Accepted Manuscript

Urbanization, inequality and property prices: Equilibrium pricing and transaction in the Chinese housing market

Teng Ge, Tao Wu

PII: S1043-951X(16)30043-8
DOI: doi: 10.1016/j.chieco.2016.04.005
Reference: CHIECO 934

To appear in: China Economic Review

Received date: 15 January 2016
Revised date: 23 April 2016
Accepted date: 23 April 2016

Please cite this article as: Ge, T. & Wu, T., Urbanization, inequality and property prices: Equilibrium pricing and transaction in the Chinese housing market, China Economic Review (2016), doi: 10.1016/j.chieco.2016.04.005

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2016, Elsevier. Licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International http://creativecommons.org/licenses/by-nc-nd/4.0/
Urbanisation, Inequality and Property Prices: 
Equilibrium Pricing and Transaction in the Chinese 
Housing Market 

Teng Ge¹
Hull University Business School

Tao Wu²
Jiangxi University of Finance and Economics

Abstract
The particularly overheated Chinese housing market, with its soaring property prices, has attracted a large amount of research. We point out three of its striking empirical features, which current literature leaves unexplored: co-existence of steady growth of real transaction price and excess supply, accelerations in price-to-income ratio, and significantly strong positive correlation between real transaction prices and income inequality. A search-equilibrium model is built to explain these facts. Heterogeneous buyers and homogeneous sellers randomly search for partners to trade in a frictional property market. The search equilibrium of the property market is either a high-price-and-low-transaction elitist matching equilibrium, or a low-price-and-high-transaction pooled matching equilibrium. The terms of trade determine which equilibrium arises. Empirical observations argue for the development of China’s property market through evolution from a pooled matching equilibrium to an elitist matching equilibrium. We set out to show that the market equilibrium is always inefficient, due to crowding out externalities and market incompleteness. Policy experiments support redistributive tax, as a means to improve social welfare.

JEL: C78, D31, R13, R31

Key Words: Housing market; Search and match; Inequality; Optimal pricing.

¹ Corresponding author. Address: Hull University Business School, University of Hull, Cottingham Road, Hull, United Kingdom, HU6 7RZ. Email address: t.ge@hull.ac.uk.
² Address: School of Economics, Jiangxi University of Finance and Economics, 169 Shuanggang East Road, Nanchang, China, 330013. Email address: roaroa.wu@gmail.com.
1 Introduction

In the last two decades there has been a large amount of research on the fast-rising property prices in China. The property market in China is now arguably regarded as being overheated (Dreger and Zhang 2013, Wu et al. 2012); it is widely accepted that speculative investments and accelerating residential demands are the main contributors to property market bubbles (Linchetberg and Ding 2009, Zheng and Kahn 2008, Hanink et al. 2012). In this paper, we claim that the soaring property prices are an equilibrium phenomenon which is a consequence of China’s economic development. Specifically, we argue that increases in property prices are the consequences of the evolution of the market from a pooled matching equilibrium to an elitist matching equilibrium. Whereas in the first equilibrium, transaction prices are lower and volume of trade is higher, in the second equilibrium transaction prices are higher but volume of trade is lower. Our model’s predictions are consistent with the empirical evidence as documented in Sato (2006) and Zhang (2015). Furthermore, we claim that policies such as direct distributive tax could reduce the transaction price and potentially correct market failures caused by crowding out externalities and market incompleteness.

Our theoretical model is motivated by three stylized facts about the Chinese property market: first, a steadily increasing property price coexists with excess supplies of residential buildings; second, according to the house affordability index (price-to-income ratio), property transaction prices are growing at a higher rate than the average income; finally, both income and income inequality are strongly positively correlated with transaction price. Literature based on a conventional model of residential investment and asset pricing, including Bertaut (2002), Case et al. (2005), Muellbauer and Murphy (1997), Hongyu et al., (2002), Wang (2011), and Ren et al. (2012) cannot explain the first two facts; on the other hand, search-and-bargaining literature like Wheaton (1990), Carrillo (2006), and Albrecht et al. (2007, 2016), Genesove and Han (2012) are silent on the third factor above. Therefore, building an equilibrium model to investigate the Chinese housing market is a contribution to research on housing market in developing countries overall, and not just China.

We built a general equilibrium model within the framework of a random search. Ex-ante heterogeneous buyers are differentiated by their disposable wealth, while the homogeneous sellers endogenously choose certain types of houses to build, before contacting any buyers in the market. Sellers commit to the price they set for trade after the investment has been made. Then buyers and sellers randomly search for potential partners to trade. Hence, given the distribution of the buyers’ wealth, sellers choose the optimal amount of investment in housing, and determine the asking prices in order to maximize their expected payoffs through a direct mechanism.

Due to market friction, the seller’s optimal strategy trades off a higher profit margin, through charging a high price, against a higher probability of trade. We show that there exist two market equilibria: in the elitist matching equilibrium, sellers build high-quality houses (high investment) and sell these houses only to wealthy buyers at a high price, hence excluding less wealthy buyers from the trade; in the pooled matching equilibrium, sellers build medium-quality houses (low investment) and sell them to both types of

---

3 Zhang (2015) had documented the relationships between income inequality and access to housing with Chinese urban household survey (UHS) from 2002 to 2009. His findings provided some sound empirical evidences for our theory.
buyers at a price also affordable to the less wealthy ones; hence, in the market, every contact is consummated into a trade. The first equilibrium arises when either the proportion of the wealthy buyers is sufficiently high, or the wealth inequality is sufficiently large. The second equilibrium arises when national wealth is relatively equally distributed among buyers or there is a relatively low proportion of wealthy buyers. We argue that the current development of the Chinese property market could be characterized by the elitist matching equilibrium. Furthermore, the model predicts that the increase in the income of wealthy households, supported by incessant economic growth, will result in even higher property prices in the future.

Neither of the above market equilibria are efficient. The volume of trade is inefficiently low in the elitist matching equilibrium, due to the crowding out externality exerted by wealthy buyers on the less wealthy ones. On the other hand, the market is incomplete for borrowing and lending. Such market incompleteness causes under-investment in properties in the pooled matching equilibrium, but over-investment in the elitist matching equilibrium. Through comparative statistical studies, our findings sheds light on implications of the policy regulating the housing market in China. Since the distribution of wealth across the buyers plays a significant role in determining property prices, government policies aimed at curbing property prices need to take into consideration the reducing of the wealth inequality between the rich and the poor.

Previous research, including Wheaton (1990), Carrillo (2006), Genesove and Han (2012), Clayton et al. (2010), and Albrecht et al. (2007, 2016), has regarded the housing market as a typical market with trade frictions, where searching buyers and sellers coexist. These papers mainly concern stock-stock matching, i.e. trading with existing properties. Hence, there is a trade-off between time to trade and transaction price. However, inflows of newly completed constructions account for a large proportion of existing housing stock (according to Zheng and Kahn 2008, this ratio was as high as 13.1% in 2005) and the price determination in an equilibrium stock-flow matching model remains to be solved. In our work, the equilibrium transaction price is determined by the distribution of types of buyers, and the evolution of price is associated with the development of the reallocation of wealth among buyers. Furthermore, we allow the quality of the housing to be endogenously determined, thus differing from the above search models, which assume ex-ante homogenous, but ex-post heterogeneous matching specific house qualities. Consequently, our model implies the positive sorting in elitist matching equilibrium, which is analogous to search-and-matching models of labour market like Acemoglu (1999) and Albrecht et al. (2007, 2016).

The remainder of this paper is organized into six sections; Section Two briefly discusses the recent findings in the Chinese housing market, which motivate our study; Section Three presents the model and derives market equilibria; Section Four shows a social welfare study; Section Five describes the policy experiment study and the policy implications; Section Six concludes and discusses potential future research.

2 Stylized Facts about the Chinese Property Market

This section illustrates three striking features of the Chinese property market. We use annual market data ranging between 1991 and 2011, from the China Economic Information Network Statistical Database. This data includes information on the supply
side of the market, for example annually completed new residential buildings (measured in square metres), and the demand side of the market, such as the annual average transaction price, annual transaction quantity (measured in square metres), and household income. We also use the rates of inflation and interest during the same period, published by the National Bureau of Statistics of China. All prices and incomes have been deflated, by taking 1999 as the reference base year.

**Fact 1. Constant increases in transaction price coexist with excess supply.**

![Figure 1. Transaction price and excess supply.](image)

The upper panel in Figure 1 demonstrates the relationship between the net supply and the real transaction price (taking 1999 as the base year) from 1991 to 2011 in China, and the lower panel shows the relationship between the growth rates of inflows of supply and real transaction prices from the same period. From the upper panel, over the last two decades, a steady increase in real property transaction prices can be observed at a time when the Chinese property market was experiencing excess supplies. As is shown, the real transaction price has been increasing since 1991, except for two slight drops in 1994 and in 2008. Meanwhile, the net inflow of stock, or excess supply, measured in square metres of completed constructions minus square metres of transacted constructions in the same period, rose until 1999 and has been declining since then. However, the net supply

---

4 "Excess supply" is calculated by subtracting the square metres of transacted residential buildings from the square metres of the currently completed residential buildings. A positive value of the series implies that the current supply of new buildings is greater than demand; otherwise, the net inflow of stock is negative.
of newly constructed square metres was always positive, except from 2009 to 2011. In other words, over the 20 years of the sample period, in the Chinese property market, excess supplies coexisted with steady increases in real transaction prices. Essentially, it implies that the price mechanism failed to clear out supply and demand in the market. On the other hand, the lower panel of Figure 1 shows that the growth rate of square metres of completed residential buildings has increased with the growth rate of transaction prices. It turns out that a higher transaction price drives up the inflows of completed square metres, i.e. a positive supply relationship to the market. This reinforces the argument we claimed regarding the upper portion of Figure 1. Although one needs to read these graphs with caution, our data argues for the theory that explains the coexistence of excess supply and soaring property prices, over the last two decades.

Fact 2. The property transaction price grows at a higher rate than the average income.

![Figure 2. Growth of per capita income and price-to-income ratio.](image)

Though the steady growth in per capita income may drive property prices up, balanced growth implies that on the equilibrium path, per capita income, per capita residential square metres, and transaction prices should grow at the same rate, i.e. the

---

5 The effect of stock-flow, i.e. trade of stock vs. trade of inflows, may also contribute to the property price adjustment during the same period, if one had detailed empirical data. However, it is reasonable to argue that the stock-flow effects are relatively weak, because we have observed a mostly positive net inflow of supply over the sample period.

6 Per capita income is measured as the urban per capita annual disposable income.
price-to-income ratio is a constant. However, we found that for the last two decades the transaction price has been growing much faster than the average income. As it is shown in Figure 2, a steady increase in the average real household income is matched by a soaring price-to-income ratio. Compared to a constant growth in real income, this increase in price-to-income ratio is rather discrete and sharp. Specifically, this ratio sharply increased from 6.6 in 2003 to 7.9 in 2005, and saw a jump from 7.2 in 2008 to 8.5 in 2009. Such trends are stronger in more economically developed areas, like Beijing, Shanghai and Guangdong. The ratios of these locations were generally higher than the national average. For instance, it took nearly 20 years for a household with an average income to purchase an average sized apartment in Beijing in the year 2010. These findings argue for the consideration of other factors, besides income growth, to explain increases in transaction prices.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>9.1</td>
<td>8.1</td>
<td>10.0</td>
<td>11.2</td>
<td>15.0</td>
<td>16.2</td>
<td>19.3</td>
<td>16.7</td>
</tr>
<tr>
<td>Shanghai</td>
<td>6.3</td>
<td>8.8</td>
<td>9.6</td>
<td>9.4</td>
<td>9.1</td>
<td>13.9</td>
<td>14.4</td>
<td>13.1</td>
</tr>
<tr>
<td>Guangdong</td>
<td>6.6</td>
<td>6.4</td>
<td>7.9</td>
<td>8.2</td>
<td>9.0</td>
<td>9.4</td>
<td>9.9</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Source: Authors’ calculation. Units: year.

Table 1. Regional Price-to-Income Ratios

Fact 3. Both income and income inequality are strongly positively correlated with transaction price.

Further investigations into the relationship between property prices and household income are captured in the correlation matrix of Table 2. We use the 90:10 ratio to measure income inequality. Table 2 shows that the real transaction prices have a significant and strong correlation with average income and income inequality. As predicted, a higher income drives up the demand, hence a higher transaction price. However, there is a surprisingly positive correlation between income inequality and property prices (0.90) and it is almost as strong as the correlation between income and price (0.97). As we argued in the introduction, current theories give satisfactory explanations regarding the correlation between income and price, and between income and inequality, but leave the correlation between price and inequality unexplored. This paper devotes a theory to explain this fact through sorting and matching. Note that the negative correlation between price and net inflows does not contradict the claim of fact 1; but rather, it portrays the demand function of the property market.

These stylized facts imply that to investigate the Chinese property market, the

---

7 "Price-to-income ratio" is calculated with the following formula:

\[
\text{per capita residential m}^2 \times \frac{\text{transaction price per m}^2}{\text{per capita income}}
\]

8 Due to the lack of some provincial data, we can only calculate the regional price-to-income ratios from 1999 to 2011.

9 One can also use the Gini coefficient as an indicator of inequality. We are unable to find the Gini coefficient after 2004. However, based on the limited data set, we have found that the Gini coefficient shows a similar trend as when we use the 90:10 ratio.
competitive models fail to capture essential facts to match the above empirical observations. Due to market frictions, supply-and-demand analyses based on a competitive market are unable to explain these stylized facts, and hence are misleading. In the following section, we build a search-and-matching model to characterize recent developments in our target property market.

<table>
<thead>
<tr>
<th></th>
<th>ineq</th>
<th>price</th>
<th>grprice</th>
<th>grsupply</th>
<th>rint</th>
<th>avincome</th>
<th>infnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>ineq</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>price</td>
<td>0.90*</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>grprice</td>
<td>-0.24</td>
<td>-0.22</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>grsupply</td>
<td>-0.46</td>
<td>-0.48</td>
<td>0.12</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rint</td>
<td>0.16</td>
<td>0.43</td>
<td>0.01</td>
<td>-0.49*</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>avincome</td>
<td>0.93*</td>
<td>0.97*</td>
<td>-0.32</td>
<td>-0.51*</td>
<td>0.32</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>infnet</td>
<td>-0.54</td>
<td>-0.46*</td>
<td>-0.17</td>
<td>0.01</td>
<td>0.32</td>
<td>-0.52*</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: ineq stands for inequality, price for transaction price, grprice for growth rate of price, grsupply for growth rate of supply of new housing, rint for real interest rate, avincome for average income, and infnet for the net inflow of housing. Values with * are significant at 95% level.

Table 2. Correlation Matrix

3 The Model
3.1 Model setting

In this section we consider a simple one-shot game in a property market. There are two types of agents: ex-ante homogeneous sellers and ex-ante heterogeneous buyers. The measure of buyers is fixed at a unit mass and that of sellers is s < 1. Proportion \( \lambda \) of the buyers are the \( H \)-type buyers, i.e. with an endowment of wealth \( y^H \), and proportion \( (1 - \lambda) \) are the \( L \)-type buyers, i.e. with an endowment \( y^L \).\(^{10}\) We assume that \( y^H > y^L \). Although the distribution of buyers’ wealth is common knowledge, the type of a particular buyer is private information, hence non-observable.

The market is frictional. The buyers and sellers contact each other through random searches. The aggregate number of contacts is determined by the number of sellers and buyers, specifically through a constant-return-to-scale matching function, \( \alpha(1,s) \). Following the standard argument, on average a buyer will contact a seller following a Poisson process at a rate \( \alpha(s) \), where \( s \) is the seller-to-buyer ratio, i.e. the market tightness. Similarly, the contacting rate for a seller, denoted by \( \mu(s) \), is just \( \mu(s) = \alpha(s)/s \). We also assume that \( \alpha \) and \( \mu \) map the positive real numbers \( (0,\infty) \) onto themselves; note that \( \alpha' > 0 \) and \( \mu' < 0 \), so \( \alpha(0) = \mu(\infty) = 0 \) and \( \alpha(\infty) = \mu(0) = \infty \). In other words, if there are very few buyers per seller, buyers could find a seller

\(^{10}\) Although, we assumed two-point distribution for simplicity, our theory prevails with any general case in which distribution of buyers’ types are discrete. If the distribution of the types of buyers is continuous, there will be continuum equilibria of the market, and the submarket for each type of buyers is infinitely divisible. Empirically, it is less appealing and not relevant to our theory.
instantly but it takes an extremely long time for a seller to contact a buyer; the opposite is true if there are many buyers for each seller.

Despite the difference of wealth endowment, buyers obtain payoffs from strictly concave preferences on housing, i.e. \( u(k) \), such that \( u'(k) > 0 > u''(k) \), and \( u(0) = 0 \), with the standard INADA conditions \( u'(0) \to \infty \) and \( u'(\infty) \to 0 \), where \( k \) denotes the quality of a house endogenously determined by the sellers. Furthermore, we assume that buyers also obtain linear utility directly from their wealth as there are no diminishing returns to marginal utility of wealth. This implies that the buyers’ preference is quasiconcave, defined through the quality of the house as well as his or her wealth.\(^{11}\)

Risk neutral sellers enter the market, and each develops a unit of house by making an irreversible investment \( k \) in the quality of the house before contacting a potential buyer.\(^{12}\) Since the investment \( k \) is made, the set up of the property is completed immediately as one unit of indivisble goods for trade,\(^{13}\) and profit maximizing sellers determine an asking price for the house they possess. The price \( p \), is a take-it or leave-it offer, under which the seller is committed to trade with the buyers. Due to the asymmetric information, sellers cannot price discriminate buyers based on \( y^H \) and \( y^L \). Hence, the price is type independent, in the sense that both the \( H \)-type buyers and the \( L \)-type buyers are charged the same asking price.\(^{14}\) After contacting a seller, the quality of the house \( k \) is observed, and the buyer decides whether to trade, following individual rationality. In this one-shot game, contact is consummated into a transaction of a house from seller to buyer (match of a buyer and a seller) upon a mutual agreement, so the game ends. Otherwise, either party could refuse to trade and the game ends. Unmatched buyers obtain utility from endowment of wealth, whereas unmatched sellers with leftover properties withdraw from the market and obtain zero payoff.\(^{15}\)

### 3.2 Search Equilibrium of the Property Market

In this part, first we define the search equilibrium of the market; then we show the existence of market equilibria followed by characterizations. Finally, we show which specific equilibrium arises depending on the terms of trade.

After contacting a seller, the buyer observes the type of house \( k \) and the asking price \( p \). Given a contact, let \( \beta^i \in \{0,1\} \) denote the probability that a buyer agrees to trade. Accordingly, the buyer could either choose to accept the house offer \( \{k, p\} \), i.e. \( \beta^i = 1 \), or to reject it, i.e. \( \beta^i = 0 \). For simplicity, mixed strategy equilibria are not considered here (though they often exist). Let \( V^H \) denote the value function of an \( H \)-type buyer, hence the buyer chooses \( \beta^H \) to maximize the expected payoff \( V^H \),

---

\(^{11}\) A specific functional form of the utility function of the buyers will be given below.

\(^{12}\) Sellers and developers are interchangeable in this model, though, in general, it is not necessarily the case that sellers and developers share the same interests.

\(^{13}\) We admit that the assumption that the setup of a property is completed immediately may change the implications in a multi-shot dynamic model, in which case stock-flow effects play an important role in deciding investment and price.

\(^{14}\) Another method of modelling the slipping rule in search literature is bargaining. However, as it is difficult to observe wealth of buyers due to asymmetric information, the issue of adverse selection may arise. In fact, the price a house is traded at is very close to the asking price that the agent sets.

\(^{15}\) Withdrawal of unmatched properties is assumed to render market equilibrium consistent with the static framework. In a multi-shot dynamic setup, free entries ensure that the market is cleared out in equilibrium.
At a rate $\alpha$, an $H$-type buyer meets a seller with a house of type $k$ and an asking price $p$. As the buyer also obtains linear utility from their endowed wealth, the buyer enjoys $y^H$ if she fails to contact a seller, occurring at a rate of $1 - \alpha$, or if the buyer contacts a seller, but refuses to trade, with a probability of $\alpha(1 - \beta^H)$. If the buyer agrees to trade, i.e. $\beta^H = 1$, then they obtain payoff $u(k)$ by paying the price $p$ that is no higher than the level of their wealth endowment. The probability that the $H$-type buyer is met with a seller and chooses to trade is $\beta^H$; otherwise the buyer just enjoys the linear utility from their wealth as equation (1) shows. Individual rationality implies that $\beta^H = 1$ iff $u(k) \geq p$. Therefore, $\beta^H$ is consistent with the willingness to pay of the $H$-type buyer to pay for a house of a certain quality. Note that in equation (1), $y^H$ is the reservation value which marks the lower boundary of the buyer’s utility. Similarly, the value of an $L$-type buyer is:

$$V^L = \max_{\beta^L} \alpha \beta^L [u(k) - p] + y^L,$$

s.t. $p \leq y^L$.  

On the other hand, given the value of $\beta^L$, sellers make decisions on ex-ante investment $k$ and price $p$, to maximize the expected payoff $V^S$, which is defined below:

$$V^S = \max_{k,p} -k + \mu(1 - \lambda)\beta^H p(k, \beta^L) + (1 - \lambda)\beta^L p(k, \beta^L),$$

s.t. $p \leq u(k), p \leq y^H$ if $\beta^H = 1$,

$p \leq u(k), p \leq y^L$ if $\beta^H = 1$.

The ex-ante investment decision of the seller is made before contacting any buyer. Then the seller sets the price and contacts a buyer at a rate $\mu$. The buyer encountered is an $H$-type with a probability $\lambda$ and $\beta^H$ is her choice of trading, so trade occurs and the seller obtains a revenue $p$ with a probability $\mu \beta^H$. Similarly, with a probability $\mu(1 - \lambda)\beta^L$, the seller trades with an $L$-type buyer and obtains revenue $p$. It is intuitive to see that the transaction price $p$ cannot be higher than the endowed wealth level of the buyer $y^l$. At the same time, to conduct a successful trade, the seller has to invest in $k$, that is consistent with the buyer’s optimal choice $\beta^H = 1$. The utility that buyers obtain from purchasing a house $u(k)$ is not lower than the direct linear utility from transferred wealth $p$, which is paid to the seller. Hence, the price announced by the seller influences the transaction probability $\beta^H$ as well as her revenue per transaction.\(^{16}\)

In a market equilibrium, sellers maximize their expected profit and buyers maximize their expected utility. The search equilibrium of the housing market can be characterized recursively with those value functions:

**Definition 1** A search equilibrium of the housing market is a set $\{k, p, \beta^H, \beta^L\}$ such that $k$, $p$ solve the seller’s problem in (3) and $\beta^H, \beta^L$ solve the buyer’s problem in (1) and (2) respectively.

Definition 1 implies that given the space of trade, in any equilibrium, the seller’s

\(^{16}\) For a study on housing market with directed search equilibrium, please see Albrecht et al. (2009).
investment $k$ and listing price $p$ must be consistent with the buyers’ choice of $\beta^i$; on the other hand, buyer’s choice of $\beta^i$ must be the best response to seller’s offer $\{k, p\}$, hence they are consistent with the equilibrium. Based on the choice of $\beta^i$, there are $2^2$ potential pure-strategy Nash equilibria. However, it is not necessary to analyze all four candidate outcomes; the following proposition shows that only two pure-strategy Nash equilibria exist in this market.\footnote{We are only interested in the pure-strategy equilibrium in this model, since the mixed-strategy equilibrium is less empirically appealing and adds little in the way of new insights.}

**Proposition 1** The equilibrium in the housing market is either a pooled matching equilibrium $\{k_P, p_P, 1,1\}$, or an elitist matching equilibrium $\{k_E, p_E, 1,0\}$.

**Proof.** Depending on the choice of $\beta^i$, the four possible outcomes of equilibria are $\beta^H = \beta^L = 0, \beta^H = \beta^L = 1, \beta^H = 1, \beta^L = 0,$ and $\beta^H = 0, \beta^L = 1$. Obviously, $\{0,0,0,0\}$ is a Nash equilibrium, as sellers invest nothing and buyers are not going to make any transactions with sellers, and the market shuts down. Not trading is also consistent with a Nash equilibrium, i.e. a degenerate search equilibrium. For the rest of the three candidate cases, in which the market is active, we solve for $k$ and $p$ for a given set of $\beta^i$. To justify an equilibrium, we then show that given the $k$ and $p$ solution, $\beta^i$ is the optimal response.

We start with the case of $\beta^H = \beta^L = 1$. In a pooled matching equilibrium, both types of buyers are willing to trade with sellers. Following equation (3) a price-setting seller’s problem can be written as:

\[
V^S = \max_{k,p} -k + \mu p, \quad (4)
\]

s.t. $u(k) \geq p$, (UC)

\[
p \leq y^H, \quad (BCH)
\]

\[
p \leq y^L, \quad (BCL)
\]

For any given level of investment $k$, if the seller sets her price such that $p < u(k)$, by increasing the price she could increase her profits from trade. On the other hand, due to market friction, further increases in $p$ do not necessarily lead to a decrease in the volume of trade. Random search implies the existence of local monopoly powers among sellers, which alleviate price competition between them. Hence, the pricing strategy $p < u(k)$ is dominated by the strategy $p = u(k)$. The pricing strategy $p > u(k)$ will be such that no buyer wants to trade with the seller, which contradicts $\beta^H = \beta^L = 1$. Hence, the optimal price for the seller is $p = u(k)$, such that the utility constraint (UC) is binding. Recalling the assumption $y^L < y^H$, then the budget constraint (BCH) is automatically satisfied if (BCL) holds. The seller’s value function (4) can be rewritten as:

\[
V^S = \max_{k} -k + \mu u(k), \quad (5)
\]

s.t. $u(k) \leq y^L$. (BCL)

Therefore, for any given form of concave function $u$, we can solve for an optimal investment $k_p$, so that the optimal price is $p_p = u(k_p) \leq y^L < y^H$.\footnote{The optimal solution could either be a corner solution or an interior solution; we will provide more details on this in the following Lemma 1.} Hence, the pooled matching equilibrium $\{k_p, p_p, 1,1\}$ exists.

To prove that the elitist matching equilibrium $\beta^H = 1, \beta^L = 0$ exists, similar
arguments apply. In an elitist matching equilibrium, only $H$-type buyers are willing to trade, hence the seller’s problem is:

$$V^S = \max_{p,k} -k + \mu \lambda p,$$

s.t. $u(k) \geq p$, \hspace{1em} (UC)

$$p \leq y^H,$$ \hspace{1em} (BCH)

$$p > y^L,$$ \hspace{1em} (BCL)

For similar principles as in the pooled matching equilibrium, the seller has the local monopoly power. The seller will collect all the consumer surplus, so utility constraint (UC) must bind at the optimum, i.e. $p = u(k)$. Combining the budget constraints (BCL) and (BCH), the seller’s problem (6) can be rewritten as:

$$V^S = \max_k -k + \mu \lambda u(k),$$

s.t. $u(k) \leq y^H$, \hspace{1em} (BCH)

$$u(k) > y^L.$$ \hspace{1em} (BCL)

Therefore, for any given form of concave function $u$, we can solve for an optimal investment $k_{E^*}$, so that the optimal price is $p_E = u(k_{E^*}) \in (y^L, y^H]$. Hence, the pooled matching equilibrium $\{k_{E^*}, p_E, 1, 0\}$ exists.

Finally, $\beta^H = 0, \beta^L = 1$ cannot be an equilibrium. As $\beta^L = 1$, it implies that $y^L \geq u(k) \geq p$, whilst the utility constraint and the budget constraint for the $L$-type buyers are both satisfied. However, given the same preference, $\beta^H = 0$ if, and only if, the price is beyond the budget constraint of the $H$-type buyers, so that $p > y^H$. Hence $y^L > y^H$, which is a contradiction. The equilibrium in which only $L$–type buyers are served does not exist. ■

Given the existence of the search equilibria, we can further study the properties of price and volume of transactions in both equilibria. There are several interesting aspects of these equilibrium results which are worth emphasizing.

**Lemma 1** Assuming that market frictions are sufficiently low, specifically when $\mu > 1/\lambda u'(k_{E^*})$, sellers set a house price $p_p = y^L$ and an investment $k_{p^*}$ such that $u(k_{p^*}) = y^L$ in the pooled matching equilibrium; and a higher price $p_E = y^H$ and a higher investment $k_{E^*}$, such that $u(k_{E^*}) = y^H$ in the elitist matching equilibrium. However, the volume of trade in the elitist matching equilibrium, $\lambda \alpha$, is lower than the one in the pooled matching equilibrium, $\alpha$.

**Proof.** (First Part) To analyze the pooled matching equilibrium, let us recall the seller’s value function (5); the Kuhn-Tucker Lagrangian problem can be written as:

$$L = -k + \mu u(k) + \gamma [y^L - u(k)].$$

The first order conditions are:

$$\frac{\partial L}{\partial k} = -1 + \mu u'(k) - \gamma u'(k) \leq 0, \hspace{1em} k \geq 0, \hspace{1em} [-1 + \mu u'(k) - \gamma u'(k)]k = 0, \hspace{1em} (8)$$

$$\frac{\partial L}{\partial \gamma} = y^L - u(k) \geq 0, \hspace{1em} \gamma \geq 0, \hspace{1em} [y^L - u(k)]\gamma = 0.$$

Any active seller will choose an investment $k > 0$, so that $\frac{\partial L}{\partial k} = -1 + \mu u'(k) - \gamma u'(k) = 0$. If the budget constraint (BCL) binds, i.e. $p = y^L = u(k)$, then the optimal investment in $k$ is tied down by a corner solution $k_{p^*} = u^{-1}(y^L)$, whereas $\gamma > 0$ which requires that
\[ u'(k_p) > \frac{1}{\mu} \] 

(9)

On the other hand, if (BCL) does not bind, the optimal solution is determined by the first order condition (8) by substituting \( \gamma = 0 \), hence the outcome is an interior solution \( k_p \), such that \( u'(k_p) = 1/\mu \).

The proof of the elitist matching equilibrium is similar to that of the pooled matching equilibrium, so we have reserved the second part for the Appendix. ■

The intuition behind this result can be explained as follows: to build a better house by increasing \( k \), sellers increase buyers’ utility from house consumption. Hence, sellers can request a higher asking price from those buyers who are willing to trade. Note that in the necessary condition (9), as (UC) constraint binds, \( u'(k_p) \) captures a seller’s marginal revenue from an increasing \( k \), whereas \( 1/\mu \) measures this seller’s marginal cost. In the case of large values of \( \mu \), the marginal cost of investment in \( k \) is lower than marginal revenue, so that it is profitable to invest more and charge a higher price up until (BCL) binds. However, if \( \mu \) is relatively low, the marginal revenue is lower than the marginal cost of investment. Therefore it is not optimal for a seller to charge \( y^L \) and invest up to \( k_p^* \), as in Figure 3, where the standard tangent condition holds for optimality. However, in either case, the choices of \( k \) and \( p \) are consistent with the pooled matching equilibrium, i.e. \( \beta^H = \beta^L = 1 \).

In fact, those interior solutions are not consistent with the matching equilibria of Proposition 1. As the buyers’ optimal choices are not constrained by their wealth, equilibrium investment and prices are always in the buyers’ budget set. The \( H \)-type and \( L \)-type buyers are perfectly substitutable, hence the elitist matching equilibrium does not exist.\(^{19}\)

To sum up, the house investment and asking price in a pooled matching equilibrium should be either an interior solution \( \{k_p, p_p\} \) such that \( u'(k_p) = \frac{1}{\mu}, p_p = u(k_p) \) if \( u(k_p) \leq y^L \), or a corner solution \( \{k_p^*, p_p\} \) such that \( k_p^* = u^{-1}(y^L), p_p = y^L = u(k_p^*) \) if \( u(k_p) > u(k_p^*) = y^L \), as described in Figure 3.

As shown in Figure 3, in a market with high matching friction, i.e. \( \mu > 1/\lambda u'(k_p^*) \) fails, investment and price are not consistent with the elitist matching equilibrium. Specifically, as these tangent conditions imply \( k_p^* < k_p \), in the elitist matching equilibrium both investment and transaction prices are lower than those of the pooled matching equilibrium. Given the same preference, \( L \)-type buyers are willing to trade in a market with a higher transaction price but not in a market with a lower price, as the \( H \)-type buyers did. This contradicts individual rationality.\(^{20}\)
Figure 3. Corner solutions of investment in the pooled and elitist matching equilibria: $k_p^*$ and $k_E^*$, and interior solutions in the pooled and elitist matching equilibria: $k_p$ and $k_E$.

Consider the case where market frictions are sufficiently low and the results of two equilibria are both corner solutions. Following Lemma 1, the budget constraint (BCL) binds in the pooled matching equilibrium, i.e. $p_p = y^L$; and the budget constraint (BCH) binds in the elitist matching equilibrium, i.e. $p_E = y^H$. As the seller’s marginal revenue from building an affordable house is always higher than the marginal cost, $u'(k_p^*) > 1/\mu$ and $u(k_E^*) > 1/\lambda \mu$, the investment choices are tied down by $k_p^* = u^{-1}(y^L)$ and $k_E^* = u^{-1}(y^H)$, as summarized in Lemma 1. By facing the housing offer $\{k_i', p_i\}$, buyers are willing to spend all their wealth to obtain such a property.\(^{21}\) We are only interested in the case where market frictions are sufficiently low (i.e. $\mu > 1/\lambda u'(k_E^*) > 1/u'(k_p^*)$), hence at least one of two buyer’s budget constraints binds and the wealth inequality matters.\(^{22}\)

Given that two search equilibria exist, sellers will pick the one which generates the higher expected profit. In the pooled matching equilibrium, the buyers’ choices are $\{\beta^H = 1, \beta^L = 1\}$, and the value functions of two types of buyers are respectively:

\[
V_p^L = \alpha[u(k_p^*) - p_p] + y^L, \\
V_p^H = \alpha[u(k_p^*) - p_p] + y^H, 
\]

where the equilibrium price chosen by sellers is $p_p = u(k_p^*) = y^L$. Following Lemma 1, the value function of a seller in the pooled matching equilibrium is:

\[
V_p^S = -k_p^* + \mu y^L. 
\]

In the elitist matching equilibrium, the buyers’ choices are $\{\beta^H = 1, \beta^L = 0\}$, and the value function of the $L$-type buyers is just $y^L$ since they choose not to buy a

---

\(^{21}\) In fact, this result is consistent with the observations in the Chinese housing market. As an example, Pugh (2009) indicates that many parents in China spend all their wealth, and even borrow from relatives and friends, in order to purchase a house for the marriage of their children.

\(^{22}\) We need to point out that the equilibrium prices set at $y^H$ and $y^L$ respectively are not the result of the one-period game setting in the model, but a result of the searching between buyers and sellers. This finding is consistent with the Diamond’s (1971) dilemma, which states that when sellers have full bargaining power, market friction does not distort the incentives for them to charge the monopoly price.
house. The value function of the $H$-type buyers is:
\[ V^H_E = au(k^*_E) - p_E + y^H, \]
where the optimal pricing choice of the $H$-type sellers is $p_E = u(k^*_E) = y^H$. The value function of sellers when they choose to exclusively serve $H$-type buyers is:
\[ V^S_E = -k^*_E + \mu \lambda y^H. \]  
(11)

From equations (10) and (11), the sellers’ profits increase with the buyers’ wealth. This implies that there is an indifference condition for sellers between investing $k^*_E$ in the elitist matching equilibrium and investing $k^*_P$ in the pooled matching equilibrium. Sellers will choose the elitist matching equilibrium if, and only if $V^S_E \geq V^S_P$. Specifically, by equating (10) and (11):
\[ k^*_E - k^*_P = \mu (\lambda y^H - y^L). \]

Let $\underline{y}^H$ denote the threshold level of wealth of an $H$-type buyer, which renders sellers indifferent between two equilibria, that is:
\[ \underline{y}^H = \frac{k^*_E - k^*_P}{\lambda \mu} + \frac{y^L}{\lambda}. \]  
(12)

Sellers choose the elitist matching equilibrium if, and only if, the $H$-type buyers’ wealth level is higher than the threshold level of wealth $\underline{y}^H$. Otherwise, choosing the pooled matching equilibrium is strictly better for the sellers. By equation (12), we can replace the condition $V^S_E \geq V^S_P$ by $y^H \geq \underline{y}^H$. The following proposition states the existence and uniqueness of $\underline{y}^H$.

**Proposition 2** For any given $\mu$, $\lambda$ and $y^L$, there exists a unique threshold level of wealth $\underline{y}^H$, at which sellers are indifferent between the elitist matching equilibrium and the pooled matching equilibrium. The sellers choose the elitist matching equilibrium in the decentralized market if, and only if $y^H \geq \underline{y}^H$. Furthermore, this threshold level of wealth $\underline{y}^H$ is decreasing in $\lambda$ and increasing in $y^L$.

**Proof.** See Appendix. $\blacksquare$

The intuition of the proposition can be explained in the following way: by choosing the elitist matching equilibrium rather than the pooled matching equilibrium, sellers trade off the rate of matching for a higher profit margin. By choosing $k^*_E$, the matching rate for a seller is lower as the transaction price $p_E$ is so high that the less wealthy buyers will be excluded from the trade. Hence, sellers have fewer chances to sell the property. $\underline{y}^H$ is the threshold level of wealth of the $H$-type buyers which makes the sellers’ expected profits indifferent between two equilibria. If the $H$-type buyers’ wealth is above $\underline{y}^H$, then the sellers are better off in the elitist matching equilibrium than in the pooled matching equilibrium. For a given $\lambda$, if $y^H \geq \underline{y}^H$, the expected profit is higher in the elitist matching equilibrium, as increases in the profit margin are sufficiently high to cover reductions in the matching rate. In addition, as this threshold $\underline{y}^H$ is decreasing in $\lambda$, the larger the proportion of the $H$-type buyers, the higher the chance a seller could meet an $H$-type buyer to trade, hence, the higher expected profit for the sellers to achieve in the elitist matching equilibrium. The threshold $\underline{y}^H$ is increasing in $y^L$ because, as the wealth of the $L$-type buyers increases, sellers are more likely to choose to serve both
types of buyer, so that they can collect more revenue in a pooled matching equilibrium, where every contact is consummated into a trade. It requires a high $y^H$ to maintain the profit margin such that the elitist matching equilibrium is more profitable than the pooled matching equilibrium.

If we apply the specific form of utility function $u(k) = 2k^{1/2}$, then $y^H$ can be given explicitly as $y^H = 2\mu\lambda - \sqrt{4\lambda^2\mu^2 + (y^L)^2} - 4\mu y^L$. For any given $\mu$, $\lambda$ and $y^L$, an elitist matching equilibrium arises if and only if $y^H \geq y^H$, otherwise a pooled matching equilibrium does. This is illustrated clearly in Figure 4. The solid line is the threshold level of $y^H$, above which is the combination of parameters causing the elitist matching equilibrium to render the sellers better off. However, the area below shows where the sellers are better off in the pooled matching equilibrium. With a higher $\lambda$, $ceteris paribus$, the threshold $y^H$ is driven down. This is reflected by the dashed line in the graph. Furthermore, if the distribution of individual wealth becomes more imbalanced, either because of a lower $y^L$, causing in turn a lower $y^H$, or because of a higher $y^H$, the condition is more likely to be satisfied, hence an elitist matching equilibrium arises.

![Figure 4](image)

Figure 4. Threshold level of type-H buyers’ wealth $y^H$ as a function of the type-L buyers’ wealth, $y^L$, and the proportion of type-H buyers, $\lambda$.

The immediate implication of the model’s result is that, for a given growth rate of aggregate income, when the wealth gap between the rich and the poor broadens, it is more likely that the housing market ends up in the elitist matching equilibrium. The soaring prices in the Chinese property market are a consequence of a transition from pooled matching equilibrium to elitist matching equilibrium.

Rapid growth in China transmits wealth to individuals, thus enlarging the wealthy

---

23 In this particular numerical example, we chose $\mu(s) = 7$, which represents the average duration of the sale of a house as $12/7 = 1.7$ months. The proportion of $H$-type buyers is $= 0.3$, $0.4$. Hence, $L$-type buyers are always in the majority. Then we created the state space based on $y^L$ with a value between 0 and 0.65, in a range which guarantees $y^H$ has a real root, at a step of 0.01.
class. Our model insists upon that inequality as a fundamental factor in explaining sharp increases in property price. Theories argue that a steady growth in income, hence a higher demand, are the driving forces behind the prediction of a relatively stable price-to-income ratio for property prices. However, this prediction is inconsistent with stylized fact 2 we have shown previously. As one can see from Figure 2, spikes during 1995-1997, 2003-2005, and 2008-2010 imply a faster growth rate of property prices. Considering big cities like Beijing, Shanghai, and Guangzhou, property prices have been mounting much faster than income. Furthermore, periods characterized by increases in price-to-income ratio are also accompanied by increases in Villa-to-affordable-house ratio. Specifically, this ratio increased from 0.36 to 0.88 during 2002-2005, and from 0.79 to 1.53 from 2008-2010.\textsuperscript{24} This data shows that the Chinese property market has evolved from a pooled matching equilibrium to an elitist matching equilibrium, thus there are more luxury houses/villas than economic/affordable houses. Consequently, soaring property prices are observed.

4 Welfare Consequences

The preceding section characterizes the search equilibria of property markets with a welfare study of the decentralized market.

A benevolent social planner maximizes the social welfare by increasing the sum of the values of agents, based on an utilitarian welfare function. Specifically, the social planner decides to invest $k$ to maximize social welfare, subject to the matching constraints. The planner’s problem can be written as follows:

$$SW = \max_k \lambda V^H + (1 - \lambda)V^L + sV^S$$

$$= \lambda[u(k) - p] + y^H + (1 - \lambda)[\alpha(u(k) - p) + y^L] + s(-k + \mu p)$$

$$= \alpha u(k) + \lambda y^H + (1 - \lambda)y^L - sk,$$

s.t. $\alpha[y^H + (1 - \lambda)y^L] \geq sk$. (RC)

Since the price $p$ is a pure transfer from buyers to sellers, it becomes irrelevant in the target function (RC). The planner is concerned with the trade-off between utility gains from relocating wealth to investing in housing, and its opportunity cost of forgoing payoffs. The planner chooses social optimal investment level $k$ for sellers in the property market, and then assigns these completed properties to the buyers, subject to matching constraint (RC). The planner also transfers wealth from buyers to sellers to finance these investments. During this process, the ex-ante investment $k$ cannot exceed the total endowment of wealth disposable for transfer. This characterizes the resource constraint for a social planner.\textsuperscript{25}

If the social marginal benefit from investment is higher than the social marginal cost of $k$, the planner chooses to reallocate more wealth to house investment; otherwise, she will reduce investment $k$ in order to improve social welfare. Since the INADA condition implies that one of the corner solutions where $k = 0$ cannot be an optimum, the constrained efficiency indicates that it is either an inner solution, where the social

\textsuperscript{24} We only have data on annually completed “villas and luxury houses” and “economic and affordable houses” from 1997. The data is from China Internet Economic Data Base and is measured in ten-thousand square meters.

\textsuperscript{25} Borrowing and lending are not allowed in this model.
marginal benefit equals the social marginal cost, or the other corner solution, where the planner invests all the possible disposable wealth. We summarize the features related to welfare consequences in the following proposition:

**Proposition 3** The social optimum (constrained efficiency) is such that any contact is consummated into a trade, i.e. $\beta^H = \beta^L = 1$; the optimal investment level is the average wealth, i.e. $k_{SW}^* = \lambda y^H + (1 - \lambda)y^L$.

**Proof.** See Appendix. ■

The INADA condition implies that a closed market cannot be the social optimum. At zero investment, a social marginal gain from investments in houses is infinite. On the other hand, the social marginal cost is the foregone utilities obtained from the wealth, discounted by market frictions (market friction implies leakages from transfers of wealth from buyers to sellers; some of the wealth, which generates direct utilities, is allocated to house investment but may not be consumed). If market friction is low, the social marginal gains from investment are above the social marginal cost. Hence, the resource constraint binds, while buyers should always trade.

Furthermore, although the planner is utilitarian, the sum of quasiconcave utility functions of buyers implies that the more equally the aggregate wealth is distributed, the better off is the whole of society. Therefore, the social optimum is such that the aggregate wealth should be equally invested in houses for all the buyers in the economy.

The implication of the social optimality is that market equilibrium cannot be efficient. Specifically, the following proposition summarizes our argument:

**Proposition 4** The decentralized market is inefficient. When the elitist matching equilibrium occurs, there are over-investments in houses and a low transaction volume. When the pooled matching equilibrium occurs or takes place, there is an efficient volume of transaction, albeit with under-investment in the housing market.

The investment in the pooled matching equilibrium is inefficiently low. To see that, suppose that the seller invests in the social optimal investment $k_{SW}^*$. Recall Lemma 1 showing that the price charged by a seller cannot be above $y^L$, where $y^L < k_{SW}^*$; this implies that this seller will make a loss. Hence, the house investment in the pooled matching equilibrium in the decentralized market must be lower than $k_{SW}^*$.

On the other hand, the elitist matching equilibrium in the decentralized market incurs an inefficiently high investment. Assuming that the seller chooses to invest $y^H$, where $y^H > k_{SW}^*$, then the asking price is $p = u(k_{SW}^*) < u(y^H)$, given the fact that the $H$-type buyer’s budget constraint binds, where the marginal revenue of investment is above the marginal cost of investment. This implies that choosing $k$ above $k_{SW}^*$ can be more profitable to a seller, which will lead to over-investment.

The elitist matching equilibrium is inefficient due to the crowding out externality exerted by $H$-type buyers on $L$-type buyers. The decentralized market converges to the elitist matching equilibrium if there is a sufficiently high proportion of potential $H$-type buyers who possess more wealth than $L$-type buyers. To extract these surpluses from trade, sellers need to increase investment $k$, which in turn drives up the willingness-to-pay of $H$-type buyers. As in Lemma 1, sellers will invest in $k$ up to the (UC) constraint. Under this condition, the transaction price will be $y^H$, which is above
the budget set to the $L$-type buyers, for those who are excluded from the market. The crowding out externality entails that the volume of trade is inefficiently low, whereby these $L$-type buyers who are willing to trade are excluded.

Though the volume of trade is efficient in the pooled matching equilibrium, the investment $k$ is inefficiently low. It is worth pointing out that the under-investment in a pooled matching equilibrium is not due to the typical hold-up problem of ex-ante investment. Instead, it is a result of the incomplete market for borrowing and lending. To understand this, consider an alternative where sellers invest after contacting a potential buyer. In this case, buyers will truthfully review their willingness to pay, given the (UC) constraint is satisfied. Hence, $H$-type buyers will get better housing than $L$-type buyers and there is no hold-up problem. When compared with the planner’s solution, social welfare could be improved, whereas $L$-type buyers could borrow from $H$-type buyers to make sellers invest more than $y^L$. Although $H$-type buyers obtain less utility from trade with sellers, $L$-type buyers are better off through borrowing. Due to the concavity of $u$, the social marginal welfare gains are higher than the social marginal welfare losses. However, if there was a market and a rate of returns on borrowing and lending, $L$-type buyers could borrow from $H$-type ones. This could relax the wealth constraint of $L$-type buyers, and hence increase investment in the pooled matching equilibrium, along with social welfare. Furthermore, the market friction and ex-ante investment exaggerate this problem by lowering $k$ even further.26 The inefficient results of the decentralized market indicate that a housing market with search friction, along with unequal distribution of individual wealth, cannot reach the social optimum. This may establish the need for government intervention. In the next section, we will investigate the impacts of hypothetical government policies on the equilibrium investment and prices in the housing market.

5 Policy Experiments

The decentralized market is inefficient, due to crowding out externalities and the lack of market for borrowing and lending. Findings in the previous section argue for policy intervention: we begin with a direct tax (e.g. redistributive taxation), then an indirect tax which is based on the volume of transaction, and the property tax.

5.1 Redistributive Tax

Suppose that the government observes the wealth of buyers and uses lump-sum transfers from $H$-type buyers to $L$-type buyers. A benevolent government redistributes wealth from the rich to the poor, due to inequality aversion.27 Specifically, we assume that the government imposes a lump-sum tax $t$ on the $H$-type buyers, and transfers $\tau$ to the $L$-type buyers. We also assume that $y^H - t \geq y^L + \tau$; the $H$-type buyers are still richer than the $L$-type buyers after such a tax. The balanced budget implies that:28

---

26 A similar principle explains the over-investment in the elitist matching equilibrium.
27 Instead of formally modelling the welfare function of the government, we only consider the static analysis of the policy.
28 In this one period game, we assume that the government is always in fiscal balance.
\[ \lambda t = (1 - \lambda)\tau. \]

Under such a policy, the buyer’s problem becomes:

\[ V^L = \max_\beta \alpha \beta^L[u(k) - p] + y^L + \tau, \quad V^H = \max_\beta \alpha \beta^H[u(k) - p] + y^H - t, \]

(14)

\[ s.t. p \leq y^L + \tau, y^H - t. \]

The seller’s value function has a similar structure as it did in the previous section, but with a new budget constraint on buyers. Following procedures similar to those used in previous sections to solve for the equilibrium under the tax schedule, the effects of the direct tax on the decentralized market is summarized by the following proposition:

**Proposition 5** A direct redistributive taxation decreases the transaction price \( p_E \) and the investment level \( k_E \) in the elitist matching equilibrium, but increases the transaction price \( p_P \) and the investment level \( k_P \) in the pooled equilibrium. Furthermore, the threshold \( y^H \) goes up under this tax policy.

**Proof.** See Appendix. ■

When the government imposes a lump-sum redistributive tax on the \( H \)-type buyers, the \( H \)-type buyers face a more rigid budget constraint. As previously discussed, sellers will charge the monopoly price in the search equilibrium, and a tighter budget constraint will decrease the monopoly price charged in the elitist equilibrium. On the contrary, a subsidy to buyers will relax the \( L \)-type buyers’ budget constraint, thus offering sellers a chance to charge a higher price to \( L \)-type buyers. As long as \( H \)-type buyers have a higher budget constraint than \( L \)-type ones, there exist two search equilibria under direct tax.

Accordingly, with a lower price in the elitist matching equilibrium, sellers invest less than when there is no direct redistributive tax, as the investment up to \( k_E^* \) will lead to them incurring a loss. However in the pooled matching equilibrium, as the equilibrium price is higher than when there is no direct redistributive tax, the investment becomes higher, due to a relaxed budget constraint faced by the \( L \)-type buyers. The changes in investment in the two equilibria, due to a direct tax, indicate a possibility to improve social welfare, because the optimal investments with this direct tax move towards the social optimal investment level.

Furthermore, the threshold level of wealth \( y^H \) increases when a direct tax is imposed, meaning sellers are more likely to serve both types of buyer, due to the reduced wealth gap. Hence the imposition of a wealth redistributive tax could make sellers better off in the pooled equilibrium, but worse off in the elitist matching equilibrium.29

### 5.2 Entry Tax

Despite the finding that a direct tax on buyers could improve social welfare, in

---

29 We can also show that, given all the constraints being satisfied, the social optimal redistributive tax then is \( t^* = (1 - \lambda)(y^H - y^L) \), and both types of buyer will have the same disposable wealth \( \lambda y^H + (1 - \lambda) y^L \). Notice that, in this case, the seller’s investment level is \( k_P^* = u^{-1}[\lambda y^H + (1 - \lambda) y^L] \), which is less than the most efficient investment level of the social planner \( k_{SW} = \lambda y^H + (1 - \lambda) y^L \). Because of the seller’s incentive of profit maximizing, the first best outcome cannot be sustained. However, the discussion of an optimal tax policy is beyond the scope of this paper.
practice it is difficult to implement because the information on individual wealth is hard to observe. We now consider a tax on sellers, for example an entry tax or a licence charge to sellers. Specifically, let the government charge a one-off tax \( t \) on sellers who sell houses in the market and transfers \( \tau \) to buyers who decide to purchase a house (\( \beta_i = 1 \)). The problem of the seller now becomes:

\[
V^S = \max_{k,p} -k - t + \mu p, \quad \text{(15)}
\]

\[
\text{s.t. } u(k) \geq p, \quad \text{(UC)}
\]

\[
p \leq y^L, \quad \text{(BCL)}
\]

\[
p \leq y^H. \quad \text{(BCH)}
\]

Then a buyer’s disposable wealth becomes \( y^l + \tau \), and the problems of buyers can be written as:

\[
V^L = \max_{\beta^L} \alpha \beta^L [u(k) - p + \tau] + y^L, \quad V^H = \max_{\beta^H} \alpha \beta^H [u(k) - p + \tau] + y^H, \quad \text{(16)}
\]

\[
\text{s.t. } p \leq y^L + \tau, y^H + \tau.
\]

The balanced budget implies that:

\[
st = \alpha [\lambda \beta^H \tau + (1 - \lambda) \beta^L \tau].
\]

Solving the equilibrium under such a tax policy, we get the following proposition:

**Proposition 6** The entry tax on sellers is welfare neutral. It only increases house prices in both equilibria; however, it has no effect on equilibrium investment and threshold level of wealth.

**Proof.** See Appendix. ■

Imposing a tax on sellers drives the house prices up in both equilibria. In the elitist matching equilibrium, the house price is increased by \( t/\lambda \mu \), and in the pooled matching equilibrium by \( t/\mu \). The intuition is that, as sellers have local monopoly power, they could share some of the tax burden with buyers, given that the price is still in the buyers’ budget set. This is possible because the policy is a pure transfer. However, to balance the government’s budget, the effective cost of taxation is to be determined by the market friction, i.e. \( t/\mu \). The parameter of matching friction plays a role in splitting the burden of tax between sellers and buyers. If the market friction is relatively low, e.g. \( \mu \) is high, sellers pass a smaller proportion of the burden of tax on to buyers. Specifically, in an elitist matching equilibrium proportion \( 1/\lambda \mu \) of tax is borne by buyers and \( 1 - 1/\lambda \mu \) is shared by sellers. On the other hand, in the pooled matching equilibrium, buyers take \( 1/\mu \) while the sellers bear \( 1 - 1/\mu \) proportion of the taxation.

Given the entry tax, the disposable wealth of an \( H \)-type buyer in the elitist matching equilibrium is \( y^H + t/\lambda \mu \); the disposable wealth of an \( L \)-type buyer in the pooled matching equilibrium is \( y^L + t/\mu \). Hence, the revenue margin between two equilibria still equals to \( \alpha(\lambda y^H - y^L) \). Yet, given the same level of taxation, when the contacting rate is higher, sellers will not raise prices as much as when it is low. Therefore, sellers also could benefit from a greater probability to sell houses in this market. In equilibrium, the revenue losses from lower pricing are just offset by a higher contact rate. This leaves the expected revenue of a seller unchanged, which contributes to the result of welfare neutrality.

### 5.3 Property Tax
Finally, we consider the case that the government charges a proportional tax \((t)\) on the house price for each buyer who decides to purchase a house \((\beta_l = 1)\), and make a transfer \(\tau\) to each seller as a subsidy. This policy is equivalent to a property tax. Under this policy, the buyers’ problems are:

\[
V^L = \max_{\beta^L} \alpha \beta^L [u(k) - p - pt] + y^L, \quad V^H = \max_{\beta^H} \alpha \beta^H [u(k) - p - pt] + y^H, \tag{17}
\]

subject to \(\beta^L p (1 + t) \leq y^L, \quad \beta^H p (1 + t) \leq y^H\).

whereas the seller’s problem is:

\[
V^S = \max_{k,p} - k + \tau + \mu p
\]

subject to \(u(k) \geq p(1 + t), \quad p(1 + t) \leq y^L, \quad p(1 + t) \leq y^H\). \tag{18}

The balanced budget implies that:

\[
a[\lambda \beta^H pt + (1 - \lambda) \beta^L pt] = st.
\]

The effects of property tax on the market equilibria can be summarized as:

**Proposition 7** The proportional property tax on buyers has no impact on investment level and the threshold level of wealth \(y^H\), but decreases transaction prices in both equilibria.

**Proof.** See Appendix. ■

Under the property tax, buyers have to pay extra units \(pt\) to the government if the transaction price is \(p\). Hence, the budget constraint of a consumer is now lower. Given that \((BCH)\) or \((BCL)\) binds in equilibrium, it implies that the price that a seller can charge is lower. Specifically, the equilibrium price is now only \(1/(1 + t)\) proportion of the previous price. Hence, the tax is effectively shared among buyers and sellers as sellers bear \(1/(1 + t)\) proportion of the taxation and buyers bear the rest.

Although the property tax leaves sellers with a lower revenue, because part of the disposable wealth is taxed by the government, they are subsidized by transfers. The effect on the seller’s profit is just offset by the transfer; on average, the amount of the subsidy is equal to the seller’s expected loss of revenue caused by taxation. Hence, such a policy has no effect on the seller’s profit margin. It implies that the same investment level \(k\) maximizes the seller’s expected profit, as the taxation is a pure transfer.

Accordingly, as there is no change in the seller’s profit as well as the investment decision in the both equilibria, the threshold level of wealth must also be the same. Under the property tax, sellers are facing the same trade-off between two equilibria. The policy only affects the transaction price and is neutral on social welfare.

### 6 Conclusion

In this paper, we built a search model to study equilibrium prices and transactions in the Chinese housing market. We found out that multiple equilibria arise in the housing market depending on the terms of trade. There exist two market equilibria: the elitist matching equilibrium and the pooled matching equilibrium. Both equilibria are not
socially optimal, from the total social welfare point of view. Our results indicate the fast-rising house prices in China are mostly caused by the unequal distribution of individual wealth. If high-wealth buyers benefit more from China’s economic growth, this will drive the Chinese housing market to an equilibrium where over-investment in houses is accompanied by very high house prices. These results can be further supported by the recent empirical findings related to China.

Furthermore, this paper provides an comparative static study on government policies. We show that the impact of a redistributive tax among buyers will depend on which equilibrium the market is in, yet still having the potential to improve social welfare. An entry tax on sellers will increase house prices, and a property tax on buyers will decrease house prices; however, both of them will have no impact whatsoever on house investment and social welfare. As the equilibrium results are mostly affected by wealth distribution across individuals, it is implied that government policies need to take into consideration the reduction of this inequality.

For future research, one interesting area is to extend the study to a long-run search model. Allowing buyers and sellers to interact more than once can give a clearer picture of the trend of the changing house prices. In addition, allowing free entry of sellers will reduce the market power of sellers in determining prices, thus providing an ability to further investigate the market structure in the Chinese housing market.

Appendix

A.1 Proof of Lemma 1 (Second Part)

Proof. In an elitist matching equilibrium, only the $H$-type buyers are willing to trade. Recall the seller’s value function (7)

$$V^S = \max_k -k + \mu \lambda u(k), \quad (A.1)$$

s.t. $p = u(k) \leq y^H$, \quad (BCH)

$$p = u(k) > y^L. \quad (BCL)$$

Assume that the budget constraint (BCL) holds, $p = u(k) > y^L$, the Kuhn-Tucker Lagrangian problem can be written as

$$L = -k + \mu \lambda u(k) + \gamma y^H - u(k),$$

the first order conditions imply that

$$\frac{\partial L}{\partial k} = -1 + \mu \lambda u'(k) - \gamma u'(k) \leq 0, \quad k \geq 0, \quad [-1 + \mu \lambda u'(k) - \gamma u'(k)]k = 0,$$

$$\frac{\partial L}{\partial p} = y^H - u(k) \geq 0, \quad \gamma \geq 0, \quad [y^H - u(k)]\gamma = 0.$$  

If (BCH) binds, i.e. $p = y^H = u(k)$, then the optimal investment in $k$ is tied down by $k^*_E = u^{-1}(y^H)$, whereas $\gamma > 0$ which requires that

$$\mu \lambda - \frac{1}{u'(k^*_E)} > 0,$$  \quad (A.2)

otherwise (BCH) does not bind; the optimal solution is determined by the first order condition (8) by substituting $\gamma = 0$, hence

$$u'(k^*_E) = \frac{1}{\lambda \mu}.$$  

It is straightforward to see that if (BCH) binds, $p = y^H > y^L$, despite the (UC) constraint being satisfied, the $L$-type buyers are excluded from the market, hence the optimal
choice of $p$ and $k$ are consistent with the elitist matching equilibrium. Following similar methods, the choice of a set up cost $k$ and an asking price $p$ consistent with the elitist matching equilibrium must be: $k^*_E$ that satisfies $u'(k^*_E) = 1/\lambda \mu$, $p = u(k^*_E)$ if $u(k^*_E) < y^H$, or $k^*_E = k^*_E = u^{-1}(y^H)$, $p = u(k^*_E)$ if $u(k^*_E) = y^H$.

Recall the necessary conditions (9) and (A.2) for the budget constraints (BCH) and (BCL) bind in the elitist equilibrium and the pooled equilibrium respectively. For these two conditions to hold, the equilibrium prices will be $y^H$ and $y^L$ respectively, as the (UC) constraint also binds. This implies that

$$u(k^*_p) = p = y^L < y^H = p = u(k^*_E).$$

As $u$ is concave, the inequality implies that $k^*_p < k^*_E$, hence $u'(k^*_p) > u'(k^*_E) > \lambda u'(k^*_E)$. Therefore, since the matching frictions are sufficiently small ($\mu$ is large enough), the necessary conditions for (BCH) and (BCL) are satisfied, i.e. $\mu > 1/\lambda u' > 1/u'$. Hence, when facing the offer $\{k^*_E, p_E\}$, only $H$-type buyers would choose to accept, whilst $L$-type ones would not, and the volume of trade in the elitist matching equilibrium is $\lambda a$. When faced with the offer $\{k^*_p, p_p\}$, both types of buyers would choose to accept, so the volume of trade in the pooled matching equilibrium is $\alpha$.

### A.2 Proof of Proposition 2

**Proof.** Recall the binding budget constraints in each equilibrium, $y^H = u(k^*_E)$, $y^H = u(k^*_E)$, then:

$$k^*_E = u^{-1}(y^H) = v(y^H), k^*_p = u^{-1}(y^L) = v(y^L). \tag{A.3}$$

$v$ is the inverse function of $u$. Substituting equations (A.3) into the right-hand side of equation (12):

$$v(y^H) - v(y^L) = \frac{v(y^H) - v(y^L)}{\lambda \mu} - \frac{v(y^L)}{\lambda \mu} + \frac{y^L}{\lambda} = f(y^H). \tag{A.4}$$

For any given $\lambda$, $\mu$ and $y^L$, we can define $v(y^H)/\lambda \mu - v(y^L)/\lambda \mu + y^L/\lambda$ as a function $f(y^H)$. Because the utility function $u$ is a continuous monotonically increasing function, as is $v$. Therefore, $f$ is a continuous monotonically increasing transform from $y^H$ to itself, i.e. $f: \mathbb{R}^+ \to \mathbb{R}^+$. Following Brouwer’s Fixed Point Theorem, we can conclude that there must exist a threshold level of wealth $y^H$ such that $f(y^H) = y^H$, and $y^H$ is unique.

Rearranging equation (12):

$$\mu y^H - v(y^H) = \mu y^L - v(y^L). \tag{A.5}$$

Recall the assumption that market frictions are sufficiently low (i.e. $\mu$ is sufficiently high); the inequality $u' > 1/\lambda \mu > 1/\mu$ must hold, then we can derive that $v' < \mu < \lambda \mu$. Hence, the left-hand side of equation (A.5), $\mu y^H - v(y^H)$, is increasing in $y^H$, and the right-hand side, $\mu y^L - v(y^L)$, is increasing in $y^L$. Given $\mu$, $y^L$ fixed, an increase in $\lambda$ does not affect the right-hand side of equation (A.5). However, to make the equation hold, $y^H$ should be reduced. Therefore, $y^H$ is decreasing in $\lambda$.

Similarly, given $\mu$, $\lambda$ fixed, an increase in $y^L$ increases the right-hand side of

---

30 On the other hand, if (BCH) does not bind, where the price is set by $p = u(k_E)$, and $k^*_E$ is determined by the first order condition of (8), which is consistent with elitist matching equilibrium if $y^k$ is lower than $p$. Therefore, we assume that this condition holds.
equation (A.5) since $v' < \mu$. To make the equation hold, $y^H$ should be increased since $v' < \lambda \mu$. Therefore, $y^H$ is increasing in $y^L$.

### A.3 Proof of Proposition 3 (Welfare Consequences)

**Proof.** The optimization problem can be solved by using Kuhn-Tucker Lagrangian method:

$$L = \alpha u(k) + \lambda y^H + (1 - \lambda) y^L - sk + \gamma [\lambda y^H + (1 - \lambda) y^L - k],$$

whereas $\gamma$ is the Kuhn-Tucker multiplier, and the resource constraint (RC) is simplified by using the fact $\alpha = s \mu$. The first order condition can be written as

$$\frac{\partial L}{\partial k} = \alpha u'(k) - s - \gamma \leq 0, \quad k \geq 0, \quad \frac{\partial L}{\partial k} k = 0,$$

$$\frac{\partial L}{\partial y} = \lambda y^H + (1 - \lambda) y^L - k \geq 0, \quad y \geq 0, \quad \frac{\partial L}{\partial y} \gamma = 0.$$  

Since an active property market implies that $k > 0$, the complementary slackness conditions requires that $\partial L / \partial k = \alpha u'(k) - s - \gamma = 0$. Recall the assumption that market frictions are sufficiently low (i.e., $\mu > 1/u'$), so we can derive that $\alpha u'(k) = s \mu u'(k) > s$. To make sure the equation $\partial / \partial k = \alpha u'(k) - s - \gamma = 0$ holds, we must have $\gamma > 0$. Therefore $\partial / \partial y = \lambda y^H + (1 - \lambda) y^L - k = 0$ holds because of the complementary slackness, and the social optimal investment is a corner solution $k^*_{SW} = \lambda y^H + (1 - \lambda) y^L$. It implies that (RC) constraint is binding, and all the disposable wealth of the buyers is transferred to the sellers to invest.

### A.4 Proof of Proposition 5 (Redistributive Tax)

**Proof.** For the pooled matching equilibrium, where the buyers’ choice is $\{\beta^H = 1, \beta^L = 1\}$, the expected value functions are,

$$V_p^H = \alpha [u(k_p) - p_p] + y^H - t,$$

$$V_p^L = \alpha [u(k_p) - p_p] + y^L + \frac{\lambda}{1 - \lambda} t,$$

where the optimal pricing choice of sellers is $p_p = u(k_p) = y^L + \lambda t / (1 - \lambda)$. Then the expected value function of sellers when they choose to serve both types of buyer is

$$V_p^S = -k_p + \mu (y^L + \frac{\lambda}{1 - \lambda} t),$$

(A.6)

$$\frac{\partial V_p^S}{\partial t} > 0,$$

which implies that the house quality in a pooled matching equilibrium is better.

For the elitist matching equilibrium, where the buyers’ choice is $\{\beta^H = 1, \beta^L = 0\}$, the value function of the $L$-type buyers is just $y^L + \lambda t / (1 - \lambda)$ since they choose not to buy, while the expected value function of the $H$-type buyers is,

$$V_E^H = \alpha [u(k_E) - p_E] + y^H - t,$$

where the optimal pricing choice of sellers is $p_E^* = u(k_E^*) = y^H - t$. Then the expected value function of sellers, when they choose to serve the $H$-type buyers exclusively is

$$V_E^S = -k_E^* + \mu \lambda (y^H - t),$$

(A.7)
which implies that the house quality in an elitist matching equilibrium is worse. By combining equations (A.6) and (A.7), the threshold level of wealth is
\begin{equation}
y^1 = \frac{1}{\mu \lambda} [k_E^{**} + \mu \lambda t - (p_E^{**} - \frac{\mu \lambda t}{1-\lambda})] + \frac{y^L}{\lambda}
\end{equation}
Recall that
\begin{equation}
u(k_E^{**}) - \nu(k_E^{*}) = \frac{\lambda}{1-\lambda} t,
\end{equation}
u(k_E^{**}) - u(k_E^{*}) = t.
Since we have the assumption that \(\mu\) is large enough (\(\mu > 1/\lambda u' > 1/u'\) for all \(k_p, k_E\)), then we can take the derivative respect to \(t\) in the equation (A.7),
\begin{equation}
u'(k_p^{**}) \frac{\partial k_p^{**}}{\partial t} - u'(k_p^{*}) \frac{\partial k_p^{*}}{\partial t} = \frac{\lambda}{1-\lambda}
\end{equation}
where \(u' > 1/\mu\) for \(k_p^{**}\) and \(k_p^{*}\). Hence,
\begin{equation}
\frac{\mu \lambda}{1-\lambda} > \frac{\partial k_p^{**}}{\partial t} \frac{\partial k_p^{*}}{\partial t}
\end{equation}
Similarly, we can show that \(k_E^{*} - k_E^{**} < \mu \lambda t\), so that
\begin{equation}
k_E^{**} + \mu \lambda t - (k_p^{**} - \frac{\mu \lambda t}{1-\lambda}) > k_E^{*} - k_p^{*}
\end{equation}
Comparing (A.8) to the benchmark model without tax policy, the threshold level of wealth is \(y^1 > y^H\), which tells us that sellers are more likely to serve both types of buyers, resulting in a pooled equilibrium.

### A.5 Proof of Proposition 6 (Entry Tax)

**Proof.** Now we consider the case that the government charges a lump-sum entry tax \(t\) on sellers, and transfers \(\tau\) to each buyer who decides to buy a house (\(\beta_i = 1\)). The cost of building a house becomes \(k_1 + t\), and the buyer’s disposable wealth becomes \(y^i + \tau\). In the pooled equilibrium, the buyers’ choice is \(\beta^H = 1, \beta^L = 1\), with both types of buyer receiving a transfer \(\tau^P\), such that \(st = \alpha \tau P\). The expected value functions are
\begin{align*}
V_p^H &= \alpha [u(k_p^{**}) + \tau_p - p_p^{**}] + y^H, \\
V_p^L &= \alpha [u(k_p^{*}) + \tau_p - p_p^{*}] + y^L,
\end{align*}
Following individual rationality, the (UI) constraints implies that the optimal pricing choice of sellers is \(p_p^{**} = u(k_p^{**}) + \tau_p = y^L + \tau_p = y^L + t/\mu\), which implies that \(k_p^{*} = u^{-1}(y^L)\). This proves that under the entry tax scheme, investment in \(k\) equals to the investment level in a laissez-faire market. This result can also be found through the value function of sellers, when they choose the pooled matching equilibrium, which is
\begin{equation}
V_p^S = -(k_p^{**} + t) + \mu (y^L + \frac{t}{\mu}) = \mu y^L - k_p^{**}
\end{equation}
which essentially implies the same investment level of \(k\).

In the elitist equilibrium, the buyers’ choice is \(\beta^H = 1, \beta^L = 0\), and only the \(H\)-type buyer receives a transfer \(\tau_E\), such that \(st = \lambda \alpha \tau E\). The expected value function
of an \( H \)-type buyers is,
\[
V_E^H = \alpha [u(k_E^{**}) + \tau_E - p_E^{**}] + y^H,
\]
where the optimal pricing choice of sellers is \( p_E^{**} = u(k_E^{**}) + \tau_E = y^H + \tau_E = y^H + t/\lambda \mu \). Similarly, in an elitist matching equilibrium, the investment is such that \( k_E^{**} = u^{-1}(y^H) \), which is the same level as in a laissez-faire market. Then the expected value function of sellers, when they choose to serve the \( H \)-type buyers exclusively is
\[
V_E^S = -(k_E^{**} + t) + \lambda \mu (y^H + \frac{t}{\lambda \mu}) = \lambda \mu y^H - k_E^{**} 
\]  
(A.11)
By combining equations (A.10) and (A.11), the threshold level of wealth is
\[
\underline{\gamma}^2 = \frac{k_E^{**} - k_p^{**} + y^L}{\lambda \mu} + \frac{y^L}{\lambda}.
\]
As argued above, both \( k_E^{**} \) and \( k_p^{**} \) are at the same level in a laissez-faire market, hence \( \underline{\gamma}^2 = y^H \). In other words, other than pushing up the transaction price, the entry tax is welfare neutral.

A.6 Proof of Proposition 7 (Property Tax)

Proof. In the pooled matching equilibrium where the buyers’ choice is \( \{\beta^H = 1, \beta^L = 1\} \), the expected value functions are
\[
V_E^H = \alpha [u(k_E^{**}) - (1 + t)p_p] + y^H,
\]
\[
V_E^L = \alpha [u(k_p^{**}) - (1 + t)p_p] + y^L,
\]
where the optimal pricing choice of sellers is \( p_p^{**} = u(k_p^{**})/(1 + t) = y^L/(1 + t) < p_p^* \), which means that \( u(k_p^{**}) = y^L \), so that \( \partial k_p^{**}/\partial t = 0 \).

Furthermore, note that \( \alpha t y^L/(1 + t) = st \), when \( \beta^H = \beta^L = 1 \), so that
\[
\tau = \frac{\mu t}{1 + t} y^L.
\]
The expected value function of sellers, when they choose to serve both types of buyer is
\[
V_E^S = -k_p^{**} + \tau + \frac{\mu y^L}{1 + t} = -k_p^{**} + \mu y^L. \quad \text{(A.12)}
\]
Although, compared with a laissez-faire market, the seller’s revenue is lower under the policy, their set up cost is also lower due to subsidies. The losses in the expected revenue are just offset by cost reduction, which results in equivalent revenues in a laissez-faire market and in one with a property tax. It implies that the same investment level in \( k \) maximizes the seller’s expected profit.

The same principle applies in the elitist equilibrium, i.e. the buyers’ choice is \( \{\beta^H = 1, \beta^L = 0\} \). The expected value function of the \( H \)-type buyers is
\[
V_E^H = \alpha [u(k_E^{**}) - (1 + t)p_E] + y^H,
\]
where the optimal pricing choice of sellers is \( p_E^{**} = u(k_E^{**})/(1 + t) = y^H/(1 + t) < p_E^* \), which means that \( u(k_E^{**}) = y^H \), so that \( \partial k_E^{**}/\partial t = 0 \). The budget balance of the government tells us that \( \lambda at y^H/(1 + t) = st \), and
\[
\tau = \frac{\lambda \mu t}{1 + t} y^H.
\]
Then the expected value function of sellers when they choose to serve the \( H \)-type buyers exclusively is
\[
V_E^S = -k_E^{**} + \tau + \frac{\lambda \mu y^H}{1 + t} = -k_E^{**} + \lambda \mu y^H. \quad \text{(A.13)}
\]
As the expected revenue for the sellers is the same as in a laissez-faire market, the
investment in $k_p^*$ is the same under property tax.

It is straightforward to see that the property tax will not influence the threshold $y^H$, as the expected revenues and investments in both equilibria are not changed.

References

[17] Ren, Y, Xiong, C and Yuan, Y., 2012, House price bubbles in China, China
Economic Review, 23, 786-800.


Urbanisation, Inequality and Property Prices
Equilibrium Pricing and Transaction in the Chinese Housing Market

Highlights

1. Three striking empirical facts characterises China’s housing market.
2. A search model with endogenous setup cost is solved.
3. Through urbanization, evolution from pooled- to elitist-matching equilibrium drives up property price.
4. China’s high growth in income and high inequality result in elitist-matching equilibrium.
5. Redistributive tax may decrease housing prices and increase social welfare.