Integrated fault estimation and fault-tolerant control for uncertain Lipschitz non-linear systems

Jianglin Lan, Ron J. Patton

Abstract

This paper proposes an integrated fault estimation and fault-tolerant control (FTC) design for Lipschitz non-linear systems subject to uncertainty, disturbance, and actuator/sensor faults. A non-linear unknown input observer without rank requirement is developed to estimate the system state and fault simultaneously, and based on these estimates an adaptive sliding mode FTC system is constructed. The observer and controller gains are obtained together via H_∞ optimization with a single-step linear matrix inequality (LMI) formulation so as to achieve overall optimal FTC system design. A single-link manipulator example is given to illustrate the effectiveness of the proposed approach.

Index Terms

Integrated design, adaptive sliding mode fault-tolerant control, non-linear unknown input observer, H_∞ optimization, Lipschitz non-linear systems

I. Introduction

Lipschitz non-linear systems are one of the most important classes of systems studied by the control community. Indeed, almost all real dynamic systems can be represented as Lipschitz non-linear systems (at least locally). Automatic dynamic systems sometimes might suffer from faults (malfunctions), e.g., offsets of actuators/sensors, deviations of component parameter, loss of actuator effectiveness, etc. These faults may degrade system performance or even cause safety problems [1]. In order to provide guaranteed performance in the presence of certain bounded faults, tolerating faults as well as having maintained stability properties and good robustness to uncertainty, fault-tolerant control (FTC) systems (including FTC Lipschitz non-linear systems) have been researched extensively [1]–[3]. The FTC systems can be achieved using *passive* FTC designs without fault information (location, magnitude and time occurrence) or *active* FTC designs with control redesign based on fault information [1]. The *passive* FTC approach is known to be limited since it treats faults in the way of perturbations using optimization methods. Therefore, in this work we consider the less conservative active FTC problem for Lipschitz nonlinear systems.

The traditional method for obtaining fault information for non-linear systems is the fault diagnosis approach, which includes the procedures of fault detection and fault isolation (FDI). Many FDI methods involve residual generator designs, as well as isolation filters for fault location [4]–[9]. In contrast to these classical methods fault estimation (FE) directly reconstructs the fault shape (the magnitude with respect to time) without any of the aforementioned complex designs. The FE signals are thus conveniently available for use in a compensation scheme to robustly compensate the fault effects within all control loops. Significant literature on the subject of FE design methods for Lipschitz non-linear FTC systems has been established, e.g., for the adaptive observer (AO) [10], [11], the sliding mode observer (SMO) [12], the extended state observer (ESO) [13], and the non-linear unknown input observer (NUIO) [14]–[16]. However, in the AO faults are estimated with zone convergence, and a proportional-integral (PI) structure with carefully chosen learning rate is implemented for time-varying fault estimation. The canonical form SMO proposed in [12] requires several state transformations as well as *a priori* knowledge of the fault upper bounds. The ESO reconstructs the faults in polynomial form with an assumption of their orders. The NUIO approach can obtain asymptotic state and fault estimations with a comparatively simple design. Nevertheless, the NUIOs proposed in [14] and [15] are designed with rank requirement on system coefficient matrices in order to decouple the disturbance completely, which limits the applicability to real systems. Although [16] releases this rank requirement by considering partially decoupled disturbance, the effect of system uncertainties are not taken into account. A novel NUIO without rank requirement for Lipschitz non-linear systems subject to faults and both disturbance and uncertainty is of great interest in this paper.

The FE-based FTC for Lipschitz non-linear systems has attracted great attention in the past decade, see for example, [10], [11], [13], [14], [17], [18] and the references therein. Without considering disturbance and uncertainty, AO-based FTC designs for Lipschitz non-linear systems and Lipschitz non-linear sampled-data systems with actuator faults are proposed in [10] and [18], respectively. Considering disturbance, [13] proposes an ESO-based FTC for Lipschitz non-linear descriptor systems with actuator fault, and [17] proposes a mixed H_2/H_∞ approach to FTC design for Lipschitz non-linear systems with actuator faults using FE obtained by a dynamic compensator which is kind of AO. Further, [11] proposes an AO-based FTC design for discrete-time Lipschitz non-linear systems subject to actuator fault along with matched uncertainty and disturbance. [14] addresses the FTC design for Lipschitz non-linear systems subject to lumped uncertainty and disturbance along with sensor faults using a NUIO.

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However, it can be concluded that as a currently acceptable FTC strategy in the literature for Lipschitz non-linear systems the FE and FTC modules are designed separately, assuming that the FE process offers a robust and accurate estimation of faults. Moreover, most of the existing works do not take into account the effect of system uncertainty (especially unmatched uncertainty).

Actually, in the presence of system uncertainty, interaction exists between the FE and FTC system functions in the sense that there is a complex uncertainty coupling between their roles. Due to this interaction, the separately designed FE and FTC functioning models might not fit each other when assumed together. Hence, this "bi-directional" robustness interaction between the FE and FTC functions should be taken into account *a priori* along with the development of a robust approach to integrate their designs into one overall design. An early form of integrated design concept was introduced in [19] to achieve FDI and control objectives simultaneously by taking account of the effect of the control system uncertainty on the FDI performance. A number of studies followed this theme as summarised in the review [20]. However, these early studies did not consider the FTC problem. Although very complex some effort has also been made to integrate together the designs of the FDI and FTC modules [21], [22], within the context of the residual design approach to FDI. Recently, a bi-directional integrated FE/FTC strategy was proposed for uncertain linear systems with disturbance and actuator/sensor faults [23]. For this case the FE function is affected by the uncertainty of the FTC system and also the estimation error in the FE system affects the FTC sysem performance. The work in [23] is based on the use of a single-step LMI formulation and in this work detailed discussion about the bi-directional robustness interaction as well as the basic idea of integrated design can be found.

No studies have described the use of integrated design in FE/FTC schemes for Lipschitz non-linear systems. Hence, this paper provides an extension of the earlier study [23] applicable to Lipschitz non-linear systems. Both matched and unmatched uncertainties along with disturbance and actuator/sensor faults are considered. The main contributions of this paper are: (i) For Lipschitz non-linear systems subject to uncertainty, disturbance and actuator/sensor faults, a new type of NUIO is proposed to estimate the state and fault simultaneously, without any rank requirement; (ii) An adaptive sliding mode FTC controller using the NUIO based FE is constructed. Although sliding mode FTC has recently been researched extensively (e.g. [23]–[27]), few works consider unmatched system uncertainty and FE design for Lipschitz non-linear systems; (iii) A novel integrated design of FE/FTC with a single-step LMI formulation is proposed for Lipschitz non-linear systems.

The paper is organized as follows. Section II formulates the problem. Section III proposes an integrated FE/FTC strategy and it is extended in Section IV for the case of simultaneous actuator and sensor faults. Section V provides an illustrative example and Section VI concludes the study.

In the paper, † denotes the pseudo-inverse, $\|\cdot\|$ denotes the Euclidean norm of a vector and the induced norm of a matrix, $\mathrm{He}(W) = W + W^\top$, $\star$ denotes the symmetric part of a matrix.

II. Problem statement

Consider a non-linear system

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + f(x,t) + F_a f_a(t) + d_0(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^p$ are the state, control input, and system output, respectively. $f_a \in R^q$ denotes the actuator fault. $d_0 \in R^l$ denotes the external disturbance. $f(x,t) \in R^n$ is a continous non-linear function. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $F_a \in R^{n\times q}$, and $C \in R^{p\times n}$ are known constant matrices. $\Delta A(t)$ represents the modelling uncertainty related to the matrix A and satisfies $\Delta A(t) = M_0F_0(t)N_0$ with $F_0^\top(t)F_0(t) \le I$ and appropriate dimension matrices M_0 and N_0. Similarly, $\Delta B(t)$ represents the uncertainty associated with B. The following Assumptions on the system (1) are made.

Assumption 1: The pair (A, C) is observable, the pair (A, B) is controllable, and $\mathrm{rank}(B, F_a) = \mathrm{rank}(B) = m$.

Assumption 2: The disturbance d and fault f_a belong to $\mathcal{L}_2[0,\infty)$, and f_a is continuously smooth with bounded first time derivative.

Assumption 3: The non-linear term $f(x,t)$ satisfies a Lipschitz constraint

$$\|f(\hat{x},t) - f(x,t)\| \le L_f\|\hat{x} - x\|, \ \forall\ x, \hat{x} \in R^n$$

where L_f is the Lipschitz constant independent of x and t.

Assumption 4: The control input uncertainty ΔB and external disturbance d_0 satisfy the matching condition: $\Delta Bu+d_0 = Dd$, where $D \in R^{n\times l}$ is a known constant matrix and $d \in R^l$ is defined as a lumped perturbation.

Remark 1: Assumptions 1 and 2 are usually made for FE-based FTC systems with actuator faults, and Assumption 3 is a general assumption corresponding to the Lipschitz non-linearity. When compared with the matching condition normally made in sliding mode control theory [28], it is not restrictive to specify Assumption 4 including both matched and unmatched uncertainties. Moreover, it is not unusual for external disturbance and uncertainties to have a common distribution matrix in [11], [29].

The FE and FTC considered in this work are defined as follows.

Definition 1: **FE**: reconstruction of fault shape (the magnitude with respect to time). **FTC**: compensation of fault effect through control redesign.

In keeping with the above the aim of this study is to develop an integrated procedure for combining the FE and FTC system functions into one (single step) design procedure for the system (1). This is a single-step FTC scheme based on FE rather than the use of residual-based FDI. The method ensures robust stability of the closed-loop system in the presence of uncertainty, disturbance, and actuator faults for which the system non-linearity satisfies a Lipschitz structure.

III. Integrated FE/FTC for Lipschitz non-linear systems

A. NUIO-based FE design

Define an actuator fault to be represented as an auxiliary state system, then (1) is augmented into

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \Delta\bar{A}\bar{x} + \bar{f}(A_0\bar{x},t) + \bar{B}u + \bar{D}\bar{d} \\ y &= \bar{C}\bar{x} \end{aligned} \tag{2}$$

where

$$\bar{x} = \begin{bmatrix} x \\ f_a \end{bmatrix},\ \bar{A} = \begin{bmatrix} A & F_a \\ 0 & 0 \end{bmatrix},\ \Delta\bar{A} = \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix},\ \bar{f}(A_0\bar{x},t) = \begin{bmatrix} f(x,t) \\ 0 \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \bar{D} = \begin{bmatrix} D & 0 \\ 0 & I_q \end{bmatrix},\ \bar{d} = \begin{bmatrix} d \\ \dot{f}_a \end{bmatrix},\ \bar{C} = [C \quad 0],\quad A_0 = [I_n \quad 0].$$

Remark 2: It can be verified below that the observability of systems (1) and (2) are equivalent. Since the pair (A, C) is observable,

$$\text{rank} \begin{bmatrix} sI_n - A \\ C \end{bmatrix} = n,\ \forall s \in \mathcal{C},\ Re(s) \geq 0$$

which leads to

$$\text{rank} \begin{bmatrix} sI_{n+q} - \bar{A} \\ \bar{C} \end{bmatrix} = \text{rank} \begin{bmatrix} sI_n - A & F_a \\ 0 & sI_q \\ C & 0 \end{bmatrix} = n+q,\ \forall s \in \mathcal{C},\ Re(s) \geq 0.$$

The augmented state $\bar{x}$ is estimated by a NUIO in the form of

$$\begin{aligned} \dot{z} &= Mz + Gu + N\bar{f}(A_0\hat{\bar{x}},t) + Ly \\ \hat{\bar{x}} &= z + Hy \end{aligned} \tag{3}$$

where $z \in R^{n+q}$ is the observer system state and $\hat{\bar{x}} \in R^{n+q}$ is the estimate of $\bar{x}$. The matrices $M \in R^{(n+q)\times(n+q)}$, $G \in R^{(n+q)\times m}$, $N \in R^{(n+q)\times(n+q)}$, $L \in R^{(n+q)\times p}$, and $H \in R^{(n+q)\times q}$ are to be designed.

Defining the estimation error as $e = \bar{x} - \hat{\bar{x}}$, it follows that the error dynamics are

$$\begin{aligned} \dot{e} &= \dot{\bar{x}} - \dot{\hat{\bar{x}}} \\ &= (\Xi\bar{A} - L_1\bar{C})e + (\Xi\bar{A} - L_1\bar{C} - M)z + \Xi\bar{f}(A_0\bar{x},t) - N\bar{f}(A_0\hat{\bar{x}},t) + (\Xi\bar{B} - G)u \\ &\quad + [(\Xi\bar{A} - L_1\bar{C})H - L_2]y + \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d} \end{aligned} \tag{4}$$

where $\Xi = I_{n+q} - H\bar{C}$ and $L = L_1 + L_2$. The matrices M, N, G, and L_2 are defined as

$$M = \Xi\bar{A} - L_1\bar{C},\ N = \Xi,\ G = \Xi\bar{B},\ L_2 = (\Xi\bar{A} - L_1\bar{C})H \tag{5}$$

and the matrices L_1 and H are to be determined.

With the definition given in (5), the error dynamics (4) become

$$\dot{e} = (\Xi\bar{A} - L_1\bar{C})e + \Xi\Delta\bar{f} + \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d} \tag{6}$$

where $\Delta\bar{f} = \bar{f}(A_0\bar{x},t) - \bar{f}(A_0\hat{\bar{x}},t)$.

Now a sufficient condition for the existence of a robust NUIO (3) is given in Theorem 1.

Theorem 1: There exists a robust NUIO (3) if the error system (6) is robustly asymptotically stable.

Proof: With (5), the error system (6) is equivalent to the original error system (4). Therefore, if (6) is robustly asymptotically stable, then (4) is also robustly asymptotically stable, indicating that $\lim_{t\to\infty} e(t) = 0$ in the presence of uncertainty and disturbance. ■

Note that the design matrices M, N, G, and L can be derived once the matrices L_1 and H are obtained. Thus, the main task of obtaining the NUIO (3) as described in the sequel is to design L_1 and H such that (6) is robustly asymptotically stable.

Remark 3: Note from (6) that the disturbance/uncertainty are completely decoupled from the state/fault estimation if the following two conditions hold: **(C1)** $\mathrm{rank}(CD) = \mathrm{rank}(D) + q$, and **(C2)** $\mathrm{rank}(\Delta A) = \mathrm{rank}(C\Delta A)$. On the one hand, similar to that in [14] and [15], the disturbance can be completely decoupled by designing H such that $(I_{n+q} - H\bar{C})\bar{D} = 0$ with the rank requirement $\mathrm{rank}(\bar{C}\bar{D}) = \mathrm{rank}(\bar{D})$, i.e., **(C1)**. On the other hand, the uncertainty effect can be totally eliminated only when $\mathrm{rank}(\Delta\bar{A}) = \mathrm{rank}(\bar{C}\Delta\bar{A})$, i.e., **(C2)**.

These rank requirements are conservative and limit the applicability of the proposed NUIO. Considering the case of partially (rather than totally) decoupled disturbance, [16] proposes a robust NUIO based on H_∞ optimization. However, satisfaction of a rank requirement similar to **(C1)** is still required and the effect of system uncertainties on the estimation is not considered. Taking account of both disturbance and uncertainty, a robust NUIO (3) is proposed in this paper using H_∞ optimization which does not impose a rank requirement and is thus applicable to many more real engineering systems.

Remark 4: It should be noted that unlike the traditional FDI, which includes the residual-generator based processes of fault detection and isolation with residual threshold design, the proposed NUIO directly estimates the fault shape once a fault occurs. The observer can naturally estimate the faults by considering them as new states in an augmented system. Moreover, for the FDI approach the sensitivities of the residual generator to the faults and to the disturbance should be simultaneously maximized and minimized, respectively, which is a very challenging and difficult problem to solve.

In the proposed NUIO, the faults are assumed to be bounded and continuously smooth, and it is seen from (6) that the FE performance is affected not only by the non-linearity and uncertainty but also by the fault modelling error $\dot{f}_a$. The augmented perturbation $\bar{d}$ in 2 is not totally decoupled, but attenuated through robust design. Therefore, the robust FE performance depends on the robustness of the error system (4).

Remark 5: Although the actuator fault considered is assumed to be continuously smooth with bounded first time derivative, it is not required to be everywhere differentiable. According to [30], for any piecewise continuous fault $f_0 \in R^{q_0}$ and a stable matrix $A_f \in R^{q_0 \times q_0}$, there always exists an input vector $w \in R^{q_0}$ such that: $\dot{f}_0 = A_f f_0 + w$. Thus, following a similar design procedure, the proposed observer can also be used to estimate piecewise continuous faults.

B. Adaptive sliding mode FTC design

A sliding surface for the system (1) is designed as [31]

$$s_1 = N_1 \hat{x} \tag{7}$$

where $s_1 \in R^m$, $\hat{x} \in R^n$ is the estimate of the system state x, and $N_1 = B^\dagger - Y_1(I_n - BB^\dagger)$ with $B^\dagger = (B^\top B)^{-1}B^\top$ and an arbitrary matrix $Y_1 \in R^{m\times n}$. Define the state estimation error as $e_x = x - \hat{x}$. Differentiating s_1 with respect to time gives

$$\dot{s}_1 = N_1(A + \Delta A)x + u + N_1 F_a f_a + N_1 f(x,t) + N_1 Dd - N_1 \dot{e}_x. \tag{8}$$

Design the control input as

$$u = u_l + u_n \tag{9}$$

where the linear feedback component is $u_l = -K\hat{\bar{x}}$ with a design matrix $K = [K_x \ E_1]$. $K_x \in R^{m\times n}$ is to be determined while E_1 is chosen as $E_1 = B^\dagger F_a$. The non-linear component u_n is designed as

$$u_n = \begin{cases} -\rho_{s_1}(t)\frac{s_1}{\|s_1\|}, & s_1 \neq 0 \\ 0, & s_1 = 0 \end{cases}$$

with $\rho_{s_1}(t) = \hat{\eta}_{s_1} + \varphi_{s_1} + \varepsilon_{s_1}$. $\varphi_{s_1}, \varepsilon_{s_1} > 0$ are design constants. The scalar $\hat{\eta}_{s_1}$ is introduced to estimate η_{s_1} which is defined as $\eta_{s_1} = \|N_1\| f_0 + \|N_1 D\| d_0 + \|E_1\|(\bar{f}_a + \|\hat{f}_a\|) + \|K_x\| e_{x0} + N_1 \bar{e}_{x0}$, where f_0, e_{x0} and $\bar{e}_{x0}$ are unknown scalars assumed to be the upper bounds of $f(x,t)$, $\|e_x\|$ and $\|\dot{e}_x\|$, respectively. The update law of $\hat{\eta}_{s_1}$ is

$$\dot{\hat{\eta}}_{s_1} = \sigma_1 \|s_1\|, \ \hat{\eta}_{s_1}(0) \geq 0$$

with a learning rate $\sigma_1 > 0$ to be designed.

Define the estimation error of η_{s_1} as $\tilde{\eta}_{s_1} = \eta_{s_1} - \hat{\eta}_{s_1}$. Consider a Lyapunov function

$$V_{s_1} = \frac{1}{2}(s_1^\top s_1 + \frac{1}{\sigma_1}\tilde{\eta}_{s_1}^2).$$

It follows from (8) and (9) that

$$\begin{aligned} \dot{V}_{s_1} &= s_1^\top \dot{s}_1 - \frac{1}{\sigma_1}\tilde{\eta}_{s_1}\dot{\hat{\eta}}_{s_1} \\ &\leq (\omega_{s_1}\|x\| + \eta_{s_1} - \rho_{s_1}(t))\|s_1\| - \tilde{\eta}_{s_1}\|s_1\| \\ &\leq (\omega_{s_1}\|x\| - \varphi_{s_1} - \varepsilon_{s_1})\|s_1\|. \end{aligned}$$

where $\omega_{s_1} = \|N_1A - K_x\| + \|N_1M_0\|\|N_0\|$. By choosing $\varphi_{s_1} > \omega_{s_1}\phi_{s_1}$ with some scalar $\phi_{s_1} > 0$, it follows that the reaching and sliding conditions are satisfied, i.e., $s_1^\top \dot{s}_1 \le -\varepsilon_{s_1}\|s_1\|$, in the subset $\Omega_{s_1} = \{x : \|x\| \le \phi_{s_1}\}$. Thus, the controller (9) ensures that if $x(0) \in \Omega_{s_1}$, then for all $t > \|s_1(0)\|/\varepsilon_{s_1}$, $s_1 = \dot{s}_1 = 0$.

Consider next the system stability analysis corresponding to the sliding mode. Suppose that the system has already been controlled to remain within the sliding mode (7). Substituting the equivalent control

$$u_{eq} = -(N_1Ax + N_1f(x,t) + N_1Dd) + u_l \tag{10}$$

into (1) gives the closed-loop system

$$\dot{x} = (\Theta A - BK_x)x + \Delta Ax + BKe + \Theta f(x,t) + \Theta Dd \tag{11}$$

where $\Theta = I_n - BN_1$.

Thus, by designing K_x such that (11) is robustly stable, then the system (1) is maintained on the sliding mode with the equivalent control (10).

C. Integrated synthesis of FE/FTC

The augmented closed-loop system composed of (6) and (11) is

$$\begin{aligned}
\dot{x} &= (\Theta_1A - BK_x)x + BKe + \Theta f(x,t) + \Delta Ax + D_1\bar{d} \\
\dot{e} &= (\Xi\bar{A} - L_1\bar{C})e + \Xi\Delta\bar{f} + \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d} \\
z_1 &= C_xx + C_ee
\end{aligned} \tag{12}$$

where $z_1 \in R^r$ is the measured output used to verify the closed-loop system performance with matrices $C_x \in R^{r\times n}$ and $C_e \in R^{r\times(n+q)}$, and $D_1 = [\Theta D\ 0]$.

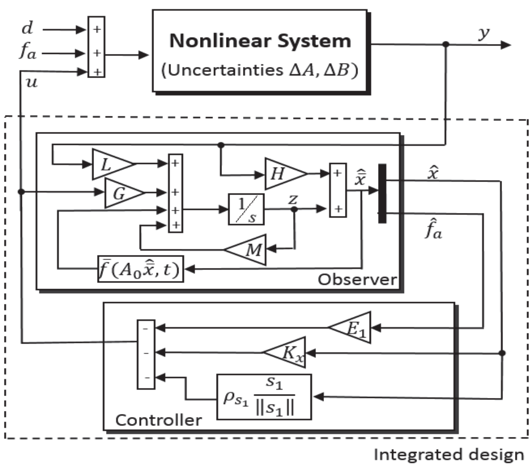

Fig. 1. Integrated FE/FTC for Lipschitz nonlinear systems

It is seen from (12) that the system uncertainty and disturbance affect the state/fault estimation, and in turn the estimation error has an immediate effect on the control system. This shows that a bi-directional robustness interaction exists between the FE and FTC models, which breaks down the so-called Separation Principle and gives rise to an integrated FE/FTC design approach to achieve optimal and robustness of the overall FTC system performance.

Now the proposed integrated design problem (see Figure 1) can be stated as follows: design the controller gain K_x and the observer gains H and L_1 to ensure the robust stability of the augmented closed-loop system (12).

This integrated design is virtually an observer-based robust control problem, and to solve it a single-step LMI formulation is proposed in Theorem 2. This single-step approach is modified from the method used in [32], which focuses on an observer-based robust control design for nominal (fault free) discrete-time Lipschitz non-linear systems.

Theorem 2: Given positive scalars γ_1, ϵ_1, ϵ_2, ϵ_3, ε_1, and ε_2, the closed-loop system (12) is stable with H_∞ performance $\|G_{z_1\bar{d}}\|_\infty < \gamma_1$, if there exist three symmetric positive definite matrices $Z \in R^{n\times n}$, $Q \in R^{n\times n}$, and $R \in R^{q\times q}$, and matrices $M_1 \in R^{m\times n}$, $M_2 \in R^{n\times p}$, $M_3 \in R^{n\times p}$, $M_4 \in R^{q\times p}$, and $M_5 \in R^{q\times p}$ such that

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \star & \Pi_3 \end{bmatrix} < 0 \tag{13}$$

with $\Pi_1 = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} \\ \star & J_{2,2} \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} \Xi_{1,3} & \Xi_{1,4} & 0 & \Xi_{1,6} & \Xi_{1,7} & 0 & 0 & \Xi_{1,10} \\ J_{2,3} & J_{2,4} & J_{2,5} & 0 & 0 & I & J_{2,9} & 0 \end{bmatrix}$,
$\Pi_3 = -\mathrm{diag}\left\{\gamma_1^2 I, I, \epsilon_1 I, (\epsilon_1+\epsilon_2)^{-1} I, \epsilon_3^{-1} Z, \epsilon_3 Z, \varepsilon_1 I, (\varepsilon_2 L_f^2)^{-1} I\right\}$,
$J_{2,2} = \begin{bmatrix} \Xi_{2,2} & \Xi_{2,3} \\ \star & \Xi_{3,3} \end{bmatrix}$, $J_{2,3} = \begin{bmatrix} QD - M_2CD & 0 \\ -M_4CD & R \end{bmatrix}$, $J_{2,4} = \begin{bmatrix} C_{ex}^\top \\ C_{efa}^\top \end{bmatrix}$,
$J_{2,5} = \begin{bmatrix} QM_0 - M_2CM_0 \\ -M_4CM_0 \end{bmatrix}$, $J_{2,9} = \begin{bmatrix} Q - M_2C & 0 \\ -M_4C & R \end{bmatrix}$,
$\Xi_{1,1} = \mathrm{He}(\Theta AZ - BM_1) + \epsilon_2^{-1} M_0 M_0^\top + \varepsilon_2^{-1}\Theta\Theta^\top, \Xi_{1,2} = [0\ F_a], \Xi_{1,3} = [\Theta D\ 0],\ \Xi_{1,4} = ZC_x^\top$,
$\Xi_{1,6} = ZN_0^\top,\ \Xi_{1,7} = BM_1,\ \Xi_{1,10} = Z,\ \Xi_{2,2} = \mathrm{He}(QA - M_2CA - M_3C) + \varepsilon_1 L_f^2 I_n$,
$\Xi_{2,3} = QF_a - M_2CF_a - A^\top C^\top M_4^\top - C^\top M_5^\top,\ \Xi_{3,3} = \mathrm{He}(-M_4CF_a)$,
then the gains are given by

$$K_x = M_1 Z^{-1}, H_1 = Q^{-1}M_2, H_2 = R^{-1}M_4, L_{11} = Q^{-1}M_3, L_{12} = R^{-1}M_5.$$

Proof: Consider a Lyapunov function $V_e = e^\top P_1 e$ with a symmetric positive definite matrix P_1. Denote $\chi_1 = x^\top \Delta\bar{A}^\top \Xi^\top P_1 e + e^\top P_1 \Xi \Delta A x$ and $\bar{M}_0 = [M_0^\top\ 0]^\top$, it follows that

$$\begin{aligned} \chi_1 &= -\left[\sqrt{\epsilon_1}^{-1}\bar{M}_0^\top \Xi^\top P_1 e - \sqrt{\epsilon_1} F_0 N_0 x\right]^\top \left[\sqrt{\epsilon_1}^{-1}\bar{M}_0^\top \Xi^\top P_1 e - \sqrt{\epsilon_1} F_0 N_0 x\right] \\ &\quad + \epsilon_1^{-1} e^\top P_1 \Xi \bar{M}_0 \bar{M}_0^\top \Xi^\top P_1 e + \epsilon_1 x^\top N_0^\top F_0^\top F_0 N_0 x \\ &\leq \epsilon_1^{-1} e^\top P_1 \Xi \bar{M}_0 \bar{M}_0^\top \Xi^\top P_1 e + \epsilon_1 x^\top N_0^\top N_0 x. \end{aligned}$$

According to Assumption 3, for some positive scalar ε_1,

$$2e^\top P_1 \Xi \Delta\bar{f} \leq \varepsilon_1^{-1} e^\top P_1 \Xi\Xi^\top P_1 e + \varepsilon_1 L_f^2 \|A_0 e\|^2.$$

The time derivative of V_e along (6) is

$$\begin{aligned} \dot{V}_e &= \dot{e}^\top P_1 e + e^\top P_1 \dot{e} \\ &= e^\top \mathrm{He}\left[P_1(\Xi\bar{A} - L_1\bar{C})\right] e + 2e^\top P_1 \Xi \Delta\bar{f} + \mathrm{He}(e^\top P_1 \Xi \bar{D}\bar{d}) + \chi_1 \\ &\leq e^\top \left[\mathrm{He}(P_1(\Xi\bar{A} - L_1\bar{C})) + \varepsilon_1^{-1} P_1 \Xi\Xi^\top P_1 + \varepsilon_1 L_f^2 A_0^\top A_0 I_{n+q} + \epsilon_1^{-1} P_1 \Xi \bar{M}_0 \bar{M}_0^\top \Xi^\top P_1\right] e \\ &\quad + \mathrm{He}(e^\top P_1 \Xi \bar{D}\bar{d}) + \epsilon_1 x^\top N_0^\top N_0 x. \end{aligned} \tag{14}$$

Consider another Lyapunov function $V_x = x^\top P x$. Assume $f(0,t) = 0$, then Assumption 3 indicates that $\|f(x,t)\| \leq L_f\|x\|, \forall x \in R^n$. It thus holds that, for some positive scalar ε_2,

$$2x^\top P\Theta f(x,t) \leq \varepsilon_2^{-1} x^\top P\Theta\Theta^\top P x + \varepsilon_2 L_f^2 \|x\|^2.$$

Denote $\chi_2 = x^\top \Delta A^\top P x + x^\top P \Delta A x$, then for some positive scalar ϵ_2,

$$\begin{aligned} \chi_2 &= -\left[\sqrt{\epsilon_2}^{-1} M_0^\top P x - \sqrt{\epsilon_2} F_0 N_0 x\right]^\top \left[\sqrt{\epsilon_2}^{-1} M_0^\top P x - \sqrt{\epsilon_2} F_0 N_0 x\right] \\ &\quad + \epsilon_2^{-1} x^\top P M_0 M_0^\top P x + \epsilon_2 x^\top N_0^\top F_0^\top F_0 N_0 x \\ &\leq \epsilon_2^{-1} x^\top P M_0 M_0^\top P x + \epsilon_2 x^\top N_0^\top N_0 x. \end{aligned}$$

Then the time derivative of V_x along (11) is

$$\begin{aligned} \dot{V}_x &= \dot{x}^\top P x + x^\top P \dot{x} \\ &= x^\top \mathrm{He}\left[P(\Theta A - BK_x)\right] x + 2x^\top P f(x,t) + \mathrm{He}(x^\top PBKe) + \chi_2 + \mathrm{He}(x^\top P D_1 \bar{d}) \\ &\leq x^\top \left[\mathrm{He}(P(\Theta A - BK_x)) + \varepsilon_2^{-1} P\Theta\Theta^\top P + \varepsilon_2 L_f^2 I_n + \epsilon_2^{-1} P M_0 M_0^\top P + \epsilon_2 N_0^\top N_0\right] x \\ &\quad + \mathrm{He}(x^\top PBKe) + \mathrm{He}(x^\top P D_1 \bar{d}). \end{aligned} \tag{15}$$

The H_∞ performance $\|G_{z_1\bar{d}}\| < \gamma$ can be represented as

$$J = \int_0^\infty (z_1^\top z_1 - \gamma_1^2 \bar{d}^\top \bar{d}) dt \ < \ 0. \tag{16}$$

Under zero initial conditions, it follows that

$$\begin{aligned} J &= \int_0^\infty (z_1^\top z_1 - \gamma_1^2 \bar{d}^\top \bar{d} + \dot{V}_x + \dot{V}_e) dt - \int_0^\infty (\dot{V}_x + \dot{V}_e) dt \\ &= \int_0^\infty (z_1^\top z_1 - \gamma_1^2 \bar{d}^\top \bar{d} + \dot{V}_x + \dot{V}_e) dt - (V_x(\infty) + V_e(\infty)) + (V_x(0) + V_e(0)) \\ &\leq \int_0^\infty (z_1^\top z_1 - \gamma_1^2 \bar{d}^\top \bar{d} + \dot{V}_x + \dot{V}_e) dt. \end{aligned}$$

A sufficient condition for satisfaction of (16) is

$$J_1 = z_1^\top z_1 - \gamma_1^2 \bar{d}^\top \bar{d} + \dot{V}_x + \dot{V}_e < 0. \tag{17}$$

Substituting (14) and (15) into (17) yields

$$J_1 = \begin{bmatrix} x \\ e \\ \bar{d} \end{bmatrix}^\top \begin{bmatrix} J_{11} & PBK & PD_1 \\ \star & J_{22} & P_1\Xi\bar{D} \\ \star & \star & -\gamma_1^2 I \end{bmatrix} \begin{bmatrix} x \\ e \\ \bar{d} \end{bmatrix} < 0 \tag{18}$$

where $J_{11} = \mathrm{He}(P(\Theta A - BK_x)) + \varepsilon_2^{-1} P\Theta\Theta^\top P + \varepsilon_2 L_f^2 I_n + (\epsilon_1 + \epsilon_2) N_0^\top N_0 + \epsilon_2^{-1} P M_0 M_0^\top P$
$+C_x^\top C_x,\ J_{22} = \mathrm{He}(P_1(\Xi\bar{A} - L_1\bar{C})) + \varepsilon_1^{-1} P_1 \Xi\Xi^\top P_1 + \varepsilon_1 L_f^2 A_0^\top A_0 I_{n+q} + \epsilon_1^{-1} P_1 \Xi \bar{M}_0 \bar{M}_0^\top \Xi^\top P_1$
$+C_e^\top C_e$.

Define $Z = P^{-1}$. Pre- and post-multiplying both sides of (18) with $\mathrm{diag}(Z, I, I)$ gives

$$\begin{bmatrix} J_{11} & BK & D_1 \\ \star & J_{22} & P_1\Xi\bar{D} \\ \star & \star & -\gamma_1^2 I \end{bmatrix} < 0 \tag{19}$$

where $J_{11} = \mathrm{He}((\Theta A - BK_x)Z) + \varepsilon_2^{-1}\Theta\Theta^\top + \varepsilon_2 L_f^2 ZZ + (\epsilon_1+\epsilon_2) Z N_0^\top N_0 Z + \epsilon_2^{-1} M_0 M_0^\top + Z C_x^\top C_x Z$, $J_{22} = \mathrm{He}(P_1(\Xi\bar{A} - L_1\bar{C})) + \varepsilon_1^{-1} P_1\Xi\Xi^\top P_1 + \varepsilon_1 L_f^2 A_0^\top A_0 I_{n+q} + \epsilon_1^{-1} P_1 \Xi \bar{M}_0 \bar{M}_0^\top \Xi^\top P_1$
$+C_e^\top C_e$.

It follows from the Young relation [33] that for some positive scalar ϵ_3,

$$\mathrm{He}\left\{ \begin{bmatrix} BK_x \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}^\top \right\} \le \epsilon_3 \begin{bmatrix} BK_x Z \\ 0 \\ 0 \end{bmatrix} Z^{-1} \begin{bmatrix} BK_x Z \\ 0 \\ 0 \end{bmatrix}^\top + \epsilon_3^{-1} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} Z^{-1} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}^\top .$$

Further define

$$P_1 = \begin{bmatrix} Q_{n\times n} & \\ & R_{q\times q} \end{bmatrix},\ L_1 = \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix},\ H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

and $M_1 = K_x Z$, $M_2 = QH_1$, $M_3 = QL_{11}$, $M_4 = RH_2$, and $M_5 = RL_{12}$. Using the Schur complement repeatedly, (19) can be finally reformulated into (13). ■

Remark 6: In previous work by the authors [23], the integrated FE/FTC problem is solved via a single-step LMI formulation by introducing an equality constraint to linearize the non-linear term PBK_x in (18). This equality constraint imposes some restriction on the systems (e.g., the matrix B has to be full-column) as well as some requirement on the Lyapunov matrix P [23]. Here the requirement for an equation constraint is removed by using the Young relation, which thus offers more flexibility to achieve an optimal design. However, it should also be noted that this LMI formulation has one more prescribed design scalar ϵ_3 for solving the LMI (13).

IV. Extension to simultaneous actuator and sensor faults case

The FE/FTC design problem for actuator faults in Lipschitz non-linear systems has been outlined in Section III. However, the possibility of the presence of sensor faults should also be considered for real applications. It is well known that measurement noise also affects the FE/FTC performance and can be regarded as a bias type of sensor fault, estimated and compensated within FTC system design [34]. With these considerations, the integrated FE/FTC strategy proposed in Section III is extended here for Lipschitz non-linear systems with simultaneous actuator and sensor faults.

Consider the non-linear systems

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + f(x,t) + F_a f_a(t) + d_0(t) \\ y(t) &= Cx(t) + F_s f_s(t) \end{aligned} \tag{20}$$

where $f_s \in R^{q_1}$ denote the sensor fault with a known constant distribution matrix $F_s \in R^{p\times q_1}$, and the other terms are defined the same as those in (1). f_s is also assumed to be bounded and continuously smooth with bounded first time derivative.

Using a similar augmentation method in Section III-A, the system (20) is represented as

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \Delta\bar{A}\bar{x} + \bar{f}(A_0\bar{x}, t) + \bar{B}u + \bar{D}\bar{d} \\ y &= \bar{C}\bar{x} \end{aligned} \tag{21}$$

where

$$\bar{x} = \begin{bmatrix} x \\ f_a \\ f_s \end{bmatrix}, \; \bar{A} = \begin{bmatrix} A & F_a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \Delta\bar{A} = \begin{bmatrix} \Delta A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \bar{f}(A_0\bar{x}, t) = \begin{bmatrix} f(x,t) \\ 0 \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \; \bar{D} = \begin{bmatrix} D & 0 & 0 \\ 0 & I_q & 0 \\ 0 & 0 & I_{q1} \end{bmatrix}, \; \bar{d} = \begin{bmatrix} d \\ \dot{f}_a \\ \dot{f}_s \end{bmatrix}, \; \bar{C} = [C \; 0 \; F_s], \quad A_0 = [I_n \; 0 \; 0].$$

Following a similar proof as that in Remark 2, it is verified that this augmented system is observable. Thus, the following NUIO is proposed,

$$\begin{aligned} \dot{z} &= Mz + Gu + N\bar{f}(A_0\hat{\bar{x}}, t) + Ly \\ \hat{\bar{x}} &= z + Hy \end{aligned} \tag{22}$$

where $z \in R^{n+q+q_1}$ is the observer system state and $\hat{\bar{x}} \in R^{n+q+q_1}$ is the estimate of $\bar{x}$. The design matrices M, G, N, L, and H are of compatible dimensions.

The adaptive sliding mode FTC design is the same as the one constructed in Section III-B, except that the linear feedback component gain becomes $K = [K_x \; E_1 \; 0]$. Finally, the obtained augmented closed-loop system is

$$\begin{aligned} \dot{x} &= (\Theta_1 A - BK_x)x + BKe + \Theta f(x,t) + \Delta A x + D_1\bar{d} \\ \dot{e} &= (\Xi\bar{A} - L_1\bar{C})e + \Xi\Delta\bar{f} + \Xi\Delta\bar{A}\bar{x} + \Xi\bar{D}\bar{d} \\ y_c &= y - F_s\hat{f}_s \\ z_2 &= C_x x + C_e e \end{aligned} \tag{23}$$

where y_c is the system output after sensor compensation, $z_2 \in R^r$ is the measured output with matrices $C_x \in R^{r\times n}$ and $C_e \in R^{r\times(n+q+q_1)}$, and $D_1 = [\Theta D \; 0 \; 0]$.

A single-step LMI formulation is outlined in Theorem 3 to solve the gains K_x, L_1 and H from the augmented closed-loop system (23).

Theorem 3: Given positive scalars γ_2, ϵ_1, ϵ_2, ϵ_3, ε_1, and ε_2, the closed-loop system (23) is stable with H_∞ performance $\|G_{z_2\bar{d}}\|_\infty < \gamma_2$, if there exist four symmetric positive definite matrices $Z \in R^{n\times n}$, $Q \in R^{n\times n}$, $R \in R^{q\times q}$, and $S \in R^{q_1\times q_1}$, and matrices $M_1 \in R^{m\times n}$, $M_2 \in R^{n\times p}$, $M_3 \in R^{n\times p}$, $M_4 \in R^{q\times p}$, $M_5 \in R^{q\times p}$, $M_6 \in R^{q_1\times p}$, $M_7 \in R^{q_1\times p}$ such that

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \star & \Pi_3 \end{bmatrix} < 0 \tag{24}$$

with $\Pi_1 = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} \\ \star & J_{2,2} \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} \Xi_{1,3} & \Xi_{1,4} & 0 & \Xi_{1,6} & \Xi_{1,7} & 0 & 0 & \Xi_{1,10} \\ J_{2,3} & J_{2,4} & J_{2,5} & 0 & 0 & I & J_{2,9} & 0 \end{bmatrix}$,
$\Pi_3 = -\mathrm{diag}\left\{\gamma_2^2 I, I, \epsilon_1 I, (\epsilon_1+\epsilon_2)^{-1}I, \epsilon_3^{-1}Z, \epsilon_3 Z, \varepsilon_1 I, (\varepsilon_2 L_f^2)^{-1}I\right\}$,
$J_{2,2} = \begin{bmatrix} \Xi_{2,2} & \Xi_{2,3} & \Xi_{2,4} \\ \star & \Xi_{3,3} & \Xi_{3,4} \\ \star & \star & \Xi_{4,4} \end{bmatrix}$, $J_{2,3} = \begin{bmatrix} QD - M_2CD & 0 & -M_2F_s \\ -M_4CD & R & -M_4F_s \\ -M_6CD & 0 & S - M_6F_s \end{bmatrix}$,
$J_{2,4} = \begin{bmatrix} C_{ex}^\top \\ C_{efa}^\top \\ C_{efs}^\top \end{bmatrix}$, $J_{2,5} = \begin{bmatrix} QM_0 - M_2CM_0 \\ -M_4CM_0 \\ -M_6CM_0 \end{bmatrix}$, $J_{2,9} = \begin{bmatrix} Q - M_2C & 0 & -M_2F_s \\ -M_4C & R & -M_4F_s \\ -M_6C & 0 & S - M_6F_s \end{bmatrix}$,
$\Xi_{1,1} = \mathrm{He}(\Theta AZ - BM_1) + \epsilon_2^{-1}M_0M_0^\top + \varepsilon_2^{-1}\Theta\Theta^\top, \Xi_{1,2} = [0 \; F_a \; 0], \Xi_{1,3} = [\Theta D \; 0 \; 0]$,
$\Xi_{1,4} = ZC_x^\top$, $\Xi_{1,6} = ZN_0^\top$, $\Xi_{1,7} = BM_1$, $\Xi_{1,10} = Z$, $\Xi_{2,2} = \mathrm{He}(QA - M_2CA - M_3C) + \varepsilon_1 L_f^2 I_n$, $\Xi_{2,3} = QF_a - M_2CF_a - (M_4CA + M_5C)^\top$, $\Xi_{2,4} = M_3F_s - (M_6CA + M_7C)^\top$,
$\Xi_{3,3} = \mathrm{He}(-M_4CF_a)$, $\Xi_{3,4} = -M_5F_s - (M_6CF_a)^\top$, $\Xi_{4,4} = \mathrm{He}(-M_7F_s)$,
then the gains are given by: $K_x = M_1Z^{-1}, H_1 = Q^{-1}M_2, H_2 = R^{-1}M_4, H_3 = S^{-1}M_6, L_{11} = Q^{-1}M_3, L_{12} = R^{-1}M_5, L_{13} = S^{-1}M_7$.

Proof: Define

$$P_1 = \begin{bmatrix} Q_{n\times n} & & \\ & R_{q\times q} & \\ & & S_{q_1\times q_1} \end{bmatrix}, \; L_1 = \begin{bmatrix} L_{11} \\ L_{12} \\ L_{13} \end{bmatrix}, \; H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix},$$

and $M_1 = K_xZ$, $M_2 = QH_1$, $M_3 = QL_{11}$, $M_4 = RH_2$, $M_5 = RL_{12}$, $M_6 = SH_3$, and $M_7 = SL_{13}$. According to the proof of Theorem 2, the proof of Theorem 3 is straightforward and thus is omitted here. ■

Remark 7: In the proposed method the Lipschitz constant is explicitly required to allow the use of H_∞ optimization to suppress the effect of the non-linearity on the control system and the observer. More details can be found in the proof of

Theorem 3.2. Larger values of the Lipschitz constant would increase the value of the H_∞ performance index, resulting in less robust observer/control designs. Considering this, for non-linear systems with relatively large Lipschitz constants the proposed H_∞ optimization design might not be able to reach the required robust FE/FTC performance. Therefore, a new method for handling large Lipschitz constants can be a research topic to follow this current work.

V. APPLICATION TO A SINGLE-LINK MANIPULATOR

Adopted from [10] the single-link manipulator

$$\begin{aligned} \dot{x} &= (A+\Delta A)x + (B+\Delta B)u + f(x,t) + d_0 \\ y &= Cx \end{aligned} \tag{25}$$

where $x = [x_1\ x_2\ x_3\ x_4]^\top = [\theta_m\ w_m\ \theta_1\ w_1]^\top$ are the angular positions and angular velocities of the motor and the link, respectively. The system matrices are defined as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 1.25 \\ 0 & 0 & 0 & 1 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},\ D = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1\sin(t) & 0 & 0.1\sin(t) & 0 \end{bmatrix}, \tag{26}$$

$$\Delta B = \begin{bmatrix} 0 \\ 0.1\cos(t) \\ 0 \\ 0 \end{bmatrix},\ f(x,t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333\sin(x_3) \end{bmatrix},\ d_0 = \begin{bmatrix} 0 \\ 0.1\sin(10t) \\ 0 \\ 0 \end{bmatrix}.$$

It follows from (1) that

$$\Delta Bu + d_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} (0.1\cos(t)u + 0.1\sin(10t)) = Dd,\ M_0 = I_4,$$
$$F_0(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin(t) \end{bmatrix},\ N_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0.1 & 0 \end{bmatrix}.$$

To demonstrate better the performance of the proposed integrated FE/FTC design, two simulation cases are studied, i.e., the system (25) has (i) actuator faults alone or (ii) both actuator and sensor faults. Comparative simulations with the same system parameters and initial conditions are performed, using the proposed integrated design, the normal control (without actuator/sensor fault compensation) and the separated design proposed in [14].

A. Actuator fault case

Suppose that the system has the following actuator fault and distribution matrix

$$F_a = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},\quad f_a(t) = \begin{cases} 0, & 0 \le t \le 1 \\ t-1, & 1 < t \le 2 \\ \sin(1.5\pi t), & 2 < t \le 5 \\ 1, & 5 < t \le 6 \\ 2, & 6 < t \le 7 \\ 1, & 7 < t \le 8 \\ 1.5\sin(\pi(t-8)), & 8 < t \le 12 \\ \sum_{k=0}^{100} 0.5^k \cos(3^k \pi t), & 12 < t \le 20 \end{cases}.$$

Although the faults studied are assumed to be differentiable, a Weierstrass function type fault (smooth but non-differentiable) is used in the time period $t \in (12, 20]s$ to further verify the capability of the proposed method. Given $C_x = C_{ex} = I_4$,

$C_{efa} = C_{efs} = [1\ 1\ 1\ 1]^\top$, $L_f = 0.333$, $\epsilon_1 = 0.1$, $\epsilon_2 = 1000$, $\epsilon_3 = 0.01$, $\varepsilon_1 = 10$, $\varepsilon_2 = 100$, and $Y_1 = [0.1\ 0.1\ 0.1\ 0.1]$. Solving Theorem 2 gives $\gamma_1 = 0.2$ and the controller and observer gains

$$K_x = \begin{bmatrix} 111.8034 \\ 4.5655 \\ -37.5276 \\ 76.0131 \end{bmatrix}^\top, \ H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2.3696 & 0 & 0 \end{bmatrix}, \ G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -51.1825 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.5219 & 0 & 0.0047 & -0.0072 & 0 \\ 0 & -0.5001 & 0 & 0 & 0 \\ 0.0047 & 0 & -0.5253 & -0.0037 & 0 \\ -0.0072 & 0 & -0.0037 & -0.5232 & 0 \\ -3.9498 & -12.3294 & -4.5633 & -3.8218 & -51.1825 \end{bmatrix},$$

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2.3696 & 0 & 0 & 1 \end{bmatrix}, \ L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 115.1605 & -118.3179 & -115.1605 & 0.00001 \end{bmatrix}.$$

The sliding surface and the non-linear component of the sliding mode controller are designed as

$$s_1 = [-0.1\ 0.0463\ \ -0.1\ \ -0.1]\hat{x}, \ \dot{\hat{\eta}}_{s_1} = 5\|s_1\|, \ \rho_{s_1}(t) = \hat{\eta}_{s_1} + 1.2$$
$$u_n = \begin{cases} -\rho_{s_1}(t)\frac{s_1}{\|s_1\|+0.01}, & s_1 \neq 0 \\ 0, & s_1 = 0 \end{cases}$$

where a continuous approximation of the function $\mathrm{sign}(s) = s_1/\|s_1\|$ is used here.

Simulations are performed with initial conditions: $x(0) = [\pi/15\ \ 0.2\ \ \pi/12\ \ 0]^\top$, $z(0) = 0$, and $\hat{\eta}(0) = 0$. The H_∞ attenuation levels of the integrated/separated designs are listed in Table I. Compared with the separated design, the proposed integrated design loses a certain degree of FTC robustness resulting from the sharing of the common Lyapunov matrices in the observer and controller designs.

TABLE I
H_∞ ATTENUATION LEVEL

	Integrated design	Separated design	
		Observer	Controller
$\gamma_{1\,\min}$	0.2	0.1	0.1

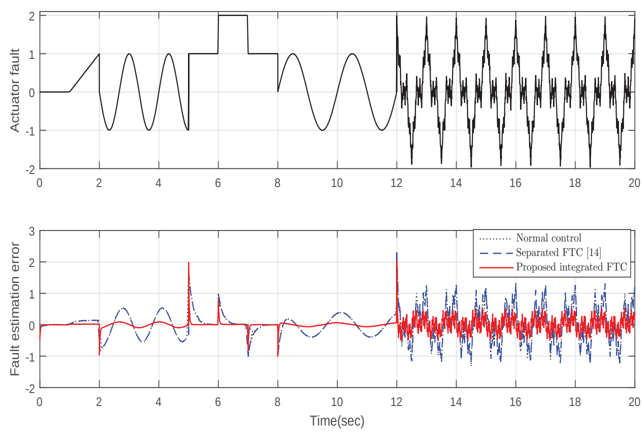

Fig. 2. Actuator fault estimation performance: actuator fault case

In the presence of uncertainty, disturbance, and actuator fault, the results in Figures 2 - 6 show that the normal control and the separated FTC in [14] cannot ensure the stability of the closed-loop system and the faults are not well estimated. When compared with this the proposed integrated design approach estimates the actuator fault with good accuracy and ensures that the closed-loop system is robustly stable. Although the proposed integrated design is developed for the faults assumed to be

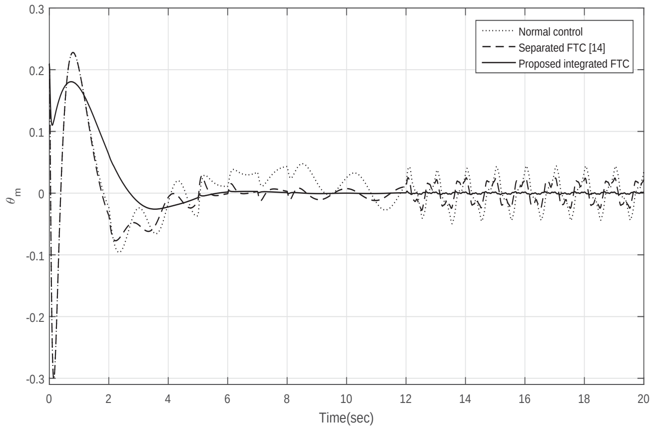

Fig. 3. Motor angular position: actuator fault case

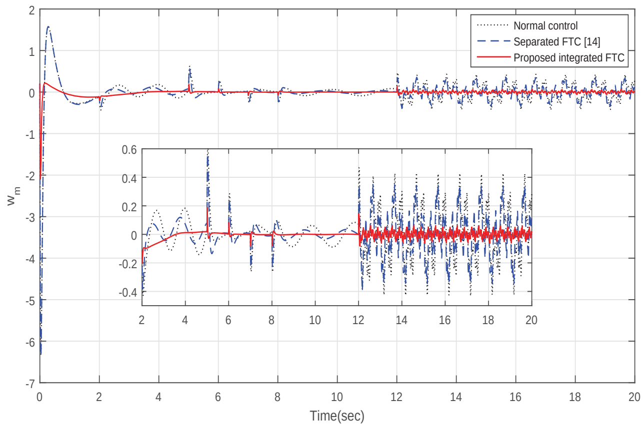

Fig. 4. Motor angular velocity: actuator fault case

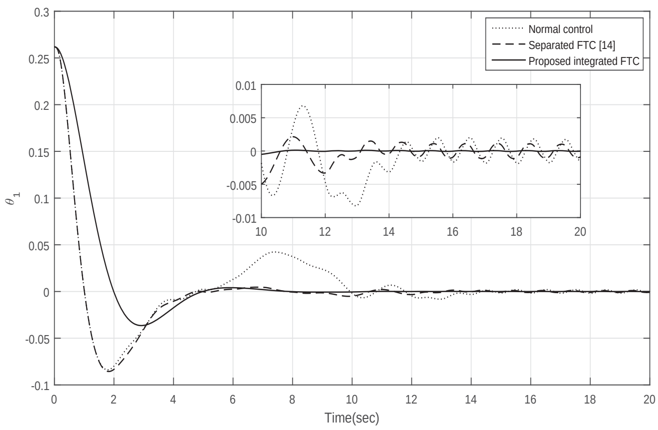

Fig. 5. Link angular position: actuator fault case

smooth with first time derivative, it is verified in the results that it is also able to achieve acceptable FE/FTC performance for the non-differentiable fault case.

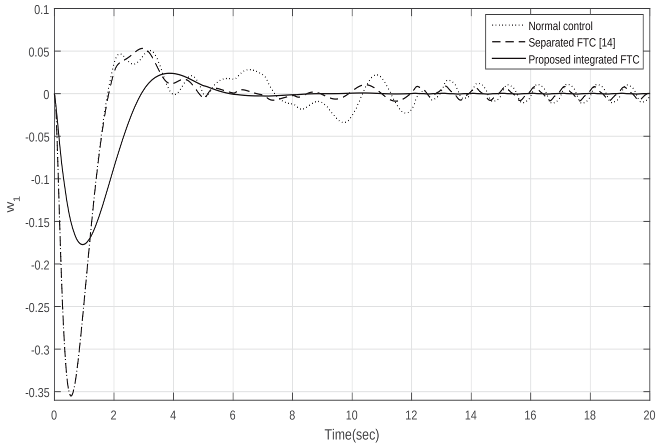

Fig. 6. Link angular velocity: actuator fault case

B. Actuator and sensor faults case

Consider the situation that the system (25) suffers from simultaneous actuator and sensor faults, which are assumed to be

$$F_a = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \; f_a(t) = \begin{cases} 1, & 0 \le t \le 3 \\ 2, & 3 < t \le 6 \\ 1, & 6 < t \le 9 \\ 0.5, & 9 < t \le 12 \\ -1, & 12 < t \le 15 \end{cases}, \; F_s = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; f_s(t) = 0.1 \sin(\pi t).$$

Given $C_x = C_{ex} = I_4$, $C_{efa} = C_{efs} = [1\ 1\ 1\ 1]^\top$, $L_f = 0.333$, $\epsilon_1 = 10$, $\epsilon_2 = 100$, $\epsilon_3 = 0.01$, $\varepsilon_1 = 1$, $\varepsilon_2 = 100$, and $Y_1 = [0.1\ 0.1\ 0.1\ 0.1]$. Solving Theorem 3 gives $\gamma_2 = 0.25$ and the controller and observer gains

$$K_x = \begin{bmatrix} 49.2502 \\ 4.6288 \\ -1.5937 \\ 55.3266 \end{bmatrix}^\top, \; H = \begin{bmatrix} 0.6895 & 0.0318 & 0.3100 & 0.3112 \\ 0.3105 & 0.9682 & -0.3100 & -0.3112 \\ 0.3105 & -0.0318 & 0.6900 & -0.3112 \\ 0.3105 & -0.0318 & -0.3100 & 0.6888 \\ 0.7993 & 2.4914 & -0.7975 & -0.8018 \\ -0.3092 & 0.0361 & 0.3095 & 0.3087 \end{bmatrix},$$

$$M = \begin{bmatrix} -116.9957 & 113.8338 & 134.1462 & 128.3954 & -0.6875 & 495.2110 \\ 116.4494 & -114.3354 & -134.1383 & -128.4409 & 0.6875 & -495.2040 \\ 116.4494 & -113.8260 & -134.6669 & -128.4470 & 0.6875 & -495.2292 \\ 116.3993 & -113.8292 & -134.1477 & -129.0052 & 0.6875 & -495.2215 \\ 14.7328 & -27.4614 & -23.0327 & -19.7599 & -53.8134 & 157.1884 \\ 73.1861 & -70.7960 & -78.6252 & -76.0187 & -0.7791 & -296.3691 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.3105 & -0.0318 & -0.3100 & -0.3112 & 0 & 0.0366 \\ -0.3105 & 0.0318 & 0.3100 & 0.3112 & 0 & -0.0366 \\ -0.3105 & 0.0318 & 0.3100 & 0.3112 & 0 & -0.0366 \\ -0.3105 & 0.0318 & 0.3100 & 0.3112 & 0 & -0.0366 \\ -0.7993 & -2.4914 & 0.7975 & 0.8018 & 1 & -0.0928 \\ 0.3092 & -0.0361 & -0.3095 & -0.3087 & 0 & 0.0365 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.6875 \\ 0.6875 \\ 0.6875 \\ 0.6875 \\ -53.8134 \\ -0.7791 \end{bmatrix}, \; L = \begin{bmatrix} 0.4411 & 0.7946 & -0.0398 & -0.3930 \\ -0.4411 & -0.7946 & 0.0398 & 0.3929 \\ -0.4412 & -0.7947 & 0.0398 & 0.3930 \\ -0.4411 & -0.7946 & 0.0398 & 0.3930 \\ 4.6438 & -123.3795 & -4.7294 & 118.9216 \\ -0.5446 & -2.7707 & 0.3007 & 1.7396 \end{bmatrix}.$$

The sliding surface and the non-linear component of the sliding mode FTC controller are designed as

$$s_1 = [-0.1 \;\; 0.0463 \;\; -0.1 \;\; -0.1]\hat{x},\ \dot{\hat{\eta}}_{s_1} = 5\|s_1\|,\ \rho_{s_1}(t) = \hat{\eta}_{s_1} + 1.1$$
$$u_n = \begin{cases} -\rho_{s_1}(t)\frac{s_1}{\|s_1\|+0.01}, & s_1 \neq 0 \\ 0, & s_1 = 0 \end{cases}.$$

Simulations are performed with initial conditions: $x(0) = [\pi/10 \;\; 0.2 \;\; \pi/2 \;\; 0]^\top$, $z(0) = 0$, and $\hat{\eta}(0) = 0$. Similarly, it is observed from Table II that the proposed integrated design loses a certain degree of FTC robustness compared with the separated design.

TABLE II
H_∞ ATTENUATION LEVEL

	Integrated design	Separated design	
		Observer	Controller
$\gamma_{2\,\min}$	0.25	0.1	0.12

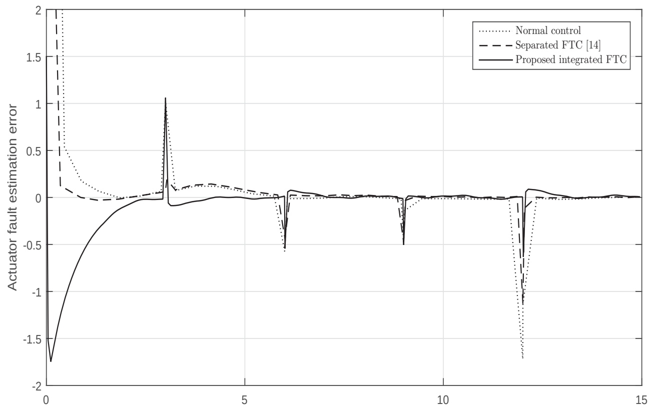

Fig. 7. Actuator fault estimation performance: actuator/sensor faults case

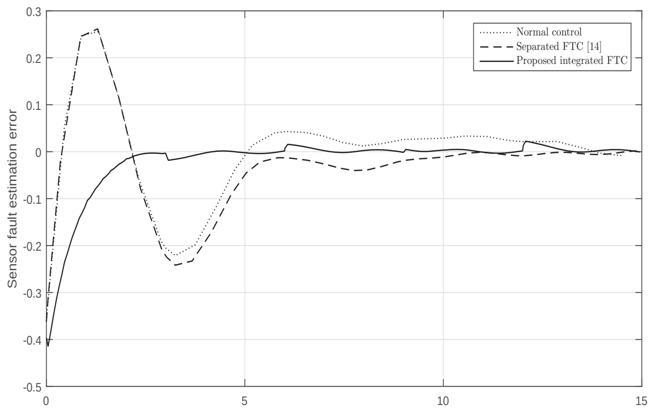

Fig. 8. Sensor fault estimation performance: actuator/sensor faults case

In the presence of uncertainty, disturbance, and actuator/sensor fault, the results in Figures 7 - 14 show that the normal control and the separately designed FE and FTC functions cannot stabilize the closed-loop system and the faults are not well estimated. Moreover, the sensor fault effect in the system output is not compensated well using the separated design. However, the proposed integrated design estimates the faults with good accuracy and ensures the robust stability of the closed-loop system with the sensor fault well compensated.

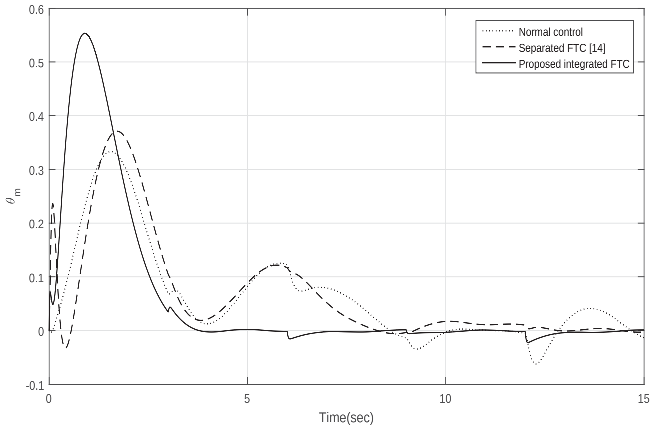

Fig. 9. Motor angular position: actuator/sensor faults case

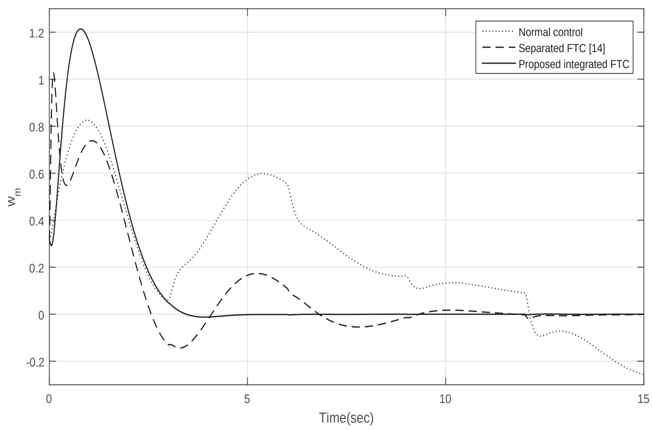

Fig. 10. Motor angular velocity: actuator/sensor faults case

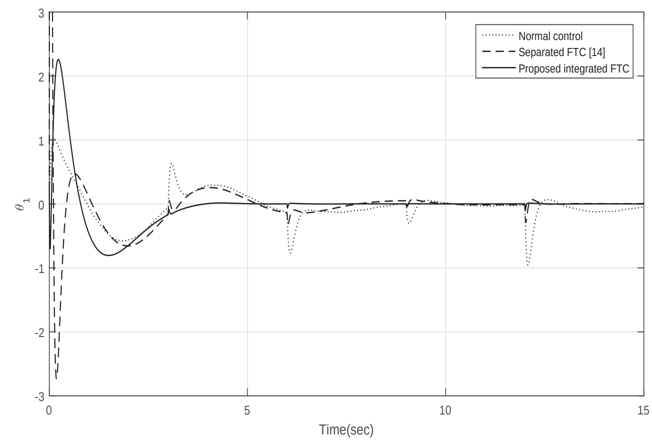

Fig. 11. Link angular position: actuator/sensor faults case

VI. Conclusion

An FE-based integrated FTC design for uncertain Lipschitz non-linear systems subject to bounded disturbance and actuator/sensor faults is proposed. A NUIO without rank requirement is proposed to estimate simultaneously the system states and faults, and using the obtained estimates an adaptive sliding mode FTC controller is constructed. The integrated FE/FTC design problem is formulated as an observer-based robust control problem solved using H_∞ optimization in a single-step LMI

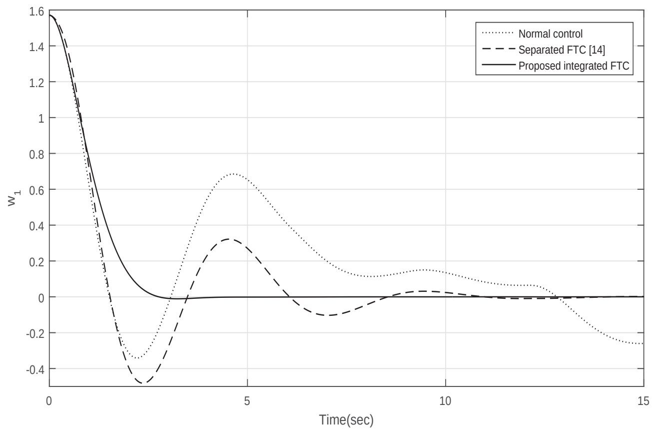

Fig. 12. Link angular velocity: actuator/sensor faults case

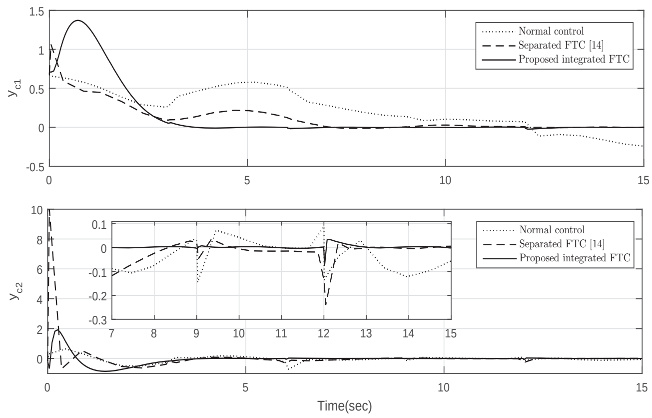

Fig. 13. Compensated system output: actuator/sensor faults case

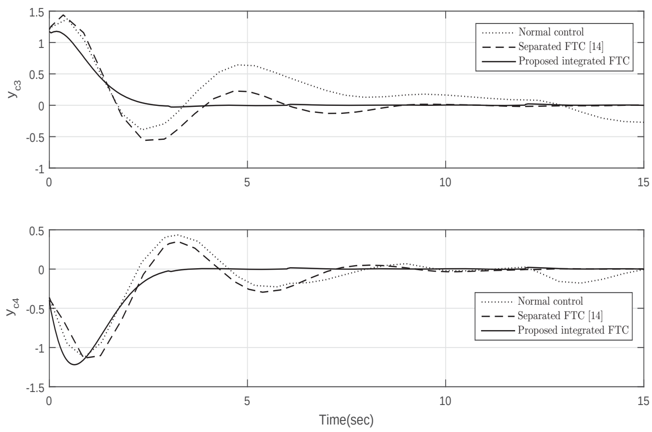

Fig. 14. Compensated system output: actuator/sensor faults case

formulation. By taking account of the bi-directional robustness interactions between the FE and FTC models the integrated design achieves good FE/FTC performance as shown in the comparative simulations of a single-link manipulator. This is demonstrated with the proposed FTC design and the normal control design as well as with the existing separated FTC design. The future research should focus on an extension of the proposed design to address the output tracking control problem for Lipschitz non-linear systems.

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