

Sliding Mode Control of Rigid-link Anthropomorphic Robotic Arm

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Abstract—Increasing demands of accuracy, productivity, reliability and repeatability in today’s robotic applications have highlighted significance of modern control techniques. The associated control law must be able to handle perturbation forces, joint friction and parameter variations which can degrade the robot performance. This paper proposes non-linear Sliding Mode Control (SMC) for a 6 Degree of Freedom (DOF) human arm like robotic manipulator AUTonomous Articulated Robotic Educational Platform (AUTAREP). The proposed control algorithm is compared with a trivial linear control strategy Proportional-Integral-Derivative (PID). Simulation results depicted the efficiency of SMC over PID in term of tracking response for various desired trajectories.

Keywords—SMC; robot control; robotic arm; PID control

I. INTRODUCTION

Advancements in the field of robotics have greatly influenced productivity and efficiency of automation industry. Robots are deployed in industries to perform various jobs like cutting, welding, assembling, pick and place etc. [1]. Robot also finds its applications in other areas like medicine [2, 3], warfare [4], space exploration [5] and nuclear plants [6]. In these applications, mostly robotic systems are centered on human arm like robotic manipulator. Robots are supposed to work at high speed with accurate response. Moreover, safety is another concern, raises in robot deployment in the environment where they have to work along human. Highly non-linear nature of robots makes it challenging to achieve such requirements. This, in turn, needs a sophisticated control strategy that provides high speed and precise robotic link movements.

The research community has played its active role in designing control strategies ranging from simple classical control to robust control [7, 8]. Proportional Integral Derivative (PID) is most widespread feedback control due to its simplicity and less computational requirements [9, 10]. It is a flexible method to control the system autonomously [11]. David has proposed a PID control law for two link robotic arm [12]. Rocco has provided a theoretical proof of PID stability for an industrial robotic manipulator [13]. The theoretical results have been verified on simple two Degree Of Freedom (DOF) arm. Modelling compensation based PID control has been proposed by Cervantes and Ramirez in [14].

Despite of simplicity and easiness of PID implementation, there are serious disadvantages and drawbacks associated with it. PID loses its significance and effectiveness for complex systems, especially in presence of internal or external disturbances. Furthermore, PID does not remain suitable for high speed dynamics, particularly in the existence of model imprecision, payload variation and uncertainties. These issues urge the need of more sophisticated control algorithm which can robustly cope both uncertainties and disturbances.

Sliding Mode Control (SMC) belongs to the class of robust control and eminent for its precision [15]. It is capable of handling parameter variations, joint friction, inertia and both matched and unmatched disturbances. In [16, 17], researchers proposed SMC based control strategies to overcome the effect of disturbances and model imprecision. A combination of discretized SMC with an estimator is presented by Corradini *et al.* to improve robustness of robotic manipulator [18]. Ahmad and Osman have proposed Proportional Integral (PI) SMC to handle the non-linear dynamics of robotic manipulator for accurate tracking [19]. Multi-Input Multi-Output (MIMO) SMC has been proposed by Hacioglu *et al.* [20]. The algorithm is implemented on two link robotic arm system. Fuzzy logic is used to tune the controller gains which increased the system speed significantly. Ajwad *et al.* combined SMC with Disturbance Observer Based Controller (DOBC) to increase robustness of the system [1]. In [21] comparison of PID and SMC controllers is provided for a two DOF robotic manipulator. Results suggest that better performance and robustness is achieved in case of SMC.

This article presents a performance comparison of traditional PID with non-linear SMC. A custom developed robotic manipulator, AUTonomous Articulated Robotic Educational Platform (AUTAREP) [22], shown in Fig. 1, is considered for the implementation of the control laws.

The platform is based upon 6-DOF (Degree Of Freedom) robotic manipulator, having five revolute joints e.g. from waist to wrist, replicating a human arm. Specifications of each link is depicted in Table I.

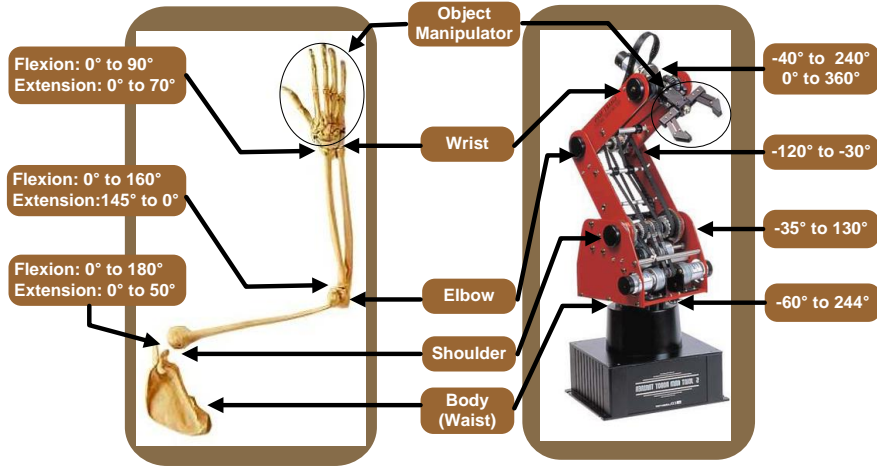


Fig. 1. AUTAREP – An anthropomorphic robotic arm

TABLE I. LINK SPECIFICATIONS

Links	DOF	Link Lengths [mm]	Range of Motion
Base	1	385	310°
Shoulder	1	220	90°
Elbow	1	220	172°
Wrist	2	155	Pitch 260°, Roll 360°

The remaining paper is structured as follows. The mathematical model of the robotic arm is described in Section II. Section III presents the derivation of control laws and simulation results. Finally, Section IV concludes the paper.

II. ROBOT MODEL

Application of control algorithms needs mathematical model of the system. Robot model includes both kinematics and dynamics. Kinematics describes the relationship between robot joint angles and position and orientation of end-effector [23]. Kinematic model comprises of Forward Kinematics (FK) and Inverse Kinematics (IK).

A. Forward Kinematics

Determining the position and orientation of end-effector through joint angles is known as FK. FK model of AUTAREP is formulated using Denavit–Hartenberg (DH) parameters based approach. Fig. 2 illustrates the schematic diagram of AUTAREP for its kinematic modelling.

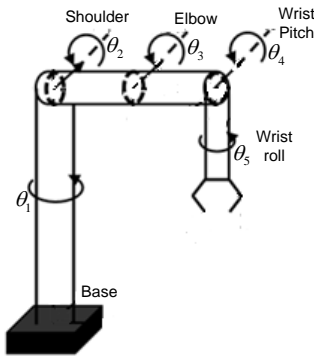


Fig. 2. Kinematic representation of AUTAREP

The overall transformation from end-effector to base is represented by homogenous transformation matrix (1)

$${}^0_6T = \begin{bmatrix} c_1 c_{234} c_5 + s_1 s_5 & -c_1 c_{234} s_5 + s_1 c_5 & -c_1 s_{234} & c_1 A \\ s_1 c_{234} c_5 - c_1 s_5 & -s_1 c_{234} s_5 - c_1 c_5 & -s_1 s_{234} & s_1 A \\ -s_{234} c_5 & s_{234} s_5 & -c_{234} & B \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where

$$A = -l_4 s_{234} + l_3 c_{23} + l_2 c_2$$

$$B = l_1 - l_4 c_{234} - l_3 s_{23} - l_2 s_2$$

and

$$c_1 = \cos(\theta_1), c_{23} = \cos(\theta_2 + \theta_3), c_{234} = \cos(\theta_2 + \theta_3 + \theta_4)$$

$$s_1 = \sin(\theta_1), s_{23} = \sin(\theta_2 + \theta_3), s_{234} = \sin(\theta_2 + \theta_3 + \theta_4)$$

Detailed derivation of FK is presented in [24].

B. Inverse Kinematics

In IK, joint angles are calculated for desired position and orientation of end-effector. IK model AUTAREP has been formulated using Geometric and Algebraic methods. Joint angle equations have been derived algebraically while orientation of the end-effector is computed geometrically. Consider the following generalized form of transformation matrix for 6 DOF robotic arm.

$${}^0_6T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$${}^0_1T^{-1} * {}^6_1T = {}^0_1T^{-1} * \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where 0_1T is transformation matrix from shoulder to base joint.

$${}^0T^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & -l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Putting (1) and (4) in (3)

$$\begin{bmatrix} c_{234}c_5 & -c_{234}s_5 & -s_{234} & -s_{234}l_4 + c_{23}l_3 + c_2l_2 \\ -s_5 & -c_5 & 0 & 0 \\ -s_{234}c_5 & s_{234}s_5 & -c_{234} & -c_{234}l_4 - s_{23}l_3 - s_2l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1n_x + s_1n_y & c_1o_x + s_1o_y & c_1a_x + s_1a_y & c_1p_x + s_1p_y \\ -s_1n_x + c_1n_y & -s_1o_x + c_1o_y & -s_1a_x + c_1a_y & -s_1p_x + c_1p_y \\ n_z & o_z & a_z & p_z - l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Comparing both sides of above equation:

$$c_1p_x + s_1p_y = -s_{234}l_4 + c_{23}l_3 + c_2l_2 \quad (5)$$

$$-s_1p_x + c_1p_y = 0 \quad (6)$$

$$p_z - l_1 = -c_{234}l_4 - s_{23}l_3 - s_2l_2 \quad (7)$$

From (6), base joint angle is computed as:

$$\theta_1 = \text{Atan2}\left(\frac{p_y}{p_x}\right) \quad (8)$$

Solving (5) and (7) simultaneously to calculate θ_2 and θ_3

$$\begin{aligned} (c_1p_x + s_1p_y + s_{234}l_4)^2 + (p_z - l_1 + c_{234}l_4)^2 - l_3^2 \\ - l_2^2 = 2l_2l_3c_3 \\ s_3 = \pm\sqrt{1 - c_3^2} \\ \theta_3 = \text{Atan2}\left(\frac{s_3}{c_3}\right) \end{aligned} \quad (9)$$

Employing sine and cosine identities in (5) and (7)

$$c_1p_x + s_1p_y + s_{234}l_4 = (c_2c_3 - s_2s_3)l_3 + c_2l_2 \quad (10)$$

$$p_z - l_1 + c_{234}l_4 = -(s_2c_3 + c_2s_3)l_3 - s_2l_2 \quad (11)$$

Simultaneously solving (10) and (11)

$$\frac{c_2}{(c_3l_3 + l_2)(c_1p_x + s_1p_y + s_{234}l_4) - s_3l_3(p_z - l_1 + c_{234}l_4)} = \frac{c_2}{(c_3l_3 + l_2)^2 + (s_3l_3)^2}$$

$$\begin{aligned} s_2 \\ = -\frac{(c_3l_3 + l_2)(p_z - l_1 + c_{234}l_4) + s_3l_3(c_1p_x + s_1p_y + s_{234}l_4)}{(c_3l_3 + l_2)^2 + (s_3l_3)^2} \\ \theta_2 = \text{Atan2}\left(\frac{s_2}{c_2}\right) \end{aligned} \quad (12)$$

$$\theta_4 = \theta_{234} - (\theta_2 + \theta_3) \quad (13)$$

In IK problem, the orientation of end-effector depends on θ_{234} and it is known. Fig. 3 shows the angles involved in orientation of end-effector.

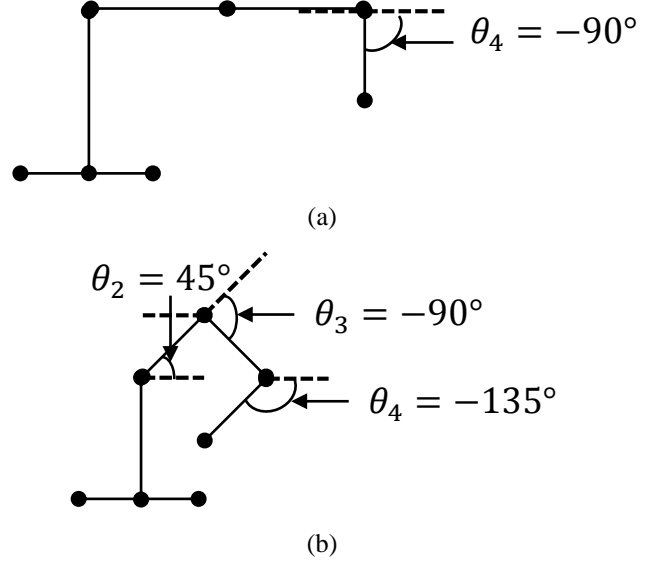


Fig. 3. Orientation of end-effector (a) home position (b) shift in orientation due to angles variation

C. Dynamic Model

Dynamics deals with torques and forces causing robot motion. The model of the platform in the present research is based on Euler-Lagrange method.

$$\tau = M(q)\ddot{q} + V(q, \dot{q}) + G(q) \quad (13)$$

where τ and q represents the input joint torque and the position of robot link respectively. $V(q, \dot{q})$ represents the matrix comprising of Coriolis and Centrifugal forces, $M(q)$ and $G(q)$ are inertia matrix and gravity matrix respectively. The derived model is provided in [25]. The dynamic model has been used to derive control laws for AUTAREP.

III. CONTROLLER DESIGN AND RESULTS

Current research work compares a traditional PID and the robust control strategy SMC. Generally, PID control is simpler and easy to implement which indeed is the reason behind its vast use in industry. But when it comes to fast movements and narrow error margin is required, a more sophisticated and robust controller e.g. SMC is more suitable.

A. PID Control

PID is a simple but efficient solution for many industrial processes. This law ensures the stability by appropriately controlling the input torque τ [4]. Fig. 4 shows PID functional block diagram. PID control law is described by the differential equation (14).

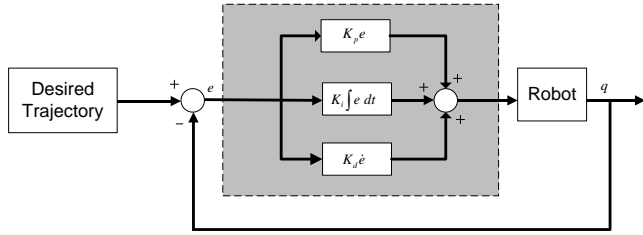


Fig. 4. Functional block diagram of PID

$$\tau = K_p e + K_d \dot{e} + K_i \int e dt \quad (14)$$

where $e = q_d - q$ is 4×1 error matrix, q_d is the desired joint position. K_p , K_d and K_i are proportional, derivative and integral gains respectively. These gains can be defined as [9]:

$$K_p = \text{diag}\{K_{p1}, K_{p2}, K_{p3}, K_{p4}\}$$

$$K_d = \text{diag}\{K_{d1}, K_{d2}, K_{d3}, K_{d4}\}$$

$$K_i = \text{diag}\{K_{i1}, K_{i2}, K_{i3}, K_{i4}\}$$

By comparing (13) and (14), closed-loop system equation can be formulated as

$$\ddot{q} = M^{-1}(q)[-V(q, \dot{q}) - G(q) + K_p e + K_d \dot{e} + K_i \int e dt] \quad (15)$$

B. SMC Control

Robot dynamics is inherently complex and highly nonlinear in nature. Dynamics of one joint affects other joints as well [6]. SMC has proved itself useful in achieving system stability and robustness against matched uncertainties as reported in [7]. In this research work, SMC is designed so that the robot tracks the predefined trajectory more precisely. Block diagram of SMC is shown in Fig. 5.

In SMC, first of all, a sliding surface is defined and all states of the system are supposed to move towards the defined sliding surface and remain there as time approaches to infinity. Sliding surface based on error signal is described in (16).

$$S = \dot{e} + \lambda e + I \int e dt \quad (16)$$

where λ and I are arbitrary positive constants. Overall SMC law comprises of two parts; equivalent (u_{eq}) and discontinuous (u_{dis}) controller.

$$u = u_{eq} + u_{dis} \quad (17)$$

u_{eq} takes the system states to the sliding surface from their initial condition and u_{dis} ensures that the states remain on sliding surface for all future time even in the presence of bounded uncertainties. u_{eq} is computed by putting $S = 0$ whereas u_{dis} is a switching function as given in (18).

$$u_{dis} = -k \text{sign}(S) - \zeta S \quad (18)$$

where k and ζ are positive gains. The term ζS has been included to achieve strong reachability. The resulting control input torque is given by (19),

$$\tau = M(q)[\lambda(\dot{e}) + I(e) + \ddot{q}_d] + V(q, \dot{q}) + G(q) - k \text{sign}(S) - \zeta S \quad (19)$$

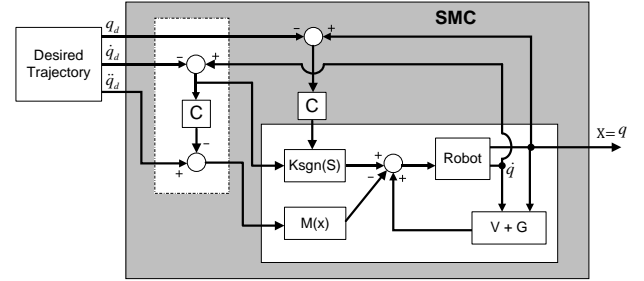
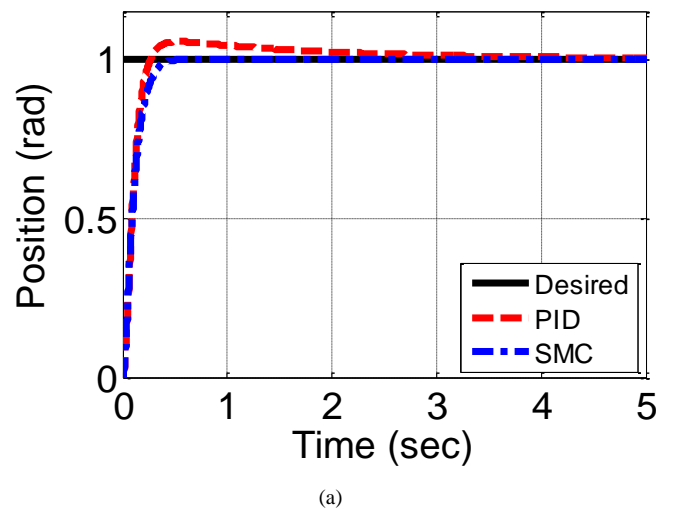


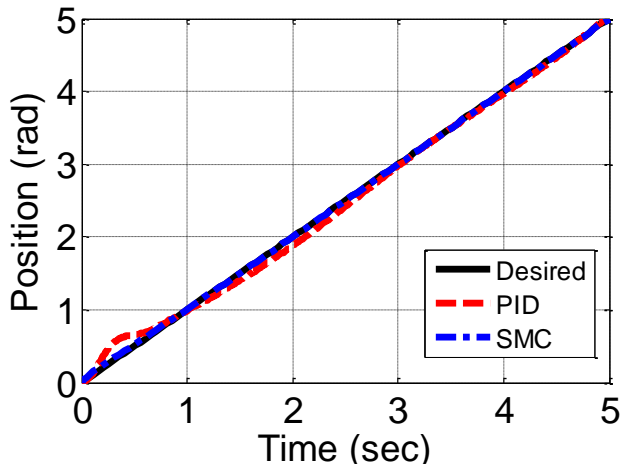
Fig. 5. Functional block diagram SMC controller

C. Simulation Results

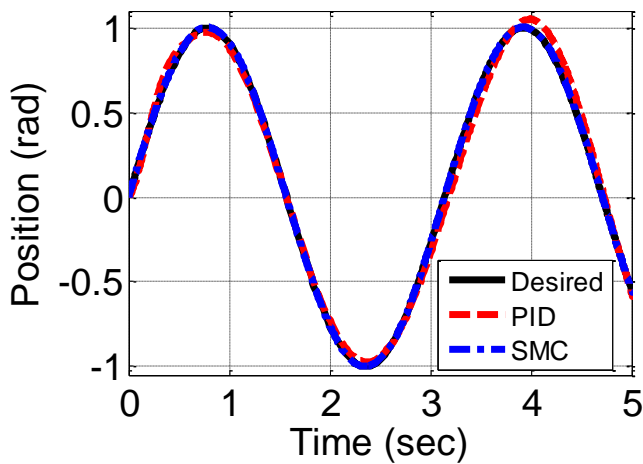
The simulation results of both of the above techniques are presented in Fig. 6. A comparison of step responses of base joint illustrated in Fig. 6a reveals that there is no overshoot in case of SMC while PID exhibits prominent overshoot. Secondly settling time is relatively much lesser in the response of SMC.

Fig. 6b represents comparison for ramp responses in case of shoulder joint while Fig. 6c depicts sinusoidal responses of elbow joint. These responses demonstrate superiority of SMC in achieving the desired trajectory and tracking it for all future time. On the other hand, PID has shown perturbations during the tracking process.





(b)



(c)

Fig.6. Simulation results (a) Step response of base joint (b) Ramp response of shoulder joint (c) Sinusoidal response of elbow joint

IV. CONCLUSION

This paper presents a comparison of SMC and PID for a 6 DOF robotic arm. Various input trajectories are applied to characterize the response of both techniques. Simulation results demonstrate that SMC has superior performance over PID for all the given trajectories. Comparison of both techniques on the basis of hardware results is anticipated in near future.

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