Money and Economic Growth

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Abstract

We study results of the cash in advance and money in utility models about the nature of fluctuations in economic activities and welfare in three interdependent economies. When the money is exogenously introduced in the form of cash in advance, it serves as a medium of exchange the rate of return in real and nominal assets become equal. Idiosyncratic technological shocks generate fluctuations in the growth rates of capital, output, prices, money, consumption, investment, labour supply and lifetime utilities of households. When households have money in their utility functions, the stock of money in excess of that required for transactions causes inflation and reduces the amount of capital stock and output in these economies. Both CIA and MIU models support for a steady growth rate of money according to the smooth growth rate of output. While the inflation targeting by manipulating the interest rates for macroeconomic stability are theoretically prudent policy moves it is impossible for central banks to eliminate business cycles that arise from shocks to production technology or structural features of the economy.

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1 Money in Growth Models

It has been agreed for long that money serves economic purposes as a medium of exchange and unit of standard as well as a standard of differed payment and means of a store. Yet there remains a substantial debate about the neutrality and non-neutrality of money in the long run growth. While Tobin (1965) viewed that money in the form of public debt could be instrumental in channeling savings to investment and hence lead to the higher growth rate, Friedman (1968) opined the growth rate of money should not be greater than that of output for a smooth functioning economy. In theory Sidrauski (1967) showed the role of money in growth, putting money in the utility (MIU) function of an intertemporally optimising representative household and came to the conclusion that money is super neutral, will not have any real impacts and higher rate of growth of money only causes inflation. Similarly Brock (1973) had provided more extensive perfect foresight model to show contribution of money in economic growth. These early views on relations between money and growth are endorsed in subsequent works by Hayakawa (1986), Gomme (1993), Balasko (2003), Berentsen et al. (2012), Aruoba et al (2011). Hower there seem to be no explicit numerical analysis on showing fluctuations in macro economic variables under these theoretical exercises. Purpose of this paper is to assess whether to illustrate how these theoretical propositions may brought into numerical analysis and whetehr conclusions reached in those studies are robust enough to the way money is introduced in these model. Growth of money is exogenous in models with cash in advance (CIA) constraints or is endogenous in the models with money in the utility (MIU) functions. Is the super- neutrality proposition of money independent of the way money is introduced in the model? If so super-neutrality should hold in both of the CIA and MIU models. This issue is illustrated with simulations of popular CIA and MIU models discussed in Williamson (2008) and Walsh (1998) to the reasonable set of parameters characterising the three economies. These simulations provide some insights on the role of money in the growth of the economy.

2 Friedman Rule with Cash in Advance Constraint

How the financial sector can contribute most to the economic growth when stock of money grows according to the growth rate of output can be shown by solutions based on optimal conditions in a cash in advance monetary economy where households maximise lifetime utility $U(\bullet)$ from consumption (C_t) but experience disutility from labour from efforts put in work, $V(L_t)$. The problem of the economy is to maximize this utility (1) with technology (2), cash in advance (3) and lifetime budget constraints (4)as:

$$\max \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(L_t)\right] \tag{1}$$

Subject to the technology constraint:

$$Y_t = zL_t \tag{2}$$

and the cash in advance constraint

$$P_tC_t + q_tB_{t+1} + P_ts_tX_{t+1} + P_tT_t = M_t + B_t + P_tX_t \tag{3}$$

where P_tC_t is consumption expenditure, P_t price of goods, C_t consumption, B_{t+1} is the amount of nominal bonds, q_t is the price of nominal bonds, X_{t+1}real bonds, s_t prices of real bonds, T_t lump sum tax payment and M_t the stock of money. Budget constraint of the consumer include income from production and allocation of money for the next period.

$$P_tC_t + q_tB_{t+1} + P_ts_tX_{t+1} + P_tT_t + M_{t+1} = M_t + B_t + P_tX_t + P_tzL_t \tag{4}$$

Government controls the money supply and engages itself in inflationary tax. Its budget constraint for a particular time t is:

$$\overline{M}_{t+1} - \overline{M}_t = -P_tT_t \tag{5}$$

The stock of money grows at a constant rate α, thus $\overline{M}_{t+1} = (1+\alpha)\,\overline{M}_t$. With this provision, $\alpha\overline{M}_t = -P_tT_t$. Normalising the cash in advance and budget constraints by $\frac{1}{M_t}$ and denoting the real values in small case letters, the cash in advance constraint and budget constraints become

$$p_tC_t + q_tb_{t+1}\,(1+\alpha) + p_ts_tX_{t+1} + p_tT_t = m_t + b_t + p_tX_t \tag{6}$$

and

$$p_tC_t + q_tb_{t+1}\,(1+\alpha) + p_ts_tX_{t+1} + p_tT_t + m_{t+1}\,(1+\alpha) = m_t + b_t + p_tX_t + p_tzL_t \tag{7}$$

The representative agent in the economy chooses C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1} from $t = 0, 1, 2, ...$ to ∞. The Bellman value function for this problem is:

$$v\,(m_t, b_t, X_t, p_t, q_t, s_t)\underset{C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}}{max}\,[U(c_t) - V(L_t)] + \beta v\,[m_{t+1}, b_{t+1}, X_{t+1}, p_{t+1}, q_{t+1}, s_{t+1}] \tag{8}$$

3 Dynamic optimisation in CIA Model

It is easier to solve this problem if it is written in a Lagrangian constrained optimisation problem as:

$$\begin{aligned} \mathcal{L}\,(C_t, L_t, b_{t+1}, X_{t+1}, m_{t+1}, \lambda_t, \mu_t) \;\; = \;\; & \sum_{t=0}^{\infty} \beta^t\,[U(C_t) - V(L_t)] \\ & + \lambda_t \left[\begin{array}{c} m_t + b_t + p_tX_t - p_tC_t \\ -q_tb_{t+1}\,(1+\alpha) - p_ts_tX_{t+1} - p_tT_t \end{array} \right] \\ & + \mu_t \left[\begin{array}{c} m_t + b_t + p_tX_t + p_tzL_t - p_tC_t - \\ q_tb_{t+1}\,(1+\alpha) - p_ts_tX_{t+1} - p_tT_t - m_{t+1}\,(1+\alpha) \end{array} \right] \end{aligned} \tag{9}$$

This CIA model is solved analytically with the first order conditions for optimisations as:

$$C_t : U'(C_t) - (\lambda_t + \mu_t)\,p_t = 0 \tag{10}$$

$$L_t : -V'(L_t) + \mu_tp_tz = 0 \tag{11}$$

$$b_{t+1}: -q_t\left(1+\alpha\right)\left(\lambda_t+\mu_t\right)+\beta\frac{\partial v}{\partial b_{t+1}}=0 \tag{12}$$

$$X_{t+1}: -p_t s_t\left(\lambda_t+\mu_t\right)+\beta\frac{\partial v}{\partial X_{t+1}}=0 \tag{13}$$

$$m_{t+1}: -\left(1+\alpha\right)\mu_t+\beta\frac{\partial v}{\partial m_{t+1}}=0 \tag{14}$$

By the envelop theorem on differentiating the Bellman equation:

$$\frac{\partial v}{\partial b_t}=\left(\lambda_t+\mu_t\right) \tag{15}$$

$$\frac{\partial v}{\partial X_t}=p_t\left(\lambda_t+\mu_t\right) \tag{16}$$

$$\frac{\partial v}{\partial m_t}=\left(\lambda_t+\mu_t\right) \tag{17}$$

Combining above last three and the first two first order conditions, the middle three first order conditions can be expressed as:

$$-\frac{q_t\left(1+\alpha\right)U'(C_t)}{p_t}+\beta\frac{U'(C_{t+1})}{p_{t+1}}=0 \tag{18}$$

$$-sU'(C_t)+\beta U'(C_{t+1})=0 \tag{19}$$

$$\frac{-\left(1+\alpha\right)V'(L_t)}{p_t z}+\beta\frac{U'(C_{t+1})}{p_{t+1}}=0 \tag{20}$$

Higher productivity lowers the level of employment:

$$\frac{dL}{d\alpha}=\frac{-V''}{\left(1+\alpha\right)V''-\beta z^2 U''}<0 \tag{21}$$

Here α can be set to achieve the optimal inflation in inflation targeting regimes to maximize the level of welfare in the economy, $\max\limits_{\{C_t,L_t\}_{t+0}^{t-\infty}}\sum\limits_{t=0}^{\infty}\beta^t\left[U(C_t)-V(L_t)\right]$. The optimal employment (L^*) is obtained implicity

$$zU^{'}(zL^*)-V^{'}(L^*)=0 \tag{22}$$

The optimal growth rate of money supply is given by the Friedman rule is $\alpha=\beta-1$ where the nominal interest rate is zero $\left[R=\frac{1}{q}-1=0\implies q=1\right]$, the real interest rate is $r=\frac{1}{\beta}-1$, cash in advance constraint does not bind $\alpha=\beta-1$ because $\lambda=0$.

$$\begin{aligned}\lambda &= \frac{U'(C)}{p} - \mu = \frac{C.U'(C)}{1+\alpha} - \frac{V'(L)}{pz} = \frac{C.U'(C)}{1+\alpha} - \frac{\beta}{1+\alpha}\frac{U'(C)}{p} \\ &= \frac{C.U'(C)}{1+\alpha} - \frac{\beta}{1+\alpha}\frac{C.U'(C)}{1+\alpha} = \frac{C.U'(C)}{1+\alpha}\left(1 - \frac{\beta}{1+\alpha}\right) = \frac{C.U'(C)}{1+\alpha}(1-q) \qquad (23)\end{aligned}$$

4 Steady State in the CIA Model

With the first order conditions for dynamic optimisation, as given above, the steady state levels of prices and quantities are obtained in terms of parameters α, β and z. First simplify the steady state with $m_t = 1, b_t = 0,\ \ X_t = 0$.Then the above equilibrium conditions, the budget constraint becomes:

$$p_t C_t = 1 + \alpha \qquad (24)$$

This shows that in CIA model like this money is held only for consumption which equals total output, $C_t = zL_t$. Setting steady state variables to constant values, $C_t = C$, $L_t = L$, $p_t = p$, $q_t = q$, $s_t = s$, analytical solutions for prices and quantities are then expressed in terms of subjective discount factor (β) and the growth rate of money supply (α).

Price of nominal bond from (18) is given in terms of β and α:

$$q = \frac{\beta}{1+\alpha} \qquad (25)$$

Price of real bond from (19) is:

$$s = \beta \qquad (26)$$

The level of employment is given implicitly by (20)

$$(1+\alpha)\, V'(L_t) - \beta z U'(zL) = 0 \qquad (27)$$

Given the steady state (C) the price of commodity is directly proportional to the growth rate of money supply and inversely to the level of output and the productivity of the labour:

$$p = \frac{1+\alpha}{C} = \frac{1+\alpha}{zL} \qquad (28)$$

Nominal interest rate depends on the price of nominal bonds, directly on the growth rate of money and inversely on the discount factor.

$$R = \frac{1}{q} - 1 = \frac{1+\alpha}{\beta} - 1 \qquad (29)$$

Real interest rate inversely relates to the price of real bond and the subjective rate of time preference:

$$r = \frac{1}{s} - 1 = \frac{1}{\beta} - 1 \qquad (30)$$

Inflation rate equals the growth rate of money supply in the steady state:

Table 1: Parameters of CIA Model

Parameters	α	β	L_0	z	m	b	X
CIA	0.03	0.99	100	$(1, 0.05)$	1	0	0

$$i = \frac{P_{t+1}}{P_t} - 1 = \frac{p_{t+1}\overline{M}_{t+1}}{p_t\overline{M}_t} - 1 = 1 + \alpha - 1 = \alpha \tag{31}$$

Fisher equation implies gross real interest rate to be inverse of the discount factor:

$$1 + r = \frac{1+R}{1+i} = \frac{1+\alpha}{\beta} \div 1 + \alpha = \frac{1}{\beta} \tag{32}$$

Thus the prices q, s, p, R, r, i and λ are all solved in terms of growth rate of money (α) and the discount rate (β). From the equilibrium condition it is clear that $Y = C = zL = \frac{1+\alpha}{P}$ and $L = \frac{1+\alpha}{zP}$. Thus the level of output, consumption and employment increase with α and decline with inflation. While the greater liquidity helps to mobilise resources, the higher rate of inflation distorts the intertemporal decisions. Higher growth rate of money supply lowers the level of employment by causing distortions through inflation.

Now let us perturb this model around this steady state and show how the shocks in growth rate of money supply or the level of technology can impact on the transitional dynamics of the economy. These are shown in a series charts that represent solutions of this model to the shocks in α or z for given values of parameters in Table 1, as shown in Figures 1 to 5.

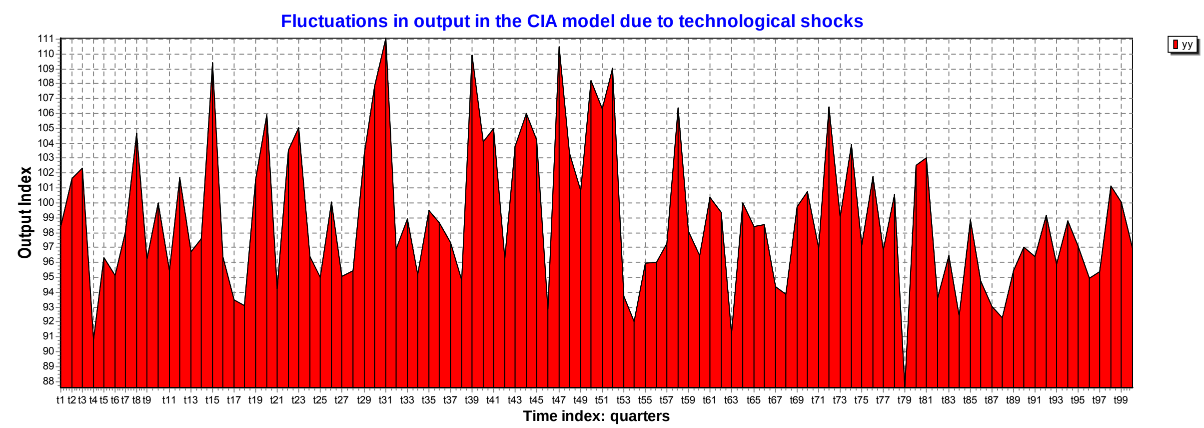

Figure 1: There are direct and indirect effects of technological shocks in output causing such pattern; Direct effect is due to production function, indirect through cap

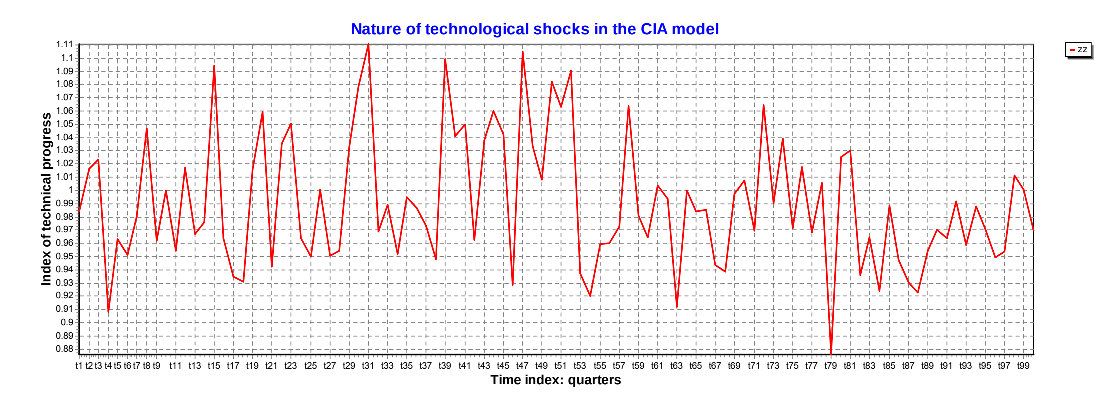

Figure 2: There are idiosyncratic shocks; unpredictable but with far reaching consequences in the economy.

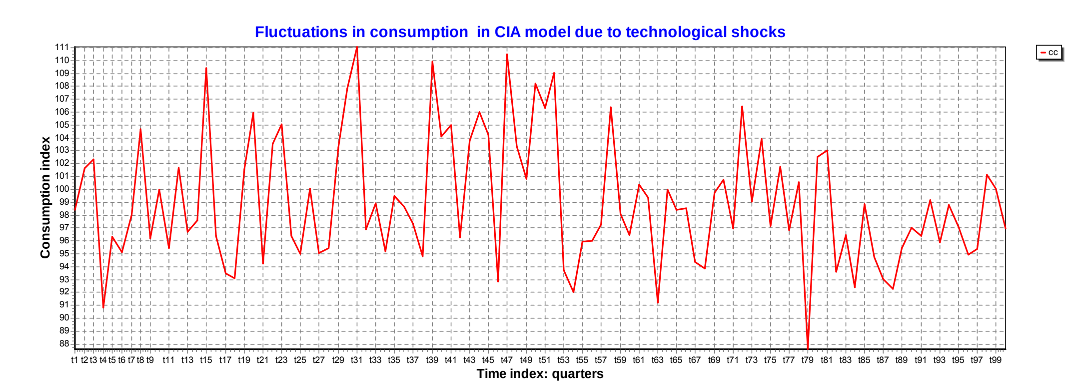

Figure 3: consumption follows output and income. when the former fluctuates later one responds to it.

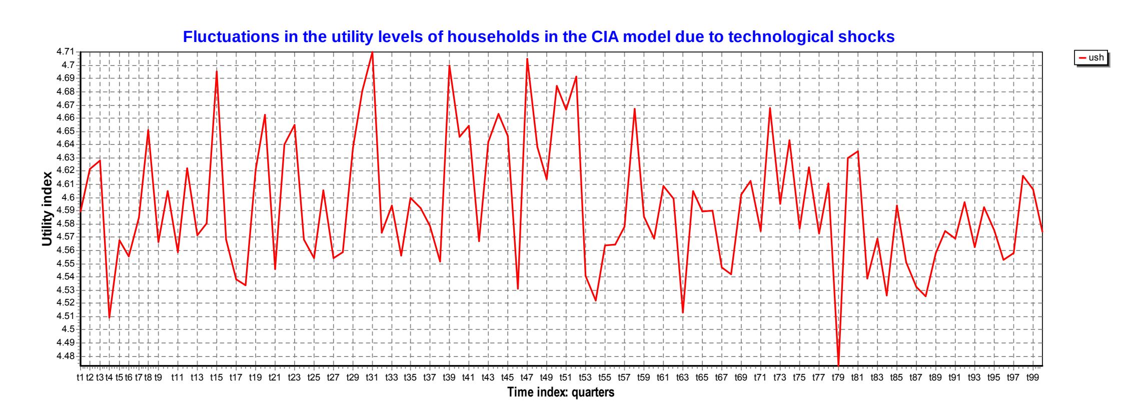

Figure 4: Utility measures the living standard of households; it comes from consumption and liesure. Both of these vary following techical shocks.

This means under the Friedman rule the cash in advance constraint does not bind. There are no distortions between the real and nominal assets; the rate of return in all assets are equal in equilibrium.

With parameter sets in Table 2, a simple three country version of this model is solved subject

Table 2: Parameters of CA Model

Parameters	δ	α	β	g_y	L_0	v	g_m	$ln\,(z)$	M_0
Country 1	0.05	0.5	0.95	0.01	100	1	0.01	$(1, 0.05)$	100
Country 2	0.05	0.4	0.99	0.02	100	2	0.02	$(1, 0.05)$	100
Country 3	0.05	0.45	0.98	0.015	100	3	0.015	$(1, 0.05)$	100

to idiosyncratic technological shocks for 15 years to generate time profiles of capital, output, prices, money, consumption, investment, labour supply and lifetime utilities of households as shown by multiple bars for three interdependent economies, $i = 1, 2, 3$. The fluctuations in these economies originate in financial sector and can have significant consequences in the level of welfare in the economy.

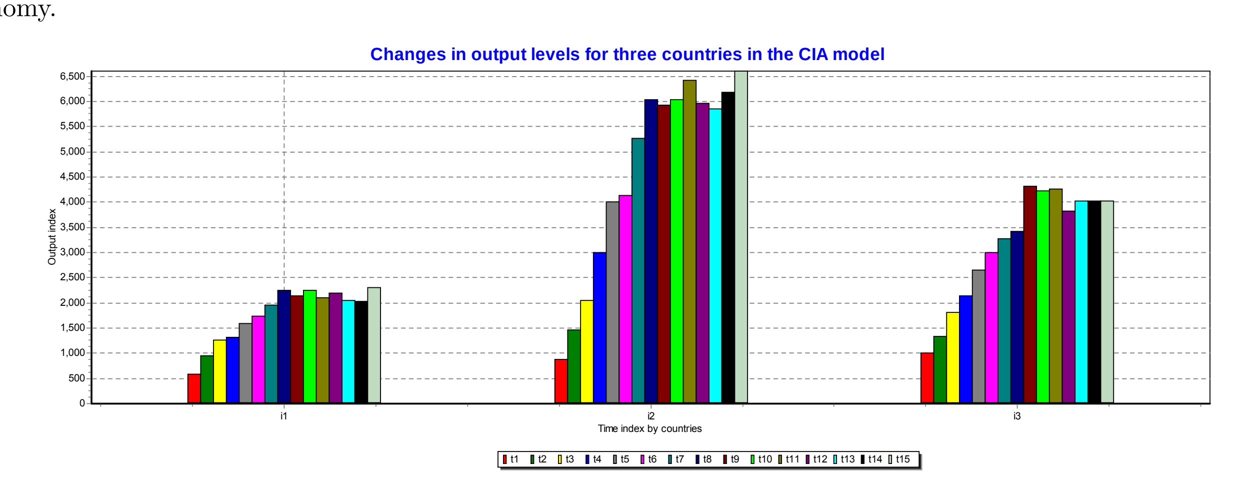

Figure 5: Three country cas is presented here to show that country which discounts less its future accumulates more capital and is better off in the long run.

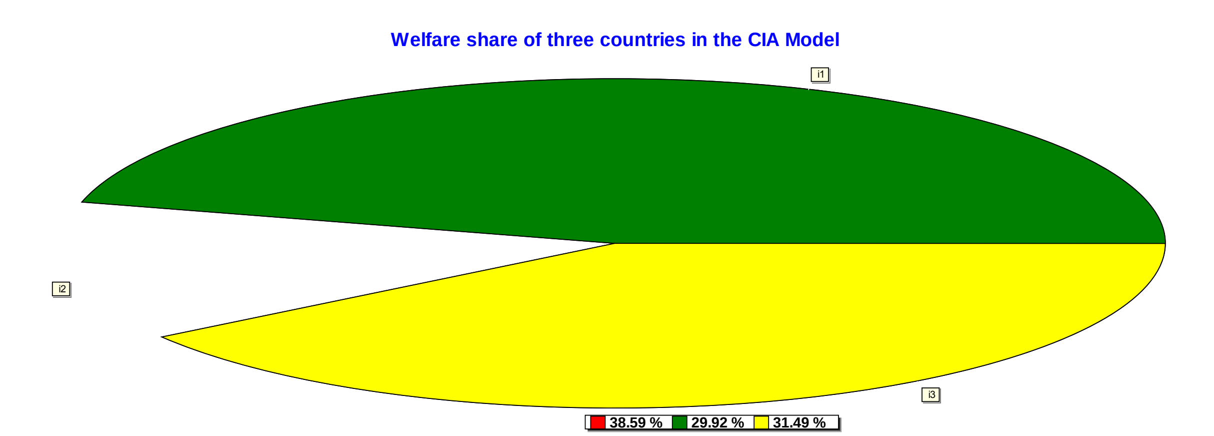

Figure 6: Small countries gain more than larger countries when risks are pooled together through a coalition.

Main lessons that can be drawn from the CIA model is that the financial crises occur because of shifts in the investor and consumer confidences, changes in perceptions and beliefs and technological shocks that hit the system. Impacts of such changes can be very sudden which affects the velocity of circulation of money, technological progress, discount factors or the beliefs in the underlying growth rates of the economy. These factors impact on prices, trend of output, prices and other features of the economy as shown by the path of model variables and welfare solutions as presented in above figures. It is clear that a balanced path of financial depth enhances welfare of households but this depends on the attitude of the consumers towards the future of the economy. These features are not typical of an economy with exogenous money but can persist even with the endogenous growth rate of money. This is shown using a solution of the money in utility function model in the next section.

5 Money in the Utility Function and Growth

Role of money was for pure exchange in the cash in advance model and the growth rate of money α was exogenous. There are circumstances when household prefer to store more or less cash depending on expected utilities from it. Thus the stock of money they like to keep is endogenous and is a part of utility maximising choice of a household. This feature is captured by the money in the utility function model of Sidrauski (1967). When this desire is excessive it causes a crisis in the system as observed during the recession that started in 2008. The problem of household as in the CIA model is to maximise the lifetime welfare (W) from consumption (c_t) and possessing the stock of money (m_t).

$$\max W = \sum_{t=0}^{\infty} \left[\beta^t U(c_t, m_t)\right] \tag{33}$$

subject to the production (Y_t) technology constraint with capital (K_t) and labour (L_t) inputs and technological shock (z_t):

$$Y_t = z_t F\left(K_t, L_t\right) \tag{34}$$

Under constant returns to scale $y_t = f\left(k_t\right)$ where $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$. Economy wide budget constraint is given by

$$Y_t + \tau_t L_t + (1-\delta) K_{t-1} + \frac{M_{t-1}}{P_{t-1}} = C_t + K_t + \frac{M_t}{P_t} \tag{35}$$

where Y_t is output, P_t price of goods, C_t consumption, $K_{t+1}$is capital stock, τ_t is net transfer for each individual, M_t money, L_t employment and δ is the rate of depreciation of capital. In per capita terms:

$$\omega_t = f\left(k_{t-1}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right) k_{t-1} + \frac{m_{t-1}}{\left(1+\pi_t\right)\left(1+n\right)} = c_t + k_t + m_t \tag{36}$$

The recursive dynamic program of this household is:

$$V\left(\omega_t\right) = u\left(c_t, m_t\right) + \beta V\left(\omega_{t+1}\right) \tag{37}$$

$$V\left(\omega_t\right) = \max\left\{u\left(c_t, m_t\right) + \beta V\left[f\left(\omega_t - c_t - m_t\right) + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)\left(\omega_t - c_t - m_t\right) + \frac{m_t}{\left(1+\pi_{t+1}\right)\left(1+n\right)}\right]\right\} \tag{38}$$

6 Dynamic optimisation in the MIU model

Again using the Lagrange multiplier (λ_t) to simplify this constrained optimisation problem:

$$\mathcal{L}(c_t, m_t, \lambda_t) = \sum_{t=0}^{\infty}\left[\beta^t u(c_t, m_t)\right] +$$
$$\sum_{t=0}^{\infty}\lambda_t\left[f(\omega_t - c_t - m_t) + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)(\omega_t - c_t - m_t) + \frac{m_t}{(1+\pi_{t+1})(1+n)}\right] \tag{39}$$

As before solving MIU model explicitly means expressing the prices and quantities like y_t, k_t, c_t, m_t in terms of the preference and technology parameters as β, δ, α and n. In other words the optimal values of variables are determined by subjective discount factor(β), depreciation (δ), productivity of capital (α) and growth rate of population (n). This is done using the first order conditions:

$$c_t : u_c(c_t, m_t) - \beta\left[f_k(k_t) + \left(\frac{1-\delta}{1+n}\right)\right]V_\omega(\omega_{t+1}) = 0 \tag{40}$$

Here marginal utility of holding capital $\beta\left[f_k(k_t) + \left(\frac{1-\delta}{1+n}\right)\right]V_\omega(\omega_{t+1})$ should equal the marginal utility of consumption $u_c(c_t, m_t)$.

$$m : u_m(c_t, m_t) - \beta\left[f_k(k_t) + \left(\frac{1-\delta}{1+n}\right)\right]V_\omega(\omega_{t+1}) + \frac{\beta V_\omega(\omega_{t+1})}{(1+\pi_{t+1})(1+n)} = 0 \tag{41}$$

Transversality conditions

$$\lim_{t\to\infty}\beta^t\lambda_t k_t = 0; \lim_{t\to\infty}\beta^t\lambda_t m_t = 0 \tag{42}$$

By envelop theorem:

$$\lambda_t = V_\omega(\omega_t) = u_c(c_t, m_t) \tag{43}$$

7 Steady state in the MIU model

Dynamic optimisation with the first order conditions:

$$u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+\pi_{t+1})(1+n)} = u_c(c_t, m_t) \tag{44}$$

Left hand side gives the total marginal benefit of holding money; the first term in it is the direct utility of money and the second term denotes the real balance effect of holding money m_t at time t for $t+1$. Thus the marginal utility of holding money should equal to marginal utility of consumption. By constant returns to scale assumption the income of households is function of capital stock $r^k k + w = f_k k + (f(k) - f_k k) = f(k)$. Financial crises promote hoarding of money, this means less capital, more inflation and less growth. Consider a steady state with $n = 0$ and $V_\omega(\omega_t) = V_\omega(\omega_{t+1}) = V_\omega(\omega^{ss})$. From the first first order conditions $1 - \beta\left[f_k(k^{ss}) + (1-\delta)\right] = 0$

Table 3: Parameters of MIU Model

Parameters	α	β	δ	z
MIU	0.3	0.99	0.05	$(1, 0.05)$

$$zf_k\left(k^{ss}\right) + (1-\delta) = \frac{1}{\beta}$$

Assuming a Cobb-Douglas production function $f(k) = zk^{\alpha}$ this condition converts to $\alpha zk^{\alpha-1} + (1-\delta) = \frac{1}{\beta}$

$$k^{ss} = \left[\frac{\alpha z\beta}{1+\beta\left(\delta-1\right)}\right]^{\frac{1}{1-\alpha}} \tag{45}$$

Consumption in the steady state:

$$c^{ss} = zf\left(k^{ss}\right) - \delta k^{ss} = \left[\frac{\alpha z\beta}{1+\beta\left(\delta-1\right)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha z\beta}{1+\beta\left(\delta-1\right)}\right]^{\frac{1}{1-\alpha}} \tag{46}$$

Steady state inflation rate equals growth rate of money supply:

$$\frac{\Delta m^{ss}}{m^{ss}} = \frac{\theta^{ss} - \pi^{ss}}{(1+\pi^{ss})} = 0$$

where $\Delta m^{ss} = 0$ implies growth rate of money supply, $\theta^{ss} = \frac{\Lambda M^{ss}}{M^{ss}}$, and equal inflation, $\theta^{ss} = \pi^{ss}$. Stock of money in excess of the amount required for transactions reduces the amount of capital stock, hence output in the economy. Excessive supply of money manifests in inflation and is not good for the economy. As in the CIA model the transitional dynamics of the MIU model is found numerically for the set of parameters in Table 3. The response of $y_t, r_t,\ z_t, c_t,\ u_t$ to shocks are represented in Figures 7 to 11.

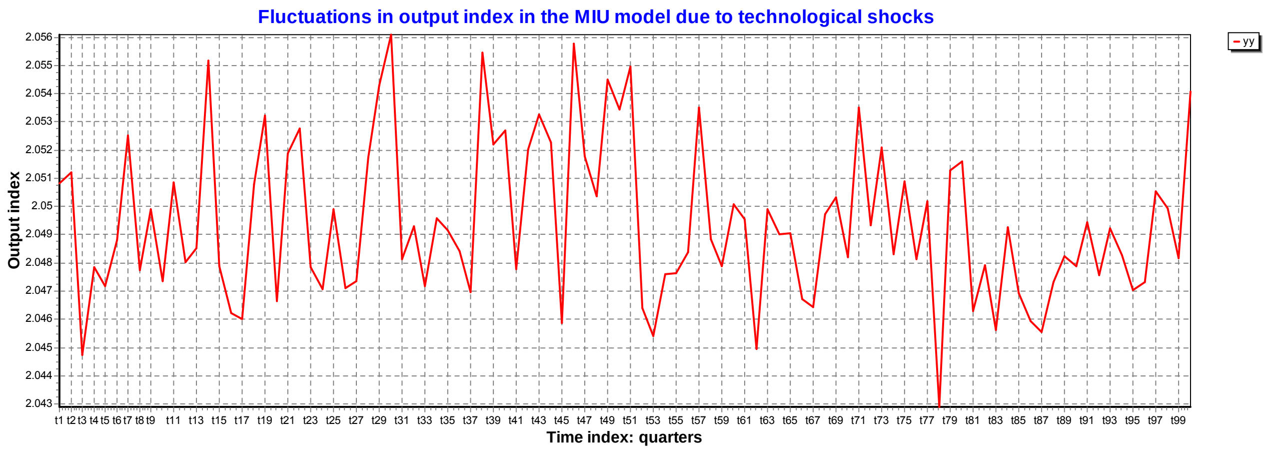

Figure 7:There are direct and indirect effects of technological shocks in output causing such pattern; Direct effect is due to production function, indirect through capit

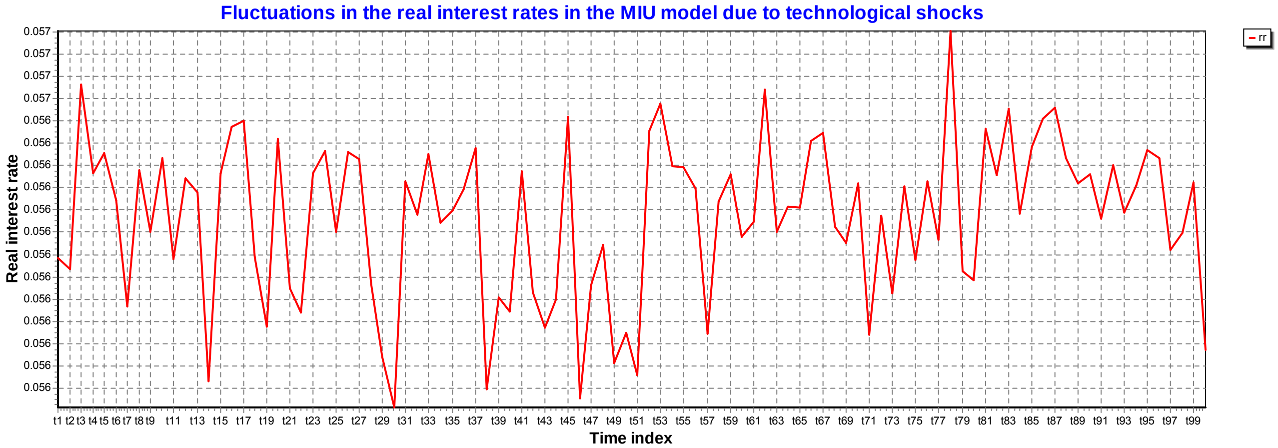

Figure 8: differences in the marginal productivities in addition to technological shocks lead to fluctuations in the interest rate

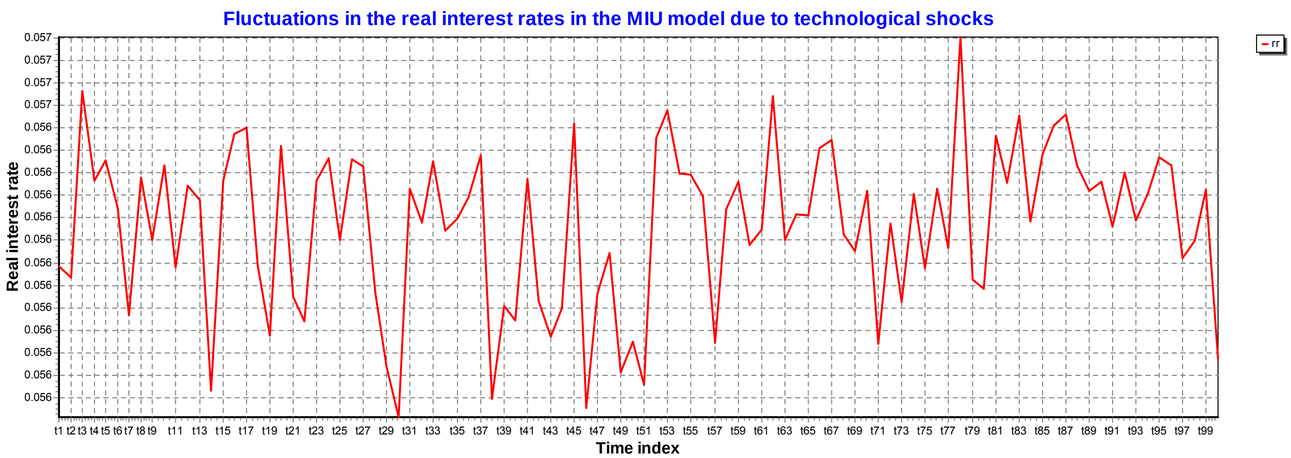

Figure 9: There are direct and indirect effects of technological shocks in output causing such pattern; Direct effect is due to production function, indirect through cap

Table 4: Parameters of MIU Model

Parameters	δ	α	β	g_y	L_0	v	g_m	$ln\,(z)$	M_0
Economy 1	0.05	0.5	0.95	0.01	100	1	0.01	$(1, 0.05)$	100
Economy 2	0.05	0.45	0.99	0.02	100	2	0.02	$(1, 0.05)$	100
Economy 3	0.05	0.45	0.98	0.015	100	1.5	0.015	$(1, 0.05)$	100

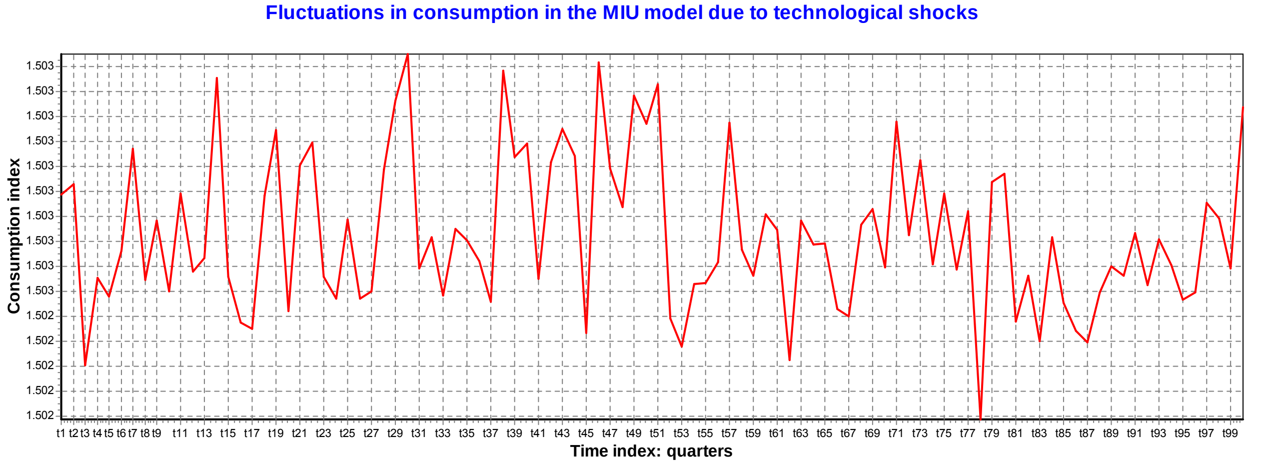

Figure 10:consumption follows output and income. when the former fluctuates later one responds to it.

Again scenarios are derived for three economies with a set of plausible parameters as given in Table 3. The time path of variables $y_t, k_t, c_t, m_t,\ u_t$ are easily computed based on model solutions, (Fig 11 and 12).

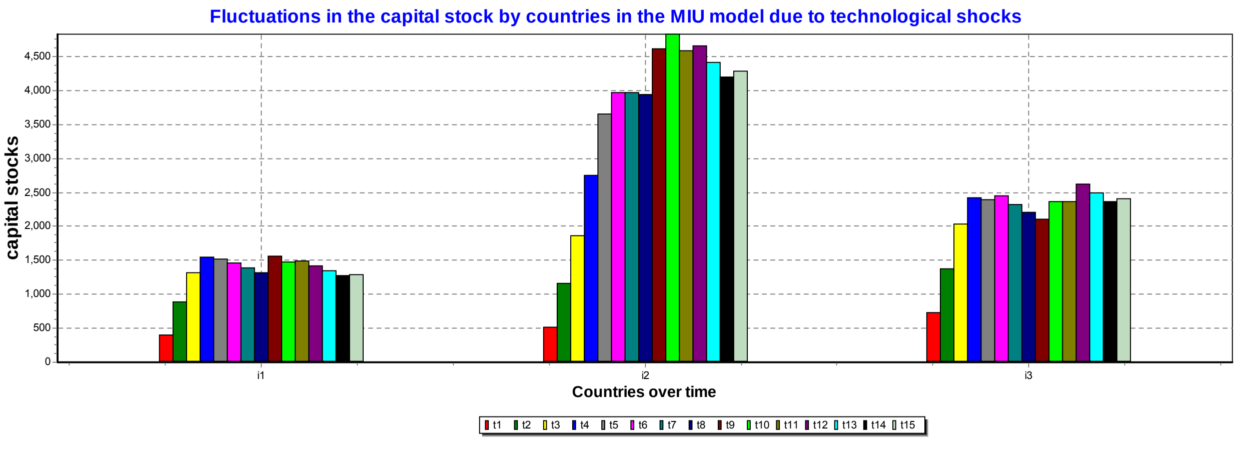

Figure 11: Three country cas is presented here to show that country which discounts less its future accumulates more capital and is better off in the long run.

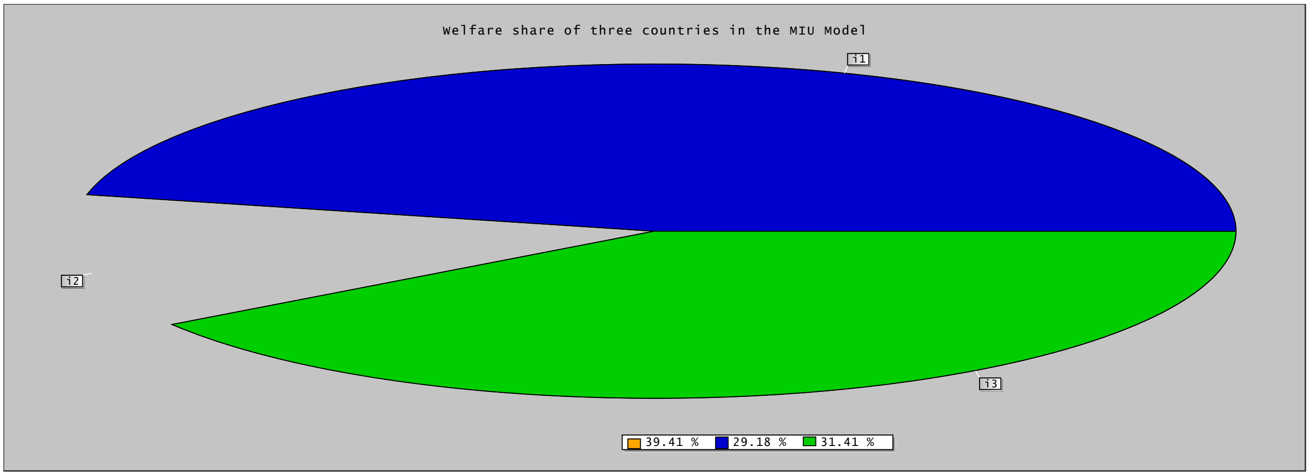

Figure 12: Small countries gain more than larger countries when risks are pooled together through a coalition.

The CIA and MIU models provide intuition about the nature of fluctuations that affect interdependent economies and allocation of welfare. Policy analyses should be based in more detailed assessment of the structural features of the economy as found in the micro-consistent dataset for consumption, production and trade.

8 Conclusion

We study results of the cash in advance and money in utility models about the nature of fluctuations in economic activities and welfare in three interdependent economies. When the money is exogenously introduced in the form of cash in advance, it serves as a medium of exchange the rate of return in real and nominal assets become equal. Idiosyncratic technological shocks generate fluctuations in the growth rates of capital, output, prices, money, consumption, investment, labour supply and lifetime utilities of households. When households have money in their utility functions, the stock of money in excess of that required for transactions causes inflation and reduces the amount of capital stock and output in these economies. Both CIA and MIU models support for a steady growth rate of money according to the smooth growth rate of output. While the inflation targeting by manipulating the interest rates for macroeconomic stability are theoretically prudent policy moves it is impossible for central banks to eliminate business cycles that arise from shocks to production technology or structural features of the economy.

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