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Hierarchical Structure-based Fault Estimation and Fault-tolerant Control for Multi-agent Systems

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***Abstract*—This paper proposes a hierarchical structure-based fault estimation and fault-tolerant control design with bi-directional interactions for nonlinear multi-agent systems with actuator faults. The hierarchical structure consists of distributed multi-agent system hierarchy, undirected topology hierarchy, decentralized fault estimation hierarchy and distributed fault-tolerant control hierarchy. The states and faults of the system are estimated simultaneously by merging the unknown input observer in a decentralized fashion. The distributed constant gain-based and node-based fault-tolerant control schemes are developed to guarantee the asymptotic stability and H-infinity performance of multi-agent systems, respectively, based on the estimated information in the fault estimation hierarchy and the relative output information from neighbors. Two simulation cases validate the efficiency of the proposed hierarchical structure control algorithm.**

***Index Terms*—Hierarchical structure control, fault estimation, fault-tolerant control, nonlinear multi-agent systems, actuator faults.**

I. Introduction

MULTI-AGENT systems (MASs) are attracting considerable attention in various control fields because of their potential applications in the formation of unmanned aerial vehicles[1], multi-robot coordination[2] and optimal scheduling in wireless networks[3], [4]. MASs are complex large-scale systems composed of a large number of distributed, autonomous or semi-autonomous agents that are connected by mechanical interconnections[5] or communication networks[1], [2]. Existing control approaches for individual agents are not suitable for MASs because of the inherent interconnected characteristics among the agents. Thus, positive effects can be obtained by specifying the concepts of MASs to achieve the satisfactory local performance of each agent and the global property of MASs.

However, faults may occur in one or more agents and degrade the performance of the system or may even lead to a catastrophic consequence in MASs, such as actuator and sensor faults of individual agent[6], [7], mechanical hinge coupling faults and network communication faults[8]. Therefore, MASs are required to perform safely and healthily, and fault-tolerant control (FTC) is regarded as a fruitful approach that can guarantee the stability and satisfactory properties of MASs with unpredictable faults[9], [10]. A distributed adaptive observer-based fault estimation (FE) design was proposed for leader-following linear MASs in the presence of additive faults, modeling uncertainties and external disturbances[11]. The FTC system compensated uncertain dynamics, time-varying faults and external disturbances simultaneously based on a fault-tolerant consensus control design with robust adaptive strategy[12]. However, most studies focused on separated FE and FTC designs[13], [14] and did not consider the bi-directional interactions between the FE and FTC hierarchies and the direct use of estimated fault signals from the FE to compensate the effects of faults. Lan and Patton[6], [15] proposed the integrated FE and FTC protocols for uncertain Lipschitz nonlinear systems with disturbances and simultaneous actuator and sensor faults. The effects of mutual couplings from the disturbances and nonlinearities between the FE and FTC systems were handled simultaneously. Studies on the integrated FE and FTC approach, especially in the application of MASs, are limited, and few findings are attributed to the hierarchical structure-based FE and FTC designs with bi-directional interactions for nonlinear MASs.

Their approaches[6], [15] used the so-called integrated FE/FTC design, which are known as decentralized control in MASs under the known interconnection topology and cannot be applied in distributed MASs with complex and strong couplings that are significant in this current study. MASs have three main FTC structures, namely, (i) centralized[16], (ii) decentralized[17] and (iii) distributed[18]. First, each agent can obtain state and fault information from all agents through a centralized monitor. This centralized structure is expensive to implement and only applicable to small-scale MASs. Second, the decentralized FTC controller of each agent is only developed based on its states and faults. This structure does not need any neighboring information interactions and leads to a simple FTC design. However, this structure is not well-suited for MASs with strong constraints in coupling characteristics[19]. Third, each agent has its own monitor that is only equipped with information interactions from its coupled agents in a distributed fashion. This structure is low cost and easy to implement, resulting in a wide range of applications in large-scale MASs[20], [21]. Therefore, it is the motivation of devising the distributed FTC strategies based on the estimated states and faults in the FE and the information exchange from

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the neighboring agents.

This paper addresses a hierarchical structure-based FE and FTC design for a class of nonlinear MASs with actuator faults. The so-called hierarchical structure consists of distributed MASs hierarchy, undirected topology hierarchy, decentralized FE hierarchy and distributed FTC hierarchy. The estimated information in the decentralized FE hierarchy and the constant gain-based and node-based designs in the distributed FTC hierarchy are developed to guarantee the asymptotic stability and H_∞ performance of MASs. The major contributions of this paper can be summarized as follows: (i) The unknown input observers in the decentralized FE hierarchy are developed to estimate the faults and states without prior information requirements of unknown nonlinear interactions and disturbances based on the previous works[6], [15]. (ii) This paper considers the decentralized FE and distributed FTC protocols in the hierarchical structure to overcome the limitation and high cost of the coupling constraints in MASs compared with the integrated FE/FTC designs[6]. Mutual effects with bi-directional interactions between the FE and FTC hierarchies are considered. (iii) In comparison with FTC designs based on their own estimated information[6], [15] or the state information of the neighbours[22], [23], the FTC strategies proposed in this paper are implemented in a fully distributed fashion not only based on the estimated information in the FE but also on the output information of the neighbors instead of using any of the global information of the communication topology.

The remainder of this paper is organized as follows. Section II introduces the problem formulation including graph theory and the distributed MASs hierarchy description. Section III is devoted to the decentralized FE hierarchy design. The distributed FTC hierarchy designs including the constant gain-based and node-based schemes are presented in Section IV to guarantee the robust stability of the hierarchical structure system. Simulations in Section V validate the efficiency of the proposed control design. Finally, conclusions follow in Section VI. The symbol † denotes the pseudo inverse, $\otimes$ denotes the kronecker product, $\mathrm{He}\,(X) = X + X^T$, and $\star$ represents the symmetric part of the specific matrix.

II. Problem Formulation

A. Graph theory

An undirected graph $\mathcal{G}$ is a pair (ν, ς), where $\nu = \{\nu_1, \cdots, \nu_N\}$ is a nonempty finite set of nodes and $\varsigma \subseteq \nu \times \nu$ is a set of edges. The edge (ν_i, ν_j) is denoted as a pair of distinct nodes (i, j). A graph is said to be undirected with the property $(\nu_i, \nu_j) \in \varsigma$ that signifies (ν_j, ν_i) for any $\nu_i, \nu_j \in \nu$. Node j is called a neighbor of node i if $(\nu_i, \nu_j) \in \varsigma$. The set of neighbors of node i is denoted as $\mathcal{N}_i = \{j \mid (\nu_i, \nu_j) \in \varsigma\}$. The adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ is a constant matrix represented as the graph topology. a_{ij} is the weight coefficient of the edge (ν_i, ν_j) and $a_{ii} = 0, a_{ij} = 1$ if $(\nu_i, \nu_j) \in \varsigma$, otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{N\times N}$ is defined as $l_{ij} = \sum_{i\neq j} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. If there is a path between two arbitrary nodes, the undirected graph is said to be connected.

B. Distributed MASs hierarchy

Consider a group of N agents with nonlinearities and actuator faults. The i-th $(i = 1, 2, \cdots, N)$ agent is described as

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + Ff_i + \xi_i(x, t) \\ y_i &= Cx_i \end{aligned} \tag{1}$$

where $x_i \in R^n, u_i \in R^m, y_i \in R^p$ are the system state, input and output vectors, respectively. $f_i \in R^q$ denotes the actuator fault and $\xi_i(x, t) \in R^n$ denotes the nonlinear interaction term with $x = [x_1^T, \cdots, x_N^T]^T$, which might be viewed as mechanical interconnections in the distributed MASs hierarchy. A, B, F and C are known constant matrices with compatible dimensions.

Assumption 2.1: (1) The pairs (A, B) and (A, C) are controllable and observable, respectively. (2) The matching condition for the actuator fault f_i is satisfied with $\mathrm{rank}(B, F) = \mathrm{rank}(B) = m$. The actuator fault f_i belongs to $\mathcal{L}_2\,[0, \infty)$ and is continuously smooth with bounded first-order time derivative.

Assumption 2.2: The nonlinear interaction term $\xi_i(x, t)$ in the distributed MASs hierarchy satisfies with the matrix inequality $\xi_i^T(x, t)\xi_i(x, t) \leq \alpha_i x^T E_i^T E_i x$, where E_i is a known constant matrix and α_i is a positive scalar as the upper bound for the corresponding interaction.

Remark 2.1: Assumption 2.1 provides the controllable and observable conditions for the described control systems and guarantees the actuator fault f_i to be constrained in a given compensation range by the designed input u_i. The interaction term $\xi_i(x, t)$ in Assumption 2.2 might be described as the specific hinge mechanism connection between the rigid and flexible parts of multiple spacecrafts and the transmission links of smart grids[5]. Furthermore, define $\xi(x, t) = [\xi_1^T, \cdots, \xi_N^T]^T$ as the interaction term of overall MASs, then it follows that the augmented interaction term $\xi^T(x, t)\xi(x, t) \leq x^T E^T E x$ with $E = [\sqrt{\alpha_1}E_1^T, \cdots, \sqrt{\alpha_N}E_N^T]^T$.

Definition 2.1[24]: Let $\gamma > 0$ and $\epsilon > 0$ be given constants, the closed-loop system can achieve a H_∞ performance index no larger than γ, i.e. $\|G_{zd}\| < \gamma$ if the following form holds:

$$\int_0^\infty z^T(t)\, z(t)\, dt \leq \gamma^2 \int_0^\infty d(t)^T d(t)\, dt + \epsilon \tag{2}$$

Lemma 2.1[25]: There exists a zero eigenvalue for the Laplacian matrix $\mathcal{L}$ with 1_N as a corresponding right eigenvector and all nonzero eigenvalues have positive real parts in the undirected graph $\mathcal{G}$. Assume that λ_i denotes the i-th eigenvalue of $\mathcal{L}$, thus, $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$. Furthermore, if $1_N^T X = 0$, then $X^T \mathcal{L} X \geq \lambda_2 X^T X$.

Figure 1 outlines a general diagram of the hierarchical structure-based FE and FTC design with bi-directional interactions for nonlinear MASs. The structure comprises (i) distributed MASs hierarchy, (ii) undirected topology hierarchy, (iii) decentralized FE hierarchy and (iv) distributed FTC hierarchy. First, the mechanical interconnection $\xi_i(x, t)$ in the i-th agent shows the distributed fashion in the existing MASs hierarchy. Second, the undirected topology $\mathcal{G}$ can be described with the Laplacian matrix $\mathcal{L}$ in the applications of flight and

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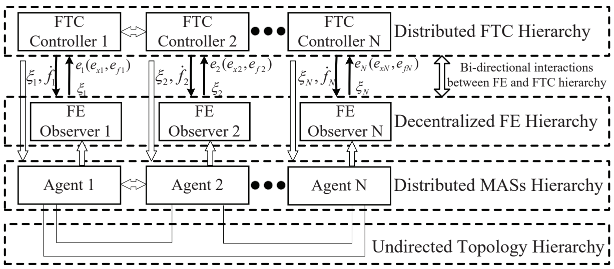

Fig. 1. Hierarchical structure-based design with bi-directional interactions between the FE and FTC hierarchy.

multi-robot formation[1], [2], which implies that agents are connected to one another through communication networks. Third, each observer in the decentralized FE hierarchy[6] is designed independently to reconstruct the shape of the fault. Fourth, compared with the decentralized FTC design[15], the distributed FTC design is constructed to not only directly use the fault estimation information in the FE process but also the output information from the neighboring agents to compensate the effects of faults. In comparison with the FTC design based on fault detection and identification (FDI) approaches[9], [26], the FE design is introduced in the distributed FTC system by estimating the fault instead of the threshold setting and fault isolation in the FDI procedure, which can significantly take full advantage of the fault information in the FTC system.

III. Decentralized Fault Estimation Hierarchy

Augment the i-th dynamic model (1) with $\bar{d}_i = \dot{f}_i$ into

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}\bar{x}_i + \bar{B}u_i + \bar{\xi}_i\left(x,t\right) + \bar{D}\bar{d}_i \\ y_i &= \bar{C}\bar{x}_i \end{aligned} \tag{3}$$

where

$$\bar{A} = \begin{bmatrix} A & F \\ 0_{q\times n} & 0_{q\times q} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0_{q\times m} \end{bmatrix}, \bar{D} = \begin{bmatrix} 0_{n\times q} \\ I_q \end{bmatrix}$$
$$\bar{C} = \left[C \; 0_{p\times q}\right], \bar{x}_i = \left[x_i^T, f_i^T\right]^T, \bar{\xi}_i\left(x,t\right) = \left[\xi_i^T, 0_{q\times 1}^T\right]^T$$

The designed observer in the decentralized FE hierarchy only requires the information from the corresponding agent rather than its neighboring observer. Then, the state $\bar{x}_i$ of the i-th augmented system is estimated by the i-th unknown input observer[6], [15] expressed as

$$\dot{z}_i = Mz_i + Gu_i + Jy_i, \quad \hat{\bar{x}}_i = z_i + Hy_i \tag{4}$$

where $z_i \in R^{n+q}$ is the state of the i-th unknown input observer. $\hat{\bar{x}}_i = [\hat{x}_i^T, \hat{f}_i^T]^T$ is the estimate of the augmented state $\bar{x}_i$. $\hat{x}_i \in R^n$ and $\hat{f}_i \in R^q$ are the estimates of the system state x_i and actuator fault f_i, respectively. Matrices M, G, J and H are of appropriate dimensions to be devised.

Define the estimation error as $e_i = \bar{x}_i - \hat{\bar{x}}_i = [e_{xi}^T \; e_{fi}^T]^T$ with $e_{xi} = x_i - \hat{x}_i$ and $e_{fi} = f_i - \hat{f}_i$. Then, we can obtain the following form with the definition of $\Gamma = I_{n+q} - H\bar{C}$.

$$\begin{aligned} \dot{e}_i = &\left(\Gamma\bar{A} - J\bar{C}\right)\bar{x}_i + \left(\Gamma\bar{B} - G\right)u_i \\ &+ \Gamma\bar{\xi}_i\left(x,t\right) + \Gamma\bar{D}\bar{d}_i - Mz_i \end{aligned} \tag{5}$$

Unfortunately, it cannot be completely decoupled due to the independent item Mz_i. Furthermore, according to the definition of $J = J_1 + J_2$, it follows that the estimation error is described as

$$\begin{aligned} \dot{e}_i = &\left(\Gamma\bar{A} - J_1\bar{C}\right)e_i + \left(\Gamma\bar{A} - J_1\bar{C} - M\right)z_i + \Gamma\bar{\xi}_i\left(x,t\right) \\ &+ \left(\left(\Gamma\bar{A} - J_1\bar{C}\right)H - J_2\right)y_i + \left(\Gamma\bar{B} - G\right)u_i + \Gamma\bar{D}\bar{d}_i \end{aligned} \tag{6}$$

Then, the matrices M, G, J_1, J_2 and H are designed in the following forms in order to decouple the estimation error e_i and additional items z_i, y_i and u_i.

$$\Gamma\bar{A} - J_1\bar{C} = M, M \text{ is Hurwitz} \tag{7}$$
$$\left(\Gamma\bar{A} - J_1\bar{C}\right)H = J_2 \tag{8}$$
$$\Gamma\bar{B} = G \tag{9}$$

Hence, the i-th estimation error dynamics are derived as

$$\dot{e}_i = \left(\Gamma\bar{A} - J_1\bar{C}\right)e_i + \Gamma\bar{\xi}_i\left(x,t\right) + \Gamma\bar{D}\bar{d}_i \tag{10}$$

and the designed matrices M, G and J_2 in the unknown input observer can be obtained with the derived matrices J_1 and H. Furthermore, define $e = [e_1^T, \cdots, e_N^T]^T, \bar{d} = [\bar{d}_1^T, \cdots, \bar{d}_N^T]^T$ and $\bar{\xi}(x,t) = [\bar{\xi}_1^T(x,t), \cdots, \bar{\xi}_N^T(x,t)]^T$, and it follows that

$$\begin{aligned} \dot{e} = &\left(I_N \otimes \left(\Gamma\bar{A} - J_1\bar{C}\right)\right)e + \left(I_N \otimes \Gamma\right)\bar{\xi}\left(x,t\right) \\ &+ \left(I_N \otimes \Gamma\bar{D}\right)\bar{d} \end{aligned} \tag{11}$$

Hence, the objective of obtaining unknown input observers is to design H and J_1 such that the estimation error dynamics (11) are robustly asymptotically stable.

Remark 3.1: The i-th estimation error dynamics can be completely decoupled when the terms $\Gamma\bar{\xi}_i(x,t) = 0$ and $\Gamma\bar{D}\bar{d}_i = 0$ are satisfied. The required condition of the Hurwitz matrix M guarantees that (11) is robustly asymptotically stable. However, it is shown from (10) and (11) that the performance of FE is influenced by the nonlinearity $\xi_i(x,t)$ and the fault modeling error $\dot{f}_i$ in Figure 1.

Remark 3.2: The proposed unknown input observer (4) has two major advantages. On the one hand, the prior information of the unknown nonlinear interactions $\bar{\xi}(x,t)$ and disturbances $\bar{d}$ does not need to be obtained. This positive effect is evident compared with the assumptions of the bounded disturbances and nonlinearities[27], [28]. On the other hand, the disturbances in the estimation error dynamics can be decoupled with the rank requirement of $\mathrm{rank}(\bar{C}\bar{D}) = \mathrm{rank}(\bar{D})$[29], i.e., $(I_{n+q} - H\bar{C})\bar{D} = 0$. Neither the bounded condition nor the rank requirement of the disturbances in this paper is required in the designed unknown input observer (4).

IV. Distributed Fault-tolerant Control Hierarchy

In this section, the communication topology $\mathcal{G}$ is undirected. Each agent in the undirected topology hierarchy can receive the relative output information rather than the state information of its neighboring agents. On the basis of the estimated information in the unknown input observer (4) of the augmented system (3) and the relative output information of neighbors, two distributed FTC protocols are proposed, namely, the constant gain-based and node-based FTC designs.

A. Distributed constant gain-based FTC design

The distributed constant gain-based fault-tolerant controller for agent i is designed as follows:

$$u_i = -K\hat{\bar{x}}_i + gK_g \sum_{j=1}^{N} a_{ij} (y_i - y_j) \tag{12}$$

where $K = [K_x \; K_f]$ denotes the augmented controller gain with the state feedback gain $K_x \in R^{m\times n}$ and actuator fault compensation gain $K_f \in R^{m\times q}$. a_{ij} denotes the (i,j)-th entry of the adjacency matrix $\mathcal{A}$ involved with the undirected topology $\mathcal{G}$. $K_g \in R^{m\times p}$ denotes the distributed constant gain and g is a positive scalar.

Then, we can obtain the closed-loop system with the actuator fault compensation gain $K_f = B^{\dagger}F$ with $B^{\dagger} = (B^TB)^{-1}B^T$.

$$\begin{array}{c} \dot{x}_i = (A - BK_x)\, x_i + BKe_i + \xi_i(x,t) \\ +gBK_gC\sum_{j=1}^{N} a_{ij}(x_i - x_j) \end{array} \tag{13}$$

The objective of obtaining the distributed constant gain-based fault-tolerant controller is to design K_x and K_g such that the closed-loop system is robustly stable. Then, it follows that the hierarchical structure-based FE/FTC model with the distributed constant gain-based controller (12) is derived as

$$\left\{ \begin{array}{c} \dot{x} = (I_N \otimes (A - BK_x) + g\mathcal{L} \otimes BK_gC)\, x \\ +\, (I_N \otimes BK)\, e + \xi(x,t) \\ \dot{e} = \left(I_N \otimes \left(\Gamma\bar{A} - J_1\bar{C}\right)\right) e + (I_N \otimes \Gamma)\, \bar{\xi}(x,t) \\ +\left(I_N \otimes \Gamma\bar{D}\right)\bar{d} \\ z_c = C_{xc}x + C_{ec}e \end{array} \right. \tag{14}$$

where $\xi(x,t) = [\xi_1^T, \cdots, \xi_N^T]^T, \mathcal{L}$ is the Laplacian matrix corresponding to the undirected graph $\mathcal{G}, z_c \in R^{r_c}$ is the measured output vector for verifying the hierarchical structure-based FE/FTC system performance with matrices $C_{xc} \in R^{r_c\times nN}$ and $C_{ec} \in R^{r_c\times(n+q)N}$.

Notably, the estimation error dynamics e and nonlinear interactions $\xi(x,t)$ affect the closed-loop FTC system. It is shown from (14) and Figure 1 that the disturbances and nonlinear interactions influence FE performance. The nonlinearities and estimated errors in FE also influence FTC performance. Hence, the issue of the coupling items both in FE and FTC systems is challenging, which motivates the introduction of the hierarchical structure-based FE/FTC strategy. The proposed hierarchical structure provides a distinct advantage of the integrated consideration of FE and FTC simultaneously in marked contrast to previous works in independent FE and FTC strategies (i.e., the derivative and proportional observer for estimation error dynamics with a discrete-time fault-tolerant design[13] and the descriptor sliding mode approach based on state estimations[14]).

Hence, the objective of the proposed hierarchical structure-based FE/FTC design is to devise the state feedback gain K_x, the distributed constant gain K_g, and the unknown input observer gains H and J_1 to guarantee the robust stability of the hierarchical structure system (14).

Theorem 4.1. Given positive scalars $\gamma_c, \varepsilon_{1c}$ and ε_{2c}, matrices $C_{xc0} \in R^{r_c\times n}, C_{ecx} \in R^{r_c\times n}$ and $C_{ecf} \in R^{r_c\times q}$, the hierarchical structure-based FE/FTC system (14) with the distributed constant gain-based controller (12) is stable with H_∞ performance $\|G_{z_c\bar{d}}\| < \gamma_c$, if there exist symmetric positive definite matrices $Q_{c0} \in R^{n\times n}, P_{c1} \in R^{n\times n}$ and $P_{c2} \in R^{q\times q}$, and matrices $X_1 \in R^{m\times n}, X_2 \in R^{m\times p}, X_3 \in R^{n\times p}, X_4 \in R^{n\times p}, X_5 \in R^{q\times p}$ and $X_6 \in R^{q\times p}$ such that

$$\begin{bmatrix} \bar{\Omega}_{c11} & \bar{\Omega}_{c12} & 0 & \bar{\Omega}_{c14} & 0 & \bar{\Omega}_{c16} \\ \star & \bar{\Omega}_{c22} & \bar{\Omega}_{c23} & 0 & \bar{\Omega}_{c25} & \bar{\Omega}_{c26} \\ \star & \star & -\gamma_c I_{qN} & 0 & 0 & 0 \\ \star & \star & \star & -\varepsilon_{2c}I_{nN} & 0 & 0 \\ \star & \star & \star & \star & \bar{\Omega}_{c55} & 0 \\ \star & \star & \star & \star & \star & -I_{r_c} \end{bmatrix} < 0 \tag{15}$$

with

$$\begin{array}{c} \bar{\Omega}_{c11} = I_N \otimes \mathrm{He}\left(Q_{c0}A - BX_1\right) + \mathcal{L} \otimes \mathrm{He}\left(gBX_2C\right) \\ +\left(\varepsilon_{1c} + \varepsilon_{2c}\right)E^TE, \bar{\Omega}_{c12} = I_N \otimes \left[\begin{array}{cc} BX_1 & Q_{c0}BK_f \end{array}\right] \\ \bar{\Omega}_{c14} = I_N \otimes Q_{c0}, \bar{\Omega}_{c16} = I_N \otimes C_{xc0}^T, \bar{\Omega}_{c55} = -\varepsilon_{1c}I_{(n+q)N} \\ \bar{\Omega}_{c22} = I_N \otimes \begin{bmatrix} \bar{\Omega}_{c1} & \bar{\Omega}_{c2} \\ \star & \bar{\Omega}_{c3} \end{bmatrix}, \bar{\Omega}_{c23} = I_N \otimes \begin{bmatrix} 0 \\ P_{c2} \end{bmatrix} \\ \bar{\Omega}_{c25} = I_N \otimes \begin{bmatrix} \bar{\Omega}_{c4} & 0 \\ -X_5C & P_{c2} \end{bmatrix}, \bar{\Omega}_{c26} = I_N \otimes \begin{bmatrix} C_{ecx}^T \\ C_{ecf}^T \end{bmatrix} \\ \bar{\Omega}_{c1} = \mathrm{He}(P_{c1}A - X_3CA - X_4C) \\ \bar{\Omega}_{c2} = P_{c1}F - X_3CF - A^TC^TX_5^T - C^TX_6^T \\ \bar{\Omega}_{c3} = \mathrm{He}(-X_5CF), \bar{\Omega}_{c4} = P_{c1} - X_3C \end{array}$$

Then, the gains are obtained as $K_x = \hat{Q}_{c0}^{-1}X_1, K_g = \hat{Q}_{c0}^{-1}X_2, H_1 = P_{c1}^{-1}X_3, J_{11} = P_{c1}^{-1}X_4, H_2 = P_{c2}^{-1}X_5$, and $J_{12} = P_{c2}^{-1}X_6$ with $Q_{c0}B = B\hat{Q}_{c0}$.

Proof: Consider a Lyapunov function $V_{ec} = e^TP_ce$ with a symmetric positive matrix P_c, and the time derivative of V_{ec} is obtained in the following form with a positive scalar ε_{1c}.

$$\begin{array}{c} \dot{V}_{ec} \leq e^T(\mathrm{He}\left(P_c\left(I_N \otimes \left(\Gamma\bar{A} - J_1\bar{C}\right)\right)\right) \\ +\varepsilon_{1c}^{-1}P_c\left(I_N \otimes \Gamma\right)\left(I_N \otimes \Gamma\right)^TP_c)e + \varepsilon_{1c}x^TE^TEx \\ +\mathrm{He}(e^TP_c(I_N \otimes \Gamma\bar{D})\bar{d}) \end{array} \tag{16}$$

Then, consider anther Lyapunov function $V_{xc} = x^TQ_cx$ with a symmetric positive matrix Q_c, and the time derivative of V_{xc} is derived with a positive scalar ε_{2c} as follows:

$$\begin{array}{c} \dot{V}_{xc} \leq x^T(\mathrm{He}\left(Q_c\left(I_N \otimes (A - BK_x) + g\mathcal{L} \otimes BK_gC\right)\right) \\ +\varepsilon_{2c}^{-1}Q_cQ_c^T + \varepsilon_{2c}E^TE)x + \mathrm{He}\left(x^TQ_c\left(I_N \otimes BK\right)e\right) \end{array} \tag{17}$$

According to Definition 2.1, the sufficient condition for a H_∞ performance $\|G_{z_c\bar{d}}\| < \gamma_c$ can be represented as

$$\begin{array}{c} \int_0^\infty \left(z_c^Tz_c - \gamma_c\bar{d}^T\bar{d} + \dot{V}_{ec} + \dot{V}_{xc}\right) \\ -\left(V_{ec}(\infty) - V_{ec}(0) + V_{xc}(\infty) - V_{xc}(0)\right) < 0 \end{array} \tag{18}$$

Based on zero initial conditions, it follows that the sufficient condition of achieving (18) is $z_c^Tz_c - \gamma_c\bar{d}^T\bar{d} + \dot{V}_{ec} + \dot{V}_{xc} < 0$. Hence, according to the definition of $\zeta = [x^T \; e^T \; \bar{d}^T]^T$, it is derived as

$$\zeta^T \begin{bmatrix} \Omega_{c11} & \bar{\Omega}_{c5} & 0 \\ \star & \Omega_{c22} & P_c\left(I_N \otimes \Gamma\bar{D}\right) \\ \star & \star & -\gamma_cI_{qN} \end{bmatrix} \zeta < 0 \tag{19}$$

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where $\Omega_{c11} = \mathrm{He}(Q_c(I_N \otimes (A - BK_x) + g\mathcal{L} \otimes BK_gC)) + \varepsilon_{2c}^{-1}Q_cQ_c^T + (\varepsilon_{1c} + \varepsilon_{2c})E^TE + C_{xc}^TC_{xc}, \Omega_{c22} = \mathrm{He}(P_c(I_N \otimes (\Gamma\bar{A} - J_1\bar{C}))) + \varepsilon_{1c}^{-1}P_c(I_N \otimes \Gamma)(I_N \otimes \Gamma)^TP_c + C_{ec}^TC_{ec}, \bar{\Omega}_{c5} = Q_c(I_N \otimes BK) + C_{xc}^TC_{ec}$.

Furthermore, denote $C_{xc} = I_N \otimes C_{xc0}$ and $C_{ec} = I_N \otimes [C_{ecx} \ \ C_{ecf}]$. Denote $Q_c = I_N \otimes Q_{c0}, P_c = I_N \otimes \mathrm{diag}\{P_{c1}, P_{c2}\}$ with symmetric positive definite matrices Q_{c0}, P_{c1} and P_{c2}. Define $H = [H_1^T \ \ H_2^T]^T$ and $J_1 = [J_{11}^T \ \ J_{12}^T]^T$ with matrices $H_1 \in R^{n\times p}, H_2 \in R^{q\times p}, J_{11} \in R^{n\times p}$ and $J_{12} \in R^{q\times p}$.

Since the graph $\mathcal{G}$ is undirected, the matrix $\mathcal{L}$ is symmetric. According to the condition $Q_{c0}B = B\hat{Q}_{c0}$ and $K_x = \hat{Q}_{c0}^{-1}X_1, K_g = \hat{Q}_{c0}^{-1}X_2, H_1 = P_{c1}^{-1}X_3, J_{11} = P_{c1}^{-1}X_4, H_2 = P_{c2}^{-1}X_5$, and $J_{12} = P_{c2}^{-1}X_6$, the Schur lemma is used here to transfer the inequality (19) into the LMI formulation (15). This completes the proof.

Remark 4.1: Note that the inequality (15) needs to be solved with the equality constraint $Q_{c0}B = B\hat{Q}_{c0}$ by the LMI toolbox. The equality constraint needs to be transferred into the following optimization problem.

Minimize η subject to LMI formulation (15) with

$$\begin{bmatrix} \eta I & Q_{c0}B - B\hat{Q}_{c0} \\ \star & \eta I \end{bmatrix} > 0 \tag{20}$$

Remark 4.2: In comparison with the separate FE and FTC functions without considering the effect of system and estimation uncertainties in [13], [14], the distributed constant gain-based FTC design is proposed in Theorem 4.1 with the consideration of the resulting bi-directional robustness interactions between the FE and FTC systems by a single-step LMI formulation with equality constraint techniques.

Remark 4.3: The major difference of this paper with other works[6], [15] lies in the collection of data of the distributed information from the neighboring agents. Figure 1 shows that the closed-loop FTC system avoids the limitation of the only existing simple and known coupling interactions in the decentralized structure because of the distributed fashion and the collected output information from the neighbours[15]. Moreover, the only distributed item $gK_g\sum_{j=1}^{N} a_{ij}(y_i - y_j)$ without the estimated state item $K\hat{\bar{x}}_i$ in (12) cannot achieve the acceptable performances of FE and FTC due to the mutual influences introduced by the nonlinearities and disturbances in their procedures[11], [30]. Although the increase of the dimension adds to the complexity of the calculation of LMI, the difference of the topological structure also affects the feasible solution of the LMI directly. The dimension of the formulation of LMI should not be reduced with the individual agent because the existence of the topological structure plays a role in representing the global LMI problem, i.e., $\mathrm{He}(Q_c(gL \otimes BK_gC))$ in (17).

B. Distributed node-based FTC design

The distributed node-based fault-tolerant controller for the i-th agent is modified in the following form with the updated coupling weigh $d_i(t)$ corresponding with the i-th agent.

$$u_i = -K\hat{\bar{x}}_i + L_d\sum_{j=1}^{N} d_i(t)\,a_{ij}(y_i - y_j) \tag{21}$$

$$\dot{d}_i(t) = -\tau_i\sum_{j=1}^{N} y_i^T\Xi_d a_{ij}(y_i - y_j) \tag{22}$$

where $d_i(t)$ denotes the coupling weight for agent i and τ_i is a positive scalar. Matrices $L_d \in R^{m\times p}$ and $\Xi_d \in R^{p\times p}$ are the distributed feedback gains. The augmented controller gain K is defined in the distributed constant gain-based FTC design.

According to the dynamic model (1) and the distributed node-based fault-tolerant controller (21), it follows that the closed-loop system is derived as

$$\begin{aligned} \dot{x}_i = (A - BK_x)\,x_i + BKe_i + \xi_i(x,t) \\ + BL_dC\sum_{j=1}^{N} d_i(t)\,a_{ij}(x_i - x_j) \end{aligned} \tag{23}$$

The objective of obtaining the distributed node-based fault-tolerant controller is to design K_x and L_d such that the closed-loop system is robustly stable with the updated law (22). Then, it follows that the hierarchical structure-based FE/FTC model with the distributed node-based controller (21)-(22) is derived as

$$\left\{ \begin{aligned} \dot{x} &= (I_N \otimes (A - BK_x) + (\Delta_d(t)\,\mathcal{L} \otimes BL_dC))\,x \\ &\quad + (I_N \otimes BK)\,e + \xi(x,t) \\ \dot{e} &= \left(I_N \otimes \left(\Gamma\bar{A} - J_1\bar{C}\right)\right)e + (I_N \otimes \Gamma)\,\bar{\xi}(x,t) \\ &\quad + \left(I_N \otimes \Gamma\bar{D}\right)\bar{d} \\ z_d &= C_{xd}x + C_{ed}e \end{aligned} \right. \tag{24}$$

where $\Delta_d(t) = \mathrm{diag}(d_1(t), \ldots, d_N(t)), z_d \in R^{r_d}$ is the measured output vector with matrices $C_{xd} \in R^{r_d\times nN}$ and $C_{ed} \in R^{r_d\times(n+q)N}$.

Coupling is observed between the estimation error dynamics and closed-loop system, and the FE and FTC performances are influenced by nonlinearities and disturbances. Hence, the objective of the proposed hierarchical structure-based FE/FTC design is to devise the state feedback gain K_x, the distributed node-based gain L_d, and the unknown input observer gains H and J_1 to guarantee the robust stability of the hierarchical structure system (24) in simultaneously handling the coupling weights $\Delta_d(t)$. Furthermore, the existing equality constraint $Q_{c0}B = B\hat{Q}_{c0}$ in Theorem 4.1 needs to be managed for further flexibilities in achieving acceptable H_∞ performance.

Here, we will give the following theorem in a single-step LMI formulation without equality constraints, i.e. $Q_{c0}B = B\hat{Q}_{c0}$ in Theorem 4.1.

Theorem 4.2. Given positive scalars $\gamma_d, \varepsilon_{1d}, \varepsilon_{2d}$ and α, matrices $C_{xd0} \in R^{r_d\times n}, C_{edx} \in R^{r_d\times n}$ and $C_{edf} \in R^{r_d\times q}$, the hierarchical structure-based FE/FTC system (24) with the distributed node-based controller (21) and the updated law (22) is stable with H_∞ performance $\|G_{z_d\bar{d}}\| < \gamma_d$, if there exist symmetric positive definite matrices $Q_{d0} \in R^{n\times n}, P_{d1} \in R^{n\times n}$ and $P_{d2} \in R^{q\times q}$, and matrices $K_x \in R^{m\times n}, L_d \in R^{m\times p}, X_3 \in R^{n\times p}, X_4 \in R^{n\times p}, X_5 \in R^{q\times p}$ and $X_6 \in R^{q\times p}$ such that

$$\left[\begin{array}{cccccc} \bar{\Omega}_{d11} & 0 & 0 & \bar{\Omega}_{d14} & 0 & \bar{\Omega}_{d16} \\ \star & \bar{\Omega}_{d22} & \bar{\Omega}_{d23} & 0 & \bar{\Omega}_{d25} & \bar{\Omega}_{d26} \\ \star & \star & -\gamma_d I_{qN} & 0 & 0 & 0 \\ \star & \star & \star & -\varepsilon_{2d}I_{nN} & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon_{1d}I_{(n+q)N} & 0 \\ \star & \star & \star & \star & \star & -I_{r_dN} \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \end{array} \right.$$

$$\begin{bmatrix} \bar{\Omega}_{d17} & \bar{\Omega}_{d18} & 0 & \bar{\Omega}_{d110} \\ 0 & 0 & \bar{\Omega}_{d29} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -I_{2nN} & 0 & 0 & 0 \\ \star & -I_{nN} & 0 & 0 \\ \star & \star & -I_{(n+q)N} & 0 \\ \star & \star & \star & -I_{2nN} \end{bmatrix} < 0 \quad (25)$$

with

$$\bar{\Omega}_{d22} = I_N \otimes \begin{bmatrix} \bar{\Omega}_{d1} & \bar{\Omega}_{d2} \\ \star & \bar{\Omega}_{d3} \end{bmatrix}, \bar{\Omega}_{d23} = I_N \otimes \begin{bmatrix} 0 \\ P_{d2} \end{bmatrix}$$

$$\bar{\Omega}_{d25} = I_N \otimes \begin{bmatrix} \bar{\Omega}_{d4} & 0 \\ -X_5 C & P_{d2} \end{bmatrix}, \bar{\Omega}_{d26} = I_N \otimes \begin{bmatrix} C_{edx}^T \\ C_{edf}^T \end{bmatrix}$$

$$\bar{\Omega}_{d29} = I_N \otimes \begin{bmatrix} K_x^T B^T & 0 \\ K_f^T B^T & 0 \end{bmatrix}$$

$$\bar{\Omega}_{d1} = \mathrm{He}\left(P_{d1} A - X_3 C A - X_4 C\right)$$

$$\bar{\Omega}_{d2} = P_{d1} F - X_3 C F - A^T C^T X_5^T - C^T X_6^T$$

$$\bar{\Omega}_{d3} = \mathrm{He}\left(-X_5 C F\right), \bar{\Omega}_{d4} = P_{d1} - X_3 C$$

$$\bar{\Omega}_{d11} = I_N \otimes \mathrm{He}\left(Q_{d0} A\right) + \left(\varepsilon_{1d} + \varepsilon_{2d}\right) E^T E$$

$$\bar{\Omega}_{d14} = \bar{\Omega}_{d18} = I_N \otimes Q_{d0}, \bar{\Omega}_{d16} = I_N \otimes C_{xd0}^T$$

$$\bar{\Omega}_{d17} = I_N \otimes \begin{bmatrix} Q_{d0} & K_x^T B^T \end{bmatrix}$$

$$\bar{\Omega}_{d110} = \mathrm{diag}\left(\lambda_1, \lambda_2, \cdots, \lambda_N\right) \otimes \begin{bmatrix} Q_{d0} & \alpha C^T L_d^T B^T \end{bmatrix}$$

Then, the gains are obtained as $H_1 = P_{d1}^{-1} X_3, J_{11} = P_{d1}^{-1} X_4, H_2 = P_{d2}^{-1} X_5, J_{12} = P_{d2}^{-1} X_6$, and $\Xi_d = C^{T\dagger} Q_{d0} B L_d$.

Proof: Consider a Lyapunov function $V_{ed} = e^T P_d e$ with a symmetric positive matrix P_d, and the time derivative of V_{ed} is obtained with a positive scalar ε_{1d} in the same process of proof in Theorem 4.1.

$$\begin{aligned} \dot{V}_{ed} \leq\ & e^T (\mathrm{He}\left(P_d \left(I_N \otimes \left(\Gamma \bar{A} - J_1 \bar{C}\right)\right)\right) \\ & + \varepsilon_{1d}^{-1} P_d \left(I_N \otimes \Gamma\right) \left(I_N \otimes \Gamma\right)^T P_d) e \\ & + \varepsilon_{1d} x^T E^T E x + \mathrm{He}\left(e^T P_d \left(I_N \otimes \Gamma \bar{D}\right) \bar{d}\right) \end{aligned} \quad (26)$$

Furthermore, a Lyapunov function V_{xd} is formed with a positive matrix $Q_d = I_N \otimes Q_{d0}$ and $Q_{d0} \in R^{n \times n}$.

$$V_{xd} = x^T Q_d x + \sum_{i=1}^{N} \frac{\left(d_i - \alpha\right)^2}{\tau_i} \quad (27)$$

The time derivative of V_{xd} associated with a positive scalar ε_{2d} is described as

$$\begin{aligned} \dot{V}_{xd} =\ & x^T \mathrm{He}\left(Q_d \left(I_N \otimes \left(A - B K_x\right)\right)\right) x \\ & + \mathrm{He}\left(x^T Q_d \left(I_N \otimes B K\right) e\right) + \mathrm{He}\left(x^T Q_d \xi\left(x, t\right)\right) \\ & + 2 \textstyle\sum_{i=1}^{N} x_i^T Q_{d0} B L_d C \sum_{j=1}^{N} d_i\left(t\right) a_{ij} \left(x_i - x_j\right) \\ & - 2 \textstyle\sum_{i=1}^{N} \left(d_i - \alpha\right) \sum_{j=1}^{N} y_i^T \Xi_d a_{ij} \left(y_i - y_j\right) \end{aligned} \quad (28)$$

Since $Q_{d0} B L_d = C^T \Xi_d$ is satisfied, it follows that

$$\begin{aligned} \dot{V}_{xd} \leq\ & x^T (\mathrm{He}\left(Q_d \left(I_N \otimes \left(A - B K_x\right) + \alpha \mathcal{L} \otimes B L_d C\right)\right) \\ & + \varepsilon_{2d}^{-1} Q_d Q_d^T + \varepsilon_{2d} E^T E) x + \mathrm{He}\left(x^T Q_d \left(I_N \otimes B K\right) e\right) \end{aligned} \quad (29)$$

Overall, the distributed node-based FTC design is switched to the constant gain-based FTC design. According to Definition 2.1, the sufficient condition for a H_∞ performance $\|G_{z_d \bar{d}}\| < \gamma_d$ is $z_d^T z_d - \gamma_d \bar{d}^T \bar{d} + \dot{V}_{ed} + \dot{V}_{xd} < 0$. It also follows that

$$\zeta^T \begin{bmatrix} \Omega_{d11} & \bar{\Omega}_{d5} & 0 \\ \star & \Omega_{d22} & P_d \left(I_N \otimes \Gamma \bar{D}\right) \\ \star & \star & -\gamma_d I_{qN} \end{bmatrix} \zeta < 0 \quad (30)$$

where $\Omega_{d11} = \mathrm{He}(Q_d (I_N \otimes (A - B K_x) + \alpha \mathcal{L} \otimes B L_d C)) + \varepsilon_{2d}^{-1} Q_d Q_d^T + \left(\varepsilon_{1d} + \varepsilon_{2d}\right) E^T E + C_{xd}^T C_{xd}, \Omega_{d22} = P_d (I_N \otimes (\Gamma \bar{A} - J_1 \bar{C})) + \varepsilon_{1d}^{-1} P_d (I_N \otimes \Gamma)(I_N \otimes \Gamma)^T P_d + C_{ed}^T C_{ed}, \bar{\Omega}_{d5} = Q_d (I_N \otimes B K) + C_{xd}^T C_{ed}$.

Furthermore, denote $C_{xd} = I_N \otimes C_{xd0}$ and $C_{ed} = I_N \otimes [C_{edx}\ C_{edf}]$. Denote $P_d = I_N \otimes \mathrm{diag}\{P_{d1}, P_{d2}\}$ with symmetric positive definite matrices $P_{d1} \in R^{n \times n}$ and $P_{d2} \in R^{q \times q}$. The inequality (30) is quite similar to the inequality (19) in the proof of Theorem 4.1. Note that the Young Inequality is used in order to avoid the equality constraint $Q_{d0} B = B \hat{Q}_{d0}$. It follows that the former two items in the first row of the matrix inequality (30) are described as

$$\mathrm{He}\left(\begin{bmatrix} I \\ 0_{5\times1} \end{bmatrix} \begin{bmatrix} -K_x^T B^T Q_{d0} \\ 0_{5\times1} \end{bmatrix}^T\right)$$

$$\leq \begin{bmatrix} Q_{d0} Q_{d0} + K_x^T B^T B K_x & 0 \\ 0 & 0_{5\times5} \end{bmatrix}$$

$$\mathrm{He}\left(\begin{bmatrix} I \\ 0_{5\times1} \end{bmatrix} \begin{bmatrix} \alpha C^T L_d^T B^T Q_{d0} \\ 0_{5\times1} \end{bmatrix}^T\right)$$

$$\leq \begin{bmatrix} Q_{d0} Q_{d0} + \alpha^2 C^T L_d^T B^T B L_d C & 0_{1\times5} \\ 0_{5\times1} & 0_{5\times5} \end{bmatrix}$$

$$\mathrm{He}\left(\begin{bmatrix} I \\ 0_{5\times1} \end{bmatrix} \begin{bmatrix} 0 \\ \left[Q_{d0} B K_x \quad Q_{d0} B K_f\right]^T \\ 0_{4\times1} \end{bmatrix}^T\right)$$

$$\leq \begin{bmatrix} Q_{d0} Q_{d0} & 0 & 0_{1\times4} \\ 0 & \begin{bmatrix} K_x^T B^T B K_x & K_x^T B^T B K_f \\ K_f^T B^T B K_x & K_f^T B^T B K_f \end{bmatrix} & 0_{1\times4} \\ 0_{4\times1} & 0_{4\times1} & 0_{4\times4} \end{bmatrix}$$

According to the definitions $H = [H_1^T\ H_2^T]^T$ and $J_1 = [J_{11}^T\ J_{12}^T]^T$ with $H_1 = P_{d1}^{-1} X_3, J_{11} = P_{d1}^{-1} X_4, H_2 = P_{d2}^{-1} X_5$, and $J_{12} = P_{d2}^{-1} X_6$, the Schur lemma is applied in transferring the inequality (30) into the LMI formulation (25). This completes the proof.

Remark 4.4: The existing nonlinear constraints in Theorem 4.1 are transformed into linear items by introducing equality constraints. Although this method facilitates the solution of the considered H_∞ problem, the equality constraints with additional conservativeness impose restrictions on the controlled systems. Meanwhile, the distributed node-based FTC design in Theorem 4.2 is proposed without equality constraint with the application of the Young Inequality. Thus, solving the H_∞ problem in a simple way with reduced design complexity by decoupling the estimation error from the FTC system is important and can be an interesting research direction.

Note that the undirected topology will play a role in the description of LMI formulations, that is, $\mathrm{He}\left(Q_c \left(g L \otimes B K_g C\right)\right)$

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in (17) and $\mathrm{He}\left(Q_d\left(\alpha L\otimes BL_dC\right)\right)$ in (29). In order to avoid the requirement of the global information in the undirected graph, we will derive the following corollary.

Corollary 4.1. Given positive scalars $\gamma_d, \varepsilon_{1d}, \varepsilon_{2d}$ and α, matrices $C_{xd0}\in R^{r_d\times n}, C_{edx}\in R^{r_d\times n}$ and $C_{edf}\in R^{r_d\times q}$, the hierarchical structure-based FE/FTC system (24) with the distributed node-based controller (21) and updated law (22) is stable with H_∞ performance $\|G_{z_d\bar{d}}\|<\gamma_d$, if there exist symmetric positive definite matrices $Q_{d0}\in R^{n\times n}, P_{d1}\in R^{n\times n}$ and $P_{d2}\in R^{q\times q}$, and matrices $K_x\in R^{m\times n}, L_d\in R^{m\times p}, X_3\in R^{n\times p}, X_4\in R^{n\times p}, X_5\in R^{q\times p}$ and $X_6\in R^{q\times p}$ such that

$$\left[\begin{array}{ccccccccc}
\bar{\Omega}_{d11} & 0 & 0 & \bar{\Omega}_{d14} & 0 & \bar{\Omega}_{d16} & \bar{\Omega}_{d17} & \bar{\Omega}_{d18} & 0\\
\star & \bar{\Omega}_{d22} & \bar{\Omega}_{d23} & 0 & \bar{\Omega}_{d25} & \bar{\Omega}_{d26} & 0 & 0 & \bar{\Omega}_{d29}\\
\star & \star & -\gamma_d I_{qN} & 0 & & 0 & 0 & 0 & 0\\
\star & \star & \star & -\varepsilon_{2d} I_{nN} & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -\varepsilon_{1d} I_{(n+q)N} & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -I_{r_dN} & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & -I_{2nN} & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -I_{nN} & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & -I_{(n+q)N}
\end{array}\right]<0 \quad (31)$$

with

$$\bar{\Omega}_{d22}=I_N\otimes\begin{bmatrix}\bar{\Omega}_{d6} & \bar{\Omega}_{d7}\\ \star & \bar{\Omega}_{d8}\end{bmatrix}, \bar{\Omega}_{d23}=I_N\otimes\begin{bmatrix}0\\ P_{d2}\end{bmatrix}$$

$$\bar{\Omega}_{d25}=I_N\otimes\begin{bmatrix}P_{d1}-X_3C & 0\\ -X_5C & P_{d2}\end{bmatrix}$$

$$\bar{\Omega}_{d26}=I_N\otimes\begin{bmatrix}C_{edx}^T\\ C_{edf}^T\end{bmatrix}, \bar{\Omega}_{d29}=I_N\otimes\begin{bmatrix}K_x^TB^T & 0\\ K_f^TB^T & 0\end{bmatrix}$$

$$\bar{\Omega}_{d11}=I_N\otimes\mathrm{He}\left(Q_{d0}A\right)+\left(\varepsilon_{1d}+\varepsilon_{2d}\right)E^TE$$

$$\bar{\Omega}_{d14}=\bar{\Omega}_{d18}=I_N\otimes Q_{d0}, \bar{\Omega}_{d16}=I_N\otimes C_{xd0}^T$$

$$\bar{\Omega}_{d17}=I_N\otimes\left[\sqrt{2}Q_{d0}\quad K_x^TB^T\quad \alpha\lambda_2C^TL_d^TB^T\right]$$

$$\bar{\Omega}_{d6}=\mathrm{He}\left(P_{d1}A-X_3CA-X_4C\right), \bar{\Omega}_{d8}=\mathrm{He}\left(-X_5CF\right)$$

$$\bar{\Omega}_{d7}=P_{d1}F-X_3CF-A^TC^TX_5^T-C^TX_6^T$$

Then, the gains are obtained as $H_1=P_{d1}^{-1}X_3, J_{11}=P_{d1}^{-1}X_4, H_2=P_{d2}^{-1}X_5, J_{12}=P_{d2}^{-1}X_6$, and $\Xi_d=C^{T\dagger}Q_{d0}BL_d$.

Proof: Since the undirected graph $\mathcal{G}$ is connected, it follows from Lemma 2.1 that $x^T\left(\mathcal{L}\otimes BL_dC\right)x\geq\lambda_2x^T\left(I_N\otimes BL_dC\right)x$, where λ_2 is the smallest nonzero eigenvalue of $\mathcal{L}$. Therefore, it is derived as

$$\begin{aligned}\dot{V}_{xd}\leq\ & x^T(\mathrm{He}\left(Q_d\left(I_N\otimes\left(A-BK_x-\alpha\lambda_2BL_dC\right)\right)\right)\\ & +\varepsilon_{2d}^{-1}Q_dQ_d^T+\varepsilon_{2d}E^TE)x+\mathrm{He}(x^TQ_d(I_N\otimes BK)e)\end{aligned} \quad (32)$$

According to the definitions $H=[H_1^T\ H_2^T]^T$ and $J_1=[J_{11}^T\ J_{12}^T]^T$ with $H_1=P_{d1}^{-1}X_3, J_{11}=P_{d1}^{-1}X_4, H_2=P_{d2}^{-1}X_5,$

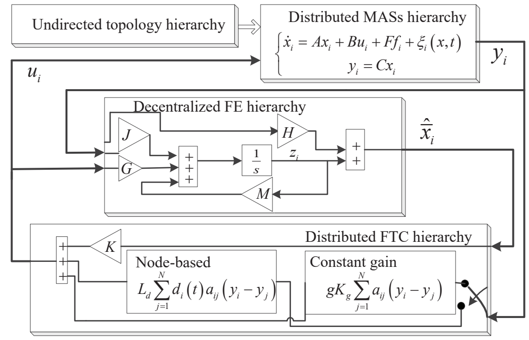

Fig. 2. The hierarchical structure-based FE and FTC algorithm.

and $J_{12}=P_{d2}^{-1}X_6$, the Schur lemma is applied and the proof of Corollary 4.1 is straightforward and thus is omitted here.

Remark 4.5: In comparison with the previous works where the controllers are designed with the relative state information of neighboring agents[23], [25], [30], the proposed distributed protocols (12) and (21)-(22) rely on the relative output information of neighbors. The controllers and updated laws based on the output information can be constructed and improved by each agent in a fully distributed pattern without utilizing any global information of the communication topology.

Remark 4.6: (i) All the required parameters in the proposed hierarchical structure-based FE and FTC design are pre-determined off-line mainly by solving single-step LMIs in Theorems 4.1 and 4.2. The algorithm implementation procedure is relatively straightforward and easy to follow (Figure 2). Thus, the proposed approach does not require online computation with acceptable complexities and can be applicable in practice. (ii) The hierarchical structure-based FE/FTC design based on the H_∞ approach using a single-step LMI formulation has considerable design and computational complexity, especially in the case of multiple unmanned aerial vehicles[1] and large-scale systems[2]-[4].

V. Simulation Results

In this section, two simulation cases of the hierarchical structure FE and FTC design are put forward to validate the effectiveness of the proposed control scheme, i.e., case 1 for 3 quadrotors with voltage faults under the undirected communication topology and case 2 for a 3-machine power system with nonlinear interconnections and steam valve aperture faults.

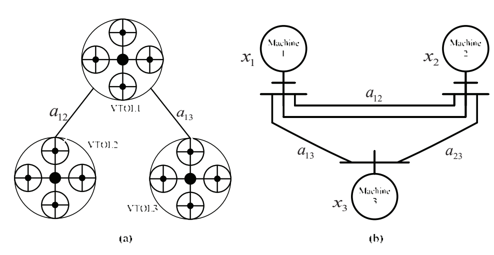

Fig. 3. (a) The topology of the three quadrotors and (b) the 3-machine power system.

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A. Case 1 for 3 quadrotors

The linear model of the i-th quadrotor $(i = 1, 2, 3)$ is given by

$$\begin{aligned} J_\psi \ddot{\psi}_i &= K_c\left(V_{fi} + V_{bi}\right) + K_n\left(V_{ri} + V_{li}\right) \\ J_\theta \ddot{\theta}_i &= lK_p\left(V_{fi} - V_{bi}\right) \\ J_\phi \ddot{\phi}_i &= lK_p\left(V_{ri} - V_{li}\right) \end{aligned} \tag{33}$$

where ψ_i, θ_i and ϕ_i denote the yaw, pitch and roll angles, respectively. V_{fi}, V_{bi}, V_{ri} and V_{li} denote the voltages of the front, behind, right and left motor, respectively. J_ψ, J_θ and J_ϕ denote the equivalent moments of inertia. l represents the distance from the pivot to each motor and K_c, K_n and K_p are the propeller constants. Define $x_i = [\psi_i^T, \theta_i^T, \phi_i^T, \dot{\psi}_i^T, \dot{\theta}_i^T, \dot{\phi}_i^T]^T, u_i = [V_{fi}^T, V_{bi}^T, V_{ri}^T, V_{li}^T]^T$ and $y_i = [\psi_i^T, \theta_i^T, \phi_i^T]^T$. Hence, the linear quadrotor model is characterized by the following matrices[11] and the Laplacian matrix $\mathcal{L}$ is defined as shown in Figure 3(a).

$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} I_3 & 0_{3\times3} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0326 & -0.0326 & 0.0326 & 0.0326 \\ 0.4235 & -0.4235 & 0 & 0 \\ 0 & 0 & 0.42365 & -0.4235 \end{bmatrix}$$

It follows from (1) that the quadrotor dynamics are modeled without mechanical interconnections, which requires the information exchange among multiple quadrotors through communication networks. To demonstrate the performance of the proposed hierarchical structure-based FE and FTC algorithm in Theorem 4.1, the quadrotor dynamic system is proposed with the following voltage faults in the control inputs, i.e. $f_i = [f_{i1}^T, f_{i2}^T, f_{i3}^T, f_{i4}^T]^T, i = 1, 2, 3$, and the fault distribution matrix is satisfied with $F = B$.

$$\begin{aligned} f_{11} &= f_{13} = f_{14} = f_{21} = f_{22} = f_{23} \\ &= f_{24} = f_{31} = f_{32} = f_{33} = 0 \end{aligned}$$
$$f_{12} = \begin{cases} |0.1\sin(0.5t)|, t \le 8 \\ 0.05\text{sat}\left(0.1\sin(0.5t)\right), t > 8 \end{cases}$$
$$f_{34} = \begin{cases} |0.2\sin(0.5t)|, t \le 20 \\ 0.1\text{sat}\left(0.2\sin(0.5t)\right), t > 20 \end{cases}$$

Simulation parameters are designed as $\gamma_c = 0.1, g = \varepsilon_{1c} = \varepsilon_{2c} = 1, C_{xc0} = [1\ 0\ 1\ 1\ 0\ 1], C_{ecx} = [1\ 1\ 1\ 0\ 1\ 1], C_{ecf} = [1.5\ 1.5\ 0\ 0.5]$, and we can derive the unknown input observer and distributed constant gain-based FTC gains by solving Theorem 4.1.

$$K_x = \begin{bmatrix} 0.0344 & 0.1587 & 0.0516 & -0.3533 & 3.4160 & 0.3700 \\ -0.5903 & -0.1130 & -0.5768 & -0.8790 & -3.1430 & 0.0057 \\ 1.1491 & -0.1178 & 1.3891 & 1.8240 & -0.8156 & 3.0232 \\ -0.5929 & 0.0720 & -0.8636 & -0.5913 & 0.5423 & -3.3977 \end{bmatrix}$$

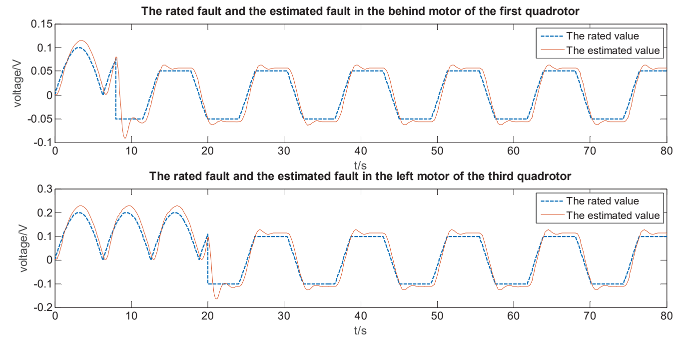

Fig. 4. The rated faults and the estimated faults in the first and third quadrotors.

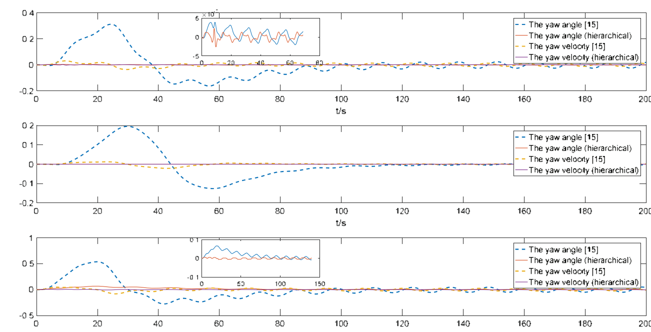

Fig. 5. The yaw angles and yaw velocities in the three quadrotors.

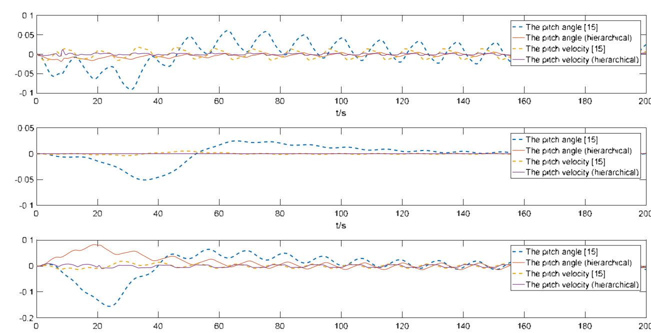

Fig. 6. The pitch angles and pitch velocities in the three quadrotors.

$$K_g = \begin{bmatrix} 0.2917 & -0.1152 & 0.2727 \\ 0.4990 & 0.1092 & -0.4821 \\ -0.6530 & -0.0167 & -0.8455 \\ -0.1378 & 0.0227 & 0.0905 \end{bmatrix}$$

In the presence of the time-varying and additive voltage faults in the input channels, the results in Figures 4-7 show the effectiveness of the hierarchical structure-based FE and distributed constant gain FTC designs. Figure 4 presents the good tracking trajectories of the rated and estimated faults in the behind motor of the first quadrotor and left motor of the third quadrotor, respectively. In comparison with the previous work[15], the proposed algorithm of Theorem 4.1 shows rapid convergence of the angles ψ, θ, ϕ and the angular velocities $\dot{\psi}, \dot{\theta}, \dot{\phi}$, and guarantees the robust stability of the quadrotor systems in Figures 5-7.

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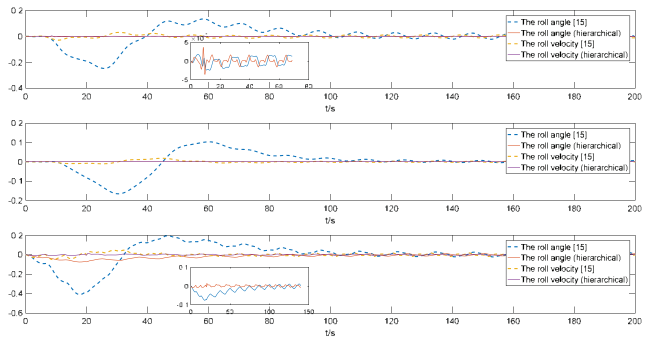

Fig. 7. The roll angles and roll velocities in the three quadrotors.

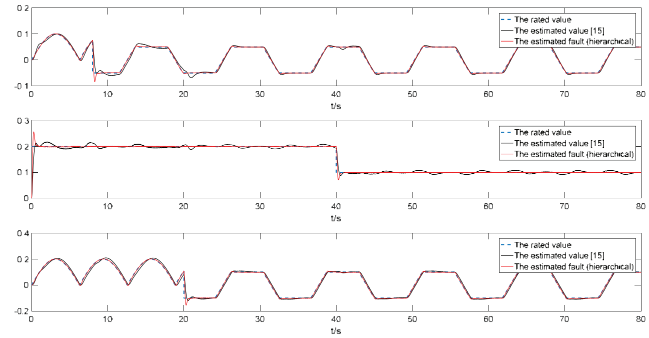

Fig. 8. The rated faults, the estimated faults[15] and the estimated faults (hierarchical) in the three machines.

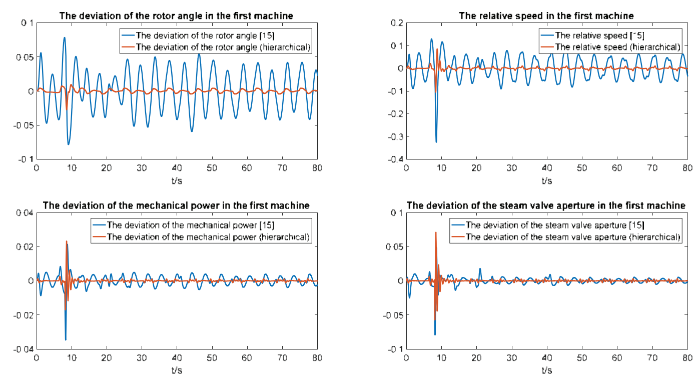

Fig. 9. The respective states[15] and states (hierarchical) in the first machine.

B. Case 2 for a 3-machine power system

The state of the nonlinear model of the i-th machine ($i = 1, 2, 3$) in the power system is described as $x_i = [\Delta\sigma_i{}^T, \Delta\omega_i{}^T, \Delta P_{mi}^T, \Delta X_{e_i}^T]^T$, where $\Delta\sigma_i$ denotes the deviation of the rotor angle, $\Delta\omega_i$ denotes the relative speed, ΔP_{mi} represents the deviation of the per unit mechanical power and ΔX_{e_i} represents the deviation of the per unit steam valve aperture. Hence, the nonlinear power model is characterized by the following matrices[15] and the Laplacian matrix $\mathcal{L}$ is defined as shown in Figure 3(b).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.2941 & 30.7999 & 0 \\ 0 & 0 & -2.8571 & 2.8571 \\ 0 & 0.6366 & 0 & -10 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

The nonlinear interconnection $\xi_i = \sum_{j=1}^{3} \alpha_{ij} \sin(\Delta\sigma_i - \Delta\sigma_j)$ satisfies that $\xi^T(x,t)\xi(x,t) \le x^T(L_n^T L_n \otimes C_n^T C_n)x$.

$$L_n = \begin{bmatrix} 0.6598 & -0.3299 & -0.3299 \\ -0.2772 & 0.5544 & -0.2772 \\ -0.3299 & -0.2772 & 0.6071 \end{bmatrix}, C_n = \begin{bmatrix} 1 & 0_{1\times 3} \end{bmatrix}$$

It follows from (1) that the 3-machine power system is modeled with nonlinear interconnections. To demonstrate the performance of the proposed hierarchical structure-based FE and FTC algorithm in Theorem 4.2 and Corollary 4.1, the machine power dynamic system is proposed with the faults in the steam valve control inputs and the fault distribution matrix is satisfied with $F = B$.

$$f_1 = \begin{cases} |0.1\sin(0.5t)|, t \le 8 \\ 0.05\text{sat}(0.1\sin(0.5t)), t > 8 \end{cases}, f_2 = \begin{cases} 0.2, t \le 40 \\ 0.1, t > 40 \end{cases}$$

$$f_3 = \begin{cases} |0.2\sin(0.5t)|, t \le 20 \\ 0.1\text{sat}(0.2\sin(0.5t)), t > 20 \end{cases}$$

Simulation parameters are designed as $\gamma_d = 0.1, \alpha = \varepsilon_{1d} = \varepsilon_{2d} = 1, C_{xd0} = [0.01 \;\; 0.1 \;\; 0.15 \;\; 1], C_{edx} = [1 \;\; 1 \;\; 0.1 \;\; 10], C_{edf} = [0.1]$, and we can derive the unknown input observer and distributed node-based FTC gains by solving Theorem 4.2.

$$K_x = \begin{bmatrix} 0.2097 & 0.0856 & 1.9222 & -0.6742 \end{bmatrix}$$

$$\Xi_d = \begin{bmatrix} 0.0474 & 0.9518 \\ 0.0456 & 0.9153 \end{bmatrix}, L_d = \begin{bmatrix} -0.0022 & -0.0451 \end{bmatrix}$$

In the presence of the time-varying and additive faults both in the first and third machines and the time-invariant and additive faults in the second machine, the results in Figures 8-11 indicate the effectiveness of the hierarchical structure-based FE and distributed node-based FTC designs. The curves in Figure 8 simulated by both the approach[15] and proposed hierarchical algorithm show the good tracking properties of the rated and estimated faults in the 3-machine power system. The respective deviations of the rotor angle $\Delta\sigma$, the speed $\Delta\omega$, the mechanical power ΔP_m, and the steam valve aperture ΔX_e in the three machines in Figures 9-11 show the robust stability of the 3-machine power system. Note that the first machine fails at $t = 8s$, the second machine fails at $t = 40s$ and the third one suffers a failure at $t = 20s$. Compared with the previous study[15], the proposed algorithm of Theorem 4.2 shows smaller amplitudes of the oscillations for several states in the convergence process to an extent, as shown in Figures 9-11.

VI. Conclusions

In this paper, a hierarchical structure-based FE/FTC design was developed for a class of nonlinear MASs with additive actuator faults. The graph theory and distributed MASs hierarchy description were introduced, and unknown input observers in the FE hierarchy were presented to track actuator faults. Subsequently, two distributed FTC designs, namely, constant gain-based and node-based FTC protocols, were proposed by employing the estimated system states and actuator faults,

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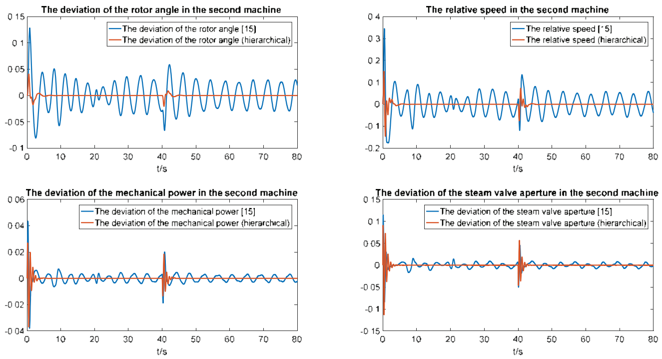

Fig. 10. The respective states[15] and states (hierarchical) in the second machine.

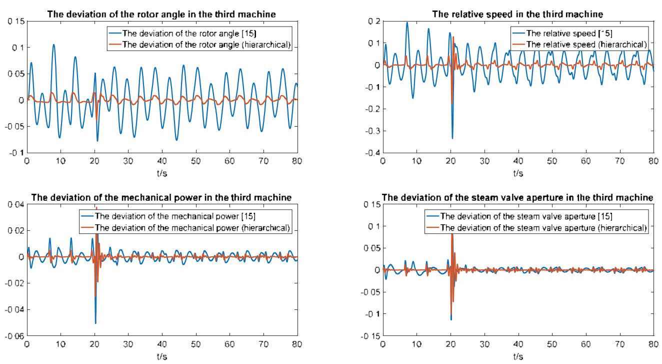

Fig. 11. The respective states[15] and states (hierarchical) in the third machine.

respectively. Moreover, the optimization of H_∞ in a single-step LMI formulation was used to guarantee the asymptotic stability and H_∞ performance of MASs. Simulation results of the quadrotors and the 3-machine power system verified the effectiveness of the proposed hierarchical structure control scheme. Current investigations focus on the extensions of the proposed method to nonlinear MASs with disturbances, uncertainties and simultaneous actuator/sensor faults.

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