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Discussion Paper Series Editor: Dr M Visvalingam



**C.I.S.R.G. DISCUSSION PAPER 15**

Algorithms for Sketching Surfaces

by

M Visvalingam and K Dowson

## Discussion Papers

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## Special Issues

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2. Visvalingam, M. and Kirby, G. H. (1987)  
Director of Research and Development based on Ordnance Survey Small Scales Digital Date, sponsored by the Ordnance Survey, 38 pp.
3. Visvalingam, M. and Sekouris, N. M. (1989)  
Management of Digital Map Data Using a Relational Database Model, sponsored by the Ordnance Survey, 50 pp.



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## 1. Introduction

Until relatively recently, mainstream computer graphics has focused mainly on the challenge of simulating photorealism. Now that this aim has been realised to a large extent, a growing band of researchers (Sasada, 1987; Dooley and Cohen, 1990; Saito and Takahashi, 1990; 5D Solutions, 1994; Hsu and Lee, 1994; Leister, 1994; Salisbury et al, 1994; Winkenbach and Salesin, 1994) have found the emulation of sketching and illustration techniques equally challenging. This study explores the problem of sketching surfaces, such as terrain. In particular, it considers how a sketch may be abstracted from a Digital Elevation Model, or a DEM, which is basically a grid of height values. DEMs are becoming increasingly available. They are used within a variety of applications of Geographical Information Systems (Burrough, 1986, Catlow, 1986, DoE, 1987).

Cartographers have used a wide variety of techniques, including landscape sketches, for representing terrain (Imhof, 1982; Dowson, 1994). A sketch is a generalised depiction which only shows elements of interest. Superfluous details are omitted to reduce distracting clutter. Cartographers also include some view-dependent features, such as occluding contours, to bring life into what could easily become a dull drawing (Lobeck, 1924). However, the aesthetic dimension of sketching is outside the scope of this feasibility study, which is primarily concerned with assessing the value of Visvalingam's line generalisation algorithm (see Visvalingam and Whyatt, 1993) for locating view-independent shape-defining convexities and concavities within DEMs. A simple sketching technique (similar to the profile-based sketching technique of cartographers like Tanaka (1932), Raisz (1938), and Robinson and Thrower (1957)), was initially investigated. The core cells, selected using Visvalingam's algorithm, are extended to form different types of profile-strokes, called P-strokes, which comprise the sketch.

The sketching strategy and the algorithms used in this feasibility study were designed by Visvalingam and were implemented and tested by Dowson (1994); the sketching parameters used in this paper differ from those used by Dowson. These sketches resemble the landscape drawings of artists and the field sketches made by geographers. As Raisz noted, this type of sketch does not require a great deal of artistic ability. However, it took a great deal of experimentation to identify the sketching parameters presented in this paper. The results are promising and open up directions for future research.

## 2. Background

Although, there are a multiplicity of trends and concerns within digital cartography, the advent of direct manipulation has been a boon to the freelance cartographic consultant (Visvalingam, 1990). Developments in Human Computer Interaction and Graphical User Interfaces are enabling a new breed of cartographers to develop their own individualistic styles based on the principles of traditional cartography (Visvalingam, 1994). Alan Collinson, winner of several British Cartographic Society Design Awards, has stressed that cartography, without art, becomes an impoverished discipline,

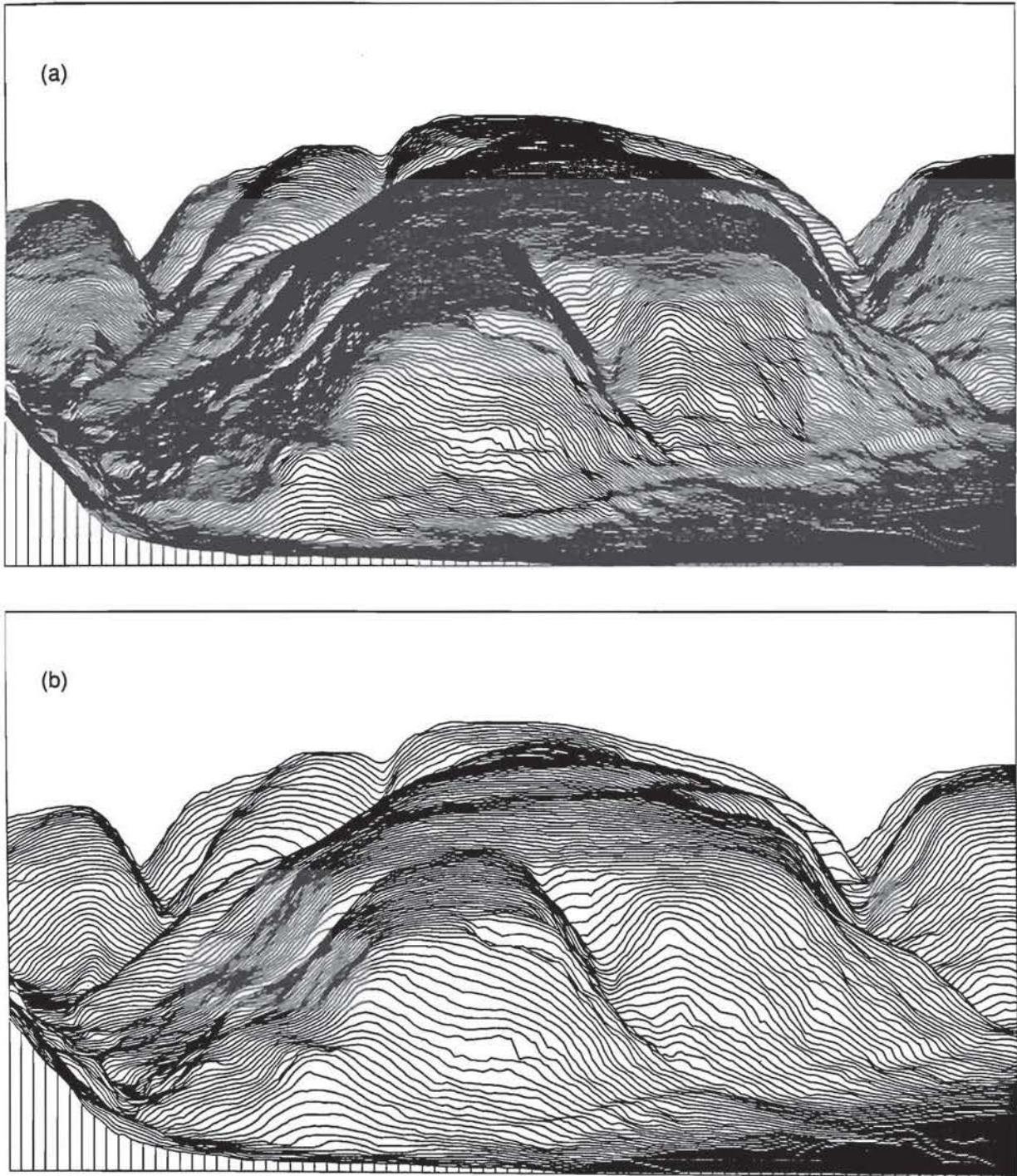


Figure 1 : Data; (a) all profiles; (b) alternate profiles only  
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cography. The research reported in this paper forms a part of a longer term programme of research which seeks to innovate new techniques and tools for accelerating the revival of the art forms of cartography.

Monkhouse and Wilkinson (1976, p 87) noted that the depiction of relief on a plane surface has been one of the major problems for cartographers since the earliest days of map making. Cartographers tend to distinguish between quantitative and qualitative representations of relief. Computer generated displays of terrain tend to be quantitative and include contours, profiles, wire frames of DEMs and Triangulated Irregular Networks (TINs), and photorealistic depictions (McLaren & Kennie, 1989). These tend to display all visible data and, as such, may be considered as being data diagrams. Qualitative depictions of terrain provide a visual impression of relief through a variety of pictorial methods (Raisz, 1938). The skilled drawing of block diagrams and physiographic maps was appreciated as a scientific art; the text by Lobeck (1924) contains several splendid examples and guidance for their graphic construction. The construction of pictorial techniques involves the recognition of 'skeletal lines', which bring out relief features such as watersheds, stream networks, breaks of slope, ridges and edges of terraces (Imhof 1982, p 105). The identification of surface-specific features forms the subject of continuing research; this paper explores how DEMs may be sketched without such semantic knowledge. Visvalingam's algorithm for line generalisation, reported in Visvalingam and Whyatt (1993), is useful for eliminating unnecessary height information and for identifying DEM cells which lie on significant relief features. Rough, but pleasing, sketches may be created by extending these cells in profile.

### **3. Data**

The ideas described here were successfully tested using two different DEMs (Dowson, 1994). The test DEM used in this paper (Figure 1) represents only a quarter of a small 1:10 000 sample dataset released by the Ordnance Survey of Great Britain. This subset consists of a grid of 250 x 250 cells of 10m resolution. Even so, the display of all data results in illegibility and a loss of information in places. Some of the clutter may be eliminated by systematic data reduction, for example, by plotting only alternate lines but this is not entirely adequate. As the resolution and coverage of DEMs continue to increase there will be a growing need to generalise and symbolise relief elements, particularly for viewing at small scales. This is especially necessary in applications which use the DEM as a base for 'mapping' foreground information (Visvalingam, 1990).

### **4. P-strokes for sketching terrain**

Like a map, a sketch is a graphic precis of reality. The cartographer brings to field sketching his skills in cartographic generalisation which includes the processes of selection, simplification, classification, symbolisation, displacement and exaggeration (Robinson et al, 1984). Lobeck (1924, p 165) advised that the field sketch "should be left unfinished, with just enough detail suggested here and there to make it possible to complete the drawing" later; the aim is "to select from the landscape those critical

and important lines which give it its character, that is, to lay out its major elements" and then only "to put in some details, not too accurately, but in such a way as to explain better the larger features".

The sketches essentially consist of vertically displaced subsections of vertically exaggerated profiles which give the impression of an oblique parallel projection. The various design decisions are discussed below.

#### 4.1 Profile-based sketching style

Sketching may be based on a variety of styles. It is relatively easy to compute the view-dependent occluding contours in Figure 1, which are immediately seen. While there is a need to include them in the final sketch, this study is more concerned with abstracting the major shape-defining convexities and concavities which define the form of surface features (Marr, 1982). The technique of drawing sections of profiles (P-strokes) was adopted instead for a variety of reasons. Profiles :

- **may be easily located on the DEM.** In interactive applications, the user can easily see these P-strokes superimposed on quantitative visualisations (fishnet, contour and other plots) to assist visual analysis of the data and/or touching-up of the image. It is also possible to highlight (via line style, weight or colour) the index profiles so that it is easy to determine the 3D location of features of interest and evaluate the results.
- **are well established in theatre and art.** The geographical field sketch uses the coulisse metaphor to emphasise the effect of ridges and hills appearing behind each other (Raisz, 1931). Coulisses are grooves into which stage props (profiles) are slotted to provide an illusion of depth. Vertically stacked profiles also feature in Chinese landscape paintings. Arnheim (1956, p 201) noted that 'the volume of a mountain is often conceived as a skeleton of echelons or slices in staggered formation. The complex curvature of the solid is thus obtained through a kind of 'integral' based on the summation of frontal planes.'
- **form a set of neat lines.** Many texts in geomorphology and geology show authors' sketches which look untidy, partly because the lines are not neatly organised. Cartographers have used P-strokes very effectively within a number of techniques (Tanaka (1932), Raisz, (1938), and Robinson and Thrower (1957)).
- **can be processed in parallel.**
- **are sufficient for this study.** As previously stated, the emphasis in this feasibility study is primarily on assessing the utility of Visvalingam's algorithm for identifying significant cells in the DEM and less on the stylistic conventions for depicting them. P-strokes are sufficient for this feasibility study.

## 4.2 Vertical oblique projection

The projections used in computer graphics are well documented (Foley et al, 1990; Hopgood and Duce, 1991). The cartographer includes an element of linear perspective, when needed, by using graphical techniques. The construction of block diagrams relies on the use of vanishing points for one or two point perspective depending on the purpose of the resulting map-like sketch (Lobeck, 1924); the graphical methods are similar to those developed during the Renaissance by Alberti. Drawing machines have been in use since the Renaissance and Albrecht Durer devised several such machines for enforcing the laws of linear perspective when sketching a scene (Arnheim, 1956; Cole, 1992).

Gombrich (1960, p 243) described perspective as "the most important trick in the armoury of illusionist art", but he later noted that "perspective is merely one way of describing space and has no absolute validity" (p 247). This study is mainly concerned with evaluating the utility of Visvalingam's algorithm for abstracting relief features. Perspective is unhelpful since much of the distant information will become reduced and hidden. In the sketches included in this paper, each profile is displaced vertically up by 2 metres from the one in front of it. Such vertical oblique projections are often employed in Oriental art, such as in the vertically stacked Chinese landscape paintings, as noted earlier. An image produced using the vertical oblique projection is more similar to a map than the corresponding image shown in perspective since distances and heights may be easily determined. It is possible to produce map-like displays by increasing the vertical displacement (Dowson, 1994). It is also possible to use cabinet projections by some displacement of the profiles in both vertical and horizontal directions. These result in larger sketches requiring reduction and are not necessary here.

## 4.3 Viewpoint and Vertical exaggeration

Although the focus of this exploratory study is on detecting and sketching the view-independent structure of the environment, the sketch is inevitably view dependent to some extent. Since profile strokes are two dimensional entities which possess no depth, they need to be perpendicular to the line of sight. Cartographers use a vertical exaggeration of two to four for general purpose profiles although more extreme values may be used depending on the type of environment and the purpose of the display (Dickinson, 1969; Monkhouse and Wilkinson, 1976). Equally, the vertical exaggeration can also be reduced to achieve map-like displays. A vertical exaggeration of 2.5 is used in most of the sketches shown in this paper.

Dowson (1994) illustrated the effect of varying the vertical exaggeration and the vertical displacement of successive profiles and also included sketches of DEMs as seen from the four cardinal directions, using different sketching parameters to those used in the sketches presented in this paper.



## 5. The abstraction of P-strokes

The main aims of the feasibility study were to abstract the view-independent structural elements in the surface, to generate the polylines making up the P-strokes, and to find effective ways for symbolising them. These aspects of the research are discussed below.

### 5.1 Visvalingam's algorithm for line generalisation

The strokes which form the artist's sketch convey the structure and form of landscape elements. Peucker and Douglas (1975) used local operators, similar to those in image processing, to filter, connect and infer a variety of features, such as ridges, channels, peaks, pits and passes from DEMs. A similar image processing approach has been adopted by others (see Dowson, 1994). Heller (1990) suggested that it would be a difficult and needless detour to try to algorithmically extract structures from a DEM, especially since Fowler and Little (1979) and Heller (1990), amongst others, have noted that the abstracted surface specific features do not always follow significant relief features. But, this could be a reflection of the current state of methodology.

Since some line generalisation algorithms can be used to extract the global structure of lines, Dowson (1994) evaluated the utility of two line generalisation algorithms, namely the algorithms by Douglas and Peucker (1973) and that by Visvalingam (reported in Visvalingam and Whyatt, 1993) respectively. The Douglas-Peucker algorithm is also known as Ramer's (1972) algorithm within the field of Pattern Recognition; Visvalingam's algorithm has also been considered in Pattern Recognition (Pikaz and Dinstein; 1995). Both these algorithms were compared using 1:50 000 coastlines by Visvalingam and Whyatt (1993) and 1:1250 boundaries of road polygons by Visvalingam and Williamson (1995). These studies indicated that the Douglas-Peucker algorithm was preferable for minimal simplification of lines but that Visvalingam's algorithm was more effective for caricatural generalisation. Dowson (1994) demonstrated that Visvalingam's algorithm was more appropriate for tracking large-scale features and for producing more pleasing sketches. The design of the algorithm was based on the premise that it is difficult to select automatically the shape-preserving points on lines; it is much easier to remove unimportant points. This algorithm has been described in detail elsewhere and only the gist of the algorithm is presented below.

In brief, the algorithm makes multiple passes over the line. On each pass, it eliminates the point which it regards as least important. A variety of metrics may be used to measure the importance of points. Visvalingam and Whyatt tested the concept of effective area. This is the area of the triangular feature formed by connecting the point with its two neighbours; it measures the area by which the current line would be displaced as a result of removing that single point. When a point is removed the effective areas of adjacent points need to be recalculated before the next pass. Note that the area values are signed to differentiate between positive convexities and negative concavities. The magnitude of the cell's value corresponds, in general, to the size of the feature on which the cell is located. Note,

however, that the algorithm evaluates the significance of a point within the current part generalised and not the original line. The gist of the algorithm may be summarised by the following pseudocode:

```

CLEAR the output list
LET E be the effective area
CALCULATE E for all intermediate points
WHILE (there are intermediate points)
  { ELIMINATE the point with smallest E
  ADD its sequence number and E to output list
  RECOMPUTE E for the neighbouring points
  WHILE (the smaller of the recomputed E is less than
    that of the last eliminated point)
    { RESET its E to that of the last eliminated point
    ELIMINATE this point
    ADD its sequence number and E to output list
    RECOMPUTE E for its neighbouring points
    }
  }
}

```

Note that all points are tagged with E and that their elimination sequence is recorded. The tagged points may then be filtered at runtime by interactive fine-tuning of the tolerance parameter for E. The theoretical ideas underpinning this algorithm, the special cases and implementation issues are discussed in detail by Visvalingam and Whyatt.

Visvalingam's algorithm is used to generalise the polylines, representing the surface profiles, for all the rows and columns of the DEM. Dowson also explored the use of perpendicular distance, instead of area, but concluded that it was difficult to choose between sketches produced using effective area and perpendicular distance. Effective area produces more pleasing sketches since it includes an element of subtle randomness present in manual sketches and is illustrated in this paper. Current research is exploring the use of other metrics for landform analysis.

## 5.2 The sketching pipeline and parameters

Visvalingam's algorithm is applied to the rows and columns of the DEM to derive and associate two area values with each cell. User supplied tolerances are used to filter sets of cells. These core cells indicate the positions where marks should appear. These cells are then extended along the profiles to form polylines representing the P-strokes. The visual impact of varying the length and direction of these strokes was studied. The colour, style and width of the strokes were then determined. These stages, illustrated in Figure 2, are discussed below.

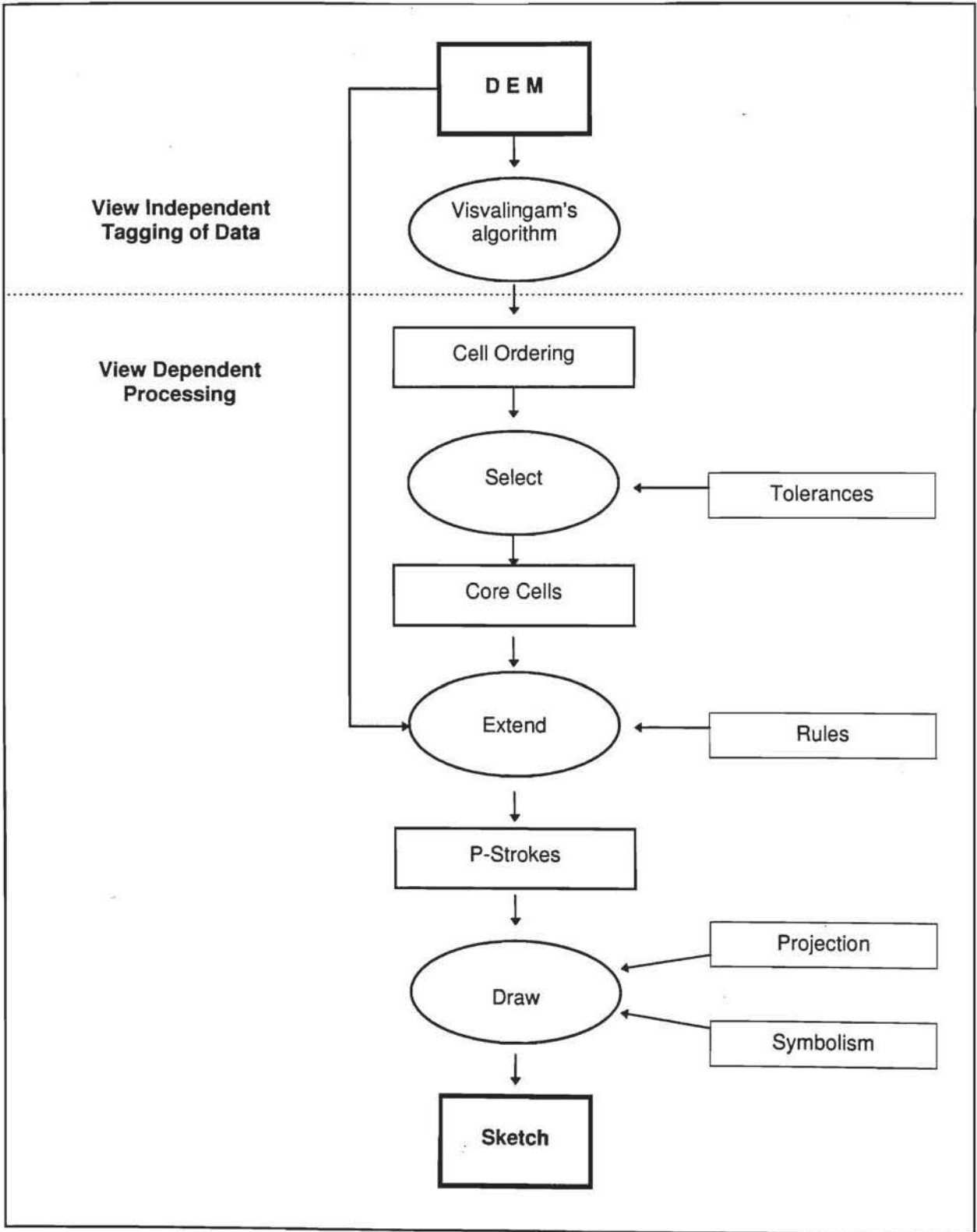


Figure 2 : The Sketching Pipeline

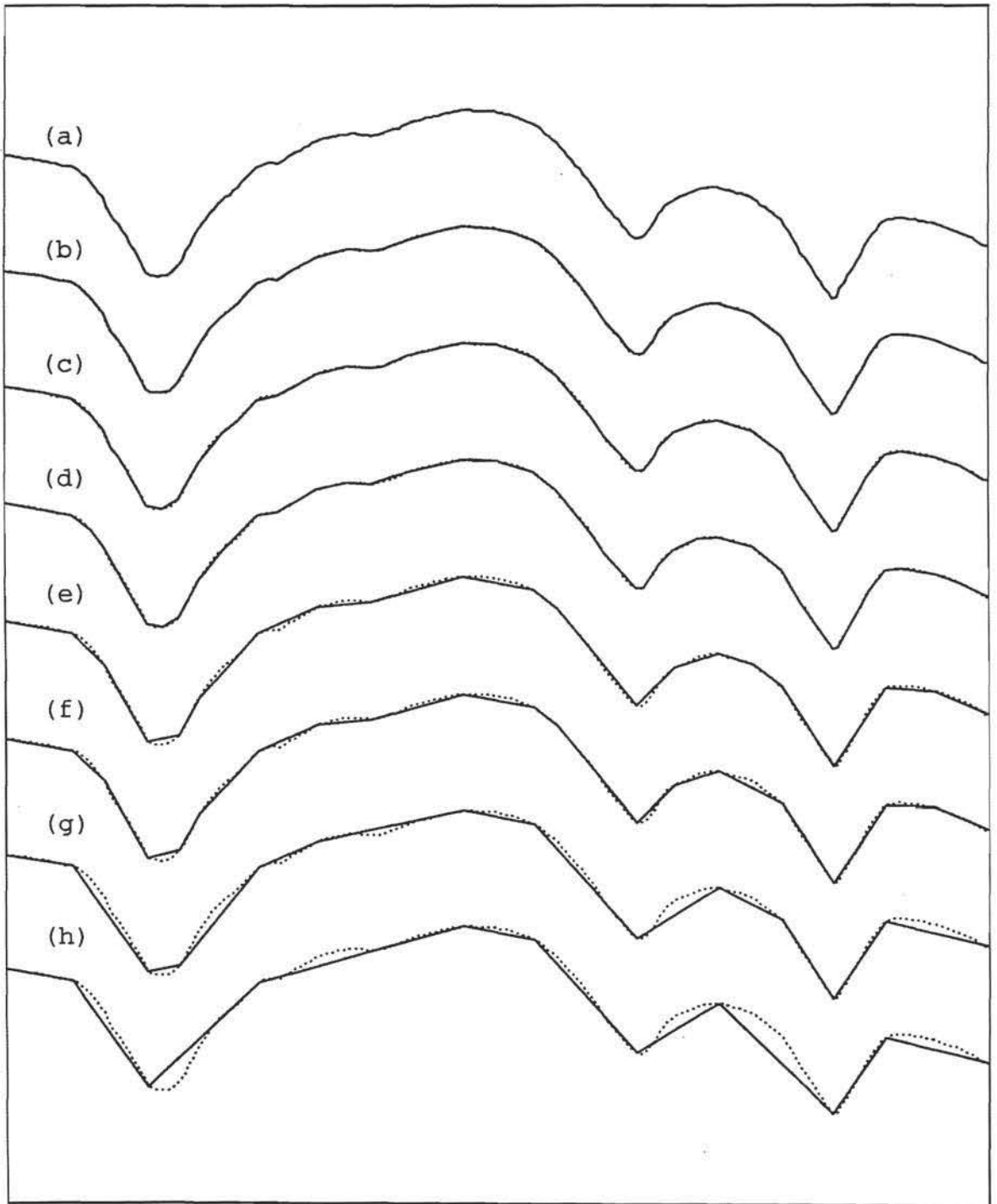


Figure 3 : Visvalingam's algorithm applied to a single profile section

Subfigures	Tolerance	Points Retained	
		#	%
(a)	0	250	100.0
(b)	15	76	30.4
(c)	50	50	20.0
(d)	100	37	14.8
(e)	400	26	10.4
(f)	800	18	7.2
(g)	2000	13	5.2
(h)	4000	11	4.4

### 5.2.1 Cell ordering and filtering

Visvalingam's algorithm is applied to the row and column profiles of the DEM to derive two area values for each cell. Depending on the view direction, these two cell values are interpreted differently as being either:

- parallel to the direction of viewing. These are used for detecting the convexities and concavities which form the silhouettes of hills, the transverse valleys and other breaks of slope
- orthogonal to the viewing direction. These are necessary for emphasising the form.

The area values are then used to identify core cells to seed the generation of P-strokes. The choice of tolerance values for selecting core cells, like the other sketching parameters, depends on :

- the purpose of the sketch and the viewer's preferences
- the scale of depiction
- the nature of the surface being sketched
- the direction of viewing
- the type of feature - parallel or orthogonal to the view direction and whether convex or concave

Only the last three factors are pertinent to the aims of this study. In this study, the tolerance values were selected through a tedious process of interactive evaluation of the completed sketch. Such interactive assessment is still common practice in Digital Cartography (Robinson et al, 1995), in other artistic applications of computer graphics (Saito and Takahashi, 1990) and in image processing. Figures, such as Figure 3, were used to guide the selection of tolerances.

Figure 3 illustrates the application of Visvalingam's algorithm to one of the profile lines in Figure 1. It shows the effect of filtering the line with different area tolerance values. Initially, surface roughness is eliminated. The original (dotted) line can be approximated with just 20 to 30% of the points. After this, the terrain becomes noticeably faceted even with 15% of the points. When only 10% of points are retained, one valley becomes truncated; Visvalingam and Williamson (1995) pointed out this tendency of the algorithm to pull away from curves and discussed the implication of this in the context of generalisation of road and coastline data. Whereas some algorithms, e.g. the numerous variations of the algorithm by Peucker and Douglas (1975), tend to focus on the river channels, Visvalingam's algorithm is helpful for locating the valley sides and it is relatively easy to locate the channel within the truncated section. The channel is not abstracted here since the long term aim is to drape the topographic detail onto the sketch. The valley sides help to anchor streams and other features which can otherwise appear to float in space. With less than 7% of the points, the profiles begin to look over generalised. However, it is possible to use much higher tolerances for the sketch since the P-strokes help to visually project the form.

Global tolerances, such as those used in this feasibility study, are not ideal given the nature of terrain. Nevertheless, the use of separate tolerances for the four types of geometric features (see Table 1), yields quite effective sketches.

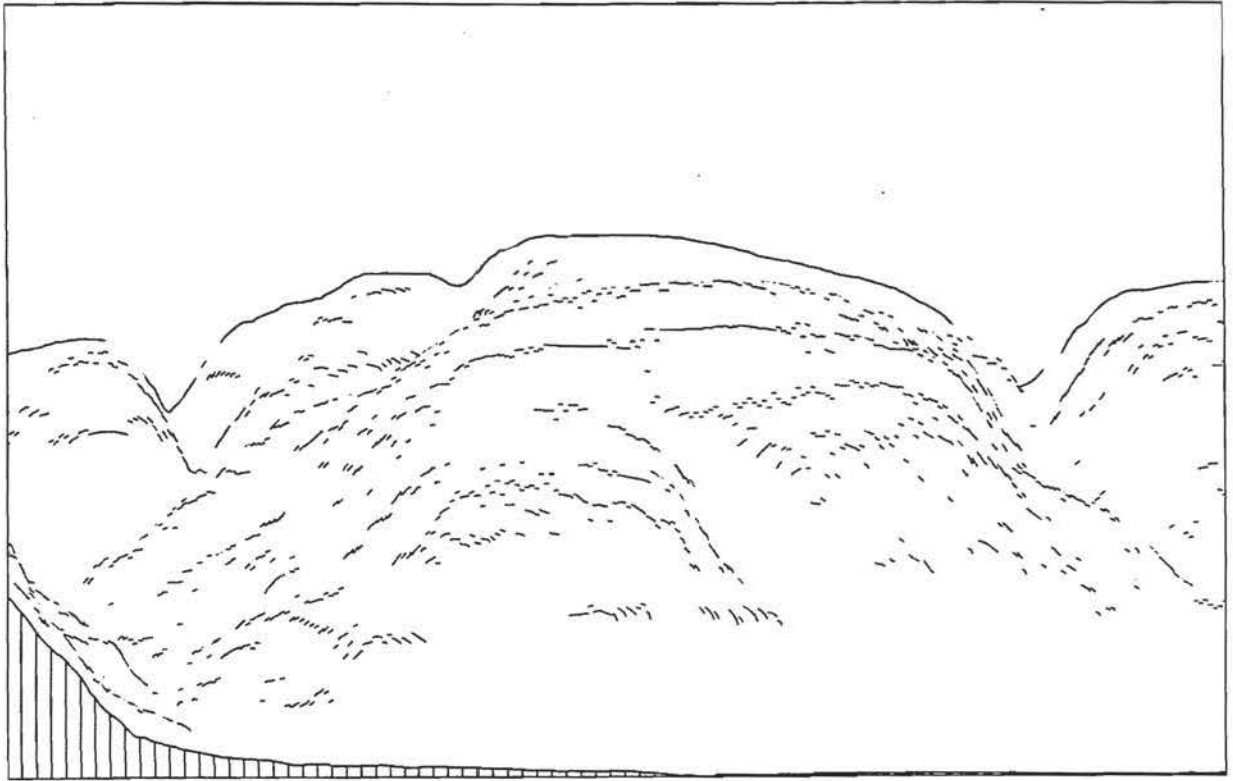


Figure 4 : The Unextended Core Cells for Convex Features Parallel to the Line of Sight

Key	Line of Sight	Features	sq m	Core Cells Number	P-strokes Number	Extension		Style
						Right	Left	
(a)	Parallel :	convex	1000	3495	6082	full	none	solid
(b)		concave	4000	1397	2886	minimal	full	dotted
(c)	Orthogonal	convex	2000	3075	2862	none	none	solid
(d)		concave	1000	4089	5362	minimal	none	dotted
TOTAL				12056 (19.3%)	17192 (27.5%)			

Table 1 : Sketching Parameters and Sketch Statistics

The tolerances used for the convex and concave features, listed in Table 1, were optimal for the view illustrated in this paper. In this terrain, the convex features, defining elements of the plateau, are much larger than the concavities which relate to the incised valleys. Given the perceptual importance of convex forms, a lower tolerance was selected to emphasise these. The valleys running parallel to the line of sight are also emphasised to bring out the form. In contrast, the transverse valleys and breaks of slope are given much less emphasis. The totals of ~20% of points for core cells and ~28% for P-strokes include some double counting since the same cells can have high values in both row and column directions. In addition, this figure includes the hidden cells as well. Dowson filtered about 15% as core cells but selected different extension rules.

### 5.2.2 Extending core cells into P-strokes

The effective areas for the core cells indicate significant, scale-related, convexities and concavities on the surface. The next step is to:

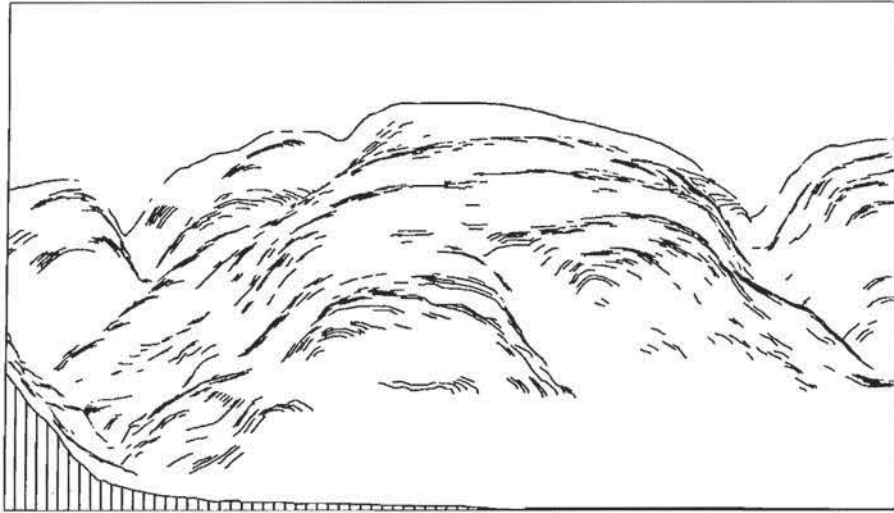
- form the polylines for the P-strokes
- select the appearance of the various types of P-strokes

At present, only the direction and length of the polylines may be varied. The core cells, which exceed the tolerance, may form strings of one or more adjacent cells (Figure 4 shows the core cells for convex features along sections parallel to the line of sight). These tend to form short runs when there is *no extension*. The artist does not always draw continuous lines but the strokes must be long enough to enable the visual system to fill in the missing parts and to 'close' and perceive meaningful forms (McKim, 1972, p 57). Indeed, it is the suggestive, rather than explicit, rendering which partly adds the aesthetic quality to drawings. Core cells may be extended to the right and/or to the left. The extension of core-cells may be *minimal* (adding only the adjoining cell on the selected side(s)), or *full* (when the string is extended until the effective area of the next cell is zero or of the opposite sign). Extending strings in this way joins disconnected groups of core cells into longer, perceptually more appropriate, P-strokes.

Table 1 lists the extension rules used for each of the layers used to construct the sketch; these are shown in Figure 5. The P-strokes abstracted from orthogonal profiles are used to pick out the form of the land by suggesting the effect of illumination. Only the impact of lighting from the right or left was considered. Cartographers use the convention of illumination from the north west. However, artists and cartographers do not always adhere rigidly to an algorithm based on light sources (Imhof, 1982). Table 1 shows the parameters used for illumination from the left.

### 5.2.3 Depiction of the P-strokes

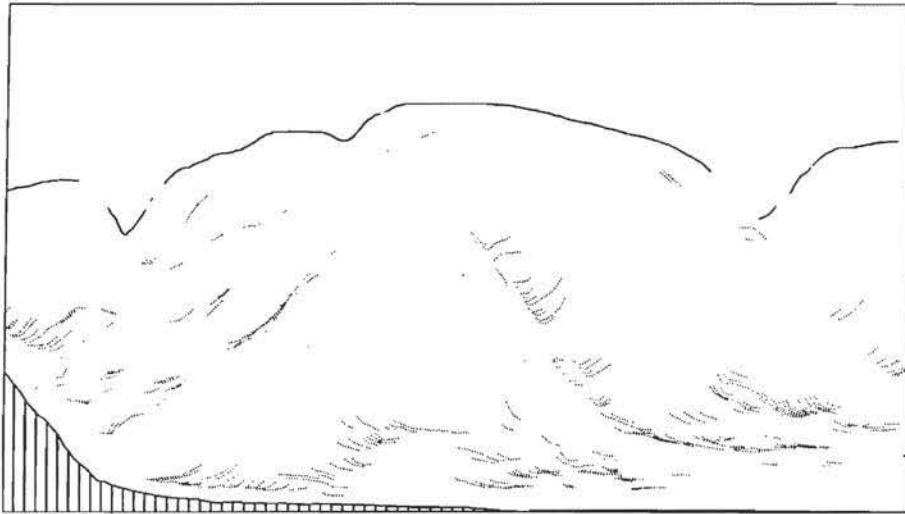
The line attributes for the P-strokes, shown in Table 1, were found by a process of trial and error. Grey-scale values have not been provided since these are printer/plotter dependent. It was particularly difficult to find suitable symbolism for the valleys, especially for the transverse valleys. This could explain why they are often shrouded in mist and haze in photorealistic renderings and in



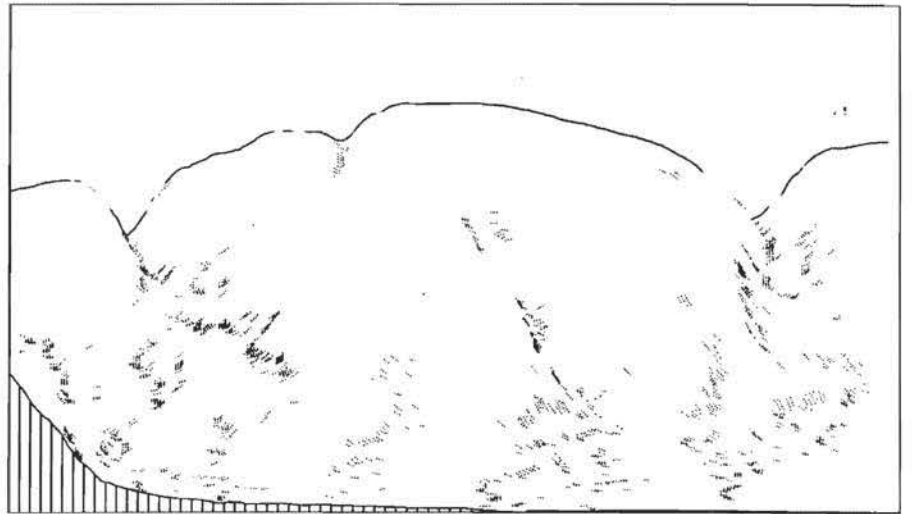
(a)



(c)



(b)



(d)

Figure 5 : The Four Types of P-strokes; see Table 1 for descriptions of subfigures (a) - (d)





Figure 6 : A Sketch of the DEM from the West

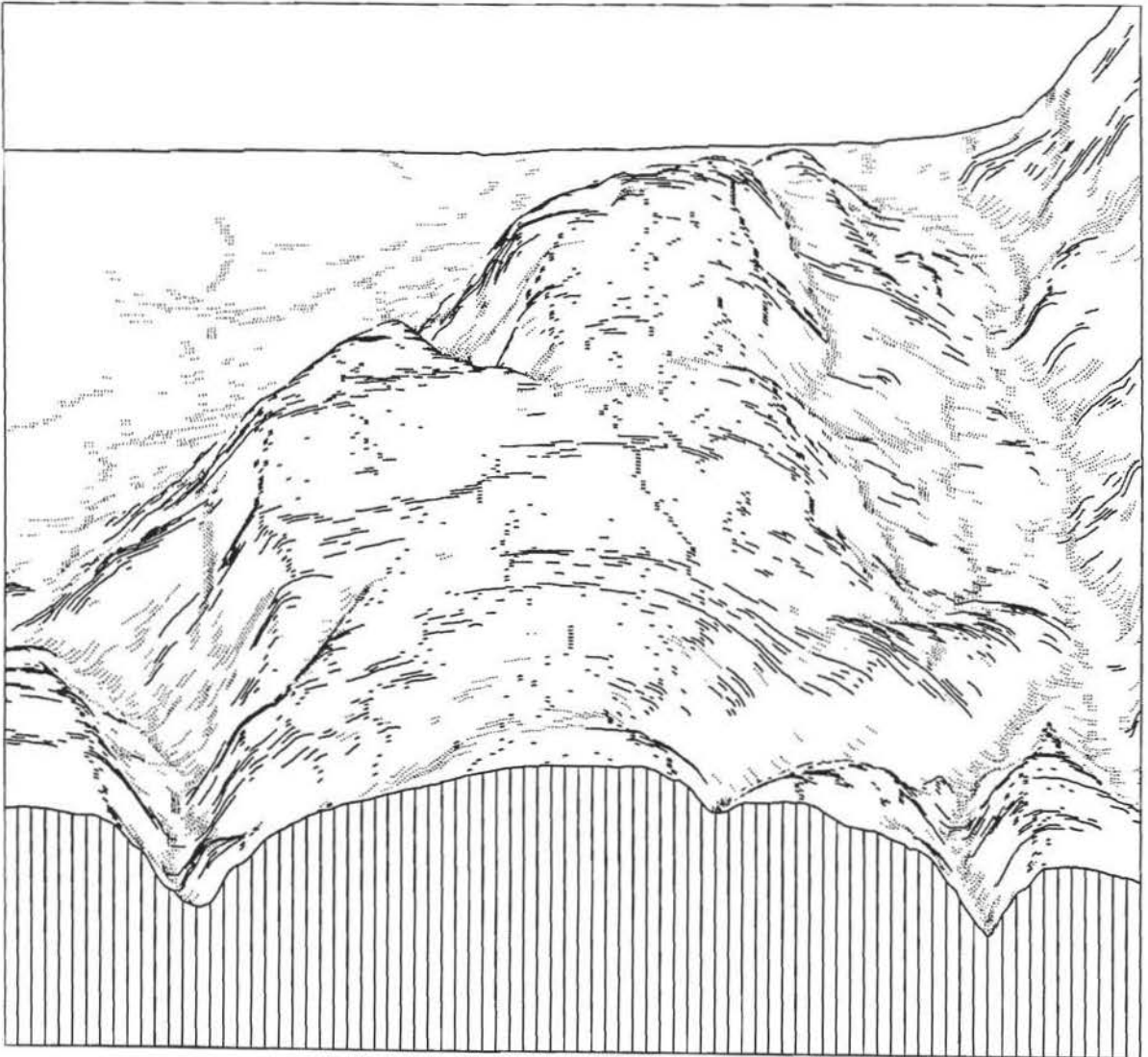


Figure 7 : Another sketch from the East

many Oriental paintings. Here, the concavities are given less visual weight, as in areal perspective, to make them recede. However, if the visual weight of the various lines are not carefully balanced, they will tend to be perceived as distinct layers and the sketch will look less effective.

## 6. Discussion of results and further work.

Currently, only these four types of P-strokes are superimposed to form the sketch shown in Figure 6. These exploratory attempts at automated sketching resemble field sketches produced by artists, geographers and cartographers, and surpass the manual sketches (drawn by less skilled illustrators) in some textbooks. The results suggest that it is possible to detect significant points and features on surfaces and that it may be possible to automatically emulate the variety of sketching styles employed by artists and cartographers in the pre-computer age.

Cartographers could enhance the artistic rendering of the display using image mapping systems in much the same way in which they currently touch up and adjust scanned images and computer-generated maps. P-stroke sketching may also be evolved, to bring out the relief more sharply in 3D visualisations of simulated textured terrain, or of real terrain draped with map or remote-sensed data.

Also, P-stroke sketches may be varied to communicate more information. For example, Figure 7 shows the view from the East with a vertical displacement of 6 metres between sections. There is more detail visible and the image provides scope for inclusion of other topographic and cultural features. Different tolerances and line widths may be used on the sunlit and dark sides to vary the sketch as manually done by Robinson and Thrower (1957); this was explored by Whelan (1997). The expressiveness of the sketches may also be enhanced by colour washes/hill shading and by exploring the hairy brush strokes of Strassman (1986), the vocabulary of lines proposed by Dooley and Cohen (1990) and or the skeletal strokes of Hsu and Lee (1994) and other techniques.

Even as it stands, the P-stroke technique picks out many relief forms - breaks of slope and features on the plateau and the valleys - which are not immediately apparent in Figure 1. Indeed, they also focus attention on spurious features; for example, runs of vertically aligned convexities can be seen on the plateau summit, which fall on sections of the original contours from which the DEM was interpolated. Some sketching parameters amplify these artefacts, which are much more obvious on some US Military DEMs obtained through the Internet see Dowson, 1994). However, Visvalingam's algorithm is not as sensitive as some other techniques, which have been explored for filtering core cells, which also filter out some of the lines of the triangulation used to derive the gridded height field from the initial contours.

P-strokes are by definition only extended along the profile lines and this is not sufficient if we increase the vertical displacement and/or reduce the vertical exaggeration excessively. On the other hand, unconstrained strokes can easily generate untidy and ineffective sketches. Cartographers include some strokes in other directions even within profile-based sketches to emphasise features, such as ridges, spurs and other generalisations. Current research is focusing on abstracting the elements in the landscape and their structure with a view to developing more varied sketching styles. Visvalingam's

line generalisation algorithm also appears to be a simple and flexible tool for identifying a greater variety of surface-specific features than is currently possible.

## 7. Conclusion

This paper has described the P-stroke technique for sketching surfaces. This technique does not rely on high level semantic knowledge. Instead, it uses Visvalingam's two-dimensional line generalisation algorithm to filter core-cells, located on the perceptually significant concavities and convexities. These cells are connected and extended along profile sections to form "profile strokes" or P-strokes which simulate the pen strokes of an artist. The long term aims of this research are two fold. One aim is to supplement the quantitative clichés currently used in digital cartography with the qualitative art forms of conventional cartography. Another aim is to investigate the utility of Visvalingam's algorithm for recognising the surface specific features and for analysing landforms.

This programme of research falls within a new agenda in computer graphics which is quite different to that of photorealism. Research into landform analysis and sketching will benefit from multidisciplinary input; it is of interest to a variety of disciplines, including cartography, geography, geology, scientific visualisation, psychology, and several applications, such as the military, civil engineering, architecture, land use planning, tourism, skiing and orienteering. The computer screen is more than a window through which we could view a virtual photorealistic world; it is also a mirror onto which we can project and reflect on the mental images which arise from our observations and experience.

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