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Explorations in Digital Cartography

DISCUSSION PAPER 2

The Visvalingam algorithm
metrics, measures and heuristics

by

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List of publications

1.	Visvalingam, M and Whelan, J.C (November 2014) "Implications of Weighting Metrics for Line Generalisation with Visvalingam's Algorithm" https://hydra.hull.ac.uk/resources/hull:10064	24 pp
2.	Visvalingam, M (March 2015) "The Visvalingam algorithm: metrics, measures and heuristics" https://hydra.hull.ac.uk/resources/hull:10596	12 pp
3.	Visvalingam, M (May 2015) "Testing Implementations of Visvalingam's Algorithm for Line Generalisation" https://hydra.hull.ac.uk/resources/hull:10874	16 pp
4	Visvalingam, M (April 2016) "Geometric data for testing implementations of point reduction algorithms : case study using Mapshaper v 0.2.28 and previous versions" http://hydra.hull.ac.uk/resources/hull:13115	25 pp
5	Visvalingam, M (July 2016) Bloch's function D for Weighted Effective Areas : Impact of tuning parameters	18 pp

ABSTRACT

This paper provides the background necessary for a clear understanding of forthcoming papers relating to the Visvalingam algorithm for line generalisation, for example on the testing and usage of its implementations. It distinguishes the algorithm from implementation-specific issues to explain why it is possible to get inconsistent but equally valid output from different implementations. By tracing relevant developments within the now-disbanded Cartographic Information Systems Research Group (CISRG) of the University of Hull, it explains why a) a partial metric-driven implementation was, and still is, sufficient for many projects but not for others; b) why the Effective Area (EA) is a measure derived from a metric; c) why this measure (EA) may serve as a heuristic indicator for in-line feature segmentation and model-based generalisation; d) how metrics may be combined to change the order of point elimination; and e) how Tobler's rule-of-thumb is useful for scale-related filtering of EA. The issues discussed in this paper also apply to the use of other metrics. It is hoped that the background and guidance provided in this paper will enable others to participate in further research based on the algorithm.

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1. Introduction

In the 1980s, researchers were becoming increasingly aware that the RDP algorithm (named after Ramer (1972) and Douglas and Peucker, 1973) was insufficient for generalising complex boundaries. This algorithm, initially designed for weeding, remains the preferred choice for curve approximation and minimal simplification not just in cartography but also in Computer Graphics and Pattern Recognition. However, it was not optimal for generalisations requiring the elimination of features.

Visvalingam designed her algorithm to complement the RDP algorithm – not to replace it. Since in-line feature segmentation was a problem two decades ago, Visvalingam proposed that the iterative elimination of triangular features would result in the elimination and generalisation of scale-related features. Although Visvalingam and Whyatt (1993, p 47) demonstrated that the algorithm is capable of generalising complex coastlines, they regarded it "*as only a step towards the evolution of a more intelligent system for line generalisation".*

Visvalingam's algorithm has been selected as a classic (Field and Kent, 2014) and is growing in popularity. It is not the aim of this paper to compare it with other algorithms. Those wishing to do so, can use our free-to-download data at https://hydra.hull.ac.uk/resources/hull:9040 and publications, a resource which will continue to grow.

This paper limits itself to providing a historical trace of how this algorithm was developed and applied within the now-disbanded Cartographic Information Systems Research Group of the University of Hull (CISRG, 2014). This trace is used a) to point to implementation issues discussed more fully elsewhere (Visvalingam, 2015), b) to explain concepts such as areal displacement and effective area, c) to indicate how the algorithm can be imaginatively applied to a variety of data, d) to highlight some issues which currently limit the usefulness of this algorithm, and e) to encourage others to address some of these problematic issues which will be discussed more fully in a forthcoming paper. The paper explains why a) a partial metric-driven implementation was, and still is, sufficient for many projects but not for others; b) why the Effective Area is a measure derived from a metric; c) why this measure serves as a heuristic indicator to facilitate model-based generalisation; and, d) how metrics may be combined to derive measures to change the order of point elimination. The issues discussed in this paper also apply to the use of other metrics.

It is hoped that the background and guidance provided in this paper will enable others to participate in further research and in the testing of the growing number of open source and commercial implementations of the algorithm. In case they wonder why their results are not identical to ours, this paper explains why variations in implementation are inevitable and sometimes necessary to suit different purposes. Given the allowance for variability, the testing should not be concerned with pedantic details but with whether the implementation is consistent with the intent of the algorithm.

2. Background

2.1 The Visvalingam Algorithm

The Visvalingam algorithm is very simple. It consists of repeated elimination of the point which is least significant in a given line and treating the remainder as the new input line. An algorithm is an abstract statement and it does not specify implementation details. For example, this algorithm does not specify the measure of significance nor how least should be defined. These decisions can be varied to meet different purposes so long as the implementation is consistent with the specification.

2.2 Metrics, Measures and Heuristics

There is a great deal of confusion over the terms metrics and measures. Even within the National Institute of Standards and Technology, DARPA ICV (1999) and SAMATE (2014) offer opposite meanings. Such confusion arises partly because of varying usage of these words within different disciplines. Software Engineering is a relatively new discipline compared with Statistics. This paper favours the definitions provided in DARPA ICV (1999) since they correspond to usage in statistics and its applications.

Chirhocub (2010) explained that "*A metric is the 'how' of measurement, that is,… the method by which one assigns numerical values based on the concept of distance, i.e. length, area, volume, progress, etc".* He noted that distance is the metric but the value is not. He also explained why **all metrics are measures** – but not all measures are metrics, using concepts in basic statistics.

The word measure is used rather loosely to refer to the taking of measurements, to the metrics; and in a more technical sense. In statistics, measures are derived through interpretation of one or more qualitative or quantitative metrics and/or other measures. The DARPA Report (1999, p 35) proposed that "*a metric is an observable value, while a measure associates meaning to that value by applying human judgement. "*

In statistics, we have Measures of Central Tendency (e.g. mean and median), Measures of Dispersion (variance and standard deviation) and Measures of Significance whose interpretation depends on associated probability distributions.

The word heuristic also has multiple meanings. In computing, heuristic refers to a rule of thumb, used often to speed up processing when exhaustive searching is impractical. More generally, it serves *to indicate or point out; stimulating interest as a means of furthering investigation … based on experimentation, evaluation, or trial-and-error methods…* it *refers to experience-based techniques for problem solving, learning, and discovery that give a solution which is not guaranteed to be optimal* (The Free Dictionary, undated). This meaning is also used in other disciplines, e.g. in medicine.

2.3 Effective Area versus Least Areal Displacement

Areal displacement (**AD**) is the metric used to assess importance	
Compute AD for all internal points of the input line	[1]
While there are more than the required number of points {	[2]
Flag the point with lowest AD for removal	[3]
Recalculate AD for the two neighbouring points	[4]
}	[5]
Output the co-ordinates of points, which have not been flagged for deletion	[6]

Pseudocode 1: Expression of the Visvalingam algorithm for filtering individual coastlines

As stated in Visvalingam and Whyatt (1993), any metric can be used to assess the points' importance. This was demonstrated by Visvalingam and Brown (1999) who used the algorithm for deriving decogons (decorative patterns) from fractal Koch curves. Visvalingam chose to iteratively drop the point which resulted in the least areal displacement (AD) but pointed out that this was one of many options. Areal displacement was chosen because some other metrics, such as for shape, only start to have an impact when the size of a feature exceeds a perceptual limit. It also takes account of the relationship of the distance between the points and their angles.

Visvalingam and Whyatt (1993) only focused on the filtering of individual coastlines and used Whyatt's program (Whyatt, 1991), which corresponds to Pseudocode 1. This approach has some merits and some limitations as explained in Section 3 below. So, in the paper by Visvalingam and Whyatt (1993), Visvalingam provided a fuller specification which corresponds to Pseudocode 2. Visvalingam coined the term Effective Area (EA) to stress that it is a heuristic measure rather than a prescriptive metric, as explained fully in Section 3.3.2.

Statements 5 and 6 in Pseudocode 2 stipulate that if a point's EA is less than or equal to (<=) that of the last eliminated point, then its EA should be set to that of the latter. This conditional operator gives better results with some types of data. It makes no difference to the coastline data used in previous projects but produces more consistent results when using fractals to test implementations of the algorithm. Statement 5 ensures that points will be filtered in the same order as in Pseudocode 1, i.e. in a way which is consistent with the algorithm. The EA of the dropped point is recorded (statement 7) so that the polyline can be filtered repeatedly on this value (or its corresponding rank) in subsequent interactive applications. It is useful to note the sign of EA since this can be exploited in some applications, such as in terrain sketching (See Section 3.3.1). The modification of the EA in statement 5 is quite important since it can lead to a cascade of points being eliminated with the same EA or rank on thin elongated features.

Let previous = 0.0 [1]

Let EA = AD for all internal points of the input line [2]

While there are internal points { [3]

  Find the point with the least EA [4]

  [The condition in statement 5 below is preferable to the condition used in past papers, which was:

  ***If** (EA of this point < previous)…]*

  If (EA of this point <= previous) EA = previous [5]

    Else previous = EA [6]

  Record the EA of this point and note that it has already been dropped. [7]

  Recalculate the EA = AD for the two neighbouring points [8]

} [9]

Pseudocode 2: Original specification of the Visvalingam algorithm

Lines 5 to 7 in Pseudocode 2 are not an integral part of the basic generalisation algorithm. However, they facilitate the filtering of both single, and also multiple, independently processed polyline(s). The various implications and applications of the original specification (Pseudocode 2) are explained by tracing its use within the CISRG, using illustrations based on data for Humberside derived from Ordnance Survey 1:50000 maps. This free-to-use data is available at https://hydra.hull.ac.uk/resources/hull:9040.

2.4 The least important point

Section 2.3 explained that the algorithm may be driven with any metric. Experience favoured the use of EA for generalising coastlines, but Visvalingam and Brown pointed out that the offset value was better for approximating lines; they also demonstrated the effect of using other metrics. In addition, the algorithm and Pseudocodes 1 & 2 are not prescriptive as to the precise method to use when choosing the least important point for the following reasons. Visvalingam and Whyatt (1990) scrutinised the RDP algorithm. They found that different implementations, and even the same program run on different computers, could produce different results depending on the specific test condition used, the direction of parsing, machine rounding errors and other factors. They also pointed out that cartographic data are inexact and are only representative. Given that digitising errors are much larger than rounding errors, they can make prescriptive stipulations of how to choose a point from a set of equal-valued candidates somewhat pedantic.

Visvalingam and Whelan (2014, Appendix 1), noted that the output from Mapshaper's Visvalingam option is not identical to ours. Visvalingam and Brown (1999) had previously demonstrated how Visvalingam's algorithm is sensitive to the direction in which a line is parsed. Indeed, if the same line is input in start-to-end and then in end-to-start order, it is possible to get different results. All versions should output similar, even if not identical, generalisations of coastlines. The only restriction is that the output must not be so inconsistent with the intent of the algorithm that it looks inappropriate for its intended purpose. This can happen if the point is picked from a candidate set with equal-values without due regard to its position along the line. This may not matter when coastlines are only simplified to a modest extent.

Visvalingam's implementation searches the array of EA from start to end, and eliminates the point chosen with the test condition ***If (EA < minEA) minEA = EA***. This is no more and no less arbitrary than using ***If (EA <= minEA) minEA = EA***. It is being stated here for the benefit of those who wish to compare their results with our published figures and for no other reason.

3. Implementations to suit different purposes

The basic algorithm and the specification as stated in Pseudocode 2 were developed on an ICL Perq personal workstation running the PNX operation system. This platform enabled speedy, suck-it-and-see interactive development of ideas and heuristics, using C, Fortran, and the Graphical Kernel System (see Visvalingam 1987a and b). The Graphical Kernel System (GKS) was then the international standard for 2D graphics (Hopgood et al, 1984). Visvalingam was fortunate to have sole personal use of this system through the award of a UK Science and Engineering Research Council (SERC) grant from 1983-86 to Graham Kirby (in Computer Science) and herself (then an Honorary Research Associate in Geography) at the University of Hull. When this project commenced, powerful UNIX-based single user workstations with high resolution A4 bit-mapped screens were just becoming widely available. It was still not possible to undertake explorations of multivariate data through cross-linked graphical interfaces as anticipated by Visvalingam and Kirby (1984) and Visvalingam (1985). However, the ability to run concurrent interactive programs enabled Visvalingam to manually cross-reference data and displays to study the impact of changes to specification. Other CISRG members, including research students, initially had to rely on batch processing on a multiuser mainframe computer and use a Calcomp pen plotter and later Postscript for quality figures.

The various implementations used within the multidisciplinary CISRG reflected the requirements of projects allocated to students, which took account of their disciplinary strengths, skills in programming, the computing resources at their disposal and their desire to learn new programming languages and other skills.

3.1 Whyatt's implementation

Whyatt's implementation, which corresponds to Pseudocode 1, continues to provide an excellent starting point for short undergraduate student projects since it is easy to implement. It focuses on the gist of Visvalingam's algorithm; namely the iterative elimination of the least important point. Even in the late 1980s, it was not unusual to run the input data repeatedly through filter programs to calculate and output the required subset specified by a filter value or the number (percentage) of points to be retained. Whyatt only had access to a Sequent Symmetry multitasking computer, running DYNIX (a version of Unix). His research focused on comparing algorithms using a few carefully selected isolated coastlines. Each coastline was repeatedly run thorough his Fortran program to retain a specified number of points for offline drawing of maps on a Calcomp plotter (Whyatt, 1991, Appendix of Program Listings). Note that Pseudocode 1 uses the metric AD, which is the calculated Areal Displacement. AD is not the same as EA as explained in Section 3.3. Figure 1a shows 8 out of 2227 points in the Humberside coastline.

3.2 The rank order of points

A minor modification could be made to Pseudocodes 1 & 2 to attribute a rank to each point to indicate the order in which points are eliminated by Visvalingam's algorithm. The tagged data output can then be repeatedly filtered on this rank value by a separate process. Figure 1a shows that filtering on rank gives the same output as the process described in section 3.1 using Pseudocode 1.

The rank attribute is much more efficient and convenient for retaining the number (or percentage) of points in some applications. It was only mentioned briefly by Visvalingam and Whyatt (1993) although it has been widely used within the CISRG. It is especially useful for comparing algorithms, metrics, weights etc. All CISRG projects which compared the RDP, Visvalingam and Bendsimplify algorithms used the rank filter since the metrics themselves are not directly comparable. The Bendsimplify algorithm was studied by Visvalingam and Herbert (1999). The comparison of weighted and unweighted EAs by Vinsvalingam and Whelan (2014) also compared subsets filtered with rank.

When (***EA of a point <= previous***) in Statement 5 of Pseudocode 2, the rank can also be reset, if needed, to that of the previously eliminated point so that individual lines could be filtered on rank or on EA in a consistent way. Ranks could be specific to individual lines and/or be global to a set of lines, to suit the implementation and its target applications. Rank is not used in applications which rely on multiple scale-related filters (See Section 3.3.1). Like the choice of the metric and the least important point, this is a purpose-driven implementation issue. Within the CISRG, ranks were specific to separately processed individual polylines since EA was used to filter multiple lines across all applications.

3.3 Why and how the EA measure was conceived and used for filtering

Visvalingam had already noted that some in-line shapes can result in the lack of a monotonic relationship between rank and AD; ADs can sometimes become smaller with increasing rank. Visvalingam was impressed by Wade's solution to a similar problem in his implementation of the RDP algorithm and adapted his approach in her implementation. Wade's Fortran program is listed in Whyatt and Wade (1988). He wrote this in his first year of postgraduate research (1983/84) on behalf of the Market Analysis Division of CACI, the collaborating partner part-funding his SERC CASE (Science and Engineering Research Council Collaborating Awards in Science and Engineering) studentship. In Section 2.4, we explained how different implementations of the RDP algorithm can produce different results for a variety of reasons, including the treatment of special geometric cases. Peucker (1975, p 511) had already noted that offset values can increase with recursive segmentation of a line by the RDP algorithm. This was ignored in earlier implementations by others since the recursive selection of points was terminated when the required number of points was ouptut or when offset values fell below a specified threshhold.

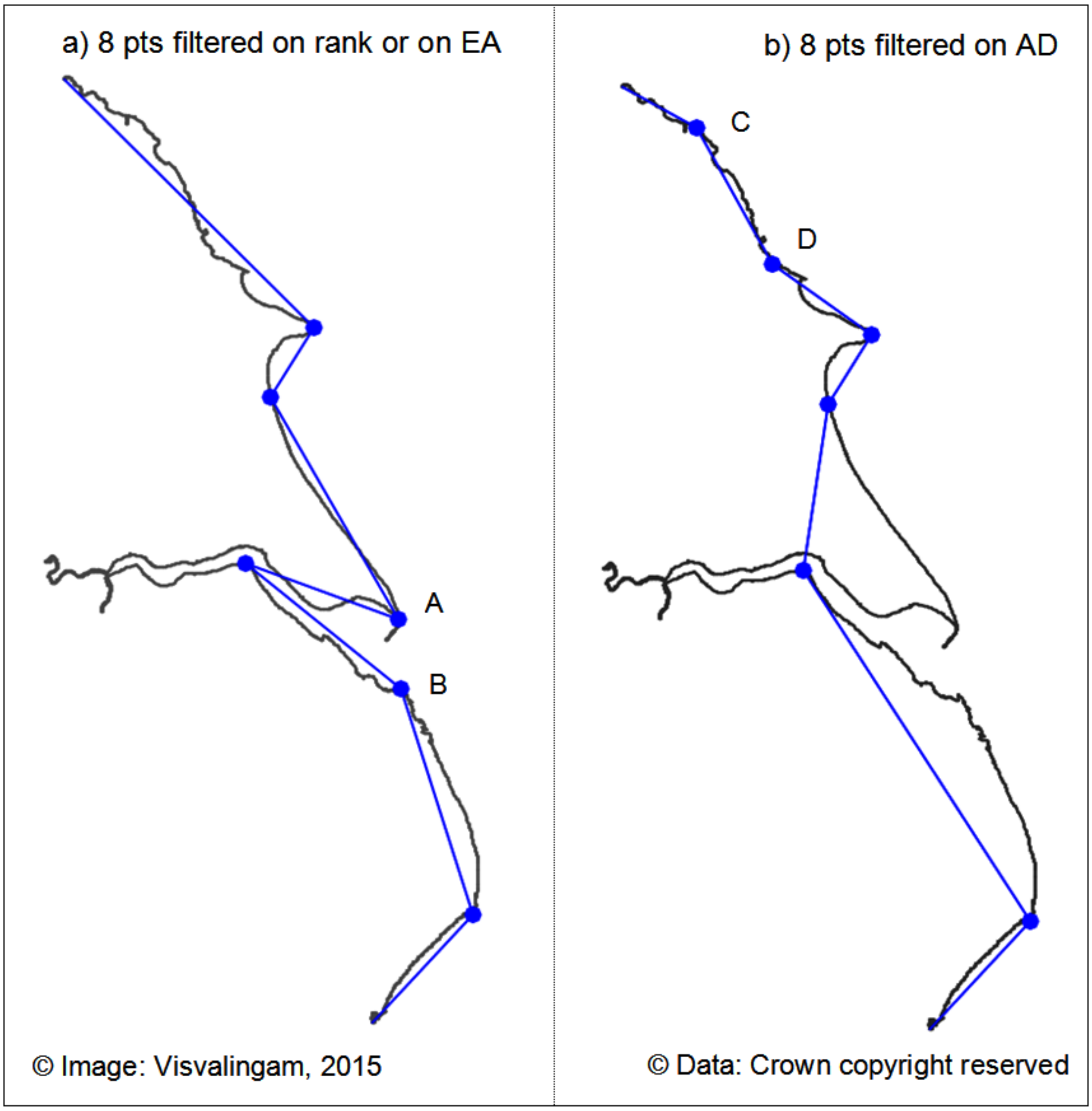

Figure 1 : The effective area (EA) is not the same as areal displacement (AD)

The GIMMS mapping package (Waugh and McCalden, 1983) was the first to tag points. Whyatt and Wade (1988) noted how its GENERAL command avoided the need for repeated calculation of offsets. Using the GENERAL command, it was possible to assign points to nominal, scale related classes for subsequent filtering. However, GIMMS did not address the lack of a monotonic relationship between the rank and offset values.

In his independent implementation of the RDP algorithm, Phil Wade tagged each point with the offset distance which led to its selection, instead of to a class, to enable subsequent interactive filtering by an independent process. When an offset distance was greater than that of the previously chosen point, Wade demoted the former's value to that of the latter as explained with illustrations in Visvalingam and Whyatt (1990). This ensured that his output was consistent with the spirit of the original specification of the RDP algorithm.

With his and the sponsor's permission, his program was used in undergraduate projects to evaluate the RDP algorithm. These preliminary investigations, especially by Ian Jenkinson, revealed that the RDP algorithm had significant limitations, and this was confirmed by Whyatt's PhD project, which compared output from Wade's implementation of the RDP algorithm and from other algorithms.

Returning to Visvalingam's algorithm, maps filtered using AD (Section 3.1) and rank (Section 3.2) can differ. As already noted, filtering individual lines on rank will give identical results consistent with the algorithm and Pseudocodes 1 & 2. Filtering on AD can remove points in the wrong order and produce unacceptable results. So, Visvalingam adapted Wade's technique and promoted AD to EA when the AD of the current point is smaller than that of the last point to be eliminated. As illustrated in Figure 1, points C & D have larger ADs than points A and B. So, filtering on AD gives Figure 1b which is inappropriate. The intended output can be obtained by filtering on EA.

Visvalingam coined the term EA to stress that it is not AD, but a **heuristic** measure based on the metric AD, but not always equivalent to it. It was derived through step-by-step interactive scrutiny of how data was being processed by the algorithm. Appendix 1 illustrates why and how EA differs from AD. The concept of the EA indicates the presence of a potentially meaningful geometric entity, such as an inflection which may occur when bays alternate with headlands. As is true of many heuristics, it does not guarantee optimal results in every case. For those used to filtering data on a distance metric, the square root of EA facilitates filtering on a scale-related metric. EA also a) facilitates the filtering of maps consisting of several independently tagged lines; and b) the segmentation of lines to facilitate model-based filtering as explained below.

3.3.1 Filtering maps and terrain with several polylines

Visvalingam produced a Fortran version of her program for use by Peter Williamson in his four-month Dissertation project in part fulfilment of an MSc conversion course in IT and Manufacture in 1991/92. His project work was also done on the mainframe computer running Unix. It involved the generalisation of data for a set of road line segments on a 1:1250 large-scale Ordnance Survey plan. Figures with single and multiple lines were used to compare the performance of the RDP and Visvalingam algorithms (Visvalingam and Williamson, 1995). They pointed out that the RDP algorithm was better for approximation and minimal simplification but that it can distort roads at higher levels of generalisation.

Since EA is based on AD, which is a scale-related metric, it can be used to filter multiple lines. Programs written by Wade and Visvalingam were used by Williamson to filter the lines and the results were compared using Williamson's C program using GKS. Alan Whitaker wrote Postscript programs for producing the high quality figures for publication. They show that Visvalingam's algorithm can be used for generalising road outlines.

Visvalingam's implementation was adapted by Dowson for the research on terrain sketching undertaken by Visvalingam and Dowson (1998). This project demonstrated that it is possible to use EA to identify important breaks of slope in multiple terrain profile sections for producing algorithmic sketches of the terrain. EA is the **heuristic measure** of significance (i.e. the indicator of significant convex (+EA) and concave (-EA) breaks of slope). Different EA values were used to filter core cells for different types of breaks of slope. The core cells were then extended to form the profile stokes which made up the P-stroke sketch. Ranks have little use in this sort of application. Terrain sketching and coastline generalisation use Visvalingam's algorithm in different ways. This is another reason for avoiding pedantic specifications in the algorithm and in the Pseudocode(s)

Other research students translated Dowson's version into programming languages of their choice. Brown's C translation was used in undergraduate and MSc coursework and included metrics other than EA as reported in Visvalingam and Brown (1999). As acknowledged in Visvalingam and Brown (1999), Michael Harasimiuk and Roger Whyatt (students on the taught MSc Programme on Computer Graphics and Virtual Environments in 1995/96) were the first to use an albeit erroneous triadic Koch curve to compare algorithms using Brown's and Wade's programs. Whelan's Java translation of Dowson's program was used by Whelan and Visvalingam (2003) for the P-strokes which complemented the novel Formulated Silhouettes. This exploratory strand of research on algorithmic sketching of terrain was regarded as pioneering by members of the Mountain Cartography Group. It has encouraged research on non-photorealistic, artistic visualisation of terrain by Way *et al* (2001), Premoze (2002), Buchin *et al* (2004), Jenny and Patterson (2007) and Bratkova *et al* (2009).

Visvalingam ensured that all implementations were fit for their purpose and that they produced consistent results within the scope for variation as noted in Sections 2.4 and 3.3. Variations in output do not imply a lack of conformance with Visvalingam's specification nor that an implementation is inappropriate. Visvalingam (2015) shows the difficulties involved in testing and certifying that an implementation conforms to the specification, given the allowances for variation.

3.3.2 Model-based filtering

Cartographers eliminate features – not points. The detection of features within curves, without recourse to supplementary information, remains a research challenge. So, Visvalingam started with triangles and found that repeated elimination of the smallest triangle using EA can point to the presence of inline structures, such as inflections and thin elongated features like rivers and spits. Unfortunately, points with the same offset value in Wade's implementation of the RDP algorithm did not depict features in a manner which aided their delineation. Sections of lines belonging to features can be eliminated or segmented with EA. It looked as if the visual clues provided by EA could be heuristically exploited by a separate process to delineate the implied geomorphic features. As noted in Visvalingam (2015), an arbitrary choice of a point for elimination from a tie-break can lead to unbalanced generalisations and impede line segmentation.

If lines could be segmented into their constituent inline features, they can be structured for intelligent feature-based filtering as suggested in Visvalingam and Whyatt (1993). The feasibility of doing this without recourse to the Delaunay Triangulations, was only demonstrated in year 2000 as described in Visvalingam and Whelan (2014, Section 5.5). Visvalingam anticipated that long lines could be segmented and topologically structured to be processed by the Disassociative Area Model; Wade (1986) conceived and programmed this model for extracting the hierarchic area topology from link-and-node structured Ordnance Survey topographic data for the Administrative Areas of England Wales (see also, Wade et al, 1986; Visvalingam et al, 1986; Kirby et al, 1989). Linear data can be topologically structured on other criteria and not just on administrative boundaries. Figure 1 in Visvalingam (1990) represented the model of Digital Cartography within which linear data was to be topologically structured during the Digital Mapping phase for subsequent Visual Mapping both offline and interactive. The automation of this remains a research challenge.

Ariza-Lopez et al (2005) used Visvalingam's algorithm to tag vertices of roads with EA. They then traced the profile of EA against the distance between vertices and used the RDP algorithm to segment the trace and the corresponding roads. Garcia-Balboa and Ariza-Lopez (2009) suggested parameter values for automating this process. Zhou (2014) used the RDP algorithm to segment coastlines, for generalisation using Visvalingam's algorithm with one filter value and then merging the segments with another cut-off. As he concluded, the results were unsatisfactory and line segmentation remains a research challenge.

1. Complex measures, such as weights

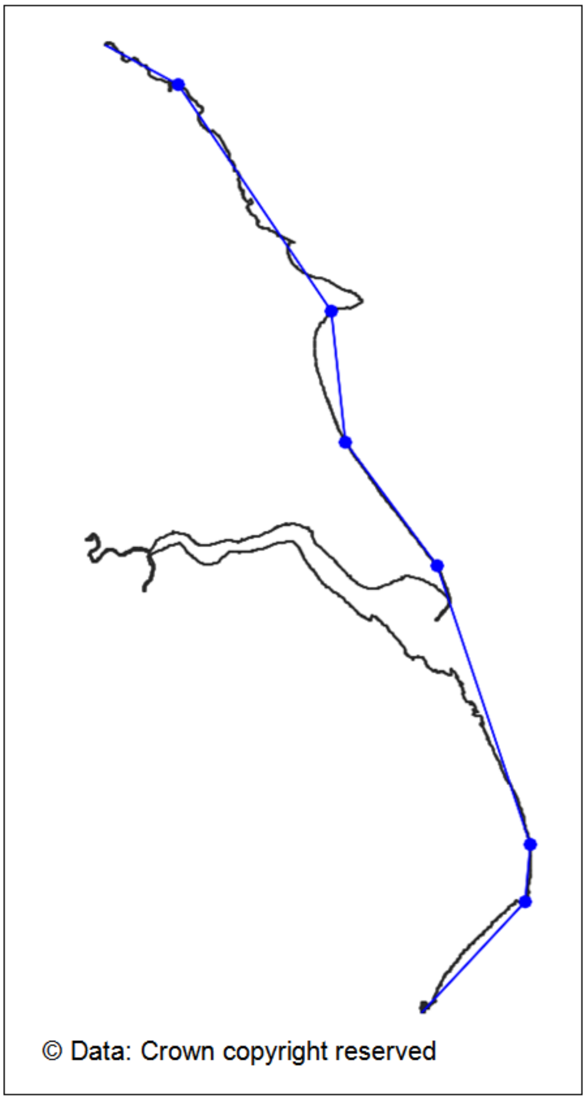

Figure 2: Last 8 points output by Mapshaper v 0.2.0 using EA = EA * (1-cosine (theta))

As we noted in Section 2.2, measures can be derived through interpretation of several qualitative or quantitative metrics and/or measures. Visvalingam and Whyatt suggested that the EA measure could be weighted to change the rank order of points, for example to take account of shape. AD iand EA in Pseudocodes 1 & 2 respectively have a standard weight of 1. Zhou and Jones (2004) proposed a complex formula for using weighted EAs (which they shortened to WEA) to take account of shape. In Mapshaper 0.2.0 Bloch (2014a) used a simpler scheme and down weighted the EA of acute angles only with ***if (theta < 90) EA = EA * (1 – cosine (theta))*** to eliminate spiky detail. Visvalingam and Whelan (2014) found the results promising and cited Bloch's reasons for using this. They also demonstrated (see their Figure 7) how WEA for spike removal can amplify the angular truncation of elongated features, producing inappropriate caricatural generalisations of this coastline. Zhou (2014) was unsuccessful in his attempt to correct a similar problem with his use of shape weightings.

When only the general shape of the coast needs to be shown, Figure 2 provides a much more pleasing depiction of the shape of the coast, compared with Figure 1a. The same figure was also output for the last 8 points by the v0.30.10 version (October 2015) of Mapshaper, which uses Bloch's (2014c) optionD for weighting. Visvalingam and Whelan pointed out that the output of the original EA would also require post processing given the tendency towards spikiness with some types of outlines, such as creeks. Despite these reservations, their view was that the results from both weights were promising and encouraged further research on shape weighting.

5. Filter tolerances

The choice of filter tolerances (also known as cut-offs) depends upon the reasons for point reduction. So far, the figures in this paper have been designed to illustrate the effect of using different heuristics with the Visvalingam algorithm rather than with mapping *per se*. Filtering a given number (or percentage) of points is appropriate for such comparisons. When it comes to mapping, the selection of filter values will depend on the map's purpose, its scale and other considerations, such as line width and spacing, bearing in mind the scope for post-processing.

Some reasons for filtering include the removal of superfluous points which have become coincident on scale reduction, the removal of clutter and preservation of the legibility and quality of line work without incurring topological errors. The automation of all this, let alone the aesthetic quality of the output, is still a challenge. If we just focus on the problem of choosing cut-off values for scale related displays, Nagi (2010) provided a succinct description of Tobler's (1987) rule-of-thumb. It can be expressed as:

resolution = (denominator of the map scale) / (1000 * k), where k = 2

The term, resolution, refers to the size of a feature which can be seen (or resolved) at a given scale. This is dependent on display resolution, such as pixel size. Tobler's rule can be used to select the RDP offset distance. Like the RDP offset distance, the EA is also a metric though based on area (see Section 2.2). Weifang and Li (2012) used Visvalingam's algorithm, in preference to the RDP algorithm, for progressive transmission of data packages using tolerances based directly on EA. The EA avoids computationally demanding square root transformations. However, those preferring to work with linear distances can use the square root of abs(EA) as an indicator of triangle size. This √|EA)| transformation is monotonic and does not change the rank of points.

Mapshaper uses a different formula. Matthew Bloch (personal communication, November 2015) stated that "Users may set the resolution in Mapshaper with the 'interval' option. At present, Mapshaper converts the interval parameter provided by the user to an areal measure using the equation, interval = sqrt (area) * 0.65 (where area is either triangle area or weighted area). The 0.65 factor is based on his subjective judgment, after comparing the effects of Visvalingam and RDP simplification on several sample datasets. He is exploring better solutions".

As Tobler noted, rules-of-thumb only provide a quick first cut at the problem and his formula is used here for purposes of illustration. The SWURCC data used in this study was said to be digitised from a 1:50 000 source and has a precision of 10 metres. A linear cut-off of √|EA)| = 10 metres eliminates only 2.05% of points, mainly duplicate and collinear points and detail in The Scalp area in Lincolnshire, which appears as a blotch at the southern extreme of the coastline. A cut-off of 50 metres loses 30.47% points with a loss of some very minor features. The effects of scale-related filtering become obvious with cut-offs of 100 metres and 1 km (see Figure 3a). None of the maps in Figure 3 have been drawn to scale. The displacement is more noticeable after 250m (not shown).

Between 500m and 800m, the contorted River Ouse starts to cross itself (see Figure 7b in Visvalingam and Whyatt (1993). The problem of self intersection is outside the scope of this paper but is briefly addressed by Visvalingam and Whelan (2014). With a 1 km cut-off, which corresponds to 1: 2 000 000 scale, both the rivers Ouse and Trent drop out (Figure 3c). Figure 3d shows the whole of this coastline at the same level of detail. The AA Road Atlas Britain 2015 retains and exaggerates the Humber on A3 (1: 2 000 000), A4 and smaller formats.

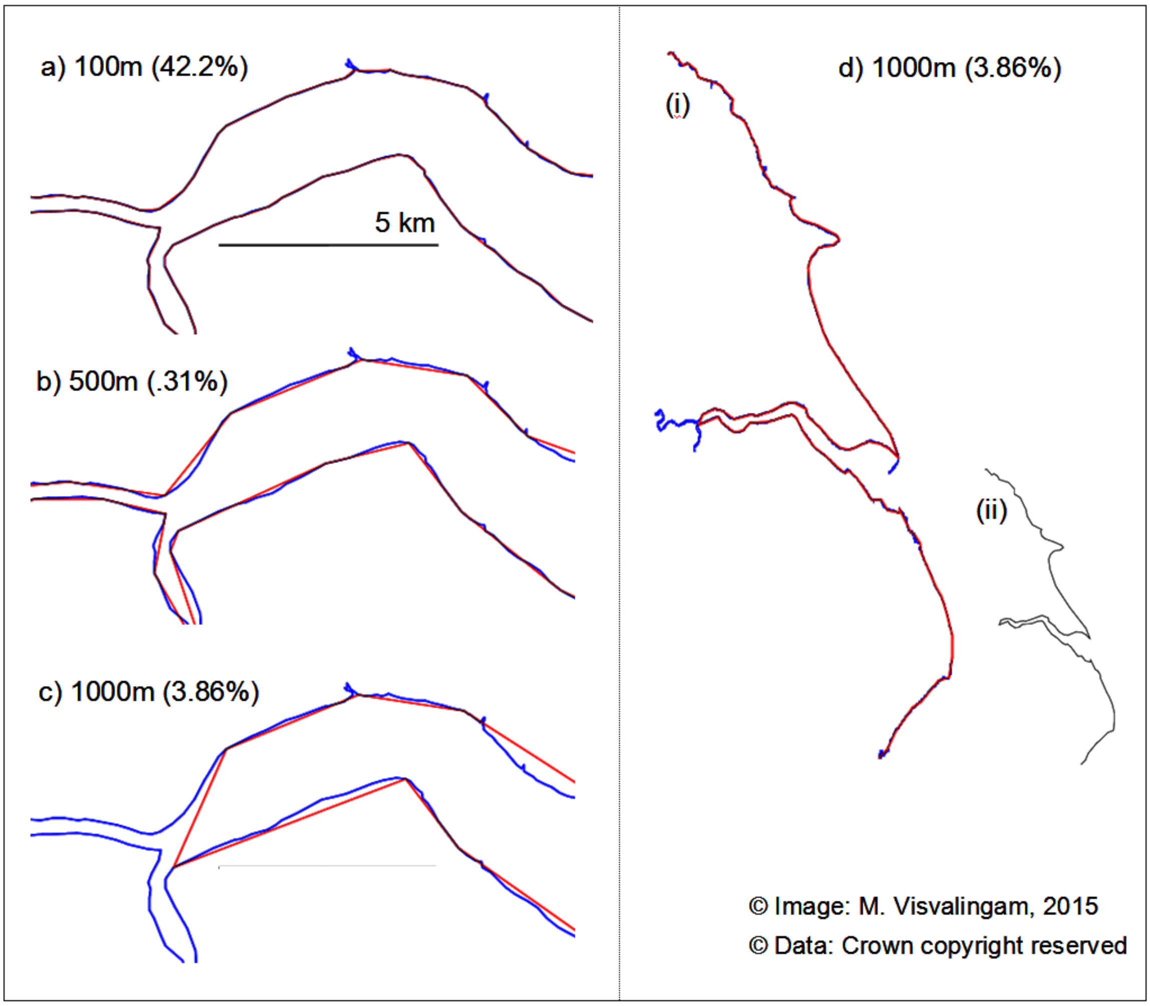

Figure 3: Cut-offs (with % points) for (a) 1:200 000, (b) 1: 1000 000, (c & d) 1:2000 000

Tobler's formula appears to provide a good rule of thumb for filtering EA. However, as Raposo (2010, Section 1.3) noted, Tobler's rule is based on the assumption that "*the smallest mark that can be made on a map is approximately a half-millimetre"*. Given the higher resolution of the line widths that can be output today, the constant k in Tobler's equation can be increased. The output in Figure 3d (i) can be used at the scale of 1: 2000 000. The coastline is still clear in Figure 3d (ii), which is smaller. Raposo (2010) also noted that Tobler had suggested that the value of k be increased to 5 as a rule of thumb. Figure 3d (ii) will print well at 1:5000 000. This shows that the same level of detail (% points) can be used at a range of scales, in much the same way in which the same map of Great Britain has been printed at a range of small scales in the AA Road Atlas Britain 2015.

While Tobler's rule is helpful, it can miss some other useful depictions of polylines, such as those in Visvalingam and Whyatt (1993) and Visvalingam and Whelan (2014). They were found through interactive exploration; this is necessary especially when the changes in feature size are less distinct as in The Scalp data used by Visvalingam and Whelan (2014).

Visvalingam's algorithm was designed to achieve purpose-oriented typification and caricature through the elimination of scale-related features for scale-independent generalisation and not just reduced-scale displays. Visvalingam and Whyatt (1993, p 48) stressed that "*the selection of cut-off values remains subjective, particularly at higher caricatural levels of generalisation. … Since caricatures consist of a minimal set of points, the inclusion or omission of even one point can alter the shape of the feature*."

Although it can be used as a point reduction algorithm, Visvalingam's algorithm was designed to facilitate research into line segmentation and model-based generalisation of curves, as noted in Section 3.3.2. Zhou (2014) has also highlighted the need for research into line segmentation. The real challenge is to find an intuitive and elegant solution, that is transparent and which does not incur excessive computation.

6. Conclusion

This paper has explained why the specification of Visvalingam's algorithm and sample Pseudocode(s) avoided implementation-specific details since these can be purpose-oriented. It explains why Pseudocode 1 is sufficient for expressing the algorithm but goes on to demonstrate why Pseudocode 2 with the concept of EA is more useful. When EA is used for filtering multiple lines, with one or more filter values, it is just an implementation-specific driver. The scope it offers for inferring geometric features makes it a **heuristic indicator** – a rule-of-thumb based on a metric. The subsequent detection of in-line structures (still a research challenge) can support intelligent model-based generalisation. The paper reviewed how the EA-driven algorithm could be used to filter map layers consisting of several lines and to detect significant curvatures in digital terrain models for algorithmic generation of P-stroke sketches. This has inspired others to explore other techniques for non-photorealistic visualisation of terrain. The ideas underpinning P-stroke sketches can be developed further. The paper illustrated how the heuristic driving the algorithm may be based on multiple metrics, for example when EA is weighted to eliminate spiky detail in simplification and typification. The paper also explained why variations in implementation are inevitable and sometimes necessary to suit different purposes.

Although the Visvalingam algorithm is growing in use, there are unresolved issues. For example, there is a need for more research on weighted EAs to suit different purposes and different types of features. The utility of the algorithm is limited by the fact that filter values have to be finalised manually. The problem of resolving self intersections also merits further research. Pertinent background and test data have been provided to enable others to participate in research based on Visvalingam's algorithm. Other research topics will be addressed in forthcoming papers.

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Appendix1: Reason for introducing the concept of the Effective Area

Visvalingam's algorithm consists of the iterative elimination of the least important points and treating the rest as forming the new input line. The area of a triangle provides a scale related measure of '*least important'*. Areal displacement was chosen because it would result in the progressive displacement of the line being processed leading to the elimination of scale-related features. This appendix explains the difference between the Areal Displacement (AD) and the Effective Area (EA) of a point.

What is the Areal Displacement?

For each intermediate point, the area of the triangle it subtends with its two neighbours is calculated. This metric is called the areal displacement to stress that the algorithm was designed to generate progressive displacements of the line using this area metric. Visvalingam's algorithm repeatedly removes the point with the smallest AD and treats what remains as the new line.

The AD was sufficient for Whyatt's (1991) project. His program was run every time the input data had to be filtered to retain the required subset of points. There was no tagging of vertices by a one-off application of Visvalingam's algorithm for subsequent filtering by a separate process. Figure A1 (a) shows the correct order in which his program filteres 6 points. This order equates to the rank of the point. At higher levels of simplification, it is difficult to pre-select cut-off values since the shape can be distorted by the inclusion/omission of even a single point as noted in Section 5. There is a need to explore suitable cut-offs, for example through an interactive program. Visvalingam's algorithm can be used just once to tag points in the database to facilitate this. If points in the database were assigned their rank, this tag may be used to filter points in the right order as in A1 (a).

However, if a point was assigned the AD which led to its removal, the order in which the line is filtered will be incorrect as shown in Figure A1 (b). For when a point is removed, the AD of the two neighbours are recalculated and they can become smaller than the area of the last removed point. In Figure A1 (a) the AD of points 6, 8 and 3 demonstrate this possibility. So, filtering on AD will result in the points being removed in the wrong order with periodic shape distortions as shown in Figure A1 (b).

In Field and Kent (2014, p 158) Visvalingam was cited as saying that step-by-step visualisation was needed for the birth of new insights. It was through such step-by-step visualisation of how the algorithm was processing the data that It became apparent that when the calculated AD is smaller than the previous value, it usually occurs when there is an inflection in the line. Cartographers would normally apply the give-and-take rule to remove such inflections as scale changes. In Visvalingam's algorithm this is achieved by using the Effective Area; i.e. the area value in effect or in use, which need not be the calculated AD. When the calculated AD of a point is less than that of the last eliminated point, the point acquires the value of the latter. This adjustment also has the effect of removing elongated features. If some other metric is used to drive the algorithm, such as offset distances which give similar results to RDP, they too must be adjusted in this way for run-time filtering of tagged datasets.

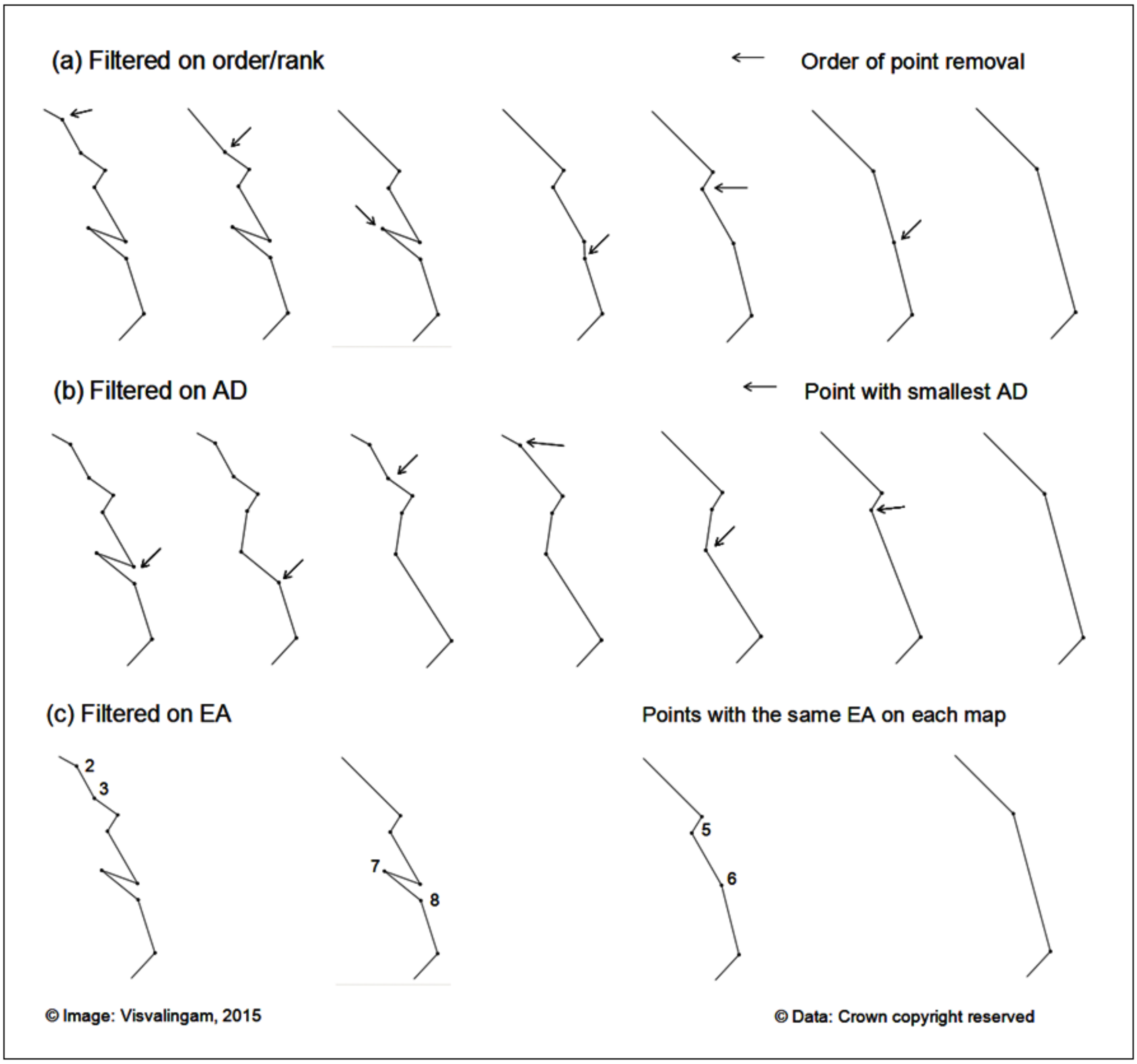

Figure A1: Filtering on (a) Rank, (b) areal displacement (AD), (c) effective area (EA)