDF) still had a mass of observations at the upper bound<sup>28</sup>. Therefore, the EDF bootstrap estimates, as described above, are inconsistent. In the case of DEA, this inconsistency mainly arises near the upper bound of the distribution where a large number of DMUs are seemingly efficient. To address this problem, Simar & Wilson (1998) proposed smoothing the bootstrap DEA by replacing the EDF with a kernel density estimate (KDE). The Simar & Wilson (1998) proposal for smoothing is based on a Gaussian KDE represented as

$$\hat{F}_h(t) = \frac{1}{nh} \sum_{i=1}^n \phi\left(\frac{t - \hat{\theta}_i}{h}\right) \quad (3.11)$$

where  $h$  (a choice variable for the researcher) is the smoothing parameter and  $\phi$  is the density of the standard normal variate. The smoothing parameter,  $h$ , is also known as the bandwidth of the kernel density estimator,  $\hat{F}_h(t)$ . The value of  $h$  must be chosen with care, a high value tends to over-smooth while smaller values lead to multi-modal distributions and at any rate tend to place too much weight near the upper limit 1.

Silverman (1986), in some detail, discusses methods for determining the parameter  $h$  under various conditions and leaves the choice to the researcher to decide depending on the purpose to which the KDE is to be put. A simple (it is automatically related to the statistics of the sample) and effective method he proposed is the “automatic robust rule”

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<sup>28</sup> The empirical density function is a non-parametric estimate of the underlying density function of the random variable.



for selecting bandwidth, given by

$$h = 0.9 \frac{1}{\sqrt{n}} \min\left(\hat{\sigma}_{\hat{\theta}_i}^2, \frac{R_{13}}{1.34}\right) \quad (3.12)$$

where  $R_{13}$  and  $\hat{\sigma}_{\hat{\theta}_i}^2$  respectively denote the inter-quartile range and the variance of the empirical distribution,  $\hat{\theta}_i$ .

A number of bootstrap algorithms, for the DEA, have been proposed in the literature<sup>29</sup>. In the present study, the Simar & Wilson (1998) bootstrap algorithm is adopted. The practical implemented is as follows.

Estimate DEA efficiency scores,  $\hat{\theta}_i$  for the sample  $i = 1, \dots, n$ . Transform the original input and output vectors using the estimated DEA score,  $\hat{\theta}_i$ . This produces a pseudo-frontier  $(\hat{x}_i^f, y_i^*) = (x_i \cdot \hat{\theta}_i, y_i)$  for which  $\hat{x}_i^*$  is an estimated efficient input vector to produce the output vector,  $y_i$ .

Smoothed re-sampled pseudo efficiencies denoted by  $\theta_i^*$  are obtained. In obtaining  $\hat{x}_i^f$  in Step 1, there still remains a problem with  $\hat{F}_h(t)$  in that it does not integrate to 1 i.e. the boundary condition that  $t < 1$  is sometimes violated and  $\hat{F}_h(t)$  is still biased and inconsistent<sup>30</sup>. This problem is addressed in two steps. First, a reflection method

<sup>29</sup> Ferrier & Hirschbeger (1997), Simar & Wilson (1998) and Löthgren & Tambour (1999).

<sup>30</sup> This problem is the source of controversy in some empirical studies as will be noted later in this Chapter, particularly in the review of studies by Ferrier & Hirschberg (1997) and Löthgren & Tambour (1999).

proposed by Silverman (1986) overcomes the problem of violating the bounded nature of the efficiency score.

Hence for each point  $\hat{\theta}_i^* \leq 1$ , there is a symmetric reflection given by  $2 - \hat{\theta}_i^* \geq 1$ ,  $i = 1, \dots, n$ . The reflection method generates a new sequence of efficiency estimates given

$$\text{by } \tilde{\theta}_i^* = \begin{cases} \hat{\theta}_i^* + h\rho_i^* & \text{if } \hat{\theta}_i^* + h\rho_i^* \leq 1 \\ 2 - \hat{\theta}_i^* - h\rho_i^* & \text{otherwise} \end{cases} \quad (3.13)$$

where  $\rho_i^*$  is a random normal deviate drawn from a standard normal distribution (Simar & Wilson, 1998).

Therefore, if  $\hat{\theta}_i^* + h\rho_i^* > 1$ ,  $\tilde{\theta}_i^*$  is set equal to the symmetric image of  $\hat{\theta}_i^* + h\rho_i^*$  (which is a reflected on the boundary).

Second, a correction of the re-sampled estimated efficiencies, denoted by

$$\theta_i^* = \bar{\theta}_i^* + \frac{(\hat{\theta}_i^* - \bar{\theta}_i^*)}{\sqrt{\frac{1+h^2}{\hat{\sigma}_\theta^2}}}$$

is generated where  $\bar{\theta}_i^* = \sum \theta_i^*$  and  $\hat{\sigma}_\theta^2$  is the sample variance of

the original, i.e. non-corrected DEA efficiency scores. The correction is based on the use of the KDE in (3.10) rather than the empirical density function (EDF), which generates  $\hat{\theta}_i^*$ . This correction guarantees that  $\theta_i^*$  has the same first two moments as the original efficiency estimates,  $\hat{\theta}_i^*$  (Simar & Wilson, 1998).

The bootstrap pseudo-data is now given by  $(x_i^*, y_i^*) = (\frac{\hat{x}_i^f}{\theta_i^*}, y_i)$ .

Bootstrap efficiencies, using the pseudo-data in step 3, are estimated using the linear programmes (3.8) and (3.9) to generate

$$\hat{\theta}_i^b = \min_{\theta, \lambda} \left\{ \theta : -y_i - \sum_{i=1}^n y_{ri} \lambda_i, \theta x - \sum_{i=1}^n x_{si}^* \lambda_i \geq 0, \sum_{i=1}^n \lambda_i = 1, \lambda_i \geq 0 \right\}$$

Steps 2-4 are repeated B times (where B is normally 1000) to generate B DMU specific efficiency estimates.

Simar & Wilson (1998) have indicated that DEA estimators, in common with other nonparametric estimators, are biased in small samples and they proposed a method for correcting this bias. An estimate of the bias of the efficiency score is denoted by

$$bias^* = \frac{1}{B} \sum_{b=1}^B \hat{\theta}_i^b - \hat{\theta}_i$$

that is the difference between the mean of the smoothed

bootstrap scores and the original unsmoothed DEA scores. To correct for the bias of the smoothed bootstrap DEA, the sequence of B bootstrap DEA scores is ordered in terms

of size. Bias-correction takes the form  $\left[ \hat{\theta}_{iB}^{*b\alpha} - 2 \cdot bias^*, \hat{\theta}_{iB}^{*b(1-\alpha)} - 2 \cdot bias^* \right]$

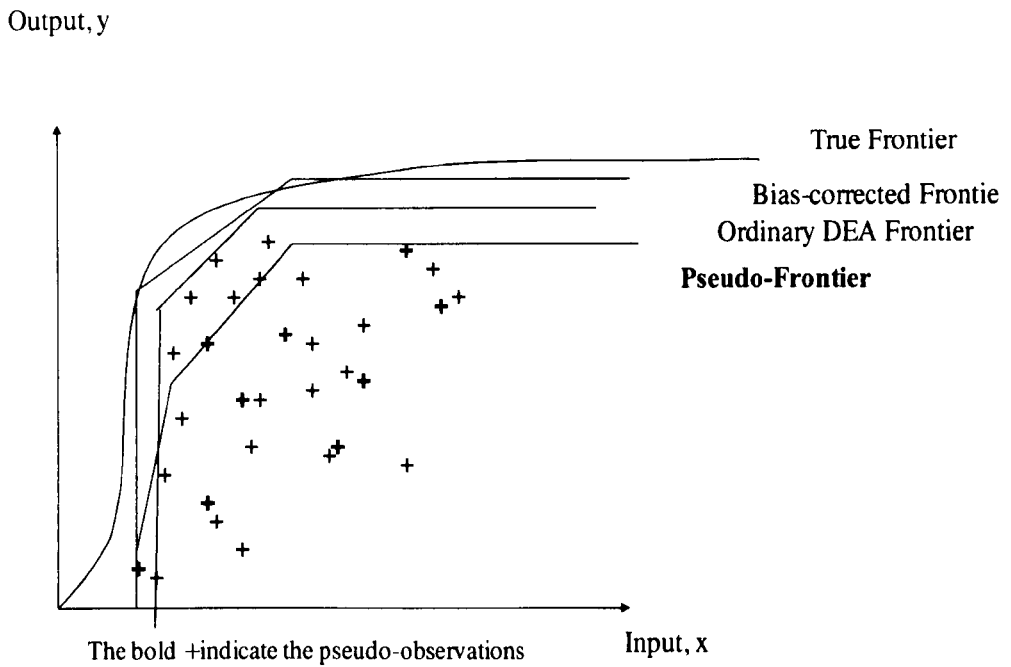
where  $\hat{\theta}_{iB}^{*b\alpha}$  is the  $\alpha^{th}$  percentile of the estimated distribution  $\hat{F}$ , that is by shifting the bounds in the intervals by the factor  $2bias^*$  (Simar & Wilson, 1998).

The statistical properties of nonparametric efficiency estimates can now be approximated, “correcting” the common misconception that methods such as DEA, are

deterministic. With the bias-corrected estimated efficiency scores hypothesis tests, such as testing the difference between means or point estimates, can be carried out.

Using data for a sample of Illinois power stations analysed by Färe et al (1989), Simar & Wilson (1998) illustrated an implementation of the above smoothed bootstrap and obtained smoothed efficiency estimates. Their work, although more illustrative than anything else, is particularly important in showing how smoothed estimated DEA efficiencies can be obtained in practice. Their methodology is the one which is largely followed in the next two chapters for two samples gold mines.

The relationship between the true unobserved frontier, the bias-corrected DEA frontier and the pseudo-frontier described by the bootstrap algorithm above, is illustrated in Figure 3.5.



**Figure 3.5 Variable Returns to Scale DEA Technology<sup>31</sup>**

The main points illustrated in Figure 3.5 are that the bias-corrected frontier can lie both above and below the true frontier while the pseudo-frontier always lies below both which is why the bold + are never above the pseudo- frontier. A point on the true frontier is obtained by shifting the pseudo-frontier by twice the bias estimate (Førsund et al, 2006).

<sup>31</sup> This diagram is adopted from Førsund et al (2006).

One of the major uses of the bootstrap results is the realisation of statistical estimates of parameter such statistics as the standard deviation. With the bias-corrected efficiency estimates calculated, there are some statistical tests which can be performed. Banker (1993) and Banker & Natarajan (2004) proposed some useful statistical tests of DEA results. In the context of this dissertation, the most relevant are the hypothesis tests which compare two groups of DMUs, particularly whether the differences in efficiencies between them are statistically significant or not. The two tests discussed below depend on certain assumed distributional properties.

The first test involves dividing the sample into two sub-samples, one of size  $m_1$  and the other  $m_2$ , and assuming an exponential distribution for the inefficiencies of each. The

test statistic,  $\left[ \frac{\sum_{i=1}^{m_1} (1 - \theta_i^*)}{m_1} \right] / \left[ \frac{\sum_{i=1}^{m_2} (1 - \theta_i^*)}{m_2} \right]$  is assumed to be asymptotically the ratio of two  $\chi^2$  variates

divided by their sub-sample sizes. The null hypothesis to be tested is formally stated as

$$H_0: \sigma_1 = \sigma_2$$

$$H_1: \sigma_1 > \sigma_2.$$

This is the F-test familiar from intermediate statistics. The test statistics has an F-distribution with  $(2m_1, 2m_2)$  degrees of freedom.

If the inefficiency results are assumed to follow a half-normal distribution<sup>32</sup> with

standard deviations  $\sigma_1$  and  $\sigma_2$ , then  $\sum_{i=1}^{m_i} \left( \frac{(1 - \theta_i)}{m_i} \right)^2$ ,  $i=1,2$ , is a  $\chi^2$  with  $m_i$  degrees of

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<sup>32</sup> There are no negative efficiency estimates so the negative part of the normal distribution falls away.

freedom. Hence the test can be conducted based on the statistic  $\left[ \frac{\sum_{i=1}^{m_1} (1 - \theta_i^*)^2}{m_1} \right] / \left[ \frac{\sum_{i=1}^{m_2} (1 - \theta_i^*)^2}{m_2} \right]$  which

obeys the F-distribution with  $(m_1, m_2)$  degrees of freedom.

Finally, an important point which seems to be misunderstood by a number of researchers, needs to be re-emphasised. The bootstrap addresses sample variability but not noise arising from mis-specification or errors in measurement. The bootstrap, in the words of Coelli et al (2005) provides “an indication of the degree to which the efficiency estimates are likely to vary when a different sample is randomly selected from the population”

### 3.5 Review of Selected Literature

Following Efron’s (1979) article, there has been considerable interest in the use of the bootstrap. Most of this interest has centred on theoretical discussion on the feasibility and relevance of the procedure. Initially there was much scepticism, as the discussions following DiCiccio & Efron (1996) in the same issue of *Statistical Science* indicate. Hence, until recently, examples of practical implementation of the bootstrap in economics were conspicuous by their rarity.

As noted earlier, the bulk of the implementation of the bootstrap has been in medicine and environmental analysis, where assaying and measurement are notoriously difficult and expensive, and simulation and extrapolation methods find ready acceptance.

Although, there is widespread acceptance of the bootstrap, there still have not been many studies which have applied the bootstrap to DEA; most of the work has been from

a theoretical perspective. In this review, focus will be centred on two types of studies; DEA studies done on mining and some bootstrap DEA studies.

In a study of banking efficiency, Ferrier & Hirschberg (1997), used the bootstrap DEA to obtain the statistical properties of the efficiency scores of Italian banks for the year 1986. Their data set consisted of ninety-four DMUs, using five inputs and four outputs. Using a modification of the conventional bootstrap-- sampling with replacement in a sub-sample of  $n-1$  observation (i.e. an observation was deleted) — they obtained a bootstrapped empirical distribution of efficiency estimates. The main weakness of the above approach, highlighted by Simar & Wilson (1999), was that they did not address the biased nature of DEA in constructing the empirical distribution of score. A curious part of their results was the presence of negative biases, so that that bias-correction may, in principle result in efficiency scores higher than 1 (although it did not).

Löthgren & Tambour (1999) considered and estimated VRS DEA scale efficiencies of Swedish eye hospitals. Their main objective was to test firm-specific scale efficiency, that is, test the hypothesis whether the scale efficiencies were significantly different from 1. Their data came from twenty-nine public ophthalmology departments operating in 1993, which represented 85 per cent of the total number of ophthalmology departments in Sweden. The production technology was defined over three inputs and four output variables. Two categories of labour, corresponding to two types of physicians and the number of available beds (representing maximum capacity) as a proxy for capital were the inputs. Output was represented by three types of ophthalmologic procedures and the number of visits to the eye clinics. An important methodological proposition was the calculation of the bootstrap scale efficiencies. To



obtain these, they computed bootstrap CRS and VRS efficiency scores and, for each iterative computation, calculated the bootstrap scale efficiency which is significant difference from what has been done in the present study as described in equations (3.11) - (3.13) above. The mean scale efficiency for each DMU, the standard deviation and other statistics were then calculated over these replications.

The main findings were that the DEA identified a third of the DMUs as scale efficient. Of the remaining, eight were operating under increasing returns to scale and eleven decreasing returns. In the context of their study, they noted that neither large nor small eye clinics were scale efficient; rather it was among the medium-sized units which were scale efficient. They noted that five clinics at large hospitals operated in a region of decreasing returns to scale while all departments at other hospitals operated in a region of increasing returns to scale which is contrary to conventional neo-classical expectations.

Using the bootstrap DEA results, they could not reject the hypothesis that the eye clinics were “scale inefficient” and in fact were exhibiting decreasing returns to scale for eleven units. This confirmed the DEA results. However, only seven were deemed scale efficient, compared with ten in the DEA. Five eye clinics were deemed to be operating under increasing returns to scale compared to eight in the DEA. Of the remaining six, no decision on the nature of returns to scale could be made, even though the DEA had categorised them as scale-inefficient. The overall conclusion that they made was that that bootstrap altered one third of the initial DEA results, changing them from scale inefficient to scale efficient. A drawback with their work was that they used the naïve bootstrap, neither correcting for bias nor smoothing to remove inconsistency of results

at the upper bound (Simar & Wilson, 1998). They did acknowledge that their procedure differed from Simar & Wilson (1998) and hence expected different results.

Gonzalez & Miles (2002) carried out a study of efficiency in the Spanish public sector. The objective of their study was to investigate how DEA efficiency scores and conclusions drawn from them are affected by bootstrapping and correcting for bias. They applied the bootstrap to two data sets, one for the high courts and the other for fire services. They applied two different algorithms, one by Löthgren & Tambour (1999) and the other due to Simar & Wilson (1998) to each data set. Their main conclusion was that bootstrap DEA scores gave greater scope for improved performances as they tended to be lower than ordinary DEA scores.

Pedraja-Chaparro & Salinas-Jimenez (1996) applied the DEA to twenty-one high courts circuits in Spain for the year 1991. The DEA scores were then bias-corrected and confidence intervals constructed. Testing the hypothesis of difference in efficiency score between different production units, they could not reject the null hypothesis of no significant differences, in efficiency, in the majority of cases. This was because of the overlapping intervals.

Hawdon (2003) made a study of efficiency and regulation of the gas industries in thirty-three countries for a two year-period (1998-1999) using a DEA model specified for two outputs and two inputs. The primary objective of his study was to measure the relative performances of the gas suppliers and to assess how far reforms such as deregulation, particularly in the United Kingdom, had affected the efficiency of the gas suppliers. For the second part of his study he employed a nesting of DEA models by progressively

removing variables and testing the effects of the removal of those variables. That part of the analysis will not be reviewed here. However, there is enough work done on the application of the bootstrap which justifies the inclusion of this study here.

The two inputs were labour and capital, with the head count of employees representing labour and the length of pipelines representing capital. The two outputs were gas consumption (measured in sales) and the number customers served by the gas supplier.

The results of the analysis are divided into DEA and smoothed bootstrap DEA. The smoothing procedure was based on the smoothed kernel density function following Simar & Wilson (1998). Hawdon (2003) found that, although the bias between DEA and bootstrap DEA was mainly positive, indicating that DEA over-estimates efficiency, there are cases of negative bias. He concluded from this that bias is not stable, particularly when calculating over a number of years. Hawdon (2003) was also able to identify the best-performing countries using bias-corrected DEA. He also noted that using bias-correction reduced the variability of the efficiencies, in that the differences among the various countries became much smaller than in DEA. Finally, he noted that confidence intervals overlapped therefore making it difficult to establish the statistical significance of the differences between countries.

Boame (2004) carried out a study of the determinants of efficiency in the Canadian urban transit system with the objective of estimating efficiency. His study involved two-stage analysis where the second stage was an application of Tobit regression using the results of DEA and bootstrap DEA as the dependent variable. The second part of the analysis will not be reviewed here. Instead the focus will be put on the DEA and the

application of the bootstrap. The bootstrap DEA was based on the Simar & Wilson (1998) algorithm for smoothing the bias-corrected DEA. The sample consisted of 30 transit systems covering the period 1990-98. He specified a VRS production technology with three inputs and one output using annual data. The inputs were fleet size (to capture capital flows), fuel (both diesel and petrol) and labour measured in paid employee hours for each year. The output variable was represented by revenue kilometres (the total distance for total fare passengers carried).

There were two sets of results for the first stage; the DEA and bias-corrected DEA. The DEA results reflected high efficiency scores with two transit systems achieving full efficiency over the whole study period. Over the whole sample, the mean efficiency was 0.8609, implying potential savings of inputs of over 13 per cent without any reduction in output. He, however, observed dramatic reductions in efficiencies once he had corrected for bias with the sample mean dropping to 0.7844. This implies that the potential input savings has actually risen to just under 22 per cent, suggesting that the use of the bootstrap in efficiency analysis is justified.

Boame (2004) also constructed confidence intervals for each estimated of the efficiency score. He noted that the 95 per cent confidence intervals for the bias-corrected scores contained the bootstrap DEA scores from which he concluded that, given the confidence intervals, 95 per cent of the time these intervals will contain the true technical efficiency estimates.

Finally, he observed that most transit systems (56 per cent) experienced increasing returns to scale which he took to imply the case for providing subsidies to urban transit

systems. The main benefit derived from this study is the improvement in terms of accuracy of bias-corrected efficiency estimates over DEA.

In a study of Chinese rural credit cooperatives (RCC), Dong & Featherstone (2006) applied the bootstrap on a sample of one hundred and forty-five observations over a five year period (1991-1995). The objective was to estimate the efficiencies of and identify the technology of providing rural credit. In addition, they also wanted to decompose overall efficiency into technical efficiency and scale efficiency and identify the primary cause of overall inefficiency.

The main findings were that the DEA estimated efficiencies of the different RCC were quite close to each other, indicating the effect of central mandates. There were also some dynamic effects which were identified by the varying nature of returns to scale over the five year period. Still, they were able to identify one clear best-practice RCC after bias-correction.

Førsund et al (2006) carried out a study of the efficiency and productivity development of local Norwegian tax offices. The analysis had two main parts, one concerned with efficiency using DEA and the other on productivity using the Malmquist productivity indices. This review will concentrate on the DEA section of the work. The objectives of the study were to provide explanations of the differences in performance among ninety-eight Norwegian tax offices over a three-year period (2002 to 2004). From these explanations, the tax authorities would then implement reforms to reduce in efficiencies.

Førsund et al (2006) specified a technology with six outputs, all justified by the types of activities in which the tax offices were involved which is processing tax returns. The six

outputs were (i) the number of people who have moved locations, (ii) number of false registrations detected by internal control activities, (iii) number tax returns from employees and pensioners, (iv) number of complaints on tax assessment, (v) number of returns from non-incorporated businesses and (vi) number of corporate tax returns. Only one input, the total operational expenditure of each office, was specified. It is quite clear that outputs and inputs are defined by the context of the study and in particular, the type of industry being investigated. The multiple outputs were the major reason why DEA rather than SFA was adopted.

The efficiency analysis used the bootstrap to improve the quality of the policy recommendations. In particular, they identified the inherent bias as a key problem with DEA results. The methodology applied, the DEA and bootstrap algorithm, were based on Simar & Wilson (1998). There were two key but general results. The first was that the large tax offices (measured by one of the output dimensions) tended to be located in the middle of the distribution. On the other hand, the smaller and medium offices were mostly located in the right side of the distribution, implying that they (the latter) tended to be more efficient than the large ones. The second key result was the rejection of CRS on favour of VRS in terms of being better suited to explain scale efficiency scores.

There were further profound results from bias-corrected efficiency estimates in that they were able to characterise a best-practice tax office in relation to the sample mean. The first observation was that the average best-practice unit used 46 per cent more of the input than the sample average. It also produced 75 per cent more registration of relocated people, 69 per cent more tax returns from employees and 62 per cent more tax returns from firms. Of the remaining outputs it would register 28 per cent more

registration of non-incorporated businesses, handle 10 per cent more the number of complaints and detected 5 per cent less false registrations.

Another observation stemmed from the confidence intervals constructed from the bias-corrected scores and using the statistics generated by the bootstrap. They noted that the confidence intervals tended to be wider for those DMUs with an original DEA score of or close to 1, that is, the efficient units. With a smoothed bootstrap correction, these intervals became markedly narrower but still wider than for the least efficient DMUs. They also noted that the left-hand part of the [ranked] distribution, containing the least efficient units, was almost exclusively populated by the smallest tax offices, with the large units located in the middle of the distribution. The right-hand part, containing the most efficient units, interestingly also contained small-sized units and medium sized units. The final observation was that the [smoothed] bias-correction had a systematic downward effect on the efficiency estimates, confirming the conventional view that DEA scores have a positive bias and that DEA, on its own, seems to provide an overly optimistic picture of efficiency. This is profound result which justifies the use of the bootstrap approach in DEA efficiency estimation.

The above reviews focussed on the application of the bootstrap to DEA. The next set look at the application of DEA to mining. There have been very few studies applying DEA to mining—most have been concerned with agriculture and finance. There are three studies, all on coal which are reviewed here. None of them applied bootstrap DEA. Their main relevance is in setting the context such as the specification of mining technology and how certain inputs are defined

Byrnes, Färe & Grosskopf (1984) carried out a study of the efficiency of fifteen Illinois coal mines by applying DEA. The bootstrap was not applied; rather the results of the nonparametric results-- they did not call their procedure DEA but attributed their model to Cooper, Charnes & Rhodes (1978)-- were reported as they were. They specified one output, the total tonnage of the coal produced and eight inputs. The eight inputs included labour, as measured by miner days. The other inputs were three capital variables and four geological inputs. The three capital variables were represented by the sizes (in cubic feet terms) of three types of capital equipment, shovels, draglines and excavators. The geological variables, non traditional inputs in the sense of the production function in that they are not paid for, were justified by the fact they distinguished the different qualities of ore across the observations, an important consideration when comparing mining operations. This incorporation of geological information was repeated in a latter study by Byrnes & Färe (1987) and is an example of how the production technology can be defined by the context of the study. This is pertinent for the purposes of this dissertation which is similarly based on defining a mining technology.

The main finding from their study was that inefficiency mainly was a result of mines deviating from the optimal scale rather than from technical inefficiency, with six mines out of fifteen being scale inefficient. Output could have increased by 36 per cent had the operations been scale efficient. All were technical efficient with only one was exhibiting congestion inefficiency. They also observed that both the larger and smaller mines (defined inn terms of output) were all scale efficient. They also attempted to distinguish some mine characteristics based on their inputs and output. In this regard, they noted



that efficient mines generally had low labour-output ratios, a point which will be relevant for this study. Finally, they noted that, contrary to their and maybe conventional expectations, the mines with increasing returns to scale were not the small ones but medium sized ones. Again, this would be an interesting characteristic to explore for in this dissertation.

Byrnes & Färe (1987) estimated the efficiencies of US interior coal mines<sup>33</sup>. The study was motivated by official concern at low and declining productivity among US coal mines. The main objective of the study was to estimate the relative efficiency of a cross section of US coal mines. The secondary objective was to decompose overall efficiency into technical, scale and congestion components. They subdivided the mines into five different categories; by location, age, union status, the amount of the captive coal produced<sup>34</sup> and ratio of area reclaimed to area stripped.

The sample consisted of one hundred and eighty-six mines with a wide range of sizes, in terms of output, from two thousand and thirty to over six million short tonnes of coal for the year 1978. They specified nine input variables, a labour input, six types of capital and two variables to represent the geological characteristics of the mines.

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<sup>33</sup> Arkansas, Illinois, Indiana, Iowa, Kansas, Kentucky, Missouri, Oklahoma and Texas.

<sup>34</sup> The captive coal category distinguishes between mines which sell *all* their output on the open market and those which do not.

The data were confidential so the results for individual mines were not reported. All the mines were categorised according to geological characteristics. In addition, the relative efficiencies were in relation to all the other mines, that is one technology was assumed.

The results indicated that efficiency was difficult to achieve with only fourteen mines (7.5 per cent of total sample) having a score of 1. In terms of the components of overall efficiency, about 80 per cent purely technical efficient, while 25 per cent exhibited congestion (efficiency could be increased by reducing usage of some inputs). This indicated a departure from free disposability assumption. Only fifteen mines were scale efficient. The rest were scale inefficient, one hundred and sixty four operating under increasing returns to scale and seven exhibiting decreasing returns. There were some cross state variations in efficient mines, with 88 per cent of the observations for Texas being efficient. There were other results which were correlated to the categories outlined above.

For the purposes of this study, the most interesting point is how the production technology was specified. In particular the incorporation of the geological characteristics as represented by seam thickness and inverses of overburden extracted would justify the choice to be made in Chapter 5 to include grade of ore as an input. Additionally, when representing capital, Byrnes & Färe (1987) used the value of annual services, as in repairing and maintaining equipment, (in thousands of 1981 US\$) to represent two of the capital categories. This method of proxying for capital will be used in Chapter 4, where the annual cost of servicing capital services is used as proxy for capital services.

Finally, one practical implementation of the hypothesis tests, which were first suggested in Banker (1993), was carried out in Banker et al (2004). The objective of the study was to analyse the trend of both technical allocative efficiency in Texas public schools over the period 1993-99 and test for the presence of allocative efficiency. This was a panel data sample, containing three thousand five hundred and ten observations for five hundred and eighty-five districts. Hence the efficiencies were being estimated for each district. Again, as with the Byrnes et al (1984) study, there was no implementation of the bootstrap. What is of relevance to this study is the application of the statistical tests proposed in Banker (1993). The Banker (1993) tests were used to test the whether any observed differences among the districts, now divided into three regions, were statistically significant. The results of the statistical analysis showed the presence of significant inefficiencies over time. The differences among the regions were also found to be significant and, hence, were able to report which, among them, was the most allocatively inefficient region.

The challenges which other researchers have encountered and how they addressed them will be useful in the analyses which follows in the next two chapters. In particular, the way some have approached the problem of accounting for capital services is informative as this is an awkward variable to capture. The other important lesson is the possibility of modification of the specification of the mining production technology to take into account of “non-paid for” but important variables which represent the different qualities of the different mines.

Finally, the practical implementation of the Banker (1993) tests is of taken up in the hypothesis testing in the following chapters.

## **CHAPTER 4 DEA AND BOOTSTRAP DEA ON ZIMBABWEAN GOLD MINES**

### **4.0 Introduction**

In Chapter 2 the importance of gold to the Zimbabwean economy was discussed. Also highlighted was its capacity to earn foreign currency, its being the basis for infrastructure such as electricity generation, and of gold mines as sites for most of the modern industrial cities. The problems that the gold mining sector has faced and continues to face, since 1965, were also discussed. These included particularly the tendency of successive political regimes, for one reason or another, to regard gold mining as a “cash cow” seemingly to be continually exploited, while being offered little support through, for example, the promotion of investment, access to foreign currency and adoption of new technology. These points, the economic importance of gold mining and political neglect, highlight the need for a well-performing gold mining sector, with its own internally-driven initiatives. An important exercise in pursuing this need is to analyse the performance of gold mines in Zimbabwe with a view to measuring the efficiency of individual gold mines. This estimated efficiency can then be used, in conjunction with other variables, to characterise efficiency in gold mining as in Byrnes et al (1984).

There are two primary objectives in carrying out this study. The first is to estimate empirically the relative technical efficiency of individual gold mines in Zimbabwe. This will reveal, within the local Zimbabwean context, the performance pattern of gold mining through the measurement of efficiency scores. The estimated technical efficiency scores are also decomposed into technical and scale efficiencies in order to

distinguish the nature of the inefficiency identified. The decomposition of overall efficiency into technical and scale efficiency allows the primary causes of inefficiency in the sample to be properly attributed. Byrnes et al (1984) observed that the relationship between nature of returns to scale and size does not always correspond to expectations. Hence the smaller mines do not necessarily exhibit increasing returns to scale, for example. As Cooper et al (2000) ask, is inefficiency “caused by the inefficient operation of the DMU itself or by the disadvantageous conditions under which the DMU is operating?” This dissertation will attempt to investigate whether the first part of this question is true. A secondary objective to the estimation of efficiency, linked to the notion of scale efficiency, is to investigate whether any link exists between mine size and efficiency score. The conventional wisdom would seem to suggest that for capital intensive projects, such as those found in mining, economies of scale are best captured by investing in large operations. An analysis of a possible relationship between size and efficiency is carried out and a hypothesis that size and efficiency are positively correlated is tested. In addition, using some of the techniques proposed by Byrnes et al (1984), this study will also use the input-output properties of individual mines to characterise the mines, such as whether inefficient mines have high labour-output ratios etc.

The second objective is to estimate the statistical properties of the efficiency estimates, using bootstrap DEA. From these, estimated properties, such as bias of the DEA, corrections are made to the point estimates of the efficiency. To this end, KDE-based efficiency scores are then derived. The KDE-based scores are then used to check whether the differences in efficiencies, represented by the point estimates, between the

different mines are statistically significant. From the bootstrap, confidence intervals of the efficiency scores are constructed and the resultant distribution for each mine estimated. The main question being asked in these tests of significance is whether the identified differences in efficiency are statistically significant, particularly between the least efficient and the most efficient.

To achieve these objectives, the chapter proceeds as follows. The data used in this study are discussed in Section 4.1. The DEA estimates of efficiency are reported and then analysed in Section 4.2.1. In addition, the separation of technical efficiency into scale and technical efficiency is performed. In Section 4.2.2, the estimation of the statistical properties using the bootstrap is undertaken. Confidence intervals are then constructed from these statistical estimates and an analysis of the main differences between DEA and bias-corrected DEA is conducted using the confidence intervals and also the Banker (1993) statistics. The main results and conclusions drawn from them are discussed in Section 4.3.

Finally, it must be emphasised that the efficiency scores are to be estimated from the input orientation. A key characteristic of a mine is that mine capacity and therefore maximum output is fixed at the design stage and can only change, and then only gradually, in the long term. Hence, output maximisation strategies can only occur when the mine is assumed to be operating below capacity. Even more importantly, the mine faces an externally determined output in that the price of its product is determined by the laws of world supply and demand through the trading which takes place at the London Metal Exchange (LME), New York Mercantile Exchange (NyMEX), the Chicago Board of Trade (CBOT) among many others. The one ever-present pressure in

the global mining industry is the necessity of keeping costs under tight control (Smith, 2004). The logical objective for mine management would seem to be to minimise costs for a given output. It therefore seems reasonable given the conditions of Zimbabwean gold mines that cost-cutting strategies are a more plausible objective than output maximisation. At any rate, it may be easier for management to alter the input mix in order to change performance than adjust the scale of the operation.<sup>35</sup>

The key results of ordinary DEA are that Zimbabwean mines suffer primarily from technical inefficiency as opposed to scale inefficiency. In addition, there is evidence that small mines generally exhibit increasing or constant returns to scale as opposed to relatively larger mines which tend to operate under decreasing returns to scale. When using bias-corrected results, however, the predominant component is scale inefficiency. The results also challenge the use of DEA whether ordinary or bias-corrected without correcting for context-specific issues such as the geology and, in Zimbabwe's case, the socio-political climate.

#### **4.1 Data**

All gold produced in Zimbabwe is sold to the central bank, the Reserve Bank of Zimbabwe (RBZ), which maintains and operates the country's sole gold refinery. It has already been mentioned that the price at which the gold is sold to RBZ is externally determined. Therefore, the mines face a single, internationally determined output price.

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<sup>35</sup> Under constant returns to scale, input minimisation and output maximisation based efficiency measures coincide and produce the same efficiency estimates even though the adjustment path is different. This was one of the observations of Farrell (1957).

This allows the use of revenue as the output variable rather than actually physical output. The data for this study were obtained from the Central Statistical Office of Zimbabwe for the year 1995, the only year for which a sufficiently large sample was available for the required variables. The data after 1995 were not deemed reliable, as political instability affected data surveys. An assumption is made here that the performance of mines in 1995 captures their performance prior to that date and before the disruption caused by the political problems beginning in 1998 and subsequently until the present time. These data, which are collected in connection with the *Census of Production*, are unpublished and confidential. To maintain the confidentiality of the mines, the mines were made anonymous and some observations whose identities could be inferred from the sizes of some of the variables, such as the labour force and output, were removed. The data set comprises observations of thirty-four gold mines each employing over ten employees. The reason for imposing this minimum number of employees is that the operations are expected to be broadly similar for mines with over ten employees, but less so when those with fewer than ten are included, as most of the small operations involve little more than gold panning. Despite differences in scale, all the mines in the sample have a mill for crushing the ore and a mineral processing plant for gold extraction. They also operate some type of earth-moving equipment for transporting the ore from the rock-face to the mills and processing plants on the surface. Together the thirty-four gold mines account for about 75 per cent of Zimbabwe's total gold output for the year 1995<sup>36</sup>.

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<sup>36</sup> The initial data set, comprised ninety mines but some of them had missing data for one or more relevant variables. Also excluded from the ninety are those that had less than 10 employees.



Given these characteristics, a production technology relating five inputs and one output is specified. The inputs are labour, electricity, fuel (petrol and diesel), materials and service charges on capital equipment. These five inputs represent the relevant resources necessary to produce the gold output. The single output is represented by value of at-mine output given the price prevailing at the LME.

Labour is represented by the number of full-time employees, comprising specialist staff such as mining and mineral process engineers and geologists, and non-mining specific ones such as plant fitters, plant operators and office workers including administrators and bookkeepers. In economic theory, the number of man-hours per time period is “a more accurate and preferred measure of labour input” as this takes into account the contribution of part-time employees as well (Coelli et al, 2005). However, in the absence of data on man-hours, using a head count of full-time employees to represent labour has been a common practice in efficiency studies; see for example, Hawdon (2003), Ferrier & Hirschberg (1997) and Murillo-Zamorano (2001).

The other four inputs are all measured in expenditure form. The use of value variables is based on an assumption that the mines face the same prices for these inputs, in which case the expenditures reflect the actual amounts committed to the production process. This is not an unreasonable assumption to make, as will soon become clear.

Energy is divided into two components, expenditures on electricity and on oil-based fuels such as diesel and petrol. The two are treated as distinct inputs mainly because they tend serve different purposes. Electricity is used to power heavy fixed plant and equipment such as mills and concentrators while diesel and petrol power motorised

plant and equipment and light vehicles. Electricity is supplied by a state-owned utility, the Zimbabwe Electricity Supply Authority (ZESA)—in fact, the renamed ESC mentioned in Chapter 2. ZESA is responsible for all the generation and distribution of electricity in Zimbabwe. The prices charged by ZESA are regulated by the state through an energy regulator so the mines face regulated prices for electricity and are assumed to pay the same unit price. Petroleum fuels are imported by another state-owned monopoly, National Oil Company of Zimbabwe (NOCZIM). Although NOCZIM is only responsible for the importation of fuel, the retail prices which the end consumers eventually pay are also regulated and capped by an administrative unit under the Ministry of Energy which also includes representatives of the petroleum retailers. Hence the assumption of a single price is a reasonable one to make.

Materials inputs are represented by expenditure on chemicals, explosives and other consumables which are mainly used in the transformation from gold ore to gold metal. Although the manufacturing and selling of these are carried out by a large number of companies, the assumption of a single price for each appears to be reasonable as Zimbabwe is a relatively small market for the global and multinational business of mining supplies.

The last input is capital services, the measurement of which has been the source of much difficulty in empirical analysis. The main reason for this difficulty is that capital is a durable input, paid for in one period but its services are not. Instead they are used over several subsequent production periods (unlike labour services and materials which are consumed in the production process during the period for which they are paid). There are many approaches to measuring capital and they essentially depend on the context

and data availability. Where data on depreciation and current investment are available, the perpetual inventory method has been used to derive an estimate of the capital stock, although what ought to be measured is the flow of “capital services” rather than the physical stock. Often what is used is the capital stock, to infer capital services, measured in a variety of ways.

Maximum capacity, measured or proxied in various ways, has been the most commonly used measure of capital stock. Fried *et al* (2000), for example, in a study of nursing homes used the bed capacity of each home as a proxy. Löthgren & Tambour (1999), in their study of efficiency of Swedish eye clinics, also used the bed capacity of the eye hospitals as a proxy for the capital stock of each clinic studied. Simar & Wilson (1998) used the maximum capacity of the power stations in their study as a proxy for capital. Unfortunately for this study, figures for maximum capacity were not available and therefore an alternative measure is implemented.

In this study, a different approach is adopted. Instead of capacity, the expenditure on repairing and servicing plant and equipment is used. This measure captures the costs of maintenance of plant and equipment and other fixed structures. The justification for this measure is that the level of expenditure on repair and maintenance is roughly proportional to capital stock and therefore broadly reflects the size of the stock. In addition, servicing also depends on capacity utilisation and this also reflects the flow of capital services.

The use of capital service costs, that is, repair and maintenance, as a proxy for the flow of capital services has been employed before. For example, in two studies of mining

efficiency, Byrnes et al (1984) and Byrnes & Färe (1987) used the cost of annually servicing the capital equipment (in dollars) as a proxy for one measure, among many other measures of capital services flows. These two studies are also much closer in context to this dissertation than the others mentioned above, as they involve mining. Caution is advised though, as the cost of service could reflect the vintage as well size of the capital stock. Hence older machinery will need more regular servicing than that which is newer. This is a potential distortion of the measure. This measurement of capital, at least, suffices in at least capturing the expenditure on capital equipment, whether that expenditure reflects vintage or usage. Those mines with older and, therefore, costly-to-maintain capital stock would find themselves facing high servicing charges.

Having discussed the variables and the production technology, the next step is to illustrate the main characteristics of the sample. These are summarised by the descriptive statistics shown in Table 4.1 while the full raw data set is presented in appendix A.

**Table 4.1: Descriptive Statistics of the Data Set**

Statistic	Labour	Materials	Electricity	Services	Fuel	Output
Mean	476	6432569	1010305	1036598	312511	18462684
Standard Deviation	625	17634051	1970171	2406494	465810	37100756
Coefficient of Vanation	1.31	2.74	1.95	2.32	1.49	2.00
Minimum	12	13388	600	2200	1887	56819
1st Quartile	74	95150	19615	16828	17502	609845
Median	269	2636965	485198	691556	91520	9215733
3rd Quartile	729	6005415	1392904	1122162	508845	23060077
Maximum	3348	103405000	11326000	14187000	2228000	214695000

The key impression conveyed by Table 4.1 is of a relatively heterogeneous data set, characterised by large IQR and standard deviations. The relatively large standard deviations, which are larger than the means, are indications of wide dispersion of the data, with a large number of observations located “far” from the mean. The substantial variability in the data is also indicated by the coefficients of variation— all are greater than 1-- with materials having the greatest degree of dispersion.

## **4.2 Results and Analysis**

Efficiency estimates were derived using CRS and VRS DEA and the data described in Section 4.1, using the DEA programme routines written by Zhu (2002). The DEA efficiency scores are presented in Section 4.2.1. The second estimation procedure employed the bootstrap, bias-correction and smoothing of the DEA. The results of this are presented and discussed in Section 4.2.2.

### **4.2.1 DEA Estimates**

The full individual results are detailed in Table 4.2 and the summary of the results of the DEA are presented in Tables 4.3.

In Table 4.2, Column 1 presents the overall efficiency estimate (CRS), Column 2 the technical efficiency (VRS). Column 3 presents scale efficiency for comparison purposes since it can be deduced from the ratio of CRS to VRS efficiency scores, and Column 4

the output variable, an indicator of mine size, which is one of the most common methods of measuring size in economics<sup>37</sup>.

**Table 4.2: Ordinary DEA and Scale Efficiency Scores**

DMU	(1) CRS DEA	(2) VRS DEA	(3) Scale Efficiency	(4) Output (Proxy for Size)
1	1.0000	1.0000	1.0000	214695000
2	0.5765	0.5837	0.9877	35040000
3	0.7642	0.7982	0.9574	33385946
4	1.0000	1.0000	1.0000	39112673
5	0.5866	0.5872	0.9990	23658000
6	0.4723	0.5018	0.9412	21266308
7	0.7528	0.7533	0.9993	33870309
8	1.0000	1.0000	1.0000	29323000
9	0.7768	0.7776	0.9990	31564000
10	1.0000	1.0000	1.0000	18221000
11	1.0000	1.0000	1.0000	37385000
12	0.4110	0.4121	0.9973	14825000
13	0.5425	0.5450	0.9954	15231000
14	0.8295	0.8687	0.9549	17353684
15	0.9977	1.0000	0.9977	11974146
16	0.8931	0.9036	0.9884	9922572
17	0.8495	0.8538	0.9950	7823101
18	0.7034	0.7413	0.9489	1608697
19	0.8528	0.8542	0.9984	11831478
20	1.0000	1.0000	1.0000	799277
21	0.6965	0.7211	0.9659	1446963

<sup>37</sup> Other measures would be some dimension of inputs such the size of the labour force. Førsumd et al (2006), for example, used the single input as a measure of size. Previous studies on mining, such as Byrnes & Färe (1987) used output and this is the convention followed here, although a potential weakness is that it may reflect the level of capacity utilisation.

DMU	(1) CRS DEA	(2) VRS DEA	(3) Scale Efficiency	(4) Output (Proxy for Size)
22	1.0000	1.0000	1.0000	1430838
23	1.0000	1.0000	1.0000	5007668
24	1.0000	1.0000	1.0000	8508894
25	0.9902	1.0000	0.9902	561431
26	0.8568	0.8799	0.9737	305556
27	0.7941	1.0000	0.7941	276000
28	0.3606	0.9020	0.3998	82908
29	0.6238	1.0000	0.6238	89267
30	0.9881	0.9957	0.9924	755085
31	1.0000	1.0000	1.0000	163660
32	0.5576	0.9851	0.5660	69887
33	0.3192	1.0000	0.3192	56819
34	1.0000	1.0000	1.0000	86076

Table 4.3 shows the descriptive statistics (over the sample) of the results of the two DEA programmes.

**Table 4.3 Descriptive Statistics of DEA Efficiency Scores**

	(1) CRS DEA Estimates	(2) VRS DEA Estimates	(3) SCALE Efficiency Estimates
Mean	0.7999	0.8725	0.9231
Std. Dev.	0.2151	0.1724	0.1747
Median	0.8512	0.9904	0.9964
Minimum	0.3192	0.4121	0.3192
Maximum	1.0000	1.0000	1.0000

A comparison of the two sets of DEA efficiency scores shows that they identify different “least-efficient” mines. Hence, the most inefficient mine using the overall

inefficiency measure is not necessarily the one identified as least technically efficient these being mine 33 when CRS is imposed and mine 12 under VRS. This difference can be explained by noting, from Chapter 3 and also Ganley & Cubbin (1992), that overall efficiency, which is obtained assuming CRS, is decomposable into technical efficiency and scale efficiency<sup>38</sup>. Hence, the reasons for overall inefficiency can be traced to either a non-optimal input or simply a poorly run operation, operating at a disadvantageous scale or a mixture of both.

From Table 4.3, the mean efficiencies indicate that the average potential saving as measured by overall inefficiency is 20 per cent. The corresponding figure for adjusting the input mix (as measured by technical efficiency) is 12.75 per cent. Adjusting the scale of operations will potentially realise an improvement of 7.69 per cent in scale efficiency. An observation made by Simar & Wilson (1998) was that DEA (as with other nonparametric estimation methods) tends to place a large mass of ostensibly efficient DMUs at the upper bound of the distribution.

As mentioned earlier, the overall efficiency score can be decomposed into two sub-categories, technical efficiency and scale efficiency. The next step is to separate the two components which make up overall efficiency and identify the characteristics of the estimated inefficiency, that is, if it is primarily due to scale or technical inefficiency.

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<sup>38</sup> Ganley and Cubbin (1992) note that, with more appropriate information such as prices etc., overall productive efficiency (as opposed to just overall technical efficiency) can be decomposed into its allocative, technical, scale and congestion components. This is a suggestion worth pursuing were a more comprehensive data set to become available.



A DMU is judged to be primarily experiencing technical inefficiency, if the technical efficiency score is greater than the scale efficiency score. Of the sixteen mines identified as *technically efficient*, the estimates imply that five of them could increase their overall efficiency by adjusting the scale of their operations. However, of the rest of the mines which are identified as having technical inefficiency, that is are technically inefficient, eight suffer more from technical inefficiency than from scale inefficiency. The other ten are characterised by scale inefficiency. Hence, slightly more of the inefficient mines suffer more scale inefficiency than from technical inefficiency.

An important observation from table 4.3, again with respect to the sample as a whole, is that mean scale efficiency, at 0.9231, is higher than mean technical efficiency at 0.0.8725. This implies that, in general, when considering performance, the gold mines in the sample tend to suffer more from technical inefficiency than scale inefficiency.

It must be noted that scale adjustments normally can generally made in the longer term, when investment in capacity augmentation can be done. At any rate, scale adjustments tend to be more difficult to implement as mine sizes take time to adjust, especially upwards and as such that they can only be implemented in the long run. In addition, there is also the issue of whether the deposit of ore can support the expanded capacity. Hence a [feasible] strategy of improving efficiency would involve identifying and addressing the causes of technical inefficiency in the short term.

A number of previous studies-- Byrnes et al (1984, 1987) and Førsund et al (2006) using nonparametric estimation, and Lundvall & Battese (2000), Mlambo (1993), Zhou (2000) using parametric methods -- have investigated the association between efficiency

and DMU size. Given their findings, it would also be of interest to investigate the relationship between efficiency score and mine size. This investigation is also justified by the results of visual analysis which indicates that both DMU 1, the largest mine, and DMU 33, the smallest one, are both technically efficient. In terms of overall efficiency, however, the largest mine is among the most efficient and the smallest is least efficient. A nonparametric statistic, the Spearman rank correlation coefficient between two mine characteristics, mine size and labour-output ratio, and the three estimates of efficiency is reported in Table 4.4.

**Table 4.4: Spearman Correlation:  
(Mine Characteristics and DEA Efficiency)**

	(1) OUTPUT	(2) LABOUR-OUTPUT RATIO
SCALE-OUTPUT	0.2129	-0.48715
CRS-OUTPUT	-0.0431	-0.40243
VRS-OUTPUT	-0.7421	-0.55062

The null hypothesis to be tested is that there is no correlation between the estimated efficiency and the size of the mine. The critical [absolute] value for the degrees of freedom (32) at the 5 per cent level of significance is 0.43. From Table 4.4, the decision then is to not reject the null hypothesis that there is a no statistical relationship between the overall and scale measures of efficiency and output. The null hypothesis is rejected for the case of technical efficiency, however. In addition, the relationship between technical efficiency and output is observed to be negative. This infers that the smaller mines have higher technical efficiency than the larger mines. This result mirrors, to a

great extent, the findings of Førsund et al (2006), where smaller DMUs were generally found to be more efficient than the larger ones. A possible explanation may be higher rates of capacity utilisation, although this can not be confirmed in the present study owing to lack of additional information.

Column 2 of Table 4.4 reports the relationship with respect to the labour output ratio, again a min-level characteristic. The purpose of this test is to check whether high labour intensive mines are less efficient than those that have relatively lower ratios. This follows the method used by Byrnes et al (1984) and is presented here for comparative purposes with this study. This time, the null hypothesis is not rejected only for overall efficiency. Hence there is no statistically significant relationship between the estimated overall efficiency and the labour-output ratio. The null hypothesis is, however, rejected for technical and scale efficiency. This implies that there are statistically significant relationships between both technical and the labour-output ratio. This relationship is negative, meaning that those gold mines with a low labour-output ratio are expected to be more efficient than those with high labour-output ratios. This result ought to be read with some caution as the labour-output ratio completely masks the role of capital and the capital-labour substitutability possibilities.

A further test following on from Byrnes et al (1984) is to group the mines into those that are overall efficient and those that are not. A common characteristic which can be determined from the data set is the labour-output ratio. An analysis of the labour-output ratio shows that the labour output ratio of the inefficient mines is almost twice that of the efficient mines (1.85). Hence the locally inefficient mines are generally associated with a high labour-output ratio.

#### 4.2.2 Bootstrap Analysis

Having estimated the relative efficiencies of the gold mines, a reasonable question to ask is whether the differences in efficiency discussed in Section 4.2.1 are statistically significantly different from each other. This question is motivated in great part by the large number of technically mines. It has already been noted in Chapter 3 that the DEA estimators, in common with other nonparametric estimators, are biased (upwards). The estimated efficiencies, therefore, are expected to be biased, generally upwards<sup>39</sup>, and any conclusions drawn from them must be tempered with this knowledge. It has also been noted in Section 4.2.1 that the main inefficiency is technical rather than scale inefficiency. An interesting and worthwhile exercise, after correcting for bias, is to check whether this preponderance of technical inefficiency still holds.

The investigation of bias and differences in efficiencies observed, require knowledge of the statistical properties of the estimates. It has already been noted that the statistical properties of the results obtained by DEA are unknown and as a result no inferences can be made on the significance of any differences between or among the estimated efficiencies. Estimates of the statistical properties can however be made and the bootstrap is one way to approximate the distribution and estimate the statistical properties of the indices obtained from a seemingly deterministic analysis. As mentioned in the literature review, the use of the bootstrap is fraught with difficulties and potential controversies, so again most conclusions must be used with caution. A

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<sup>39</sup> See Ferrier & Hirschberg (1997), Gonzalez & Miles (2002) and Hawdon (2003) for examples where some observations have negative biases.

reflection of these difficulties is the very slow rate at which the bootstrap has been implemented in relation to the amount of DEA work done.

The bootstrap and smoothing routines used in this study were implemented using a set of Visual Basic macros<sup>40</sup>. To test the sensitivity of the smoothed bootstrap method, the automatic bandwidth selection method proposed by Silverman (1986) was employed. Tables 4.5 and 4.6 report the results for individual mines and summary statistics of the both the original and the bootstrap efficiencies.

**Table 4.5: DEA and Bias-corrected DEA Efficiency Scores**

DMU	(1) CRS DEA Estimate	(2) BC <sup>41</sup> - CRS	(3) CRS Bias	(4) VRS DEA Estimate	(5) BC-VRS	(6) VRS Bias	(7) BC - Scale Estimate
1	1.0000	0.7837	0.2163	1.0000	0.8854	0.1146	0.8433
2	0.5765	0.5288	0.0477	0.5837	0.5540	0.0297	0.9211
3	0.7642	0.7049	0.0593	0.7982	0.7593	0.0389	0.9255
4	1.0000	0.7326	0.2674	1.0000	0.7826	0.2174	0.7459
5	0.5866	0.5374	0.0492	0.5872	0.5582	0.0290	0.9222
6	0.4723	0.4449	0.0274	0.5018	0.4885	0.0133	0.9439
7	0.7528	0.6675	0.0853	0.7533	0.7149	0.0384	0.8899
8	1.0000	0.6693	0.3307	1.0000	0.7967	0.2033	0.6980
9	0.7768	0.7145	0.0623	0.7776	0.7389	0.0387	0.9254
10	1.0000	0.8879	0.1121	1.0000	0.9293	0.0707	0.8942
11	1.0000	0.7758	0.2242	1.0000	0.8143	0.1857	0.7957
12	0.4110	0.3809	0.0301	0.4121	0.3911	0.0210	0.9310
13	0.5425	0.4986	0.0439	0.5450	0.5198	0.0252	0.9219

<sup>40</sup> The macros were written by Christopher Hammond of the Business School, University of Hull, United Kingdom.

<sup>41</sup> BC- bias-corrected.

DMU	(1) CRS DEA Estimate	(2) BC <sup>41</sup> - CRS	(3) CRS Bias	(4) VRS DEA Estimate	(5) BC-VRS	(6) VRS Bias	(7) BC - Scale Estimate
14	0.8295	0.8007	0.0288	0.8687	0.8504	0.0183	0.9523
15	0.9977	0.9257	0.0720	1.0000	0.9604	0.0396	0.9304
16	0.8931	0.7915	0.1016	0.9036	0.8558	0.0478	0.8861
17	0.8495	0.8013	0.0482	0.8538	0.8270	0.0268	0.9488
18	0.7034	0.6555	0.0479	0.7413	0.7132	0.0281	0.9239
19	0.8528	0.8113	0.0415	0.8542	0.8226	0.0316	0.9512
20	1.0000	0.7342	0.2658	1.0000	0.8085	0.1915	0.7575
21	0.6965	0.6721	0.0244	0.7211	0.7017	0.0194	0.9518
22	1.0000	0.8412	0.1588	1.0000	0.8531	0.1469	0.8344
23	1.0000	0.8858	0.1142	1.0000	0.9033	0.0967	0.9010
24	1.0000	0.8900	0.1100	1.0000	0.9021	0.0979	0.9013
25	0.9902	0.8813	0.1089	1.0000	0.9104	0.0896	0.8966
26	0.8568	0.7772	0.0796	0.8799	0.8182	0.0617	0.9009
27	0.7941	0.7392	0.0549	1.0000	0.9050	0.0950	0.7810
28	0.3606	0.3395	0.0211	0.9020	0.8530	0.0490	0.4034
29	0.6238	0.5773	0.0465	1.0000	0.7828	0.2172	0.5797
30	0.9881	0.9383	0.0498	0.9957	0.9451	0.0506	0.9476
31	1.0000	0.7715	0.2285	1.0000	0.8262	0.1738	0.7977
32	0.5576	0.5183	0.0393	0.9851	0.9260	0.0591	0.5763
33	0.3192	0.2995	0.0197	1.0000	0.8474	0.1526	0.3010
34	1.0000	0.6906	0.3094	1.0000	0.7675	0.2325	0.7205

The most obvious observations in Table 4.5, compared to Table 4.2, are the reductions in efficiency scores for all the efficient mines, with some dramatic changes in some cases where mines which once were fully efficient are now less efficient than mines which were not.

Given these apparent changes in efficiency scores after bias-correction, a good starting point is to test whether the bias in the DEA is statistically significant or not. Following

the work by Boame (2001), the statistical significance of these changes can be analysed by testing the difference between the mean of the DEA estimates, denoted  $\theta_{DEA}$  below, and the mean of the bias-corrected bootstrap DEA scores, denoted  $\theta_{BC}$ .

Formally, this test is stated as follows:-

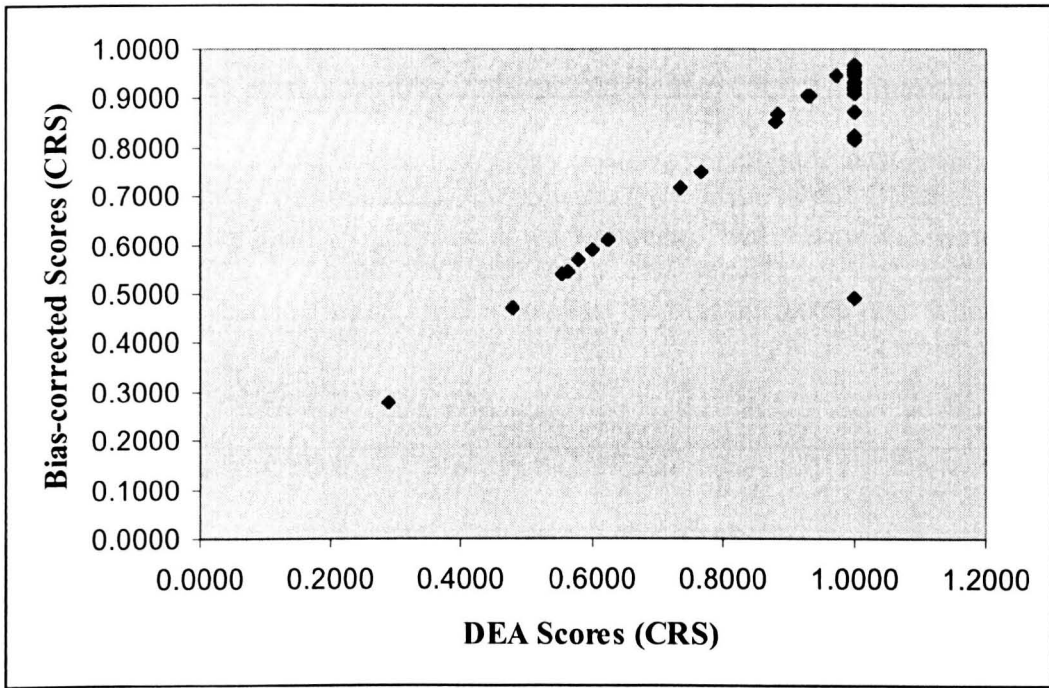
$$H_0: \theta_{DEA} - \theta_{BC} = 0$$

$$H_1: \theta_{DEA} - \theta_{BC} \neq 0$$

$\alpha = 0.05$ , 2 tailed test.

The critical value of the  $t$  statistic is 2.042 at the 5 per cent level of significance. With a calculated  $t$ -value of 2.4159, the null hypothesis that the means are the same is rejected. Hence, the conclusion is that the bias, on average across the sample, is significant and the bias-corrected DEA efficiency score are statistically different from the DEA estimates.

Another point which requires attention, given the changes caused by bias-correction, is whether the ranking of the DMUs, in terms of efficiencies, is maintained between DEA and bias-corrected DEA. Førsund et al (2006), for example, observed that bias-correction resulted in some significant changes in rankings, this change resulting from the different biases of the scores for each DMU which tended to be higher for some of the efficient DMUs. To this end, this study will check the correlation between the rankings of the two sets of results, that is, the DEA and bias-corrected DEA. A rank correlation coefficient (the Spearman coefficient) and a scatter plot are employed to investigate this. Figure 4.1 shows a plot of DEA against bias-corrected DEA.



**Figure 4.1: Scatter Plot of Bias-corrected DEA against DEA (CRS).**

The Spearman rank correlation coefficient is 0.9338 and indicates a strong and positive relationship between the two sets of results. This positive relationship is significant at the 5 per cent level (critical value of 0.43 for 32 degrees of freedom).

From Figure 4.1, it is obvious that most of the misalignment of the ranks occurs at the upper bound of the distribution, confirming the assertion by Simar & Wilson (1998) that it is at the upper bound that the bias (and inconsistency) of DEA occurs. In particular, the relationship can be split into two, with a strongly correlated part and a sub-sample at the upper bound where the relationship breaks down. Førsund et al (2006) did not measure the degree of correlation between the DEA and the bias-corrected DEA but did note these large changes at the upper bound.



The next point to consider is how the rankings compare when VRS is imposed. After all, VRS results in even more fully efficient DMUs than CRS. The Spearman rank coefficient is now 0.8982, which indicates a positive but slightly weaker relationship. This again is statistically significant at the 5 per cent level. Figure 4.2 shows the relationship between the initial DEA scores and the bias-corrected ones when VRS is imposed.

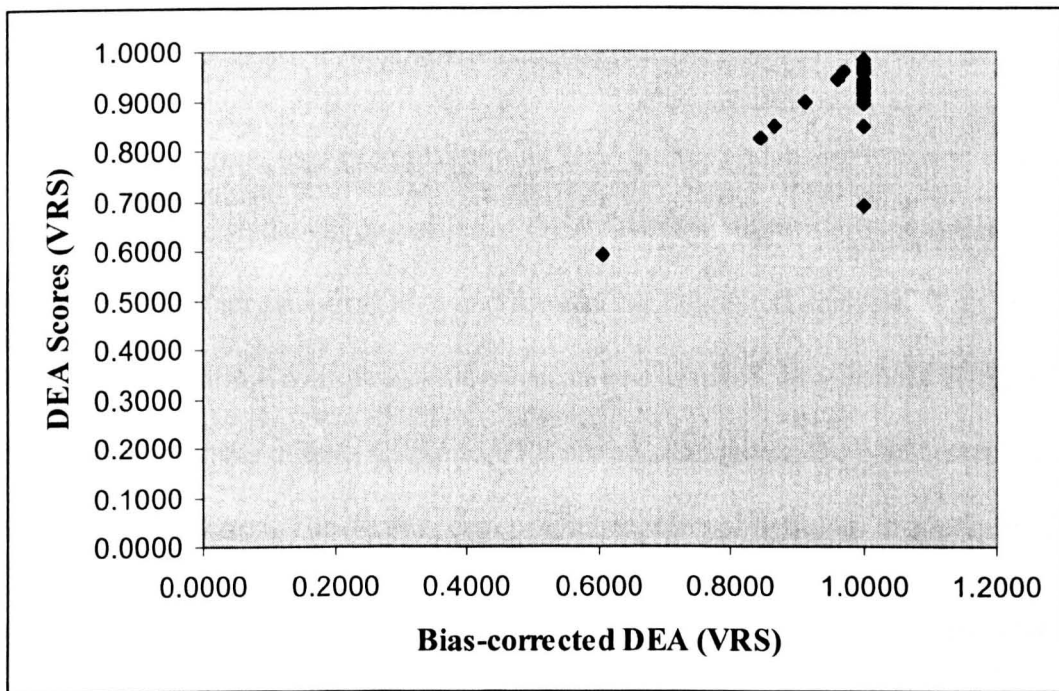


Figure 4.2: Scatter Plot of DEA against Bias-corrected DEA (VRS)

Under VRS, ranking the mines by their bias-corrected efficiencies also results in similar distributional changes at the upper end. Elsewhere, particularly at the lower end of the distributions, that is, for the relatively more inefficient DMUs the rankings are maintained. Again, as with CRS, the large number of efficient units in the initial DEA run causes the breakdown of the linear relationship between the two sets of results. As a

result of bias-correction, some mines are ranked higher than mines which were earlier ranked higher than them.

Another observation from Figure 4.2 which is worth mentioning, especially when compared to Figure 4.1, is that the observations are grouped relatively more closely together in Figure 4.2. This means that the VRS distribution of results is less widely dispersed than the CRS. This is to be expected as the VRS technology also more tightly envelops the observations than CRS.

One of the important uses of bootstrapping DEA scores and bias-correction is the estimation of the statistical properties of DEA. These properties are important when carrying out further statistical tests and for making inferential analysis. With the relevant information, confidence intervals can be estimated. In appendix B, the 95 confidence intervals of both CRS and VRS scores centered on the bias-corrected point estimate are reported. The 90 per cent confidence interval is shown in appendix C.

The first point to note is that the confidence intervals are generally wider the higher estimated efficiency scores, that is, the relatively efficient mines tend to have wide confidence intervals with the reverse being true for the lower the efficiency scores. In fact the upper bound for some of them is greater than 1. Similar value confidence intervals for efficient DMUs were observed by Simar & Wilson (1998) in their application of the bootstrap to a sample of Illinois power stations.

Ferrier & Hirschberg (1997) observed some negative biases in the results after the bias-correction, indicating the potential for the upper bounds of the confidence intervals being greater than 1 for efficient DMUs. The widths of the confidence intervals have

implications for statistical precision. Wider confidence interval imply lower statistical precision and, conversely, narrower ones higher precision.

In terms of hypothesis testing, there have been some difficulties in past studies in making clear inferences about the efficiency estimates where some of the observations have such wide intervals. This is particularly so when the intervals of more than one DMU overlap; in which case the hypothesis that the point estimates of two or more different DMUs are statistically the same cannot be rejected. For example, in the study of tax offices in Norway, Førsund et al (2006) could only reject a null hypothesis of “no differences between the DMUs” when comparing the lowest quarter and the highest quarter of the efficiency scores. This meant that for the other half of the sample, the null hypothesis could not be rejected because of the overlapping intervals. A similar finding was made by Gonzalez & Miles (2002) in a study of Spanish public services. They found that since confidence intervals of most observation overlapped, the null hypothesis could not be rejected, despite having statistically significantly different bias-corrected point estimates

An explanation of the confidence interval has already been given in Chapter 3 but it may help to refresh this in the light of appendix B. The 95 per cent confidence interval for the point estimate of DMU 27, with a bias-corrected point VRS estimated score of 0.9050, is given by [0.8113,1.2224]. This means that in repeated sampling, we are 95 per cent confident that this interval will contain the point true estimate.

A hypothesis that there is no difference in performance between, for example, DMU 27, and DMU 15 with a bias-corrected local technical efficiency score of 0.9604, cannot be

rejected. This is because their intervals overlap (0.8113, 1.2224) and (0.9161, 1.1285); that is, the lower end of the interval for DMU 15 falls in the interval of DMU 27.

In the literature, some researchers have, as noted above, extended the analysis of confidence intervals by inspecting and taking note of which mines can be considered significantly less efficient than the VRS best-performing mine, in this case DMU 15. That is, which mines do not have confidence intervals which overlap with that of DMU 15? Using this method, the seven worst-performing mines can be regarded as significantly less efficient than the DMU 15. In addition, DMU 14 which lies somewhat in the middle of the distribution can also be considered among those that are performing significantly worse than DMU 15. Out of a sample of thirty-four mines, just under 21 per cent can be inferred to be statistically less efficient than the best-performing mine. The rest cannot be regarded as statistically less efficient than mine 15, using the confidence intervals for inference, however.

With the results ranked in ascending order of the bias-corrected VRS score, another observation is that the confidence intervals are much wider for the middle part of the distribution and much narrower for the lower end (the least efficient mines) of the distribution. The likelihood of committing inferential errors, both type I and type II are highest in this middle band of results. The upper part in turn contains markedly narrower confidence intervals than the middle part of the distribution although not as narrow as the lower part. The mines which have DEA efficiency scores equal to 1 (VRS), which are mostly located in the middle of the distribution, also have the widest confidence intervals. For example, DMU 1 has an interval of (0.5689, 1.9110). This is in comparison to the least efficient DMU, DMU 12, which has an interval of (0.3707,

0.4540) or the best performing DMU, DMU 15, which has an interval of (0.9223, 1.1262). Similar distributional characteristics and confidence interval widths were observed by Førsund et al (2006) in the tax offices study.

The next point to consider and discuss is scale efficiency. As noted, this is obtained as ratio of overall to technical efficiency and by construction cannot be greater than 1.

Table 4.6 shows a decomposition of the bias-corrected DEA scores into pure and scale efficiencies. In Column 4 output is reported. Table 4.7 reports the summary statistics of the results in Table 4.6

**Table 4.6: Decomposing Technical of Efficiency**

DMU	(1) BC-CRS DEA	(2) BC-VRS DEA	(3) BC-SCALE	(4) OUTPUT
1	0.7837	0.8854	0.8433	214695000
2	0.5288	0.5540	0.9211	35040000
3	0.7049	0.7593	0.9255	33385946
4	0.7326	0.7826	0.7459	39112673
5	0.5374	0.5582	0.9222	23658000
6	0.4449	0.4885	0.9439	21266308
7	0.6675	0.7149	0.8899	33870309
8	0.6693	0.7967	0.6980	29323000
9	0.7145	0.7389	0.9254	31564000
10	0.8879	0.9293	0.8942	18221000
11	0.7758	0.8143	0.7957	37385000
12	0.3809	0.3911	0.9310	14825000
13	0.4986	0.5198	0.9219	15231000
14	0.8007	0.8504	0.9523	17353684
15	0.9257	0.9604	0.9304	11974146
16	0.7915	0.8558	0.8861	9922572
17	0.8013	0.8270	0.9488	7823101

DMU	(1) BC-CRS DEA	(2) BC-VRS DEA	(3) BC-SCALE	(4) OUTPUT
18	0.6555	0.7132	0.9239	1608697
19	0.8113	0.8226	0.9512	11831478
20	0.7342	0.8085	0.7575	799277
21	0.6721	0.7017	0.9518	1446963
22	0.8412	0.8531	0.8344	1430838
23	0.8858	0.9033	0.9010	5007668
24	0.8900	0.9021	0.9013	8508894
25	0.8813	0.9104	0.8966	561431
26	0.7772	0.8182	0.9009	305556
27	0.7392	0.9050	0.7810	276000
28	0.3395	0.8530	0.4034	82908
29	0.5773	0.7828	0.5797	89267
30	0.9383	0.9451	0.9476	755085
31	0.7715	0.8262	0.7977	163660
32	0.5183	0.9260	0.5763	69887
33	0.2995	0.8474	0.3010	56819
34	0.6906	0.7675	0.7205	86076

**Table 4.7 Descriptive Statistics of the Bias-Corrected DEA Estimates**

	Bias-corrected CRS	Bias-corrected VRS	Bias-corrected Scale
Mean	0.6961	0.7857	0.8295
Std. Dev.	0.1689	0.1385	0.1580
Median	0.7334	0.8204	0.8988
Minimum	0.2995	0.3911	0.3010
Maximum	0.9383	0.9604	0.9523

The mean of the bias-corrected bootstrap technical efficiency score is 0.7857 while the median is 0.8204. All the results are lower after correcting for bias. Bias-correction has

changed the estimates so much that one can infer higher potential savings on input usage than using DEA.

From table 4.6, it is observed that with bias-correction, just as was observed with the results reported in Table 2.5, the observed maximum scale efficiency score is reduced from 1 to 0.9383 and 0.9604 for overall and technical efficiency respectively. The minimum efficiency score shows marginal declines, from 0.3192 in Table 4.2 to 0.3010 in Table 4.7.

Whereas, with ordinary DEA, efficient mines with an estimated efficiency score of 1 were identified, with bias-correction, there no longer are any fully efficient mines. Consequently, instead of being technically efficient units, they will now be referred to as best-performing units (Førsund et al, 2006).

The initial observation is that the mean of bias-corrected technical efficiency is higher (0.9604) than that of the bias-corrected scale efficiency (0.9525). This implies that , in terms of performance, the primary cause of low technical efficiency is operating at the “wrong scale”, a change from DEA where the implication was that the primary reason for low overall efficiency was technical inefficiency.

The result of correction for bias also some causes significant individual changes. Out of the thirty-four mines, sixteen now have individual technical efficiency scores which are greater than the corresponding scale efficiency scores, implying that individually for most mines, scale inefficiency is still the primary cause of overall inefficiency. It must be recalled that the corresponding figure was five mines in Section 4.2.1. However, for

the remaining eighteen mines, local technical inefficiency is now the primary cause of technical inefficiency compared to only one in the initial DEA run.

Another characteristic of the productive DMU worth analysing is the relationship between mine size and estimated efficiency score. The justification for investigating the relationship between efficiency DMU size is that if a statistically significant relationship between the size of an operation and its efficiency exists, larger units are probably expected to be more efficient than smaller units. This would seem to be a logical outcome for operations taking place at depths and necessitating movements of large amounts of rock. Although caution needs to be taken when making unqualified comparisons between the nonparametric results of different samples, it is a commonly observed feature of VRS DEA that correcting for bias tends to result in the relatively smaller DMU being regarded as better performing than larger ones, as has been reported by Førsund et al (2006) and Veiderpass (1993), among others.

Table 4.8 shows the Spearman rank correlation coefficients of the efficiency estimates. In addition to the size characteristic, the relationship between efficiency estimates and the labour-output ratio is also measured and tested.

**Table 4.8: Spearman Correlation:  
(Mine Characteristics and Bias-corrected Efficiency)**

	OUTPUT	LABOUR-OUTPUT
SCALE-OUTPUT	0.0503	-0.3470
CRS-OUTPUT	-0.3622	0.0828
VRS-OUTPUT	-0.7736	0.0553



The results are somewhat different from those reported in Table 4.4. At the 5 per cent level of significance, the null hypothesis of the Spearman coefficients being statistically equal to zero is not rejected, except for that between technical efficiency and output. This confirms the results reported in Førsund et al (2006) and Veiderpass (1993), although they did not report the Spearman coefficients but indicated there was a correlation between output and technical efficiency. There is no significant correlation between any of the three efficiency scores and the labour-output ratio, again, a significant departure from findings reported in Table 4.4. Therefore, the inference to be drawn is that the smaller mines are more likely to have higher technical efficiency than large ones.

Following Førsund et al (2006), a sub-sample of the best-practice mines is selected. The characteristics, in this instance the mean technical efficiencies, of this sub-sample are then compared to the sample characteristics. Førsund et al (2006) selected the upper decile of VRS bias-corrected estimates but also suggested an alternative of selecting the upper third. Given the sample size in this study compared to that in Førsund et al (2006) study (thirty four as opposed to ninety-eight), the upper third (eleven mines) is selected as the best-practice sub-sample.

Using the bias-corrected VRS scores, it is observed that the best practice mine, on average, uses 2 per cent more labour than the sample average. It also spends 42 per cent more on materials and 13 per cent less on fuel. However, it spends 21 per cent less on electricity and 40 per cent less capital services. At the same time it produces 25 per cent more output. Given the grouped nature of the results as illustrated in Figure 4.2, an

obvious question to ask is whether the differences between these two groups is statistically significant.

Using the Banker (1993) test, the  $F$ -statistics for the difference between the estimated bias-corrected technical efficiency scores between the two groups of mines is calculated. The test is based on the assumption of a half-normal distribution, i.e. it considers the positive half of the normal distribution, the so-called sum of squares ratio test (Banker et al, 2004). The motivation for this assumption is that by using the bootstrap which re-samples a thousand times, the size of the sample increasingly gets large enough to justify the approximation to a normal distribution. The critical value at 5 per cent and (34,11) degrees of freedom is 2.16. The calculated  $F$ -statistic is 2.7617. Hence the null hypothesis that variance of the estimated efficiency of the best-practice sub-sample is not significantly different from the whole sample is rejected. The alternative that the mean efficiency score of the best-practice sub-sample is higher than that of the whole sample is accepted. The corresponding  $F$ -statistics is 2.1975 for overall efficiency (CRS) which is also significant at the 5 per cent level. Hence there is a statistically significant difference in estimated efficiency between the upper third of the distribution of results and the whole sample.

Another test, this time based on the nature of returns to scale is tested. Table 4.9 reports the results of testing the null hypothesis that there is no difference between the two groups as identified by the nature of returns to scale against the alternative that they are different.

**Table 4.9 Hypothesis Testing: Nature of Returns to Scale**

	BC-CRS	BC-VRS
CRS-IRS (11,14)	11.4504	5.0750
CRS-DRS (11,9)	10.0287	20.8466
IRS-DRS (14,9)	0.8758	4.1077

The null hypothesis is denoted by the order of the relationship in the first column, with the degrees of freedom in parenthesis. Hence, for CRS-IRS tests, the test is stated thus:-

$$H_0: \sigma_{CRS} = \sigma_{IRS}$$

$$H_1: \sigma_{CRS} \neq \sigma_{IRS}$$

$$\alpha = 0.05$$

The null hypothesis is rejected for all the cases except for the difference between IRS and DRS when CRS is imposed. Hence, CRS technology has allowed the distinction between constant and non-constant returns to scale. It has not been able to differentiate between increasing and decreasing returns to scale, regarding the efficiency scores as not statistically different.

It has already been mentioned that the fundamental contribution of the bootstrap is the ability to correct for the bias in results that are inherent in nonparametric methods and the estimation of statistics such as standard deviations. The standard deviation and the point estimates can be used to test for difference between means. With the bias having been corrected, a pertinent question may now be are there any significant differences between the best-performing DMUs and others, at the lower end, the middle and upper

end of the distribution? The relevant standard deviations have already been reported in table 4.5. This study implements a set of alternative tests of hypothesised differences in efficiency.

DMUs 1 and 31 are mines which were judged efficient in the initial DEA run with VRS imposed but whose estimated technical efficiencies significantly reduced from 1.000 to 0.8505 and 0.8944, respectively. The test here is to check whether this reduction is statistically significant by comparing their post bias-correction efficiencies to that of the best-performing DMU, mine 15. Using VRS results, the test was conducted for differences in the point estimates of a number of mines in relation to mine 15, the best-performing mine in terms of bias-corrected score.

Formally, this test, for mine 1, is as follows.

$$H_0: \theta_{15} = \theta_1$$

$$H_0: \theta_{15} \neq \theta_1$$

$$\alpha = 0.05, 2 \text{ tailed test, } t_{0.05} = 1.96$$

$$\theta_{15} = 0.9604, \theta_1 = 0.8854.$$

The calculated  $t$ - statistic is 6.9489. The null hypothesis is thus rejected in favour of the alternative. Hence mine 15 is statistically more efficient than mine 1, despite the two having been identified as similarly efficient in the initial DEA run.

Following the same scheme of hypothesis testing, the test is repeated for four more DMUs each with different characteristics. Mine 12 is the poorest performing mine with a technical efficiency score of 0.3809, while mines 28, 30 and 32 have a DEA

efficiency score less than 1, but relatively high at 0.9020, 0.9957 and 0.9851 respectively. It must be noted that using the confidence intervals in Table 4, almost all the DMUs would have been deemed as well-performing as mine 14. The corresponding *t*-statistics are listed in Table 4.10.

**Table 4.10: Testing Hypothesis  
(Comparing VRS Efficiency of Best Performing Mine with Others)**

DMU	Bias-corrected DEA Efficiency Score	t-statistic
12	0.3809	286.2844
28	0.9020	37.1129
30	0.9957	5.8153
32	0.9851	9.0617

The main observation is that, at the 5 per cent level of significance, the estimated efficiency score of mine 15 is statistically different from the mines listed. Hence, mine 15 is significantly more efficient than all the other mines listed in Table 4.10.

Finally, a comparison of the eleven best practice mines is also made. Specifically, the aim is to find how many of the best practice units in sub-sample chosen above are not peers in the DEA, that is have a DEA score of 1. Using VRS, three out of the eleven are not peers. These are mines 3, 5 and 11. As Førsund et al (2006) noted, the implementation of bias-correction identifies a different set of DMUs which ought to be role models.

A final question to consider, given that scale efficiencies have been computed and their impact on the overall efficiency discussed, is whether it is possible to identify the nature of returns to scale? It has already been shown, for example, that sixteen mines (after bias correction) are primarily scale inefficient. What is unknown, at this stage, is the nature of the returns to scale. In other words where on Figure 3.4, for example, the individual mines are likely to be located, or in the context of the four production possibility sets, in which does each individual DMU appear?

In Table 4.11 the nature of returns to scale is reported in column 4. As discussed in Chapter 3, the nature of returns to scale is inferred from the CRS DEA programme. Recall that when CRS is imposed, the  $\sum \lambda_i$  constraint is non-binding. It only becomes binding under variable returns to scale (VRS), non-increasing returns to scale (NIRTS) and non-decreasing return to scale (NDRTS).

**Table 4.11: Nature of Returns to Scale**

DMU	(1) $\sum \lambda_i$ CRS	(2) $\sum \lambda_i$ NIRTS	(3) $\frac{\sum \lambda_i CRS}{\sum \lambda_i NIRTS}$	(4) RETURNS TO SCALE	(4) OUTPUT
33	0.0840	0.0840	1.0000	IRTS	56819
32	0.4165	0.4165	1.0000	IRTS	69887
28	0.0644	0.0644	1.0000	IRTS	82908
34	1.0000	1.0000	1.0000	CRTS	86076
29	0.0024	0.0024	1.0000	IRTS	89267
31	1.0000	1.0000	1.0000	CRTS	163660
27	0.0702	0.0702	1.0000	IRTS	276000
26	1.2601	1.0000	1.2601	DRTS	305556
25	0.1323	0.1323	1.0000	IRTS	561431
30	0.9474	0.9474	1.0000	IRTS	755085
20	1.0000	1.0000	1.0000	CRTS	799277

DMU	(1) Σλ <sub>i</sub> CRS	(2) Σλ <sub>i</sub> NIRS	(3) $\frac{\sum \lambda_i CRS}{\sum \lambda_i NIRS}$	(4) RETURNS TO SCALE	(4) OUTPUT
22	1.0000	1.0000	1.0000	CRTS	1430838
21	0.0379	0.0379	1.0000	IRTS	1446963
18	0.0430	0.0430	1.0000	IRTS	1608697
23	1.0000	1.0000	1.0000	CRTS	5007668
17	0.2044	0.2044	1.0000	IRTS	7823101
24	1.0000	1.0000	1.0000	CRTS	8508894
16	0.3220	0.3220	1.0000	IRTS	9922572
19	0.9114	0.9114	1.0000	IRTS	11831478
15	0.3104	0.3104	1.0000	IRTS	11974146
12	1.0228	1.0000	1.0228	DRTS	14825000
13	0.4209	0.4209	1.0000	IRTS	15231000
14	1.8784	1.0000	1.8784	DRTS	17353684
10	1.0000	1.0000	1.0000	CRTS	18221000
6	2.1367	1.0000	2.1367	DRTS	21266308
5	0.6538	0.6538	1.0000	IRTS	23658000
8	1.0000	1.0000	1.0000	CRTS	29323000
9	0.9076	0.9076	1.0000	IRTS	31564000
3	1.0490	1.0000	1.0490	DRTS	33385946
7	1.3243	1.0000	1.3243	DRTS	33870309
2	7.7929	1.0000	7.7929	DRTS	35040000
11	1.0000	1.0000	1.0000	CRTS	37385000
4	1.0000	1.0000	1.0000	CRTS	39112673
1	1.0000	1.0000	1.0000	CRTS	214695000

It can be seen that from Table 4.11, that the number of mines operating under constant returns to scale is eleven. Of the remaining twenty-three mines which are not operating under constant returns to scale, seven are under decreasing returns to scale and sixteen are under increasing returns to scale.

Now that the nature of the returns to scale has been identified for each mine, a useful exercise is to check if there is a pattern to the characteristics of the individual mine and the nature of returns to scale. Specifically, this involves the investigating whether, as Byrnes et al (1984) found, there is a relationship between the nature of the returns to scale and the size of the mine.

From Table 4.11, it can be observed that the smaller mines are generally operating under increasing or constant returns to scale. The smallest mine, DMU 33, is under increasing returns to scale. Of the next eight smallest, five are operating under IRTS and three CRTS. There is one relatively small DMU, mine 26, which exhibits decreasing returns to scale. This is different from the results obtained by Byrnes & Färe (1987) where it was the middle-sized mines rather than the “low output” ones which were experiencing increasing returns to scale. The larger mines tend to exhibit either constant or decreasing returns to scale, although of there are three relatively large mines which are operating under increasing returns to scale. The largest mine, DMU 1, is identified as operating under decreasing returns to scale.

The bootstrap is not only useful for enabling the calculation of bias-corrected efficiency scores. The issue of statistical precision has already been briefly commented on. It has been noted that there are too many overlapping intervals to reject a null hypothesis of “no difference” in efficiency scores between a large number of mines. Two other reviewed studies, Gonzalez & Miles (2004) and Førsund et al (2006) commented on this result and inferred that where the confidence intervals overlap, the null hypothesis of no statistical difference in efficiency between the two DMUs could not be rejected.



### 4.3 Concluding Remarks

Two sets of conclusions can be drawn from these results. The first set arises from the initial DEA run and the second from bias-corrected DEA.

First, having estimated the efficiencies using DEA, it can be seen that there is a wide range of inefficiency in gold mining in Zimbabwe, with a minimum overall efficiency estimate of 0.3192 and a corresponding value of 0.4121 for technical efficiency. Both DEA programmes identify a number of fully efficient mines, 32 per cent are deemed fully efficient by CRS and 47 per cent by technical efficiency measure. The mean values 0.7799 for overall and 0.8725 for technical efficiency indicate potential input savings of 22 per cent and 12.75 per cent respectively. In terms of the components of overall efficiency, the scale efficiency estimates are generally higher than the technical efficiency estimates, implying that a typical Zimbabwean gold mine principally suffers from technical inefficiency.

Second, although it has been suggested above that, using DEA, overall the mines tend to suffer primarily from technical inefficiency, both sources (technical and scale) of inefficiency are present. There is therefore a case for gradual adjustments in the operations of gold mining to achieve efficiency, first by addressing technical inefficiency in the short run and scale inefficiency in the longer term.

Third, it has been noted that there is some evidence of a negative relationship between the scale of operations, as measured by output, and the estimated technical efficiency scores. There is no statistically significant correlation with respect to overall or scale efficiency. This negative relationship between mine size and technical efficiency

implies, therefore, that many small mines are judged as more efficient or better – performing than larger ones. It needs qualifying that output is but one way of measuring the size of an operation. In fact, output may be misleading as there may exist short-term problems in achieving maximum capacity. A plausible extension of the analysis would be to gather more information about the geological and, possibly, regional characteristics of the mines. This would help identify whether there are any mine-specific factors which help determine overall efficiency. In particular, how has the political problems in Zimbabwe, as described in Chapter 2, impacted on management decisions and choices. Clearly more data would need to be collected to answer this question but it is a valid question nevertheless.

It must be noted in passing that Byrnes et al (1984) also used the labour-output ratio to represent a mine-specific characteristic and declared that high labour-output mines had low efficiencies. There are potential pitfalls with using this approach. Recall that the objective is to measure efficiency which is a multi-input concept. The labour-output ratio is a partial measure of a firm's performance and as such only gives a partial, and probably not very accurate, insight. Hence, while the labour-output ratio was analysed in Tables 4.4 and 4.8, mostly as comparative analysis to Byrnes et al (1984), the limited usefulness of the approach means that for the purposes of this dissertation, this will not be pursued and no inferences will be made on the basis of the results thereof.

A common policy pursuit by the authorities in Zimbabwe, although only perfunctorily mentioned in the policy strategies referred to in Chapter 2, is to encourage the existing gold mines to expand their existing operations to compete (in size) with those in South Africa and elsewhere in Africa. The results from this study imply that, in fact, what the

authorities and the industry may need to do is adopt a mix of policies. In particular, given that it is the smaller mines which are generally more efficient than larger ones, the solution to improving performance may lie elsewhere. There may be mine-specific characteristics such as the geology, which may favour smaller operations in Zimbabwe. In this case, the focus of mining policy may be to encourage investment in technology which is suitable for smaller scale mining, relatively to other mining economies.

There are those mines, however, which primarily suffer from scale inefficiency. Measures will have to be adopted to adjust the scale of operations. In many cases, this may require investment in capital if the adjustment is to expand the scale and this can only be done in the medium to long term. In some cases, particularly where there are contiguous mining operations, the merging or combining of operations could equally be effective. In this case, it has been observed that most of the small mines are operating under increasing returns to scale. This implies that the scope for improving scale efficiency lies in increasing the scale of operation for small mines and reducing them for the larger ones. Again, there are some exceptions to the rule, as reported in table 4.9.

The second set of observations is drawn from the bootstrap and bias-corrected estimates. First, it is interesting to note that, although there no longer are efficient mines when bias-correction takes place, there still remain best-practice mines of which mine 15 is the most efficient. This point was noted by Førsund et al (2006) who noted that the concept of the peer is uniquely defined in the original DEA. Rather, the bias-corrected DEA routinely identifies best-performing units rather than peers.

Second, given a best-performing unit, the bootstrap allows the testing of whether the mines identified as least efficient are significantly less efficient than the best-performing mine. Two methods have been used in previous studies, the confidence intervals method (Førsund et al, 2006; Gonzalez & Miles, 2002) and the hypothesis test method (Banker, 1993). Using the confidence intervals method, it can be concluded that given the individual 95 per cent confidence intervals, only thirteen mines can conclusively be deemed less efficient than the best performing mine when using the overall efficiency measure (CRS). This number falls to eight mines when the criterion is technical efficiency (VRS). All the other mines have intervals which overlap the interval of DMU 15 so can not conclusively be judged to be less efficient than the best-performing mine. This method suggests caution in the way that DEA estimated efficiencies are to be used and what inferences can be drawn from them.

Third, if the width of the confidence interval is to be used as a measure of the precision of the point estimates, the precision of the efficiency scores of the least efficient mines is higher than of the efficient ones. High efficiency scores are associated with wide intervals. This observation was also made by Gonzalez & Miles (2004) and Førsund et al (2006). Gonzalez & Miles (2004) indicate in their discussion that increasing the sample size does somewhat increase the number of units which are statistically significantly different from each other.

When the DMUs can be grouped into different categories, however, according to identifiable characteristics, it is possible to test the differences in estimated efficiency. Using statistics derived from methods suggested by Banker (1993), a different conclusion is reached which is that a best-performing sub-sample can be identified. The

mean efficiency of this best-performing sub-group is significantly higher than the total sample.

Comparing DEA results to bias-corrected DEA reveals, as shown from previous studies, marked changes in the estimates, with bias-correction lowering the efficiency scores for all individual mines<sup>42</sup>. Since the bias-corrected estimates point to significantly lower efficiencies, the potential for resource-savings identified by these (bias-corrected) estimates is much higher than suggested by the DEA. The fundamental result from bias-correction is that the potential savings identified by using this estimate are much higher than those implied by DEA. This has important policy implications, particularly over the question of which DEA method to use<sup>43</sup>. Policy recommendations based on DEA tend, therefore, to under-estimate the scope for efficiency improvements by generally over-stating the number of fully efficient mines. As a result, the unknown technology frontier can more plausibly be inferred from the bias-corrected results.

Finally, it can be seen that scale inefficiency is the predominant inefficiency when bias-corrected DEA is used. This set of conclusions is different from those obtained under DEA. Hence, again, bias-corrected DEA results in different policy implications.

Using bias-corrected results, the question to be asked is why scale inefficiency is predominant over technical efficiency. Also, given that Zimbabwe has been a gold

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<sup>42</sup> In some studies, Ferrier & Hirshberger (1997) and Gonzalez & Miles (2002), negative biases were observed. Manly (1997) also described negative biases for a zoological study on jackals.

<sup>43</sup> A large number of studies still use and draw conclusions based on the DEA without correcting for bias.

mining country for a considerable time now, are there other causes for the scale inefficiency which have afflicted its gold mines, in particular give that most of them are on the increasing returns to scale part of the VRS frontier? Could the political environment, for example be a reason why the scale efficiency is rather low? Or are there other features, geological or otherwise which contribute to low scale efficiencies? Question such as these require further data and investigation and would be interesting to follow after this dissertation.

There are weaknesses to this analysis, however. One concerns the variables which have been used. In particular, the proxy for capital services can and may contain distortions as a result of the age of the capital stock rather than the reflecting the actual size of the capital stock.

The other weaknesses which arise from this study are in comparison to other studies which have implemented DEA and bootstrap DEA. First of all, comparing the results here with those obtained in DEA study of mining shows that there are peculiarities in mining which need to be taken into account. The first is that mining can only take place where mineral deposits occur. This differentiates an efficiency study into gold mining from that into banking and finance, for example, where the latter economic activity could quite conceivable in any country.

Byrnes & Färe (1987) and Byrnes et al (1984) attempted to account for these mining-specific peculiarities by introducing mine-specific variables. Their non-discretionary variables included the degree of unionisation, the degree of labour intensity and geological characteristic such as thickness of the coal seams. While the labour intensity

issue is included, its weakness has already been noted—it is a partial productivity measure which ignores the role of capital, at least, and also capital-labour substitution. However, the results for gold mining have made the case for introducing the other non-discretionary variables which would capture the unique features for each mine. In particular, it would have been of great interest to distinguish between the performances of gold mine depending on the degree of unionisation.

In addition, it would seem that the results and certainly the explanations for the types of efficiencies which pre-dominate (technical or scale efficiency), would benefit from including geological characteristics such as whether the mining of gold involves the mining of other minerals such copper and silver. It would also have helped to include, in addition to the geological characteristics, other mine-specific characteristics such the mineralogical complexity of the ore and therefore distinguishing between oxide and sulphide ores, and, open-pit and underground mines. A major conclusion that can be drawn is that, to obtain meaningful results from the application of DEA in efficiency studies, the context in which the DMUs are operating is equally as important. The case for gold in Zimbabwe has shown the inadequacy of ignoring these contextual issues. It is therefore not been possible to attribute the different efficiencies to a particular source of set of sources.

## **CHAPTER 5 ZIMBABWEAN GOLD MINING IN A GLOBAL CONTEXT: INTERNATIONAL BENCHMARKING OF GOLD MINES**

### **5.1 Introduction**

In Chapter 4, an analysis of efficiency of Zimbabwean gold mining was undertaken. The results implied a relatively efficient gold mining sector in Zimbabwe when analysed in isolation, although scale and technical inefficiencies were identified. Given the way Zimbabwean gold mines perform relative to each other, a pertinent question to ask is how they compare with gold mines outside Zimbabwe. In this chapter such a wider perspective is adopted with efficiency being estimated using a second data set for an international sample. A warning is in order, however. The sources and dates for the two sets of data are different as are the variables used Chapter 4 was based on an anonymous sample from Zimbabwe census data. There is no [known] direct relationship between the Zimbabwean mines in this Chapter and those in Chapter 4 (which were, anyway, anonymous). In addition, the data sources and collection methods are totally different. For this chapter, data was obtained from a commercial database. In addition, the identities of the DMUs are known and extensive use is made of this knowledge in the analysis. Hence, the approach in this chapter, while based on the same DEA methodology, is slightly different from that adopted previously.

The objectives of the analysis in this chapter are to carry out an inter-country comparison of the productive efficiency of gold mines. In so doing, the assumption made is that there exists, and is being employed, a common technology across all the



countries in the sample<sup>44</sup>. This allows the identification of the most efficient gold mines in the sample and by inference in the world. In Chapter 2, it was noted that Zimbabwe had experienced periods of political and economic shocks. These shocks resulted in many constraints being faced by gold mines in Zimbabwe, constraints which were probably not faced by mines elsewhere.

In order to assess the impact of these shocks, a secondary objective is to examine how the performances of Zimbabwean gold mines in the sample compares with others elsewhere in the world, as measured by the estimated efficiency score. A further objective is to identify and analyse peer influences on the individual, inefficient mines in the sample. This is achieved by investigating (a) which efficient mines (peers) exercise the most influence on the subset of inefficient mines, (b) whether any one country has a preponderance of peers and (c) the international nature of peers in their reference set. The statistical properties of the estimated efficiency score are also investigated as in Chapter 4 to determine whether the differences in performance using the bootstrap method are statistically significant. Hypotheses on the characteristics of groups of mines and their efficiencies are also tested using non-parametric methods outlined in Chapter 3.

The chapter is organised as follows. The next section provides a description of the data. Section 5.3 reports the results of the DEA computations of efficiency and includes the decomposition of the overall efficiency score into technical efficiency and scale

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<sup>44</sup> Fundamentally, the technology as it is modelled in terms of the inputs and outputs is deemed the same. How the inputs are combined will necessarily differ from mine to mine.

efficiency components. This allows the identification of the nature of inefficiency. In Section 5.3.2, peer influence is discussed and analysed. An international comparative element is introduced with the aim of investigating whether there are any truly international peers; mines referenced by mines from every country and whether there are mines which are only referenced by mines from their own country. In Section 5.3.3, the bias-corrected DEA estimates are analysed. Section 5.4 provides some concluding remarks together with a summary and discussion of the main findings.

## **5.2 Data and Model Specification**

The data were extracted from a commercially available database provided by the Raw Materials Group (RMG)<sup>45</sup>. Their data have been compiled from a variety of sources including trade literature, company annual reports and press and media releases. The sample consists of fifty-nine gold mines from fifteen countries for the year 2003, the sample being determined by the availability of data for the key variables<sup>46</sup>. Already, it can be seen that this is for a different time period from that used in Chapter 4. The

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<sup>45</sup> The RMG is a Swedish research and consultancy group which has been collecting world mining data for more than 25 years. However their data tends to be used for specific commercial purposes and they ignore virtually most of the mining operations in Zimbabwe, except the very large-scale ones.

<sup>46</sup> The initial data set comprised about five hundred gold mines. However, there are missing data points for many of the sample. This is a result of the use to which the original data is put—mainly to monitor structural changes in the global mining industry, not just gold. The sample covers most of the largest primary gold producers in the world, and excludes all those which produce gold as a by-product of, say, copper (or other base metal) mining, platinum mining or those which, for a variety of reasons, did not operate for the full year.

sample also includes at least one gold mine from each major gold-producing region of Africa, Australasia, Central Asia, North America, South America and South-East Asia. For example, there are three Zimbabwean, nine South African, seven US mines. From Australia and Canada there are nineteen and sixteen mines respectively. There is at least a mine from three other African countries (Ghana, Mali and Tanzania), Russia, South-East-Asia and South America.

The DEA analysis is performed using a model comprising five variables -- one output and four inputs. The inputs comprise (a) the grade of gold, (b) the recovery rate of gold, (c) the maximum physical amount of gold ore that could be processed (rated capacity) in million tonnes per year, and (d) the labour force, which is represented by the total number of full-time employees.

A brief discussion on the grade and recovery rate is necessary in order to justify their inclusion in the analysis. The grade of gold in the ore produced is measured in grammes per tonne and the recovery rate is measured as a percentage of the indicated gold content extracted from each tonne of ore which is mined. The grade of ore and recovery rate are treated as non-purchased inputs which represent the characteristics of the ore. The grade variable differentiates between an occurrence of a mineral and a deposit<sup>47</sup>, and hence the quality of the mineral material being extracted. The recovery rate is a technical attainment indicator, representing the percentage of the metal contained in the ore produced which is recovered. This rate must not be confused with “efficiency” as

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<sup>47</sup> An ore deposit is one which can economically be exploited while an occurrence is one which, with current technology and prices, cannot.

the process of recovering gold is a function of factors such as the mineralogical and physical properties of the gold ore, for which the recovery rate is a proxy. There are also costs and benefits considerations when extracting metal at the margins as noted in the discussion on gold mining and processing in Chapter 1.

In discussing inputs, the issue of capital flows arises, once more. It has already been acknowledged in Chapter 4 that data for capital input are notoriously difficult to obtain and how, in that chapter, the service costs of capital was used in place of capital flows. Here as there, this problem of capturing capital flows is encountered. In this context, given the data available, the variable most likely to capture the differences in capital stock is the maximum amount of raisable ore. This is a key departure from Chapter 4 where the cost of servicing the capital stock was the proxy for capital. The key here is that the variable used is the expected annual output of ore (which is the maximum capacity of the mine). The case for using capacity as a proxy for capital is supported by several previous studies, including Simar & Wilson (1998), Löthgren & Tambour (1999) and Fried *et al* (2000), where the production capacity of a DMU is used in place of capital. Again, the shortcomings from the use of stock rather than the flow of capital services were documented in Chapter 4. Finally, with reference to Chapter 4, there is no energy and materials which further limit the comparability of ten results of the two sets of results. It must be borne in mind, however, that the research questions being addressed in this chapter do differ from those in Chapter 4.

The output variable is the total gold metal produced in tonnes per annum for the year 2003. The choice of the output variable is informed by fact this represents the main activity of gold mining.

The data set is briefly described with an overview of the main characteristics given by the descriptive statistics for the sample presented in Table 5.1.

**Table 5.1 Descriptive Statistics of the Data Set**

	Labour	Ore Production	Grade in Ore	Recovery	Gold Metal
Minimum	55.00	0.15	0.20	56.00	0.72
Standard Deviation	1713.61	8.68	7.43	8.91	9.23
Mean	1111.12	3.93	6.93	89.71	9.20
Median	400.00	1.32	5.81	92.00	6.13
1st Quartile	195.50	0.52	2.74	88.50	3.03
3rd Quartile	967.50	3.09	7.82	96.00	11.57
Maximum	7100.00	57.09	49.10	99.00	48.51
Coefficient of Variation	1.54	2.21	1.07	0.10	1.00

There is much variability in the data with the coefficient of variation being greater than 1 for all the variables. The variability indicated by these descriptive statistics is not surprising, as this is a sample of mines located across the world, the sizes of which are invariably determined by geological and other conditions. The most significant variation is in the maximum capacity of the mines as shown by the quantity of raisable ore.

The data are reported in appendix E. The regional dimensions are interesting. It can be seen that Australian, Canadian, South African and US mines are particularly large as measured by gold output. A large number of mines in these countries produce more than the mean annual gold output of 6.13 tonnes. The largest mine is in the US, producing over 48 tonnes of gold per annum. There are twenty-three other mines from Australia, Canada and South Africa which produced more than sample average of 6.13 tonnes. The three Zimbabwe mines are the smallest here, already illustrating some mine

characteristics in relation to the rest of the sample. Still, caution needs to be applied as there are only three Zimbabwe mines out of at least thirty-four (in comparison to Chapter 4). Freda-Rebecca, producing 1.59 tonnes of gold is the largest Zimbabwean mine while smallest mine (and in the sample as well) is Renco, producing 0.72 tonnes of gold.

The other variable for which there is notable variability is the grade of ore with the lowest grade being 0.2 grammes per tonne and the highest being 49.1 grammes per tonne. It can be observed (from appendix E) that a number of US mines have grades at below 1 gramme per tonne, an indication of the variation in the geological quality of the ore. Additionally the Zimbabwean mines generally have lower grade deposits than the mean grade at 7.43 grammes per tonne, with Renco grading the highest at 3.59.

Given the strong similarities between South Africa and Zimbabwe, particularly with regard to geological conditions and the ownership of mines<sup>48</sup>, this size distribution begs a question to be asked. Why are do mines from countries with such similar geology and political and economic history differ so much? Recall also from Chapter 2 that even in the early days, one of the biggest disappointments of Cecil John Rhodes's prospectors was the absence of any deposits comparable in size, but not geology, to those found in South Africa. Equally valid, is the possibility that the deeper-lying deposits in Zimbabwe have not yet been discovered and this is a reflection of the level of exploration investment. However, since this does not lie within the scope of this

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<sup>48</sup> The mines captured by the database tend to be the large ones, so the three mines would typically be among the largest in Zimbabwe.

research, it must be left for another study. This study will be focused on efficiency estimation given the observed path of development of the Zimbabwean gold mining industry.

### 5.3.1 Results: DEA Estimates

Estimates of efficiency were obtained using both CRS and VRS DEA (Equations 3.9 and 3.10). The DEA routines outlined in Chapter 3 were employed and the individual DEA scores are given in Table 5.2 together with the summary statistics.

**Table 5.2: DEA Results and Scale Efficiency Scores**

MINE	COUNTRY	(1) CRS DEA	(2) VRS DEA	(3) SCALE EFFICIENCY
Cerro Vanguardia Mine	Argentina	0.6228	0.8409	0.7406
Super Pit Mine	Australia	1.0000	1.0000	1.0000
Granny Smith Mine	Australia	0.4545	0.8746	0.5197
Peak Mine	Australia	0.5221	0.9817	0.5318
Plutonic Mine	Australia	0.4559	0.8723	0.5226
Challenger Mine	Australia	0.4983	1.0000	0.4983
Thunderbox Mine	Australia	0.4512	0.9279	0.4863
Kirkalocka Mine	Australia	0.2595	1.0000	0.2595
Gidgee Mine	Australia	0.4194	0.9360	0.4481
Norseman Mine	Australia	0.5632	0.9209	0.6116
Darlot Mine	Australia	0.5453	0.9175	0.5943
Lawlers Mine	Australia	0.4066	0.9244	0.4399
Henty Mine	Australia	0.6599	0.9780	0.6747
Sao Bento Mine	Brazil	0.5537	0.9277	0.5969
Crixas (Serra Grande) Mine	Brazil	0.6514	0.8802	0.7401
Kemess South Mine	Canada	0.7125	1.0000	0.7125
Troilus Mine	Canada	0.4377	0.9262	0.4726
Laronde Mine	Canada	0.3725	0.8783	0.4241

MINE	COUNTRY	(1) CRS DEA	(2) VRS DEA	(3) SCALE EFFICIENCY
Holloway Mine	Canada	0.4388	0.9568	0.4586
Joe Mann Mine	Canada	0.4487	1.0000	0.4487
Eskay Creek Mine	Canada	1.0000	1.0000	1.0000
Holt McDermott Mine	Canada	0.4529	0.9423	0.4806
Seabee Mine	Canada	0.4530	1.0000	0.4530
Golden Giant Mine	Canada	0.8267	0.9540	0.8666
Campbell Mine	Canada	0.9091	1.0000	0.9091
Musselwhite Mine	Canada	0.5073	0.8969	0.5656
Doyon Mine	Canada	0.5097	0.8714	0.5849
Sleeping Giant Mine	Canada	0.5328	0.9461	0.5632
Beaufor Mine	Canada	0.4604	0.9363	0.4917
Bibiani Mine	Ghana	0.3477	0.9993	0.3479
Iduapnem Mine	Ghana	0.4782	0.8864	0.5395
Kumtor Mine	Kyrgyzstan	0.6630	0.9478	0.6995
Penjom Mine	Malaysia	0.5923	0.9803	0.6042
Sadiola Mine	Mali	0.6435	1.0000	0.6435
Morila Mine	Mali	0.8828	0.9371	0.9421
Orcopampa Mine	Peru	0.8539	0.9875	0.8647
Julietta Mine	Russia	0.5878	1.0000	0.5878
Kubaka Mine	Russia	0.5255	0.8702	0.6039
Ergo Gold Tailings Mine	South Africa	0.7103	1.0000	0.7103
Petrex Mines	South Africa	0.3212	0.8703	0.3691
Tau Lekoa Mine	South Africa	0.4581	0.8098	0.5657
South Deep Mine	South Africa	0.6715	0.8164	0.8225
Great Noligwa Mine	South Africa	0.9849	0.9909	0.9939
Savuka (West) Mine	South Africa	0.5095	0.7765	0.6561
Tautona Mine	South Africa	1.0000	1.0000	1.0000
Kopanang Mine	South Africa	0.6919	0.8184	0.8454
Mponeng (South) Mine	South Africa	0.8054	0.8723	0.9233
Geita Mine	Tanzania	0.7463	0.9074	0.8225
Chatree Mine	Thailand	0.3314	0.9424	0.3517
Round Mountain Mine	USA	1.0000	1.0000	1.0000
Montana Tunnels Gold Mine	USA	0.2725	1.0000	0.2725



MINE	COUNTRY	(1) CRS DEA	(2) VRS DEA	(3) SCALE EFFICIENCY
Fort Knox Mine	USA	0.8332	1.0000	0.8332
Betze Post Mine	USA	1.0000	1.0000	1.0000
Cortez Mine	USA	1.0000	1.0000	1.0000
Meikle (Purple Vein) Mine	USA	0.6943	1.0000	0.6943
Bald Mountain Mine	USA	0.4828	1.0000	0.4828
Renco Mine	Zimbabwe	0.2288	1.0000	0.2288
Blanket Mine	Zimbabwe	0.1195	1.0000	0.1195
Freda Rebecca Mine	Zimbabwe	0.1651	0.9593	0.1721
Mean		0.5886	0.9434	0.6236
Std. Dev.		0.2272	0.0609	0.2293
Median		0.5328	0.9540	0.5943
Minimum		0.1195	0.7765	0.1195
Maximum		1.0000	1.0000	1.0000
Coefficient of Variation		0.3861	0.0646	0.3677

The sample is characterised by some low efficiency scores with a minimum of 0.1195, a median of 0.5328 and a mean of 0.5886 when analysing the CRS frontier (overall efficiency). The mean indicates an estimated average overall efficiency of 41.14 per cent. Out of the fifty-nine mines, twenty-four have estimated an overall efficiency score of less than 0.5. Analysing technical efficiency, the mean increases to 0.9434 and of the mines have an estimated technical efficiency score of less than 0.5. This implies that on average, technical efficiency falls short by about 5.66 per cent. Both measures exhibit low variance with the coefficient of variation being lower than 1 in both cases.

The analysis proceeds to look at individual efficiency scores. Assuming CRS technology, it can be seen that six mines are judged efficient, that is have an overall efficiency score of 1. These mines represent what can be termed the “advanced gold

mining economies”; three are American, one Australian, one Canadian and one South African<sup>49</sup>. Therefore the peers on the overall efficiency frontier are from those economies. The implication is that the overall efficiency frontier is defined by the mines from these developed mining economy countries. Hence, a tentative conclusion is that the optimal scale of operations for the mines in this sample are defined by these countries.

It is observed that no Zimbabwean mines are overall efficient. In fact, the three least efficient mines in the sample are Zimbabwean with Blanket Mine, with an estimated efficiency score of 0.1195, being the most inefficient. Of the three, Renco is the most efficient with an estimated efficiency score of 0.2288. The least efficient is Blanket, which also happens to be the oldest among these three mines having started operations in the 1960s and the other two having started after 1980.

A legitimated question would be why do the Zimbabwean mines have such obviously low overall efficiency scores? This may be an indication of a uniquely Zimbabwean characteristic and needs to be used to qualify any inferences on Zimbabwean mines. The next most inefficient mines are Kirkalocka (Australia) and Montana Tunnels (USA) with scores of 0.2595 and 0.2725 respectively. There are also mines from Canada and South Africa which have poor overall efficiency estimates, indicating this is not

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<sup>49</sup> Advanced here refers to the level of development, similar in fact to economic development. These are the largest gold producers and the major technological inventions and innovations in gold mining have mainly come from these countries.

confined to Zimbabwe alone but extends over mines from the developed mining economies.

Turning to the technical efficiency measures, twenty-one mines are now judged efficient, as opposed to six. The minimum efficiency score rises from 0.1195 (for Blanket) to 0.7765 for Savuka of South Africa. Hence, the two frontiers identify different inefficient mines. The average efficiency score rises to 0.9434 from 0.5886.

In terms of cross-country distribution, the largest number of efficient mines comes from the USA with all the American DMUs having technical efficiency scores of 1.

However, there now are some mines from other countries as well, such as from Mali, Russia and Zimbabwe. Two of the three mines from Zimbabwean, (Renco and Blanket), are now identified as being fully efficient. Given the low overall efficiency scores, the implication is that these mines have low scale efficiency scores.

Scale efficiency estimates are presented in Column 3 of Table 5.2. It is observed that for most of the mines, the scale efficiency score in Column 3 is lower than the technical efficiency score in Column 2, indicating that most of the mines suffer from scale inefficiency. Of the thirty-eight mines deemed to have technically inefficient, only five have a scale efficiency score higher than the technical efficiency score. This implies that, in this sample, scale inefficiency largely dominates technical inefficiency. The levels of scale efficiency mirror those of the overall efficiency in size. Hence, there are a large number of mines (twenty) with a scale efficiency score of less than 0.5. These, as with overall efficiency, include mines from both developed and developing economies. An observation can therefore be made that using DEA, scale efficiency

dominates local technical efficiency in gold mining. Having the frontier defined by six mines and with such a large fraction of the sample exhibiting such low efficiency scores is an indication that a large proportion of the sample is well away from the overall efficient frontier. The other implication is that in a sample of world mines, Zimbabwean DMUs suffer from scale inefficiencies.

Given this observation about the nature of overall efficiency, it is pertinent to analyse the nature of return to scale. These are reported in Table 5.3.

**Table 5.3: Nature of Returns to Scale**

MINE	COUNTRY	$\sum \lambda_i$ CRS	$\sum \lambda_i$ NIRS	$\frac{\sum \lambda_i CRS}{\sum \lambda_i NIRS}$	RTS
Cerro Vanguardia Gold Mine	Argentina	0.3624	0.3624	1.0000	IRS
Super Pit Gold Mine	Australia	1.0000	1.0000	1.0000	CRS
Granny Smith Gold Mine	Australia	0.1896	0.1896	1.0000	IRS
Peak Gold Mine	Australia	0.1885	0.1885	1.0000	IRS
Plutonic Gold Mine	Australia	0.2949	0.2949	1.0000	IRS
Challenger Gold Mine	Australia	0.0857	0.0857	1.0000	IRS
Thunderbox Gold Mine	Australia	0.2047	0.2047	1.0000	IRS
Kirkalocka Gold Mine	Australia	0.0652	0.0652	1.0000	IRS
Gidgee Gold Mine	Australia	0.1106	0.1106	1.0000	IRS
Norseman Gold Mine	Australia	0.2490	0.2490	1.0000	IRS
Darlot Gold Mine	Australia	0.2353	0.2353	1.0000	IRS
Lawlers Gold Mine	Australia	0.1161	0.1161	1.0000	IRS
Henty Gold Mine	Australia	0.2195	0.2195	1.0000	IRS
Sao Bento Gold Mine	Brazil	0.1875	0.1875	1.0000	IRS
Crixas (Serra Grande) Gold Mine	Brazil	0.3458	0.3458	1.0000	IRS
Kemess South Gold Mine	Canada	0.3568	0.3568	1.0000	IRS
Troilus Copper/Gold Mine	Canada	0.1884	0.1884	1.0000	IRS
Laronde Gold Mine	Canada	0.2152	0.2152	1.0000	IRS

MINE	COUNTRY	$\Sigma \lambda_i CRS$	$\Sigma \lambda_i NIRS$	$\frac{\Sigma \lambda_i CRS}{\Sigma \lambda_i NIRS}$	RTS
Holloway Gold Mine	Canada	0.1355	0.1355	1.0000	IRS
Joe Mann Gold Mine	Canada	0.4487	0.4487	1.0000	IRS
Eskay Creek Gold Mine	Canada	1.0000	1.0000	1.0000	CRS
Holt McDermott Gold Mine	Canada	0.1663	0.1663	1.0000	IRS
Seabee Gold Mine	Canada	0.1129	0.1129	1.0000	IRS
Golden Giant Gold Mine	Canada	0.4530	0.4530	1.0000	IRS
Campbell Gold Mine	Canada	0.4527	0.4527	1.0000	IRS
Musselwhite Gold Mine	Canada	0.2790	0.2790	1.0000	IRS
Doyon Gold Mine	Canada	0.2786	0.2786	1.0000	IRS
Sleeping Giant Gold Mine	Canada	0.1545	0.1545	1.0000	IRS
Beaufor Gold Mine	Canada	0.1155	0.1155	1.0000	IRS
Bibiani Gold Mine	Ghana	0.1756	0.1756	1.0000	IRS
Iduapriem Gold Mine	Ghana	0.1770	0.1770	1.0000	IRS
Kumtor Gold Mine	Kyrgyzstan	0.4598	0.4598	1.0000	IRS
Penjom Gold Mine	Malaysia	0.2376	0.2376	1.0000	IRS
Sadiola Gold Mine	Mali	0.3245	0.3245	1.0000	IRS
Moria Gold Mine	Mali	0.9038	0.9038	1.0000	IRS
Orcopampa Gold Mine	Peru	0.4184	0.4184	1.0000	IRS
Julietta Gold Mine	Russia	0.3350	0.3350	1.0000	IRS
Kubaka Gold Mine	Russia	0.2724	0.2724	1.0000	IRS
Ergo Gold Mine	South Africa	0.2583	0.2583	1.0000	IRS
Petrex Gold Mines	South Africa	0.1301	0.1301	1.0000	IRS
Tau Lekoa Gold Mine	South Africa	0.2975	0.2975	1.0000	IRS
South Deep Gold Mine	South Africa	0.5169	0.5169	1.0000	IRS
Great Noligwa Gold Mine	South Africa	1.0147	1.0000	1.0147	DRS
Savuka (West) Gold Mine	South Africa	0.2636	0.2636	1.0000	IRS
Tautona Gold Mine	South Africa	1.0000	1.0000	1.0000	CRS
Kopanang Gold Mine	South Africa	0.5556	0.5556	1.0000	IRS
Mponeng (South) Gold Mine	South Africa	0.6695	0.6695	1.0000	IRS
Geita Gold Mine	Tanzania	0.4459	0.4459	1.0000	IRS
Chatree Gold Mine	Thailand	0.1526	0.1526	1.0000	IRS
Round Mountain Gold Mine	USA	1.0000	1.0000	1.0000	IRS

MINE	COUNTRY	$\Sigma \lambda_i CRS$	$\Sigma \lambda_i NIRS$	$\frac{\Sigma \lambda_i CRS}{\Sigma \lambda_i NIRS}$	RTS
Montana Tunnels Gold Mine	USA	0.0649	0.0649	1.0000	IRS
Fort Knox Gold Mine	USA	0.4616	0.4616	1.0000	IRS
Betze Post Gold Mine	USA	1.0000	1.0000	1.0000	CRS
Cortez Gold Mine	USA	1.0000	1.0000	1.0000	CRS
Meikle (Purple Vein) Gold Mine	USA	1.0000	1.0000	1.0000	CRS
Bald Mountain Gold Mine	USA	0.0987	0.0987	1.0000	IRS
Renco Gold Mine	Zimbabwe	0.0400	0.0400	1.0000	IRS
Blanket Gold Mine	Zimbabwe	0.0299	0.0299	1.0000	IRS
Freda Rebecca Gold Mine	Zimbabwe	0.0443	0.0443	1.0000	IRS

Six mines are confirmed to be operating under constant returns to scale the same mines which with overall efficiency scores of 1. Of the fifty-three mines which are deemed globally inefficient, it is observed that fifty-two mines are operating under increasing returns to scale. Only one mine, Great Nologwa, is exhibiting decreasing returns to scale.

It can therefore be concluded that the predominant reason for the low overall inefficiencies is operating at the wrong scale. Specifically, most mines are operating under IRTS, implying that their performance can be improved by upward adjustments in scale. Given that the scale inefficiencies are relatively large for a significant proportion of the sample, these scale adjustments would necessarily need to be large, too. For those mines deemed to have low overall efficiency scores, only one of them, Great Nologwa (South Africa) could benefit from a reduction in scale.

In terms of figure 3.4, the increasing returns production possibility part of the VRS frontier is a large distance from the CRS frontier, explaining the large divergences between the two efficiency measures.

### 5.3.2 Analysis of Peer Influence

One of the benefits of using DEA is that it identifies peers for inefficient mines. In considering behavioural and organizational changes for inefficient mines, the characteristics of the peers may help in defining the reference DMU which these inefficient mines emulate. The peers and the number of mines which refer to them are given in Table 5.4, which shows the technically efficient peers which are referenced by relatively *inefficient* units. This excludes self-evaluators of more of which there were three.

**Table 5.4: Gold Mining Peers: VRS DEA**

MINE	COUNTRY	PEER INFLUENCE (%)	COUNT AS PEER
Challenger Mine	Australia	23.3201	28
Kirkalocka Mine	Australia	3.7816	5
Kemess South Mine	Canada	1.8487	3
Joe Mann Mine	Canada	3.9919	5
Eskay Creek Mine	Canada	2.7254	16
Campbell Mine	Canada	1.7241	6
Sadiola Mine	Mali	3.2684	2
Julietta Mine	Russia	2.2289	3
Ergo Gold Tailings Mine	South Africa	1.7407	5
Tautona Mine	South Africa	1.7610	4
Round Mountain Mine	USA	3.2409	3
Montana Tunnels Gold Mine	USA	1.7241	23
Betze Post Mine	USA	8.6101	17

Cortez Mine	USA	1.7241	38
Meikle (Purple Vein) Mine	USA	3.6703	9
Bald Mountain Mine	USA	9.9420	2
Renco Mine	Zimbabwe	2.4797	39
Blanket Mine	Zimbabwe	1.8084	9

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The peers subset includes mines from both the developing economies such as Mali and Zimbabwe and from the advanced mining economies. Out of the fifteen countries in the sample, seven provide peers.

It has already been seen from Table 5.2 that the USA provides the largest number of efficient mines with all the seven mines from that country adjudged fully efficient under VRS technology (and three under CRS). Of the seven, six are referred to by other mines, implying the other one, Fort Knox, is a self-evaluator. The next highest number of peers is provided by Canada with five of which one, Seabee is another self-evaluator.

Zimbabwe along with Australia and South Africa provides two peers. In the context of the VRS DEA frontier, this is defined by the mines predominantly from the USA and Canada, but also from Australia, Mali, Russia, South Africa and Zimbabwe.

An important point, when looking at peers is also to determine how influential the individual peers are. This can be assessed by looking at the number of times a peer is referenced by inefficient mines (as reported in Table 5.4). From Table 5.4, it is can be seen that the most referenced mines are Renco (Zimbabwe) and Cortez (USA), in that they appear the most often in the reference set of inefficient mines', with a frequency (count) of thirty-nine and thirty-eight respectively. Using another measure of peer influence which takes into account the weights ( $\lambda_i$ ), Challenger (Australia), however, is



the most influential peer with a peer influence index of 23 per cent. It can be seen that there is divergence between peer influences as measured by the peer count and by peer influence and which measure one adopts depends entirely on the context.

Table 5.5 reports the DMUs which reference the peers

**Table 5.5: Peers and Referencing Mines**

Cerro Vanguardia Mine	0.2141	0.0000	0.0000	0.0000	0.0109	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1843	0.0000	0.0000	0.5908	0.0000
Granny Smith Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5938	0.0000	0.0101	0.2114	0.0000	0.0000	0.0636	0.1210
Peak Mine	0.8308	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0306	0.0000	0.0000	0.0671	0.0000	0.0000	0.0715	0.0000
Plutonic Mine	0.0120	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3050	0.0000	0.0000	0.2883	0.0000	0.0000	0.3947	0.0000
Thunderbox Mine	0.2026	0.3022	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2980	0.0000	0.0000	0.1483	0.0000	0.0489	0.0000	0.0000
Gidgee Mine	0.9469	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0033	0.0000	0.0000	0.0044	0.0000	0.0000	0.0454	0.0000
Norseman Mine	0.7983	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0015	0.0000	0.0000	0.0787	0.0000	0.0000	0.1215	0.0000
Darlot Mine	0.7733	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0312	0.0000	0.0000	0.1004	0.0000	0.0000	0.0950	0.0000
Lawlers Mine	0.8792	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0621	0.0000	0.0000	0.0427	0.0000	0.0000	0.0161	0.0000
Henty Mine	0.7176	0.0000	0.0000	0.0949	0.1316	0.0000	0.0558	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sao Bento Mine	0.4138	0.0000	0.0000	0.0000	0.0823	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0164	0.0264	0.0000	0.4612	0.0000
Crixas (Serra Grande) Mine	0.4302	0.0000	0.0000	0.0000	0.0613	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1131	0.0258	0.0000	0.3696	0.0000
Troilus Mine	0.0000	0.0000	0.0723	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8603	0.0000	0.0605	0.0000	0.0000	0.0000	0.0069	0.0000
Laronde Mine	0.1942	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2512	0.0000	0.0000	0.1895	0.0000	0.0000	0.2712	0.0939
Holloway Mine	0.8757	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0196	0.0000	0.0000	0.0225	0.0000	0.0000	0.0822	0.0000

Joe Mann Mine	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Holt McDermott Mine	0.8223	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0186	0.0000	0.0000	0.0363	0.0000	0.0000	0.1228	0.0000
Golden Giant Mine	0.4817	0.0000	0.0000	0.0000	0.0489	0.0000	0.1796	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2702	0.0000	0.0195	0.0000
Musselwhite Mine	0.5854	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0942	0.0000	0.0000	0.1696	0.0000	0.0000	0.1508	0.0000
Doyon Mine	0.4507	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0766	0.0000	0.0000	0.1693	0.0000	0.0000	0.3035	0.0000
Sleeping Giant Mine	0.1329	0.0000	0.0000	0.7451	0.0711	0.0000	0.0000	0.0000	0.0096	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0412	0.0000
Beaufor Mine	0.5053	0.0000	0.0000	0.4752	0.0133	0.0000	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Bibiani Mine	0.0517	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4549	0.0000	0.0000	0.1660	0.0000	0.0000	0.3274	0.0000
Iduapriem Mine	0.0000	0.3848	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4106	0.0000	0.1218	0.0000	0.0000	0.0000	0.0000	0.0828
Kumtor Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2928	0.0000	0.0214	0.0000	0.0000	0.0000	0.0000	0.3417	0.0000	0.0000	0.0000	0.3442	0.0000
Penjom Mine	0.7572	0.0000	0.0000	0.0000	0.0045	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0718	0.0000	0.0000	0.1665	0.0000
Morila Mine	0.0322	0.0000	0.0000	0.0000	0.0500	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7228	0.0000	0.0000	0.1949	0.0000
Orcopampa Mine	0.2301	0.0000	0.0000	0.0000	0.0892	0.0000	0.6541	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0138	0.0000	0.0128	0.0000
Kubaka Mine	0.5564	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0285	0.0000	0.0000	0.1196	0.0000	0.0000	0.2954	0.0000
Petrex Mines	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0597	0.0256	0.0000	0.0000	0.1290	0.7858
Tau Leko Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0846	0.1566	0.0000	0.0000	0.3192	0.4395
South Deep Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0182	0.3738	0.0000	0.0000	0.6080	0.0000
Great Noligwa Mine	0.0000	0.0000	0.0000	0.0000	0.0176	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5748	0.0000	0.0000	0.0000	0.4076	0.0000	0.0000	0.0000	0.0000
Savuka (West) Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0091	0.1442	0.0000	0.0000	0.8467	0.0000

Kopanang Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0221	0.4217	0.0000	0.0000	0.5561	0.0000
Mponeng (South) Mine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3049	0.0000	0.0000	0.0000	0.0000	0.2224	0.1020	0.0000	0.3707	0.0000
Geita Mine	0.0000	0.2371	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2821	0.0000	0.4009	0.0000	0.0000	0.0000	0.0000	0.0799
Chatree Mine	0.6310	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1718	0.0000	0.0000	0.0860	0.0000	0.0000	0.1007	0.0105
Freda Rebecca Mine	0.0000	0.2692	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0057	0.0000	0.0000	0.0092	0.7159
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

**Key:** Each number in the bottom row corresponds to a peer in table below.

Challenger Mine	1
Kirkalocka Mine	2
Kemess South Mine	3
Joe Mann Mine	4
Eskay Creek Mine	5
Seabee Mine	6
Campbell Mine	7
Sadiola Mine	8
Julietta Mine	9

Ergo Gold Tailings Mine	10
Tautona Mine	11
Round Mountain Mine	12
Montana Tunnels Gold Mine	13
Fort Knox Mine	14
Betze Post Mine	15
Cortez Mine	16
Meikle (Purple Vein) Mine	17
Bald Mountain Mine	18
Renco Mine	19
Blanket Mine	20

Table 5.5 is read with the bottom row (numbered 1 to 20) containing the peers and the key to interpreting them is given in the table below it. Hence, 1 stands for Challenger mine and the non-zero values in that whole column indicating the  $\lambda_i$  where the constraint is binding. Therefore the mines in that row reference Challenger. The rows sum to 1 (the VRS constraint,  $\sum \lambda_i = 1$ ). The sizes of the  $\lambda_i$  are an indication of the influence the peer on the reference DMU for the inefficient mine. It can be observed that Challenger does exert the most influence on those inefficient mines which reference it. Hence, it exerts its greatest influence on Gidgee (Australia) and the least on Plutonic (Australia).

Turning to the Zimbabwean peers, it can be seen that Renco exerts its highest influence on Savuka (South Africa). Most South African mines also reference Renco. Blanket is the most influential peer in Freda Rebecca's reference set.

From Table 5.5, it can be seen that all the peers are truly international in nature. There are no "national peers" which are only referenced by mines from their own countries. The most international peer is Renco which is referenced by mines from ten other countries. As can be deduced, there is no peer which is referenced by mines from each country which would have been truly international.

This exercise has also illuminated number of interesting points. For a variety of reasons, it is expected that mines from Australia, the USA, Canada and South Africa would dominate the peers subset and this has proved to be the case. The main reason why this dominance is expected is that these countries also have longer mining (in the modern era) traditions, have pioneered modern mining and mineral processing technologies.

They have also heavily invested in a large number of mining schools, professional courses and have active government departments which spend large amounts of money on R & D. Whether using the peer count or the percentage peer index, the most influential mines come from these four countries. In particular, an Australian mine, Challenger, is the most influential peer, as judged by the percentage influence it exerts on the location and shape of the VRS frontier. The two next influential mines are from the USA, Bald Mountain and Betze Post. Hence, as both Tables 5.4 and 5.5 show, the most influential peers (whichever method is used to measure influence) are mainly drawn from the advanced mining economies. It needs to be borne in mind that these countries also provide the largest number of DMUs to the sample and therefore the inference must be tampered by this qualification.

Further, given the economic and political shocks which often taken place in Africa, in particular in Zimbabwe, and also the pressures under which the mining sectors Has tended to operate, it is slightly surprising that mines from Zimbabwean are part of this peers' subset.

Recall from Chapter 2 the challenges which the Zimbabwean mining sector has faced, such as shortages of foreign currency, political instability and intermittent access to technology since the 1960s. The sample provides evidence that wherever gold mining is taking place, technical efficiency is determined by other factors other than the usual political environmental ones. In particular, these results show that a mine can be technically efficient despite encountering external constraints. An interesting aspect of all this is that Freda-Rebecca, the newest mine, although with a relatively high technical efficiency score, does not appear as a peer. It is the relatively older Zimbabwean mines,

Renco and Blanket, which are regarded as peers. Renco happens to be the most influential peer using the peer count. The fact that its influence is less under the peer index method is an indication of how small its limited influence is on each of the in each of the thirty-nine referencing sets where it appears as a peer. Being a peer tends to imply that it has some distinct features. These, unfortunately cannot be identified from the present data set.

Finally, it was observed that there are three mines which are self-evaluators. Efficient DMUs which self-evaluate are arguably unique in some way and may have certain characteristics unaccounted for by the available data such as different mine-specific characteristics (some gold may be mined in association with copper, silver or lead-zinc), different ownership and (therefore objectives), geological conditions among many others. Not enough data are available to pursue this line of enquiry in this study. As a result, a discussion on self-evaluators would be a convenient way to conclude this section. Bias-correction which will be implemented later has been shown to change the positions of so many of the efficient DMUs such that those forming the bias-corrected frontier may differ from those defining the DEA frontier.

### **5.3.3 Bias-corrected DEA Analysis**

In this section, the results of the bootstrap and bias-correction are analysed. In Table 5.6, the individual bias-corrected efficiency scores are reported while the summary statistics of both the bias-corrected CRS and VRS scores with comparisons to the original DEA result are given in Table 5.7



**Table 5.6: Bias-corrected Bootstrap DEA Efficiency**

(1) MINE	(2) COUNTRY	(3) Bias-corrected CRS	(4) CRS Bias	(5) Bias-corrected VRS	(6) VRS Bias	(7) Bias-corrected Scale
Cerro Vanguardia Mine	Argentina	0.5809	0.0419	0.8166	0.0243	0.7685
Super Pit Mine	Australia	0.5865	0.4135	0.9305	0.0695	0.6629
Granny Smith Mine	Australia	0.3431	0.1114	0.8446	0.0300	0.6259
Peak Mine	Australia	0.4776	0.0445	0.9693	0.0124	0.5699
Plutonic Mine	Australia	0.3570	0.0989	0.8484	0.0239	0.6192
Challenger Mine	Australia	0.4151	0.0832	0.9371	0.0629	0.5482
Thunderbox Mine	Australia	0.2922	0.1590	0.8925	0.0354	0.6328
Kirkalocka Mine	Australia	0.1750	0.0845	0.9341	0.0659	0.3234
Gidgee Mine	Australia	0.3905	0.0289	0.9090	0.0270	0.4657
Norseman Mine	Australia	0.5295	0.0337	0.9086	0.0123	0.6397
Darlot Mine	Australia	0.5014	0.0439	0.9064	0.0111	0.6345
Lawlers Mine	Australia	0.3036	0.1030	0.8930	0.0314	0.5337
Henty Mine	Australia	0.5321	0.1278	0.9446	0.0334	0.7783
Sao Bento Mine	Brazil	0.5257	0.0280	0.8997	0.0280	0.6088
Crixas (Serra Grande) Mine	Brazil	0.6147	0.0367	0.8606	0.0196	0.7649
Kemess South Mine	Canada	0.4786	0.2339	0.9416	0.0584	0.8244
Troilus Mine	Canada	0.3223	0.1154	0.8929	0.0333	0.5769
Laronde Mine	Canada	0.2918	0.0807	0.8595	0.0188	0.5052
Holloway Mine	Canada	0.4149	0.0239	0.9322	0.0246	0.4716
Joe Mann Mine	Canada	0.4169	0.0318	0.9362	0.0638	0.4527
Eskay Creek Mine	Canada	0.8532	0.1468	0.9341	0.0659	0.5661
Holt McDermott Mine	Canada	0.4304	0.0225	0.9290	0.0133	0.4976
Seabee Mine	Canada	0.4085	0.0445	0.9431	0.0569	0.4715
Golden Giant Mine	Canada	0.7579	0.0688	0.9261	0.0279	0.9068
Campbell Mine	Canada	0.7976	0.1115	0.9533	0.0467	0.9068
Musselwhite Mine	Canada	0.4393	0.0680	0.8850	0.0119	0.6331
Doyon Mine	Canada	0.4625	0.0472	0.8579	0.0135	0.6295
Sleeping Giant Mine	Canada	0.4821	0.0507	0.9140	0.0321	0.5969
Beaufor Mine	Canada	0.4311	0.0293	0.9021	0.0342	0.5050
Bibiani Mine	Ghana	0.2787	0.0690	0.9790	0.0203	0.4088

Iduapriem Mine	Ghana	0.4217	0.0565	0.8578	0.0286	0.5846
Kumtor Mine	Kyrgyzstan	0.4750	0.1880	0.9212	0.0266	0.8286
Penjom Mine	Malaysia	0.5644	0.0279	0.9677	0.0126	0.6246
Sadiola Mine	Mali	0.5233	0.1202	0.9703	0.0297	0.7405
Morila Mine	Mali	0.5891	0.2937	0.9028	0.0343	0.7572
Orcopampa Mine	Peru	0.7379	0.1160	0.9564	0.0311	0.9068
Julietta Mine	Russia	0.2851	0.3027	0.9358	0.0642	0.7429
Kubaka Mine	Russia	0.4929	0.0326	0.8587	0.0115	0.6330
Ergo Gold Tailings Mine	South Africa	0.3412	0.3691	0.9340	0.0660	0.7538
Petrex Mines	South Africa	0.2887	0.0325	0.8440	0.0263	0.3947
Tau Lekoa Mine	South Africa	0.3806	0.0775	0.7871	0.0227	0.6435
South Deep Mine	South Africa	0.5994	0.0721	0.7965	0.0199	0.8762
Great Noliwa Mine	South Africa	0.8171	0.1678	0.9505	0.0404	0.8686
Savuka (West) Mine	South Africa	0.4731	0.0364	0.7518	0.0247	0.6818
Tautona Mine	South Africa	0.8428	0.1572	0.9405	0.0595	0.8894
Kopanang Mine	South Africa	0.6051	0.0868	0.7975	0.0209	0.8984
Mponeng (South) Mine	South Africa	0.7212	0.0842	0.8455	0.0268	0.9307
Geita Mine	Tanzania	0.6119	0.1344	0.8768	0.0306	0.8765
Chatree Mine	Thailand	0.2585	0.0729	0.9243	0.0181	0.4210
Round Mountain Mine	USA	0.9118	0.0882	0.9365	0.0635	0.5174
Montana Tunnels Gold Mine	USA	0.2160	0.0565	0.9343	0.0657	0.3094
Fort Knox Mine	USA	0.6293	0.2039	0.9535	0.0465	0.8691
Betze Post Mine	USA	0.3473	0.6527	0.9353	0.0647	0.5844
Cortez Mine	USA	0.3547	0.6453	0.9339	0.0661	0.5529
Meikle (Purple Vein) Mine	USA	0.8004	0.1006	0.9425	0.0575	0.8485
Bald Mountain Mine	USA	0.3903	0.0925	0.9349	0.0651	0.5412
Renco Mine	Zimbabwe	0.2146	0.0142	0.9326	0.0674	0.2281
Blanket Mine	Zimbabwe	0.1069	0.0126	0.9379	0.0621	0.1246
Freda Rebecca Mine	Zimbabwe	0.1487	0.0164	0.9219	0.0374	0.1822

**Table 5.7 Descriptive Statistics: DEA and Bias-corrected DEA Estimates**

	CRS DEA	Bias –corrected CRS	VRS DEA	Bias –corrected VRS	SCALE	Bias-corrected SCALE
Mean	0.5886	0.4493	0.9434	0.9062	0.6236	0.6261
Standard Deviation	0.2272	0.1815	0.0609	0.0499	0.2293	0.1911
Median	0.5328	0.4311	0.9540	0.9261	0.5943	0.6259
Minimum	0.1195	0.1069	0.7765	0.7518	0.1195	0.1246
Maximum	1.0000	0.8428	1.0000	0.9790	1.0000	0.9307

The most visible changes are the obvious reductions in the bias-corrected efficiency scores when compared with the DEA. This is expected, given the method. As expected, the mean efficiency is downwardly adjusted, falling from 0.5886 to 0.4493 for CRS DEA and from 0.9434 to 0.9062 for the VRS DEA. Another point to note in passing is that the biases are all positive, as indicated by columns 4 and 6 in Table 5.6. This may be compared to findings in earlier studies, such as Ferrier & Hirschberg (1997) and Gonzalez & Miles (2002), where some biases were negative. Although this may have something to do with the way the former applied the bootstrap, that is, they did not smooth their results, there is no indication that the latter committed the same oversight.

Using the overall efficiency measure, the best-performing mine after bias-correction is identified as Round Mountain (USA) with a score of 0.9118. From Table 5.2, it can be seen that Round Mountain was also deemed DEA efficient using the overall efficiency measure. Although Table 5.2 indicates that Round Mountain retains its top status after bias-correction, some efficient mines see their scores drastically reduced. Of the six mines considered overall efficient using DEA, only three remain in the top ten best-performing sub-set. The rest see their rankings go down after bias-correction. Two of

them, Betze Post and Cortez (all USA), are 15<sup>th</sup> and 16<sup>th</sup> best-performing, respectively. The magnitude of the biases is, therefore, observed to be quite large at the top end of the distribution of results. There is little change at the bottom end of the ranked distribution with the two Zimbabweans mines still the worst-performing. It can thus be concluded that most of the adjustments in rank as a result of bias-correction take place at the top end of the distribution of efficiency scores but the lower end remains relatively unchanged. This confirms the premise of Simar & Wilson that the inconsistency of non-parametric estimation is at the upper bound where there is a mass of ostensibly efficient DMUs.

As with overall efficiency, after bias-correction, there now are no longer any fully efficient mines using the technical efficiency measure; rather there remain best performing mines such as Bibiani (Ghana) with a technical efficiency score of 0.9790 and Sadiola (Mali) with a score of 0.9703. Although Sadiola was deemed efficient under DEA, Bibiani was not. So part of the change, which has mostly taken place at the upper bound of the distribution, is the displacement of efficient mines by mines which were deemed less efficient under DEA. Of the ten most efficient mines, using the technical efficiency measure, only four were fully efficient under DEA. The other six had technical efficiency scores less than 1. However, the top twenty-seven best performing mines includes all the twenty-one technically efficient mines.

The main effect of correcting for bias, therefore, is that it allows the identification of mines which lie at the upper bound in DEA and allows a proper performance ranking and identification of the best-practice in the absence or reduction of sampling errors.

It was noted in Chapter 4 that using confidence intervals to test for differences of means leads to unsatisfactory inferences. Primarily this is because no differences can be inferred when the confidence intervals of different DMUs overlap. However, the benefit of the bootstrap approach is in the testing of differences between the efficiency scores. This is made possible by the availability of standard deviations, which then enable the significance of sampling variations to be tested. Based on tests of the differences between means, several hypotheses can be tested. For example, it is observed that after bias-correction, Holloway (Canada) which was not judged efficient under DEA becomes better-performing than some mines such as Super Pit (Australia), which had originally been judged “efficient” under VRS DEA.

Hence the first step is to check whether the bias-corrected DEA scores of Holloway and Super Pit are significantly different or is the observed difference because of random sampling variations. This hypothesis is tested only to give an idea of the significance of the difference and is motivated by the seemingly close efficiency scores for the two mines.

Formally, this test is stated as follows.

$$H_0: \theta_{\text{Holloway}} = \theta_{\text{Super Pit}}$$

$$H_1: \theta_{\text{Holloway}} \neq \theta_{\text{Super Pit}}$$

$$\alpha = 0.05, 2 \text{ tailed test, } t_{0.05} = 1.96$$

$$\theta_{\text{Holloway}} = 0.9322, \text{ std error} = 0.0196; \theta_{\text{Super Pit}} = 0.9305, \text{ std error} = 0.0541$$

There are two ways of approaching this test. The first is to test the hypothesis that Super Pit comes from a distribution centred on Holloway. This test will use the standard error of Holloway. The other test uses a pooled standard error based on the two observed standard errors. Using the first method, the calculated  $t$  statistic is 0.0843 which is less than the critical value. Hence the null hypothesis cannot be rejected. The  $t$  value is 0.9075 for the second variant of the test and, again, at the 5 level of significance, the null hypothesis cannot be rejected. Hence the difference between the two mines is statistically insignificant and may merely be due to sampling variation. The conclusion here is that after bias-correction, what had initially seemed to be a more efficient mine (Super Pit) turns out not to be significantly different from another mine deemed less efficient (Holloway). In particular the differences between the two after bias-correction, noted by visual analysis, turns out to be insignificant. This is a profound result and sets the tone for further tests.

A second set of test is now performed which are centred on the Zimbabwean mines in the sample. The first set of tests solely concerns the comparisons among Zimbabwean mines. Renco and Blanket are judged equally (and fully) efficient by VRS DEA. After bias-correction, Blanket seems to be more efficient than Renco. The aim here is to check whether the differences in their point estimates are statistically significant. A relevant question, then, is whether they are really that different in terms of performance.

$$H_0: \theta_{\text{Blanket}} = \theta_{\text{Renco}}$$

$$H_1: \theta_{\text{Blanket}} \neq \theta_{\text{Renco}}$$

$$\alpha = 0.05, t_{0.05} = 1.96$$

$$\theta_{\text{Blanket}} = 0.9379, \text{std error} = 0.0466; \theta_{\text{Renco}} = 0.9326, \text{std error} = 0.0519$$

The calculated  $t$  statistic (using the pooled standard error) is 0.1130. Hence the null hypothesis that there is no difference in performance between Renco and Blanket cannot be rejected. The inference is that the difference between the two mines point estimates is merely a result of sampling variations. For the sake of completeness, a test of the difference between Blanket and Freda Rebecca produced a calculated  $t$ -statistic of 0.3434 which also implies that the difference between them is statistically insignificant and that Freda-Rebecca is just as efficient as Blanket. Therefore, on the basis of the  $t$ -statistic, there are no significant differences among the three Zimbabwean mines in terms of technical efficiency.

A second set of tests on Zimbabwean mines compares them to mines from other comparative countries. These hypothesised similarities are (a) similar geology and (b) similar political (geographical experiences). These tests will use the Banker (1993) statistics, outlined in Chapter 3 and discussed in the review of literature.

The null hypothesis is that there is no difference in the average efficiency scores of the Zimbabwean mines and (a) Australian (b) Canadian (c) South African (d) Ghanaian and (e) Malian mines. Tanzania which only has one mine is excluded as there are not

enough degrees of freedom. Instead, Blanket will be compared to Geita using the difference of means method. Table 5.8 reports the results.

**Table 5.8: Test of Technical Efficiency Differences:**

**(Zimbabwe against Comparative Countries)**

COUNTRY	Sum ratio Test	Sum of Squares Ratio Test
Ghana	1.1792	2.1385
Mali	1.0904	0.9348
South Africa	2.1724	5.7021*
Australia	1.3033	1.9440
Canada	1.3265	1.8146
USA	18.1954**	1.2705

\* significant at the 10 per cent level of significance

\*\* significant at the 5 per cent level of significance

From Table 5.8, the null hypothesis that country specific effects, in relation to Zimbabwe, do not exist, cannot be rejected except with respect South Africa and USA and then only for the specific tests indicated.

Table 5.9 reports the results for overall efficiency, again applying the same Banker (1993) test.



**Table 5.9: Test of Overall efficiency Differences: (Zimbabwe against Comparative Countries)**

COUNTRY	Sum ratio Test	Sum of Squares Ratio Test
Ghana	0.7706	0.5994
Mali	1.9000	3.6001
South Africa	0.5180	0.3200
Australia	0.7013	0.5093
Canada	0.6398	0.4214
USA	2.7594	2.3333

Here the null hypothesis of country-specific differences between Zimbabwe and the above-listed countries cannot be rejected by any of the measures. The implication is that Zimbabwean mines as a group are as efficient as, say, those from Ghana, with which it (as a country) competes for direct foreign investment in mining. It can also be concluded that the overall efficiency of Zimbabwean mines as a group, despite the prima facie evidence, is not statistically different from that of countries with similar gold belts such as Australia, Canada and the USA. Neither is it significantly less efficient than South Africa with which it also shares a common geology and many other cultural and political characteristics.

A technique adopted in previous studies (Byrnes et al, 1984; Førsund et al, 2006) has been to try to correlate DMU characteristics and efficiency. The two-stage estimation process, as described in Chapter 3, has been one such method. Others, in the absence of additional information on the observations have made use of the characteristics of the DMUs in an effort to explain the causes of inefficiency. Hence, in a study of Illinois

coal mines, Byrnes et al (1984) noted that there was a strong, negative relationship between labour-output ratio and efficiency. Instead of using partial measure such as labour-output ratio, this study will work on the DMU characteristics, two inputs and one output. The two inputs are the labour input and the grade of ore which denotes the geological characteristics. Labour is a proxy for size in much the same way as output was in Chapter 4.

The non-parametric statistic of the Spearman rank correlation coefficient is applied and the results are reported in Table 5.10.

**Table 5.10 Spearman Rank Coefficient of Efficiency**

Variable	Spearman Rank Correlation Coefficient
LABOUR	-0.3192*
GRADE	0.0784
RECOVERY	-0.4174*
ORE PRODUCTION	-0.1094

\* significant at the 5 per cent level using the *t*-test.

The Spearman coefficient is significant at the 5 per cent level of significance for labour and the recovery rate. The correlation in both cases is negative. There are two implications here, first in relation to the non-significant variables and with respect to the significant ones.

First, there appears to be no correlation of any statistical significance between technical efficiency and maximum capacity. Recalling that maximum capacity is a proxy for capital, this implies that the size of the mines in terms of capital stock is not a

significant factor in determining its technical efficiency. Neither is the geological characteristic, proxied by the grade of ore.

Turning to labour and the recovery rate, both have a negative correlation with efficiency. The implication is that large mines (as measure by the size of the labour force) are generally less efficient than those without. It must also be noted that there is a distinct possibility that the part of the labour forces is a direct substitute for capital for entirely logical economic reasons, such as being relatively cheap and labour may not be such a good indicator of mine size after all. Unless information on input prices is available and, hence, allocative efficiency calculated, this conclusion on the correlation between labour and efficiency must be tampered with caution.

The other negative correlation is with respect to the recovery rate. The implication here is that those mines with mineralogically simple tend to have than those with simpler ores. A plausible explanation for this is that those mines with complex ores do not spend as much on crushing, for example, and therefore save on milling costs. The downside of that is that they recover less gold than is available.

#### **5.4 Concluding Comments**

The main aim in this chapter was to benchmark the performance of gold mines from different countries, by estimating their efficiencies in using three inputs to achieve two objectives, production of gold metal and a high recovery of metal from the ore. Another was to decompose overall efficiency and determine which was the more predominant.

Based on DEA, a number of findings, on the nature of the efficiencies of the sample, have been made. Decomposing the efficiency scores to determine scale and technical efficiency components showed that a significant number of mines were found to be inefficient mainly because they were operating at the non-optimal scales. That is scale inefficiencies dominated. A large number of mines had very low overall efficiency scores, with some scores as low as 0.1195. Only six mines out of a total of fifty-nine mines are judged efficient under CRS technology, about 10 per cent of the total. Twenty-four mines, about 41 per cent of the total sample had an overall efficiency score of less than 0.5 per cent. The result was that many of these mines had low scale efficiencies with, in particular, the three Zimbabwean mines the least scale efficient. An implication of the relatively low scale efficiencies may be that gold mining operations are not easily adjustable in size. This could arise from the fact the maximum capacity is already determined by the size of the deposit. However, given that the orientation in this study is from the input side, it would seem that there may be an issue which prevents reduction in inputs which would push the inefficient gold mines closer to the frontier.

Given the low scale efficiency estimates, the next step was to investigate the nature of the returns to scale. Fifty-two mines, over 88 per cent of the sample, were deemed to be operating under increasing returns to scale. Only one, Great Noligwe (South Africa) was operating under decreasing returns to scale.

The general performance of the mines in the sample improves significantly for the three Zimbabwean mines under VRS technology. This however is maybe simply be consequence of only the Zimbabwean mines being isolated at that [low efficiency] end of the distribution with the VRS frontier which inevitably has to envelop them. The

mean technical efficiency is now 0.9434 as opposed to 0.5886 for overall efficiency. The implication, particularly for Zimbabwean mines, is that the VRS frontier, in the increasing returns to scale region is quite far from the CRS frontier. In terms of Figure 3.4, the distance between  $P^{\text{REF}}$  and  $P^{\text{CRS}}$  is relatively large for a large number of mines, particularly the three Zimbabwean mines. In the long term all three mines, as with the other mines in the sample, will need to embark on long term adjustment plans to improve scale efficiency and hence overall efficiency. Since the three mines are operating under increasing returns to scale, the adjustments required are of an input reducing nature. Yet, without any further information it is not clear how this adjustment would take place.

The results of the bootstrap and bias-correction show that applying the naïve bootstrap, that is, the non-smoothed DEA leads to potentially misleading conclusions. In particular, as noted by Simar & Wilson, there is a mass of ostensibly efficient mines at the upper bound (sixteen with VRS technology), that is, at efficiency score of 1. In this case, the study is in agreement with other DEA studies which show the justification for correcting bias. Correcting for bias, which is often (but may not always be) positive bias, see for example Hawdon (2003), Gonzalez & Miles (2002) and Ferrier & Hirschberg (1997), provides a more reliable picture of efficiency. Since the technology frontier is unknown, the use of the bias-corrected results where none of the DMUs attain full efficiency seems to be more plausible.

Bias-correction therefore indicates that the scope for savings is much higher than indicated by DEA results. The performance of the mines is significantly altered by bias-correction. Hence, for the Zimbabwean mines, it is noted that Renco and Blanket which

were the most efficient under DEA, are respectively ranked 14<sup>th</sup> and 26<sup>th</sup> when bias-correction is carried out. Freda Rebecca which was 28<sup>th</sup> becomes ranked 32<sup>nd</sup>. Hence at the top of the distribution of results, there are some major changes. The best-performing mine is one that was not deemed fully efficient in DEA.

Another outcome of the analysis is the use of Spearman rank correlation to investigate any correspondence between certain identifiable mine characteristics. Two of the inputs, ore grade and recoveries are what are termed non-purchased inputs. Rather they denote certain geological and other mine characteristics which are deemed to affect the attainment of the DMUs' objectives, in this case the production of gold with the minimal use of inputs.

First, it was observed that there was a statistically significant correlation between technical efficiency and recoveries. The correlation was negative in nature implying that mines with low recoveries generally tend to be more efficient. Although, this may seem perverse, there are perfectly logical reasons why this may be a reasonable result. Those mines with relatively more complex ores, for that is the major reason for low recoveries, tend to be more efficient in the use of other resources. In point of fact, that may mean that the level of fineness to which the ore is ground is much lower, that is the ore being fed into the mineral processing cycle is of a much coarser texture than for those mines with higher recoveries. This means in the process of extraction, savings of other inputs such as energy, cyanide and activated carbon can be made.

An example of this is Renco and Freda Rebecca, where the use of low-grade ores and low recoveries was synonymous with their operations when they started were being

brought on stream. Freda Rebecca, for example was low grade, high tonnage (ore) operation with most of the savings taking place at the heap leach stage of the gold extraction process. The implication is that mines in the sample with a low recovery rate have a higher probability of being efficient than those with higher rates.

There, however, is no statistically significant correlation between the grade of ore and efficiency. In this regard, the geological conditions do not seem to play a notable role in the performance of the mines.

On the other question of how Zimbabwean gold mines compare with those from other countries, the prima facie evidence based on the DEA, is that they perform comparatively well. The Banker tests, for example, showed no country-specific effects which would have confirmed negative expectations given the political and economic environments under which they operate. The low overall efficiencies and, as consequence, the degree of scale inefficiency, however, is such that this problem of low efficiencies for Zimbabwean mines cannot be ignored. One way of improving efficiency would be seem to be the reduction of inputs usage. One important question, potentially being posed by the results of this dissertation, is whether the unstable political environment in Zimbabwe has contributed to this apparent inability to achieve scale efficiency? A legitimate question would be whether the results are pointing to something else other than poor optimisation decisions by the mines. It has already been noted in Chapter 2 that the political and economic environment in Zimbabwe is rather unstable. Whether this could this contributory factor is a question worth investigating further. Compared to other African countries such as Ghana, Mali and Tanzania, the

answer does not seem to be in the affirmative. Yet it is a major observation, however, that all three Zimbabwean mines have poor overall efficiencies.

To answer this question requires more information and, preferably more [Zimbabwean] observations would be required. This also means identifying and collecting data on the so-called environmental variables which capture cross-country differences should they exist. Hence a contribution of this chapter is to challenge the application of DEA and bootstrap DEA across countries without acknowledging characteristics which may be peculiar to individual countries and be part of what Cooper et al (2000)

“disadvantageous conditions under which the DMU is operating”. In this study a common technology has been assumed but nothing else such as a common socio-political climate or even specific geological differences. As with the results in Chapter 4, the inter-country comparisons highlight peculiarities to gold which have not arisen in other studies such as banking, libraries and electricity distribution.

Given the nature of the performance of Zimbabwean mines, there are some possible policy recommendations. Another question which was implicitly being asked is why total gold output in Zimbabwe has not risen in line with the boom experienced by countries with similar gold belts must have been caused by other factors. In terms of performance, the political and economic shocks to which the Zimbabwe has been subjected do not seem to have caused the gold mines to lag behind their competitors. However, it is possible that the relatively hostile political environment may have had a large role in deterring investment in more gold mines in Zimbabwe.



This study, while not in a position to answer this question, has at least pointed in the direction in which the answers may be found. This involves a more detailed study of inter-country differences and then analysing if and to what extent they explain the relatively low growth rates for total gold output in Zimbabwe.

In relation to the above point, a series of statistical tests were carried out, making use of the bias-corrected estimates. They help answer important questions about prima facie differences between groups of DMUs. Using these tests, it was concluded that in comparison to two groups of mines, Zimbabwean mines are only statistically different from the mines in South Africa and the USA, among countries with similar gold belts judged by technical efficiency scores. Even the apparently low overall (and scale) efficiencies for the Zimbabwean mines have not been deemed significantly different from some comparator mines. There is no significant difference with two African countries with which Zimbabwe is deemed to compete for foreign direct investment. Statistical tests show that the differences in efficiencies are not as significant as visual analysis may imply. Hence, a test of the difference in performance between Zimbabwean and other groups of comparator mines shows that, as a group, there are no statistically significant differences, except with South Africa and the USA. Therefore the observed divergences in the growth of the gold output between Zimbabwe on the one hand and Ghana and Mali on the other is as result of other factors not considered in this analysis. One such factor may be the investment climate. However, this will need to be supported by more investigations and more data than has been available to this dissertation.

Another test was done on the difference between means. Using this test, a variety of conclusions were reached with regards to Zimbabwean mines and also about the sample itself. A fundamental conclusion reached is that there are only two mines which are being judged to be statistically similar to the best-performing mine. The rest would be located some distance away from the best-practice frontier defined by these three.

There are some general implications from this study, too. The next point worth discussing is the network of peers. The results show a subset of international peers, with peers from many different countries being referenced by mines from countries other than their own. In this regards, Australia, Canada and the USA are the most influential in terms of the location of the frontier. In terms of providing the most numbers of peers, Australia is the relatively dominant country followed by Canada. Two Zimbabwean mines are identified as peers. One of them, Renco, is judged the most influential peer using the peer count measure. Given that in the overall efficiency measures, Zimbabwean mines did not perform very well, Renco's being a peer reinforces scepticism of not taking into account contextual issues in applying DEA to efficiency studies in gold mining, in particular, and cross-country analysis in general.

## **CHAPTER 6 SUMMARY AND CONCLUSIONS**

### **6.1 Introduction**

This chapter summarises the main findings of this research. In particular, it attempts to present some common issues arising from the results from Chapters 4 and 5, bearing in mind that these are two separate studies and therefore comparisons must be treated with caution.

This is the first study which has looked at multi-factor analysis of gold mining in Zimbabwe. It also the first study to study the performance of gold mining whether in Zimbabwe or in a cross-country setting using DEA. In addition, this study has applied the DEA technique to a sample comprising both developed and developing economies and analysed the relative performances of individual mines in that context

Chapter 1 gives a brief summary of the objectives and poses the research questions. Primarily these are to use the DEA method to investigate and measure the efficiency of gold mining, applied on two different a samples; the first a sample of anonymous Zimbabwean mines and the second a sample of mines from different countries. A general history of gold mining and the evolution of mining and mineral processing technology are also given. The main activities in the extraction of gold are highlighted as are the major producers.

In Chapter 2, a history of gold mining in Zimbabwe is given particular attention, tracing the interaction of gold mining and economic development. The importance of gold

mining to Zimbabwe, in particular during periods of crises are *highlighted*. The problems and challenges that have confronted and continue to face the gold mines, such as wars, foreign currency shortages and political instability, are also identified and discussed.

Chapter 3 lays the theoretical foundations of this dissertation. The point of departure from the basic theory of production found in intermediate microeconomics to frontier analysis is identified. In particular the influence on nonparametric analysis, of which data envelopment analysis (DEA) is part, of the seminal work by Farrell (1957) is discussed. A brief synopsis, given that this approach is not used in this study, of stochastic frontier analysis (SFA) is carried out with a look at the major milestones achieved in its development. A more extensive discussion of the DEA is carried out, including modifications which have made the original, deterministic work more amenable to statistical analysis. Attention is drawn to the fact that the bias-correction, in particular, is applied to sampling variation only and not errors in or omissions of variables.

In this regard the work done by Efron (1979), in the development of the bootstrap, and by Simar & Wilson (1998), in applying the bootstrap to efficiency analysis, is particularly emphasised. Finally a selective review of previous studies is carried out highlighting methodological problems, results and analyses. Particular attention is paid to studies of mining and of DEA analysis where the bootstrap has been applied.

The core of this dissertation has been the application of DEA and bias-corrected DEA in estimating the efficiency scores of gold mines. The DEA estimates are biased, and have

been shown to be so by, among many, Simar & Wilson (1998). Since this technique is still widely used, it has formed an important part in analysing the performance of the gold mines and in testing some hypotheses using tests which have long existed in nonparametric statistical analysis and formalised in the efficiency literature by Banker (1993). Given that the DEA estimates are largely biased and inconsistent, particularly at the upper bound, the bootstrap is particularly useful in the correction of this bias which, arises from sampling variation, and in adjusting the calculated efficiency estimates. The results of bias-correction demonstrate the upward biased nature of the DEA identified and proved by Coelli et al (2005) and Simar & Wilson (1998, 2007) among others.

Two empirical studies are carried out. The first is on Zimbabwe, the results of which are reported in Chapter 4. The main findings in are that the technical inefficiency in Zimbabwean gold mining is a mixture of both scale and technical inefficiency. Over the whole sample scale inefficiency was the more predominant cause of overall inefficiency. In addition, it was noted that the majority of mines were operating under increasing returns to scale implying that the mines were mainly smaller than optimal and that there was scope for reducing inputs usage. This poses a number of questions as to the reason for this state of affairs. Why have these problems not been addressed? The results therefore point to certain characteristics of Zimbabwean mining which may require further data and investigation. These include the socio-political environment in which the mines operate. They may also include any geological features which may contribute to these results being observed.

The second set of empirical results is reported in Chapter 5. Two broad conclusions can be drawn. The first set of conclusions, based on DEA, shows that the most influential

mines in the location of the frontier are from Australia, Canada, South Africa and the USA. An analysis of the most influential mines shows that Australia provides the most number of peers, although the most influential peers judged by how times it is referenced by inefficient mines are from Canada and the USA. The results also identify the nature of returns to scale of the different mines and show that all three types of returns to scale are exhibited. Hence the reference technology is mainly determined by Australian gold mines.

The second set of conclusion is based on the results of bias-corrected efficiency estimates. Correction for bias changes the order in which some of the mines, previously judged as efficient by DEA, are ranked. Bias-correction also serves as a “tie-breaker”, distinguishing among the many efficient mines. In this regard the best-performing mine is identified as from Malaysia. By testing the difference of means to distinguish which among the fifty-nine mines can be classified as statistically from this best-performing mine, only two mines, one from Australia and one from South Africa are judged as not being significantly different from it. Then rest are judged to be different and hence less efficient.

The third observation is that all three Zimbabwean mines perform relatively poorly in overall efficiency terms when compared to mines from the rest of the world. There are possible explanations for this and they will be explored in more detail below. Suffice it to say that the small number of Zimbabweans mines must necessarily lead to some caution in making inferences not supported by data and evidence.

A key characteristic of the studies cited in the review in Chapter 3, with the exception of Hawdon (2003) has been that they have they been applied not only to a single industry but also to a single country. Yet there have been studies which have used a cross-country analysis have focused on developed economies only, and none of these have focused on mining: for example, Edvardsen & Førsund (2003) on electricity distribution, Homburg (2001) on libraries and Cherchye et al (2001) on banking. Their findings seem to imply a common technology across countries. Additional studies include Berg et al (1993) , Bergendahl (1998) and Molyneaux & Casu (2003) banking, Reichmann & Sommersgutter-Reichmann (2006) on university libraries, Edvardsen & Førsund (2003b) on electricity distribution, Goto & Tsutsui (1998) on electricity generation. Key to this ability to conduct a cross-country analysis was an assumption of a common underlying technology. It is important, however, to note that the emphasis in all these studies has been on measuring efficiency based on “controllable” inputs.

In this study, the cross-country analysis also assumed a single underlying random process and technology. Given the description of the gold production process in Chapter 1, this is not an implausible assumption to make. As was argued in Chapter 1, the technology for producing gold is fairly straightforward and easily transferrable; not only that, but the analysis of peers also supports the assumption of a common gold production technology.

The results of Chapter 5 seem to indicate that gold mining technology is quite mobile as mines from Zimbabwe and Australia are found in the same reference set for a mine from Thailand, for example. In fact the results of the inter-country comparison seem to indicate that mines in the developed mining economies define the overall efficiency

reference frontier, as indicated by the overall efficiency estimates. Relative to this frontier, Zimbabwean mines are the least efficient. Even taking into account that there are only three mines from to which to draw inferences, the question again needs to be asked; are there any Zimbabwe-specific issues which cause these, seemingly, unique low scores? A Banker (1993) test does not seem to support, except in two cases of South Africa and the USA, any significant difference between Zimbabwean mines and those from other countries.

The inter-country results seem to be challenging the methodological assumption of a common technology, though. In particular can one merely apply DEA and bootstrap DEA on cross-sectional data from different countries without controlling for inter-country differences? Hence, is it a plausible expectation that an USA gold mine can easily be “imposed” in Zimbabwean conditions (as suggested by the peers for the Zimbabwean mines with low efficiencies). At best the results seem to indicate that this may not be entirely possible. A possibility would be that more and better data would be required robustly to answer this question.

## **6.2 Reflections and Conclusions**

By estimating the efficiency of gold mining in Zimbabwe, the study enables the identification of how much potential there is for improving current gold mining operations. Apart from this, the results also allow the identification the direction mines might take to improve this performance, that is, by better using current levels of inputs to increase local technical efficiency and also by adjusting the scale of operations to improve overall efficiency. It has been noted that scale inefficiency is the more



dominant source of technical inefficiency which points to possible policy remedies in addressing this. The policy issue could point to the turbulent political environment prevailing in Zimbabwe since 2000. Additionally, there could also be geological and other physical features which would favour the relatively small operations. Whichever route one takes—and a prudent approach is not to exclude any credible reasons, there are enough indications to suggest justify further investigation supported, at the very least, by collecting more data. These include more observations and, also, additional variables such as non-discretionary inputs to control for country-specific characteristics.

The second contribution comes from the second set of results in Chapter 5 which allow the comparison of the performance of Zimbabwe mines with mines from other countries. Given that, in relation to other countries with similar gold belts, the total output from Zimbabwe has been seriously lagging behind, the results indicate that in terms of overall efficiency, Zimbabwean mines do not, from initial observations, compare favourably with mines from elsewhere. In particular, when corrected for sampling bias, they seem to suffer more from scale inefficiency than mines from other countries. This seems to reinforce the findings in Chapter 4, particularly with respect to scale inefficiency as the predominant source of overall inefficiency, even though caution needs to be exercised as the context were different.

On this issue of the relatively poor performance of Zimbabwean mines, a question which can legitimately be asked is whether there are country-specific factors which have possibly affected efficiency. Clearly, as noted in Chapter 2, mines in Zimbabwe in 2003 were operating in a very tough economic environment, arguably tougher than in other countries. There were frequent shortages of foreign currency and energy. This is

likely to be a reason why the glaringly obvious steps of expanding or merging contiguous mines rarely ever took place. These questions can be answered provided certain conditions are met. These “uncontrollable” characteristics, however, if properly defined and measured, can in principle be included as environmental variables in the DEA. Fundamentally, therefore, when cross-country studies are carried out, answers to the above questions and further insight may be obtained by including environmental and category variables which take into account the possibility of more than one distinct random process giving rise to the technology frontier. Equally as important, is the identification and measurement of these variables, whether as inputs or outputs. Byrnes et al (1984) attempted to account for different environmental features by incorporating geological characteristics of the different mines. This approach can to reflect different political and economic environments which may exist in different countries.

However, some caution ought again to be exercised here. There are only three Zimbabwean mines in the sample used and they may or not be capturing any Zimbabwe-specific characteristics, where they happen to be present. In addition, although the Zimbabwean operation come least of all the mines in overall efficiency, there are some mines from other countries which lie close to them. Specifically, these are Granny Smith, Lawlers and Plutonic in Australia, Troilus in Canada, Bibiani in Ghana, Julietta in Russia, Chatree in Thailand and Montana Tunnles in the United States. A second and equally reasonable question to ask, then, is whether there are any other characteristics which other inefficient mines share with the poorly performing Zimbabwean mines and, hence, which would explain the relatively low overall efficiency scores? Specifically could, as already mentioned, ask if there are any

geological and other physical features of these mines which would separate them from the rest. Without further information, one cannot conclusively explain the relatively poor performance. There are, however, suggestions on how to pursue analyses which incorporate these extra, no-controllable features, of gold mines.

A number of studies have attempted to estimate efficiency scores in the presence of so-called environmental variables or non-discretionary inputs (or outputs) in a variety of different ways. The main problem has always been how to measure them. This issue will briefly be discussed below.

The third contribution is the use of bias-corrected scores which show the potential pitfalls of using the DEA in measuring performance of productive organisations. From Chapter 3 it was declared that nonparametric estimation methods tend to produce biased and inconsistent results. Hence a key observation is that the potential for improvements to a DMU's performance are underestimated if the correction of bias is not implemented.

The pooling together of the results and conclusions from the two samples necessarily presents some difficulties. First, the data in Chapter 4 are anonymous. Hence the three Zimbabwean mines in the second sample cannot be identified from the first sample. Secondly, conventional wisdom and practice caution against making cross-sample comparisons particularly those for different years. Finally, the specifications of the two models are different. In Chapter 4, energy is explicitly included but is not in Chapter 5. In addition, there is a different proxy for capital services in each study. This limits the scope for making inter-sample comparisons.

However, there are still some common observations which will be highlighted. Both sets of results indicate that scale efficiencies dominate technical efficiency. Hence, the mines in both samples have potential further improve their performances by making adjustments to the scale of their operations. In the general context of this study, this would entail reductions in inputs usage without reducing output for a many of the mines. In addition, the nature of the returns to scale indicates that most the mines are operating under increasing returns to scale. Hence, the nature of the necessary adjustments is predominantly upwards. This is more relevant in the international sample, where over 88 per cent of the observations were observed to operate under increasing returns to scale, than in the Zimbabwe-only sample. Whether this is a feature of gold mining is something that can only be uncovered through further investigations with a large data set.

On the evidence presented so far, therefore, the indications are that not many gold mines operate at their optimal scale, as judged using overall efficiency and scale efficiencies. The degree of scale inefficiencies in gold mining, with a mean bias-corrected scale efficiency score of 62.61 per cent, is so large that steps need to be taken to address.

A possible suggestion to the problem of small scale operations may be to merge those mines which lie on contiguous claims, of which Zimbabwe has many. This necessitates the question of why merging of contiguous properties or expansion of existing properties has not up to now been undertaken.

A possible explanation for the state of affairs may be an unstable economic environment, certainly when compared to other gold mining economies to. To test this

would require not only identification of these latent variables which are deemed to affect efficiency but also their measurement. However, the discussion in Chapter 2 also indicates that the relatively small scale of Zimbabwean undertakings has confronted politicians since the times of the Pioneer Column and solutions have seemingly remained elusive. This is a reason why one hesitates attributing this poor performance to the unstable political environment of the 21<sup>st</sup> century, at least without additional data and analysis.

Another result, given the negative correlation between low recoveries and efficiency is that there is also scope for developing the mineralogically complex ores which are found in one or two areas in Zimbabwe. The Copper Queen deposit in Sanyati (see map in Chapter 2) comes to mind. In this regard, government policy as it pertains to investment incentives could be of great assistance through the granting of tax and other fiscal incentives on technology adoption and foreign investment. In particular, the tenure of properties, the encouragement of exploration, among others, could be improved. Again, whether this could take place in the current political climate is doubtful. The scope of improvements is so large that on average, the Zimbabwean mines could improve their mean performance, by expanding the scale of their operations, by about 82 per cent as opposed to the, still large, sample average of about 37 per cent.

### **6.3 Contribution to the Literature and Possible Directions for Further Study**

The study has also provided another framework analysing cross-country performances in a resource-based industry such as gold using observations from both developing and

developed countries. Most previous cross-country studies on mining have tended to compare partial productivity measures such as labour productivity (Tilton, 2001; Kuby & Xie, 2000). Here, performance has been analysed using more a more general multi-input measure. This is therefore the first study which has applied the DEA to gold mining, first in a single country context but also, and equally as importantly, in a multi-country setting which encompassed both developed and developing economies. In doing so, the study has highlighted some important issues in DEA methodology, particularly with respect to the local context in which the DMUs operate which precludes the straightforward application of DEA without accounting for mine—specific peculiarities. It validates the previous studies on coal mining which advocated for including non-discretionary inputs. Therefore, following from the studies by Byrnes, Färe & Grosskopf (1984) and Byrnes & Färe (1987), the inclusion of geological and other mine-specific characteristics may add to understanding the sources of the differences in efficiencies.

With respect to bias-corrected DEA, specifically that obtained through the bootstrap, it is important to heed the words of Coelli et al (2005);

“The DEA bootstrapping methods are designed to deal with *sampling variability*. That is, they provide an indication of the degree to which efficiency estimates are likely to vary when a different sample is randomly selected from the population. However, these methods do not attempt to account for random noise such as that which may result from measurement error or specification error. The DEA method assumes that data noise does not exist.”

This study has also indicated that bias introduced as a result of some heterogeneity of the observation for which this heterogeneity must be controlled can lead to erroneous results and conclusions. Hence, there is no substitute for removing or minimising misspecification or other measurement error bias. This may involve more observation upon which inferences can be made. In this case, while there is not enough evidence to support the case for a distinctly different random process for Zimbabwe-- as opposed to all mines with a low overall efficiency score-- at the very least, the result illuminates some unanswered questions about Zimbabwe in comparison with the result of the world. One could, therefore, argue for a different technology for all the mines with low overall efficiency or for Zimbabwean mines only, and hence justify the subdivision of the sample into two or more subsamples.

The possibility that such cross-country factors exist and contribute to the differences in overall efficiency justifies further research using additional non-discretionary variables, be they geological or political risk. The key challenge is whether such data exists as to enable this to be done and how it can be measured.

Finally, it was argued in Chapter 3 that overall efficiency is but one aspect of economic efficiency, the other component being allocative efficiency. An interesting further study would investigate economic efficiency and the input and output choices of firms when facing different relative prices. It has been noted that there are differences in labour-output ratio and therefore an analysis which takes into account input substitutability through prices would further enrich the results.

## APPENDIX A: ZIMBABWEAN GOLD MINING DATA

DMU	LABOUR	MATERIALS	ELECT	SERVICES	FUEL	OUTPUT
1	4291	132543372.18	14517540.10	18184737.89	2855825.48	214695000
2	1074	5352882.78	1809057.34	984438.94	948001.97	35040000
3	1147	11788444.83	2439232.66	804603.31	569382.12	33385946
4	2664	27420871.05	5316082.35	5060845.69	375736.20	39112673
5	730	4184587.23	1340257.77	771627.08	415696.07	23658000
6	534	4769008.77	890984.69	1074965.39	530604.18	21266308
7	1395	6141753.72	3197841.31	1682492.23	1889623.48	33870309
8	1910	16882374.03	3230933.68	404704.46	1172064.86	29323000
9	837	18810561.76	1840810.18	785640.94	785344.99	31564000
10	684	5806867.07	865544.70	915814.03	104807.33	18221000
11	889	4292812.26	1992149.48	1255757.90	822230.48	37385000
12	226	4800943.08	628224.72	817629.39	242510.84	14825000
13	345	2724440.85	735620.48	1087345.72	258074.82	15231000
14	422	3780584.44	626826.89	1213271.34	357210.83	17353684
15	508	2949630.31	797527.21	846686.57	96979.98	11974146
16	365	4225391.52	557865.30	1539684.28	45476.66	9922572
17	280	1545936.25	433868.60	650082.83	76304.25	7823101
18	199	138011.06	206189.83	51368.64	39911.58	1608697
19	185	5912088.87	498673.06	1429002.74	275770.85	11831478
20	334	203075.87	25338.58	19095.74	34519.23	799277
21	152	279292.17	135331.30	548850.38	16366.10	1446963
22	190	759276.42	119386.93	11119.26	87050.21	1430838
23	153	768384.70	179394.47	309140.93	155208.51	5007668
24	126	3400674.76	396801.06	830843.21	162956.99	8508894
25	127	60290.90	22319.02	62566.54	35181.53	561431
26	73	101482.92	6183.69	11275.88	14017.05	305556
27	62	34005.25	39672.79	7934.56	6801.05	276000
28	27	20527.60	3119.10	37419.23	1986.46	82908
29	77	32106.32	25197.27	12026.68	4525.29	89267
30	34	220783.61	28096.06	17038.93	17243.40	755085
31	41	103649.42	2730.41	23323.34	36988.40	163660



32	14	25163.93	1248.18	15905.58	3954.12	69887
33	5	9928.86	1826.91	2184.35	1866.63	56819
34	20	64866.98	1008.29	3697.08	9074.65	86076

**APPENDIX B: DEA AND BOOTSTRAP DEA RESULTS: 95% CONFIDENCE INTERVAL)**

Mine	CRS Original Score	VRS Original Score	CRS Bias-corrected Efficiency Score	VRS Bias-corrected Efficiency Score	CRS		VRS	
					2.5%	97.50%	2.5%	97.50%
1	1.0000	1.0000	0.7837	0.8854	0.7724	1.1031	0.5689	1.9110
2	0.5765	0.5837	0.5288	0.5540	0.4820	0.6792	0.5250	0.6776
3	0.7642	0.7982	0.7049	0.7593	0.6468	0.8580	0.7212	0.9069
4	1.0000	1.0000	0.7326	0.7826	0.4665	1.3588	0.5664	1.8361
5	0.5866	0.5872	0.5374	0.5582	0.4890	0.6791	0.5300	0.6655
6	0.4723	0.5018	0.4449	0.4885	0.4182	0.5120	0.4758	0.5380
7	0.7528	0.7533	0.6675	0.7149	0.5835	1.0489	0.6774	0.9473
8	1.0000	1.0000	0.6693	0.7967	0.3400	1.8353	0.5945	1.7565
9	0.7768	0.7776	0.7145	0.7389	0.6532	0.8748	0.7013	0.8447
10	1.0000	1.0000	0.8879	0.9293	0.7774	1.2424	0.8599	1.2129
11	1.0000	1.0000	0.7758	0.8143	0.5534	1.2646	0.6297	1.5603
12	0.4110	0.4121	0.3809	0.3911	0.3514	0.4552	0.3707	0.4540
13	0.5425	0.5450	0.4986	0.5198	0.4555	0.6172	0.4952	0.6210
14	0.8295	0.8687	0.8007	0.8504	0.7730	0.9024	0.8330	0.9188
15	0.9977	1.0000	0.9257	0.9604	0.8549	1.1431	0.9223	1.1262
16	0.8931	0.9036	0.7915	0.8558	0.6912	1.2587	0.8092	1.1358
17	0.8495	0.8538	0.8013	0.8270	0.7542	0.9422	0.8013	0.9298
18	0.7034	0.7413	0.6555	0.7132	0.6086	0.8416	0.6859	0.8342
19	0.8528	0.8542	0.8113	0.8226	0.7708	0.9744	0.7919	0.9389
20	1.0000	1.0000	0.7342	0.8085	0.4699	1.2579	0.6182	1.4362
21	0.6965	0.7211	0.6721	0.7017	0.6485	0.7628	0.6831	0.7719

22	1.0000	1.0000	0.8412	0.8531	0.6844	1.5458	0.7076	1.3813
23	1.0000	1.0000	0.8858	0.9033	0.7734	1.0527	0.8080	1.0730
24	1.0000	1.0000	0.8900	0.9021	0.7817	1.1061	0.8055	1.1348
25	0.9902	1.0000	0.8813	0.9104	0.8323	1.1661	0.7640	1.1633
26	0.8568	0.8799	0.7772	0.8182	0.6991	1.0321	0.7578	1.0936
27	0.7941	1.0000	0.7392	0.9050	0.6853	0.8968	0.8113	1.2224
28	0.3606	0.9020	0.3395	0.8530	0.3189	0.3817	0.8050	1.0691
29	0.6238	1.0000	0.5773	0.7828	0.5317	0.6966	0.5673	1.8435
30	0.9881	0.9957	0.9383	0.9451	0.8903	1.0784	0.8957	1.0973
31	1.0000	1.0000	0.7715	0.8262	0.5449	1.3886	0.6537	1.4340
32	0.5576	0.9851	0.5183	0.9260	0.4799	0.6461	0.8680	1.2299
33	0.3192	1.0000	0.2995	0.8474	0.2803	0.3474	0.6960	1.4042
34	1.0000	1.0000	0.6906	0.7675	0.3826	1.5381	0.5364	1.9025

## APPENDIX C: DEA AND BOOTSTRAP DEA RESULTS: (90% CONFIDENCE INTERVAL)

Mine	CRS Original Score	VRS Original Score	CRS Bias-corrected Efficiency Score	VRS Bias-corrected Efficiency Score	CRS		VRS	
					5%	95%	5%	95%
1	1.0000	1.0000	0.7837	0.8854	0.4386	0.7692	0.5696	1.5452
2	0.5765	0.5837	0.5288	0.5540	0.6728	0.8700	0.5254	0.6512
3	0.7642	0.7982	0.7049	0.7593	0.8823	0.9170	0.7218	0.8690
4	1.0000	1.0000	0.7326	0.7826	0.6870	0.4980	0.5673	1.5404
5	0.5866	0.5872	0.5374	0.5582	0.6860	0.8761	0.5307	0.6462
6	0.4723	0.5018	0.4449	0.4885	0.5277	0.6215	0.4761	0.5245
7	0.7528	0.7533	0.6675	0.7149	0.8700	0.8983	0.6779	0.9013
8	1.0000	1.0000	0.6693	0.7967	0.7227	0.5573	0.5957	1.5009
9	0.7768	0.7776	0.7145	0.7389	0.8983	0.8981	0.7018	0.8255
10	1.0000	1.0000	0.8879	0.9293	0.3094	0.7743	0.8611	1.1471
11	1.0000	1.0000	0.7758	0.8143	0.8486	0.5559	0.6306	1.4267

12	0.4110	0.4121	0.3809	0.3911	0.4718	0.5756	0.3711	0.4444
13	0.5425	0.5450	0.4986	0.5198	0.6311	0.7929	0.4956	0.6062
14	0.8295	0.8687	0.8007	0.8504	0.8878	0.9831	0.8335	0.8935
15	0.9977	1.0000	0.9257	0.9604	0.5777	0.8644	0.9232	1.0923
16	0.8931	0.9036	0.7915	0.8558	0.5181	0.9676	0.8098	1.0456
17	0.8495	0.8538	0.8013	0.8270	0.9342	0.9234	0.8018	0.9083
18	0.7034	0.7413	0.6555	0.7132	0.7994	0.9690	0.6864	0.7991
19	0.8528	0.8542	0.8113	0.8226	0.9230	0.9496	0.7928	0.9167
20	1.0000	1.0000	0.7342	0.8085	0.6816	0.4690	0.6191	1.3837
21	0.6965	0.7211	0.6721	0.7017	0.7460	0.8602	0.6836	0.7510
22	1.0000	1.0000	0.8412	0.8531	0.9822	0.7163	0.7085	1.2137
23	1.0000	1.0000	0.8858	0.9033	0.4905	0.7698	0.8088	1.0593
24	1.0000	1.0000	0.8900	0.9021	0.4538	0.7781	0.8067	1.1003
25	0.9902	1.0000	0.8813	0.9104	0.5494	0.8744	0.7647	1.1344
26	0.8568	0.8799	0.7772	0.8182	0.8877	0.9257	0.7585	1.0064
27	0.7941	1.0000	0.7392	0.9050	0.8990	0.9198	0.8124	1.1345
28	0.3606	0.9020	0.3395	0.8530	0.4034	0.4662	0.8056	1.0132
29	0.6238	1.0000	0.5773	0.7828	0.7177	0.8826	0.5680	1.5234
30	0.9881	0.9957	0.9383	0.9451	0.7643	0.9418	0.8964	1.0752
31	1.0000	1.0000	0.7715	0.8262	0.7610	0.5531	0.6545	1.3646
32	0.5576	0.9851	0.5183	0.9260	0.6372	0.8035	0.8688	1.1388
33	0.3192	1.0000	0.2995	0.8474	0.3591	0.4262	0.6968	1.2482
34	1.0000	1.0000	0.6906	0.7675	0.6156	0.4878	0.5370	1.5288

## APPENDIX D: DEA AND BOOTSTRAP RESULTS: STANDRAD ERRORS

Mine	CRS Original Score	VRS Original Score	CRS Bias- corrected Efficiency Score	VRS Bias- corrected Efficiency Score	CRS Standard Deviations	VRS Standard Deviations
1	1.0000	1.0000	0.7837	0.8854	0.0982	0.3365
2	0.5765	0.5837	0.5288	0.5540	0.0593	0.0491
3	0.7642	0.7982	0.7049	0.7593	0.0624	0.0484
4	1.0000	1.0000	0.7326	0.7826	0.2816	0.3316

5	0.5866	0.5872	0.5374	0.5582	0.0582	0.0405
6	0.4723	0.5018	0.4449	0.4885	0.0275	0.0171
7	0.7528	0.7533	0.6675	0.7149	0.1264	0.0756
8	1.0000	1.0000	0.6693	0.7967	0.4302	0.3191
9	0.7768	0.7776	0.7145	0.7389	0.0677	0.0482
10	1.0000	1.0000	0.8879	0.9293	0.1314	0.0985
11	1.0000	1.0000	0.7758	0.8143	0.2163	0.2736
12	0.4110	0.4121	0.3809	0.3911	0.0324	0.0252
13	0.5425	0.5450	0.4986	0.5198	0.0519	0.0366
14	0.8295	0.8687	0.8007	0.8504	0.0343	0.0227
15	0.9977	1.0000	0.9257	0.9604	0.0912	0.0576
16	0.8931	0.9036	0.7915	0.8558	0.1501	0.0881
17	0.8495	0.8538	0.8013	0.8270	0.0572	0.0341
18	0.7034	0.7413	0.6555	0.7132	0.0652	0.0441
19	0.8528	0.8542	0.8113	0.8226	0.0538	0.0417
20	1.0000	1.0000	0.7342	0.8085	0.2733	0.2592
21	0.6965	0.7211	0.6721	0.7017	0.0330	0.0245
22	1.0000	1.0000	0.8412	0.8531	0.2456	0.1918
23	1.0000	1.0000	0.8858	0.9033	0.0926	0.0954
24	1.0000	1.0000	0.8900	0.9021	0.0984	0.0990
25	0.9902	1.0000	0.8813	0.9104	0.0964	0.1332
26	0.8568	0.8799	0.7772	0.8182	0.0988	0.0883
27	0.7941	1.0000	0.7392	0.9050	0.0650	0.1133
28	0.3606	0.9020	0.3395	0.8530	0.0211	0.0711
29	0.6238	1.0000	0.5773	0.7828	0.0554	0.3297
30	0.9881	0.9957	0.9383	0.9451	0.0539	0.0606
31	1.0000	1.0000	0.7715	0.8262	0.2347	0.2509
32	0.5576	0.9851	0.5183	0.9260	0.0461	0.1055
33	0.3192	1.0000	0.2995	0.8474	0.0190	0.1957
34	1.0000	1.0000	0.6906	0.7675	0.3591	0.3553

## APPENDIX E: WORLD GOLD MINING DATA

Mine	Country	Labour	Ore Hoisted	Grade	Recovery	Au prod
Cerro Vanguardia Gold Mine	Argentina	791	1.000	7.150	95	7.030
Super Pit Gold Mine	Australia	560	13.008	2.430	85	27.120
Granny Smith Gold Mine	Australia	400	3.995	2.500	89	8.710
Peak Gold Mine	Australia	150	0.637	6.400	90	3.800
Plutonic Gold Mine	Australia	640	2.730	4.220	90	10.390
Challenger Gold Mine	Australia	55	0.298	4.190	90	1.770
Thunderbox Gold Mine	Australia	180	2.516	2.700	93	6.610
Kirkalocka Gold Mine	Australia	120	0.993	1.640	94	2.250
Gidgee Gold Mine	Australia	106	0.344	5.930	95	1.860
Norseman Gold Mine	Australia	210	0.591	7.690	96	4.110
Darlot Gold Mine	Australia	196	0.797	6.240	96	4.820
Lawlers Gold Mine	Australia	100	0.731	4.390	96	3.090
Henty Gold Mine	Australia	93	0.289	11.400	96	3.180
Sao Bento Gold Mine	Brazil	549	0.374	8.500	89	2.960
Crixas (Serra Grande) Gold Mine	Brazil	514	0.748	8.200	95	5.890
Kemess South Copper/Gold Mine	Canada	350	18.633	0.700	67	9.150
Troilus Copper/Gold Mine	Canada	285	5.980	1.030	81	5.100
Laronde Gold Mine	Canada	525	2.221	3.770	91	7.360
Holloway Gold Mine	Canada	148	0.461	4.870	92	2.390
Joe Mann Gold Mine	Canada	160	0.166	7.340	93	1.330
Eskay Creek Gold Mine	Canada	135	0.249	49.100	93	10.950
Holt McDermott Gold Mine	Canada	195	0.507	5.830	93	2.780
Seabee Gold Mine	Canada	108	0.209	7.950	95	1.580
Golden Giant Gold Mine	Canada	265	0.653	11.760	95	7.140
Campbell Gold Mine	Canada	332	0.363	17.600	96	6.130
Musselwhite Gold Mine	Canada	293	1.331	5.500	96	6.930
Doyon Gold Mine	Canada	462	1.278	5.600	96	6.760
Sleeping Giant Gold Mine	Canada	191	0.203	10.500	97	2.070
Beaufor Gold Mine	Canada	115	0.251	6.800	99	1.710
Bibiani Gold Mine	Ghana	479	2.591	3.290	78	6.620
Iduapriem Gold Mine	Ghana	1306	3.754	2.000	95	7.580

Kurmtor Gold Mine	Kyrgyzstan	1600	5.631	4.500	82	21.070
Penjom Gold Mine	Malaysia	238	0.524	7.280	89	3.890
Sadiola Gold Mine	Mali	1159	5.071	2.770	76	14.050
Morila Gold Mine	Mali	500	2.735	13.400	92	24.700
Orcopampa Mine	Peru	265	0.356	17.300	95	5.640
Julietta Gold Mine	Russia	400	0.145	27.900	88	3.680
Kubaka Gold Mine	Russia	428	0.883	6.420	97	5.210
Ergo Gold Tailings Mine	South Africa	1850	30.905	0.200	56	6.310
Petrex Gold Mines	South Africa	3800	1.844	2.640	89	4.700
Tau Lekoa Gold Mine	South Africa	4252	2.363	4.240	97	10.010
South Deep Gold Mine	South Africa	4730	1.958	7.200	97	13.710
Great Noligwa Gold Mine	South Africa	7100	2.389	10.570	97	25.260
Savuka (West) Gold Mine	South Africa	3229	1.003	5.810	98	5.830
Tautona Gold Mine	South Africa	5498	1.663	12.090	98	20.110
Kopanang Gold Mine	South Africa	6312	2.184	7.070	98	15.450
Mponeng (South) Gold Mine	South Africa	5876	1.733	8.960	99	15.520
Geita Gold Mine	Tanzania	2256	5.704	3.600	92	20.560
Chatree Gold Mine	Thailand	220	1.324	3.900	91	4.350
Round Mountain Gold Mine	USA	650	57.087	0.550	66	24.430
Montana Tunnels Gold/ Mine	USA	200	4.230	0.530	75	1.740
Fort Knox Gold Mine	USA	400	13.685	1.070	84	12.190
Betze Post Gold Mine	USA	988	9.107	6.480	85	48.510
Cortez Gold Mine	USA	370	3.452	6.530	89	33.140
Meikle (Purple Vein) Gold Mine	USA	521	1.471	13.200	90	17.160
Bald Mountain Gold Mine	USA	105	4.125	0.700	91	2.820
Renco Gold Mine	Zimbabwe	988	0.234	3.590	74	0.720
Blanket Gold Mine	Zimbabwe	661	1.200	1.630	77	1.100
Freda Rebecca Gold Mine	Zimbabwe	947	1.197	1.750	85	1.590

## BIBLIOGRAPHY

- AFONSO, A. & ST. AUBYN, M. (2006), "Cross-country Efficiency of Secondary Education Provision: A Semi-parametric Analysis with Non-discretionary Inputs", *Economic Modelling*, Vol. 23.(3), pp 476-491.
- AIGNER, D.J. & CHU, S.F. (1968). "On Estimating The Industry Production Function" *American Economic Review*, Vol. 58 (4), pp 826-839.
- AIGNER, D., LOVELL, C.A.K. & SCHMIDT, P (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models", *Journal of Econometrics*, Vol. 6(1), pp 21-37.
- ALI, A.I. & SEIFORD, L.M. (1993), "The Mathematical Programming Approach to Efficiency Analysis", in Fried, H.O., LOVELL, C.A.K. & SCHMIDT, S.S. eds., *The Measurement of Productive Efficiency: Techniques and Applications*, Oxford University Press, New York, pp 120-159.
- ANDERSEN, P. & PETERSEN, N.C., (1993) " A Procedure for Ranking Efficient Units in Data Envelopment Analysis. ", *Management Science*, Vol. 39(10), pp 1261-1264.
- ANDREWS, D.W.K. (2000), "Inconsistency of the Bootstrap when a Parameter Is on the Boundary of the Parameter Space", *Econometrica*, Vol. 68(2), pp 399-405.
- APPA, G. & YUE, M. (1999), "On Setting Scale Efficient Targets in DEA ", *Journal of the Operational Research Society*, Vol. 50(1), pp. 60-69.
- ARRIGHI, G. (1973), "The Political Economy of Rhodesia" in Arrighi, G. & Saul, J.S. ed. *Essays on the Political Economy of Africa*, Monthly Review Press, New York.
- BAA (2000), "Potential Problems in the Use of Benchmarking", Extract from an OXERA report commissioned by BAA in December 2000, accessed 30 September 2006.
- BARNEKOV, C. (1969). "Sanctions and the Rhodesian Economy", *Rhodesian Journal of Economics*, Vol. 3(1), pp 43-75.
- BANKER, R.D. (1984), "Estimating the Most Productive Scale Using Data Envelopment Analysis", *European Journal of Operational Research*, Vol. 17(), pp 35-44.
- BANKER, R.D., CHARNES, A. & COOPER, W.W. (1984), "Some Models for Estimating Technical and Scale Efficiencies in Data Envelopment Analysis", *Management Science*, Vol. 30 (9), pp 1078-1092.
- BANKER, R.D., (1993), ".Maximum Likelihood, Consistency and Data Envelopment Analysis: A Statistical Foundation", *Management Science*, Vol. 39(10), pp 1265-1273.

- BANKER, R.D., COOPER, W.W., SEIFORD, L.M., THRALL, R.M., & ZHU, J., (2004), "Returns to Scale in Different DEA Models", *European Journal of Operational Research*, Vol. 154(2), pp 345-362.
- BANKER, R.D., JANAKIRAMAN, S. & NATARAJAN, R., (2004), "Analysis of Trends in Technical and Allocative Efficiency: An Application to Texas Public School Districts", *European Journal of Operational Research*, Vol. 54 (2) pp. 477-491.
- BANKER, R. D. & NATARAJAN, R., (2004), "Statistical Tests Based on DEA Efficiency Scores", in COOPER, W.W., SEIFORD, L. & J. ZHU, eds. *Handbook of Data Envelopment Analysis*, Kluwer Academic Publishers, Boston, pp 299-321.
- BANÑOS-PINO, J, FERNANDEZ-BLANCO & RODRIGUEZ-ALVAREZ, A (2002), "The Allocative Efficiency Measure by Means of a Distance Function: The Case of Spanish Public Railways", *European Journal of Operational Research*, 137 (1):pp 191-205.
- BARBER, W.J. (1961), *The Economy of British Central Africa*, Oxford University Press, London.
- BARKER, J.G. (1981), "Sources of Deep Coal Mine Productivity Change, 1962-1975", *Energy Journal*, Vol. 2(2), pp 95-106
- BATTESE, G. AND CORRA, G. 1977, "Estimation of a Production Frontier Model with the Application of the Pastoral Zone of Eastern Australia", *Australian Journal of Agricultural Economics*, Vol. 21(3), pp 167 - 179.
- BERG, A.S., FØRSUND, F.R, HJALMARSSON, L. & SUOMINEN, M., (1993), "Banking Efficiency in the Nordic Countries", *Journal of banking and Finance*, Vol. 17(2/3), pp 371-388
- BERGENDAHL, G (1998), "DEA and Benchmarks- An Application to Nordic Banks", *Annals of Operations Research*, Vol. 82, pp233-249.
- BERAN, R. & DUCHARME, G. (1991), *Asymptotic Theory for Bootstrap Methods in Statistics*, Montréal: Centre de Reserches Mathematiques, University of Montreal.
- BISHOP, P. & BRAND, S., (2003), "The Efficiency of Museums: A stochastic Frontier Production Function Approach", *Applied Economics*, Vol. 35(17), pp 1853-1858
- BJUREK, H., (1994), *Essays on Efficiency and Productivity Change with Applications to Public Service Production*, Unpublished PhD Dissertation, University of Gothenburg, Sweden.
- BLANK, J.L.T & EGGINK, E., (2004), "The Decomposition of Cost Efficiency: An Empirical Application of the Shadow Cost Function Model to Dutch General Hospitals", *Health Care Management Science*, Vol. 7(2), pp 79-88



- BOAME, A.K. (2004), "The Technical Efficiency of Canadian Urban Transit System", *Transportation Research Part E*, Vol. 40(5), pp 401-416.
- BOAME, A.K. (2001), "The Sources of Efficiency Change in the Canadian Urban Transit Systems: A Data Envelopment Approach", *PhD Dissertation*, University of Manitoba.
- BOLUNČIĆ, V. (2006), "Sensitivity Analysis of an Efficient DMU in DEA Model with Variable Returns to Scale (VRS)", *Journal of Productivity Analysis*, Vol. 25(1), pp 173-192.
- BOS, J.W.B & KOLARI, J.W. (2005), "Large Bank Efficiency in Europe and the United States: Are There Economic Motivations for Geographic Expansion in Financial Services?", *The Journal of Business*, Vol. 78(4), pp 1555-1592.
- BOSSOFIANE, A. DYSON, R.G. & THANASSOULIS, E. (1991), Applied Data Envelopment Analysis", *European Journal of Operational Research*, Vol. 52(1), pp 1-15.
- BRIEC, W. (2000), "An Extended Färe-Lovell Technical Efficiency Measure", *International Journal of Production Economics*, Vol. 65(2), pp 191-199.
- BYRNES, P, FÄRE, R. & GROSSKOPF, S (1984), "Measuring Productive Efficiency: An Application to Illinois Strip Mines", *Management Science*, Vol. 30(6), pp 671-681.
- BYRNES, P & FÄRE, R. (1987), "Surface Mining of Coal: Efficiency of US Interior Mines", *Applied Economics*, Vol. 19(12), pp-1665-1673.
- CARPENTER, J. (1998), "Assessing Parameter Uncertainty via Bootstrap Likelihood Ratio Confidence Regions", *Journal of Applied Statistics*, Vol. 25(5), pp 639-649.
- CEBENOYAN, A.S., (1988) "Multiproduct Cost Functions and Scale Economies in Banking". *The Financial Review*, Vol. 23(4), pp 499-512
- CHAMBERS, R.G. (1988), *Applied Production Analysis: A Dual Approach*, Cambridge University Press, Cambridge.
- CHARNES, A., COOPER, W.W. & RHODES. E., 1978. "Measuring the Efficiency of Decision Making Units." *European Journal of Operations Research* 2(6), pp 429-44.
- CHARNES, A. & RHODES, E. (1981), "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Flow Through", *Management Science*, Vol. 27() pp 668-697.
- CHIANG, A.C. (1984), *Fundamental Methods of Mathematical Economics*,
- COELLI, T., RAO, D.S.P. & BATTESSE, G.E. (1998), *An Introduction to Efficiency and Productivity Analysis*, Kluwer Academic Publishers, Dordrecht.

COELLI, T., RAO, D.S.P., O'DONNELL, C.J. & BATTESSE, G.E. (2005), *An Introduction to Efficiency and Productivity Analysis*, Springer, New York.

COFTUS, M. (1997), *Essays on Productivity and Efficiency in the Romanian Cement Industry*, Unpublished PhD Dissertation, University of Gothenburg, Sweden.

COOPER, W.W., SEIFORD, L.M. & TONE, K. (2000), *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References, and DEA Solver Software*, Kluwer Academic Publications, Dordrecht.

COOPER, W.W., SEIFORD, L.M. & ZHU, L. (2004), *Data Envelopment Analysis: History, Models and Interpretations*, <http://www.deafrontier.com/hbchapter1.pdf> accessed 9 March 2006.

DALL, V. & HIDALGO, J., (2005), "A Parametric Bootstrap for Cycles", *Journal of Econometrics*, Vol. 129(1/2), pp 219-261.

DICICCIO, T.J. & EFRON, B., (1996), "Bootstrap Confidence Intervals", *Statistical Science*, Vol. 11( 3), pp 189-212.

DIXON, P.M. (undated), *Bootstrap Resampling*, [http://media.wiley.com/product\\_data/excerpt/76/04718999/0471899976-9.pdf](http://media.wiley.com/product_data/excerpt/76/04718999/0471899976-9.pdf), (accessed 20 October 2004).

DONG, F. & FEATHERSTONE, A. (2006), "Technical Efficiencies for Chinese Rural Credit Cooperatives: A Bootstrapping Approach in Data Envelopment Analysis", *Journal of Chinese Economic and Business Studies*, Vol. 4(1), pp 57-75.

DUCKWORTH W.M. & STEPHENSON W.R., (no date) *Resampling Methods: Not Just for Statisticians Anymore*, <http://www.public.iastate.edu/~wrstephe/JSM2003/Bootstrap.proc.pdf>, (accessed 17 August 2004).

DUSANSKY, R & WILSON, P.W. (1994), "Technical Efficiency in the Decentralized Care of the Developmentally Disabled", *The Review of Economics and Statistics*, Vol. 76(2), pp340-345.

EDVARDBSEN, D.G., (2004), *Four Essays on the Measurement of Productive Efficiency*" Unpublished PhD Dissertation, University of Gothenburg, Sweden.

EDVARDBSEN, D.G., FØRSUND, F.R. & KITTELSEN, S. A.C (2003a), "Far Out or Alone in the Crowd: Classification of Self-Evaluators in DEA", Working Paper 2003:7, Health Economics Research Programme, University of Oslo.

EDVARDBSEN, D.G. & FØRSUND, F.R. (2003b), "International Benchmarking of Electricity Distribution Utilities", *Resource and Energy Economics*, Vol. 25(4), pp 353-371.

EFRON, B. (1979), "Bootstrap Methods: Another Look at the Jackknife", *The Annals of Statistics*, Vol. 7(1), pp1-26.

EFRON, B. (1981), "Nonparametric Estimates of Standard Error: The Jackknife, the Bootstrap and Other Methods", *Biometrika*, Vol.68 (3), pp 589-599

EFRON, B. (1987), "Better Bootstrap Confidence Intervals", *Journal of the American Statistical Association*, Vol 82(), pp 171-185.

EFRON, B. & TIBSHIRANI, R.J. (1993), *An Introduction to the Bootstrap*, Chapman and Hall, London.

EMROUZNEJAD, A (1995-2001), "Ali Emrouznejad's DEA HomePage", Warwick Business School, Coventry CV4 7AL, UK, <http://www.deazone.com/tutorial/index.htm>, (accessed 15 August 2005)

FARRELL, M.J. (1957), "The Measurement of Productivity Efficiency", *Journal of Royal Statistical Society, Series A*, 120(111), pp. 253-281.

FÄRE, R. & PRIMONT, D. (1995), *Multi-output Production and Duality : Theory and Applications*, Kluwer Academic Publishers, Dordrecht..

FÄRE, R., GROSSKOPF, S & LOVELL, C.A.K (1994), *Production Frontiers*, Cambridge University Press, Cambridge.

FÄRE, R., GROSSKOPF, S. & KOKKELENBERG. E.C. (1989), "Measuring Plant Capacity, Utilization and Technical Change: A Nonparametric Approach", *International Economic Review*, Vol. 30(3), pp 655-666.

FERRIER, G.D. & HIRSCHBERG, J.G. (1997), "Bootstrapping Confidence Intervals for Linear Programming Efficiency Scores: With an Illustration Using Italian Bank Data", *Journal of Productivity Analysis*, Vol. 8(1), pp 19-33.

FERRIER, G.D. & LOVELL, C.A.K. (1990), "Measuring Cost Efficiency in Banking" Econometric and Linear Programming Evidence", *Journal of Econometrics*, Vol. 46(1-2), pp 229-245.

FONSECA, G.L. & USSHER, L., *The History of Economic Thought Website*, <http://cepa.newschool.edu/het/essays/product/cost.htm>, Accessed 6 April 2002.

FØRSUND, F.R. & HJALMARSSON, L. (2004), "Calculating Scale Elasticity in DEA Models", *Journal of the Operational Research Society*, Vol. 55(10), pp1023-1038.

FØRSUND, F.R, KITTELSEN, S. A.C, LINDSETH, F. & EDVARDSEN, D.G (2006), "The Tax Man Cometh – But is He Efficient?" *National Institute Economic Review*, Vol. 12(1), pp 106-119.

- FRIED, H.O., LOVELL, C.A.K., SCHMIDT, S.S. & YAISAWARNG, S. (2002), "Accounting for Environmental Effects and Statistical Noise in Data Envelopment Analysis", *Journal of Productivity Analysis*, Vol. 17(1-2), pp 157-174.
- FUKUYAMA, H. & WEBER, W.L. (2002), "Estimating Allocative Efficiency and Productivity Change: Application to Japanese Banks", *European Journal of Operational Research*, Vol. 137(1), pp 177-190.
- GANLEY, J.A. & CUBBIN, J.S. (1992), *Public Sector efficiency Measurement: Applications of Data Envelopment Analysis*, Elsevier Science Publishers, Amsterdam.
- GIBSON, C.A. (1972), "The Rhodesian Mining Industry as an Exporter", *The Rhodesian Journal of Economics*, Vol. 6(4).
- GOLANY, B. & YUNG, G. (1997), "Estimating Returns to Scale in DEA", *European Journal of Operational Research*, Vol. 103(1), pp28-37.
- GOLD SHEET MINING DIRECTORY, "World Gold Production", <http://www.goldsheetlinks.com/production.htm>, accessed 31 October 2006.
- GONZALEZ, X.M. & MILES, D. (2002), "Statistical Precision of DEA: A Bootstrap Application to Spanish Public Services", *Applied Economic Letters*, Vol. 9(0), pp127-132.
- GOTO, M. & TSUTSUI, M. (1998), "Comparison of Productive and Cost Efficiencies among Japanese and US Electric Utilities", *Omega, International Journal of Management Science*, Vol. 26(2), pp 177-194.
- GOVERNMENT OF RHODESIA (1979), *Economic Survey of Rhodesia 1979*, Government Printers, Salisbury.
- GOVERNMENT OF ZIMBABWE (1981), *Growth with Equity*, Government Printers, Harare.
- GOVERNMENT OF ZIMBABWE (1985), *First Five Year National Development Plan 1986-1990*, Government Printers, Harare.
- GOVERNMENT OF ZIMBABWE (1987), *Foreign Investment Policy Guidelines and Procedures*, Government Printers, Harare.
- GOVERNMENT OF ZIMBABWE (1991), *Second Five Year National Development Plan 1991-1995*, Government Printers, Harare.
- GOVERNMENT OF ZIMBABWE (1991), *A Framework for Economic Reform (1991-1995)*, Government Printers, Harare.
- GOVERNMENT OF ZIMBABWE (1995), *Zimbabwe Programme for Economic and Social Transformation (1996-2000)*, Government Printers, Harare.

- HAMMOND, C.J. (2002a), "Efficiency in the Provisions of Public Services: A Data Envelopment Analysis of UK Public Library Systems", *Applied Economics* Vol. 34(5), pp 649-657.
- HAMMOND, C.J. (2002b), "Estimating the Statistical Cost Curve: An Application of the Stochastic Frontier Technique", *Applied Economics* Vol. 18 (9), pp 971-984.
- HAMMOND, C.J., Johnes, G. & Robinson, T. (2002), "Technical Efficiency under Alternative Regulatory Regimes: Evidence from Inter-War British Gas Industry", *Journal of Regulatory Economics*, Vol. 22(3), pp 251-270.
- HALL, P. (1986). "On The Number of Bootstrap Simulations Required to Construct a Confidence Interval", *The Annals of Statistics* 14(4), pp 1453-1462.
- HALL, P. (1988), "Theoretical Comparison of Bootstrap Confidence Intervals", *The Annals of Statistics*, Vol. 16(3), pp 927-953.
- HALL, P. (1992a), *The Bootstrap and Edgeworth Expansion*, Springer-Verlag, New York.
- HALL, P. (1992b), "On the Removal of Skewness by Transformation", *Journal of the Royal Statistical Society: B*, Vol. 54(1), pp 221-228.
- HARDMAN, D.R. (1996), "Coal-mining Productivity in South Africa Compared with Australia and the USA", *Journal of the South African Institute of Mining and Metallurgy*, Vol. 96(7), pp 297-301.
- HAWDON, D. (2003), "Efficiency, Performance and Regulation of the International Gas Industry-- A Bootstrap DEA Approach", *Energy Policy*, Vol. 31(11), pp 178-1178.
- HAWKINS, A.M. (1967), "The Rhodesian Economy under Sanctions", *The Rhodesian Economic Journal*, Vol. 1(1), pp 49-58
- HELBRONER, R.L & MALONE, L. J., (1986) eds., *The Essential Adam Smith*, Oxford University Press, Oxford.
- HELVOIGT, T.L. & GROSSKOPF, S. (2005), "Productivity Growth, Technical Efficiency, and Returns to Scale in the Washington Sawmill Industry", *International Journal of Information Technology & Decision Making*, Vol. 4(3), pp 477-490.
- HOLLANDER, S., (1979), *The Economics of David Ricardo*, Heinemann Educational Books, London.
- HOROWITZ, J.L., (2003), "The Bootstrap in Econometrics", *Statistical Science*, Vol. 18(2), pp 211-218.
- HOMBURG, C. (2001), "Using Data Envelopment Analysis to Benchmark Activities", *International Journal of Production Economics*, Vol. 73(1), pp 51-58.

- HORSKY, D. & NELSON, P. (2006) "Testing the Statistical Significance of Linear Programming Estimators", *Management Science*, Vol. 52 (1), pp 128-135
- IWI, G., MILLARD, R.K., PALMER, A.M., PREECE, A.W. & SAUNDERS, M. (1999), "Bootstrap Resampling: A Powerful Method for Assessing Confidence Intervals for Doses from Experimental Data", *Physics in Medicine and Biology*, Vol. 44(4), pp N65-N62.
- JONDROW, J., LOVELL, C.A.K., MATEROV, I.S., & SCHMIDT, P. (1982), "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model," *Journal of Econometrics*, 19(2-3), pp 223-238.
- JOHNES, J. (2006), "Data Envelopment Analysis and Its Application to the Measurement of Efficiency in Higher Education", *Economics of Education Review*, Vol. 25(3), pp 273-288
- JOURDAN, P.P (1990), *The Minerals Industry of Zimbabwe*, Open Report No. 107, Institute of Mining Research, University of Zimbabwe.
- KEH, H.T. & CHU, S. (2003) "Retail Productivity and Scale Economies at The Firm Level: A DEA Approach", *Omega*, Vol. 31(2) pp. 75-82
- KOKSAL, G. & NALCACI, B. (2006), "The Relative Efficiency of Departments at a Turkish Engineering College: A Data Envelopment Analysis", *Higher Education*, Vol. 51(2), pp173-189.
- KOOPMANS, T.C. (1951): An Analysis of Production as an Efficient Combination of Activities, in KOOPMANS, T.C (ed), *Activity Analysis of Production and Allocation*", *Cowles Commission for Research in Economics*, Monograph No. 13, Wiley, New York.
- KRUVANT, W.J., MOODY, C.E. Jnr, VALENTINE, P.L. (1982), "Sources of Productivity Decline in US Coal Mining, 1972-1977", *Energy Journal*, Vol. 3(3), pp 53-70
- KUBY, M. (1998), *Analysis and Forecast of Labor Productivity in US Coal Mining*, Report to the Energy Information Administration. Office of Integrated Analysis and Forecasting, Washington.
- KUBY, M. & XIE, Z. (2000), "The effect of Restructuring on US Coal Mining Labor Productivity, 1980-1995", *Energy*, Vol. 24(11), pp1015-1030.
- KULSHRESHTHA, M. & PARIKH, J.K. (2002), "Study of Efficiency and Productivity Growth in Opencast and Underground Coal Mining in India: A DEA Analysis", *Energy Economics*, Vol. 24(5), pp439-453.
- KUMBHAKAR, S.C. & LOVELL, C.A.K. (2000), *Stochastic Frontier Analysis*, Cambridge University Press, Cambridge.

KUOSMANEN, T., POST, T. & SCHOLTES, S. (2005 in print), "Non-parametric Tests of Productive Efficiency with Errors-in-Variables", *Journal of Econometrics*, (in press, corrected proof)

KUOSMANEN, T., PEMSL, D & WESSELE, J. (2006), "Specification and Estimation of Production Functions Involving Damage Control Inputs: A Two-Stage, Semiparametric Approach", *American Journal of Agricultural Economics*, Vol. 88(2), pp 499-511

LEVICH, R.M. & THOMAS (III), L.R. (1993), "The Significance of the Technical Trading-Rule Profits in the Foreign Exchange Markets: A Bootstrap Approach", *Journal of International Money & Finance*, Vol. 12(5), pp 451- 474.

LÖTHGREN, M (1998), "How to Bootstrap DEA Estimators: A Monte Carlo Comparison", *Working Paper Series in Economics and Finance, No. 223*, Department of Economics, Stockholm School of Economics.

LÖTHGREN, M & TAMBOUR, M. (1999), "Bootstrapping the Data Envelopment Analysis Malmquist Productivity Index", *Applied Economics*, Vol. 31 (11), pp 417-425.

LOVELL, C.A.K (1993), "Production Frontiers and Productive efficiency", in Fried, H.O., LOVELL, C.A.K. & SCHMIDT, S.S. (eds), *The Measurement of Productive Efficiency: Techniques and Applications*, Oxford University Press, New York, p3-67.

LOVELL, C.A.K. AND WOOD, L.L. (1992), "Monitoring the performance of Soviet Cotton-Refining Enterprises: Sensitivity of Findings to Estimation Techniques", *Atlantic Economic Journal* 20(1), 25-31.

LUNDVALL, K. & BATTESE, G.E. (2000), "Firm Size, Age and Efficiency: Evidence from Kenyan Manufacturing Firms", *Journal of development Studies*, Vol. 36(3), pp 146-63.

MALMQUIST, S. (1953), "Index Numbers and Indifference Curves". *Trabajos de Estatistica*, Vol.4(1), pp209-42.

MANLY, B.F.J. (1997), *Randomization, Bootstrap and Monte Carlo Methods in Biology*, Chapman & Hall, London.

MEEUSEN, W. & VAN DEN BROECK, J. (1977), "Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error", *International Economic Review*, Vol. 18 (), pp435-44.

MILES, X. M & GONZÁLEZ, D. (2002) "Statistical Precision of DEA: A Bootstrap Application to Spanish public services", *Applied Economics Letters*, Vol. 9(2), pp 127-135

ZIMBABWE MINISTRY OF MINES (1991), *Annual Report to Parliament*, Government Printers, Harare.

ZIMBABWE MINISTRY OF MINES (1995), *Annual Report to Parliament*, Government Printers, Harare.

ZIMBABWE MINISTRY OF MINES (2005), *Annual Report to Parliament*, Government Printers, Harare.

MLAMBO, K (1993), *Total factor Productivity Growth: An Empirical analysis of Zimbabwe's Manufacturing Sector Based on Factor Demand Modelling*, PhD dissertation, Department of Economics, School of Economics and Commercial Law, Gothenburg University.

MOJIRSHEIBANI, M (1998), "Iterated Bootstrap Prediction Intervals", *Statistica Sinica*, Vol 8(), pp489-504. <http://www.stat.sinica.edu.tw/statistica/oldpdf/A8n212.pdf> accessed on 25 August 2004.

MOLYNEUX, P. & CASU, B. (2003), "A Comparative Study of Efficiency in European Banking", *Applied Economics*, 35(17), pp 1865 -1876 .

MORRISON PAUL, C., NEHRING, R., BANKER, D AND SOMWARU, A. (2004), "Scale Economies and Efficiency in U.S. Agriculture: Are Traditional Farms History?", *Journal of Productivity Analysis*, Vol. 22 (3), pp. 185–205, 2004

MOSLEY, P. (1983), *The Settler Economies: Studies in the Economic History of Kenya and Southern Rhodesia 1900-1963*, Cambridge University Press, Cambridge.

MURILLO-ZAMORANO, L.R. & VEGA-CERVERA, J.A. (2001), "The Use of Parametric and Non-Parametric Frontier Methods to Measure Productive efficiency in the Industrial Sector: A Comparative Study", *International Journal of Production Economics*, Vol. 69(3), pp 265-275.

MURILLO-ZAMORANO, L.R. (2004), "Economic Efficiency and Frontier Techniques", *Journal of Economic Surveys*, Vol. 8(1), pp 35-77.

NATIONAL MINING ASSOCIATION, "The History of Gold", [http://www.nma.org/pdf/gold/gold\\_history.pdf](http://www.nma.org/pdf/gold/gold_history.pdf), accessed 16 May 2006.

ODECK, J. (1993), *Measuring Productivity Growth and Efficiency with Data Envelopment Analysis: An Application on the Norwegian Road Sector*, Unpublished PhD dissertation, University of Gothenburg.

OFFICER, L. H. (2002), "What Was the Gold Price Then?", *Economic History Services*, <http://eh.net/hmit/goldprice/> accessed on 25 August 2006.

OLIVEIRA, M. A. & SANTOIA, C. (2005), "Assessing School Efficiency in Portugal Using FDH and Bootstrapping", *Applied Economics*, Vol. 37(8), pp 957–968.

OLLMAN, B., (1975), *Alienation : Marx's Conception of Man in Capitalist Society*, Cambridge University Press, Cambridge.



- PANAGIOTIDIS, T (2005), "Market capitalization and efficiency. Does it matter? Evidence from the Athens Stock Exchange", *Applied Financial Economics*, 2005, Vol. 15(10), 707-713.
- PARSONS, N. (1983), *A New History of Southern Africa*, Holmes & Meier, New York.
- PASTOR, J.T., RUIZ, J.L & SIRVENT, I. (2002), "A Statistical Test for Nested Radial DEA Models", *Operations Research*, Vol. 50(4), pp 728-735.
- PEDRAJA-CHAPPARO, F. & SALINAS-JIMENEZ, J. (1996), "An Assessment of the Efficiency of Spanish Courts Using DEA", *Applied Economics*, Vol. 28(11), pp1391-1403.
- PEREIRA, A. (2006), "Economies of Scale in Blood Banking: A Study Based on Data Envelopment Analysis", *Vox Sanguinis*, Vol. 90(4), pp 308-315.
- PORTER, R.C. (1978), " A Model of the Southern African-Type Economy", *American Economic Review*, Vol. 68(5), pp 743-755.
- POST, T, CHERCHYE, L. & KUOSMANEN, T. (2002), "Nonparametric Efficiency Estimation in Stochastic Environments", *Operations Research*, Vol. 50(4), pp 645-655.
- THE PRIVATEER MARKET LETTER (1996-2006), "History Gold 1933-1993", <http://www.the-privateer.com/gold.html>, accessed 24 August 2005.
- REICHMANN, G. & SOMMERSGUTER-RECIHMANN, M. (2006), "University Library Benchmarking: An International Comparison Using DEA", *International Journal of Production Economics*, Vol. 100(1), pp131-147.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, December 1980, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, December 1988, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, March 1991, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, September 1994, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, December 1995, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE, *Quarterly Bulletin of Statistics*, March 2000, Reserve Bank Publications, Harare.
- RESERVE BANK OF ZIMBABWE (2004), *Bank Annual Report 2004* Reserve Bank Publications, Harare.

RESERVE BANK OF ZIMBABWE, *Monthly Review*, March 2005, Reserve Bank Publications, Harare.

RESTI, A. (1997). "Evaluating the Cost-Efficiency of the Italian Banking System: What Can Be Learned from the Joint Application of Parametric and Nonparametric Techniques", *Journal of Banking and Finance*, Vol. 21(2) pp 221-250.

RICHMOND, J. (2005), "Slack and Net Technical Efficiency Measurement: A Bootstrap Approach", *International Journal of Information Technology & Decision Making*, Vol. 4(3), pp 395-410.

RUGIERO, J. (2006), "Measurement Error, Education Production and Data Envelopment Analysis", *Economics of Education Review*, Vol. 25(3), pp 327-333.

SALAS, O. (1994), Efficiency and Productivity Change: A Micro Data Study of the Colombian Cement Industry, Unpublished PhD Dissertation, University of Gothenburg, Sweden.

SAMPAIO DE SOUSA, M.C. & STOSIC, B. (2005), "Technical Efficiency of the Brazilian Municipalities: Correcting Nonparametric Frontier Measurements for Outliers", *Journal of Productivity Analysis*, Vol. 24(2), pp157-181.

SCHMIDT, P (1986), "Frontier Production Functions", *Econometric Reviews*, Vol. 4(2) pp 289-328.

SENGUPTA, J. K. (1997), "New Efficiency Theory: Extensions and

New Applications of Data Envelopment Analysis", Department of Economics, University of California, <http://www.econ.ucsb.edu/papers/wp06-97.pdf>, accessed 3 May 2006.

SEIFORD, L.M. & THRALL, R.M. (1990), "Recent Developments in DEA: The Mathematical Programming Approach to Frontier Analysis", *Journal of Econometrics*, Vol. 46(1/2), pp 7-38.

SEXTON, R.F & LEWIS, H.F. (2003), "Two-Stage DEA: An Application to Major League Baseball", *Journal of Productivity Analysis*, Vol. 19(2/3), pp 227-249.

SHAO, B.B.M. & LIN, W.T. (2002) "Technical Efficiency Analysis of Information Technology Investments: A Two-Stage Empirical Investigation", *Information & Management*, Vol. 39(5), pp 391-401.

SHEBEB, B., (2002), "Productivity Growth and Capacity Utilization in the Australian Gold Mining Industry: A Short-Run Cost Analysis", *Economic Issues*, Vol. 7 (2), pp71-81

SHEPHARD, R.W. (1953), *Cost and Production Functions*. Princeton University Press, Princeton.

SHEPHARD, R. W. (1970), *The Theory of Cost and Production Functions*, Princeton University Press, Princeton.

- SILVERMAN, B.W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman & Hall, London.
- SIMAR, L. & WILSON, P.W. (1998), "Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models", *Management Science*, Vol. 44(1), pp 49-61.
- SIMAR, L. & WILSON, P.W. (1998), "Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models", *Management Science*, Vol. 44(1), pp 49-61.
- SIMAR, L. & WILSON, P.W. (1999), "Some Problems with the Ferrier/Hirschberg Bootstrap Idea", *Journal of Productivity Analysis*, Vol. 11 pp 67-80.
- SIMAR, L. & WILSON, P.W. (2000), "Statistical Inference in Nonparametric Frontier Models: The State of the Art", *Journal of Productivity Analysis*, Vol. 13(1) pp 49-78.
- SIMAR, L. & WILSON, P.W. (2000), "A General Methodology for Bootstrapping Non-Parametric Frontier Models", *Journal of Applied Statistics*, Vol. 27(6), pp 779-802.
- SIMAR, L. & WILSON, P.W. (2007): "Estimation and Inference in Two-Stage, Semi-Parametric Models of Production Processes", *Journal of Econometrics*, Vol. 136(1), pp 31-64.
- SMITH, J., (2004), "Productivity Trends in the Gold Mining Industry in Canada", *Centre for the Study of Living Standards*, CSLS Research Report 2004-08.
- STAAT, M., (2002), "Bootstrapped Efficiency Estimates for a Model for Groups and Hierarchies in DEA", *European Journal Of Operational Research*, Vol. 138(1), pp 1-8
- STAAT, M., (2006), "Efficiency of Hospitals in Germany: a DEA-bootstrap Approach", *Applied Economics*, Vol. 38(19), pp 2255-2263.
- STATSOFT, "Distribution Tables", <http://www.statsoft.com/textbook/sttable.html#f10>, accessed 31/01/2007
- THOMPSON, R. G., DHARMAPALA, P. S., GATEWOOD, E. J., MACY, S. & THRALL, R. M. (1996), "DEA/Assurance Region SBDC Efficiency and Unique Projections", *Operations Research*, Vol. 44 (4 ), pp. 533-542.
- THOMPSON, C.H. & WOODRUFF, H.W. (1953), *Economic Development in Rhodesia and Nyasaland*, Dennis Dobson, London.
- TILTON, J.E. (2001), "Labor Productivity, Costs, and Mine Survival During a Recession", *Resources Policy*, Vol. 27(2), pp107-117
- TONE, K. & SAHOO, B.K. (2003), "Scale, Indivisibilities and Production Function in Data Envelopment Analysis", *International Journal of Production Economics*, Vol. 84(2), pp 165-192.

TORGERSEN, A.M., FØRSUND, F.R. & KITTELSEN, S.A.C. (1996), "Slack-adjusted Efficiency Measures and Ranking of Efficiency Units", *Journal of Productivity Analysis*, Vol. 7(4), pp 379-398.

TSAI, H-C., CHEN, C-M. & TZENG, G-H (2006), "The Comparative Productivity Efficiency for Global Telecoms", *International Journal of Production Economics*, article in press.

UNITED STATES GEOLOGICAL SURVEY (1998), "The Mineral Industry of Zimbabwe", <http://minerals.usgs.gov/minerals/pubs/country/1998/9246098.pdf>, accessed 20 June 2006

UNITED STATES GEOLOGICAL SURVEY (2003), "The Mineral Industry of Zimbabwe", <http://minerals.usgs.gov/minerals/pubs/country/2003/zimyb03.pdf>, accessed 20 June 2006

VAN DEN EECKAUT, P., TULKENS, H. & JAMAR, M.A. (1993), "A Study of Cost-Efficiency and Returns of Scale for 235 Municipalities in Belgium", in FÅRE, R, GROSSKOPF, S. & LOVELL, C.A.K. (eds)., *The Measurement of Productive Efficiency*, Oxford University Press, Oxford.

VARIAN, H.R. (1992), *Microeconomic Analysis*, W.W. Norton & Co., New York.

VEIDERPASS, A. (1993), *Swedish Retail Electricity Distribution: A Non-Parametric Approach to Efficiency and Productivity Change*, Unpublished PhD Dissertation, University of Gothenburg, Sweden.

VIEWING, K.A., PHIMISTER, G. & JOURDAN, P (1987), "The Development of the Mining Industry in Zimbabwe", *Institution of Mining and Metallurgy Conference on African Mining*, Harare, Zimbabwe, August September 1987.

WANG, F-C, (2006), "Measuring the Cost Efficiency of International Tourist Hotels in Taiwan", *Tourism Economics*, VOL. 12(1), PP 65-85

WATERMAN, M.S. & WHITEMAN, D.E., (1978), "Estimation of Empirical Densities by Empirical Density Functions", *International Journal Of Mathematical Education In Science And Technology*, Vol. 9(2), pp 127-137.

WHEELOCK, D.C. & WILSON, P.W. (2000), "Why Do Banks Disappear? The Determinants of US Bank Failures and Acquisitions", *Review of Economics and Statistics*, Vol. 82(1), pp 127-138.

WILSON, P.W. & Carey, K. (2004), "Nonparametric Analysis of Returns to Scale in the US Hospital Industry", *Journal of Applied Econometrics*, Vol. 19(4), pp 505-524.

WORLD GOLD COUNCIL, "World Production through History", [http://www.gold.org/discover/knowledge/aboutgold/gold\\_prod/index.html](http://www.gold.org/discover/knowledge/aboutgold/gold_prod/index.html), accessed 26 July 2006.

WORLD GOLD COUNCIL, "Gold Jewellery",  
[http://www.gold.org/discover/knowledge/aboutgold/gold\\_jewellery/index.html](http://www.gold.org/discover/knowledge/aboutgold/gold_jewellery/index.html), accessed  
26 July 2006.

WORLD GOLD COUNCIL, (2005) "The Value of Gold to Society",  
[http://www.gold.org/pr\\_archive/pdf/The\\_Value\\_of\\_Gold\\_to\\_Society.pdf](http://www.gold.org/pr_archive/pdf/The_Value_of_Gold_to_Society.pdf), accessed 26 July  
2006.

YOUNG, G.A. (1994), "Bootstrap: More than a Stab in the Dark?", *Statistical Science*,  
Vol. 9(3), pp382-415.

ZELNYUK, V & ZHEKA, V. (2006), "Corporate Governance and Firm's Efficiency:  
The Case of a Transitional Country, Ukraine", *Journal of Productivity Analysis*, Vol.25  
(1), pp 143-157.

ZHOU, H. (2000), An Analysis of the Characteristics of the Production Technology in  
Zimbabwe's Engineering Industry, PhD Dissertation, University of Hull.

ZHU, J. (2002), Quantitative Models for Performance Evaluation and Benchmarking: Data  
Envelopment Analysis with Spreadsheets and DEA Excel Solver, Kluwer Academic  
Publishers, Dordrech.