An Empirical Study of the Cross Market Efficiency of the Index Options Market: A **Case Study from the Italian Derivatives Market**

Izidin El Kalak¹

Cardiff University Business School Email: elkalaki@cardiff.ac.uk

Robert Hudson

Hull University Business School Email: robert.hudson@hull.ac.uk

Abstract

This study examines the cross-market efficiency of the FTSE/MIB index options contracts traded on the Italian derivatives market (IDEM) between 1st October 2007 and 31st December 2012, a period including the financial crisis, using daily option prices. Two fundamental no-arbitrage conditions are tested: the lower boundary condition (LBC) and the put/call parity (PCP) condition while taking into account the role of transaction costs in mitigating the number of violations reported. Ex-post tests of LBC and PCP revealed a low incidence of mispricing in this market. Furthermore, to check the robustness of the results obtained by the ex-post tests, exante tests were applied to PCP violations occurring within a one-day lag. The results showed a significant drop in the number of profitable arbitrage strategies. Overall, the number and monetary value of the violations reported declined during the post financial crisis period compared to those during the financial crisis period. The findings obtained from these tests generally support the cross-market efficiency of the Italian index options market during the sample period, though some violations were occasionally reported.

JEL classification: G01, G14

Keywords: Cross-market efficiency; index options market; no-arbitrage conditions; financial crisis

¹ Corresponding Author: elkalaki@cardiff.ac.uk

1. Introduction

The rapidly-growing success of index options markets in both Europe and the US has drawn significant attention from many researchers trying to investigate the efficiency of these markets. The work of Stoll (1969) is considered to be the seminal research that paved the way for future empirical investigations on the efficiency of the derivatives markets.

The majority of these studies have focused on studying the efficiency of the US derivative market (e.g. Evnine and Rudd (1985); Kamara and Miller (1995); Ackert and Tian (2001)). The empirical literature on the European markets is not only very limited but also mainly relates to the pre-Euro period, for example, the studies of Mittnik and Rieken (2000a) on the DAX index from 1992 to 1995; Cavallo and Mammola (2000) on the MIB30 index from 1996 to 1997; Chesney et al. (1995) on the Swiss index; and Puttonen (1993) on the Finnish index. To our knowledge, only two major investigations have been conducted on the Italian derivative market by Cavallo and Mammola (2000) and Torricelli and Brunetti (2003).

Therefore, one of the main motivations in conducting this study is to provide a new insight into this market to capture the recent changes that occurred in the Italian market such as (i) the development and fast growth of the Italian market making it the fourth biggest derivative market in Europe, which has led to the creation of a wider range of options than studied in the previous studies, e.g. only one-month at-the-money options were previously considered; (ii) the introduction of the Euro and the subsequent financial crisis have led to new market rules affecting the efficiency of the Italian market which were not considered in previous studies; (iii) The Italian stock market known as "Borsa Italiana" has merged with the London Stock Exchange effective from the 1st October 2007. Another motivation for this study is the sample period chosen where the efficiency of the Italian options market is being tested both during the peak of financial crisis period (2007-2009) and also during the period following this crisis (2010-2012).

This paper aims to test the efficiency of the Italian derivative market, in particular, the cross-market efficiency of the FTSE/MIB index options contracts for the period between 1st October 2007 and 31st December 2012. The sample period is then divided into the crisis period from 1st October 2007 to 31st December 2009 and the post crisis period from 1st January 2010 to 31st December 2012. After applying various filtering criteria, our sample includes 61030 pairs of matched call and put options for the whole period. The cross-market

efficiency of the FTSE/MIB index options market is tested by the calculation of both the lower boundary condition and the put-call parity (PCP) relationship.

The lower boundary conditions were economically tested on an ex-post basis with respect to the frequency and size of the violations. In the first scenario tested no market frictions were considered and the frequency of violations observed showed a relatively high percentage of mispricing compared to other studies. When both bid/ask spread and transaction costs were considered, the percentage of violations dropped. Furthermore, the mean level of violations was moderate and did not exceed 200 Euros per contract for the whole sample. In addition, throughout the whole period in all the scenarios, the put options violated the lower boundary condition more often than the calls. Moreover, the results showed that investors can make extra profits when implementing a short strategy compared to a long one. This can be related to put options being overpriced relative to calls. These findings are quite different from the study conducted by Torricelli and Brunetti (2003) perhaps due to a recent rule partially limiting short selling in the Italian index market.

Furthermore, in order to check the robustness of the results reported relating to the PCP tests and to find out to what extent the ex-post arbitrage opportunities can be exploited, ex-ante tests were carried out. Results showed that the incidences of PCP violations dropped remarkably even in the absence of transaction costs. When transaction costs are included, the percentage of violations decreased, even more, to be almost negligible for retail investors. These findings are in line with results reported in the study of Torricelli and Brunetti (2003). In conclusion, it can be said that the Italian index options market was efficient during the sample period studied.

The contributions of this paper are two-fold. The first contribution is providing new evidence about the efficiency of the Italian options market through a more comprehensive and updated sample compared to previous studies and after taking into account the different economical changes that have happened in this market. The second contribution is related to the financial crisis where we compare the market efficiency of the Italian options market during the crisis with that during the post crisis period.

The remainder of the paper is organized as follows: Section 2 covers the prior literature and provides the theoretical framework and main hypotheses. The dataset and methodology are outlined in section 3. Section 4 provides the empirical findings. Section 5 concludes.

2. Literature review and hypotheses development

Put-call parity and lower boundary conditions as evidence of cross-market efficiency have long been studied in the literature since the seminal work of Stoll (1969) and Merton (1973). The Lower Boundary Condition (LBC) was first investigated by Merton (1973) who tested this condition on European call options traded on individual stocks. However, this conditional test remained exclusive to European calls until Galai (1978) who modified it to accommodate American calls. The LBC is a weaker form of test for cross-market efficiency compared to the PCP and the occurrence of violations only supports the weak form of market inefficiency but does not indicate any higher form of market inefficiency as tested by PCP. The majority of empirical tests on the LBC have investigated the US markets such as Galai (1978), Bhattacharya (1983), Figlewski (1989), and Evnine and Rudd (1985). In addition, several studies have been conducted on European markets. The majority of these papers resonate with studies on the American market showing that a significant number of violations are reported while testing the LBC. The main studies are Trautmann (1985) on the German market, the DAX, Puttonen (1993) on the Finnish market and Chesney et al. (1995) on the Swiss market. However, the numbers of these mispricings were reduced remarkably when transaction costs are taken into account.

Put-Call parity was first outlined in the seminal paper of Stoll (1969) in which he found a relationship between the prices of a Call and its equivalent Put - traded on the same underlying asset with the same strike and expiration date, and that this relationship must hold in order to avoid any risk-free profit. The concept of the PCP was modified and extended from its basic form by Merton (1973) which provided an inequality formula for American options. Furthermore, Klemkosky and Resnick (1992) and Cox et al. (1979) studied the effect of dividend payments on the PCP for both European and American options. Finally, Cusack (1997) incorporated different components of transaction costs such as Brokerage commissions, index trading fees, short selling, taxes and bid/ask spreads into the calculation of PCP.

According to Fama (1970), market efficiency is based on the assumption that market prices fully incorporate expectations and information from all market participants. Three forms of market efficiency are commonly discussed in the literature known as the weak, semi-strong, and strong market efficiency. When 'efficiency' is mentioned in this study, it

refers to the weak form of market efficiency since we consider the historical market prices of both options and their underlying index.

The empirical literature on market efficiency indicates three distinctive ways to test the efficiency of the options market (Capelle-Blancard and Chaudhury, 2001). The first approach is based on a time series study of the predictability of implied volatility and whether a profitable arbitrage could be generated by implying historical volatility or the GARCH model to delta-neutral volatility trading. The pioneering investigations in this field are Latane and Rendleman (1976); Whaley (1982); Day and Lewis (1992). The second approach consists of comparing observed options prices with estimated prices from theoretical options pricing models. This approach was developed by Cohen et al. (1972) and has been used in many subsequent studies (e.g. Black and Scholes (1973); Galai (1978); Finnerty (1978)). The previous two approaches are based on the principle of jointly testing the efficiency of the options market, the level of synchronisation between prices in the realised and estimated options markets and the validity of the theoretical option pricing models (Capelle-Blancard and Chaudhury, 2001) and they both share a major disadvantage concerning the estimation of the standard deviations of the underlying prices. This standard deviation is not observable in the market, and price volatility estimation needs to be done, a method which is intensively debated. The final approach examines the market prices of the options and their underlying and simply tests for any mispricing or violations in the no-arbitrage relationships. This approach avoids the dilemma of jointly testing the hypotheses of market efficiency and model specification. Moreover, it does not require any restrictions upon the stochastic process underlying the stock. The essential principle that lies behind the no-arbitrage pricing relationships is that the future cash flows from identical investment strategies should have the same price. Using this method eliminates dependency on any assumptions related to market price dynamics and traders' risk performances (Mittnik and Rieken, 2000b). In this paper, the third approach is used with a focus on cross-market efficiency tests. To implement this approach we apply two fundamental no-arbitrage strategies which are the lower boundary condition and the PCP condition.

2.1. The lower boundary condition

The basic principle of the lower boundary condition is that the price of an option can never be less than its intrinsic price. Therefore, under the assumptions of a frictionless market², the market is considered to be efficient with respect to this condition at any time t if the following condition holds:

$$C_{t} \ge \max\left(0, I_{t} - Ke^{-rT}\right) \tag{1}$$

Where (C) is the price of a European call option with an exercise price of (K), the time to maturity is (t), the index value is (I_t) , and (r) is the risk free rate.

In case this condition is violated, arbitrageurs will be able to generate a riskless free profit by buying the call $(-C_t)$, short selling the index (I_t) and lending the amount (Ke^{-rT}) at the risk free rate. This violation can lead to an immediate cash inflow of $I_t - Ke^{-rT} - C_t > 0$. By holding this position till the option's maturity, two possibilities arise:

- **a.** If $I_T > K$, the call is exercised $(I_T K)$ and the index is bought and returned to the owner $(-I_T)$, and since the investment has grown to (K), that leads to a cash flow equal to zero.
- **b.** If $I_T < K$, the option will not be exercised and expires worthless (a value of zero), and the index is bought at the current price $(-I_T)$. That leads to a profit of $K I_T > 0$. Therefore, since this strategy leads to a positive cash inflow at time t and a nonnegative payoff at time t, an exploitable arbitrage opportunity arises.

On the other hand, the lower boundary condition for the European put option is:

$$P_{t} \ge \max\left(0, Ke^{-rT} - I_{t}\right) \tag{2}$$

Another arbitrage strategy can be performed if a violation occurs for this condition. Borrowing (Ke^{-rT}) at the risk free rate and buying the index and the put. This leads to an immediate cash inflow of $Ke^{-rT} - I_t - P_t > 0$. Again, the terminal payoff depends on the status of the index price and whether it is higher or lower than the exercise price. If $I_T < K$, then the put is exercised $(K - I_T)$, the debt is repaid (-K) and the index is sold (I_T) leading to a zero payoff. If $I_T > K$, the put will be worthless at expiration, the debt is paid (-K) and the index is sold (I_T) leading to a cash inflow of $I_T - K > 0$.

The testing of the lower boundary conditions was through implementing the following equations:

² e.g. no dividend payments on the underlying assets, the lending and borrowing rates are equal, no commission fees ...etc.

$$\varepsilon_t^C = I_t - Ke^{-rt} - C_t - T_t^C \le 0 \tag{3}$$

$$\varepsilon_t^P = Ke^{-rt} - I_t - P_t - T_t^P \le 0 \tag{4}$$

where ε_t^C : Ex-post profits from call option; ε_t^P : Ex-post profits from put option; T_t^C : Transaction costs for call options; T_t^P : Transaction costs for put options.

2.2.Put-Call parity

A theoretical relationship exists between the put price, call price and other relevant variables such as the exercise price, underlying price, time to maturity, and risk-free rate. When these variables are given for a certain put (call) price, there will exist a unique theoretical call (put) price. An arbitrage opportunity occurs when the realised put (call) price is different from its theoretical put (call) price. Therefore, a trader can profit by generating a riskless return.

Consider the construction of a portfolio consisting of a European call option with an underlying asset and another portfolio consisting of an identical (same exercise price and maturity date) put option with the same underlying asset forming two sets of securities. If these two portfolios have the same payoffs then they must have the same value and the call (Put) option can be replicated by the combination of its identical put (call) with the same underlying asset. This relationship gives rise to the Put-Call parity condition for European options which states that at any time (t) the relationship below must hold:

$$C_t = (P_t + I_t - Ke^{-rT})$$
(5)

This equation clearly states that buying a call directly is equal to buying its synthetic call. That is, buying the identical put (P_t) and the asset (I_t) and borrowing $(-Ke^{-rT})$ at the same time using a risk free rate. There exist two scenarios for this equation. If $I_T > K$ at maturity, the terminal call value will be $I_T - K$. On the other side of the equation, the put option will expire as worthless with a zero cash flow, liquidating the debt (-K) and selling the index (I_T) will lead to the same cash flow of $I_T - K$. In contrast, if $I_T < K$ the call will expires as worthless with a zero payoff while the put value will be $K - I_T$ and liquidating the portfolio will result in $I_T - K$ leading to the same payoff of the call. Therefore, a call can be replicated with its identical put at any time without any risk.

The same situation can be conducted with respect to a 'long' put. This can be converted at any time into a call by buying a call $(-C_t)$, lending (Ke^{-rT}) and shorting the asset (I_T) . Again, the payoff of both portfolios at time T is identical, regardless of the asset value at maturity and whether it is higher or lower than the exercise price.

However, in the case of options mispricing in the market, this equation will not hold. For example, if the call option is priced too high relative to its identical put price, the trader can exploit a riskless arbitrage by writing the overpriced call and simultaneously buying an underpriced synthetic call, generating an immediate cash inflow of $C_t - P_t - I_t + Ke^{-rT} > 0$ and zero payoff at maturity. This strategy is known as a conversion strategy. The opposite scenario is when the put is priced relatively higher than the call price (this will motivate traders to buy the underpriced call) and simultaneously writing a synthetic call, leading to an immediate cash flow of $P_t - C_t + I_t - Ke^{-rT} > 0$ and a zero payoff at maturity. This strategy is known as a reversal strategy.

The option of choosing any of these strategies depends on whether the overpriced option is a call or a put. A conversion strategy is taken when the call is overvalued relative to the put price, leading to an immediate cash inflow and a zero cash at time T, according to the following formula:

$$\operatorname{Long} C_t + \operatorname{X} e^{-rt} - P_t + I_t \ge 0 \tag{6}$$

In contrast, the reversal strategy is taken when the put is overpriced relative to the call price, leading to an immediate cash inflow and zero cash at time T, according to the following formula:

Short
$$P_t + I_t - C_t - Xe^{-rt} \ge 0$$
 (7)

According to Jensen (1978) and Galai (1978), as long as there is no investor able to constantly make a risk-free profit after imposing all market restrictions such as taxes and transaction costs, this market is considered to be an efficient market. Under the framework of this statement, if at any point in time the PCP holds, more precisely, if both call and put options are accurately priced relative to each other, then the options market will be considered an efficient one.

The FTSE/MIB index option is represented in index points with a notional value of 2.5 Euros per each point, which means that if the index value is 25,000, each contract has an

underlying value of 62,500 Euros. The mean $(\mbox{\ensuremath{\mathfrak{E}}})$ value of violations reported for call options is calculated as the average of the arbitrage profit $(\mbox{\ensuremath{\mathfrak{E}}})$ for each LBC deviation and PCP deviation is calculated as per equation (6) for each call and equation (7) for each put, multiplied by the monetary contract size of $\mbox{\ensuremath{\mathfrak{E}}}2.5$.

2.3. Hypotheses

Based on the theoretical foundations presented above, the following hypotheses are designed to be tested in order to draw conclusions related to the efficiency of the Italian index options market:

 H_1 : There are no significant violations reported when testing the lower boundary condition.

Failing to reject H_1 implies that the Italian index options market is efficient.

 H_2 : There are no significant violations reported when testing the put call parity relation.

Failing to reject H_2 implies that the market is efficient.

 H_3 : The number of violations occurring in both "PCP and LBC" tests will be reduced through the implementation of transaction costs in each scenario.

Testing these hypotheses confirms the importance of market frictions in determining the efficiency of the index options market.

3. Data and Methodology

3.1.Market and contract specification

This study is conducted on the Italian Derivatives market "IDEM", in particular, the FTSE/MIB option index. The FTSE/MIB is a portfolio that equally weights the performance of 40 Italian listed equities, which represent approximately 80% of the whole domestic market capitalisation. It is comprised of highly liquid, pioneering firms across diversified economic sectors by adhering to the Industry Classification benchmark (ICB) traded on the Borsa Italia (BIT) main equity market, after adjustments for rights issues, stock splits and so forth. The 10 economic sectors that underlie the FTSE/MIB are; Utilities, Financial, Healthcare, Consumer service, Oil & Gas, Industrials, Technology, consumer goods, Materials, and Telecommunications.

The FTSE/MIB index option is based on European style options contracts which are exercisable only on their day of expiry. This index option is represented in index points with a notional value of 2.5 Euros per point, which means that if the index value is 25,000, each contract has an underlying value of 62,500 Euros.

The expiration date for each option is the third Friday of the expiration month if the Exchange is open; otherwise, it is the previous day when the Exchange is open. At expiration in the money (ITM), options are automatically exercised. Everyday options with six different expiration dates are available: four quarterly ending in March, June, September, and December and the two nearest non-quarterly months. The strike prices of an option contract have integer multiples of 500 index points. In each option series (calls and puts), at any given time, there are at least nine different strikes for each expiration, quoted as follows: one at the money, four in, and four out of the money. The contracts are cash settled by the Italian Clearing House, Cassa di Compensazione e Garanzia (CC&G), which also calculates and manages the margins.

3.2.Data set

Our sample is limited to option contracts traded on the FTSE/MIB option index during the period between 1st October 2007 and 31st December 2012. The data is extracted from the DataStream database resulting in a final sample of 61,030 pairs of matched call and put options.

Three types of data are used to construct the market efficiency tests: (i) data associated with the underlying asset which consists of the daily closing prices of the FTSE/MIB index and its annual dividend yield; (ii) data associated with option contracts including option name, daily closing prices of both call and put options, strike prices, maturity dates, option types, deal dates and number of contracts; (iii) the risk-free rate of return defined as the monthly risk free interest rates which have been collected from the LIBOR website.

In order to conduct this study we have to apply some filtering to the original dataset, therefore, a discussion of the following criteria will be further explained:

3.2.1. Price synchronicity

The high number of violations reported in some studies can be attributed to non-synchronous data. A number of studies have used weekly and even monthly closing prices, which leads to

more violations compared to studies using high-frequency data. Recently, empirical researchers have improved their data set quality by using tick-by-tick data. In other words, matching call and put option contracts within a 15, 10, 5 or even 1-minute interval leading to a high level of synchronisation between the option prices and the underlying index. Among those studies, Cavallo and Mammola (2000) used intra-day quotes captured every 15 minutes and Capelle-Blancard and Chaudhury (2001) used even higher frequency data captured every minute. It is worth noting that using a wider interval in the data set will permit use of a larger sample size. However, increasing the interval spread between prices to 30 minutes or 1-hour intervals will raise the chances of stale prices.

Since this study will focus on the daily closing prices of the options and their underlying index, following the studies conducted by Ackert and Tian (2001) and Kamara and Miller (1995), we will retain only those pairs of put/call options traded on the same day then match each pair with the same day FTSE/MIB index quotation.

In addition, to mitigate the bias arising from the non-synchronous trading between options and the index, at any given day, any option price (Call or Put) with a value of zero will be directly eliminated from our sample (Dixit et al., 2009).

3.2.2. Maturity and strike matching

The matching process is carried out as follows: First, each call is matched with a put that has been traded on the same day, if there is no match for the call in that day the call will be eliminated. Each of these pairs of call/put must have the same strike and maturity.

3.2.3. Index adjustment for dividend

When the asset underlying an option is a stock index, the PCP relation has to be adjusted for dividends paid on the relevant stocks during the lifetime of the option. There are two cases when accounting for the dividend payments. Firstly, the index may be a performance index, adjusted for dividend payments, e.g. the DAX index (Mittnik and Rieken, 2000a). Secondly, if the index is a capitalised index and not a performance one, it is not adjusted for dividends and an estimation of the dividend payments should be applied.

Cavallo and Mammola (2000) used options with a maturity of one month, and during the life of these options they did not record any dividend payments on the underlying stocks since dividend payments occur after the expiration day of the option, so they do not affect the options with 1-month maturity used for the empirical tests. However, Capelle-Blancard and Chaudhury (2001) assumed that arbitrageurs know exactly the amount of dividends paid on the CAC40.

Incorporating transaction costs and dividend payments in equation (3) in order to make the PCP as realistic and testable as possible leads to the following equations:

$$(C^{ask} + TC^c) - (P^{bid} - TC^p) + Xe^{-rt} - (\gamma(I_t^{bid} - D) - TC^s) \ge 0$$
(8)

$$(P^{ask} + TC^p) - (C^{bid} - TC^c) + (I_t^{ask} - D + TC^s) - Xe^{-rt} \ge 0$$

$$(9)$$

where $I_o^{bid/ask} = bid/ask$ index price; $C/P^{ask} = ask$ call/put price; $C/P^{bid} = bid$ call/put price; D present value of the dividends paid on the index up to time T; $TC^c = call$ transaction costs; $TC^p = cost = cost$

Since the FTSE/MIB is not a performance index, an adjustment for dividends should be conducted. The dividend yield for an index is the total dividend amount for the index expressed as a percentage of the total market value of the constituents of that index. The following method used is determined by the source index agency in the (IDEM).

$$DY_{t} = \frac{\sum_{1}^{n} (D_{t} * N_{t})}{\sum_{1}^{n} (P_{t} * N_{t})} * 100$$
 (10)

where DY_t is the aggregate dividend yield on day t, D_t is the dividend per share on day t, N_t is the number of shares in issue on day t, P_t is the unadjusted share price on day t, and n is the of constituents in the index.

By taking the weighted average of aggregated dividend yields for each day t which are derived from DATASTREAM, a percentage of ($\delta = 3.97\%$) will be used as the constant dividend yield, hence the price of the underlying index (I_t) will be replaced by ($I_t e^{-\delta T}$) in this study.

3.2.4. Risk-free rate

Most studies conducted in the European markets rely on the interbank bid and offer rates (Mittnik and Rieken, 2000a) or on the interbank offer rate adjusted for bid/ask spread (Cavallo and Mammola, 2000). Moreover, it is worth mentioning that the risk-free rate used in the PCP to test for arbitrage opportunities is also used in calculating the short selling costs in checking the short strategy.

In line with studies such as Torricelli and Brunetti (2003) and Cavallo and Mammola (2000) the Euribor rate has also been used for 1, 2, 3, 6 and 12 months in order to facilitate the comparability with other studies and because choosing another rate (e.g. IRS rate) will not materially affect the results.

3.2.5. Estimation of transaction costs

Shleifer and Vishny (1997) stated that a precise estimation of the accurate costs for different strategies is very challenging and sometimes might be impossible. However, it is very important to consider transaction costs when studying the efficiency of options markets. Transaction costs include several types of costs such as bid/ask spread, commission costs, clearing fees, short selling costs, etc. The main costs taken into account in the most recent studies are the bid/ask spread and the commission costs, whereas the other costs are estimated to be negligible and therefore eliminated from the study.

The main papers studying efficiency in European markets such as Mittnik and Rieken (2000a); Capelle-Blancard and Chaudhury (2001); Cavallo and Mammola (2000) state that arbitrage profits are eliminated when accounting for transaction costs in the arbitrage strategies. The main difficulties in calculating these costs is that they tend to vary over time, market liquidity (positive bid/ask spread vs. zero bid/ask spread), agent type (retail investors vs. market makers), depend on the strategy used (short vs. long) and the size of the transaction (Berkman, 1996; Torricelli and Brunetti, 2003). According to these difficulties, researchers make different assumptions regarding transaction costs. Some apply a single scenario for all costs (Mittnik and Rieken, 2000a) while others base their studies on multiple scenarios such as Cavallo and Mammola (2000) and Capelle-Blancard and Chaudhury (2001).

It should be noted that in general, bid/ask spreads are not readily available, however, they are observable. Therefore, most studies apply the estimation method based on a predetermined sample of bid/ask spread then assume this spread is constant over time (Stoll, 1969; Phillips and Smith, 1980; Capelle-Blancard and Chaudhury, 2001). On the other hand, commission costs are observable but difficult to set since they vary dramatically from one investor to another and depend on the size and credibility of the traders (Torricelli and Brunetti, 2003).

This study will be following Torricelli and Brunetti (2003) in the estimations of transaction costs. In the period of this study, both Repo and Euribor's interest rates are very

similar and low. Therefore, short selling costs, which are affected mainly by the offer rates of the risk-free rate, are considered to be negligible. The same applies to the clearing fees in the Italian market (Cavallo and Mammola, 2000; Torricelli and Brunetti, 2003). Therefore, both short selling costs and clearing fees are eliminated and the main attention will be on the bid/ask spread and the commission costs.

As mentioned before, commission costs are observable but difficult to set. This fact is particularly true in the Italian options market where trading is extremely diversified. The commission fees vary according to the type of arbitrageurs and the means of trade. For example, market makers making high numbers of transactions face very low commissions compared to a higher commission charged for retail investors making a lower number of transactions.

In order to facilitate the implementation of PCP tests, an assumption that four different types of traders exist with four different commissions' levels will be introduced as follows: (i) high: equal to 40 Euros per option traded, representing retail investors who occasionally trade options; (ii) medium high: equal to 25 Euros per option traded, representing retail investors who trade online; (iii) medium low: equal to 10 Euros per option traded, representing professional investors with low volume of transactions or active retail investors; (iv) minimum: equal to 1 Euro per option traded, representing arbitrageurs who realise high volume of transactions.

For the period under investigation, unfortunately, bid/ask prices for both options and the underlying index are not readily available. Therefore, an estimation process will be conducted. For the option spread quotation, if the bid/ask spread for a certain period is not available, an estimation of the average bid/ask spread can be made based on the bid/ask quotations for another sample in another time period. Then, the average derived will be assumed as a constant number during the period studied (Phillips & Smith, 1980). This method is widely used in the recent literature (Capelle-Blancard and Chaudhury, 2001; Torricelli and Brunetti, 2003) and will be used in this study.

Using the daily bid and ask quotations for the FTSE/MIB options available on Yahoo Finance for the whole sample period, resulted in a value of 0.879 (1.104) of the trading price, as the mean bid (ask) price. These numbers multiplied by the daily closing prices, provides an estimation of the bid/ask option quotations. Despite the bias that may occur using this

method, there are two supportive arguments for the choice of this bid/ask spread. First, the results derived are in line with the asymmetry of bid/ask spread, as described in Nordén (2003) and Chan and Chung (2012). Second, the spread obtained falls within the maximum and minimum bounds allowed, imposed by the Boras Italia on the market makers. Regarding the FTSE/MIB bid/ask quotations they are considered as negligible and therefore ignored in this study, following the assumptions of Torricelli and Brunetti (2003).

4. Results

This section presents the impact of arbitrage profitability on the efficiency of Italian option index FTSE/MIB by using the LBC and PCP for the period between the 1st October 2007 to the 31st December 2012. The sample period is then divided into the crisis period from 1st October 2007 to 31st December 2009 and the post crisis period from 1st January 2010 to 31st December 2012. For each test, the analysis is divided into three different scenarios to better emphasise the role of transaction costs in determining the number of violations. The first scenario (scenario A) excludes transaction costs and the bid/ask spread. The second scenario (scenario B) includes only the bid/ask spread. The third scenario (scenario C) includes transaction costs (both bid/ask spreads and commission levels).

4.1. Lower boundary condition test

4.1.1. Scenario A

This is a preliminary scenario where the mispricing of options is tested without the implementation of transaction costs in the dataset. The first set of results are presented in Table 1, which shows that the number of violations in call prices are 1,323 (5.28% out of 25,044) for the crisis period then it drops during the post crisis period to 492 (1.37% out of 35,986) indicating a higher efficiency in pricing these options. The total call violations for the whole sample are 1,815 (2.97% out of 60,030). On the other hand, the frequency of put violations shows more dramatic mispricing with a total number of 5,496 (8.83%) for the whole sample, which is more than triple that of the call options, 2,623 (10.47%) violations occurred during the crisis period and 2,873 violations during the post crisis period; however, the overall percentage of violations during the post crisis period is lower than the crisis period (7.69%).

[Insert Table 1 here]

These results indicate that arbitrageurs can profit from the mispricing of both call and put options to create synthetic portfolios and generate risk free profits, especially from put options that have a higher percentage of mispricing. Despite the relatively high percentage of violations in both call and put options the size of mispricing is low with a mean of 58.05 and 64.05 Euros for the whole sample of call and put options, respectively.

4.1.2. Scenario B and C

In these scenarios both bid/ask spread and transaction costs are taken into account. In Table 2, the test results are presented for both calls and puts, covering the four suggested commission levels.

The effect of transaction costs implementation is to cause an enormous decrease in the number of violations. For both cases (calls and puts) the percentage of violations are no more than 1.5% of the total observations. In addition, there are only moderate differences in the number of violations between the four commission levels for both calls and puts across the six years. Furthermore, the numbers of violations recorded for put options for all the commission levels in the whole sample are higher than that of call options, indicating that put options are more exposed to violations than call options. Regarding the size of violations, the mean value is relatively small, below 200 Euros in all the observations. However, the mean value is higher for this scenario (with high transaction costs) where the mean of the whole sample is 60.5 (87.17) Euros for calls (puts) compared to that of scenario A with only 58.05 (64.05) Euros. On the other hand, the maximum values of exploitable profits have decreased substantially relative to scenario A. These results can be explained by the inclusion of transaction costs that lead to the elimination of the majority of small profits, leaving only the high profit opportunities exploitable and at the same time reducing their values.

[Insert Table 2 here]

To conclude, it is clearly observed that the inclusion of transaction prices into the LBC test has severely decreased the exploitable arbitrage opportunities resulting in a negligible number of violations. Therefore, it can be said that the Italian options market is efficient with regard to the LBC test.

4.2. PCP Ex-post tests

4.2.1. Scenario A

Tables 3 and 4 contain the ex-post test results with no bid/ask spread or transaction costs. In the analysis of this scenario, the focus will be on following points:

Firstly, due to the intrinsic nature of the strategies both long and short strategies are perfectly symmetrical and the numbers of profitable short strategies must be exactly identical to the number of non-profitable long ones. Therefore, only results with positive profits are reported. Secondly, as shown in Table 3, the short strategy is more profitable relative to the long one during all periods. In addition, the percentage of PCP violations to the whole violations in each year has experienced a dramatic decrease from the crisis period to the post-crisis period. The year 2007 had 49.47% (56.97) long (short) violations which dropped to 15.42% (19.31%) for long (short) violations in 2012.

The long and short strategies reported in Table 3 for the whole sample indicate a total number of violations of 15,083 (24.71% out of 61,030) and 23,641 (38.74% out of 21,470) respectively.

[Insert Table 3 here]

Even though the percentage of violations is quite high, that does not indicate the inefficiency of the Italian market since the transaction costs are not yet taken into account.

Finally, as can be derived from Table 4, despite the high frequency of violations, 91.12% and 87.53% of the total number of violations for long and short strategies, respectively have a profitability of below 200 Euros. However, some observations have profits above 200 Euros, including some exceptional cases where the maximum profits are 1629.15 and 1890.79, for long and short, respectively. For the whole sample of violations, the mean values are 373.99 Euros and 463.96 Euros for long and short strategies respectively which show that short strategies are more profitable than long ones. Furthermore, the mean Euro values of these violations are reduced during the post crisis period compared to the crisis period indicating better market efficiency after the crisis period.

[Insert Table 4 here]

4.2.2. Scenario B

In this scenario, the bid/ask spread, which is the most important part of the transaction costs (Demsetz, 1968), is included in the figures. Given that the index's bid/ask spread is negligible, only the bid/ask spreads for option prices are considered.

In this regard, the results are summarised in Tables 5 and 6. It should be noted that both positive and negative mispricings are reported in Table 5 because the symmetrical behaviour of both strategies no longer exists; this is due to the asymmetric nature of the bid/ask spreads included. In table 5, the number of violations reported shows a dramatic decrease in exploitable opportunities, compared to scenario A, dropping from 15038 (24.71%) and 23641 (38.74%) to 2223 (3.64%) and 3254 (5.33%) for long and short strategies, respectively. The percentage of violations reported experienced a gradual decline throughout the sample period for both strategies. For the long strategy the percentage of violations dropped from 6.6% in 2007 to 2.03% in 2012. Similarly, there was a drop for the percentage of violations for the short strategy from 8.76% in 2007 to 3.7% in 2012.

In line with the findings in scenario A, a short strategy is still more profitable than a long one during all periods because the option prices are, on average, closer to the bid than the ask spread and it can be seen that the sample contains more 'in the money put options' than 'in the money call options'.

[Insert Table 5 here]

Table 6 reports the size of the profits from the violations incurred for this scenario. Similarly to the previous findings reported in table 4, the majority of violations are still less than 200 Euros. For both long and short strategies the post crisis period experienced a decline in the average monetary value of violations compared to the crisis period. In addition, the whole sample mean values for both long and short strategies have decreased in this scenario compared to the findings in scenario A.

[Insert Table 6 here]

4.2.3. Scenario C

In this scenario all transaction costs are included (bid/ask spread and commission fees represented by four different levels as discussed earlier).

The results obtained from this scenario reflect the most realistic numbers of arbitrage opportunities available for the market investors since all the costs that can minimise the chances of risk-free profits are included.

Tables 7 and 8 present the frequency and size of violations during the sample period for both strategies. In Table 7, it is clear that the level of violations is reduced as the commission costs increase. In addition, it can be observed that the percentage of violations for the long strategy decreases significantly to range from 2072 (3.42%) to 966 (1.58%) according to investor type, for the whole sample, compared to (3.64%) in scenario B and (24.71%) in scenario A.

On the other hand, the level of violations in the short strategy are higher than those in the long strategy, ranging from 2961 (4.85%) to 1568 (2.57%) for the whole sample. However, these are still lower compared to (5.33%) in scenario B and (38.74%) in scenario A.

This huge reduction in the frequency of violations, starting with the implementation of the bid/ask spread, followed by the inclusion of transaction costs, reflects the importance of considering these costs when investigating PCP.

[Insert Table 7 here]

Table 8 shows the level of profits for the violations reported in this scenario. For the long strategy, it can be noted that in addition to the decrease of the number of violations from the crisis period to the post crisis period, the profits of these violations has declined as well. The minimum (maximum) profits during the crisis period with maximum and minimum commission levels were 176.64 (687.54) and 111.73 (1421.31) dropping to 213.64 (334.73) and 5.94 (1267.9) during the post crisis period indicating a better market efficiency. However, the situation is different for the short strategy. First, the mean monetary value of violations is higher than the reported means of the long strategy during both periods (except for the crisis period with maximum commission level and post-crisis period with medium low commission level). Second, a comparison between the means during the crisis and post crisis periods shows conflicting findings across the different commission levels. For example, despite the lower number of violations during the post crisis period compared to the crisis one, the means for medium low (394.67), medium high (409.53), and maximum (425.34)

commission levels are higher than the values during the crisis period with (356.79), (384.3), and (219.93), respectively.

[Insert Table 8 here]

To conclude, throughout the results obtained in each of the three scenarios, it is clear that the role of bid/ask spread and transaction costs are very important in determining the true level of market efficiency. Furthermore, the results reported in the last scenario together with the findings from the LBC tests show that the cross-market efficiency in the Italian index options market is substantial.

4.3. PCP Ex-ante test

Ex-post tests make the assumption that an arbitrageur creates a position (short or long) at prices prevailing at the same instant that the arbitrage opportunity exists (Klemkosky and Resnick, 1979). In other words, ex-post tests assume the ability to simultaneously execute trades to create arbitrage profits the instant they occur, which is practically unrealistic. Expost tests are unable to respond to the question of how fast mispriced option prices are adjusted by market forces. Capelle-Blancard and Chaudhury (2001), Ackert and Tian (2001), and Mittnik and Rieken (2000a) emphasise the important use of Ex-ante tests as an essential measure of how long the detected arbitrage persists to be exploited by investors or corrected by market forces. In order to establish an Ex-ante test, a time lag needs to be identified to eliminate the arbitrage opportunities.

The procedure of an Ex-ante test with a lag of h (could be seconds, minutes, days) can be explained as follows: For each Ex-post pair of options indicating a profitable trade, a scanning process for the whole sample is conducted to allocate a profitably matched pair available at the time (h). If no matched pair can be found in the sample within the next predefined execution lag, then the Ex-post profitable options pair will be excluded. In contrast, if a matched pair is found, the Ex-post options pair will be included in the sample, the profit or loss realised will be calculated using the new option prices and index level at lag (h).

Since the time lagged set of matched pairs might not be available for each violation, it should be noted that it is not necessary for the number of Ex-ante observations to match with that of Ex-post observations (Lee and Nayar, 1993). In addition, a major difference between Ex-post and Ex-ante tests is that the Ex-ante tests are not riskless arbitrage tests. Due to the

movements in market prices, execution of arbitrage opportunities might cause a loss rather than a profit (Li, 2006). Therefore, the Ex-ante observations will be defined as the total number of potential arbitrage opportunities that exist for the predefined execution lag, whereas the Ex-ante violations will be limited to the profitable ones only. The length of the execution lag depends on the study. For example, Mittnik and Rieken (2000a) and Li (2006) took multiple lags, ranging from 1 minute to 1 day, whereas Capelle-Blancard and Chaudhury (2001) took a 15-minute lag.

Ex-ante tests are another way to prove the efficiency of the Italian index options market and they can incorporate a parameter to test for the robustness of the ex-post tests carried out previously. So far, the ex-post tests applied can be used as an indication of any instantaneous possibility of exploitable arbitrage opportunities in the index options market. On the other hand, ex-ante tests indicate to what extent realising these opportunities, is possible. In testing the three scenarios with the ex-ante tests two key statistics have been calculated, known as percentage (a) and percentage (b);

$$a = ex - ante\ PCP\ violations\ / Total\ PCP\ observations$$
 (11)

$$b = ex - ante\ PCP\ violations\ / ex - ante\ PCP\ observations$$
 (12)

4.3.1. Scenario A

As previously explained in this scenario, no transaction costs are taken into account. Table 9 reports the arbitrage violations for both percentages a and b. Regarding percentage (a) the number of violations has dropped significantly to less than half of the numbers reported earlier. More precisely, the possibilities of any arbitrage opportunities while imposing a lag period of one day are reduced by more than one half. The potentially exploitable profit opportunities from the long (short) arbitrage strategies for the whole period dropped to 10.57% (16.63%). Similarly, percentage (b) reveals a slightly higher percentage of violations with 12.83% (20.18%) for long and short strategies – this slight increase is because of the small difference between the ex-ante and total number of observations. Furthermore, the percentages of (a) and (b) for both strategies declined substantially from 2007 to 2012 indicating a decrease of arbitrage opportunities throughout the years and especially during the post crisis period.

[Insert Table 9 here]

4.3.2. Scenarios B and C

As anticipated when testing the ex-ante violations while accounting for the bid/ask spread and the commission costs, the number of these violations are further reduced. Table 10 reports the number of instances of mispricing for both scenarios B and C. In scenario B, percentage (a) has been reduced to 3.60 (4.79%) for the long (short) strategy, while percentage (b) becomes 21.88% (29.07%) for the long (short) strategy. However, in scenario C, for percentage (a) the violations dropped significantly to range from 2.71% (4.03%) for arbitrageurs to 0.09% (0.15%) for retail investors in the long (short) strategy. On the other hand, for percentage (b) the test showed a higher number of violations for arbitrageurs with 16.47(24.49%) in the long (short) strategy to 0.56% (0.92%) in the long (short) strategy for retail investors. Therefore, it can be said that the results reported in Tables 9 and 10, for exante tests are additional proof of the validation of the efficiency results reported earlier using the ex-post tests.

[Insert Table 10 here]

5. Conclusion

This paper provides a fundamental test of the cross-market efficiency of the Italian FTSE/MIB index options contracts during the period from 1st October 2007 to 31st December 2012 which includes the period of the financial crisis. The investigation of any pricing violations is conducted using the arbitrage pricing relationships between the options and their underlying index, to be more precise, by testing the cross market efficiency with two fundamental conditions: the lower boundary condition and the PCP condition.

There are several factors that influenced the decision to test the Italian index options market. First, the limited number of studies conducted on this market. Second, the fact that the two main studies on this market, Cavallo and Mammola (2000) and Torricelli and Brunetti (2003) are relatively old, which makes it interesting to test the efficiency of this market with respect to a new set of data, taking into account the introduction of the Euro and the impact of the recent financial crisis on this market and whether the market efficiency hypothesis holds during the period of crisis and how it compares with the post crisis period. Third, it is important to consider the effect of the new rules applied to this market.

In this study, daily options' prices have been used, allowing for put and call pairs to be matched within a one-day interval. After applying various filtering criteria, our sample includes 61,030 pairs of matched call and put options.

A number of empirical tests are used in order to analyse different points. First, ex-post results for both lower boundary conditions and PCP are tested without the inclusion of any market frictions. Then the bid/ask spreads for option prices are included, followed by the inclusion of commission fees. Then, ex-ante tests are conducted in order to verify the robustness of the ex-post results.

With respect to the Lower Boundary Conditions in the first scenario, where no market frictions were considered, the frequency of violations showed a relatively high percentage of mispricing of 2.97% (8.83%) for calls and puts, respectively. The percentage of mispricing in the crisis period was substantially more than in the post crisis period. When both bid/ask spreads and transaction costs were considered, the percentages dropped significantly to between 0.41% and 0.14% (between 0.74% and 0.37%) with minimum to high commission levels.. Furthermore, the mean value of violations was moderate and did not exceed 200 Euros for the whole sample. There was somewhat higher percentage of violations in the crisis but they were still at a low level. In addition, it can be stated that throughout the whole period in all the scenarios, the put options violated the lower boundary condition more often than the calls. These results support market efficiency concerning the lower boundary conditions and therefore, the first hypothesis is confirmed.

With respect to PCP, once the bid/ask spread is included; the frequency of violations decreased severely from 24.71% (38.74%) to 3.64% (5.33%) for the long and short strategies, respectively. In this study, four different scenarios were established for estimation of commissions according to trader type, with the highest commission being applied to retail investors and the lowest to arbitrageurs. When these commissions were accounted for, the percentage of violations dropped further to 3.42% (1.58%) for the long strategy applied by arbitrageurs (retail investors). For the short strategy, the percentage was 4.85% (2.57%) for arbitrageurs (retail investors). As for the LBC tests the percentage of violations was somewhat higher during the crisis period. Furthermore, it can be noted that during the first two scenarios, a short strategy was more profitable than a long one. This can be related to put options being overpriced relative to calls. These findings are quite different from the study

conducted by Torricelli and Brunetti (2003) perhaps due to a recent rule partially limiting short selling in the Italian index market.

Even though we report a higher frequency of violations than other studies, it can be said that the Italian options market is weakly efficient since the higher percentages can be related to the use of daily data instead of the high-frequency data used in previous studies.

In order to check the robustness of the results reported for the PCP tests and to find out to what extent the ex-post arbitrage opportunities can be exploited, ex-ante tests were carried out. Results showed that the incidences of PCP violations dropped considerably, even in the absence of transaction costs, to 10.57% (16.63%) for the long (short) strategy. When transaction costs are included, the percentage of violations decreased even more so as to be almost negligible for retail investors with 0.56% (0.92%) for the long (short) strategy. These findings are in line with the results reported in Torricelli and Brunetti (2003) study. To conclude, it can be said that the Italian index options market was broadly efficient throughout the sample period studied and there is some indication that the market has become even more efficient since the financial crisis.

The efficiency analysis performed in this paper can be extended in two main directions: (i) in this paper, the main focus was on giving an overview of the frequency and level of violations in determining market efficiency through testing the relationship between exploitable arbitrage opportunities and the arbitrage strategies (short vs. long). Therefore, this study can be extended to test the relationships between arbitrage profitability and other factors such as the moneyness (in the money, out of the money, at the money) of options and the maturity of options. (ii) the market efficiency in this paper was tested by applying crossmarket efficiency tests (Lower boundary condition and PCP) further options market efficiency tests can be conducted such as call and put spreads, box spreads and put/call convexities (butterfly spreads).

References

- Ackert, L. F. & Tian, Y. S. (2001), "Efficiency in index options markets and trading in stock baskets." *Journal of Banking & Finance*, Vol. 25, No.9: pp. 1607-1634.
- Berkman, H. (1996), "Large option trades, market makers, and limit orders." *Review of Financial Studies*, Vol. 9, No.3: pp. 977-1002.
- Bhattacharya, M. (1983), "Transactions data tests of efficiency of the Chicago Board Options Exchange." *Journal of Financial Economics*, Vol. 12, No.2: pp. 161-185.
- Black, F. & Scholes, M. (1973), "The pricing of options and corporate liabilities." *The Journal of Political Economy*, Vol. 81, No.3: pp. 637-654.
- Capelle-Blancard, G. & Chaudhury, M. (2001), "Efficiency tests of the French index (CAC 40) options market," *EFMA 2002 London Meetings*, at
- Cavallo, L. & Mammola, P. (2000), "Empirical tests of efficiency of the Italian index options market." *Journal of Empirical Finance*, Vol. 7, No.2: pp. 173-193.
- Chan, K. & Chung, Y. P. (2012), "Asymmetric price distribution and bid-ask quotes in the stock options market." *Asia-Pacific Journal of Financial Studies*, Vol. 41, No.1: pp. 87-102.
- Chesney, M., Gibson, R. & Louberge, H. (1995), "Arbitrage trading and index option trading at Soffex: An empirical study using daily and intraday data." *Financial Markets and Portfolio Management*, Vol. 9, No.1: pp. 35-60.
- Cohen, J. B., Black, F. & Scholes, M. (1972), "The valuation of option contracts and a test of market efficiency." *The Journal of Finance*, Vol. 27, No.2: pp. 399-417.
- Cox, J. C., Ross, S. A. & Rubinstein, M. (1979), "Option pricing: A simplified approach." *Journal of Financial Economics*, Vol. 7, No.3: pp. 229-263.
- Cusack, A. J. (1997), Are there consistent abnormal profits arising from mispricing of options in the Australian Market. University of Melbourne, Department of Accounting and Finance.
- Day, T. E. & Lewis, C. M. (1992), "Stock market volatility and the information content of stock index options." *Journal of Econometrics*, Vol. 52, No.1: pp. 267-287.
- Demsetz, H. (1968), "The cost of transacting." *The Quarterly Journal of Economics,* Vol. 82, No.1: pp. 33-53.
- Dixit, A., Yadav, S. S. & Jain, P. (2009), "Violation of lower boundary condition and market efficiency: An investigation into the Indian options market." *Journal of Derivatives & Hedge Funds,* Vol. 15, No.1: pp. 3-14.
- Evnine, J. & Rudd, A. (1985), "Index Options: The Early Evidence." *The Journal of Finance*, Vol. 40, No.3: pp. 743-756.
- Fama, E. F. (1970), "Efficient capital markets: A review of theory and empirical work*." *The Journal of Finance*, Vol. 25, No.2: pp. 383-417.
- Figlewski, S. (1989), "Options arbitrage in imperfect markets." *The Journal of Finance*, Vol. 44, No.5: pp. 1289-1311.
- Finnerty, J. E. (1978), "The Chicago board options exchange and market efficiency." *Journal of Financial and Quantitative Analysis*, Vol. 13, No.01: pp. 29-38.
- Galai, D. (1978), "Empirical tests of boundary conditions for CBOE options." *Journal of Financial Economics*, Vol. 6, No.2: pp. 187-211.
- Jensen, M. C. (1978), "Some anomalous evidence regarding market efficiency." *Journal of Financial Economics*, Vol. 6, No.2: pp. 95-101.
- Kamara, A. & Miller, T. W. (1995), "Daily and intradaily tests of European put-call parity." *Journal of Financial and Quantitative Analysis*, Vol. 30, No.04: pp. 519-539.
- Klemkosky, R. C. & Resnick, B. G. (1979), "Put-Call Parity and Market Efficiency." *The Journal of Finance*, Vol. 34, No.5: pp. 1141-1155.
- Klemkosky, R. C. & Resnick, B. G. (1992), "A note on the no premature exercise condition of dividend payout unprotected american call options: A clarification." *Journal of Banking & Finance*, Vol. 16, No.2: pp. 373-379.

- Latane, H. A. & Rendleman, R. J. (1976), "Standard deviations of stock price ratios implied in option prices." *The Journal of Finance*, Vol. 31, No.2: pp. 369-381.
- Lee, J. H. & Nayar, N. (1993), "A transactions data analysis of arbitrage between index options and index futures." *Journal of Futures Markets*, Vol. 13, No.8: pp. 889-902.
- Li, S. (2006), The arbitrage efficiency of Nikkei 225 options market: a put-call parity analysis. Institute for Monetary and Economic Studies, Bank of Japan.
- Merton, R. C. (1973), "The relationship between put and call option prices: Comment." *The Journal of Finance*, Vol. 28, No.1: pp. 183-184.
- Mittnik, S. & Rieken, S. (2000a), "Lower-boundary violations and market efficiency: Evidence from the German DAX-index options market." *Journal of Futures Markets,* Vol. 20, No.5: pp. 405-424.
- Mittnik, S. & Rieken, S. (2000b), "Put-call parity and the informational efficiency of the German DAX-index options market." *International Review of Financial Analysis*, Vol. 9, No.3: pp. 259-279.
- Nordén, L. (2003), "Asymmetric option price distribution and bid—ask quotes: consequences for implied volatility smiles." *Journal of Multinational Financial Management*, Vol. 13, No.4: pp. 423-441.
- Phillips, S. M. & Smith, C. W. (1980), "Trading costs for listed options: The implications for market efficiency." *Journal of Financial Economics*, Vol. 8, No.2: pp. 179-201.
- Puttonen, V. (1993), "Boundary conditions for index options: evidence from the Finnish market." *Journal of Futures Markets*, Vol. 13, No.5: pp. 545-562.
- Shleifer, A. & Vishny, R. W. (1997), "The limits of arbitrage." *The Journal of Finance,* Vol. 52, No.1: pp. 35-55.
- Stoll, H. R. (1969), "The Relationship Between Put and Call Option Prices." *The Journal of Finance*, Vol. 24, No.5: pp. 801-824.
- Torricelli, C. & Brunetti, M. (2003), The Put-Call Parity in the Index Options Markets: Further results for the Italian Mib30 Options market. *Materiali di Discussione*. Dipartimento di Economia Politica, Università di Modena e Reggio Emilia
- Trautmann, S. (1985), "Distribution-Free Tests of the Efficiency of the Frankfurt Options Exchange from April 5, 1983 through September 28, 1984." *Geld, Banken und Versicherungen,* Vol. 1, No.1: pp. 87-1.
- Whaley, R. E. (1982), "Valuation of American call options on dividend-paying stocks: Empirical tests." Journal of Financial Economics, Vol. 10, No.1: pp. 29-58.

Table 1: Violations of the lo	ower boundary cor	ditions, scenario A	(No trading	costs)						
Davie d		Ca	ıll				Pu	ıt		
Period	Violations	Percentage	Mean	Min.	Max.	Violations	Percentage	Mean	Min.	Max.
1st Oct 2007 – 31st Dec 2007	243	13.39%	67.22	0.05	714.28	219	4.06%	78.34	0.7	437.99
1st Jan 2008 – 31st Dec 2008	963	53.06%	44.98	0.05	622.79	808	14.99%	55.85	0.03	623.67
1st Jan 2009 – 31st Dec 2009	117	6.45%	72.51	0.12	484.1	1596	29.60%	73.31	0.2	539.61
1st Jan 2010 – 31st Dec 2010	182	10.03%	55.36	0.03	567.21	1267	21.57%	54.02	0.07	700.24
1st Jan 2011 – 31st Dec 2011	176	9.70%	61.99	0.07	534.36	792	14.69%	66.39	0.05	433.88
1st Jan 2012 – 31st Dec 2012	134	7.38%	58.46	0.01	444.72	814	15.10%	56.56	0.01	510.07
Crisis period	1323	5.28%	62.54	0.05	714.28	2623	10.47%	67.11	0.7	623.67
Post crisis period	492	1.37%	57.55	0.01	567.21	2873	7.69%	59.75	0.01	700.24
Whole Sample	1815	2.97%	58.05	0.01	714.28	5496	8.83%	64.05	0.01	700.24

The table reports for each period and for the whole sample the number of call and put options that violate the lower boundary condition, the percentage of these violations over the subsamples' periods and descriptive statistics for the mean, minimum, and maximum value of violations reported in Euro. The mean $(\mbox{\ensuremath{\mathfrak{E}}})$ value of violations reported for call options is calculated as the average of the arbitrage profit $(\mbox{\ensuremath{\mathfrak{E}}})$ for each LBC deviation. The arbitrage profit $(\mbox{\ensuremath{\mathfrak{E}}})$ for each LBC deviation is calculated as per equation (3) for calls and equation (4) for puts, multiplied by the monetary contract size $\mbox{\ensuremath{\mathfrak{E}}}2.5$.

Period	Commission levels			Call]	Put		
renod	Commission levels	Violations	Percentage	mean	Min	Max	Violations	Percentage	mean	Min	Max
	Minimum	28	0.78%	62.72	1.99	651.24	31	0.87%	68.23	3.66	408.64
1st Oct 2007 – 31st Dec	Medium low	22	0.62%	58	2.22	623.11	23	0.64%	57.88	3.22	400.6
2007	Medium High	17	0.48%	53.11	2.47	573.64	15	0.42%	61.06	2.98	387.9
	High	13	0.36%	46.93	3.37	546.91	7	0.20%	59.11	2.41	331.6
	Minimum	143	1.26%	92.80	0.07	507	108	0.95%	175.53	7.15	612.0
1st Jan 2008 – 31st Dec	Medium low	115	1.02%	97.66	1.12	497	99	0.87%	166.49	6.70	602.0
2008	Medium High	48	0.42%	101.25	2.27	472	79	0.70%	169.86	6.20	577.0
	High	35	0.31%	114	19.30	432	69	0.61%	159.62	1.01	537.0
	Minimum	10	0.10%	89.11	1.74	359.40	46	0.45%	154.43	0.49	494.7
1st Jan 2009 – 31st Dec	Medium low	10	0.10%	76.11	4.13	349.40	45	0.44%	152.71	1.94	484.7
2009	Medium High	8	0.08%	75.78	16.74	324.40	37	0.36%	134.00	2.25	459.7
	High	8	0.08%	71.78	29.13	284.40	35	0.34%	126.16	6.19	419.
	Minimum	18	0.14%	44.64	0.33	487.64	120	0.95%	78.66	4.22	632.3
st Jan 2010 – 31st Dec	Medium low	15	0.12%	49.06	0.89	411.72	66	0.52%	69.2	5.11	622.7
2010	Medium High	15	0.12%	38.22	1.69	410	48	0.38%	51	9.2 5.11 51 5.55	598.3
	High	14	0.11%	52.88	2.46	396.33	43	0.34%	67.58	7.62	577.4
	Minimum	13	0.11%	53.76	0.1	477.77	71	0.62%	53.91	0.9	410.3
1st Jan 2011 – 31st Dec	Medium low	11	0.08%	55.08	0.93	423.62	59	0.51%	51.08	4.07	397.4
2011	Medium High	9	0.08%	45.81	1.97	399.05	42	0.37%	62.47	5.16	394.0
	High	6	0.05%	41.22	2.08	389.36	35	0.31%	58.89	8.43	380
	Minimum	21	0.18%	56.95	0.09	412.35	49	0.41%	61.66	1.19	499.1
1st Jan 2012- 31st Dec	Medium low	15	0.13%	51.64	2.19	391.61	42	0.35%	67.91	3.54	486.3
2012	Medium High	7	0.06%	48.82	2.47	366.72	31	0.26%	42.1	5.91	477.
	High	7	0.06%	39.2	3.16	321.11	24	0.20%	54.65	7.62	380.9
	Minimum	233	0.41%	66	0.07	651.24	425	0.74%	95.16	0.49	632.
Whole Sample	Medium low	188	0.33%	64.3	0.09	623.11	334	0.58%	93.66	1.01	622.
whole sample	Medium High	104	0.18%	60	0.33	573.64	252	0.44%	86.5	1.94	598.
	High	83	0.14%	60.5	2.08	546.91	213	0.37%	87.17	2.25	577.

The table reports the number of call and put options that violate the lower boundary condition including transaction costs divided over the different commission levels, the percentage of these violations over the subsamples' periods and descriptive statistics for the mean, minimum, and maximum value of violations reported in Euro. The mean (\mathfrak{E}) value of violations reported for call options is calculated as the average of the arbitrage profit (\mathfrak{E}) for each LBC deviation. The arbitrage profit (\mathfrak{E}) for each LBC deviation is calculated as per equation (3) for each call and equation (4) for each put, multiplied by monetory contract size $\mathfrak{E}2.5$.

Period	Strategy						
Terrou	Long	Short					
1 st Oct 2007 – 31 st Dec 2007	1768 (49.47%)	2036 (56.97%)					
1 st Jan 2008 – 31 st Dec 2008	4416 (39.02%)	6902 (60.98%)					
1 st Jan 2009 – 31 st Dec 2009	2982 (29.37%)	7170 (70.63%)					
1 st Jan 2010 – 31 st Dec 2010	1945 (15.38%)	2697 (21.32%)					
1 st Jan 2011 – 31 st Dec 2011	2136 (18.68%)	2537 (22.19%)					
1 st Jan 2012 – 31 st Dec 2012	1836 (15.42%)	2299 (19.31%)					
Whole Sample	15083 (24.71%)	23641 (38.74%)					

The table reports the number and the percentage of PCP violations registered for each period and over the whole sample. No cases found of perfectly null profit.

Table 4: Descriptive statistics for	the profit of long a	and short strategies,	scenario A (No tr	rading costs)						
Profit Range			Long					Short		
1 Tont Range	Mean	Max	Min	Obs.	Percentage	Mean	Max	Min	obs.	Percentage
Crisis period		Inclu	ided observations	: 9166			Include	d observations :	16108	
[0,200)	95.64	199.01	0.01	8358	91.18%	126.61	197.22	0.01	13533	84.01%
[200 , 400)	251.67	397.31	200.49	564	6.15%	276.91	391.34	204.23	2155	13.38%
[400 , 600)	484.66	576.86	403.74	134	1.46%	509.61	583.6	407.41	309	1.92%
[600,800)	691.3	789.73	607.64	78	0.85%	679.53	786.63	610.66	83	0.52%
[800, 1000)	820.69	887.36	812.25	25	0.27%	913.15	999.14	810.33	23	0.14%
[1000 , 1900	1483.16	1629.15	1318.00	7	0.08%	1817.62	1890.79	1725.69	5	0.03%
All	391.64	1629.15	0.01	9166	100%	473.64	1890.79	0.01	16108	100%
Post crisis period		Inclu	ided observations	: 5917			Includ	ed observations	: 7533	
[0,200)	87.03	185.64	0.01	5386	91.03%	93.12	173.36	0.03	6934	92.05%
[200 , 400)	220.33	365.13	200.47	243	4.11%	210.33	350.31	200.1	339	4.50%
[400 , 600)	420	490.66	410.23	196	3.31%	527.68	589.13	433.78	193	2.56%
[600 , 800)	638.66	718.94	600.31	66	1.12%	672.36	798.31	601.31	46	0.61%
[800, 1000)	836.88	873.36	829.13	22	0.37%	897.15	966.47	870.66	13	0.17%
[1000 , 1900	1240.99	1467.94	1010.36	4	0.07%	1652.37	1873.68	1567.98	8	0.11%
All	363.5	1467.94	0.01	5917	100%	426.16	1873.68	0.03	7533	100%
Whole sample		Inclu	ded observations :	15083			Include	d observations :	23641	
[0,200)	93.13	199.01	0.01	13744	91.12%	118.36	197.22	0.01	20693	87.53%
[200 , 400)	248.66	397.31	200.47	807	5.35%	260.16	391.34	200.1	2494	10.55%
[400 , 600)	440.67	576.86	403.74	330	2.19%	515.36	589.13	407.41	276	1.17%
[600,800)	673.96	789.73	600.31	144	0.95%	675.67	798.31	601.31	129	0.55%
[800, 1000)	826.14	887.36	812.25	47	0.31%	909.36	999.14	810.33	36	0.15%
[1000 , 1900	1386.36	1629.15	1010.36	11	0.07%	1713.94	1890.79	1567.98	13	0.05%
All	373.99	1629.15	0.01	15083	100.00	463.96	1890.79	0.01	23641	100.00

This table reports for each strategy (long and short) and each profit range: the mean, maximum, minimum, number of observations and percentage of the profit range over the whole sample. The mean (\mathfrak{E}) value of violations reported for call options is calculated as the average of the arbitrage profit (\mathfrak{E}) for each PCP deviation. The arbitrage profit (\mathfrak{E}) for each PCP deviation is calculated as per equation (6) for each call and equation (7) for each put, multiplied by monetory contract size $\mathfrak{E}2.5$.

able 5: Freque	ency of PCP vio	olations in scenario	B (Incorporating bio	d/ask spread)				
Strategy	Profits	1 st Oct 2007 – 31 st Dec 2007	1 st Jan 2008 – 31 st Dec 2008	1 st Jan 2009 – 31 st Dec 2009	1 st Jan 2010 – 31 st Dec 2010	1 st Jan 2011 – 31 st Dec 2011	1 st Jan 2012 – 31 st Dec 2012	Whole Sample
	Nagativa	3338	10681	9698	12363	11064	11663	58807
	Negative	93.4%	94.38%	95.53%	97.75%	96.77%	97.97%	96.36%
Lana	Danitian	236	637	454	285	369	242	2223
Long	Positive	6.6%	5.62%	4.47%	2.25%	3.23%	2.03%	3.64%
	Total	3574	11318	10152	12648	11433	11905	61030
		100%	100%	100%	100%	100%	100%	100%
	Nametina	3261	10323	9440	12328	10960	11464	57776
	Negative	91.24%	91.20%	92.99%	97.47%	95.86%	96.3%	94.67%
Cla a ut	Davidian	313	995	712	320	473	441	3254
Short	Positive	8.76%	8.79%	7.01%	2.53%	4.14%	3.7%	5.33%
	Total	3574	11318	10152	12648	11433	11905	61030
	Total	100%	100%	100%	100%	100%	100%	100%

The table reports the number and the percentage of PCP violations under scenario B where bid/ask spreads are included for each period and over the whole sample.

Profit Range			Long					Short		
From Kange	Mean	Max	Min	Obs.	%	Mean	Max	Min	obs.	%
Crisis period		Include	d observations: 13	27			Include	ed observations : 20	20	
[0,200)	117.63	191.20	0.02	1158	87.26%	126.94	181.28	0.04	1896	93.86
[200 , 400)	281.35	391.56	209.85	103	7.76%	223.63	368.22	209.45	89	4.41
[400,600)	503.64	541.00	407.22	36	2.71%	473.21	569.15	421.71	24	1.19
[600,800)	687.44	777.65	625.12	16	1.21%	657.36	755.55	609.42	10	0.50
[800, 1000)	848.54	867.51	831.21	13	0.98%	0	0	0	0	0%
[1000 , 1600)	1421.31	1421.31	1421.31	1	0.08%	1582.00	1582.00	1582.00	1	0.05
All	260.17	1421.31	0.02	1327	100%	293.16	1582.00	0.04	2020	100
Post crisis		Includ	ed observations : 89	96	Included observations: 1234					
[0,200)	92.64	185.64	0.01	673	75.11%	123.64	173.36	5.6	926	75.04
[200 , 400)	243.89	365.13	200.47	143	15.96%	306.99	350.31	269	214	17.34
[400,600)	431.61	483.69	412.46	46	5.13%	531.03	576.94	433.78	54	4.38
[600,800)	627.94	698.61	613.33	23	2.57%	669.04	746.64	620.72	26	2.11
[800, 1000)	847.64	873.36	829.13	10	1.12%	847.61	920.1	870.66	9	0.73
[1000 , 1600)	1267.9	1267.9	1267.9	1	0.11%	1473.64	1657.5	1567.98	5	0.41
All	248.61	1267.9	0.01	896	100%	253.96	1657.5	5.6	1234	100
Whole sample		Include	d observations : 22	223			Include	ed observations : 32.	54	
[0,200)	103.76	191.2	0.01	1831	82.37%	125.62	181.28	0.04	2822	86.72
[200 , 400)	253.14	391.56	200.47	246	11.07%	278.56	368.22	209.45	303	9.31
[400 , 600)	461.9	541	407.22	82	3.69%	506.32	576.94	421.71	78	2.40
[600,800)	662.28	777.65	613.33	39	1.75%	660.37	755.55	609.42	36	1.11
[800, 1000)	848.01	873.36	829.13	23	1.03%	847.61	920.1	870.66	9	0.28
[1000 , 1600)	1344.61	1421.31	1267.9	2	0.09%	1502.36	1657.5	1567.98	6	0.18
All	256.61	1421.31	0.01	2223	100%	279.41	1657.5	0.04	3254	1009

This table reports for each strategy (long and short) and each profit range: the mean, the maximum, the minimum, the number of observations and the percentage of the profit range over the whole sample. The mean (\mathfrak{E}) value of violations reported for call options is calculated as the average of the arbitrage profit (\mathfrak{E}) for each PCP deviation. The arbitrage profit (\mathfrak{E}) for each PCP deviation is calculated as per equation (6) for each call and equation (7) for each put, multiplied by monetory contract size $\mathfrak{E}2.5$.

G	D : 1		Commi	ssion levels	
Strategy	Period	Minimum	Medium low	Medium high	Maximum
	781 0 1000 0 181	213	164	143	97
	1 st Oct 2007 – 31 st Dec 2007	5.96%	4.59%	4%	2.71%
	1 st Jan 2008 – 31 st Dec 2008	629	492	309	215
	1 Jan 2008 – 31 Dec 2008	5.56%	4.35%	2.73%	1.90%
	1 st Jan 2009 – 31 st Dec 2009	443	370	248	200
	1 Jan 2009 – 51 Dec 2009	4.36%	3.64%	2.44%	1.97%
Long	1 st Jan 2010 – 31 st Dec 2010	271	220	181	133
	1 Jan 2010 – 31 Dec 2010	2.14%	1.74%	1.43%	1.05%
	1 st Jan 2011 – 31 st Dec 2011	316	308	224	175
	1 Jan 2011 – 31 Dec 2011	2.76%	2.69%	1.96%	1.53%
	1 st Jan 2012 – 31 st Dec 2012	200	183	161	146
	1 Jan 2012 – 31 Dec 2012	1.68%	1.54%	1.35%	1.23%
	Whole Sample	2072	1737	1266	966
	whole Sample	3.42%	2.85%	2.07%	1.58%
	1 st Oct 2007 – 31 st Dec 2007	246	181	163	117
	1 Oct 2007 – 31 Dec 2007	6.88%	5.06%	4.56%	3.27%
	1 st Jan 2008 – 31 st Dec 2008	891	742	562	467
	1 Jan 2006 – 31 Dec 2006	8.78%	7.31%	5.54%	4.60%
	1 st Jan 2009 – 31 st Dec 2009	705	612	503	402
	1 Jan 2007 31 Dec 2007	6.94%	6.03%	4.95%	3.96%
Sh4	1 st Jan 2010 – 31 st Dec 2010	294	257	226	184
Short	1 Jan 2010 31 Dec 2010	2.32%	2.03%	1.79%	1.45%
	1 st Jan 2011 – 31 st Dec 2011	452	389	314	223
	1 Jan 2011 31 Dec 2011	3.95%	3.4%	2.75%	1.95%
	1 st Jan 2012 – 31 st Dec 2012	373	299	213	175
	1 Jan 2012 - 31 Dec 2012	3.13%	2.51%	1.79%	1.47%
	Whole Sample	2961	2480	1981	1568
	Whole Sample	4.85%	4.06%	3.25%	2.57%

	Commission levels	Minimum Profit	Maximum Profit	Mean Profit	Total observations
	Crisis period				
	Maximum	176.64	687.54	331.6	512
	Medium high	164.47	768.31	290.65	700
	Medium low	156.89	823.31	273.2	1026
Long	Minimum	111.73	1421.31	239.46	1285
8	Post crisis period				
	Maximum	213.64	334.73	303.85	454
	Medium high	104.75	469.99	323.75	566
	Medium low	96.85	863.4	294.75	711
	Minimum	5.94	1267.9	56.72	711 787 986
	Crisis period				
	Maximum	143.94	634.73	219.93	986
	Medium high	143.06	703.97	384.3	1228
	Medium low	84.68	735.15	356.79	1535
Short	Minimum	10.58	1582	686.46	1842
	Post crisis period				
	Maximum	278.54	912.85	425.34	582
	Medium high	169.56	1571.36	409.53	753
	Medium low	79.64	1596.47	394.67	945
	Minimum	73.43	1657.5	165.7	1119

This table reports for each strategy (long and short) and each commission level: the minimum, the maximum, and the mean profit realized and the number of observed profitable portfolios over the whole sample. The mean (\mathfrak{C}) value of violations reported for call options is calculated as the average of the arbitrage profit (\mathfrak{C}) for each PCP deviation. The arbitrage profit (\mathfrak{C}) for each PCP deviation is calculated as per equation (6) for each call and equation (7) for each put, multiplied by monetary contract size $\mathfrak{C}2.5$.

	Ex-ante		Available observation	ons	Percentage					
	Long	Short	Total	Ex-ante	Long		Short			
	Long	Short	Total	Ex-ante	a	b	a	b		
1st Oct 2007 – 31st Dec 2007	844	915	3574	3418	23.61%	24.69%	25.6%	26.77%		
1st Jan 2008 – 31st Dec 2008	1702	3124	11318	9972	15.04%	17.07%	27.60%	31.33%		
1st Jan 2009 – 31st Dec 2009	1187	2691	10152	8629	11.69%	13.76%	26.51%	31.19%		
1st Jan 2010 – 31st Dec 2010	876	1210	12648	9845	6.93%	8.9%	9.57%	12.29%		
1 st Jan 2011 – 31 st Dec 2011	967	1164	11433	9436	8.46%	10.24%	10.18%	12.34%		
1st Jan 2012 – 31st Dec 2012	876	1048	11905	8997	7.36%	9.74%	8.8%	11.65%		
Whole Sample	6452	10152	61030	50297	10.57%	12.83%	16.63%	20.18%		

The table reports, for each period and for the whole sample, the number of long and short ex-ante violations, the number of available observations both in the total (ex-post) sample and in the ex-ante one and the percentage a and b for the two strategies recorded in scenario A.

		Scenario B		Scenario C]	
		TC=0		TC>0								Available (Observation
				1		2		3		4			
		L	S	L	S	L	S	L	S	L	S	Ex-ante	Total
	No. of violations	346	451	184	216	113	153	34	74	13	25		
urt o accompany	Percentage a	9.68%	12.62%	5.15%	6.04%	3.16%	4.28%	0.95%	2.07%	0.36%	0.7%	975	3574
1 st Oct 2007 – 31 st Dec 2007	Percentage b	35.49%	46.27%	18.87%	22.15%	11.59%	15.69%	3.49%	7.59%	1.33%	2.56%		
	No. of violations	412	815	348	752	102	301	42	180	4	9		
15 I 2000 215 D	Percentage a	3.64%	7.20%	3.07%	6.64%	0.90%	2.66%	0.37%	1.59%	0.04%	0.08%	2015	1131
1 st Jan 2008 – 31 st Dec 2008	Percentage b	20.45%	40.45%	17.27%	37.32%	5.06%	14.94%	2.08%	8.93%	0.20%	0.45%		
	No. of violations	415	648	399	502	99	246	39	176	5	6		
1st 1 2000 21st D	Percentage a	4.09%	6.38%	3.93%	4.94%	0.98%	2.42%	0.38%	1.73%	0.05%	0.06%	1785	10152
1 st Jan 2009 – 31 st Dec 2009	Percentage b	23.25%	36.30%	22.35%	28.12%	5.55%	13.78%	2.18%	9.86%	0.28%	0.34%		
	No. of violations	296	333	243	241	131	180	61	73	7	14		12648
S I 2010 215 D	Percentage a	2.34%	2.63%	1.92%	1.91%	1.04%	1.42%	0.48%	0.58%	0.056%	0.11%	1874	
1 st Jan 2010 – 31 st Dec 2010	Percentage b	15.8%	17.77%	12.97%	12.86%	6.99%	9.61%	3.26%	3.9%	0.37%	0.75%		
	No. of violations	418	441	308	410	217	302	97	167	17	21		
	Percentage a	3.66%	3.86%	2.69%	3.59%	1.9%	2.64%	0.85%	1.46%	0.15%	0.18%	1676	1143
1 st Jan 2011 – 31 st Dec 2011	Percentage b	24.94%	23.31%	18.38%	24.46%	12.95%	18.02%	5.79%	9.96%	1.01%	1.25%		
	No. of violations	312	233	173	340	124	202	57	73	10	17		
ort	Percentage a	2.62%	1.96%	1.45%	2.86%	1.04%	1.70%	0.48%	0.61%	0.08%	0.14%	1724	1190
st Jan 2012 – 31st Dec 2012	Percentage b	18.10%	13.52%	10.03%	19.72%	7.19%	11.72%	3.31%	4.23%	0.58%	0.99%		1190
	No. of violations	2199	2921	1655	2461	786	1384	330	743	56	92		
	Percentage a	3.60%	4.79%	2.71%	4.03%	1.29%	2.27%	0.54%	1.22%	0.09%	0.15%	10049	6103
Whole Sample	Percentage b	21.88%	29.07%	16.47%	24.49%	7.82%	13.77%	3.28%	7.39%	0.56%	0.92%		

The table reports, for each period and for the whole sample, the percentage a and b for the two strategies recorded in scenarios B and C and, in the last column, the number of available observations both in the total (ex-post) sample and in the ex-ante one.