

THE UNIVERSITY OF HULL

**Relationships between Number Skills and Cognitive Abilities in
people with Specific Arithmetic Difficulties and people with
Dyslexia.**

**being a Thesis submitted for the degree of Doctor of Philosophy
in the University of Hull**

by

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1 Introduction

1.1 Aims and rationale for the studies

The overall aim of this thesis was to analyse the relationships between cognitive abilities and number skills in children and adults. Examining the links between number skills and cognitive abilities is important both to improve our theoretical knowledge and to inform practitioners who are assessing and teaching children who have number skills difficulties. One important theoretical debate that can be informed by this work is whether normally developing individuals solve problems involving numbers using distinct cognitive modules that are specialised for such work or whether they utilise more general-purpose cognitive systems. If weaknesses in particular number skills are associated with particular cognitive deficits, it will support the hypothesis that people utilise their general cognitive architecture. Although research into the interactions between children's cognitive profiles and their responses to different teaching programmes is in the early stages, some studies have suggested that tailoring teaching to a child's cognitive profile can be effective. Therefore identifying groups of children with number skills difficulties that have homogeneous cognitive profiles may help in the design of future intervention strategies.

1.2 Scope

Three main areas of investigation were conducted, all of which examined the links between cognitive abilities and number skills.

- An examination of the relationships between three number skills (number fact recall, counting speed and place value understanding) and three cognitive abilities (non-verbal reasoning, auditory-verbal-sequential short-term memory and visual-spatial short-term memory) in normally developing children.

- An examination of the cognitive and number skills profiles of children with specific arithmetic abilities (SAD). These children had poor arithmetic attainment, but much better reading attainment. The assessment of these children's cognitive and attainment profiles was comprehensive. The children's verbal, non-verbal and spatial abilities were assessed as well as their psychomotor, visual-spatial memory and auditory-verbal memory abilities. Particular attention was paid to the balance of verbal and spatial abilities in these children as previous research has indicated that children with specific arithmetic difficulties share a homogenous ability profile with poor spatial ability, but better verbal ability.
- An examination of the number skills profiles of children and adults with dyslexia. A wealth of previous research has indicated that dyslexic individuals have working memory weaknesses (Hulme, 1981; Shankweiler, Liberman, Mark, Fowler & Fisher, 1979). Three number skills (number fact recall, counting speed and place value understanding) were assessed in dyslexic children, to determine whether a diagnosis of dyslexia was associated with a particular number skills profile. As children with dyslexia had a specific difficulty with number fact recall, the number fact recall of dyslexic adults was compared with non-dyslexic adults, to determine whether this difficulty persisted into adulthood.

1.3 Structure of the thesis

The thesis is divided into nine chapters. Chapter 1 describes the aims of the thesis and gives an outline of its content. Chapter 2 describes and evaluates the two major models of normal adult numerical processing. Chapter 3 describes current knowledge about how children develop number skills; particular emphasis is placed on the interplay between conceptual understanding and procedural skills. Chapter 4 describes and evaluates previous research into the attainment, cognitive and psychosocial strengths and weaknesses of children with arithmetic difficulties. The limitations of the various research methodologies utilised in previous studies are examined. Chapter 5 provides an overview

of how dyslexia is defined; current knowledge about the cognitive profiles of dyslexic individuals is also discussed. Research into the number skills of dyslexic children is described and evaluated. Chapter 6 describes and evaluates Study 1, which had three main aims: to produce norms for some new computerised tests of number skills; to examine how place value understanding, counting speed and number fact recall develop in junior age children; to examine the relationships between cognitive and number skills junior aged children. Chapter 7 reports the results of Studies 2 and 3. The aim of Study 2 was to examine the ability profiles of children with specific arithmetic abilities. The results indicated that children with large verbal/spatial ability discrepancies were over-represented in the group with specific arithmetic difficulties. The cognitive profiles of the children with specific arithmetic abilities were examined in Study 3. The children were divided into four groups: low general conceptual ability; non-verbal learning difficulty; low verbal reasoning; and specific memory weakness. An illustrative case study of a child in each group is provided. Chapter 8 describes and evaluates Study 4, in which the counting speed, number fact recall and place value understanding of children with SAD and children with dyslexia was compared to a randomly selected sample of children attending mainstream schools. The children with dyslexia showed weaknesses on the test of number fact recall and one test of counting speed, but they had unimpaired place value understanding. In contrast the children with specific arithmetic difficulties were impaired both on the tests of place value understanding and number fact recall. Chapter 9 describes and evaluates Study 5, in which the number fact recall of a group of dyslexic students was compared to a group of non-dyslexic students who were matched on intellectual ability. The adults with dyslexia were slower and less accurate at recalling number facts. Chapter 10 draws together the results of the five studies. The findings are discussed in reference to models of adult numerical processing and Rourke's non-verbal learning difficulty classification (Rourke & Del Dotto, 1994). A multiple-route model of arithmetic difficulties is proposed and methods that could be used to evaluate the model are

described. Recommendations for the diagnostic assessment of children with arithmetic difficulties and for cognitively tailored teaching are made.

2 Acquired dyscalculia and models of normal adult numerical processing

In this chapter models of adult numerical processing are described and evaluated. These models are relevant as they have been used both to classify children with arithmetical difficulties and to explain the causes of these difficulties. An evaluation of the model's conceptual basis and empirical support is therefore relevant

2.1 Assumptions of studies of acquired dyscalculia

Psychologists have based their models of adult numerical processing largely on studies of adult brain-damaged patients with acquired dyscalculia. Acquired dyscalculia is the loss of numerical skills (e.g. number reading or long multiplication) following brain damage. McCloskey (1992) outlines the fundamental assumptions of such studies. A reasonable level of premorbid homogeneity is assumed. For example, an adult who, after brain injury, cannot perform multi-digit addition is assumed to have had this skill before the damage occurred. In some studies evidence such as formal qualifications and/or occupational status is reported to support the case for premorbid ability. The patients selected are assumed to have selective damage to components of the cognitive system with little or no reorganisation. The preservation of one skill when another is damaged is used as evidence for the independence of the skills. For example, if arithmetic fact retrieval is intact whilst calculation procedures are impaired it is concluded that these abilities are distinct cognitive systems.

2.2 Distinctions between primary and secondary dyscalculia

Hartje (1987) reviews early studies of dyscalculia (e.g. Berger, 1926; Geller, 1952; Henschen, 1920). The results of these early studies suggested that dyscalculia could result from lesions in the left hemisphere, where language functions are located and the right hemisphere where spatial functions are located. Dyscalculia was sometimes accompanied by spatial or language difficulties. Hartje (1987) suggests that these findings have been

used to support a distinction between primary dyscalculia, which is a fundamental loss of arithmetical concepts, and secondary dyscalculia where arithmetic skills are impaired as a consequence of other cognitive deficits.

Keller and Sutton (1991) assert that individual's with primary dyscalculia, "demonstrate deficits in fundamental arithmetic operations and in the understanding of the concept of numbers yet maintain adequate language memory and visual-spatial skills" (p. 552). In contrast individuals with secondary dyscalculia retain a fundamental understanding of the concept of numbers, but damage to other cognitive abilities impairs their abilities to perform numerical tasks. Kellor and Sutton (1991) describe two main types of secondary dyscalculia. Individuals with dyscalculia secondary to language difficulties have problems understanding and comprehending language (including mathematical language) that interfere with their ability to carry out mathematical tasks. In contrast individuals with language difficulties secondary to spatial difficulties have problems with mathematical tasks with heavy spatial demands such as writing and aligning numbers or understanding place value.

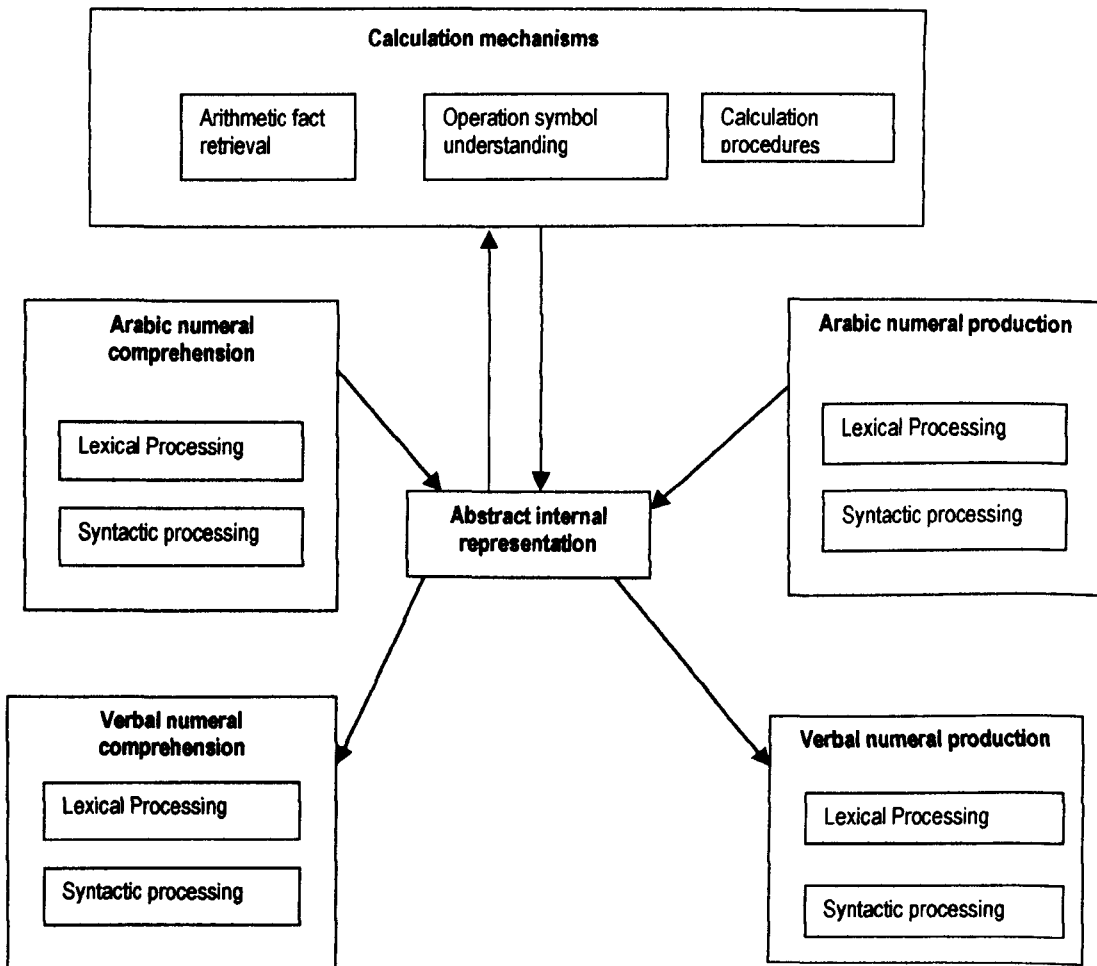
Whilst it is well documented that dyscalculia can co-exist with spatial or language deficits, such case studies cannot prove that the cognitive difficulties cause the arithmetic difficulties. There have been reports of other symptoms occurring with dyscalculia. One well established pattern of symptoms is Gerstmann syndrome. Gerstmann syndrome was identified by Josef Gerstmann, who reported that a small number of his patients displayed an unusual quartet of symptoms (Gerstmann, 1940). These patients had finger agnosia (a difficulty identifying and naming their own fingers), right-left confusion, agraphia (a difficulty in producing writing) and dyscalculia. Whilst Gerstmann argued that the patients dyscalculia was caused by their finger agnosia and right-left confusion, this assertion cannot be proved by the case study reports. Further cases of patients who display the quartet of symptoms have been reported (e.g. Butterworth, 1999; Kinsbourne and Warrington, 1962). However, finger agnosia and right-left confusion cannot be the only

cause of acquired dyscalculia because case studies of patients without these symptoms have been reported (Critchley, 1953).

2.3 The modular view of numerical processing

McCloskey & Caramazza (1985) produced a parsimonious model of number processing that is illustrated in Figure 1. This model proposes that numerical values are stored as abstract codes. Each component or module in the model (illustrated in Figure 1 by separate boxes) is functionally distinct and can be selectively impaired by brain injury. Numerical comprehension mechanisms translate verbal or Arabic numbers into abstract numerical codes. Calculation procedures operate on the abstract numerical codes, not verbal or visual codes. The abstract codes can then be converted back into a verbal or Arabic format and outputted. In addition, McCloskey & Caramazza (1985) assert that different calculation operations can be independently damaged, i.e. subtraction can be selectively impaired whilst division, multiplication and addition are preserved. It should be noted that McCloskey and Caramazza (1985) do not view damage to the number comprehension or production modules as dyscalculia secondary to language difficulties. These modules deal specifically with numerical language and are independent of generalist language systems.

Figure 1. A schematic diagram illustrating McCloskey & Caramazza's 1985 model of number processing.



McCloskey, Sokol & Goodman (1986) expanded on the McCloskey & Caramazza (1985) model, giving further details of the number production mechanisms. It was hypothesised that the number production module utilised the abstract numerical codes to produce a syntactic frame of the correct magnitude. The syntactic frame indicates the number of digits required. The abstract representation is then referred to again to determine which digits fit into which slots in the syntactic frame. Finally, if a verbal

number is required the numerical value produced is matched with the correct word, e.g. 1 ten and 6 units would be matched with 'sixteen'.

McCloskey & Caramazza (1985) cite case studies of adults with acquired dyscalculia to support their assertion that each component of the model is functionally distinct. Benson & Denkla (1969) described a patient with intact numeral comprehension (illustrated by his ability to answer written calculations when multiple choice answers were presented) but impaired number production. He was unable to produce Arabic numbers when presented with written arithmetic problems or dictated numbers. Singer & Low (1933) reported a similar case that supported the numeral comprehension production distinction. Their patient could indicate which of two Arabic numbers was larger, but could not write aurally presented numbers.

McCloskey & Caramazza (1985) report the results of HY, which support their disassociation between Arabic and verbal comprehension mechanisms. HY could judge which of two Arabic numbers was larger, but he could not reliably judge which of two verbal numbers was larger. A patient described by Berger (1926) displayed an Arabic/verbal dissociation in number production. This man could answer arithmetic questions presented verbally, but was unable to translate his answers into Arabic numerals.

Case studies also suggest that lexical and syntactic processes can be disturbed independently. A patient described by Benson & Denkla (1969) produced syntactically correct answers (i.e. they were approximately the correct magnitude and well formed) but these were lexically incorrect because they contained the wrong individual digits. Singer & Low (1933) presented the results of a patient whose syntactical skills were damaged but whose lexical skills were intact. When asked to write Arabic numbers, he used the correct digits in the wrong order. Similarly, patient VO studied by McCloskey & Caramazza (1985) produced numbers containing the correct digits, but of the wrong magnitude, e.g. five thousand and seventeen written '500017'.

McCloskey & Caramazza (1985) outline three components of the calculation system that can be selectively impaired: operation symbol processing, calculation procedures and arithmetic fact store. They acknowledge that impaired number processing will affect calculation indirectly. For example, if a patient cannot understand Arabic numbers they will not be able to answer problems presented in that format. The independence of each component of the calculation system is supported by case studies. Ferro & Botelho (1980) describe a patient who could not understand written operation symbols (he often performed an alternative operation correctly). However, the patient could perform verbally presented arithmetic problems. Ferro & Botelho (1980) described DRC, a physician who could comprehend and produce numerals and define the four basic arithmetic operations. However, he had problems retrieving arithmetic facts. He made errors and had long retrieval times. McCloskey & Caramazza (1985) describe a patient MW who had an arithmetic retrieval deficit, despite accurate calculation procedures and a good understanding of the operation symbols. He could retrieve some arithmetic facts, but had considerable difficulty with multiplication facts. Some multiplication facts were retrieved correctly on all trials, but others were retrieved correctly on less than 50% of the trials. Interestingly, his incorrect responses were nearly all products in the 1 to 10 times tables, 85% were products of one of the multipliers in the question. Although MW often indicated he could not retrieve an answer, he could sometimes reconstruct it from known facts, e.g. $7 \times 6 = (10 \times 6) - (3 \times 6)$.

Patients with the reverse pattern (good retrieval but poor procedures) have been studied. McCloskey & Caramazza (1985) describe patient 1373 in the Vietnam Head Injury Study who had excellent arithmetic fact retrieval, but poor long multiplication fact retrieval. The patient could correctly answer long division problems. He is therefore also illustrative of McCloskey & Caramazza's final distinction between different operations. Patient 1373's multiplication is impaired but his division is intact.

2.4 The encoding complex view of numerical processing

Since its publication in 1985, McCloskey & Caramazza's model has been challenged. Campbell & Clark (1988) and Clark & Campbell (1991) proposed an alternative encoding complex view of numerical processing. Several factors distinguish the encoding complex view from the modular view. Campbell and Clark (1988) and Clark and Campbell (1992) propose that format specific numerical representations are processed rather than abstract codes. Examples of formats include analogue, phonological, visual and articulatory. These specific representations are connected in networks with excitatory and inhibitory links. Specific numerical representations in one format can excite or inhibit numerical representations in the same format or in another format. For example, seeing the number 7 could activate a visual representation of the number and a phonological representation of the number 7. Visual codes for visually dissimilar digits would be inhibited. Comprehension, production and calculation systems can all operate on these specific codes, rather than on the abstract representations proposed by McCloskey & Caramazza (1985). Clark and Campbell (1992) argue that numerical processing is not conducted using specific modules designed only for that task, but by co-opting the general cognitive architecture of the brain. For example, number facts would be stored and retrieved using the general memory mechanisms rather than a specific number fact module. If the encoding complex view of numerical processing is applied all individuals with dyscalculia will be viewed as having secondary dyscalculia. All arithmetic tasks are tackled using general cognitive architecture. There are no arithmetic specific codes or modules to be damaged.

The encoding complex view of numerical processing is best explained through an example. Consider adults' ability to solve verbally presented single digit multiplication problems. Clark and Campbell (1991) would envisage this knowledge being stored in an associative network. Each possible question number would be linked to each possible answer number. For example, associative links would connect the phonological

representation for the number 7 with the phonological representations for all the answers to the questions in the 7 times table. The phonological representation for the number 2 would connect with the phonological representations for all the answers in the 2 times table.

When the question 2×7 is presented verbally all these answers would be excited, but the phonological code for 14 would receive excitation from both 2 and 7. It would therefore have the largest amount of excitation and be chosen as the correct response.

Clark and Campbell (1992) present empirical and conceptual arguments that casts doubt on the modular view of numerical processing presented by McCloskey & Caramazza (1985) and supports their own encoding complex view of numerical processing. They argue that abstract codes are not necessary for a comprehensive model of numerical processing unless there is compelling evidence for their existence, but the processing of specific codes must be included (because they are the obvious output and input of numerical tasks). McCloskey (1992) argues that there must be an “internal semantic representation” (p. 119) to give us a sense of the magnitude of the numerical value. However, this does not necessitate that the magnitude representation is abstract as McCloskey (1992) suggests. Clark and Campbell (1992) argue that magnitude can be represented in format specific codes such as visual spatial codes.

Clark and Campbell (1991) argue that the abstract numerical codes McCloskey & Caramazza (1985) propose are not only superfluous to a comprehensive account of numerical processing, but also conceptually flawed. McCloskey & Caramazza represent abstract numerical codes using power of ten operators, e.g. the abstract numerical code for three hundred and twenty two is $\{3\}10\text{EXP}2, \{2\}10\text{EXP}1, \{2\}10\text{EXP}0$. This method of conceptualising the abstract code tells us nothing extra about what the abstract code actually consists of. There have been no attempts to explain how it is stored within the brain. The choice of using the power of ten is also arbitrary and no reasons are given to explain why this was used instead of another base such as binary numbers. The representations of the abstract codes are, in fact, circular. It uses numbers and a power

operator to explain and abstract representation of other numbers. This does not add to our understanding of what the elusive abstract codes actually are. In summary, Clark and Campbell (1992) argue that unless there is compelling evidence for the existence of abstract codes (and they do not believe McCloskey & Caramazza supply it), it is more parsimonious to construct a model without them.

Clark and Campbell (1991) present empirical evidence that suggests numerical processing operates on format specific representations. For example, digit size has been found to interfere with performance on magnitude comparison tasks (e.g. Besner & Coltheart, 1979; Pavio, 1975). Participants are slower to identify which number is of greater magnitude if it is physically smaller and quicker to identify it if it is physically larger. If, as McCloskey et al suggest, these digits are being converted into abstract numerical codes before comparison, the physical size of the digit should not affect the speed of processing. Clark and Campbell (1991) suggest that that the size incongruence effect can be explained by the physical size of the digit interfering with the analysis of the visual-spatial magnitude code. A further study by Foltz, Poltrock & Potts (1984) suggests that adults process number representations in different formats differently. For digits the effect of digit size incongruence had less of an impact when the magnitude gap was larger. The effect of size incongruence was stable for number words.

Neuropsychological evidence is also presented to support the importance of format specific codes in number processing. Left hemisphere damage in dyscalculic patients appears to disrupt numerical processing more than right hemisphere damage (Boller & Grafman, 1983). Patients with left hemisphere damage have been found to have difficulties both with aurally presented computations (Jackson & Warrington, 1986) and number comprehension tasks (Dahmen, Hartje, Bussing & Strum 1982). Furthermore, the vast majority of dyscalculics with left hemisphere damage also have aphasic dysfunctions (Benton, 1987). Although McCloskey & Caramazza (1985) assert such patients have damaged the relevant numerical processing modules, Clark and Campbell (1991) argue it

can also be validly interpreted as the disruption of all verbal codes including the codes for number words. They also argue that the dyscalculic deficits of patients with right hemisphere damage and visual-spatial impairments can be interpreted as damage to the specific visual spatial codes.

Clark and Campbell (1991) present evidence which suggests that adults co-opt more general cognitive systems when they attempt numerical tasks. The working memory system has been found to be particularly important in numerical processing. Baddeley (Baddeley, 1997; Baddeley, 1986; Baddeley, 1996; Baddeley & Hitch, 1974; Baddeley, Lewis, & Vallar 1984; Baddeley & Lieberman, 1980) conceptualised working memory as a number of interacting subsystems: the visual spatial sketch pad (for the temporary store of visual and spatial material), the phonological loop (for the temporary store of auditory verbal material), and the central executive (which controls the flow of information). Studies have indicated that numerical information presented in different formats is stored in different working memory subsystems. For example, concurrent articulation (which interferes with phonological loop processing) has been found to disrupt both counting (Logie & Baddeley, 1987) and mental addition (Hitch, 1978).

The evidence suggests that specific verbal and visual numerical representations rather than abstract numerical codes are stored in working memory. Clark and Campbell (1991) regard abstract codes that cannot be stored even briefly as difficult to conceptualise. Furthermore, a model of mental arithmetic that is consistent both with the empirical evidence concerning working memory and the modular model of arithmetic would require numerous translations between specific and abstract codes. Clark and Campbell (1991) illustrate this point with the example $4 \times 36 = 144$. First 4 and 6 would have to be transformed into abstract codes; the calculation system would then operate on them. Following this the number production system would transform the resulting abstract code into a specific representation of 24 that could be stored in working memory. The same transformations would then have to be performed on 4 and 30. Finally 120 and 24 would

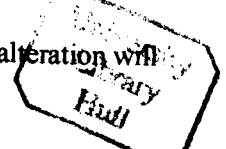
have be turned back into abstract codes for the final answer to be computed. This is an incredibly convoluted process, Clark and Campbell (1991) argue that it is equally valid and more parsimonious to hypothesise that the calculation system can operate on the same specific representations that are stored by the working memory system.

Clark and Campbell (1991) also attack the modularity of the model put forward by McCloskey & Caramazza (1985). Clark and Campbell (1991) assert that the processes carried out by each module are not adequately explained. For example, stating that the calculation module operates on abstract codes does not have a high level of explanatory power, unless these processes are explained more fully. The strong modularity assumption of the McCloskey et al (1985) model rejects direct associations between specific representations without transformation into abstract codes. For example, question representations and specific answer representations cannot be directly linked (e.g. there could be no direct link between the phonological code for 4×2 and the phonological code for 8). Neither can specific representations of the same magnitude in different formats be directly connected (e.g. there can be no direct link between the visual code for 2 and the phonological code for 2). Clark and Campbell (1991) argue that artificially limiting the connection between specific number representations is done to preserve the strong modular nature of the McCloskey & Caramazza (1985) model rather than because there is empirical evidence that suggests these links do not exist.

Clark and Campbell (1991) also argue that it is unclear which module performs which arithmetical task. The McCloskey & Caramazza (1985) model does not explain counting ability. In fact, Sokol, Goodman-Schulman & McCloskey (1989) assert that counting is not a normal function of the calculation system. However, this view disregards the evidence that both children (see section 3.3) and some adults (Svenson, 1985) use counting strategies to solve arithmetic problems. Calculation can therefore involve verbal number production, which challenges the strong modular assumptions of the model. Furthermore, Clark and Campbell (1991) suggest that different modules or more general

cognitive processes could be involved in the various module testing tasks used by modular theorists. For example, McCloskey & Caramazza (1985) and McCloskey & Caramazza (1986) use arithmetic verification tasks (e.g. $4 + 2 = 6$; true or false?) as a test of the calculation system. However, other researchers (e.g. Campbell, 1987; Starzyk et al, 1982) have suggested that people first generate a format specific answer and then check it against the given answer. This model would involve both the number production and calculation systems. Another example presented by Clark and Campbell (1991) illustrates the variety of processes that could be used to solve a magnitude comparison task. McCloskey & Caramazza (1986) view magnitude comparison tasks as tests of the integrity of the number comprehension module. However, generating a counting string could solve these judgement tasks (the number later in the counting string is larger) or they could be solved by examining calculation relationships (e.g. $5 + 1 = 6$, therefore 6 is larger. Clark and Campbell (1991) argue that modular theorists assign particular arithmetical tasks to particular modules to fit their theory, discarding other possible explanations without rigorous empirical testing.

Clark and Campbell (1991) present positive evidence in support of the use of associative networks in arithmetic tasks. They propose these networks connect format specific representations. When an individual is presented with numerical task (e.g. they are asked to read the digit 7) the relevant network links are either excited or inhibited. The specific representation with the highest level of excitation is the chosen response (e.g. the phonological code 'seven' is excited most and therefore selected). Specific representations that are physically similar to the correct answer (e.g. they look or sound similar) will also be excited, but to a lesser extent. The connection strengths in associative networks are altered with practice. As a child learns and receives more practice on a specific numerical task the appropriate connections become stronger. Correctly answering the question $5 \times 5 = 25$ will strengthen the excitatory connections between this question and the format specific representations of 25. However, part of this process of network alteration will



involve strengthening the connections between questions and incorrect answers.

Therefore, other specific representations that are related to the presented question will also be excited; when the network is optimally developed the correct answer will have the highest level of excitation.

Studies examining the error patterns and reaction times of normal adults attempting mental arithmetic tasks are consistent with the associative networks proposed by Clark and Campbell (1991). Incorrect responses to multiplication questions tended to be close in magnitude to the actual answer (Campbell & Graham, 1985). Clark and Campbell (1991) suggest that this may be due to indirect activation of associated questions (particularly those that share an operand) and therefore associated responses. Direct evidence of priming effects has been consistently reported. Priming related answers to multiplication questions reduces the likelihood of the correct answer being generated (Campbell, 1987a; Campbell, 1987b; Campbell, 1991). Campbell (1987a, 1987b) found that when related problems were introduced into a problem set the reaction times and accuracy levels for a target problem deteriorated. Clark and Campbell suggest that related responses are excited by the primes, making it more difficult to determine which response is excited the most - either the previously primed response or the correct answer. Unrelated questions do not produce similar interference effects because the response to these questions would be inhibited by the target question. Campbell (1990a, 1990b) has presented evidence that suggests number facts are stored as format specific networks. Multiplication and addition problems were presented in blocks containing both word and digit problems. Incorrect responses for word problems were more likely to be responses to previous word problems rather than responses to previous digit problems; in contrast, incorrect responses to digit problems were more likely to be responses to previous digit problems.

Clark and Campbell (1991) suggest that impaired inhibitory processes could account for many of the numerical deficits of dyscalculic patients. They emphasise the importance of inhibition in associative networks. A small number of number

representations are associated with a large number of answer representations. If the example of the multiplication table associative network is reconsidered it is clear that format specific number representations in the multiplication sums connect to numerous format specific answer representations. An inability to inhibit competing responses will have serious affect on numerical performance. Campbell and Clark (1988) reanalysed the performance data for HY, a patient with acquired dyscalculia. HY had difficulty reading Arabic numerals, which McCloskey & Caramazza (1985) attributed to an impaired numeral production mechanism. HY's errors were not random. He tended to produce answers with the correct syntactical frame. Nor were his lexical confusions random: they tended to match the odd/even nature of the correct answer, were likely to be numerically close to the correct answer, and tended to be visually similar to the correct answer. Neither the McCloskey & Caramazza (1985) model nor the further details provided by McCloskey & Caramazza (1986), account for this error pattern. HY is hypothesised to have selective damage to the number production system. According to McCloskey & Caramazza (1985) and McCloskey et al (1986) an abstract numerical representation is inputted into the number production system. There is no mention of multiple activation in the McCloskey & Caramazza model, so it cannot account for the increased likelihood of errors being of a similar magnitude to the correct answer. The input to the number production system is not format-specific, therefore, even if multiple activation was included in the McCloskey & Caramazza (1985) model, visually similar numbers are no more likely to be activated.

One further criticism of the McCloskey & Caramazza (1985) model is the problem of how the different modules evolved. Specialist neurological mechanisms develop in response to evolutionary pressures that exist for tens of thousands of years. By contrast, the tools necessary for number fact recall and multi-digit arithmetic have been developed relatively recently in human history. For example, the place value system of numerical notation developed as recently as the first century (Butterworth, 1999).

2.5 Conclusions

McCloskey (1992) defends the modular account of number processing. He evaluates three types of evidence that has been cited in support of the encoding complex view: arithmetic with Arabic and roman numerals, bilingual arithmetic and numerical comparison tasks. The results of studies by Gonzalez & Kolers (1982) and Gonzalez & Kolers (1987) were that adults took longer to verify simple sums presented in Roman numerals rather than in Arabic numerals. Marsh & Maki (1976) and McClain & Huang (1982) reported similar results for bilingual subjects who responded more quickly to mental addition sums presented in their own language. If these results are interpreted in terms of calculation processes operating on different format specific number codes they support the encoding complex view. However, McCloskey (1992) and Sokol et al. (1989) rightly point out that it may simply be the case that the number comprehension system takes longer to convert less familiar codes into abstract representations. McCloskey (1992) argues that the results of numerical comparison tasks favour the modular rather than the encoding complex view of numerical processing. It has been widely reported that adults take longer to judge which of two numbers is bigger if they are closer in magnitude. This finding is consistent in many formats including Arabic digits (Moyer & Landauer, 1967), written number words (Sekuler, Rubin & Armstrong, 1971), dot patterns (Buckley & Gilliman, 1974) and Japanese numerals (Takahashi & Green, 1983). McCloskey (1992) asserts that the consistency of this effect across numerals indicates that magnitude comparison is always executed using abstract numerical codes. However, the evidence does not have to be interpreted in this way; it could be explained by comparison to format specific visual spatial codes or even by reference to counting strings. Conversion to abstract codes can not explain the interaction effects between digit size, digit magnitude and time taken to identify the larger number (see the discussion of Besner & Coltheart, 1979; and Pavio, 1975 in section 2.4).

The main criticism of the encoding complex model is its vagueness. McCloskey (1992) states that the encoding complex view “has not yet been developed into a specific model capable of generating clear predictions” (p. 123). It is not clear which codes can operate during which arithmetical tasks or, in fact, the range of codes available. How magnitude comparison is conducted is not precisely specified: do we all use a visual-spatial code for all tasks or do different people use different methods in different situations? The encoding complex model needs further refinement and precision. However, Campbell & Clark (1988) and Clark & Campbell (1991) have highlighted many conceptual and empirical weaknesses in the modular model. It is not yet clear which model will finally prove to be more accurate.

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3 Number skills development in normally developing children

Number skills can be seen to develop in two interacting areas: conceptual understanding and procedural skills. Conceptual understanding is defined by Rittle-Johnson & Siegler (1998) as “understanding of the principles that govern the domain and of the interrelations between pieces of knowledge in a domain” (p. 77). Conceptual understanding of number enables us to comprehend the principles that underlie our system of representing quantities, e.g. knowing that any given number in the counting sequence can be broken down into smaller numbers that precede it, but not larger numbers that are subsequent to it. Procedural skills are defined by Rittle-Johnson and Siegler (1998) as “action sequences for problem solving” (p. 77). Arithmetical procedural skills enable children to compute the answer to numerical problems, e.g. quantifying a set by following counting rules, subtracting one number from another by correctly carrying out an algorithm.

Although certain individuals have large discrepancies between their conceptual understanding and procedural skills (see Dowker, 1992; Dowker, 1999) the two domains clearly interact. If a child understands the principles behind a procedure their chances of carrying it out correctly are improved. Efficient and accurate procedural skills will enable a child to successfully answer arithmetic problems and may therefore allow them to identify patterns underlying number work. In turn, identifying patterns in their answers may accelerate the development of children’s conceptual understanding. This chapter examines the development of children’s number skills in six areas: subitising, counting, single digit addition and subtraction, single digit multiplication, comprehending and producing Arabic numerals and multi-digit addition and subtraction. Story problems, measuring, graph reading and other topics that involve applied number skills are not included, as they are outside the scope of this thesis. The more advanced skills of multi-

digit multiplication and subtraction are also excluded because of the scarcity of psychological research in these areas.

3.1 Subitising

Subitising is defined by Starkey & Cooper (1995) as “the ability to enumerate small sets of objects without counting them” (p. 399). The time taken to enumerate larger sets increases in a linear manner, smaller sets (of approximately one to five objects) are enumerated in approximately the same time. If the time available to enumerate the objects is constrained by the experimenter adults make accurate judgements of arrays up to about five objects, errors are made when larger arrays are presented. These findings have led to the conclusion that small sets are not enumerated by counting (discussed in the subsequent section), but rather by the quicker and more automatic process of subitising.

There is growing evidence that subitising develops in infants and young children before verbal counting. Studies suggest that infants under the age of 12 months can detect changes in the numerosity of objects (Starkey, Spelke & Gelman, 1990) and events (Wynn, 1995). Furthermore, Starkey & Cooper (1995) have demonstrated that pre-school children can use subitising to determine which set has a greater numerosity, regardless of the length, density spatial configuration or object composition of the array. The experimenters presented two arrays ranging in size from one to seven to children aged two to five years. The presentation duration was only 200ms, which eliminated the possibility of covert counting. The 2-year-old children accurately chose the larger array when the array numerosity was three or less, the 3-year-olds were accurate when the array numerosity was four or less and the 4- and 5-year-olds were accurate when the array numerosity was five or less. It was concluded that the size of array that young children can subitize increases as they get older. As counting ability develops between 3 and 5 years (see section 3.2) it was also concluded that young children who were not yet capable of counting were able to subitize small groups.

As the results of Starkey & Cooper (1995) indicated that subitising preceded counting developmentally it was argued that young children's subitising ability helped their counting ability to develop. Specifically they proposed that if a young child used count words to label an array which they could subitize, the semantic understanding of numerosity they gained from subitising would be linked to the count words. The developmental primacy of subitising, means that Starkey & Cooper's (1995) model of development could be correct. However, as yet there is no direct evidence to suggest that young children's subitising ability promotes their semantic understanding of counting.

3.2 Counting

Fuson (1992) defines counting as "the method used in all cultures to differentiate and label quantities not easily or accurately differentiated by perceptual means" (p. 48). Children's counting ability develops as they grow older. Fuson, Richards & Briars (1982) describe five stages of counting development. To begin with children simply say the number sequence without clearly differentiating the different words; this stage is called *string counting*. This develops into the *unbreakable list* stage, which consists of three sub-stages. Children at the *sequence* sub-stage say each word separately. At the *sequence-count* sub-stage children start pairing the number words with objects. The final sub-stage *sequence-count-cardinal* is reached when the child associates the final word said with the total number of objects in the set. Once a child achieves the *breakable chain* stage they can make a cardinal count transition. This enables children to start counting at any given point as long as concrete objects are available. For example, if a child is told that two objects are hidden behind a screen and two more are shown on a table he can calculate the total number by saying, 'three, four'. If a child is able to solve this problem when presented verbally (without physical support) she is at the *numerable chain* stage. Solving the problem mentally is more demanding. The child must keep track of the total count and the number of objects they still have to count. The most advanced counting stage is *bi-directional chain/truly directional counting*. At this stage the child understands the

following: each number word refers to both all the previous number words and itself; number words are part of the sequence and cardinal amounts; each subsequent number word is $n + 1$ and each previous number word is $n-1$; and addition and subtraction are inverses. A child who is at this stage can count forwards and backwards from any given point in the number sequence.

Early counting research concentrated on the development of young children's ability to determine the numerosity of a set. Gelman & Gallistel (1978) argued that once children can apply three fundamental 'how-to-count' rules they can determine the numerosity of a set if asked to do so. The *one-to-one correspondence* rule states that each number word must be matched to a single object. The *stable order* rule states that the number words must always be recounted in a correct order. The *cardinality* rule states that the last word said indicates the number of items in the set. A child who can apply all three rules will be at or above the *sequence-count-cardinal* stage described by Fuson, Richards and Briars (1982).

Gelman & Meck (1983) studied children between the ages of 2 and 6 years to determine when they could understand and apply the three 'how-to-count' rules. The children were asked to count sets of objects ranging in size from two to nineteen. The objects were arranged in straight rows. In a separate task the children were asked to judge whether a puppet had counted correctly. On some trials the puppet broke the cardinality rule by stating a different number than the last one said in the sequence, on other trials he did not keep one-to-one correspondence or he said the number words in the wrong order. 50% of children under 5 years could count small sets correctly and even larger proportions were able to detect the puppet's mistakes. Gelman and Meck (1983) asserted that counting principles come before counting procedures. In other words, the children understood the 'how-to-count' rules before they could apply them accurately. Since children who had not commenced formal schooling and could not count accurately detected the puppet's errors,

Gelman and Meck (1983) concluded that children could develop an understanding of counting principles without formal schooling.

Nunes & Bryant (1996) challenge these conclusions. They agree that by the age of 5 years most children have a firm grasp of the 'how-to-count' rules but assert that the knowledge of 3 and 4 year olds is much less sound. Gelman and Meck (1983) may have over-estimated the abilities of very young children by presenting them with very simple counting tasks. They engaged in discussion with the children about the puppet's error, recording the child's 'best' answer. As there are only two possible answers and children tend to change their answer when challenged by an adult (see Rose & Blank, 1974; Samuel & Bryant, 1984), it is unsurprising that high success rates are reported. Gelman and Meck replicated the study in 1986, this time they reported both the children's best and first answers (Gelman & Meck, 1986). The 3 and 4 year olds first judgements were still overwhelmingly correct when the puppet counted correctly (in a conventional manner) and when the puppet flouted the 'one-to-one correspondence' rule. However, when the puppet counted correctly but unconventionally (counting from the middle out, instead of in a straight line) only about two-thirds of the children's first judgements were correct.

More recent studies indicate that young children learn counting procedures before they fully understand the how-to-count principles. Frye, Braisby, Love, Maroudas & Nicholls (1989, experiment 2) tested the conceptual understanding of children aged 3 years 4 months and 4 years 11 months who could accurately count small sets. On some trials the experimenter counted correctly in the standard manner (left to right) in others he counted correctly in a non-standard manner (the sequence of pointing was not left to right, but the essential principles were adhered to). Frye et al (1989) also included trials where the stable order or one-to-one principles were violated. Frye demonstrated a count and then said 'I think I counted right/wrong. Did I?' The question was varied so the children had to contradict the experimenter on half the trials, thus avoiding response bias. Correct judgements for the non-standard correct trials were given on only 53% of trials; 39% of

trials that violated the stable order or one-to-one correspondence principles were accepted. As chance response rates were 50%, these results indicate that young children who are able to count nevertheless have a very limited understanding of the principles underlying counting. Furthermore, the children frequently accepted the experimenter's total, even when the last word he said in the count sequence was different from the total claimed. This suggests the children also had a limited understanding of cardinality.

Other studies have indicated that knowledge of mechanical counting procedures proceeds a full understanding of the cardinality principle. Wynn (1990) and Frye et al. (1989, experiment 1) found that although children aged 3 years 4 months and 4 years 11 months could accurately count small sets when asked, they did not use counting to decide how many objects to pass to the experimenter when asked to "give x objects". This suggests that the children did not know that counting could be used to determine the numerosity of a set.

The results of Fuson & Willis (1988) indicate that young children have difficulty maintaining one-to-one correspondence when objects are not placed in straight lines. Fuson (1988) studied children aged 3 years 6 months to 6 years. They were asked to count blocks in sets of four and above. When the blocks were arranged in shapes or scattered randomly rather than ordered in straight lines many of the 3- and 4-year olds failed to keep one-to-one correspondence. These children may have had difficulties because they did not understand the one-to-one principle or because they could not apply the principle accurately. If the children did not understand the principle they tended to succeed only when the objects were placed in straight lines because the position of the blocks coincides with repetitive and rhythmic movements they associate with counting.

Older children spontaneously use strategies that maintain one-to-one correspondence. Fuson (1988) noted that the vast majority of the children aged 5 years 6 months to 6 years in her study moved the blocks from one pile to another. Younger children are less likely to employ this strategy. Fuson (1988) and Herscovics, Bergeron, &

Bergeron (1986) reported that only half of children aged 5 years to 5 years 6 months used this strategy. Young children can be taught strategies to help them maintain one-to-one correspondence. Fuson (1988) asked three-year-olds to count a circle of dots; one was red and the others were green. About half of three-year-olds spontaneously used the red dot to make sure they stopped in the right place on all three trials. Fuson then demonstrated how to use the dot as a place marker to the same children. Although the percentage of correct trials increased, the number of children who used the technique consistently did not increase significantly. It is possible that after the demonstration the children were simply copying the adult and did not understand that the procedure was necessary to maintain one-to-one correspondence.

3.3 Single digit addition and subtraction

Young children solve single digit addition and subtraction problems using counting strategies. Carpenter and his colleagues (Carpenter, Hiebert & Moser, 1981; Carpenter & Moser, 1979; Carpenter & Moser, 1982) conducted a large-scale survey of 6 to 8 year old children's arithmetic strategies. They described four strategies that were used to solve addition problems. Addition problems require the child to find the total of two or more addends. *Count all* strategies require the child to count through both addends, e.g. for the problem $4 + 2$, the child would say '1, 2, 3, 4 ... 5, 6'. The child may use concrete objects if provided or his fingers. If this is the case the child may count out the two addends separately and then recount them altogether, e.g. '1, 2, 3, 4 ... 1, 2... 1, 2, 3, 4, 5, 6'. *Count on* strategies require the child to count through only one addend, e.g. '4 ... 5, 6'. Fingers or concrete objects may still represent the counted addend. *Derived fact* strategies involve the sum being broken down into smaller sums for which the child can retrieve the number facts, e.g. for the sum $7 + 5$ the child could decompose the 5 into 3 and 2 if they know the two number facts $7 + 3 = 10$ and $10 + 2 = 12$. The *recalled fact* strategy requires the child to retrieve the answer to the given sum from memory.

Since this early work many researchers have attempted to determine the order in which strategies develop and how children come to choose more advanced strategies in preference to less efficient strategies. Fuson (1992) provides a review of research into children's addition and subtraction skills. She highlights the fact that much of the reported data has limitations. Many researchers employ poor experimental controls, e.g. a lack of counterbalancing. There is also poor reporting of the range of strategies used by individual children. Studies that record all the strategies children use indicate they often use more than one. Siegler (1987) reported the strategy choice of 5 and 6 year old children when solving subtraction problems. Only 12% of the children used only one strategy, 12% used two strategies, 33 % used three strategies and 42% used four strategies. Similarly, Canobi, Reeve & Pattison (1998) reported multiple strategy use in her sample of 5- and 6-year-old children. It is clear that reporting only the most popular strategy does not fully describe children's behaviour.

Although children utilise more than one strategy at a given point in time, Fuson (1992) reports that the results of the studies suggest that strategies are acquired and discarded in a similar sequence. The sequence has three broad levels. She describes Level 1 as *the single representation of the addend or sum*. Addition strategies at this level would correspond to the *count all* strategies described above. Subtraction procedures at this level require the use of fingers or concrete objects. *Take away a* involves the minuend (the initial number in a subtraction problem) being counted out then the subtrahend (the number that is taken away) being counted and removed before the remaining objects are recounted.

Level 2 is described as *abbreviated sequence counting* procedures. The addition strategy at this level corresponds to the *count on* procedures previously described. Two subtraction procedures are possible which can be executed with or without concrete objects. *Counting down* involves saying the minuend and then counting backwards the same number of words as the subtrahend. *Counting up* involves saying the subtrahend and

then counting forward until the minuend is reached, whilst keeping track of the number of words said.

The use of the *count up* strategy as opposed to the *count down* is a good example of how conceptual understanding can improve procedural accuracy. *Counting down* has been found to be more difficult than counting up (Baroody, 1983; Baroody, 1984). However, it is likely that a higher level of conceptual understanding is required to implement this strategy. Subtraction problems require the number to be reduced, which strongly suggests that backward counting is required. Baroody & Gannon (1984) and Woods, Resnick & Groen (1975) found *count down* strategies preceded *count up* strategies. Increasing children's knowledge so that they could employ a *count up* strategy drastically improved their procedural accuracy (Baroody & Gannon, 1984; Fuson & Willis, 1988).

Groen & Resnick (1977) suggested that there are two *count on* stages: *count on from first* (where the child counts on from the first addend regardless of its size), and *count on from larger* (where the child counts on from the larger addend regardless of its position in the problem). *Count on from larger* is more efficient, but Groen and Resnick (1977) suggested that it developed later because it requires understanding of the commutativity of addition (if an operation is commutative the order of the digits in the problem is irrelevant; both addition and multiplication are commutative, i.e. $x + y = y + x$ and $a \times b = b \times a$). Empirical evidence has not always supported this proposal. Baroody (1987) and Siegler & Jenkins (1989) found few children used *count on from first*, and Carpenter and Moser (1984) found most children used both procedures with no strong preference for one or the other.

However, Rittle-Johnson and Siegler (1998) cite studies which suggest that an understanding of the commutativity of addition is related to use of the *count on from larger* strategy. Baroody (1984) found that kindergarteners who succeeded on two tasks designed to assess understanding of the commutativity of addition used the *count on from larger* strategy on 43% of addition sums presented, whilst the children who failed on the tasks

used it on only 24% of the sums. The smaller addend was presented first on all the trials. Similarly, Cowan, Dowker, Christakis & Bailey (1996) found kindergarteners who passed an addition commutativity task used the *count on from larger* strategy on 61% of trials whilst the children who failed used it on 21% of trials. Again, addition trials with the smaller addend first were used to test the children. A recent study by Canobi et al. (1998) found only these children who demonstrated an understanding of commutativity consistently used the *count on from larger strategy*.

Although the studies of Cowan and Renton (1996) and Baroody and Gannon (1984) demonstrate a relationship between an understanding of commutativity and use of the *count on from larger* strategy, they do not support the hypothesis that an understanding of commutativity is sufficient for the *count on from larger* strategy to be the preferred strategy for solving addition problems. A third factor, such as motivation or intelligence, may influence the development of commutativity understanding and the *count on from larger* strategy independently. Furthermore, a large proportion of the children who succeeded on the tasks in both studies did not generate the *count on from larger* strategy. Rittle-Johnson and Siegler (1998) suggest this may be because the commutativity tasks overestimated the children's understanding, i.e. the children who did not use the *count on from larger* strategy but passed the commutativity tasks did not fully understand the commutativity tasks. Alternatively, the children may appreciate that changing the order of the addends does not violate the addition procedure but fail to appreciate that it is more efficient to do so. Also it appears that the *count on from larger* strategy can be generated without understanding of commutativity. Some children who used the procedure failed on the commutativity tasks.

Level 3 is described as *derived fact and known fact procedures*. At this stage children can retrieve some number facts without the need to use counting strategies. Fuson (1992) suggests that the transition to Level 3 occurs during third grade (approximately 8 years old). Some facts (e.g. doubles problems such as $6 + 6 = 12$) are acquired before

others. In Japan, Korea, mainland China and Taiwan, derived fact strategies are systematically taught and this appears to accelerate learning (Fuson, 1990; Fuson, 1992). Children from these countries are typically taught the number pairs that make ten and then the *up over ten* procedure. The smaller addend is broken into two parts; one part is the number that makes ten with the larger addend, the remainder is then added to ten. This procedure is slightly more efficient in these Asian cultures because their number words are more transparent. Instead of special words for ten plus units quantities (e.g. 'twelve') they simply say the ten and the unit (e.g. 'ten two') (Fuson, 1992). Similar procedures are used with subtraction problems with a minuend (starting number) of 10 or more. *Down over ten* is the reverse of up over ten, whilst *subtract from ten* requires the subtrahend (the number taken away) to be subtracted from ten and the remainder added to the minuend e.g. $13-5=(10-5)+3=5+3=8$. Such derived fact strategies have been taught to U.S. children and have been more effective than traditional textbook instruction (Steinberg, 1984; Thornton, 1990; Thornton & Smith, 1988). However, the subtraction versions of these derived fact strategies were harder for children to learn and did not produce such a high level of success as teaching the *count up* strategy (Fuson, 1986; Fuson & Willis, 1988).

Some studies have focused less on the mechanics of addition and subtraction; instead they have examined the development of children's conceptual knowledge. Children's understanding of the commutativity of addition has already been discussed in this section. Other principles children must grasp include inversion, i.e. the knowledge that the addition of one number can be cancelled out by the subtraction of another number. Researchers have tapped children's understanding by comparing children's performance on problems where knowledge of inversion would help (e.g. $4 + 2 - 2 = 4$) with their performance on similar sized problems where a knowledge of inversion would not help (e.g. $3 + 2 - 1 = 4$). If children's performance was significantly faster or more accurate when tackling the problems involving inversion, researchers concluded that they could apply the inversion principle.

Early studies indicated that young children's understanding of addition was limited. Starkey & Gelman (1982) reported that the percentage of inversion problems solved by 3-, 4- and 5-year-old children was not significantly higher than the percentage of control non-inversion problems. However, the problems in this study used very small numbers and the percentage of problems solved in both groups was quite high. It would therefore be difficult to detect differences. Bryant, Christie & Rendu (1999) cites a study reported by Bisanz, LeFevre, & Gilliland (1989), who examined inversion understanding in children aged 6 to 9 and 11 years as well as the inversion understanding of a group of young adults. Children and adults who understood the inversion principle could solve small and large number inversion problems more quickly than small and large number control problems. The number of children classified as inversion understanders was very small for 6-, 7-, 8- and 9- year old children, but increased steeply thereafter.

The study by Bryant et al (1999) suggests that young children have a better understanding of the inversion principle than the previous studies would suggest. 38 children aged 5 and 6 were presented with inversion and non-inversion control problems in six formats. Some formats involved the use of blocks, others placed the problem in a story and the abstract format simply presented the sum in the traditional form. Both 5- and 6-year-old children solved significantly more inversion rather than control problems in all formats.

It is not yet fully understood how children automate number facts or the decision processes involved in strategy choice. Models of strategy choice have been developed from computer simulations of children's behaviour. Models of the memory organisation of number facts have suggested a semantic network in which activation of addends automatically activates the sum (Ashcraft, 1983; Ashcraft, 1987; Bisanz & LeFevre, 1990). Siegler and his colleagues developed a *distributions of associations model* (Siegler & Shrager, 1984). This model proposes that the child sets an accuracy threshold of certainty for accepting an answer: If the strength of the association between the addends

and the sum is above this threshold the answer is retrieved from memory; if it is weaker another strategy is employed. Obtaining the correct solution when the sum has previously been calculated will increase the association strength. Siegler & Jenkins (1989) developed a modified version of the old model, known as the *strategy choice model*. In this version the solution procedure is chosen from all available procedures. The strategy with the highest association strength with the addend pair is utilised. The association between a particular number pair and a particular strategy depend on many factors, including past speed and accuracy of the particular number pair, past speed and accuracy when used with similar number pairs, and past speed and accuracy on all pairs with which that strategy has been used.

3.4 Single digit multiplication

The ability to recall multiplication facts rapidly develops over time and is not perfect even in adults. Campbell & Graham (1985) tested adult graduate and undergraduate students on their oral recall of all the single digit multiplication problems from 0×0 to 9×9 . They made errors on 7.65 % of trials. As part of the same experiment the children in Grades 3 to 5 were tested on the same set of problems. Error rates declined with age. Children in Grade 3 made errors on just under a third of the problems whilst children in Grade 5 made errors on less than one fifth of the problems. Children's reaction times also decreased with age. Cooney, Swanson & Ladd (1988) reported higher error rates for children attempting the same set of questions; students in Grade 3 made mistakes on over half of the problems whilst children in Grade 4 made mistakes on just less than half of the problems.

Fewer studies have focused on the development of single digit multiplication than on the development of single digit addition and subtraction. Research into single digit multiplication focuses on two major issues: firstly, why some problems are harder to learn than others, and secondly, how the strategies children use to answer single digit addition change over time. Early studies indicated that children and adults found problems with a

larger product more difficult than problems with a smaller product. When tackling multiplication problems with larger products individuals required more trials to learn, gave more incorrect responses, and took longer to answer (Clapp, 1924; Miller, Perlmutter, & Keating, 1984; Norem & Knight, 1930; Parkman, 1972).

Counting methods are encouraged when children first learn multiplication.

Heddens (1984) reviewed American elementary mathematics programmes. Multiplication tended to be introduced by using skip counting or repeated addition of equal addends. The National Numeracy Strategy in England and Wales also introduces multiplication in the context of multiple addition of equal addends (DfEE, 1999). It explicitly states that in Year 3 children should “Understand multiplication as repeated addition” (section 3, p.14). Attainment targets for skip counting are introduced in Year 1, whilst targets for rapid recall of multiplication facts are only introduced in Year 2. Without a solid knowledge of single digit addition children will struggle to master multiplication. It has been suggested that adults use a similar, but subconscious counting procedure (see Baroody, 1983; Parkman, 1972). Cooney et al. (1988) found that 9- and 10-year-old children use a variety of strategies when solving multiplication problems including repeated addition.

Campbell & Graham (1985) present two alternative explanations of the *problem size* effect. Multiplication problems can be solved by a repeated addition procedure. More counting is required to solve problems with larger products, which would increase the time taken to achieve the answer. However, the *problem size* effect can be explained without reference to counting strategies. Theoretical models suggest that multiplication problems and their answers are stored in associative-network models (e.g. Siegler, 1988; Siegler & Shipley, 1995) in a similar manner to addition and subtraction facts. The problems and answers are connected so that activation of the problem nodes automatically activates the answer nodes. It is hypothesised that the connections between problems and answers that are larger are longer and therefore it takes longer to produce the answer.

There are exceptions to the *problem size* effect. Some problems are answered faster than their problem size would predict, these include tie problems (e.g. $5 \times 5 = 25$, $3 \times 3 = 9$) (Miller et al., 1984; Starzyk, Ashcraft & Hamann, 1982). Furthermore, one study published by Miller et al. (1984) indicates that non-tie combinations do not conform to the problem size effect. The average reaction times were calculated for the 2 to 9 multiplication tables. Only the reaction times of the 4, 8 and 9 multiplication tables were in the order one would predict from the problem size effect. The problems in the 5 multiplication table were answered more quickly than those in the 2, 3 and 4 multiplication tables. The problems in the 7 multiplication table were answered more quickly than those in the 3 or 6 multiplication table.

These exceptions to the problem size effect have been explained in terms of accessibility. More practised and familiar problems are more accessible than less familiar problems (Miller et al., 1984; Starzyk et al., 1982). Siegler (1988) analysed the problems presented in two popular American textbooks. Tie problems were featured more often than non-tie problems. However, this finding is disputed by Baroody & Gannon (1984) who re-analysed the data. When combinations that are commonly solved by using rules (e.g. $0 \times n$ and $1 \times n$) are removed the difference between tie and non-tie combinations is no longer significant. Furthermore, large tie combinations (e.g. $7 \times 7 = 49$) are featured less often than similar sized non-tie sums.

Miller et al. (1984) assert that the accessibility of multiplication answers is reduced if there are more competing confusion products. Campbell & Graham (1985) expand on this idea, explaining that as all multiplication problems are made up of the same ten digits, learning one problem can interfere with learning another. If this hypothesis is true the answers to problems with a multiplier in common are more likely to be confused. For example, people are more likely to confuse the answer of 7×8 with 7×9 rather than with 9×6 . Evidence supporting the interference effect has been gathered. Starzyk, Ashcraft & Hamann (1982) found that participants took significantly longer to determine whether a

multiplication sum was right or wrong if the false answer was from the same multiplication table. Miller et al. (1984) found that 41% of adult errors to multiplication questions were answers to neighbouring problems (i.e. they differed from the correct answer only by the size of one of the multipliers).

In the earlier section on single digit addition and subtraction the move from models describing cognitive links between problems and answers to models describing links between problems and strategies was described. Siegler's *strategy choice* model can be applied to single digit multiplication (see Siegler, 1988). This model would predict that children's choice of strategy is dependent on estimated speed and accuracy (based on success when using strategies previously) and on their decision regarding the level of accuracy required. If children are very concerned about accuracy they may choose a slower but more reliable procedure; alternatively if they are more concerned about speed they may choose a less reliable but quicker procedure. Children can choose between all the strategies they know for solving multiplication problems.

A study conducted by Cooney et al. (1988) suggests that children progress from using simple reconstructive strategies to automated fact retrieval. 9- and 10-year-olds in Grades 3 and 4 completed two separate multiplication tasks. They were asked to calculate as quickly as possible all the single digit multiplication sums from 0×0 to 9×9 . They were then asked to answer a further 10 single digit multiplication problems and report afterwards how they had calculated the answer. Their mental multiplication strategies were classified into five categories. *Memory retrieval* was recorded when the children simply remembered the answer to the problem presented. *Counting* was recorded when the child used a skip counting method, e.g. 3×2 is calculated by saying '2,3,6'. Sometimes this strategy is called 'repeated addition' because the child successively adds the same number. As discussed earlier in this section, a repeated addition strategy is emphasised when children first learn arithmetic. Some multiplication sums can be calculated using *rules*. A *rules* classification was recorded if the child stated that a particular class of sums

always resulted in the same answer, e.g. any number multiplied by zero equals zero or any number multiplied by one equals itself. A *derived fact* strategy was recorded when children recalled a known multiplication fact and modified it to fit the question, e.g. answering 7×6 by recalling $6 \times 6 = 36$ and adding 6. Finally, idiosyncratic strategies were recorded as *other*. Strategies in this category were special methods taught to the children to remember particular sums. For example, some children answered 2×9 by recalling that $9 \times 9 = 81$ and that this number reversed is 18.

Children in Grade 4 made fewer mistakes and had faster reaction times when completing the timed multiplication tasks. When analysing the children's verbal reports the experimenter divided the questions into three groups: problems involving 0, problems involving 1 and all other problems. The analysis of problems involving 0 indicated that the grades did not differ significantly. The *rules* strategy was recorded for over half of the responses for both grades. The analysis of problems involving 1 indicated that the 3rd grade children were more likely to use a counting strategy, whilst the 4th grade children were more likely to use a *rules* strategy. On the remaining multiplication problems a higher percentage of 3rd graders used counting than 4th graders (18.75% and 11.25% respectively). The 3rd graders used retrieval less than the 4th graders (55% and 73.75% respectively). When analysed using chi-squared the significance level approached but did not exceed the conventional level of statistical significance ($p < 0.08$).

Cooney et al. (1988) also analysed the timed multiplication data using regression vectors. A mental counting model NSTEP was constructed to represent the process of repeated addition. A mental retrieval model ASSOC was also constructed. The proportion of variance in individual children's latency times was calculated and compared to their verbal reports. None of the children in Grade 4 had a significant proportion of the variance in their reaction time scores explained by NSTEP. Eight out of ten children in the Grade 4 had a significant proportion of the variance in their reaction times explained by ASSOC. These eight children all reported using retrieval on the majority of trials. The results from

the children in Grade 3 were much more mixed. Five children had a significant proportion of the variance in their reaction times explained by NSTEP. Two of these children reported counting as their primary strategy and three retrieval. Three students had a significant proportion of their reaction times explained by ASSOC. Two of these children reported retrieval as their primary strategy, one reported a mixture of counting and retrieval. Two students in Grade 3 did not have a significant proportion of the variance in their reaction times explained by ASSOC or NSTEP.

Overall, the results of Cooney et al. (1988) support the hypothesis that children originally use reconstructive strategies to solve multi-digit addition, but later develop automatic retrieval. This developmental pathway can be accommodated by Siegler's *strategy choice model* (Siegler, 1988). As children grow older they have more practice at retrieving multiplication facts and therefore their accuracy increase. This means they are more likely to chose retrieval over skip counting. The findings of the Cooney et al. (1988) study highlight the inconsistencies between reaction time measures and children's verbal reports. The Grade 3 children's reports and the regression analyses did not always match. Cooney (1988) noted that the children's instructional programme emphasised the use of retrieval strategies, which they consequently reviewed as the 'best' method of calculating the answers. The children may therefore have been reticent to admit to using a counting strategy.

More recently research into single digit multiplication has investigated the importance of conceptual understanding. Siegler's *strategy choice model* asserts that practice of a multiplication combination only influences future answers to that combination (Siegler, 1988). It does not recognise that an understanding of conceptual links between combinations could mean that practice on a particular sum could influence performance on the commuted version of that sum. Baroody (1999) investigated the importance of commutativity in multiplication fact development. He found that children in Grade 3 with negligible knowledge of large single digit multiplication problems not only showed a

significant increase in their mastery of practised multiplication problems but also in their mastery of the unpractised commuted versions of these problems.

3.5 Comprehending Arabic numbers

As discussed in the preceding section on counting, understanding the value of numbers requires the knowledge that number words which are earlier in the number sequence are smaller than number words that are later. However, one does not have to memorise all the number words one comprehends. The base system allows us to understand and generate numbers with which we are unfamiliar. Base systems count units of different sizes together. Dickson, Brown & Gibson (1984), Fuson (1990) and Nunes & Bryant (1996) all provide descriptions of the nature of the base system, which provides the structure for our Arabic notation system. The Arabic notation system uses units of ten. The digit to the extreme right represents the number of ones, the next digit to the left is the number of tens and this is followed by digit representing the number of hundreds and so on. The size of the units increases by the power of ten as you move left. Zeros indicate that there is none of a particular size of unit. They must be included to ensure the other digits are in the correct place for their unit size. It is only by understanding that the digits in different positions represent the number of different sized units that one can comprehend the magnitude of the number. This is known as understanding place value.

Translation between Arabic numbers and spoken or written words requires more than just saying the digits aloud. A rule system must be followed. In general the number of units of different sizes is articulated e.g. 4300 is said 'four thousand three hundred'. Two numbers are not broken down into their units at all when they are spoken (11 is said 'eleven' and 12 is said 'twelve'). All units of ten have special words one does not say 'four ten six', but 'forty six'. The largest units are always said first, with one exception. 'Teen' represents the larger unit (ten), but it is said second e.g. 'eighteen'. Deciding the unit words for large numbers is complex. Each digit place does not have a special word; after the thousand position multiples of a hundred and ten are introduced e.g. 'ten

thousand' or 'seven hundred million'. To assign the correct unit word (or multiple of that word) one must break the number down from right to left into groups of three digits. The number making up each group must be said followed by the appropriate unit (with the exception of the final group) e.g. 56,311,017 is said 'fifty six million three hundred and eleven thousand and seventeen'.

Children's understanding of place value and hence their understanding of the relative magnitude of large numbers develops slowly. An unpublished study cited by Nunes & Bryant (1996), demonstrated that the production and reading of Arabic numbers was difficult for 5- and 6-year-old children. Less than 50% of the sample could read or write numbers over 100. The children's attempts were not unsystematic; they resulted from a limited understanding of the place value system. The children tended to translate the number words directly into separate numerals, e.g. 'four thousand five hundred and two' would be written as 40005002. This is known as 'concatenation' (Nunes & Bryant, 1996) (p. 71). Poor understanding of place value continues in middle childhood for many children. (Brown, 1981) reports that many secondary school children had problems reading and writing numbers over 1000. Over 40% of secondary aged children could not write the number 400,073. Over 30% of 12 year olds could not write the number that comes after 6399. More recent figures indicate that many primary school aged children still have a very limited understanding of place value. Minnis, Felgate & Schagen (1999) report the results of children who had been taking part in the National Numeracy Project for three years. Only 39% of Year 6 children could correctly write the number 10 less than 7004. A third of Year 4 children could not correctly ring the number with 7 tens given the following choice: 7, 69, 78, 107, 707.

3.6 Multi-digit addition and subtraction

Multi-digit arithmetic requires knowledge of single digit arithmetic. It is possible to answer some multi-digit problems correctly (even those presented in a written format) using the same strategies described for single digit arithmetic, e.g. $22 - 16 = 6$ could be

solved by counting down 16 from 22 or the answer to $100 + 100$ could be retrieved from memory. However, pencil and paper algorithms are taught because for many problems they enable the correct answer to be achieved quickly and accurately. All algorithms for multi-digit addition and subtraction are based on six principles described by Fuson (1992):

- Calculation starts with the digits in the right-most position.
- Only like units are added and subtracted.
- Unit marks can only have a value of 9 or less.
- A number borrowed from left to right during subtraction (because the lower unit mark is larger than the upper unit mark) must be recognised as 10 times larger than the unit marks in the column it moves into.
- A number carried in addition because the unit mark is larger than 9, is added to the column on the immediate left.
- Subtraction is not commutative and must be calculated in the direction specified.

Many of these principles are apparent to a person with an understanding of the base 10 system. In England and Wales the National Numeracy Strategy introduces pencil and paper multi-digit addition in Year 3. However, as discussed in the previous section many children do not develop a full understanding of place value until secondary school, long after multi-digit arithmetic has been introduced. Without an understanding of the base ten system that underpins these procedures, children must learn and accept them in a rote fashion.

It is therefore unsurprising that many primary aged children have problems with multi-digit arithmetic. Brown et al. (1989) reported that one third of U.S. third grade children answered a two-digit subtraction problem (with one carry) incorrectly and half answered a three-digit subtraction problem (with one carry) incorrectly. Minnis et al. (1999) reported the results of children who had taken part in the National Numeracy Project for three years. These confirm that British children continue to have problems in

this area. 59% of Year 3 children answered a two-digit addition problem (with one carry) correctly, 23% could answer a three-digit addition problem (two carry) and only 5% of these children could answer 3000–1997 correctly. 75% of Year 4 children answered a two-digit addition problem (one carry) correctly, 49% could answer a three-digit addition problem (two carry) and only 15% of these children could answer 3000–1997 correctly. 73% of Year 6 children answered a three-digit addition problem (two carry) correctly and 55% of these children could answer 475–396 correctly. Over a quarter of the children in this sample about to enter secondary children could not competently add three digit numbers and almost a half could not subtract three digit numbers.

Many of the errors children make when attempting multi-digit arithmetic suggest that they do not fully understand the multi-unit meanings of the digits involved. Ginsberg (1977) found that many third graders aligned digits to the left instead of the right when attempting subtraction. Labinowicz (1985) found third graders often identified a traded ten as a one. Children who understand addition and subtraction algorithms by rote can develop systematic errors, known as ‘bugs’ (Brown & Burton, 1978; Brown & VanLehn, 1982). Examples of common ‘bugs’ include: *smaller from larger* (the smaller number in a column is subtracted from the larger regardless of the correct direction of the algorithm), *zero instead of borrow* (a zero is written at the bottom of every column which the bottom digit is larger than the top digit) and *borrow from zero* (the student does not borrow from the digit to the left of the zero, the zero is simply crossed out and replaced with a nine).

‘Bugs’ are thought to develop when children are faced with an arithmetic problem they cannot solve because their knowledge of the standard algorithm is incomplete (Young & O’Shea, 1981). Brown & VanLehn (1982) suggest that children adjust or ‘repair’ their incomplete version of the standard algorithm so that an answer can be obtained. The theories of Young & O’Shea (1981) and Brown & VanLehn (1982) suggest that children implement procedures that violate the logic of the place value system because they do not relate the number marks to the multi-unit values they represent. For example, when

attempting $302 - 199$ a child who understands that the 0 in 302 represents ‘no tens’, cannot borrow a ten from the 0. Children who do not fully understand place value may develop the *borrow from zero* and attempt to borrow a ten from the zero.

Fuson (1990, 1992) agrees with Brown & VanLehn (1982) and Young & O’Shea (1981) that children’s ‘bugs’ result from an incomplete understanding of place value. She asserts that children view multi-digit arithmetic as a series of single digit sums placed next to each other. Multi-digit number marks are simply as concatenated single digit (CSD) numbers. Any digit is treated the same regardless of whether it is the units, tens or hundreds position. ‘Bugs’ that violate the logic of the place value system therefore develop.

Knowledge of the base system alone is not enough: children must be able to relate it to written number marks and to addition and subtraction algorithms. Resnick (1982) reports the cases of children in Grades 2 and 3 who understood concrete representations of the base system (e.g. they could identify the correct concrete representations of multi-digit numbers and could count a large number of objects by tens) but had a weak understanding of the multi-unit values of multi-digit written numbers. These children had ‘bugs’ that were characteristic of a CSD conceptual framework, they did not relate their knowledge of the base system to the number marks. Similarly, Davis & McKnight (1980) found that children in Grades 3 and 4 could pass tasks designed to assess their understanding of the base system (e.g. representing multi-digit numbers with Dienes blocks), but they could not answer multi-digit subtraction problems that required borrowing across zero.

Although conceptual knowledge of the base system does not appear to be sufficient for accurate multi-digit procedures to develop, the two constructs are positively related. Cauley (1988) used a puppet to assess children’s procedural and conceptual knowledge of multi-digit subtraction procedures. Children’s procedural knowledge was assessed by asking them if the puppet had used the correct procedure and by asking them to demonstrate to the puppet how the question should be answered. Conceptual

understanding was assessed by asking questions about the procedures, e.g. 'How much did you borrow?', 'What did you do with it?'. Children who had better conceptual knowledge were more likely to have better procedural knowledge.

Similar results were reported by Hiebert & Wearne (1996), who conducted a longitudinal study that followed children from the 1st to the 4th Grade. This study examined the relationships between accurate use of taught multi-digit procedures, ability to invent legitimate procedures before multi-digit procedures were taught and conceptual understanding of the base ten system. Children's procedural knowledge was assessed several times a year using aurally presented multi-digit addition and subtraction problems. The children were given writing materials so that written procedures could be utilised. Tasks to assess conceptual understanding included representing multi-digit numbers using concrete materials and specifying the number of tens in a multi-digit number. In all grades children with higher levels of conceptual understanding were more likely to use and explain taught procedures accurately and invent legitimate procedures before they were taught. Furthermore, high levels of conceptual understanding in the 1st Grade predicted later procedural accuracy. Children classified as having high conceptual understanding in the 1st Grade steadily increased in procedural skill, whilst those classified as having poor conceptual understanding had a flatter learning curve. 86% of the children classified as having high procedural skills in the 4th Grade were classified as having high conceptual understanding in the 1st Grade.

Rittle-Johnson & Siegler (1998) note that whilst longitudinal studies such as Hiebert & Wearne (1996) show that conceptual understanding predicts later procedural accuracy, they do not rule out the possibility that an unmeasured third variable may account for the relationship. For example, highly motivated or intelligent children may develop good procedural skills and conceptual understanding without the factors being directly related. Trying to untangle the relationship between a number of factors that influence development is complex. For example, if one wanted to know how much

conceptual understanding contributed to procedural skills development over and above general intelligence it would be entered into a regression equation after IQ scores. If a significant proportion of the total variance in procedural understanding was not explained by conceptual understanding this could be because intelligence influences procedural skills and conceptual understanding independently or because intelligence influences the development of conceptual understanding, which, in turn, influences the development of procedural skills. If the first hypothesis is correct training in base ten concepts should have no effect on multi-digit calculation skills. However, if the second hypothesis is correct, training in base ten concepts may boost the multi-digit skills of children who do not understand this concept, even children with low IQs.

Training studies indicate that intervention that focuses on the link between base ten concepts and multi-digit procedures can increase algorithm accuracy. Resnick (1982) conducted a pilot training experiment that emphasised “a mapping at the operational level between block subtraction and written subtraction” (p. 149). After the training the children not only discarded their ‘bugs’ and developed correct algorithms, but they could also explain why the procedures they were using worked. More recent training experiments have successfully used the same technique of connecting concrete representations to written arithmetic algorithms to teach larger groups of children. Fuson (1986) reported an experiment where above average 1st Graders and average ability 2nd Graders who were taught multi-digit addition and subtraction procedures by using physical embodiments of the first four places of the base ten system. These children made substantial improvements in their arithmetic performance on tests when the blocks were not available. Many children could apply the procedures they learnt to problems involving larger numbers than those which they manipulated physically. Often the children who made errors on these tests could self correct if asked to ‘think about the blocks’.

Hiebert & Wearne (1996) compared conventional classroom instruction (which emphasised learning the correct algorithms) with alternative instruction (which emphasised the base ten system, representing and solving problems using concrete representations and inventing viable procedures before traditional written methods were introduced). The children who received alternative instruction solved subtraction problems more accurately, demonstrated the subtraction procedure more effectively, were more likely to be classified as good conceptual understanders before they used the correct addition and subtraction procedures and scored higher on conceptual knowledge (but only after the end of Grade 3). Rittle-Johnson & Siegler (1998) highlight several problems with this study. Group assignment was not random. Normal classroom teachers did not teach the children who received alternative instruction. The instructors may have been more enthusiastic or knowledgeable than their normal classroom teachers. Furthermore, the conceptual assessment tasks were very similar to alternative teaching methods. Therefore actual conceptual understanding may not have increased; the alternative instruction group may simply have had greater familiarity with the task.

A study conducted by Fuson & Briars (1990) has reported conceptual and procedural gains using a programme delivered by normal classroom teachers to children in Grades 1 and 2. However, this study lacked any control groups. 1st Grade children received alternative instruction in addition and 2nd Grade children also received alternative instruction in subtraction. The alternative instruction programme required the children to complete each step of the algorithm using blocks then immediately write that step down, before completing the next step using the blocks. The results of second graders on a test of conceptual understanding rose from just above 0% before the alternative teaching to over 90% after. 100% of 2nd Graders correctly aligned horizontally presented uneven problems (e.g. $296+41$) after receiving the alternative instruction. Furthermore, after receiving the alternative instruction the children were more likely to identify the correct value of a traded digit (e.g. they would say they were carrying a 'ten' rather than a 'one'). The children's

procedural skills also improved. Rittle-Johnson & Siegler (1998) highlight that these results compare very favourably with the results of studies examining the procedural and conceptual performance of US children in higher grades whom have received traditional instruction.

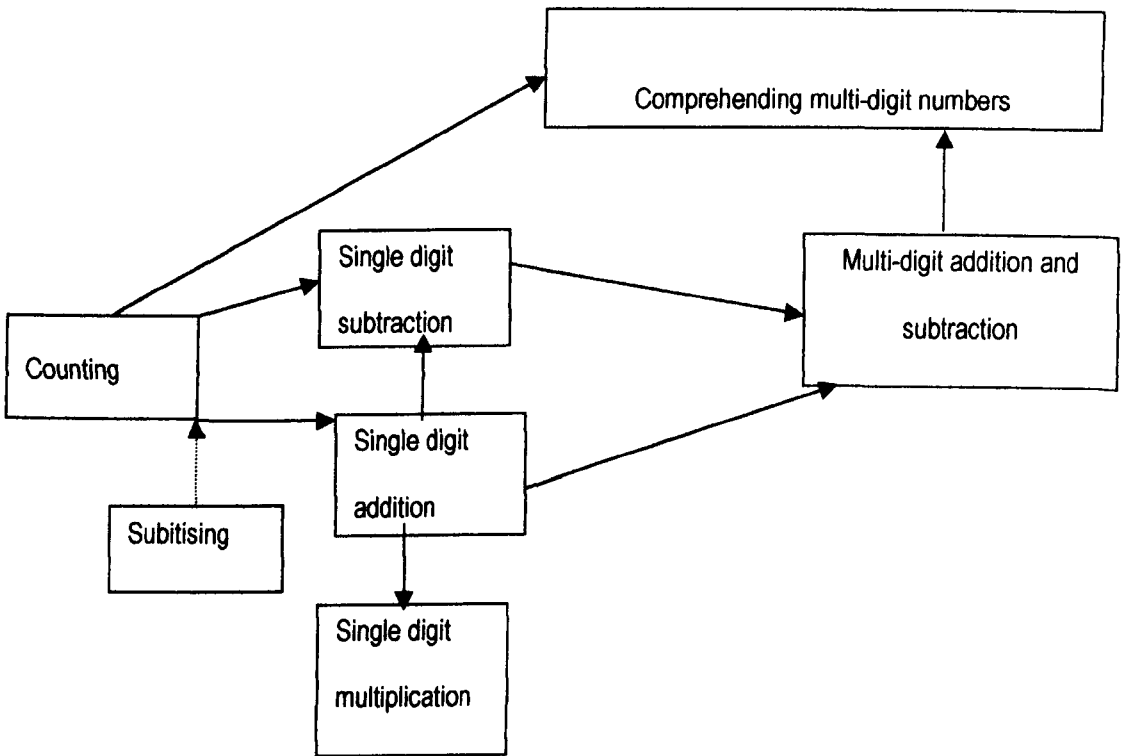
Fuson (1986) and Swart (1985) also reported arithmetic performance gains following block based conceptual instruction. Fuson (1990) argues that it is not the use of the blocks per se that enables children to construct accurate multi-digit algorithms, it is the “tight link” (p.389) between the manipulation of the blocks and the marks on the paper that is important. Fuson argues that children should be encouraged to verbalise as they write it down. Using blocks when teaching addition and subtraction procedures appears to emphasise the multi-unit values of the different digits and therefore avoid errors typical of children with a CSD conceptual structure.

A study by Resnick & Omanson (1987) suggests that block-based instruction is most effective if it is introduced before bad habits develop. Children in grades 4 to 6 who were identified as having ‘buggy’ subtraction procedures received two 40 minute individual tuition sessions. ‘Buggy’ subtraction procedures are incorrect procedures containing ‘bugs’, they are utilised by children who have incomplete knowledge of the standard procedure. In the first session they learnt to subtract using Dienes blocks in the second they noted down each procedural step after they had completed it using the blocks. Although the children showed superior conceptual understanding after the tuition they did not show significant procedural gains (eight of the nine children still used ‘buggy’ subtraction procedures). Rittle-Johnson & Siegler (1998) suggest that this intervention may have been less effective than those reported by Fuson (1986) and Hiebert & Wearne (1996) because it was very brief rather than because of the age of the children. Labinowicz (1985) found children discarded the *Smaller from larger* bug after block based instruction. This suggests that conceptual block based instruction can be effective after ‘bugs’ have developed.

3.7 Conclusion

The studies reviewed in this chapter show that children's arithmetic skills develop slowly over time in several interactive domains. Figure 2 illustrates the relationships between different arithmetical skills. The relationship between subitising and counting is indicated by a dotted line as there is yet no direct evidence that subitising influences the development of counting ability. Single digit addition and subtraction development is based on counting skills. Skill in executing single digit addition can aid subtraction development if commutativity is understood. Single digit addition can also aid the development of multiplication as early reconstructive strategies emphasise repeated adding. Multi-digit arithmetic draws on knowledge both of single digit operations and place value. It is clear that a weakness in basic arithmetical skill such as counting will make learning more advanced arithmetical skills harder.

Figure 2. *The relationships between arithmetical skills*



The arrows link the earlier number skills that contribute to the development of later number skills e.g. young children use counting skills to calculate single digit addition and subtraction problems. The arrow between subitising and counting is shown as a dotted line, because the relationship between these two skills is not yet clear.

When analysing models of arithmetic development environmental and cultural factors must be taken into account. Both the transparency of the number system and the emphasis on learning derived fact strategies in SE Asian countries such as Taiwan enable the children to abandon counting strategies for addition and subtraction earlier than US children (Fuson, 1992). The finger counting methods used in Sweden (Neuman, 1987) and Korea (Fuson, 1992) appear to differ from those used by most English speaking children.

Furthermore, the multi-digit training studies using Dienes blocks not only accelerated arithmetic learning, but also changed the order in which children acquired conceptual understanding and procedural knowledge indicating that educational methods can alter the pattern of arithmetical development (Hiebert & Wearne, 1996). Therefore the findings outlined in this chapter must not be viewed as the inevitable pattern of development, but rather the one which exists in the culture and educational system studied. The results of American studies are believed to be broadly applicable to British children as the same language and number system is used. However, caution must be used when evaluating the results as there are some differences in the educational systems.

4 Academic, cognitive and psychosocial aspects of arithmetic difficulties

The term ‘arithmetic difficulties’ is used to describe children who do not achieve age-appropriate arithmetic attainment regardless of their intelligence as well as children whose arithmetic attainment is significantly below the level that would be expected on the basis of their intelligence. Different researchers use the same labels to identify different groups; therefore the specific criteria used in specific studies is outlined in the text.

American psychologists tend to describe children who have difficulty with one or more academic subject despite an adequate IQ as ‘learning disabled’. In the UK the term ‘learning disability’ is used to describe a global developmental delay. Therefore, for clarity such children will be described in this chapter as having a specific learning difficulty. The study of children with arithmetic difficulties has focused on four key areas:

- What aspects of arithmetic do children with arithmetic difficulties find difficult? Are all aspects of arithmetic impaired or might some areas be relative strengths?
- Are the strategies of children with arithmetic difficulties atypical? For example, do they fail to learn the *count-on* strategy and persist with an immature *count-all* strategy?
- Are their neuropsychological/cognitive profiles atypical? For example, do they show significant deficits in memory or spatial skills?

- What are the social and emotional repercussions of arithmetic difficulties?

Research in this area can not only shed new light both on the development of skills in the normal population but also has important educational implications. By identifying the atypical cognitive features of children with arithmetic difficulties we can infer the features that are important in normal cognitive development. Teachers want to better understand the cognitive bases of arithmetic difficulties to permit more accurate and earlier identification and to tailor interventions to children's cognitive profiles. Effective intervention may be informed by knowledge of the aetiology of a learning problem. This is not to say that all children with a particular learning difficulty are identical, only that their shared cognitive features often make similar interventions suitable (the issue of cognitively tailored educational programmes is discussed in section 10.5). However, important methodological issues must be acknowledged when considering the conclusions of previous research and planning new studies.

4.1 The traditional methodology and its problems

The traditional methodology (described by Ginsberg, 1997) identifies children who are doing poorly at arithmetic (generally through the use of a norm referenced test), excludes children with identified neurological deficits, behavioural and emotional problems and below average IQs and compares them to a group of peers with no arithmetic difficulties. The problems inherent in this approach will be discussed and major research findings in each of the three areas outlined above will be reviewed in the light of these criticisms.

Some children who have not achieved age-appropriate arithmetic skills are not cognitively atypical. Ginsberg (1997) describes a variety of non-cognitive reasons why children may not achieve their full arithmetical potential, including poor motivation, low expectations and stereotypes, classroom disruption, poor teaching and inadequate textbooks. Such children can have difficulties with arithmetic without any significant cognitive weaknesses. It is assumed that social and emotional problems require different

types of intervention to cognitive problems. Psychologists have attempted to include only children with arithmetic difficulties who are cognitively atypical in their experimental groups. This is an attempt to prevent characteristic cognitive weaknesses being harder to identify because there is a large number of children without such weaknesses but who have other social or emotional problems within the experimental group.

Different researchers have employed different methods to try to exclude children whose difficulties are not due to a cognitive impairment. Identifying only children who have lower achievement in arithmetic than other subjects (e.g. Hitch & McAuley, 1991; Ward, 1992) would exclude children who have generally poor motivation, teaching or environment. However, one cannot rule out the possibility that arithmetic-specific non-cognitive factors play a role in the production of this profile, e.g. parents who think 'girls are no good at maths', poor relationship with the maths teacher or mathematics anxiety (see section 4.5.1). This method also excludes children who have good reasoning ability but find aspects of many subjects difficult, e.g. poor single word reading and mechanical arithmetic, but good reading comprehension and mathematical reasoning.

Numerous studies have indicated that children who are poor at both reading and arithmetic have cognitive profiles that differ from their normally achieving peers and from children who are specifically poor at arithmetic (e.g. Fletcher, 1985; Rourke & Finlayson, 1978; Rourke & Strang, 1978; Share, Mofitt & Silva, 1988; Siegel & Ryan, 1989). Studies that include children who are *only* poor at mechanical arithmetic must acknowledge that they are a selected sample of children whose cognitive profile affects their arithmetic performance. Similarly, when discussing the results of studies that do not separate children according to their literacy achievements (e.g. Garnett & Fleischner, 1983; Geary, 1990; Goldman, Mertz & Pellegrino, 1989; Shalev, Weirtman & Amir, 1988) one must recognise that the experimental groups have heterogeneous cognitive profiles.

The findings of different studies may be inconsistent because different criteria have been used by researchers, both for selecting participants and for assigning them to different groups. Early research described clinical referrals and did not discuss the problems of co-morbidity (e.g. Slade & Russell, 1971). More recent research generally lists behaviour problems, cultural deprivation, overt neurological damage, sensory deficits and English as a second language as exclusionary criteria. However, the methods of identifying these problems are often not fully expanded upon. It is frequently assumed that if none of these problems have come to the attention of teachers or parents they do not exist. Shalev & Gross-Tsur (1993) found evidence of undiagnosed medical conditions that had a direct impact on arithmetic abilities in children attending remedial classes in mainstream schools. After treatment some of the children improved their arithmetic skills. Direct investigation of exclusionary criteria (e.g. administering hearing and sight tests, investigating the socio-economic background of the children) is rare. Examples of thorough investigation of background factors can be found in the work of Shalev & Gross-Tsur (1993) and Share et al. (1988). Furthermore, there is an argument that emotional and behavioural difficulties should not always be grounds for exclusion. Rourke (1993), Rourke & Del Dotto (1994) and Semrud-Clikeman & Hynd (1990) present a powerful argument that some of the behavioural difficulties of some children with learning disabilities are not secondary to their experience of failure but a direct result of their cognitive deficits (see section 4.5 for further discussion of this issue).

The type of standardised test used to determine group assignment also differs. Although most studies used the WRAT tests (Jastak & Wilkinson, 1984) to measure reading, spelling and arithmetic performance, some researchers use instruments that sample a far wider range of skills. Share et al. (1988) did not use measures of mechanical arithmetic and single word reading, but measures of reading comprehension and mathematics. Jordan & Oettinger Montani (1997) used composite reading (which included word decoding, reading vocabulary and reading comprehension) and mathematics (which

included mathematical applications, mathematics and number concepts) scores from the Stanford Achievement Test (Gardner, Rudman, Karlsen & Merwin, 1982) to identify their subtypes. Rourke & Del Dotto (1994) identify poor reading comprehension as a weakness of children with good single word reading but poor arithmetic. It is therefore likely that children with different cognitive characteristics are given similar labels by these different researchers. Some researchers ignore spelling when dividing the children into groups (e.g. Hitch & McAuley, 1991; Jordan & Oettinger Montani, 1997), whereas others have created additional subtypes using spelling. For example, Fletcher (1985) separated children who were arithmetic and spelling poor, specifically arithmetic poor, reading and spelling poor and reading, spelling and arithmetic poor.

Sub-typing children with arithmetic difficulties is further complicated by developmental factors. The characteristic neuropsychological profiles of 9-14 year olds with specifically poor arithmetic (see Rourke & Finlayson, 1978; Rourke & Strang, 1978) has not been consistently found in younger children with a similar pattern of academic strengths or weaknesses (see Ozols & Rourke, 1988; Ozols & Rourke, 1991). Older children with poorer reading and spelling than arithmetic showed poorer verbal, auditory-perceptual skills and superior visual-spatial skills than the children with better reading and spelling. Older children with poorer arithmetic than reading and spelling showed poor visual spatial and tactile-perceptual skills and superior verbal and auditory perceptual skills. However, the psychomotor and tactile perceptual skill deficits found in older children with better reading and spelling than arithmetic were not found in the younger children with the same academic profile. Furthermore, finding younger specific learning disabled children with better reading but poorer reading is reported to be difficult by many researchers (e.g. Ackerman & Dykman, 1996; Hitch & McAuley, 1991; Siegel & Ryan, 1989).

An alternative approach to academic sub-typing is to select only children who do not respond to intensive teaching; the rationale being that only those children with cognitive deficits will fail to improve. Geary, Brown & Samaranayake (1991) compared the performance of children who did not make significant gains in arithmetic after a year of remedial teaching with their normally achieving peers. It is still possible that non-cognitive problems are resistant to remedial teaching. Similarly, it is not an *a priori* assumption that children with cognitive deficits cannot improve with intensive teaching. A variation on this method is to select children with persistent deficits regardless of the help they receive. Badian (1999) utilised this approach, selecting children who achieved a score below the 20th percentile on a test of academic skills for at least 7 years.

Even with clear subtyping and rigorous exclusionary criteria, group comparison studies cannot identify causal factors. Gathercole & Baddeley (1993) provide a review of the relative merits of different research approaches to identifying the cognitive antecedents of specific reading difficulties. Their criticisms are applicable to the study of arithmetic difficulties. The vast majority of current research into arithmetic difficulties conforms to what Gathercole and Baddeley (1993) term 'chronological age match' studies, i.e. children of the same age but differing academic abilities are compared on a variety of cognitive tests. Any cognitive differences between the groups may be the result not the cause of academic difficulties.

The three alternative approaches that Gathercole & Baddeley (1993) describe are 'reading age match' studies, longitudinal correlations and training studies. In the case of arithmetic difficulties an 'arithmetic age match' study would be appropriate. Younger children (who were achieving age appropriate levels in academic work) would be compared with older children with arithmetic difficulties who were matched on arithmetic ability. Using this method one can determine whether the children with arithmetic difficulties are cognitively atypical or whether their arithmetical strategies and cognitive profile closely resembles that of younger children of a similar attainment level. If the

cognitive profiles of the children with arithmetic difficulties differ from those of the younger comparison group it can be concluded that the differences are not the result of the older children's level of arithmetic attainment. However, Gathercole & Baddeley (1993) stress that one should be cautious when evaluating studies where the groups do not differ. The older children will differ from the younger children in many ways (e.g. higher reasoning ability, more experience of testing). They may be able to do better than the younger children on tests of cognitive abilities by using advanced compensatory strategies. For example, older children with learning difficulties may have a smaller short term memory storage capacity than their younger arithmetic matched controls, however they may nevertheless achieve higher scores on digit span tests because they are familiar with 'chunking' strategies. It is therefore possible that a cognitive ability that is lower than chronological age matched controls but similar to arithmetic age matched controls could cause arithmetic difficulties.

Longitudinal correlations can also be used to determine the direction of the relationship between cognitive deficits and arithmetic difficulties. If children's cognitive abilities are measured before arithmetic instruction commences, the results of these tests may be correlated with later arithmetic achievement. If pre-school cognitive deficits predict poor attainment one can exclude the possibility that later attainment caused the cognitive deficits. Longitudinal correlations and arithmetic age match studies alone can not be used to determine causal links. An unmeasured third variable could cause both the cognitive deficits and the attainment deficits. The relationship between attainment and cognitive ability would exist but there would be no causal link. Causal connections can be verified using training studies. Children deficient in the cognitive ability hypothesised to cause the achievement deficits are trained in the ability. A control group with similar deficits undergoes a placebo-training programme. If the training results in ability and attainment benefits whilst the placebo programme does not, a causal link is established.

Most of the studies reviewed in this chapter, regardless of their design, suffer from two further weaknesses. Firstly, children's skills are often only examined in one or two areas. For example, only children's memory abilities or only children's spatial abilities are examined. In such studies the pattern of children's abilities cannot be examined. For example, it cannot be determined whether children with arithmetic difficulties have an isolated visual-spatial memory deficit, or whether it is in the context of a broader spatial ability weakness. Secondly, only sample level statistics are reported. In studies that do not examine individual scores, only the deficit of the majority of the children will be revealed.

4.2 The areas of arithmetic affected

The most basic level of research establishes what children with arithmetic difficulties can and cannot do relative to their normally achieving peers. Shalev et al. (1988) examined the arithmetic attainment of 11 children (aged 9-15 years) referred to a Neuropediatric Diagnostic Unit because of "selective deficit in learning arithmetic" (p. 555) using a comprehensive arithmetic battery. The usual exclusionary criteria were used. The children were described as being of 'normal' intelligence (however, two had full scale IQs of below 85). Although reading scores were not reported it is stated that all the children were one to two years behind their class level in reading. The children were compared to controls that were judged by their teachers to be normal arithmetic achievers. The test battery used is based on the McCloskey & Caramazza (1985) model of numerical processing, which was discussed in section 2.3. Different sections of the test assess the functioning of different parts of the model. The arithmetic battery consisted of three sections: number comprehension, number production and calculation. The control and the experimental group differed significantly only on the calculation section. However, the mean scores masked the significant difficulties in other areas shown by a minority of the children. One child had problems both with comprehending verbal concepts of 'more' and 'less', had difficulties judging relative quantities and made syntactical errors writing numbers. Another child also had syntactical problems writing numbers. One child had

impaired counting because she could not synchronise moving her finger and saying the count words.

Gross-Tsur, Manor & Shalev (1996) administered the same arithmetic battery, which had been standardised by Shalev, Manor, Amir & Gross-Tsur (1993) to a larger, more representative, sample of children with arithmetic difficulties. Over 3000 children (aged 11-12 years) underwent a standardised arithmetic test; the lowest 20% were tested on the individually administered arithmetic battery. The children whose mean score was equal or below the median score for children aged two years younger and had an IQ of 80 or above were used as the experimental group. 140 children were available and had consent to participate. Again it was reported that the mean differences between groups were more marked for the calculation section of the test than for the numerical comprehension and production sections. However, it is possible that individual children may have had difficulties in these areas as no case studies are described.

Sokol, Macaruso & Gollan (1994) administered the Johns Hopkins University Dyscalculia Battery to 20 students who had been diagnosed with developmental dyslexia. Teachers selected students who ranged in age from 12 to 20 years because they had poor basic math skills. Sokol et al. (1994) do not report the results of standardised reading, arithmetic or intelligence tests for these children, but describes them as “within the normal range on standard intelligence tests” (p.423). One would assume that as they were diagnosed as dyslexic they have some form of reading difficulty. The test battery used is similar to those used by Shalev et al. (1988) because it is based on the McCloskey & Caramazza (1985) model of numerical processing. There were no norms, so it is not possible to compare the performance of these children and adolescents with their normally achieving peers.

Sokol et al. (1994) examined the pattern of the children’s performance. Three of the dyslexic students performed poorly on all areas of the test; in contrast, six students performed well, producing less than 10% errors in all sections. Sokol et al. (1994) describe

the remaining 11 students as having selective disturbances in number processing: they produced more than 10% errors on three to five sections of the test. Descriptions of students with poor calculation skills, but good numeral processing and vice versa are given. Some students had even more specific deficits that only affected a specific area of calculation or numeric functioning, e.g. poor fact retrieval but good calculation procedures or specific difficulties understanding Arabic but not verbal numbers.

Dowker (1999) also argued that arithmetical ability was composed of a number of distinct arithmetical abilities. Dowker focused on the distinction between conceptual and procedural mathematical abilities. She reported the case studies of ten children aged between 6 and 9 years. A battery of tests were administered including a mental calculation task (designed to assess procedural skills) and a use of principles task (designed to assess conceptual skills). The use of principles task required the child to determine the answer to a sum they could not calculate mentally by reference to a related correctly answered sum. Two of the children she studied had a general arithmetical deficit (i.e. they had difficulty with both conceptual and procedural tasks). Four of the children had better conceptual skills than procedural skills, whilst one showed the reverse pattern. One boy had a particularly idiosyncratic pattern: his calculation skills were better than his conceptual skills for addition, but his procedural skills were poorer than his conceptual skills for subtraction. The remaining two children were good at both procedural and conceptual skills.

Russell & Ginsberg (1984) administered a different battery of tests assessing arithmetic to three groups of children who were of normal intelligence (they achieved a stanine score of 4 or above on the Cognitive Abilities Test; Thorndike, Hagen & France, 1986). The 4th Grade children identified as having 'maths difficulties' achieved a math achievement score of 3 or below, which was also at least 2 stanines below their intelligence score. They were matched with normally achieving 4th Grade children on intelligence. The normally achieving children had a maths score that was no more than one stanine

above or below their intelligence score. Randomly selected 3rd Graders were used as an arithmetic age match group. The children with arithmetic difficulties achieved significantly poorer results than the normally achieving chronological age match group on seven of the tests: mental addition, counting large numbers, multiples of large numbers, accuracy and bugs in written addition and subtraction, monitoring errors, addition facts and story problems. They achieved significantly lower scores than the ability match group on the addition facts and story problems.

The results of Gross-Tsur, Auerbach, Manor & Shalev (1997), Russell & Ginsberg (1984) and Shalev et al. (1988) suggest that children with arithmetic difficulties have greater problems with calculation and number fact recall than with comprehending and producing numbers. However, Shalev et al. (1988) and Macaruso & Sokol (1998) described children who did have problems comprehending and producing numbers. Since Russell & Ginsberg (1984) and Shalev et al. (1988) only report mean scores it is therefore possible that some of the children in these studies presented a pattern of strengths and weaknesses that differed from the majority. It is also important to note that the suitability of the tasks used was questionable. Sokol et al. (1994) used an unstandardised test of arithmetic to examine the number skills profiles of students ranging in age from 13 to 20 years. Without a normative sample with which to compare the children to it is difficult to assess the severity of these children's difficulties. Although Gross-Tsur et al. (1996) used an arithmetic battery that had been trialed on normal children (see Shalev et al., 1993). It had clear ceiling effects in the number processing section. 200 Israeli children in Grades 3 (mean age 9:6), 4 (mean age 10.6), 5 (mean age 11:7) and 6 (mean age 12:6) completed the battery. The median score for the number processing section was 38 or 39 out of 40 for each grade. At least 15 of the 40 questions involved quantities of 14 or less. Children were not asked to write numbers larger with more than four digits. As discussed in Chapter 3, a full understanding of place value develops slowly throughout middle childhood. The items in the number processing section of the test could have been made harder and hence make

the test more discriminating. The batteries used do not test the full range of number skills. In particular, skill efficiency (a trade off between speed and accuracy) was not assessed. Counting speed or speeded number fact knowledge were also not tested.

No attempt was made in any of these studies to compare children with and without literacy difficulties. It may be that children who have arithmetic difficulties and literacy difficulties have a different pattern of achievement to those who have literacy difficulties alone. Exclusionary criteria were not applied in either the Gross-Tsur et al. (1997) and Shalev et al. (1988) study. The areas of arithmetic that are weak in children who have atypical and normal cognitive profiles may differ. When groups are heterogeneous reporting means can mask important individual differences.

Single case studies of children with specific numerical processing deficits have been recorded. Temple (1989) reports the case of Paul whose single word reading, non-word reading, reading comprehension and spelling were all within the average range. However, Paul had severe difficulties reading and writing number words. Paul could produce the write syntactical frame (i.e. if asked to write three thousand and two he would write a number between 1000 and 9999), but he used the wrong digits. Case studies also suggest that the ability to execute accurate pencil and paper algorithms and the ability to recall number facts can be impaired independently. Temple (1991) reported the case of SW who suffered from tuberous sclerosis and possibly myoclonic epilepsy. Despite an IQ within the average range and an unimpaired short term auditory verbal memory SW had always experienced difficulties acquiring arithmetic skills. He was motivated and had received remedial lessons. Performance was good on tests of number processing and number fact recall, however his procedural skills were severely impaired.

In the same paper Temple discussed the case of HM, who displayed the reverse pattern. HM was described as a 19-year-old young woman of good intelligence who had unimpaired number processing skills. Her knowledge of arithmetical procedures was good, but her retrieval of multiplication facts was very poor. Temple (1994) reports the

cases of 12-year-old identical twin boys who displayed profiles similar to HM. They were of above average intelligence with literacy skills that were one to two years behind their chronological age. Both had some degree of dysgraphic difficulties. Although number processing and procedural skills were intact, both boys had considerable difficulties recalling multiplication facts. Their reaction times were much longer than a control group of children of a similar age. Instead of recalling the facts they used elaborate reconstructive strategies.

Ta'ir, Brezner & Ariel (1997) described the case of a profound developmental dyscalculic, YK, who had fundamental arithmetic deficits. YK was 11 years old, he had an IQ in the normal range and achieved A grade marks in all subjects, except maths and geometry. His problems with maths were severe. YK had difficulty counting sets of identical objects that were larger than 15. He could not match the appropriate numeral to sets of dots above ten. YK was unable to write numbers with more than three digits to dictation or convert Arabic numerals with more than four digits to number words. He could not answer written or oral sums when the answer was greater than 10.

4.3 Strategy differences

The developmental sequence of strategies used by children to solve arithmetic problems was discussed in section 3.3. Some researchers have compared the strategies used by children with arithmetic difficulties with their normally achieving peers. Geary (1990) and Geary et al. (1991) studied children receiving remedial help for mathematics (LD) during first grade and second grade. Many of the children in the LD group were also receiving remedial help for reading. The mathematics and reading scores of the LD group were significantly below those of the normally achieving children. At the end of first grade the LD children were divided into two groups, those who were moved out of the remedial classes because their scores had improved (LD-improved) and those who had not (LD-no-change). At this stage the three groups all used the same strategies i.e. retrieval,

finger and verbal counting (including both *counting all*, *counting on*). Overall, the LD-no-change group was the least accurate.

After ten more months the performance of the LD-no-change group was again compared to the normally achieving children (this group now included the LD-improved). The normal children had increased their use of the memory retrieval strategy and had become more accurate when using it. The LD-no-change group's use of the retrieval strategy did not increase significantly, neither did they become more accurate when using it. However, despite their persisting retrieval difficulties, the LD-no-change group's counting strategies had developed. They had almost completely abandoned *counting all* in favour of *counting on*. The LD-no-change group's accuracy when using counting strategies was not significantly different from their normally achieving peers.

The finding that children with arithmetic difficulties continue to use less sophisticated counting strategies when older children have moved on to using fact retrieval has been replicated. Bull & Johnston (1997) reported that 7-year-old children with high mathematics ability use memory retrieval significantly more often than low mathematics ability children, even when differences in reading ability are controlled for. Jordan & Oettinger Montani (1997) compared the performance of children (mean age 8.6 years) on timed and untimed arithmetic tasks. Children in the MD-specific group (poor maths but better reading) performed worse than normally achieving controls only in timed conditions, whilst children in the MD-general group (poor maths and reading) performed worse in both conditions. The control children made significantly less use of back-up strategies (fingers, verbal counting or pencil and paper) than either of the impaired groups. When using back-up strategies, the MD-general group made more errors than the control children did, but the MD-specific children performed at a similar level of accuracy. These results suggest that the MD-specific group performed worse than the controls in the timed condition because they could not use the quicker retrieval strategy, but performed at a similar level in the untimed condition because they could use back-up strategies accurately.

However, the MD-general children performed more poorly in both conditions because they were inaccurate even when they used back-up strategies.

Some studies have not found that children with arithmetic difficulties have a particular problem with retrieval. Russell & Ginsberg (1984) did not find that the strategy mix of 4th Grade children with mathematics difficulties differed from their age-matched controls. However, both the size of the integers (some answers were above 100) and the classification of strategies differed considerably from the previous studies (*counting all* and *counting on* were not distinguished, and retrieval was not a separate category). As noted in the previous section, the impaired children performed worse than age- and ability-matched controls on a simple speeded number fact test. Geary, Hoard & Hamson (1999) found that 1st Grade children with either specific mathematics difficulties or mathematics and reading difficulties made significantly greater use of retrieval than controls. However, the impaired children made high numbers of errors when using retrieval, which suggests they were guessing. It can be concluded that children with arithmetic difficulties have a particular problem retrieving number facts quickly and accurately.

4.4 Cognitive weaknesses

4.4.1 Visual-spatial analysis

Visual-spatial deficits could cause arithmetic difficulties in children by directly interfering with children's calculation procedures or by thwarting early numerical development. Adult lesion patients who suffer from a particular form of acquired dyscalculia known as spatial dyscalculia have spatial skill deficits that directly interfere with their ability to calculate. For example, writing numbers legibly and keep numbers correctly aligned when calculating is difficult for them (Geary, 1993, and Hartje, 1987, provide reviews of the appropriate studies of acquired dyscalculia). Children with developmental spatial skill deficits may suffer from similar problems when attempting arithmetic problems. Studies indicate that visual-spatial skills are involved in normal children's arithmetic development. Young children's early addition strategies involve

finger counting (see section 3.3). Being able to identify the number from the finger pattern rather than counting each finger individually will increase the child's calculation speed. Geary & Burlingham-Dubree (1989) found that 5-year-old children's arithmetic strategy choice was significantly correlated with their performance IQ but not their verbal IQ. The children with higher performance IQs used more advanced calculation strategies. Children with spatial skill deficits may therefore fall behind with their arithmetic in their early school years.

If visual-spatial skills are associated with arithmetic development and/or are required to effectively carry out learnt arithmetical procedures it is logical to suggest that poor visual-spatial skills could be one cause of arithmetic difficulties. Studies have examined the IQ profiles of children who have arithmetic difficulties without any obvious neurological damage. Rourke & Finlayson (1978) compared the IQ profiles of children with specific learning difficulties (aged 9-14 years) with different academic profiles. All the children exhibited poor performance in at least one subject and had an IQ within the average range. The children's academic skills were tested using the WRAT reading, spelling and arithmetic tests (Jastak & Wilkinson, 1984). Three groups of 15 children were compared: group 1 had uniformly poor reading, spelling and arithmetic scores whilst the other two groups had distinctive academic profiles. Group 2 children had reading and spelling scores that were below centile 15; this group's arithmetic scores were at least 1.8 years above their reading and spelling scores. Group 3 had arithmetic scores that exceeded their spelling and reading scores by at least two years. Only six girls were included in the sample two in group 1 and four in group 3. Full scale IQ did not differ significantly across the groups. The mean arithmetic scores for groups 2 and 3 did not differ significantly, but the mean reading scores did.

All but one of the children in group 2 had a higher performance IQ than verbal IQ (one child's performance and verbal IQs were equal), in contrast all of the children in group 3 had verbal IQs that were superior to their performance IQs. This result suggests

that children who have significant arithmetic difficulties, but better reading have visual-spatial skill deficits. It should be noted that Rourke was interested in the pattern of the sub-types performance on the IQ tests not their absolute scores. The mean verbal IQ for group 2 children and the mean performance IQ for children in group 3 were within the average range. Although their verbal or performance IQs were discrepant they were not always below average for the general population.

Researchers have attempted to replicate Rourke & Finlayson (1978) findings. Ozols & Rourke (1988) examined the visual spatial and auditory-verbal skills of 7-and 8-year-old learning-disabled children. The children with poorer reading than arithmetic had auditory-verbal skills that were poorer than their visual spatial skills whilst the children whose reading was superior to their arithmetic showed the reverse cognitive pattern. Share et al. (1988) found that arithmetic disabled boys had higher verbal than performance IQs and performed better on verbal than non-verbal neuropsychological tests; the reverse pattern was found in arithmetic and reading disabled boys. von-Aster (1996) compared children with specific arithmetic difficulties (their arithmetic was significantly poorer than would be predicted on the basis of their IQ) with children with a mixed disorder of scholastic skills (their reading and arithmetic was poorer than would be predicted on the basis of their IQ). All the children with specific arithmetic difficulties who had a significant IQ discrepancy had a higher verbal IQ (with the exception of one girl who had elective mutism). In contrast, all the children with a mixed disorder of scholastic skills and a significant IQ discrepancy showed the reverse pattern.

However, not all results have been consistent. Share et al. (1988) found no differences between the verbal/performance IQ patterns of specifically arithmetic disabled girls and their controls. Slade & Russell (1971) found no differences in the performance and verbal IQ's of 4 children with dyscalculia and good reading skills. Ackerman & Dykman (1996) found both children with reading and arithmetic difficulties and children with specific arithmetic difficulties had performance IQs that were higher than verbal IQs.

Partial confirmation of Rourke's pattern of results was reported by Davis, Parr & Lan (1997). He separated the children with specific learning difficulties into groups according to their performance on the Woodcock-Johnson Psycho-Educational Battery-Revised (WJ-R) (Woodcock & Johnson, 1989). Children who performed one standard deviation higher on the Basic Maths Skills sub-tests than the Basic Reading and Basic Writing sub-tests had significantly lower verbal comprehension indexes than perceptual organisation indexes. A child's verbal comprehension index is derived from four verbal sub-tests of the WISC; their perceptual organisation index is derived from four performance sub-tests from the WISC. Although the children with specific learning difficulties who had Basic Maths scores at least one standard deviation below their Basic Reading and Writing scores had a significantly lower perceptual organisation index than the other sub-group of children with specific learning difficulties, their mean verbal comprehension index was not significantly higher than their perceptual organisation index. Rosenberger (1989) compared children who had been referred to a learning disorders clinic because of school difficulties. The children were divided into two groups: dyscalculics (who had a reading standard score greater than 100 and a maths score at least 20 points lower) and 'dyslectics' (who had a maths standard score greater than 100 and a reading standard score at least 20 points lower). The groups did not differ on full scale IQ. There were no significant differences in performance and verbal IQ between the groups. However, the dyscalculiacs performance on the Bender Visuomotor Gestalt test was significantly poorer than the 'dyslectics'.

There are two possible reasons for the inconsistencies in the results. Firstly, the criteria for selecting the children differed. The original Rourke & Finlayson (1978) study used a traditional definition of learning difficulties. Slade & Russell (1971) did not exclude children with concurrent socio-emotional problems. Ackerman & Dykman (1996) did not exclude children with below average full scale IQs, and Rosenberger (1989) did not stipulate that the children had to have a significant discrepancy between their IQ and arithmetic ability. Secondly, different measures of arithmetic and reading were used.

Although most studies use the WRAT reading and arithmetic tests (Jastak & Wilkinson, 1984) that assess only mechanical skills, Rosenberger (1989) and Share et al. (1988) used much broader measures of reading and mathematics. Davis et al. (1997) used a more stringent method when selecting children with specific learning difficulties who primarily had literacy problems than when selecting children with specific learning difficulties who primarily had maths problems. After applying the one standard deviation literacy/maths difference rule to all his participants with specific learning difficulties, he had considerably more participants in the literacy problems group. To obtain two groups of equal size he excluded the 18 children from the literacy problems group who had the smallest literacy maths discrepancy. Rourke's predictions may therefore have only been confirmed in the primary literacy problems group, because that group contained more extreme cases.

4.4.2 Psychomotor and tactile perceptual skills

Psychomotor tasks (also known as visual-motor tasks) involve controlled movement (e.g. a pegboard test, a test of scissors use or a copying test) whilst tactile perceptual tests involve the identification of objects by touch (e.g. identifying an object in a bag without looking). Rourke and his colleagues hypothesised that the children with poor arithmetic but better reading, identified in their Rourke & Finlayson (1978) study, would perform poorly on such tests. Rourke & Strang (1978) found that the children with poor arithmetic but better literacy skills did perform worse than the children with poor literacy skill but better arithmetic. Ozols & Rourke (1988) and Ozols & Rourke (1991) reported less unambiguous results when children aged between 7 and 8 years were tested. The psychomotor and tactile perceptual skill deficits found in older children with better reading and spelling than arithmetic were not found in the younger children with the same academic profile.

Badian (1999) conducted a prospective longitudinal study involving 1075 children, which examined the cognitive skills of pre-school children in relation to their later academic achievement on Stanford Achievement Test (Gardner et al., 1982). Children

were classified as having a persistent difficulty if they scored below the 20th percentile for seven consecutive years on at least one sub-test of the Stanford Achievement Test. They were divided into three groups LA (low in arithmetic only), LR (low in reading only) and LAR (low in both arithmetic and reading). The controls surpassed the three groups with persistent difficulties on all academic measures. The LAR group scored more poorly than all the other groups on all academic measures. The LA group achieved higher scores than the LR group on the word reading, reading comprehension and spelling measures. On the mathematical sub-tests (arithmetic and number concepts) the LR group achieved higher scores than the LA group.

The pre-school cognitive tests were divided into three groups: pre-academic skills (letter naming, colour naming and visual matching), language measures (WPPSI Similarities, sentence memory and telling a story about a picture), and visual-motor skill (name writing, form copying, draw-a-person and pencil and scissors use). All three groups had lower pre-school visual-motor skills than the controls (but the groups with academic difficulties did not differ from each other). Only the LAR and LR had lower pre-school verbal skills than controls, the LA group and the controls did not differ. This suggests that a selective deficit in visual-motor skills predicts a selective deficit in arithmetic. Boys in the LA group had considerably stronger pre-school language skills than pre-school visual-motor skills. However, a large discrepancy between the visual-motor and language cognitive measures was not found in LA girls. This study and the study of Share et al (1988) suggests that selective sparing of verbal skills in comparison to visual-spatial and visual-motor is typical of boys with better reading and spelling than arithmetic but not girls with the same academic profile. Badian (1999) notes that that most of the children studied by Rourke were boys. This may be due to referral bias in his clinic based samples. The less marked differences between verbal and visual-spatial/visual-motor skills found by some researchers may in part be due to the gender mix of the sample.

A particular class of tactile perceptual skills concerns the tactile perception of one's own body, e.g. identifying which fingers have been touched when one's eyes are closed. Fayol, Barrouillet & Marinthe (1998) reported that four tests of body tactile perception administered at 5:9 predicted arithmetic achievement 8 months later. The tests used were: simultagnosia, identifying which two body parts (e.g. shoulder and elbow) had been touched; digital agnosia identifying the number of the finger touched (e.g. first, second, etc.); digital discrimination (pointing to the fingers that have been touched); and graphisthesia (identifying a pattern drawn on the back of the hand with a pointer). All tests were conducted with the subjects eyes closed. These neuropsychological tests predicted arithmetic achievement over and above age and developmental level (assessed using drawing tests). As these tactile perceptual tests predicted future arithmetic achievement Rourke et al's argument that tactile perceptual deficits are associated with arithmetic weaknesses is strengthened.

4.4.3 Memory

The model of human memory by Atkinson & Shiffrin (1968) distinguishes between our long-term store of learnt material, which does not have space restrictions, and our short-term store, which has a limited capacity and decays over time if the information is not rehearsed. Information is transferred from the short-term memory to the long-term memory after it has been rehearsed. Baddeley and his colleagues rejected the concept of a unitary short-term storage system; and developed the 'working memory' model that consisted of a number of interacting sub-systems (see, for example, Baddeley, 1986; Baddeley, 1996; Baddeley & Hitch, 1974; Baddeley et al., 1984; Baddeley & Lieberman, 1980). The working memory model proposes that a central executive directs attention, plans action, retrieves items from long term memory and supervises at least two different slave systems that retain information temporarily. The phonological loop retains information that is presented aurally or re-coded in an aural form. The visual-spatial sketchpad retains information that is presented or re-coded in a visual or spatial form. This

'working memory' model is accepted because it parsimoniously explains many facets of human short-term memory that have been identified in the laboratory, Baddeley (1997) provides a review of this evidence.

Both the long-term and short-term store are utilised when arithmetic problems are solved. Children must retrieve number facts and procedures from their long-term memory. It is logical to suggest that impairments in either of the working memory slave systems will affect a child's ability to master mathematical tasks. If an arithmetic problem is presented aurally a child must retain the problem information using the phonological loop whilst they compute the answer. If a child is using the *counting on* strategy they must keep track of the number of steps they have counted on. A derived fact strategy requires that the known fact is recalled and held in the phonological loop whilst it is combined with the final stage of the calculation. For example, a child calculating 7 multiplied by 6 might recall 6 multiplied by 6 is 36 and hold this in the phonological loop whilst they add 6 to it. The phonological loop and the visual-spatial sketchpad could both contribute to the development of children's number fact stores. The associative network models of number fact stores (reviewed in section 3.4) suggest that by correctly answering a sum, the associative link between the sum and the answer is strengthened. A stronger link will mean that the answer can be retrieved more easily on future occasions. For these links to be strengthened the sum will still have to be retained (either in the phonological loop or the visual spatial sketchpad) when the answer is calculated.

Adams & Hitch (1997) found that English and German children had shorter addition spans when multi-digit mental addition sums were presented aurally rather than visibly. This suggests that storing the aurally presented material draws on memory resources that can be used for other processes when the child is provided with a visual reminder of the sum. Heathcote (1994) asserted that digits are retained in the visual-spatial sketchpad during multi-digit calculations. He found that adults made more mistakes on problems that involved visually similar digits and performed more problems when there

was visual-spatial interference. Performance on problems involving carrying was particularly badly affected, which suggests that the visual-spatial sketchpad is utilised during the carrying procedure.

The role of the central executive is much wider than the slave systems. A long list of difficulties have been associated with central executive dysfunction, including, “disrupted organisational and planning skills, generalised memory deficits, difficulties with mental flexibility ... distractibility and problems with sustained attention” (Bull, Johnston & Roy, 1999) (p.425). The frontal lobes are thought to control central executive function. As the scope of central executive functioning is so wide a weakness in this area would affect arithmetic performance, however one would also expect it to affect many other aspects of academic performance. Ashcraft (1995) argues that the central executive has a particularly important role to play in simple arithmetic because it is responsible for retrieving learnt facts and procedures.

The working memory model would suggest that short term aural or visual-spatial memory could be impaired independently. The link between phonological awareness/auditory-verbal short-term memory and reading ability has been established. Evidence from ability match studies (e.g. Olson, Wise, Rack & Faulkner, 1989 and Rack, 1985), longitudinal correlations (e.g. (Jorm, Share, MacLean & Mathews, 1984; Mann & Liberman, 1984; Singleton, Thomas & Horne, 2000) and training studies (e.g. Bradley & Bryant, 1983; Bradley & Bryant, 1985; Lundberg, Frost, & Peterson, 1988) indicates that phonological memory deficits make learning to read more difficult (see section 5.2 for further discussion of this issue). Children with dyslexia, who have phonological memory deficits, have been found to have difficulties with mental arithmetic (see section 5.4). One possibility is that that children with both arithmetic and reading difficulties have phonological memory weakness, whilst children whose difficulties are limited to arithmetic have a visual-spatial memory weakness.

Researchers have tested this hypothesis by comparing the simple memory spans of children with different types of learning difficulties. Simple memory span tasks require the child to recall information presented to them, without the distraction of a concurrent task. An example of a simple auditory verbal memory span test is a forward digit span; an equivalent visual-spatial task would be the serial recall of the position of dots. Fletcher (1985) found that reading-spelling-arithmetic disabled children performed worse than controls on both verbal and non-verbal simple memory span tasks. Children who were only arithmetic disabled or spelling and arithmetic disabled performed worse than controls only on the non-verbal memory task. Similarly, Siegel & Linder (1984) found that children with specific arithmetic difficulties performed worse than controls on a letter memory task only when presentation was visual. Brandys & Rourke (1991) reported that children with specific learning difficulties whose arithmetic was poorer than their reading performed more poorly than controls and children with specific learning difficulties whose reading was better than their arithmetic on both the copying and recall components of the Rey-Osterrieth complex figure task (Osterrieth, 1944). This task assesses visual-spatial memory.

More recently McClean & Hitch (1999) found that the performance of 9-year-old children with difficulties specific to arithmetic did not differ from normal age-matched controls on a phonological memory task. The performance of the children with specific arithmetic difficulties on a simple spatial memory task was significantly poorer than the age-matched control children. Similarly, Bull & Johnston (1997) found that two groups of 7-year-old children who differed in mathematics ability did not differ in digit span after reading ability had been controlled for. In contrast, Siegel & Ryan (1989) found that children with reading ability in the average range but below average arithmetic abilities performed worse than controls on simple auditory memory span tests. This evidence suggests that although a phonological memory deficit is associated with arithmetic difficulties in children who also have reading difficulties, it is not necessary for arithmetic

difficulties to develop. In the studies where either phonological or visual-spatial memory is found to be weaker in the arithmetic difficulties group we do not know whether the deficit is universal; the individual results of the children are not reported.

Although simple visual-spatial memory deficits have been associated with arithmetic difficulties (particularly in children who do not also have reading difficulties), it is not yet clear whether they can cause them. The evidence described above is limited to age-match comparisons. McClean & Hitch (1999) did include an ability-match comparison. The children with arithmetic difficulties did not perform more poorly than the younger arithmetic matched controls on the spatial memory task. However, negative results on ability match comparisons do not prohibit the measure tested being a causal factor. Singleton, Thomas, Horne & Simmons (in preparation) found that children's performance on visual memory tasks at school entry could predict their mathematical performance 9 months and 2 years later, even after IQ had been controlled for. This study supports the hypothesis that a visual-spatial memory deficit can make learning arithmetic more difficult.

Baddeley's model of working memory emphasises the utility of short-term memory; information must be stored whilst processing is executed. The information is therefore available when it is needed. He states that tasks that require the use of working memory require "simultaneous processing and storage of information" (Baddeley, 1986) (p. 34). It has been asserted that processing and storage both draw on the same cognitive resources (e.g. Carpenter & Just, 1988; Daneman, 1987; Daneman & Carpenter, 1980; Daneman & Tardif, 1987; Just & Carpenter, 1992). This model proposes that the storage capacity of working memory is dependent on the amount of cognitive resources required to carry out the concurrent task. The 'structural capacity' of working memory is the amount of storage space available when there is no concurrent task; the 'functional capacity' is the storage space left during concurrent processing. When the demands of the concurrent task are higher the functional capacity of working memory capacity is smaller.

Some studies use complex memory span tasks, which measure the functional capacity of working memory. Complex memory span tasks require participants to process and store information simultaneously. For example, a child might be asked to count the number of dots on four cards and remembering the total dots on each previous card as they count subsequent ones or decide which was the odd word out of set of words whilst remembering the correct word for three previous sets. Children with arithmetic difficulties may perform more poorly than controls on complex span tasks when the concurrent task is numerical because they must devote a greater proportion of their cognitive resources to processing. Fewer cognitive resources can therefore be utilised for storage. Differences in the functional working memory capacity of control children and children with arithmetic difficulties could be found using complex span tasks, even if the two groups do not differ in structural capacity.

Siegel & Ryan (1989) found reading and arithmetic disabled children performed more poorly than controls on both visual counting and sentence working memory tasks, whilst specifically arithmetic impaired children performed more poorly only on the visual counting working memory task. Hitch & McAuley (1991) found that children with better reading than arithmetic performed more poorly than normal controls only on complex working memory span tests when the concurrent task was counting. When the concurrent task was comparison the children with arithmetic difficulties performed at a similar level to controls. This difference did not depend on the presentation modality. The children with arithmetic difficulties performed more poorly on an auditory-verbal memory task when the concurrent task was counting but at a level similar to controls on a visual-spatial memory task when the concurrent task was comparison. A second experiment found that the children with arithmetic difficulties were significantly slower than the controls at counting the numbers 1-20, counting from 2-20 in twos, and counting spots. They also had lower digit spans. This would suggest that they were doubly impaired, counting was less efficient and their auditory verbal memory capacity was smaller.

Swanson (1993) challenges the hypothesis that different sub-types of children with specific learning difficulties have different working memory deficits. He studied a group of clinically referred children diagnosed with specific learning disabilities. All the specific learning disabled children had full scale IQs of 90 or above. They were split into two groups according to their primary area of academic difficulty. The LD-math children had a score on the WRAT arithmetic test that was at least 1.5 standard deviations below their score on the WRAT word reading test. The LD-reading children had to have a discrepancy of the same magnitude, but with their arithmetic score being higher. Over 70% of the children in each group were boys. The children were compared both to age and younger ability matched controls on a variety of tasks to assess working memory. Some of the tasks were auditory-verbal whilst others were visual-spatial. In all the tasks the stimulus was presented, then a question about the stimulus was asked, finally the children were then asked to recall the stimulus. On some tasks the children had to select a strategy (from a range illustrated on cards) to help them remember, before they were shown the stimulus. After the stimulus was removed and the process question asked, the children were asked to indicate which strategy they used (prospective tasks). On the other tasks the children were not given suggested strategies and were asked to recall the stimulus immediately after the process question was asked (retrospective tasks).

Swanson (1993) found that both the LD-math and the LD-reading children performed more poorly on the working memory tasks than the age-match control children, but the performance of both LD groups was superior to the ability matched controls. The performance of the two different groups did not differ significantly on either the visual-spatial or the auditory-verbal working memory tasks. Swanson (1993) concluded that the specific learning disabled children share a problem that is related to working memory. He suggests that specific learning disabled children's difficulties are unlikely to be due to weaknesses in the phonological loop or visual-spatial sketchpad, but are probably related

to a central executive deficit. Swanson does not speculate on why children who share the same cognitive weakness should develop such different academic profiles.

Swanson (1993) acknowledges that his conclusions are inconsistent with the findings of some previous studies and puts forward possible explanations to account for the differences. He suggests that the specificity of a child's academic weakness is inversely related to the specificity of their cognitive weakness. A child with a small discrepancy between their reading and arithmetic scores would therefore be more likely to have difficulties only in visual-spatial or auditory verbal memory tasks. Swanson cites some empirical support for this explanation. The specific learning disabled children Siegel & Ryan (1989) studied had limited working memory deficits that were related to their academic weaknesses. These children had an average discrepancy of 8 standard score points between their reading and arithmetic scores. In contrast, the specific learning disabled children in the Swanson (1993) study had average discrepancies of 25 (LD-math) and 37 (LD-reading) standard score points. This argument appears somewhat illogical: one would expect a generalised cognitive deficit to produce a more not less generalised academic deficit.

An alternative explanation suggested by Swanson concerns the measures used. He states that much of the previous research relied on measures of short-term memory, without the use of a concurrent processing task (e.g. Fletcher, 1985). The results are therefore not strictly comparable with his study. The differences between the task used to measure memory do seem a likely explanation. The tasks used by Swanson differ greatly from those used previously. On all the prospective tasks the children are actively encouraged to rehearse the stimulus. On all the visual-spatial measures there is a time lapse (ranging between 5s and 30s) between the presentation of the stimuli and recall. During this rehearsal the child may transfer the information into the long-term store. Cognitive resources may not have to be used to stop decay once this has occurred. If the stimuli are transferred to the long-term store attention does not have to be divided between

rehearsal and solving the process question. Most other studies do not allow this extended rehearsal period. The poor auditory verbal short-term-memory of children with specific reading difficulties have been found on tests where isolated words, letters or digits must be recalled in the right sequence. Only two auditory verbal tasks in the Swanson study required the sequential recall of isolated words, other tasks allowed free recall or were such that semantic information could assist a child when recalling (e.g. the information took the form of a story or it had to be recalled in semantic categories). Although both the LD-math and LD-reading children were poorer than the age-matched controls on these tasks they are different to the previous simple auditory-verbal/visual-spatial memory span tasks on which the two groups of learning-disabled groups have been found to differ from each other.

Furthermore, it is likely that the results of Swanson (1993) do not concur with the complex span tests discussed previously, because the concurrent processing requirements are different. Only one auditory-verbal processing question required phonological analysis. Therefore the auditory-verbal processing task does not tap the phonological skills in which children with specific reading difficulties have been found to be deficient. Similarly, concurrent counting tasks, which Hitch & McAuley (1991) found reduced the functional capacity of children with arithmetic difficulties, were not used by Swanson (1993). It appears that the performance of different sub-types of specific learning disabled children on complex span tasks differs only when the concurrent task taps a process in which one group is particularly weak.

The central executive function of children with arithmetic difficulties has only recently been investigated. McClean & Hitch (1999) found that the performance 9-year-old children with poor arithmetic but better reading differed from normal age-matched controls on four tasks designed to measure central executive function. Three of these tasks were variations of the making trails task (Reitan, 1958). This task was chosen because it taps the ability to switch retrieval strategies, which Baddeley (1996) attributed to the central executive. In the trails tasks the children had to connect together circles (printed in a

random pattern) in a prescribed order. In written trails half the circles contained the numbers 1-11 and the others contained the letters A-K. The children had to connect alternate numbers and letters together in numerical and alphabetical order (e.g. 1-A-2-B ... 11-K). Verbal trails used the same task and stimuli, but the children had to respond verbally. Colour trails used different stimuli. The numbers 1-11 were printed in both 11 yellow circles and 11 pink circles. The child had to connect alternate colours in numerical order (e.g. yellow 1-pink 1-yellow 2-pink 2 ... yellow 11-pink-11).

The atypical children differed from both the age and ability matched controls on a missing item task. This task was designed to “measure the capacity to hold and manipulate information accessed in long-term memory” (p. 250), which McClean & Hitch (1999) also attributed to central executive function. This task presented incomplete addition items that the children had to fill in (e.g. $2+3=4+?=?$). The speed taken to complete the 15 questions was the dependant measure. When considering the findings of McClean & Hitch (1999), the extent to which the tasks assess central executive function needs to be considered. The trails task requires the children to retrieval numerical and alphabetical sequences from LTM. Studies have indicated that children with arithmetic difficulties are significantly slower at counting than typical children (e.g. Geary et al., 1991; Hitch & McAuley, 1991) and have problems learning sequences (e.g. Ward, 1992). These results suggest that the children in the McClean and Hitch (1999) study may perform more poorly on the making trails task because they have a specific difficulty recalling either the number sequence or sequences in general. Furthermore, there is evidence that children with arithmetic difficulties are slower at recalling number facts than ability-matched controls (see, for example, Russell & Ginsberg, 1984). This could account for the time differences on the missing item task. Further studies are required to determine whether children with arithmetic difficulties perform more poorly than ability and age matched controls on non-numerical tests of central executive function. It is also worrying to note that the central executive tasks in the McClean and Hitch (1999) study do not all correlate with each other.

One would expect strong correlations between tasks that all measure the same cognitive construct.

Sequencing ability is the ability to learn and recall material in a fixed temporal order and sort information in accordance with an already learnt sequence. It is a particular aspect of memory function. Ward (1992) investigated the factors that best predicted the mathematical ability of 6- and 7-year-old children. Sequencing ability was measured both by asking the children to recite verbal strings (e.g. the alphabet, the months of the year) and asking questions about them (e.g. What letter comes before k?). Sequencing ability predicted a significant unique proportion of the variance in maths scores. It was the second best predictor overall (reading ability being the best predictor).

Ward (1992) conducted further experiments examining the sequencing ability of children with arithmetic difficulties (defined as having a maths quotient at least 0.7 standard deviations below the regression line and a reading quotient above 85). The controls had maths quotients that were 'close' to the regression line and were matched with the impaired children on reading ability and non-verbal ability (measured using Ravens Matrices; Raven, 1965). There were no significant differences between the two groups in their ability to recite the days of the week forwards or backwards. However, the groups differed significantly in their ability to learn novel sequences of either unfamiliar words or spatial locations. The difference in their ability to learn a sequence of familiar words approached significance. The children with arithmetic difficulties were not poorer at tasks involving delayed recall of the sequences learnt or immediate free recall. The children with arithmetic difficulties in Ward's experiments had problems learning serially ordered material (both verbal and non-verbal material) but did not have difficulties accessing it once it was learnt.

Bull & Johnston (1997) compared the sequencing abilities of two groups of 7-year-olds, those with high and low mathematics ability. The children were asked to place words into alphabetical order and numbers into numerical order. The two groups did not differ on

these measures after reading ability had been controlled for. This is consistent with Ward's finding that children with arithmetic difficulties do not have problems accessing already learned sequences.

It is difficult to make general statements about the memory deficits of children with arithmetic difficulties. A wide variety of different methods have been used to assess the children. Even small differences in the design of a task can produce different results. For example, the same child can perform differently on different memory measures that are both visual-spatial (e.g. Henderson, 1991). As several studies report that children with arithmetic difficulties, but a normal reading score, do not have significant impairments in retaining auditory verbal information for a short period of time (e.g. McClean & Hitch, 1999) it is unlikely that an impaired phonological store is *necessary* for arithmetic difficulties to develop. Some studies report that children with arithmetic difficulties (particularly those who also have reading difficulties) have a weak phonological short-term store, hence such a deficit may be *sufficient* for an arithmetic difficulty to develop. However, this hypothesis has not been tested rigorously with longitudinal correlations and training studies in the same manner as the relationship between phonological awareness/memory and reading has been studied. There appears to be a link between difficulties in retaining visual-spatial information for a short period of time and arithmetic difficulties. Whether a weak visual-spatial short-term memory can lead to the development of arithmetic difficulties needs further investigation. The functional capacity of working memory is reduced when children with arithmetic difficulties are assessed using tasks that use counting as the concurrent task. Learning novel sequential material appears to be more difficult for children with arithmetic difficulties.

4.4.4 Processing speed and counting speed

Bull & Johnston (1997) describe two alternative ways of conceptualising processing speed: global or domain specific. If processing speed is domain specific the same child may differ considerably in the speed that they complete different cognitive and

psychomotor tasks. For example, a child may have a slow speech rate but be a fast counter. A child's processing speed will increase in a specific domain as their experience of the information for a specific task increases. For example, as a child becomes more familiar with the counting sequence it will be represented more strongly in the memory store and therefore retrieved more quickly when the child counts. An alternative global view has been proposed by Kail & Salthouse (1994). This global view suggests that a child's information-processing speed on all tasks will be similar. Individual differences in processing efficiency (such as the ability to process different information in parallel) limit processing speed on all speeded tasks. This global view of processing speed was supported by Kail (1992), who found that children's processing speed on psychomotor and cognitive tasks was related.

One domain specific form of processing speed that has been investigated is counting speed. Young children use counting strategies to solve arithmetic problems. Slower counting will increase the chance of the memory trace of the problem integers decaying before the count is completed and hence the solution being associated with the sum (Baddeley, 1986). This suggests that children who have slow and laborious counting are likely to have greater difficulty automating basic number facts. The findings on counting speed have been mixed. Geary (1990) found that the counting speeds of children enrolled in a remedial maths programme were not slower than normally achieving children. Similarly, Geary (1991) found no significant counting speed differences between 4th Grade mathematically disabled children and their normally achieving controls. Bull & Johnston (1997) found no significant difference between the counting speed of two groups of 7-year-old children who differed in arithmetic ability. The two groups did not differ regardless of whether or not reading ability was controlled for. Some studies have identified counting speed differences. Geary et al. (1991) followed up the children studied by Geary (1990) when they reached 2nd Grade. At this older age they were significantly

slower counters than the controls. Hitch & McAuley (1991) found that children with arithmetic difficulties were slower counters than controls.

Geary (1993) suggested that the contradictory results may be due to the groups being heterogeneous. Some of the children may have had impaired counting speed, whilst others did not. A significant result would be dependent on the proportion of slow counters in the group. Geary's studies included children with reading and maths difficulties whilst Hitch & McAuley (1991) included only children whose reading was better than their arithmetic. Further studies, which report individual results, are required to determine whether children with arithmetic difficulties but better reading are slower counters than children who have both arithmetic and reading difficulties. Another factor to consider is the method used to determine counting speed. Geary (1990) and Geary et al. (1991) assessed children's counting speed whilst they used counting strategies. In contrast, Bull & Johnston (1997) and Hitch & McAuley (1991) asked children to count dots (numbering no more than 10). Although Bull & Johnston (1997) found their groups did not differ on the dot counting task they did differ in the speed at which they executed counting strategies when calculating arithmetic problems. Children with arithmetic difficulties may not differ from normal children on simple counting tasks, where they can start at the number one and they only have to count up to a relatively low number. However, on more demanding counting tasks such as those required for calculation where they may begin at a number other than one (if they use counting on) or when they have to count relatively large numbers, differences may be apparent.

Bull et al. (1999) examined the relationship between a measure of global processing speed and arithmetic difficulties. Their global processing speed measure was the mean of each child's scores on three different tests: a cross-out task, a visual number matching task and a pegboard test. The two groups of 7-year-olds that differed in mathematical ability obtained significantly different scores on all three tests. The differences on the visual number matching task and the pegboard test remained significant

after reading ability had been controlled for. The difference for the cross-out task was verging on significance ($p = 0.053$). These measures correlated significantly, which supports the global view of processing speed. The measure of processing speed correlated significantly with mathematical achievement, before and after reading ability had been controlled for. The composite measure of processing speed was the only cognitive factor to account for a significant proportion of the variance in mathematical ability once reading ability had been entered into the regression equation.

The studies reviewed in this section have identified a variety of cognitive weaknesses that are associated with arithmetic difficulties. One possibility is that children's arithmetic difficulties have many different causes, different children being affected by one or more of the deficits identified. Many different cognitive skills are used when children solve arithmetic problems. The hypothesis that there are a variety of routes to learning difficulties is explored by Lyytinen, Ahonen & Raesaesen (1994). They suggest that damage to different areas of the cognitive architecture could result in different pathways to mathematical difficulties. They suggest that deficits in memory, approximate numerical perception, spatial and verbal abilities could all impact on mathematical development. In order to rigorously examine this hypothesis the individual cognitive profiles of children with the same academic profile need to be examined. If children with heterogeneous cognitive profiles have specific arithmetic difficulties it can be concluded that many different cognitive abilities contribute to arithmetic skill development and that a deficiency in any one area can disrupt arithmetic acquisition.

4.5 Psychosocial aspects

Children with learning difficulties are more likely to display socially undesirable or maladaptive behaviour (see Rourke & Fuerst, 1991, for a review). Rourke & Del Dotto (1994) present possible reasons for this phenomenon. The undesirable or maladaptive behaviour of the children may be a reaction to the consequences of their specific learning disability. For example, the specific learning disabled children may be aggressive because

they are frustrated with their failure at school or they may become socially isolated and withdrawn if their peers avoid them because they think they are odd or different. An alternative explanation is that the specific learning disabled children's socially undesirable or maladaptive behaviour is caused directly by their cognitive deficits. Rourke proposes that specific learning disabled children with different academic profiles have different cognitive profiles and therefore different undesirable and maladaptive behaviours.

Rourke & Del Dotto (1994) review several studies, known collectively as the 'Windsor taxonomic research', that investigated the relationship between psychosocial functioning and learning disabilities. Four studies (Fuerst, Fisk & Rourke, 1989; Fuerst, Fisk & Rourke, 1990; Fuerst & Rourke 1993; Porter & Rourke, 1985) examined specific learning disabled children's profiles on the Personality Inventory for Children (PIC, Wirt, Lachar, Klinedinst, & Seat, 1977). Multi-variant statistical sub-typing revealed seven distinct personality profiles: normal, mild hyperactive, mild anxiety, somatic concern, conduct disorder, internalised psychopathology and externalised psychopathology. Although all the profiles were not found in all the studies, the normal, internalised psychopathology and externalised psychopathology were. The mild hyperactive, mild anxiety, somatic concern and conduct disorder profiles indicate mild to moderate elevations on some scales. Rourke and Del Dotto (1994) argue that only the internalised psychopathology and externalised psychopathology describe frank maladjustment.

Rourke and Del Dotto (1994) draw several conclusions from the Windsor taxonomic research. Firstly, many specific learning disabled children had normal PIC profiles, which indicates specific learning disability does not always lead to psychosocial maladjustment. The Fuerst and Rourke (1993) study was a cross sectional study, which indicated that older children were not more likely to suffer more severe forms of maladjustment. Rourke (1994) suggests that this challenges the hypothesis that the psychosocial difficulties of specific learning disabled children are a reaction to the consequences of their specific learning disability. If this hypothesis is correct one would

expect that as older children are likely to have experienced more frustration, depression and parent/teacher conflicts they would experience more psychosocial problems. The results of Fuerst and Rourke (1993) support the alternative hypothesis that the cognitive deficits of specific learning disabled children impact directly on their psychosocial functioning. The results indicate that specific learning disabled children who had better reading and spelling than arithmetic not only showed significantly higher levels of frank maladjustment, but they were also more likely to have a particular type of maladjustment – internalised psychopathology. Rourke (1994) argues that if different subtypes of children have different types and levels of psychosocial problems (as these studies indicate) the problems cannot only be due to the children's reaction to academic failure because all the specific learning disabled children experience a similar amount of academic failure. Instead Rourke (1994) points to their differing cognitive deficits as reasons for their maladjustment. Specifically he suggests that the specific learning disabled children with poorer arithmetic than reading and spelling have difficulty processing visual-spatial information, which results in them having difficulty interpreting body language and other non-verbal social cues.

Several recent studies have reported on the psychosocial problems of children with specific arithmetic difficulties, which also suggests they have lower social competence than normally achieving children. Silver, Elder & DeBolt (1999) used the Social Skills Rating System (Gresham & Elliott, 1990) to compare the social competence of children with specific arithmetic difficulties (SAD) with their normally achieving peers. The teachers and parents of SAD and the teachers and parents of control children were asked to rate the children's' social competence. The SAD children received lower ratings than the controls. The SAD children did not self-report their social difficulties. Davis et al. (1997) reported that children with poor arithmetic and better reading (SAD) were more likely to have counselling specified in their Individual Education Plans than children with poorer reading than arithmetic (RAD). Loveland, Fletcher & Bailey (1990) compared RAD and

SAD children's understanding of stories presented verbally and non-verbally (using puppets). The specific learning disabled children were more likely to misinterpret affect and motivation than the normally achieving controls. The SAD children had greater difficulty understanding the non-verbally presented stories, whilst the RAD children had greater difficulty understanding the verbal stories. This supports Rourke and Del Dotto's (1994) proposal that specific learning disabled children with better reading and spelling than arithmetic have difficulty interpreting non-verbal social cues. Research into the psychosocial aspects of arithmetic difficulties is still relatively limited, but preliminary studies indicate that *some* children with arithmetic impairments have psychosocial problems that *may* be caused by their inability to detect non-verbal cues.

4.5.1 Mathematics anxiety

Whilst the cognitive deficits of children with arithmetic difficulties may affect their social functioning, it is also possible that a child's emotional state may affect their arithmetic functioning. Some individuals suffer from mathematics anxiety, which is defined as "a feeling of tension, apprehension, or fear that interferes with maths performance" (p.176) (Ashcraft, Kirk, & Hopko, 1998). In their review of studies of mathematics anxiety, Ashcraft et al. (1998) conclude that high scores on scales of mathematics anxiety are associated with lower mathematics achievement and an avoidance of courses involving mathematics. The traditional explanation of such findings suggests that if parents, teachers and peers attitudes to mathematics are negative, mathematics anxiety is likely to develop. Mathematics anxiety will lead to a reduced involvement in mathematics based activities (e.g. paying less attention in class, putting less effort into homework, avoiding subjects that have a mathematical content) that increase mathematics attainment (Fennema, 1989).

Ashcraft et al. (1998) challenge this indirect model of the relationship between mathematics anxiety and mathematics performance. They instead propose that the experience of mathematics anxiety directly degrades mathematics performance. Ashcraft,

Kirk & Hopko (1998) state that the experience of mathematics anxiety is “one of intrusive thoughts and worry, with attention devoted to those thoughts” (p. 191). Arithmetical tasks involving working memory resources cannot be performed as efficiently if attention is devoted to anxious thoughts. It is not yet possible to determine whether mathematics anxiety affects arithmetic and mathematics performance directly or indirectly. However, it is possible that some individuals have specific difficulties with arithmetic for affective and not cognitive reasons.

4.6 Classifying arithmetic learning difficulties: Non-verbal learning disability and the alternatives

Throughout this chapter numerous pieces of evidence indicate that children with arithmetic difficulties form a heterogeneous group. Researchers have sought to classify them into more homogeneous sub-groups, so that effective remediation can be designed and in order to shed light on normal numerical processing. One of the most influential classifications is Rourke’s distinction, between children with poor arithmetic and better reading and spelling described as Non-verbal learning disability (NLD) and children with poor arithmetic and even weaker reading and spelling described as basic phonological processing disorder (BPPD). Many of the studies by Rourke and his colleagues, which support this distinction have been reviewed in this chapter; a comprehensive review is presented in Rourke & Del Dotto (1994). In Rourke & Del Dotto (1994) the profiles of the two specific learning disability sub-groups are described. The sub-type descriptions are based on the findings of the various studies that are summarised. NLD children are described as good at word decoding, spelling, rote memory and graphomotor skills (in later childhood), whilst being weak in reading comprehension, mathematics, mechanical arithmetic and science. Rourke and Del Dotto (1994) also argued that NLD children are susceptible to socio-emotional deficits including poor social competence and difficulty adapting to novel situations. BPPD children are reported to have good mathematics, science and reading comprehension (in later childhood), but poor graphomotor skills, word

decoding, spelling, rote memory, mechanical arithmetic and reading comprehension (in early childhood). This profile closely matches the descriptions of children diagnosed as dyslexic.

Rourke & Del Dotto (1994) argue that the pattern of weaknesses found in NLD children is consistent with a right hemisphere impairment. The IQ profile of the NLD children is similar to the IQ profiles of children who have suffered early right hemisphere brain lesions. Woods (1980) found that children who suffered a right hemisphere lesion before their first birthday exhibited depressed performance IQ scores but intact verbal IQ scores twelve years after the damage occurred. Taylor (1976) reported similar results with children who had either left or right temporal-lobe damage. Children with damage to the right side of their brain had higher verbal than performance IQs. Children with damage to the left side of their brain did not exhibit verbal/performance IQ discrepancies. The results of many studies reviewed in this chapter have supported this distinction as they identified psychological differences between sub-groups of children with developmental arithmetic difficulties divided according to their literacy skills (e.g. Brandys & Rourke, 1991; Fletcher, 1985; Siegel & Ryan, 1989). Rourke's dichotomy has lead many researchers to study children with poor arithmetic but better reading separately (e.g. Geary et al., 1999; Hitch & McAuley, 1991; McClean & Hitch, 1999).

If Rourke's dichotomy is valid, one would expect the number skills affected in the two sub-groups to differ, as the different cognitive deficits would impact on different areas of arithmetic. Studies that examine children's number skill difficulties in relation to their reading ability are scarce. Most studies concentrate solely children's cognitive profiles or their number skills profiles. Raesaenen & Ahonen (1995) used written tests to compare the errors of children who had arithmetic difficulties with or without reading difficulties. Reading ability (both accuracy and rate) was inversely related to fact retrieval errors in normally achieving children and in children with arithmetic difficulties. Furthermore, children with arithmetic and reading difficulties made more multiplication fact retrieval

errors than control children. The children who were only poor at arithmetic did not make significantly more multiplication fact retrieval errors than controls. Procedural errors were subdivided into rule errors (which were errors in the carrying or borrowing procedures) and algorithm errors (other errors in completing the correct procedure). The children who had problems solely with arithmetic were the only group to make significantly more rule and algorithm errors on addition and subtraction questions. These results support Rourke's dichotomy. Children with poor reading and arithmetic make a large number of fact retrieval errors, which is consistent with a poor rote memory. Children without reading difficulties made a similar number of fact retrieval errors to the normally achieving control children, which is consistent with their hypothesised good rote memory.

Jordan, Levine & Huttenlocher (1995) also found that children's number skills differed according to their cognitive profiles. She divided kindergarten and first grade children into four groups according to their performance on two ability tests. Language abilities were assessed using a short form of the Test of Language Development, Primary (TOLD-P) (Newcomer & Hammill, 1988). This included orally presented tests of picture vocabulary and grammatic completion. Spatial abilities were assessed using the Test of Non-verbal Intelligence—Second Edition (TONI-2) (Brown, Sherbenou & Johnsen, 1990). This test consisted of a series of non-verbal matrices that the child had to complete. Children were assigned to the non-impaired group (NA) if they scored above the 30th percentile on both tests. They were described as having a general cognitive delay (delayed) if they scored below the 30th percentile on both tests. Children who scored below the 30th percentile on the TONI-2 but above the 30th percentile on the TOLD-P were assigned to the low spatial group (LS). These children had a cognitive profile that is similar to the children described by Rourke & Del Dotto (1994) as non-verbal specific learning disabled. Children who scored below the 30th percentile on the TOLD-P but above the 30th percentile on the TONI-2 were assigned to the low language group (LL).

These children had a cognitive profile that is similar to the children with poor reading and better arithmetic described by Rourke & Del Dotto (1994).

Attainment in addition and subtraction was assessed using identical sums presented in three different question formats. Counters were used for the non-verbal format: the examiner or the child did not use number words. In addition problems the examiner laid the counters representing the first addend on a mat, they were then covered before the counters representing the second addend were placed under the mat. The child then had to place the same number of counters on their own mat. The subtraction procedure was similar; the only difference being the examiner removed the counters representing the subtrahend from under the mat one by one. The story format placed the sums in simple vignettes, which were presented orally. The number fact problems were presented aurally e.g. 'four take away two'.

When the scores for the three formats were combined all the impaired groups scored significantly worse than the NA children did. The LL children scored more poorly on the story and number fact problems than the non-verbal problems. All the other groups scored at a similar level on each of the three formats. The different groups' scores for each format were compared. On the non-verbal problems the LL and NA children achieved significantly higher scores than the delayed children. The NA children performed significantly better than the LL and delayed children on the story problems. The groups number fact scores did not differ significantly.

Strategies were classified as either fingers (physical movements such as finger counting or head nodding observed), counting (verbal counting observed) or unobserved (no overt strategy). The frequencies of the fingers and unobserved strategies did not differ significantly between the groups in 1st Grade. The delayed children used unobserved strategies significantly more frequently than the other groups. Delayed, LL and LS children were significantly less accurate than NA children when using unobserved strategies. Accuracy when using finger strategies did not differ significantly between the

groups. Verbal counting was rarely used on story or number fact problems. The groups' accuracy when using verbal counting for non-verbal problems did not differ significantly.

The results of Jordan et al. (1995) broadly support the validity of the dichotomy suggested by Rourke and his colleagues. Children's cognitive profile was related to their number skills profile. In particular, the children with poor language abilities had a distinctive number skills profile. Their scores when the problems were presented in a non-verbal context were superior to their scores when the problems were presented in a verbal context or as number facts. The LL children may find story problems more difficult than non-verbal problems because they cannot understand the vignette. An alternative explanation is poor verbal memory. They do better at the non-verbal problems when the addends are concretely represented. The memory load is increased in the two aurally presented contexts. Rourke & Del Dotto (1994) identified poor verbatim memory as a characteristic of the children with poor reading but better arithmetic (these children had a similar ability profile to the LL children). The LS children perform more poorly on the arithmetic tasks than the NA children overall, but there are no significant differences in their performance on the sums in different formats. Jordan et al. (1995) suggests that their relative success on the non-verbal problems may be due to verbal mediation. The counters can be verbally counted. They suggest the cognitive problems may cause greater difficulties when they tackle more advanced arithmetic that has higher visual spatial demands.

Shalev, Manor & Gross-Tsur (1997) compared children with reading and arithmetic difficulties with children who had arithmetic difficulties without reading and spelling difficulties. The children, all in 5th Grade, were administered an arithmetic battery based on the McCloskey & Caramazza (1985) model, which had been standardised on 200 5th Grade children, see Shalev et al., 1993 for standardisation details). The children with arithmetic and reading difficulties performed more poorly than the children with arithmetic-only difficulties overall. In the number facts section the children with

arithmetic and reading difficulties made more multiplication and division errors. The children with arithmetic and reading difficulties also made more difficulties on complex multiplication and division questions. The two groups did not differ on the number production and number comprehension sections. Shalev, Manor and Gross-Tsur (1997) concluded that the areas of arithmetic affected did not differ significantly, but the children with reading and arithmetic difficulties had more severe arithmetic deficits. However, interpretation of this study is difficult for two reasons. The children with reading and arithmetic difficulties had significantly poorer IQ scores than the children with arithmetic-only difficulties, therefore the poorer performance of the group with reading and arithmetic difficulties may have been due to lower reasoning ability. There were also ceiling effects on the number comprehension and production sections of the arithmetic battery used; it may therefore not have been sensitive enough to detect differences in these areas (see section 4.2 for a discussion of the Shalev et al., 1993 arithmetic battery).

4.6.1 Applying McCloskey's modular model of arithmetical processing to children with arithmetic difficulties

Rourke's classification of children with arithmetic learning difficulties is not universally accepted. Macaruso & Sokol (1998) argues that "... little is gained from a left/right dichotomy when examining numeric processing errors" (p. 220). They believe that children's arithmetic studies can be better understood using the McCloskey & Caramazza's (1985) model of arithmetical processing rather than Rourke's dichotomy. McCloskey & Caramazza's (1985) model is discussed in Chapter 3. It is based on the study of adult neurological patients who have lost mathematical skills after their brain was damaged. Arithmetic skills are viewed as independent modules (number production, number comprehension, fact retrieval and calculation procedures). Macaruso & Sokol (1998) cite two studies in support of their argument: Sokol et al. (1994) and Shalev, Manor, Amir, Wertman-Elad & Gross-Tsur (1995).

The difficulties of students studied by Sokol et al. (1994) are discussed in an earlier section of this chapter. All of these students were diagnosed as dyslexic. One would therefore expect them to have significant reading and spelling difficulties. It would be likely that they would fit into Rourke's better arithmetic than reading and spelling group. As the students had heterogeneous arithmetic difficulties but would fit into the same group according to Rourke's classification system Macaruso & Sokol (1998) argued that the right/left hemisphere dichotomy is not very useful. Some of the students had number skills difficulties limited to one module of the McCloskey & Caramazza (1985) numerical processing model, which suggests that this model has greater validity.

Shalev et al. (1993) used an arithmetic battery based on McCloskey & Caramazza (1985) model of arithmetical processing to compare children with left and right hemisphere dysfunction. Children were assigned to the left hemisphere dysfunction group if they had a combination of right body neurological signs, abnormal performance on language related tasks, a higher performance than verbal IQ and normal visual-spatial function. Children were assigned to the right hemisphere dysfunction group if they had left body neurological signs, a higher verbal than performance IQ, impaired visual spatial function and unimpaired verbal/language function. All of the children with left hemisphere dysfunction had reading scores one or two standard deviations below the mean; none of the right hemisphere children had significant reading impairment. However, because group assignment was based on a mixture of neurological and psychological signs the performance/verbal IQ discrepancy varied greatly between individuals. 8 of the 25 participants had a performance/verbal IQ discrepancy of less than 10. The right hemisphere dysfunction group had significantly lower full scale IQs. When completing the assessment battery children in both specific learning disability groups were significantly more likely to make number fact and calculation errors than normal children; they were not more likely to make errors on the number processing section. Overall the children in the left hemisphere dysfunction group performed worse than the right hemisphere dysfunction

group. Macaruso & Sokol (1998) argued that as the two specific learning disability sub-groups pattern of performance did not differ on this arithmetic battery, their ability profiles did not influence their arithmetic skills.

Several factors must be borne in mind when judging the Macaruso & Sokol (1998) argument for the acceptance of McCloskey & Caramazza's modular model and the rejection of Rourke's dichotomy. The supporting studies have methodological weaknesses. The limitations of the tests used have already been discussed in an earlier section of this chapter. Sokol et al.'s (1994) study sample was highly selected, the children were chosen on the basis of standardised test scores (that are not reported) and teacher referrals. 11 of the 20 in the selected sample had selective deficits in arithmetic; the other 9 either performed reasonably well on the battery or had a generalised arithmetic deficit. It is widely reported that children with specific reading difficulties/dyslexia have arithmetic difficulties (see sections 5.3 and 5.4.) If many of the dyslexic children at the school had mild to moderate arithmetic difficulties it is likely that students with profound or atypical difficulties were selected. The sample was not representative of the population of children with reading and arithmetic difficulties. If the majority specific learning disabled children with poor reading and better arithmetic difficulties primarily have fact retrieval problems (as the study of Raesaenen & Ahonen (1995) suggests) Rourke's classification is still useful even if a minority have problems primarily in other areas of arithmetic. Larger scale studies that have not separated children into groups according to their reading ability have indicated that the majority of children have problems with calculation and fact retrieval; in contrast, number comprehension and production deficits appear rare (Russell & Ginsberg, 1984; Shalev et al., 1988).

Case studies of children with developmental disorders have been put forward as support for the use of the McCloskey & Caramazza (1985) method of classifying children with arithmetic difficulties. Temple has interpreted case studies of children with isolated arithmetical weaknesses (Temple 1989, 1991 and 1994 previously reviewed in section 4.2)

using the McCloskey & Caramazza (1985) model. She interprets their isolated weaknesses as impaired number skills modules. However, their weaknesses could be explained as being the result of specific cognitive weaknesses (see section 10.4 for further discussion of this issue). Even if an impaired number skills module can result in arithmetic difficulties, if it is a relatively rare occurrence, the model has little utility when assessing the majority of children with arithmetic difficulties.

It is important to note that the McCloskey & Caramazza (1985) modular model of arithmetical processing has not been accepted by all psychologists even as an explanation for adult arithmetical processing. It has been seriously challenged by the encoding complex view put forward by Campbell & Clark (1988) and Clark & Campbell (1991) (see section 2.3 for a further discussion of this issue). Furthermore, McCloskey & Caramazza's (1985) model of numerical processing is derived from studies of adult neurological patients with dyscalculia. Applying a model derived from adults who have lost acquired function to children who never gained the function at a normal rate is questionable. Children with developmental arithmetic difficulties and adults with acquired dyscalculia with the same cognitive deficits (e.g. poor visual-spatial skills) may have weaknesses in different areas of arithmetic as they acquire the deficits at different stages of development. Rourke & Conway (1997) highlight the difference in the cognitive skills required to develop a yet unlearned skill and execute that skill once learnt. Adults with acquired spatial dyscalculia have problems with the spatial organisation of numbers when executing already learnt mathematical skills, e.g. misalignment of digits in columns, difficulties maintaining the decimal place. These types of errors may not be the most prominent feature of children with developmental arithmetic difficulties and poor spatial skills such as the NLD children described by Rourke & Del Dotto (1994). Their spatial and tactile-perceptual weaknesses may affect early arithmetical development and therefore compromise very basic number skills such as counting fluency and single digit arithmetic. McCloskey & Caramazza's (1985) model proposes separate modules that are independent. Although these modules

may be independent in adulthood the studies reviewed in Chapter 3 indicate that in childhood they are inter-dependent. For example, single digit arithmetic skill and place value understanding influence multi-digit arithmetic performance. Therefore a cognitive weakness that disrupts the development of more basic arithmetic skills will indirectly influence more complex arithmetic skills. A wider range of arithmetical skills may be affected if a specific cognitive weakness is present from birth than if the same specific weakness is acquired later in life.

4.6.2 Developmental genetic disorders affecting arithmetic: Support for the NLD syndrome

It may be more useful to derive our theoretical models of developmental arithmetic difficulties from genetic developmental disorders that affect arithmetic attainment. Children who suffer from such disorders have a genetic deficit present from birth that affects their cognitive and therefore educational development throughout their life. Children with these genetic disorders are more similar to children with developmental arithmetic difficulties than adults with arithmetic difficulties caused by an acquired brain injury. Children with a genetic disorder that affects arithmetic attainment and children with a developmental arithmetic difficulty are both affected by cognitive weaknesses throughout their life span. Adults with acquired arithmetic difficulties are only affected by a cognitive weakness after arithmetic skills have been established.

Turner's Syndrome

Turner's syndrome (TS) is a genetic abnormality affecting only females. The second X chromosome is either deleted or functionally ineffective. This results both in physical deformities and sexual retardation (Temple & Marriott, 1998). Studies reviewed by Temple & Marriott (1998) suggest that TS females have a distinctive cognitive profile, with verbal skills being superior to non-verbal skills. Impairments in spatial processing have been reported in numerous studies (e.g. Lewandowski, Costenbader & Richman, 1985; McGlone, 1985; Money, 1973; Rovet & Netley, 1980; Rovet & Netley, 1982;

Schucard, Schucard, Clopper & Schacter, 1993; Waber, 1979). Pennington, Bender, Puck, Salbanblatt & Robinson (1982) reported lower performance than verbal IQ scores in TS girls. However, this IQ profile is not universal; some girls with TS do not have a superior verbal IQ (see O'Connor, Fitzgerald, & Hoey, 2000; Temple & Carney, 1993).

Impairments that are associated with poor central executive functioning (e.g. low Wisconsin Card Sorting Test scores, poor verbal fluency, difficulty with the Tower of Hanoi task) have also been noted (Romans, Roeltgen, Kushner & Ross, 1997; Waber, 1979).

Reports of arithmetic difficulties in TS girls date back to the 1960's (e.g. Shaffer, 1962; Tsuboi & Nielsen, 1985; Waber, 1979). Rovet's (1993) study confirmed the earlier findings. TS girls were about 2 grades below their overall grade placement in arithmetic. They also performed more poorly than controls on two standardised tests of arithmetic, the WRAT-R Arithmetic test and the Keymath diagnostic arithmetic test. Rovet (1993) reported that TS girls were more impaired in conceptual-factual areas than computational areas of mathematics. Rovet, Szekely & Hockenberry (1994) found that TS girls had poorer procedural skills than controls. The TS girls' arithmetic fact retrieval was adequate in untimed conditions, but poor in time limited conditions.

Temple & Marriott (1998) conducted a detailed analysis of the arithmetical skills of eleven TS girls. The TS girls did not differ significantly from the age matched control girls on any of the number processing tasks (reading number words, reading Arabic numbers, writing Arabic numbers, writing number words, copying Arabic numbers and magnitude judgements). However, it should be noted that there were ceiling effects on all the number processing tasks. The groups' accuracy and speed when answering oral addition and multiplication problems were compared. The addition accuracy of the groups did not differ significantly, but the TS girls were significantly slower. The accuracy difference between the groups was not significant for the multiplication questions. Neither was there any evidence that the TS girls were slower. However, the pattern of the TS girls'

multiplication errors was distinctive. They tended to make more shift errors (this is when either the number in the units or tens column is correct, e.g. $6 \times 2 = 22$ or $6 \times 4 = 14$) and more consistent errors (this is when the same answer is given for both permutations of the multiplication sum, e.g. $5 \times 4 = 17$, $4 \times 5 = 17$). The multiplication results should be viewed with caution. The standard deviations for the multiplication results of both the control and TS girls were very large. This suggests a large variation in multiplication skill for both groups. There was also a trend approaching significance for the TS girls to perform more poorly on the multiplication questions ($p = 0.09$). The TS girls performed more poorly on the written arithmetic section than the control girls. The TS girls made significantly more procedural errors.

Temple and Marriott (1998) conclude that TS girls arithmetical procedures are "... not simply a consequence of the spatial deficit." (p. 63). This suggests that Temple and Marriott (1998) presume that the spatial deficit would have to work directly in a manner similar to adults with acquired spatial dyscalculia, i.e. the spatial difficulties would affect the girls current performance. Examples of direct effects would include difficulty understanding spatial information such as graphs or geometrical diagrams, mis-aligning numbers in written calculations or problems writing numerals accurately. Temple and Marriott (1998) found no evidence for these kinds of direct effects. However, poor spatial abilities could have affected arithmetic performance indirectly, by disrupting the development of the TS girls' early number skills, which in turn resulted in inaccurate fact development and slow inefficient procedures.

The cognitive profile of TS girls is similar to the cognitive profile of children with Non-verbal learning difficulties studied by Rourke (e.g. Rourke, 1982; Rourke, 1993; Rourke & Conway, 1997; Rourke & Del Dotto, 1994; Rourke & Finlayson, 1978; Rourke & Strang, 1978). NLD and TS children have poor arithmetic skills, tend to have better verbal than spatial skills, and poor performance on tests of executive function such as the WCST. If the two groups' cognitive profiles are similar it is more likely that the same

neurological abnormalities cause their arithmetic difficulties. One recent study suggests that the two groups may respond to similar intervention strategies. Williams, Richman & Yarbrough (1993) taught children with NLD and TS to use verbal mediation when attempting a spatial matching task. The performance of both the TS and the NLD children improved after this intervention. The level of performance gain did not differ between groups. This suggests that both groups will respond positively to cognitive intervention programmes that emphasise verbal mediation.

Research reviewed in the psychosocial section of this chapter (e. g. Fuerst et al., 1989; Fuerst et al., 1990; Fuerst & Rourke, 1993; Loveland et al., 1990; Silver et al., 1999) suggested that NLD children with poor arithmetic but better reading are prone to poor social skills, withdrawal and internalised psychopathology. Their psychosocial difficulties may be due to their poor understanding of non-verbal social cues. Recent studies have suggested that TS girls have a similar psychosocial profile. Mazzocco, Baumgardner, Freund & Reiss (1998) found that TS girls had higher rates of social and attention problems than their sisters did. Williams (1994) compared TS, NLD and normally achieving girls using a parental behaviour rating scale. TS and NLD girls displayed higher levels of social isolation than the controls. The TS girls also had problems with impulsiveness and medical non-compliance.

Velo-Cardio-Facial Syndrome

Velo-Cardio-Facial syndrome (VCF) is a genetic abnormality affecting both males and females. The major physical features are a cleft palate or velo-pharyngeal insufficiency, cardiac anomalies and a characteristic facial appearance (Swillen et al., 1999). These symptoms are caused by the deletion of a gene on chromosome 22q11 (Scambler, Kelly & Lindsay, 1992). Swillen, Devriendt, Legius, Eyskens & Fryns (1997) reviewed the cognitive profiles of children diagnosed with VCF. 45% of the children were defined as mentally retarded, as their full scale IQs were below 70. There was a high incidence of specific learning difficulties in the children without general mental

retardation. A review of recent studies by Swillen et al. (1999) indicates that VCF children with specific learning difficulties display a distinctive pattern of deficits that are similar to the NLD children Rourke identified (see Rourke & Del Dotto, 1994). Verbal IQ is usually superior to performance IQ in specific learning disabled VCF children. Other cognitive areas in which deficits have been reported for VCF children include attention, concentration, visual-spatial skills and motor abilities (Golding-Kushner, Weller & Shprintzen, 1985; Moss et al., 1995; Swillen et al., 1997).

Swillen et al. (1999) used a wide battery of tests to determine whether the psychological profiles of VCF and NLD children were similar as earlier studies indicated. Seven of the nine children studied were not old enough to complete standardised reading and arithmetic tests. Six of the seven children had single word reading test scores within the average range. None of the VCF children had an arithmetic test score in the average range. The difference between the mean single word reading and mean arithmetic scores was significant. This suggests that the academic profiles of this group of VCF children and NLD children are similar. However, contrary to previous findings there was no evidence of significant verbal IQ superiority in VCF children. A statistical comparison of verbal and performance IQ was not statistically significant. Verbal IQ was only statistically significantly better in two of the nine cases studied. The difference between the IQ scores was not statistically significant in any of the other cases. Swillen et al (1999) describe three of the nine cases as having a "clinically significant" verbal IQ superiority because it is 10 standard score points higher than performance IQ. The remaining six children had small verbal/performance IQ differences, two of these children had a higher performance IQ. However, the performance of the VCF children was similar to the NLD children in other respects. All the VCF children had poor psychomotor scores (all their z scores were -1 or less).

4.6.3 Geary's three subtypes

The cognitive profiles of VCF and Turner's syndrome children adds strength to the argument that developmental arithmetic difficulties without reading difficulties can be associated with visual-spatial skill deficits. Geary (1993) expands on Rourke's classification system to include an additional arithmetic difficulties subtype that need not be associated with reading difficulties or visual spatial difficulties. Subtype 1 (semantic memory) is characterised by difficulties quickly and accurately retrieving arithmetic facts. Geary asserts that this subtype tends to be associated with left hemisphere dysfunction and reading disabilities, in particular those reading disabilities which are associated with phonetic deficits. This subtype is clearly similar to Rourke's BPPD subtype as both emphasise poor reading, poor memory and poor arithmetic fact retrieval. Geary's subtype 3 (visuo-spatial) is characterised by difficulties in the spatial representation of numerical information and right hemisphere dysfunction. Geary asserts that this subtype is not associated with reading disability. This subtype is clearly similar to Rourke's NLD subtype as both emphasise poor visual-spatial skills and unimpaired word reading.

Geary's subtype 2 (procedural) is not included in Rourke's classification system. Subtype 2 is characterised by the use of developmentally immature procedures, errors in executing the procedures and a potential developmental delay in the understanding of concepts underlying procedural use. Geary (1993) suggests that children classified in the procedural sub-type have difficulties using both counting procedures to solve single digit problems, and algorithms to solve multi-digit problems. He suggests that some children with procedural difficulties may simply be at the lower end of the normal distribution and be slower than average at learning arithmetic facts. Others have intractable difficulties in learning procedural skills. One example of an individual who had severe procedural difficulties is SW, who was studied by Temple (1991). SW's inability to perform multi-digit procedures was selective; he had good arithmetic fact recall. However, such case studies are rare and there are no studies examining how children who have inaccurate

single digit arithmetic procedures progress when learning multi-digit arithmetic procedures. Geary's own studies (Geary, 1990; Geary et al., 1991) indicated that the majority of MD children tackling single digit problems had inaccurate counting procedures in Grade 1, but that their counting procedures became more accurate in 2nd Grade. Their primary difficulty was an inability to use retrieval (which is more associated with sub-type 1). Geary (1993) concedes that these three subtypes are a "best guess" (p. 358) formulated using the evidence available at the time. The procedural subtype is particularly unclear with its relationship with reading disability unexplored.

4.7 Conclusions

Present evidence (e.g. Ozols & Rourke, 1988; Rourke & Fuerst, 1991; von-Aster, 1996) suggests that some children with arithmetic difficulties but better literacy skills have severe deficits in visual-spatial skills and relatively unimpaired verbal skills. Rourke (1994) described such children as non-verbal learning disabled. However, many studies (e.g. Ackerman & Dykman, 1996; Davis et al., 1997; Share et al., 1988) indicate that not all children with this academic profile (especially girls) have this pattern of cognitive skills. Other children with arithmetic difficulties, particularly those with co-morbid reading difficulties have short-term memory problems and slow inaccurate fact retrieval (see Geary, 1993). Other cognitive factors such as processing speed and sequencing ability have been associated with arithmetic difficulties. Individual cases of children with atypical arithmetic difficulties, e.g. a selective deficit in procedural skills (Temple, 1991) or a profound inability to comprehend numerical concepts (Ta'ir et al., 1997) have been reported.

5 Dyslexia and arithmetic

5.1 Defining dyslexia

Dyslexia is a specific learning difficulty characterised by difficulties with reading and spelling. Despite adequate instruction dyslexic children have problems achieving competent literacy skills. Researchers and practitioners are still not agreed on how dyslexia should be defined or diagnosed. Older definitions of dyslexia were based on exclusionary criteria. An example of a definition of dyslexia incorporating exclusionary criteria is given below.

“... a disorder manifested by difficulty in learning to read despite conventional instruction, adequate intelligence and socio-cultural opportunity. It is dependant upon fundamental cognitive disabilities which are frequently of constitutional origin”
(Critchley, 1970) (p. 11)

Exclusionary criteria were used to distinguish between children with low intellectual ability who had general learning difficulties (including difficulties learning to read) and dyslexic children with average or above average intellectual ability who were thought to have unique cognitive difficulties that affected limited aspects of their learning (including literacy skills). An extension of exclusionary criteria is the discrepancy definition. If a discrepancy definition is used children are identified as dyslexic, if there is a statistically significant difference between their literacy attainment and the literacy attainment predicted on the basis of their age and IQ, taking into account regression effects.

Exclusionary criteria and discrepancy definitions of dyslexia are becoming increasingly controversial (for reviews see T. Miles & E. Miles, 1999; Morrison & Siegel, 1991; Reason, Fredrickson, Hefferman, Martin & Woods, 1999; Stanovich, 1991). At a fundamental level discrepancy definitions and exclusionary criteria require an acceptance that IQ tests accurately and validly measure children's intellectual potential. Some

commentators have argued IQ tests cannot validly measure potential (e.g. Fletcher & Morris, 1986; Howe, 1997). The issue is further confused by the reciprocal relationships between reading ability, reading experience and verbal intellectual ability. Verbal ability tends to decrease if a child has less reading experience due to low reading ability (Stanovich, 1986b; Stanovich, 1993). Morrison and Siegel (1991) assert that for discrepancy criteria to be valid, IQ tests scores must be independent of reading ability. If some children would have got a higher score on an IQ test if their reading ability and experience were greater it is not truly measuring their 'potential'.

The use of discrepancy criteria implies that children with low intellectual ability are not expected to achieve adequate reading skills. However, empirical evidence does not support this hypothesis. Research has shown that adequate word level reading skills can be achieved by children with low IQ scores (see Siegel, 1988; Siegel, 1992; section 7.3). The average correlation between reading skill and general intellectual ability in the general population is low. Stanovich (1986a) estimated the correlation to be 0.31 however, others would dispute this estimate (e. g. Rayner & Pollastsek, 1989 estimate the correlation to be 0.7). If a high IQ was sufficient for good reading skills to develop, a much higher correlation would be found than Stanovich estimated. Furthermore, experimental comparisons have indicated that children with specific reading difficulties (whose reading ability is discrepant from their IQ) and general reading difficulties (whose reading ability is commensurate with their IQ) differ on few cognitive, phonological or developmental measures except IQ (Felton & Wood, 1992; Fletcher et al., 1989; Friedman & Stevenson, 1988; Rutter & Yule, 1975; Share, McGee, McKenzie, Williams, & Silva, 1987; Shaywitz, Fletcher, Holahan & Shaywitz, 1992). A study by Vellutino et al. (1996), suggests that poor readers who are readily remediated differ from those who are difficult to remediate not in IQ levels, but on tasks that tap phonological skills such as short-term memory and auditory-verbal awareness. As children with specific and general reading difficulties appear to have similar cognitive weaknesses and IQ level is not a good predictor of the

success of remedial teaching, it seems unnecessary to split the two groups when implementing word level reading intervention programmes.

Partly in response to these criticisms, a working party of the Division of Educational and Child Psychology of the British Psychological Society formulated a new definition of dyslexia:

“Dyslexia is evident when accurate and fluent word reading and/or spelling develops very incompletely or with great difficulty. This focuses on literacy learning at the ‘word level’ and implies that the problem is severe and persistent despite appropriate learning opportunities. It provides the basis for a staged process of assessment through teaching.” (p. 18) (Reason et al., 1999)

The move to abandon discrepancy definitions has not been universally welcomed (see Nicolson, 1996, 2001). Exclusionary definitions have been useful in research. Examining a child with a high IQ, but poor reading has helped psychologists identify the cognitive weaknesses that impact specifically on reading. These weaknesses would be more difficult to determine in children with general learning difficulties. Children with general learning difficulties may have poor reading for the same reasons as children with specific reading difficulties, but it is more difficult to detect which of their cognitive deficits impacts on their reading difficulties. Furthermore, some would argue that discrepancy definitions can be useful in practice; they enable teachers and psychologists to identify bright children whose literacy skills are not commensurate with their intellectual ability. If a discrepancy definition is used a child with an above average IQ score could have a reading score within the average range, but be judged as dyslexic. Such a child may be able to partially compensate for their specific cognitive weaknesses (such as poor phonological skills) by using contextual cues. It is argued that such children require extra help with mechanical literacy skills so they can communicate their verbal ideas on paper.

Definitions that use positive indicators of dyslexia are becoming more popular. Such definitions identify the specific cognitive deficits that a dyslexic individual is expected to display. An example of a recent definition incorporating positive indicators is given below.

“Dyslexia is a specific form of language impairment that affects the way in which the brain encodes the phonological features of spoken words. The core deficit is in phonological processing and stems from poorly specified phonological representations. Dyslexia specifically affects the development of reading and spelling skills but its effects can be modified through development leading to a variety of behavioural manifestations.” (Snowling, 2000)

Definitions using positive indicators overcome the problems of discrepancy definitions (IQ levels are not a diagnostic criteria). They also identify a homogenous group of individuals as dyslexic. Whilst the definition proposed by Reason et al (1999) may include children’s whose reading problems are caused by different cognitive factors (and possibly non-cognitive factors, such as an unsupportive home background), children identified using the Snowling et al (2000) definition will all share a core problem with phonological processing. If dyslexic children are a homogenous group then they are more likely to respond similarly to interventions. However, the strength of Snowling’s (2000) definition rests on the universality of a phonological processing deficit, in children with persistent and severe reading problems, which cannot be explained by environmental factors. There is also the problem of deciding what level of phonological deficit is required for an individual to be considered dyslexic, what tests should be used to determine the severity of the deficit (e.g. short term memory tasks, spoonerisms, phoneme deletion), and whether the level of deficit required should be adjusted depending on the child’s intellectual ability. Section 5.2 examines the evidence for the phonological representations hypothesis.

5.2 The phonological representations hypothesis and the alternatives.

The phonological representations hypothesis proposes that, “dyslexic readers have poorly specified phonological representations” (p.35) (Snowling, 2000). A brief account of how the phonological representations hypothesis explains reading difficulties is given by Reason et al (1999). Snowling (2000) describes the theory and supporting evidence in more detail. At the biological level it is hypothesised that a genetic difference causes a biological abnormality. This biological abnormality results in weak representations of phonological information. Frith (1997) has suggested that abnormalities in the perisylvian region of the left hemisphere may be responsible for weak phonological representations. Phonological information comprises the sounds that make up spoken language. As dyslexic children’s phonological codes are weak, it is difficult for them to link phonemes (the sounds that make up spoken words) with graphemes (the letter sequences that make up written language). The difficulties the dyslexic child experiences when learning to read and spell are believed primarily to be the result of these weak grapheme phoneme links. Evidence for the phonological deficit hypothesis can be divided into two areas: studies that indicate phonological abilities predict reading abilities and studies that indicate dyslexic children have weak phonological representations.

5.2.1 Evidence that suggest phonological abilities predict reading ability

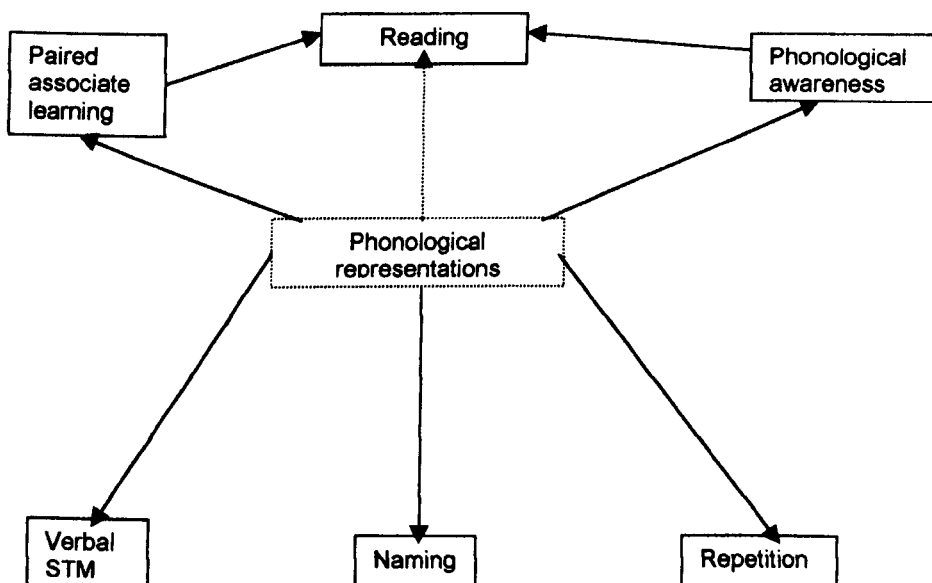
For the phonological representations hypothesis to be upheld, phonological abilities must be causally related to reading ability. If reading ability is not determined (at least in part) by phonological abilities, then the phonological representations hypothesis cannot explain the core reading deficit of dyslexic children. Several studies have indicated that phonological abilities are predictive of future reading ability, even if they are measured before schooling begins (Bradley & Bryant, 1983; Bryne & Fielding-Barnsley, 1989; Ellis & Large, 1987; Lundberg, Olofsson & Wall, 1980; Share, Jorm, Maclean & Mathews, 1984; Singleton et al., 2000). Furthermore, training in phonological skills before reading instruction begins has been shown to increase later reading skills (Bradley & Bryant, 1983;

Bradley & Bryant, 1985; Bryne & Fielding-Barnsley, 1991; Bryne & Fielding-Barnsley, 1995; Lundberg et al., 1988). These results suggest that phonological representations influence later reading ability. Therefore it is theoretically possible that weak representations could explain dyslexic children's reading difficulties.

5.2.2 Evidence that suggests dyslexic children have weak phonological representations

Snowling (2000) argues that the weak phonological representations of dyslexic children cause six areas of deficit. The links between weak phonological representations and the dyslexic individual's weaknesses are shown in Figure 3 below. In some of these areas the direct link between weak phonological representations and poor performance is clear (e.g. phonological awareness tasks). However, in other areas, such as paired associate learning, several cognitive functions could influence performance. Snowling (2000) suggests that weak phonological representations could explain dyslexic children's poor performance on these tasks.

Figure 3. Causal links between the different phonological processes and reading
(adapted from Snowling 2000, p. 59)



There is ample evidence that dyslexic children and adults phonological awareness is lower not only than age matched controls, but also than reading matched controls (Bradley & Bryant, 1978; Bruck, 1990; Manis, Custodio & Szeszulki, 1993; Swan & Goswami, 1997). Phonological awareness has been measured through tasks such as rhyme production, phoneme deletion and spoonerisms. The finding that dyslexic children's scores are worse than younger children with similar reading ability indicates that their poor phonological awareness is not a consequence of their poor reading.

Studies indicate that dyslexic children are poorer than chronological aged matched controls at paired associate learning tasks when at least one of the stimuli is verbal (Vellutino, Scalon & Spearing, 1995; Vellutino, Steger, Harding & Spearing, 1975). Dyslexic children have also been shown to be poorer than younger reading age matched controls at paired associate learning (Windfur, 1998). Snowling (2000) argues that deficits in paired associate learning can be explained in terms of weak phonological representations. If the phonological code for the verbal stimuli is weak it will be harder to link with another visual or verbal stimuli.

Studies indicate that dyslexic children's auditory-verbal short term memory is poorer than normally developing children of their own age (Hulme, 1981; Shankweiler, Liberman, Mark, Fowler & Fischer, 1979), but similar to younger reading matched controls (Johnston, Rugg & Scott, 1987). One could conclude from these results that short term memory increases with reading skill and consequently dyslexic children's memory is poorer because they are poor readers (see section 4.1, for a discussion of the difficulties in interpreting negative findings in attainment match studies). However, Snowling (2000) argues that there are no obvious reasons why reading skill should improve verbal memory. There is also evidence to suggest that dyslexic adults continue to have short-term memory problems (Snowling, Nation, Moxham, Gallagher & Frith, 1997) and that these problems persist even when dyslexic adults become competent readers (Paulseu et al., 1996). As auditory-verbal information is held in short-term memory as a phonetic code, Snowling

(2000) argues that dyslexic individual's short-term memory deficits can be explained in terms of weak phonological codes.

Experiments have indicated that dyslexic children have problems naming pictures. Snowling, Wagtendonk & Stafford (1988) found that dyslexic children performed more poorly on a picture naming task than normally developing children who were matched on a test of receptive vocabulary. This suggests that dyslexic children's difficulties are not with semantic understanding, but rather with accessing the phonological codes for the words. The dyslexic children did not perform more poorly than a group of younger children who were of similar reading ability. Swan & Goswami (1997) did find a difference between dyslexic children and younger chronologically age matched controls on a picture naming task. However, Snowling (2000) notes that this is a rare finding. Evidence has also indicated that both dyslexic adults and children are slower at naming familiar objects; this is true even if each stimulus is presented singularly, rather than asking the participant to rapidly name a sequence of stimuli (Bowers & Swanson, 1991; Felton & Wood, 1989; Pennington, Orden, Smith, Green & Haith, 1990). These results are also consistent with difficulties accessing the phonological codes for words. Another empirical finding that supports the phonological representations hypothesis is dyslexic children's difficulties repeating polysyllabic non-words. Snowling (1981) found that dyslexic children did not have marked difficulties repeating real polysyllabic words, but did perform significantly worse than a group of younger reading matched controls when repeating polysyllabic real words.

Overall, the evidence suggests that weak phonological representations tend to cause reading difficulties and that the majority of dyslexic children show evidence of weak phonological representations. However, for the phonological deficit definition of dyslexia to be accepted weak phonological representations must be universal. It is not known whether other cognitive weaknesses (e. g. visual memory difficulties) could cause significant reading difficulties. Rack (1997) highlights the case of 41-year-old man, whose

reading and spelling ability was significantly below his above average IQ. Investigations failed to reveal any phonological weaknesses. The 'Freedom from Distractibility' index for his WAIS results was above average and not significantly discrepant from his other index scores. He had no problems with spoonerisms and had no difficulties in decoding words. Furthermore a single case study reported by (Howard & Best, 1997) suggests that poor phonological awareness, does not necessarily cause reading difficulties. However, case studies of dyslexic individuals without phonological awareness difficulties or normal readers with phonological difficulties are rare.

Alternative cognitive theoretical accounts of dyslexia include the temporal processing hypothesis (Tallal, Miller, Jenkins & Merzenich, 1997), the cerebellar deficit hypothesis (Nicolson & Fawcett, 1995; Nicolson & Fawcett, 2001), the double deficit hypothesis (Wolf & Bowers, 1999; Wolf & Bowers, 2000; Wolf & O'Brien, 2001), the magnocellular hypothesis (Stein, Talcott & Witton, 2001) and the syndrome hypothesis (Miles, 1993) all acknowledge the importance of the dyslexic children's phonological deficits in learning to read. These alternative accounts differ from the phonological deficit hypothesis because they emphasise the importance of other signs and symptoms or highlight factors that may cause the weak phonological representations of dyslexics. These extra deficits may also impact on dyslexic children's reading development. However, the alternative theories do not deny the importance of weak phonological representations in causing dyslexic children's reading difficulties.

5.3 The arithmetic difficulties of children with dyslexia

Research concerning the mathematical abilities of dyslexic children is sparse. Reading and spelling difficulties are integral to a diagnosis of dyslexia. Arithmetic or mathematics difficulties are not included in definitions of dyslexia; therefore it is possible that dyslexic children are not impaired in this area. However, the empirical studies that have been conducted indicate that at least some dyslexic children have difficulties with number skills.

Steeves (1983) compared dyslexic children with normally developing children on a test of school mathematics. 54 dyslexic boys (aged 10-14 years) were split into two groups according to their score on a non-verbal reasoning test. Dyslexic boys who achieved scores above the 90th centile on a non-verbal reasoning test (DH) were compared with normally developing children in a high mathematics set (NH). Dyslexic boys who achieved a score below the 50th centile on the non-verbal reasoning test (DA) were compared with normally developing children in an average mathematics set (NA). The NH boys achieved similar scores to the DH boys on the non-verbal reasoning test. The DH boys attained lower scores than the NH boys on the school mathematics test. On average the NA children achieved slightly higher scores than the DA children on the non-verbal reasoning test. The DA boys performed more poorly than the NA children on the schools mathematics test. Both groups of dyslexic boys achieved lower scores than the NA children on the Wechsler Memory Test.

Steeves (1983) concluded that dyslexic children with above average non-verbal reasoning have lower mathematical attainment than that of non-dyslexic children of similar non-verbal ability. As the DH boys had high non-verbal ability, Steeves excluded perceptual weaknesses/confusions or spatial processing problems as possible causes of their mathematical weaknesses. Steeves (1983) attributed their difficulties to their impaired memory abilities. Although this explanation is a strong possibility, it is conceivable that other factors (that were not measured in this study) contributed to the dyslexic boys' poorer maths scores. Furthermore, this study does not provide conclusive evidence for mathematical weaknesses in dyslexic children of average non-verbal ability. Although the DA children performed more poorly than the NA children on the test of mathematical attainment this may be due to their poorer non-verbal ability.

Miles, Haslum & Wheeler (2001) also found evidence of the mathematical difficulties of dyslexic children. He identified 269 children from the 12905 children in the British Births Cohort study as dyslexic. The dyslexic children had a discrepancy of more

than 1.5 standard deviations between their estimated intelligence (derived from their scores on similarities, a test of verbal reasoning and matrices a test on non-verbal reasoning) and their single word reading or spelling score. The dyslexic children also showed deficits on at least two of the four supplementary tests administered (a recall of digits test, reciting the months forwards, reciting the months backward and left-right discrimination). These dyslexic children achieved significantly lower scores than a control group of children on an un-timed school mathematics test administered at 10 years of age. This difference remained significant even after reading ability had been partialled out. The control children did not differ from the dyslexic children in estimated intelligence.

The results of Miles et al (2001) suggest that, on average, dyslexic children perform more poorly than normally developing children, when they are matched in terms of reasoning ability. What these studies do not tell us is whether all dyslexic children have difficulties with mathematics or whether only a proportion are impaired in this way. Joffe (1981) gave a test of arithmetic to 51 dyslexics (aged 8 years to 17 years) who had been diagnosed at the University of Aston. She also gave the test to a similar number of normally developing controls. All the participants were of average or above average intelligence. Joffe (1981) found that about 10% of the dyslexics scored well above expectations, whilst about 60% performed well below expectations. Joffe concluded that only 60% of dyslexics had significant mathematical deficits. Miles (1991) criticised this conclusion; he rightly highlights the possibility that the 40% who performed reasonably well on the mathematics test could have problems with other areas of mathematics that were not assessed, e.g. reciting multiplication tables.

5.4 Are particular areas of mathematics difficult for dyslexic children?

Anecdotal reports often highlight dyslexic individuals' difficulties with mental arithmetic. They typically find it difficult to recall number facts such as $7 \times 6 = 42$ or $13 - 7 = 6$. Miles (1987) reported the case of an undergraduate who did not know that $6 + 7 = 13$, but he had a way of working it out. Empirical evidence supports the hypothesis that

dyslexic children have particular difficulties with mental arithmetic. Miles (1983) compared the percentages of normally developing and dyslexic children who stumbled, when reciting their multiplication tables. A child was only classified as able to recite correctly if he or she did it without any pauses or hesitations. In the 7- and 8-year-old age group 90% of the dyslexic children had difficulties, whilst only 71% of the normally developing children did. In the 9-to 12-year-age group 96% of the dyslexic children had difficulties, whilst only 51% of the normally developing children did. Even in the 13- to 18-year-old age group 85% of the dyslexic children had problems, in contrast only 53% of the normally developing children did. Miles reported dyslexic children having similar difficulties on a mental subtraction task.

Ackerman, Anhalt & Dykman (1986) examined the mental arithmetic skills of reading disabled children. The reading disabled children and a group of normally developing controls were asked to say whether visually presented sums were correct as quickly as they could. Out of the 24 reading disabled children 16 were classified as 'slow and inaccurate', whilst 20 of the 24 control children were classified as 'fast and accurate'. Pritchard, Miles, Chinn & Taggart (1989) reported similar results. A group of 15 dyslexic boys (aged 12 to 14 years) were compared to a group of age-matched normally developing boys. The boys all attended private schools that had specialist dyslexic units. All the boys scored above the 25th centile on a standardised test of non-verbal reasoning. The dyslexic boys attended the schools' specialist units and had spelling ages at least 2 years behind their chronological ages. The normally developing boys had spelling ages of 12 years or above. Both groups of boys were asked questions from the multiplication tables. They were asked only to answer the question if they knew the item immediately without 'working it out'. The dyslexic boys knew significantly less multiplication facts 'in one' than the control boys did.

Turner Ellis, Miles & Wheeler (1996) also studied dyslexic children's multiplication fact recall. All the children in the Turner Ellis et al (1996) study were male, attended private schools, suffered from no gross handicap or problems of social adjustment and scored above the 50th centile on a test of non-verbal ability. Three different groups of children were compared. The dyslexic children had a spelling age at least 18 months behind their chronological age and had at least four positive indicators on the Bangor Dyslexia Test (Miles, 1982, 1997). The chronological age match and the spelling age match children had a spelling age that was no more than six months behind their chronological age and had no more than three positive indicators on the Bangor Dyslexia Test. The dyslexic children were divided into three different age bands: 9 years 5 months to 11 years 4 months, 11 years 5 months to 13 years 4 months, and 13 years 5 months to 15 years 4 months. The chronological age match children were of a similar age to the dyslexic children, whilst the spelling age match children were younger as their chronological ages were similar to the dyslexic children's spelling ages.

The boys were presented with every pair of multiplication sums between 1×1 and 12×12 on a computer. The children had to type the answer as quickly as possible. After 22s the sum disappeared. The dyslexic children were slower at responding than the chronological age match children. However, they were faster at responding than the younger spelling age match children.

A study conducted by Erenberg (1995) examined both the accuracy and the strategies of children with specific learning difficulties (a proportion of whom would have been dyslexic) and non-learning-disabled children, when attempting multiplication sums. The participants were divided into three groups, which were matched for age, mental ability, socio-economic status, grade level and amount of weekly instruction. The non-learning disabled children were achieving grade level in maths and obtained average or above average standard scores on a standardised maths test. The learning-disabled children were divided into two groups: those achieving grade level in maths and those who

were achieving below grade level in maths. The learning disabled children who achieved grade level in maths achieved an average or above average score on the standardised maths test, whilst the learning-disabled students who were underachieving in maths achieved scores at least one standard deviation below the mean. Two tests of multiplication were administered, each consisted of 24 sums. In the first test the sums were presented on flash cards, in the second test the sums were presented on a written worksheet. After each sum in the flash card test the participants were asked to explain how they got to their answer. Each sum in the flash card task was timed. A child's strategy was coded as 'rapid automatic' if they produced a correct answer in 3s or less. Accurate responses that took between 3s and 4s to produce were coded as 'delayed automatic' if the child indicated that they had used a retrieval strategy. Reconstructive strategies included skip counting, repeated addition, adding one more set, building on known facts through the use of an anchor point, twice as much as a known fact, use of patterns, guessing and use of manipulatives. A child's response was coded as reconstructive if it took longer than 2.6s to produce and either the explanation they gave matched one of the above strategies or they were observed to carry out one of the above strategies (e.g. use of subvocal counting or using manipulatives). Incorrect responses could be coded as reconstructive.

When the results of both tests were combined the non-learning disabled children achieved scores of 95% or higher, the learning-disabled children who were achieving grade level in maths achieved a mean accuracy score of 81% and the learning-disabled children who were not achieving grade level in maths achieved a mean accuracy score of 49%. The non-learning disabled children used 'rapid automatic' strategies on 70% of trials. The learning-disabled children who were achieving grade level at maths used 'rapid automatic' strategies on 42% of trials and 'reconstructive strategies' on 58% of trials. The learning disabled children who were not achieving at grade level in maths used 'rapid automatic' strategies on only 10% of trials; however, instead of using systematic reconstructive strategies, they relied primarily on guessing. Overall, these results indicate that the

learning-disabled children were less accurate at recalling multiplication facts than the normally-developing controls. The learning-disabled children who were achieving grade level maths had to rely on reconstructive strategies to greater extent than the normally achieving control children, but by doing so they achieved better results than the underachieving learning-disabled children, who relied primarily on guessing.

Case reports and informal reports by specialist teachers of dyslexic children suggest that at least some dyslexic children have problems with mathematics that are not directly related to mental arithmetic. Critchley (1970) reported three case studies of dyslexic children who have problems understanding place value. Spring & Capps (1974) assert that dyslexic children find it difficult to work symbolically, i.e. without concrete aids. Kibel (1992) also asserts that dyslexic children have particular difficulties understanding procedures that are not supported by concrete aids. She uses Alex, a fourteen year old dyslexic boy, as an example. Alex could not complete subtraction tasks that involved regrouping until the process was illustrated using Dienes blocks. E. Miles (1992) argues (from her experience as a specialist teacher) that dyslexic children have particular difficulties understanding the symbolic language used in mathematics; this includes mathematical terms, Arabic numbers and mathematical symbols. However, all these reports must be treated with caution until they are empirically tested with large groups of dyslexic children.

Overall, the weight of evidence suggests that dyslexic children are less proficient than their normally developing peers at recalling number facts. There are isolated case reports of dyslexic children having difficulties with place value understanding and working without concrete materials, but these have not been confirmed by empirical studies.

5.5 Possible causes of dyslexic individuals' arithmetic weaknesses

Joffe (1990) outlines four cognitive weaknesses characteristic of dyslexic children. She believes that three of these cognitive weaknesses have a significant impact on dyslexic children's ability to learn mathematics. The four areas of cognitive weakness are: verbal

labelling, abstracting, transferring knowledge from one domain to another, and short-term memory. Verbal labelling is the ability to learn and access the verbal names for objects and symbols. Joffe (1990) argues that a deficit in verbal labelling affects dyslexic children's ability to learn the names of numbers and mathematical symbols. Abstraction is the ability to understand and apply patterns and rules. Joffe (1990, 1981) argues that a weakness in abstracting results in dyslexic children having difficulties understanding and applying the rules that govern the number system. For example, she argues that dyslexic children have difficulties understanding the base ten system and place value. This leads to difficulties with multi-digit addition. Joffe also asserts that poor abstraction leads dyslexic children to have difficulties understanding the pattern of multiplication tables. Transferring knowledge from one domain to another can help children grasp new topics. For example, if you have a firm grasp of the base ten number system, it should facilitate an understanding of the metric system of measurement. Joffe (1990) asserts that dyslexic children require "more explicit exposition and discussion of the relationships and how one situation relates to another" (p. 9). Joffe (1990) disputes the link between dyslexic children's short-term memory deficits and their difficulties learning arithmetical procedures and multiplication facts. She believes that these deficits are better explained by verbal labelling deficits.

There are several limitations to the explanations of the mathematical weaknesses of dyslexic children put forward by Joffe (1990). Firstly, they are based on clinical findings not empirical research. Joffe (1990) does not cite empirical evidence to support her claims that dyslexic children experience the difficulties she describes. For example, she does not cite any empirical studies that support her claim that place value understanding is impaired in dyslexic children. Secondly, her descriptions of dyslexic children's cognitive weaknesses are not consistent with current theory and empirical evidence. No major theoretical account of dyslexia maintains that either abstraction or transferring knowledge are core deficits in dyslexia. Empirical evidence has indicated that dyslexic children can

score highly on tests of non-verbal reasoning that require children to understand patterns and rules (e.g. Steeves, 1983).

Miles (2001) asked specialist dyslexic teachers' to decide how difficult different mathematics questions would be for dyslexic children and explain their ratings. The judges rated questions as difficult if they required procedures or number facts to be recalled or if they used unfamiliar language. Of the 72 questions in the mathematics test administered 13 were particularly difficult for the dyslexic participants (i.e. the pass rate for the dyslexic children was at least 20% lower than for normal children). Three of the particularly difficult questions concerned fractions, three multi-digit division, two multi-digit multiplication, two recognising mathematical terminology, one multi-digit subtraction, one decimals and one understanding mirror writing.

Miles et al (2001) used the specialist teachers comments, and current knowledge about the particular cognitive weaknesses of dyslexics, to formulate possible reasons for the dyslexic children's mathematical weaknesses. The specialist teachers often judged an item to be difficult if it required the child to recall number facts or procedures. This could account for the dyslexic children's particular difficulties with multi-digit arithmetic. A poor short-term memory is a well-reported empirical finding (see section 5.2.2); therefore these judgements are consistent with current research. Another factor also commented on by the specialist teachers was the familiarity of the language used. Questions that used unfamiliar language were judged to be particularly difficult for dyslexics. Miles et al (2001) suggest that dyslexic children require more pairings to link verbal labels with particular concepts. Some studies have suggested that dyslexic children are weaker at paired associate learning (Done & Miles, 1978; Vellutino, 1979; Vellutino et al., 1995; see section 5.2.2 for discussion of these findings). Miles et al (2001) also speculate that some dyslexic children will have "disorders in their awareness of space" (p. 15). Such a disability could account for the dyslexic children's particular difficulties with mirror writing and contribute to their difficulties with multi-digit arithmetic. However, no

empirical findings are reported that indicate that children with dyslexia have poorer spatial awareness than controls.

Although Miles et al (2001) link current evidence about dyslexic children's cognitive weaknesses with the questions they found particularly difficult, the mathematics test employed was not designed to measure specific number skills independently. For example, question 15, which dyslexic children found particularly difficult, consisted of a multi-digit division sum, written as a 'story problem' (i.e. using words rather than symbols). Dyslexic children may have found this question difficult because they could not read or understand the words, convert the words into an appropriate sum, recall the procedures for multi-digit division, or because they could not recall the appropriate number facts correctly. Without having tests that tap specific number skills directly, excluding other potentially confounding factors (e.g. reading) it is not possible to determine which number skills are differentially difficult for dyslexics.

A logical way to proceed is to formulate hypotheses that specify, which number skills will be difficult for dyslexic children on the basis of current descriptions of their cognitive deficits and test these empirically using specifically designed tests. In the present study the following hypotheses were formulated using the phonological representations theory as it has withstood the most empirically scrutiny. Three specific hypotheses are put forward.

1. Dyslexic children will count more slowly than their normally developing peers.

Dyslexic children perform more poorly on rapid naming tasks than their normally developing peers. Snowling (2000) argues that this is because they have greater difficulty accessing the weaker phonological representations of the names required. Counting quickly requires rapid access to the phonological codes for number words, consequently one would expect dyslexic children to be slower than normally developing children.

2. Dyslexic children will be poorer at recalling number facts than their normally developing peers. Short-term memory and counting speed contribute to the learning of number facts. Evidence has shown that dyslexic children are poorer than normally developing children on short-term memory tasks. The phonological deficit hypothesis suggests that dyslexic children will be slower counters. The studies reviewed in Chapter 3 indicated that number facts are learnt by holding the problem integers in the phonological loop, whilst the answer is calculated. For addition and subtraction sums counting methods are used. If the memory trace of the problem integers decays before the answer is calculated, the association between the problem and the answer will not be strengthened. Consequently, one would expect dyslexic children to learn number facts more slowly and be reliant on counting procedures for longer periods. Even when the number fact is learned, one would expect dyslexic children to take longer to access them, because of their difficulties accessing phonological codes from long term memory.

3. Dyslexic children will not have particular difficulties with place value understanding. Understanding place value requires children to learn particular rules, which link the positions of digits with values. Once the basic rules are understood they can be applied to new numbers. The ability to understand and apply rules is more reliant on abstract reasoning ability rather than memory, therefore one would not expect dyslexic children to be particularly weak at place value understanding tasks. Their place value understanding will be related to their abstract reasoning ability.

6 Study One - The relationship between memory, non-verbal reasoning and number skills in children aged 7-11 years

6.1 Rationale

This study examined normally achieving children's performance on *Maths Suite*, a computerised number skills assessment package. *Maths Suite* was designed by the author to assess three core number skills (counting speed, place value understanding and number fact recall). Study One had three main aims: to describe how these core number skills develop throughout Key Stage 2, to provide norms for normally developing children aged 10 and 11 years with which to compare the SAD children with and to examine the relationships between cognitive and number skills in children aged 7 to 11 years.

Computerised assessment was chosen to measure number skills in this study because the measurement of number fact recall and counting speed requires accurate time recording. The computer software allowed individually timed items to be presented to large numbers of children. This would not have been possible using conventional assessment. Teachers (who administered the tests) could not be expected to time each item. The computer software ensured accurate presentation with each item being presented to each child in the same way. Item times were recorded accurately. Furthermore, studies have reported that children prefer computerised assessment and feel less threatened by it (Singleton, 2001; Singleton, Horne, & Vincent, 1995; Watkins & Kush, 1988). For a discussion of the advantages and disadvantages of computerised assessment see Singleton (1997; 2001).

Maths Suite was administered to a large number of children in mainstream schools so that the performance of children with Specific Arithmetic Difficulties and the performance of children with dyslexia could be compared to their normally achieving peers. In order to collect sufficient data within the tight time and budget constraints of a PhD some procedures that would have been optimal could not be implemented. Informal

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pilot testing was conducted to ensure the program ran smoothly and that the items were in the appropriate difficulty range. Only one full-scale school study was viable, therefore formal pilot testing to ascertain the precise levels of item difficulty was not conducted. Instead, previous research was examined to estimate relative item difficulty. The rationale behind the item ordering is explained in section 6.2.3.

The version of *Maths Suite* sent out to schools included cut-off rules. This meant that when children got items wrong repeatedly they were not presented with further items within that sub-test. Informal pilot testing with a trials version of *Maths Suite* (that did not contain discontinuation rules) generated a great deal of anxiety and stress in some children who experienced repeated failure. It was therefore decided that sending out a version without discontinuation rules would be ethically dubious and also a disincentive for teachers to complete the trials.

6.2 Method

6.2.1 Design

Assessing the validity and reliability of Maths Suite

Concurrent validity of *Maths Suite* was assessed by examining the correlations between *Maths Suite* sub-test scores and scores on an established pencil and paper maths test (the Numeracy Progress Test; *NPT*; Vincent & Crumpler, 2000). Calculating the alpha coefficient established the reliability of the *Most* and Number Facts *Maths Suite* sub-tests. The composition of *Maths Suite* is explained in section 6.2.3. Centiles were produced for two age groups (9:6-10:5 and 10:6 to 11:5) for the Number Facts test and for one age group for the *Most* test (9:6-11:5).

Examining number skills development throughout Key Stage two

The development of children's number skills was examined by comparing the performance of the children in different age bands on the three *Maths Suite* sub-tests.

The relationships between number and cognitive skills

The relationships between the three core number skills: place value understanding, counting speed and number fact recall, and the three cognitive abilities: non-verbal reasoning, auditory-verbal sequential short-term memory, and visual-spatial short term memory were examined.

6.2.2 Participants

The author planned to test 20 children from each of 20 schools in England and Wales (400 in total). Notices calling for schools to participate were posted on two Special Educational Needs E-mail forums. Schools who had visited an educational software exhibition and had shown an interest in the educational software developed by the Hull University dyslexia team were contacted directly. As the schools contacted via these methods tended to be in economically disadvantaged areas, schools in more economically advantaged areas of East Yorkshire were contacted directly. The schools were offered the incentive of £100 worth of educational software, to encourage them to participate in the study¹. As data from only 12 schools was returned in the first phase of testing, further schools were recruited in the second phase. Three of the schools in the second phase of testing had responded to the previous requests for assistance, but they had not been selected. To ensure a reasonable socio-economic mix of schools in phase two, an inner-city school was contacted directly.

¹ The author is grateful to Lucid Research Ltd. for generously donating this software.

Table 1. Participant school characteristics

ID	Type	LEA/County	Location	Employment status of families	Pupils per year group	Students on SEN register	% of free school meals
Phase 1 Schools							
1	Primary	Derby	Suburban	Unskilled/Unemployed	52	115	56
2	Junior	Kirklees	Suburban	Unskilled/unemployed	140	70	65
3	Junior	York	Suburban	Unskilled/Unemployed	75	100	27
4	Junior	East Yorkshire	Suburban	Skilled/Professional	70	25	3
5	Primary	East Sussex	Suburban/Rural	Unskilled/Skilled/Professional	30	60	6
6	Primary	Leicestershire	City/Estate	Unskilled /Skilled	39	65	11
7	Primary	Suffolk	Suburban	Skilled/Professional	30	33	2
8	Primary	Lincolnshire	Rural	Unskilled/ Skilled	40	39	28
9	Junior	Bridgend	Suburban	Unskilled/Skilled/Professional	60	44	a
10	Junior	Sheffield	Inner city	Unskilled/Unemployed	70	92	31
11	Primary	Carmarthanshire	Suburban	Skilled/Professional	30	56	a
12	Primary	Middlesex	Suburban	Unskilled/Skilled (high percentage of asylum seekers)	65	150	23
Phase 2 Schools							
13	Primary	Birmingham	Inner-city	Skilled/Unskilled	30	96	72
14	All age	Dorset	Rural	Skilled/Professional	20	10	0
15	Primary	Torfaen	Rural/Suburban	Skilled/Professional	30	25	a
16	Primary	East Yorkshire	Suburban	Skilled/Professional	10	14	4

Note. All the schools were state schools with the exception of school 14, which was an independent school.
*a*Details about percentage of pupils receiving free school meals not available for Welsh schools.

The schools that agreed to participate came from a variety of different areas in England and Wales, and were therefore believed to be broadly representative of the school population as a whole. The characteristics of the schools that participated are shown in Table 1. Each contact teacher was asked to say which of three locations best described

their school (rural/suburban/inner city). They were also given five choices to describe the employment status of the pupils' families (mainly unemployed/a mixture of unemployed and unskilled/a mixture of unskilled and skilled/a mixture of skilled and professional/mainly professional). Some teachers felt unable to use these categories so their responses were recorded verbatim. The percentage of children receiving free school meals was obtained from each school's most recent Ofsted report (reports displayed on www.ofsted.gov.uk/inspect/).

Table 2. Number of children contributed from each participating school grouped by age and sex

	Phase 1 Schools												Phase 2 Schools				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
7:5 to 8:5																	
Boys	1	2	2	1	1	1	2	2	2	2	1	1	0	0	0	0	18
Girls	1	2	2	1	2	3	3	0	2	1	2	3	0	0	0	0	22
Total	2	4	4	2	3	4	5	2	4	3	3	4	0	0	0	0	40
8:6 to 9:5																	
Boys	4	2	2	3	2	1	3	2	2	3	3	3	0	2	2	0	33
Girls	3	4	1	2	2	2	2	2	3	3	3	2	0	1	1	0	31
Total	7	6	3	5	4	3	5	4	5	6	6	5	0	3	3	0	64
9:6 to 10:5																	
Boys	1	3	4	3	2	5	3	3	1	2	2	3	3	4	4	2	43
Girls	3	2	2	4	1	2	1	3	4	1	3	2	4	1	1	0	35
Total	4	5	6	7	3	7	4	6	5	3	5	5	7	5	5	2	78
10:6 to 11:5																	
Boys	2	2	3	3	3	1	4	2	4	3	1	2	3	3	3	1	43
Girls	4	2	4	2	1	3	3	4	1	3	3	3	2	0	0	3	41
Total	6	4	7	5	4	4	7	6	5	6	4	5	5	3	3	4	84
11:6 to 12:5																	
Boys	0	0	0	0	0	1	0	1	0	0	1	0	0	1	1	0	4
Girl	0	1	0	1	2	1	0	1	1	2	0	1	0	3	3	0	13
Total	0	1	0	1	2	2	0	2	1	2	1	1	0	4	4	0	17
Grand Total	19	20	20	20	16	20	21	20	20	20	20	20	12	15	15	6	283

Scottish schools were not included in the sample, as they do not follow the National Curriculum for England and Wales. The contact teachers from the schools who participated in the first phase of testing were asked to randomly select five children from Years 3 to 6. The contact teacher was instructed to compile a list of the children in each year group in alphabetical order. They were asked to choose the first five children whose birthday fell on a randomly selected day (e.g. the 4th). They were instructed to choose no more than three children of either sex in each year group. The first boy and girl selected in each year group also completed three cognitive assessments from the *LASS Junior* computerised assessment suite (Thomas, Singleton, & Horne, 2001). To ensure that there was a large enough comparison group for Study Four, which compared normally achieving children with 10- and 11-year-old children with dyslexia and children with Specific Arithmetic Difficulties, contact teachers from the schools in the second phase of testing were asked to test five 9-year-olds, five 10-year-olds and five 11-year-olds. The same random selection method was used to select the children in phase two. The teachers from the schools who participated in the second phase of Study One only had to administer *Maths Suite*. The amount of testing required was reduced to encourage maximum participation.

The mean age and gender proportions for each of the age groups are shown in Table 1. Some selected children were absent for some tests and some contact teachers were unable to complete the testing due to time pressures, therefore the number of children completing the individual assessments is slightly below the level expected. One school tested an additional child who is included in the study. Overall 283 children participated, 141 boys and 142 girls.

6.2.3 Materials

The pencil and paper maths test

The teachers from the schools in phase one were asked to administer to all the children the appropriate version of the *Numeracy Progress Tests (NPT; Vincent & Crumpler, 2000)*, a written test of numeracy attainment. Children in Year 3 completed *NPT3*, children in Year 4 completed *NPT4*, children in Year 5 completed *NPT5*, and children in Year 6 completed *NPT6*. All versions of the *NPT* are untimed. Spoken instructions are given at the start of the test and scrap paper is provided for rough working. The *NPT* was designed to be consistent with the National Numeracy Strategy in England and Wales. All the versions used in this study include questions in four main areas: place value and counting, addition and subtraction, fractions, and multiplication and division. All versions except *NPT3* include questions involving decimals. *NPT6* has some questions involving percentages.

The computerised number skills assessment

Counting speed, place value understanding and number fact recall were measured using *Maths Suite*, a computerised number skills assessment package designed by the author and programmed by Rik Leedale of Lucid Research Ltd. The children completed a response time measure (*Numbers*) and three core sub-tests. Each core sub-test has spoken instructions (produced by the computer) and practice items that are not scored. The child responds by clicking on the screen using the mouse or by typing digits. The sub-tests are described below.

Numbers is a response time measure. The child is presented with a large digit in the centre of the screen. The child must hit that digit on the keypad as quickly as possible. Six single digits are presented. Any that are incorrectly answered are repeated at the end of the sequence. The computer calculates a mean time for the last five items correctly answered.

Spots measures counting speed and efficiency. The student is asked to count the number of red spots shown amongst yellow ones. There are 50 spots in total. Each spot is 150mm in diameter. In part one (with memory aids) the child is allowed to click on the red spots, which then turn to white. This helps the child keep track of the spots already counted. The number of spots should then be entered at the foot of the screen and an OK icon clicked. If the child wishes to change their answer they have to click on an eraser icon. In part two the student is not allowed to click on the red spots but must still attempt to count them before entering the number as before. There is one practice and three test items in both parts of the sub-test. A demonstration is shown before part one. The number of red spots in the test trials in part one are 24, 19 and 23. The number of red spots in the test trials in part two is 21, 17 and 25. The computer records speed and accuracy.

Most assesses the child's understanding of place value. The child is presented with three bags of money with different amounts shown on each bag. The objective is to click on the bag that contains the greatest sum of money. If the child changes his or her mind he or she can click on another bag. An OK icon must be clicked on after every choice to confirm it. There are two practice items followed by up to 31 test items. If a child answers a practice trial wrongly verbal instructions are delivered to correct him or her. The first 15 test items are always administered regardless of performance. In the first four items the important distinguishing factor is the first digit (e.g. £72, £79, £81). In items 5-8 the important distinguishing factor is the total number of digits (e.g. £7100, £71000, £710000). In items 9-15 the child must understand that a larger number is more than a smaller number even if the first digit is smaller (e.g. £888, £999, £1002). The final items 16-31 require the child to understand positional importance within digits (e.g. £8100, £8092, £8079). The magnitude of the items increases as the child progresses through items 16-31. After item 15 a discontinuation rule applies: if the child gets any four out of five consecutive items incorrect no further items are presented. As items 16-31 all assess the child's understanding of the same construct, it was hypothesised that the main factor influencing

difficulty would be the magnitude of the choices. The computer records the number of correct answers. The numbers used in the *Most* items are shown in Appendix 1.

Number facts assesses children's speed and accuracy when answering visually presented sums in time restricted conditions. The items are presented in three separate blocks of addition, subtraction and multiplication sums. There are two practice items and up to 12 test items in each block. If a child answered a practice question correctly, the computer told them they had given the right answer, if they answered a practice question incorrectly, the computer told them they had got it wrong and gave them the right answer.

The test items within each block were ordered according to their estimated level of difficulty. The *problem size effect* states that the greater the magnitude of the answer the harder a mental arithmetic sum is to compute. Various studies have supported the *problem size effect* for addition, multiplication and subtraction sums (e. g. Ashcraft & Battaglia, 1978; Clapp, 1924; Janssen, De-Boeck, Viaene & Vallaey, 1999; Miller et al., 1984; Norem & Knight, 1930; Parkman, 1972). Adults and children require more trials to learn problems with larger answers and are more likely to make errors on such problems. Retrieval times are longer for problems with larger answers. Children's counting strategies for single digit addition and subtraction (reviewed in section 3.3) are consistent with the *Problem Size effect*. If the digits involved in a sum are larger there is a greater chance of a counting error; larger digits also increase the chance of a child running out of time.

The items in each block were therefore presented in approximate order of answer size. However each item was not always larger than the preceding item because this pattern may have helped children to identify the answer.

No tie questions (e.g. $4 + 4$ or 3×3) were included in the test blocks as they have been found to be easier than non-tie problems (Groen & Parkman, 1972; Miller et al., 1984; Parkman & Groen, 1971; Starzyk et al., 1982). Problems involving 0 and 1 were also excluded from the test blocks because adults and children tend to solve these problems

by rules as opposed to using retrieval or counting strategies (Ashcraft, 1983; Baroody, 1983) (see section 3.4 for further discussion of the problem size effect).

The addition block consists of six items where the sum was less than 10, followed by six items where the sum was between 10 and 20. The subtraction block consists of six items where the subtrahend (first number) was less than 10 followed by six items where the subtrahend was greater than 10. Items where crossing a 10 is not required (e.g. $19 - 5 =$) are given before sums where crossing a ten is required (e.g. $13 - 7 =$) because current knowledge about children's subtraction strategies indicates that crossing a 10 is difficult. When crossing a 10 is not required children who understand tens and units will simply be able to subtract the units. However, when crossing a 10 is required the child (even a child who understands tens and units) must resort to counting, retrieval or more advanced derived fact strategies (e.g. $10 - 7 = 3$, $3 + 3 = 6$) (see section 3.3 for a full discussion of children's subtraction strategies). The items in the multiplication block each consisted of the multiplication of two numbers between 2 and 9. The items used in the addition, subtraction and multiplication fact tests are shown in appendix 2.

In all blocks each sum is presented in a horizontal format (e. g. $2 + 1 =$). The sum is displayed in large type. The child must type in a response and then press the ENTER key within a timed period of 7s. Once the time limit is up the sum disappears. After the sum is removed the child must press the SPACE BAR to access the next sum. The child has to click on an eraser to change an answer. The verbal instructions encourage the child to answer quickly. If the child does not answer within the 7s verbal instructions encourage them to answer more quickly next time. Once a child does not give the correct answer on four consecutive occasions either by giving the wrong answer or by running out of time, the computer moves onto the next block of questions.

The computerised cognitive assessment

LASS Junior (Thomas et al., 2001) is a multifunctional computerised assessment suite for children aged 8 to 11 years. It includes standardised measures of literacy, memory, phonological processing and non-verbal reasoning. In this study three different *LASS* tests were used: *Reasoning*, *Mobile* and *Cave*. *Reasoning* is an adaptive test that estimates non-verbal reasoning ability using a matrix task. The child is shown a matrix containing different shapes; one shape is missing. The child must choose the shape that fits best from six possibilities. They must click on the shape they choose. Each item has a time limit of 60s. The first few items are probe items, which are not scored. The set of test items administered is dependent on the child's performance on the probe items; a poorer performance will result in an easier question set being presented. If the child fails repeatedly on the set of items presented they will also be presented with an easier set, similarly, if they answer the vast majority of the presented set correctly they will be presented with a more difficult set. The computer generates a projected score that estimates the score the child would have achieved if they had attempted all the test items, based on normative data obtained in standardisation. The projected score represents the mean difficulty level of the last three correct items based on normative pass rates.

LASS Mobile is a test of auditory-verbal sequential memory. A picture of a mobile phone is shown on screen. The child hears a string of spoken digits and then has to input them by clicking on the buttons on the mobile phone keypad or by pressing the appropriate number keys on the computer keyboard. The child's answer is recorded as correct if the right digits are inputted in the right order. *Mobile* is a computerised version of forward digit span tests. The first items in the test require the child to recall two digits, the number of digits increases progressively in the later items. *Cave* is a test of visual-spatial memory. A picture of a cave is shown on screen. The cave is divided into eight compartments. The child sees various 'phantoms' (pictures of spiders, daggers, etc.) appear in these compartments. Each phantom disappears before the next one is shown. Once the last

phantom has disappeared, the child must select the ‘phantoms’ they saw (from an array of ‘phantoms’ that include distracter items) and put them back in the correct compartments. There is a time limit for each trial. The temporal order the ‘phantoms’ appeared in does not have to be replicated, but the correct ‘phantom’ must be put in the correct compartments. Scores for both *Mobile* and *Cave* are produced in z score format, which was converted into standard score format (mean 100, standard deviation 15) for the purposes of this study.

6.2.4 Procedure

Phase one testing took place during the summer term of 2001. The contact teachers were asked to conduct testing within an eight-week period. Although in normal circumstances a counter-balanced design would have been appropriate, in this study all schools conducted the computerised assessments before the pencil and paper test. This is because the *NPT* is designed for use in June or July, thus prohibiting its use in April when the materials were sent out. Each school was sent the appropriate *NPT* test booklets, a *LASS Junior* CD and a *Maths Suite* CD. They were also sent two floppy disks to download the data onto, and the appropriate instruction booklets for the assessment materials. They were asked to administer the assessments to the randomly selected children in the standard manner prescribed in the instruction booklets. The downloaded data from the computerised assessments and the completed *NPT* test booklets were returned to the University. The author marked the *NPT* tests. The order of testing for the *LASS Junior* and *Maths Suite* tests was not specified as long as they were completed within the eight week period and before the *NPT*. Phase two testing took place in the autumn term of 2001. The *NPT* could not be used as it is designed for use in the summer term. Teachers were not asked to administer the *LASS Junior* tests, to reduce the overall workload and encourage participation. The procedure for administering *Maths Suite* was the same as in phase one.

6.3 Results

6.3.1 The development of number skills in Key Stage 2

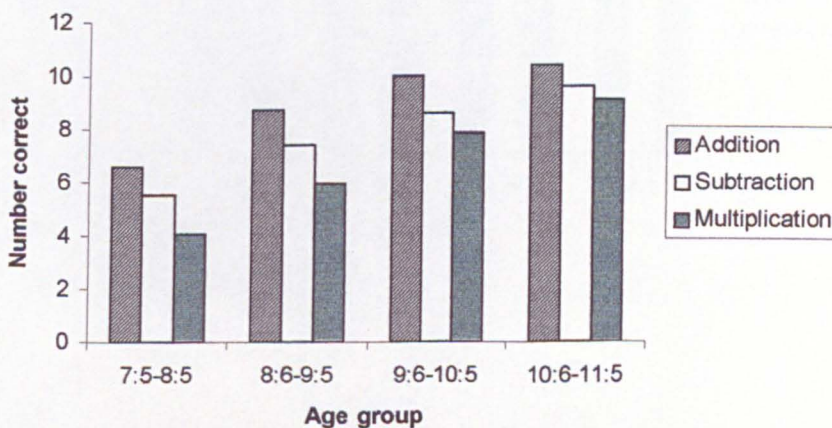
For the purpose of this analysis the children were divided into the five age bands shown in Table 1. Only children in the four younger age bands were considered in the analysis as the top band, containing only 17 children, was small and therefore not a representative sample. If children did not attempt a single item in the addition facts test, their score was excluded from any analysis involving number fact scores. It is likely that these 22 children were simply not pressing the Enter key after their answer, as they did not produce even one wrong answer.

Table 3. Number Fact Mean scores for the four age bands

Test by Age Band	<i>n</i>	<i>M</i>	<i>SD</i>
Addition Facts			
7:5-8:5	37	6.59	3.83
8:6-9:5	58	8.74	2.76
9:6-10:5	73	9.97	2.12
10:6-11:5	76	10.42	1.72
Subtraction Facts			
7:5-8:5	37	5.51	3.36
8:6-9:5	58	7.41	3.39
9:6-10:5	73	8.63	2.96
10:6-11:5	76	9.62	2.17
Multiplication Facts			
7:5-8:5	37	4.05	3.57
8:6-9:5	58	5.91	3.10
9:6-10:5	73	7.90	2.80
10:6-11:5	76	9.14	2.32
Number Facts			
7:5-8:5	37	16.62	9.72
8:6-9:5	58	22.07	8.25
9:6-10:5	73	26.51	6.88
10:6-11:5	76	29.18	5.25

Note. Number facts score equals the total of addition, subtraction and multiplication facts scores.

Figure 4. Number fact scores for the different age groups



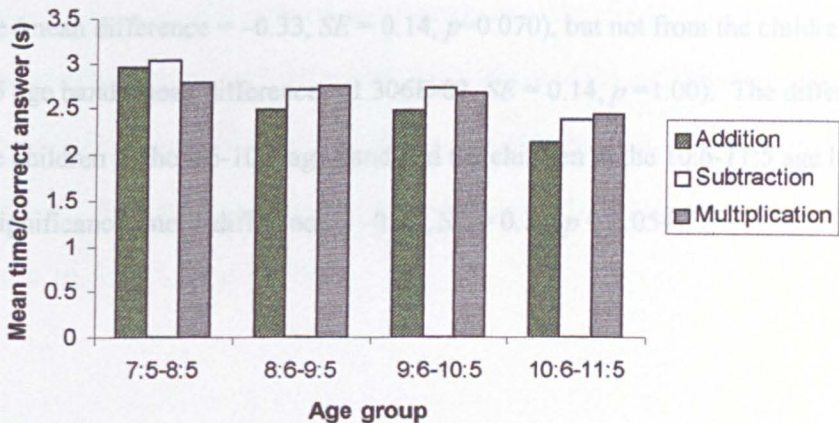
The mean scores shown in Table 3 and indicate that the number of addition, subtraction and multiplication facts recalled increases as children get older. Overall, addition facts are the easiest to recall subtraction facts somewhat harder and multiplication facts the most difficult. A repeated measures ANOVA with one within-participants variable (number fact type) and one between-participants (age band) variable indicated that there was a significant difference between number fact types $F(2,480) = 100.02, p < 0.001$ and between age bands $F(3,240) = 2334.91, p < 0.001$. The interaction between number fact type and age band was statistically significant $F(6,480) = 3.15, p = 0.005$. The younger two groups of children found the multiplication facts particularly hard. Post Hoc Tukey *a* tests indicated that the children in the 7:5-8:5 age band differed significantly from the children in the 8:6-9:5 age band (mean difference = $-1.97, SE = 0.51, p = 0.001$), the children in the 9:6-10:5 age band (mean difference = $-3.45, SE = 0.49, p < 0.001$) and the children in the 10:6-11:5 age band (mean difference = $-4.34, SE = 0.49, p < 0.001$). The children in the 8:6-9:5 age band differed significantly from the children in the 9:6-10:5 age band (mean difference = $-1.47, SE = 0.43, p = 0.003$) and the children in the 10:6-11:5 age band (mean difference = $-2.37, SE = 0.42, p < 0.001$). The children in the 9:6-10:5 age band did not differ significantly from the children in the 10:6-11:5 age band (mean difference = $-0.89, SE = 0.40, p = 0.112$).

The mean times in Table 4 and Figure 4 are for the number facts that the children got correct. The results indicate that addition, subtraction and multiplication facts are

Table 4. Mean answer times (seconds) for correct addition, subtraction and multiplication facts

Test by Age Band	<i>n</i>	<i>M</i>	<i>SD</i>
Addition Facts			
7:5-8:5	36	2.95	1.30
8:6-9:5	57	2.48	0.79
9:6-10:5	72	2.46	0.76
10:6-11:5	76	2.11	0.70
Subtraction Facts			
7:5-8:5	33	3.04	1.25
8:6-9:5	55	2.71	0.98
9:6-10:5	72	2.72	0.88
10:6-11:5	76	2.37	0.70
Multiplication Facts			
7:5-8:5	31	2.79	1.27
8:6-9:5	55	2.74	0.85
9:6-10:5	73	2.67	0.75
10:6-11:5	76	2.43	0.68

Figure 5. Mean answer times (s) for correct addition, subtraction and multiplication facts



The mean times in Table 4 and Figure 4 are for the number facts that the children got correct. The results indicate that addition, subtraction and multiplication facts are recalled more quickly as children get older. Addition facts are recalled the quickest. Subtraction facts are recalled quicker than multiplication facts by children in the 7:5-8:5 band and by children in the 9:6-10:5 age band, however the reverse pattern is shown by children in the 8:6-9:5 age band and the children in the 10:6-11:5 age band.

A repeated measures ANOVA with one within-participants variable (number fact type) and one between-participants (age band) variable was calculated using the analysis that included only the correct response times. It indicated that there was a significant difference between number fact types $F(2,456) = 16.10, p < 0.001$ and between age bands $F(3,228) = 4.240, p = 0.006$. The interaction between number fact type and age band was not statistically significant $F(6,456) = 0.546, p = 0.773$. Post Hoc Tukey *a* tests indicated that the children in the 7:5-8:5 age band differed significantly from the children in the 10:6-11:5 age band (mean difference = 0.51, $SE = 0.17, p = 0.011$), but not from the children in the 9:6-10:5 age band (mean difference = -0.19, $SE = 0.17, p = 0.667$) or the children in the 8:6-9:5 age band (mean difference = 0.17, $SE = 0.18, p = 0.743$). The difference between the children in the 8:6-9:5 age band and the children in the 10:6-11:5 age band verged on significance (mean difference = -0.33, $SE = 0.14, p = 0.070$), but not from the children in the 9:6-10:5 age band (mean difference = 1.306E-02, $SE = 0.14, p = 1.00$). The difference between the children in the 9:6-10:5 age band and the children in the 10:6-11:5 age band verged on significance (mean difference = -0.32, $SE = 0.13, p = 0.054$).

Table 5. Most scores for the different age groups

Age Band	<i>n</i>	<i>M</i>	<i>SD</i>
7:5-8:5	40	20.95	5.54
8:6-9:5	64	22.44	6.07
9:6-10:5	77	25.91	4.39
10:6-11:5	83	25.81	4.92

The results in Table 5 indicate that as children get older their place value understanding increases. A one-way ANOVA with one between-participants variable indicated that there was a significant difference between the scores for the age bands $F(3,264) = 352.92, p < 0.001$. Post Hoc Tukey *a* tests indicated that the children in the 7:5-8:5 age band differed significantly from the children in the 9:6-10:5 age band (mean difference = $-4.96, SE = 1.00, p < 0.001$) and the children in the 10:6-11:5 age band (mean difference = $-4.86, SE = 1.00, p < 0.001$), but they did not differ significantly from the children in the 8:6-9:5 age band (mean difference = $-1.49, SE = 1.04, p = 0.483$). The children in the 8:6-9:5 age band differed significantly from the children in the 10:6-11:5 age band (mean difference = $-3.47, SE = 0.88, p < 0.001$) and the children in the 9:6-10:5 age band (mean difference = $-3.37, SE = 0.86, p = 0.001$). The children in the 9:6-10:5 age band did not differ significantly from the children in the 10:6-11:5 age band (mean difference = $-0.10, SE = 0.82, p = 0.999$).

Table 6. Time per spot for the different age groups

Test by Age Band	<i>n</i>	<i>M</i>	<i>SD</i>
<i>Spots 1</i>			
7:5-8:5	30	1.90	0.74
8:6-9:5	48	1.79	1.18
9:6-10:5	68	1.57	0.63
10:6-11:5	73	1.30	0.63
<i>Spots 2</i>			
7:5-8:5	30	0.96	0.28
8:6-9:5	48	0.73	0.244
9:6-10:5	68	0.74	0.38
10:6-11:5	73	0.68	0.42

Note. Only times for correct trials used.

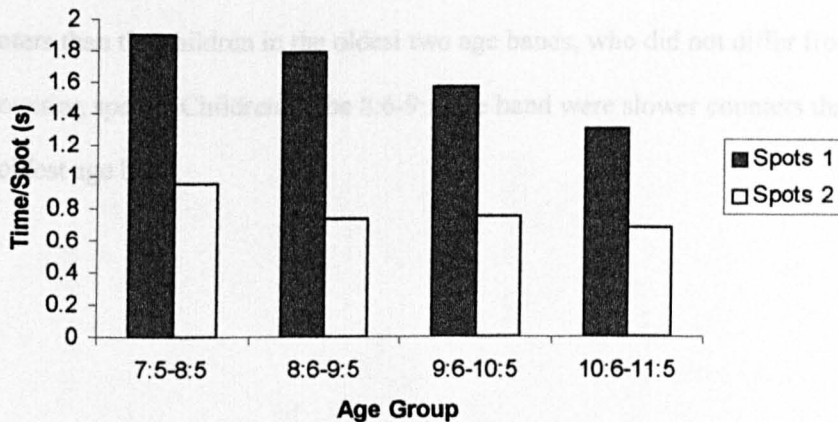


Figure 6. Time per spot for the different age bands

The results displayed in Table 6 and Figure 6 indicate that as children get older their counting speed increases. An ANOVA with one within participants variable (counting with or without memory aids) and one between-participants variable (age band) indicated that there was a significant difference between the age bands for the counting times [$F(3,215) = 7.113, p < 0.001$]. There was also a significant difference between the times for counting with and without memory aids [$F(1,215) = 183.425, p < 0.001$]. Post Hoc Tukey *a* tests indicated that the children in the 7:5-8:5 age band differed significantly from the children in the 10:6-11:5 age band (mean difference = 0.29, $SE = 0.11, p < 0.001$) and the children in the 9:6-10:6 age band (mean difference = 0.45, $SE = 0.10, p = 0.034$), but they did not differ significantly from the children in the 8:6-9:5 age band (mean difference = 0.18, $SE = 0.11, p = 0.346$). The children in the 8:6-9:5 age band differed significantly from the children in the 10:6-11:5 age band (mean difference = 0.27, $SE = 8.968E-02, p < 0.015$), but not the children in the 9:6-10:5 age band (mean difference = 0.10, $SE = 9.098E-02, p = 0.689$). The children in the 9:6-10:5 age band did not differ significantly from the children in the 10:6-11:5 age band (mean difference = -0.16, $SE = 8.134E-02, p = 0.169$). In summary, the children in the youngest age band were slower counters than the children in the oldest two age bands, who did not differ from each other in counting speed. Children in the 8:6-9:5 age band were slower counters than children in the oldest age band.

6.3.2 Formulating a normative comparison group

Figure 7. Histogram showing Number JGD scores for children 10:6 - 11:3

Figure 7. Histogram showing Most score for children 9:5 - 11:6

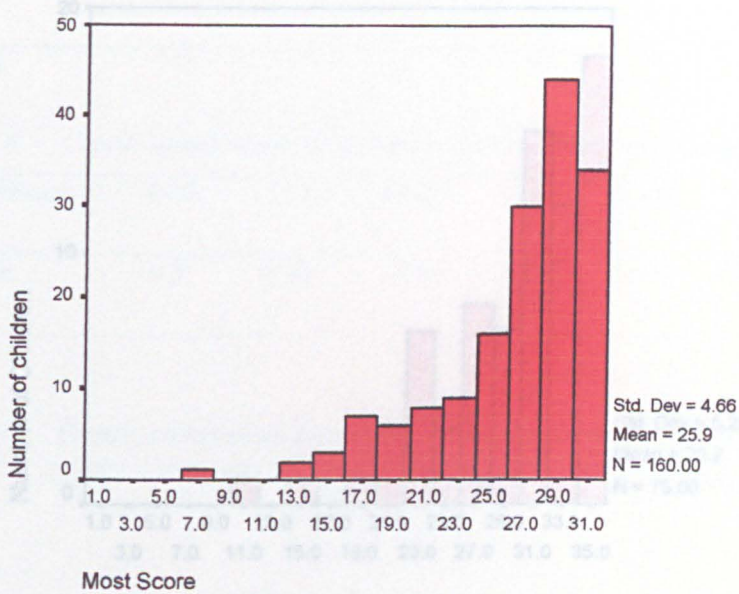


Figure 8. Histogram showing Number fact scores for children 9:6 -10:5

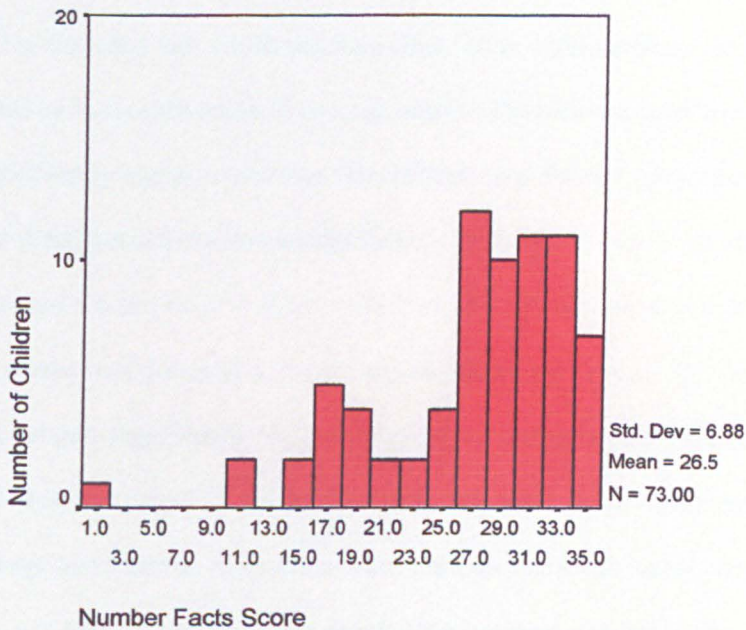
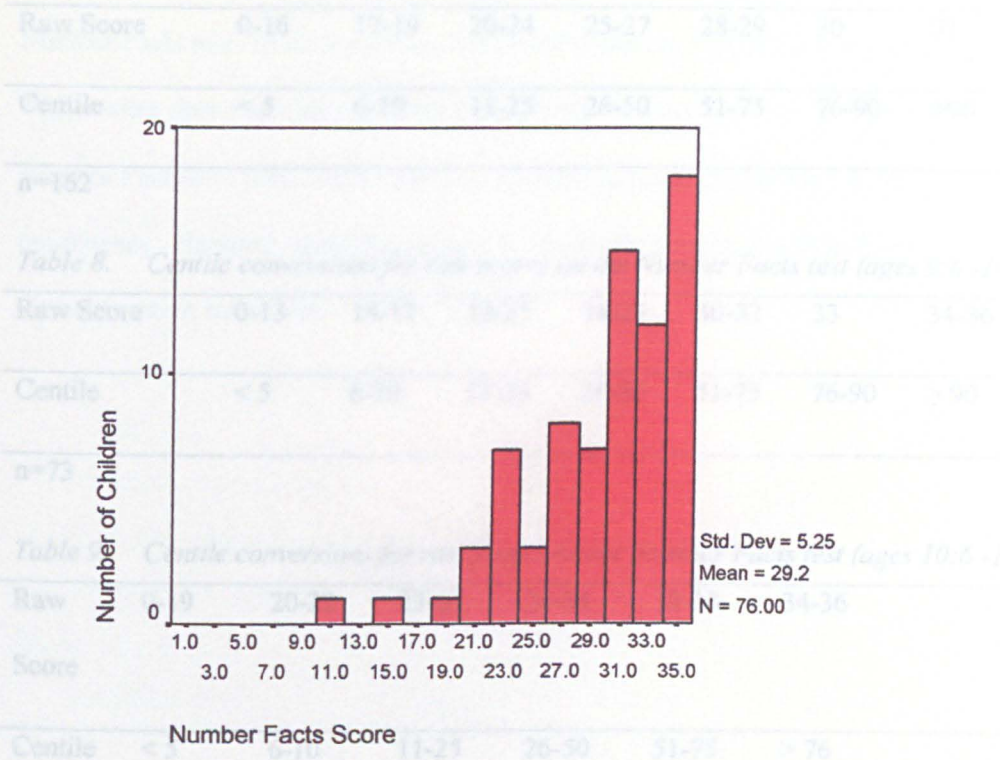


Figure 9. Histogram showing Number fact scores for children 10:6-11:5



For the *Mixar* test, which assesses place value understanding, the centile scores were calculated on the combination of two age bands. The children aged 10:6 to 11:5 did not gain significantly higher scores than the children aged 9:6 to 11:5 on the *Mixar* test (see 6.3.1 for details) and there was no significant correlation between age and *Mixar* score in children aged 9:6 to 11:5 ($r = .02$, $p = .81$, $n = 163$). It was therefore decided that more reliable norms would be gained by pooling the oldest age bands. The children aged 10:6 to 11:5 did not gain significantly higher scores than the children aged 9:6 to 11:5 on the Number Facts test (see 6.3.1 for details). However, there was a significant correlation between age and Number Facts score when the two oldest age bands were pooled ($r = .21$, $p = .011$, $n = 149$). It was therefore decided that separate centiles would be assigned to children aged 9:6-10:5 and children aged 10:6-11:5.

Table 7. Centile conversions for raw scores on the *Most* test (ages 9:6 -11:5)

Raw Score	0-16	17-19	20-24	25-27	28-29	30	31
Centile	< 5	6-10	11-25	26-50	51-75	76-90	>90

n=162

Table 8. Centile conversions for raw scores on the *Number Facts* test (ages 9:6 -10:5)

Raw Score	0-13	14-17	18-23	24-29	30-32	33	34-36
Centile	< 5	6-10	11-25	26-50	51-75	76-90	> 90

n=73

Table 9. Centile conversions for raw scores on the *Number Facts* test (ages 10:6 -11:5)

Raw Score	0-19	20-22	23-26	26-31	32-33	34-36
Centile	< 5	6-10	11-25	26-50	51-75	> 76

n=76

For the *Most* test, which assesses place value understanding, the centile scores were calculated on the combination of two age bands. The children aged 10:6 to 11:5 did not gain significantly higher scores than the children aged 9:6 to 11:5 on the *Most* test (see 6.3.1 for details) and there was no significant correlation between age and *Most* score in children aged 9:6 to 11:5 ($r = .02, p = .81, n = 162$). It was therefore decided that more reliable norms would be gained by pooling the oldest age bands. The children aged 10:6 to 11:5 did not gain significantly higher scores than the children aged 9:6 to 11:5 on the *Number Facts* test (see 6.3.1 for details. However, there was a significant correlation between age and *Number Facts* score when the two oldest age bands were pooled ($r = .21, p = .011, n = 149$). It was therefore decided that separate centiles would be computed for children aged 9:6-10:5 and children aged 10:6-11:5.

Figure 7, Figure 8 and Figure 9 show the distribution of scores for the *Most* and Number Facts test. In all three cases the distributions are strongly negatively skewed. The centiles shown in Table 7, Table 8 and Table 9 reflect this. For *Most* $\alpha = .887$, for Number Facts $\alpha = .938$. These alpha coefficients indicate that the tests have good internal consistency. However, these figure may be somewhat inflated because of the discontinuation rules used.

6.3.3 Relationships between the cognitive variables and the number fact scores

Table 10. The mean scores and number of children tested for cognitive and number skills measures. Standard deviations in months for age are given in brackets.

Test	<i>n</i>	<i>M</i> age	<i>M</i> score
Standardised tests			
<i>Numeracy Progress Test</i>	196	9:10 (12.80)	101.48 (15.58)
<i>LASS Mobile</i> (Auditory-verbal memory)	77	9:10 (12.62)	104.57 (17.68)
<i>LASS Cave</i> (Visual-spatial memory)	76	9:10 (12.69)	103.84 (15.37)
<i>LASS Reasoning</i> (Non-verbal reasoning)	74	9:10 (12.57)	0.375 (0.175)
Maths Suite tests			
Response time	203	9:10 (12.85)	1.36 (0.575)
<i>Most</i> (Place Value Understanding)	203	9:10 (12.85)	24.79 (5.17)
Addition Facts	203	9:10 (12.85)	8.80 (3.58)
Subtraction Facts	203	9:10 (12.85)	7.83 (3.81)
Multiplication Facts	202	9:10 (12.95)	6.90 (3.81)
Number Facts (Total of above three tests)	202	9:10 (12.95)	23.49 (10.50)

Note The sample consisted of the children aged 8 years or more who completed the appropriate tests. Children who did not attempt any *Number Facts* questions were excluded from the appropriate analyses.

The analysis of the relationships between the cognitive and number fact variables was carried out for the children who were aged 8 years or older and completed at least one standardised test in addition to *Maths Suite*. These 203 children (99 boys and 104 girls) all attended the schools who participated in phase one of Study One. Table 10 shows the

number of children who completed each test and their mean ages. The mean age of the children was similar for all the tests. The mean score of 101.48 for the *NPT* indicates that the sample has mathematical ability that is broadly in line with the general population of this age range. The mean *NPT* score for children who completed at least one *LASS* test was 101.12 indicating that this sub-sample also has mathematical ability, which is broadly in line with the general population of this age range.

Table 11. The Relationship between number skills and reasoning and memory abilities

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. NPT	—	.48*** (192)	.49*** (176)	.34*** (176)	.43*** (176)	.53*** (176)	-.23** (176)	-.33** (170)	-.05 (169)	-.25** (185)	-.00 (177)	.58*** (68)	.20† (71)	.25* (70)
2. Place Value Score		—	.42*** (244)	.31*** (244)	.39*** (244)	.40*** (244)	-.21 (241)	-.27*** (238)	-.04 (237)	-.10 (238)	-.05 (226)	.56*** (70)	.13 (72)	.20† (73)
3. Number Facts Score			—	.84*** (246)	.92*** (246)	.87*** (246)	-.44*** (243)	-.44 (240)	-.22 (239)	-.19** (219)	-.18** (211)	.42*** (69)	.22† (70)	.18 (69)
4. Addition Facts Score				—	.69*** (246)	.57*** (246)	-.33*** (243)	-.29*** (240)	-.13* (239)	-.13* (203)	-.19** (211)	.31** (67)	.14 (70)	-.01 (69)
5. Subtraction Facts Score					—	.71*** (246)	-.40*** (243)	-.36*** (240)	-.17** (239)	-.13* (219)	-.17** (211)	.37** (67)	.21† (70)	.22† (69)
6. Multiplication Facts Score						—	-.42 (243)	-.45*** (240)	-.20 (239)	-.20** (219)	-.16* (211)	.42*** (67)	.21† (70)	.22† (69)
7 Addition time							—	.73*** (238)	-.67*** (236)	.06 (219)	.20** (209)	-.33** (65)	-.01 (68)	-.10 (67)
8 Subtraction time								—	.65*** (238)	.13† (215)	.17* (205)	-.28* (65)	.00 (68)	-.11 (67)
9 Multiplication time									—	.10 (212)	.16* (205)	-.21* (65)	.05 (68)	-.09 (67)
10 Counting time 1										—	.16* (205)	-.23 (69)	-.17 (71)	-.30** (70)
11 Counting time 2											—	.05 (66)	.01 (69)	.19 (68)
12 Reasoning ability score												—	.12 (70)	.38*** (69)
13 Auditory Verbal Memory													—	.13 (72)
14 Visual spatial memory														—

Note The sample consisted of the children aged 8 years or more who completed the appropriate tests. Children who did not answer any *Number Facts* questions in the time limit were excluded from correlations involving number fact scores of times. The degrees of freedom for each correlation is shown in brackets. † $p < .1$, * $p < .05$ ** $p < .01$ *** $p < .001$

Table 12. Summary of Stepwise Regression Analysis for variables Predicting NPT score

Variable	R ²	R ² change	F	df	p	B	SE B	β	t	p
<i>Number</i>	.22	.22	53.11	1, 187	<.001	.51	.13	.30	3.98	<.001
<i>Facts</i>										
<i>Most</i>	.28	.06	15.19	1,186	<.001	.83	.21	.30	3.90	<.001

Table 13. Summary of Hierarchical Regression analysis for the two counting speeds predicting Number Facts score

Variable	R ²	R ² change	F	df	p	B	SE B	β	t	p
<i>Age</i>	.25	.25	70.70	1,214	>.001					
						0.32	0.04	0.498	8.41	>.001
<i>Spots</i>	.29	.04	5.36	2, 212	.005					
1						-1.19	0.59	-.12	-2.01	.046
2						-3.38	1.55	-.132	-2.18	.030

The relationships between the Number Skill Measures

The correlations between the number skills and cognitive measures are shown in Table 11. The *NPT* scores correlated significantly with all the number skills measures, except multiplication fact time and counting time without memory aids. Place value understanding correlated significantly with all the number skills measures except

multiplication fact time, counting time 1 and counting time 2. *Number Facts* score correlated with all the number skills measures except multiplication and subtraction times. Subtraction and addition fact scores correlated with all the other number skills measures. Multiplication fact scores correlated with all the number fact measures, except counting time one. Neither counting time measure correlates with either place value understanding, but both correlated with the *Number Facts* measure. Counting time two correlates with addition, subtraction and multiplication time, whilst counting time one only correlates marginally with counting time two.

Number Facts score, *Most* score and counting times one and two were entered into a Stepwise multiple regression analysis to determine whether these number skills measures accounted for a significant proportion of the variance in the children's general numeracy attainment. *Number Facts* score predicted 22% of the variance in *NPT* scores. *Number Facts* and *Most* predicted 28% of the variance in *NPT* scores. The summary of the stepwise regression analysis in 0 indicates that both variables explained independent proportions of the variance in *NPT* scores.

The results of hierarchical regression analyses with *Number Facts* as the dependent variable and age and counting speed as the predictors is shown in Table 13. 25% of the variance in *Number Facts* scores is accounted for by age. An additional 4% is accounted for by counting speed, the increase in the amount of variance explained is significant $F(2,212) = 5.36, p=0.005$. Both counting speed one and counting speed two accounted for significant proportions of the variance.

The relationships between the cognitive variables

Auditory-verbal memory did not correlate with reasoning ability or visual-spatial memory. There was a significant correlation between reasoning ability and visual-spatial memory.

The relationships between the cognitive and number skills variables

Reasoning ability correlated with all the number fact measures except counting time one (where the correlation verged on significance) and counting time two. Visual-spatial memory correlated with counting time one and *NPT* score. Although the correlation between visual-spatial memory and *NPT* score was not significant when age and reasoning ability was partialled out, the correlation between counting time one and *NPT* score remained significant ($r = -.24$, $n = 69$, $p = .045$). Auditory-verbal memory correlated with *NPT* score, *Number Facts* score, and subtraction facts score at a level that verged on statistical significance. Non-verbal reasoning was not partialled out of the auditory-verbal memory correlations, because auditory-verbal memory and non-verbal memory did not correlate with non-verbal reasoning ability.

6.4 Discussion

6.4.1 Evaluating of *Maths Suite* as a tool to measure children's number skills

The *Number Facts* and *Most* tests in *Maths Suite* provide a reasonable, but not an optimal, measure of children's number fact recall and place value understanding. Stepwise multiple regression indicated that *Number Facts* scores and *Most* scores accounted for an independent proportions of the variance in *NPT* scores. This supports the validity of the *Maths Suite* test as the skills they assess independently contribute to children's numeracy attainment. Furthermore, the sampling method used to identify the children seems to have obtained a representative sample of abilities, as the sub-sample that was administered the *NPT* obtained a mean score very close to the population mean. The alpha coefficients calculated indicated that both the *Most* and *Number Facts* tests had adequate internal consistency.

Discontinuation rules are incorporated into both *Number Facts* (items are no longer presented if the child is unable to answer four items consecutively) and *Most* (items are no longer presented if the child answers four of any five consecutive items incorrectly). These rules were incorporated to reduce testing time and to avoid stress to the children.

However, it is possible that the item order is not completely correct and that consequently some children's scores may have been underestimated. It would also have been desirable to have a larger number of children completing the tests to obtain more robust norms. However, being able to compare the SAD children's scores to a randomly selected group of normal children, will be a step forward from previous studies that have inferred children's abilities from unstandardised tasks (e.g. Sokol et al., 1994; Ta'ir et al., 1997; Temple, 1989; Temple, 1991).

6.4.2 The development of number fact recall, place value understanding and counting speed

Overall, children scored better on addition facts than subtraction facts and better on subtraction facts than multiplication facts. The children aged 7:5 to 9:6 found multiplication facts particularly difficult to recall. As the age of the children increased, they were able to recall more number facts, more quickly. The trend that older children are faster and more accurate at recalling number facts is consistent with previous research (Campbell & Graham, 1985; Cooney et al., 1988). The two younger groups of children did not differ in their place value understanding; their understanding was poorer than the older two groups. The older two groups did not differ in place value understanding. The trend for increased place value understanding as children get older has been reported previously (Brown, 1981; Minnis et al., 1999; Nunes & Bryant, 1996).

When counting without memory aids, the children in the youngest age group differed from the three older groups, but the three older groups did not differ significantly from each other. When counting with memory aids, the children who were in the two younger groups did not differ significantly from the children aged 9:6-10:5, but they did differ significantly from the children aged 10:6-11:5. The children in the 9:6-10:5 age band did not differ significantly from the children in the 10:6-11:5 age band. Hitch et al. (1987) reported a similar trend with older children counting faster than younger children. Hitch et al. (1987) used a pointer analogy to describe the differences between 'ballistic

counting', counting a visual array, and using a counting on strategy. Ballistic counting is simply counting up to a predetermined point. Only one 'pointer' is required, which moves step by step through the number series. Counting a visual array is more complex because the individual requires two 'pointers', one to run through the number series and one that selects items from the visual array. These pointers must be co-ordinated, and additionally there must be some way of signalling to the visual array counter which items have already been counted. In Study One the counting speed measure in part one of *Spots* is ballistic counting; the pointer is physically represented and the previously counted *Spots* are identified for the child, as they turn white. The additional complexity of visual array counting is largely removed. However, part two of *Spots* involves the more complex task of array counting. The age groups may have differed more on *Spots* part two, because it is a more complex counting task, which takes longer to develop.

6.4.3 The relationship between number skills and cognitive measures

The results of this study indicate that non-verbal reasoning is related to place value understanding, number fact knowledge and *NPT* score in children aged 8 to 11 years. However, whilst auditory-verbal memory ability correlates with number fact knowledge and *NPT* score at a level that verges on statistical significance, it does not correlate with place value understanding. Visual-spatial memory was correlated with *NPT* score and with place value understanding at a level that verged on statistical significance. However, neither of these relationships with visual-spatial memory remained significant after non-verbal reasoning ability had been partialled out. Therefore, there may not be a direct relationship between visual-spatial memory and non-verbal reasoning - the relationship may be indirect (i.e. visual spatial memory being related directly to non-verbal reasoning, which is related directly to the number skills measures). The counting time measures did not correlate with the short-term memory measures. Counting time one correlated with reasoning ability, but counting time two did not.

It is tentatively concluded from these results that different cognitive abilities influence different number skills. In particular that auditory-verbal memory is related to number fact recall and not place value understanding. Although the relationships between number fact recall and auditory-verbal memory is weak, it must be taken in the context of the study. The sample of 71 children who completed the *Number Facts* test and the auditory-verbal memory test came from 16 different schools in very different socio-economic areas. Environmental differences will therefore account for much of the variation in number fact scores.

McClelland and Hitch (1999) found a significant correlation ($r=.36$) between digit span and a speeded calculation test in a sample of 122 9-year-old children who attended only five different primary schools in the same locality, digit span did not correlate significantly with the pencil and paper maths test they used. The greater strength of the relationship between digit span and speeded calculation in the McClelland and Hitch (1999) study may have been due to the sample having environmental influences that varied less than the sample in the present study. McClelland and Hitch (1999) also found a significant correlation between a visual-spatial memory task and a pencil and paper maths test, however as reasoning measures were not used, it is not possible to determine whether this relationship would have persisted if reasoning ability had been partialled out. As in this study, visual-spatial memory measures did not correlate with speeded calculation. In contrast to the two previous studies, Bull and Johnston (1997) reported a correlation of 0.38 between verbal short-term memory (made up of counting span, digit span and word span) and a pencil and paper maths test in a sample of 7-year-old children. The difference between the correlations between auditory-verbal memory measures and pencil and paper arithmetic tests found by McClelland and Hitch (1999), Bull and Johnston (1997) and the present author may be due to differences in the age of the children studied, the pencil and paper maths test used or the exact auditory-verbal memory measure used.

The lack of a relationship between visual-spatial memory and number skills and the strong relationship between non-verbal reasoning and number skills is in stark contrast to the findings of Singleton, Thomas, Horne and Simmons (in preparation). In a longitudinal study of children's mathematical ability, they found that verbal reasoning, but not non-verbal reasoning, assessed at 5:3 predicted number skills ability at 5:11 and 7:11. Visual memory measures taken at 5:3 contributed to a significant proportion of the variance over and above the reasoning measures at 5:11 and 7:11. The conflicting results may be due to developmental differences; visual-spatial memory may be important in early numerical development, but becomes less important once very basic skills such as counting are well established.

Non-verbal reasoning correlated significantly with all the number skills measures. Place value understanding requires the ability to understand the link between a digit's position and its value and apply these rules for unfamiliar numbers. Therefore the link between non-verbal reasoning (which requires rules to be determined and applied) is clear. It is not possible to determine in this study whether this is because the non-verbal test correlates with a general reasoning ability factor, or whether non-verbal reasoning accounts for an independent proportion of the variance that cannot be explained by verbal reasoning measures. The relationship between number fact recall and reasoning ability is less transparent; recalling number facts appears to be a fairly mechanical activity. However, in sections 3.3 and 3.4 the relationship between conceptual understanding and single digit arithmetic performance was discussed. Some studies have indicated that children with greater conceptual understanding can utilise more economical and reliable procedures (Baroody & Gannon, 1984; Canobi et al., 1998; Cowan et al., 1996) and answer addition problems more quickly (Bisanz et al., 1989). An understanding of the commutativity principle can help children generalise their knowledge of learnt multiplication facts to unpractised multiplication facts (Baroody, 1999). If more intelligent children develop conceptual understanding more quickly, they may be able to use more flexible and reliable

strategies for answering single digit arithmetic problems and therefore build up a store of number facts more quickly.

It is important to consider the exact nature of the measures used when interpreting the results. The memory measures did not simply differ in modality. *LASS Mobile* is an auditory-verbal sequential measure (the child only receives credit if the digits are recalled in the correct order) whilst *LASS Cave* is a non-sequential visual spatial measure (the child receives credit if the phantoms are placed in the right position, regardless of the temporal order in which this is done). It may be the sequential aspects of *LASS Mobile* that mediates its relationship with number fact recall rather than its auditory-verbal nature. A study conducted by Ward (1992) has indicated that children with arithmetic difficulties have difficulties learning novel verbal and visual sequences. Future studies should compare the predictive capabilities of verbal and visual-spatial memory measures that require both sequential and free recall.

The finding that the measures of counting speed correlate with number fact recall, but not place value understanding, is consistent with the finding that children who have arithmetic difficulties are slower counters than normally developing children (Geary, 1991; Hitch & McAuley, 1991). If children are slower counters they are less likely to be able to associate a single digit arithmetic question with its answer because their memory for the sum will decay before they reach the answer (see sections 3.3 for a discussion of children's counting strategies when answering addition and subtraction questions).

6.5 Conclusions from Study One

Number fact recall, place value understanding and counting speed all improve as children progress through Key Stage 2. Non-verbal reasoning ability is correlated with these three number skills. Auditory-verbal sequential memory correlates weakly with number fact recall at a level that verges on statistical significance. The correlation between visual-spatial memory and place value understanding was not significant when non-verbal

reasoning ability was partialled out. Counting speed one and counting speed two are both correlated with number fact recall, but not place value understanding.

7 Study Two and Study Three - The ability, cognitive and number skills profiles of children with Specific Arithmetic Difficulties.

7.1 Rationale (Study Two)

This study aimed to examine the ability profiles of children with specific arithmetic difficulties, i.e. those who have a deficit in arithmetic but not in reading skills. Such children are unusual, as arithmetic difficulties tend to coexist with reading difficulties. An epidemiological study of over 1000 9- and 10-year-old children in British schools, indicated that 2.3% of children with normal intelligence have arithmetic and reading difficulties and 4.0% of children with normal intelligence have reading difficulties alone (Lewis, Hitch & Walker, 1994). Children with arithmetic difficulties without a reading deficit were rarer, 1.3% of children in this age group being affected. The specific focus of Study Two was whether or not children with arithmetic difficulties but better reading share a homogeneous ability profile; in particular, whether they share a fundamental deficit in spatial processing. If children with specific arithmetic difficulties (SAD) share a homogenous profile it could aid our understanding of the core abilities underlying arithmetic, and inform strategies for identifying specific arithmetic difficulties early in childhood.

Spatial deficits could cause arithmetic difficulties in children by directly interfering with their calculation procedures or indirectly by hindering early numerical development. Section 4.4.1 provides a discussion of the direct and indirect effects that spatial difficulties could have on the development and performance of number skills.

If spatial skills are associated with arithmetic development it is logical to suggest that poor spatial skills could be one cause of arithmetic difficulties. Studies by Rourke and his colleagues (reviewed in Rourke & Del Dotto, 1994) have indicated that children with specific arithmetic difficulties (weak arithmetic, but better reading) have poor non-verbal, spatial and psychomotor skills. However, these findings have not been consistently

replicated (see sections 4.4.1 and 4.6 for a discussion of Rourke's studies and the NLD profile).

The British Ability Scales, Second Edition (BAS II) (Elliott, Smith & McCulloch, 1997) was employed in Study Two to determine whether children with SAD in English primary schools share a core spatial deficit. The BAS II core scales were chosen in preference to WISC III (Wechsler, 1991), because the BAS II global ability measure (GCA) is divided into three clusters: verbal reasoning, non-verbal reasoning and spatial ability. This allows a more fine-grained analysis than the WISC III, in which the global ability measure (IQ) is divided into only two clusters: verbal and performance. The Performance IQ scale of the WISC III includes a variety of non-verbal sub-tests, whilst the BAS II separates the spatial tests (involving an understanding of directional and space relationships) from the other non-verbal tests.

The BAS II *Word Reading* and *Number Skills* tests were chosen in preference to their WRAT-3 (Wilkinson, 1993) equivalents because the BAS II manual provides detailed information on the statistical significance and frequency of any achievement-ability discrepancies found on the ability and achievement tests of the scales. The BAS II tests are also more appropriate because they have been standardised more recently on British populations. The BAS II achievement tests are similar to the WRAT-3 tests because they both assess mechanical skills (i.e. single word reading and calculations rather than reading comprehension or mathematical understanding). This should make the results comparable with the majority of previous studies, which used the WRAT-3.

There is controversy about the validity of discrepancy definitions in the specific learning disability field (see section 5.1 of this thesis and Berninger, 1998; Morrison & Siegel, 1991; Reason et al., 1999 for reviews). A recent study by Gonzalez & Espinel (1999) indicated that there were no significant cognitive differences between children whose arithmetic difficulties were commensurate or discrepant with their IQ. It was therefore decided that all children would be included who met the achievement criteria,

and the pattern of abilities of children who did and did not meet the traditional criteria for specific learning disability would be compared.

7.2 Method (Study Two)

7.2.1 Materials

The tests employed in Study Two were the *Core*, *Word Reading* and *Number Skills* tests from the school age battery of the British Ability Scales II (*BAS II*) (Elliott, Smith & McCulloch, 1997). The *BAS II* is a standardised ability test with six core scales. The *Word Definitions* scale requires the child to define given words. The *Verbal Similarities* scale requires the child to give the link between three words. The *Matrices* scale consists of a number of incomplete patterns; the child must identify the relationship that connects the shapes and use it to choose the right shape to complete the pattern. The *Quantitative Reasoning* scale requires the child to identify the relationship that connects a pattern of numbers and hence to identify the number that completes the pattern. The *Pattern Construction* scale uses coloured blocks with which, the child has to replicate patterns under timed conditions. The *Recall of Designs* scale requires the child to draw designs from memory. *Word Definitions* and *Verbal Similarities* are both presented and answered orally; the remaining four core scales require non-verbal responses from the child. The *Word Reading* test consists of a page of single words increasing in difficulty that the child reads aloud. The *Number Skills* test involves number identification and a range of increasingly difficult arithmetic problems. There are no time constraints on either of the two achievement scales.

At the most general level of interpretation all the core scales can be used to calculate a child's General Conceptual ability (GCA). GCA is a measure of psychometric g or "the ability of an individual to perform complex mental processing that involves conceptualisation and the transformation of information" (Elliott, Smith & McCulloch, 1997) (p. 18). All six core scales correlate with each other in the general population and have high g loadings. At a more specific level of analysis the core scales can be separated

into three distinct clusters. The verbal ability cluster consists of *Verbal Similarities* and *Word Definitions*, the non-verbal reasoning ability cluster consists of *Matrices* and *Quantitative Reasoning*, and the spatial ability cluster consists of *Pattern construction* and *Recall of Designs*. Information in the manual allows the administrator to calculate whether any discrepancies between these clusters are statistically significant and the frequency of the discrepancy in the general population.

7.2.2 Participant selection and procedure

The head teachers of all 196 primary schools in the Hull and East Yorkshire Local Education Authorities were invited to participate in the study. The head teachers were asked if they had any children aged nine years or above whose arithmetic skills were very weak and, in the head teacher's opinion considerably poorer than their reading and spelling skills. The head teachers of 32 schools referred a total of 55 children who they identified as having significant difficulties with number skills but relatively unimpaired reading and spelling skills. All these children were assessed individually by the author in their schools in the quietest room available.

Children were only included in the specific arithmetic difficulties group (SAD) if they scored at or below the 25th centile on the *BAS II Number Skills* test and achieved a score on the *BAS II Word Reading* test at least 15 standard score points higher. Of the 55 children identified by teachers, 34 were excluded from the SAD group. 19 of the excluded children scored above the 25th centile on the number skills test and the discrepancy between their reading and number scores was too small. 12 of the excluded children scored too highly on the number skills test despite having an adequate discrepancy and three children were excluded because despite having a low enough number score they had too small a discrepancy. This stringent method of participant selection, which uses information from two sources (teachers and standardised tests), should ensure that the children in the sample had significant and persistent number difficulties. In addition to the 21 children, identified by contacting the schools directly one girl who was referred to the

author's supervisor for psychological assessment was included in the sample. She was identified by her teacher as having considerable maths difficulties and met the standardised test criteria and was added to the SAD group. All children participated with parental permission.

7.2.3 Participant characteristics

The 22 SAD children were split equally into girls (11) and boys (11). There were 13 children with in Year 5, four in Year 4 and five in Year 6. The mean age of the children was 10 years and 3 months ($SD = 9$ months, range 9 years 5 months to 11 years 4 months). The SAD children attended 18 different schools in areas that had very different levels of economic advantage. Some schools were situated within large estates of social housing whilst others were located in prosperous villages. Ten of the children attended schools in Hull, four children attended schools that were in towns in East Yorkshire and eight children attended village schools in rural areas. The most recent Ofsted report for each school was consulted [www.ofsted.gov.uk/inspect/]. No school was identified as having serious weaknesses.

7.3 Results (study two)

Table 14. Achievement standard scores for the children with specific arithmetic difficulties (SAD) ($n=22$).

	BAS Word reading	BAS Number skills
Mean	107.73	79.77
Median	108.25	81.00
SD	12.68	8.20

The mean and median achievement test scores for the children with SAD are shown in Table 14. The medians are shown as they are not as strongly affected by extreme scores, e.g. child 1 had scores on many measures that were more than two standard deviations below the sample mean. As expected, the mean *Number Skills* score for the children with specific arithmetic difficulties was substantially below the population mean of 100. Two children with SAD had *Number Skills* scores of 70 or below, 15 had scores of 85 or below and five had scores between 85 and 90. Twelve children had a *Number Skills*

score that was significantly below the level expected on the basis of age and GCA. It is important to note that having a below average GCA did not preclude a child having a number skills score that was below the expected level. Five of the twelve children had GCAs that were below the normal range. Child 1, who had the lowest GCA, had a *Number Skills* score below the expected level.

The mean *Word Reading* score for the children with arithmetic difficulties is slightly above the mean for the general population (100). Eight SAD children had a *Word Reading* standard score more than one standard deviation above the population mean, 13 had *Word Reading* scores within the normal range. None of the SAD children had a reading score below the level that would be expected on the basis of their age and GCA. Only one boy (child 1) had a reading score below the normal range. His reading score was 81; this was 26 points higher than his number skills score. To be included in the study the children had to have a number skills score that was at least 15 points lower than their reading score. The difference in the majority of children was much larger, the mean difference being almost two standard deviations (median=26.00, mean=27.95, *SD*=9.84, min.=15, max=47).

Table 15. The average standardised scores for the SAD children on BAS II (n=22).

	GCA	Verbal ability	Non-verbal ability	Spatial ability
Mean	83.91	88.50	84.73	86.23
Median	86.67	90.67	90.20	89.50
<i>SD</i>	12.60	13.50	13.42	15.35

Table 16. The individual ability scores for the SAD children on BAS II, noting any statistically significant discrepancies.

ID No.	GCA	Verbal ability (V)	Non-verbal ability (NV)	Spatial ability (S)	Significant V/S difference	Frequency of V/S difference	Significant V/NV difference?	Frequency of V/NV difference	Significant S/NV difference	Frequency of S/NV difference
1	49	56	54	63	No	N/A	No	N/A	No	N/A
2	70	71	66	89	Spatial	25 %	Non-verbal	25 %	Spatial	10 %
3	75	98	70	67	Verbal	10 %	Verbal	5 %	No	N/A
4	95	106	97	84	Verbal	15 %	No	N/A	No	N/A
5	60	91	58	49	Verbal	<1%	Verbal	2%	No	N/A
6	89	90	93	91	No	N/A	No	N/A	No	N/A
7	94	92	98	95	No	N/A	No	N/A	No	N/A
8	82	85	91	79	No	N/A	No	N/A	No	N/A
9	80	86	79	87	No	N/A	No	N/A	No	N/A
10	84	74	84	104	Spatial	5 %	No	N/A	Spatial	25 %
11	98	108	94	91	Verbal	25 % +	Verbal	25 % +	No	N/A
12	91	96	92	90	No	N/A	No	N/A	No	N/A
13	86	72	95	99	Spatial	10 %	Non-verbal	10 %	No	N/A
14	85	105	81	77	Verbal	10 %	Verbal	10 %	No	N/A
15	94	90	91	105	No	N/A	No	N/A	No	N/A
16	96	82	97	112	Spatial	5 %	Non-verbal	25 %	No	N/A
17	97	110	89	92	Verbal	25 %	Verbal	15%	No	N/A
18	95	96	91	101	No	N/A	No	N/A	No	N/A
29	88	75	101	94	Spatial	25 %	Non-verbal	10 %	No	N/A
20	65	78	68	65	No	25 %	No	N/A	No	N/A
21	88	93	91	87	No	N/A	No	N/A	No	N/A
22	81	93	84	76	Verbal	25 %+	No	N/A	No	N/A

Note. The ability cluster reported for the significant differences is the *higher* score. The *p* level for a significant discrepancy is 0.05

The mean and median ability scores for the SAD group are shown in Table 15, the children's individual scores are shown in Table 16. The SAD children have a mean GCA score that is below the mean score for the general population. One boy (child 1) had a GCA more than three standard deviations below the general population mean. Two children had a GCA that was more than two standard deviations below the population mean. Seven children had GCA scores more than one standard deviation below the mean and the remaining 12 children had GCA scores within the average range. No SAD child had a GCA score more than one standard deviation above the mean.

The mean and median verbal ability score for the SAD children is below that of the general population. Some children had verbal ability scores within the upper half of the average range. This suggests that intact verbal ability alone is not sufficient for competent number skills to develop.

The median non-verbal ability score is within the normal range, whilst the mean is just outside it. The highest non-verbal reasoning ability score was 101. This is more than 10 standard score points below the maximum score for either the spatial or verbal ability

clusters. It is tempting to attribute the lack of above average non-verbal ability in the SAD group to the presence of the *Quantitative Reasoning* scale (which draws heavily on number fact knowledge) in the cluster. However, an examination of the individual scale scores does not support this hypothesis; the maximum *Matrices* and *Quantitative Reasoning* t-scores are the same: 54. Both the mean and the median spatial ability scores are below the population mean. Four children have spatial ability scores that are more than two standard deviations below the population mean whilst another four have scores above the population mean. Hence, below average spatial ability is not necessary for children to have difficulty learning number skills despite having adequate reading skills.

Table 17. The intercorrelations of the ability and achievement scales for the SAD children (in bold type) and children in the BAS II standardisation sample (in normal type).

	1	2	3	4
1. Verbal	–			
2. Non-verbal	.56 .38 p=.083	–		
3. Spatial	.48 .03 p=.90	.57 .76** p<.001	–	
4. Number	.54 .26 p=.247	.66 .72** p<.001	.44	–
5. Reading	.65 .45* p=.037	.51 .72** p<.001	.35 .45* p=0.035	.55 .63** p=0.002

Note: The data for the typical children is taken from the technical data gathered during the standardisation of the *BAS II* (Elliott, Smith and McCulloch, 1997, p.115).

Table 17 shows the intercorrelations of the ability and achievement measures for the SAD children in bold type. The same intercorrelations for a sample of children drawn from the general population are shown in normal type. All the *BAS II* correlations for typical children are significant at the 5% level or above. A comparison of the correlations in Table 17 suggests that SAD children's non-verbal and spatial ability scores are less closely associated with their verbal ability scores than those of typical children. In the sample of children drawn from the general population the correlation between spatial and verbal ability is 0.48, the correlation between spatial and non-verbal ability is 0.57 and the correlation between non-verbal ability and verbal ability is 0.56. In contrast, the correlations reported for the SAD children are not statistically significant for spatial/verbal or non-verbal/verbal relationships. However, the correlation between the SAD children's

non-verbal and spatial ability is statistically significant. As would be expected, the ability and attainment scores of the typical children all correlate significantly. Similarly, verbal ability and non-verbal ability and spatial ability are significantly correlated with both reading and number skills in the SAD group.

If the sample level statistics (shown in Table 15) are examined, the SAD children appear to have a smooth ability profile with the median verbal, non-verbal and spatial cluster scores all falling in the lower half of the normal range. However, closer examination of the individual ability scores (shown in Table 16) reveals that this conclusion is erroneous. Of the 22 children with SAD 12 had statistically significant discrepancies between their verbal reasoning and spatial ability cluster scores. The frequency of these discrepancies in the general population can be calculated using the data provided in the BAS II Technical Manual. The statistical frequency of the discrepancies ranged from <1% to >25 % of the general population. This suggests that large verbal/spatial ability discrepancies are over-represented in SAD children. However, the direction of the discrepancy varies. Five children with arithmetic difficulties with a significant discrepancy had better spatial skills, while seven had better verbal skills.

There are nine statistically significant discrepancies between the SAD children's non-verbal and verbal cluster scores. This suggests that large verbal/non-verbal ability discrepancies are also over-represented in SAD children. Again, the direction of the discrepancy varies. Four SAD children with a significant discrepancy had better non-verbal abilities and five had better verbal abilities. There are only two statistically significant non-verbal/ spatial discrepancies, suggesting that this type of discrepancy is not over-represented in SAD children.

The data were analysed to determine whether having a verbal ability score that is statistically discrepant from spatial ability is associated with having arithmetic difficulties, which conformed to traditional criteria for having specific learning difficulties (see Kaufman, 1994; Pumfrey & Reason, 1991; Turner, 1997). One of the children in the

'traditional criteria' group had spatial abilities that were significantly higher than their verbal abilities, two children had verbal skills that were significantly higher than their spatial skills and four did not have a significant verbal spatial difference. Of the four 'traditional criteria' children who did not have a statistically significant discrepancy three had spatial skills that were higher than their verbal skills. Inferential statistical analysis has not been conducted in regard to the 'traditional criteria' because the sample size is too small for chi-squared tests to be reliable.

Although girls with significant verbal/spatial discrepancies outnumbered boys, gender was not statistically associated with having a significant verbal/spatial discrepancy (chi-squared = 1.636, d.f. = 1, $p = 0.201$). Of the 11 SAD boys, two had significantly higher verbal ability and two had significantly higher spatial ability; the remainder did not have a significant verbal/spatial discrepancy. Four of the eleven girls had significantly higher verbal ability and three had significantly higher spatial ability; the remainder did not have a significant verbal/spatial discrepancy.

7.4 Discussion (Study Two)

The results of this study do not suggest that children with poor number skills but better reading necessarily have poor spatial skills. Only seven of the 22 children with SAD had spatial ability scores that were below the normal range; the remaining 16 children had spatial ability scores within the normal range. Many of the children in this study had large statistically significant discrepancies between their verbal and spatial abilities. However, it was not always the case that the SAD children's spatial ability was the poorer of the two abilities as Rourke and Finlayson (1978) would predict. Twelve of the 22 children had a statistically significant discrepancy and eight of those children had poorer spatial skills. The lack of a significant correlation between verbal and non-verbal or spatial ability in the SAD children suggests that these abilities are less strongly associated in SAD than they are in the normal population.

The conclusion that poor spatial abilities do not appear to be at the root of many of the SAD children's arithmetic difficulties appears to be robust. There is no evidence to suggest that the difference between these results and those of Rourke and Finlayson (1978) is due to the different way that arithmetic difficulties was defined in this study. If only those children who conform to the traditional definition of specific learning disability are considered, a homogenous pattern of abilities is still not found. Only four of the children with specific learning difficulties had an NLD profile, two showed no significant discrepancy between their verbal and spatial abilities and one had significantly higher spatial ability. This study needs to be replicated with a larger sample of children who meet the traditional specific learning difficulty criteria so that the results can be tested statistically. Spatial/verbal ability discrepancies were not significantly associated with gender. Both boys and girls had statistically significant verbal/spatial and verbal/non-verbal discrepancies in both directions. This finding is different to that of Share et al (1988), who reported that only boys with SAD displayed a low spatial but high verbal ability profile.

The findings of the Study Two may be divergent from the results of Rourke and Finlayson (1978) because of the differences between the tests included in the verbal sections of *WISC III* (Wechsler, 1991) and *BAS II* (Elliott, Smith, & McCulloch, 1997). Rourke and his colleagues conducted a whole series of studies into the strengths and weakness of children with better reading and spelling than arithmetic (see Rourke & Del Dotto, 1994). Rourke and Del Dotto (1994) concluded that whilst children with specific arithmetic difficulties are "average to superior in the more rote aspects of psycholinguistic skills", they perform poorly on tests of "semantic-acoustic processing" (p. 29). *WISC III* includes more verbal tests that require rote knowledge (e.g. *Digit Span* and *Information*) than the *BAS II* verbal cluster. The SAD children's verbal scores may have been higher if they had been assessed using *WISC III*.

The results of Study Two have important implications for research and practice in educational psychology. The study illustrates the dangers of examining only sample level statistics when investigating clinical populations. Although the mean and median ability cluster scores for the SAD children were similar, large statistically significant discrepancies were far commoner in this sample of children with SAD than one would expect in the general population. As the discrepancies existed in both directions, they cancelled each other out in the sample level statistics. The results of the present study also raise questions about how participants should be selected for future investigations of arithmetic difficulties. All but one of the SAD children in this study had a single word reading score within the normal range, but many had GCAs that were below the normal range. It should not be assumed that children with single word reading abilities within the average range but below average arithmetic have average intellectual abilities.

The most notable finding is the over-representation of statistically significant discrepancies in the SAD group. Previous studies of SAD children (e.g. Geary, 1991; Hitch & McAuley, 1991; Siegel & Linder, 1984; Siegel & Linder, 1988) have used solely verbal (e.g. PPVT, Dunn, 1965; BPVS, Dunn, Dunn, Whetton & Burley, 1997) or non-verbal (e.g. Coloured Progressive Matrices, Raven, 1965) tests to estimate the global reasoning abilities of children with arithmetic difficulties. The dissociation between BAS II ability clusters in the SAD children would suggest that either a non-verbal or a verbal test could drastically under- or over-estimate these children's global reasoning abilities. When carrying out future research on children with arithmetic difficulties a balanced intellectual ability test including both verbal and non-verbal elements is strongly recommended. If time considerations preclude the use of either a full BAS II or a WISC III, a short form WISC III would be preferable to a verbally or non-verbally biased test. Balanced intellectual ability tests are also recommended for teachers or educational psychologists when forming a profile of children identified as having arithmetic difficulties. It should

not be assumed that SAD children's primary weakness is spatial as many SAD children have significant verbal reasoning deficits.

The present results are similar to those of Dowker (1992, 1999) who found that both children and adults with large discrepancies between their mathematical reasoning and arithmetic skills had large performance/verbal IQ discrepancies. These discrepancies did not tend to be in a particular direction. Dowker (1992) suggested that an uneven cognitive profile is related to poor mechanical arithmetic skills because cognitively atypical children do not respond well to the structured and often inflexible manner in which arithmetic is taught.

Another possible reason why different ability profiles may result in poor arithmetic skills, is the fact that arithmetic draws on a variety of cognitive skills. Lyytinen, Ahonen and Raesaesen (1994) provide a review of the many different cognitive processes involved in calculation. They emphasize the variety of deficits that could make learning arithmetic hard (e.g. slow conceptual development, poor attention allocation, short memory span, slow speed of processing). Children with arithmetic difficulties but better reading may share a strength (e.g. good phonological awareness that boosts their reading performance), but have poor arithmetic performance for many different reasons. The possibility that there are multiple routes to arithmetic difficulties is discussed further in section 10.3.

It is possible that the SAD children could share a cognitive weakness that is responsible for their arithmetic difficulties that was not assessed in Study Two. A poor auditory-verbal short-term memory is unlikely as it is so closely associated to poor reading performance. McClean & Hitch (1999) found that children with poor arithmetic but better reading did not differ from ability- or age- matched controls. Similarly, Bull & Johnston (1997) found that two groups of seven-year-old children who differed in mathematics ability did not differ in digit span after reading ability had been controlled for. A visual-spatial central executive memory deficit is a more likely possibility. The results of the studies reviewed in section 4.4.3 suggest that children with SAD have poor visual-spatial

memory (e.g. Brandys & Rourke, 1991; Fletcher, 1985; Siegel & Linder, 1984) and central executive weaknesses (McClellan & Hitch, 1999).

The results of Study Two suggest it is not possible to predict a child's ability profile from their academic profile. If the SAD children have heterogeneous memory profiles as well as heterogeneous ability profiles, individualised assessment and remediation will be required. A common intervention strategy may not be suitable for all SAD children if they have different cognitive profiles. However, the effectiveness of intervention tailored to children's cognitive profiles has not yet been thoroughly empirically tested. This issue is discussed further in section 10.5.

7.5 Rationale (Study Three)

Analysing the *BAS II* profiles of the SAD children revealed that they had heterogeneous ability profiles, and that uneven profiles were significantly over-represented. In Study Three further psychometric testing was carried out to determine whether any aspects of the SAD children's cognitive profiles were shared and to gain a more detailed picture of their abilities.

The additional tests covered four main areas: reading comprehension, short-term memory, psychomotor skills and number skills. A reading comprehension test was included to determine whether the children had simply good mechanical reading skills, or whether they could also adequately understand the text they read. Tests of memory were included because children with specific arithmetic difficulties have been found to have memory weaknesses (see section 4.4.3). Tests of psychomotor skills were included, because poor psychomotor abilities are an integral part of Rourke's non-verbal learning disability profile (see section 4.4.2). These tests helped to determine whether any of the children with specific arithmetic difficulties fitted this profile.

7.6 Method (Study three)

7.6.1 Participants

Approximately four months after the assessment carried out in Study Two all 23 children with SAD were traced. Permission was received to assess the 17 children who remained at their original primary school. Permission was gained to test only one of the six children who had now moved on to secondary school. Of the 18 children who participated in Study Three eight were boys and ten were girls. Their mean age was 10 years and 8 months (standard deviation 6.7 months).

7.6.2 Materials

The children had already completed the *BAS II* core and attainment scales, which are described in section 7.2.1. All the children were administered the following additional tests (with the exception of the *BAS II Recall of Objects* test, which was omitted in the case of three children due to time constraints).

Wechsler Objective Reading Dimensions-Reading Comprehension (Rust, Golombok, & Trickey, 1993)

The *WORD Reading Comprehension* test provides an assessment of children's ability to comprehend small pieces of text. The child is asked to read silently or aloud a short piece of text. They are then asked a question about the text. Their score is determined only by their responses to the questions, it is not directly affected by their reading accuracy or rate.

BAS II Recall of Objects (Elliott et al., 1997)

The *BAS II Recall of Objects* test provides a measure of a child's short-term memory. The child is shown a card containing twenty pictures of common objects. They are asked to try and remember as many pictures as they can. The child is given two minutes in which to study the pictures. On the verbal trial the child must say the names of the pictures that they can remember. On the spatial trial, they must place cards representing the pictures they saw in the correct positions on a matrix. The authors of the

test suggest that the child's score on the verbal trial reflects their visual-verbal recall, verbal working memory and their ability to integrate visual and verbal information. The spatial trial is intended to measure their visual-spatial working memory. Only the immediate recall version of this test was used.

LASS Junior (Thomas et al., 2001)

The *LASS Mobile* test was used to assess auditory verbal memory and the *LASS Cave* test was used to assess visual spatial memory. See section 6.2.3 for details of these tests.

Wide Range Assessment of Visual Motor Abilities (WRAVMA) (Adams & Sheslow, 1995)

The WRAVMA consists of three tests that are designed to give a complete assessment of a child's visual-motor abilities. The WRAVMA Matching test is designed to test visual-spatial ability. It consists of increasingly difficult visual spatial tasks. The child has to decide which of the four response pictures 'goes best' with the stimulus picture. Some of the items require attention to small details, but the majority of the items require mental rotation skills. The WRAVMA technical manual reports significant correlations between the WRAVMA Matching test and all the WISC-III (Wechsler, 1991) sub-tests and IQ scores. The highest correlation was between WRAVMA Matching and WISC-III Block Design ($r=0.61$). The WRAVMA Drawing test is designed to measure visual-motor integration. The child has to copy drawings that are increasingly difficult. The WRAVMA Pegboard test is designed to measure fine motor skills. The child has to insert as many pegs as they can into a pegboard in a 90-second period. Scores are obtained both for the child's dominant and non-dominant hand. Both the WRAVMA Pegboard and Drawing tests yield significant correlations with WISC-III IQ scores, but they are slightly lower than the correlations between the WISC-III and the WRAVMA Matching test.

Maths Suite

The children with SAD completed *Spots* (which assess counting speed), *Number Facts* (which assesses recall of addition, subtraction and multiplication facts) and *Most*

(which assess place value understanding). These tests are described in detail in section 6.2.3.

7.6.3 Procedure

The children's schools were contacted approximately six months after the original testing. Testing was completed at the child's school in two sessions of about an hour in length. The assessment took place in the quietest room available. During the first session the children completed the computerised assessments *LASS Junior* and *Maths Suite*. These tests were completed on a laptop computer. During the second session they completed *WORD Reading Comprehension*, the *WRAVMA* tests and the *BAS II Recall of Objects* test. Teachers were supplied with a questionnaire when the children were tested, it asked for details of the child's social and academic development. A copy of the teacher questionnaire is shown in Appendix 3. The additional information obtained from the questionnaire is included in the case studies.

7.7 Results (Study Three)

Table 18. Standardised ability, attainment, memory and psychomotor scores for the children with specific arithmetic difficulties

	Group 1			Group 2					Group 3			Group 4					Mean		
	Bob	Lee	Dave	Jack	Bill	Joy	Mark	Gail	Lucy	Lily	Beth	Eve	Kate	Fay	Tony	Kay		Tom	Colin
Ability																			
GCA	49	70	65	60	85	97	101	81	75	86	96	88	94	89	88	94	82	80	82.22 (13.88)
Verbal	56	71	78	91	105	110	118	93	98	72	82	75	90	90	93	92	85	86	88.06 (14.84)
Non-Verbal	54	66	68	58	81	89	91	84	70	95	97	101	91	93	91	98	91	79	83.17 (14.25)
Spatial	63	89	65	49	77	92	94	76	67	99	112	94	105	91	87	95	79	87	84.50 (16.04)
Matching	70	69	63	48	73	110	99	54	63	99	108	99	101	96	92	80	89	102	84.17 (19.42)
Attainment																			
Word Reading	81	93	106	86	124	122	134	108	90	113	102	124	116	124	108	100	108	104	107.78 (14.27)
Spelling	79	86		89		102		106	89					112		99	94	114	96.44 (12.14)
Reading Comp.	77	74	84	77	96		110	90	74	79	76	80	91	86	94	94	78	79	85.61 (10.43)
Number Skills	55	75	77	66	79	84	88	82	75	90	81	87	85	77	89	82	79	87	79.89 (8.69)
Memory																			
Recall of Objects verbal	73	93	72	94	108	^c	112	88	108	^c	88	97	64	99	^c	115	79	79	90.30 (15.82)
Recall of Objects spatial	57	86	87	70	105 ^b	^c	84	73	^d	^c	105 ^b	105 ^b		105 ^b	^c	94	91	91	90.46 (15.20)
Mobile	81	74	104	^a	90	106	106	113	94	83	116	113	104	83	94	94	75	113	97.18 (13.13)
Cave	78	75	79	^a	88	88	110	72	82	74	100	96	65	107	108	79	116	110	88.38 (15.22)
Psychomotor																			
Drawing	81	103	87	82	76	92	115	93	93	87	135	109	104	93	93	99	86	103	96.17 (14.05)
Pegboard	58	83	100	56	78	81	111	73	86	106	109	89	105	94	68	123	112	80	89.56 (19.27)
Number below expected	Yes	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	Yes	Yes	No	Yes	Yes	No	

Note. Where appropriate t scores have been converted to standard scores, to ease comparisons across tests. ^aResults excluded due to exceptionally poor mouse control. ^bThese children scored at the ceiling of the test, so the standard score given, may be an underestimate of their true ability. ^cTests not administered due to time constraints. ^dResult invalid due to administration error.

Table 19. Maths Suite scores for the children with specific arithmetic difficulties

	Group 1			Group 2						Group 3			Group 4					
	Bob	Lee	Dave	Jack	Bill	Joy	Mark	Gail	Lucy	Lily	Beth	Eve	Kate	Fay	Tony	Kay	Tom	Colin
Age	11:0	10:7	10:8	10:1	11:2	11:1	11:8	10:2	10:9	10:8	10:7	10:11	10:2	10:9	10:0	10:3	11:1	10:7
Number Facts (raw score)	15	19	21	11	25	36	30	25	11	31	24	23	22	24	25	19	23	29
Number Facts (centile)	≤5	≤5	6-10	≤5	11-25	≥76		26-50	≤5	26-50	11-25	11-25	11-25	11-25	26-50	11-25	11-25	26-50
Most (raw score)	12	21	21	16	28	24	21	15	13	26	23	18	18	16	27	25	19	25
Most (centile)	<5	11-25	11-25	5	51-75	11-25		<5	<5	26-50	11-25	6-10	6-10	<5	26-50	26-50	6-10	26-50

The mean ability, attainment, psychomotor and memory scores for the SAD children are shown in Table 1. The *BAS II* ability and achievement scores have been discussed for all the 23 children with SAD included in the first study and will not be discussed further here. The mean reading comprehension score is below average whilst the mean single word reading score is within the average range; this suggests that the SAD children's understanding of the text they read is poorer than their word recognition skills. The mean scores for the *BAS II Immediate Recall of Objects* spatial trial and verbal trial are practically equal. This suggests that SAD children are equally weak at either visual-spatial or verbal short-term memory. However, it must be noted that the *BAS II Immediate Recall of Objects* verbal trial has a ceiling effect and that four children obtained a raw score of 20 out of 20. Comparing the mean scores of *LASS Mobile* and *LASS Cave* (that do not have ceiling effects), suggests that SAD children have a stronger auditory-verbal memory and a weaker visual-spatial memory. Examining the mean scores for the two psychomotor tests suggests that the SAD children have broadly average drawing abilities and slightly below average fine motor skills. However, as a group their fine motor skill ability is not poorer than their general intellectual ability.

The results of Study Two suggest that children with SAD have heterogeneous ability profiles; examining the individual children's memory and psychomotor scores suggests they also have heterogeneous cognitive profiles. Examining the mean scores for the SAD group is therefore of limited value. Mean scores for a group are only valuable if the majority of the individuals in the group share the same cognitive profile to a significant extent. For example, it is of little value to say that because on average the children with SAD have poorer visual-spatial memory abilities and better auditory verbal memory abilities, that SAD is characterised by a specific weakness in visual-spatial memory. Four of the 18 children have a *LASS Cave* score that is superior to their *LASS Mobile* score and seven have *LASS Cave* scores that are within the average range for the general population or above.

In this chapter an attempt has been made to divide the children with SAD into meaningful groups. In doing so the purposes of such an analysis have been considered. Firstly identifying the cognitive deficits associated with difficulties in number skills is of theoretical importance. By identifying the deficits associated with reading difficulties, the theoretical understanding of the cognitive factors associated with reading development significantly progressed (Snowling 2001; Stanovich, 2000). Similar benefits may be anticipated as a result of the same scientific exercise in relation to number skills development. Secondly understanding the cognitive basis of a child's academic difficulties could help formulate the most effective remediation strategy. Keeping these two points in mind the author has attempted to divide the children into groups that share a homogenous cognitive and ability profile. One or two case studies are included to illustrate the cognitive profile of each group.

7.7.1 A note on intellectual ability discrepancies

Ten of the eighteen children in this analysis have a *Number Skills* score that is significantly below the level that would be expected on the basis of their intellectual ability. It could be argued that only the scores of the children with GCA/Number Skills discrepancies should be analysed. The other children's *Number Skills* scores are commensurate with their GCA and their poor reasoning ability can explain their low scores. However, this view ignores the need for effective teaching strategies for low ability students. Although these students may not catch up with their higher ability peers, an examination of their cognitive profile may inform the design of an effective teaching programme. It is an empirical question whether teaching programmes should be designed according to a child's intellectual level (i. e. different programmes for students with high or low intellectual abilities, regardless of the pattern of their abilities) or should be designed to match the pattern of a child's intellectual profile (e.g. regardless of overall intellectual abilities children who have relatively strong verbal skills will benefit from a similar type of intervention). The analysis of the cognitive scores of these 18 pupils' suggests that some

pupils whose number skills are commensurate with their intellectual ability share very similar profiles to pupils whose number skills are discrepant from their intellectual ability. Each group has at least one child who has a Number Skills/GCA discrepancy therefore it can be argued that each cognitive deficit can lead to number skills difficulties that are not fully accounted for by the child's intellectual level. Group 1, which contains children who have marked difficulties in all ability and cognitive domains, is somewhat of a special case, and is discussed in section 7.7.2.

7.7.2 Group 1: Low general conceptual ability

Three of the 18 children with SAD are grouped together because they have low ability in all three clusters. The children in this category had a GCA standard score of 70 or below and each cluster score was below 90. It could be argued that these children are not so much unusually poor at number skills, but rather unexpectedly good readers. All three boys in Group 1 had *Word Reading* scores that were significantly higher than would have been predicted on the basis of their GCA.

An illustrative case study of Bob, a boy with low general reasoning ability

Case History This information is derived from a detailed report of Bob's academic and social development supplied by his school. Bob was 11 years 0 months old when this report was supplied. Bob attended a city primary school, situated in a large estate of social housing. He was in Year 6.

Bob's teacher described him as a friendly boy who is very polite and wants to make friends. However, Bob was said to have difficulty being accepted by his peers and tended to play with younger pupils. Bob was described as having poor concentration. It was also reported that he tended to be disorganised, easily distracted and that he sometimes distracted other pupils. The teacher said Bob was attention seeking and that he also seeks the approval of teachers. The teacher described Bob's ability to read aloud and his reading comprehension abilities as weak. Two tests administered by the teacher when he was 10 years 7 months old confirmed that Bob's literacy abilities were below average. He

achieved a standard score of <70 (< centile 2) on the *Optional Year 5 SAT Reading Section* and a standard score of 73 (centile 4) on the *Optional Year 5 SAT Spelling Section*. The teacher describes Bob's written arithmetic attainment and his mathematical understanding as extremely poor. His mental arithmetic is said to be weak. The teacher administered the *Optional Year 5 SAT Written Mathematics Section* when Bob was 10 years 7 months; he achieved a standard score of <70 (< centile 2). On the *Mental Mathematics section* administered at the same time Bob achieved a standard score of 79 (centile 8).

The teacher gave a description of Bob's mathematical abilities. Bob's mental mathematics is reported to be better than his written mathematics. His oral work is better than his written work in all subjects. Bob has problems remembering number facts and applying skills he has previously learnt to new situations. He often confuses different methods of calculating.

Bob is on the SEN Code of Practice at Stage 3. He has numeracy targets included in his Individual Education Plan (IEP). Bob has extra support within the classroom, from a child support assistant. The amount of support provided is variable, but it is normally for about 20 minutes two or three times a week.

Standardised Test Performance Bob completed the BAS II core and attainment scales when he was 10 years 4 months old. When he was 11 years 0 months Bob was administered tests that assessed psychomotor ability, memory and reading comprehension, he also completed the *Maths Suite* assessment program.

Intellectual Ability Bob had a general conceptual ability (GCA) of 49 (<centile 0.1), which is clearly well below the normal range. Bob's standard scores on the three ability clusters were all low: verbal ability 56 (centile 0.2), non-verbal reasoning ability 54 (centile 0.1), spatial ability 63 (centile 0.7). There was a statistically significant difference between Bob's spatial ability and his GCA. A difference of this magnitude is found in less than 15% of the general population. However, Bob's relatively high spatial ability was due to

his score on the *Recall of Designs* sub-test (t score 39, centile 14); on the other sub-test in the spatial ability cluster *Pattern Construction* Bob achieved his lowest sub-test score (t score 20; centile 1). Overall Bob's performance on the *BAS II* core scales indicates he has very poor reasoning ability in all domains. His relative strength appears to be his memory.

Achievement Scales The *BAS II* achievement scales were administered to Bob at 10 years 4 months. Bob's standard scores on the literacy scales were below average: *Word Reading* 81 (centile 10; age equivalent 7 years 1 month), *Spelling* 79 (centile 8; age equivalent 7 years 10 months). Bob's *Number Skills* score was significantly below average and below the level that would be expected on the basis of his age and GCA (standard score 55; centile 0.1). It was considerably worse than his literacy scores. An analysis of Bob's performance on the *Number Skills* test revealed that he was able to say the names of two-digit numbers, but was unable to name three-digit numbers (except 100). He was unable to answer single digit addition or subtraction sums correctly. On some sums Bob miscalculated (e.g. $2 + 3 = 6$); on others he used the wrong operation (e.g. $4 - 1 = 5$). When Bob was 11 years 1 month he completed the *WORD Reading Comprehension* test. His score was below average (standard score 77; centile 6), but not quite as poor as his *Number Skills* score. Overall Bob's results are generally consistent with the teacher-administered tests. His *Word Reading* score is somewhat higher than the reading score he achieved at school. This is probably because the SAT reading test used at school includes reading comprehension elements.

Memory Scales At 11 years 0 months Bob attempted *LASS Mobile* and *LASS Cave*. He achieved a score at centile 10 on *Mobile* and at centile 7 on *Cave*. Bob completed the *BAS II Recall of Objects-Immediate* test at 11 years 0 months. He achieved below average scores on both the verbal (t score 32; centile 4) and spatial (t score 21; centile 1) trials. Overall Bob's short-term memory is poor. However, his memory scores (with the exception of the *BAS II Recall of Objects-Immediate* spatial trial) are somewhat better than his reasoning abilities.

Psychomotor Scales At 11 years 0 month Bob was administered the *WRAVMA Drawing* test. This test requires the examinee to copy accurately line drawings of increasing complexity, and is designed to measure visual-motor skills. Bob achieved a score that was below average (standard score 81; centile 10). He also completed the *WRAVMA Pegboard* test. With his dominant right hand he achieved a score that was significantly below average (standard score 58; centile 0.6). He also achieved a significantly below average score with his non-dominant left hand (standard score 59; centile 0.7). Overall his *WRAVMA Visual-Motor-Composite*, which is made up of the *Matching, Drawing and Pegboard* tests was very low (standard score 59; centile 0.7).

Conclusions Bob is a boy whose abilities in all cognitive spheres are significantly below average. His single word reading, drawing and auditory-verbal-sequential memory scores are his highest attainments. It is likely that the combination of intensive teaching and relatively unimpaired memory skills enabled Bob to read at a relatively high level. The following argument is put forward as a tentative explanation as to why Bob has a lower *Number Skills* score than would be expected on the basis of his GCA. Study One indicated that mathematical attainment correlates highly with reasoning ability. Bob has an unusually high *Recall of Designs* score, which means that his scores on the other reasoning based tests would be lower than the majority of the other children with his GCA. If his core reasoning abilities are lower than most children with similar GCAs it is perhaps not unsurprising that his *Number Skills* score is lower than expected.

Commonalties and differences in Group 1

Both Lee and Dave share profiles that are very similar to Bob's. Their highest cognitive scores also come from the memory scales. Lee achieved a score within the average range on the *BAS II Recall of Objects Immediate* verbal trial, whilst Dave achieved a score within the average range on *LASS Mobile*. The profiles of the children in Group 1 suggest that whilst single word reading can develop reasonably well despite poor reasoning

ability, number skills fair less well. The relatively unimpaired memory scores of the boys in Group 1 may have facilitated the development of their single word reading abilities.

7.7.3 Group 2: Non-verbal learning difficulties (NLD)

Six children were placed in Group 2. They all had verbal ability scores in the average or above average range and a significant discrepancy between their verbal and spatial abilities. These children had the NLD profile described by Rourke, i. e. poor visual spatial skills, but better verbal ability and verbal memory. For a full description of the NLD profile see section 4.6. Rourke argued that all children with a specific learning difficulty whose reading and spelling skills are superior to their arithmetic skills would have a profile of non-verbal difficulties. However, the studies reported in section 4.4.1 (e.g. Ackerman & Dykman, 1996; Share et al., 1988) have disputed the assertion that this attainment profile is always associated with an NLD cognitive/ability profile.

Nevertheless, some children with SAD, including a proportion of the children in the present sample, do have significant weaknesses in visual-spatial and psychomotor skills and relatively unimpaired verbal skills.

Four of the six children in Group 2 had number skills scores below the level that would be expected on the basis of age and GCA. Two case studies have been chosen to represent Group 2. Jack was chosen because he has particularly severe non-verbal learning difficulties; Gail was chosen because she had a visual problem that may account for her difficulties.

An illustrative case study of Jack, a child with severe non-verbal learning difficulties

Case History This information is derived from detailed report of Jack's academic and social development supplied by his school. It included additional details of his early development provided by his parents. Jack was 10 years 1 month when this report was supplied. He had a younger brother. Jack attended a small rural primary school and was in Year 5. Jack was delivered early via an emergency caesarean section. His birth weight

(0.85 kg) was very low. Jack is left-handed; he has problems distinguishing left from right. He has always been physically small and has considerable difficulties with fine motor skills e.g. when working on the computer he has difficulties controlling the mouse.

Jack is described by his teacher as a timid and submissive boy who is very quiet in school. He tends to prefer the company of younger pupils. Jack's teacher described his ability to read aloud and his reading comprehension abilities as average. Two tests administered by Jack's teacher when he was 9 years 7 months confirmed that Jack's literacy abilities are average to slightly below average. He achieved a standard score of 90 (centile 25) on the *Young's Group Reading Test* and a standard score of 86 (centile 18) on the *Spar Spelling Test* (Young & O'Shea, 1981). Jack is said to enjoy stories, writing and history. He takes pleasure in supporting a younger girl at his school who has literacy difficulties. The teacher describes Jack's written arithmetic attainment and his mathematical understanding as weak. His mental arithmetic is said to be extremely poor. The teacher administered *Young's Group Maths Test* when Jack was 9 years 7 months; he achieved a standard score of 72 (centile 3).

Jack's teacher gave a description of Jack's mathematical abilities. Jack can recognise the numbers between 0 and 100. He understands the process of addition and subtraction, but has difficulty calculating and understanding division and multiplication problems. Jack's memory for number facts is described as very poor. His teacher reported that Jack could not recall any multiplication facts automatically. Jack's lack of confidence (particularly in a group setting) contributes to his difficulties with mental maths. Jack is only able to cope with one maths idea or procedure in one session; he is not able to combine procedures to solve a problem.

Jack is on the SEN Code of Practice at Stage 3 because of his mathematical and motor difficulties. For four days a week Jack works with Year 2 pupils for one hour of maths. For part of this time Jack works with a group of four children who are supported by

a classroom assistant. It is reported that Jack enjoys these sessions. Sometimes Jack completes paired work with a slightly more able child.

Standardised Test Performance Jack completed the *BAS II* core and attainment scales when he was 9 years 5 months. When he was 10 years 1 month old he was administered tests that assessed psychomotor ability, memory and reading comprehension and also completed the *Maths Suite* assessment program.

Intellectual Ability Jack had a general conceptual ability (GCA) of 60 (centile 0.8), which is clearly well below the normal range. However, Jack's scores on the three ability clusters differed widely. Jack's verbal ability was within the average range (standard score 91; centile 27), whilst his non-verbal reasoning ability (standard score 58; centile 0.3), and his spatial ability (standard score 49; centile < 0.1) were considerably below average. The difference between Jack's verbal and non-verbal reasoning ability was statistically significant, a difference of this magnitude being found in less than 2% of the general population. The difference between Jack's verbal and spatial ability was also statistically significant. A difference of this magnitude is found in less than 1% of the general population. There were no significant differences between the sub-tests that comprised the non-verbal reasoning ability and spatial ability clusters. However, Jack's Word Definitions (t score 50; centile 50) score was significantly higher than his Similarities score (t score 39; centile 14).

Achievement Scales The results of the *BAS II* achievement scales administered to Jack at 9 years 5 months confirm the results of the teacher-administered tests. Jack's standard scores on the literacy scales were only slightly below average, but still within the normal range (*Word Reading* 86, centile 18, age equivalent 7 years 1 month), (*Spelling* 89, centile 23, age equivalent 8 years 3 months). The majority of Jack's spelling errors were phonologically correct (e. g. 'frend' for friend, 'wiul' for while, and 'circul' for circle). In contrast, Jack's *Number Skills* score was significantly below average (standard score 66; centile 1) and below the level that would be expected on the basis of his GCA. An analysis

of Jack's performance on the *Number Skills* test revealed that Jack was able to say the names of two-digit numbers, but was unable to name three-digit numbers (except 100). Jack was able to answer one single digit addition sum correctly, but made an error on another single digit sum. Jack answered a single digit subtraction sum wrong because he did not correctly identify the operation sign and added the numbers together. He could not attempt any of the sums involving two-digit numbers. When Jack was 10 years 1 month he completed the *WORD Reading Comprehension* test. His score was below average (standard score 77; centile 6), but not quite as poor as his *Number Skills* score.

Memory Scales At 10 years 1 month Jack attempted the computerised assessments *LASS Mobile* and *LASS Cave*. However, testing had to be abandoned, because Jack's poor mouse control made the results unreliable. Jack did complete the *BAS II Recall of Objects-Immediate* test at 10 years 1 month. In the verbal trial he achieved a score well within the average range (t score 46; centile 34). In contrast, in the spatial trial Jack achieved a score that was significantly below average (t score 30; centile 2). These scores indicate that Jack's verbal short-term memory is superior to his spatial short-term memory.

Psychomotor Scales At 10 years 1 month Jack was administered the *WRAVMA Drawing* test. Jack achieved a score that was below average (standard score 82; centile 12). Jack also completed the *WRAVMA Pegboard* test. With his dominant left hand he achieved a score that was significantly below average (standard score 56; centile 0.4). He also achieved a significantly below average score with his non-dominant right hand (standard score 53; centile 0.1). Overall Jack's *WRAVMA Visual-Motor-Composite*, which is made up of the *Matching*, *Drawing* and *Pegboard* tests was very low (standard score 48; centile 0.05).

Conclusions Jack fits the non-verbal learning disability category described by Rourke. Jack's verbal reasoning ability and verbal memory is within the average range, whilst his non-verbal ability, spatial ability and spatial memory are considerably below average. His single word reading and spelling attainments are only slightly below average. Jack's

number skills are significantly below average and below the level that would be expected on the basis of his age and GCA. Although Jack's reading comprehension abilities are superior to his number skills, they are still somewhat below average. Jack's difficulties are not confined to academic work and this has been recognised by his teachers, as his motor difficulties are highlighted on his IEP. Jack's very low score on the WRAVMA Pegboard test confirms the severity of his fine motor difficulties. Rourke and Del Dotto (1994) identified fine motor difficulties as an integral part of the NLD profile.

An illustrative case study of Gail, a child with moderate non-verbal learning difficulties

Case History This information is derived from two detailed reports of Gail's academic and social development. Gail's teacher supplied one report and Gail's parents supplied the other. Gail was 9 years 10 months when these reports were provided. Gail had an older sister and a younger brother. Neither of her siblings had significant educational problems. Gail attended a rural primary school, and was in Year 4.

Gail was delivered two weeks postmature. Her birth weight was 3.67 kg. Gail was diagnosed with severe squints at the age of six months. This condition had a significant effect on her vision, so much so that she didn't walk or crawl until the left eye was corrected at 16 ½ months. Within days of her eye operation Gail was crawling and within a week she was walking. Gail's right eye was corrected when she was 3 years 6 months. She is minimally long-sighted and wears glasses. Gail's parents describe her as a very clumsy child who has poor hand-eye co-ordination.

Gail's health is described as good by her parents although she suffers from eczema. Gail's hearing has been tested and is within normal limits. In contrast to her delayed motor development Gail's speech development was good. She spoke her first words at 6 to 8 months and was speaking simple sentences at 12 to 14 months. Gail has never had any significant pronunciation difficulties and can clearly express her ideas orally

Gail's teacher describes her as friendly and enthusiastic. However, both Gail's parents and teacher describe her as lacking in concentration. Gail's teacher describes her

written and mental arithmetic attainment as extremely poor. The teacher administered the *QCA Written Maths Test* when Gail was 8 years 11 months; she achieved a standard score of 70 (centile 2).

The teacher gave a description of Gail's mathematical abilities. Gail is said to have difficulties with spatial concepts and have a poor short-term memory. Her performance on numeracy tasks is said to be very variable, sometimes she cannot cope with very basic concepts, but on other occasions she grasps very advanced skills.

Gail was put on the SEN Code of Practice at Stage 1, when she was 5 years old. Following an assessment by a LEA Educational Psychologist she was moved to Stage 2 when she was 8 years old. She receives some extra help for numeracy in a small group setting.

Standardised Test Performance Gail underwent two testing sessions. She completed all the *BAS II* scales when she was 9 years 10 months. Additional memory test results were available for Gail, because her parents brought her to the Psychology Department of the University of Hull for an independent assessment. She had been administered all the *BAS II* diagnostic tests as part of this assessment (the other children in the sample were only administered the *Recall of Objects Immediate* test). When she was 10 years 2 months old Gail was administered tests that assessed psychomotor ability, memory and also completed the *Maths Suite* assessment program.

Intellectual Ability Gail had a general conceptual ability (GCA) of 81 (centile 10), which is below average. However, Gail's scores on the three ability clusters differed significantly. Her verbal ability was within the average range (standard score 93; centile 32), whilst her non-verbal reasoning ability (standard score 84; centile 14), and her spatial ability (standard score 76; centile 5) were below average. The difference between Gail's verbal reasoning and spatial ability was statistically significant. A difference of this magnitude is fairly common, being found in more than 25% of the general population. However, there are very significant differences within the spatial ability cluster. Gail's *Recall of Designs*

score (t score 47; centile 38) was significantly higher than her *Pattern Construction* score (t score 26; centile 1). A difference of this magnitude is relatively rare, it is found in less than 5% of the general population.

The results obtained from the *BAS II* are consistent with the results of the *WISC-III^{UK}* administered by the LEA Educational Psychologist when Gail was 8 years old. At that age Gail's Full Scale IQ was slightly below average (standard score 84; centile 14). There was a large difference between Gail's Verbal IQ, which was within the average range (standard score 93; centile 32) and her Performance IQ, which was below average (standard score 84; centile 14).

Achievement Scales The *BAS II* achievement scales and the *WORD Reading Comprehension* tests were administered to Gail at 9 years 10 months. Gail's standard scores on the literacy scales were within the average range (*Word Reading* 108; centile 70, age equivalent 10 years 9 months), (*Spelling* 106, centile 66, age equivalent 10 years 9 months). The majority of Gail's spelling errors were phonologically correct (e. g. 'farmersist' for pharmacist). Gail's score on the *WORD Reading Comprehension* test was also within the average range (standard score 90; centile 35). In contrast, Gail's *Number Skills* score was significantly below average (standard score 82; centile 12; equivalent age 8 years 3 months). An analysis of Gail's performance on the *Number Skills* test revealed that Gail was able to answer single digit addition multiplication sums correctly, but made an error on another single digit subtraction sum. She attempted the sums involving two-digit numbers, but got them all wrong. Gail was able to identify and name three-digit numbers.

The results of the attainment tests administered to Gail by the Educational Psychologist when she was 8 years old were consistent with these more recent results. Gail achieved scores within the average range on the *WORD Basic Reading* (standard score 101; centile 53; age equivalent 8 years) and *Spelling* (standard score 93; centile 32; age equivalent 7 years 6 months) tests. Her score was below average on the *WORD*

Numerical Operations test (standard score 86; centile 18), but within the average range on the *WOND Mathematics Reasoning* test (standard score 92; centile 30; age equivalent 7 years 3 months).

Memory Scales Gail completed the *BAS II Diagnostic* scales test at 9 years 2 months. She achieved a score within the average range on the *Speed of Information Processing* test (t score 49; centile 46). On *Recall of Objects-Immediate* verbal trial Gail achieved a score that was slightly below average (t score 42; centile 21). In contrast, in the spatial trial Gail achieved a score that was significantly below average (t score 32; centile 4). The same pattern emerged in *BAS Recall of Objects-Delayed* tests. In the verbal trial she achieved a score that was below average (t score 39; centile 14), whilst in the spatial trial Gail achieved a score that was even poorer (t score 32; centile 4). Gail achieved scores well within the average range for both the *Recall of Digits Forward* (t score 50; centile 50) and the *Recall of Digits Backward* (t score 53; centile 62) tests. Her score on the *Recognition of Pictures* test was below average (t score 37; centile 12). At 10 years 2 months Gail attempted *Mobile* (a test of auditory-verbal memory) and *Cave* (a test of visual-spatial memory) from the *LASS Junior* computerised assessment suite (Thomas et al., 2001). Gail's auditory verbal memory as measured by *LASS Mobile* was well above average (centile 80), whilst her visual-spatial memory was well below average (centile 3).

Overall these tests indicate that Gail's auditory-verbal- sequential short-term memory is average to above average, whilst her short-term verbal memory for object names is slightly below average. Gail's memory for visual-spatial information is poor.

Psychomotor Scales At 10 years 2 months Gail was administered the *WRAVMA Drawing* test. She achieved a score that was within the average range (standard score 93; centile 32). Gail also completed the *WRAVMA Pegboard* test. With her dominant right hand she achieved a score that was significantly below average (standard score 73; centile 4). She achieved an average score with her non-dominant left hand (standard score 94; centile 34).

Overall her *WRAVMA Visual-Motor-Composite*, which is made up of the *Matching*, *Drawing* and *Pegboard* tests was very low (standard score 64; centile 1).

Conclusions Gail's pattern of abilities is also consistent with the non-verbal disability pattern described by Rourke, however her difficulties are less severe than those displayed by Jack. Her verbal reasoning ability is superior to her non-verbal and spatial abilities. Her single word reading and spelling attainments are superior to her number skills. Gail's reading comprehension abilities are poorer than her single word reading and spelling. Gail's verbal memory scores are considerably better than her spatial memory scores. Gail's low score on the *WRAVMA Pegboard* test indicates that she has poor fine motor control that is consistent with the NLD profile. It is surprising that Gail's fine motor score for her non-dominant left hand is so much better than for her dominant right hand. However, Gail's mother reported that Gail had been left-handed until she went to school, but she was pushed by the teacher to use her right, hand. Gail may therefore be ambidextrous or 'truly' left-handed (see Annett, 1985 for a discussion of handedness).

Gail's developmental history provides clues about why she may have difficulties with visual-spatial and visual-motor skills. Gail's squints appeared to delay her gross motor development. Her orthoptic difficulties would almost certainly have reduced her ability to explore the world and would have altered her perception. These factors could have stunted the development of her visual-spatial and visual-motor abilities.

Commonalties and differences in Group 2

Jack, Bill, Gail and Lucy clearly fit into the NLD group because their scores on all the measures of spatial and non-verbal ability are below average and significantly poorer than their verbal reasoning ability. Joy is a more marginal case because even though her *BAS* profile, memory profile and poor fine motor skills are consistent with the NLD profile, her *WRAVMA Matching* score is inconsistent; she achieved an above average score on the *Matching* test, which tests visual spatial reasoning. Mark is also a marginal case: his non-verbal and spatial ability scores, although discrepant from his verbal reasoning

ability are all within the average range. He also has psychomotor skills that are above average range. None of the children in Group 2 show evidence of significant verbal short-term memory differences; this is consistent with the NLD profile. All of the children, with the exception of Mark, show deficits on the *WRAYMA Pegboard* test, which is consistent with the psychomotor difficulties highlighted by Rourke. It is interesting to note that only two children (Jack and Bill) show deficits on the *WRAYMA Drawing* test, which draws on visual-motor skills one would expect NLD children to be poor at.

7.7.4 Group 3: Verbal reasoning weaknesses

An illustrative case study of Beth, a child with verbal reasoning weaknesses

Case History This information is derived from a detailed report of Beth's academic and social development, supplied by the school. Beth was 10 years 7 months old when this report was supplied. Beth's teacher described her as friendly and popular. She is said to be a responsive and co-operative child who works well. The teacher described Beth's written and mental arithmetic attainment as weak. The teacher administered the Maths Links Test (Level 1) when Beth was 10 years 0 months; she achieved a slightly below average standard score of 88 (centile 21). In contrast, Beth's ability to read aloud and comprehend what she reads was described as average.

The teacher gave a description of Beth's mathematical abilities. Beth is said to work well on basic problems, but finds it difficult to progress onto harder ones. She is described as lacking confidence. Beth is not on the SEN Code of Practice and does not receive any specific extra help.

Standardised Test Performance Beth underwent three testing sessions. She completed the *BAS II* core and attainment scales when she was 10 years 2 months old. Beth was administered tests, which assessed psychomotor ability, memory and completed the *Maths Suite* assessment program when she was 10 years 7 months old.

Intellectual Ability Beth had a general conceptual ability (GCA) of 96 (centile 39), which is within the average range. However, her scores on the three ability clusters differed.

Beth's verbal ability was below average (standard score, 82; centile 12), whilst her non-verbal reasoning ability was within the average range (standard score, 97; centile 42), and her spatial ability was above average (standard score 112; centile 79). The difference between Beth's verbal and non-verbal reasoning ability was statistically significant. A difference of this magnitude is fairly common, being found in 25% of the general population. The difference between Beth's verbal and spatial ability was also statistically significant. A difference of this magnitude is less common, being found in 5% of the general population. Within the spatial ability cluster, Beth's *Pattern Construction* score (t score 65; centile 93) was significantly better than her *Recall of Designs* score (t score 49; centile 46).

Achievement Scales The *BAS II* achievement scales were administered to Beth at 10 years 2 months. Beth's score on the *Word Reading* test was within the average range (standard score 102; centile 55; equivalent age 10 years 3 months). In contrast Beth's *Number Skills* score was below average (standard score 81; centile 10; age equivalent 8 years 9 months). An analysis of Beth's performance on the *Number Skills* test revealed that she was able to identify and name three digit numbers. She answered a multi-digit addition sum (with no re-grouping) correctly. She also answered a multi-digit subtraction sum (with no re-grouping) correctly; however, she used a counting strategy rather than the more conventional pencil-and-paper procedure. When she attempted to use the conventional written procedure on another multi-digit subtraction sum (with no re-grouping), she made an error. Beth answered a single digit multiplication incorrectly. Beth answered all sums involving regrouping incorrectly. She was unable to attempt multiplication or division problems involving multi-digit numbers. Beth completed the *WORD Reading Comprehension* test when she was 10 years 7 months, and achieved a standard score of 76 (centile 5), which is significantly below average.

Memory Scales At 10 years 2 months Beth completed two sets of memory tests. She achieved a score that was slightly below average (t score 42; centile 21) on the *Recall of*

Objects-Immediate verbal trial. On the spatial trial Beth attained a raw score of 20 out of 20. This produced a t score of > 32 (centile > 62). At 10 years 2 months Beth attempted *Mobile* (that measures auditory-verbal memory) and *Cave* (that measures visual-spatial memory) from the *LASS Junior* computerised assessment suite (Thomas et al., 2001). Beth's auditory verbal memory as measured by *LASS Mobile* was above average (centile 86), her visual-spatial memory as measured by *LASS Cave* was within the average range (centile 50).

Overall, these tests indicate that Beth's auditory-verbal and visual-spatial short-term memory is average to above average, whilst her verbal short-term memory for names of objects is slightly below average.

Psychomotor Scales At 10 years 7 months Beth was administered the *WRAVMA Drawing* test. She achieved a score that was above average (standard score 135; centile 99). Beth also completed the *WRAVMA Pegboard* test. With her dominant right hand Beth achieved a score that was within the average range (standard score 109; centile 73). She achieved an average score with her non-dominant left hand (standard score 101; centile 53). Overall Beth's *WRAVMA Visual-Motor-Composite*, which is made up of the *Matching*, *Drawing* and *Pegboard* tests was above average (standard score 132; centile 98).

Conclusions Beth is a girl of average general conceptual ability; she has weak verbal reasoning ability, average non-verbal reasoning ability and above average spatial ability. Whilst Beth's single word reading ability is average, her number skills are below average and significantly poorer than one would expect on the basis of her age and GCA. Beth also has reading comprehension abilities that are significantly below average. Beth has average visual spatial short-term memory and above average auditory-verbal sequential memory, her verbal short-term memory for object names is slightly below average. Beth's psychomotor abilities are average to above average.

Despite average to above average scores on tests of non-verbal and spatial abilities Beth has significant problems with mechanical number skills. Beth's only significant

cognitive deficit is her weakness in verbal reasoning. It seems unlikely that Beth's slight weakness on one memory test could account for her significant number difficulties. Beth's difficulties making generalisations and drawing inferences from verbal information, is a more likely explanation of her number skills weakness.

Commonalties and differences in the verbal reasoning weakness group

The group of children with distinct verbal reasoning weaknesses is very small, with only three members, so caution must be taken when associating this deficit with number skills difficulties. However, it is worth noting that in Study Two, of 23 children, five (over one fifth) had spatial skills that were higher than their verbal skills. Whilst all the children with significantly higher verbal skills were re-tested in this study, two of the children with significantly higher verbal skills dropped out.

Beth and Eve do not have any significant memory weaknesses, therefore verbal reasoning weaknesses appear to be the more likely cognitive cause of their academic difficulties. However, with such a small group size the possibility that all three children's specific difficulty with arithmetic may be due to environmental/emotional factors must be considered, hence their ability profiles may be coincidental. Lily has memory difficulties as well as verbal reasoning weaknesses, therefore it is impossible to say which cognitive deficit is impacting on her number skills difficulties. It is interesting to note that despite having below average scores on both visual-spatial and auditory verbal memory tests Lily still achieves an above average reading score. Auditory-verbal memory defects are usually associated with reading difficulties (e. g. Hulme, 1981; Shankweiler et al., 1979; see section 5.2.2 for further discussion of this issue).

7.7.5 Group 4: Memory weaknesses

The children in Group 4 do not have any significant discrepancies between their *BAS II* ability clusters. Hence it is unlikely that their number skills difficulties are due to a particular bias in either verbal or non-verbal processing. It is therefore suggested that these children's number skills difficulties are likely be due to memory deficits.

An illustrative case study of Fay a child with an auditory-verbal memory weakness

Case History This information is derived from a detailed report of Fay's academic and social development, supplied by the school. Fay was 10 years 9 months old when this report was supplied. The teacher describes her as friendly, popular and co-operative, but at the same time withdrawn, oversensitive, anxious and timid. Fay tends to seek the approval of teachers. Fay's teacher describes Fay's mathematical understanding, written and mental arithmetic attainment as weak. The teacher administered the NFER Maths test when Fay was 10 years 4 months; she achieved a below average standard score of 78 (centile 7). Fay's ability to read aloud was described as average, although her reading comprehension ability was described as weak. The teacher administered the NFER Reading test when Fay was 10 years 4 months; she achieved an average standard score of 99 (centile 47).

Fay's teacher gave a description of Fay's mathematical abilities. Fay is reported to lack confidence and find learning new concepts stressful. Fay's recall of multiplication tables is said to be good (however this was not confirmed by the score she achieved on the *Number Facts* test; see Table 19), but she finds it difficult to apply her knowledge in problem situations. It is reported that Fay has difficulties interpreting written problems. Fay's teacher did not state whether or not Fay is on the Code of Practice. However, she did state that Fay received some extra help in a small group from a child support assistant, once a week, so it is most likely that Fay is on the Code of Practice at Stage 2 at least. Fay was also receiving extra help from a private tutor at home.

Standardised Test Performance Fay underwent three testing sessions. She completed the *BAS II* core, diagnostic and attainment scales when she was 10 years 2 months old. Fay was administered tests assessing psychomotor ability and memory and completed the *Maths Suite* assessment program when she was 10 years 9 months old.

Intellectual Ability Fay had a general conceptual ability (GCA) of 89 (centile 23), which is slightly below average but within the normal range. Fay's scores on the three ability clusters were all within the average range and did not differ significantly (verbal ability:

standard score, 90, centile 25; non-verbal ability: standard score, 93, centile 32; spatial ability: standard score, 91, centile 27).

Achievement Scales The *BAS II* achievement scales were administered to Fay at 10 years 2 months. Her scores on the *Word Reading* test (standard score 124; centile 95; age equivalent 13 years 3 months) and the *Spelling* test (standard score 112; centile 79; age equivalent 11 years 9 months) were above average. In contrast Fay's *Number Skills* score was below average and below the level that would be expected on the basis of her age an GCA (standard score 77; centile 6; age equivalent 8 years 3 months). An analysis of Fay's performance on the *Number Skills* test revealed that she was able to identify and name three digit numbers. She answered single digit addition, subtraction and multiplication sums correctly. Fay answered a multi-digit addition sum (with no re-grouping) correctly. However, she erred on multi-digit subtraction sums that did not involve regrouping. Fay answered all sums involving regrouping or multi-digit multiplication (if the number was greater than 12) incorrectly. She was unable to attempt multiplication or division problems involving multi-digit numbers. Fay completed the *WORD Reading Comprehension* test when she was 10 years 9 months, achieving a standard score of 86 (centile 18), which is below average. This result contrasted with her above average score for single word reading.

Memory Scales At 10 years 9 months Fay completed two sets of memory tests. Fay achieved a score within the average range on the *Recall of Objects-Immediate* verbal trial (t score 49; centile 46). On the spatial trial Fay attained a raw score of 20 out of 20. This produced a t score of >53 (centile >62). At 10 years 2 months Fay attempted *Mobile* (that measures auditory-verbal memory) and *Cave* (that measures visual-spatial memory) from the *LASS Junior* computerised assessment suite (Thomas et al., 2001). Fay's auditory verbal memory as measured by *LASS Mobile* was below average (centile 13), her visual-spatial memory as measured by *LASS Cave* was within the average range (centile 68).

Overall, these tests indicate that Fay's visual-spatial short-term memory and her memory for object names is average, whilst her auditory-verbal sequential short-term memory is below average.

Psychomotor Scales At 10 years 9 months Fay was administered the *WRAYMA Drawing* test. Fay achieved a score that was within the average range (standard score 93; centile 32). Fay also completed the *WRAYMA Pegboard* test. With her dominant right hand she achieved a score that was within the average range (standard score 94; centile 34). She achieved an average score with her non-dominant left hand (standard score 104; centile 61). Overall her *WRAYMA Visual-Motor-Composite*, which is made up of the *Matching*, *Drawing* and *Pegboard* tests was average (standard score 92; centile 30).

Conclusions Fay is a girl of slightly below average general conceptual ability, she has no significant differences between her verbal reasoning, non-verbal and spatial abilities. Whilst Fay's single word reading ability is above average, Fay's number skills are below average and significantly poorer than one would expect on the basis of her age and GCA. Fay also has reading comprehension abilities that are below average. Fay has average visual spatial short-term memory and verbal short-term memory for object names. Fay's psychomotor abilities are average to above average. Fay's only cognitive deficit is her below average auditory-verbal sequential memory.

Fay's cognitive profile is surprising, as deficits in auditory-verbal memory are usually associated with poor reading (e. g. Hulme, 1981; Shankweiler et al., 1979), however Fay's reading attainment is above average. A similar disassociation was found in Paul a case study described by (Temple, 1989). Paul's reading age was very similar to his chronological age, however he had a digit span of only two and a letter span of three (see section 4.2 for further details of Paul's case).

Commonalties and differences in Group 4

All the children in Group 4, except Tony have a memory weakness; however, these weaknesses do not show a consistent pattern. Kay only has weaknesses in auditory-verbal

sequential memory. Tom has weaknesses in both visual-verbal short-term memory and auditory verbal short-term memory. Kate has weaknesses in both visual-verbal short-term memory and visual-spatial short-term memory. Colin only has weaknesses in visual-verbal short-term memory. Although the *LASS Mobile* and *LASS Cave* tests did not identify any memory weaknesses in Tony, he may have weaknesses that would have been identified by the *BAS II Recall of Objects* test.

7.7.6 The Maths Suite scores of the children in the different groups

The *Maths Suite* scores for the children in the different groups are shown in Table 19. Overall the children with SAD tended to achieve below average scores on both the *Number Facts* and the *Most* tests. The boys in Group 1, who had low general conceptual ability, achieved below average scores for both the *Number Facts* and *Most* tests. The children in Group 2, who had NLD profiles, did not display a homogenous number skills profile. Jack and Lucy achieved significantly below average scores for both *Number Facts* and *Most*. Joy and Gail were clearly stronger at recalling Number facts than understanding Place Value, whilst Bill showed the reverse pattern. In Group 3 (verbal ability weaknesses), Beth and Eve achieved below average scores for both tests; Eve's place value understanding was slightly worse than her Number Fact recall. Lily achieved average scores on both tests. In Group 4 (memory weaknesses), Kate, Fay and Tom all achieved below average scores on both tests, but they had more marked weaknesses in place value understanding. Kay achieved a place value understanding score within the average range, but a below average *Number Facts* score. Colin and Tony achieved average scores on both tests. It is not possible to reliably ascertain whether or not particular cognitive profiles are associated with particular number skills profiles in this sample because it is too small. A larger number of children in each group is required. However, examining the number skills profiles of the children with SAD as a group (which is attempted in Chapter 8) is also somewhat flawed because associations between cognitive and ability profiles cannot be determined when the SAD do not share a particular cognitive profile.

7.8 Discussion (Study Three)

7.8.1 Strengths and limitations of Study Three

The design of this study focussed on one group of children and examined their ability, attainment, memory and psychomotor skills in detail. They were selected using strict criteria, based on teacher recommendations and standardised tests. Controlled comparison studies that focus on group differences in a single domain have two major limitations; they ignore individual differences within groups that may be heterogeneous and they do not examine the inter-relationships between cognitive and ability variables. These limitations can be explored by comparing the present study with that of McClean and Hitch (1999). McClean and Hitch (1999) identified a group of 12 children who achieved a below average score on a standardised arithmetic test and an average score on a standardised reading test. These children were therefore selected using an operational definition very similar to the one in the present study. One stated aim of the McClean and Hitch (1999) study was to consider whether “... working memory deficits are responsible for children’s arithmetic difficulties in arithmetic” (p. 244). The children with SAD were compared to children matched on age and normally achieving younger children who were matched on arithmetic ability. The children with SAD performed significantly more poorly than the age matched control children on some of the tasks that assessed spatial memory, they also performed more poorly than younger ability matched children on one task designed to assess executive working memory. In view of these results McClean and Hitch (1999) concluded that, “deficits in executive and spatial aspects of working memory” (p.240) caused arithmetic difficulties in the children with SAD. McClean and Hitch (1999) do not explore the possibility of different cognitive difficulties leading to the same academic difficulty. Individual scores were not reported so we do not know whether spatial and executive working memory deficits are found in the vast majority of the sample, or whether - as in the current - study a sizeable minority of the sample did not have

significant spatial memory deficits. There is evidence that at least some of the SAD children in the McClean and Hitch (1999) study did have auditory-verbal memory weaknesses as their mean score on the Digit Span test differed from the age-matched children at a level that verged on statistical significance ($p = 0.051$).

The inter-relation between cognitive abilities was not examined in the McClean and Hitch (1999) study. They could not determine whether the working memory deficits of the SAD children in their study were part of the wider problem because ability measures were not employed. In the present study, because ability measures were also considered, it was possible to make a more balanced evaluation of the children's problems. Some children (those in Group 4) did have an isolated deficit in working memory, but other children with SAD had a working memory problem that was part of a broader ability weaknesses. The children in Group 1 did have memory deficits, but these were in the context of low verbal and non-verbal reasoning and low spatial abilities. All but one of the children in Group 2 had spatial memory weaknesses, but this was in the context of general visual-spatial processing difficulties. Focussing only on memory factors ignores these wider problems.

7.8.2 Different routes to arithmetic difficulties

The results of Studies Two and Three strongly support the model of arithmetic difficulties described by Lytinen et al. (1994) in which they propose that verbal, spatial, and memory difficulties can all contribute to poor arithmetic.

In Study One the significant relationship between reasoning ability and maths was illustrated. It could therefore be argued that the number skills difficulties of the SAD children are simply due to poor reasoning ability: the pattern of their actual ability/cognitive profiles could be dismissed as irrelevant. However, this ignores the finding that 10 of the 18 children with SAD had number skills that were significantly below the level that was expected on the basis of their GCA. These children with significant GCA/attainment discrepancies included children with significant verbal ability deficits, children with an NLD profile and children with memory weaknesses. It is

tentatively concluded that these three cognitive profiles are all associated with specific arithmetic difficulties. However, caution must be emphasised when one is examining a study with such a small sample. It is possible that some of the children may have poor number skills because of non-cognitive factors. They may, for instance suffer, from maths anxiety (see Section 4.5.1 for a discussion of mathematics anxiety), or have developed negative attitudes and poor motivation to mathematics (see Section 4.1 for a review of the non-cognitive causes of arithmetic difficulties). However, the results of Study Three suggest that different cognitive profiles can result in specific arithmetic difficulties, and that memory weaknesses are not necessarily the cause. A study with a much larger sample is required so the cognitive and ability profiles of large numbers of children with specific arithmetic difficulties can be examined. The results of Study Two and Three suggest that in future studies of arithmetic difficulties a broad range of measures, including ability, memory and psychomotor measures should be employed. In particular, a comprehensive working memory battery should be employed, in tandem with a test of intellectual ability, so that the links between deficits in the different aspects of working memory and different abilities (i.e. verbal and visual-spatial) can be examined. Particular attention needs to be paid to executive function deficits, as executive functioning was not measured in this study. The SAD children in this sample could have shared an executive deficit. A study conducted by McClean & Hitch (1999) concluded that executive function deficits were the most likely cause of specific arithmetic difficulties because the SAD children they studied differed from both age- and ability-matched controls on a test of executive function (however there were limitations to this study, which have already been discussed in Section 4.4.3 and 7.8.1).

In this study no clear links could be found between the SAD children's cognitive profile and their number skills profiles. However, with such small numbers in each SAD group this is unsurprising. The results of Study Three indicated that the vast majority of children with SAD are impaired in both number fact recall and place value understanding.

Previous research (e.g. Gross-Tsur et al., 1997; Russell & Ginsberg, 1984; Shalev et al., 1988) has suggested that the majority of children with SAD have impaired calculation and number fact recall, but adequate number comprehension and production. The results of Study Three suggest that the majority of children with SAD, do have problems comprehending large numbers, where place value understanding is required. The earlier studies limited the numbers children had to comprehend to relatively small values.

7.9 Conclusions (Studies Two and Three)

The children with SAD had average to below average GCAs. Statistically significant discrepancies between verbal ability and non-verbal reasoning ability, and between verbal ability and spatial ability were much more common in the sample of children with SAD than in the general population. These discrepancies were in both directions: some children with SAD had verbal strengths whilst others had spatial and non-verbal reasoning strengths.

The children with SAD had heterogeneous cognitive profiles: there was wide variation in their scores on all the memory and psychomotor measures used. The children with SAD were divided into four broad categories: low general conceptual ability, non-verbal learning difficulties, verbal reasoning weaknesses, and memory weaknesses. Although the results suggested that all these profiles were associated with SAD a study with a larger sample size is necessary to confirm this. These results support the hypothesis put forward by Lyytinen et al. (1994) that different routes can all lead to arithmetic difficulties.

8 Study Four: Comparing the number skills profiles of children with dyslexia and children with Specific Arithmetic difficulties

8.1 Rationale

The aim of Study Four was to determine whether children with different cognitive profiles have different number skills profiles. Children with dyslexia and children with specific arithmetic difficulties were compared with a control group of children who were randomly selected from the mainstream school population. As the two groups of children with learning difficulties had different ability and cognitive profiles it was predicted that they would have different number skills profiles.

Section 5.5 concluded with three hypotheses: that dyslexic children will be slower counters than their normally developing peers, that dyslexic children will be poorer at recalling number facts than their normally developing peers, and that dyslexic and normally developing children will not differ in place value understanding. Study Four aimed to test these hypotheses. As Study One indicated that non-verbal reasoning ability was related to both place value understanding and number fact recall, it was predicted that SAD children would be poorer than normally developing children at both these areas, because many of the children had specific or general reasoning weaknesses. As a previous study conducted by Hitch & McAuley (1991) reported that children with poor arithmetic, but better reading, were slower counters than control children, it was hypothesised that the children with SAD in the present study would be slower counters than the control children.

8.2 Method

8.2.1 Participants

All the children in Study Four were aged 10 years 0 months to 12 years 0 months. The sample of control children consisted of the children who had been administered *Maths Suite* (in Study One) and were at least 10 years old, the method of selecting these children is described in section 6.2.2. The group of children with SAD consisted of the 18 children who participated in study three. The method of selecting the children with SAD is

described in section 7.2.2: the mean ability and attainment scores for these children are shown in Table 1. The children in the dyslexic sample had all received a diagnosis of dyslexia. The headteachers of the schools listed on the Crested² and British Dyslexia Association websites (www.crested.org.uk, www.bda-dyslexia.org.uk) were contacted and asked if they would be willing to participate in the study. Teachers from seven schools administered the *Maths Suite* tests to their pupils. Six of these schools were private schools who specialised in the teaching of dyslexic pupils and one was a state school that employed specialist dyslexia tutors. The contact teachers from the private schools were asked to randomly select three 10-year-old and three 11-year-old dyslexic pupils to test. If the schools did not have enough dyslexic pupils in one age group they were asked to test additional pupils from the other age group. The contact teacher from the state school was asked to test all children who were diagnosed as dyslexic, in Year 7 and on Stage 3 or above of the SEN Code of Practice. Of the seven schools that participated, five tested six pupils, one tested seven pupils and one tested three pupils (they were unable to complete testing due to time pressures).

There were 144 control children: 71 boys and 73 girls, 40 dyslexic children: 37 boys and 3 girls and 18 children with specific arithmetic difficulties: 8 boys and 10 girls. The mean age of the control children was 10 years 9 months ($SD = 6.1$ months), the mean age of the dyslexic children was 11 years 3 months ($SD = 5.3$ months) and the mean age of SAD children 10 years 8 months ($SD = 6.7$ months). An ANOVA confirmed that there was a significant difference between the groups [$F(2,199)=38.27, p < .001$]. A Post Hoc Tukey *a* test indicated that the age of dyslexic children differed significantly from the control (mean difference = $-5.06, SE = 1.11, p = .001$) and the SAD children (mean difference = $-6.15, SE = 1.76, p < .001$). However, the SAD children did not differ significantly in age from the control children (mean difference = $-1.45, SE = 1.55, p = .616$).

² Crested is a national organisation representing schools that specialise in teaching dyslexic pupils.

As the diagnostic criteria for dyslexia was not specified, a sub-group of the children with dyslexia was formed. This sub-group consisted of the dyslexic children who fitted the discrepancy definition of dyslexia (see section 5.1 for a discussion of diagnostic criteria). Children were included in the specific reading difficulties (SRD) group if the psychometric test information supplied by the contact teacher indicated that the child's reading attainment was at least 10 standard score points below their general intellectual ability. Children with dyslexia were excluded from the SRD group, if insufficient psychometric data was supplied or the discrepancy between reading attainment and general intellectual ability was not large enough. Tests of children's general intellectual ability included both individually administered tests such as the WISC III (Wechsler, 1991) and group administered tests, such as the Cognitive Abilities Tests (Thorndike et al., 1986). If only a verbal or non-verbal reasoning measure was administered the child was not included in the SRD group. If more than one reading attainment score was recorded, the single word reading score was used to determine the size of the discrepancy. There were 15 children in the SRD group: 2 girls and 13 boys; the mean intellectual ability standard score was 107.82 ($SD = 16.39$) and the mean reading attainment standard score was 81.87 ($SD = 12.84$). The average discrepancy between reading and attainment was over 25 standard score points. Only four dyslexic children were excluded from the SRD group because the discrepancy between their general intellectual ability and their reading score was too small; the remaining 18 dyslexic children were excluded, because insufficient psychometric information was available to the author. It is likely that had further psychometric information been available many more of the dyslexic children would have been assigned to the SRD group.

The children with SRD and the children with SAD have very different profiles. None of the children with SAD had reading attainment that was significantly below their general intellectual ability, whilst all the children with SRD had reading attainment that was at least ten points below their general conceptual ability. The children with SAD were

chosen because they had significant arithmetic difficulties, whilst the children with SRD were chosen because they had reading difficulties regardless of their arithmetic attainment. The mean general intellectual ability of the children with SRD was somewhat above the population mean, but the mean general intellectual ability for the children with SAD was slightly below the average range.

8.2.2 Materials

The children were administered the *Numbers*, *Number Facts*, *Spots* and *Most* tests from *Maths Suite* which are described in Section 6.2.3. They assess response time, number fact recall, counting speed and place value understanding.

8.2.3 Procedure

The control and SAD children had been administered the *Maths Suite* tests in Studies One and Three respectively. The testing procedures for the control children are described in section 6.2.4 and for the SAD children in Section 7.6.3. The contact teachers of the schools that agreed to test some dyslexic children were sent a *Maths Suite* CD and a floppy disc. When the required number of dyslexic children had been tested, they downloaded the data onto the floppy disc and returned it to the author. The contact teachers were also asked to supply the results of any reading, spelling, arithmetic or general intellectual ability tests that had been administered recently to the dyslexic children.

8.3 Results

Table 20. Mean number of items correct for the Number Facts and Most tests (standard deviations in brackets)

	Control	Dyslexic	SRD	SAD
<i>n</i>	142 _a	40 _b	15	18
Addition Facts	10.20 (1.97)	9.03 (2.90)	8.53 (2.97)	9.67 (5.33)
Subtraction Facts _a	9.30 (2.54)	7.13 (3.01)	8.00 (3.25)	7.11 (2.03)
Multiplication Facts _a	8.75 (2.57)	6.15 (3.26)	7.46 (3.38)	6.17 (2.85)
Total Number Facts _a	28.24 (6.22)	22.31 (7.23)	23.29 (8.70)	22.94 (6.51)
<i>Most</i>	25.96 (4.68)	25.08 (5.28)	25.73 (5.74)	20.28 (5.00)

Note. Children who did not attempt any Number Facts items excluded

^a132 attempted the Number Facts test

^b39 attempted the Number Facts test

The results in Table 20 suggest that control children recall more addition, subtraction and multiplication facts than children do with dyslexia (including the dyslexic children with specific reading difficulties) and the children with specific arithmetic difficulties. Three ANOVAs were calculated to compare the children with learning difficulties to their control peers. As the dyslexic children were somewhat older than the control children a single multivariate ANOVA was not used; age had to be a covariate in all analyses and this would preclude the appropriate post-hoc tests being applied. A one-way ANOVA indicated that the dyslexic and control children differed significantly in their recall of addition [$F(1,168) = 11.69, p = .001$], subtraction [$F(1,168) = 24.24, p < .001$], and multiplication facts [$F(1,168) = 29.46, p < .001$]. The effects of age were partialled out. However, a one-way ANOVA indicated that the scores of the dyslexic and control children on the *Most* test, that measured place value understanding did not differ significantly [$F(1,181) = 1.80, p = .183$]. A second ANOVA confirmed that when compared with the control group, the sub-group of dyslexic children with SRD also recalled significantly fewer addition [$F(1,144) = 9.50, p = .002$], subtraction [$F(1,144) = 4.08, p = .045$], and multiplication [$F(1,144) = 3.909, p = 0.050$]. The sub-group of dyslexic children with SRD did not differ significantly from the control children on the test

of place value understanding [$F(1,154) = 0.066, p = .798$]. A one-way ANOVA indicated that the SAD children and the control children differed significantly in their ability to recall subtraction facts [$F(1,147) = 10.06, p = .002$] and multiplication facts [$F(1,147) = 14.26, p < .001$]. The children with SAD did not recall significantly fewer addition facts than the control children [$F(1,147) = 0.808, p < .370$]. The effects of age were partialled out. In contrast to the dyslexic children, the scores of the SAD children differed from the scores of the controls on the *Most* test that measures place value understanding [$F(1,157) = 22.27, p < .001$].

Table 21. Mean Number Facts times in seconds (standard deviations in brackets)

	Control	Dyslexic	SRD	SAD
<i>n</i>	132	39	15	18
Addition Facts _a	2.14 (0.74)	2.88 (0.82)	2.85 (0.81)	2.91 (0.67)
Subtraction Facts _a	2.41 (0.80)	2.99 (0.99)	2.86 (0.98)	3.31 (0.68)
Multiplication Facts _a	2.47 (0.70)	3.03 (0.77)	2.98 (0.76)	3.04 (0.66)

Note. Children who did not attempt any number fact items excluded.

The results in Table 21 indicate that both SAD and dyslexic children take longer to answer addition, subtraction and multiplication facts correctly than control children. A one-way ANOVA with age partialled out indicated that dyslexic children's response times to addition [$F(1,165) = 26.12, p < .001$], subtraction [$F(1,165) = 13.24, p < .001$] and multiplication [$F(1,165) = 18.19, p < .001$] sums differed significantly to those of control children. When only the dyslexic children with SRD are compared to the control children the time differences remain significant for addition [$F(1, 133) = 12.85, p < .001$], subtraction [$F(1,133) = 4.29, p = .040$], and multiplication [$F(1,133) = 7.63, p = .007$] facts. A one-way ANOVA with age partialled out indicated that the response times of SAD children to addition [$F(1,146) = 14.74, p < .001$], subtraction [$F(1,146) = 17.51, p < .001$] and multiplication [$F(1,146) = 8.05, p = .005$] sums were significantly longer than

those of control children. Overall these results indicated that when dyslexic and SAD children recalled number facts correctly, they did so more slowly than the control children

Table 22. Counting time per spot for correct trials

	Control	Dyslexic	SRD	SAD
<i>n</i>	126	37	15	14
Counting with memory aids	1.38 (0.74)	1.24 (0.38)	1.24 (0.33)	1.73 (1.07)
Counting without memory aids	0.70 (0.33)	0.81 (0.41)	0.92 (0.39)	0.69 (0.17)

Note. Three dyslexic participants, eleven control participants and four SAD participants were excluded because they did not answer any of the counting with memory aids trials correctly.

The results in Table 22 indicate that all groups counted more quickly when they did not have to click on the spots (*Spots* part two, the without memory aid condition). The control children were slower than the dyslexic children at counting the dots with a memory aid (*Spots* part two), but this difference was not statistically significant [$F(1,160) = 0.52, p = .346$]. The dyslexic children were slower than the control children at counting dots without a memory aid, this difference was statistically significant [$F(1,160) = 4.35, p = .039$]. The dyslexic children with SRD did not differ at a statistically significant level from the control children when counting dots without memory aids [$F(1,137) = 0.37, p = .543$]. The children with SRD were slower than the control children when counting without memory aids, this difference verged on statistical significance [$F(1,137) = 3.80, p = .053$]. For the SAD children, the differences counting speed with memory aids [$F(1,137) = 2.24, p = .137$] and without memory aids [$F(1,137) = 0.10, p = .750$] were not significant.

8.4 Discussion

The results of Study Four indicate that whilst children with SAD are poorer than the control children at both place value understanding and two of the three number fact recall sub-tests, children with dyslexia are only poorer at the three number fact recall sub-tests. Children with dyslexia and SAD not only recalled fewer number facts than their peers, the ones they did recall were recalled more slowly. Children with specific arithmetic difficulties did not count more slowly than the control children. Children with

dyslexia did count more slowly than the control children, but only in part two of the test, when there were no memory aids.

The finding that dyslexic children have poor number fact recall is consistent with previous research in the area (Ackerman et al., 1986; Miles, 1987; Pritchard et al., 1989; Turner Ellis et al., 1996). Whilst Turner Ellis et al. (1996) concentrated on multiplication, the present study gives clear empirical evidence that children with dyslexia can recall fewer addition, subtraction facts than control children. When they do recall addition, subtraction and multiplication facts correctly, they do so more slowly than control children. Despite undergoing specialist education programmes (mainly conducted in the private sector) the dyslexic children performed more poorly on the *Number Facts* test than a randomly selected sample of children attending mainstream schools (the vast majority of whom were educated in the state sector). The finding that dyslexic children with SRD (whose mean general intellectual ability score is above the population mean) scored more poorly than randomly selected control children, indicates that the dyslexic children's number fact recall difficulties are not due to low general intellectual ability.

The dyslexic children in this study were not matched on intelligence, therefore it is possible that they have subtle difficulties with place value understanding, which could be identified if they had been compared to children matched on intelligence. It could also be argued that the dyslexic children, who came predominately from private schools, had received more intensive tuition in smaller groups and without these advantages they may have shown difficulties with place value understanding. These questions can only be resolved by further studies that compare dyslexic children's place value understanding to children matched on intelligence and school environment. However, it should be noted that even if place value understanding difficulties exist they are less severe than their number fact recall deficits. The case reports of place value understanding difficulties identified by Critchley (1970) were probably isolated cases and not typical of dyslexic children.

The results of this study suggest that different aspects of children's number skills can be affected by different cognitive deficits. In Chapter 7 the variation in the ability and cognitive profiles of the children with specific arithmetic difficulties was discussed. Overall the children with SAD were poorer than the control children, both at recalling number facts and at understanding place value. The children with SAD tended to have low average to below average reasoning ability. As the results of Study One indicated that non-reasoning contributes to the variance in both number fact recall and place value understanding, it is unsurprising that the SAD children (many of whom had specific or general reasoning weaknesses) had deficits in both areas. The children with dyslexia, including the sub-group with SRD, were poorer than the control children only at number fact recall; their place value understanding was not significantly impaired. The results of Study One indicated that number fact recall correlated with auditory-verbal memory at a level that verged on statistical significance. McClean & Hitch (1999) also reported a correlation between a test of speeded calculation and auditory verbal memory. Therefore children with dyslexia may have number fact recall difficulties, because of their consistently reported auditory-verbal weaknesses (e.g. Hulme, 1981; Shankweiler et al., 1979). As auditory-verbal memory did not correlate with place-value understanding, and non-verbal reasoning is not specifically impaired by dyslexia, the dyslexic children's unimpaired place value understanding fits with their cognitive profile. It is logical to suggest that auditory-verbal memory contributes to number fact recall. The digits in the sum, must be held in auditory-verbal memory whilst the answer is calculated in order for the association between the sum and the answer to be strengthened.

It is interesting to note that the dyslexic children have slower counting speed scores than the control children only in part two of the test when no memory aids are available. Hitch et al. (1987) used a pointer analogy to describe the differences between 'ballistic counting' and array counting (see Section 6.4.2 for a discussion of these differences). Hitch et al. (1987) argue that the central executive co-ordinates the different processes

required for array counting. The co-ordination of counting processes required for counting a visual array is also required when using a counting strategy to solve arithmetic problems. For example, when solving $5 + 2 = 7$ using a *counting on* strategy a child must co-ordinate two counts, “1, 2” to keep track of the number of digits that need to be added on, and “5, 6”, the actual progress through the number series.

The fact that the dyslexic children’s counting speed does not differ from control children when memory aids are available suggests that their counting weaknesses are not due to weak phonological representations of the number series. The finding that the dyslexic children are slow at counting only in part two suggests their difficulties are with co-ordinating the different processes involved with array counting, and indicates that they may have executive function weaknesses. Research into the executive functioning of dyslexic children is scarce. A search of periodical articles dating back to 1987 revealed only one relevant article. van der Schoot, Licht, Horsley & Sergeant (2000) reported that dyslexic readers who read fast and inaccurately performed more poorly on all the tasks used to assess executive functioning, when compared with dyslexic readers who read slowly, but accurately.

The finding of the present study, that dyslexic children have difficulties with the co-ordination of counting procedures, suggest a possible reason for their difficulty recalling number facts. Children with dyslexia may have had difficulties co-ordinating the counting procedures required for advanced counting strategies (such as *counting on*) and hence are slower at building up the associations between sums and their answers. Dyslexic children’s difficulties learning number facts may be influenced by poor auditory-verbal memory and/or a difficulty co-ordinating counting procedures.

The hypothesis that children with SAD would be slower counters than the control children was not supported by the results of this study. The children with SAD were not significantly slower than the control children on either of the counting tasks. However, four (22%) of the 18 children with SAD were excluded from the analysis because they

were not able to complete any of the counting trials without memory aids correctly.

This indicates that almost a quarter of the children with SAD had severe problems co-ordinating their counting procedures. The counting speeds of the poorest counters in the SAD group were therefore not considered in the statistical analysis. In future it may be more appropriate to consider the counting speeds of children with SAD using a less demanding counting task. Hitch & McAuley (1991) assessed the counting speeds of children with SAD by asking them to count less than ten dots, and reported that the children with SAD were slower counters. Some of the dyslexic children also had severe difficulties co-ordinating their counting procedures; three (7.5%) of the 40 dyslexic children answered all of the counting trials without memory aids incorrectly.

Children with SAD were poorer at recalling subtraction and multiplication facts than the control children, but the two groups recalled a similar number of addition facts. The children with SAD may have found it easier to formulate strategies for calculating addition facts without retrieving them from memory. In Section 3.3 the development of children's strategies for answering single digit addition and subtraction sums are discussed. *Count all* is conceptually the easiest strategy to use and can be applied to single digit addition sums; *count on* requires more conceptual understanding, but is more efficient. Both *count all* and *count on* require the child to count forwards. However, the counting strategies that can be used to tackle single digit subtraction sums either require greater conceptual knowledge or an ability to count backwards. *Counting down* requires the child to backwards. *Counting up* requires an understanding that addition is the reverse of subtraction. Children with SAD may therefore be able to use counting strategies to answer the addition sums, but lack the conceptual knowledge or facility with backwards counting to answer the subtraction sums. The children with SAD will have a greater chance of strengthening the associations between addition sums and their answers, than strengthening the associations between subtraction sums and their answers, if they have a better grasp of the counting strategies appropriate for answering addition sums.

In summary, despite general intellectual ability that, on average, was higher than that of the general population the sub-group of children with SRD scored more poorly than a group of randomly selected children on all three number facts tests. However, the dyslexic children did not display significant deficits in place value understanding. Two possible reasons for the number fact deficit of dyslexic children were proposed; a difficulty co-ordinating the counting procedures required when using strategies to solve single digit addition and subtraction sums and poor auditory verbal memory resulting in a reduced likelihood of the associations between single digit addition and subtraction sums being strengthened. Children with SAD had deficits in subtraction and multiplication fact recall and place value understanding. These deficits may be related to the general and specific reasoning weaknesses displayed by many of the children with SAD. However, as the children with SAD have heterogeneous ability and cognitive profiles, it would be more appropriate to examine the number skills profiles of children who share homogenous ability and cognitive profiles separately. This was not possible in this study because of the small sample of children with SAD available (see Section 7.7.6 for further discussion of this issue).

9 Study Five: Number fact recall abilities of adults with dyslexia

9.1 Rationale

As the results of Study Four indicated that dyslexic children were less accurate and slower than control children at recalling number facts, the number fact recall abilities of dyslexic adults were investigated. The aim of Study Five was to determine whether dyslexic individual's difficulties with number fact recall continued into adulthood. Vogel & Walsh (1987) examined the *WAIS* profiles of adults with specific learning difficulties (a proportion of whom would have been dyslexic) who were undergraduate students. They calculated the mean score for each *WAIS* sub-test. The mean score for the *WAIS Arithmetic* sub-test was lowest of all subtests for both males and females. Two studies of adults with specific learning difficulties (Blalock, 1987; Cordoni, O'Donnell, Ramaniah, Kurtz & Rosenshein, 1981) found that the mean *WAIS Arithmetic* subtest scores were not the lowest, but the second lowest. Both these studies found *Digit Span* to be lowest mean *WAIS* subtest score. All three studies suggest that dyslexic adults have problems with mental arithmetic. However, as the *WAIS Arithmetic* test consists of orally presented problems the dyslexic adults may not have difficulties with the recall of basic number facts; their difficulties may be due to problems in other skills. For example, the dyslexic adults may have difficulties identifying the numerical operation required to solve the word problem or in storing the partial products whilst calculating multi-digit sums. The present study directly investigates the ability of dyslexic adults to solve simple addition, subtraction and multiplication sums under timed conditions. It was hypothesised that dyslexic adults would recall fewer addition, subtraction and multiplication facts than the non-dyslexic adults and that the facts that they did recall would be recalled more slowly. The dyslexic adults were also asked to complete a written arithmetic test to determine whether they also had problems with written arithmetic. A correlational analysis was planned to determine which if any of the dyslexic adults cognitive weaknesses were related to their number fact recall and written arithmetic performance.

9.2 Methodology

9.2.1 Participants

The participants were 19 students who had received a diagnosis of dyslexia and 19 students who had not been diagnosed as dyslexic the groups being matched for intelligence. All 38 students were currently studying a course at higher education level at a university. The students with dyslexia had all undergone a full psychological assessment as an adult and were diagnosed as dyslexic using the criteria outlined in the report of the National Working Party on Dyslexia in Higher Education (Singleton, 1999). The key criteria laid down in the report include a statistically significant discrepancy between the student's literacy skills and their general intellectual ability and evidence of cognitive disabilities or neurological anomalies. To be diagnosed as dyslexic using the guidelines laid down in the report the student must have a deficit in at least one of the following areas: phonological processing, memory, visual perception or motor co-ordination.

Of the 19 dyslexic students, 18 had undergone their most recent psychological assessment at the University of Hull, 15 of these students had been assessed using the most recent procedures that utilised *WAIS III^{UK}* (Wechsler, 1998a) and four had been assessed using earlier procedures that utilised *WAIS-R* (Wechsler, 1981). The remaining dyslexic student had been assessed by a psychologist at the Dyslexia Institute using the *WAIS-R* (Wechsler, 1981). Six of the 19 students had received a formal diagnosis as a child before they came to university 12 had received their first formal diagnosis as an adult. There were 12 male and 7 female students with dyslexia and 10 male and 9 female non-dyslexic students. The mean age of the dyslexic students was 21 years and 0 months ($SD = 21$ months), the mean age of the non-dyslexic students was 20 years and 10 months ($SD = 11$ months). An ANOVA indicated that the groups did not differ significantly in terms of age $F(1,36) = 0.95, p = .34$. The mean estimated IQ of the dyslexic students was 112.16 ($SD = 11.30$) and for the non-dyslexic students was 112.29 ($SD = 10.08$). An ANOVA analysis indicated that the groups did not differ significantly in terms of estimated IQ $F(1,36) <$

0.01, $p = .97$. On the *WRAT-3* Reading test the non-dyslexic students achieved a mean score of 110.00 ($SD = 7.51$) and the dyslexic students achieved a mean score of 96.53 ($SD = 11.52$). An ANOVA indicated that the dyslexic students reading attainment was significantly poorer than the non-dyslexic students reading attainment $F(1,36) = 18.22, p < .001$.

9.2.2 Materials

Tests completed by both the dyslexic and non-dyslexic students at the time of the study

Maths Suite Numbers and Number Facts. The version of *Numbers*, which measures response time, that the students completed was identical to the one completed by the children in Studies One and Four, which is described in Section 6.2.3. The version of *Number Facts* completed by the students was the same as the one that the children completed in Studies One and Four, which is described in Section 6.2.3 except that the discontinuation rules were removed. All the students completed all the addition, subtraction and multiplication items.

LADS (Singleton, Horne & Thomas, 2002). The *LADS* computerised assessment suite screens adults for dyslexia. Both the dyslexic and the non-dyslexic students completed a developmental version of *LADS*. There are three sub-tests within this version of the assessment suite. In the *Word Recognition* sub-test the examinee has to click on a real word, which is displayed with a selection of non-words as quickly as they can. In the *Word Construction* test the examinee hears a non-word and must choose the three syllables from a collection of nine that correctly spell the word. The *Memory* test is a computerised version of a backwards digit span test. The examinee receives a score on each test between one and nine, the higher the score the individual receives the more poorly the individual has performed. Only the *LADS Memory* scores will be discussed in the present study.

Tests completed by the dyslexic students at the time of the study

WRAT-3 Arithmetic (Wilkinson, 1993). The *WRAT-3 Arithmetic* test is a test of written arithmetic problems. It includes whole number addition, subtraction, multiplication sums,

problems involving fractions and decimals and some algebra. The test has a time limit of 15 minutes.

Tests completed by the non-dyslexic students at the time of the study

WRAT-3 Reading (Wilkinson, 1993). The *WRAT-3 Reading* test is a single word reading test. The adult has to read aloud a series of increasingly difficult real words.

Estimated IQ. The non-dyslexic adults were administered one verbal (*Similarities*) and one performance (*Block Design*) sub-test from the *WAIS R* (Wechsler, 1981). The scaled scores gained from these sub-tests were converted into standard scores and the mean calculated to produce an estimate of the participant's IQ.

Tests completed by the dyslexic students during their diagnostic assessment

Literacy measures. The dyslexic adults were administered the *WRAT-3 Reading* test described above and the *WRAT-3 Spelling* test (Wilkinson, 1993), which requires the examinee to spell a series of increasingly difficult single words. They also completed the *Passage Comprehension* sub-test from the *Woodcock Reading Mastery Tests – Revised* (Woodcock, 1987). This test measures reading comprehension using cloze procedure. The examinee has to supply the missing word that fits best into the sentence presented.

Phonological decoding skills were assessed using the *Word Attack* sub-test from the *Woodcock Reading Mastery Tests – Revised* (Woodcock, 1987). The examinee has to read aloud letter strings that are not real words in the English language.

Estimated IQ. Depending on the time of assessment the dyslexic students IQs were estimated in different ways. The 15 students who had been assessed most recently at the University of Hull completed six sub-tests from the *WAIS III^{UK}* (Wechsler, 1998a). They completed the *Vocabulary, Similarities and Information* sub-tests that make up the verbal comprehension index and the *Picture Completion, Block Design and Matrix Reasoning* sub-tests that make up the perceptual organisation index. The mean of the verbal comprehension index and the perceptual organisation index was calculated to produce an estimated IQ score. The four students who had been assessed earlier at the University of

Hull were administered the four sub-test short form of the *WAIS-R* (Wechsler, 1981).

They completed the *Vocabulary*, *Similarities*, *Picture Completion* and *Block Design* sub-tests, and their estimated IQs were pro-rated from these scores. The participant who had been assessed prior to coming to university had completed four verbal and three performance sub-tests from the *WAIS-R* (Wechsler, 1981). An IQ had been pro-rated from these sub-tests.

Auditory-verbal short-term memory. Depending on the time of assessment the dyslexic student's auditory-verbal sequential memory capacity was assessed in different ways. The 15 students who had been assessed most recently at the University of Hull completed the *WAIS III^{UK} Digit Span* sub-test (Wechsler, 1998a). There are two parts to this sub-test: in the first the examinee has to repeat strings of digits in the order they were presented, in the second the examinee has to repeat strings of digits in reverse order. The student whose most recent assessment had been carried out prior to coming to university completed the earlier version of the *Digit Span* sub-test from the *WAIS-R* (Wechsler, 1981). The four students who had been assessed using the earlier procedures at the University of Hull completed the *WMS-R Forwards Digit Span* test and the *WMS-R Backwards Digit Span* test (Wechsler, 1984). The mean of these two tests was calculated to produce an auditory-verbal short-term memory measure that was comparable to the ones obtained by the other students.

Visual-spatial short-term memory. Three different measures of visual-spatial short-term memory were administered at the time of assessment. Depending on the time of assessment the dyslexic students visual-spatial short-term sequential memory was measured in different ways. The students who had been assessed using the most recent procedures at the University of Hull were administered the *WMS III Spatial Span* test. In this test the examinee is shown a randomly arranged pattern of blocks. The examiner points to a sequence of the blocks. The examinee must then point to the same blocks in the same temporal order. In the second part of the test different sequences are presented and

the examinee must repeat them in the reverse temporal order. The greater the number of sequences the examinee repeats correctly the higher score they receive. The dyslexic students who had been assessed using the earlier procedures completed the forward and backward *Visual Span* tests from the *WMS-R* (Wechsler, 1984). The average of these two tests was calculated, so that an equivalent visual-spatial sequential memory score was obtained. The *WMS-R Visual Span* tests are equivalent to the *WMS-III Spatial Span* test. The only difference is that the pattern used in the *WMS-R* tests is two-dimensional squares on plastic sheet rather than the three dimensional blocks used in *WMS-III*.

All the dyslexic students completed the *Digit Symbol* sub-test: the 15 students who had been assessed most recently completed the most recent version from the *WAIS III^{UK}* whilst the other five students completed the version from the *WAIS-R* (Wechsler, 1981). In the *Digit Symbol* test the examinee is given a sheet with the digits one to nine on it. Below each number is a symbol. On the same sheet is a sequence of random digits. The examinee is allowed 90 seconds in which to copy the correct symbol under each digit. The examinee's score is based on the number of symbols produced correctly. Visual working memory is believed to contribute to performance on this test. The 15 students who had been assessed most recently completed the *WMS Visual Reproduction* test (Wechsler, 1998b). In this test the examinee is shown a line drawing, which they must then reproduce from memory.

Logical Memory. The 19 dyslexic students who were assessed at the University of Hull all completed the *Logical Memory* test: those assessed using the most recent procedures completed the version in *WMS III^{UK}* (Wechsler, 1998b), those assessed using the earlier procedures completed the version in *WMS-R* (Wechsler, 1984). In this test the examinee is told a short story, which they must then recount.

9.2.3 Procedure

The dyslexic students were recruited in two ways. Dyslexic students who had been assessed by staff at the psychology department of the University of Hull were contacted

directly, starting with the students who had been assessed most recently and working back through the records. To boost the numbers of students participating notices advertising the study were posted in the Disabilities Office of the University of Hull where students with dyslexia attend for support and advice, and on the University of Hull computer network. Students with dyslexia who contacted the author asking to participate were accepted if they could supply a full psychological report that met the criteria laid down in the report of the National Working Party on Dyslexia in Higher Education (Singleton, 1999). The dyslexic students who participated in the study were paid £10 for their time.

Undergraduate students in the Department of Psychology, University of Hull, recruited 40 non-dyslexic students, in conjunction with a research project that was a required part of their course. The non-dyslexic students did not receive any payment for participating in the study. Of the 40 non-dyslexic students recruited 20 were selected as the control group. The twenty were chosen because their estimated IQ's most closely matched those of the dyslexic students. The non-dyslexic students were not placed in the control group if they had a below average *WRAT-3* reading score. This reduced the possibility of undiagnosed dyslexic students being included in the control group. The undergraduate students administered the *LADS* computerised tests, the *WRAT-3 Reading* test and the *Similarities* and *Block Design* tests from the *WAIS-R* to the non-dyslexic students. The author administered the *Maths Suite* and *LADS* computerised tests and the *WRAT-3 Arithmetic* test to the dyslexic students. The literacy, estimated IQ and memory scores for the dyslexic students were obtained from their most recent psychological report.

9.3 Results

Table 23. Addition, subtraction and multiplication facts correct for dyslexic and non-dyslexic students

Question type	Dyslexic (<i>n</i> = 19)		Non-dyslexic (<i>n</i> = 19)	
	Mean	<i>SD</i>	Mean	<i>SD</i>
Addition	11.47	0.84	11.79	0.54
Subtraction	10.43	1.61	11.37	0.83
Multiplication	9.37	1.42	10.47	1.22

Table 24. Mean answer times for correct addition, subtraction and multiplication facts for dyslexic and non-dyslexic students (all times shown in seconds)

Question type	Dyslexic (<i>n</i> = 19)		Non-dyslexic (<i>n</i> = 19)	
	Mean	<i>SD</i>	Mean	<i>SD</i>
Addition	1.53	0.48	1.14	0.39
Subtraction	1.76	0.63	1.29	0.39
Multiplication	1.84	0.52	1.57	0.35

The mean number of addition, subtraction and multiplication facts answered correctly by the dyslexic and non-dyslexic students is shown in Table 23. An ANOVA indicated that dyslexic students recalled significantly fewer subtraction [$F(1,36) = 4.10, p = .050$] and multiplication [$F(1,36) = 6.62, p = .014$] facts than non-dyslexic students. Although the mean number of addition facts correct for dyslexic students was lower than for non-dyslexic students, the difference was not statistically significant [$F(1,36) = 1.91, p = .176$]. The mean for both groups was very close to the ceiling of the addition facts test for both groups. The mean times for answering addition, subtraction and multiplication facts correctly are shown in Table 24. An ANOVA indicated that dyslexic students answered addition [$F(1,36) = 7.93, p = .008$] and subtraction [$F(1,36) = 7.45, p = .010$] questions significantly more slowly than the non-dyslexic students. Although the mean time for answering multiplication questions was slower for the dyslexic students, this difference only verged on statistical significance [$F(1,36) = 3.57, p = .067$].

Figure 10. Boxplot showing total Number Facts scores for dyslexic and non-dyslexic students.

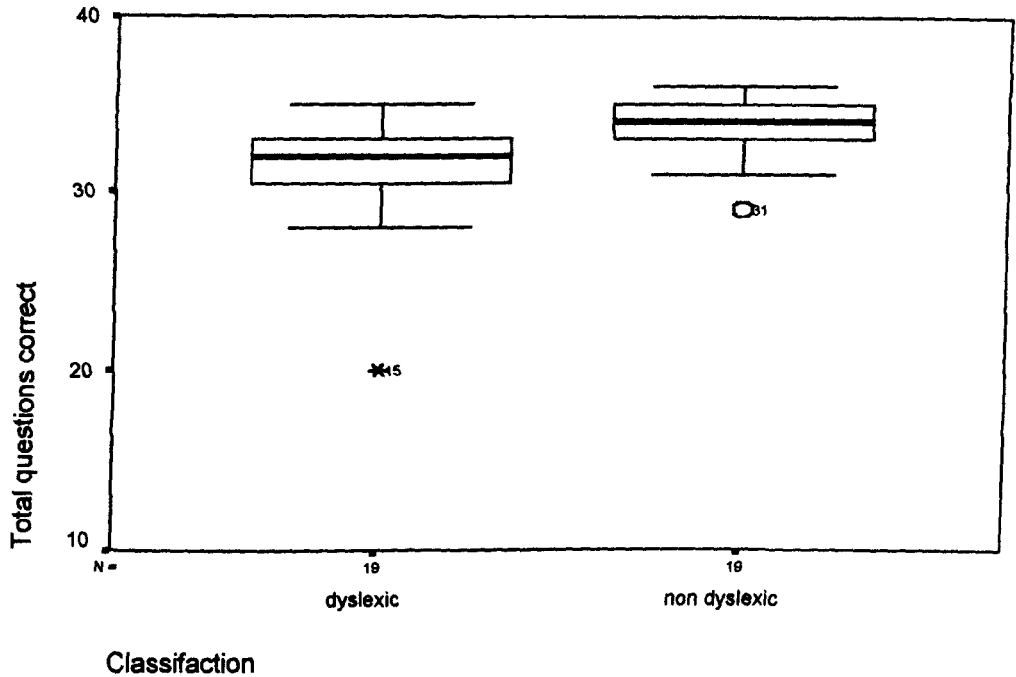


Table 25. Addition, subtraction and multiplication facts correct for dyslexic and non-dyslexic students after removal of the extreme score

Question type	Dyslexic (n = 18)		Non-dyslexic (n = 18)	
	Mean	SD	Mean	SD
Addition	11.61	0.61	11.83	0.51
Subtraction	10.83	0.92	11.39	0.85
Multiplication	9.56	1.20	10.56	1.20

Table 26. Mean answer times for correct addition, subtraction and multiplication facts for dyslexic and non-dyslexic students after removal of the extreme score (all times are in seconds)

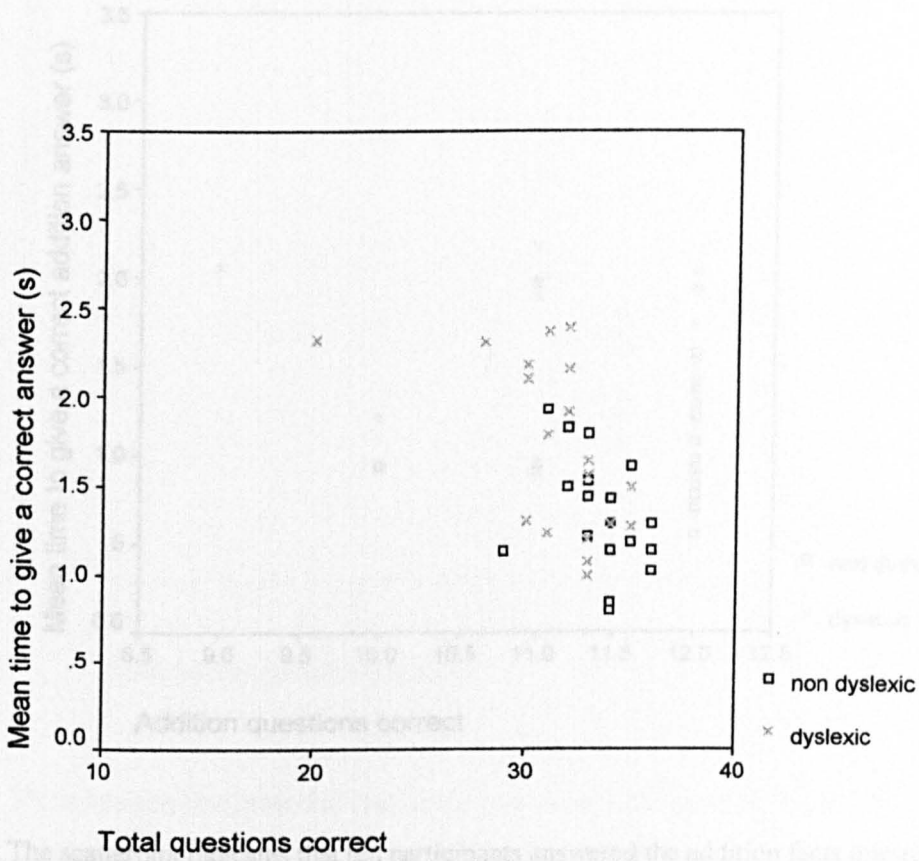
Question type	Dyslexic (n = 18)		Non-dyslexic (n = 18)	
	Mean	SD	Mean	SD
Addition	1.50	0.48	1.09	0.34
Subtraction	1.69	0.57	1.24	0.31
Multiplication	1.83	0.53	1.57	0.36

The boxplot shown in Figure 10 indicates that there is one extreme score in the dyslexic sample and one outlier in the non-dyslexic sample. To ensure that the differences between the groups were not solely due to the one extreme score in the dyslexic sample, the data was re-analysed excluding the dyslexic student with the extreme score and the

non-dyslexic student who matched his estimated IQ. The mean scores for number fact questions correct and time to answer number fact questions without the extreme score are shown in Table 25 and Table 26, respectively. After removing the extreme score, the difference between the dyslexic and non-dyslexic students on the multiplication facts questions remained significant [$F(1,34) = 6.26, p = .017$]. The difference between the dyslexic and non-dyslexic students on the subtraction facts questions verged on significance [$F(1,34) = 3.53, p = .069$]. There was no significant difference between the two groups on the addition facts questions. After removing the extreme score, the difference between the groups on the time taken to answer the addition [$F(1,34) = 9.09, p = .005$] and subtraction [$F(1,34) = 8.63, p = .006$] questions correctly remained significant. The difference between the groups on time taken to answer the multiplication questions verged on significance [$F(1,34) = 3.03, p = .091$].

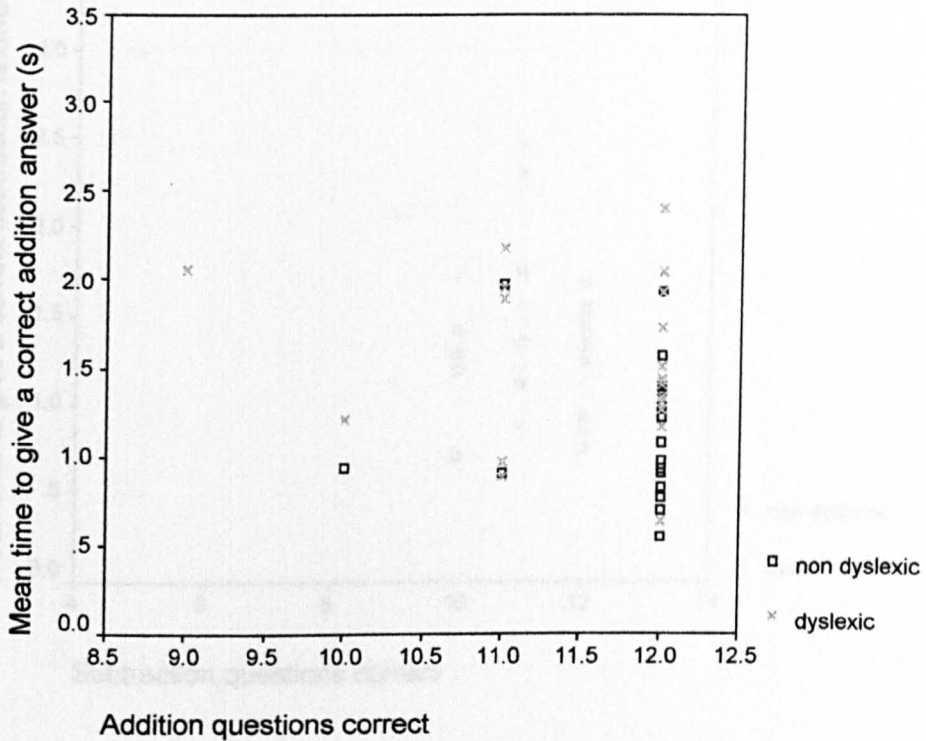
Figure 11. Scatterplot showing the number of correct addition facts plotted against the

mean time taken to give a correct answer for both the dyslexic and non-dyslexic participants



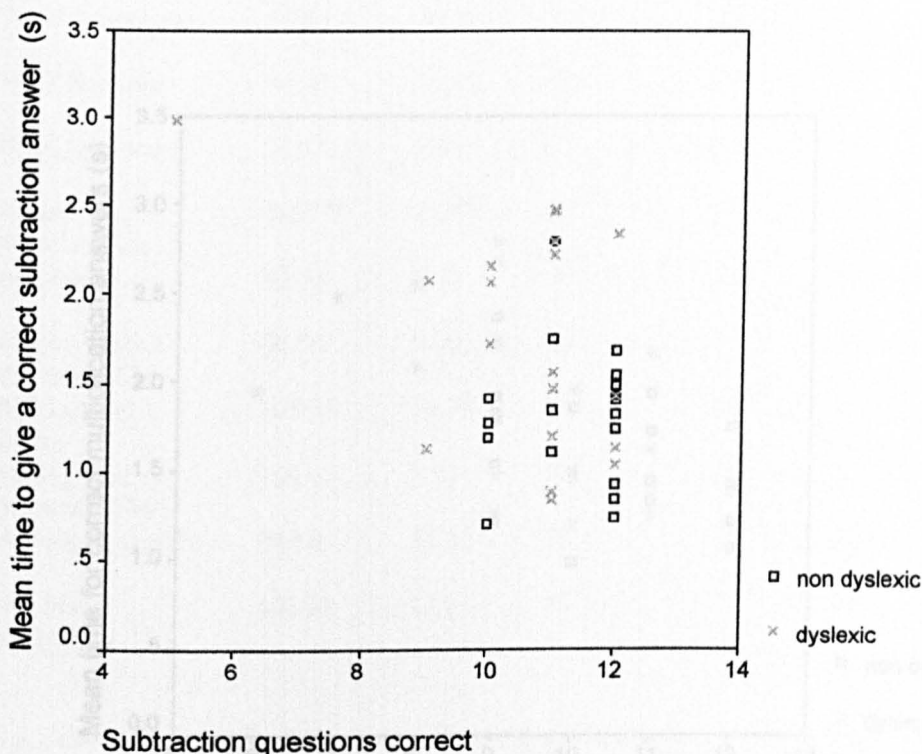
An examination of the scatterplot indicates seven dyslexic participants who answered the number facts questions more slowly than the rest of the participants. This sub-group is labelled A.

Figure 12. Scatterplot showing the number of correct addition facts plotted against the mean time taken to give a correct addition fact for both the dyslexic and non-dyslexic participants



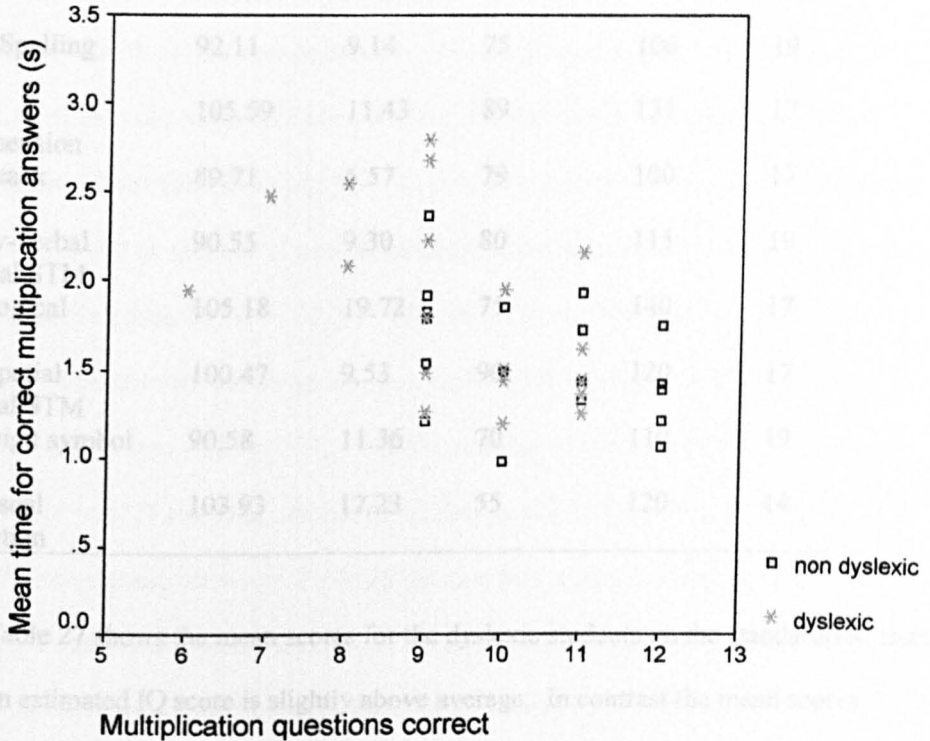
The scatterplot indicates that ten participants answered the addition facts questions less accurately than the rest of the participants who answered all the addition questions correctly. This sub-group is labelled B. Seven participants in the sub-group were dyslexic.

Figure 13. Scatterplot showing the number of correct subtraction facts plotted against the mean time taken to give a correct subtraction fact for both the dyslexic and non-dyslexic participants



The scatterplot indicates that nine of the participants answered the subtraction questions more slowly than the rest of the participants. This sub-group is labelled C. Eight of the participants in the sub-group were dyslexic.

Figure 14. Scatterplot showing the number of correct multiplication facts plotted against the mean time taken to give a correct multiplication fact for both the dyslexic and the non-dyslexic participants



The scatterplot indicates that eight participants answered the multiplication questions more slowly than the rest of the participants. This sub-group is labelled D. The eight participants in sub-group D also received poorer accuracy scores. Seven of the eight participants in sub-group D were dyslexic.

Table 27. Scores for the dyslexic students on the standardised tests

Test	<i>M</i>	<i>SD</i>	<i>Min.</i>	<i>Max.</i>	<i>n</i>
Estimated IQ	112.16	11.30	91	137	19
WRAT Arithmetic	84.53	10.38	65	109	19
WRAT Reading	96.53	11.52	70	116	19
WRAT Spelling	92.11	9.14	75	106	19
Passage comprehension	105.59	11.43	89	131	17
Word attack	89.71	6.57	79	100	17
Auditory-verbal sequential STM	90.55	9.30	80	115	19
WMS Logical memory	105.18	19.72	75	140	17
Visual-spatial sequential STM	100.47	9.53	90	120	17
WAIS Digit symbol	90.58	11.36	70	110	19
WMS visual reproduction	103.93	17.23	55	120	14

Table 27 shows the mean scores for the dyslexic students on the standardised tests. The mean estimated IQ score is slightly above average. In contrast the mean scores reading and spelling are somewhat lower, being in the lower half of the average range. This profile would be expected in a sample of high achieving dyslexic adults diagnosed using a discrepancy definition. The mean score for the *WRMT-R Word Attack* test is slightly below average, indicating that, on average, the sample have difficulties with phonological decoding. The mean score for the *WRMT-R Passage Comprehension* test is within the average range, but somewhat lower than the mean estimated IQ for the group. The mean scores for all the memory measures are within the average range, but somewhat lower than the mean estimated IQ for the group. However, by examining the maximum and minimum scores, it is possible to determine that the dyslexic students did not display a homogenous memory profile. The diagnostic criteria indicated that they had to have a

cognitive deficit, but the area of deficit differed from individual to individual. Some of the dyslexic students had a specific weakness, e.g. poor auditory-verbal sequential short-term memory, but average or above average scores on all the visual memory measures, whilst other dyslexic students had deficits on nearly all the memory measures.

On the *LADS* memory test the dyslexic students achieved a mean score on of 3.32 ($SD = 2.75$) and the non-dyslexic students achieved a mean score of 2.89 ($SD = 2.75$).

There was no significant difference between the two groups on this measure. In the total sample *LADS* memory scores did not correlate at a statistically significant level with number of correct addition facts [$r(37) = .17, p = .31$], number of correct subtraction facts [$r(37) = -.06, p = .71$], number of correct multiplication facts [$r(37) = .05, p = .78$] or total number of number facts [$r(37) = .04, p = .83$]. In contrast, estimated IQ correlated significantly with number of correct addition facts [$r(37) = .36, p = .029$], number of correct subtraction facts [$r(37) = .33, p = .045$], number of correct multiplication facts [$r(37) = .37, p = .021$] and total number of number facts [$r(37) = .43, p = .007$].

The correlations shown in Table 28 indicate that within the sample of dyslexic students all three number facts measures correlated with each other and *Number Facts* total score at a statistically significant level, the only exception was the correlation between addition facts and multiplication facts that verged on significance. *WRAT-3 Arithmetic* scores correlated with all the number fact measures at a statistically significant level. The correlation between estimated IQ and digit symbol verged on statistical significance as did the correlation between *WMS Visual Reproduction* and estimated IQ. The other correlations between the cognitive measures were not significant. Estimated IQ correlated with *WRAT-3 Arithmetic* at a statistically significant level. The correlations between subtraction facts and estimated IQ and *Number Facts* and estimated IQ verged on statistical significance. There were no other statistically significant correlations between the number skills and cognitive measures, with the exception of a negative correlation between auditory-verbal sequential short-term memory and *WRAT-3 Arithmetic* score that

verged on statistical significance. This correlation suggest that dyslexic students who achieved better auditory-verbal sequential memory scores did worse on the test of written arithmetic.

Table 28. The correlations between the number skills and cognitive abilities for the dyslexic students

Test	1	2	3	4	5	6	7	8	9	10	11
1. WRAT Arithmetic	—	.49* (18)	.46* (18)	.51* (18)	.57* (18)	.46* (18)	-.40† (18)	.07 (16)	-.33 (16)	.37 (18)	.33 (13)
2. Addition facts		—	.50* (18)	.45† (18)	.70** (18)	.34 (18)	-.33 (18)	-.24 (16)	.10 (16)	-.17 (18)	-.07 (13)
3. Subtraction facts			—	.66* (18)	.91** (18)	.43† (18)	-.20 (18)	.33 (16)	.29 (16)	.08 (18)	.23 (13)
4. Multiplication facts				—	.87*** (18)	.34 (18)	-.34 (18)	.32 (16)	.11 (16)	.20 (18)	-.00 (13)
5. Number Facts					—	.44† (18)	-.33 (18)	.24 (16)	.22 (16)	.08 (18)	.10 (13)
6. Estimated IQ						—	-.17 (18)	.01 (16)	.33 (16)	.36 (18)	.47† (13)
7. Auditory-verbal STM							—	.38 (16)	.37 (16)	-.16 (18)	.20 (13)
8. WMS Logical memory								—	.05 (16)	.06 (16)	.05 (16)
9. Visual-spatial STM									—	.23 (16)	.06 (13)
10. WAIS Digit symbol										—	.51† (13)
11. WMS Visual reproduction											—

Note. The degrees of freedom for each correlation are shown in brackets. † $p < .100$, * $p < .050$, ** $p < .010$, *** $p < .001$

9.4 Discussion

The results of Study Five indicate that dyslexic individuals continue to have difficulties recalling number facts in adulthood. Specifically, dyslexic students recalled fewer multiplication and subtraction facts than non-dyslexic students did. The difference in recall of multiplication facts remained significant and the difference in recall of subtraction facts verged on significance, even after an extreme score was removed. This indicates that the differences were not due to one atypical dyslexic individual. When the dyslexic students recalled addition and subtraction facts correctly they did so more slowly than their non-dyslexic peers. These differences remained significant after the removal of the extreme score.

These results are consistent with previous findings that dyslexic children are less accurate and slower when answering number fact questions (Pritchard et al., 1989; Turner Ellis et al., 1996). The results of the present study also suggest that a difficulty in quickly and accurately recalling basic number facts could explain (at least, in part) the particular problems that dyslexic adults have with the *WAIS Arithmetic* sub-test (Blalock, 1987; Cordoni et al., 1981; Vogel & Walsh, 1987). The lack of a statistically significant difference in addition fact accuracy appears to be due to both groups having mastered this skill; the mean scores of both groups were close to the ceiling of this test. The difference in the multiplication speed of the two groups only verged on significance. As the answer times were compared for the items that the participants got right, the mean answer time dyslexic students would be calculated on an easier set of items. This may explain why the difference did not quite reach a statistically significant level.

A rather surprising finding was the extent of dyslexic adult's difficulties with written arithmetic. The mean score of the dyslexic students on the *WRAT-3 Arithmetic* test was lower than the mean score of the dyslexic students on the *WRAT-3 Reading* test. Only three of the dyslexic students achieved a score on the written arithmetic test that was within the average range, the other 15 dyslexic students achieved below average scores.

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Dyslexia is often conceptualised as a specific difficulty with language and literacy skills; arithmetic is not identified as a primary area of difficulty. However, caution must be exercised when analysing this result. As the control group were not administered the *WRAT-3 Arithmetic* test, the dyslexic students scores were compared to the normative sample. The norms are almost ten years old and therefore written arithmetic attainment may be lower in the general population now. It is also possible that course of study was an uncontrolled extraneous variable that influenced the dyslexic students scores on the written arithmetic test. Studying a natural science subject would increase the likelihood that the dyslexic students would have studied the more advanced concepts such as algebra examined in the *WRAT-3 Arithmetic* test. A bias towards arts students in the present sample could lead to an underestimation of dyslexic students' written arithmetic abilities as a whole. However, an examination of the errors of the dyslexic students indicates that they are erring on many of the relatively easy simple arithmetic questions. Five of the 19 dyslexic students answered the question 7×6 incorrectly. Furthermore, 10 of the dyslexic students (over half of the whole group) erred on at least one simple addition (without regrouping), simple subtraction (without regrouping) or single digit multiplication sum. To confirm the written arithmetic difficulties of dyslexic adults it would be useful to repeat this study, but administer the *WRAT-3 Arithmetic* test to both the dyslexic and the control group. It would also be useful to compare the performance of dyslexic students studying natural science with dyslexic students studying the arts. The influence of course of study should have less of an effect on student's performance on the basic number fact measures, as these skills are taught to all children in primary schools.

An examination of the scatterplots indicated that some dyslexic participants had more severe difficulties than others. This raises the question of whether only a sub-group of dyslexic adults have difficulties with number fact recall. Whilst the results of this study may suggest that some dyslexic adults have unimpaired number fact recall, the finding must be considered in the context of the test used. Only small samples of the possible

addition, subtraction and multiplication questions were used. A longer test, more comprehensive test would need to be utilised to confirm that some dyslexic adults are unimpaired in this area.

In the discussion of Study Four, it was suggested that auditory-verbal memory deficits could be the cause of dyslexic children's number fact weaknesses. Memory weaknesses were also suggested as a possible cause of dyslexic individual's arithmetical weaknesses by Steeves (1983). However, the results of the present study did not support this hypothesis. There were no significant correlations between any of the memory measures and any of the number fact measures. Furthermore, there was a negative correlation between auditory-verbal sequential short-term memory and the dyslexic students' written arithmetic scores that verged on statistical significance. This correlation indicates that the students with lower auditory-verbal memory scores tended to obtain higher written arithmetic scores. Again it is important to be cautious when interpreting this result. Firstly, correlations will be difficult to detect on the number fact measure, because there is a very limited range of scores in this sample. Correlations might be revealed if there was a greater spread of scores. This could be done by increasing the number of questions with higher digits. Secondly, the heterogeneous memory profiles of the dyslexic students makes the lack of significant correlations difficult to interpret. Studies have indicated that both auditory-verbal STM (Bull & Johnston, 1997; McClean & Hitch, 1999) and visual-spatial STM (McClean & Hitch, 1999; Singleton et al., in preparation) correlate with number fact recall abilities or arithmetic attainment (which is influenced by number fact recall). Therefore the possibility exists that both auditory-verbal and visual-spatial short-term memory weaknesses lead to number fact difficulties. In the present sample of individuals with heterogeneous memory profiles, it may therefore be difficult to detect correlations. For example, the correlation between auditory-verbal sequential short-term memory and number fact recall may be disturbed if dyslexic students with higher auditory-verbal memory tend to have lower visual-spatial short-term memory, which also correlates

with number fact recall. This problem is particularly important in this sample, because the diagnostic criteria used required the dyslexic students to have at least one cognitive deficit, therefore if a dyslexic participant had relatively unimpaired memory in one area, they were likely to have relatively poor memory in another area. This study needs to be replicated, using a larger number of dyslexic students, possibly dividing them into groups according to their memory profiles.

The number fact recall difficulties of the dyslexic students could be explained by another aspect of short-term memory that was not assessed: central executive functioning. The dyslexic children in Study Four performed more poorly than the control children on a counting task that drew on central executive resources. Further studies of adults and children with dyslexia could examine their central executive functioning and the relationship between central executive and number facts.

It is important that dyslexic students and staff who support dyslexic students are aware that dyslexic individuals are likely to have difficulties with arithmetic, particularly recalling number facts. If the student's course has a numerical component it will be important for them to ensure that their basic arithmetic skills are adequate for the demands of the course. Knowledge of particular difficulties with number fact recall experienced by dyslexic adults, would also be useful for employers when they are making selection decisions. Many companies (particularly large graduate employers) employ numerical reasoning tests as part of their selection procedures. These tests are designed to assess candidates' abilities to detect patterns in number sequences. Dyslexic individuals may perform badly, not because they are unable to reason about the numbers, but because they are slow to retrieve number facts.

10 General discussion, implications and conclusions

10.1 Main findings

In Study One the relationships between number skills and cognitive abilities in children age 8 to 11 years were examined. Non-verbal reasoning ability correlated with both number fact recall and place value understanding at a statistically significant level. Auditory-verbal sequential memory correlated weakly with number fact recall at a level that verged on statistical significance. Visual-spatial memory correlated with place value understanding, but this correlation was no longer significant when non-verbal reasoning ability was partialled out. Two different measures of counting speed correlated with number fact recall, but not place value understanding.

In Study Two the ability profiles of children with specific arithmetic difficulties were examined. Children with SAD were far more likely to have statistically significant discrepancies between their verbal and spatial abilities than children in the general population. Some of the children with significant discrepancies had poorer spatial abilities whilst others had poorer verbal abilities. In Study Three the cognitive profiles of the children with specific arithmetic difficulties were examined. The children were divided into four groups; low general conceptual ability, non-verbal learning difficulties, verbal reasoning weaknesses and memory weaknesses.

In Study four the number skills profiles of children with dyslexia and children with SAD were compared to a control group of children randomly selected from the mainstream school population. The children with dyslexia performed at a similar level to the control children on the test of place value understanding, but worse than the normally developing children on the tests of addition, subtraction and multiplication fact recall. The children with dyslexia counted more slowly than the control children when there were no memory aids to help them keep track of the items they had counted. However, when there were memory aids available the children with dyslexia counted at a similar speed to the control children. In contrast the children with SAD performed worse than the normally developing

children on the subtraction and multiplication fact tests and the test that assessed place value understanding. The children with SAD did not count more slowly than the control children.

In Study Five the written arithmetic and number fact recall abilities of dyslexic adults were examined. Despite having average to above average IQ scores the dyslexic adults tended to gain below average scores on the test of written arithmetic. The mean standard score on the written arithmetic test for the dyslexic adults was below the normal range and 27 standard score points lower than the mean score for the dyslexic adults on the IQ test. The dyslexic adults recalled fewer subtraction and multiplication facts than a control group of normally developing adults matched on IQ score.

10.2 A more rounded approach to research into specific arithmetic difficulties

Most studies of specific arithmetic difficulties discussed in Section 4 focussed exclusively on one specific area, e.g. number skills, ability profile or memory. In this thesis an attempt has been made to try and take a less blinkered view of children with specific arithmetic difficulties by examining their number skills, psychomotor abilities, memory abilities and the pattern of their verbal, non-verbal and spatial abilities. Looking at one aspect of a child with SAD such as memory offers a very limited view of their strengths and weaknesses. The results of Study Three indicated that that some children with SAD have isolated memory weaknesses, but other children with SAD have a memory weakness in the context of a particular ability weakness. For example, some children with SAD had a visual-spatial memory weakness in the context of poor spatial and non-verbal reasoning. Studies that only examine the memory abilities of children with SAD may erroneously conclude that SAD is associated with memory difficulties and ignore the possibility that SAD children's difficulties may be equally due to weak spatial or verbal reasoning abilities. Ensuring that children have intellectual abilities that are within the

average range does not preclude the possibility of a weakness in a particular type of reasoning.

In Section 4.1 different methods of investigating specific learning difficulties were examined. The studies in this thesis are chronological-age-match studies, which are the weakest level of analysis. However, studies (particularly British studies) of children with weak arithmetic ability but better reading are still scarce. As yet a comprehensive picture of SAD children's cognitive and reasoning abilities has not been built up. It is therefore prudent to examine the abilities of children with SAD in different cognitive spheres, before applying a deeper level of analysis to one sphere such as memory.

A recurring theme in this thesis has been the heterogeneous ability and cognitive profiles of children with SAD. This finding suggests that an examination of the individual children's scores would be worthwhile in future studies of SAD. If, as Studies Two and Three suggest there are distinct subtypes of children with SAD, comparing the scores of SAD children as a group with normally achieving children will never offer a complete picture of the range of cognitive deficits associated with SAD. Sample level statistics will simply reveal the cognitive deficits shared by the majority of children with SAD.

10.3 Different routes to arithmetic difficulties: A weak hypothesis?

The results of Studies Two and Three indicated that children with SAD had heterogeneous ability and cognitive profiles, which were consistent with Lyytinen et al's (1994) proposal that different cognitive weaknesses can all lead to arithmetic difficulties. Logically, there are good reasons to suggest that arithmetic difficulties will have multiple causes, this may be termed the 'multiple route' hypothesis. Whilst single word reading is a discrete skill, in which visual stimuli are matched to the appropriate phonological codes, arithmetic is made up of numerous interacting sub-skills (see Section 3, particularly 3.7, for a discussion of this matter). Each sub-skill may be underpinned by different cognitive abilities. However, despite the logical arguments for the multiple route hypothesis, in an unspecific form this hypothesis is very weak and hard to falsify.

The results of Studies Two and Three can be used to make a specific, testable hypothesis. It is hypothesised that a significant weaknesses in verbal ability, visual-spatial ability, visual-spatial working memory and auditory verbal working memory can all independently cause a child to under-achieve at arithmetic even if their general intellectual ability is within the average range. The study by McClean & Hitch (1999) (discussed in Section 4.4.3) suggests that central executive functioning should also be added to the list. As a first step to testing this hypothesis a large group of children who were performing below the expected level in arithmetic (based on their age and GCA) would have to be recruited. If significant groups of children were identified with each type of weakness it would strengthen the multiple route hypothesis.

Other research strategies could be used to assess the multiple route hypotheses. One possibility would be to analyse the associations between particular cognitive profiles and particular number skills profiles. In Study Four this technique achieved some success. The children with SAD were poor at both place value understanding and two of the number fact recall sub-tests (subtraction and multiplication). Many of the children with SAD had general or specific (verbal, non-verbal or spatial) reasoning weaknesses. In comparison, the children with dyslexia (who generally have average to above-average reasoning abilities, but working memory deficits) were found to have weak number fact recall, but place value understanding that was similar to a group of control children. It is possible that the working memory weaknesses of the dyslexic children caused their number fact recall difficulties whilst their average or above average reasoning abilities allowed them to develop good place value understanding. The correlation (reported in Study One) between non-verbal reasoning and both number fact recall and place value understanding suggests that reasoning ability impacts on the development of both number skills and on place value understanding. Other studies such as Jordan et al., (1995) and Raesaenen & Ahonen, (1995) discussed in Section 4.6 have reported links between children's cognitive profile and their number skills profile.

Caution must be emphasised when attempting to link children's cognitive profiles with their number skills profiles. Firstly, and most importantly, statistical association does not provide sufficient evidence to support a causal link. Secondly, the interacting and cumulative nature of arithmetic ability must be noted. Certain cognitive deficits may impact specifically on early arithmetic skills, such as counting, but have an indirect effect on later arithmetic skills that rely on counting for their development, such as single digit arithmetic. It must not be assumed that because a child has a particular number skill deficit and a particular cognitive deficit that these are causally linked. However, if particular cognitive abilities are found to correlate only with one specific number skill, it would be hypothesised that children deficient only in that particular cognitive skill would only be deficient in the number skills with which it correlated and number skills dependent on those skills. In Study One auditory-verbal memory correlated only with number fact recall (at a level that verged on significance) and dyslexic children were found to be weak at number fact recall, but not place value understanding (which is not associated with auditory-verbal memory nor dependent on number fact recall). However, the lack of a correlation between the auditory-verbal sequential memory measure and *Number Facts* casts doubt on the hypothesis that auditory-verbal memory deficits are the cause of number fact recall difficulties in dyslexic individuals. The lack of a correlation between auditory-verbal sequential memory and number fact recall in Study Four does not preclude the possibility that the number fact difficulties of dyslexic individuals are caused by poor auditory-verbal memory. There are problems in interpreting this negative result (see Section 1.1 for a discussion). However, the dyslexic individuals' number fact difficulties may be due to a different cognitive deficit. For example, dyslexic individuals number fact recall difficulties could be caused by difficulties co-ordinating counting procedures (see Section 8.4 for a discussion of this hypothesis).

A different research methodology that could be used to test the multiple route hypothesis is longitudinal correlation. If a cognitive ability (measured in pre-school

children) predicts school age arithmetic attainment it adds strength to the hypothesis that a deficit in that area can cause arithmetic difficulties. The success of this research method can be seen in the area of dyslexia and reading difficulties. The strength of the phonological representations hypothesis comes from the converging evidence that pre-school phonological abilities predict future reading skill and that dyslexic individuals have poor phonological abilities. Dyslexic individuals have deficits in other areas (e.g. balancing while their eyes are shut, Fawcett & Nicolson, 1994), but there is no strong evidence that these abilities influence reading ability. Evidence that several different cognitive abilities contribute to unique proportions of the variance in arithmetic attainment would strengthen the multiple route hypothesis. If only some of the cognitive abilities predict future arithmetic abilities, it can be concluded that the non-predictive factors are simply correlates of specific arithmetic difficulties, in a similar way that poor balancing ability is a correlate of dyslexia. Again, developmental factors must be taken into account when considering longitudinal correlation studies. If a particular arithmetic skill is taught or develops in later childhood, the cognitive abilities that contribute to its development cannot be revealed if the arithmetic abilities of the children in the study are tested in early childhood. For example, if auditory-verbal memory accounts for a significant proportion of the variance in speeded number fact recall and the child's arithmetic is assessed when they are 6 years-old, this relationship will not be revealed as the child will not yet have developed this particular number skill. Similarly, the relationships revealed may be different depending on the content of the arithmetic test used. For example, if only written arithmetic is assessed, any cognitive abilities that contribute solely to mental arithmetic development will not be highlighted.

10.4 Links with models of adult numerical processing

If the McCloskey & Caramazza (1985) model, which states that arithmetical processes are carried out by distinct cognitive modules is used to interpret the findings of Study 4 the children with dyslexia would be regarded as having an impaired number facts

module and an unimpaired number comprehension model, whilst the children with SAD have an impaired number comprehension module and an impaired number fact module. Previous studies that have examined the validity of the McCloskey & Caramazza (1985) model with children with specific learning difficulties have tended to use tests that ask children only to comprehend relatively small numbers (e.g. the Shalev et al., 1993 arithmetic battery). These tests have ceiling effects on their number comprehension and production sections. Therefore it is unsurprising that many studies have reported no differences between children with specific learning difficulties and normally developing children on number production and comprehension (e.g. Gross-Tsur et al., 1996; Shalev et al., 1988) or between children with arithmetic difficulties who have different cognitive profiles (e.g. Shalev et al., 1997). In Studies Three and Four in which a more sensitive test of number comprehension (*Most*) was used, the children with SAD showed significant deficits in number comprehension in comparison to their normally developing peers. It might be argued that comprehending large numbers requires place value understanding, which is not a function of the number comprehension module. However, if this viewpoint were accepted the model of McCloskey & Caramazza (1985) would have to be extended to include a 'large number comprehension module'.

It is not necessary to use the model put forward by McCloskey & Caramazza (1985) to explain the differences between the dyslexic and SAD children's number skills weaknesses. Their arithmetic difficulties can be seen as the result of weaknesses in their general cognitive architecture (this possibility was discussed in section 10.3). The majority of the research studies that have examined the McCloskey & Caramazza (1985) model have been hampered by the use of unstandardised tests or tests with ceiling effects and a disregard for the developmental literature. In Study Four associations between particular cognitive profiles and particular number skills were found. This suggests that the disassociations identified in children with specific learning difficulties may be a

consequence of their specific cognitive profile, rather than the impairment of a number skills module.

The authors of the case studies discussed in section 4.2 have interpreted the children's arithmetic difficulties as the result of an impaired number skills module. However, alternative explanations, which link the children's cognitive profiles to their number skills difficulties, are possible. HM, described by Temple (1989), has a specific difficulty recalling multiplication facts. However, HM had dyslexia and deficits in the multiplication fact recall of dyslexic children (and adults) were reported in Studies Four and Five. These results are consistent with previous research (see for example Pritchard et al., 1989 and Turner Ellis et al., 1996, discussed in section 5.4). The working memory deficits of children and adults with dyslexia (including HM) may be the cause of their number fact retrieval problems.

YK, described by Ta'ir et al. (1997), has profound difficulties with arithmetic. The authors conclude that YK has an impaired 'cardinal/ordinal skills acquisition device'. However, YK has multiple cognitive deficits that may have compromised his arithmetic development. YK has a NLD profile; his performance IQ is 21 points lower than his verbal IQ. YK also has deficits on tests of executive function (such as the Wisconsin Card Sorting Test), very poor visual spatial memory (his score on a bead memory test was approximately 5 years behind his chronological age) and poor visuomotor organisation. Ta'ir et al. (1997) argue that, "... cognitive deficits such as those exhibited by YK have not been associated with developmental dyscalculia" (p. 198). However, there are numerous studies that link visual-spatial memory deficits with specific arithmetic difficulties (e.g. Brandys & Rourke, 1991; Fletcher, 1985; McClean & Hitch, 1999; Siegel & Linder, 1984). A recent study conducted by McClean & Hitch (1999) found that children with SAD performed more poorly than age-matched controls on tasks that measured central executive functioning - they even differed from arithmetic-matched controls on one of the tasks. Studies Two and Three in this thesis indicated that some, if

not all children with SAD have visual spatial abilities that are considerably poorer than their verbal abilities. Therefore, all of the cognitive deficits YK displayed *are* associated with specific arithmetic difficulties.

The cognitive and ability test results of SW described by Temple (1991) are limited to IQ, auditory-verbal memory and receptive vocabulary (which were all in the average range). However, SW's difficulties with arithmetic procedures could have been due to visual-spatial or central executive deficits. SW has a specific neurological abnormality: his EEG revealed "recurrent sharp waves in the left parieto-occipital area and a right frontal slow wave abnormality during overbreathing" and computerised tomography revealed: "a right frontal tuber³" (p. 159). As central executive functioning is associated with prefrontal lobe damage it is quite possible that SW has a cognitive deficit that interferes with his ability to learn and execute arithmetic procedures.

The most difficult case study to explain in terms of deficits in the general cognitive architecture is that of Paul described by Temple (1989). Paul did have cognitive and ability deficits. It is reported that he received a standard score on a group test of non-verbal reasoning of 85; however, it is also noted that on previous intelligence tests he had gained "unexpectedly poor" (p. 99) scores. Paul's naming ability was approximately three years below his chronological age; he also had very poor auditory-verbal and visual spatial memory. Paul was unable to recall the months of the year or the letters of the alphabet correctly. Considering these multiple cognitive difficulties it is unsurprising that Paul had arithmetic difficulties, and perhaps more surprising that his reading accuracy and comprehension abilities were age appropriate. However, the exact nature of Paul's arithmetic difficulties are hard to relate to his cognitive weaknesses. Paul's ability to read Arabic numbers, write Arabic numbers to dictation, read number words and read numeral words was severely impaired: he erred on approximately 50% of trials. Paul produced the correct syntactical frame for numbers, but used the wrong digits. This category specific

³ A growth on the brain resulting from SW's inherited disorder tuberous sclerosis (for further information on this condition see www.tuberous-sclerosis.org/)

deficit is difficult to explain in terms of weaknesses in the general cognitive architecture. Paul had good phonological skills and could read non-words and unfamiliar regular words correctly, but he could not apply these skills to number words. Paul does have the numbers stored, he can count orally and number the face of a clock, but sometimes he does not access them correctly in response to stimuli. Paul's category specific disorder presents a strong challenge to the encoding complex view of numerical processing, proposed by Campbell & Clark (1988) and Clark & Campbell (1991). If general cognitive mechanisms are used to carry out arithmetic tasks, how can Paul read at an age appropriate level, but be unable to read number words?

The McCloskey & Caramazza (1985) model could explain the heterogeneity of the sample in Studies Two and Three and the lack of correlations between the memory measures and number fact recall in Study Four. If the children's arithmetic difficulties were caused by a weakness in an arithmetic specific module, one would not expect them to have similar weaknesses to their general cognitive architecture. It is possible to explain the number fact recall difficulties of children and adults with dyslexia in a manner that is consistent with the McCloskey & Caramazza (1985) model. The neurological systems that are compromised in dyslexia may be physically close to the neurological systems involved in number fact recall. Further studies need to be conducted to determine whether specific number fact recall deficits in dyslexic individuals are caused by weaknesses in their general cognitive architecture (such as working memory impairments) or an impaired number fact module. If future studies using number fact tests, that produced a wider spread of scores, reported correlations between the extent of dyslexic individuals' working memory deficits and their number fact recall ability, this would strengthen the argument that working memory is the cause of dyslexic individuals' number fact recall difficulties. If a particular aspect of working memory predicted future number fact recall ability and dyslexic individuals were found to be deficient in that same aspect of working memory the hypothesis would be strengthened further. However, as discussed in Section 4.1 causal

links can only be truly demonstrated by training studies. Until reliable and empirically tested techniques to improve working memory have been developed such studies are not a possibility.

10.4.1 Different levels of deficit

Butterworth (1999) proposes a different modular view to McCloskey & Caramazza (1985). The model presented by Butterworth (1999) may be able to accommodate both individuals who have deficits in their general cognitive architecture, leading to specific arithmetic difficulties, and individuals with damage to a number module, leading to profound number difficulties e. g. in an individual such as Paul; Temple (1989). Butterworth's (1999) model consists of a single number module. He states that "The job of the number module is to categorize the world in terms of numerosities – the number of things in a collection." (p.9). This module allows us to determine small numerosities automatically and distinguish between sets that have different numerosities. Butterworth (1999) argues that more complex numerical abilities such as counting and simple arithmetic are built on this automatic skill, which is hard-wired into the human brain. The numerical module enables us to determine the numerosity of sets up to about four or five; this process is called subitising (see section 3.1). More complex numerical skills, such as counting, are built on this innate ability to determine numerosity and require the use of culturally determined conceptual tools, such as specific counting words or numbers.

Butterworth (1999) described an adult neurological patient that, he argues, had a compromised number module, because of her impairments on very basic number tasks. Signora Gaddi (previously described as patient CG in Cipolotti, Butterworth & Denes, 1991) experienced a stroke that damaged the left parietal lobe of her brain. After the stroke she retained good language, reasoning and both short- and long-term memory abilities, yet she was unable to answer very fundamental number questions. Signora Gaddi was unable to count above four, and was unable to subitize even two dots. She was very slow at making magnitude judgements if both numbers were below four and performed at

chance level if both numbers were above four. Signora Gaddi had great difficulty recalling everyday numerical information. If the quantity was four or less she could recall the number very slowly (for example, to recall the number of wheels on a car she had to visualise the car and count the wheels). If the quantity was greater than four she was unable to recall it (e.g. she could not say how many days were in a week). In contrast, a different patient, Mr Bell (previously described by Rossor, Warrington, & Cipolotti, 1995) suffered from Pick's disease (a degenerative neurological condition), which had resulted in impaired reasoning, language and long-term memory. Despite these cognitive deficits Mr Bell had relatively unimpaired number calculation skills. These case studies lead Butterworth (1999) to conclude that our basic ability to determine numerosities is modular and disassociated from our general reasoning and memory abilities.

It is possible that genetic abnormalities can cause the number module to be compromised, resulting in a severe form of developmental dyscalculia. Butterworth (1999) describes such a case. Charles, who was 31 years old when he was tested, had been diagnosed as dyslexic at fourteen years of age. He had experienced profound difficulties with both school arithmetic and informal arithmetic (such as money handling) throughout his education. Charles had achieved good A level results and a university degree. He scored highly on tests on general intelligence and reasoning. However, he could not read or write numbers above three digits and performed very poorly on tests of calculation (he used his fingers for all calculations). Charles was not only very slow at magnitude comparison tasks, but his performance was abnormal. Normal adults take longer to decide which of two numbers is larger if they are closer together (e.g. it takes longer to decide for 8,9 than for 2,9). However, Charles showed the reverse pattern: he took longer when the numbers were further apart. Butterworth (1999) suggested that instead of comparing internal representations of the numeral's numerosities Charles was using a counting algorithm, i.e. he counted from one digit until he got to the other digit. What was most surprising, was that Charles lacked the ability to subitize - he could not quantify a small

number of dots at a glance - an ability that is common to all humans. Butterworth (1999) found that whilst control adults could quantify one, two, three or four dots in a very similar time, Charles took longer to quantify as the number of dots increased, indicating that he could not subitize.

Butterworth (1999) concedes that whilst adults who have already acquired arithmetic skills may be able to preserve them despite poor reasoning, language and memory abilities, these abilities may be necessary for developing what he calls the conceptual tools provided by culture (such as counting skills and words, calculation procedures, etc.). This suggests that an unimpaired number module is necessary but not sufficient for number skills to develop. Some of the SAD children in Studies Two and Three may have had an impaired number module. In the future it would be interesting to carry out tests of subitising on similar children. However, it is also possible that the development of their conceptual tools was damaged either by specific cognitive weaknesses or by non-cognitive factors. Paul, described by Temple (1989) and discussed in sections 4.2 and section 10.4 is a possible candidate for an impaired number module as he has profound difficulties with arithmetic.

In summary, case studies suggest that our basic ability to conceive numerosities and subitize is carried out by a distinct number module. If this module is damaged or does not develop properly our ability to understand and manipulate numbers is profoundly impaired. Cultures have developed conceptual tools, such as Arabic numbers and calculation procedures, which develop properly only when the person has an unimpaired number module. I would argue that the singular number module theory put forward by Butterworth (1999) is more valid than the multiple module theory proposed by McCloskey & Caramazza (1985). It seems unlikely that the evolutionary pressures for number fact recall, calculation, number comprehension and number production modules to develop have existed for a long enough period of history. The cultural tools that make these procedures possible developed only recently in the history of humankind. For example,

the place value system was invented in the first century AD (see Butterworth, 1999); it is therefore impossible that a module for calculation procedures (which are dependent on place value) could have evolved. I would argue instead that children have to utilise both aspects of their general cognitive architecture (e.g. memory and reasoning) and their innate understanding of numerosities to develop a full understanding of arithmetic. Isolated areas of arithmetic weakness, such as the poor multiplication fact recall of HM (described by Temple, 1989), are caused not by impaired modules, but rather by a weakness in a cognitive ability that is specifically important for the development of that skill. Individuals with more fundamental arithmetic weaknesses, such as Charles (described by Butterworth, 1999) and possibly Paul, (described by Temple 1989), have an impaired number module. Diffuse arithmetic weaknesses that affect higher order rather than the fundamental numerical processes carried out by the number module are due either to cognitive weaknesses that impact on the development of many arithmetic skills or non-cognitive problems, such as mathematics anxiety or poor motivation. Children whose arithmetic difficulties stem from an impaired number module can be described as having primary dyscalculia, whilst those children whose arithmetic difficulties stem from weak cognitive abilities can be described as having secondary dyscalculia.

10.5 Implications for the assessment and teaching of children with SAD and dyslexia

10.5.1 The purposes of formal assessment

Formal assessment is often used for multiple purposes. Simple assessment of arithmetic attainment using standardised arithmetic tests is often used to quantify the extent of a pupil's problem. Quantifying the problem can be necessary in order to determine resource-worthiness if the child is educated in the state system or to confirm or dismiss worries that a child has significant difficulties based on less formal assessment. Simple assessment of this kind can also be used to identify areas of arithmetic where the pupil has particular difficulty, and areas, that are relative strengths. Psychological assessment

(where tests of intellectual ability and memory are usually employed) is often used to determine whether the child has a specific learning difficulty (i.e. there is a statistically significant discrepancy between the child's arithmetic attainment and their general intellectual ability). Again, the presence or absence of a specific learning difficulty can be considered important in determining whether the child receives special support and/or resources. However, the recent report on dyslexia, literacy and psychological assessment produced for the Division of Educational and Child Psychology of the British Psychological Society (Reason et al., 1999) did not endorse this practice in relation to literacy difficulties, making it less likely to be used in the future in this and other SEN contexts. A diagnosis of specific learning difficulties will also influence the expectations of both teachers and parents. It is often reported that receiving a diagnosis of a specific learning difficulty such as dyslexia can help the child's self esteem and understanding of their difficulties (e. g. T. R. Miles & E. Miles, 1999). In short, if a child receives a diagnosis of dyslexia they no longer need to worry that they are 'stupid'. On the other hand, encouraging such attitudes is not helpful to children who have general learning difficulties or to their parents. It is beyond the scope of this thesis to consider the complex social, political and psychological debates surrounding SEN diagnosis and resource-worthiness; the focus instead will be on what intervention is likely to be most effective for children with arithmetic difficulties if intervention is available and considered necessary.

Psychological assessment is often said to inform intervention approaches; for example, Rourke and Del Dotto (1994) state that a " ... comprehensive neuropsychological examination of the children with LD should provide insight into their disposition and treatment" (p. 84). Teaching can be tailored to a child's particular cognitive profile. However, empirical evidence to back up the effectiveness of specially tailored teaching approaches is extremely scarce.

10.5.2 Obtaining a valid measure of general intellectual ability

If psychological assessment of a child with arithmetic difficulties is deemed appropriate it is recommended that a full-scale intellectual ability test is used. The hazards of using a verbally or visually biased test were discussed in Section 7.4. As a large proportion of children with SAD have significant discrepancies between their verbal, non-verbal and spatial abilities, thus a balanced test is required to make an accurate assessment. A full-scale intellectual ability test may also reveal significant weaknesses that may be impacting on the child's ability to learn arithmetic. The usefulness of this psychometric information when designing teaching interventions is discussed later in this section.

10.5.3 Labelling and diagnostic categories

In Section 4.6 a number of classifications of children's arithmetic disabilities were examined. The primary practical purpose of a diagnostic label is that individuals who are given that label will share common characteristics and will respond to similar intervention techniques. If children who share the same label do not share important characteristics that go beyond the classification criteria then the label has no value. Even if children with arithmetic difficulties can be divided into sub-groups that share homogenous cognitive profiles, these groupings are only useful in a practical context if they have different responses to intervention techniques. Identifying sub-groups of children with arithmetic difficulties that have homogenous cognitive/ability profiles may still be useful if the goal of the research programme is to understand which cognitive factors influence arithmetic development. These principles will be used to evaluate the usefulness of the diagnostic categories described in this thesis.

Classification systems based on the pattern of number skills weaknesses

Classifying children using the McCloskey model (McCloskey & Caramazza, 1985) would be done by analysing a child's number skills strengths and weaknesses. Such a classification system has little educational value as it does not provide any psychological information that might influence the style of teaching that would be useful. A simple

assessment of a child's number skills might indicate that they have poor calculation procedures, but their other number skills would remain intact. However, classifying the child in such a manner only indicates which area of arithmetic requires most attention, which was possible without applying a classificatory label. These classifications may be useful when building theory, but they do provide further psychological information that might improve teaching interventions.

Non-verbal learning difficulties

The results of Study Two indicated that some children with poor arithmetic, but better reading, had an unusual balance of abilities (poor spatial and non-verbal abilities, but better verbal ability). Identifying such children as having non-verbal learning difficulties may be useful. Studies reviewed in Sections 4.4.1, 4.5 and 4.6 indicated that children who have weak visual-spatial skills, but better verbal skills, have many weaknesses that extend beyond the realm of arithmetic. Their difficulties include poor social competence and poor reading comprehension. Therefore, if a child referred for psychological assessment because of poor arithmetic is found to have a non-verbal learning disabled profile, it could trigger investigations into areas such as reading comprehension and social competence. Hartas (1998) argues that the competent mechanical skills of children with non-verbal learning difficulties such as a wide vocabulary, often masks deficits in more complex skills such as conversational ability and understanding of language. Following assessment and diagnosis, parents and teachers of children with non-verbal learning difficulties can be made aware of the underlying difficulties (such as poor reading comprehension and social skills) that these children may have. Understanding their difficulties is the first step to providing support. Hartas (1998) argues that intervention strategies are available that can improve social competence. These strategies include coaching and peer mediation and a fluid classroom environment. Children with non-verbal difficulties share cognitive characteristics; therefore, this label is an educationally useful label if children with particular cognitive profiles respond better to programmes designed specifically for their

particular cognitive profile, rather than to a general programme. Similarly, as the majority of dyslexic children share cognitive deficits (see Section 5.2), understanding their cognitive profile may prove to be educationally useful. The value of understanding the cognitive profile of children with arithmetic difficulties is discussed in the remainder of section 10.5.

10.5.4 Tentative links between profile and intervention

Designing intervention programmes for children with arithmetic difficulties is a complex issue. As already discussed, some researchers (e.g. Rourke and Del Dotto, 1994) believe that educational programmes will be most effective if the style is tailored to the cognitive profile of the child. Therefore, some suggestions will be made regarding the style of arithmetic teaching that is likely to be most appropriate for children with different cognitive profiles, specifically children with non-verbal learning difficulties and children with dyslexia. It should be noted that the style of an intervention is different to its content. The *content* of an intervention can be decided simply by examining a child's number skill profile. A different *style* may be used to teach the same content to different children. For example, multi-digit addition could be taught to children with NLD by emphasising the memorisation of standard algorithms and taught to children with dyslexia by emphasising the generation of novel procedures.

It must be stressed that the proposals put forward in the latter part of Section 10.5 are simply suggestions, which need to be tested using controlled empirical studies. It is possible that linking a child's cognitive profile to the style of intervention is not useful. Intervention techniques have been formulated and evaluated that are designed to be implemented for all children who have poor arithmetic regardless of their cognitive profile. Such approaches tailor the learning to the child's level of arithmetic attainment. For example, teachers using the maths recovery programme described by Wright, Martland, & Stafford (2000) first carefully assess a child's level of arithmetic attainment and then set activities that are just beyond the child's current level of understanding. Research reported

by Wright et al. (2000) has indicated that the majority of children who undergo the mathematics recovery programme make progress, and the case study of a child with a specific learning difficulty who made progress is reported. The intervention approaches discussed in Section 3.6 (Hiebert & Wearne, 1996; Fuson, 1986; Fuson & Briars, 1990; Swart, 1985) which improved the multi-digit arithmetic performance of normally developing children by linking multi-digit numbers to physical representations, may also help children with learning difficulties.

Furthermore, there are numerous American studies that have demonstrated that intervention techniques can raise the arithmetic attainment of students with specific learning difficulties (see Mastropieri, Scruggs & Chung, 1998 and Mastropieri, Scruggs & Shiah, 1991 for reviews). Interventions that have had positive benefits for students with specific learning difficulties included peer tutoring (Beirne-Smith, 1991), error self-monitoring (Dunlap & Dunlap, 1989), tape-recorded self-instruction (Wood, Rosenberg & Carran, 1993) and computer-assisted instruction (Bahr & Rieth, 1989; Christensen & Gerber, 1990; Kosckinski & Gast, 1993; Okolo, 1992). No attempt was made to tailor the teaching style to the children's cognitive profiles in these studies.

If generalist programmes are to be modified or replaced for children with particular cognitive or academic profiles then research must indicate that children with particular cognitive or academic profiles on specially tailored programmes make more progress than children with similar cognitive or academic profiles on general programmes. Few studies have examined the interactions between a child's cognitive profile and instructional outcomes. Lyon & Flynn (1991) provide a review of intervention studies that examines the relationship learning difficulty subtypes and treatment style. They outline two suitable methods of analysing subtype intervention style interactions. A regression design can be utilised. In this design the experimenter measures the children's cognitive abilities before intervention commenced, randomly assigns the children to two or more teaching interventions and then measures the children's skill levels using appropriate tests. A

regression analysis would be conducted to determine whether particular cognitive abilities predicted success using a particular intervention strategy. The alternative design uses an ANOVA to analyse the data. The children with specific learning difficulties are assigned to separate subtypes. This can be done by inspecting the children's scores on cognitive and academic tests and utilising predetermined categories based on existing theories. For example, children with arithmetic difficulties could be divided into groups based on the presence or absence of reading difficulties. Alternatively, the subtypes can be defined statistically, by entering the children's test scores into a factor analysis. An equal number of children from each subtype then needs to be allocated to two or more treatment conditions. An ANOVA with two independent variables (subtype and treatment type) can then be conducted. Lyon & Flynn (1991) highlight the practical difficulties that make studies examining subtype intervention interactions difficult to conduct. Firstly, large numbers of students with specific learning difficulties are required in order to conduct a powerful and reliable study. Secondly, parents and teachers may be unwilling to consent to the children undergoing experimental intervention, particularly if it reduces the time the child spends in class or being involved in other educational activities. Thirdly, there are difficulties in quantifying children's improvement (e.g. difficulty determining what tests would be valid and reliable measures). Finally, it is enormously difficult to control for the numerous extraneous variables that may influence the results, e.g. teaching style, classroom climate, previous and concurrent intervention, and departures from the planned teaching programme.

The number of studies investigating subtype-intervention interactions is therefore small. Due to the practical difficulties outlined, none of the studies reported have followed the ideal designs described by Lyon & Flynn (1991) - they all have some limitations. As studies that investigate subtype-treatment interactions are rare, studies that aimed to remediate reading rather than arithmetic skills are included to illustrate the principles involved.

Lyon (1983) examined the response to a synthetic phonics programme of different subtypes of learning disabled readers. The six subtypes had been identified by cluster analysis in previous studies (see Lyon, Rietta, Watson, Porch, & Rhodes, 1981; Lyon & Watson, 1981). The cognitive profiles of the six groups differed. Subtype 1 had a very broad range of deficits including poor language comprehension, difficulties blending phonemes, poor visual-motor integration and poor visual-spatial memory and ability. Subtype 2 had strengths in naming and auditory discrimination skills. Subtype 2 shared many of the deficits of subtype 1, but the deficits were less severe; they also had relative strengths in visual-spatial ability and memory and phoneme blending. Subtype 3 were poor at language comprehension and phoneme blending, but relatively strong in all other linguistic and visual-spatial areas. Subtype 4 had deficits only in visual-motor integration. Subtype 5 were poor at phoneme blending, auditory memory and language comprehension but better at visual-spatial tasks and visual-motor integration. Subtype 6 had a normal profile with no specific deficits. Five children from each of the six subtypes were chosen to take part in the study. They were matched on single word reading ability, age, race and sex. Their IQs ranged from 103.5-105, their single word reading centile scores ranged from 4 to 8. The children all received 26 hours of a synthetic phonics training programme. The results indicated that children in subtype 6 made significantly more progress than the children in any other group. On average children in subtype 6 gained 18 centiles on a standardised single word reading test. Subtype 4 fared better than the other four groups, but not as well as the children in subtype 6. On average the children in subtype four gained 8.2 centiles. The children in subtypes 1, 2, 3 and 5 did not differ from each other; they all made very minimal progress.

Lyon (1985b) conducted a further study, this time with younger learning disabled readers. In this later study, instead of comparing many different subtypes response to a single intervention strategy, he examined the response of one subtype to two different teaching programmes. Ten reading disabled children who were all members of a single

subtype (derived by factor analysis) participated in the study. They had deficits in a wide range of verbal, cognitive and linguistic areas. They had poor morphosyntactic skills, poor sound blending abilities, poor language comprehension, a short auditory-verbal memory span, poor auditory discrimination and poor naming ability. In contrast, they had strengths on all measures of visual-perceptual skills. Five of the children were assigned to the same synthetic phonics programme that was used in the Lyon (1983) study. The remaining five children were assigned to a combined programme that emphasised memorisation of a sight vocabulary, contextual analysis, structural analysis and analytic phonics. Prior to the intervention neither group differed in their single word reading ability; the centile range for the children in both groups was 8 to 10. Both groups received 30 hours of training. The post-intervention training revealed much larger gains for the children in the combined programme (on average, 11 centiles) as opposed to the synthetic phonics group (on average, 1 centile).

The results of the two studies by Lyon are promising; they suggest that tailoring an intervention programme to a child's cognitive strategy may be effective. The synthetic phonics programme utilised in the Lyon (1983) study was more effective for learning-disabled readers who did not have significant psycholinguistic difficulties, whilst the Lyon (1985) study suggested that children with linguistic difficulties made more progress (at least in the short-term) using a combined programme that included learning whole words, contextual analysis, structural analysis and analytic phonics.

A study that took a different approach to classifying children with specific reading difficulties also reported different subtypes had different responses to different types of intervention. Lovett, Ransby, & Barron (1988) divided 112 dyslexic children into two subtypes: accuracy disabled and rate disabled. The accuracy disabled children performed poorly on four out five of the single word reading tests administered; in contrast the rate disabled children had average decoding skills, but were slow readers. The two groups did not differ in IQ or age. The children were randomly assigned to one of three teaching

programmes. The decoding skills programme emphasised the acquisition of word recognition of high frequency words. The other experimental programme was more holistic and emphasised a variety of skills including vocabulary development, reading in context, listening and reading comprehension, syntactical elaboration and written composition. The final programme was the control condition and did not involve studying text. All children achieved 40 hours of teaching. The post-intervention tests included a standardised measure of single word reading and two experimental measures (a test of regular word reading and a test of irregular word reading). The accuracy disabled students made significantly more gains in the two experimental programmes as opposed to the control programmes on all three measures. They made greater gains on the test of irregular words in the decoding skills condition. The rate disabled children made significant gains on the standardised reading test and on the test of irregular words in both experimental conditions as opposed to the control programme. However, they did not make significant gains on the test of irregular words.

A more recent study that has linked cognitive profiles with intervention outcomes was conducted by Naglieri & Gottling (1997). The aim of this intervention programme was improvement in arithmetic attainment. Naglieri & Gottling (1997) examined the cognitive processes of children with specific learning difficulties using the PASS theory described by Das, Naglieri, & Kirby (1994). The PASS theory differentiates between four basic cognitive processes planning, attention and simultaneous and successive processing. Twelve children with specific learning difficulties took part in the study, they completed the Cognitive Assessment System (Naglieri & Das, 1997) before the study began. The 12 children with specific learning difficulties all underwent seven baseline sessions followed by 21 intervention sessions administered by their normal class teachers. At the start of each session (both baseline and intervention) the students spent 10 minutes completing an arithmetic worksheet. The intervention consisted of teacher-lead discussions about the arithmetic worksheets the students had just completed. The goal of these discussions was

to highlight the need for a planned and systematic approach when completing the worksheets. The teachers did not comment on the students' contributions, rather they asked open-ended questions such as "Let's talk about how we did the worksheets" or "What could you have done to get more correct?" The teachers did not demonstrate specific strategies to the students. The four children who achieved the lowest scores on the planning section of the Cognitive Assessment System (Naglieri & Das, 1997) were compared with four students who achieved the highest scores on the planning section. At the start of the study the two groups achieved similar arithmetic scores, however the low planning students made greater progress throughout the intervention and achieved higher arithmetic scores at the end of the intervention period. When the children were divided according to simultaneous processing scores, the groups did not differ in the benefits they achieved from the intervention. When the students were divided into groups based according to attention or successive processing scores the children who made higher scores made greater progress. This study suggests that children who have lower planning abilities benefit more from an intervention that emphasises planning than children with high planning abilities. But, this result should be interpreted with caution: the sample size was very small and no inferential statistics were conducted.

Overall, the four studies reviewed above are promising, suggesting that a child's cognitive or academic profile can interact with intervention style. However, the small sample sizes seriously limit the reliability of the results. In the remainder of this section general features of intervention programmes are discussed and tentative practical suggestions are made for teachers working with children with arithmetic difficulties with two very different cognitive profiles: NLD and dyslexia. The section is concluded by outlining a study that could test the educational value of these two diagnostic categories.

Working with strengths and attacking weaknesses

When designing intervention programmes there are two possible approaches, which would normally both be used within the programme. The first approach would be to use a

child's cognitive strengths to help them achieve important academic skills. For example, a child with dyslexia, but good general intellectual ability, may be encouraged to use their reasoning ability to determine what a word is from the context of the sentence. The second approach is to try to directly improve lower-level skills that a child finds difficult because they are directly associated with a cognitive weakness e.g. working on a dyslexic child's phonological decoding skills. Working on fundamental weaknesses can be very useful if significant progress can be made in ameliorating this weakness; however, if the fundamental weaknesses are intractable devising compensatory strategies that utilise a child's relative cognitive strengths can be helpful.

Children with NLD profiles

Children with arithmetic difficulties and an NLD profile have verbal strengths. Verbal mediation was found to improve the performance of children with a NLD profile on a mental rotation task (Williams et al., 1993). Verbal mediation may therefore improve NLD children's performance on arithmetic tasks. It may therefore help the child to have arithmetic tasks clearly broken down into sub-steps that are explained verbally. The stages could be written down until the child gains confidence with them and the child could verbalise each step as they do it. One problem with such a mechanistic approach is that the arithmetic knowledge thus gained by NLD children, may be isolated and superficial. Without explicit teaching they may not understand the numerical processes behind the verbal facts and procedures that they store, and they may not be able to connect the knowledge they acquire to other arithmetic concepts or the informal arithmetic they use everyday. This approach does not attack the fundamental weaknesses of children with NLD profiles. Hartas (1998) advocates the opposite approach, emphasising the use of concrete manipulatives and practical applications. He argues that for children with NLD to gain a full understanding of arithmetic they must be able to grasp the conceptual framework behind the mechanical skills. Their cognitive weaknesses make it hard for them to grasp these concepts. In effect, employing verbal strategies, is using verbal

strengths to compensate for non-verbal weaknesses, while the strategy recommended by Hartas (1998) is attempting to attack more fundamental weaknesses. The differences between these two approaches highlights the difficulties encountered when choosing a test to measure the effectiveness of an intervention programme. If the chosen test emphasises mathematical reasoning, a programme that uses manipulatives may result in greater improvement in children with non-verbal learning difficulties. Alternatively, if a test that emphasises arithmetical procedures is chosen, a programme that emphasises verbal mediation may result in greater improvement. Researchers must be clear about the intended goals of an educational programme before they decide on the measures used in a study. Tests that only emphasis arithmetical skills are only valid measures if the limited aim of improving only mechanical arithmetic skills is made clear.

Children with dyslexia

A large number of individuals with dyslexia have difficulties accurately and quickly recalling number facts. A difficulty in recalling number facts will impact on the ability of dyslexic individuals to carry out mental arithmetic and results in them making mistakes in written calculations. If a dyslexic child's poor recall of number facts is not recognised as part of their dyslexic symptoms by their teacher, the child may develop negative attitudes to maths or even maths anxiety because of continued failure on arithmetic problems. A dyslexic child with average or above average intellectual ability may be able to cope with the conceptual aspects of maths (e.g. deciding on the correct operations to carry out when answering a story problem or correctly factorising an equation), but they may err in the mechanical aspects of arithmetic (e.g. recalling the wrong answer when carrying out the operation they planned for the story problem).

Dyslexic children's problems with number fact recall can be tackled on three levels. The weakness can be tackled directly by providing repeated practice on addition, subtraction and multiplication number facts. Repeated practice can be presented on a computer either unadorned or in a game format. Studies of children with specific learning

difficulties (some of whom would have dyslexia) have reported positive results, with the children recalling more number facts after computer assisted practice (e.g. Bahr & Rieth, 1989; Christensen & Gerber, 1990; Kosckinski & Gast, 1993; Okolo, 1992). Secondly mental compensatory strategies can be developed. Chinn (1992) makes some helpful suggestions of compensatory strategies that can be developed. Rules can be learnt for all the multiples of 0, 1 and 10. Furthermore, if the commutativity principle is understood, the overall number of facts that have to be learnt can be reduced. In addition, a child who understands that multiplication can be treated as repeated addition, can derive unknown facts from known facts. For example, a dyslexic child who cannot recall the answer to 7×6 , but can recall the answer to 6×6 can derive $7 \times 6 = 42$ by recalling that $6 \times 6 = 36$ and adding another 6 on. Children with dyslexia who have good reasoning ability should readily be able to grasp the principles and patterns which these strategies are based on and therefore be able to reduce the amount of facts that have to be committed to memory. The study reported by Erenburg (1995) indicated that dyslexic children who utilised derived fact strategies were much more successful at recalling number facts than dyslexic children who did not. Finally, an electronic calculator can be used as a physical compensatory strategy. This would be particularly useful if a dyslexic child had a good understanding of the conceptual aspects of mathematics, but was disheartened at getting the wrong answers due to calculation errors.

Although research is still scarce, there is some evidence to suggest that dyslexic children (Miles et al., 2001; Steeves, 1983) and adults (see the results of Study Five in Chapter 9) have written arithmetic weaknesses. An understanding of the cognitive profile of dyslexic children may help to inform educational programmes that aim to improve written arithmetic performance. Specialist dyslexic teachers (e.g. Steeves, 1979) have advocated a multi-sensory approach to teaching arithmetic. A multi-sensory approach incorporates as many sensory experiences as possible (e.g. hearing, visual, tactile). Steeves (1979) argues that as dyslexic children have auditory processing problems and so

they are taught best through visually based methods. For example, using Dienes blocks when teaching multi-digit arithmetic or concrete apparatus when teaching fractions.

Conclusions

It is apparent that if a teaching intervention was based on children's cognitive strengths a child with dyslexia and a child with non-verbal learning difficulties would be taught using very different styles, even if their actual level of arithmetic attainment was very similar. However, if a teaching intervention attacked weaknesses and worked with strengths, both children would benefit from a multi-sensory method. Further research is needed to determine whether the time, effort and money spent in psychologically assessing children with arithmetic difficulties and tailoring the teaching to their cognitive profile is worthwhile or whether a systematic multi-sensory programme would be equally beneficial to all children with arithmetic difficulties.

10.6 Conclusions and future directions

The studies presented in this thesis have begun to examine the links between cognitive and number skills profiles in children and adults; however, there is much more work to be done. Studies One and Two indicated that children with arithmetic difficulties but better reading did not share a homogenous cognitive profile with spatial deficits, contrary to the predictions of Rourke and Del Dotto (1994). Some of the children with specific arithmetic difficulties did display the non-verbal learning difficulty profile. Further research is required to determine whether children with non-verbal learning difficulties would respond positively to cognitively tailored educational programmes that aimed to improve their arithmetic skills. Some children with specific arithmetic difficulties displayed unexpected cognitive profiles such as poor verbal reasoning and specific weaknesses in both verbal and visual-spatial memory. This finding led to a multiple route model of arithmetic difficulties being proposed. Larger studies are required to determine whether these cognitive profiles are reliably associated with arithmetic difficulties, and whether these abilities are related to later arithmetic achievement. Study

Four confirmed the previously reported finding that dyslexic children had particular difficulties recalling number facts, and also suggested that dyslexic children do not have particular difficulties in another area, place value understanding. The finding that dyslexic children who share a particular cognitive profile are weak at one number skill, but better at another, suggests that the case studies of children with number skills dissociations (used to support modular models of numerical functioning) may be explained by deficits in particular cognitive skills. Study Five reported a new finding, that adults with dyslexia are slower and less accurate than their peers at recalling number facts. No direct evidence was found to link dyslexic individuals' memory weaknesses to their number fact weaknesses. Further research is required to determine whether there is a relationship between any of the cognitive weaknesses reported in dyslexic individuals and number fact recall.

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Appendix 1: Speech files, rules and items for *Most* test

Speech files

- A) "You can see three bags of money on the screen. The amount of money in each bag is written on the outside. Click on the bag that contains the most money, if you change your mind you can click on a different bag. When you're sure click on okay."
- B) "Good, that's right."
- C) "That's not quite right, £9 is more than £1 or £3."
- D) "That's not quite right, £8 is more than £2 or £6."
- E) "Click on the bag that contains the most money, then click on okay."

Rules

- Practice items are not scored. The second practice item is only given if the child fails on the first.
- If practice items are correct say B), if not say appropriate correction (C, D).
- Always administer items 1-20, then test discontinued after 4 failures in 5 consecutive items i.e. if items 16-20 failed stop at 20
- The bag changes colour when you click on it.
- Say E) if no bag is chosen or okay is not clicked after 7s on any item.

Practice A	1	[9]	3
Practice B	8	2	[4]
Understanding the number with the highest first digit is most			
1	7	18	[22]
2	[34]	26	19
3	72	79	[81]
4	[78]	49	41
5	299	301	[309]
6	[80009]	70999	69999
Understanding the number with the largest number of digits is most			
7	[300]	30	3
8	4000	[400000]	40000
9	7100	71000	[710000]
10	7777	[777777]	77777
Understanding a larger number is more than a smaller number with a higher first digit			
11	900	[2000]	909
12	888	999	[1002]
13	9987	9999	[10000]
14	99874	[100345]	99899
15	3942	[34941]	9447
16	8394	[24117]	8943
17	[545333]	98542	98946
Understanding positional importance within digits			

18	1812	[1821]	1182
19	2088	2089	[2103]
20	1009	1102	[1901]
21	[8100]	8092	8079
22	10472	10099	[11091]
23	20078	20999	[21000]
24	33999	[34101]	34099
25	21982	22892	[23899]
26	[43005]	41999	42098
27	8156784	8059897	[8163245]
28	101101	[110001]	101111
29	[222112]	221112	[221121]
30	[441202]	429444	440222
31	78019867	[78029940]	78028999

Appendix 2: Speech files, rules and items for *Number Facts* test

Speech files

- A) "An addition sum will appear after each ready screen. Type in the answer as fast as you can because the sum will disappear quickly. Click on the rubber if you want to change your answer. Press enter after each answer."
- B) "A subtraction sum will appear after each ready screen. Type in the answer as fast as you can because the sum will disappear quickly. Click on the rubber if you want to change your answer. Press enter after each answer."
- C) "A multiplication sum will appear after each ready screen. Type in the answer as fast as you can because the sum will disappear quickly. Click on the rubber if you want to change your answer. Press enter after each answer."
- D) "Click on okay to start."
- E) "See if you can answer more quickly next time."
- F) "That's not quite right, 2 add 1 equals 3."
- G) "That's not quite right, 2 add 2 equals 4"
- H) "That's not quite right, 3 minus 1 equals 2."
- I) "That's not quite right, 4 minus 2 equals 2."
- J) "That's not quite right, 2 times 2 equals 4."
- K) "That's not quite right, 3 times 1 equals 3."
- L) "Good, that right."

Rules

- Each sum appears for 7s then disappears
- The time and answer are recorded
- Practice items are not scored. Both practice items are given, for both operations regardless of whether they are answered correctly.
- During practice trials if the child does not answer in time E) is said. It is also said the first time that the child does not answer within the time limit on the test items (for both operations).
- If the child answers correctly within the time limit (on the practice trials only) I) is said. If they get it wrong the appropriate correction (F to K) is said.
- If the child answers four consecutive questions incorrectly or they do not answer in 7s, testing in that particular block is discontinued and they move onto the next block.

Addition	Subtraction	Multiplication
P 4+1=[5]	P 3-1=[2]	P 2X2=[4]
P 2+2=[4]	P 4-2=[2]	P 3X1=[3]
3+2=[5]	6-2=[4]	2X3=[6]
2+4=[6]	5-4=[1]	3X4=[12]
3+6=[9]	8-3=[5]	7X3=[21]
2+7=[9]	9-5=[4]	4X5=[20]
5+3=[8]	7-4=[3]	2X9=[18]
4+3=[7]	8-2=[6]	5X6=[30]
5+8=[13]	12-3=[9]	7X4=[28]
6+5=[11]	19-5=[14]	8X6=[48]
9+7=[16]	18-6=[12]	9X3=[27]
7+8=[15]	13-7=[6]	6X7=[42]
4+8=[12]	14-6=[8]	7X8=[56]
5+9=[14]	15-8=[7]	9X7=[63]

Appendix 3: Teacher's questionnaire

Maths Research Questionnaire

To help use with our research we would be grateful if you would fill in the following questionnaire concerning

We value teacher's views on children's progress, as they see the child everyday in their normal classroom environment. If you cannot or do not wish to complete any sections of the questionnaire please return it partially completed (any extra information is useful). A prepaid envelope is enclosed. The identity of the child or your school will never be disclosed to anyone outside of the project research team.

Attainment and ability

Please record the results of any recent standardised tests of ability (e.g. Nfer nelson verbal/ non-verbal reasoning, Cognitive Ability Tests), Reading (e.g. Suffolk reading test) or mathematics (e.g. Young's group maths test).

Name of test	Date taken	Result (please give standard scores/quotients)

Teachers judgement of ability in relation to age group

	Exceptionally good	Good	Average	Weak	Extremely poor
Reading (aloud)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Reading (comprehension)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Arithmetic (written)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Arithmetic (mental)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Mathematical understanding	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Can you briefly describe the problems this child has with arithmetic/mathematics (e.g. difficulties learning tables, difficulties with story problems, poor calculation procedures)? Continue overleaf in the extra information section if necessary.

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In comparison to his/her same age peers is this child arithmetic/mathematics attainment:

Improving

Staying the same

Deteriorating

Extra help

Does this child have any extra help/support with arithmetic/mathematics? Yes/No

What extra support with arithmetic/mathematics do they receive (e.g. extra individual/group teaching, support within the classroom)? Please include the frequency and length of any extra help. Continue overleaf in the extra information section if necessary.

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Attitude and behaviour (tick all appropriate boxes)

Behaviour in class

- Normal
- Co-operative
- Friendly
- Responsive
- Disorganised

- Withdrawn
- Aggressive
- Over active
- Over sensitive
- Lacks concentration

- Attention seeking
- Anxious
- Passive
- Timid
- Disruptive

Attitude to work

- Enthusiastic
- Works well
- Seeks approval

- Easily distracted
- Distracts others
- Disinterested

- Slow
- Competent

Attitude to peers

- Normal
- Friendly
- Popular

- Dominant
- Submissive
- Withdrawn

- Prefers younger pupils
- Prefers older pupils

