

Constraining SN Ia Progenitors from the Observed Fe-peak Elemental Abundances in the Milky Way Dwarf Galaxy Satellites

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ABSTRACT

Chemical abundances of iron-peak elements in the red giants of ultra-faint dwarf galaxies (UFD) and dwarf spheroidal galaxies (dSph) are among the best diagnostics in the cosmos to probe the origin of Type Ia Supernovae (SNe Ia). We incorporate metallicity-dependent SN Ia nucleosynthesis models for different progenitor masses in our inhomogeneous galactic chemical evolution model, *i-GETOOL*, to recreate the observed elemental abundance patterns and their spread in a sample of UFD and dSph galaxies with different average metallicities and star formation histories. Observations across different environments indicate that $[\text{Mn}/\text{Mg}]$ increases on average with metallicity while $[\text{Ni}/\text{Mg}]$ remains nearly constant. Chemical evolution models assuming SN Ia progenitors with Chandrasekhar mass (M_{Ch}) produce similar to identical $[\text{Mn}/\text{Mg}]-[\text{Fe}/\text{H}]$ and $[\text{Ni}/\text{Mg}]-[\text{Fe}/\text{H}]$ patterns to those observed in the examined UFD and dSph galaxies, without needing to invoke a substantial fraction of sub- M_{Ch} progenitors that changes across the different environments, as claimed by some previous chemical evolution studies. We note though that the observed UFD sample is still statistically poor to draw firm conclusions. Sub- M_{Ch} progenitors in our dSph models systematically under produce both $[\text{Mn}/\text{Mg}]$ and $[\text{Ni}/\text{Mg}]$, with the $1 M_{\odot}$ model explaining a number of outliers in $[\text{Ni}/\text{Fe}]$, while the outliers in $[\text{Mn}/\text{Mg}]$ require higher sub- M_{Ch} progenitor masses. The average dispersion of $[\text{X}/\text{Mg}]$ from our UFD model ranges between 0.20 and 0.25 for iron-peak elements, with the exception of $[\text{Sc}/\text{Mg}]$ that has $\sigma \approx 0.39$.

Key words: stars: abundances – galaxies: abundances – galaxies: evolution – galaxies: dwarf – Local Group.

1 INTRODUCTION

Type Ia Supernovae (SNe Ia) are among the most dominant events contributing to galaxies’ chemical evolution. Such events are the primary contributors to the origin of iron-peak elements in the Universe (Tinsley 1979; Matteucci & Tornambe 1985; Matteucci & Greggio 1986). According to the main proposed scenarios, they occur through the thermonuclear explosion of an electron-degenerate carbon-oxygen (CO) white dwarf (WD), which interacts with a companion star in a binary system until the WD attains central densities $\rho_c \approx 2 \times 10^9 \text{ g cm}^{-3}$ and temperatures $T_c \approx 2 \times 10^8 \text{ K}$, corresponding to masses close to the Chandrasekhar mass (M_{Ch}). At those densities and temperatures, carbon can be ignited in the electron-degenerate core from which nuclear runaway reactions occur and the WD eventually explodes (e.g., see Arnett 1996; Rauscher 2020).

Two main scenarios have been postulated to achieve ignition condition, either via the accretion from a binary star companion onto the WD by Roche lobe overflow (single degenerate scenario; see Whelan & Iben 1973; Nomoto et al. 1984; Thielemann et al. 1986; Hachisu et al. 2012) or via the merger between two CO WDs (double degenerate scenario; see Iben & Tutukov 1984; Webbink 1984). It has been proposed that either channel can give rise to a double deto-

nation of the WD at sub-Chandrasekhar masses (sub- M_{Ch}) (Woosley & Weaver 1994); in particular, the WD can accrete He-rich material from the companion until a detonation takes place in the He shell, which causes the development of shock waves that propagate through the WD and reach the core, where a secondary detonation occurs as carbon burning ignites (Fink et al. 2007, 2010; Woosley & Kasen 2011; Flörs et al. 2020; Gronow et al. 2021).

The elemental abundance templates that result from the thermonuclear explosion of a SN Ia are expected to depend on the birth metallicity of the WD progenitor in the binary system (e.g., see Höflich et al. 1998; Timmes et al. 2003). In particular, the amounts of C, N and O at birth in the WD progenitor regulate how much ^{14}N is synthesized in the CNO cycle. ^{14}N is then used during He-burning to produce ^{22}Ne from a series of α -captures interlaced with a β -decay. The amount of ^{22}Ne in the CO core of the WD progenitor is critical because it determines the neutron excess, which is among the key factors in regulating the abundances of the synthesized nuclides under nuclear statistical equilibrium (e.g., see Hartmann et al. 1985). Since the amount of ^{22}Ne in the WD depends on the C, N, and O abundances at birth in the star, different birth metallicities give rise to different SN Ia nucleosynthetic yields (e.g., see Howell et al. 2009; Townsley et al. 2009). The elemental abundance template from a SN Ia explosion also depends on the mass and density of the WD,

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with Mn and Ni being among the chemical elements with the largest changes (e.g., see [Seitenzahl et al. 2013](#); [Lach et al. 2020](#)).

In this work, we aim to investigate the effects of metallicity-dependent M_{ch} and sub- M_{ch} SN Ia models on the chemical evolution of a sample of ultra-faint dwarf galaxies (UFD) and dwarf spheroidal galaxies (dSph) that are satellites of the Milky Way (MW), by making use of `i-GEtool`, an inhomogeneous chemical evolution model ([Alexander et al. 2023](#)). An interesting work in the past that made use of chemical evolution models to constrain SN Ia progenitors from the observed chemical abundance patterns is [Seitenzahl et al. \(2013\)](#), who tested different SN Ia models to explain the observed $[\text{Mn}/\text{Fe}]$ in the MW, concluding that ≈ 50 per cent of near- M_{ch} SN Ia progenitors are required to reproduce observations. [McWilliam et al. \(2018\)](#) explored different SN Ia progenitors to explain the observed elemental abundances of a star in the dSph galaxy Ursa Minor, finding that the observed abundances in the star indicate a diluted chemical enrichment from a single sub- M_{ch} SN Ia progenitor. Similarly, by investigating the chemical evolution of dSph galaxies with different star formation histories (SFHs), [Kirby et al. \(2019\)](#) found that the observed $[\text{Ni}/\text{Fe}]$ in ancient and short-lived dSphs are compatible with a pure chemical enrichment of only sub- M_{ch} SN Ia progenitors, while the observed $[\text{Ni}/\text{Fe}]$ in the MW and dSphs with more extended SFHs indicate that the chemical enrichment was contributed by different classes of SNe Ia; in this respect, [Eitner et al. \(2020\)](#) found that the observed abundances of $[\text{Mn}/\text{Fe}]$ in the MW are reproduced by assuming ≈ 75 per cent of sub- M_{ch} SN Ia progenitors, with the remaining fraction stemming from M_{ch} SNe Ia.

An interesting work is that of [Kobayashi et al. \(2020\)](#), who incorporated several metallicity-dependent SN Ia models into detailed chemical evolution models for the MW and its dSph satellites, finding that the observed $[\text{Ni}/\text{Fe}]$ and $[\text{Mn}/\text{Fe}]$ in the Solar neighbourhood require up to 25 per cent of sub- M_{ch} SNe Ia progenitors, with a higher percentage being present for dSph galaxies. A similar investigation is presented in [Gronow et al. \(2021\)](#), who found that sub- M_{ch} SNe Ia along with core-collapse Supernovae (CCSNe) account for more than 80 per cent of solar Mn when considering [Limongi & Chieffi \(2018\)](#) massive star yields, in line with the conclusions drawn also by [Palla \(2021\)](#), who performed one of the most systematic investigations on the effect of different SN Ia nucleosynthesis models to explain the observed chemical abundance patterns of MW stars.

The nucleosynthesis origin of the observed iron-peak elemental abundances in the MW and its main galaxy satellites in the Local Group was also investigated by [Sanders et al. \(2021\)](#), who concluded that a significant fraction of sub- M_{ch} SN Ia progenitors is required to explain the abundances in dSph galaxies. Recently, [de los Reyes et al. \(2022\)](#) aimed to constrain the SFH of the dSph galaxy Sculptor, developing detailed chemical evolution models to reproduce the observed abundance patterns of several chemical elements, including iron-peak and s -process elements. From their best fit model, the observed $[\text{Ni}/\text{Fe}]$ and $[\text{Mn}/\text{Fe}]$ in Sculptor are better reproduced by sub- M_{ch} SNe Ia models.

Our work is organised as follows. Section 2 provides a detailed description of the metallicity-dependent SNe Ia yields we use in our work. The chemical evolution models are presented in Section 3, where we discuss some of the physical parameters of the study. Section 4 summarises our observational samples with a full recollection of the data. We present our results in Section 5, where we discuss the predicted chemical abundance ratios when varying metallicity-dependent yields. Lastly, we show the conclusions of our work in Section 6.

2 TYPE IA SUPERNOVA NUCLEOSYNTHETIC YIELDS

In this section, we present the SN Ia models that are adopted in our chemical evolution calculations with `i-GEtool`. The section is split into two parts. The first part describes the SN Ia models of [Kobayashi et al. \(2020\)](#), while the second part reviews those of [Keegans et al. \(2023\)](#); both SN Ia yields are compared with the W7 SN Ia yields of [Iwamoto et al. \(1999\)](#), widely assumed by chemical evolution calculations in the literature ([Kobayashi et al. 2006](#); [Romano et al. 2010](#); [Matteucci 2012](#); [Prantzos et al. 2018](#); [Womack et al. 2023](#)).

2.1 Type Ia Supernova models of Kobayashi et al. (2020)

[Kobayashi et al. \(2020\)](#) developed SN Ia models for a variety of WD progenitor masses and metallicities by using the 2D hydrodynamical code of [Leung et al. \(2015\)](#). To have a larger number of isotopes with associated yields for their chemical evolution calculations, [Kobayashi et al. \(2020\)](#) post-processed the nucleosynthesis with a tracer particle scheme ([Travaglio et al. 2004](#)), using the so-called *Torch* nuclear reaction network ([Timmes 1999](#)). SN Ia models and yields were calculated for WD progenitor masses $M_{\text{WD}} = 0.9, 1.0, 1.1, 1.2, 1.3, 1.33, 1.37, \text{ and } 1.38 M_{\odot}$ at the following metallicities: $Z = 0, 0.002, 0.01, 0.02, 0.04, 0.06 \text{ and } 0.1$. Here, we adopt the two benchmark models of [Kobayashi et al. \(2020\)](#), that have WD progenitor masses $M_{\text{WD}} = 1.0 M_{\odot}$ (sub- M_{ch} double detonation model) and $M_{\text{WD}} = 1.38 M_{\odot}$ (near- M_{ch} deflagration detonation model), used also in their chemical evolution calculations for comparison against observational data in the MW and dSph galaxies.

In Fig. 1, we show how the total yields of Mn, Fe, Ni, and Mg change as a function of metallicity according to the benchmark models of [Kobayashi et al. \(2020\)](#). We also include the W7 SN Ia yields of [Iwamoto et al. \(1999\)](#) for each of the elements as a baseline (horizontal black lines), which are constant as a function of metallicity. As our current work focuses on Fe-peak elements, we choose to analyse Ni, Mn and Fe as they are predominantly created in SNe Ia events. We also include Mg as it is an α -element mainly produced by CCSNe, typically used as a ‘chemical clock’ when its abundance is analysed relative to an iron-peak element in chemical evolution studies ([Matteucci & Greggio 1986](#)). From the figure, higher SN Ia progenitor masses contribute more iron-peak elements to the ISM of a galaxy. Interestingly enough, the production of Ni is higher than Mn for all considered progenitor masses, including the W7 [Iwamoto et al. \(1999\)](#) yields. Finally, for sub-solar metallicities, the SN Ia yields of [Kobayashi et al. \(2020\)](#) exhibit a weak dependence on metallicity. Some change with metallicity starts appearing at super-solar metallicities, specially for Mn, which increases as Z also increases.

2.2 Type Ia Supernova models of Keegans et al. (2023)

[Keegans et al. \(2023\)](#) post-processed the $0.8 M_{\odot}$ double detonation model of [Miles et al. \(2019\)](#), the $1.0 M_{\odot}$ double detonation model of [Shen et al. \(2018\)](#) and the $1.4 M_{\odot}$ mass deflagration detonation model of [Townsend et al. \(2016\)](#). For each progenitor mass, [Keegans et al. \(2023\)](#) considered the following metallicities: $Z = 0, 10^{-7}, 10^{-6}, 10^{-5}, 0.0001, 0.001, 0.002, 0.005, 0.01, 0.014, 0.02, 0.05 \text{ and } 0.1$, developing a total number of 39 unique SN Ia models. The initial abundance of metals of the WD for each of these models is based on a uniform distribution of the mass fraction of ^{22}Ne .

Nucleosynthesis is post-processed by using the so-called *Tracer Particle Post-Processing Network-Parallel* (‘`tpnp`’ for short) of the NuGrid collaboration ([Pignatari et al. 2016](#); [Ritter et al. 2018](#); [Jones et al. 2019](#)). The adopted nuclear reaction network allowed [Keegans](#)

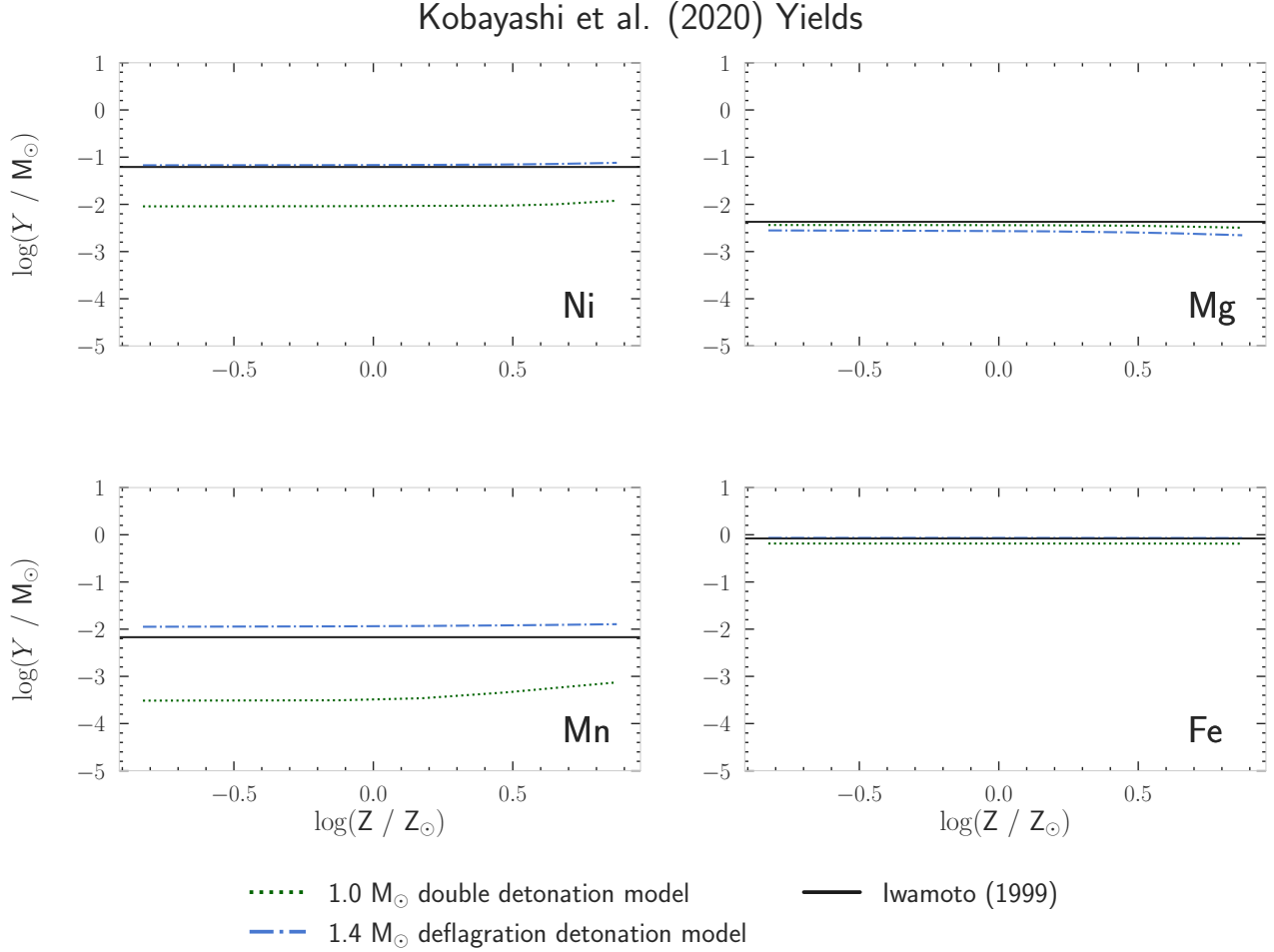


Figure 1. The SN Ia nucleosynthesis yields of Kobayashi et al. (2020) for Ni (upper left), Mg (upper right), Mn (bottom left) and Fe (bottom right) a function of metallicity. Various progenitors are shown through colours where the green dotted and the blue dot-dashed lines represent WD progenitors with masses 1.0 M_{\odot} and 1.4 M_{\odot} , respectively. The black solid line corresponds to the W7 non-metallicity dependent SN Ia yields of Iwamoto et al. (1999).

et al. (2023) to cover over 5,000 isotopes and over 70,000 reactions whereas the works of Townsley et al. (2016), Shen et al. (2018) and Miles et al. (2019) are more restricted in their reaction networks. The CO WD is initialised to be as close to the chemical make-up as seen in the original models, varying the ^{22}Ne mass fraction. One key difference between the Keegans et al. (2023) yields and the original is the under production of some of the heavy-iron group elements such as Cu, Ga and Ge. The majority of the other elements however are very similar to each other, except for N and Mg that have a higher production in Keegans et al. (2023).

Fig. 2 shows the total yields of Mn, Fe, Ni, and Mg as predicted by the 0.8 M_{\odot} (red), 1.0 M_{\odot} (green) and 1.4 M_{\odot} (blue) SNe Ia models of Keegans et al. (2023) as a function of metallicity. Like in Fig. 1, we also show the W7 yields of Iwamoto et al. (1999) as a baseline. The behaviour of the Keegans et al. (2023) yields as a function of the SN Ia progenitor mass is similar to that of Kobayashi et al. (2020), as higher progenitor masses are predicted to produce a larger amount of iron-peak elements. At very low metallicities ($[\text{Fe}/\text{H}] \lesssim -2$), the SN Ia models of Keegans et al. (2023) predict nearly constant yields as a function of Z , and the 1.4 M_{\odot} model predicts similar iron-peak elemental yields as the W7 Iwamoto et al. (1999) model. When the

metallicity passes a certain threshold ($[\text{Fe}/\text{H}] \gtrsim -2$), all progenitor masses experience an increase in the production of iron-peak elements, with the 1.0 M_{\odot} model exhibiting a stronger metallicity-dependence than the 1.4 M_{\odot} model.

Massive stars dying as CCSNe also provide some contribution to iron-peak elements, even though they mainly contribute to α -elements such as Mg. Fig. 3 details the total contribution of Mg (red), Mn (green), Fe (blue) and Ni (black) from a population of massive stars formed out of a stellar population with total stellar mass $M_{\star} = 10^4 M_{\odot}$ and varying metallicity, by assuming the Kobayashi et al. (2006) massive star yields and the IMF of Kroupa (2001) (left y-axis, filled bars), the same assumed in our calculations with i-GEtool. The filled bars in the figure are computed by varying the upper mass-limit for the chemical enrichment of massive stars from 25 M_{\odot} (lower edge of the bars) to 40 M_{\odot} (upper edge of the bars). We choose a total stellar mass of $M_{\star} = 10^4 M_{\odot}$ for the stellar population as it is similar to the predicted mass of a UFD galaxy. In the same figure, we also show the total yields from the 1.4 M_{\odot} model of Keegans et al. (2023), that correspond to the predicted chemical enrichment from a single SN Ia event (right y-axis, dotted lines).

It is clear from Fig. 3 that a single SN Ia event provides comparable

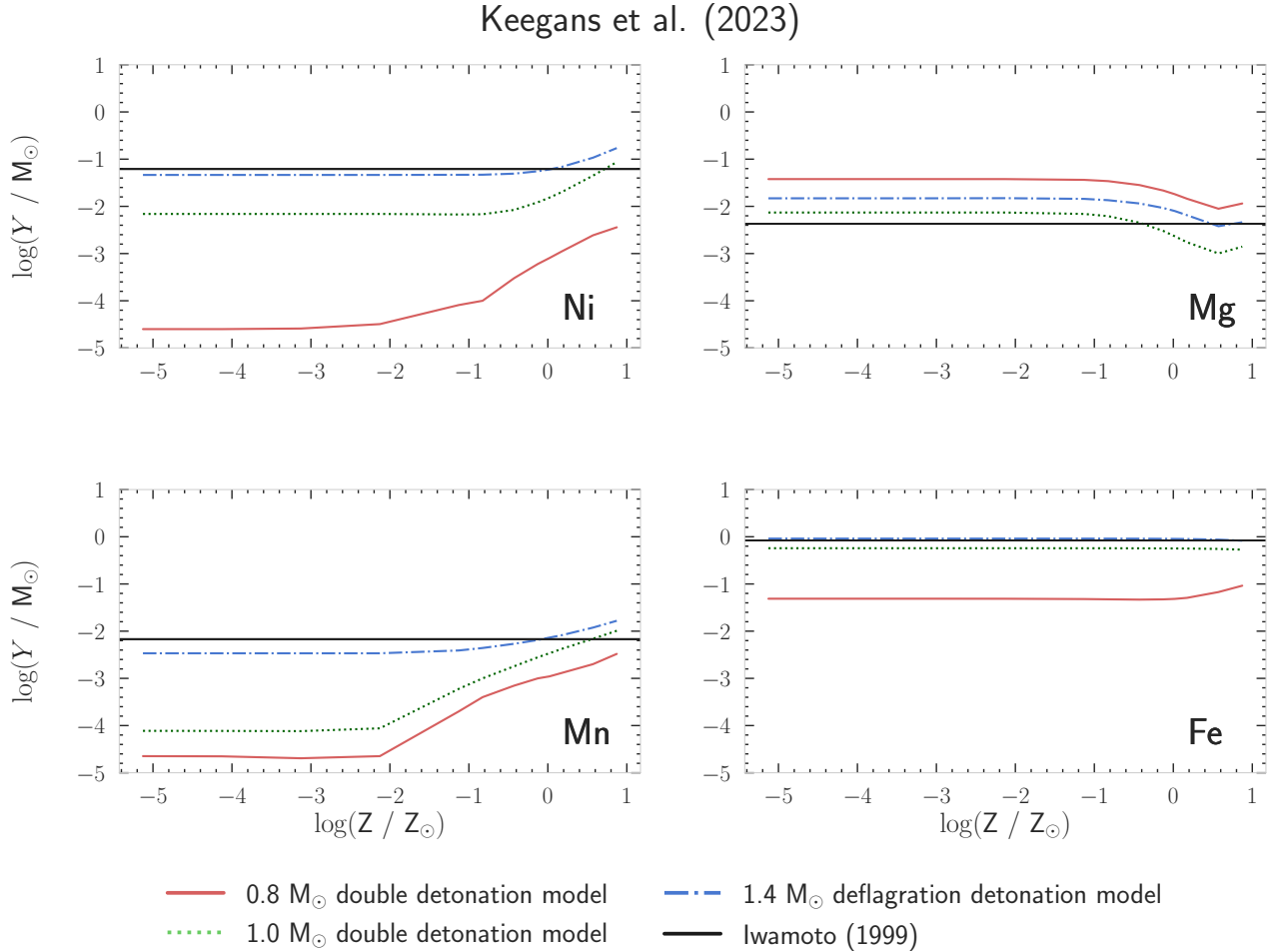


Figure 2. Same as Fig. 1, but for the SN Ia nucleosynthesis yields of Keegans et al. (2023). Red solid, green dotted and blue dot-dashed lines represent WD progenitors with masses $0.8 M_{\odot}$, $1.0 M_{\odot}$ and $1.4 M_{\odot}$, respectively. The black solid line corresponds to the W7 non-metallicity dependent SN Ia yields of Iwamoto et al. (1999).

amounts of iron-peak elements to those from massive stars in a $10^4 M_{\odot}$ stellar population. This highlights the significance of SNe Ia in the chemical enrichment and contamination of iron-peak elements in the ISM. We cannot ignore the differences between the nucleosynthesis yields of Mg from a single SN Ia and the CCSNe from a stellar population with stellar mass $M_{\star} = 10^4 M_{\odot}$ as the latter outweigh the former by ≈ 2.5 orders of magnitude.

3 INHOMOGENEOUS CHEMICAL EVOLUTION MODELS

i – GETool is an inhomogeneous galactic chemical evolution tool which tracks the elemental abundances of stars with different masses, metallicities and ages along with the chemical makeup of the local ISM of galaxies as a function of time (see Alexander et al. 2023 for more details). We use **i – GETool** to investigate how the assumption of metallicity-dependent SN Ia yields affects the chemical evolution of UFDs and dSphs. We create one UFD and two dSph toy models to cover different SFHs and metallicity ranges, from $-4 \lesssim [\text{Fe}/\text{H}] \lesssim -2$ of UFDs to $[\text{Fe}/\text{H}] \gtrsim -2$ of dSphs. One dSph model aims to reproduce the chemical abundances of Sculptor-like dwarf galaxies with a short

but intense SFH (henceforth, *dSph A*) whereas the other is set up to have a long but shallow SFH (henceforth, *dSph B*), similar to the inferred SFH of Leo I (e.g., see Weisz et al. 2014).

Initial setup — The simulation takes place within the constraint of a three-dimensional box where we place primordial gas of H and He with abundances $X = 0.74$ and $Y = 0.26$ in mass, respectively. These ratios are chosen to reproduce the conditions of the early Universe from the epoch of reionization. The box is divided into 40^3 cells, with sides 20 pc for the UFD model and 40 pc for *dSph A* and *dSph B*. The models assume the following initial gas mass density, $\rho_{\text{gas}}(r, t = 0)$, as a function of radius, r :

$$\rho_{\text{gas}}(r, t = 0) = \rho_0 b^{-\frac{r}{200 \text{ pc}}}, \quad (1)$$

where $\rho_0 = 55 M_{\odot} \text{ pc}^{-3}$ and $b = 85$, giving rise to a total initial gas mass $M_{\text{gas, tot}}(t = 0) = 3 \times 10^6 M_{\odot}$ when running the model.

Gas accretion — The SFH is driven by primordial gas accretion, where the inflow is uniformly distributed throughout the box and serves as a driving factor for star formation and subsequent chemical enrichment. The accretion rate has the same parametrization as in Kobayashi et al. (2020) as follows:

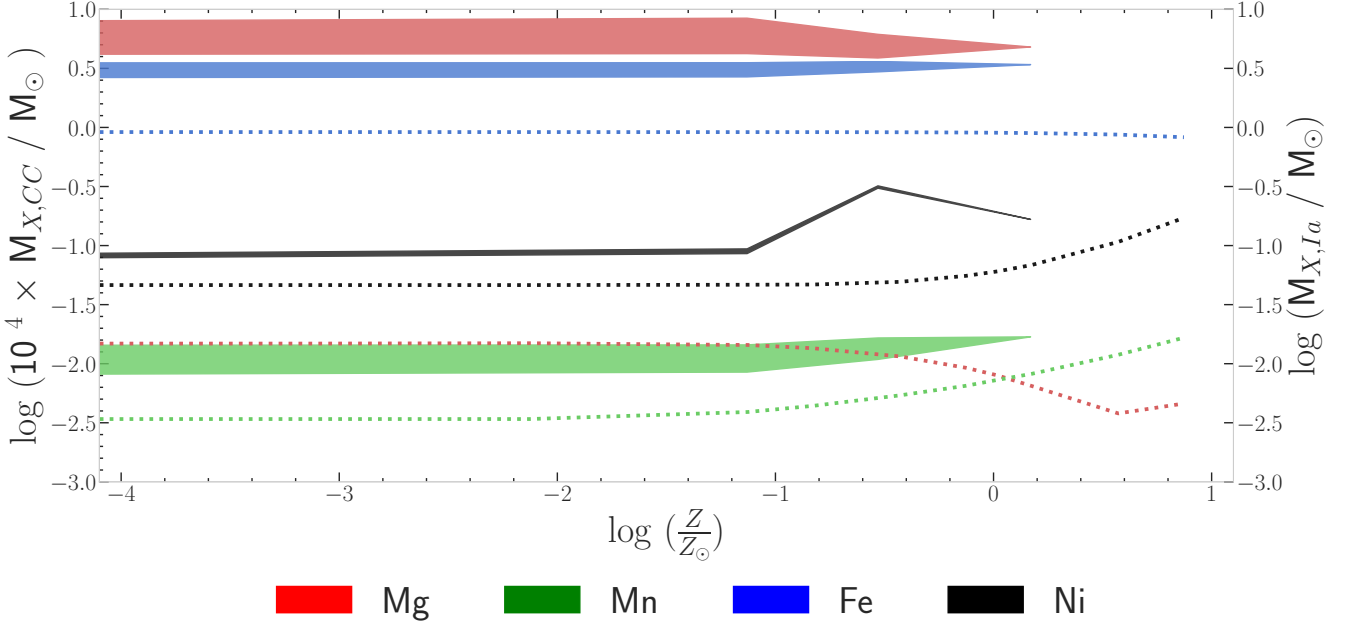


Figure 3. A comparison between the total massive star yields of Kobayashi et al. 2006 for a $10^4 M_{\odot}$ stellar population (left y-axis, filled bars) and the Keegans et al. (2023) yields from a single M_{ch} SN Ia event with progenitor mass $M_{\text{WD}} = 1.4 M_{\odot}$ (right y-axis, dotted lines). The nucleosynthesis yields of Mg are shown in red, Mn in green, Fe in blue and Ni in black. For each element, the width of the bar corresponds to the total massive-star contribution when changing the massive star cutoff for CCSNe from $25 M_{\odot}$ (lowest edge of the bar) to $40 M_{\odot}$ (highest edge).

$$\left(\frac{d\rho_{\text{gas}}(t)}{dt}\right)_{\text{infall}} = \mu \times t \times \exp\left(-\frac{t}{\lambda}\right), \quad (2)$$

where t is the simulation time, μ measures the intensity of the gas accretion and λ varies the duration of the gas accretion rate. The total gas mass accreted into the galaxy is $M_{\text{UFD, inf}} = 6 \times 10^6 M_{\odot}$, $M_{\text{dSph A, inf}} = 4 \times 10^8 M_{\odot}$, and $M_{\text{dSph B, inf}} = 9 \times 10^8 M_{\odot}$ for the UFD model, *dSph A* and *dSph B*, respectively, where these values are a constraint on μ . Table 1 contains the values used for λ and μ in our models.

Star formation — The star formation rate (SFR) is assumed to follow a Schmidt-Kennicutt relation, $\text{SFR}(t) = \epsilon \times \langle \rho(t) \rangle V$, where ϵ is the star formation efficiency (SFE; see Table 1), $\langle \rho(t) \rangle$ is the average gas mass density and V is the total volume of the simulated galaxy. At any given time step Δt , stars are formed from a random sampling of the Kroupa (2001) initial mass function (IMF) from $0.1 M_{\odot}$ to $100 M_{\odot}$. When star formation occurs within a cell, the star adopts the chemical composition of the adjacent eight cells (see Alexander et al. 2023 for more details).

Stellar yields — In our model, we adopt the same stellar nucleosynthesis yields as in Alexander et al. (2023). In particular, we assume the yields of Kobayashi et al. (2006) for high-mass stars along with the yields Karakas (2010) for asymptotic giant-branch (AGB) stars. At the end of the stars’ lifetimes, they dump enriched material into the surrounding ISM for future stellar populations to inherit (see Alexander et al. 2023 for details).

Delay-time distribution for Type Ia Supernovae — To model the delay-time between the formation of the SN Ia progenitor system and the SN Ia explosion, we assume a power law of the form, $\text{DTD}_{\text{Ia}}(\tau) = N_{\text{Ia}} \tau^{-1.1}$ which is motivated by a number of observational studies (e.g., see Maoz & Badenes 2010; Maoz et al. 2014;

Free Parameters			
Parameters	UFD	dSph A	dSph B
Volume [kpc ³]:	0.512	4.096	4.096
Infall Gas [M_{\odot}]:	6×10^6	4×10^8	9×10^8
λ [Myr]:	100	250	2000
ϵ [Gyr ⁻¹]:	0.004	0.20	0.04
Stellar Properties			
Properties	UFD	dSph A	dSph B
Number of Stars	8.4×10^4	2.4×10^7	5.3×10^7
Stellar Mass [M_{\odot}]:	3.2×10^4	9.2×10^6	1.7×10^7
SFH [Gyr]:	1.9	2.0	12.0

Table 1. Free Parameters of each model and stellar properties shown afterwards as we consider two different sets of metallicity-dependent yields for SNe Ia with different progenitor masses. The table highlights some of the differences between each model, with rows corresponding to UFD, dSph A and dSph B. *Row 1*: Total volume of the box for each model. *Row 2*: The total accreted gas in each model. *Row 3*: The peak accretion of each model as a function of time. *Row 4*: The star formation efficiency for each model. *Row 5*: The total number of stars in each model. *Row 6*: The stellar mass of each model at the point where star formation is truncated. *Row 7*: The duration of the star formation activity.

Maoz & Graur 2017; Castrillo et al. 2021), where our toy models assume $N_{\text{Ia}} = 10^{-3} M_{\odot}^{-1}$, and a minimum delay-time $\tau_{\text{min}} = 150$ Myr.

Outflows — All supernovae events are assumed to have an identical explosion energy, $E_{\text{SN}} = 10^{51}$ ergs, sweeping out $M_{\text{swept}} = 5 \times 10^4 M_{\odot}$ of the ISM mass. Galactic outflows are driven by SN explosions in the form of a SN bubble, where the swept-up mass M_{swept} is lost

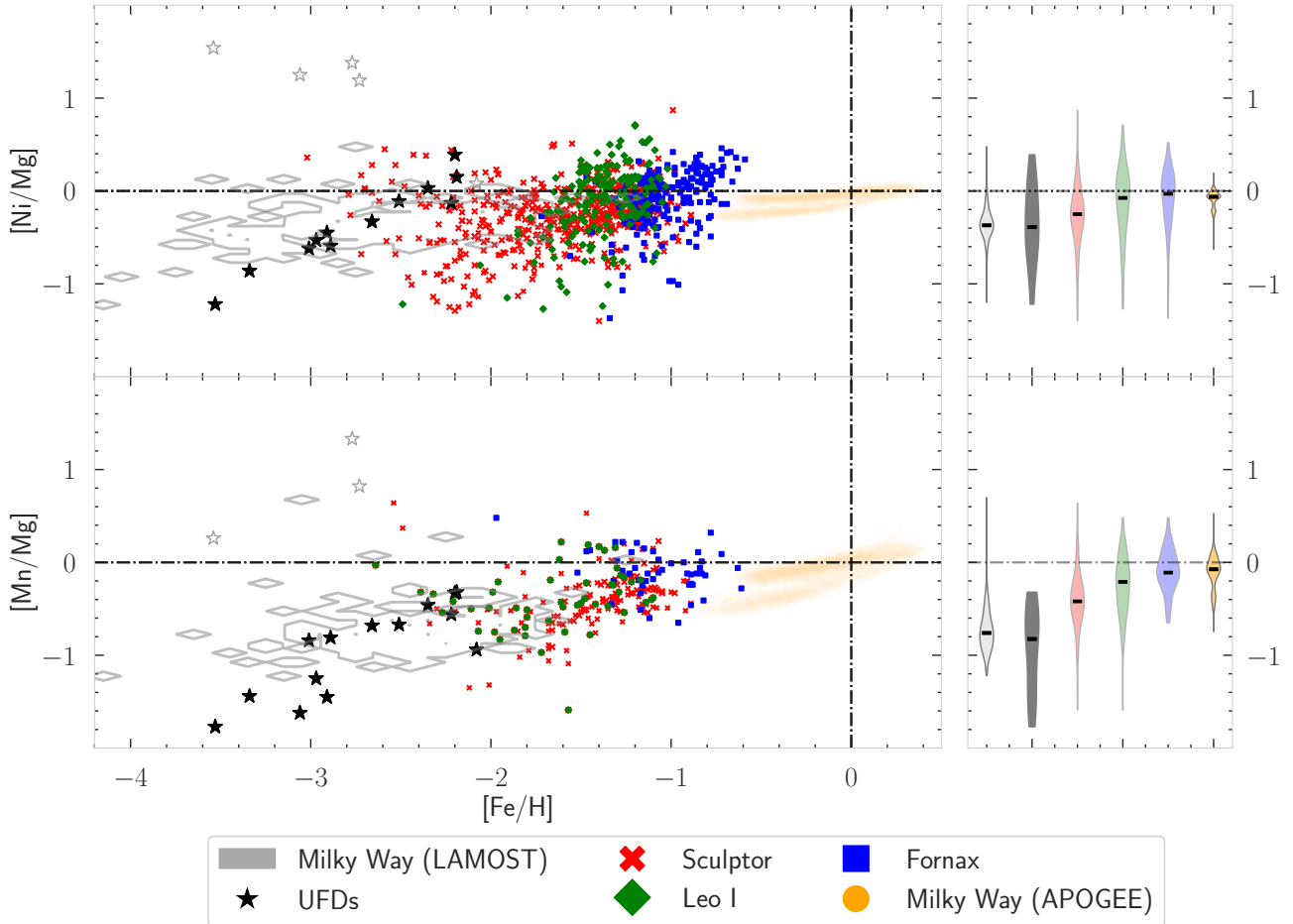


Figure 4. The observed $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (top left panel) and $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (bottom left panel) for MW halo stars (LAMOST - grey), UFD stars (black), Sculptor stars (red), Leo I stars (green), Fornax stars (blue) and MW field stars (APOGEE - orange). On the right are violin plots describing the distribution function of $[\text{Ni}/\text{Mg}]$ (top right panel) and $[\text{Mn}/\text{Mg}]$ (bottom right panel) in the different galaxies ordered from left to right according to their average stellar metallicity, with the black lines showing the mean of $[\text{X}/\text{Mg}]$ within each sample.

from the galaxy if any part of the bubble is outside the constraint of the box.

4 OBSERVATIONAL SAMPLES

Our observational samples are taken from a select catalogue of galaxies according to the following criteria. (i) The sample must cover a range of metallicities from $-4 \leq [\text{Fe}/\text{H}] \leq -1$ to test the full scope of the SNe Ia contributions in our models. (ii) The age distribution of the stars in the sample must result from a similar star formation history to the model of comparison. As such, we use these two criteria for our UFD sample from which we select Reticulum II (Ret II, Ji et al. 2016) and Carina II (Car II, Ji et al. 2020). The MIKE spectrograph was used by Ji et al. (2016) to obtain the spectra of nine Ret II candidates whereas MagLiteS obtained spectra for nine stars in Car II (Ji et al. 2020). Both of these UFDs have similar chemical abundances as well as star formation histories, making them prime candidates for our sample.

Our dSph sample consists of Sculptor (North et al. 2012; Kirby

et al. 2018; Hill et al. 2019; de los Reyes et al. 2020, 2022), Leo I (Kirby et al. 2018; de los Reyes et al. 2020) and Fornax (Kirby et al. 2018; de los Reyes et al. 2020), with the chemical abundances being taken from a variety of works in the literature, including measurements from the medium-resolution spectroscope DEIMOS (Kirby et al. 2018; de los Reyes et al. 2020, 2022) and the FLAMES/GIRAFFE spectrograph (North et al. 2012; Hill et al. 2019). There are many differences between the SFH of Sculptor and Leo I. Firstly, Savino et al. (2018) found two stellar populations in Sculptor, one which is very old and metal-poor, whose SFH could be fit with a Gaussian centered on $\mu_{1,\tau} = 12.58$ Gyr with a standard deviation of $\sigma_{1,\tau} = 0.66$ Gyr, and another relatively younger and more metal-rich with $\mu_{2,\tau} = 8.0$ and $\sigma_{2,\tau} = 0.64$ (see also de Boer et al. 2011). This is different from de los Reyes et al. (2022), who derived the SFH of Sculptor by using a one-zone GCE model to reproduce the observed chemical abundance patterns, finding that the galaxy formed the vast majority of its stars over a total star formation period of ~ 0.92 Gyr, in agreement with Vincenzo et al. (2016) who could reproduce both the observed metallicity distribution function (MDF) and the colour-magnitude diagram (CMD) of the central

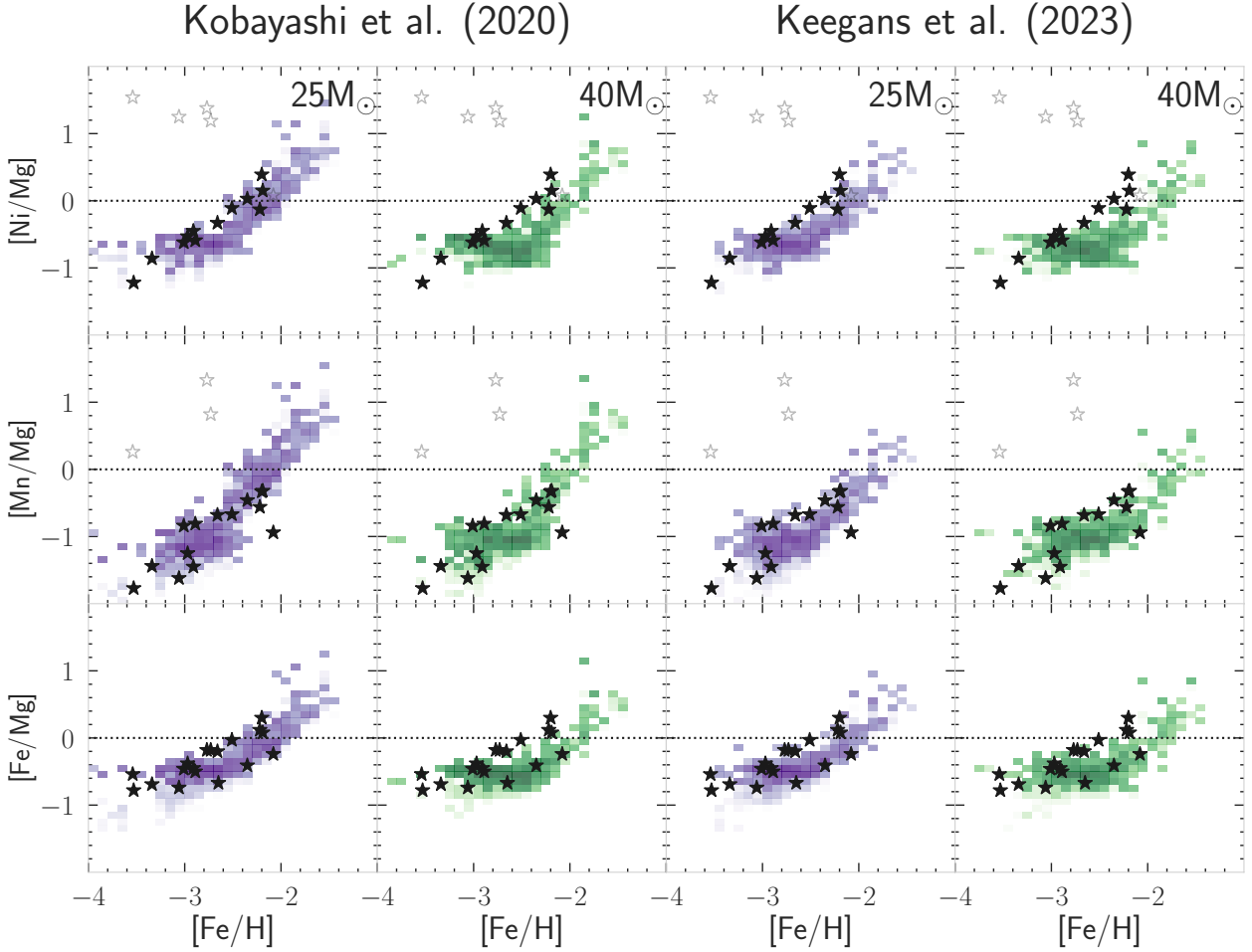


Figure 5. Chemical abundances of our favoured UFD model for [Ni/Mg] (top panels), [Mn/Mg] (middle panels) and [Fe/Mg] (bottom panels) are shown as a function of [Fe/H]. For Kobayashi et al. (2020) (left 2 panels) and Keegans et al. (2023) (right 2 panels), we change the upper mass limit for CCSNe from $25 M_{\odot}$ to $40 M_{\odot}$. Model predictions are shown as 2D histograms where each colour map corresponds to the number density of stars within each bin. The black starred symbols depict the observed chemical abundances in our UFD sample with the empty symbols showing the upper limits.

stellar populations of Sculptor. Even with such differences in the predicted SFH, by following the results of de los Reyes et al. (2022), we deem the chemical abundances of Sculptor to be a good candidate for comparison with dSph A.

Lastly, our sample of Milky Way stars is split into one that covers the iron abundance range $[\text{Fe}/\text{H}] \leq -1$ from the Large sky Area Multi-Object fibre Spectroscopic Telescope (LAMOST, Li et al. 2022), and one that covers the range $[\text{Fe}/\text{H}] \geq -1$ from the APOGEE Data Release 17 (DR17) (Abdurro’uf et al. 2022). Our MW sample from APOGEE consists of stars residing in both the thin and thick disk, by applying similar cuts as in Weinberg et al. (2022).

In Figure 4, we compare the chemical abundances of [Ni/Mg]-[Fe/H] (top left panel) and [Mn/Mg]-[Fe/H] (bottom left panel) for our chosen observational samples – UFDs (black stars), Sculptor (red crosses), Leo I (green diamonds) and Fornax (blue squares). Faint blank stars denote upper limits for UFDs. In the same figure, we show several violin plots of our galaxies, depicting their distributions of [Ni/Mg] (top right) and [Mn/Mg] (bottom right). We find that the distributions of both [Ni/Mg] and [Mn/Mg] increase as we consider galaxies with higher average [Fe/H], with [Mn/Mg] having

a stronger dependence than [Ni/Mg]. One possible explanation for this difference is Mn being a more metallicity-dependent element from SNe Ia than Ni. Interestingly enough, the steady increase in [Mn/Mg] is in line with the MW halo field stars, suggesting that the main contributors for Mn in both environments took place following a similar process.

5 RESULTS

Chemical abundance measurements are the cornerstone of chemical evolution modelling and offer insight into the evolution of galaxies, adding additional dimensions and investigation pathways. This section is split into two parts. In Section 5.1 we analyse the predicted chemical abundance patterns from our UFD models with both Kobayashi et al. (2020) and Keegans et al. (2023) SN Ia yields, including an upper mass limit sensitivity study for the progenitors of CCSNe and a model dispersion comparison between a set number of elements. In Section 5.2, we analyse the predicted chemical abundance patterns for our dSph models, comparing both sub- M_{ch} and

M_{ch} SN Ia yields from both [Kobayashi et al. \(2020\)](#) and [Keegans et al. \(2023\)](#).

5.1 Ultra-Faint Dwarf Galaxies

Fig. 5 shows a comparison between different $[X/\text{Mg}]-[\text{Fe}/\text{H}]$ chemical abundance patterns in our UFD models, where X corresponds to Ni (top panels), Mn (middle panels) and Fe (bottom panels). The first two columns show the predicted chemical abundances as purple and green 2D histograms for the $1.4 M_{\odot} M_{\text{ch}}$ SN Ia yields of [Kobayashi et al. \(2020\)](#), where we vary the upper mass limit for CCSNe ($M_{\text{up}} = 25 M_{\odot}$ in purple and $40 M_{\odot}$ in green). In short, any stars which is above the shown mass will not explode as a CCSN. The third and fourth columns show the same abundance patterns for different massive-star cutoff as the first two, but for the [Keegans et al. \(2023\)](#) $1.4 M_{\odot} M_{\text{ch}}$ SN Ia yields. Black stars in all panels denote the observed chemical abundances of our UFD sample with empty points depicting upper limits.

Both [Kobayashi et al. \(2020\)](#) and [Keegans et al. \(2023\)](#) M_{ch} yields predict similar chemical abundance patterns of $[\text{Mn}/\text{Mg}]-[\text{Fe}/\text{H}]$ and $[\text{Ni}/\text{Mg}]-[\text{Fe}/\text{H}]$, which reproduce the observed sample of RGB abundances in UFD galaxies when assuming a $25 M_{\odot}$ upper mass limit for CCSNe. A $40 M_{\odot}$ upper mass limit produces more Fe in massive stars, allowing $[\text{Fe}/\text{H}]$ to evolve further before SN Ia events start dumping Fe-peak elements into the ISM, increasing $[X/\text{Mg}]$. Note that the total amount of Fe released by a massive star population is higher by ≈ 1.5 orders of magnitude than Ni and ≈ 2.5 orders than Mn (see Fig. 3), hence the main effect of increasing the upper mass limit is shifting the predicted chemical abundance patterns to the right, keeping the same characteristic shape. In our work, we refer to this abundance pattern as an ‘elbow’. Models with the [Kobayashi et al. \(2020\)](#) and [Keegans et al. \(2023\)](#) SN Ia yields predict similar $[\text{Fe}/\text{Mg}]-[\text{Fe}/\text{H}]$ chemical abundance patterns for both 25 and $40 M_{\odot}$ upper mass limits. Our toy model however can reach higher super-solar $[\text{Mn}/\text{Mg}]$ with the [Kobayashi et al. \(2020\)](#) SN Ia yields than [Keegans et al. \(2023\)](#), indicative of the differences between their respective Mn production. For the continuation of our work, a $25 M_{\odot}$ upper mass limit for CCSNe is adopted in our reference toy models.

Fig. 6 compares the predicted UFD chemical abundance patterns that are obtained when assuming the [Kobayashi et al. \(2020\)](#), first column) and [Keegans et al. \(2023\)](#), second column) SN Ia progenitors. Chemical evolution models with the $0.8 M_{\odot}$ and $1.0 M_{\odot}$ sub- M_{ch} models are shown through red and blue contours, respectively, while the $1.4 M_{\odot} M_{\text{ch}}$ models are shown in green. Filled black stars are the observed chemical abundances in our sample of UFD stars where upper limits are represented as empty stars. Both sub- M_{ch} and M_{ch} SN Ia yields of [Kobayashi et al. \(2020\)](#) predict similar abundance patterns to observations at very low metallicity (i.e. $[\text{Fe}/\text{H}] \lesssim -2$) for all given elements; however, the UFD model with M_{ch} SN Ia progenitors predict a larger production of Fe-peak elements to the ISM than those with sub- M_{ch} progenitors, reproducing the observed patterns also at higher metallicities.

The $0.8 M_{\odot}$ sub- M_{ch} SN Ia yields of [Keegans et al. \(2023\)](#) provide negligible chemical enrichment to the ISM for all elements shown. This was eluded to in [Keegans et al. \(2023\)](#) where they suggested that their $0.8 M_{\odot}$ models would not be suitable for chemical evolution and stellar abundance studies. The $1.4 M_{\odot} M_{\text{ch}}$ SN Ia progenitors of [Keegans et al. \(2023\)](#) are found to exhibit similar $[\text{Ni}/\text{Mg}]-[\text{Fe}/\text{H}]$ and $[\text{Mn}/\text{Mg}]-[\text{Fe}/\text{H}]$ to those from [Kobayashi et al. \(2020\)](#), reproducing the abundance patterns seen within our UFD sample. As previously stated, higher $[\text{Mn}/\text{Mg}]$ are reached with the [Kobayashi et al. \(2020\)](#) M_{ch} SN Ia yields than [Keegans et al. \(2023\)](#), as Mn is produced more

in the former (see Figs 1 and 2). Interestingly, there are negligible differences in $[\text{Fe}/\text{Mg}]-[\text{Fe}/\text{H}]$ abundance ratios between [Kobayashi et al. \(2020\)](#) and [Keegans et al. \(2023\)](#) sub- M_{ch} and M_{ch} SN Ia yields.

Fig. 7 shows the average standard deviation of the chemical abundances as predicted by our UFD model assuming the [Keegans et al. \(2023\)](#) SN Ia yields. We remind the readers that a $25 M_{\odot}$ massive star cut-off for CCSN progenitors is assumed in the reference UFD model. We show more elements than the usual four to highlight the disparity between them for three SNe Ia progenitor masses: $0.8 M_{\odot}$ (red crosses), $1.0 M_{\odot}$ (green diamonds) and $1.4 M_{\odot}$ (blue triangles). The average model dispersion is estimated through the arithmetic mean of the standard deviation of $[X/\text{Mg}]$ through numerous bins of $[\text{Fe}/\text{H}]$. Elements on the x-axis are in ascending order of atomic number. We note that the dispersions in Fig. 7 are from one model, and a more accurate estimate could be obtained by averaging out the dispersion over several runs of the same model.

With the exception of Sc, the M_{Ch} model in Fig. 7 has an almost constant average dispersion that ranges between 0.20 and 0.25 for the considered chemical elements while clearer differences emerge in the sub- M_{Ch} models, specially the $0.8 M_{\odot}$ one. An interesting feature of Fig. 7 is the noticeably higher average dispersion of $[\text{Sc}/\text{Mg}]$ than other abundance ratios, with sub- M_{Ch} SNe Ia progenitor models predicting a higher dispersion than the M_{Ch} models. Even though [Hill et al. \(2019\)](#) measured weak Sc lines, they found an unusually large dispersion of $[\text{Sc}/\text{Mg}]$ at $[\text{Fe}/\text{H}] \approx -2.3$ relative to other elements they analysed. Perhaps with more chemical abundance data, Sc can be used to disentangle between different SN Ia progenitors in UFDs by analysing the observed chemical abundance dispersions.

5.2 Dwarf Spheroidal Galaxies

Fig. 8 shows a comparison between the predictions of our toy models for dSph galaxies assuming the [Kobayashi et al. \(2020\)](#) SN Ia yields and the observational data of $[\text{Ni}/\text{Mg}]-[\text{Fe}/\text{H}]$ (top panels) and $[\text{Mn}/\text{Mg}]-[\text{Fe}/\text{H}]$ (bottom panels), considering dSph galaxies with different SFHs (different columns); the *dSph A* model on the left characterises an ancient dSph galaxy like Sculptor, whereas *dSph B* on the right is for a dSph galaxy with a more extended star formation activity like Leo I. The dSph models with the [Kobayashi et al. \(2020\)](#) $1.4 M_{\odot} M_{\text{ch}}$ SN Ia yields are presented as 2D histograms, whereas the black contours correspond to the dSph models assuming the [Kobayashi et al. \(2020\)](#) $1.0 M_{\odot}$ sub- M_{ch} SN Ia yields. Our model stars are compared to the chemical abundances of Sculptor (*dSph A*) and Leo I (*dSph B*), that are shown as red crosses and green diamonds, respectively.

The *dSph A* model with the [Kobayashi et al. \(2020\)](#) $1.4 M_{\odot}$ SN Ia progenitors can reproduce the observed abundance patterns of $[\text{Ni}/\text{Mg}]-[\text{Fe}/\text{H}]$ and $[\text{Mn}/\text{Mg}]-[\text{Fe}/\text{H}]$ in Sculptor. Sculptor stars are found to have a large dispersion in $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$ at $[\text{Fe}/\text{H}] \lesssim -2$, and our *dSph A* model reproduces some of the abundances at these very low metallicities, including several stars with sub-solar $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$. *dSph A* also predicts a second, less dense sequence of stars with super-solar $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$. The *dSph B* model with the [Kobayashi et al. \(2020\)](#) $1.4 M_{\odot}$ SN Ia progenitors predicts that stars have a larger scatter in their abundances at $[\text{Fe}/\text{H}] \lesssim -1.6$ than the *dSph A* model, with a more pronounced second sequence of stars with super-solar $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$. *dSph B* also predicts a sharper increase of $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$ from $[\text{Fe}/\text{H}] \approx -1.6$, reproducing the super-solar $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$ at those higher metallicities that are observed in Leo I.

The *dSph A* model with the [Kobayashi et al. \(2020\)](#) $1 M_{\odot}$ SN Ia progenitors cannot reproduce the overall chemical abundance

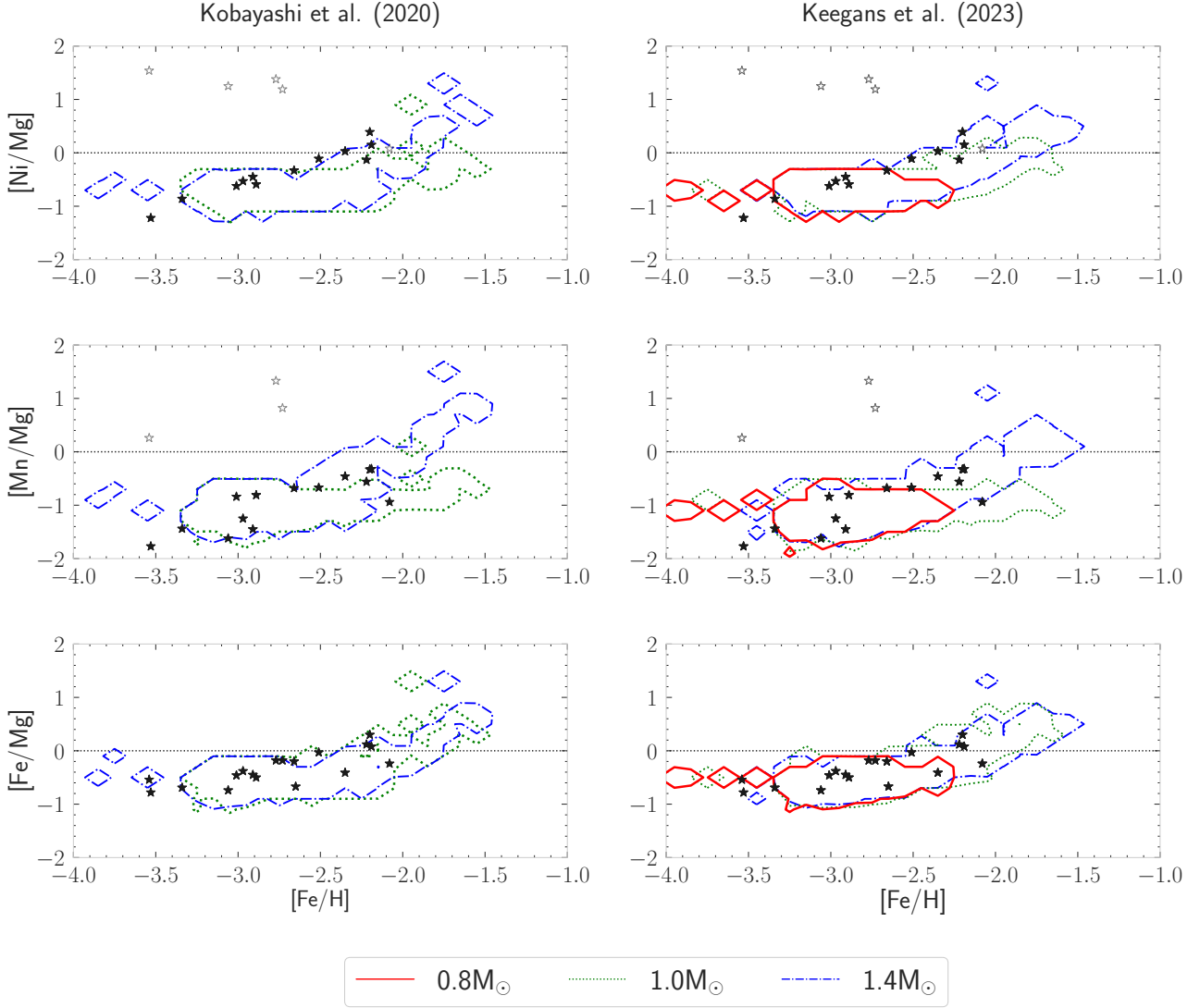


Figure 6. The $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (top panels), $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (middle panels) and $[\text{Fe}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (bottom panels) chemical abundances of our UFD toy models assuming the Kobayashi et al. (2020) (left column) and Keegans et al. (2023) (right column) SN Ia yields. The results of the models assuming SN Ia progenitors with masses $M_{\text{WD}} = 0.8 M_{\odot}$ are shown as red solid contours, $M_{\text{WD}} = 1.0 M_{\odot}$ are the green dotted contours, and $M_{\text{WD}} = 1.4 M_{\odot}$ are the blue dot-dashed contours. The chemical abundances of our UFD samples are the black starred symbols with the empty symbols showing the upper limits.

patterns observed in Sculptor, though it can explain the observed $[\text{Ni}/\text{Mg}]$ of a number of stars with iron abundances in the range $-1.5 \lesssim [\text{Fe}/\text{H}] \lesssim -0.8$ that have $[\text{Ni}/\text{Mg}]$ lower than the predictions of the *dSph A* model with $1.4 M_{\odot}$ M_{ch} SN Ia progenitors. In the observed sample, there is also a clump of stars with $[\text{Mn}/\text{Mg}] \approx -0.5$ at iron abundances $-1.2 \lesssim [\text{Fe}/\text{H}] \lesssim -0.8$, corresponding to intermediate values of $[\text{Mn}/\text{Mg}]$ between those predicted by the $1 M_{\odot}$ sub- M_{ch} SN Ia progenitors and the $1.4 M_{\odot}$ M_{ch} ones, (i) showing that the Mn yields from SNe Ia of Kobayashi et al. (2020) are more sensitive to the SN Ia progenitor mass than the Ni yields (see also Fig. 1), and (ii) suggesting the presence of some contribution from a population of sub- M_{ch} SN Ia progenitors that have masses larger than $1 M_{\odot}$ explored in this work. Similar arguments would also be valid from the analysis of the behaviour of *dSph B* with the Kobayashi et al. (2020) $1 M_{\odot}$ SN Ia progenitors, though the observed sample

shows a larger scatter in $[\text{Ni}/\text{Mg}]$, with a dozen of stars that have lower $[\text{Ni}/\text{Mg}]$ than both models in the figure.

Fig. 9 is similar to Fig. 8 but shows the predictions of our *dSph* models with the Keegans et al. (2023) SN Ia yields. Similarly to Fig. 8, the chemical abundances of our observations are shown as red crosses and green diamonds for Sculptor and Leo I, respectively. Both *dSph A* and *dSph B* models with only $1.4 M_{\odot}$ M_{ch} SN Ia progenitors reproduce better the observed $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ and $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ chemical abundance patterns, showing a similar average behaviour as the models with the Kobayashi et al. (2020) SN Ia yields but predicting (i) lower average values of $[\text{Ni}/\text{Mg}]$ and $[\text{Mn}/\text{Mg}]$ at $-1.5 \lesssim [\text{Fe}/\text{H}] \lesssim -1.0$ and (ii) a less sharp increase of $[\text{Mn}/\text{Mg}]$ in *dSph B* from $[\text{Fe}/\text{H}] \approx -1.6$ because of the lower amount of Mn from the M_{ch} SN Ia models of Keegans et al. (2023) than those of Kobayashi et al. (2020); this allows *dSph B* with Keegans et al. (2023) SN Ia yields to reproduce better the observed $[\text{Mn}/\text{Mg}]$ in Leo

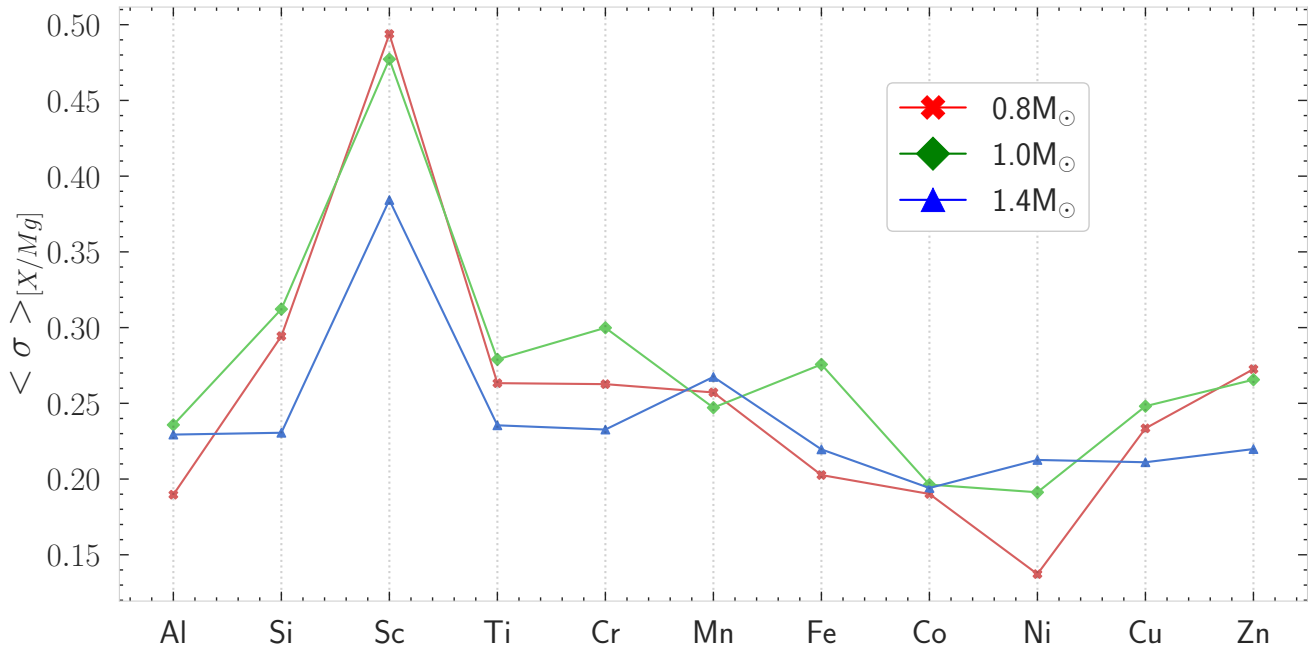


Figure 7. Distribution of the average standard deviation of $[X/Mg]$ from our UFD model where red crosses, green triangles and blue triangles correspond to the predictions of the models with SN Ia progenitors with masses $M_{WD} = 0.8 M_{\odot}$, $1.0 M_{\odot}$ and $1.4 M_{\odot}$, respectively.

I – the same SN Ia yields, however, do not allow *dSph A* to reach the high $[Mn/Mg]$ that are also observed in Sculptor, which are better reproduced by the Kobayashi et al. (2020) M_{Ch} SN Ia yields.

6 CONCLUSIONS

In our present work, we investigate the Kobayashi et al. (2020) and Keegans et al. (2023) metallicity-dependent SN Ia yields on the chemical evolution of UFDs and dSphs, exploring also the effect of M_{Ch} and sub- M_{Ch} SN Ia progenitors. Our analysis is focused on Mn and Ni, as the yields of these elements show the largest changes as a function of the WD progenitor mass. We incorporate this set of SN Ia yields within our chemical evolution tool *i-GETool* (Alexander et al. 2023) and create one UFD and two dSph models, assuming the massive star yields of Kobayashi et al. (2006) and the AGB yields of Karakas (2010), which are both kept constant as we change the SNe Ia yields. The assumed DTD for SNe Ia is $\propto \tau^{-1.1}$ as a function of the delay-time, τ , which does not depend on metallicity and is same for all progenitors (see, e.g., Kobayashi et al. 2020 and Johnson et al. 2023 for detailed discussions on this). Each model assumes the galaxy to form from a single accretion event which produces SFHs that truncate at different cosmic times. The IMF is from Kroupa (2001), and the outflow prescriptions are the same as Alexander et al. (2023). We compare our observational sample of UFD stellar abundances (Carina II from Ji et al. 2020 and Reticulum II from Ji et al. 2016) to the respective model, first examining the effect of changing the massive star cut-off and then the various SN Ia progenitor yields. We also compare the average chemical abundance dispersion of several chemical elements relative to Mg, using the Keegans et al. (2023) M_{Ch} SN Ia yields. Finally, we examine the predicted $[Ni/Mg]$ - $[Fe/H]$ and $[Mn/Mg]$ - $[Fe/H]$ from our two dSph models and compare them to observations in Sculptor and Leo I, considering both M_{Ch} and

sub- M_{Ch} SNe Ia progenitors. The following is a summary of the conclusions in this present work.

(i) M_{Ch} SN Ia progenitors contribute more Fe-peak elements to the ISM than sub- M_{Ch} . The SN Ia yields of Mn show a stronger dependence on the mass of the WD progenitor than Ni. The $0.8 M_{\odot}$ sub- M_{Ch} SN Ia model contributes the least Ni, Mn and Fe yields, but interestingly the most for Mg. For metallicities $\log(Z/Z_{\odot}) \lesssim -2$, the W7 metallicity-independent SN Ia yields of Iwamoto et al. (1999) are similar to the other M_{Ch} SN Ia yields that are examined in this work.

(ii) A single SN Ia event provides comparable amounts of iron-peak elements to the ISM as that of all massive stars forming out of a $10^4 M_{\odot}$ stellar population, which makes the stellar populations in the local UFD and dSph galaxies probably the best laboratories in the cosmos to study the SN Ia nucleosynthesis. Large differences in the IMF-averaged massive-star yields are found when changing the cutoff mass between 25 and $40 M_{\odot}$ at metallicities $\log(Z/Z_{\odot}) \lesssim -1$. Such difference decreases as Z approaches Z_{\odot} .

(iii) We compare observational samples taken from different galaxy environments, spanning more than 10,000 individual stars, focusing on the observed $[Ni/Mg]$ - $[Fe/H]$ and $[Mn/Mg]$ - $[Fe/H]$ abundance patterns. $[Mn/Mg]$ evolves more strongly than $[Ni/Mg]$ as a function of $[Fe/H]$, showing the same average behaviour across the different galaxy environments, suggesting that the origin of Mn and Ni in both the MW field stars and dSphs may be caused by similar producers.

(iv) In our toy model for UFD galaxies, a $25 M_{\odot}$ cut-off reproduces the observations of Carina II and Reticulum II while also maintaining a consistent SFH similar to the observed one, as higher upper mass limits truncate the SFH quicker, stopping any further chemical enrichment.

(v) The assumption of only M_{Ch} SNe Ia progenitors produces similar to identical chemical abundance patterns to those observed, with

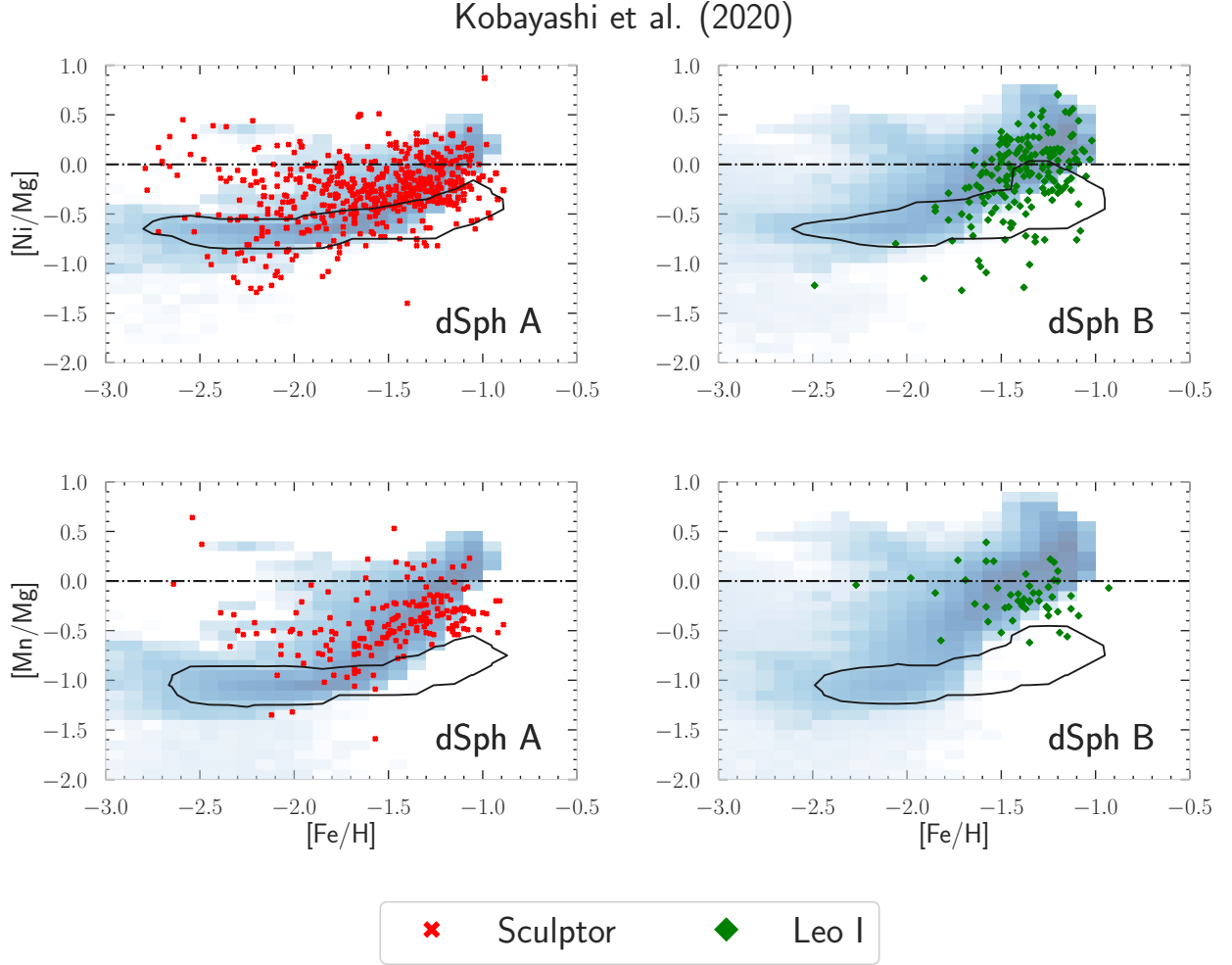


Figure 8. The chemical abundance patterns of $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (top panels) and $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ (bottom panels) for our two toy dSph models, where *dSph A* (first column) simulates the SFH and chemical enrichment of an ancient dSph galaxy like Sculptor, and *dSph B* (second column) is for a dSph galaxy with a more extended period of star formation like Leo I. The predictions of the dSph models assuming the M_{Ch} SN Ia yields of Kobayashi et al. (2020) are shown as 2D histograms, where the colour scheme represents the number of stars within each grid cell. The black contour lines show instead the predictions of the dSph models assuming the sub- M_{Ch} SN Ia yields of Kobayashi et al. (2020). Both models adopt a stellar mass cut-off of $25 M_{\odot}$ for CCSNe. Model predictions are compared to the observed samples in Sculptor (red cross) and Leo I (green diamonds).

the $1.0 M_{\odot}$ sub- M_{Ch} SN Ia model slightly under producing both $[\text{Mn}/\text{Mg}]$ and $[\text{Ni}/\text{Mg}]$.

(vi) We examined the predicted average dispersion in the chemical abundances of our model UFD stars. The dispersion of $[\text{Sc}/\text{Mg}]$ exhibits the highest values among the iron-peak elements relative to Mg, similar to the findings of Hill et al. (2019). Of the three SNe Ia progenitor models that are explored in this work, the M_{Ch} model has an almost constant dispersion when considering different chemical elements, excluding Sc.

(vii) Our toy model *dSph A* has a short but powerful SFH like in the Sculptor dSph galaxy (e.g., de Boer et al. 2011) whereas *dSph B* is characterised by a long but calm SFH, similar to the observations in Leo I (e.g., Weisz et al. 2014). When assuming only M_{Ch} SN Ia progenitors, both *dSph A* and *dSph B* provide a good match to the observed $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ and $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$. Kobayashi et al. (2020) M_{Ch} SN Ia progenitors produce more Mn than Keegans et al. (2023) M_{Ch} progenitors, resulting with higher $[\text{Mn}/\text{Mg}]$ at $[\text{Fe}/\text{H}] \gtrsim$

-1 . Models with only sub- M_{Ch} SN Ia progenitors systematically under produce both Mn and Ni in *dSph A* and *dSph B*.

(viii) While the observed $[\text{Ni}/\text{Mg}]$ of the outliers of the predicted distribution with the $1.4 M_{\odot}$ M_{Ch} SN Ia progenitors can be explained by assuming chemical enrichment from some $1 M_{\odot}$ sub- M_{Ch} SN Ia progenitors, the $[\text{Mn}/\text{Mg}]$ of the outliers requires higher WD progenitor masses in the range $1 < M_{\text{WD}} < 1.4 M_{\odot}$.

Our present work shows that M_{Ch} SN Ia progenitors in inhomogeneous chemical evolution models can reproduce the $[\text{Mn}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ and $[\text{Ni}/\text{Mg}]$ - $[\text{Fe}/\text{H}]$ as observed in a sample of red giants in local UFD and dSphs, without the need of a substantial fraction of sub- M_{Ch} SN Ia progenitors. In future work, we hope to also include neutron capture elements in our inhomogeneous chemical evolution model to further study Ret II as it is believed to be a rich source of r -process elements Ji et al. (2016). Finally, our models can be applied to the Solar neighbourhood to examine the chemical evolution of several elements and probe the star formation in the nearby volume of our Galaxy in great detail.

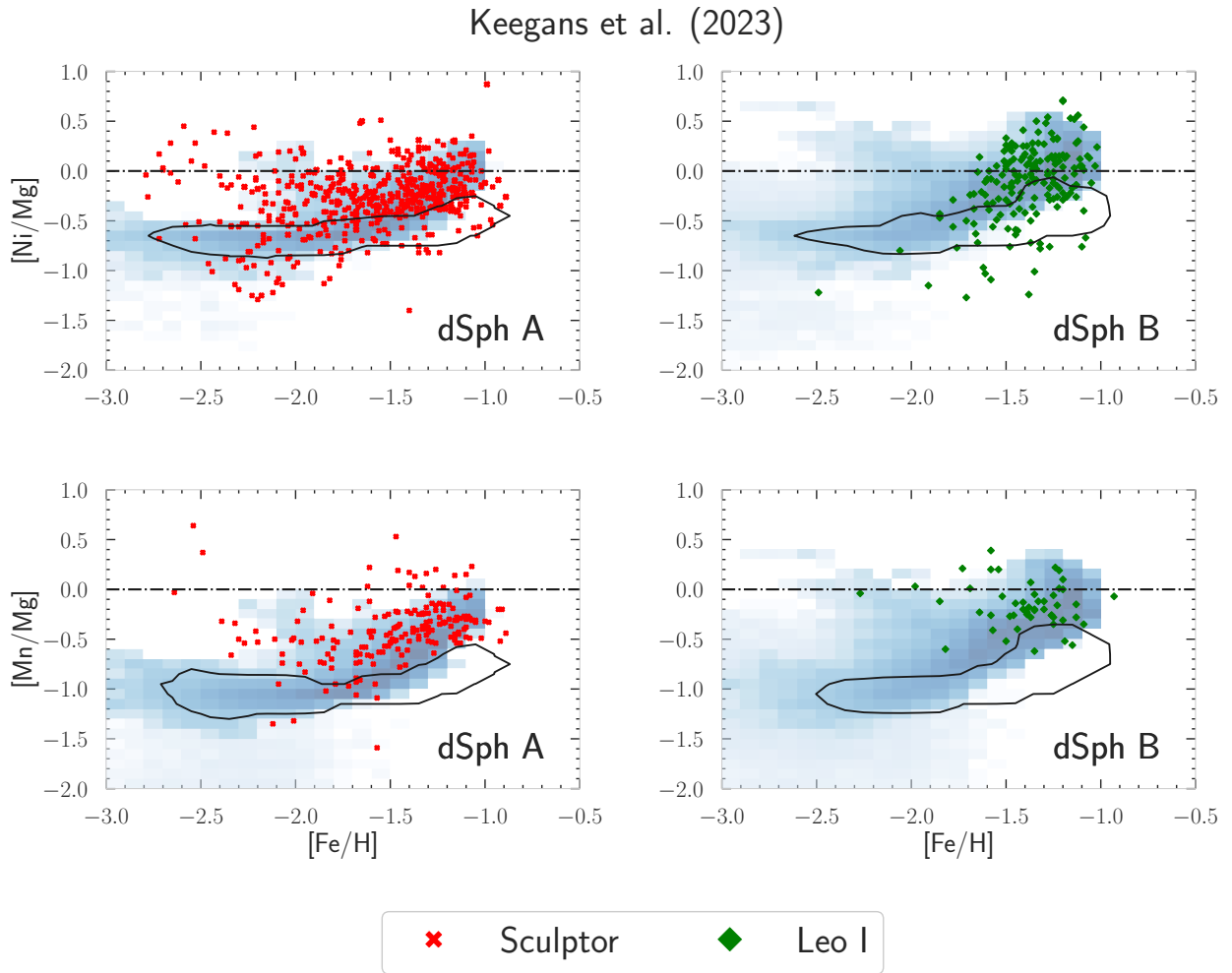


Figure 9. Similar to Figure 8, but for Keegans et al. (2023) SN Ia yields.

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DATA AVAILABILITY

The data within this work, which includes the chemical abundances as predicted by our inhomogeneous galactic chemical evolution models for UFDs and dSphs, will be shared upon reasonable request to the lead author.

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