

THE UNIVERSITY OF HULL
CARTOGRAPHIC INFORMATION SYSTEMS
RESEARCH GROUP C.I.S.R.G.

DEPARTMENT OF COMPUTER SCIENCE · HULL HU6 7RX · UNITED KINGDOM
TELEPHONE 0482 465295/465951 · FACSIMILE 0482 466666 · TELEX 9312134728 HUG

DR M VISVALINGAM
HONORARY CO-ORDINATOR

Discussion Paper Series Editor: Dr M Visvalingam



C.I.S.R.G. DISCUSSION PAPER 18

Cognitive Evaluation of
Computer-drawn Sketches

by

M Visvalingam and K Dowson

No part of this Discussion Paper should be copied, cited or distributed without written permission from the editor of the series (M.Visvalingam@dcs.hull.ac.uk).

CISRG Discussion Papers

1. Wade, P., Visvalingam, M. and Kirby, G. H. (1986)
From Line Geometry to Area Topology, 48 pp.
2. Visvalingam, M. (1987)
Problems in the Design and Implementation of a GKS-based User Interface for a Graphical Information System, 18 pp.
3. Visvalingam, M. (1988)
Standardising Basic Spatial Units: Problems and Prospects, 19 pp.
4. Whyatt, J. D. and Wade, P. R. (1988)
The Douglas-Peucker Algorithm for Line Simplification: an introduction with programs, 17 pp.
5. Visvalingam, M. (1988)
Cartography, Geographical Information Systems and Maps in Perspective, 16 pp.
6. Visvalingam, M. and Whyatt, J. D. (1989)
The Douglas-Peucker Algorithm for Line Simplification: re-evaluation through visualisation, 35 pp.
7. Visvalingam, M. (1989)
Trends and Concerns in Digital Cartography, 37 pp.
8. Visvalingam, M. and Whyatt, J. D. (1990)
Cartographic Algorithms : problems of implementation and evaluation and the impact of digitising errors, 24 pp.
9. Visvalingam, M. (1991)
Visualisation, VISC and Scientific Insight, 21 pp.
10. Visvalingam, M. and Whyatt, J. D. (1992)
Line Generalisation by Repeated Elimination of the Smallest Area, 16 pp.
11. Varley, D. A. and Visvalingam, M. (1993)
Area Topology for Road Extraction and Topographic Data Validation, 34 pp.
12. Visvalingam, M., Varley, D. A., and Wright, C. P. (1993)
Global dimensions for the recognition of prototypical urban roads in large-scale vector topographic maps, 20 pp.
13. Visvalingam, M., Williamson, P.J. (1993)
Generalising Roads on Large-Scale Maps: a comparison of two algorithms, 21 pp
14. Visvalingam, M. (1996)
Approximation, Generalisation and Deconstruction of Planar Curves, 13 pp.
15. Visvalingam, M. and Dowson, K. (1997)
Algorithms for Sketching Surfaces, 19 pp.
16. Visvalingam, M. and Herbert, S. P. (1998)
A Computer Science Perspective on Bendsimplification, 18 pp
17. Visvalingam, M. (1998)
Sources of Variability in Cartographers' Deconstruction of Fractals, 22 pp
18. Visvalingam, M. and Dowson, K. (1998)
Cognitive Evaluation of Computer-drawn Sketches, 29 pp.



Special Issues

1. Visvalingam, M. (ed) (1986)
Research based on Ordnance Survey Small-Scales Digital Data Proceedings of a meeting held as part of the 1986 Annual Conference of the Institute of British Geographers (Reading Jan 1986), **sponsored by the Ordnance Survey**, 79 pp.
2. Visvalingam, M. and Kirby, G. H. (1987)
Directory of Research and Development based on Ordnance Survey Small Scales Digital Date, **sponsored by the Ordnance Survey**, 38 pp.
3. Visvalingam, M. and Sekouris, N. M. (1989)
Management of Digital Map Data Using a Relational Database Model, **sponsored by the Ordnance Survey**, 50 pp.

Web Pages

http://www2.dcs.hull.ac.uk/CISRG/	/ HOME page
http://www2.dcs.hull.ac.uk/CISRG/publications/DiscussionPapers.html	/ Discussion Papers
http://www2.dcs.hull.ac.uk/CISRG/publications/Publications.html	/ Publications

CONTENTS

1. Introduction	1
2. Background	3
3. Rationale for exploring the Douglas-Peucker algorithm	5
4. Methods Used for Comparison and Results	7
4.1 Generalised Profiles	11
4.2 Core cells against contour maps	12
4.3 A comparison of P-stroke sketches	13
5. Discussion and Future Work	15
5.1 Subconscious visual reasoning	20
5.2 Implications for future research	24
6. Conclusion	26
Acknowledgements	27
References	28

1. Introduction

Global terrain data is becoming increasingly available in various forms, such as contours, Triangulated Irregular Networks (TINs) and grid Digital Elevation Models (DEMs). DEMs are also known as height fields and as altitude matrices. The research reported in this paper contributes to three strands of research being undertaken within the Cartographic Information Systems Research Group (CISRG) of the University of Hull. It contributes to research on a) the automatic derivation of minimalist sketches of grid DEM data, b) methodology for the evaluation of underpinning line generalisation algorithms, and c) the abstraction of subsets of points for representing the terrain as a Triangulated Irregular Network (TIN). This paper focuses on the first two aims. Field sketching of landscapes and the drawing of generalised block diagrams were valued as a scientific art and formed a part of the cartographer's training in the pre-computer era (Lobeck, 1924, Raisz, 1938). With the availability of systems for visualising terrain in various ways, the artistic methods of relief representation have largely been ignored within Geographical Information Systems (GIS). This is partly because our understanding of the cognitive processes involved in sketching is still insufficient. Yet engineering firms, such as the co-sponsors of this research project, and several other applications require artists for sketching the landscape.

As in Chinese calligraphy, sketching strives to use a minimum of graphic strokes to induce perception of the whole form, the visual impact of which is greater than the sum of the component parts. Such gestalt perception relies on the coherence of the configuration of strokes. Skilled artists study the overall structure and form of the landscape and select suitable features. In contrast, algorithmic sketching, like image analysis in pattern recognition, tends to proceed bottom up, starting with the identification of the local features. Sketch configurations with striking Gestalt qualities may not necessarily select the features which are optimal for model-based pattern recognition. This research focuses on the identification of features for human perception rather than for segmentation of the terrain.

Manual sketching is knowledge-based and involves the recognition of 'skeletal form lines' (Imhof, 1982, p 105), which bring out the relief features, such as watersheds, stream networks, breaks of slope, ridges and edges of terraces. In a feasibility study, Visvalingam and Dowson (1998; initially reported in Dowson, 1994) demonstrated that it was possible to derive sketches of DEMs without such prior knowledge. They investigated a simple profile-based sketching technique used by cartographers, such as Tanaka (1932), Raisz (1938), Robinson and Thrower (1957) and Dickinson (1969).

The generation of P-stroke sketches involves a number of processes. Visvalingam's algorithm for line generalisation was used for filtering cells of the DEM lying on global curvatures; four tolerance values were selected for selecting those on the more important convexities and concavities in the rows and columns of the DEM respectively. These selected

cells indicated where the polylines making up the sketch should be drawn. These polylines consisted of no more than subsets of the original row profiles and were called Profile-strokes, abbreviated to P-strokes. The extents of the four types of P-strokes were prescribed by rules that took account of the type of curvature (convex or concave) and the assumed view direction. The appearance of each type of P-stroke was varied so that the sketch bonded together into a visually coherent whole. The research was more concerned with the skill of 'seeing' in the process of sketching, rather than with rendering which forms a separate thread of research.

The aim of this research was to investigate whether equally effective sketches could be produced using more widely available algorithms, than Visvalingam's algorithm, for locating the features to be sketched. Visvalingam's algorithm was first evaluated by Whyatt (1991) and subsequently published by Visvalingam and Whyatt (1993). In contrast, Ramer's algorithm (1972; independently described by Douglas and Peucker, 1973), is well-known and widely available in proprietary GIS and mapping software. Visvalingam and Whyatt (1990) demonstrated that the widely cited mathematical evaluation of line generalisation algorithms by McMaster (1987) were suitable for minimal simplification (approximation) of curves even if not for their caricatural generalisation. Visvalingam and Williamson (1995) showed that the Douglas-Peucker algorithm is better than Visvalingam's algorithm for minimal simplification but that the latter produced better caricatures. The current P-stroke sketches use some 25 to 30 percent of the DEM cells. As such, they are not caricatural and fall within the limits of minimal simplification.

This study therefore was undertaken to establish whether equally effective sketches could be produced with the widely available Douglas-Peucker algorithm. This paper also includes the results obtained when Visvalingam's algorithm was used with the perpendicular distance metric, which drives the Douglas-Peucker algorithm. Dowson (1994) undertook similar comparisons prior to his selection of Visvalingam's algorithm with the area metric (explained below) for the sketches reported in Visvalingam and Dowson (1998).

In this paper, the first author re-interprets some of the previously unpublished results in Dowson (1994). Isolated profile lines and plots over contour maps do not lead to any definitive conclusions about the relative merits of the underpinning line generalisation algorithms. The P-stroke sketch, together with separations of its constituent types of strokes, provides a more useful framework for expert visual assessment of a) different algorithms for locating the global curvatures; and b) the contribution made by the different strokes to the perception of the three dimensional form of terrain. However, anecdotal comments on the sketches emphasise the subjective nature of visual assessment. This subjectivity has encouraged continuing research on objective mathematical measures of performance. These in turn have proved to be inappropriate as reviewed below. Some conjectures about the complex perceptual process

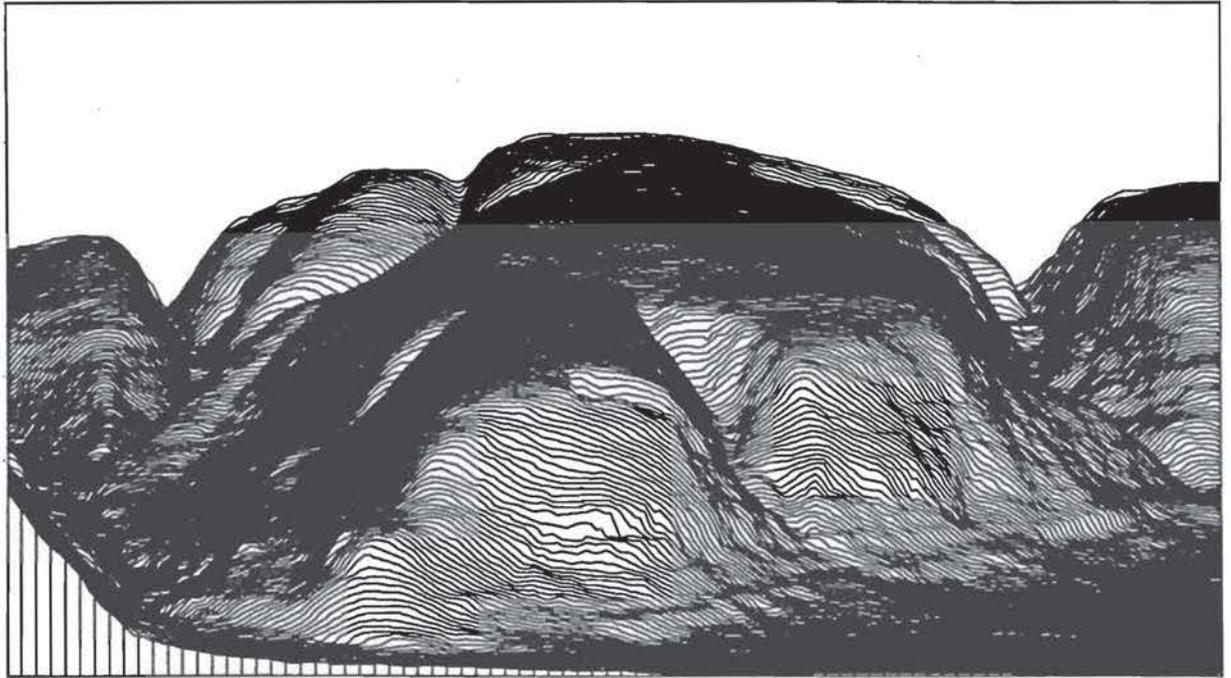
involved in seeing the P-stroke sketches were derived through scrutiny of Dowson's sketches. These provide a framework for explaining the subjective and variable visual comparisons made by people who happened to see them. These conjectures suggest that it might be possible to develop programs, based on elements of perceptual computation, for automatically and objectively evaluating the sketches.

The paper provides a brief background to the research and then provides a fuller explanation of the reasons for comparing Visvalingam's algorithm with the Douglas-Peucker algorithm. It then explains the use of profile plots to guide the selection of optimal tolerance values for conducting a fair and unbiased comparison. The relative utility of the algorithms was then evaluated by visual comparison of a) contour maps showing the filtered cells, and b) P-stroke sketches of the landscape. The sketches seemed to provide the most conclusive results. Visvalingam's algorithm locates more of the breaks of slope and curvatures that produce effective and aesthetically pleasing depictions of the landscape. Although the Douglas-Peucker algorithm selects cells which lie on more continuous tracks on planar contour maps, it does not pick out the curvatures which induce holistic vision. The resulting sketches lack coherence and appear clumsy. The conjectures they provoke about the nature of perception are then outlined with some ideas for future work. The conclusion summarises the main threads of the paper and returns to the main aims of this thread of research within the Cartographic Information Systems Research Group (CISRG) of the University of Hull.

2. Background

Visvalingam and Dowson's (1998) P-stroke sketching involves the use of Visvalingam's line generalisation algorithm for associating measures of significance to the cells of the DEM. The sketching technique treats the row and column profiles as independent lines. Visvalingam's algorithm repeatedly scans each line and eliminates the point which contributes least information to the description of the line. The significance of the point may be measured in various ways as explained by Visvalingam and Whyatt (1993). Visvalingam and Dowson (1998) used the effective area metric that was found to be the most effective for 2D lines by Visvalingam and Whyatt (1993) and Visvalingam and Williamson (1995). The effective area of a point is the area formed by the point and its two immediate neighbours. When a point is eliminated from consideration, the signed area that led to its elimination is recorded with the point. The non-boundary cells of the DEM are thus rated by two measures of significance with respect to their importance in their respective rows and columns.

The cells can be filtered at run time using tolerance values to locate cells (called core cells) lying on scale-related convexities and concavities. These core cells are then extended into P-strokes as explained in the introduction and rendered to form the sketch. Visvalingam and Whelan (1998) demonstrated how the visual impact of these sketches could be considerably improved by the inclusion of occluding contours; but noted that the latter are not sufficient on



(a) All Data
© Crown Copyright; Ordnance Survey



(b) P-stroke sketch with occluding contours
Source : Visvalingam and Whelan (1998)

Figure 1 : P-stroke sketch with occluding contours

their own to adequately define the form of the land. Figure 1b provides an example of the finished sketch.

This study investigated whether the now classic and more widely available Douglas-Peucker algorithm would provide better sketches since the method appears to be better than Visvalingam's algorithm for minimal simplification. This algorithm generalises lines by a process of progressive selection of points which are rated to best preserve the shape of the curve. We used Wade's modification of the algorithm (reported in Whyatt and Wade, 1988); and evaluated by Visvalingam and Whyatt, 1990). The algorithm recursively subdivides the line at its maximum offset (the furthest point from the line joining the current first and last points) and stores the offset value with the point. Since Visvalingam's algorithm may be used with any measure of significance, the equivalent of the test heuristic used to drive the Douglas-Peucker algorithm was also used with Visvalingam's algorithm (using the perpendicular distance from the point to the line joining its two neighbours). This provides three sets of results for comparison.

3. Rationale for exploring the Douglas-Peucker algorithm

The utility of the Douglas-Peucker algorithm for sketching was evaluated for the following reasons:

1. ***A DEM encodes the one-dimensional variation in heights over a surface*** as a matrix of height values, also referred to as a height field. The problem considered here is therefore different to the previously considered generalisation of 2D lines. The selection of extreme points, as described above, by the Douglas-Peucker algorithm makes it particularly inappropriate for generalising 2D data consisting of a hierarchy of scale-related features within features since the extreme points are often located on minor features (Visvalingam and Whyatt, 1993). The retention of these points causes shape distortion. Even surface terrain can have overhanging sections where there are cliffs as in karst landscapes. These awkward cases which occur in the real world cannot, by definition, be represented in a grid DEM. Thus, the previous conclusions about the two geometric algorithms, based on 2D lines, may not apply to grid DEMs.
2. ***The P-stroke style of sketching uses some 28 % of cells*** (Visvalingam and Dowson, 1998). This still falls within the range of minimal simplification of lines, which aims to represent the original surface with a reduced set of points. Visvalingam and Whyatt (1993) found that source coastlines could be reproduced at the original scale with just 30% of points. Since the Douglas-Peucker algorithm was found to be better for minimal simplification by Visvalingam and Williamson (1995), it could be more useful than Visvalingam's algorithm for sketching and analysis of surfaces.

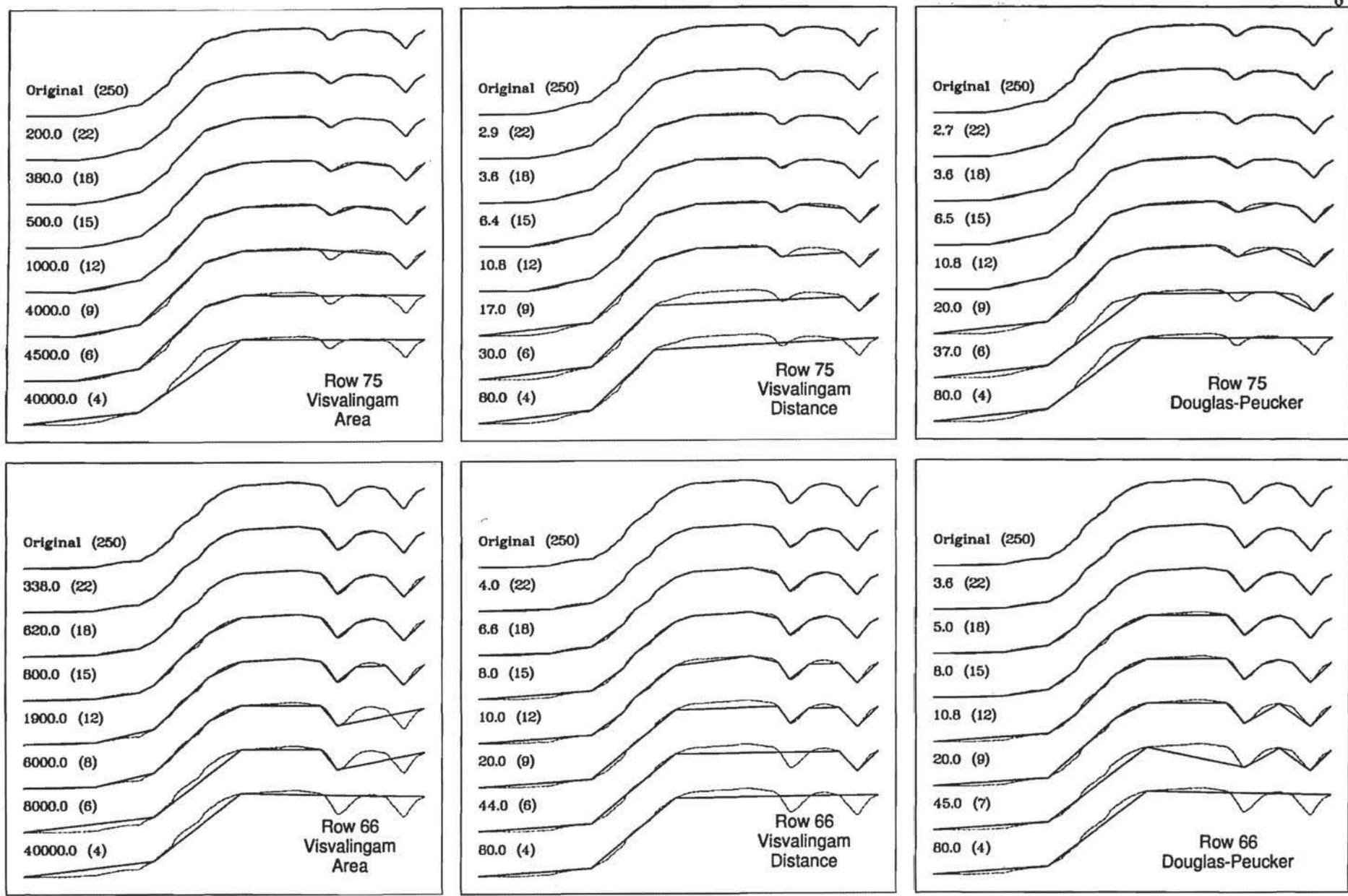


Figure 2 : Three methods (columns) for filtering two profiles (rows)

3. ***The Douglas-Peucker algorithm is more accessible to end-users.*** This algorithm is available in many Geographical Information Systems (GIS) and is better known than Visvalingam's algorithm. It is often the only line generalisation algorithm to be included in textbooks on Digital Cartography and GIS and forms the basis of recent PhD theses in GIS (João, 1988; Dutton, 1999).
4. ***It could well be that people may perceive height (a distance measure) as having more significance than area in the context of terrain;*** DEMs only represent one dimensional variations in height over a surface. A popular approach to fractal terrain simulation involves vertical displacement of midpoints of line sections and of triangular facets respectively. As noted earlier, the Douglas-Peucker algorithm relies on a distance measure. Visvalingam's algorithm can also be used with different metrics and the utility of distance, as opposed to area, was worth exploring.
5. ***P-strokes pick out scale-related convexities and concavities in the terrain.*** It would be interesting to study the visual impact of the convex and concave features selected by some other method.

This paper therefore undertook a comparison of the results obtained using the Douglas-Peucker algorithm with those obtained from Visvalingam's algorithm using effective area and perpendicular distance.

4. Methods Used for Comparison and Results

The selection of appropriate tolerance values is a continuing problem in cartographic generalisation as it is in Image Processing. Comparative studies, such as this, need to ensure that the tolerances chosen do not bias the results in favour of a specific approach. This study therefore undertook prior comparison of DEM profiles to select tolerance values which would best reveal the differences between the methods. The impact of the chosen cut-off values was then checked by plotting the filtered cells on contour plots of the study area.

These maps confirm that the selected tolerance values are appropriate for revealing differences in the output of the algorithms. These maps initially favour the output produced by the Douglas-Peucker algorithm since it outputs trackable features. Visvalingam's algorithm outputs a more dispersed set of cells, especially when used with the area metric. However, other observations did not justify the choice of one approach over others. In contrast, the P-stroke sketches showed that Visvalingam's algorithm used with area was still the most promising method for further research on sketching. However, the sketches based on the Douglas-Peucker output revealed some interesting observations relating to human vision, which merits further experimental study.

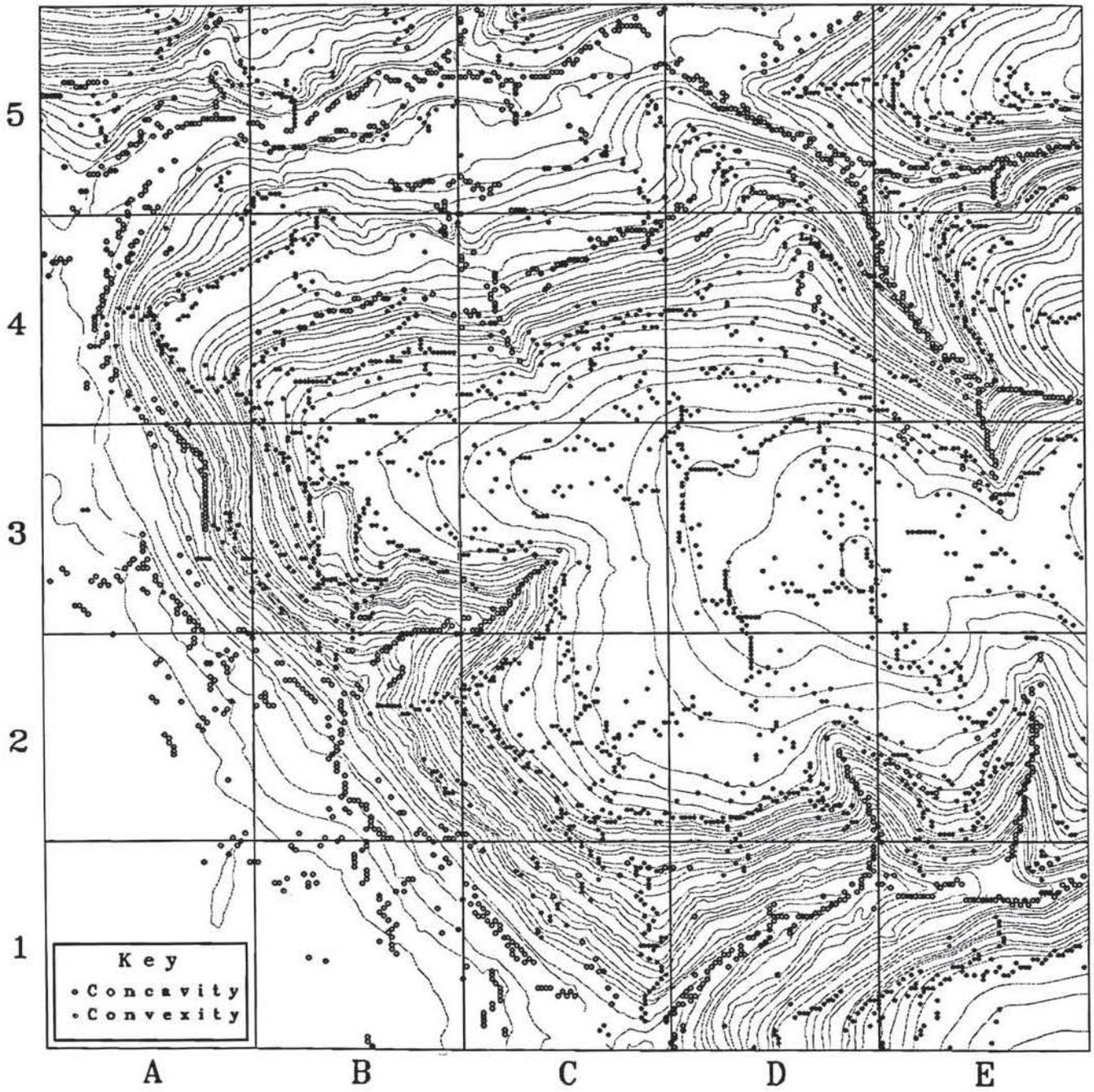


Figure 3a : Cells selected by Visvalingam's algorithm using effective area (see Table 1)

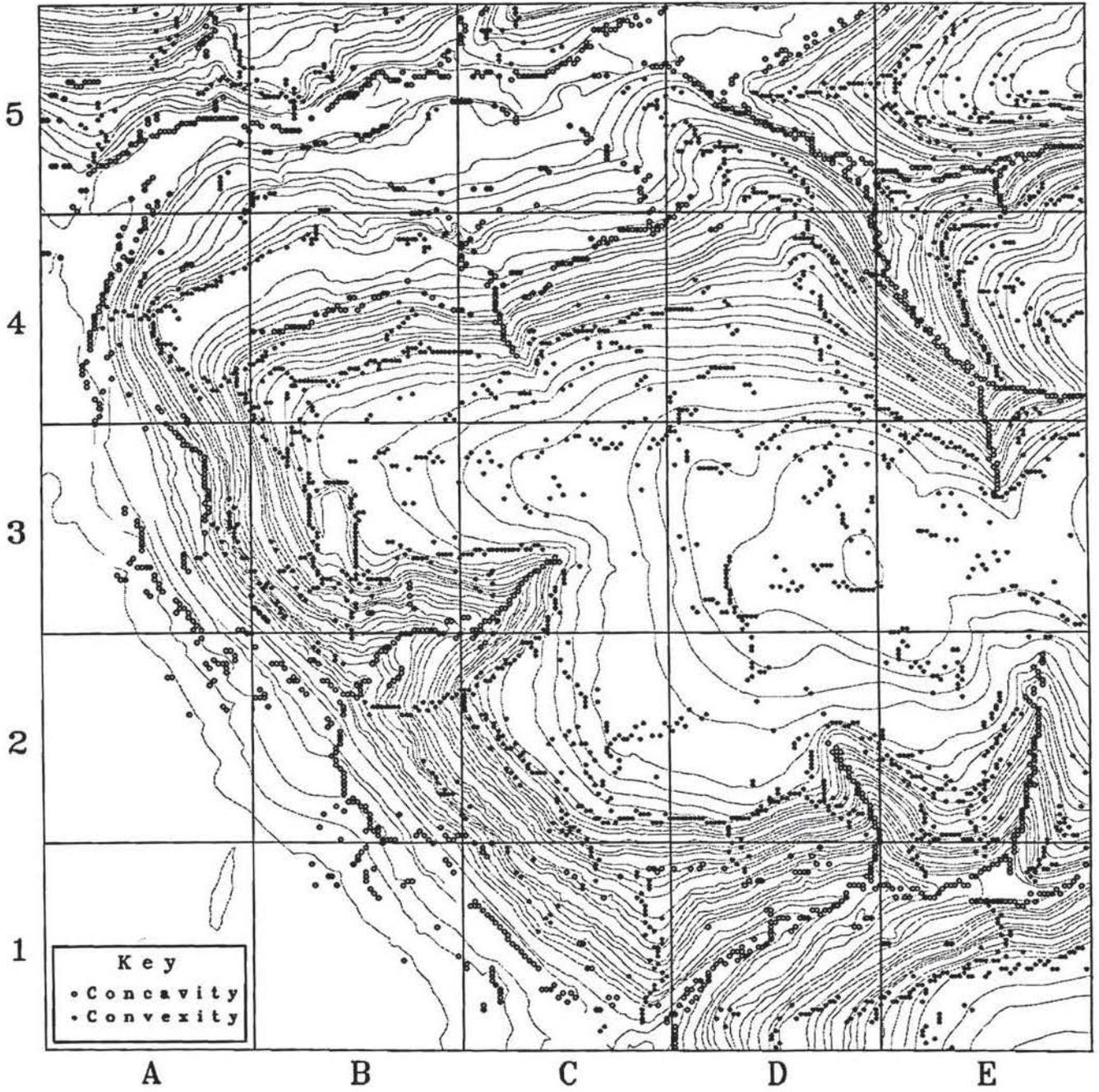


Figure 3b : Cells selected by Visvalingam's algorithm using perpendicular distance (see Table 1)

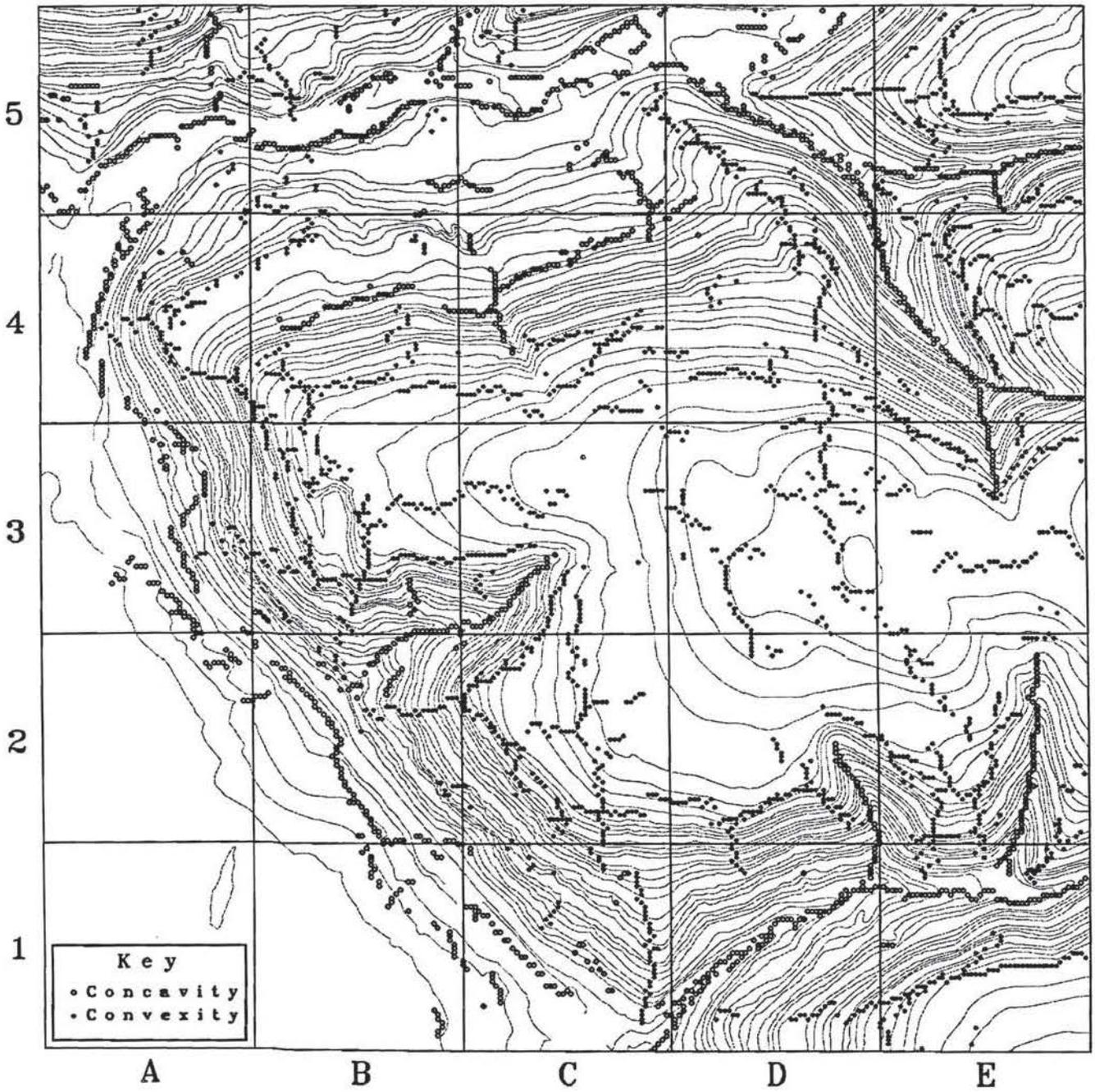


Figure 3c : Cells selected by the Douglas-Peucker algorithm (see Table 1)

4.1 Generalised profiles

Figure 1 is based on the Port Talbot DEM generated by the Ordnance Survey of Great Britain. Figure 2 shows the application of the two filtering algorithms to two profiles of this DEM. On the contour plots in Figure 3, the grid lines are 50 units apart; each unit represents 10 meters on the ground. The two profiles shown in Figure 2 would be horizontal lines running east west (at $y = 75$ and 66 units respectively) across this contour plot. Figure 2 shows 3 sets of polylines for each of the two profiles. The results for Visvalingam's algorithm appear on the left while those for the Douglas-Peucker algorithm appear on the far right. The leftmost column shows the results obtained using Visvalingam's algorithm, driven by effective area; the middle column shows the results for the same algorithm using the distance metric. Within each sub-figure, the top line shows the original profile. The other lines show the effect of progressive filtering. The tolerance value used is shown together with the number of points retained in parenthesis.

The figures for row 75 show that with this terrain, it is possible to discard almost 90% of the points before there is any significant displacement of the filtered line from the source line. At the other extreme, 4 points are clearly too few but even so all methods give a good overall impression. The distance measure (middle figure) is better at locating the edge of the plateau but it does not give as good a representation of the plateau top as the other two methods. With 6 points the area metric is better at sketching the shape of the plateau, while the other two methods select a valley point. With 9 points, the area metric retains the shape of the plateau but now includes the valley. The other two methods are not as successful in representing the plateau shape. With 12 points, all three methods are roughly similar although the Douglas-Peucker profile in the last column looks more faceted. The profiles for 15 points are even more similar.

The three lower figures for profile 66 show that the above comparison of profiles is not entirely reliable and at high levels of filtering the inclusion/omission of single points can affect the relative performance of methods. However, both these profile-based analyses and Dowson's (1994) original analyses indicate that a subset of 5 to 6 percent of the points (i.e. an average of 12 points per profile) showed up sufficient differences between the methods without over-generalisation of the results. This level of filtering was used with other visualisation techniques to study the impact of the algorithms on the whole DEM. First, the distributions of significant cells were studied against the backdrop of contour maps. Then, the selected cells were used to sketch the DEM using the technique of P-strokes described by Visvalingam and Dowson (1998).

4.2 Core cells against contour maps

Method \ Feature	Visvalingam (area)		Visvalingam (perpendicular distance)		Douglas-Peucker	
	tolerance sq m	#pts	tolerance sq m	#pts	tolerance sq m	#pts
Convex						
(east-west)	2000	1010	12	982	12	994
(north-south)	2000	1163	12	1227	12	1198
Concave						
(east-west)	4000	682	18	634	17	626
(north-south)	4000	813	18	814	17	807
TOTALS		3668 5.87%		3657 5.85%		3625 5.80%

Table 1 : Tolerance Values and Number of Points for Figures 3a - 3c

Figure 3 shows the distribution of DEM cells, which were selected using the three different filters. Table 1 provides the tolerance values and the number of points retained for each method. Figures 3a-c show the resulting distributions. These distributions, based on about 5.75 % of points, are not dissimilar to those produced by Dowson, who used a different set of tolerance values. All three methods give a good overall impression of the major features when plotted over contour data for the area. However, detailed study reveals some notable differences:

- On the whole, Figure 3c has a cleaner and crisper pattern. The Douglas-Peucker algorithm selects cells which are more connected and which may be easier to track (as in cells D1, D2, C2 and D5). Visvalingam's algorithm with effective area produces a dispersed, possibly more noisy, distribution (as in A1-3, D3 and E2). Visvalingam's algorithm with perpendicular distance (Figure 3b) outputs a distribution, which is somewhat intermediate in character.
- Visvalingam's algorithm provides a better definition of the northern side of the valley running from A5 to C5 respectively, while the Douglas-Peucker is better at picking out the southern side. The north south concavities in A5 and B3, picked out in Figures 3b and 3c, are not so clearly defined in Figure 3a.
- The northern edge of the central plateau (in B4 and C4) is located in different places by the three methods. Note also that the depiction of the convex top of the plateau in B3 and C3 is markedly different in the three maps. The distinct line of cells, picked out by the Douglas-Peucker method, running east to west through B3 and C3 in Figure 3c seems logically misplaced against the contours.

- Visvalingam's algorithm with the area measure picks the north-south convexity (in C5) while the other two maps show the corresponding concavity.
- Some other features picked out by the Douglas-Peucker algorithm are misleading. For example, the east-west line in E3 is not a distinct ridgeline but just a watershed. North-south profiles across this area show an undulating flattish area as indicated in Figure 1b.

The maps suggest that the number of core cells retained is about right. The results, however, are as confusing as those obtained by Dowson (1994). In general, the Douglas-Peucker algorithm produces the least confusing distribution. The area-based method produces a more scattered distribution while the distance metric has characteristics of both distributions. Detailed scrutiny finds elements in favour of all methods, making a choice difficult.

4.3 A comparison of P-stroke sketches

Visvalingam and Dowson (1998) provide a detailed explanation of the P-stroke technique for sketching. The resulting sketch consists of 4 types of P-strokes or filtered sections of the profile lines shown in Figure 1a. All the sketches in this paper view the DEM from the west. So, the profile lines shown in Figure 1a run from north in the left to south on the right. Sketches from other directions may be found in Dowson (1994) and Visvalingam and Dowson (1998). Different tolerances may be used for each type of P-stroke as shown in Table 2. The A & C types pick out major convexities and concavities along the line of sight. The B & D types pick out the major convex and concave curvatures running across the line of sight (i.e. those on the profile lines shown in Figure 1a). The filtered cells, called core cells, are used as measures of significance of generalised curvatures on profiles. Different sketching rules were used to extract the P-strokes for depicting these curvatures in the feasibility study (Visvalingam and Dowson, 1998).

Tables 2 & 3 list the tolerance values and other sketch parameters used for each type of P-stroke. The sketches use 28% of the DEM cells. This subset includes those which are occluded. In general, the core cells selected by the Douglas-Peucker algorithm generate shorter P-strokes. This is not surprising since extreme points are often located on minor features. To compensate for this, a larger number of core cells were retained for the Douglas-Peucker algorithm to ensure that there was an equivalent amount of P-stroke points. Figure 4 shows the P-stroke sketches output by the three methods.

- All three sketches give a good overall impression of the plateau behind Port Talbot in Wales, UK.
- However, there is noticeable variation in the quality of the sketches. Figure 4a (Visvalingam's method with area) emerges as the most effective sketch, while Figure 4c (produced by the Douglas-Peucker method) appears somewhat crude. Figure 4b again shows elements of both sketches.

Method Stroke Types	Visvalingam (area)			Visvalingam (perpendicular distance)			Douglas-Peucker		
	tol	strokes	#pts	tol	strokes	#pts	tol	strokes	#pts
Convex on:									
A : forward profiles	1000	868	6082	6.9	861	6114	5.8	895	6093
B : cross profiles	2000	954	2862	12.2	956	2868	12.4	960	2880
Concave on:									
C : forward profiles	4000	524	2962	16.2	534	2960	13.9	533	2953
D : cross profiles	1000	1334	5317	8.3	1327	5294	8.0	1332	5309
TOTALS		3680	17223 27.6%		3678	17236 27.6%		3720	17235 27.6%

Table 2 : Tolerance Values and Number of Points for P-stroke Sketches (Figures 4a - 4c)

Method Stroke Types	Extension Rules		Style
	Left	Right	
Convex			
A : forward profiles	full	none	solid
B : cross profiles	none	none	solid
Concave			
C : forward profiles	minimal	full	dotted
D : cross profiles	minimal	none	dotted

Table 3 : Sketch Parameters for P-stroke Sketches (Figures 4a - c)

- Most non-cartographers, shown the sketch, preferred Figure 4a. They were convinced that it had more strokes and found it difficult to believe that this was entirely due to the more dispersed distribution of the strokes.
- The various types of P-strokes in Figure 4a harmonise to provide a holistic integrated description of the landscape. In contrast, the configuration of convex P-strokes (B-type) on the cross-profiles in Figure 4c look computer drawn and detract from the rest of the sketch.

- The large areas of white space, which gave Figure 3c a clean and uncluttered look, create visual ambiguity in the sketch, especially at the left and right edges of the central plateau.

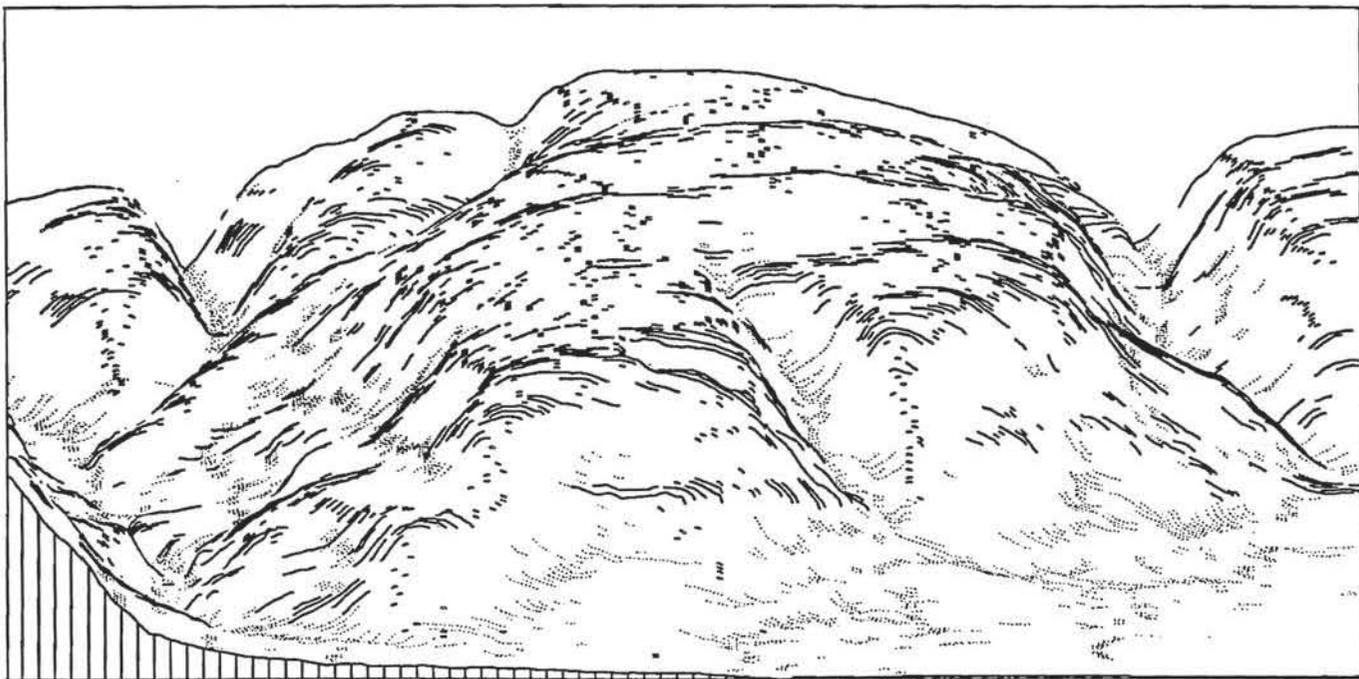
Visvalingam's algorithm with the area metric may be used with a simple algorithm to produce sketches which surpass many manual sketches of terrain in the cartographic and geographic literature. The sketches obtained using distance-based methods, although disappointing, were nevertheless valuable since they provide insights into how the cognitive system processes P-stroke sketches of terrain. Gregory (1970, p 59) proposed that perceiving is a *kind of thinking* and that it appears to be driven by concepts and hypotheses. He noted that when a visual paradox is due to incompatible information, it is not possible to resolve it without rejection of some information. Figure 4c provides additional support for Gregory's idea that perception is theory-driven. This is explored next.

5. Discussion and Future Work

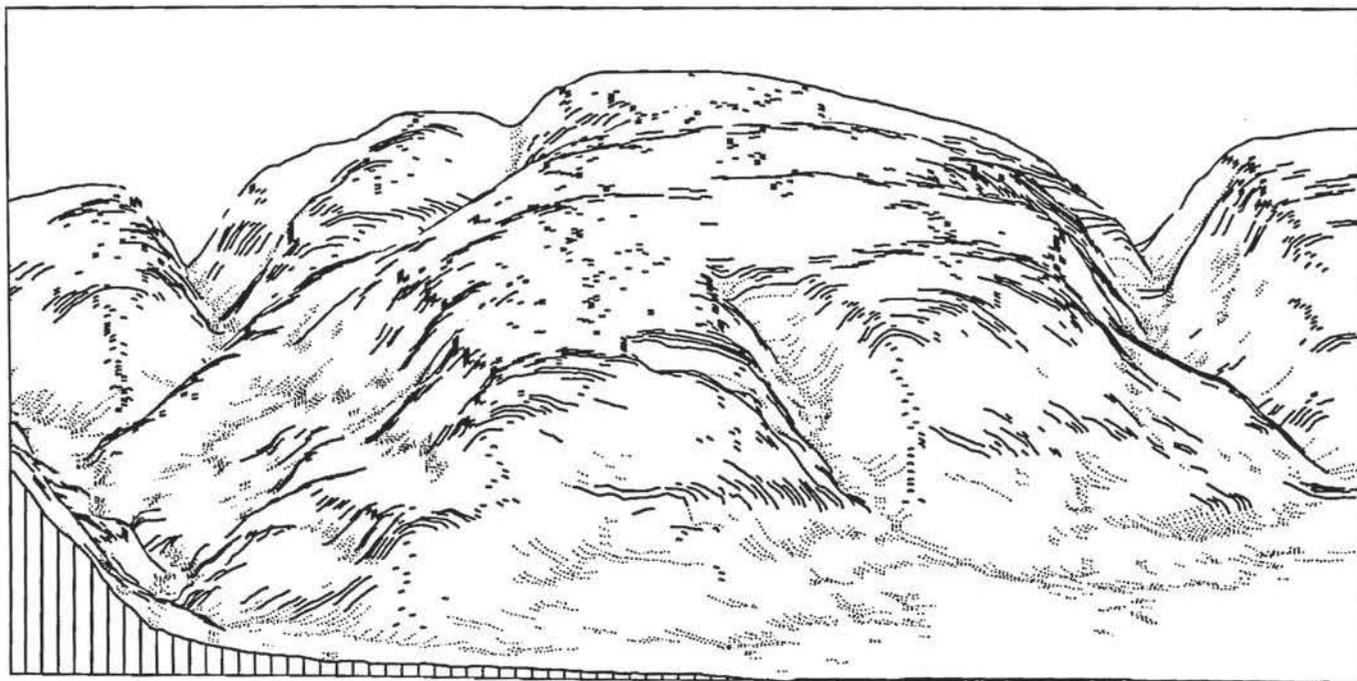
Gregory (1970) used a picture of a paved street as an illustration of the double reality of pictures, which are simultaneously seen as marks forming an image on a flat paper and also perceived as the three dimensional world, which they represent. A range of 3D cues, including those related to geometric and aerial perspective, enable the visual system to imagine the scene extending into 'the depth' of the picture. The sketching technique, which uses a parallel oblique projection, does not employ the normal depth cues in Figures 1 and 4.

The sketches in Figure 4 show only the significant curvatures in the landscape. They do not even include the occluding contours shown in Figure 1b. Sketches are generalised depictions of the surface and as such they do not need to be too precise in their location of features of interest, such as the occluding curvatures and the edges of plateaus. Deviations in the location of the form-defining breaks of slope are only of concern when they detract from the Gestalt quality of the sketch.

The anecdotal comparisons made by colleagues and students in Computer Science on Figures 4a and 4b are worth noting even though their opinions were not elicited as part of a designed experiment. A number of people (mostly males) saw the sketches as very similar and only saw the main differences when they were pointed out to them. The three secretaries (all female) were able to instantly assess that the figures were dissimilar and identify many of the differences quite quickly. They showed a preference for Figure 4a, produced using Visvalingam's algorithm. The gender difference is probably coincidental in this case. Members of the British Cartographic Society's Design Group were encouraging but not overly impressed by any of the computer-drawn sketches but preferred Figure 4a. Since perception is conditioned by preceptions, it is subjective to some extent and readers are free to come to their own conclusions. The first author ventures her personal conjectures after careful



(a) Visvalingam's Algorithm using effective area



(b) Visvalingam's Algorithm using perpendicular distance



(c) Douglas-Peucker algorithm

Figure 4: Sketches based on different methods for selecting core cells

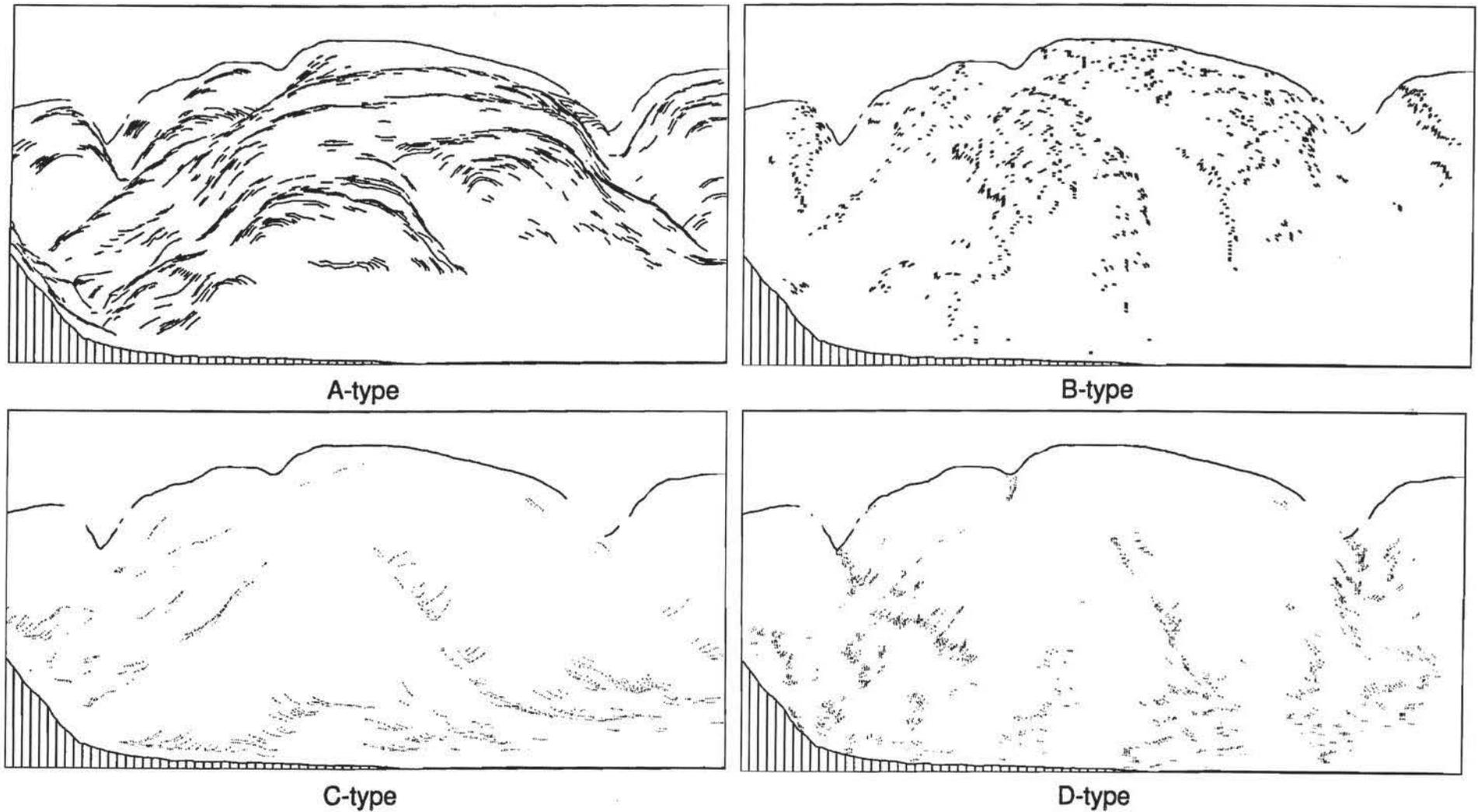


Figure 5: Types of P-strokes generated using Visvalingam's algorithm (with the area metric)

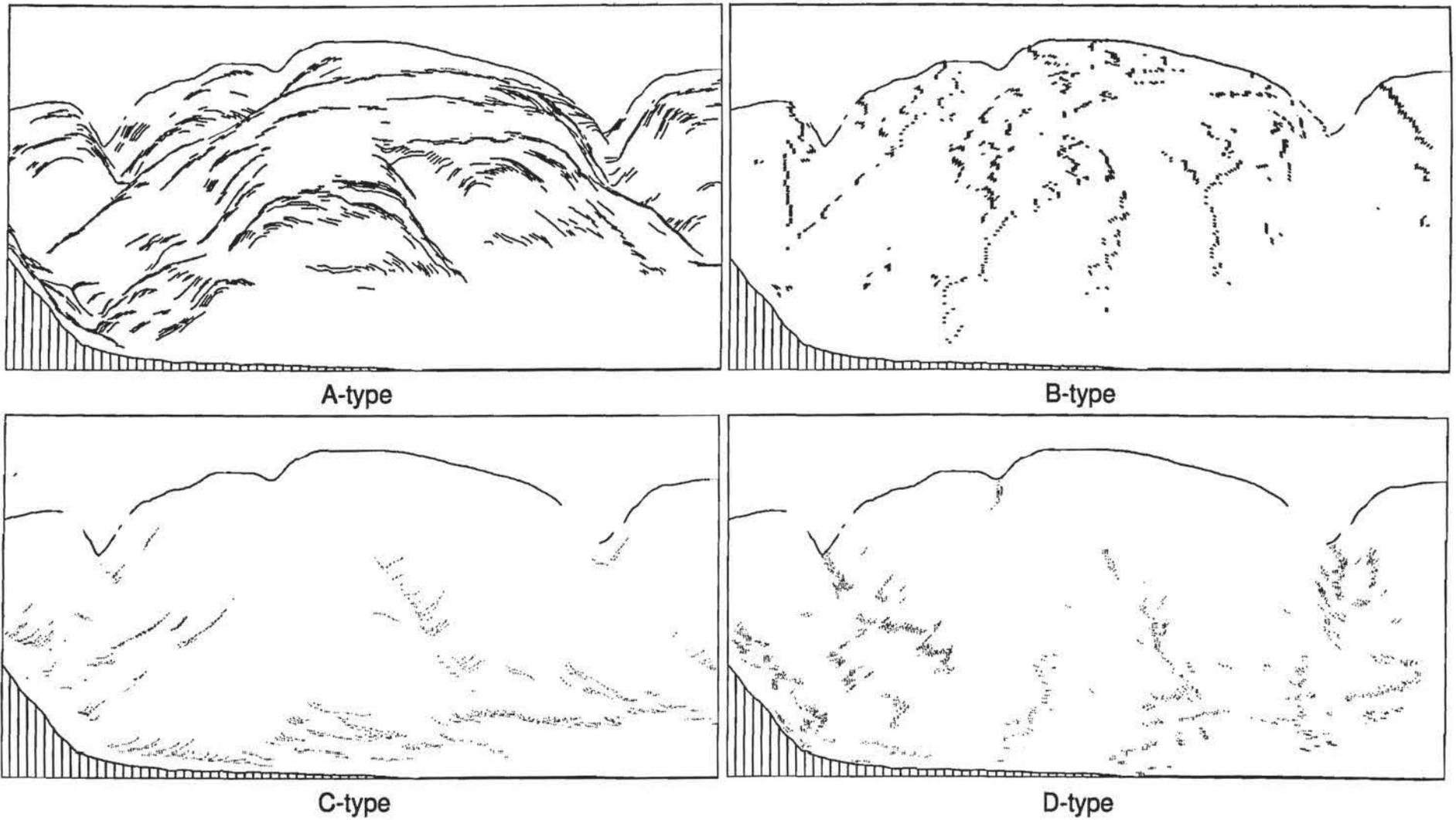


Figure 6: Types of P-strokes generated using the Douglas-Peucker algorithm

scrutiny of Figure 4 and of the four types of P-strokes in Figures 5 and 6. Although these interpretations are not based on an exhaustive literature survey, nor on a specific theory of visual cognition, they are included here because a) they seem to fit with and extend some commonly held views, and, more importantly, b) they provide yet another approach to the evaluation of line generalisation algorithms.

5.1 Subconscious visual reasoning

Hoffman and Richards (1982) noted that the visual system seems to segment figures at the extrema of concavities and recent research by Visvalingam (1999) confirms that vision is biased towards the perception of convex forms. The discussion here, therefore, focuses on the A and B type strokes in Figures 5 and 6.

The convex A-type P-strokes in Figures 5 and 6 are not occluding contours, but some of them indicate the presence of occlusion. Other P-strokes in this class sketch other convexities, such as the frontal breaks of slope on the plateau top. The A-type strokes in both these Figures were initially judged by many to be identical and the plateau region was seen as sloping into the distance. The inclination of the plateau depends on the projection used; here, an oblique parallel projection is used (details in Visvalingam and Dowson, 1998). The top of the plateau is perceived as having slightly different inclinations in the A and B type sub-figures in Figure 5. The plateau seems higher (although we know that the skyline is exactly the same) and is seen as tilted more towards the viewer in the B-type (compared with the A-type) strokes. Although this impression is lost in Figure 4a, the A and B type strokes connect mentally to convey the 3D form of the major convex forms in Figure 4a. The B-type strokes add to the height and not just the full body and 3D form of the plateau.

The B-type P-strokes in Figure 6 also look higher and seem tilted towards the viewer. Whereas the A type strokes in Figures 5 and 6 look very similar, the B strokes look distinctly different. When seen against the framework of the A-type strokes in Figure 4c, the B-type strokes (in Figure 6) look misplaced and incoherent. Also, they do not sketch the full extent of the plateau. Actually, the A-type strokes (in Figure 6) picked out by the Douglas-Peucker algorithm are also misplaced and this also adds to the lack of fit between the two types of strokes.

As observed earlier, the B-type strokes picked by the Douglas-Peucker algorithm form connected runs in Figures 3c and 4c. Where these lines are angled on the sketch, as at the summit of the plateau on the extreme right in Figure 4c, they bond reasonably well with the other strokes. However, where they occur in vertically aligned runs, as down the spur on the left, they seem out of place and appear to be detached from the surface and seem to float in front of the latter. The same effect can be noted with the vertical runs of cells at the front of the central plateau. The visual system is quite good at detecting vertical lines. Cartographers,

who are adept at visually separating out the superimposed colour layers on maps, can also instantly spot similar but shorter vertical runs in Figures 4b, and on 4a to a lesser extent, which also seem to float. As noted by Visvalingam and Dowson (1998), these appear to be located on spurious creases running along the original contours from which the DEM was created by interpolation. Dowson (1994) found that such artefacts were much more common and noticeable on DEMs produced by the US Geological Survey. Despite these artefacts and speckled random errors, Figures 4a and 4b enable the visual system to bind the four types of strokes into a coherent whole.

The trained eye sees Figure 4c as consisting of two parts, namely a) the scene (formed by A, C and D type strokes) sloping into the distance, and b) some scribbles (consisting largely of the B-type strokes), which appear to lie between the viewer and the surface. Most people can instantly tell (without having to look at the content) when OHP slides are placed upside down or the wrong way around. Sometimes, the outcome of perception is an uneasy feeling of something being amiss and the conscious mind is left to resolve the problem. Similarly, the immediate and subconscious projection of the P-strokes in Figure 4c onto two different regions of perceived space leaves the trained mind with a gut feeling that the sketch is 'not right'. Conscious investigation suggests that visual inconsistencies could have been detected at a subconscious level. A prominent example occurs at the front right of the central plateau, where a run of mislocated B-strokes, some in front of and others behind an occluding profile, is seen as one linear primitive. This prominent example is used below to speculate about the perception of the P-stroke sketch.

There are different theories about the nature of subconscious cognition; it is known to be complex and to involve sequential computation even if not deductive logic. The aim of the following speculations is to derive heuristics which may be used in the automatic evaluation of sketching techniques based on elements of perception; there is no attempt being made here to theorise about the nature of vision in general. We could speculate that the subconscious processing of P-strokes, in the prominent example, involves a train of spatial reasoning, as in:

1. *Grouping of runs of P-strokes to form linear primitives.* Gestalt laws of continuity appear to apply locally in such early detection. (The term, primitive, is used loosely here in preference to icon (as used by Nakayama, 1990), which is linked with a competing theory of vision to that proposed by Minsky and Papert, 1969).
2. *Recognition of some primitives.* The primitive formed by the A-type stroke is recognised as a shoulder on the plateau, even if it is not labelled as such. Such recognition perhaps involves the matching of the shape of convex parts (Hoffman and Richards, 1982) against generic knowledge of landforms. At this initial stage the other primitive, emerging from the run of B-strokes, may remain unclassified since it does not make immediate sense.

3. *Deduction of relative positions of primitives in the represented space.* Since the lower part of this unclassified primitive is not occluded, the lower part cannot lie beyond the occluding shoulder.
4. *Other consequences.* If the lower part of this primitive lies in front of the shoulder, then the rest of this primitive must also lie in front of the shoulder.
5. *Deductions about the pose of the tokens.* If the bottom of the primitive lies on the land surface, then the primitive must be tilted towards the viewer to be in front of the shoulder. Or else, the primitive must be detached and lie between the viewer and the surface, appearing to float.
6. *Optimising the deductions.* It is assumed that the primitive is floating since effort is required to establish whether the bottom of the primitive lies on the surface.

The primitive is thus *perceived* to be in front of the surface although we *know* that all marks lie on the terrain surface. Conscious effort is needed to offer an alternative explanation which requires splitting the primitive into 2 parts; this involves local fixation and consideration rather than holistic instantaneous perception. This is similar to the example presented by Gestalt psychologists (reported in Hochberg, 1964, p 87) where a wireframe drawing of a cube, when seen corner to corner along its diagonal, is perceived as a flat triangulated hexagon. This is because conscious effort is needed to break the continuity of lines to perceive the tridimensional cube. The rejection of the inconsistent token from the mental model of the land surface, makes them float in front of the scene like dirt and raindrops on a window. Like the latter, they might be ignored by those intent on the scene, who see the sketches as nearly identical. But the information is there, and may be studied by adjusting the focus. As with the perception of random dot stereograms and of ambiguous figures, this requires the development of latent mental skills. Cartographers have tended to think in terms of colour separations and geographers and geologists are trained to focus on different colour planes during map analysis and interpretation. This might explain the range of conflicting anecdotal comments.

This suggests that holistic (global) perception may be the outcome of a weighted integration of the outcomes of local computation. The controversy over whether vision arises as a result of the accumulation of simultaneous local computations or as a result of a sequence of subconscious attentional fixations is outside the scope of this paper. Similarly, controversies relating to the nature of cerebral computation are not the main concerns of this paper. Since, subconscious feelings of unease (albeit subjective) are entirely image based, it may be possible to train machines to classify (score) images and thus the underpinning algorithms. However, expert 'guides' are needed to train machines and the main problem here is that their judgement is very subjective in the case of P-stroke sketches. Also, studies of this kind are concerned not just with classification but also with explanations and their implications. So, the

computational approach of deductive reasoning based on explicit premises, conjectures and inputs is suggested here.

The computation in stages 2 to 6 assumes constraints derived from generic environmental knowledge, which override facts (arising from the process used to generate the sketch) about the given image. The local outcomes appear to re-inforce each other since there are other locations in Figure 4c where similar conditions apply, e.g. in the front left of the central plateau. The mental collation of such distributed conclusions sometimes leads to a re-classification of even non-intersecting tokens, such as at the distant centre of the central plateau, as also detached from the surface. The first author's perception, of the P-stroke sketch, seems to involve :

1. **local operators** which rely principally on two types of input, namely a) external input consisting of the *marks on the image*, and b) innate or acquired generic knowledge, including constraints and rules about spatial relationships.
2. **a predisposition to reject known facts** when they contradict the two types of input noted above. The case of the varying size of the moon when near the horizon and at zenith has already been cited widely to demonstrate this. Artists tend to use a range of techniques to take account of such failures and distortions in perception (Gombrich, 1960). Indeed, the processes of cartographic generalisation are designed to ensure that clutter does not encourage the emergence of artefacts.
3. **dynamic changes in the perceived location of tokens in 3D space**, depending on the field and focus of perception. If we fixate locally on them, some of the freestanding tokens (e.g. in the far centre of the central plateau) appear to lie on the terrain surface. Yet, they appear to float in a holistic view. However, this double vision does not occur even when we vary our field of vision in the case of a) the A-type strokes and b) the intersecting tokens cited above possibly because the local output are more strongly weighted. Gestalt psychologists have demonstrated that the depth effect, for example, is more acute when examples are placed in a scene with other objects with depth cues, than when they are presented in isolation (Hochberg, 1964).
4. **labelling**; this varies with the local and global view as noted above. Weighted global vision constructs the perceived terrain from the form-related (e.g. shoulder, spur or peak) and view-related (e.g. silhouette) tokens. Other primitives which do not make sense are seen as scribbles, dirt etc. The author herself has one occasion attempted to wipe a polyline off the screen thinking it to be hair!

It is possible to note other B-type tokens which do not have good configurational properties but it is not the aim of this paper to catalogue these. Instead, its aim is to note that conjectures, such as these, could provide heuristics for the automatic evaluation of generalisation algorithms, which is discussed next.

5.2 *Implications for future research*

Visvalingam and Whyatt (1990) pointed out that the results from widely cited evaluative studies, undertaken by some others, were inconclusive since the 2D test data they used were inappropriate and/or inexacting. With 1D profiles, this study indicates that isolated lines may also be insufficient for comparing algorithms. Indeed, manual generalisation of lines on maps is guided by the configuration of lines on source and derived images which indicate their semantic relationships. For example, rivers and contours are generalised with respect to each other to ensure that rivers always flow down the valley as depicted by contours. Sketches of the landscape, similarly, provide contextual information for studying the performance of line and DEM filtering algorithms.

It looks as if Visvalingam's method of iterative elimination of redundant points can be used to sketch a better configuration of lines than the extreme points selected by the Douglas-Peucker method. Even with one-dimensional lines, Figure 4 suggests that the area metric appears to be more useful than the distance measures. This has some implications for other studies focusing on the problem of levels of detail. For example, one approach to extracting a TIN from a DEM consists of a 3D extension of the Douglas-Peucker method, where the point which lies furthest from a triangular plane is selected on each iteration. This study suggests that this approach is likely to facet the landscape unduly and generate artefacts.

Researchers are divided about the visual and mathematical approaches used for evaluation of algorithms for generalisation of DEMs and 2D lines. Most evaluations still rely on visual judgement despite known reservations, including variations in people's perceptual and processing equipment, their experience and deep knowledge, their awareness of object knowledge (for comparison) and their understanding of the purpose of the exercise. As noted earlier, the type of visualisation (Figures 2 - 4) also influences conclusions. As a result, there is continuing interest in the formulation of more objective methods for comparison.

Fowler and Little (1979) measured the statistical goodness of fit of derived Triangulated Irregular Networks (TIN) to the original grid DEM. In the 2D context, McMaster (1987) also attempted to replace subjective visual analysis with mathematical measures of performance, such as areal and vector displacement of the generalised line from the source line. He concluded that the Douglas-Peucker algorithm was perceptually and mathematically superior to other algorithms he tested. Visvalingam and Whyatt (1990) demonstrated that his metrics and conclusions were only valid in the case of line approximation since cartographers deliberately omit entire features in caricatural levels of typification and generalisation. However, despite the acknowledged role of visualisation in scientific discovery, scientists generally tend to regard visual evaluation as subjective and unreliable (see, for example,

Jorgenson et al,1995). There is thus continuing research on mathematical evaluation of techniques for polygonal approximation of curves (Rosin, 1997).

Returning to the immediate problem of the evaluation of algorithms for sketching, perhaps there is a third way. While perception of a sketch may be an innate neural process as suggested by some, visual evaluation is a semi-conscious active process which engages other knowledge which perception seems to ignore. Automatic evaluation, based on explicit conjectures about the types of spatial reasoning involved, offers some scope for removing the subjective elements in human perception. Even negative results will serve to refine the conjectures about pertinent perceptual criteria. From the foregoing conjectures, it is possible to formulate one line of evaluation as follows:

1. Use the Gestalt law of continuance to group the A and B type P-strokes in the images (Figures 5 and 6) into tokens.
2. Assign a strong weight to B-type tokens which intersect the A-type ones; this can be ascertained by reference to the original DEM.
3. Add the weighted lengths of the B tokens to derive a total float score for each image. This should be standardised for the region being fixated on to facilitate comparisons (see 6)
4. Use the totals to rank the images generated by using different techniques.
5. Different scale-related totals may be calculated for sub-regions of the image, to compare the changes in perception during local and global attention.

The non-intersecting vertical runs of P-strokes could also be compared numerically. Their lengths will also need to be weighted since the visual impact of longer lines may be disproportional to their lengths; such non-uniform scaling is well known in psychophysics and in cartography. If free standing, short vertical runs of P-strokes have a lower weighting, the local part of the image fixated upon will have a low total float score, providing a heuristic for re-labelling if necessary. Even some of the strokes in Figure 4a appear to float. Some of these are located on spurious speckled errors in the DEM while others appear to pick out contours, highlighting deficiencies in interpolation of the heightfield. The sketch therefore provides a means of validating and cleaning DEMs. Although one gets the impression that there are many floating marks, the mind tends to focus on the obvious cases. It is difficult to consciously search and analyse the subconscious holistic perception, since this changes with local fixation. While the filtered strokes will not float (in much the same way that the B type strokes in Figures 5 and 6 do not), the results can be scrutinised. This approach to perceptual evaluation could also be applied to the detection of artefacts in simulated DEMs.



6. Conclusion

The research initially set out to explore whether the technique of P-stroke sketching, described by Visvalingam and Dowson (1998), could be improved upon by using distance-based filtering techniques. The latter were initially thought to be more appropriate for the one-dimensional variations in DEM profiles, especially since distance measures are widely used in simplification and simulation of terrain. It was difficult to draw definitive conclusions about the line generalisation algorithms using isolated lines. The P-stroke sketches, which are essentially a set of generalised profile parts, were much more useful in drawing attention to differences between the output of algorithms.

The results showed that all three methods were quite successful at portraying convex forms running across the line of sight. These are the dominant strokes in a sketch and as such many people saw the output of the three methods as very similar. The convex breaks of slope, elongated parallel to the line of sight, proved to be the touchstone for comparing the methods.

Despite the unintelligent rules for computer sketching and the interpolation artefacts in data, the sketches produced using Visvalingam's line filtering algorithm are relatively pleasing and provide a basis for future research on sketching. The P-stroke sketches produced using Visvalingam's algorithm and the area metric demonstrate that it is possible to automate the production of sketches, although this remains a research challenge. The sketching rules are still somewhat simplistic and do not take full advantage of the scale-dependent structural information provided by Visvalingam's algorithm. Current research (by John Whelan) is focusing on the use of the scale-related shape information provided by Visvalingam's algorithm. There are also opportunities for experimenting with different sketching styles and symbolism.

The use of the Douglas-Peucker algorithm resulted in sketches, which were perceived by some people (including the authors) to be less successful. One of the problems in studies of this kind is the subjective nature of visual evaluations. These less successful sketches were interesting because they served to demonstrate Helmholtz's thesis (see Hochberg, 1964; Gregory, 1970) that perceiving involves subconscious thinking. Knowledge of the spatial structure of landforms makes inconsistent tokens float in the foreground. The brain is known to ignore information which does not conflate with the preferred hypothesis for organising the recognised tokens into meaningful elements. Gestalt psychologists have used reversing figures (such as Rubin's vases) to demonstrate the brain's tendency to see the figure and suppress the background. In much the same way that the background is an area of mental space for placing information deemed to be less important, this paper speculates that the foreground (in front of the figure) may also be used as scrap space for inconsistent information, which may be ignored, perceived as incoherent scribbles or studied. It is well

known that major scientific breakthroughs were initiated by those experts (such as Fleming) who saw the significance of information which was being mentally ignored by others. This suggests that anomalies may be initially at least placed in the foreground (rather than background) to arrest attention. This, rather than detailed deductive logic, may be responsible for the unclassified tokens being shifted into the mental foreground. However, conjectures about subconscious spatial reasoning help us identify inconsistencies in the location of curvatures, which manifest themselves as lacking good form. This seems to offer some scope for training machines to assess the clarity of a sketch, as an alternative to objective, but often inappropriate, mathematical evaluations on the one hand, and more discerning but subjective and ad-hoc human judgement on the other.

The method of P-stroke sketching and inclusion of occluding lines as in Figure 1b has already taught some students of Digital Cartography (who claimed that they could not previously draw) to analyse the landscape and draw aesthetically pleasing sketches of their favourite places. Hopefully, this strand of research will teach us to 'see' landscapes and drawings differently and that this will stimulate interest in automating the forgotten art forms of cartography and in breeding intelligent systems capable of reasoning and debating about the relative authenticity and aesthetic quality of computer-drawn sketches.

ACKNOWLEDGEMENTS

This study would not have been possible without the award of an EPSRC CASE studentship to Kurt Dowson and the additional sponsorship received from the collaborating body involved in this CASE project, namely Scott Wilson Kirkpatrick and Partners, a UK-based consulting firm of civil engineers. We are particularly grateful to Steve Vincent, the industrial supervisor for setting up this project and for his support and encouragement. We are also grateful to the Ordnance Survey of Great Britain for permission to use their DEM in this research. Our special thanks go to present and past research students within the Cartographic Information Systems Research Group for their various contributions to this research programme; Phil Wade for permission to use his implementation of the Douglas-Peucker algorithm; Duncan Whyatt and Peter Williamson for evaluation of line generalisation algorithms; Chris Brown, John Whelan and Simon Herbert, for ensuring that software developed in-house has remained accessible and usable despite changes in computing systems; John Whelan and Simon Herbert for their assistance with the preparation of the figures and for their comments on this paper; and, John Whelan again, for permission to re-use Figure 1b.

REFERENCES CITED

- Burrough P A (1986) **Principles of Geographical Information Systems for Land Resource Assessment**, Oxford, 193pp.
- Dickinson, G C (1969) **Maps and Air Photographs**, Edward Arnold Ltd., London, 286pp
- Douglas D and Peucker T (1973) Algorithms for the reduction of the number of points required to represent a digital line or its caricature, **Canadian Cartographer** 10, 112 - 22
- Dowson, K (1994) Towards Extracting Artistic Sketches and Maps from Digital Elevation Models, Unpublished PhD Thesis, University of Hull, 203 pp.
- Dutton, G H (1999) **A Hierarchical Coordinate System for Geoprocessing and Cartography**, Lecture Notes in Earth Sciences, Springer-Verlag, Berlin.
- Fowler, R J and Little, J J (1979) Automatic extraction of Irregular network Digital Terrain Models, **Computer Graphics : A Quarterly Report of SIGGRAPH-ACM**, 13 (3), 199 - 207
- Gombrich, E H J (1960) **Art and Illusion : A Study in the Psychology of Pictorial Representation**, Pantheon Books, 466pp.
- Gregory , R L (1970) **The Intelligent Eye**, Weidenfield & Nicolson, London
- Hochberg J E (1964) **Perception**, Prentice-Hall, New Jersey
- Hoffman D D and Richards W A (1982) Parts for recognition, **Cognition** 18, 65 - 96.
- Imhof, E (1982) **Cartographic Relief Presentation**, Steward, H. J. (ed.) Walter de Gruyter, 389pp.
- João E M (1998) **Causes and Consequences of Map Generalisation**, Taylor & Francis, London.
- Jorgenson, L, Kritz R, Mones-Hattal, B Rogowitz, B and Fraccia, F D (1995) Is Visualization Struggling under the Myth of Objectivity? **Proc of IEEE Visualization '95**, 412 - 415.
<http://www.cecm.sfu.ca/projects/PhilVisMath/panel95/>
- Lobeck, A K (1924) **Block Diagrams and Other Graphic Methods used In Geology and Geography**, John Wiley and Sons, 206pp.
- McMaster R B (1987) Automated line generalisation, **Cartographica** 24 (2) 74 - 111
- Minsky M and Papert S (1969) **Perceptrons**, MIT Press, Cambridge, Mass.
- Nakayama, K (1990) The iconic bottleneck and the tenuous link between early visual processing and perception, in Blakemore, C (ed.) **Vision : coding and efficiency**, Cambridge University Press, Cambridge, 411 - 422.

- Raisz, E (1938) **General Cartography**, McGraw-Hill, New York, 370pp
- Ramer, U (1972) An iterative procedure for polygonal approximation of planar closed curves, **Computer Graphics and Image Processing 1**, 244 - 256
- Robinson, A H and Thrower, N J W (1957) A new method of terrain representation', **Geographical Review, 47** (4), 507 - 520
- Rosin, P L (1997) Techniques for assessing polygonal approximations of curves, **IEEE Trans on Pattern Analysis and Machine Intelligence 19** (6), 659 - 666.
- Tanaka, K (1932) The orthographical relief method of representing hill features on a topographical map', **Geographical Journal, 79** (3), 214 - 219
- Visvalingam, M and Dowson, K (1998) Algorithms for sketching surfaces, **Computers & Graphics 22** (3) 269 - 280.
- Visvalingam, M and Whelan, J C (1998) Occluding contours within artistic sketches of surfaces, **Proceedings of Eurographics UK '98**, 281 - 289.
- Visvalingam M and Whyatt J D (1993) Line generalisation by repeated elimination of points, **Cartographic J 30** (1) 46 - 51.
- Visvalingam M and Williamson P J (1995) Simplification and generalisation of large-scale data for roads: a comparison of two filtering algorithms, **Cartography and GIS 22** (4) 264 - 275.
- Whyatt, J D (1991) Visualisation and Re-evaluation of Line Simplification Algorithms, Unpublished Ph D Thesis, University of Hull, 259 pp.
- Whyatt J D and Wade P (1988) The Douglas-Peucker line simplification algorithm, **Bull of the Soc of University Cartographers 22** (1) 17 - 25.