A Decision Making Framework for Joint Replenishment and Delivery Scheduling Problems under Mixed Uncertainty

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Abstract

Concerning the essence of risk, a joint replenishment and delivery scheduling problem with fuzzy cost-related parameters and random number of imperfect quality items is developed to make it suitable for the inherent uncertainties of procurement-shipment process. The mathematical modelling-based decision system is formulated as a chance-constrained programming with the idea of embedding decision makers' risk tolerance. Following this notion, the model is translated into an equivalent non-linear counterpart and a neighbourhood heuristic search is designed based on the properties of the cost function. We introduce an integrated cross-entropy algorithm, incorporating the heuristic in the cross-entropy framework, to solve it. The numerical results demonstrate that ICE is quite effective in comparison to state-of-the-art algorithms. Our framework is helpful for decision makers to determine economically acceptable performance objectives in the presence of uncertain issues, and thus to build resilience in supply chain.

Keywords: supply chain resilience, joint replenishment, mixed uncertainty, chance-constrained programming, cross-entropy algorithm

1. Introduction

1.1. Background and motivation

The Shanghai Omicron outbreak, which occurred in early March 2022, caused a significant crisis in China, leading to an excess of 600,000 confirmed cases of illness. The implementation of the government's dynamic zero-COVID plan has resulted in a prolonged two-month period of lockdown and strict restrictions. This has significantly hampered financial growth and imposed severe negative effects on the nation's economic well-being. In light of the limited accessibility of infection prevention measures, such as protective clothing and N95 face masks,

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at affordable prices, it is imperative for the healthcare system to prioritize the procurement and distribution of medical supplies. This will facilitate the strengthening of market resilience. The serious concerns of addressing the spread of epidemics and managing health expenditure have become increasingly pressing due to governmental and community pressures. Fortunately, the successful implementation of vaccination programs has enabled a substantial number of individuals, amounting to hundreds of millions, to effectively evade infection stemming from the pandemic. As of April 2022, China had acquired a total of 3.2 billion vaccine doses, with an associated expenditure exceeding 120 billion US dollars. The optimization of vaccine supply chains and the development of efficient logistics and maintenance scheduling methods are crucial considerations due to the stringent storage requirements of vaccinations.

The implementation of joint replenishment, a strategic approach that facilitates the sharing of significant ordering costs across enterprises, has demonstrated its efficacy as a cost-saving measure inside logistics networks. In the context of a densely populated urban centre such as Shanghai, the establishment of a streamlined supply chain for direct distribution from suppliers to shops poses significant challenges. Typically, the coordination of procurement-shipment activities necessitates the involvement of a third-party warehouse. The joint replenishment and delivery scheduling (JRD) framework is designed to address the challenges related to vaccine supply. It enables the smooth flow of vaccines from pharmaceutical manufacturers to distribution centres, such as central pharmacies, and eventually to retailers like hospitals or vaccination units. This framework is essentially a simplified version of the centralized drug procurement systems in China (Liu et al., 2022). In this study, we examine the scheduling improvements related to replenishment and shipment within a specific scenario involving numerous suppliers, a single distribution centre, and multiple retailers in a three-stage supply chain.

In fact, the replenishment-shipment activities in supply chain, may be subject to "mixed uncertainty" due to various factors, such as the subjective ambiguity or imprecise preferences and objective operational conditions (Arlin Cooper et al., 1996). It is risky to define crisp parameters in mathematical optimization based on inadequate prior knowledge, which may result in unreliable solutions for practical systems. Firstly, it is widely acknowledged that cost coefficients in an unstable market are predominantly influenced by human subjectivity, typically derived from the expertise of experienced professionals. Consequently, this reliance on subjective judgements poses challenges in extracting pricing patterns from historical data (Maiti and Maiti, 2006; Zhao et al., 2023). A concrete illustration can be observed in the interactions between the National Medical Insurance Administration and pharmaceutical producers throughout the period spanning from 2021 to 2022. These engagements have resulted in a series of price reductions for vaccines, with the cost per dosage being successively lowered from 90 yuan to 40 yuan and then to 20 yuan. The presence of cost coefficients in this argument demonstrates the inherent traits of *fuzziness* in nature, as viewed through the lens of fuzzy set theory. Furthermore, it is well acknowledged that in the case of controlled infections such as human papilloma virus, there exists a prevailing notion that individuals may not attain flawless and long-lasting immunity with vaccination (Gandon et al., 2001; Pezzoli and Azman, 2021). This phenomenon can be attributed, in part, to the potential degradation of vaccination efficacy resulting from suboptimal handling and temperature fluctuations encountered throughout the logistic and storage phases. In this particular scenario, there exists a sufficient amount of historical data that allows for the estimation of the probability distribution function of the parameter. Consequently, the utilization of *randomness* is favoured over other alternatives (Khan et al., 2011). Chance-constrained programming (CCP) is a natural modal for dealing with uncertainty, where the vague goal is minimizing the budget under a certain chance value that could also be interpreted as the decision maker's risk tolerance (Charnes and Cooper, 1959). These requirements, necessitates extension of the JRD in a mixed uncertainty framework, something that has received little attention so far.

Motivated by the above points, this paper aims at addressing these gaps in the literature by developing an analytical decision making model for JRD with fuzzy cost-related parameters and stochastic imperfect quality rate, denoted as FSJRD for short. Then, a tailored integrated cross-entropy algorithm (ICE) ia designed by absorbing the advantage of heuristics and meta-heuristics to solve effectively.

1.2. Related literature

The classical joint replenishment problem (JRP) model was firstly proposed and heavily researched since the early works of Shu (1971) and Goyal (1974). After that, several extended JRP models have been put forward incorporating a variety of constraints/parameters into some classical assumptions. Multi-item joint replenishment has become a common practice in business operations, and the subject remains fashionable in academic research. Those who are interested in JRPs may refer to Khouja and Goyal (2008) and Bastos et al. (2017) for knowing the latest extensions and practical applications.

Following the notion of joint replenishment, the deterministic JRD with n-retailer, one-warehouse, and n-supplier was first proposed in Cha et al. (2008). Researchers have also developed several relevant extensions involved very specific problems, which presented a diversity of models for solving multi-product related problems. Wang et al. (2012) established the JRD with stochastic demand. Wang et al. (2013a) investigated a three-level JRD under fuzzy environment. Liu et al. (2017) extended the replenishment and delivery model in a multi-warehouse system. Cui et al. (2020a,b) simultaneously considered the stochastic lead-time and demand. Carvajal et al. (2020) considered the budget and storage capacity constrains in a two-echelon supply chain. Wang and Wang (2022) formulated the case that the distribution of demand is normally-distributed and its mean and variance can be evaluated. Other works with the consideration of dynamic demand can be found in e.g., Kang et al. (2017) and Baller et al. (2019). It is noteworthy to mention that previous studies have shown a positive reception for research on stochastic demand under the premise of perfect quality. However, the consistency of commodities, particularly those with stringent storage requirements like vaccinations, in terms of ideal quality does not align with the practical situation (Gandon et al., 2001; Khan et al., 2011; Pezzoli and Azman, 2021). Given the aforementioned context, it is of significance to construct a model that captures the uncertainty in demand, specifically focusing on faulty items. The exploration of various forms of uncertainty arising from diverse demand-cost patterns is a relatively nascent area of research. One potential solution to address this discrepancy is to implement a framework known as FSJRD, which allows for the utilization of accurate distributions based on historical data or personal experience.

Due to the NP-hard feature of JRPs and JRDs (Arkin et al., 1989; Cha et al., 2008), one difficult point of our FSJRD model is to find an effective solution method, since FSJRD is based on that with fuzzy costs and random number of imperfect items considered additionally. Usually, meta-heuristics seem more suitable to these inventory problems, like many other NP-hard problems. Those more sophisticated algorithms, such as evolutionary computing (e.g. Olsen, 2005), genetic algorithm (e.g. Ongkunaruk et al., 2016; Otero-Palencia et al., 2019), particle swarm optimization algorithm (e.g. Kang et al., 2017), differential evolution algorithm (e.g. Wang et al., 2013a; Cui et al., 2020c), characterized by easy-to-use features are most favorable to researchers. Whereas, heuristics and some special algorithms using the mathematical properties of the problems have been attractive recently. For classic deterministic model, Cha et al. (2008) illustrated that RAND algorithm is considered the best approach in terms of solution quality and computational time. Lee and Yao (2003) and Yao et al. (2020) proved that the JRP model is a piece-wise convex function under power-of-two (PoT) policy. Wang and Wang (2022) introduced Lipschitz optimization method using Lipschitz continuous property of the objective function of stochastic JRD. Wang et al. (2023) demonstrated the favourable performance of an iterative heuristic algorithm that is based on the RAND algorithm in addressing the location-inventory-delivery model. All of the aforementioned algorithms possess certain limitations. Meta-heuristics are not able to provide a guarantee of producing solutions of high quality, particularly when dealing with problems of a large scale. The design of heuristics involves the creation of a solving thread that is capable of providing unique solutions. This thread is tailored to address specific optimization issues and relies heavily on the qualities inherent to those problems. In nature, when flexibility and complexity are considered, heuristic coupled with meta-heuristic is a preferred solution strategy by sharing advantages of both–our focus in this paper. This insight may be a guidance for designing efficient algorithms to solve JRP- or JRD-type problems.

The cross-entropy (CE) method of Rubinstein (1999), known also as a stochastic search algorithm for rare event estimation, has numerous applications in various of constrained combinatorial (discrete) optimization problems, such as closed-loop supply chain planning (Wang et al., 2016), facility layout problem (Ning and Li, 2018), among others. Considering that the JRDs consists of only one continuous variable with limited search range, we may adopt the equally-spaced discretization method to deal with it. In this case, CE method provides a simple and efficient tool for solving such discrete problems. It essentially transforms an optimization into a rare probability event estimation problem. Due to its precise mathematical framework and tailored updating rules, Rubinstein and Kroese (2004) conducted a survey that it has a polynomial time complexity and high searching ability. To the best of our knowledge, we are the first to utilize the CE-based algorithm as a solution methodology to handle inventory problems.

1.3. Contribution and organization

Based on these descriptions, the novelty and the main contributions of this paper are summarized as follows:

1. For the family of JRD model, a novel component of it considers the mutual influence of fuzzy cost and random number of imperfect quality items, which is suitable to accommodate the inherent uncertainties in supply chain.

2. The chance-constrained programming is initially adopted in the practical inventory scenario with the consideration of decision makers' risk tolerance additionally. We then derive the tractable reformulation of FSJRD model by leverage recent advances in uncertainty theory.

3. To better hedge against model computational complexity, a neighbourhood heuristic search is designed stemming from the optimality and improvement conditions of the cost

function. An integrated cross-entropy algorithm (ICE) incorporating the heuristic is proposed to improve the quality of solutions and promote convergence speed-up.

The rest of this paper proceeds as follows. Section 2 describes the mathematical framework of the proposed FSJRD model under known distributions. Correspondingly, the CCP model and its crisp counterpart are formulated. Section 3 presents several propositions inherited in the equivalent model. We then propose a neighbourhood heuristic search approach. In Section 4, an ICE algorithm incorporating the heuristic is introduced with a redesigned solution structure. Numerical experiments and elaborate comparisons are presented in Section 5, where we illustrate the effectiveness and efficiency of the ICE compared to the state-of-the-art algorithms, and the impacts of uncertainty-related parameters. Section 6 summaries the main findings of this research, and provides several remarks and lines on the management sights.

2. Model formulation

In this section, we consider the specific situation of a three-stage supply chain consisting of multi-suppliers, one distribution centre (DC), and multi-retailers. The DC joint coordinates the scheduling of replenishment process from upstream suppliers and delivery process to downstream retailers, based on the demand and cost information. Both of them directly affect the total system cost, but of different nature, and represent uncertainties of two different types correspondingly. Also, a relevant CCP model is employed to construct a well-defined modelling-based inventory system with risk tolerance level of decision makers.

2.1. Proposed FSJRD

The classic JRD in a deterministic framework for items replenishment might lead to inferior solutions and leave the supply chain vulnerable to risk event, since it assumes away the intrinsic feature of uncertainty in practical operations. In response to this concern, we develop a new mixed-uncertainties decision model, namely FSJRD, which concentrates on fuzzy unit costs and random number of imperfect items. Before we proceed further, the necessary mathematical notations of our FSJRD model including the decision variables and relevant parameters are reported in Table 1. Note that throughout this paper, a fuzzy variable is associated with a tilde sign.

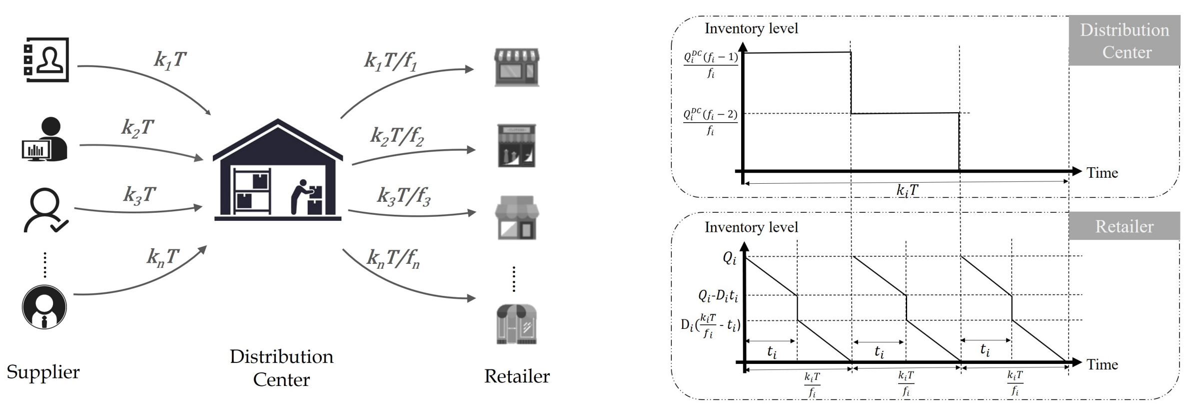
To be a little more detailed here, information uncertainties in supply chain are of diverse types. According to Sazvar et al. (2021), the occurrence of randomness is observed when a sufficient amount of data is available, whereas fuzziness is associated with a state of limited information. Using vaccine allocation as a case study, it can be observed that the pricing of vaccines is determined through a process of negotiation between governmental entities and vaccine makers. In this context, it is conceivable to establish linguistic parameters indicating that the cost of purchasing per order could range from \$7 to \$8, with the condition that it cannot be lower than \$6 or exceed \$9 (Vujošević et al., 1996; Maiti and Maiti, 2006). In many cases, the expenses related to things often encompass vague or unclear data. In contrast, the major ordering cost is not contingent upon the specific item and can be easily regulated, so it is seen as deterministic. If deemed required, it is possible to infer a broader fuzzy situation. In line with the literature Wang et al. (2013b) and Sazvar et al. (2021), triangular and trapezoidal fuzzy numbers are employed to describe the cost parameters. Further, we assume that the imperfect rate of each replenished item is a random variable and follows Beta distribution by

Table 1: Notation for the proposed FSJRD problem.

Decision variables	
T	The basic supply cycle time, $T \in (0, 1)$
k_i	A positive integer representing the replenishment frequency of item i
f_i	A positive integer representing the outbound frequency of item i
Parameters	
i	The index of item, $i = 1, 2, \ldots, n$
D_i	The demand rate for qualified items i per unit time
S	The fixed major ordering cost for each replenishment
$\tilde{s}_i^{DC}$	The minor ordering cost of item i, $(s_{i\_min}^{DC}, s_{i\_mid1}^{DC}, s_{i\_mid2}^{DC}, s_{i\_max}^{DC})$
$\tilde{s}_i$	The outbound transportation cost of item i, $(s_{i\_min}, s_{i\_mid1}, s_{i\_mid2}, s_{i\_max})$
$\tilde{h}_i^{DC}$	The holding cost for item i in DC per unit time, $(h_{i\_min}^{DC}, h_{i\_mid}^{DC}, h_{i\_max}^{DC})$
$\tilde{h}_i$	The holding cost of item i in retailer per unit time, $(h_{i\_min}, h_{i\_mid}, h_{i\_max})$
p_i	The imperfect rate of item i in each replenishment, $p_1 \sim Beta(a_i, b_i)$
x_i	The screening rate of item i per unit time
α_0	The confidence level, predetermined by the decision-maker, $\alpha_0 \in (0, 1]$

referring to the assumption of Cui et al. (2020c). The benefits of appropriately treating the uncertainties of underlying data or parameters are in terms of risk reduction.

Fig. 1(a) shows n items involved in the replenishment and delivery activities. The basis cycle time T is assumed as the benchmark period. Under indirect grouping strategy, the period between successive replenishments of item i from its supplier is k_i (replenishment frequency) multiple of T (basis cycle time), i.e., $T_i = k_i T$. The delivery of item i to the retailer who orders it, is placed f_i (outbound frequency) times during the replenishment period T_i, i.e., the outbound period of item i is defined as $k_i T / f_i$. In order to avoid shortages, the expected replenishment quantity for item i may be interfered and given by $Q_i^{DC}(k_i, f_i, T) = D_i k_i T / (1 - E[p_i])$, where $E[p_i]$ represents the expectation value of imperfect rate. Subsequently, the outbound transportation quantity of item i can be calculated by $Q_i(k_i, f_i, T) = D_i k_i T / ((1 - E[p_i]) f_i)$.



(a) The basic structure in a JRD procedure. (b) Example of the inventory profiles of item i.

Figure 1: The structure and inventory profiles for FSJRD system.

Herein, we show the coordination in our FSJRD system in Fig. 1(b). As depicted, the outbound period remains the same throughout the planning horizon, and the inventory level of DC does not change until the next period comes. By referring to Paul et al. (2014), after item i is delivered to its retailer, it will be screened at the rate x_i to identify the imperfect quality items, and so the screening time is denoted as $t_i = Q_i/x_i$. Based on this, we get the average inventory levels for DC and retailer as follows:

$$I_i^{DC}(k_i, f_i, T) = \frac{D_i k_i T(f_i - 1)}{2(1 - E[p_i])f_i} \quad \text{and} \quad I_i(k_i, f_i, T) = \frac{D_i k_i T}{2f_i} + \frac{D_i^2 E[p_i] k_i T}{x_i(1 - E[p_i])^2 f_i}. \tag{1}$$

Then, the total cost can be calculated as the sum of fixed major ordering, minor ordering, transportation, and inventory holding costs for DC and retailers, which is given as

$$\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T) = \frac{S}{T} + \sum_{i=1}^{n} \frac{\tilde{s}_i^{DC}}{k_i T} + \sum_{i=1}^{n} \frac{\tilde{s}_i f_i}{k_i T} + \sum_{i=1}^{n} \tilde{h}_i^{DC} I_i^{DC} + \sum_{i=1}^{n} \tilde{h}_i I_i, \tag{2}$$

where $\boldsymbol{k} = (k_1, k_2, \ldots, k_n)$ and $\boldsymbol{f} = (f_1, f_2, \ldots, f_n)$ are both decision vectors.

Clearly, for a given feasible solution $(\boldsymbol{k}, \boldsymbol{f}, T)$, the total cost $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ is a regular fuzzy interval. If we adopt the classic objective, the meaning of the minimum total cost (i.e., minimum a fuzzy interval) is not so clear. As indicated by Liu and Iwamura (1998), a decision-maker may concern more about the reliability of the optimal solution, which is reflected by his/her risk tolerance. In this case, a meaningful and solvable optimization model of FSJRD is formulated for the first time using the CCP model with the credibility measure[1]. Analogously to the spectrum of model (A.10), combining the constraints on k_i, f_i and T, the new optimization problem can be constructed as follows:

$$\begin{cases} \min\limits_{k,f,T} \min \ \bar{b} \\ \text{s.t.} \quad \text{Cr}\{\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T) \leq \bar{b}\} \geq \alpha_0 \\ \quad \text{Eqs. (1)} - (2) \\ \quad 0 < T < 1 \\ \quad k_i, f_i \geq 1, \quad i = 1, 2, \ldots, n, \ \text{integers.} \end{cases} \tag{M1}$$

where $\bar{b}$ can be audited as the budget to systematically exploit the positive aspects of uncertainty events and get the bad influences well under control.

2.2. Model transformation

It is known that CCP is an exceptionally complex model whose solution is very challenging to be calculated and it can be generally achieved through simulations (see, e.g. Liu and Iwamura, 1998). However, the simulation processes are usually very time-consuming especially when the number of contained fuzzy intervals increases and/or the functions involved are high dimensional, and/or most importantly, the results might not be reliable either. Aiming at appropriately dealing with these problems, this section provides the new theoretical background needed to transform model (M1) into its crisp non-linear counterpart. Given feasible solution $(\boldsymbol{k}, \boldsymbol{f}, T)$, the value of "$\min \bar{b}$" in model (M1) is exactly the α-pessimistic value[2] of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$, i.e., $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0)$. This section is to introduce the reader to the underlying mathematics and the analytic expression of "$\min \bar{b}$", i.e., $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0)$.

[1]See appendix 1, Definition A.4 for the concept of credibility measure

[2]See appendix, Definition A.5.

Lemma 1. *For any given feasible solution* $(\boldsymbol{k}, \boldsymbol{f}, T)$*, the fuzzy objective function* $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ *given by Eq. (2) is continuous, and increasing with respect to* $\tilde{s}_i^{DC}, \tilde{s}_i, \tilde{h}_i^{DC}$ *and* $\tilde{h}_i$*, respectively.*

Proof: The proof is provided in the Appendix B.1 of the appendix.

Proposition 1. *For any given feasible solution* $(\boldsymbol{k}, \boldsymbol{f}, T)$*, the* α*-pessimistic value of the fuzzy objective function* $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ *is given by*

$$\begin{aligned}\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha) = \frac{S}{T} + \sum_{i=1}^{n} \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha)}{k_i T} + \sum_{i=1}^{n} \frac{(\tilde{s}_i)_{\inf}(\alpha) f_i}{k_i T} + \sum_{i=1}^{n} \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha) D_i k_i T (f_i - 1)}{2(1 - E[p_i]) f_i} \\ + \sum_{i=1}^{n} \frac{(\tilde{h}_i)_{\inf}(\alpha) D_i k_i T}{2 f_i} + \sum_{i=1}^{n} \frac{(\tilde{h}_i)_{\inf}(\alpha) D_i^2 E[p_i] k_i T}{x_i (1 - E[p_i])^2 f_i},\end{aligned} \tag{3}$$

wherein if $0 < \alpha \leq 0.5$*,*

$$\begin{aligned}&(\tilde{s}_i^{DC})_{\inf}(\alpha) = (1-2\alpha) s_{i\_min}^{DC} + 2\alpha s_{i\_mid1}^{DC}, \quad (\tilde{s}_i)_{\inf}(\alpha) = (1-2\alpha) s_{i\_min} + 2\alpha s_{i\_mid1}, \\ &(\tilde{h}_i^{DC})_{\inf}(\alpha) = (1-2\alpha) h_{i\_min}^{DC} + 2\alpha h_{i\_mid}^{DC}, \quad (\tilde{h}_i)_{\inf}(\alpha) = (1-2\alpha) h_{i\_min} + 2\alpha h_{i\_mid}^{DC},\end{aligned} \tag{4}$$

and if $0.5 < \alpha \leq 1$*,*

$$\begin{aligned}&(\tilde{s}_i^{DC})_{\inf}(\alpha) = (2-2\alpha) s_{i\_mid2}^{DC} + (2\alpha - 1) s_{i\_max}^{DC}, \quad (\tilde{s}_i)_{\inf}(\alpha) = (2-2\alpha) s_{i\_mid2} + (2\alpha - 1) s_{i\_max}, \\ &(\tilde{h}_i^{DC})_{\inf}(\alpha) = (2-2\alpha) h_{i\_mid}^{DC} + (2\alpha - 1) h_{i\_max}^{DC}, \quad (\tilde{h}_i)_{\inf}(\alpha) = (2-2\alpha) h_{i\_mid} + (2\alpha - 1) h_{i\_max}.\end{aligned} \tag{5}$$

Proof: The proof is provided in the Appendix B.2 of the appendix.

Until now, the arithmetic operation of the equivalent analytical formula has been derived, and thus its α-pessimistic value is explicitly and directly computed. The formula is utilized to implement the fuzzy to crisp and deterministic conversion process. Based on Proposition 1, the reformulated model is demonstrated by

$$\begin{cases} \min \ \widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0) \\ \text{s.t.} \quad \text{Eqs. } (3) - (5) \\ \quad 0 < T < 1 \\ \quad k_i, f_i \geq 1, \quad i = 1, 2, \ldots, n, \text{ integers.} \end{cases} \tag{M2}$$

Thus, the optimal solution of k_i, f_i and T can be obtained via the deterministic mixed-integer model (M2) with a non-linear objective function. As a result, it is now possible to solve the CCP model without simulation, and so with lower computational complexity and higher accuracy. However, it has been determined that the model (M2) is a highly complex problem that falls within the category of NP-hard problems. This suggests that finding an analytical solution for it may be infeasible. This study presents the introduction of a meta-heuristic approach, namely an integrated cross-entropy algorithm, as a means to address the aforementioned crucial problem and achieve a solution of superior quality.

3. Neighbourhood heuristic search

Meta-heuristic algorithms, such as the Genetic Algorithm (GA), have demonstrated satisfactory efficacy in addressing deterministic JRDs. Nevertheless, a significant proportion of these algorithms exhibit elevated computational cost and reduced stability, hence posing a notable limitation when applied to larger-scale problems. Taking into consideration the aforementioned points, there is an expectation for the creation of a heuristic methodology that can ensure improved initial feasible solutions and convergence. In this part, we provide a neighbourhood heuristic search algorithm that exhibits qualities of reliability, accuracy, and time-efficiency. The unique heuristic approach is developed based on the premise of determining the optimality conditions of choice variables and developing the improvement conditions of current solutions. The fundamental characteristics and principles for the development of these algorithms are outlined below:

For our purpose, the mathematical structure of model (M2) is restated as follows. For each retailer i, the cost function $\Theta_i(k_i, f_i, T)$ for item i is given by

$$\Theta_i(k_i, f_i, T) = \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha_0)}{k_i T} + \frac{(\tilde{s}_i)_{\inf}(\alpha_0) f_i}{k_i T} + \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha_0) D_i k_i T (f_i - 1)}{2(1 - E[\,p_i\,]) f_i} + \frac{(\tilde{h}_i)_{\inf}(\alpha_0) D_i k_i T}{2 f_i} + \frac{(\tilde{h}_i)_{\inf}(\alpha_0) D_i^2 E[\,p_i\,] k_i T}{x_i (1 - E[\,p_i\,])^2 f_i}.$$

Hence, we have

$$\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0) = \frac{S}{T} + \sum_{i=1}^{n} \Theta_i(k_i, f_i, T). \tag{6}$$

3.1. Identifying optimality conditions

From Eq. (6), an explicit formula for the best outbound frequency of items i that achieves the lowest cost is deduced, which is a function of k_i and T.

Proposition 2. *For given fixed values of k_i and T, there is a function δ such that the best value of f_i is uniquely given by*

$$f_i^*(k_i, T) = \left\lfloor \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\delta(k_i, T)} \right\rfloor, \tag{7}$$

where $\lfloor x \rfloor$ represents that the floor function maps x to the largest integer smaller than or equal to x and

$$\delta(k_i, T) = k_i^2 T^2 D_i \frac{(\tilde{h}_i)_{\inf}(\alpha_0) x_i (1 - E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha_0) D_i E[p_i] - (\tilde{h}_i^{DC})_{\inf}(\alpha_0) x_i (1 - E[p_i])}{2(\tilde{s}_i)_{\inf}(\alpha_0) x_i (1 - E[p_i])^2}.$$

Proof: The proof is provided in the Appendix B.3 of appendix.

It is noticed that the cost function is convex in T, and thus the best value of T can be derived by the first order derivative of Eq. (6).

Proposition 3. *For given fixed values for $\boldsymbol{k}$ and $\boldsymbol{f}$, the best value of T is given by*

$$T^*(\boldsymbol{k}, \boldsymbol{f}) = \sqrt{A(k, f)/B(k, f)}, \tag{8}$$

wherein

$$A(\boldsymbol{k},\boldsymbol{f}) = S + \sum_{i=1}^{n} \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha_0) + (\tilde{s}_i)_{\inf}(\alpha_0) f_i}{k_i} \tag{9}$$

and

$$B(\boldsymbol{k},\boldsymbol{f}) = \sum_{i=1}^{n} D_i k_i \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha_0) x_i (1-E[p_i])(f_i-1) + (\tilde{h}_i)_{\inf}(\alpha_0) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha_0) D_i E[p_i]}{2x_i(1-E[p_i])^2 f_i}. \tag{10}$$

Proof: The proof is provided in the Appendix B.4 of appendix.

3.2. Constructing improvement conditions

We now construct a conditional improvement for the existing solutions to identify a higher-performance population such as to make the whole procedure computationally efficient.

Proposition 4. *For a given solution $(\boldsymbol{k},\boldsymbol{f},T)$, if the basic replenishment cycle T is not larger than the critical replenishment cycle T_i^{1*} for item i, where*

$$T_i^{1*} = \sqrt{\frac{2x_i f_i (1-E[p_i])^2 \left((\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i\right)}{k_i(k_i+1)D_i \left((\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i-1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]\right)}}, \tag{11}$$

then a vector $\hat{\boldsymbol{k}} = (k_1,\ldots,k_i{+}1,\ldots,k_n)$ exists such that $\widetilde{TC}(\hat{\boldsymbol{k}},\boldsymbol{f},T)_{\inf}(\alpha_0) \le \widetilde{TC}(\boldsymbol{k},\boldsymbol{f},T)_{\inf}(\alpha_0)$.

Proof: The proof is provided in the Appendix B.5 of appendix.

Proposition 5. *For a given solution $(\boldsymbol{k},\boldsymbol{f},T)$, if the basic replenishment cycle T is not less than the critical replenishment cycle T_i^{2*} for item i, where*

$$T_i^{2*} = \sqrt{\frac{2x_i f_i (1-E[p_i])^2 \left((\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i\right)}{k_i(k_i-1)D_i \left((\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i-1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]\right)}}, \tag{12}$$

then a vector $\hat{\boldsymbol{k}} = (k_1.\ldots,k_i{-}1,\ldots,k_n)$ exists such that $\widetilde{TC}(\hat{\boldsymbol{k}},\boldsymbol{f},T)_{\inf}(\alpha_0) \le \widetilde{TC}(\boldsymbol{k},\boldsymbol{f},T)_{\inf}(\alpha_0)$.

Proof: The proof is provided in the Appendix B.6 of appendix.

3.3. Heuristic design

On the solid foundations of the above Propositions 2-5, a neighbourhood heuristic search can now be formulated. The idea stems from a mechanism of adding (or subtracting) 1 for each element of the existing solutions. To do so, we assume that the values of k_i and T are known and serve as inputs. Then, from Proposition 2, the best values of $f_1, f_2, \ldots, f_n$ are calculated via Eq. (7). Thus, a feasible solution $\boldsymbol{\vartheta} = (k_1,\ldots,k_n,f_1,\ldots,f_n,T)$ is obtained. According to Eqs. (11) and (12), by just adding (or subtracting) 1 now from k_i, the new solution is always better than the previous one. In addition, if the value of k_i is revised, a direct way to further improve the quality of the solution by utilizing Proposition 2 is also possible. Finally, from Proposition 3, the optimal value of T is precisely calculated. Algorithm 1 describes the detailed steps of the heuristic.

Algorithm 1: Neighbourhood heuristic search

Input: The set $\omega = (k_1, \ldots, k_n, T)$, and the lower and upper bounds of k_i, i.e., k_i^{low} and k_i^{up}.
Output: The high-quality solution $\hat{\boldsymbol{\vartheta}}$.
According to the set ω, compute the best value of f_i using Eq. (7).
for $i = 1, \ldots, n$ **do**
  Compute the thresholds T_i^{1*} and T_i^{2*} using Eqs. (11) and (12);
  if $T \leq T_i^{1*}$ *and* $k_i + 1 \leq k_i^{up}$ **then**
    Set $\hat{k}_i = k_i + 1$;
  else if $T \geq T_i^{2*}$ *and* $k_i - 1 \geq k_i^{low}$ **then**
    Set $\hat{k}_i = k_i - 1$;
  Compute the best value of f_i, denoted as $\hat{f}_i$, using Eq. (7).
Compute the best value of T, denoted as $\hat{T}$, using Eq. (8).
Return the high-quality solution $\hat{\boldsymbol{\vartheta}} = (\hat{k}_1, \ldots, \hat{k}_n, \hat{f}_1, \ldots, \hat{f}_n, \hat{T})$.

4. Integrated cross-entropy framework

In this section an ICE solution framework is developed by absorbing the exploration effectiveness of CE method in solving integer programming problems. Considering only one continuous variable T involved in model (M2), an equally-spaced discretization approach is employed to divide its search range into several sub-intervals. We then iteratively optimize the quality of each solution by using the proposed neighbourhood heuristic search.

Before we proceed further, it should be mentioned that each sample in the ICE is stated through an $(n+1)$-ary vector ω, where n represents the number of items. The vector contains values of replenishment frequencies k_i and the basic cycle time T, i.e., $\omega = (k_1, \ldots, k_n, T)$. In the following, the structure and detailed steps of ICE with examples are described.

4.1. Sample generation mechanism

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_d)$ which consists all the possible values, and $\boldsymbol{p} = (p_1, p_2, \ldots, p_d)$, $p_1 + p_2 + \cdots + p_d = 1$ which represents the corresponding probability of a random variable. The combination of $\boldsymbol{x}$ and $\boldsymbol{p}$ can be regarded as the distribution law of the discrete variable.

For one thing, if the decision variable refers to the replenishment frequency (k_i) with the lower and upper bounds k_i^{low} and k_i^{up}, the elements in $\boldsymbol{x}$ are exactly all the integers in the interval $[k_i^{low}, k_i^{up}]$. For another, considering that the special variation range of the continuous variable, T, is smaller than 1, the equally-spaced discretization method is adopted to handle it. Denote T^{low} and T^{up} as the lower and upper bounds of T. The operation process divides the range (T^{low}, T^{up}) into m equally-spaced sub-intervals [3]. In this case, the elements in $\boldsymbol{x}$ are essentially m sub-intervals rather than real numbers. It is a rather novel idea to generate random samples for continuous decisions.

We define a matrix P^G in the G^{th} iteration, which comprises a family of probability density functions. Note that the initial policy for each decision variable is assumed to be an uniformly random one. Subsequently, a set of discrete probability distributions forms the matrix P^G, i.e., $P^G = (\boldsymbol{p}_{k_1}^G, \ldots, \boldsymbol{p}_{k_n}^G, \boldsymbol{p}_T^G)$. Based on the distribution laws for k_i and T, the selected values of the

[3] The value of m is determined by the decision makers.

k_i and T can be randomly constructed in a straightforward way, i.e., Monte Carlo simulations[4]. In this regard, the vector $\omega = (k_1, \ldots, k_n, T)$ is obtained. At last, the correspondingly high-quality solution $\hat{\boldsymbol{\vartheta}}$ is generated through the neighbourhood heuristic search in Algorithm 1.

4.2. Elite samples retention mechanism

For each individual $\hat{\boldsymbol{\vartheta}}$ in the population, the fitness is calculated on the basis of the objective function in model (M2), denoted as $S(\hat{\boldsymbol{\vartheta}})$. To ensure that the elite samples of the next generation are not worse than the last ones, a certain number of best samples is maintained and copied to the next generation. We then sort the fitness of all individuals in a non-decreasing order, and return $\lceil \rho K \rceil$ elite values and their corresponding samples, where K is the number of population, ρ is the percentage of elite individuals, and $\lceil x \rceil$ represents that the ceiling function maps x to the minimum integer larger than or equal to x. Clearly, the ρ sample quantile of the performances can be roughly set as the fitness of the $\lceil \rho K \rceil^{th}$ sample, i.e., $S(\hat{\boldsymbol{\vartheta}}_{\lceil \rho K \rceil})$. In this way, the objective value varies monotonically during the iteration process.

4.3. Parameter update mechanism

In the G^{th} iteration, the new matrix $\hat{P}^G$ is built using the characteristics of elite individuals with the conditions that each row sums up to 1. We could obtain the elements in $\hat{P}^G = (\hat{\boldsymbol{p}}_{k_1}^G, \ldots, \hat{\boldsymbol{p}}_{k_n}^G, \hat{\boldsymbol{p}}_T^G)$ as

$$\hat{\boldsymbol{p}}_{k_i,j}^G = \frac{\sum_{k=1}^{\lceil \rho K \rceil} I_{\{\hat{\boldsymbol{\vartheta}}_k(k_i) = k_i^{low}+j-1\}}}{\lceil \rho K \rceil}, j = k_i^{low}, \ldots, k_i^{up} \text{ and } \hat{\boldsymbol{p}}_{T,j}^G = \frac{\sum_{k=1}^{\lceil \rho K \rceil} I_{\{T_j^{low} < \hat{\boldsymbol{\vartheta}}_k(T) \leq T_j^{up}\}}}{\lceil \rho K \rceil}, j = 1, \ldots, m, \tag{13}$$

where T_j^{low} and T_j^{up} are respectively the lower and upper bounds for the j^{th} sub-interval. Using a smoothed updating procedure, the matrix in the next generation is updated as

$$P^{G+1} = \lambda \hat{P}^G + (1 - \lambda) P^G, \tag{14}$$

where $\lambda \in (0, 1)$ represents the transition probability. Meanwhile, since less information is obtained in the early stage of iteration, the smooth parameter is increasingly changed. Instead of taking a predetermined value, the smoothing parameter is defined by $\lambda = 1 - 0.8e^{1 - \frac{G_{max}^2}{G_{max}^2 + 1 - G^2}}$.

In terms of parameter updating for the continue variable (T), the search range that will be sought in the next generation is also need to reconstruct via a sufficient small factor ϵ. The idea is to reduce the length of the search range of T. In other words, a sub-interval will not be considered if its corresponding probability is less than ϵ, and the new search range will be divided again into a new set of m sub-intervals. The detailed steps are displayed in Algorithm 2.

[4]The choosing possible value is a sub-interval for T, and we uniformly take a value from that range to get a specific number.

Algorithm 2: Reconstruction process for the continuous variable T

Input: The original distribution law of T in generation G, $\boldsymbol{p}_T^G$, the number of sub-intervals $|m|$ and a tolerance factor ϵ.

Output: The reconstructed distribution law of T in generation G, $\boldsymbol{p}_T^G$.

Find the probability degree greater than ϵ and corresponding sub-interval in $\boldsymbol{p}_T^G$. Denote m_e as the number of eligible sub-intervals.

if $m_e < m$ **then**

  Each eligible sub-interval and corresponding probability are equally divided into m shares;

  Every m_e shares constitute a group;

  The elements in each group constitute a new sub-interval;

  The probability degree of the new sub-interval is the sum of the probabilies in each group;

  Normalize the probability degrees to sum up to 1.

The combination of the new sub-intervals and normalized probability degrees forms $\boldsymbol{p}_T^G$.

5. Numerical experiments

In this section, numerous experiments are performed to illustrate the superiority of MRAND and ICE[5]. Meanwhile, sensitivity analysis is conducted to reveal the effects of the uncertainty-related parameters involved, i.e., confidence level and imperfect rate, on the objective value and the solution. All algorithms are encoded in Matlab2017 on a Windows 10 platform, and the computational experiments are run on Intel Core i7 with 2.60 GHz and 8.00 GB of RAM. By referring to Cha et al. (2008) and Carvajal et al. (2020), and combining the result of joint replenishment, the bounds of k_i and f_i are set to $[1, 10]$, and $(0, 1)$ for T, accordingly. In the rest of paper, the confidence level α_0 is generally set as 0.8. All algorithms run ten times to obtain reliable results.

5.1. Experiment 1: Validity of the improvements for CE

The proposed ICE for tackling FSJRD combines an equally-spaced discretization method with a neighbourhood heuristic approach. To assess the effects of the improvements on the searching performance, three algorithms, including CE, CE plus discretization method (DCE) and CE plus heuristic approach (HCE), are compared with ICE. The value of ratio ρ, i.e., the percentage of elite samples, is 0.2 for all the above algorithms. The smooth parameter for CE and HCE is set as a constant number 0.4, while for DCE and ICE, it is an increasing function.

The general testing instance contains 20 items, and Table C.1 in the appendix shows the detailed data. We then conduct four sub-experiments based on the data in Table C.1, i.e., the problem scales are set to 5 (items 1 to 5), 10 (items 1 to 10), 15 (items 1 to 15), 20 (items 1 to 20), respectively. Meanwhile, the fixed major ordering cost, i.e., S, is constantly set as 200. Table 2 reports the average-found target values, average error and average computational time for each algorithm.

[5] Due to space limitation, some details and complementary results of the numerical experiments conducted here are moved in appendix.

Table 2: Computational results showing the different improvements for CE.

Item num	CE Ave. $\bar{b}_{\min}(\alpha_0)$	CE Ave. Err.	CE Ave. Time	DCE Ave. $\bar{b}_{\min}(\alpha_0)$	DCE Ave. Err.	DCE Ave. Time	HCE Ave. $\bar{b}_{\min}(\alpha_0)$	HCE Ave. Err.	HCE Ave. Time	ICE Ave. $\bar{b}_{\min}(\alpha_0)$	ICE Ave. Err.	ICE Ave. Time
5	7441.94	0.2538	1.6941	7423.09	0.0000	1.3751	7423.09	0.0000	0.5617	7423.09	0.0000	0.4920
10	13363.17	5.5933	3.3281	13080.80	3.3621	2.2479	12655.32	0.0000	1.4450	12655.32	0.0000	0.9961
15	18033.23	8.1394	4.1826	17570.58	5.3651	3.3419	17006.00	1.9794	2.6936	16675.91	0.0000	1.3094
20	20788.25	9.5774	5.6129	20342.60	7.2283	4.6831	19362.74	2.0634	3.1266	18971.29	0.0000	1.6387

$\bar{b}_{\min}(\alpha_0)$: the objective value under the confidence level α_0.

As shown in Table 2, referring to the average error, ICE gets the lowest error, while CE performs the highest. Generally, as the problem scale increases, the searching efficiency of all the algorithms is getting worse, but ICE retains the lowest increment in terms of the computational time. Through the comparison between CE and DCE (or HCE and ICE), it is found that the discretization method may slightly reduce the running time. In addition, the algorithms with the proposed discretization method are more likely to obtain higher-quality solution, causing about 2% cost savings. With regard to the improvement of neighbourhood heuristic search now, the solution accuracy is also improved significantly. Especially when the item number is reaching 20, the computational target values show a difference of about 7% in the view of CE and HCE (or DCE and ICE). Although the neighbourhood heuristic approach has shown significant improvements, it has the tendency to converge towards local optima, hence limiting the effectiveness of the HCE conclusion. Overall, from Table 2, we may draw a conclusion that the improving methods for CE indeed show a good performance, but have some differences in the effect of influence.

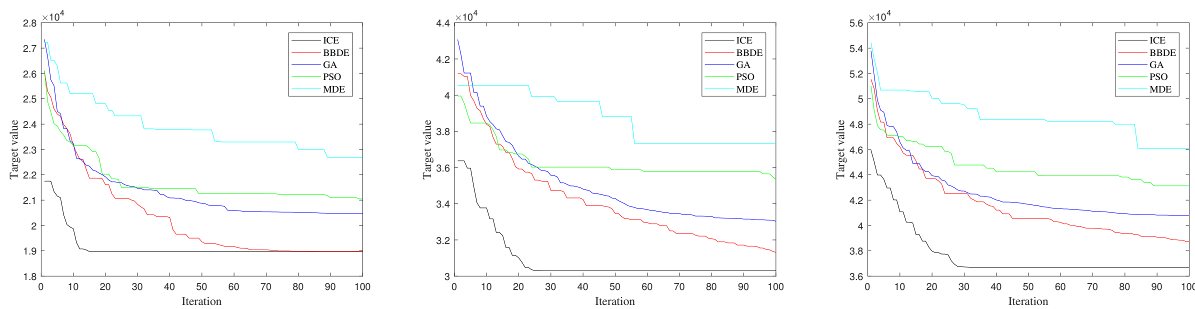
5.2. Experiment 2: Comparison on alternative meta-heuristics

As long as acceptable performance has been obtained in solving JRP- or JRD-type problems via meta-heuristics, in this paper, several state-of-the-art algorithms such as the genetic algorithm (GA) (Otero-Palencia et al., 2019), particle swarm optimization algorithm (PSO) (Kang et al., 2017), modified differential evolution algorithm (MDE) (Wang et al., 2013a) and barebones differential evolution algorithm (BBDE) (Cui et al., 2020c) are designated, and then, compared with the newly proposed ICE approach.

In order to provide convincing evidence regarding versatility and superiority of ICE, a set of various numerical experiments are devised across many sizes. In this part, we initially examine an issue scale of 20 by utilizing the data shown in Table C.1. Furthermore, to ensure the validity of the findings, additional experiments were done using randomly generated instances consisting of 30 and 40 elements within a broader framework. The range (or rule) of the relevant parameters is displayed in Table C.2. To facilitate a direct comparison of all the algorithms, following previous works, the parameters used for GA, PSO, MDE and BBDE are calibrated accordingly. In specific, for GA, the probability of crossover, P_c, and the probability of mutation, P_m, are given by 0.7 and 0.2, respectively. For PSO, the combination of the inertia factor, ω_n, and the acceleration constants ψ_1 and ψ_2 are correspondingly given by 0.73, 1.49, and 1.49. In the case of MDE, the scaling factor F and the crossover factor C_R are accordingly equal to 1.2 and 0.6. Moreover, the predefined tolerance factors for the degree of population diversity are settled to 0.02. The unique factor for BBDE is the crossover probability which is 0.7. The computational results of the above-mentioned five meta-heuristic algorithms are presented in Table 3.

Table 3: Computational results of the five algorithms.

Item num	Algorithm	Best $\bar{b}_{\min}(\alpha_0)$	Ave. $\bar{b}_{\min}(\alpha_0)$	Worst $\bar{b}_{\min}(\alpha_0)$	Ave. Err. (%)	Std. Dev.	Ave. Time (s)
20	MDE	22681.91	23097.38	23407.62	21.7491	240.12	11.2538
	PSO	21047.96	21550.34	21899.54	13.6451	216.51	6.4843
	GA	20557.56	20764.60	21094.40	9.3001	175.66	16.6399
	BBDE	18971.29	19252.06	19981.27	1.4800	358.68	12.6494
	ICE	18971.29	18971.29	18971.29	0.0000	0.00	1.6387
30	MDE	37331.27	38441.10	38914.96	26.8897	427.06	14.1256
	PSO	35341.57	35866.16	36368.50	18.3901	373.20	9.2625
	GA	33029.95	33857.21	34536.95	11.7588	462.55	25.2963
	BBDE	31309.90	31979.24	32566.95	5.5598	377.50	15.0187
	ICE	30294.89	30294.89	30294.89	0.0000	0.00	1.8451
40	MDE	46092.31	47272.42	47955.23	28.8544	599.11	15.5078
	PSO	43127.01	44047.06	44956.48	20.0628	579.47	10.5814
	GA	40761.66	41412.57	42217.79	12.8817	424.02	35.0017
	BBDE	38714.75	39458.63	40081.51	7.5557	347.50	15.6979
	ICE	36686.69	36687.06	36687.16	0.0010	0.19	2.1805



(a) $n = 20$ (b) $n = 30$ (c) $n = 40$

Figure 2: The convergence graphs of the best-found solutions of five algorithms.

According to the findings presented in Table 3, the following conclusions can be derived: In terms of the searching quality, the target values of ICE and BBDE are evidently lower than those obtained from the other three algorithms, in which MDE presents the biggest value yielding the largest error, noticeably reaching 28.8544% in the case of scale of 40. In terms of the searching robustness, except for ICE, the standard deviations of the other four algorithms are significantly larger. Apparently, the standard deviation increases substantially with respect to problem scale. However, ICE has much lower standard deviation (only 0.19) even when the problem size reaches the scale of 40. In terms of the computational time, the ICE is the most efficient one, followed by PSO, while GA is the most time-consuming. The best-found solution of each algorithm will be selected to show its evolution over iterations, which is illustrated in Fig. 2. In terms of the convergence speed, from Fig. 2, we observe that only ICE converges in all the three cases, while BBDE does not converge when the problem scale reaches 30. GA generally shows moderate performance in convergence. Furthermore, PSO and MDE converge prematurely into the local optimum. Unstable and slow evolving process are also observed in PSO and MDE. In summary, ICE provides a reliable solution in shorter times during the experimentation phase. MDE, PSO, GA and BBDE fall behind ICE in terms of robustness, and their target values are much larger.

5.3. Experiment 3: Comparison on alternative heuristics

In the existing literature, there has been a notable increase in the attention given to the utilization of heuristics in the context of JRDs. Cha et al. (2008) pointed out that the optimality conditions can be utilized such as to construct a RAND-type algorithm for solving the deterministic JRDs. The computational findings offer empirical support for the assertion that the RAND method exhibits superior efficiency compared to alternative solution approaches. Following the same idea, Wang and Wang (2022) proposed a RAND-type algorithm for the stochastic JRD problem and Wang et al. (2023) presented the iteratively search RAND to solve the location-inventory JRD model. Both studies demonstrate remarkable comparative effectiveness. Considering the above, in alignment with our proposed framework, we have developed a modified RAND algorithm (MRAND). The details of MRAND may be found in Appendix D. The primary method involves iteratively determining the optimal value of T until it reaches a point of convergence when no further changes occur.

In this section, several instances of different scales (i.e., n from 40 to 2000) through random generating from Table C.2 are conducted. Noting that the major ordering cost S is changed from n (the problem scale) to $10 \times n$ for our purpose to test the search ability of proposed algorithms. The value of M in MRAND algorithm is set to 50[6] following the similar experience of Wang and Wang (2022) and Wang et al. (2023). Computational results are shown in Table 4.

Table 4: Computational results of MRAND and ICE under different settings.

Item num	S	MRAND			ICE		
		Ave. $\bar{b}_{\min}(\alpha_0)$	Ave. Err. (%)	Ave. Time (s)	Ave. $\bar{b}_{\min}(\alpha_0)$	Ave. Err. (%)	Ave. Time (s)
40	40	34039	0.217	0.5039	33906	0.2286	2.0283
	200	35228	0.2121	0.3847	35154	0.0000	1.9954
	400	36806	0.3264	0.3835	36687	0.0010	1.9950
100	100	82427	0.6389	0.6231	82018	0.1399	3.7264
	500	85458	0.2036	0.3087	85284	0.0000	3.2252
	1000	89251	0.1045	0.4932	89158	0.0000	3.2332
500	500	410121	0.5765	0.7629	408135	0.0896	5.7196
	2500	425849	0.3199	0.7624	425040	0.1293	5.6709
	5000	444646	0.1940	0.6647	445491	0.3846	6.1321
1000	1000	825342	0.5272	0.7347	821921	0.1105	9.5424
	5000	856833	0.2543	0.7304	855505	0.0989	10.6226
	10000	898846	0.6118	0.7280	896047	0.2985	9.7908
2000	2000	1666072	0.5330	1.2131	1659266	0.1223	13.6152
	10000	1737151	0.6238	1.1133	1736381	0.5792	12.7428
	20000	1817196	0.8006	0.9121	1806994	0.2347	12.8124

[6] We also try to increase the number of M, but the effect does not have great improvement.

Based on the findings presented in Table 4, it is evident that the time consumption associated with the MRAND algorithm is significantly lower compared to that of the ICE algorithm. The MRAND algorithm is a straightforward approach to iterative search that does not use mechanisms such as crossover or mutation. Therefore, the higher speed aligns with our intuitive understanding. Meanwhile, the computational time grows linearly with dimensionality. The search efficiency of MRAND is notably better in smaller scenarios, particularly when the major ordering cost is higher. The potential explanation for the observed phenomenon is that lower ordering costs result in a larger search range. However, the iterative MRAND limits the search for the fundamental cycle time T. While it is possible to get satisfactory performance with the MRAND approach, it is important to note that this method may result in the loss of crucial information necessary for obtaining the ideal outcome. On the other hand, the utilization of the neighbourhood heuristic facilitates a seamless exploration for the most favourable region. The experimental results indicate that the error rate of the ICE algorithm, when utilizing neighbourhood heuristics, is generally lower compared to the error rate of the MRAND algorithm throughout the majority of testing situations. The analysis of efficacy and efficiency reveals that both MRAND and ICE algorithms are valuable for JRDs, albeit with distinct application contexts. The ICE demonstrates superior accuracy compared to MRAND, and it also offers ease of use for other constrained JRD problems. In comparison, MRAND has superior speed capabilities relative to ICE, rendering it a more appropriate choice for customers with time constraints.

5.4. Experiment 4: Uncertainty-related parameters analyses

In order to check the impact of uncertainty-related parameters to the secure budget of model (M2), two sensitivity parameters are analyzed with different confidence levels and imperfect rates. In this direction, Table C.3 on the appendix presents complementary data as a benchmark.

5.4.1. Sensitivity for different confidence levels

The confidence level in our case may be relevant to the risk. The higher the α_0, the lower the risk tolerance of decision makers. Also, we should emphasis that the best-case ($\alpha_0 = 0$), most-likely ($\alpha_0 = 0.5$), and worst-case ($\alpha_0 = 1$) are provide planners with the ability to coordinate procurement-shipment policy for any risk events. We could separate the confidence level into higher risk scenario, $\alpha_0 \in (0, 0.5]$, and lower risk scenario, $\alpha_0 \in (0.5, 1]$. The main comparative results are reported in Table 5 including the solutions, target values. Accordingly, the replenishment and distribution quantities of each item are given in Table 6. Fig. 3 gives an illustration of the relationships of the changes between distribution quantity, replenishment quantity and outbound frequency, taking item 1 as an example. The replenishment frequencies keep stable, and thus they are omitted.

According to Table 5, in both higher and lower risk scenario, with an increment of the confidence level, the basic cycle time T is gradually compressed the basic cycle time T is gradually compressed, and the target value increases because of the added value of cost factors. In details, with the increasing of α_0, when the replenishment frequency of an item remains unchanged, DC tends to mitigate this by reducing the basic cycle time, which results a reduction of replenishment quantity Q_i^{DC} (see the green bar chart in Fig. 3). While this may indicate that DC would pay more on ordering to hold less inventory to keep a balance. Interestingly, if the outbound frequency, f_i, of an item drops to the next level, the outbound quantity, Q_i,

Table 5: The optimal solutions under different confidence levels.

α_0	Solution													Best $\bar{b}_{\min}(\alpha_0)$
	k_1	k_2	k_3	k_4	k_5	k_6	f_1	f_2	f_3	f_4	f_5	f_6	T	
0.0^+	1	1	1	2	2	4	7	5	4	4	3	4	0.2533	3791.30
0.1	1	1	1	2	2	4	6	4	3	4	3	3	0.2283	4103.85
0.2	1	1	1	2	2	4	6	4	3	4	3	3	0.2147	4392.72
0.3	1	1	1	2	2	4	5	4	3	3	3	3	0.2003	4664.96
0.4	1	1	1	2	2	4	5	4	3	3	3	3	0.1910	4923.26
0.5	1	1	1	2	2	4	5	3	3	3	2	3	0.1801	5168.44
0.5^+	1	1	1	2	2	4	5	3	3	3	2	3	0.1823	5231.21
0.6	1	1	1	2	2	4	4	3	2	3	2	2	0.1709	5464.88
0.7	1	1	1	2	2	4	4	3	2	3	2	2	0.1650	5690.15
0.8	1	1	1	2	2	4	4	3	2	3	2	2	0.1597	5908.93
0.8	1	1	1	2	2	4	4	3	2	2	2	2	0.1540	6121.37
1.0	1	1	1	2	2	4	4	3	2	2	2	2	0.1498	6327.99

a^+: the nearest value larger than a.

Table 6: The optimal replenishment and distribution quantities under different confidence levels.

α_0	Replenishment quantity						Distribution quantity					
	Q_1^{DC}	Q_2^{DC}	Q_3^{DC}	Q_4^{DC}	Q_5^{DC}	Q_6^{DC}	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
0.0^+	2814.19	1407.10	844.26	562.84	337.70	225.14	402.03	281.42	211.06	140.71	112.57	56.28
0.1	2536.91	1268.46	761.07	507.38	304.43	202.95	422.82	317.11	253.69	126.85	101.48	67.65
0.2	2385.26	1192.63	715.58	477.05	286.23	190.82	397.54	298.16	238.53	119.26	95.41	63.61
0.3	2226.05	1113.03	667.82	445.21	267.13	178.08	445.21	278.26	222.61	148.40	89.04	59.36
0.4	2122.13	1061.06	636.64	424.43	254.66	169.77	424.43	265.27	212.21	141.48	84.89	56.59
0.5	2001.46	1000.73	600.44	400.29	240.18	160.12	400.29	333.58	200.15	133.43	120.09	53.37
0.5^+	2025.77	1012.88	607.73	405.15	243.09	162.06	405.15	337.63	202.58	135.05	121.55	54.02
0.6	1898.89	949.45	569.67	379.78	227.87	151.91	474.72	316.48	284.83	126.59	113.93	75.96
0.7	1833.38	916.69	550.02	366.68	220.01	146.67	458.35	305.56	275.01	122.23	110.00	73.34
0.8	1774.81	887.40	532.44	354.96	212.98	141.98	443.70	295.80	266.22	118.32	106.49	70.99
0.8	1711.49	855.75	513.45	342.30	205.38	136.92	427.87	285.25	256.72	171.15	102.69	68.46
1.0	1664.12	832.06	499.24	332.82	199.69	133.13	416.03	277.35	249.62	166.41	99.85	66.56

shows a large number of increment, otherwise if the outbound frequency remains the same, it has an adverse effect on the outbound quantity of that item (see the orange bar chart in Fig. 3). The discontinuity point 0.5 only affects the ordering and outbound cost factors as these two are trapezoidal fuzzy numbers. In this case, DC slightly increases the basic cycle time, which means more item quantities and time are processed for inventory holding to alleviate high costs burden of ordering and outbound.

5.4.2. Sensitivity for different imperfect rates

Examining now how the effect of the objective function is influenced by the imperfect rate. Similarly, the solutions and target values are reported in Table 7. The replenishment and distribution quantities of each item are given in Table 8. Fig. 4 gives an illustration of the relationships of the changes between distribution quantity, replenishment quantity and outbound frequency, taking item 1 as an example.

In a similar manner, the following observations hold: As the rate of imperfect quality item is increasing, the target value shows an increasing trend. It is true that if the imperfect quality

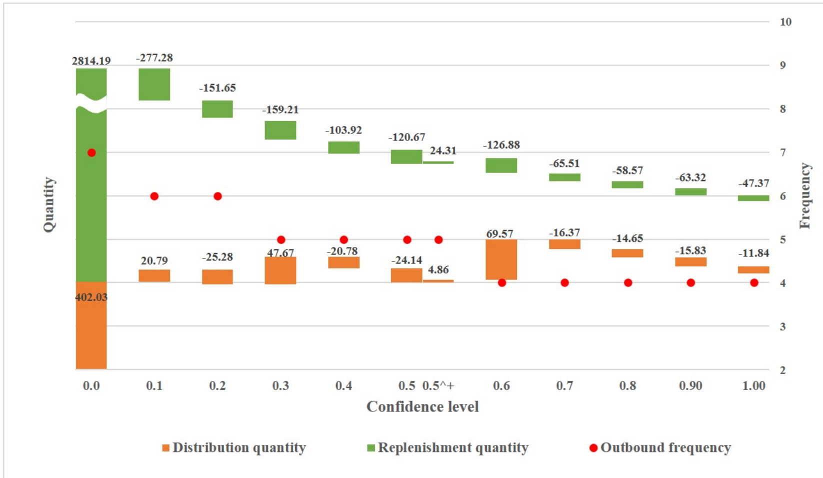


Figure 3: Changes of the optimal results of item 1 under different confidence levels.

Table 7: The optimal solutions under different imperfect rates.

$E[p_i]$	Solution													Best
	k_1	k_2	k_3	k_4	k_5	k_6	f_1	f_2	f_3	f_4	f_5	f_6	T	$\bar{b}_{\min}(\alpha_0)$
0.00	1	1	1	2	2	4	4	3	2	3	2	2	0.1881	4828.89
0.02	1	1	1	2	2	4	4	3	2	3	2	2	0.1857	4891.12
0.04	1	1	1	2	2	4	4	3	2	3	2	3	0.1838	4956.47
0.06	1	1	1	2	2	4	5	3	3	3	2	3	0.1853	5024.71
0.08	1	1	1	2	2	4	5	3	3	3	2	3	0.1827	5094.76
0.10	1	1	1	2	2	4	5	3	3	3	2	3	0.1801	5168.44
0.12	1	1	1	2	2	4	5	3	3	3	2	3	0.1775	5246.01
0.14	1	1	1	2	2	4	5	4	3	3	2	3	0.1766	5327.25
0.16	1	1	1	2	2	4	5	4	3	3	3	3	0.1748	5411.61
0.18	1	1	1	2	2	4	5	4	3	3	3	3	0.1720	5500.49
0.20	1	1	1	2	2	4	5	4	3	3	3	3	0.1691	5594.36

units exist, the cost is further increased, as the DC will add the number of item ordered which leads to higher inventory holding cost. Also, Table 7 gives evidence that the replenishment frequency, k_i, of each item remains unchanged with respect to $E[p_i]$. Interestingly, comparing with the results in the previous section, it is found an opposite conclusion that the outbound frequencies f_i are smoothly increasing. When the data details in Table 8 are examined, changes to the imperfect quality rates of all involved items follow a special rule to affect the replenishment and distribution quantities. In the first stage, i.e., the effect of imperfect quality rate is less severe, the order and outbound frequencies do not change much, and DC would slightly increase the replenishment quantities to compensate the imperfect items to meet the real demand from retailer, while the basic supply time, T, is reducing to avoid large-increment of the inventory holding cost. In the second stage, as the imperfect rates keeps increasing, further reducing on T may be not plausible, since that may cause great increases of ordering and outbound costs. In terms of the distribution quantity, from Fig. 4, if the outbound frequency of an item remains the same, the distribution quantity of that item increases, otherwise if the outbound frequency is increasing, the distribution quantity decreases sharply.

Table 8: The optimal replenishment and distribution quantities under different imperfect rates.

$E[p_i]$	Replenishment quantity						Distribution quantity					
	Q_1^{DC}	Q_2^{DC}	Q_3^{DC}	Q_4^{DC}	Q_5^{DC}	Q_6^{DC}	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
0.00	1881.39	940.69	564.42	376.28	225.77	150.51	470.35	313.56	282.21	125.43	112.88	75.26
0.02	1895.35	947.68	568.61	379.07	227.44	151.63	473.84	315.89	284.30	126.36	113.72	75.81
0.04	1914.58	957.29	574.38	382.92	229.75	153.17	478.65	319.10	287.19	127.64	114.88	51.06
0.06	1971.11	985.55	591.33	394.22	236.53	157.69	394.22	328.52	197.11	131.41	118.27	52.56
0.08	1986.27	993.13	595.88	397.25	238.35	158.90	397.25	331.04	198.63	132.42	119.18	52.97
0.10	2001.46	1000.73	600.44	400.29	240.18	160.12	400.29	333.58	200.15	133.43	120.09	53.37
0.12	2016.68	1008.34	605.00	403.34	242.00	161.33	403.34	336.11	201.67	134.45	121.00	53.78
0.14	2053.94	1026.97	616.18	410.79	246.47	164.32	410.79	256.74	205.39	136.93	123.24	54.77
0.16	2081.06	1040.53	624.32	416.21	249.73	166.49	416.21	260.13	208.11	138.74	83.24	55.50
0.18	2097.37	1048.69	629.21	419.47	251.68	167.79	419.47	262.17	209.74	139.82	83.89	55.93
0.20	2113.74	1056.87	634.12	422.75	253.65	169.10	422.75	264.22	211.37	140.92	84.55	56.37

Figure 4: Changes of the optimal results of item 1 under different imperfect rates.

6. Conclusion

This paper presents a study into the analysis of uncertain factors within the context of inventory systems in a three-stage supply chain. The organizational framework of the JRD policy pertaining to centralized medication procurement in China provides significant impetus for investigating the allocation strategies of vaccines in an environment characterized by uncertainty. In a more realistic scenario, the consideration of fuzzy costs and a random number of imperfect goods is incorporated into the analysis of JRD. In order to accomplish this objective, a framework based on chance-constrained programming is utilized. This framework enables the planner, such as a government entity, responsible for policy-making related to procurement and shipment, to actively engage in risk management strategies. The perception of non-for-profit government is manifested in the degree of risk as indicated by the CCP model. Based on a comprehensive review of existing literature in this field, it is evident that no prior investigations have been conducted to examine the phenomenon of JRD in the context of mixed uncertainty. We anticipate that our research study will offer profound insights into previously unexplored aspects of knowledge, namely in the realm of logistics modelling, by identifying various forms of uncertainty.

This study also holds significant implications for the management of vaccination supply chains. a) The concept of JRD under conditions of randomization and fuzziness contributes to the effective management of uncertain factors in centralized medication procurement systems, particularly in the context of vaccine procurement and shipment. Historical data can be employed to derive a probability distribution for the uncertainty resulting from objective factors, such as transportation losses. In the context of imprecisely calculated parameters, such as linguistic phrases used to describe price, the fuzzy set proves to be a useful approach, particularly when dealing with confined intervals. Various activity habits exhibit distinct forms of uncertainty. (b) The allocation of funds for vaccines within the government's budget is determined by extensive negotiations between non-for-profit healthcare systems and for-profit pharmaceutical firms. In light of the urgent need to combat the epidemic, centralized medication procurement systems are compelled to seek an optimum vaccine supply in order to achieve a positive external effect. The consumers' perception of the manufacturers significantly impacts the pricing of the goods. A higher level of confidence, characterized by a tough attitude, has the potential to save costs but also presents a potential increase in supply chain risk. (c) In addition to the inherent cost variability associated with products, the numerical experiment demonstrates the considerable potential for cost savings through the reduction of the major ordering cost S. Since it is unrelated to products (vaccines), it is less unclear and easier to govern. A decrease in the value of variable S corresponds to an increase in the frequency at which replenishment occurs. In this scenario, the ability to manage the budget becomes not only advantageous but also adaptable to fluctuations in demand through the modification of the allocation plan. (d) The study revealed that the solution exhibits a high degree of robustness with regards to replenishment. The implementation of our treatment significantly enhances the company's competitiveness, hence increasing its long-term viability. However, it is imperative for the managerial team to allocate greater focus towards crucial factors that can potentially lead to alterations in outbound frequencies. This is due to the high sensitivity of these frequencies to item ordering quantities, and any incorrect resolution could result in substantial financial losses. (e) Both the MRAND and ICE decision-support tools have been offered as valuable tools for identifying solutions that are close to optimal. However, it is important to note that both tools have distinct applications. It is advisable for individuals seeking to make prompt decisions to utilize the MRAND algorithm, particularly during the initial phases of an epidemic, as it is imperative for the healthcare system to promptly react. In terms of cost-saving, the ICE algorithm exhibits superior performance and may be effectively employed in a broader range of constrained cases.

Future research can be implemented on constructing a JRD problem with multi-warehouse and/or multi-distribution centras with some special replenishment policies, defining the objectives containing the relative costs, time and customer satisfactory, and designing accordingly adaptive intelligent algorithms for the new JRD problem. Also, the coordination of procurement-shipment problem and vehicle routing problem is also an interesting topic to analyse. From the computation perspective, establishing some theoretical guarantees to addresses the constrained version of JRDs is needed to be practical.

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References

Arkin, E., Joneja, D., Roundy, R., 1989. Computational complexity of uncapacitated multi-echelon production planning problems. Operations Research Letters 8, 61–66. doi:10.1016/0167-6377(89)90001-1.

Arlin Cooper, J., Ferson, S., Ginzburg, L., 1996. Hybrid processing of stochastic and subjective uncertainty data. Risk Analysis 16, 785–791. doi:10.1111/j.1539-6924.1996.tb00829.x.

Baller, A.C., Dabia, S., Dullaert, W.E., Vigo, D., 2019. The dynamic-demand joint replenishment problem with approximated transportation costs. European Journal of Operational Research 276, 1013–1033. doi:10.1016/j.ejor.2019.01.070.

Bastos, L.d.S.L., Mendes, M.L., Nunes, D.R.d.L., Melo, A.C.S., Carneiro, M.P., 2017. A systematic literature review on the joint replenishment problem solutions: 2006-2015. Production 27, e20162229. doi:10.1590/0103-6513.222916.

Carvajal, J., Castaño, F., Sarache, W., Costa, Y., 2020. Heuristic approaches for a two-echelon constrained joint replenishment and delivery problem. International Journal of Production Economics 220, 107420. doi:10.1016/j.ijpe.2019.06.016.

Cha, B.C., Moon, I.K., Park, J.H., 2008. The joint replenishment and delivery scheduling of the one-warehouse, n-retailer system. Transportation Research Part E: Logistics and Transportation Review 44, 720–730. doi:10.1016/j.tre.2007.05.010.

Charnes, A., Cooper, W.E., 1959. Chance-constrained programming. Management Science 6, 73–79.

Cui, L., Deng, J., Liu, R., Xu, D., Zhang, Y., Xu, M., 2020a. A stochastic multi-item replenishment and delivery problem with lead-time reduction initiatives and the solving methodologies. Applied Mathematics and Computation 374, 125055. doi:10.1016/j.amc.2020.125055.

Cui, L., Deng, J., Zhang, Y., Tang, G., Xu, M., 2020b. Hybrid differential artificial bee colony algorithm for multi-item replenishment-distribution problem with stochastic lead-time and demands. Journal of Cleaner Production 254, 119873. doi:10.1016/j.jclepro.2019.119873.

Cui, L., Deng, J., Zhang, Y., Zhang, Z., Xu, M., 2020c. The bare-bones differential evolutionary for stochastic joint replenishment with random number of imperfect items. Knowledge-Based Systems 193, 105416. doi:10.1016/j.knosys.2019.105416.

Duijzer, L.E., van Jaarsveld, W., Dekker, R., 2018. Literature review: The vaccine supply chain. European Journal of Operational Research 268, 174–192. doi:10.1016/j.ejor.2018.01.015.

Gandon, S., Mackinnon, M.J., Nee, S., Read, A.F., 2001. Imperfect vaccines and the evolution of pathogen virulence. Nature 414, 751–756. doi:10.1038/414751a.

Goyal, S.K., 1974. Determination of optimum packaging frequency of items jointly replenished. Management Science 21, 436–443. doi:10.1287/mnsc.21.4.436.

Kang, H.Y., Lee, A.H., Wu, C.W., Lee, C.H., 2017. An efficient method for dynamic-demand joint replenishment problem with multiple suppliers and multiple vehicles. International Journal of Production Research 55, 1065–1084. doi:10.1080/00207543.2016.1218564.

Khan, M., Jaber, M.Y., Guiffrida, A.L., Zolfaghari, S., 2011. A review of the extensions of a modified EOQ model for imperfect quality items International Journal of Production Economic 132, 1–12. doi:10.1016/j.ijpe.2011.03.009.

Khouja, M., Goyal, S., 2008. A review of the joint replenishment problem literature: 1989-2005. European Journal of Operational Research 186, 1–16. doi:10.1016/j.ejor.2007.03.007.

Lee, F.C., Yao, M.J., 2003. A global optimum search algorithm for the joint replenishment problem under power-of-two policy. Computers and Operations Research 30, 1319–1333. doi:10.1016/S0305-0548(02)00073-4.

Liu, B., 2002. Theory and practice of uncertain programming. Physica-Verlag Heidelberg.

Liu, B., Iwamura, K., 1998. Chance constrained programming with fuzzy parameters. Fuzzy Sets and Systems 94, 227–237. doi:10.1016/S0165-0114(96)00236-9.

Liu, R., Liu, S., Zeng, Y.R., Wang, L., 2017. Optimization model for the new coordinated replenishment and delivery problem with multi-warehouse. International Journal of Logistics Management 28, 290–310. doi:10.1108/IJLM-11-2015-0217.

Liu, R., Zhou, Z., Qin, Q., Bi, F., 2022. Centralized drug procurement operation scheduling with a capacitated joint replenishment and delivery strategy: Evidence from China. Computers and Industrial Engineering 172, 108584. doi:10.1016/j.cie.2022.108584.

Maiti, M.K., Maiti, M., 2006. Fuzzy inventory model with two warehouses under possibility constraints. Fuzzy Sets and Systems 157, 52–73. doi:10.1016/j.fss.2005.06.021.

Ning, X., Li, P., 2018. A cross-entropy approach to the single row facility layout problem. International Journal of Production Research 56, 3781–3794. doi:10.1080/00207543.2017.1399221.

Olsen, A.L., 2005. An evolutionary algorithm to solve the joint replenishment problem using direct grouping. Computers and Industrial Engineering 48, 223–235. doi:10.1016/j.cie.2005.01.010.

Ongkunaruk, P., Wahab, M.I., Chen, Y., 2016. A genetic algorithm for a joint replenishment problem with resource and shipment constraints and defective items. International Journal of Production Economics 175, 142–152. doi:10.1016/j.ijpe.2016.02.012.

Otero-Palencia, C., Amaya–Mier, R., Yie-Pinedo, R., 2019. A stochastic joint replenishment problem considering transportation and warehouse constraints with gainsharing by Shapley Value allocation. International Journal of Production Research 57, 3036–3059. doi:10.1080/00207543.2018.1526418.

Paul, S., Wahab, M.I., Ongkunaruk, P., 2014. Joint replenishment with imperfect items and price discount. Computers and Industrial Engineering 74, 179–185. doi:10.1016/j.cie.2014.05.015.

Pezzoli, L., Azman, A.S., 2021. Moving forward with an imperfect vaccine. The Lancet Infectious Diseases 21, 1339–1341. doi:10.1016/S1473-3099(20)30851-3.

Rubinstein, R.Y., 1999. The cross-entropy method for combinatorial and continuous optimization. Methodology and Computing in Applied Probability 1, 127–190. doi:10.1023/A:1010091220143.

Rubinstein, R.Y., Kroese, D.P., 2004. The cross entropy method: A unified approach to combinatorial optimization, Monte-Carlo simulation and machine learning. Springer-Verlag NewYork, Inc.

Sazvar, Z., Tafakkori, K., Oladzad, N., Nayeri, S., 2021. A capacity planning approach for sustainable-resilient supply chain network design under uncertainty: A case study of vaccine supply chain. Computers and Industrial Engineering 159, 107406. doi:10.1016/j.cie.2021.107406.

Shu, F.T., 1971. Economic ordering frequency for two items jointly replenished. Management Science 17, B406–B410. doi:10.1287/mnsc.17.6.b406.

Vujošević, M., Petrović, D., Petrović, R., 1996. EOQ formula when inventory cost is fuzzy. International Journal of Production Economics 45, 499–504. doi:10.1016/0925-5273(95)00149-2.

Wang, L., Dun, C.X., Bi, W.J., Zeng, Y.R., 2012. An effective and efficient differential evolution algorithm for the integrated stochastic joint replenishment and delivery model. Knowledge-Based Systems 36, 104–114. doi:10.1016/j.knosys.2012.06.007.

Wang, L., Dun, C.X., Lee, C.G., Fu, Q.L., Zeng, Y.R., 2013a. Model and algorithm for fuzzy joint replenishment and delivery scheduling without explicit membership function. International Journal of Advanced Manufacturing Technology 66, 1907–1920. doi:10.1007/s00170-012-4469-5.

Wang, L., Fu, Q.L., Lee, C.G., Zeng, Y.R., 2013b. Model and algorithm of fuzzy joint replenishment problem under credibility measure on fuzzy goal. Knowledge-Based Systems 39, 57–66. doi:10.1016/j.knosys.2012.10.002.

Wang, L., Wang, S., Gong, Y., Peng, L., 2023. Optimizing a multi-echelon location-inventory problem with joint replenishment: A lipschitz ϵ-optimal approach using lagrangian relaxation. Computers and Operations Research 151, 106128. doi:10.1016/j.cor.2022.106128.

Wang, S., Wang, L., 2022. Efficient methods for stochastic joint replenishment and delivery problem. International Transactions in Operational Research 29, 2288–2315. doi:10.1111/itor.12905.

Wang, Z., Soleimani, H., Kannan, D., Xu, L., 2016. Advanced cross-entropy in closed-loop supply chain planning. Journal of Cleaner Production 135, 201–213. doi:10.1016/j.jclepro.2016.04.006.

Yao, M.J., Lin, J.Y., Lin, Y.L., Fang, S.C., 2020. An integrated algorithm for solving multi-customer joint replenishment problem with districting consideration. Transportation Research Part E: Logistics and Transportation Review 138, 101896. doi:10.1016/j.tre.2020.101896.

Zhao, M., Zhou, J., Wang, K., Pantelous, A.A., 2023. Project scheduling problem with fuzzy activity durations: A novel operational law based solution framework. European Journal of Operational Research 306, 519–534. doi:10.1016/j.ejor.2022.07.047.

Appendix A. Preliminaries

Definition A.1. *A fuzzy interval* $\widetilde{M}$ *is a quantity with a quasi-concave membership function* $\mu_{\widetilde{M}}$*, i.e., a convex fuzzy subset of the real line* $\mathbb{R}$ *such that*

$$\mu_{\widetilde{M}}(z) \geq \min\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{M}}(y)\} \quad \forall x, y \in \mathbb{R}, z \in [x, y]. \tag{A.1}$$

Definition A.2. *A fuzzy interval* $\widetilde{M}$ *is of LR-type if there exist shape functions* L *(for left),* R *(for right), and four parameters:* $(\underline{m}, \overline{m}) \in \mathbb{R}^2 \bigcup \{-\infty, +\infty\}$*,* $\gamma > 0, \beta > 0$ *with membership function*

$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\dfrac{\underline{m} - x}{\gamma}\right), & \text{if } x \leq \underline{m} \\ 1, & \text{if } \underline{m} < x \leq \overline{m} \\ R\left(\dfrac{x - \overline{m}}{\beta}\right), & \text{if } x > \overline{m}. \end{cases} \tag{A.2}$$

Provided that the shape functions L *and* R *are continuous and strictly decreasing with respect to* x *at which* $\{x \mid 0 < L(x) < 1\}$ *and* $\{x \mid 0 < R(x) < 1\}$*, respectively, the LR fuzzy interval* $\widetilde{M}$ *is said to be regular.*

Definition A.3. *A random variable* ξ *has a beta distribution if its probability density function satisfies:*

$$f(x) = \frac{x^{a-1}(1-x)^{b-1}}{\int_0^1 u^{a-1}(1-u)^{b-1} du}, \tag{A.3}$$

where $a, b > 0$*. Its expected value can be deduced as* $E[\xi] = \frac{a}{a+b}$*.*

Definition A.4. *Suppose that* $\widetilde{M}$ *represents a fuzzy interval with membership function* $\mu_{\widetilde{M}}$*. Then, the credibility distribution of* $\widetilde{M}$ *is*

$$\mathrm{Cr}\{\widetilde{M} \leq x\} = \frac{1}{2}\Big(\sup_{y \leq x} \mu_{\widetilde{M}}(y) + 1 - \sup_{y > x} \mu_{\widetilde{M}}(y)\Big), \quad \forall x \in \mathbb{R}. \tag{A.4}$$

which can be denoted by $\Phi_{\widetilde{M}}(x)$*.*

Definition A.5. *Suppose that* $\widetilde{M}$ *represents a fuzzy interval and* Cr *is the credibility measure. Then, the* α*-pessimistic value of* $\widetilde{M}$ *is*

$$\widetilde{M}_{\inf}(\alpha) = \inf\{x \mid \mathrm{Cr}\{\widetilde{M} \leq x\} \geq \alpha\}, \quad \forall \alpha \in (0, 1]. \tag{A.5}$$

Definition A.6. *Suppose that* $\widetilde{M}$ *represents a regular fuzzy interval with credibility distribution* $\Phi_{\widetilde{M}}$*. Then, the inverse function* $\Phi^{-1}_{\widetilde{M}}$ *is called the inverse credibility distribution of* $\widetilde{M}$*. If required, we may extend the domain via*

$$\Phi^{-1}_{\widetilde{M}}(0) = \lim_{\hat{\alpha} \to 0^+} \Phi^{-1}(\hat{\alpha}), \quad \Phi^{-1}_{\widetilde{M}}(1) = \lim_{\hat{\alpha} \to 1^-} \Phi^{-1}(\hat{\alpha}). \tag{A.6}$$

Let $\widetilde{M} = (a, \underline{c}, \overline{c}, b)$ be a trapezoidal fuzzy number with $a < \underline{c} < \overline{c} < b$. The shape functions are generally assumed as $L = R = \max\{1 - x, 0\}$, and so $\widetilde{M}$ is clearly a *regular fuzzy interval.* Its α-pessimistic value and inverse credibility distribution can be directly deduced as

$$\widetilde{M}_{\inf}(\alpha) = \begin{cases} a + 2\alpha\gamma, & \text{if } 0 < \alpha \leq 0.5 \\ b - (2 - 2\alpha)\beta, & \text{if } 0.5 < \alpha \leq 1, \end{cases} \tag{A.7}$$

and

$$\Phi^{-1}_{\widetilde{M}}(\alpha) = \begin{cases} a + 2\alpha\gamma, & \text{if } 0 \leq \alpha < 0.5 \\ [a + \gamma, b - \beta], & \text{if } \alpha = 0.5 \\ b - (2 - 2\alpha)\beta, & \text{if } 0.5 < \alpha \leq 1, \end{cases} \tag{A.8}$$

respectively, where $\gamma = \underline{c} - a$ and $\beta = b - \overline{c}$ are the left and right spreads.

Remark A.1. *The triangular fuzzy number could be viewed as a special type of trapezoidal fuzzy interval. In other words, when the condition '$\underline{c} = \overline{c}$' is satisfied, the analytical formulations in Eqs. (A.7) and (A.8) hold for triangular case.*

Definition A.7. *A real-valued function $f(t_1, t_2, \cdots, t_n)$ is said to be increasing with respect to $t_1, t_2, \ldots, t_n$ if*

$$f(t_1, t_2, \ldots, t_n) \leq f(s_1, s_2, \ldots, s_n),$$

whenever $t_i \leq s_i$, for $i = 1, 2, \ldots, n$.

Theorem A.2. *Suppose that $\widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_n$ are independent regular fuzzy intervals with inverse credibility distributions $\Phi^{-1}_{\widetilde{M}_1}, \Phi^{-1}_{\widetilde{M}_2}, \ldots, \Phi^{-1}_{\widetilde{M}_n}$, respectively. If the function $f(t_1, t_2, \ldots, t_n)$ is continuous and increasing with respect to $t_1, t_2, \ldots, t_n$, then $\widetilde{M} = f(\widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_n)$ is a regular fuzzy interval with inverse credibility distribution*

$$\Phi^{-1}_{\widetilde{M}}(\alpha) = f\left(\Phi^{-1}_{\widetilde{M}_1}(\alpha), \Phi^{-1}_{\widetilde{M}_2}(\alpha), \cdots, \Phi^{-1}_{\widetilde{M}_n}(\alpha)\right), \quad \forall \alpha \in [0, 1]. \tag{A.9}$$

Under the seminal work of Liu and Iwamura (1998), a minimin chance-constrains programming (CCP) is constructed in Liu (2002) by employing the credibility measure, which is an appropriate tool to formulate a fuzzy decision system with minimal objective function. The typical single-objective minimin CCP model is given as

$$\begin{cases} \min\limits_{\boldsymbol{x}} \min \ \overline{f} \\ \text{s.t.} \\ \quad \mathrm{Cr}\{f(\boldsymbol{x}, \widetilde{\boldsymbol{M}}) \leq \} \geq \alpha \\ \quad \mathrm{Cr}\{g_j(\boldsymbol{x}, \widetilde{\boldsymbol{M}}) \leq 0, j = 1, 2, \ldots, q\} \geq \beta, \end{cases} \tag{A.10}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$ and $\widetilde{\boldsymbol{M}} = (\widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_n)$ represent the decision vector and fuzzy vector, respectively, $f(\boldsymbol{x}, \widetilde{\boldsymbol{M}})$ denotes the cost function under the constraint functions $g_j(\boldsymbol{x}, \widetilde{\boldsymbol{M}})$, and α and β are the confidence levels arbitrarily determined by decision makers. Note that if the decision vector x is given, the value of '$\min \overline{f}$' is the α-pessimistic value of $f(\boldsymbol{x}, \widetilde{\boldsymbol{M}})$ in the view of Definition A.5.

Appendix B. Proofs

Appendix B.1. Proof of Lemma 1

According to the aforementioned notation, it should be noted that the decision variables k_i, f_i and T are all positive. Then, it follows immediately from Definition A.7.

Appendix B.2. Proof of Proposition 1

It is not hard to find that $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ is a regular fuzzy interval and following from Definition A.2-A.5, for $\alpha \in (0, 1]$ and $\alpha \neq 0.5$, we have $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha) = \Phi^{-1}(\alpha)$, where Φ^{-1} represents the inverse credibility distribution of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$. In this regard, the argument may break down into two cases.

Case I: $\alpha \in (0, 0.5) \cup (0.5, 1]$. In accordance with Theorem A.2 and the monotonicity property of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ proven in Lemma 1, its inverse credibility distribution is derived as

$$\begin{aligned}\Phi^{-1}(\alpha) = \ & \frac{S}{T} + \sum_{i=1}^{n} \frac{\Phi^{-1}_{\tilde{s}_i^{DC}}(\alpha)}{k_i T} + \sum_{i=1}^{n} \frac{\Phi^{-1}_{\tilde{s}_i}(\alpha) f_i}{k_i T} + \sum_{i=1}^{n} \frac{\Phi^{-1}_{\tilde{h}_i^{DC}}(\alpha) D_i k_i T (f_i - 1)}{2(1 - E[p_i]) f_i} \\ & + \sum_{i=1}^{n} \frac{\Phi^{-1}_{\tilde{h}_i}(\alpha) D_i k_i T}{2 f_i} + \sum_{i=1}^{n} \frac{\Phi^{-1}_{\tilde{h}_i}(\alpha) D_i^2 E[p_i] k_i T}{x_i (1 - E[p_i])^2 f_i},\end{aligned} \tag{B.1}$$

where $\Phi^{-1}_{\tilde{s}_i^{DC}}, \Phi^{-1}_{\tilde{s}_i}, \Phi^{-1}_{\tilde{h}_i^{DC}}$ and $\Phi^{-1}_{\tilde{h}_i}$ are the inverse credibility distributions of $\tilde{s}_i^{DC}, \tilde{s}_i, \tilde{h}_i^{DC}$ and $\tilde{h}_i$, $i = 1, 2, \ldots, n$, respectively. Additionally, according to Eqs. (A.7) and (A.8), if $0 < \alpha < 0.5$, we have

$$\Phi^{-1}_{\tilde{s}_i^{DC}}(\alpha) = (1 - 2\alpha) s^{DC}_{i\_min} + 2\alpha s^{DC}_{i\_mid1} = (\tilde{s}_i^{DC})_{\inf}(\alpha),$$

and if $0.5 < \alpha \leq 1$, we have

$$\Phi^{-1}_{\tilde{s}_i^{DC}}(\alpha) = (2 - 2\alpha) s^{DC}_{i\_mid2} + (2\alpha - 1) s^{DC}_{i\_max} = (\tilde{s}_i^{DC})_{\inf}(\alpha).$$

Combining the above analyses, we could substitute $\Phi^{-1}_{\tilde{s}_i^{DC}}(\alpha)$ in Eq. (B.1) with $(\tilde{s}_i^{DC})_{\inf}(\alpha)$. In a similar manner, the process can be implemented for $\tilde{s}_i, \tilde{h}_i^{DC}$ and $\tilde{h}_i$, respectively. Besides, the α-pessimistic value of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$, i.e., $\inf\{x \mid \Phi(x) \leq \alpha\}$, absolutely equals to $\Phi^{-1}(\alpha)$ in this case. That is Eq. (3).

Case II: $\alpha = 0.5$. On the basis of Definition A.5, it is easy to obtain $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(0.5)$ by taking advantage of the mathematical property of its inverse credibility function, that is the left end point of the $\Phi^{-1}(0.5)$ which can be calculated by the infimum of $\Phi^{-1}_{\tilde{s}_i^{DC}}(0.5), \Phi^{-1}_{\tilde{s}_i}(0.5), \Phi^{-1}_{\tilde{h}_i^{DC}}(0.5)$ and $\Phi^{-1}_{\tilde{h}_i}(0.5)$. In that way, the α-pessimistic value of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)$ can be regarded as a function of $(\tilde{s}_i^{DC})_{\inf}(0.5), (\tilde{s}_i)_{\inf}(0.5), (\tilde{h}_i^{DC})_{\inf}(0.5)$ and $(\tilde{h}_i)_{\inf}(0.5)$ in the view of Eqs. (A.7) and (A.8). That is Eq. (3).

Appendix B.3. Proof of Proposition 2

It is clear that for given fixed values k_i and T, the best f_i^* may satisfy the following conditions:

$$\Theta_i(k_i, f_i^*, T) - \Theta_i(k_i, f_i^* - 1, T) \leq 0 \quad \text{and} \quad \Theta_i(k_i, f_i^*, T) - \Theta_i(k_i, f_i^* + 1, T) \leq 0,$$

which leads to

$$-D_i k_i T \frac{\frac{(\tilde{h}_i)_{\text{inf}}(\alpha_0)}{2} + \frac{(\tilde{h}_i)_{\text{inf}}(\alpha_0) D_i E[p_i]}{x_i(1-E[p_i])^2} - \frac{(\tilde{h}_i^{DC})_{\text{inf}}(\alpha_0)}{2(1-E[p_i])}}{f_i(f_i-1)} + \frac{(\tilde{s}_i)_{\text{inf}}(\alpha_0)}{k_i T} \leq 0$$

and

$$D_i k_i T \frac{\frac{(\tilde{h}_i)_{\text{inf}}(\alpha_0)}{2} + \frac{(\tilde{h}_i)_{\text{inf}}(\alpha_0) D_i E[p_i]}{x_i(1-E[p_i])^2} - \frac{(\tilde{h}_i^{DC})_{\text{inf}}(\alpha_0)}{2(1-E[p_i])}}{f_i(f_i+1)} - \frac{(\tilde{s}_i)_{\text{inf}}(\alpha_0)}{k_i T} \leq 0.$$

Denote $\delta(k_i, T) = k_i^2 T^2 D_i \frac{(\tilde{h}_i)_{\text{inf}}(\alpha_0) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\text{inf}}(\alpha_0) D_i E[p_i] - (\tilde{h}_i^{DC})_{\text{inf}}(\alpha_0) x_i (1-E[p_i])}{2(\tilde{s}_i)_{\text{inf}}(\alpha_0) x_i (1-E[p_i])^2}$. Therefore, we have $f_i^*(f_i^*-1) \leq \delta(k_i, T) \leq f_i^*(f_i^*+1)$, where $\delta(k_i, T)$ depends on the predetermined confidence level α_0 and is larger than zero. Since f_i^* must be a positive integer in practice, it is not hard to get

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\delta(k_i, T)} \leq f_i^* \leq \frac{1}{2} + \frac{1}{2}\sqrt{1+4\delta(k_i, T)}.$$

Consider that these two inequalities differ 1, which indicates if the values of k_i and T are ascertained, there exists an integer number in this interval, or two integer numbers in the end-point of this interval. Note that the latter case implies when f_i^* takes either of these two values, the objective function would be minimized. Clearly, we have $f_i^*(k_i, T) = \lfloor \frac{1}{2} + \frac{1}{2}\sqrt{1+4\delta(k_i, T)} \rfloor$.

Appendix B.4. Proof of Proposition 3

Apparently, for given vectors $\boldsymbol{k}$ and $\boldsymbol{f}$, the objective function

$$\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\text{inf}}(\alpha_0) = \frac{A(k, f)}{T} + B(\boldsymbol{k}, \boldsymbol{f})T$$

is a convex function of T, where A and B are declared in Eqs. (9) and (10). By taking the partial derivative of $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\text{inf}}(\alpha_0)$ with respect to T and equal to zero, i.e.,

$$\frac{\partial \widetilde{TC}(k, f, T)_{\text{inf}}(\alpha_0)}{\partial T} = -\frac{A(k, f)}{T^2} + B(k, f) = 0,$$

denote $T^*(\boldsymbol{k}, \boldsymbol{f}) = \sqrt{\frac{A(k,f)}{B(k,f)}}$. Clearly, $T^*(\boldsymbol{k}, \boldsymbol{f}) \in (0, 1)$. It is also found that

$$\frac{\partial^2 \widetilde{TC}(k, f, T)_{\text{inf}}(\alpha_0)}{\partial T^2} = 2\frac{A(k, f)}{T^3} > 0,$$

which implies that $\widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\text{inf}}(\alpha_0)$ is strictly decreasing in $\{T \mid 0 < T \leq T(\boldsymbol{k}, \boldsymbol{f})^*\}$ and strictly increasing in $\{T \mid T^*(\boldsymbol{k}, \boldsymbol{f}) \leq T < 1\}$, which indicates that the value of $T^*(\boldsymbol{k}, \boldsymbol{f})$ achieves the lowest objective value.

Appendix B.5. Proof of Proposition 4

Assume that a movement of adding 1 in k_i may result in solution quality improvements, i.e., $\widetilde{TC}(\hat{\boldsymbol{k}}, \boldsymbol{f}, T)_{\text{inf}}(\alpha_0) \leq \widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\text{inf}}(\alpha_0)$. To prove this, the difference (D_1) on the above-mentioned function is computed first. Then, we have

$$D_1(k_i) = \Theta_i(k_i, f_i, T) - \Theta_i(k_i + 1, f_i, T).$$

That is,

$$D_1(k_i) = -D_i T \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i - 1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]}{2x_i(1-E[p_i])^2 f_i} + \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i}{k_i(k_i+1)T}.$$

Therefore, the first-order partial derivative of the right-side equality with respect to T, i.e.,

$$\frac{\partial D_1(k_i)}{\partial T} = -D_i \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i - 1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]}{2x_i(1-E[p_i])^2 f_i} - \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i}{k_i(k_i+1)T^2}.$$

is absolutely less than zero. In this regard, we may conclude that the function $D_1(k_i)$ is decreasing with respect to T. Let $D_1(k_i) = 0$. The critical value is exactly T_i^{1*} in Eq. (11). Clearly, when $T \leq T_i^{1*}$, we have $D_1(k_i) \geq 0$ and so $\widetilde{TC}(\hat{\boldsymbol{k}}, \boldsymbol{f}, T)_{\inf}(\alpha_0) \leq \widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0)$.

Appendix B.6. Proof of Proposition 5

Assume that a movement of subtracting 1 in k_i may result in solution quality improvements, i.e., $\widetilde{TC}(\hat{\boldsymbol{k}}, \boldsymbol{f}, T)_{\inf}(\alpha_0) \leq \widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0)$. In a similar manner, we denote

$$D_2(k_i) = \Theta_i(k_i, f_i, T) - \Theta_i(k_i - 1, f_i, T).$$

That is,

$$D_2(k_i) = D_i T \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i - 1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]}{2x_i(1-E[p_i])^2 f_i} - \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i}{k_i(k_i-1)T}.$$

Therefore, the first-order derivative of the left-side inequality with respect to T, i.e.,

$$\frac{\partial D_2(k_i)}{\partial T} = D_i \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha) x_i (1-E[p_i])(f_i - 1) + (\tilde{h}_i)_{\inf}(\alpha) x_i (1-E[p_i])^2 + 2(\tilde{h}_i)_{\inf}(\alpha) D_i E[p_i]}{2x_i(1-E[p_i])^2 f_i} + \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha) + (\tilde{s}_i)_{\inf}(\alpha) f_i}{k_i(k_i-1)T^2}.$$

is absolutely greater than zero. In this regard, we may conclude that the function $D_2(k_i)$ is increasing with respect to T. Let $D_2(k_i) = 0$. The critical value is exactly T_i^{2*} in Eq. (12). Clearly, when $T \geq T_i^{2*}$, $D_2(k_i) \geq 0$ and so $\widetilde{TC}(\hat{\boldsymbol{k}}, \boldsymbol{f}, T)_{\inf}(\alpha_0) \leq \widetilde{TC}(\boldsymbol{k}, \boldsymbol{f}, T)_{\inf}(\alpha_0)$.

Appendix C. Experiment data

Table C.1: The problem parameters involving 20 items for experiments.

NO.	D_i	x_i	a_i	b_i	$\tilde{s}_i^{DC}$	$\tilde{s}_i$	$\tilde{h}_i^{DC}$	$\tilde{h}_i$
Item 1	10000	12500	10.5	100	(45.1,45.6,46.2,47.3)	(4.0,4.2,4.3,4.7)	(0.42,0.61,0.91)	(1.03,1.38,1.69)
Item 2	9500	11875	13.0	100	(45.7,46.0,46.7,47.4)	(4.6,4.9,5.2,5.5)	(0.51,0.72,0.79)	(1.02,1.41,1.74)
Item 3	9000	11250	13.5	100	(42.1,43.4,43.6,44.7)	(5.2,5.3,5.6,6.2)	(0.47,0.74,1.05)	(0.97,1.25,1.49)
Item 4	8500	10625	20.3	100	(44.2,45.1,45.6,46.0)	(4.9,5.1,5.4,5.8)	(0.56,0.64,0.95)	(1.14,1.39,1.68)
Item 5	8000	10000	3.3	100	(41.5,42.9,43.2,44.4)	(3.5,3.7,3.9,4.5)	(0.51,0.67,0.76)	(1.23,1.41,1.84)
Item 6	7500	9375	6.0	100	(43.9,44.7,45.1,45.9)	(4.7,5.1,5.2,5.6)	(0.65,0.74,1.02)	(1.13,1.28,1.83)
Item 7	7000	8750	23.2	100	(44.6,45.9,46.2,46.6)	(3.9,4.1,4.3,4.8)	(0.72,0.92,1.24)	(1.03,1.42,1.67)
Item 8	6500	8125	19.7	100	(42.9,43.8,44.6,45.4)	(4.8,4.9,5.3,5.5)	(0.48,0.53,0.74)	(1.10,1.35,1.66)
Item 9	6000	7500	11.0	100	(44.4,45.1,45.6,46.1)	(3.5,4.0,4.1,4.6)	(0.77,0.90,1.04)	(1.14,1.31,1.78)
Item 10	5500	6875	9.1	100	(41.7,42.8,43.3,44.2)	(4.4,4.7,5.1,5.2)	(0.83,0.88,1.02)	(1.20,1.37,1.76)
Item 11	5000	6250	7.9	100	(44.1,44.4,44.9,45.5)	(4.1,4.5,4.6,4.9)	(0.64,0.75,1.00)	(1.12,1.30,1.69)
Item 12	4500	5625	24.1	100	(42.9,43.9,44.5,45.2)	(4.0,4.4,4.6,4.8)	(0.76,0.89,1.19)	(1.10,1.42,1.75)
Item 13	4000	5000	2.9	100	(41.8,42.9,43.3,44.5)	(3.7,4.0,4.3,4.7)	(0.58,0.73,1.06)	(1.12,1.38,1.43)
Item 14	3500	4375	15.6	100	(43.1,44.5,45.0,45.5)	(3.6,3.8,4.2,4.4)	(0.83,0.90,1.06)	(1.07,1.27,1.61)
Item 15	3000	3750	13.8	100	(42.1,43.2,43.5,44.4)	(3.9,4.3,4.4,5.2)	(0.72,0.98,1.17)	(1.02,1.38,1.81)
Item 16	2500	3125	7.2	100	(41.9,42.4,43.1,43.4)	(4.8,4.9,5.4,5.5)	(0.51,0.60,0.89)	(1.07,1.27,1.73)
Item 17	2000	2500	22.5	100	(44.8,45.8,45.8,46.6)	(4.0,4.2,4.5,5.1)	(0.55,0.61,0.94)	(1.20,1.28,1.65)
Item 18	1500	1875	8.2	100	(44.0,45.1,45.8,46.0)	(4.9,5.2,5.4,5.8)	(0.57,0.75,0.87)	(1.01,1.31,1.73)
Item 19	1000	1250	1.2	100	(44.0,44.5,45.4,46.1)	(5.0,5.3,5.5,6.0)	(0.55,0.70,0.92)	(1.27,1.36,1.82)
Item 20	500	625	5.5	100	(43.4,44.2,44.6,44.9)	(4.5,4.7,5.0,5.5)	(0.43,0.60,0.78)	(1.27,1.34,1.77)

Table C.2: Problem parameter settings of the large-scale cases.

Parameter	Data range	Parameter		Data range	Parameter		Data range
D_i	$U[100, 10000]$		$s_{i\_min}^{DC}$	$U[40, 46]$		s_{i_min}	$U[3, 5.5]$
S	$10 \times item\, num$	$\tilde{s}_i^{DC}$	$s_{i\_mid1}^{DC}$	$s_{i\_min}^{DC} + 1.5 \times rand()$	$\tilde{s}_i$	s_{i_mid1}	$s_{i\_min} + 0.5 \times rand()$
x_i	$1.25D_i$		$s_{i\_mid2}^{DC}$	$s_{i\_mid1}^{DC} + 1 \times rand()$		s_{i_mid2}	$s_{i\_mid1} + 0.5 \times rand()$
a_i	$U[0, 25]$		$s_{i\_max}^{DC}$	$s_{i\_mid2}^{DC} + 1.5 \times rand()$		s_{i_max}	$s_{i\_mid2} + 0.5 \times rand()$
b_i	100		$h_{i\_min}^{DC}$	$U[0.4, 0.9]$		h_{i_min}	$U[0.9, 1.4]$
α_0	0.8	$\tilde{h}_i^{DC}$	$h_{i\_mid}^{DC}$	$h_{i\_min}^{DC} + 0.5 \times rand()$	$\tilde{h}_i$	h_{i_mid}	$h_{i\_min} + 0.5 \times rand()$
			$h_{i\_max}^{DC}$	$h_{i\_mid}^{DC} + 0.5 \times rand()$		h_{i_max}	$h_{i\_mid} + 0.5 \times rand()$

Table C.3: The problem inputs for parameter analysis ($S = 200$).

NO.	D_i	x_i	$E[p_i]$	$\tilde{s}_i^{DC}$	$\tilde{s}_i$	$\tilde{h}_i^{DC}$	$\tilde{h}_i$
Item 1	10000	12500	0.1	(43.5,45,46,47.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)
Item 2	5000	6250	0.1	(44.5,46,47,48.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)
Item 3	3000	3750	0.1	(45.5,47,48,49.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)
Item 4	1000	1250	0.1	(42.5,44,45,46.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)
Item 5	600	750	0.1	(43.5,45,46,47.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)
Item 6	200	250	0.1	(45.5,47,48,49.5)	(4.5,5.0,5.5,6.0)	(0.50,1.00,1.50)	(1.00,1.50,2.00)

Appendix D. Modified RAND algorithm

In this part, we construct a modified RAND algorithm (MRAND) according to the optimality conditions of k_i, f_i and T, respectively. In the main body of the paper, the conclusions and proofs of f_i and T are given in Propositions 2-3. Here, we supplement the best value of k_i for given f_i and T as follows:

Proposition D.1. *For given fixed values of f_i and T, there are functions γ_1 and γ_2 such that the best value of k_i is uniquely given by*

$$k_i^*(f_i, T) = \left\lfloor \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma_1(f_i, T)/\gamma_2(f_i, T)} \right\rfloor, \tag{D.1}$$

where

$$\gamma_1(f_i, T) = \frac{(\tilde{s}_i^{DC})_{\inf}(\alpha_0) + (\tilde{s}_i)_{\inf}(\alpha_0) f_i}{T}$$

and

$$\gamma_2(f_i, T) = D_i T \frac{(\tilde{h}_i^{DC})_{\inf}(\alpha_0)(f_i - 1)x_i(1 - E[\,p_i\,]) + (\tilde{h}_i)_{\inf}(\alpha_0)x_i(1 - E[\,p_i\,])^2 + 2(\tilde{h}_i)_{\inf}(\alpha_0)D_i E[\,p_i\,]}{2x_i(1 - E[\,p_i\,])^2 f_i}.$$

Proof: It is clear that for given fixed values f_i and T, the best k_i^* may satisfy the following conditions:

$$\Theta_i(k_i^*, f_i, T) - \Theta_i(k_i^* - 1, f_i, T) \le 0 \quad \text{and} \quad \Theta_i(k_i^*, f_i, T) - \Theta_i(k_i^* + 1, f_i, T) \le 0,$$

which leads to

$$-\frac{\frac{(\tilde{s}_i^{DC})_{\inf}(\alpha_0)}{T} + \frac{(\tilde{s}_i)_{\inf}(\alpha_0) f_i}{T}}{k_i(k_i - 1)} + \Big(\frac{(\tilde{h}_i^{DC})_{\inf}(\alpha_0) D_i T (f_i - 1)}{2(1 - E[\,p_i\,]) f_i} + \frac{(\tilde{h}_i)_{\inf}(\alpha_0) D_i T}{2 f_i} + \frac{\tilde{h}_i)_{\inf}(\alpha_0) D_i^2 T E[\,p_i\,]}{x_i(1 - E[\,p_i\,])^2 f_i}\Big) \le 0$$

and

$$\frac{\frac{(\tilde{s}_i^{DC})_{\inf}(\alpha_0)}{T} + \frac{(\tilde{s}_i)_{\inf}(\alpha_0) f_i}{T}}{k_i(k_i - 1)} - \Big(\frac{(\tilde{h}_i^{DC})_{\inf}(\alpha_0) D_i T (f_i - 1)}{2(1 - E[\,p_i\,]) f_i} + \frac{(\tilde{h}_i)_{\inf}(\alpha_0) D_i T}{2 f_i} + \frac{\tilde{h}_i)_{\inf}(\alpha_0) D_i^2 T E[\,p_i\,]}{x_i(1 - E[\,p_i\,])^2 f_i}\Big) \le 0.$$

Therefore, we have $k_i^*(k_i^* - 1) \le \gamma_1(f_i, T)/\gamma_2(f_i, T) \le k_i^*(k_i^* + 1)$, where $\gamma_1(f_i, T)$ and $\gamma_2(f_i, T)$ are declared in the above, and depend on the predetermined confidence level α_0. Since k_i^* must be a positive integer in practice, it is not hard to get

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma_1(f_i, T)/\gamma_2(f_i, T)} \le f_i^* \le \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma_1(f_i, T)/\gamma_2(f_i, T)}.$$

Consider that these two inequalities differ 1, which indicates if the values of f_i and T are ascertained, there exists an integer number in this interval, or two integer numbers in the end-point of this interval. Note that the latter case implies when k_i^* takes either of these two values, the objective function would be minimized. Clearly, we have $k_i^*(f_i, T) = \lfloor \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma_1(f_i, T)/\gamma_2(f_i, T)} \rfloor$.

The MRAND is in essential an iteratively search between k_i, f_i and T. Firstly, the range of T is divided into M equally-spaced values. Then, for given vectors $\boldsymbol{k} = (1, 1, \cdots, 1)$ and $\boldsymbol{f} = (1, 1, \cdots, 1)$, update the optimal value for k_i, f_i and T until the value of T does not change, accordingly. The detailed steps of MRAND are shown as follows:

Algorithm D.1: Modified RAND algorithm

Input: The initial sets $\boldsymbol{k} = (1, 1, \cdots, 1)$, $\boldsymbol{f} = (1, 1, \cdots, 1)$.

Output: The best result and solution.

```
1  Divide the range of [0,1] into M equally-spaced values T_1, ..., T_m, ..., T_M. Set m = 1.
2  Set T^s = T_m and T^s ≠ T^e.
3  while T^s ≠ T^e do
4      Reset T^s = T^e;
5      For given (f_i, T), compute the best of k_i, denoted as k̂_i, using Eq. (D.1);
6      For given (k̂_i, T), compute the best of f_i, denoted as f̂_i, using Eq. (7);
7      For given (k̂_1, ..., k̂_n, f̂_1, ..., f̂_n), compute the best of T, denoted as T̂, using Eq. (8);
8      Reset T^e = T̂.
9  Given solution (k̂_1, ..., k̂_n, f̂_1, ..., f̂_n, T̂), calculate the cost TC_m using Eq. (6).
10 Reset m = m + 1.
11 If m ≤ M, repeat the steps 2 to 10. Otherwise, stop and return the best solution with the
   corresponding minimum TC_m.
```