Robust GDI-based Adaptive Recursive Sliding Mode Control (RGDI-ARSMC) for a highly nonlinear MIMO system with varying dynamics of UAV

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Abstract The novelty of the proposed work lies in the control technique, referred to as the Robust Generalized Dynamic Inversion based Adaptive Recursive Sliding Mode Control (RGDI-ARSMC), for addressing various challenges to control a highly coupled and perturbed system called Twin Rotor MIMO Systems (TRMS) UAV. The continuous disturbances, varying parameter values, actuator failure, and unmodeled states are the challenges related to the proposed controller. The method aims to effectively mitigate unwanted signals, including coupling effects, unknown states, gyroscopic disturbance torque, parametric uncertainties, and other disturbances. The control design process is divided into two phases: the first involves estimating the deviation between the actual and desired output angles and conducting a stability phase analysis. The confined stability-based Lyapunov stability was verified. While the second phase involves the addition of a robust term and the use of an adaptive recursive design procedure to determine the controller parameters for pitch and yaw angles. The proposed control strategy is compared with other techniques such as classical sliding mode control, backstepping, and RGDI-SMC controls. The proposed strategy is also implemented in real-time to characterize its performance. On the basis of obtained results, the considered perturbations were effectively addressed by the augmentation of adaptation laws and recursive control design.

1. Introduction

The control community field has long been interested in Variable Structure Systems (VSS) control because of their highly nonlinear behavior, varying dynamics, coupling effects, and sensitivity to parametric disturbance during considered controller. The control of Unmanned Aerial Vehicles (UAVs) is particularly challenging due to matched and mismatched perturbations. These systems are of interest because of their increasing use in various environments for common security services and defense operations [1, 2]. Twin Rotor MIMO Systems (TRMS) are a type of UAV system that has gained attention due to their ability to tilt their angle of flight, hover, take-off, and land in irregular locations [3]. TRMS have high coupling and nonlinear dynamics, uncertainties, and gyroscopic torque, which make their control a challenging problem for control researchers. These systems have expanding applications in various fields [4]. The main challenges in TRMS include propeller rotation, coupling between rotors, changing propeller rotation speed, sensitivity to parametric perturbations, and the time-varying nature of the system [5,6]. The controller task must guarantee the following challenges :(i) the varying dynamics of TRMS ;(ii) the varying dynamics of the system include unmatched disturbances and varying parametric dynamics; (iii) the unavailability of time-varying unmodeled states of the system.

To address these issues, researchers have proposed a variety of linear, nonlinear, and intelligent control strategies such as robust observer [7, 8], adaptive SMC strategy [9], learning-based adaptive model predictive control (MPC), linear MPC [10, 11], nonlinear techniques based on online adaptive laws for UAV in [12], validation of adaptive RBFNN [13], adaptation laws based hierarchical SMC approach

[7, 14], and terminal SMC [15]. The adaptive laws-based neural networks (NN) backstepping control strategy in [16], and adaptive nonlinear recursive control based on fuzzy logic control are discussed respectively in [17]. Type-2 fuzzy adaptive backstepping control for the nonlinear coupled systems is also elaborated in [18], to evaluate the significance of emerging research in control, the integration of nonlinear control techniques with traditional control methods is being studied in [19,20]. The goal of these controllers is to ensure that the TRMS system is stable and able to handle internal and external disturbances, parametric uncertainties, and unmodeled states. One of the first steps in designing a controller for a non-linear system is to ensure its stability. Higher order coupled non-linear systems must be analyzed using mathematical tools [21, 22]. Nonlinear Dynamic Inversion (NDI), which is a feedback linearization approach, is employed at UAV to make the mathematical model simpler. However, this method will ignore the important terms of nonlinearities, singularities, and matrix inversions. To overcome these limitations, Generalized Dynamic Inversion (GDI) is used to solve non-square inversions due to inverse problems in matrix [23]. The left inversion approach is used to establish linear differential equations and is inverted using the Moore-Penrose Generalized Inverse (MPGI) technique, which is based on the Greville method [24]. This approach aims to avoid inversion issues and also avoid ignoring important square or high-degree terms. Robust Generalized Dynamic Inversion (RGDI) controllers have been designed for various aerospace and robotic applications and have been shown to be effective [25]. In previous research, RGDI-based control theory, mixed-optimization control, and adaptations laws-based recursive sliding control are applied in [26-28], at UAV, and its performance was characterized through an experimental test of the prototype. Three different methods (linear and nonlinear control) are elaborated on the basis of their technical strength and a detailed comparison is provided in Appendix, Section 5. However, it is important to address the chattering effect, which is a problem that can occur in higher-order complex systems with fast switching of control inputs, in any control strategy. The RGDI-ARSMC directly depends on the mathematical analysis-based model of the considered system [29]. In research [30], a robust adaptation laws-based controller was developed to address parametric uncertainties and loss of thrust anomalies. The controller uses an adaptive law to follow the reference point of the vehicle in both vertical and horizontal positions. This control approach offers finite-time convergence, reduces the problem, and provides a law called parameter-tuning law to eliminate disturbances. An adaptive recursive method with a finite-time convergence technique was employed to create a control law for stabilizing a nonlinear system. The controller used a hybrid approach for full system trajectory tracking and ensured stability in closed-loop through hybrid Lyapunov analysis. The adaptive law was used to calculate the controller coefficients, and the global stability of the closed-loop system was proven through Lyapunov analysis. When the system experiences perturbations, the relative degree of the higher-order system can be affected, leading to singularity issues. This may affect the stability and convergence time of the closed-loop system. These perturbations can be time-varying or state-dependent, and they may render the system unstable, causing a loss of solution uniqueness. Further studies are needed to understand these effects. The contribution outline of the paper is enlisted as

- Our research illustrates our ability to proficiently mitigate the consequences of parametric alterations and unaccounted-for system states by achieving an improved finite-time convergence. Furthermore, we establish stability criteria for the perturbed system by employing a piecewise linear Lyapunov function.
- The Euclidean error norm method is utilized to compute the discrepancy between the control angles and the desired angles, serving as an estimation of the system's state error.
- The adaptive recursive controller structure, constructed upon the mathematical model of the analyzed system, accentuates its remarkable resilience in the face of contemplated parametric variations and state-dependent unmodeled states.
- A control methodology is applied to achieve stabilization of a nonlinear system through the utilization of an adaptive recursive approach coupled with a finite-time convergence technique. This approach not only mitigates the inherent challenges but also incorporates a parameter-tuning law to nullify disturbances. Consequently, the outcome is a control law guaranteeing finite-time convergence.
- The efficacy of the dynamic approach's robustness was showcased under demanding conditions, including noise, concurrent parametric fluctuations, and the application of disturbance torque to both rotors. The effectiveness of this approach

is evident through a comparative analysis conducted via MATLAB simulations.

- Based on experimental validation, suggestions were provided for control engineers to gain a better understanding of the control design and system behavior.

The remainder of this paper has the following sections: the mathematical modeling in section 2, while the inclusion of robust terms is provided in section 3. The experimental setup with a detailed description of the simulation response is provided in section 4. Finally, the conclusion based on validated results is presented in section 5.

2. Twin Rotor MIMO System (TRMS) UAV

Before understanding the mathematical modeling of the TRMS, it is important to understand the various parameters and control outputs of the system [30]. The TRMS is a laboratory tool used to study the flight control of helicopters [31]. It has two rotors, as shown in Figure 1. The design of these rotors is

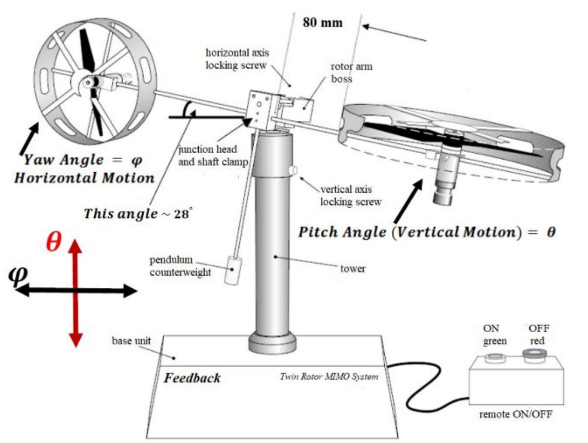

Fig. 1. Block diagram of UAV

important as they are influenced by various forces such as gravity, propulsion, centrifugal force, friction, and disturbance torque. To counteract these forces, the control input is provided through motors. To understand and simplify the mathematical model, it is important to understand the mathematical assumptions that are used in the NDI process. The TRMS has two degrees of freedom, allowing for movement in two directions, the horizontal plane and the azimuthal plane, which are derived in the model.

$$\frac{d\dot{\theta}}{dt} = \frac{a_1}{I_1}\tau_1^2 + \frac{b_1}{I_1}\tau_1 - \frac{M_g}{I_1}\sin(\theta) + \frac{0.0326}{2I_1}\sin(2\theta)\,\dot{\varphi}^2 - \frac{B_{1\theta}}{I_1}\theta - \frac{k_{gy}}{I_1}\cos(\theta)\dot{\varphi}(a_1\tau_{1^2} + b_1\tau_1)$$

$$\frac{d\dot{\varphi}}{dt} = \frac{a_2}{I_2}\tau_2^2 + \frac{b_2}{I_2}\tau_2 - \frac{B_{1\varphi}}{I_2}\dot{\varphi} - \frac{k_c}{I_2}1.75(a_1\tau_1^2 + b_1\tau_1)$$

The principle of momentum conservation is also applied to the rotor, resulting in similar momentum equations. Differential equations for both rotors are derived as follows:

$$\dot{\tau}_1 = \frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1$$

For tail motor:

$$\dot{\tau}_2 = \frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}}u_2$$

where k_1 and k_2 are the motor gains, T_{10}, T_{11} and T_{20}, T_{22} are the motor parameters, τ_1, τ_2 are rotors momentum, u_θ, u_ϕ are controlled actions of the vertical plane and horizontal plane respectively. The nomenclature, parametric values with units are given in section 1 of supporting material. The block diagram in Figure 2 represents two output states

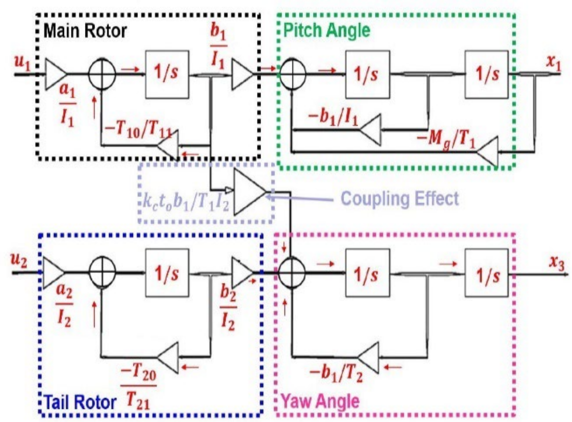

Fig. 2. Basic schematic sketch of TRMS [32].

(pitch angle and yaw angle), which are likely related to the control of the aircraft. The coupling effect is evident in a Figure which illustrates both rotors and angles with labeled blocks. This interaction between these two systems may affect the overall performance and stability of the aircraft. The NDI control method, although effective in linearizing nonlinear systems is explained in detail in section 1 of supporting material. Nonlinear Dynamic Inversion (NDI), which is a feedback linearization approach, is employed at UAV to make the mathematical model simpler. However, this method will ignore the important terms of nonlinearities, singularities, and matrix inversions. To overcome these limitations, Generalized Dynamic Inversion (GDI) is used to solve non-square inversions due to inverse problems in matrix. The left inversion approach is used to establish linear differential equations and is inverted using the Moore-Penrose Generalized Inverse (MPGI) technique, which is based on the Greville method. Singularity based issue is being resolved by designing RGDI control provided with details in [26].

3. Inclusion of Adaptive Recursive Robust Term

The RGDI-based ARSMC law is a control method that combines the conventional Generalized Dynamic Inversion (GDI) method with an adaptive RSMC term. This method is specifically for controlling MIMO systems like TRMS. The implementation of ARSMC for twin-rotor MIMO systems can be challenging as it requires knowledge of the system's dynamics and the design of a suitable Lyapunov function. Additionally, the performance of the ARSMC may be affected by the choice of adaptation laws and the design of the sliding surface. To address these problems, a hybrid controller is developed to enable the full system to track a trajectory and maintain stability in a closed-loop configuration. The stabilizing functions counteract nonlinearities that impact the system's stability. Previous research [26-28] has shown that the inclusion of sliding mode control as a robust term, while ARSMC design can ensure stability of the complex system with a sharp response towards convergence.

The sliding surface of TRMS with recursive backstepping can be defined as:

$$s = \dot{e}_\xi + c_{\xi_a} e_\xi(t) + c_{\xi_a} \int \quad e_\xi(t)dt$$

here c_{ξ_a}, c_{ξ_b} are the gain constants to enforce sliding, e_ξ is an error tracking state which will be calculated via adaptive backstepping. The adaptive backstepping method is chosen for its ability to provide stable robustness in desired position tracking problems and its capability to control TRMS position in the presence of uncertainties and disturbances. In this section, an adaptive backstepping for position trajectory tracking control is implemented, taking the output vector for TRMS position as $[x, y]$. The variables used in the design procedure for the MIMO system are:

$$\begin{bmatrix} \theta = x_1 & \text{and } e_{\xi_\theta} = x_1 - x_{1_d}, & e_{\xi_\theta} \in e_{\xi_1}, e_{\xi_2}, e_{\xi_3} \\ \phi = x_3 & \text{and } e_{\xi_\phi} = x_3 - x_{2_d}, & e_{\xi_\phi} \in e_{\xi_4}, e_{\xi_5}, e_{\xi_6} \end{bmatrix}$$

Step 1: The first step of backstepping control design is to define the position tracking errors as:

$$[e_{\xi_z}] = \begin{bmatrix} e_{\xi_\theta} \\ e_{\xi_\phi} \end{bmatrix} => \begin{bmatrix} x_1 - x_{1_d} \\ x_3 - x_{2_d} \end{bmatrix}$$

$$=> \begin{bmatrix} \text{Tracking error of pitch position (angle)} \\ \text{Tracking error of yaw position (angle)} \end{bmatrix}$$

Step 2 : Introducing new arbitrary control input

$$[e_{\xi_\theta}] = \begin{bmatrix} e_{\xi_1} \\ e_{\xi_2} \\ e_{\xi_3} \end{bmatrix} => \begin{bmatrix} x_1 - x_1 d \\ x_2 - \alpha_1 \\ x_5 - \alpha_2 \end{bmatrix}$$

$$[V_{\xi_\theta}] = \begin{bmatrix} V_{\xi_1} \\ V_{\xi_2} \\ V_{\xi_3} \end{bmatrix} => \begin{bmatrix} 1/2e_{\xi_1}^2 \\ 1/2e_{\xi_2}^2 \\ 1/2e_{\xi_3}^2 \end{bmatrix}$$

Step 3: The required condition for Lyapunov function to fulfill the asymptotic stability as:

$$\dot{V} = -V_{\xi_1}^2 - V_{\xi_2}^2 - V_{\xi_3}^2$$

$$[e_{\xi_\phi}] = \begin{bmatrix} e_{\xi_4} \\ e_{\xi_5} \\ e_{\xi_6} \end{bmatrix} => \begin{bmatrix} x_3 - x_{2_d} \\ x_4 - \alpha_3 \\ x_6 - \alpha_4 \end{bmatrix}$$

$$[\dot{V}_{\xi_\phi}] = \begin{bmatrix} \dot{V}_{\xi_4} \\ \dot{V}_{\xi_5} \\ \dot{V}_{\xi_6} \end{bmatrix} = \begin{bmatrix} e_{\xi_4}\dot{\xi}_4 \\ e_{\xi_5}\dot{\xi}_5 \\ e_{\xi_6}\dot{\xi}_{\xi_6} \end{bmatrix} => \begin{bmatrix} e_{\xi_4}(\dot{x}_3 - \dot{x}_2 d) \\ e_{\xi_5}(\dot{x}_4 - \dot{\alpha}_3) \\ e_{\xi_6}(\dot{x}_6 - \dot{\alpha}_4) \end{bmatrix} \quad (1)$$

The arbitrary control laws for the pitch and yaw position are formulated as follows:

$$\begin{cases} \alpha_{\xi_1} = -c_{\xi_1} e_{\xi_1} + \dot{x}_{1_d} \\ \alpha_{\xi_2} = -\hat{c}_{\xi_1} e_{\xi_1} + x_1 + x_2 - c_{\xi_1}(\hat{c}_{\xi_2} e_{\xi_2} + \alpha_{\xi_1})\dot{x}_{1_d} \\ \alpha_{\xi_3} = -c_{\xi_4} e_{\xi_4} + \dot{x}_{1_d} \\ \alpha_{\xi_4} = -e_{\xi_4} - \hat{c}_{\xi_5} e_{\xi_5} + x_4 - x_5 - c_{\xi_4}(-\hat{c}_{\xi_4} e_{\xi_4} - \dot{x}_{2_d})\ddot{x}_{2_d} \end{cases} \quad (2)$$

where $\hat{c}_{\xi_2}$ and $\hat{c}_{\xi_5}$ are the estimate of c_{ξ_2} and c_{ξ_5} respectively.

Theorem 1: [29], If the TRMS position system is governed by equation (1) while being accompanied by the adaptation law given by equation (2), the assurance of convergence is established. Additionally, the adaptation laws for parametric stability are provided as follows:

$$\begin{cases} \dot{\hat{\imath}}_{\xi_2} = n_1 c_{\xi_2}^2, \\ \dot{\hat{\imath}}_{\xi_5} = n_2 c_{\xi_2}^2 \end{cases}$$

Here, n_1 and n_2 represent positive constants. The application of [Barbalat's Lemma] is employed to assess the significance of the theorem under consideration [33]. Above lemma must require as:

Lemma 1: [29] The function $f(t)$ is uniform function and $\lim_{t\to+\infty} \int_0^t f(\tau)d\tau$ validated, then $f(t)$ must be converged at zero (origin) asymptotically.

Proof: To provide confined convergence of the system with explaining $\hat{c}_{\xi_2}$, and $\hat{c}_{\xi_5}$ as the parameters of a system, The Lyapunov stability analysis is applied. For $\hat{c}_{\xi_2}$: the candidate function is introduced for the considered subsystem.

$$v_{\xi_2} = v_{\xi_1} + \frac{1}{2n_1}\tilde{c}_{\xi_2}^2 \tag{3}$$

where $\tilde{c}_{\xi_2}$ denotes the error. The time derivative of equation (3) is

$$\begin{aligned} \dot{v}_{\xi_2} &= -c_{\xi_1}e_{\xi_1}^2 - \hat{c}_{\xi_2}e_{\xi_2}^2 + \frac{1}{n_1}\tilde{c}_{\xi_2}\dot{\tilde{c}}_{\xi_2} \\ &= -c_{\xi_1}e_{\xi_1}^2 - \left(c_{\xi_2} - \tilde{c}_{\xi_2}\right)e_{\xi_2}^2 - \frac{1}{n_1}\tilde{c}_{\xi_2}\dot{\hat{c}}_{\xi_2} \\ &= -c_{\xi_1}e_{\xi_1}^2 - c_{\xi_2}e_{\xi_1}^2 + \tilde{c}_{\xi_2}\left(e_{\xi_2}^2 - \frac{1}{n_1}\dot{\hat{c}}_{\xi_2}\right) \end{aligned} \tag{4}$$

In the equation (4), the mentioned term $\tilde{c}_{\xi_2}\left(e_{\xi_2}^2 - \frac{1}{n_1}\dot{\hat{c}}_{\xi_2}\right)$ will be equal to zero. By taking c_{ξ_2} as positive constant, the derivative of $\tilde{c}_{\xi_2}$ can be expressed as $\dot{\tilde{c}}_{\xi_2} = 0 - \dot{\hat{c}}_{\xi_2}$. Now the candidate function will be elaborated in equation (4), can be written as:

$$\dot{v}_{\xi_2} = -c_{\xi_1}e_{\xi_1}^2 - c_{\xi_2}e_{\xi_2}^2 \le 0 \tag{5}$$

Thus, the stability condition is satisfied through equation (5). To guarantee the stability of the positioning system, the Lyapunov candidate function for the system's position is chosen:

$$v_{\xi} = \frac{1}{2}\left(e_{\xi_1}^2 + e_{\xi_2}^2 + \frac{1}{n_1}\tilde{c}_{\xi_2}^2 + e_{\xi_4}^2 2e_{\xi_5}^2 + n_2\tilde{c}_{\xi_5}^2\right) \tag{6}$$

The time derivative of the Lyapunov position is

$$\dot{v}_{\xi} = \left(-c_{\xi_1}e_{\xi_1}^2 - c_{\xi_2}e_{\xi_2}^2 - c_{\xi_4}e_{\xi_4}^2 - c_{\xi_5}e_{\xi_5}^2\right) \le 0 \tag{7}$$

where c_{ξ_1}, c_{ξ_2}, and c_{ξ_5}, c_{ξ_4} are the parameters of pitch angle as well as yaw angle respectively. Therefore, the system's stability is ensured by equations (6) and (7), providing the capability of flight trajectory tracking. In this section, an adaptive recursive method for the trajectory tracking of a considered system is developed. Adaptive laws are employed to calculate the parameter of the proposed controller. By taking a derivative of sliding surface,

$$\dot{s} = \ddot{e}_{\xi} + c_{\xi}\dot{e}_{\xi}(t) + c_{\xi}e_{\xi}(t) \tag{9}$$

By using equation (9) and rearranging equations to get ensure the negative definite provided in [26,27]. Thus, it is not possible to ensure the finite-time closed-loop stability of sliding mode dynamics. Nevertheless, it is possible to attain semi-global practical stability of the ARSMC through appropriate design of the SMC gain.

Theorem 2: The real integers $C^* > 0$ for all real integers $\rho \in (0,1)$ which will provide the negative definite of $\dot{V}$ with sliding dynamics elaborated by in section 3 of supporting material for all values of $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right) > \rho^*$ and $C > C^*$.

Proof: Let ρ^* be a real scalar number as a constant entry in the coverage of $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right), \rho^*$, i.e., $\rho \in (0,1)$. Also, define $C\left(e_{\xi_z}; e_{\xi_r}; t\right)$ as:

$$\bar{C}\left(e_{\xi_z}; e_{\xi_r}; t\right) = -\frac{\rho^* - 1}{\rho^*}B\left(e_{\xi_z}, e_{\xi_r}, t\right)$$

It follows that $\bar{C}\left(e_{\xi_z}; e_{\xi_r}; t\right) > C\left(e_{\xi_z}; e_{\xi_r}; v; t\right)$ whenever $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right) > \rho^*$. Accordingly, let D be a neighborhood of $\left(e_{\xi_z}; e_{\xi_r}\right) = (0_2; 0_2)$, and choose a sliding gain constant C^* such that

$C^* > \max_{D}\bar{C}\left(e_{\xi_z}; e_{\xi_r}; t\right)$ Then the negative definite value of $\dot{V} < 0$ ensures along any closed-loop track that starts within D whenever $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right) \ge \rho^*$ and $C > C^*$. The finite number C^* is ensured for any range D because of $B\left(e_{\xi_z}, e_{\xi_r}, t\right)$, which is globally bounded by virtue of implementing the DSGI A^*, which results in globally bounded trajectories. The controller design steps with its variable representation via synoptic scheme is described in figure 3, to control the highly coupled nonlinear system.

Remark 1: According to the statement of theorem 2, the gain must be positive and gain C increased in a way that the positive bound ρ^* will be obtained to ensure of the situation (condition) is $\dot{V} < 0$ will hold D for $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right) < \rho^*$. A specific domain must be defined by $\rho\left(e_{\xi_z}; e_{\xi_r}; v; t\right) < \rho^*$ will be followed by state trajectory e_a which should be in this range. The driving ρ^* is close to the zero for driving e_a which is also close to zero for uniformly bounded. The following condition makes the attitude error trajectory $e_{\xi_z} = 0_{2x1}$ stable. Due increase in C^* will affect the enlargement of D, it must be followed as semi-global stability. In the SMC method, the system's trajectory is guided along a manifold by the use of multiple control structures that follow a speci-

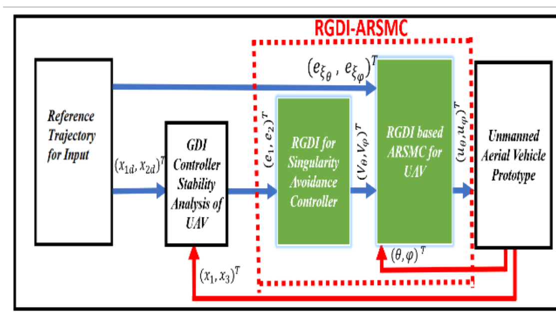

Fig. 3. Block diagram of controller for UAV

fied switching condition. The system's structure is defined by switching functions, which can be either scalar or vector. The switching surface, represented by $s(x) = 0$, is a line on the phase plane. To verify its effectiveness, we conducted a comparison study and evaluated the performance of the controller in terms of vertical stability (pitch angle) and horizontal stability (yaw angle). Additionally, the controller was tested in the presence of disturbances, such as external disturbances, parametric uncertainties, coupling effects, and noise signals section 4 & 5 of supporting material. The step input tracking response in the simulation confirms that the control theory regarding the convergence time of the angles, specifically the pitch angle and yaw angle, is logical and results in a sharp stability response. While the detailed response of different control strategies (SMC, Backstepping, RGDI based SMC) in section 4 of supporting material and for the worth of the proposed strategy is validated by providing different reference inputs in section 5 of supporting material.

4. Hardware and System Setup

The realization process and key ideas for implementation are briefly elaborated with system interconnection details. The real-time implementation of the prototype can be seen in figure 4 with a laboratory setup and other components. It tests the effectiveness of the simulation results by applying various disturbances to each rotor of the highly coupled system. The controller is designed to handle disturbances such as noise, unmodeled states, uncertainties, and coupling effects. The experimental response of the pitch angle with a sinusoidal input as a reference input is applied in figure 5 and figure 6 represents the response of the yaw angle under all applied disturbances (parametric disturbances, coupling effects, and noise signal). Some important points that can be verified from the obtained results and some observations as a control engineer are also described here to understand issues related to hardware implementation. The pitch angle response of the prototype elaborates that the controller performs very well against all disturbances and the convergence time also verifies the logical time to converge the main rotor before the tail rotor (yaw angle) response. A subplot also shows the attenuation due to the applied noise signal and also the range of disturbance being tackled by an efficient controller. Other subplots verify how the controller can manage the considered disturbance over time. The system response becomes more stable over time, which can make a remarkable difference with respect to other applied control methods to date.

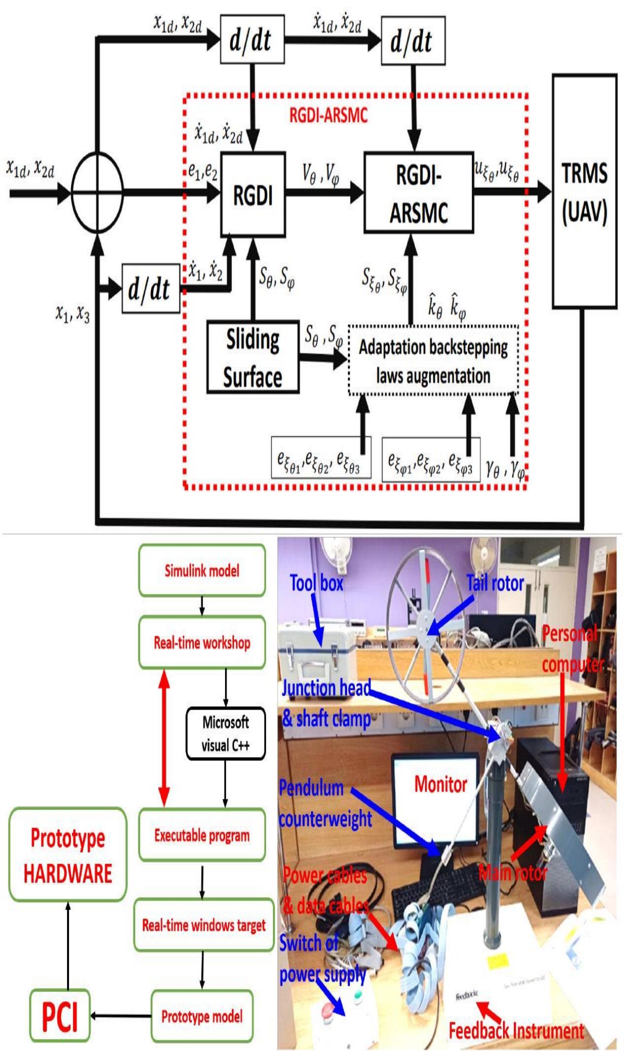

Fig. 4 Experimental apparatus (prototype) with real-time workshop

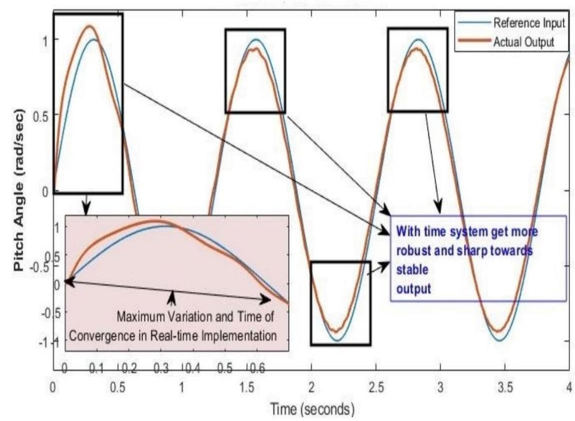

Fig. 5. Pitch angle-Experimental sinusoidal response.

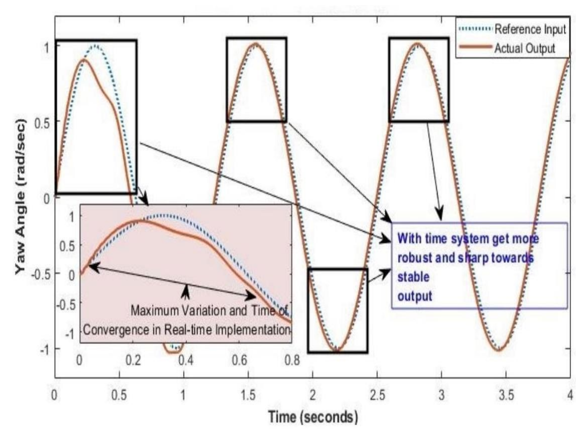

Fig. 6. Yaw angle-experimental sinusoidal response.

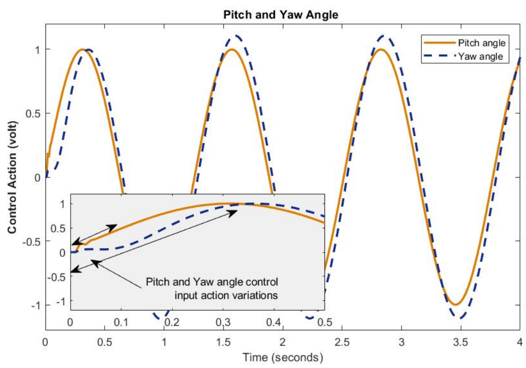

Fig. 7. Control actions under sinusoidal input.

The convergence time and effect of attenuation are more in the yaw angle. The reason behind this change is that the coupling effect due to disturbance torque is more due to the weighted rotor as well as the blades of the main rotor. The sudden and sharp variation in yaw angle is generated by different factors created by the main rotor. Noise (disturbance) can greatly impact the accuracy of actuators and input control signals, causing errors. To address this issue, a first-order filter based on the Butterworth filter is employed to filter out noise and obtain the actual actuator input. The control actions of both angles are represented in figure 7 under sinusoidal input.

5. Conclusion

The objective of this study was to design an adaptive recursive technique based on Generalized Dynamic Inversion for a highly nonlinear and cross-coupled multiple input multiple output (MIMO) system. This technique was tested on a UAV (TRMS) for flight path tracking and stabilization and used to develop novel robust controllers for controlling the UAV in the presence of uncertainties. The control strategy consisted of two phases: understanding the behavior of the system, which is challenging due to high coupling and disturbance torque, and designing time-varying dynamic constraints. Nonlinear Dynamic Inversion (NDI) was used to provide a simplified model of TRMS. GDI was used to address the limitations of NDI and singularity issues. In the second phase, output states were tracked by reference trajectories, and sinusoidal reference tracking of states ensured robustness and stability validation against the nonlinear behavior of the coupled system with uncertainties. The inclusion of a robust term in previous research controllers was developed to increase robustness against external perturbations and unmodeled states. The RGDI-ARSMC method is based on the mathematical model of the TRMS system which addresses varying parametric uncertainties and loss of thrust anomalies. The controller uses an adaptive law to track the desired trajectory of the vehicle in both vertical and horizontal angles (positions). This control approach offers finite-time convergence, reduces the problem, and provides a parameter-tuning law to eliminate external perturbations. A novel reaching law based on an adaptive recursive approach with a finite-time convergence technique was used to generate a control law for stabilizing the nonlinear system. The hybrid controller was designed for full system trajectory tracking and stability in closed-loop and provided by using hybrid Lyapunov analysis. The adaptive law was used to estimate the controller coefficients and the global stability of the closed-loop system was proved using Lyapunov analysis. Accurate fast-tracking and error convergence performance in all cases of perturbations (noise matrix, parametric disturbance) reveal the effectiveness of the applied controller. Numerical simulation and real-time experiments were conducted to evaluate the performance of the developed control system. The experimental results also provide some suggestions for control engineers to consider.

- Experimental testing has confirmed that the robust control system's performance during real-time implementation is greatly influenced by the particular adaptation law applied for parameter estimation.
- Substantial uncertainties in the physical parameters can introduce nonlinearity in the system's behavior, necessitating the use of a recursive adaptation law to address this issue accurately. Additionally, the presence of high-amplitude noise signals can severely disrupt input actuators and the high-voltage range.

5.1 Insights of Future Guidance

There are some general insights into future offers and guidelines regarding proposed research.

- Future research is likely to focus on developing advanced control algorithms that integrate the principles of GDI-based adaptive control and recursive sliding mode control. These algorithms should be tailored to address the specific challenges posed by highly nonlinear MIMO UAV systems with varying dynamics.
- Guidelines will emphasize the importance of robust parameter estimation techniques that can adapt to the changing dynamics of UAVs in real-time. This might

include machine learning-based approaches for online parameter identification.

- Future work may involve the development of more accurate nonlinear dynamic's models for UAVs. This could include accounting for aerodynamic effects, wind disturbances, and variations in vehicle configurations.
- Researchers and practitioners will likely be encouraged to conduct extensive experimental validations to demonstrate the effectiveness of RGDI-ARSMC in real-world scenarios. This could involve test flights with actual UAV platforms to showcase the control system's robustness.
- Guidelines may stress the importance of designing control systems that are not only robust but also fault-tolerant and resilient. This is crucial for UAVs operating in challenging environments where failures or disturbances can occur.
- Future offers might include the development of open-source software and resources for researchers and engineers working on RGDI-ARSMC for UAVs. This can promote collaboration and accelerate advancements in the field.

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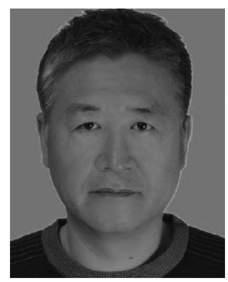

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Section 1: See reference [26]
Section 2: See reference [26,27]
Section 3: See reference [26-28]
Section 4: CASE 1:

In our novel methodology, we have compared the design pattern with our previous lab work, specifically the nonlinear paper and RGDI paper. This comparison demonstrates how the RGDI-ARSMC method is superior to other techniques such as SMC, Backstepping, and the RGDI controller. The novel RGDI-ARSMC strategy effectively addresses key issues such as finite-time convergence, robustness for parametric perturbations, and singularity. To verify its effectiveness, we conducted a comparison study and evaluated the performance of the controller in terms of vertical stability (pitch angle) and horizontal stability (yaw angle). Additionally, the controller was tested in the presence of disturbances, such as external disturbances, parametric uncertainties, coupling effects, and noise signals. Figure 5 and Figure 6 represent the brief response with their convergence time in subplots. The figure that shows the step input for both the pitch angle and yaw angle of the TRMS illustrates that the pitch angle has a faster convergence time than the yaw angle. This delay is caused by the fact that the main rotor must be stabilized first in order to counteract the disturbance generated by the tail rotor, such as the gyroscopic torque effect and coupling effect. As a result, the convergence time of the tail rotor cannot be faster than that of the main rotor. The step input tracking response in the simulation confirms that the control theory regarding the convergence time of the angles, specifically the pitch angle and yaw angle, is logical and results in a sharp stability response.

Section 5: CASE 2: The first case study is aimed at understanding the behavior of the controller by applying three different reference inputs, including ramp and sinusoidal inputs. The simulation response of the highly coupled system, with both matched and mismatched perturbations, can be seen in the figures. To further

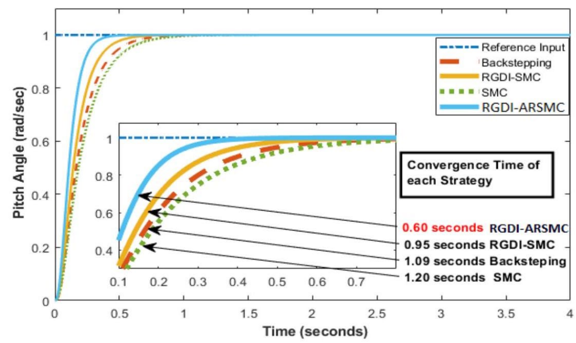

Fig. 5: Step response of pitch angle.

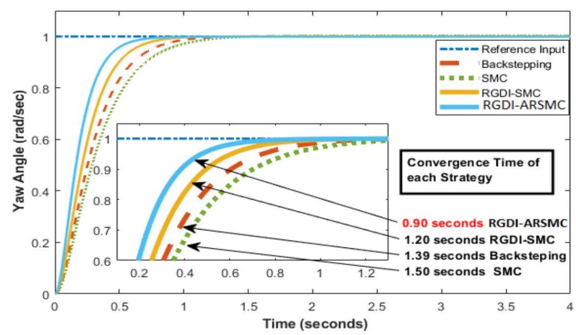

Fig. 6: Step response of yaw angle.

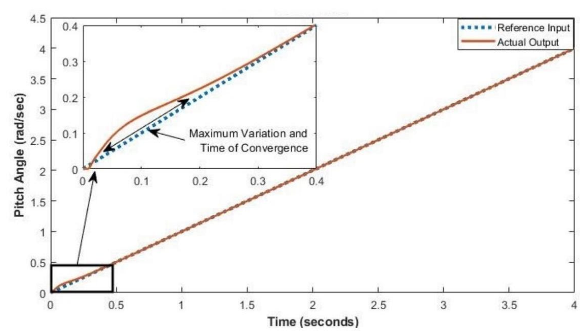

Fig. 7: Pitch angle- ramp response of the proposed RGDI-ARSMC strategy Input.

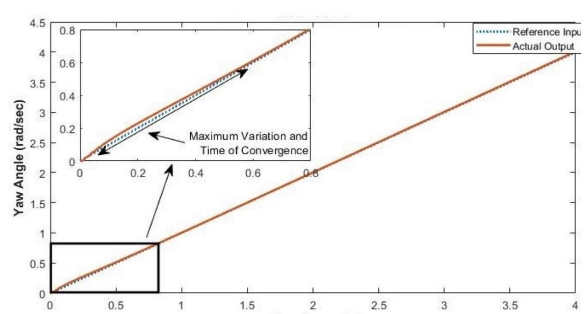

Fig. 8: Yaw angle- ramp response of the proposed RGDI-ARSMC strategy Input.

demonstrate the effectiveness of the controller, a different input, specifically a ramp input and a sinusoidal input are, is applied as shown in figures 7,8,9 and 10 respectively. The subplot in the simulation response of the angles is provided to understand the exact time of convergence as well as the confined stability tackled by RGDI-ARSMC. It is worth noting that the remarkable difference in the convergence time of both outputs is due to the recursive structure of the controller, which is designed based on the arbitrary controller for each state of the system.

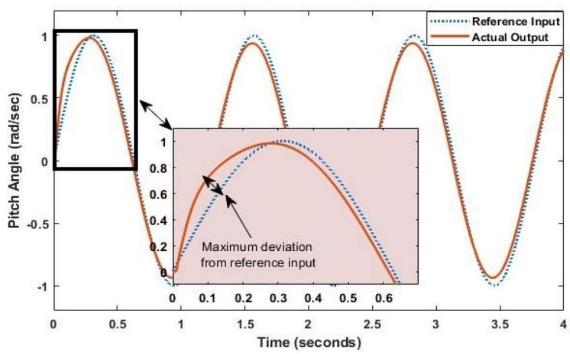

Fig. 9: Pitch angle- sinusoidal response of the proposed RGDI-ARSMC strategy Input.

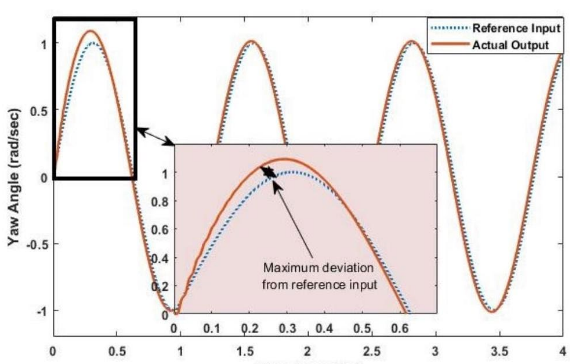

Fig. 10: Yaw angle- sinusoidal response of the proposed RGDI-ARSMC strategy Input.

A detailed comparison regarding previous research [26-28] is comprehensively provided in Table 2 and we can notice the efficient convergence time of proposed controller.

Table 2: Previous research vs proposed strategy

Control strategies	Settling time pitch angle (rad/s)	Settling time yaw angle (rad/s)	Robust ag ai-nst pert urb-ations.
Mixed optimization with μ-synthesis	3.4	3.6	good
RGDI based optimization	4	4.3	good
ARSMC	3.2	3.4	good
Proposed strategy	**0.8**	**1**	**good**