# Theories of Redistribution and Share of Labour Income

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#### Abstract

We review the functional and size theories of income distribution in the context of growing inequality around the world. The causes for the observed patterns on the shares of labour and capital income in classical, neoclassical, Marxian, structuralists or general equilibrium theories are reviewed. Each of these theory is found helpful in explaining the inequality of in income observed in the OECD, emerging and developing economies. Significant empirical evidence exists for declining labour share as shown by panel data estimation of 127 countries from 1990 to 2011.

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#### 1 Introduction

Who benefits from the process of economic growth? Do low income people (or poor) benefit as much as rich people? Do capitalists gain more than workers? These issues of size and functional distribution of income is discussed widely in the literature on distribution of income (Picketty (2014), Jenkins (1995), Atkinson (1970), Kuznet (1955)). Share of labour income has gradually declined in the global economy (Figure 1) and as discussed in Karabarbounis and Neiman (2014). Bachman et al. (2016) apply a dynamic CGE model of the US economy to investigate how tax and transfer system affects the distribution of income in the US economy under Trump and Cruz tax-transfer scenarios and find the share of labour income not only depends on tax and transfer system but also on the substitutability of capital and labour in production.



Gini coefficient an indicator of the inequality of income. Its value range between zero and one; zero for perfect equality and one for perfect inequality. In majority of countries in the Western Europe it has increased from around 0.29 in 1960 to 0.39 in recent years mainly because of declining share of labour. OECD (2015) provides a time series evidence on growing inequality among the OECD and emerging economies and its adverse impacts on economic growth (Figure 2).



Should there be more redistribution or more growth? It is obvious that no one solution fits to all circumstances. Institutions and culture vary by countries as do the endowments of labour and capital as well as of the natural resources. Policies should be designed according to economic and social institutions of a country (see debates in the World Economic Forum in Davos, http://www.weforum.org/). See Penn World Tables for the dataset at:

http://www.rug.nl/research/ggdc/data/penn-world-table. Picketty suggests global tax on property to reduce such inequality. This idea is close to socialists' approach to income distribution. Capitalists would argue for more equality by raising productivity of workers through additional accumulation of physical capital, development of human capital by investing in education and skills. They favour taxes on consumption rather than in income, indirect tax than the direct tax.



Objective of this paper is to provide a general overview of these theories with recent evidences on the share of labour in income and to show how these theories enhance our understanding on the role of income distribution in production, consumption and social welfare in an economy.

#### 2 Theories on share of labour in total income

The shares of capital and labour in national income vary considerably both over time and across countries. Picketty (2014) has formed dataset on income distribution between capital and labour for last 200 years for the advanced countries of Europe as well as the US (see http://topincomes.parisschoolofeconomics.eu/). He found that trade openness and technological innovation have a positive and significant effect on labour shares. Similarly foreign direct investments (FDI) inflows and mechanization seem to be negative drivers in it . He also looks into a number of variables including the level of economic development, education, and the strength of the regulations in the labour market.

Share of labour in income was an important issue in the functional distribution of income. It was widely discussed by classical and neoclassical economists, Marx, Kaldor, Hansen, Hahn, Hicks in terms of marginal productivity theory of distribution. Factors are paid according their marginal productivity in their theories. Recently there are new theories of bargaining of income and wages (Mortensen and Pissarides (1994)). Bhattarai, Haughton and Tuerck (2015a, 2015b) find significant impact of fair taxes and corporate income taxes on growth and distribution of income in the US economy.

It was believed that the share of labour was relatively constant, between 60 to 70 percents of GDP up to 1980s (Parente and Prescott (2002)). However many recent studies find that there is a general pattern of reduction in the share of GDP going to labour around the world, in particular from the mid-1980s onwards as shown in Figure 1 above. Seminal works on labour share are include Hicks (1932), Kuznet (1955), Kennedy (1964), Hahn (1972), Cowling, Molho and Oswald (1981), Lavoie and Stockhammer (2014), O'Mahony and Timmer (2009), Stockhammer, Onaran and Ederer (2009), Stockhammer and Onaran (2004) and Elsby, Hobijn, Şahin (2013) and Picketty (2014). What is the optimal amount labour share  $(1 - \alpha)$  that maximises the economic growth? This issue is yet far from settled. A sort summary on important theories is provided here for a concise understanding of these topics.

## 2.1 Neoclassical theory of functional distribution of income

The neoclassical theory of functional distribution can simply be represented in a employment-wage diagram. Market sets the wage rate (w) where the demand for labour intersects to the supply of labour. Area of rectangle represents wage bill. The area below the demand curve is part of production that goes to employers as profit.



Figure 3: Wage and profit in the neoclassical distribution model

Marginal productivity declines with more employment due to the law of diminishing returns to labour. Upward sloping labour supply function represents psychological and other costs as working more hours becomes more difficult. Workers are ready to put in more efforts only if they are paid more. The equilibrium point shown by the intersection between the demand and supply of labour also shows the functional distribution of income in wages and profits. It is obvious when profits are high when wages are low. There are always incentives among owners to pay low wage in order to minimise cost and to raise their profits. Given the  $\alpha$  and  $\beta$  shares of capital and labour in total output (*Y*), division of national income between capital (*rK*) and labour (*wL*) occurs according to the marginal productivity of these inputs as shown in following equations:

$$\frac{Y}{A} = K^{\alpha} L^{\beta} \Longrightarrow Y = A K^{\alpha} L^{\beta}$$
(1)

$$Y = rK + wL \Longrightarrow \frac{rK}{Y} + \frac{wL}{Y} = 1$$
<sup>(2)</sup>

$$\frac{rK}{Y} + \frac{wL}{Y} = \frac{\alpha A K^{\alpha - 1} L^{\beta} K}{A K^{\alpha} L^{\beta}} + \frac{\beta A K^{\alpha} L^{\beta - 1} L}{A K^{\alpha} L^{\beta}} = (\alpha + \beta) = 1$$
(3)

Technology of production is characterised by the elasticity of substitution ( $\sigma$ ) between labour and capital that measures the degree of response of capital labour ratio to the wage rental ratios. In a simple Cobb-Douglas function this elasticity of substitution is 1.

$$\sigma = \frac{d\left(\frac{K}{L}\right) / \left(\frac{K}{L}\right)}{d\left(\frac{w}{r}\right) / \left(\frac{w}{r}\right)} = \frac{d\left(\frac{K}{L}\right) / \left(\frac{K}{L}\right)}{d\left(\frac{(1-\alpha)AK^{\alpha}L^{-\alpha}}{\alpha AK^{\alpha-1}L^{1-\alpha}}\right) / \left(\frac{(1-\alpha)AK^{\alpha}L^{-\alpha}}{\alpha AK^{\alpha-1}L^{1-\alpha}}\right)} = \frac{d\left(\frac{K}{L}\right) / \left(\frac{K}{L}\right)}{d\left(\frac{K}{L}\right) / \left(\frac{K}{L}\right)} = 1$$
(4)

A CES production function allows other values of  $\sigma$ , a higher value of  $\sigma$  represents more capital intensive technology and a lower value of  $\sigma$  indicates more labour intensive technology. Productivity of labour rises with more capital, whether wage rate rises depends on how the distribution of income occurs between capital and labour. Thus rising income inequality among the global economy reflects more capital intensity production and more return to capital than to the labour. Increase in human capital through education can enhance the labour share and more investment in education and skills is important in raising the value of  $\beta$ , share of labour in the national income.

#### 2.2 Marxian theory the surplus value (S)

The notion of surplus value (S) is a key concept in the Marxian theory. This theory attributes that all value is created by labour. Only labour generates value. Capital is made by labour in the past. The capitalists own the capital and they pay only

the subsistence wage to the labour. Each unit of labour creates more output than requires for its subsistence but the surplus value, the gap between the output and wage. goes to capitalists who own the firm and employ the labour. In other words the surplus value (S) per unit of output represents the amount by which price (P) of a commodity is above the wage (W):

$$S = P - W$$

Wage share in output is denoted simply by the ratio wage to the price  $(\frac{W}{P})$  is the output minus the the share of the surplus value  $s = \frac{S}{P}$ . Thus:

$$\frac{W}{P} = (1-s); \quad s = \frac{S}{P}$$

Capitalists squeeze on wages by increasing the surplus value ratio (*s*); more strictly following the iron law of wage. More is the surplus value less is paid to the labour. More unequal becomes the income distribution. Labour produces more than the iron wage requited for its subsistence but the capitalists do not pay more that what is necessary for survival. Development of economies in advanced economies avoided class between capitalists and workers by adopting more egalitarian social security system funded by the tax-transfer system and provision for pensions and tax credits to low income groups in the last century, particularly after the World War II. Waves privatisation and deregulation since mid 1980s have gradually caused increases in the surplus value and reductions in the share of labour. This has resulted in significant increase in inequality in each country among the OECD and other economies as shown in Figure 1 - 3 above.

#### 2.3 Kaldorian theory of functional distribution

In Kaldorian theory of functional distribution, the share of labour is a function of saving rates of workers ( $s_w$ ) and capitalists ( $s_p$ ) and investment ratios ( $\frac{I}{Y}$ ). Let the output (Y) be total of wage (W) and profits (P):

$$Y = W + P; \quad \frac{W}{Y} = 1 - \frac{P}{Y} \tag{5}$$

Investment (*I*) equals saving (*S*). Saving occurs from profits and wages as:

$$I = S \text{ and } S = S_p + S_w S_p = s_p P \text{ and } S_w = s_w W; \quad s_p > s_w$$
(6)

It is assumed that owners of firms save more than workers,  $s_p > s_w$ :

$$I = s_p P + s_w W = s_p P + s_w (Y - P) = (s_p - s_w) P + s_w Y$$
(7)

$$\frac{I}{Y} = \left(s_p - s_w\right)\frac{P}{Y} + s_w; \ \frac{P}{Y} = \frac{1}{\left(s_p - s_w\right)}\frac{I}{Y} - \frac{s_w}{\left(s_p - s_w\right)} \tag{8}$$

Profit increases by more investment but decreases by more saving by workers. The share of labour in this model increases when saving from profit increases relative to that from wage and reduced by higher rate of investment.

$$\frac{W}{Y} = 1 - \frac{P}{Y} = 1 - \left[\frac{1}{(s_p - s_w)}\frac{I}{Y} - \frac{s_w}{(s_p - s_w)}\right] = \frac{s_p}{(s_p - s_w)} - \frac{1}{(s_p - s_w)}\frac{I}{Y}$$

Wage share can increase if the capitalists save more than workers but reduces if they invest more. Why investment reduces labour share is counter intuitive.

#### 2.4 Hahn's Dynamic Theory of Wage Share

Hahn (1972) adopts Hicks' dynamic theory of wage share. He shows interdependence between the wage share and the marginal propensity to consume. The interaction between supply and demand factors in the economy. By acceleration principle change in output at time period t, ( $\Delta Y_t$ ) responds to the level of investment in t - 1 period as:

$$\Delta Y_t = \alpha I_{t-1}$$

Here  $\alpha$  is the measure of acceleration effect of investment in income. Level of investment ( $I_t$ ) on the other hand is responds to lagged change in demand, ( $Y_{t-1} - Y_{t-2}$ ):

$$I_{t} = \nu \left( Y_{t-1} - Y_{t-2} \right) = \nu \Delta Y_{t-1}$$
(9)

These two are combined into a second order difference equation:

$$\Delta Y_t = \alpha \nu \Delta Y_{t-2} \tag{10}$$

This equation has two roots  $\lambda_1$  and  $\lambda_2$ . Let  $\Delta Y_t = \lambda^t A_t$ 

$$\Delta Y_t - \alpha \nu \Delta Y_{t-2} = \lambda^t A_1 - \alpha \nu \lambda^{t-2} A_2 = 0 \Longrightarrow \lambda^2 - \alpha \nu = 0$$
(11)

 $\lambda_1 = \sqrt{\alpha \nu}$  and  $\lambda_2 = -\sqrt{\alpha \nu}$ . The transitional path of income can be found as:

$$\Delta Y_t = \lambda_1 A_1 + \lambda_2 A_2; \quad \Delta Y_t = \lambda_1 A_1 \quad \because A_2 = 0 \tag{12}$$

Let the demand consist of consumption  $(cY_{t-1})$  and investment  $(\nu (Y_{t-1}^d - Y_{t-2}^d))$ , with *c* propensity to consume and *v* as coefficient for investment demand:

$$Y_t^d = cY_{t-1} + \nu \left(Y_{t-1}^d - Y_{t-2}^d\right)$$
(13)

In equilibrium demand and supply equal:

$$Y_t = cY_{t-1} + \nu \left( Y_{t-1} - Y_{t-2} \right)$$
(14)

Take the one period difference:

$$Y_{t} - Y_{t-1} = \Delta Y_{t} = c \Delta Y_{t-1} + \nu \left( \Delta Y_{t-1} - \Delta Y_{t-2} \right)$$
(15)

$$Y_t = \gamma_1^t R_1 + \gamma_2^{t-2} R_2; \ \Delta Y_t = \gamma_1^t R_1 \quad \because R_2 = 0$$
 (16)

Dynamic condition for equilibrium between demand and supply implies  $\lambda_1 = \gamma_1$  Then

$$-\nu + \sqrt{\frac{(\alpha\nu + v)^2}{\alpha\nu}} = c$$

Let marginal propensity to consume (*c*) relates to wage share  $\left(\frac{W}{Y}\right)$  as:

$$c = z \frac{W}{Y} + k$$

Then wage share also depends on the marginal propensity to consume:

$$\frac{W}{Y} = \frac{1}{z} \left[ c - k \right] = \frac{1}{z} \left[ -\nu + \sqrt{\frac{\left(\alpha\nu + v\right)^2}{\alpha\nu}} - k \right]$$

Proof of wage share:

$$\lambda_{1} = \sqrt{\alpha \nu} = \gamma_{1} = \frac{(c+v) + \sqrt{(c+v)^{2} - 4v}}{2} \Longrightarrow \left[ 2\sqrt{\alpha \nu} - (c+v) \right]^{2} - (c+v)^{2} + 4v = 0$$

$$4\alpha \nu - 4\sqrt{\alpha \nu}(c+v) + (c+v)^{2} - (c+v)^{2} + 4v = 0 \implies \alpha \nu + v = \sqrt{\alpha \nu}(c+v)$$

$$\implies \frac{(\alpha \nu + v)^{2}}{2} = (c+v)^{2}$$

$$\Rightarrow \frac{-v}{\alpha \nu} = (c + v)$$
$$\Rightarrow c = -v + \sqrt{\frac{(\alpha \nu + v)^2}{\alpha \nu}}$$

Thus there is interdependence between the wage share and the marginal propensity to consume (c) as well as the supply factors as captured by the acceleration coefficient  $(\alpha, \nu)$  in the economy. This theory implies the increase in inequality among the OECD economies are due to the cyclical factors of demand and supply among OECD economies.

## 2.5 Labour Market and Search and Matching Model

Producers use labour to produce goods and services. A production function shows how labour complements with other inputs in production and the marginal productivity of labour shows the additional unit of output produced by each additional unit of labour. Thus demand for labour is derived from the demand for output. On the supply side every working age person has 168 hours a week, 720 hours per months or 8760 hours per year of time endowment which can be allocated between work and leisure. How many hours does one work and how much is spent in free time really depends upon the preference between consumption and leisure on one hand and the job vacancies on the other. In theory, flexibility of real wages guarantees equality between demand and supply in the labour in a competitive labour market. However, the labour market is far from a perfectly competitive market. Firms exercise monopoly powers, acting as monopsonists in the labour market or use their market power in order to retain more effective workers. Hiring decisions of firms also are dependent on the aggregate demand. Firms hire more workers during expansion but are reluctant of recruit any workers during the contraction. A significant number of workers become unemployed as a consequence.

Given a production function that related output  $(Y_t)$  to capital  $(K_t)$ , technology  $(A_t)$  and labour  $(L_t)$ 

$$Y_t = K_t^{\alpha} \left( A_t L_t \right)^{1-\alpha} \qquad 0 < \alpha < 1 \tag{17}$$

Wage rate should be paid according to the marginal productivity of labour as:

$$w_t = (1 - \alpha) K_t^{\alpha} (A_t L_t)^{-\alpha} A_t$$
(18)

Supply of labour occurs through the utility maximising behavior of the household.

$$\max \sum_{t=0}^{\infty} \beta^t U\left(c_t, 1 - l_t\right) \tag{19}$$

subject to

$$c_t + k_{t+1} = w_t l_t + (1 + r_t) k_t \tag{20}$$

This results in labour supply to be:

$$\overline{L}_t - L_t = \overline{L}_t - \frac{(1-\alpha)}{(1-\alpha) + (1-\alpha\beta)b}$$
(21)

Income patterns over time are different for different individuals. Some people start at a very low level of earnings and experience a rapid rise in income as they gain more job specific experience. Others may have a steady and stagnant income process over years. Still others may even have to face declining growth in income. What are the factors that lead to higher income growth rates and what are the factors that setback the process of income growth has been an issue of great interest among the labour economists.

The years of schooling and job market experience are the most important factors associated with higher income levels. Given other things constant, generally it is believed that an individual with greater number of schooling years earns more than a person with a few years or no schooling. Similarly a person with greater experience earns higher income. Both schooling and experience are perceived to be the main factors enhancing productivity of an individual.

There are a number of factors that set back the income process. Gender bias has been an area of continuous research in the labour economics. Females earn less than male either because of a structural breaks in their career for family reasons or due to gender discrimination in the labour market. Similarly there are cross regional variation in the income process.

As discussed in Pissarides (2013) and in Bhattarai and Dixon (2014) "the phenomenon of equilibrium unemployment results from the interaction among Nnumber of firms and unions (representing H number of households) which bargain over wages and employment".

Matching and bargaining functions across all *N* industries are key elements determining equilibrium unemployment. The Matching function (Beveridge curve) gives equilibrium conditions in the labour market balancing entry and exit from unemployment by aggregating sector and skill specific vacancies  $\left(V_{i,t}^{h}\right)$  and unemployment  $\left(UN_{i,t}^{h}\right)$  with job creation as:

$$M_t = M\left(V_t, UN_t\right) = V_t^{\gamma_t} UN_t^{(1-\gamma_t)}$$
(22)

where  $M_t$ ,  $V_t$  and  $UN_t$  denote the aggregate number of matching, vacancies and unemployment respectively among job seekers at time t and aggregate variables

are geometric means of household level variables<sup>1</sup>. The matching parameter  $\gamma_t$  is between zero and one and varies over time. It can be adjusted for prosperous period when there are more vacancies than job seekers or in recession when there are more unemployed than vacancies. In steady state it should be about 0.5 to reflect the balance between job creation and job destruction. Heterogeneity in the labour market is reflected by sector and skill specific  $M_{i,t}^h$ ,  $V_{i,t}^h$  and  $UN_{i,t}^h$ . These capture the labour market conditions where production sectors suffer from shortages of certain skills while facing abundance of other skills. In each case job seekers and employers bargain over expected earnings by maximising the Nashproduct  $(NP_{i,t}^h)$  of the bargaining game over the difference between the earnings from work  $(W_{i,t}^h)$  than in being unemployed  $(UN_{i,t}^h)$  and earnings to firms from filled  $(J_{i,t}^h)$  and vacant jobs  $(V_{i,t}^h)$ .

$$NP_{i,t}^{h} = \left(W_{i,t}^{h} - UN_{i,t}^{h}\right)^{\theta_{b}^{h}} \left(J_{i,t}^{h} - V_{i,t}^{h}\right)^{1-\theta_{b}^{h}}$$
(23)

Market imperfections in the labour market create opportunity of gains from bargains which is divided between firms and workers as indicated by parameter  $\theta_b^h$  that can assume any value between zero and one, reflecting the relative strength of unions (workers) over firms in such bargains. Symmetric solution of this satisfies joint profit maximisation condition as:

$$\left(W_{i,t}^{h}-UN_{i,t}^{h}\right)=\theta_{b}^{h}\left(J_{i,t}^{h}+W_{i,t}^{h}-V_{i,t}^{h}-UN_{i,t}^{h}\right)$$
(24)

In aggregate the job search model can be explained using three simple equations as summarised by Pissarides (1979, 2000).

First, for each skill type *h* the dynamics of unemployment depends on the rate of job destruction,  $\lambda_t^h (1 - un_t^h)$ , and the rate of job creation,  $\theta_t^h q \left(\theta_t^h\right) un_t^h$  as  $\Delta un_t^h = \lambda_t^h (1 - un_t^h) - \theta_t^h q \left(\theta_t^h\right) un_t^h$ . The steady state equilibrium implied by this is:

$$un_t^h = \frac{\lambda_t^h}{\lambda_t^h + \theta_t^h q\left(\theta_t^h\right)}; \quad un_T = \frac{\lambda_T}{\lambda_T + \theta_T q\left(\theta_T\right)}$$
(25)

where  $\lambda_t^h$  is the rate of idiosyncratic shock of job destruction of household type *h* and  $\theta_t^h$  is the ratio of vacancy to the unemployment and  $q\left(\theta_t^h\right)$  is the probability

$${}^{1}V_{t} = \prod_{i=1}^{N} V_{i,t}^{h}; UN_{t} = \prod_{i=1}^{N} UN_{i,t}^{h}; M_{t} = \prod_{i=1}^{N} M_{i,t}^{h} = \prod_{i=1}^{N} M\left(V_{i,t}^{h}, UN_{i,t}^{h}\right).$$

of filling a job with a suitable candidate through the matching process explained in (22). Then  $un_T$  is the equilibrium unemployment rate average across all households expressed in terms of averages of  $\lambda_t^h \theta_t^h$  and  $q(\theta_T)$  given by  $\lambda_T$ ,  $\theta_T$  and  $q(\theta_T)$ respectively.

Secondly the upward sloping wage curve in  $(\theta_t^h, w_t^h)$  space shows positive links between the reservation wage  $(z_t^h)$  the price of product p and cost of hiring  $(\theta_t^h c_t^h)$ implying higher wage rates for tighter labour markets as:

$$w_{i,t}^{h} = z_{t}^{h} \left( 1 - \theta_{b}^{h} \right) + \theta_{b}^{h} p_{t} \left( 1 + \theta_{t}^{h} c_{t}^{h} \right)$$

$$(26)$$

Finally there is a downward sloping job creation curve  $w_t^h = p_t - (r_t + \lambda_t^h) \frac{p_t c_t^h}{q(\theta_t^h)}$ , where  $p_t$  is the price of product,  $w_t^h$  the wage rate, and  $(r_t + \lambda_t^h) \frac{p_t c_{h_t}}{q(\theta_t^h)}$ , is the cost of hiring and firing. It shows the possibility of job creation at lower wage rates and creation of fewer jobs at higher wage rates. The optimal job creation (demand for labour curve) occurs when firms balance the marginal revenue product of labour to wage and hiring and firing costs (see some details in Bhattarai and Dixon (2014)).Following the market signals of demand and relative prices and costs of inputs, profit maximising firms create vacancies for specific tasks and hire workers when they find suitable candidates for these jobs. Similarly there are workers seeking jobs that match their skills and others who quit jobs and join the pool of unemployed who may choose to quit jobs and become unemployed. Market specific idiosyncratic shocks cause such entries and exits in the labour market. Equilibrium unemployment and wage rates result from a Nash-bargain between workers and firms. Whether the rate of unemployment falls or rises depends on the relative proportion of entry and exit into the labour market.

#### 2.6 Human capital theory of income share

Recently authors economists Becker, Mincer, Lucas, Aghion, Helpman, Jones, Weale, Temple and Blanchard have emphasized on the human capital theory of income distribution. Education provides skills and make people more productive. Higher productivity translates into higher wage rates. Individuals who invest more on education and skills earn more than others who have not invested in them. This can be illustrated with a simple model of life time income (LI) with and without university education as given below:

$$LI = \left[1 + (1+g) + (1+g)^2 + \dots + (1+g)^n\right] Y_0$$
  
=  $Y_0 \left[\frac{1 - (1+g)^{n+1}}{1 - (1+g)}\right]$  (27)

Life time income with university education

$$LI = Y_0 \left[ \frac{1 - (1 + g)^{n+1}}{1 - (1 + g)} \right] - 3 \times C$$
  
= 30000  $\left[ \frac{1 - (1.04)^{42+1}}{1 - (1.04)} \right] - 3 \times 15000 = 3,255,371$  (28)

Life time income without university education

$$Y_0\left[\frac{1-(1+g)^{n+1}}{1-(1+g)}\right] = 17000\left[\frac{1-(1.02)^{45+1}}{1-(1.02)}\right] = 1,263,620$$
 (29)

Extra life time income comes from the university education. Difference in income made the university level education in the life time of an individual thus is the difference between these two levels of income; £3,255,371-£1,263,620=£1,991,751. Thus university education makes one better off by nearly 2 million pounds. Studies of Jenkins (1995, 1996) illustrate on such differences. Econometrically these studies estimate a standard earning function from the labour market dataset such as the Annual Population Survey (APS). In Mincerian tradition earnings depend on qualifications and status of health and many other conditions as shown in a regression table below.

$$w_{i,t} = a_i + \beta_i S_{i,t} + \gamma_i A_{i,t} + \psi_i A_{i,t}^2 + \lambda_i G_{i,t} + \delta_i R_{i,t} + \pi_i P_{i,t} \pi + \theta_t \Delta_t + \varepsilon_{i,t}$$

where  $w_{i,t}$  is the wage rate of individual *i* in year *t*;  $S_{i,t}$  is years of schooling;  $A_{i,t}$  is age of individual *i* in time *t*;  $G_{i,t}$  is the gender of an individual,  $R_{i,t}$  is regional location,  $\Delta_t$  is wave t,  $P_{i,t}$  is professional background of individual *i*. Coefficients of such earning functions are estimated using cross section or panel dataset.

Bargaining between unions of workers and firms also is important as taxes on income and consumption and unemployment benefits (see Mirrlees et al. (2010)

Source	SS	df	MS		Number of obs	= 368525
Model   Residual	139225393 3.4450e+113	17 8189 868507 9348	9728.98 364.212		Prob > F R-squared	= 0.0000 = 0.0004 = 0.0004
Total	3.4464e+113	368524 9351	198.879		Root MSE	= 966.88
GRSEXP	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
ILLDAYS2	.5135184	3.158899	0.16	0.871	-5.67783	6.704866
ILLDAYS1	2.180592	2.085396	1.05	0.296	-1.906723	6.267907
ILLDAYS3	-4.435447	3.853808	-1.15	0.250	-11.9888	3.117903
ILLDAYS4	8.066717	4.485219	1.80	0.072	7241797	16.85761
ILLDAYS5	-5.238917	3.738412	-1.40	0.161	-12.56609	2.08826
ILLDAYS6	-1.988442	6.544168	-0.30	0.761	-14.81482	10.83793
ILLDAYS7	1315924	8.270102	-0.02	0.987	-16.34075	16.07756
QUAL_14	4.705906	5.354454	0.88	0.379	-5.788664	15.20048
QUAL_15	-6.997297	16.48718	-0.42	0.671	-39.31169	25.3171
QUAL_16	-12.89791	11.85998	-1.09	0.277	-36.14313	10.3473
QUAL_17	97.47657	34.51317	2.82	0.005	29.83178	165.1214
QUAL_18	-13.84225	32.57728	-0.42	0.671	-77.69276	50.00826
QUAL_19	3.716753	4.476542	0.83	0.406	-5.057138	12.49064
QUAL_2	13.91868	12.39392	1.12	0.261	-10.37304	38.2104
QUAL_20	32.4236	7.656584	4.23	0.000	17.41692	47.43028
QUAL_21	43.351	4.285564	10.12	0.000	34.95142	51.75058
QUAL_22	-1.453274	6.644386	-0.22	0.827	-14.47607	11.56953
_cons	-5.248379	52.42735	-0.10	0.920	-108.0044	97.50769

or the Green Budgets from the IFS for the UK for more extensive analysis on these issues).

Thinks of millions of workers in the economy. They work for earnings; in Mincerian traditions earnings depend on qualifications and status of health and many other conditions as shown in a regression table below.

$$w_{i,t} = a_i + \beta_i S_{i,t} + \gamma_i A_{i,t} + \psi_i A_{i,t}^2 + \lambda_i G_{i,t} + \delta_i R_{i,t} + \pi_i P_{i,t} \pi + \theta_t \Delta_t + \varepsilon_{i,t}$$

where  $w_{i,t}$  is the wage rate of individual i in year t;  $S_{i,t}$  is years of schooling;  $A_{i,t}$  is age of individual i in time t;  $G_{i,t}$  is the gender of an individual,  $R_{i,t}$  is regional location,  $\Delta_t$  is wave t,  $P_{i,t}$  is professional background of individual i. Coefficients of such earning functions are estimated using cross section or panel dataset. For instance using the cross section of the APS:

In addition to above variable earning differ by location of the labour markets. Local, regional, national, urban, rural, global labour markets function differently. Earning also vary by professions. Teachers, lawyers, doctors, engineers, scientists, artists have different levels of earning. Skilled workers are paid more than unskilled or semi-skilled workers. Labour market institutions mater. Job prospects are less in the rigid and opaque labor markets than in flexible and transparent labour markets. Labour earning also vary by the term of employment. Earnings are less in short term compared to medium term and long term employments. There are professions where labour supply occurs in inter-generational setting.

# 2.7 Two sector model of necessity and luxury goods (income distribution)

Let us assume that workers and capitalists dwell in an economy. Workers consume only necessities and capitalists consume necessities and luxury goods. Workers supply all labour and but capitalists do not work. Total endowment of labour supply is 50 and wage rates are same across necessity and luxury sectors.

$$LS = 50; \quad w_1 = w_2 = w \tag{30}$$

Production function of sector i is

$$Q_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i} \tag{31}$$

For simplicity assume that capital share ( $\alpha_i$ ) is 50 percent in both sectors. endowment of capital is 100 for the necessity sector and 144 for the luxury sector. Technology  $A_i = 1$  in both sectors.

	α	Κ	Α
Necessity sector	0.5	100	1
Luxury sector	0.5	144	1

Table 1: Parameters in production of the two sector model

Worker spend all of their income in necessity goods and capitalists. Capitalists save 20 percent of their income and spend on investment. Of the remaining 80 percent, spend 20 percent is spent on necessities and 60 percent on luxury goods.

Table 2: Parameters in consumption of the two sector model

	$\theta_1$	$\theta_2$	$\theta_3$
Workers	1	0	0
Capitalist	0.2	0.6	0.2

$$\pi_1 = P_1 Q_1 - wL_1 - rK_1 = P_1 A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} - wL_1 - rK_1$$
(32)

$$\frac{\partial \pi}{\partial L_1} = (1 - \alpha_1) A_1 K_1^{\alpha_1} L_1^{-\alpha_1} = P_1 \times 0.5 \times 100^{0.5} \times L^{-0.5} - w = 0$$
(33)

Thus labour demand in necessity goods sector

$$L_1^{0.5} = \frac{5P_1}{w}; \quad L_1 = 25\left(\frac{P_1}{w}\right)^2$$
 (34)

Supply of necessity goods

$$Q_1 = A_1 K_1^{\alpha 1} L_1^{1-\alpha 1} = 10 L_1^{0.5} = 10 \times \left\{ 25 \left(\frac{P_1}{w}\right)^2 \right\}^{0.5}; \quad Q_1 = 50 \frac{P_1}{w}$$
(35)

Demand for labour and supply function in the luxury goods sector

$$\pi = P_2 Q_2 - wL_2 - rK_2 = P_2 A_2 K_2^{\alpha 2} L_2^{1 - \alpha 2} - wL_2 - rK_2$$
(36)

$$\frac{\partial \pi}{\partial L_2} = (1 - \alpha_2) A_2 K_2^{\alpha_2} L_2^{-\alpha_2} = P_2 \times 0.5 \times 144^{0.5} \times L^{-0.5} - w = 0$$
(37)

Thus labour demand in luxury goods sector

$$L_2^{0.5} = \frac{6P_2}{w}; \quad L_2 = \left(\frac{6P_2}{w}\right)^2 = 36\left(\frac{P_2}{w}\right)^2$$
 (38)

Supply of luxury goods

$$Q_2 = A_2 K_2^{\alpha 2} L_2^{2-\alpha 2} = 12 L_2^{0.5} = 12 \times \left\{ 36 \left( \frac{P_2}{w} \right)^2 \right\}^{0.5}; \quad Q_2 = 72 \frac{P_2}{w}$$
(39)

Income of workers come from wages in necessity and luxury goods sectors as:

$$Y_L = wL_1 + wL_2 = 50w (40)$$

Income of capitalists (from the production function capitalist get the same as the labour)

$$Y_K = Y_L = 50w \tag{41}$$

Demand for necessities and luxury goods

$$P_1 Q_1^d = Y_L + 0.2Y_K = 50w + 0.2(50w) = 60w$$
(42)

$$Q_1^d = 60 \frac{w}{P_1} \tag{43}$$

Demand for luxury goods

$$P_2 Q_2^d = 0.6Y_K + I = 0.6(50w) + 0.2(50w) = 40w$$
(44)

$$Q_2^d = 40 \frac{w}{P_2} \tag{45}$$

Market clearing conditions in goods and labour markets

$$Q_1 = 50\frac{P_1}{w} = Q_1^d = 60\frac{w}{P_1} \tag{46}$$

$$Q_2 = 72\frac{P_2}{w} = Q_2^d = 40\frac{w}{P_2} \tag{47}$$

$$L_1 + L_2 = LS = 25\left(\frac{P_1}{w}\right)^2 + 36\left(\frac{P_2}{w}\right)^2 = LS = 50$$
(48)

$$I = S \tag{49}$$

Walras' Law: sum of excess demand is zero; when two markets clear third market automatically clears.

Let the necessity goods be a numeraire,  $P_1 = 1$ . From necessity goods market:

$$50\frac{P_1}{w} = 60\frac{w}{P_1} \Longrightarrow 5\frac{1}{w} = 6\frac{w}{1} \Longrightarrow w^2 = \frac{5}{6} \Longrightarrow w = 0.913$$
(50)

From luxury goods market:

$$72\frac{P_2}{w} = 40\frac{w}{P_2} \Longrightarrow P_2^2 = \frac{5}{9}w^2 = \frac{5}{9} \times \frac{5}{6} = \frac{25}{54} = 0.463 \Longrightarrow P_2 = 0.680$$
(51)

Allocations:

$$Q_1 = 50\frac{1}{0.913} = 54.8 = Q_1^d = 60\frac{w}{P_1} = 60 \times \frac{0.913}{1} = 54.8$$
(52)

$$Q_2 = 72\frac{P_2}{w} = 72 \times \frac{0.680}{0.913} = 53.63 = Q_2^d = 40\frac{w}{P_2} = 40 \times \frac{0.913}{0.680} = 53.7$$
(53)

$$L_1 = 25 \left(\frac{P_1}{w}\right)^2 = 25 \times \left(\frac{1}{0.913}\right)^2 = 29.97$$
(54)

$$L_2 = 36 \left(\frac{P_2}{w}\right)^2 = 36 \left(\frac{0.680}{0.913}\right)^2 = 19.97$$
(55)

$$L_1 + L_2 = 29.97 + 19.9 \approx 50 \tag{56}$$

Table 3: Allocation and distribuion in the two sector model

	P	Q	L	W
Necessity sector	1	54.8	29.97	29.97
Luxury sector	0.680	53.7	19.97	19.97

Consumption:

workers' demand for necessity good

$$Y_L = C_{1,L}; \quad 50w = 50 \times 0.913 = 45.65$$
 (57)

Capitalist's demand for necessity good

$$0.2Y_K = C_{1,K}; \ 0.2 (50w) = 0.2 (50 \times 0.913) = 9.13$$
(58)

Total demand for necessity good

$$C_{1,L} + C_{1,K} = 45.65 + 9.13 \approx 54.8 \tag{59}$$

workers do not consumer luxury good  $C_{2,,L} = 0$ ;

Table 4: Parameters	in consumption	of the two sec	ctor model

	$C_1$	$C_2$	$I_2$
Workers	45.65	0	0
Capitalist	9.13	40.23	13.43

Capitalist's demand for luxury good

$$C_{2,K} = \frac{0.6Y_K}{P_2} = \frac{27.39}{0.681} = 40.23 \tag{60}$$

Investment demand by capitalist for luxury good

$$\frac{I}{P_2} = \frac{0.2(50 \times 0.913)}{P_2} = \frac{9.13}{0.680} = 13.43$$
(61)

Market clearing for the luxury goods market:

$$C_{2..K} + I = 40.23 + 13.43 = 53.7 \tag{62}$$

It is possible to increase the amount available to workers by altering the proportions saved and invested and by changing the consumption structure or endowments of capital and labour in this economy. This simple example illustrates that distribution should not be looked into in isolation to the consumption, production and investment and tax and spending activities of the economy but simultaneously. A multi-household multisectoral general equilibrium model is a tool to achieve this objective. One illustration is provided below where the factors or production are owned by both households engaging on producing two goods which both of them consume.

# 3 A simple computable general equilibrium model with labour leisure choice

Consider an economy with two individuals, i = 1, 2 and two commodities x (goods) and y (services). Both households are endowed with given amount of capital stock  $(\overline{k_1}, \overline{k_2})$  and time  $(\overline{L_1}, \overline{L_2})$ , which they spend either working or in the form of leisure. Households and firms optimize taking prices of commodities  $(p_x, p_y)$  and factors  $(p_L, p_k)$  as given. Competition between suppliers and consumers or producers sets the equilibrium price of commodities and income of households  $(I_1, I_2)$  More specifically the problems of households and firms can be stated as:

#### Household's problem:

$$\max U_1 = x_1^{a_1} y_1^{b_1} \left( \bar{L_1} - LS_1 \right)^{g_1}; \ a_1 > 0, \ b_1 > 0, \ g_1 > 0.$$
(63)

subject to:

$$I_1 = p_x x_1 + p_y y_1 + p_L \left( \bar{L_1} - LS_1 \right); \quad I_1 = p_L \bar{L_1} + p_k \bar{K_1}$$
(64)

$$x_1 \ge 0, \ y_1 \ge 0, \left(\bar{L_1} - LS_1\right) \ge 0.$$
  
 $\max \ U_2 = x_2^{a_2} y_2^{b_2} \left(\bar{L_2} - LS_2\right)^{g_2}; \ a_2 > 0, b_2 > 0, g_2 > 0.$  (65)

subject to:

$$I_2 = p_x x_2 + p_y y_2 + p_L \left( \bar{L_2} - LS_2 \right); \quad I_2 = p_L \bar{L_2} + p_k \bar{K_2}$$
(66)

 $x_2 \ge 0, \ y_2 \ge 0, (\bar{L_2} - LS_2) \ge 0.$ Firm's problem:

$$\max \ \pi_x = p_x x - p_k k_x - p_L LS1_x - p_L LS2_x \tag{67}$$

subject to:

$$x = k_x^{\theta_x} LS1_x^{\theta_x} LS2_x^{\theta_x}$$
(68)

$$\max \ \pi_{y} = p_{y}y - p_{k}k_{y} - p_{L}LS1_{y} - p_{L}LS2_{y}$$
(69)

subject to:

$$y = k_y^{\theta_y} LS1_y^{\theta_y} LS2_y^{\theta_{2y}}$$
(70)

**Equilibrium conditions:** 

$$x = x_1 + x_2 \tag{71}$$

$$y = y_1 + y_2$$
 (72)

$$k_x + k_y = \overline{k_1} + \overline{k_2} \tag{73}$$

$$L_1 + LS1_x + LS1_y = \bar{L_1}; \quad LS1 = LS1_x + LS1_y$$
 (74)

$$L_2 + LS2_x + LS2_y = \bar{L_2}...LS2 = LS2_x + LS2_y$$
(75)

#### **Price normalisation:**

$$p_x + p_y + p_L + p_k = 1 (76)$$

Questions that arise here are what are the demand for *x* and *y* and leisure (or labour supply) by households 1 and 2 i. e. determine  $x_1, x_2, y_1, y_2, LS1, LS2$ . How should we determine the demand for labour and capital by firms supplying *x* and *y*, i,e, evaluate  $k_x, k_y, LS1_x, LS1_y, LS2_x, LS2_y$ . How can we compute the equilibrium relative price system for this economy that are consistent to optimisation

problems of households and firms. What are the optimal allocations of resources in this economy? Given the optimal demand for x and y and leisure by both households what is the optimal levels of their welfare. Is this Pareto optimal allocation? How can tax and transfer scheme in this economy in order to improve the distribution system. How can we apply notions of Hicksian equivalent and compensating variations in order to evaluate the welfare consequences of tax and welfare reforms proposed above.

First assign values for behavioral parameters.

	Tuble 6. Fullunceerb for the 2 by 2 model whit feibure												
$a_1$	<i>a</i> <sub>2</sub>	$b_1$	$b_2$	<i>g</i> 1	<i>8</i> 2	$\theta_x$	$\theta_y$	$\theta 1_x$	$\theta 1_y$	$\overline{L_1}$	$\overline{L_1}$	$\overline{k_1}$	$\overline{k}_2$
0.5	0.4	0.3	0.4	0.2	0.2	0.2	0.8	0.4	0.1	24	24	40	10

Table 5: Parameters for the 2 by 2 model with leisure

Solve this CGE (computable general equilibrium) model. We find solution to this model using GAMS as presented below:

Table 6: Efficient allocation in the 2 by 2 model with	ı leisure
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$I_1$	I <sub>2</sub>	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>	X	У	<i>u</i> <sub>1</sub>	$u_2$
3.42	2.01	5.9	2.8	1.7	1.3	8.7	3.1	4.6	2.4
$p_x$	$p_y$	$p_k$	$p_l$	$k_x$	k <sub>y</sub>	$L_1$	<i>L</i> <sub>2</sub>	$ls_1$	$ls_2$
0.289	0.599	0.047	0.064	35.3	14.7	10.7	6.3	13.3	17.7
$ls_{11}$	$ls_{12}$	<i>ls</i> <sub>21</sub>	ls <sub>22</sub>	$\overline{L_1}$	L <sub>2</sub>	$\overline{k_1}$	$\overline{k}_2$		
6.7	8.9	6.7	8.9	24	24	40	10		

 $I_1 = p_L \bar{L_1} + p_k \bar{K_1} (=0.064 \times 24 + 0.047 \times 40) = 3.42; I_2 = p_L \bar{L_2} + p_k \bar{K_2} (= 0.064 \times 24 + 0.047 \times 10) = 2.01.$ 

Equilibrium conditions are satisfied in both goods and factor markets:

$$x = x_1 + x_2 = 5.9 + 2.8 = 8.7 \tag{77}$$

$$y = y_1 + y_2 = 1.7 + 1.3 = 3.1 \tag{78}$$

$$k_x + k_y = 35.3 + 14.7 = \overline{k_1} + \overline{k_2} = 44 + 10 = 50 \tag{79}$$

$$L_1 + LS1_x + LS1_y = 10.7 + 6.7 + 6.7 = 24 = \bar{L_1}; \quad LS1 = LS1_x + LS1_y$$
 (80)

$$L_2 + LS2_x + LS2_y = 6.3 + 8.9 + 8.9 = 24 = \bar{L_2}...LS2 = LS2_x + LS2_y$$
 (81)

Alternative scenarios of this model could be found for varies rates of VAT in commodities x and y or for different rates of taxes in labour and capital inputs. Also these scenarios could be computed with a revenue target and do equal yield tax reforms finding model solution when all taxes or only a labour income tax or the capital income tax are changed. The it is also possible to compute the optimal tax rates that maximise revenue (hint make tax rates endogenous and solve the maximisation routine). Thus a general equilibrium model like this is appropriate to study both the functional and size distribution of income.

# 3.1 Global Empirical Evidence on Declining Labour Share

We construct panel data set for 127 countries for year 1990 to 2011 for labour income share (labshare), consumption share (consshare), capital share (capshare), government consumption share (Govconshare), import share (impshare), exports share (expshare) and real trade share (Rtrdshare). It is clear that average share of labour is declining for each decade as shown in Table 7. Labour shae in income was about 59 percent of GDP and it has declined by 9 pecent point to 51.4 percent by 2011. The dispersion in these shares have increased as the standard deviation has reduced from 0.116 to 0.137. Maddison project have more data investigate. EU KLEMs dataset also provides such information.

Years	Average share	Stand Dev	Countries			
1950	0.587773529	0.116112995	48			
1960	0.570278757	0.128005175	87			
1970	0.55353534	0.146336493	107			
1980	0.547496719	0.136702051	109			
1990	0.548241206	0.138618056	127			
2000	0.530072014	0.133766665	127			
2011	0.513952814	0.136878886	127			
Data source: Penn World Tables v8; Maddison dataset						

Table 7: Average labour share by decades

We estimate the fixed effect and random effect panel data models and results are reported in Table 8. As is clear the labour share is dealing as the shares of capital and exports are rising. Increase in private and public consumption, imports have positive and significant impacts. The process of substitution of labour by capital as discussed in Karabarbounis and Neiman (2014) and Picketty (2014) have increased the capital share causing reduction in labour share of about ten percent magnitude.

Dep Variable: labour share	Fixed Effect	Random Effect				
Consumption share	0.559***	0.548***				
Capital share	-0.004***	-0.104***				
Gov cons share	0.088***	0.088***				
Import share	0.034***	0.033***				
Exports share	-0.033***	-0.033***				
Real Trade share	0	0				
Constant	0.520***	0.520***				
Tests	F(1,2270) = 58.21(0.000)	Wald: $\chi^2(2) = 290.7 (0.000)$				
Sample	<i>N</i> =127; NT= 2794	<i>N</i> =127; NT= 2794				
Within	0.0986	0.0986				
Between	0.0160	0.0160				
Overall	0.0218	0.0218				
Hausman Test for random effect model $\chi^2(2) = 24.46 (0.000)$						

Table 8: Static Panel Regression Estimates for the OECD countries (1990:1-2014:4)

#### 3.2 Social Welfare Function

Distribution of income can be a result of the social choice. If the policy makers assign different weights to utility of different types of individuals in the economy it results in patter of income distribution that is different when policy makers tread every individuals equally. In general it is good to reward more productive workers than to lazier one. For instance, consider an economy that is inhabited by type 1 and type 2 people. The type 1 is more productive than the type 2. Policy makers encourage productive people by assigning a greater weight to the utility of more productive people. Let us assume that they want to maximise the social welfare function:  $W = U_1^{\frac{3}{4}} U_2^{\frac{1}{4}}$  where W is the index of the social welfare,  $U_1$  represents the utility of type 1 people and  $U_2$  is the utility of type 2 people. Utility of more productive type is three times worth more than less productive ones. For simplicity assume that resources of this economy produce a given level of output Y. It is consumed either by 1 or by 2 type people. Market clearing condition implies:  $Y = Y_1 + Y_2$ . If the preferences for type 1 are given by  $U_1 = \sqrt{Y_1}$  and for type 2 by  $U_2 = \sqrt{Y_2}$  and the total output, Y, is 1000 billion pounds. Four scenarios are

considered and the optimal allocations and social welfare are presented in Tables below.

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$ ; $\alpha_2 = \frac{1}{4}$ ; Economy 1				
	Income	Utility	U. function: $\sqrt{Y_i}$		
Type 1 individuals	750	$\sqrt{750} = 27.4$	$\sqrt{Y_1} = \sqrt{750} = 27.4$		
Type 2 individuals	250	$\sqrt{250} = 15.8$	$\sqrt{Y_2} = \sqrt{250} = 15.8$		
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 27.4^{\frac{3}{4}} \times 15.8^{\frac{1}{4}} = 23.9$				
	Y=1000; $\alpha_1 = \frac{1}{2}$ ; $\alpha_2 = \frac{1}{2}$ ; Economy 2				
Type 1 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_1} = \sqrt{500} = 22.4$		
Type 2 individuals	500	$\sqrt{500} = 22.4$	$\sqrt{Y_2} = \sqrt{500} = 12.8$		
Social Welfare	$W = (U_1)^{\frac{1}{2}} (U_2)^{\frac{1}{2}} = 22.4^{\frac{1}{2}} \times 22.4^{\frac{1}{2}} = 22.4$				

Table 9: Parameters in consumption of the two sector model

Table 10: Parameters in consumption of the two sector model

Output (Y) and weight	Y=1000; $\alpha_1 = \frac{3}{4}$ ; $\alpha_2 = \frac{1}{4}$ ; Economy 3 (20 percent tax away)		
	Income	Utility	U. function: $\sqrt{Y_i}$
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	250	$\sqrt{0.8 \times 250} = 14.1$	$\sqrt{Y_2} = \sqrt{200} = 14.1$
Social Welfare	$W = (U_1)^{rac{3}{4}} (U_2)^{rac{1}{4}} = 24.4^{rac{3}{4}}  imes 14.4^{rac{1}{4}} = 21.3$		
	Y=1000; $\alpha_1 = \frac{3}{4}$ ; $\alpha_2 = \frac{1}{4}$ ; Economy 3 (Tax revenue to poor)		
Type 1 individuals	600	$\sqrt{0.8 \times 750} = 24.4$	$\sqrt{Y_1} = \sqrt{600} = 24.4$
Type 2 individuals	400	$\sqrt{400} = 20$	$\sqrt{Y_2} = \sqrt{400} = 20$
Social Welfare	$W = (U_1)^{\frac{3}{4}} (U_2)^{\frac{1}{4}} = 24.4^{\frac{3}{4}} \times 20^{\frac{1}{4}} = 23.2$		

Let us consider four scenarios of social welfare. It is maximized at 23.9 when policy makers put weight  $\alpha_1 = \frac{3}{4}$ ;  $\alpha_2 = \frac{1}{4}$  and there are not taxes. Social welfare index diminishes to 22.2 in economy 2 where policy makers put equal weight to productive and non-productive workers. Social welfare decreases even further to 21.1 if 20 percent tax is imposed and no transfer is returned any of these households. It slightly improves to 22.4 if all tax revenue is given back to the poor household. Tax economy is Pareto inferior to the no tax economy. More elaborated analysis is in Bhattarai, Haughton and Tuerck (2015). This is more comprehensive theory of income distribution and welfare that can accommodate wide ranging concerns relating to social justice and inequality.

# 4 Conclusions

We review the functional and size theories of income distribution in the context of growing inequality around the world. The causes for the observed patterns on the shares of labour and capital income in classical, neoclassical, Marxian, structuralists or general equilibrium theories are reviewed. Each of these theory is found helpful in explaining the inequality of in income observed in the OECD, emerging and developing economies. Significant empirical evidence exists for declining labour share as shown by panel data estimation of 127 countries from 1990 to 2011.

# References

- [1] Atkinson, A.B (1970) On the measurement of inequality, *Journal of Economic Theory* 2(3):244-263.
- [2] Paul Bachman, Keshab Bhattarai, Frank Conte, Jonathan Haughton, Michael Head & David G. Tuerck (2016) Trump v. Cruz: The Comparative Economic Effects of Two Tax Proposals, Department of Economics and Beacon Hill Institute at Suffolk University, 8 Ashburton Place, Boston, MA 02108.
- [3] Bertola, Giuseppe, "Finance, Labour, Capital, and International Integration", CESifo Working Paper No. 5820, March 2016
- [4] Bhattarai K, J Haughton M Head and D. Tuerck (2015) Simulating Corporate Income Tax Reform Proposals with a DCGE Model, Department of Economics and Beacon Hill Institute at Suffolk University, 8 Ashburton Place, Boston, MA 02108.
- [5] Bhattarai K, J Haughton and D. Tuerck (2015) Fiscal Policy, Growth and Income Distribution in the UK, Applied Economics and Finance, 2:3:20-36
- [6] Bhattarai K, J Haughton and D. Tuerck (2015) The Economic Effects of the Fair Tax: Analysis of Results of a Dynamic CGE Model of the US Economy, International Economics and Economic Policy, forthcoming
- [7] Cartelier J (1991) Marx's theory of value, exchange and surplus value: a suggested reformulation Cambridge Journal of Economics 1991, IS, 257-269
- [8] Cowling, Keith. Molho, Ian and Oswald (1981) Wage share, concentration, and unionism, Warwick economic research papers, 188
- [9] Elsby MWL, Bart Hobijn, Ayşegül Şahin (2013) The Decline of the U.S. Labor Share Brookings Papers on Economic Activity (FALL 2013), pp. 1-52
- [10] Foley, Duncan K. and Thomas R. Michl. 1999. Growth and distribution. Cam-

bridge, MA: Harvard University Press.

- [11] Guerriero M and Kunal Sen (2015) What Determines the Share of Labour in National Income? A Cross-Country Analysis, IZA Discussion Paper No. 6643 and ASSA/AIEFS 2015.
- [12] Hahn, Frank H. (1972) The share of wages in the national income : an enquiry into the theory of distribution LSE, Redwood Press Limited
- [13] Hansen Gary D.and Edward C. Prescott.2002. "Malthus to Solow." American Economic Review, 92(4): 1205-1217
- [14] Haughton J and D Haughton (2011) Living standard analytics, Dordrecht : Springer.
- [15] Hicks, John R. (1932) The theory of wages. London: Macmillan.
- [16] Jenkins, S.P. (1995) Accounting for inequality trends, Economica, 62:29-63.
- [17] Jenkins, S.P. (1996) Recent trends in UK income distribution, Oxford Review of Economic Policy, 12:1:29-46.
- [18] Karabarbounis, Loukas; Neiman, Brent (2014) The Global Decline of the Labor Share, Quarterly Journal of Economics, 129, 1, 61-103
- [19] Kennedy, Charles. (1964) Induced bias in innovation and the theory of distribution. Economic Journal 74: 541-547.
- [20] Lavoie Marc and Engelbert Stockhammer (2014) Wage-led Growth An equitable strategy for economic recovery, ILO
- [21] Mortensen D T and C. A. Pissarides (1994) Job Creation and Job Destruction in the Theory of Unemployment, Review of Economic Studies, 61:3:397-415.
- [22] OECD (2015), In It Together: Why Less Inequality Benefits All, OECD Publishing, Paris. DOI: http://dx.doi.org/10.1787/9789264235120-en
- [23] O'Mahony Mary and Marcel P. Timmer (2009) Output, Input And Productivity Measures At The Industry Level: The Eu Klems Database, The Economic Journal, 119 (June), F374–F403
- [24] Parente, S.L. and E.C. Prescott (2002). Barriers to Riches. MIT Press, Cambridge.
- [25] Piketty T (2014) Capital in the twenty-first century, Cambridge Mass:, Harvard University Press.
- [26] Piketty T (2014) Income inequality in the United States, 1913–1998, Quarterly Journal of Economics, 98:1,1-39
- [27] Stockhammer, Engelbert, Özlem Onaran and Stefan Ederer. 2009. Functional income distribution and aggregate demand in the Euro area. Cambridge Journal of Economics 33 (1): 139-159.
- [28] Stockhammer, E. and Onaran, O<sup>"</sup>. 2004. Accumulation, distribution and employment: a structural VAR approach to a Kaleckian macro-model, Structural

Change and Economic Dynamics, 15, 4, 421–47

[29] Tytell, Irina and Jaumotte, Florence (2008) How Has the Globalization of Labor Affected the Labor Income Share in Advanced Countries?, IMF Working Papers, 1-54, 2008.