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#### ORIGINAL ARTICLE

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### An options-pricing approach to forecasting the French presidential election

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#### ABSTRACT

A subjective probability argument suggests vote-share estimates from polling companies can be interpreted as market prices. The corresponding election constitutes the price at a known future date. This makes an options-pricing approach particularly attractive. In this setting, vote-share estimates, the probability of winning the popular vote and the second-round qualification probability all have a convenient representation in terms of binary options prices. In this article, we develop options-pricing, vote-transfer, and Monte Carlo methods to forecast the French presidential election. The approach fits well with the proportional and regimented two-stage nature of the French election but applies more broadly. Unusually for a French system characterised by uncertainty and constant flux the incumbent President Macron appears in a dominant position throughout the 2017 and 2022 elections albeit with no chance of an outright win in the first round.

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#### 1. Introduction

Beyond the raw technical challenge (Easaw et al., 2023) the forecasting of elections is important for several reasons. The use of predictive analytics in election campaigns is now well-established (Kenett et al., 2018; Quinlan & Lewis-Beck, 2021). Polls underpin policy formation well in advance of elections as well as feeding back once campaigns are under way. The increased emphasis on predictive analytics also occurs amid suggestions of increased electoral volatility (Quinlan & Lewis-Beck, 2021). More generally, it is important to have as informed an electorate as possible (Pajala et al., 2018). This is especially true given recent concerns over the manipulation of Social Media (Reisach, 2021). Information supplied to the electorate in-turn affects both voting levels and post-election satisfaction levels (Bunker, 2020). Election forecasts can have significant economic impacts, notably on stock prices (Reade & Vaughan Williams, 2019) and prospective capital investments. Successful forecasting work can also yield improved understanding of the underlying political environment (Quinlan & Lewis-Beck, 2021).

Outside of the U.S. forecasting elections using statistical models is arguably most extensive in France (Lewis-Beck, 2005). National election forecasting in France has been the subject of scientific forecasting efforts for a number of years (Dufresne et al., 2022). The record of forecasting models in France is mixed. For a discussion and overview see Foucault and Nadeau (2012).

As a case study France is important due to its unique political economy and wider European influence. France has been labelled a "state influenced market" (Hastings & Heyes, 2018; Schmidt, 2012) reflecting its strong interventionist traditions, welfare state and efforts to protect industry and culture. In recent years the French economy has been characterised by high state spending and a relatively robust response to the COVID-19 pandemic. France has a traditionally high electoral turnout and a politically active population. Recent examples include strong engagement from younger voters partly driven by populist policies from far-right candidates. The dynamics of French politics thus makes it an exciting case study, including the potential for policy transfer and influence across Europe.

In this article, we propose an options-pricing approach based on opinion-poll data that offers certain theoretical advantages and is particularly wellsuited to the French system. The French presidential election follows a regimented two-stage structure:

- 1. First open stage. All candidates compete in an open vote. Candidates can only win at this stage if they secure more than 50% of the votes cast.
- 2. Second run-off stage. If no candidate achieves over 50% of the vote in the first stage the top

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two candidates run off. The winner of this second stage wins the presidency.

This two-stage structure lends itself to the following questions:

- 1. What is the estimated share of the vote?
- 2. What is the probability that any of the candidates can win outright in the first round?
- 3. What is the probability that a given candidate reaches the second round of the vote?
- 4. What is the overall win probability of the respective parties given the first round of polling?
- 5. What is the estimated vote share in the second round of polling?
- 6. What is the conditional probability, given the final two candidates, of winning the second round of the election?

Questions 1–3 and 5–6 above can be solved using a financial options-pricing argument (see Section 3). Question 4 is more involved and has to be solved *via* a combination of Monte Carlo simulation (Section 5) and a new vote-transfer model (Section 6). The above thus leads to the development of new techniques that, whilst well suited to the French context, apply more generally.

The importance of our contribution is threefold. First, the options-pricing, vote transfer and simulation methods developed are innovative and have a broad range of application beyond France (see e.g. Fry & Burke, 2020; Taleb, 2018). This notwithstanding our approach fits particularly well to the French system. Second, we contribute to wider debates surrounding the French system (Dufresne et al., 2022; Jérôme et al., 2022; Mongrain, 2022) and the raw forecasting challenge (see e.g. Lewis-Beck et al., 2016; Murr, 2016, 2021). Third, we contribute to the literature on political and electoral forecasting (see Section 2).

The layout of this article is as follows. Section 2 reviews the related literature. An options-pricing approach to estimating vote shares is developed in Section 3. An empirical statistical approach to estimating the same quantities based entirely on the sampling variability in polls is given in Section 4. A Monte Carlo simulation algorithm is outlined in Section 5. A new vote-transfer model for multi-stage elections is developed in Section 6. An empirical application to the 2022 French presidential election is contained in Section 7. Additional robustness checks are performed in Section 8. Section 9 concludes and explores the opportunities for further research.

#### 2. Related literature

Models using opinion-poll data make up much prior political forecasting work though other approaches

are possible (see below). Polling data inevitably involves nontrivial statistical modelling (Bunker, 2020) with imperfect data (Pekar et al., 2022). Potential problems with polls include the difficulty of obtaining a representative sample, ensuring respondents reveal their intentions truthfully and hoping that respondents do not systematically change their intentions before the election (Khan & Lieli, 2018). Further limitations include sampling variability and the sparsity of polling data (Pekar et al., 2022). In the face of such uncertainties generating accurate forecasts remains problematic (Liu et al., 2021). An overview, including a review of recent forecasting failures, is given by Kenett et al. (2018). Sections 3-8 below therefore reflect a continued need to develop models that incorporate polling data as efficiently as possible.

Alternative approaches to opinion polls include fundamental political economy models based on relevant fundamentals. These typically include variables such as economic growth (Nadeau & Lewis-Beck, 2020) or incumbent time in office (Abramowitz, 2012). Further alternatives include synthetic models (based on a synthesis of fundamental and polling information) (Erikson & Wlezien, 2014) and political stock markets/prediction markets (Bunker, 2020). Recent evidence suggests both structural models (Quinlan & Lewis-Beck, 2021) and prediction markets (Reade & Vaughan Williams, 2019) include useful predictive information over and above polls.

The options-pricing approach outlined in Section 3 below is noteworthy for its simplicity and elegance. Both facets emerge as key themes in the literature. Green and Armstrong (2015) emphasise the utility of simple models in practical forecasting applications. The elegance of our approach also appeals to a high degree of technical sophistication associated with much political modelling. This includes computational allocation problems (Wang et al., 2015), a political travelling salesman problem (Shahmanzari et al., 2022) advanced Bayesian statistical modelling of U.S. elections (Rigdon et al., 2015) and SDE modelling of opinion-poll data (Levene & Fenner, 2021).

# 3. An options-pricing approach to estimating election outcomes

In this section, we lay out a financial optionspricing model to make opinion-poll forecasts. The model is, then, further adapted to some of the intricacies of the French system. This underscores the potential versatility of the approach.

Let  $P_{i,t}$  denote the subjective probability of event  $E_{i,t}$  at time t. In this context,  $P_{i,t}$  may represent the

probability that a randomly chosen constituent will vote for Party *i* at time *t*. This probability can, in turn, be thought of as representing the price of a wager that pays \$1 if event  $E_{i,t}$  occurs and 0 otherwise (see e.g. Lad, 1996). This formulation also provides an important philosophical link between subjective probabilities and financial betting (Taleb, 2017, 2018).

The importance of this approach for political applications is threefold. First, forecasts of vote share made by polling companies, based on moderated use of raw polling numbers, corresponds naturally to the subjective probabilities outlined above. Second, under this framework, the actual vote share in the election corresponds to an asset price at a known future date. This renders analysis particularly amenable to an options-pricing approach (Fry & Burke, 2020; Taleb, 2018). Third, if  $P_{i,t}$  and  $P_{j,t}$  denote asset prices as time *t* the ratio  $Q_t := P_{i,t}/P_{j,t}$  represents the price of asset *i* in units of asset *j*. These  $Q_t$  values prove particularly useful in the analysis of proportional vote systems (see below).

Consider, first, the univariate problem of predicting the vote share for one party with  $P(t) = P_{i,t}$ . Here, we will estimate the relevant probabilities by appealing to the Binary Options pricing formula (Hull, 2008, chapter 24 with Q = 1) in a standard Black–Scholes setting:

Binary call option Payoff:

$$I(P_T > K_i)$$
 Price:  $e^{-r(T-t)}\Phi(d_2)$ 

Binary put option Payoff:

$$I(P_T < K_i) \text{ Price: } e^{-r(T-t)} \Phi(-d_2),$$
(1)

where  $\Phi(\cdot)$  denotes the standard normal CDF,  $I(\cdot)$  denotes the indicator function, T is the expiration date,  $K_i$  is the strike price, and  $d_2 = (\ln (P_t/K_i) + (r - \sigma^2/2)(T - t))/\sigma\sqrt{T - t}$ . As is common in empirical options-pricing applications (see e.g. Dowd & Blake, 2022) r is the risk-free interest rate. This formulation thus makes an explicit link between (subjective) probabilities and the price of an associated bet (Taleb, 2017, 2018).

The probability that the party wins the presidency in the first round is now given by the Binary call option price in Eq. (1) with  $K_i = 0.5$  (reflecting the 50% share of the vote that parties need to win outright in the first round):

First-stage win probability

$$=e^{-r(T-t)}\Phi\left(\frac{\ln\left(2P_t\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right).$$
(2)

Now, suppose we want to estimate the vote share at time t < T, where T is the known election date.

Suppose we use the median price of the bet to estimate the probability that a randomly chosen constituent votes for a given party. Using the binary call option price in Eq. (1) gives

$$e^{-r(T-t)}\Phi(d_2) = \frac{1}{2}$$
$$\ln K_i$$

$$= \ln P_t + \left(r - \frac{\sigma^2}{2}\right)(T - t) - \sigma\sqrt{T - t}\Phi^{-1}\left(\frac{1}{2}e^{r(T - t)}\right),$$
  
$$K_i = P_t e^{\left(r - \frac{\sigma^2}{2}\right)(T - t)}e^{-\sigma\sqrt{T - t}\Phi^{-1}\left(\frac{1}{2}e^{r(T - t)}\right)}.$$
 (3)

Equation (3) therefore means that the  $K_i$  would be the estimated vote share, where  $P_t$  is the polling percentage at time t, r is the interest rate and  $\sigma$  is an estimate of the underlying volatility associated with polling numbers. In the French system, Eq. (3) can apply in both the first and second round of the vote.

An options-pricing argument is particularly attractive for proportional systems like France. For two parties *i* and *j* define  $Q_t = P_{i,t}/P_{j,t}$ . Financially,  $Q_t$  gives the price of asset *i* in units of asset *j*. The probability that *i* beats *j* can be calculated as the price of a binary option with strike price K = 1. This gives

Pr(i beats j in the popular vote)

$$=e^{-r(T-t)}\Phi\left(\frac{\ln\left(Q_t\right)+\left(r-\frac{\sigma_Q^2}{2}\right)(T-t)}{\sigma_Q\sqrt{T-t}}\right),\quad(4)$$

where  $\sigma_Q$  denotes the volatility associated with  $Q_t$ . In the second round of the vote, Eq. (4) gives the win probability for Party *i* versus Party *j*. In the first round of the vote, if  $P_{j,t}$  denotes the vote share of the party currently ahead in the polls Eq. (4) gives the probability that *i* wins the popular vote. Similarly, if  $P_{j,t}$  denotes the vote share of the party currently second in the polls Eq. (4) gives the probability that *i* qualifies for the second round of the election.

The above, thus, requires a plug-in estimator of the volatility  $\sigma$ . Following Fry and Burke (2020) we estimate  $\sigma$  as the residual standard error of the following weighted regression model:

$$\Delta X_t = \mu \Delta t + u_t, \tag{5}$$

where  $\Delta X_t$  is the first-difference of the log-price,  $\Delta t$  is the first difference of the polling time, measured in days, and  $Var(u_t) = \sigma^2 \Delta t$ .

#### 4. Forecasting election outcomes based on sampling variability

In this section, we detail how election outcomes can also be modelled without recourse to an optionspricing argument. Consider the random-walk model in Eq. (5). Let t denote the current polling date and T denote the known future election date. The electoral calculus works as follows. First, the probability that i wins the first-stage is given by

First stage win probability = 
$$Pr(X_T > 0.5)$$

$$= Pr(\ln X_T > \ln 0.5)$$
$$= \Phi\left(\frac{\mu(T-t) + \ln X_t - \ln 0.5}{\sigma\sqrt{T-t}}\right).$$
(6)

Similarly, the predicted share of the vote during an election can be calculated as

Median 
$$P_T = \exp(\mu(T-t) + X_t)$$
. (7)

To calculate the probability that Party *i* beats Party *j* set  $Q_t := P_{i,t}/P_{j,t}$ . Then use

$$Pr(i \text{ beats } j) = Pr(Q_T > 1) = Pr(\ln Q_T > 0)$$
$$= \Phi\left(\frac{\ln Q_t + \mu_Q(T-t)}{\sigma_Q\sqrt{T-t}}\right).$$
(8)

#### 5. A Monte Carlo simulation algorithm

In this section, we develop a Monte Carlo simulation algorithm. This enables second-stage election outcomes to be simulated on the basis of first-round polling data. Let Party 1 and Party 2 denote the two parties who quality for the second round of the election. We assume that those who voted for either Party 1 or Party 2 in the first round continue to do so. Those voting for parties eliminated in the first round are assumed to transfer their vote to either Party 1 or Party 2.

The above suggests that election-winning probabilities can be obtained *via* Monte Carlo simulation as follows:

- 1. Simulate the vote share percentage in the first round.
- 2. Assume that the vote percentages of the two leading parties remain fixed at Round 1.
- 3. Simulate the transfer of votes between the first and second rounds from the eliminated candidates to the final two candidates.
- 4. The projected vote share of the final two parties is obtained by adding the vote shares at Stages 2–3.

To be precise Eq. (3) suggests election outcomes can be simulated as follows.

- 1. Given  $K_1$ ,  $K_2$ , ...  $K_n$  simulate percentage vote shares  $p_1$ ,  $p_2$ , ...  $p_n$ .
- 2. Let the two biggest parties be Party 1 and 2 with  $p_1 > p_2$ . If  $p_1 > 0.5$  Party 1 wins. Else set

Second-round vote share Party 1

$$= p_1 + q_{31}p_3 + \ldots + q_{n1}p_n, \tag{9}$$

where  $q_{j1}$  denotes the probability that an initial vote for candidate *j* in the first round is transferred to Party 1 in the second round. Similarly, set

Second-round vote share Party 2

$$= p_2 + (1 - q_{31})p_3 + \dots + (1 - q_{n1})p_n.$$
(10)

Estimation of the above vote-transfer probabilities is discussed in Section 6.

Constructing a simulation algorithm also requires specification of a statistical model for the proportion  $p_i$  who vote for candidate *i*. It is natural to use the Dirichlet distribution in this context (see e.g. Ghosal & Van Der Vaart, 2017). Equation (3) suggests  $E[p_i] = K_i$ . Applying the constraint  $\sum_{i=1}^{n} K_i = 1$  it follows from Narayanan (1992) that given an associated variance  $s_i^2$  the vote share can be modelled as a Dirichlet distribution with parameters ( $\alpha_1$ ,  $\alpha_2$ , ...  $\alpha_n$ ) where

$$\alpha_{i} = \frac{(K_{1} - K_{i}^{2} - s_{1}^{2})K_{i}}{K_{1}^{2}} \qquad (i < n)$$

$$\alpha_{n} = \frac{(K_{1} - K_{1}^{2} - s_{1}^{2})(1 - \sum_{i=1}^{n-1}K_{i})}{K_{1}^{2}} \qquad (i = n).$$
(11)

To estimate the associated variance  $s_i^2$  we can use Eq. (3) in conjunction with the delta method (Bingham & Fry, 2010, chapter 7) to convert uncertainty in the estimated volatility  $\sigma$  to the associated plug-in estimator for  $K_i$ . This gives

$$s_{i}^{2} \approx \operatorname{Var}(\hat{\sigma}) \left(\frac{\partial K_{i}}{\partial \sigma}\right)^{2}$$

$$= \operatorname{Var}(\hat{\sigma}) P_{t}^{2} e^{2r(T-t)} \left(\sqrt{T-t} \Phi^{-1} \left(\frac{1}{2} e^{r(T-t)}\right) + \hat{\sigma}(T-t)\right)^{2} e^{-2\hat{\sigma}\sqrt{T-t} \Phi^{-1} \left(\frac{1}{2} e^{r(T-t)}\right) - \hat{\sigma}^{2}(T-t)}.$$
(12)

Note that Eq. (12) relies, in a financial context, on an estimate of the volatility of volatility (see e.g. Huang et al., 2019).

Here, we use the residual standard error from a regression model to estimate  $\sigma$  (see Eq. (5)). A classical result in regression states that the residual standard deviation estimator has a chi-squared distribution:

$$\hat{\sigma} \sim \sigma \sqrt{\chi^2_{df}},$$
 (13)

where df denotes the residual degrees of freedom. See e.g. Bingham and Fry (2010) for details. It follows from (13) that

$$\operatorname{Var}(\hat{\sigma}) \approx \hat{\sigma}^2 \left( df - \frac{\left(\Gamma(df + 1/2)\right)^2}{2\left(\Gamma(df)\right)^2} \right), \quad (14)$$

where  $\Gamma(\cdot)$  denotes the Gamma function.

#### 6. A simple vote-transfer model

Whilst complex data on party positioning exists (Jolly et al., 2022) such data lags behind the constant birth and death of parties within the French system. In this section, we, therefore, construct a theoretical vote transfer model between the first and second rounds of the French presidential election based on the simple division of parties from left to right. This enables second-stage election outcomes to be simulated on the basis of first-round polling. Table 1 gives an outline description of the parties involved. Figure 1 gives a diagrammatic representation showing the respective distances between the parties. This left-right dichotomy was, historically, the conventional way of looking at French elections. However, this traditional hegemony has since been challenged by the emergence of parties in the centre and from the far right (Mongrain, 2022). Nonetheless, this approach remains convenient for forecasting applications and a related left-to-right listing is also considered in Dufresne et al. (2022).

A theoretical vote-transfer model can be constructed as follows. Suppose a constituent votes for Party A in the first round but is left with a choice of Party B and Party C in the second round. Let  $P_{AB}$  denote the probability the voter transfers to Party B. Let  $P_{AC}$  denote the probability the voter transfers to Party C. We assume that the switching probability is proportional to the distance between the parties:

$$\frac{P_{AB}}{P_{AC}} = \frac{\text{Distance}(A, C)}{\text{Distance}(A, B)}.$$
 (15)

Equation (15), therefore, imposes a physically realistic mechanism whereby vote-transfer between adjacent parties is more likely. The model reduces

Table 1. Outline description of major French politicalparties.

Left to right	Symbol		Anal	icised	name			Catego	risatio	n
iigiit	Symbol		Aligi	iciscu	nume			cutego	illutio	
Point 1	LO	Work	ers sti	ruggle			Far le	eft		
Point 2	NPA	New	Antica	apitalis	t Party	<i>'</i>	far le	ft		
Point 3	PCF	Fren	ch Cor	nmuni	st Part	у	Left wing to far left			eft
Point 4	LFI	La Fi	ance l	nsoun	nise		Left	wing t	o far l	eft
Point 5	EELV	Euro	pe Ecc	logy -	- the G	ireens	Cent	re left	to left	wing
Point 6	PS	Socia	list Pa	rty			Cent	re Left		
Point 7	LREM	La R	epubli	c en N	larche!		Centre			
Point 8	Res	Résis	tons!				Centre			
Point 9	LR	The	The Republicans			Centre Right				
Point 10	) DLF	Debo	out La	France	5		Right	wing	to far	right
Point 1	1 NR	Natio	onal Ra	ally			Far r	ight -		
Point 12	2 REC	Reco	nqête				Far r	ight		
	2 0		-	0	_		0	10	44	10
1 1	2 3	4	- 5	6	7	8	- 9	10	11	12



Figure 1. Schematic diagram of French political parties on the left-right political spectrum.

to a special case of a network-flow model discussed elsewhere in Fry and Binner (2016). The model also gives some theoretical support for the notion that political parties may strategically aim for the middle ground to secure election-winning votes. This is consistent with parties positioning themselves where they anticipate the majority of voters' preferences to be (Akdağ, 2016).

From Eq. (15)

$$P_{AB} = \frac{\text{Distance}(A, C)}{\text{Distance}(A, B)} P_{AC}; P_{AC} \left[ 1 + \frac{\text{Distance}(A, C)}{\text{Distance}(A, B)} \right] = 1$$
(16)

Equations (15) and (16), therefore, lead to the solutions

$$P_{AB} = \frac{\text{Distance}(A, C)}{\text{Distance}(A, B) + \text{Distance}(A, C)};$$
$$P_{AC} = \frac{\text{Distance}(A, B)}{\text{Distance}(A, B) + \text{Distance}(A, C)}.$$
 (17)

Throughout the first round of the 2022 presidential election voting three parties LREM, NR and DLF persistently dominate polls. Projected votetransfer probabilities based on the original vote cast are shown below in Table 2.

#### 7. Empirical results

The layout of this section is as follows. The polling data used in this study is discussed in Subsection 7.1. Subsection 7.2 covers the first round of the presidential election. A Monte Carlo simulation exercise is outlined in Subsection 7.3. Subsection 7.4 covers the second round of the presidential election.

#### 7.1. The data

The polling data used in this article comes from the first and second round of polling for the 2017 and 2022 French presidential elections. The 2022 election is analysed in Subsections 7.2–7.4 and 8.2. The

 Table 2. Projected vote-transfer probabilities under three scenarios.

First round	Scen	Scenario 1		Scenario 2		Scenario 3	
vote	LREM	NR	LREM	LFI	NR	LFI	
LO	0.625	0.375	0.333	0.667	0.231	0.769	
NPA	0.643	0.357	0.286	0.714	0.182	0.818	
PCF	0.667	0.333	0.2	0.8	0.111	0.889	
LFI	0.7	0.3	0	1	0	1	
EELV	0.75	0.25	0.333	0.667	0.143	0.857	
PS	0.833	0.167	0.667	0.333	0.286	0.714	
LREM	1	0	1	0	0.429	0.571	
Res	0.75	0.25	0.8	0.2	0.571	0.429	
LR	0.5	0.5	0.714	0.286	0.714	0.286	
DLF	0.25	0.75	0.667	0.333	0.857	0.143	
NR	0	1	0.636	0.364	1	0	
REC	0.167	0.833	0.615	0.385	0.889	0.111	

Scenario 1s round is between LREM vs. NR. Scenario 2s round is between LREM and LFI. Scenario 3s round is between NR and LFI.

Table 3. Summary statistics of 2022 and 2017 presidential election polls.

Election	First day of polling	Last day of polling	No. of polls	No. of days	No. of polling companies
2017 First round	March 19	April 21	88	34	9
2017s round	April 24	May 5	36	13	8
2022 First round	March 7	April 8	118	32	14
2022s round	April 10	April 22	54	13	3

2017 election is analysed in Section 8.1. In the light of recent controversies, particularly in regard to the U.S. presidential election, some comments on data coverage are in order. Following a similar approach in Wlezien (2013) Table 3 summarises the number of the polls, the number of polling days and the number of different polling companies involved.

#### 7.2. First-round results

Following the official announcement of the full list of presidential candidates the first round of polling was held from 3 March 2022 to 8 April 2022. The round of the election itself was on 10 April 2022. When there are multiple polls in one day we take an average over multiple polls to obtain a single value for each party for each polling day. We choose the value of the risk-free interest rate to be 4% per annum. This is consistent with both recent applied options-pricing work (see eg. Dowd & Blake, 2022) and available Treasury Bill data at the time of writing.

Estimates of the share of the vote based on the entirety of the first round of polling are shown in Table 4. These include an estimated share of the vote obtained from Eq. (3) together with an associated estimated standard deviation based on Eq. (14). According to Eq. (2) the probability that Macron wins the election outright in the first round is 0.00 (to two decimal places). It, therefore, seems extremely unlikely that any of the parties can secure an outright election victory in the first round.

Equation (4) enables real-time estimation of several key quantities of interest. Figure 2 plots the estimated probability President Macron wins the popular vote during the first round of the election. The options-pricing method also seems to out-perform an approach based on sampling variability in polls outlined in Section 4. Figure 3 plots the estimated probability that the four major parties qualify for the second round of the election.

# 7.3. Monte Carlo simulation of election outcomes given the first round of polling

Table 4 summarises estimation results on the basis of the first round of polling. As discussed in Section 3 these results are first calibrated to a Dirichlet distribution using the method of Narayanan (1992).

**Table 4.** Estimated values of  $K_i$  and  $s_i$  obtained after the first round of polling.

Party	Estimated vote share $(K_i)$	Standard error (s <sub>i</sub> )
LO	0.005117346	0.004397742
NPA	0.009503101	0.003780203
PCF	0.02677225	0.002903718
LFI	0.1732441	0.002826978
PS	0.02042434	0.002270473
EELV	0.04615099	0.002225763
LREM	0.2627287	0.001436373
LR	0.08540158	0.002565392
RES	0.02373998	0.004231739
DLF	0.02206708	0.006069075
RN	0.2301469	0.004782486
REC	0.08893775	0.002844208

Simulated results from the Dirichlet distribution obtained are then combined with the vote-transfer model in Section 4 to simulate second-stage outcomes. Table 5 outlines the simulation results. It is estimated that there is a 96% probability that the incumbent President Macron wins. There is a roughly 4% probability that the far-right RN party wins, with only a remote possibility of an LFI victory. Results also underscore the dominant position of the three major parties.

#### 7.4. Second-round results

As the 2022 French presidential election reached its second stage a second round of opinion polling was conducted from 10 April to 22 April 2022. The vote itself occurred on 24 April 2022. As before, we take an average over multiple polls to obtain a single value for each party for each polling day. A plot of the estimated vote share for the incumbent LREM party is shown in Figure 4. A plot of the estimated win probability for the LREM party is shown in Figure 5. Results show the level of support for President Macron seemingly increasing over time. Results reinforce the dominant position of the incumbent president at the end of the first round of polling. The options-pricing and purely statistical approaches lead to similar results in each case. However, there is some suggestion of more consistent results under the options-pricing approach.

#### 8. Additional robustness checks

Additional robustness checks performed in this section are as follows. Subsection 8.1 applies our options-pricing method to the 2017 French



Figure 2. Estimated probability that President Macron wins the popular vote during the first round of the 2022 French presidential election according to Eq. (4) (solid line) and Eq. (8) (dashed line).



Figure 3. Estimated probability of qualification for the second round of the 2022 French presidential election according to Eq. (4).

**Table 5.** Estimated probability of election victory after first round of polling obtained by Monte Carlo simulation (results based on 1,000,000 simulations).

Party	Probability of winning
LO	0
NPA	0
PCF	0
LFI	0.000682
PS	0
EELV	0
LREM	0.961071
LR	0
RES	0
DLF	0
RN	0.038247
REC	0

presidential election. Subsection 8.2 compares our options-pricing model to the raw vote-intention polls.

## 8.1. Application to the 2017 French presidential election

Figures 6–9 repeat the analysis in Section 7 for the 2017 French presidential election. Figure 6 shows that Macron remains the consistent favourite to win the popular vote in the first round. The options-pricing approach also seems to out-perform the purely statistical approach. Second-round qualification probabilities in Figure 7 show a more congested presidential race in 2017 though the stage seems set for a showdown between Macron (EM) and Le Pen (FN) towards the end of polling. Figures 8 and 9 show Macron in an even more dominant position in the second round in 2017 both in terms of the estimated vote share and the estimated probability of winning.



Figure 4. Real-time model estimated vote share for Macron for the second round of the 2022 French presidential election based on Eq. (4) (solid line) and Eq. (7) (dashed line).



Figure 5. Real-time estimated probability of a Macron victory in the second round of the 2022 French presidential election based on Eq. (4) (solid line) and Eq. (8) (dashed line).

#### 8.2. Comparison with raw vote-intention polls

Tables 6 and 7 show a comparison with the options-pricing model with the raw-voting intention polls. We compare forecasts using the RMSE and the MAE defined *via* 

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (A_t - F_t)^2}$$
$$\text{MAE} = \frac{1}{n} \sum_{t=1}^{n} |A_t - F_t|,$$

where  $F_t$  denotes the forecast value and  $A_t$  denotes the actual value. Results in Table 6 suggest that the options-pricing approach marginally out-performs the raw voting-intention polls during the first round of polling. Results in Table 7 suggest the raw voteintention polls marginally out-perform the optionspricing method during the second stage of polling.

Tables 8 and 9 repeat Tables 6 and 7 for the 2017 French Presidential election. Table 8 suggests the raw voting-intention polls marginally outperform the options-pricing approach during the first round of polling. Table 9 suggests the options-pricing method marginally out-performs the raw voting-intention polls during the second round of polling.

#### 9. Conclusions and further work

Political forecasting is arguably more important than ever before given the use of predictive analytics in election campaigns (Kenett et al., 2018; Quinlan & Lewis-Beck, 2021), concerns raised over the



Figure 6. Estimated probability that President Macron wins the popular vote during the first round of the 2017 French presidential election according to Eq. (4) (solid line) and Eq. (8) (dashed line).



Figure 7. Estimated probability of qualification for the second round of the 2017 French presidential election according to Eq. (4).

manipulation of Social Media (Reisach, 2021) and recent suggestions of increased electoral volatility (Quinlan & Lewis-Beck, 2021). It is important to have as informed an electorate as possible (Pajala et al., 2018). Forecasting work can also impact on the economy (Reade & Vaughan Williams, 2019), increase voter turnout and satisfaction (Bunker, 2020) and help explain shifts in the underlying political environment (Quinlan & Lewis-beck, 2021). Models presented in this article, therefore, constitute a partial mathematical solution to a serious practical problem as well as building on much prior academic forecasting work.

Allied to the above the French presidential election (Baujard & Lebon, 2022) and its associated forecasting (Mongrain, 2022) is interesting and important in its own right. Moreover, outside of the

United States, forecasting using statistical models is most extensive in France (Lewis-Beck, 2005). This article develops an elegant options-pricing approach for opinion-poll data that serves two key purposes. First, we develop an options-pricing approach that enables principled estimation of vote share together with a series of adjustments that prove particularly well-suited to the multistage and proportional nature of the French system. Second, we develop a Monte Carlo simulation algorithm that can be applied to both single-stage and multistage elections. Overall, the approach is simple enough to be operationalised in forecasting applications (Green & Armstrong, 2015) whilst remaining in keeping with the technical traditions of the area (Levene & Fenner, 2021; Shahmanzari et al., 2022; Wang et al., 2015).



Figure 8. Real-time model estimated vote share for Macron in the second round of the 2017 French presidential election based on Eq. (4) (solid line) and Eq. (7) (dashed line).



Figure 9. Real-time estimated probability of a Macron victory in the second round of the 2017 French presidential election based on Eq. (4) (solid line) and Eq. (8) (dashed line).

**Table 6.** Comparison with the options-pricing approach and raw vote-intention polling numbers for the 2022 French presidential election after the first round of polling.

Party	Options-pricing model	Raw polling numbers	Actual
LO	0.005117346	0.005714286	0.0056
NPA	0.009503101	0.01000000	0.0077
PCF	0.02677225	0.02714286	0.0228
LFI	0.1732441	0.1735714	0.2195
PS	0.02042434	0.02071429	0.0175
EELV	0.04615099	0.04642857	0.0463
LREM	0.2627287	0.2628571	0.2785
LR	0.08540158	0.08571429	0.0478
RES	0.02373998	0.02428571	0.0313
DLF	0.02206708	0.02285714	0.0206
RN	0.2301469	0.2307143	0.2315
REC	0.08893775	0.08928571	0.0707
RMSE	0.01876124	0.01876718	
MAE	0.01146484	0.01151905	

A subjective probability argument suggests voteshare estimates from polling companies can be interpreted as asset prices. The corresponding

**Table 7.** Comparison with the options-pricing approach and raw vote-intention polling numbers for the second round of the 2022 French presidential election.

	•	
Date	Options-pricing model	Raw polling numbers
Apr 13	0.5361323	0.5370000
Apr 14	0.5337876	0.5341667
Apr 15	0.5388382	0.5390000
Apr 16	0.5547051	0.5550000
Apr 18	0.5456631	0.5460000
Apr 19	0.5547377	0.5550000
Apr 20	0.5573433	0.5575000
Apr 21	0.5515747	0.5516667
Apr 22	0.5599435	0.5600000
RMSE	0.0385388	0.03822347
MAE	0.03741939	0.03712962

election therefore constitutes an asset price at a known future date. This renders an options-pricing approach particularly attractive (Taleb, 2018). Rearranging standard binary options-pricing formulae (Hull, 2008) leads to principled estimates of vote

 
 Table
 8. Comparison with the options-pricing approach and raw vote-intention polling numbers for the 2017 French presidential election after the first round of polling.

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Party	Options-pricing model	Raw polling numbers	Actual
LO	0.003026144	0.003333333	0.0064
NPA	0.01269327	0.01333333	0.0109
LFI	0.1897006	0.19	0.1958
PS	0.07488916	0.075	0.0636
EM	0.2399117	0.24	0.2401
Res	0.007769815	0.008333333	0.0121
LR	0.1914305	0.1916667	0.2001
DLF	0.04128121	0.04166667	0.0470
UPR	0.007300721	0.008333333	0.0092
FN	0.2282986	0.2283333	0.2130
S&P	0.003089621	0.003333333	0.0018
RMSE	0.00704024	0.006947207	
MAE	0.005449996	0.005278781	

**Table 9.** Comparison with the options-pricing approach and raw vote-intention polling numbers for the second round of the 2017 French presidential election.

Date	Options-pricing model	Raw polling numbers
Apr 27	0.6010233	0.6012500
Apr 28	0.5950509	0.5950000
Apr 29	0.6000272	0.600000
Apr 30	0.6001014	0.600000
May 1	0.5934568	0.5933333
May 2	0.5963662	0.5962500
May 3	0.6067211	0.6066667
May 4	0.6170280	0.6170000
May 5	0.6266846	0.6266667
RMSE	0.05789811	0.05793319
MAE	0.05694896	0.05698148

share. The approach also allows us to calculate the probability of an outright win during the first round of the election. Allied to the above the ratio of the vote share of two parties can also be interpreted as an asset price (albeit one expressed in a different currency). Applying our options-pricing approach to these ratios yields closed-form expressions for the probability of winning the popular vote, the probability of qualifying for the second round of the election and the probability of winning the election in the second round.

Development of the Monte Carlo simulation algorithm solves problems far beyond the French context. The options-pricing approach outlined above leads to point estimates of vote share. This requires volatility values derived from a residual standard deviation parameter from a regression model (Fry & Burke, 2020). This standard error characterises the uncertainty associated with polling and, in a financial context, refers to the volatility of volatility (Huang et al., 2019). The nature of the French system necessitates modelling the transfer of votes at the end of the first round. Here, we develop a statistical vote-transfer model that accounts for an incomplete data record and relies only a schematic listing of parties from left to right.

The practical implications of our study are as follows. We present new ways of analysing political systems. Our approach is particularly well suited to those systems defined by either constant flux, see e.g., the constant birth/death of parties within the French system, a relatively incomplete historical data record, proportional-vote systems and systems that are multistage in nature. Results in Sections 6 and 7 show that an options-pricing approach appears to lead to less-volatile estimates compared to an equivalent approach based on sampling variability in polls. Results in Section 8 show an options-pricing approach can be successfully applied to different elections and may lead to some marginal improvements with respect to raw votingintention polls. Results also highlight two important wider themes. First, the utility of options-pricing methods in practical applications (see e.g., Dowd et al., 2019). Second, the continued relevance of additional political forecasting work including comparisons of different methods using Brier scores and related criteria (Hanretty, 2021).

Forecasting work is particularly relevant given recent suggestions of enhanced electoral volatility (Quinlan & Lewis-Beck, 2021) and manipulation of social media, please see e.g., Reisach (2021). Polling can variously impact upon voter behaviour (Bunker, 2020) and the economy at large (Reade & Vaughan Williams, 2019). Successful forecasting work can also improve our understanding of the underlying political environment (Quinlan & Lewis-Beck, 2021). There is, therefore, a continued need to develop new forecasting methods. This includes citizen forecasting approaches (Dufresne et al., 2022), election stock markets (Graefe, 2017) and combining different methods (Graefe, 2023). Wlezien et al. (2013) present an interesting approach whereby regression methods are used to convert raw polling at various times during the electoral campaign to a projected share of the vote at the actual election date. However, the constant flux within the French political system means that such methods may be difficult to apply here. France, itself, is a particularly interesting country to study. Notwithstanding the specifics of the French political economy (Hastings & Heyes, 2018) and its electoral landscape, we also stress the versatility of this approach and encourage its application with respect to other country case studies. The sophisticated evolution of forecasting research appears especially relevant and valuable at a time when multiple crises have affected economies and societies throughout the globe, stirring the creation of new political parties, governance strategies and (subsequent) voting outcomes.

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